Marcus Domingo
Project 1
MATH-446-001

$$f(x) = (432x^4 + 72x^2 + 16x + 5)e - 8e^{6x}$$

1. IVT: f is a continuous function on the interval [0,1]. $f(0) = 5.591409142295227$, $f(1) = -1.800332388000882 * 10^3$, therefore $f(0)f(1) < 0$ and thus satisfies the IVT.

   For 10 correct decimal places we do:

   $tol = 0.5 * 10^{-10}$, but for rounding sake this sometimes yields a 4 in the 11th decimal place depending on the starting interval, so I am going to use $tol = 0.5 * 10^{-11}$

   $xc = bisect(f, 0,1, (0.5 * 10^{-11}))$, the approximate root to 10 correct places and rounded is 0.4738158381.

   Now we do the same thing for the interval [0,1.2]:

   $xc = bisect(f, 0,1.2, (0.5 * 10^{-11}))$, the approximate root to 10 correct places and rounded is 0.4738158381.

   Once more for the interval [0,1.4]:

   $xc = bisect(f, 0,1.4, (0.5 * 10^{-11}))$, the approximate root to 10 correct places and rounded is 0.4738158381.

   When using $tol = 0.5 * 10^{-10}$ for some the new intervals I was getting 0.47381583804 instead of the expected 0.47381583806 or 0.47381583807. So up to the 10th decimal place was correct across the different starting intervals but the rounding wasn't so I wanted to make sure all the rounding was the same so I used $tol = 0.5 * 10^{-11}$ to fix the rounding issue.

2. The backward error is $|f(r)| = |f(0.4738158381)| =$

$$2.932125653387629 * 10^{-9}$$

Since the backward error is so small, the estimated root is close to 0 which gives confidence that the root is accurate up to 10 decimal places rounded.

3. $f_{new}(x) = (432x^4 + 72x^2 + 16x + 3)e - 8e^{6x}$

For 10 correct decimal places we do:

$$tol = 0.5 * 10^{-10}$$

$xc = bisect\big(f, 0,1, (0.5 * 10^{-10})\big)$, the approximate root to 10 correct places and rounded is 0.1669921875.

Now we do the same thing for the interval [0,1.2]:

$xc = bisect\big(f, 0,1.2, (0.5 * 10^{-10})\big)$, the approximate root to 10 correct places and rounded is 0.1669921875.

Once more for the interval [0,1.4]:

$xc = bisect\big(f, 0,1.4, (0.5 * 10^{-10})\big)$, the approximate root to 10 correct places and rounded is 0.1667968750.

For both intervals [0,1] and [0,1.2] the roots are the same but for [0,1.4] the root is different. They are only the same up to 3 decimal places.

The backward error for [0,1] and [0,1.2] is $|f(r)| = |f(0.1669921875)| = 0$

The backward error for [0,1.4] is $|f(r)| = |f(0.1667968750)| = 0$

The backward error for both of the points is 0 which means that accuracy of these roots must be contingent only upon the first 3 decimal places and that must be the 3 decimal places are the rounded exact root where the exact root is 1/6 (0.1666666…).

4. The main difference between f and f_{new} is that all the intervals for f get the same rounded 10 decimal place estimated root and f_{new} intervals differ. Although f_{new} has different estimated roots, their backwards errors are all 0. I believe that the estimated rounded root of f is correct to 10 decimal places. With f_{new} since the estimated roots are different after 3 decimal places depending on the interval one can't really tell which is truly the correct 10 decimal place estimated root, but with the backwards error being 0 for each estimated root it seems that only the correct 3 decimal places are all we need.

bisect.m

```
function xc=bisect(f,a,b,tol)
if sign(f(a))*sign(f(b)) >= 0
   error('f(a)f(b)<0 not satisfied!')
end
fa=f(a);
fb=f(b);
while (b-a)/2>tol
   c=(a+b)/2;
   fc=f(c);
   if fc == 0
      break
   end
   if sign(fc)*sign(fa)<0
      b=c;fb=fc;
   else
      a=c;fa=fc;
   end
end
xc=(a+b)/2;
```

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<u>MatLab Session</u>

```
f=@(x)(432*x.^4+72*x.^2+16*x+5)*exp(1)-8*exp(6*x);
xc=bisect (f,0,1,0.5*10.^(-11))

xc =

   0.473815838067821

xc=bisect (f,0,1.2,0.5*10.^(-11))

xc =

   0.473815838071459

xc=bisect (f,0,1.4,0.5*10.^(-11))

xc =

   0.473815838069640

abs(f(0.4738158381))

ans =

     2.932125653387629e-09

f=@(x)(432*x.^4+72*x.^2+16*x+3)*exp(1)-8*exp(6*x);
xc=bisect (f,0,1,0.5*10.^(-11))

xc =

   0.166992187500000

xc=bisect (f,0,1.2,0.5*10.^(-11))

xc =

   0.166992187500000

xc=bisect (f,0,1.4,0.5*10.^(-11))

xc =

   0.166796875000000

abs(f(0.1669921875))
```

```
ans =

     0

abs(f(0.1667968750))

ans =

     0

diary end
```