Marcus Domingo
Project 2
MATH-446-001

$$x^3 + 2x + 2 = e^x + 4x^2$$

1. For the left root, we use the function $g(x) = \frac{e^x - 2}{x^2 - 4x + 2}$ with an initial guess of $-\frac{1}{2}$. The resulting fixed point rounded to 10 decimal places is -0.3637607483. To get this number I initially used 50 steps, but upon further analysis of the input/output I realized that it only takes 28 steps to get to the same input as output.

   For the right root, we use the function $g(x) = \frac{\sqrt{x^3 + 2x + 2 - e^x}}{2}$ with an initial guess of $\frac{1}{2}$. The resulting fixed point rounded to 10 decimal places is 0.6419146345. To get this number I initially used 50 steps, but upon further analysis of the input/output I realized that it only takes 26 steps to get to the same input as output.

2. $g(x) = \frac{e^x - 2}{x^2 - 4x + 2} \rightarrow g'(x) = \frac{x^2 - 4x + 2(e^x) - (e^x - 2)(2x - 4)}{(x^2 - 4x + 2)^2} = \frac{(x^2 - 6x + 6)e^x + 4x - 8}{(x^2 - 4x + 2)^2}$

   $S = |g'(r)| = |g'(-0.3637607483)| = 0.285621709630925$

   $g(x) = \frac{\sqrt{x^3 + 2x + 2 - e^x}}{2} \rightarrow g'(x) = \frac{3x^2 + 2 - e^x}{4\sqrt{x^3 + 2x + 2 - e^x}}$

   $S = |g'(r)| = |g'(0.6419146345)| = 0.260168505440198$

3. For $g(x) = \frac{e^x - 2}{x^2 - 4x + 2} \rightarrow \lim_{i \to \infty} \frac{e_{i+1}}{e_i} \approx 0.28562170 \approx S = 0.285621709630925$

   For $g(x) = \frac{\sqrt{x^3 + 2x + 2 - e^x}}{2} \rightarrow \lim_{i \to \infty} \frac{e_{i+1}}{e_i} \approx 0.26016850 \approx S = 0.260168505440198$

   Both of these approximations were obtained from the matlab session where the right column is the ratio. In the middle of the ratios you can see the ratios get close and repeat for the first couple of decimal places for about 3 or 4 ratios, that is where the approximation was grabbed from.

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fpi.m

```
function fpi(g, x0, k)
steps=k;
x=zeros(steps+1,1);
x(1)=x0;
for i=1:steps
  x(i+1)=g(x(i));
end
r=x(steps+1)
e=abs(x-r);
rat=zeros(steps+1,1);
for i=1:steps
  rat(i)=e(i+1)/e(i);
end
rat(steps+1)=0;
[x e rat]
```

MatLab Session

```
g=@(x) (exp(x)-2)/(x.^2-4*x+2);
fpi(g,-0.5,50)

r =

  -0.363760748308309


ans =

  -0.500000000000000   0.136239251691691   0.263401398433784
  -0.327875138891145   0.035885609417164   0.292176930819170
  -0.374245695528391   0.010484947220082   0.283767479979815
  -0.360785461257945   0.002975287050364   0.286152767628319
  -0.364612134932259   0.000851386623950   0.285470146794675
  -0.363517702843791   0.000243045464518   0.285665008946343
  -0.363830177893105   0.000069429584796   0.285609343192675
  -0.363740918570197   0.000019829738112   0.285625241812394
  -0.363766412182052   0.000005663873743   0.285620700771036
  -0.363759130588721   0.000001617719588   0.285621997767021
  -0.363761210364609   0.000000462056300   0.285621627507022
  -0.363760616335036   0.000000131973273   0.285621733309486
  -0.363760786002744   0.000000037694435   0.285621705152268
  -0.363760737541960   0.000000010766349   0.285621705553282
  -0.363760751383412   0.000000003075103   0.285621708332725
  -0.363760747429993   0.000000000878316   0.285621649949688
  -0.363760748559175   0.000000000250866   0.285622107619885
  -0.363760748236656   0.000000000071653   0.285620986935072
```

```
  -0.363760748328774   0.000000000020466   0.285622838543433
  -0.363760748302463   0.000000000005845   0.285607110976050
  -0.363760748309978   0.000000000001669   0.285685785536160
  -0.363760748307832   0.000000000000477   0.285498137802607
  -0.363760748308445   0.000000000000136   0.286180187525479
  -0.363760748308270   0.000000000000039   0.284900284900285
  -0.363760748308320   0.000000000000011   0.290000000000000
  -0.363760748308306   0.000000000000003   0.275862068965517
  -0.363760748308310   0.000000000000001   0.312500000000000
  -0.363760748308309   0.000000000000000   0.200000000000000
  -0.363760748308309   0.000000000000000   2.000000000000000
  -0.363760748308309   0.000000000000000                   0
  -0.363760748308309                   0                 Inf
  -0.363760748308309   0.000000000000000                   0
  -0.363760748308309                   0                 Inf
  -0.363760748308309   0.000000000000000                   0
  -0.363760748308309                   0                 Inf
  -0.363760748308309   0.000000000000000                   0
  -0.363760748308309                   0                 Inf
  -0.363760748308309   0.000000000000000                   0
  -0.363760748308309                   0                 Inf
  -0.363760748308309   0.000000000000000                   0
  -0.363760748308309                   0                 Inf
  -0.363760748308309   0.000000000000000                   0
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  -0.363760748308309   0.000000000000000                   0
  -0.363760748308309                   0                 Inf
  -0.363760748308309   0.000000000000000                   0
  -0.363760748308309                   0                 Inf
  -0.363760748308309   0.000000000000000                   0
  -0.363760748308309                   0                   0

gPrime=@(x) ((x.^2-6*x+6)*exp(x)+4*x-8)/((x.^2-4*x+2).^2);
abs(gPrime(-0.3637607483))

ans =

   0.285621709630925

g=@(x) sqrt((x.^3+2*x+2-exp(x)))/2;
fpi(g,0.5,50)

r =
```

0.641914634501946

ans =

0.500000000000000	0.141914634501946	0.242424463916516
0.607511055310904	0.034403579191043	0.255540006441178
0.633123143653868	0.008791490848079	0.258967705350723
0.639637922290408	0.002276712211539	0.259856378504229
0.641323016311760	0.000591618190187	0.260087319901771
0.641760762112456	0.000153872389491	0.260147384893564
0.641874605002213	0.000040029499733	0.260163010633570
0.641904220306782	0.000010414195165	0.260167075881748
0.641911925071243	0.000002709430704	0.260168133536996
0.641913929594417	0.000000704907529	0.260168408676790
0.641914451107276	0.000000183394670	0.260168480719687
0.641914586788434	0.000000047713513	0.260168500972517
0.641914622088393	0.000000012413553	0.260168507407111
0.641914631272331	0.000000003229616	0.260168558790952
0.641914633661702	0.000000000840244	0.260168633458017
0.641914634283341	0.000000000218605	0.260168885959063
0.641914634445072	0.000000000056874	0.260169283084575
0.641914634487149	0.000000000014797	0.260176021728855
0.641914634498097	0.000000000003850	0.260179951551505
0.641914634500945	0.000000000001002	0.260252715584128
0.641914634501686	0.000000000000261	0.260647359454855
0.641914634501878	0.000000000000068	0.261437908496732
0.641914634501929	0.000000000000018	0.262500000000000
0.641914634501942	0.000000000000005	0.285714285714286
0.641914634501945	0.000000000000001	0.333333333333333
0.641914634501946	0.000000000000000	0.500000000000000
0.641914634501946	0.000000000000000	0.500000000000000
0.641914634501946	0.000000000000000	0
0.641914634501946	0	Inf
0.641914634501946	0.000000000000000	0
0.641914634501946	0	Inf
0.641914634501946	0.000000000000000	0
0.641914634501946	0	Inf
0.641914634501946	0.000000000000000	0
0.641914634501946	0	Inf
0.641914634501946	0.000000000000000	0
0.641914634501946	0	Inf
0.641914634501946	0.000000000000000	0
0.641914634501946	0	Inf
0.641914634501946	0.000000000000000	0
0.641914634501946	0	Inf

```
   0.641914634501946   0.000000000000000                   0
   0.641914634501946                   0                 Inf
   0.641914634501946   0.000000000000000                   0
   0.641914634501946                   0                 Inf
   0.641914634501946   0.000000000000000                   0
   0.641914634501946                   0                 Inf
   0.641914634501946   0.000000000000000                   0
   0.641914634501946                   0                 Inf
   0.641914634501946   0.000000000000000                   0
   0.641914634501946                   0                   0

gPrime=@(x) (3*x.^2+2-exp(x))/(4*sqrt(x.^3+2*x+2-exp(x)));
abs(gPrime(0.6419146345))

ans =

   0.260168505440198

diary off
```