$$f(x) = (432x^4 + 72x^2 + 16x + 5)e - 8e^{6x}$$

- 1. $f'(x) = (1728x^3 + 144x + 16)e 48e^{6x}$ therefore, $x_{i+1} = x_i \frac{f(x_i)}{f'(x_i)}$ for 5 steps gives a root r = 0.473815838069242. To check the accuracy of root I repeated with 6 steps and 7 steps getting a root of r = 0.473815838069241 and r = 0.473815838069242 respectively. The root continues to switch between 1 and 2 for the last decimal places so I believe the root is correct up until that point r = 0.47381583806924.
- 2. $f'(r) = -95.327948195875706 \neq 0$ therefore, Newton's Method converges to the root quadratically. $\lim_{i \to \infty} \frac{e_{i+1}}{e_i^2} = 7.278903995234184$, theoretical value: $M = \frac{f''(r)}{2f'(r)}$ $f''(r) = (5184x^2 + 144)e 288e^{6x} \to M = 7.283386597094728$. If rounded they match couple of decimal places ≈ 7.28 .
- 3. $f_{new}(x) = (432x^4 + 72x^2 + 16x + 3)e 8e^{6x}$ $f'_{new}(x) = f'(x) = (1728x^3 + 144x + 16)e 48e^{6x} \text{ therefore, } x_{i+1} = x_i \frac{f(x_i)}{f'(x_i)} \text{ for 5}$ steps gives a root r = 0.283196592326605. To check the accuracy of the root I repeated with 15 steps and 30 steps getting a root of r = 0.179524666382757 and r = 0.167127502576928 respectively. Knowing that the root is exactly $\frac{1}{6}$ then the root is correct up until the second decimal place...the third if rounded, r = 0.16 or 0.167.

$$f'(r) = 0, f''(r) = 0, f'''(r) = 0, f''''(r) = 0$$

 $f'''''(r)=-1.690988759847803*10^5$ therefore, Newton's Method converges to the root linearly with a multiplicity of 5. $\lim_{i\to\infty}\frac{e_{i+1}}{e_i}\approx 0.799841624652284$, theoretical value: $\frac{m-1}{m}=\frac{4}{5}=0.8$. If rounded they match a decimal place ≈ 0.8 .

```
newt.m (for finding root)
function newt(f, fp, k)
g=@(x) x-f(x)/fp(x);
steps=k;
x=zeros(steps+1,1);
x(1)=0.5;
for i=1:steps
x(i+1)=g(x(i))
end
r=x(steps+1)
newt.m (for finding quadratic convergence)
function newt(f, fp, k)
g=@(x) x-f(x)/fp(x);
steps=k;
x=zeros(steps+1,1);
x(1)=0.5;
for i=1:steps
x(i+1)=g(x(i))
end
r=x(steps+1)
e=abs(x-r);
rat=zeros(steps+1,1);
for i=1:steps
rat(i)=e(i+1)/e(i)^2;
end
rat(steps+1)=0;
[x e rat]
<u>newt.m</u> (for finding linear convergence)
function newt(f, fp, k)
g=@(x) x-f(x)/fp(x);
steps=k;
x=zeros(steps+1,1);
x(1)=0.5;
for i=1:steps
x(i+1)=g(x(i))
end
r=x(steps+1)
e=abs(x-r);
rat=zeros(steps+1,1);
for i=1:steps
 rat(i)=e(i+1)/e(i);
end
```

```
rat(steps+1)=0;
[x e rat]
MatLab Session
f=@(x) (432*x.^4+72*x.^2+16*x+5)*exp(1)-8*exp(6*x);
fp=@(x) (1728*x.^3+144*x+16)*exp(1)-48*exp(6*x);
newt(f, fp, 5);
x =
 0.5000000000000000
 0.478047251377279
          0
          0
          0
           0
\mathbf{x} =
 0.5000000000000000
 0.478047251377279
 0.473942618700500
          0
          0
          0
\mathbf{x} =
 0.5000000000000000
 0.478047251377279
 0.473942618700500
 0.473815955037586
          0
          0
\mathbf{x} =
 0.5000000000000000
 0.478047251377279
 0.473942618700500
 0.473815955037586
 0.473815838069341
          0
\mathbf{x} =
```

```
0.473815955037586
 0.473815838069341
 0.473815838069242
\mathbf{r} =
 0.473815838069242
newt(f, fp, 6);
\mathbf{x} =
 0.5000000000000000
 0.478047251377279
          0
          0
          0
          0
          0
x =
 0.5000000000000000
 0.478047251377279
 0.473942618700500
          0
          0
          0
          0
\mathbf{x} =
 0.5000000000000000
 0.478047251377279
 0.473942618700500
 0.473815955037586
          0
          0
          0
\mathbf{x} =
 0.5000000000000000
 0.478047251377279
 0.473942618700500
 0.473815955037586
 0.473815838069341
          0
          0
```

```
0.5000000000000000
 0.478047251377279
 0.473942618700500
 0.473815955037586
 0.473815838069341
 0.473815838069242
          0
\mathbf{x} =
 0.5000000000000000
 0.478047251377279
 0.473942618700500
 0.473815955037586
 0.473815838069341
 0.473815838069242
 0.473815838069241
\mathbf{r} =
 0.473815838069241
newt(f, fp, 7);
\mathbf{x} =
 0.5000000000000000
 0.478047251377279
          0
          0
          0
          0
          0
          0
\mathbf{x} =
 0.5000000000000000
 0.478047251377279
 0.473942618700500
          0
          0
          0
          0
          0
```

 $\mathbf{x} =$

 $\mathbf{x} =$

0.5000000000000000 0.478047251377279 0.473942618700500 0.4738159550375860 0 0 0 $\mathbf{x} =$ 0.5000000000000000 0.478047251377279 0.473942618700500 0.473815955037586 0.473815838069341 0 0 0 $\mathbf{x} =$ 0.5000000000000000 0.478047251377279 0.4739426187005000.473815955037586 0.473815838069341 0.4738158380692420 0 $\mathbf{x} =$ 0.5000000000000000 0.478047251377279 0.473942618700500 0.473815955037586 0.473815838069341 0.473815838069242 0.473815838069241 0 $\mathbf{x} =$ 0.5000000000000000 0.478047251377279 0.473942618700500 0.473815955037586 0.473815838069341

```
0.473815838069242
 0.473815838069241
 0.473815838069242
\mathbf{r} =
 0.473815838069242
newt(f, fp, 5);
\mathbf{x} =
 0.5000000000000000
 0.478047251377279
           0
           0
           0
           0
\mathbf{x} =
 0.5000000000000000
 0.478047251377279
 0.473942618700500
           0
           0
           0
\mathbf{x} =
 0.5000000000000000
 0.478047251377279
 0.473942618700500
 0.473815955037586
           0
           0
\mathbf{x} =
 0.5000000000000000
 0.478047251377279
 0.473942618700500
 0.473815955037586
 0.473815838069341
           0
\mathbf{x} =
```

```
0.478047251377279
 0.473942618700500
 0.473815955037586
 0.473815838069341
 0.473815838069242
\mathbf{r} =
 0.473815838069242
ans =
 0.5000000000000000 \quad 0.026184161930758 \quad 6.171746669725795
 0.478047251377279 \quad 0.004231413308037 \quad 7.080794895271423
 0.473942618700500 \quad 0.000126780631258 \quad 7.277170009868958
 0.473815955037586 \quad 0.000000116968344 \quad 7.278903995234184
 0.473815838069341  0.0000000000000100
                                                     0
 0.473815838069242
                                             0
                                0
fpp=@(x) (5184*x.^2+144)*exp(1)-288*exp(6*x);
fpp(0.47381583806924)/(2*fp(0.47381583806924))
ans =
 7.283386597094728
f=@(x) (432*x.^4+72*x.^2+16*x+3)*exp(1)-8*exp(6*x);
newt(f, fp, 5);
\mathbf{x} =
 0.5000000000000000
 0.438579819183476
           0
           0
           0
           0
\mathbf{x} =
 0.5000000000000000
 0.438579819183476
 0.387587511880733
           0
           0
           0
\mathbf{x} =
```

```
0.438579819183476
 0.387587511880733
 0.345586289965882
           0
           0
\mathbf{x} =
 0.5000000000000000
 0.438579819183476
 0.387587511880733
 0.345586289965882
 0.311204672896067
           0
\mathbf{x} =
 0.5000000000000000
 0.438579819183476
 0.387587511880733
 0.345586289965882
 0.311204672896067
 0.283196592326605\\
\mathbf{r} =
 0.283196592326605
newt(f, fp, 15);
\mathbf{x} =
 0.5000000000000000
 0.438579819183476
           0
           0
           0
           0
           0
           0
           0
           0
           0
           0
           0
           0
           0
           0
```

0 0 x =0.5000000000000000 0.438579819183476 0.387587511880733 0.3455862899658820.311204672896067 0.283196592326605 0 0 0 0 0 0 0 0 0 0 x =0.50000000000000000.438579819183476 0.3875875118807330.3455862899658820.311204672896067 0.2831965923266050.2604670866894820 0 0 0 0 0 0 0 0 $\mathbf{x} =$ 0.50000000000000000.438579819183476 0.387587511880733 0.345586289965882

0.311204672896067 0.283196592326605 0.260467086689482

0 0 0 0 0 0 0 0 $\mathbf{x} =$ 0.50000000000000000.438579819183476 0.3875875118807330.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.242076245291962 0.2272307525307930 0 0 0 0 0 0 x =0.50000000000000000.438579819183476 0.3875875118807330.3455862899658820.311204672896067 0.283196592326605 0.260467086689482 0.242076245291962 0.2272307525307930.215269283335350 0 0 0 0 0 0 $\mathbf{x} =$

0.242076245291962

0.438579819183476 0.387587511880733 0.3455862899658820.311204672896067 0.283196592326605 0.260467086689482 0.242076245291962 0.227230752530793 0.215269283335350 0.205645634503915 0 0 0 0 0 $\mathbf{x} =$ 0.5000000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.242076245291962 0.2272307525307930.215269283335350 0.205645634503915 0.197911843682273 0 0 0

x =

 $\begin{array}{c} 0.5000000000000000\\ 0.438579819183476\\ 0.387587511880733\\ 0.345586289965882\\ 0.311204672896067\\ 0.283196592326605\\ 0.260467086689482\\ 0.242076245291962\\ 0.227230752530793\\ 0.215269283335350\\ 0.205645634503915\\ 0.197911843682273\\ 0.191702490389894\\ 0\\ 0\\ \end{array}$

0

 $\mathbf{x} =$

0.5000000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.242076245291962 0.2272307525307930.215269283335350 0.205645634503915 0.197911843682273 0.191702490389894 0.186720721881015 0 0

 $\mathbf{x} =$

 $\begin{array}{c} 0.5000000000000000\\ 0.438579819183476\\ 0.387587511880733\\ 0.345586289965882\\ 0.311204672896067\\ 0.283196592326605\\ 0.260467086689482\\ 0.242076245291962\\ 0.227230752530793\\ 0.215269283335350\\ 0.205645634503915\\ 0.197911843682273\\ 0.191702490389894\\ 0.186720721881015\\ 0.182726164066434\\ 0\end{array}$

 $\mathbf{x} =$

0.500000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.242076245291962 0.227230752530793

 $\mathbf{x} =$

0.500000000000000 0.438579819183476

 $\mathbf{x} =$

 $\mathbf{x} =$

 $\mathbf{x} =$

0.50000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067

 $\begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$

 $\mathbf{x} =$

 $\mathbf{x} =$

 $\mathbf{x} =$

0.500000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.242076245291962

 $\mathbf{x} =$

 $\mathbf{x} =$

 $\mathbf{x} =$

 $\begin{array}{c} 0.5000000000000000\\ 0.438579819183476\\ 0.387587511880733\\ 0.345586289965882\\ 0.311204672896067\\ 0.283196592326605\\ 0.260467086689482\\ 0.242076245291962\\ 0.227230752530793\\ 0.215269283335350\\ 0.205645634503915\\ \end{array}$

x =

0.50000000000000000.438579819183476 0.3875875118807330.3455862899658820.311204672896067 0.283196592326605 0.2604670866894820.242076245291962 0.2272307525307930.2152692833353500.2056456345039150.1979118436822730.191702490389894

 $\mathbf{x} =$

0.5000000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.2831965923266050.260467086689482 0.242076245291962 0.2272307525307930.215269283335350 0.205645634503915 0.197911843682273 0.191702490389894 0.186720721881015 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

 $\mathbf{x} =$

0.500000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.242076245291962 0.227230752530793 0.215269283335350 0.205645634503915 0.197911843682273 0.191702490389894 0.186720721881015

x =

0.50000000000000000.438579819183476 0.3875875118807330.3455862899658820.311204672896067 0.2831965923266050.2604670866894820.242076245291962 0.2272307525307930.2152692833353500.205645634503915 0.1979118436822730.1917024903898940.186720721881015 0.1827261640664340.1795246663827570 0 0 0 0 0 0 0 0 0 0 0 0

 $\mathbf{x} =$

0.5000000000000000 0.438579819183476 0.3875875118807330.345586289965882 0.311204672896067 0.283196592326605 0.2604670866894820.242076245291962 0.227230752530793 0.215269283335350 0.205645634503915 0.1979118436822730.191702490389894 0.1867207218810150.1827261640664340.179524666382757 0.176959723446675 0 0 0 0 0 0 0 0 0 0 0 0 0 0

 $\mathbf{x} =$

0.5000000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.2420762452919620.227230752530793 0.2152692833353500.205645634503915 0.197911843682273 0.191702490389894 0.186720721881015 0.182726164066434 0.179524666382757 0.176959723446675

0 0 0 0 0 0 0 0 0 0 0 x =0.5000000000000000 0.438579819183476 0.387587511880733 0.3455862899658820.311204672896067 0.283196592326605 0.2604670866894820.242076245291962 0.2272307525307930.2152692833353500.205645634503915 0.197911843682273 0.1917024903898940.186720721881015 0.182726164066434 0.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0 0 0 0 0 0 0 0 0 0

0.174905372494976 0

 $\mathbf{x} =$

0.5000000000000000

 $0 \\ 0$

0.438579819183476 0.387587511880733 0.3455862899658820.311204672896067 0.283196592326605 0.2604670866894820.242076245291962 0.227230752530793 0.2152692833353500.205645634503915 0.197911843682273 0.191702490389894 0.186720721881015 0.1827261640664340.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0.171943364986582 0 0 0 0 0 0 0 0 0 0 0

 $\mathbf{x} =$

0.5000000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.242076245291962 0.227230752530793 0.215269283335350 0.2056456345039150.197911843682273 0.191702490389894 0.1867207218810150.182726164066434 0.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0.171943364986582

0.170889143088191 0 0 0 0 0 0 0 0 0 0 $\mathbf{x} =$ 0.5000000000000000 0.438579819183476 0.387587511880733 0.3455862899658820.311204672896067 0.283196592326605 0.2604670866894820.242076245291962 0.2272307525307930.2152692833353500.205645634503915 0.197911843682273 0.1917024903898940.186720721881015 0.182726164066434 0.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0.171943364986582 0.170889143088191 0.1700453640732850 0 0 0 0 0 0 0 0 $\mathbf{x} =$ 0.5000000000000000 0.438579819183476 0.387587511880733 0.345586289965882

0.311204672896067 0.283196592326605 0.260467086689482 0.242076245291962 0.227230752530793 0.2152692833353500.205645634503915 0.197911843682273 0.1917024903898940.186720721881015 0.1827261640664340.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0.171943364986582 0.170889143088191 0.170045364073285 0.169370084280272 0 0 0 0 0 0 0 0

 $\mathbf{x} =$

0.5000000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.2420762452919620.227230752530793 0.215269283335350 0.205645634503915 0.197911843682273 0.191702490389894 0.186720721881015 0.182726164066434 0.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0.171943364986582 0.170889143088191 0.170045364073285 0.169370084280272

0 0 0 0 0 x =0.5000000000000000 0.438579819183476 0.387587511880733 0.3455862899658820.311204672896067 0.283196592326605 0.260467086689482 0.242076245291962 0.227230752530793 0.2152692833353500.205645634503915 0.197911843682273 0.1917024903898940.186720721881015 0.182726164066434 0.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0.171943364986582 0.170889143088191 0.170045364073285 0.169370084280272 0.168829704576515 0.1683973016855520 0 0 0 0 0

0.168829704576515 0 0

 $\mathbf{x} =$

0.50000000000000000.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482

0.242076245291962 0.227230752530793 0.2152692833353500.205645634503915 0.197911843682273 0.1917024903898940.186720721881015 0.182726164066434 0.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0.171943364986582 0.170889143088191 0.170045364073285 0.169370084280272 0.168829704576515 0.168397301685552 0.168051321916618 0 0 0 0 0

 $\mathbf{x} =$

0.5000000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.2831965923266050.260467086689482 0.242076245291962 0.2272307525307930.215269283335350 0.205645634503915 0.1979118436822730.191702490389894 0.186720721881015 0.182726164066434 0.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0.171943364986582 0.170889143088191 0.170045364073285 0.169370084280272 0.168829704576515 0.168397301685552 0.168051321916618

x =

0.5000000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.2604670866894820.242076245291962 0.227230752530793 0.215269283335350 0.205645634503915 0.197911843682273 0.191702490389894 0.186720721881015 0.182726164066434 0.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0.171943364986582 0.170889143088191 0.170045364073285 0.169370084280272 0.168829704576515 0.1683973016855520.168051321916618 0.167774421795670 0.1675527087665680 0 0

 $\mathbf{x} =$

0.500000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.242076245291962 0.227230752530793 0.215269283335550

0.205645634503915 0.197911843682273 0.191702490389894 0.186720721881015 0.1827261640664340.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0.171943364986582 0.1708891430881910.170045364073285 0.169370084280272 0.168829704576515 0.168397301685552 0.168051321916618 0.1677744217956700.167552708766568 0.167376182797637 0 0

x =

0.5000000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.2420762452919620.227230752530793 0.215269283335350 0.2056456345039150.197911843682273 0.191702490389894 0.186720721881015 0.182726164066434 0.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0.171943364986582 0.170889143088191 0.170045364073285 0.169370084280272 0.168829704576515 0.1683973016855520.168051321916618 0.167774421795670 0.167552708766568 0.167376182797637

 $\mathbf{x} =$

0.5000000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.2420762452919620.227230752530793 0.2152692833353500.205645634503915 0.197911843682273 0.1917024903898940.1867207218810150.182726164066434 0.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0.171943364986582 0.170889143088191 0.1700453640732850.169370084280272 0.168829704576515 0.1683973016855520.168051321916618 0.167774421795670 0.167552708766568 0.167376182797637 0.167237022214275 0.167127502576928

r =

0.167127502576928

fp(1/6)

ans =

0

fpp(1/6)

ans =

```
fppp=@(x) 10368*x*exp(1)-1728*exp(6*x);
fppp(1/6)
ans =
   0
fpppp=@(x) 10368*exp(1)-10368*exp(6*x);
fpppp(1/6)
ans =
   0
fppppp=@(x) -62208*exp(6*x);
fppppp(1/6)
ans =
  -1.690988759847803e+05
newt(f, fp, 30);
\mathbf{x} =
 0.5000000000000000
 0.438579819183476
          0
          0
          0
          0
          0
          0
          0
          0
          0
          0
          0
          0
          0
          0
          0
          0
          0
          0
          0
          0
          0
          0
          0
          0
```

 $\mathbf{x} =$

 $\mathbf{x} =$

 $\mathbf{x} =$

 $\mathbf{x} =$

 $\begin{array}{c} 0.5000000000000000\\ 0.438579819183476\\ 0.387587511880733\\ 0.345586289965882\\ 0.311204672896067\\ 0.283196592326605\\ 0.260467086689482\\ &0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ \end{array}$

 $\mathbf{x} =$

 $\mathbf{x} =$

0.50000000000000000.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.2420762452919620.2272307525307930.2152692833353500 0 0 0 0 0 0 0

 $\mathbf{x} =$

0.5000000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.2604670866894820.2420762452919620.2272307525307930.2152692833353500.205645634503915

 $\mathbf{x} =$

 $\begin{array}{c} 0.50000000000000000\\ 0.438579819183476 \end{array}$

 $\mathbf{x} =$

0.50000000000000000.438579819183476 0.387587511880733 0.3455862899658820.311204672896067 0.283196592326605 0.260467086689482 0.242076245291962 0.2272307525307930.215269283335350 0.205645634503915 0.1979118436822730.1917024903898940 0 0 0 0 0 0 0

 $\mathbf{x} =$

0.5000000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.2420762452919620.227230752530793 0.2152692833353500.2056456345039150.197911843682273 0.191702490389894 0.1867207218810150 0 0 0 0 0 0 0 0 0 0 0 0 0

 $\mathbf{x} =$

0.50000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067

 $\mathbf{x} =$

0.5000000000000000 0.438579819183476 0.387587511880733 0.3455862899658820.311204672896067 0.2831965923266050.2604670866894820.242076245291962 0.227230752530793 0.215269283335350 0.205645634503915 0.1979118436822730.191702490389894 0.186720721881015 0.1827261640664340.179524666382757 0 0 0 0 0 0 0 0

 $\mathbf{x} =$

0.5000000000000000 0.438579819183476 0.3875875118807330.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.2420762452919620.227230752530793 0.215269283335350 0.205645634503915 0.197911843682273 0.1917024903898940.1867207218810150.1827261640664340.179524666382757 0.176959723446675 0 0 0 0 0 0 0 0 0 0 0 0 0 0

 $\mathbf{x} =$

0.50000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.242076245291962 0.227230752530793 0.215269283335350 0.2056456345039150.197911843682273 0.1917024903898940.186720721881015 0.182726164066434 0.179524666382757 0.176959723446675 0.174905372494976 0 0 0 0 0 0 0 0 0 0 0 0 0

x =

0.50000000000000000.438579819183476 0.387587511880733 0.3455862899658820.311204672896067 0.283196592326605 0.2604670866894820.242076245291962 0.2272307525307930.2152692833353500.205645634503915 0.197911843682273 0.191702490389894 0.186720721881015 0.182726164066434 0.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0 0 0 0 0 0 0 0

 $\mathbf{x} =$

0.5000000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.2831965923266050.260467086689482 0.2420762452919620.227230752530793 0.215269283335350 0.205645634503915 0.197911843682273 0.191702490389894 0.1867207218810150.182726164066434 0.179524666382757 0.1769597234466750.174905372494976 0.173260357957501 0.171943364986582 0 0 0 0 0 0 0 0 0 0 0

x =

0.500000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.242076245291962 0.227230752530793 0.215269283335350 0.205645634503915 0.197911843682273 0.191702490389894 0.1867207218810150.182726164066434 0.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0.1719433649865820.170889143088191 0 0 0 0 0 0 0 0 0 0

x =

0.5000000000000000 0.438579819183476 0.387587511880733 0.3455862899658820.311204672896067 0.283196592326605 0.2604670866894820.242076245291962 0.2272307525307930.2152692833353500.205645634503915 0.197911843682273 0.1917024903898940.186720721881015 0.182726164066434 0.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0.171943364986582 0.170889143088191 0.170045364073285 0 0 0 0 0 0 0

0.5000000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.242076245291962 0.2272307525307930.215269283335350 0.205645634503915 0.1979118436822730.191702490389894 0.186720721881015 0.1827261640664340.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0.171943364986582 0.170889143088191 0.170045364073285 0.1693700842802720 0 0 0 0 0 0 0

 $\mathbf{x} =$

0.500000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.242076245291962 0.227230752530793 0.215269283335550 0.205645634503915 0.197911843682273 0.191702490389894 0.186720721881015 0.182726164066434 0.179524666382757 0.1769597234466750.174905372494976 0.173260357957501 0.171943364986582 0.170889143088191 0.170045364073285 0.169370084280272 0.168829704576515 0 0 0 0 0 0 0

x =

0.5000000000000000 0.438579819183476 0.387587511880733 0.3455862899658820.311204672896067 0.283196592326605 0.2604670866894820.242076245291962 0.227230752530793 0.2152692833353500.205645634503915 0.197911843682273 0.1917024903898940.186720721881015 0.1827261640664340.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0.171943364986582 0.170889143088191 0.170045364073285 0.169370084280272 0.168829704576515 0.168397301685552 0 0 0 0 0 0

0.5000000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.2604670866894820.242076245291962 0.227230752530793 0.215269283335350 0.205645634503915 0.197911843682273 0.191702490389894 0.1867207218810150.182726164066434 0.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0.171943364986582 0.170889143088191 0.170045364073285 0.1693700842802720.168829704576515 0.168397301685552 0.168051321916618 0 0 0 0 0

 $\mathbf{x} =$

0.5000000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.2604670866894820.242076245291962 0.227230752530793 0.215269283335350 0.2056456345039150.197911843682273 0.1917024903898940.186720721881015 0.182726164066434 0.179524666382757 0.176959723446675

```
0.174905372494976
0.173260357957501
0.171943364986582
0.170889143088191
0.170045364073285
0.169370084280272
0.168829704576515
0.168051321916618
0.167774421795670
0
0
0
0
0
```

```
0.5000000000000000
0.438579819183476
0.387587511880733
0.345586289965882
0.311204672896067
0.283196592326605
0.260467086689482
0.242076245291962
0.227230752530793
0.215269283335350
0.205645634503915
0.197911843682273
0.191702490389894
0.186720721881015
0.182726164066434
0.179524666382757
0.176959723446675
0.174905372494976
0.173260357957501
0.171943364986582
0.170889143088191
0.170045364073285
0.169370084280272
0.168829704576515
0.168397301685552
0.168051321916618
0.167774421795670
0.167552708766568
        0
        0
        0
```

 $\mathbf{x} =$

0.5000000000000000

0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.2604670866894820.242076245291962 0.2272307525307930.215269283335350 0.205645634503915 0.197911843682273 0.191702490389894 0.186720721881015 0.1827261640664340.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0.171943364986582 0.1708891430881910.170045364073285 0.169370084280272 0.168829704576515 0.168397301685552 0.168051321916618 0.167774421795670 0.167552708766568 0.167376182797637 0 0

 $\mathbf{x} =$

0.5000000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.242076245291962 0.227230752530793 0.2152692833353500.205645634503915 0.197911843682273 0.191702490389894 0.1867207218810150.182726164066434 0.179524666382757 0.176959723446675 0.174905372494976 0.173260357957501 0.171943364986582

0.170889143088191 0.170045364073285 0.169370084280272 0.168829704576515 0.168397301685552 0.168051321916618 0.167774421795670 0.167552708766568 0.167376182797637 0.167237022214275 0

x =

0.5000000000000000 0.438579819183476 0.387587511880733 0.345586289965882 0.311204672896067 0.283196592326605 0.260467086689482 0.242076245291962 0.227230752530793 0.2152692833353500.205645634503915 0.197911843682273 0.1917024903898940.186720721881015 0.182726164066434 0.179524666382757 0.176959723446675 0.174905372494976 0.1732603579575010.171943364986582 0.170889143088191 0.170045364073285 0.169370084280272 0.168829704576515 0.168397301685552 0.168051321916618 0.167774421795670 0.167552708766568 0.167376182797637 0.167237022214275 0.167127502576928

r =

0.167127502576928

ans =

```
0.5000000000000000 \quad 0.332872497423072 \quad 0.815484363256178
0.438579819183476 \quad 0.271452316606547 \quad 0.812150038208540
0.387587511880733 0.220460009303804 0.809483715221244
0.345586289965882 \quad 0.178458787388953 \quad 0.807341417181777
0.311204672896067 \quad 0.144077170319139 \quad 0.805603618481521
0.283196592326605 0.116069089749677 0.804172620926524
0.260467086689482 0.093339584112553 0.802968466461740
0.242076245291962 0.074948742715034 0.801924725840784
0.227230752530793  0.060103249953865  0.800984652167319
0.215269283335350 \quad 0.048141780758422 \quad 0.800097780351597
0.205645634503915 \quad 0.038518131926987 \quad 0.799216876968454
0.197911843682273 0.030784341105345 0.798295072448322
0.191702490389894 \quad 0.024574987812965 \quad 0.797282971336816
0.186720721881015 \quad 0.019593219304087 \quad 0.796125498694924
0.182726164066434 \quad 0.015598661489505 \quad 0.794758179358516
0.179524666382757 \quad 0.012397163805829 \quad 0.793102440505241
0.176959723446675 \quad 0.009832220869746 \quad 0.791059316210090
0.174905372494976  0.007777869918048  0.788500636445666
0.173260357957501 \quad 0.006132855380572 \quad 0.785256150815043
0.171943364986582  0.004815862409654  0.781093850132003
0.170889143088191 \quad 0.003761640511262 \quad 0.775688555995957
0.170045364073285 \quad 0.002917861496357 \quad 0.768570306076289
0.169370084280272  0.002242581703343  0.759036782048627
0.168829704576515 0.001702201999587 0.745974396065897
0.168397301685552 \quad 0.001269799108624 \quad 0.727531885489433
0.168051321916618 0.000923819339690 0.700265940479678
0.167774421795670 \quad 0.000646919218741 \quad 0.657278648278577
0.167552708766568 0.000425206189640 0.584846191724103
0.167376182797637  0.000248680220708
                                         0.440403491018328
0.167237022214275 \quad 0.000109519637347
                                                  0
                                          0
0.167127502576928
                             0
```

diary off