4.5a Integration by Substitution

Ex1: $\int (x^2+1)^2 \cdot 2x\,dx$ *PATTERN RECOGNITION*

$f(g(x)) \cdot g'(x) \rightarrow F(g(x))$

$= \frac{1}{3}(x^2+1)^3 + C$

Ex2: $\int 5 \cdot \cos(\underbrace{5x}_{g(x)})\,dx$

$f(g(x)) \cdot g'(x) \rightarrow F(5x) + C = \sin(5x) + C$

Ex3: $\int x(x^2+1)^2\,dx$ $g(x) = x^2+1$ $g'(x) = 2x\,dx$

$\int (x^2+1)^2 \cdot x\,dx \cdot 2 \cdot \frac{1}{2}$

$\frac{1}{2}\int (x^2+1)^2 \cdot 2x\,dx = \frac{1}{2} \cdot \frac{1}{3}(x^2+1)^3 + C = \frac{1}{6}(x^2+1)^3 + C$

$f(g(x)) \cdot g'(x)$

Ex4: $\int \sqrt{2x-1}\,dx$ *CHANGE OF VARIABLES*

$u = 2x-1$

$du = 2dx$

$\frac{1}{2}du = dx$

$\int \sqrt{u} \cdot \frac{1}{2}du$

$\frac{1}{2}\int u^{1/2}du = \frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = \frac{1}{3}(2x-1)^{3/2} + C$

Ex5: $\int x\sqrt{2x-1}\,dx$

$u = 2x-1 \longrightarrow u+1 = 2x$

$du = 2dx$ $\frac{1}{2}(u+1) = x$

$\frac{1}{2}du = dx$

$\int \frac{1}{2}(u+1)\sqrt{u} \cdot \frac{1}{2}du = \frac{1}{4}\int (u^{3/2} + u^{1/2})du$

$= \frac{1}{4}\left[\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right] + C$

$= \frac{1}{10}(2x-1)^{5/2} + \frac{1}{6}(2x-1)^{3/2} + C$

Ex6: $\int \sin^2(3x)\cos(3x)\,dx$ $\qquad u = \sin(3x)$

$\qquad \frac{1}{3}du = \cancel{3}\cdot\cos(3x)\,dx$

$\frac{1}{3}\int u^2 \cdot du$

$\frac{1}{3}\cdot\frac{1}{3}u^3 + C = \frac{1}{9}\sin^3(3x) + C$

Exploration

a) $\int 2x(x^2+1)^4$ $\qquad g(x) = x^2+1$ $\qquad \frac{1}{5}(x^2+1)^5 + C$

b) $\int 3x^2\sqrt{x^3+1}\,dx$ $\qquad g(x) = x^3+1$ $\qquad \frac{2}{3}(x^3+1)^{3/2} + C$

c) $\int \sec^2 x\cdot(\tan x + 3)\,dx$ $\qquad g(x) = \tan x + 3$ $\qquad \frac{1}{2}(\tan x + 3)^2 + C$

d) $\int x(x^2+1)^4\,dx$ $\qquad u = x^2+1$ $\qquad du = 2x\,dx$ $\qquad \frac{1}{2}du = x\cdot dx$

$\frac{1}{2}\int u^4\cdot du = \frac{1}{2}\cdot\frac{1}{5}u^5 + C = \frac{1}{10}(x^2+1)^5 + C$

e) $\int x^2\sqrt{x^3+1}\,dx$ $\qquad u = x^3+1$ $\qquad du = 3x^2dx$ $\qquad \frac{1}{3}du = x^2dx$

$= \frac{1}{3}\int u^{1/2}du = \frac{1}{3}\cdot\frac{2}{3}\cdot u^{3/2} + C = \frac{2}{9}(x^3+1)^{3/2} + C$

f) $\int 2\cdot\sec^2 x\cdot(\tan x + 3)\,dx$ $\qquad u = \tan x + 3$ $\qquad du = \sec^2 x\,dx$

$= 2\int u\cdot du = 2\cdot\frac{1}{2}u^2 + C = (\tan x + 3)^2 + C$