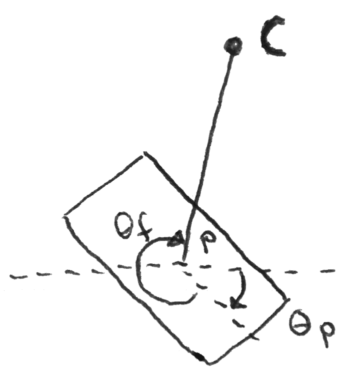
# Mass object

$\frac{dx}{dt} = v$
$\frac{\nabla x}{\nabla t} = v_{new} * k + v_{old} * (1-k)$
$\nabla x = v_{new} * k * \nabla t + v_{old} * (1-k) * \nabla t$
$x_{new} - x_{old} = v_{new} * k * \nabla t + v_{old} * (1-k) * \nabla t$
$x_{new} - k * \nabla t * v_{new} = x_{old} + v_{old} * (1-k) * \nabla t$
$\frac{dy}{dt} = v$
....
$\frac{dv}{dt} = \frac{1}{m} \sum F$
$\frac{\nabla v}{\nabla t} = \frac{1}{m} \sum F_{new} * k + \frac{1}{m} \sum F_{old} * (1-k)$
$\nabla v = \frac{1}{m} \sum F_{new} * k * \nabla t + \frac{1}{m} \sum F_{old} * (1-k) * \nabla t$
$v_{new} - v_{old} = \frac{1}{m} \sum F_{new} * k * \nabla t + \frac{1}{m} \sum F_{old} * (1-k) * \nabla t$
$v_{new} - \frac{1}{m} \sum F_{new} * k * \nabla t = v_{old} + \frac{1}{m} \sum F_{old} * (1-k) * \nabla t$
...
$\frac{d\emptyset}{dt} = w$
$\frac{\nabla \emptyset}{\nabla t} = w_{new} * k + w_{old} * (1-k)$
$\emptyset_{new} - \emptyset_{old} = w_{new} * k * \nabla t + w_{old} * (1-k) * \nabla t$
$\emptyset_{new} - w_{new} * k * \nabla t = \emptyset_{old} + w_{old} * (1-k) * \nabla t$
$\frac{dw}{dt} = \frac{1}{I} \sum \tau$
$\frac{\nabla w}{\nabla t} = \frac{1}{I} \sum \tau_{new} * k + \frac{1}{I} \sum \tau_{old} * (1-k)$
$\nabla w = \frac{1}{I} \sum \tau_{new} * k * \nabla t + \frac{1}{I} \sum \tau_{old} * (1-k) * \nabla t$
$w_{new} - w_{old} = \frac{1}{I} \sum \tau_{new} * k * \nabla t + \frac{1}{I} \sum \tau_{old} * (1-k) * \nabla t$
$w_{new} - \frac{1}{I} \sum \tau_{new} * k * \nabla t = w_{old} + \frac{1}{I} \sum \tau_{old} * (1-k) * \nabla t$

# 1 Compound pendlum




## 1.1 Physic model

Object variables: $p = \{p_x, p_y\}, \Theta_p$
Forces: $F = \{F_x, F_y\}, \tau$
Constants: $c = \{c_x, c_y\}, \Theta_f, l$
Model:

$$\Theta_{sum} = \Theta_f + \Theta_c$$

$$p = c + l \cdot e^{i \cdot \Theta}$$

Force prependicular to pendlum angle equals torque / l

$$F_x \cdot \cos(\Theta_{sum} + \pi/42) + F_y \cdot sin(\Theta_{sum} + \pi/2) = \frac{\tau}{l}$$

rewritten:

$$\begin{aligned}
p_x &= c_x + l \cdot \cos\Theta_{sum} \\
p_y &= c_y + l \cdot \sin\Theta_{sum} \\
F_x \cdot \sin(\Theta_{sum}) - F_y \cdot \cos(\Theta_{sum}) &= \frac{\tau}{l}
\end{aligned}$$

implementation:
_next is next newton rhapsson iteration, _curr is current value

$$\begin{aligned}
\cos(\Theta + \Delta\Theta) &\approx \cos(\Theta) - \sin(\Theta) \cdot \Delta\Theta \\
\sin(\Theta + \Delta\Theta) &\approx \sin(\Theta) + \cos(\Theta) \cdot \Delta\Theta \\
\Delta\Theta &= \Theta_{next} - \Theta_{curr}
\end{aligned}$$

Eq1

$$\begin{aligned}
p_{x_{next}} &= c_x + l \cdot \cos(\Theta_{sum} + \Delta\Theta) \Longrightarrow \\
p_{x_{next}} &= c_x + l \cdot \cos(\Theta_{sum}) - l \cdot \sin(\Theta_{sum}) \cdot \Delta\Theta \Longrightarrow \\
p_{x_{next}} &= c_x + l \cdot \cos(\Theta_{sum}) - l \cdot \sin(\Theta_{sum}) \cdot \Theta_{next} + l \cdot \sin(\Theta_{sum}) \cdot \Theta_{curr} \Longrightarrow \\
p_{x_{next}} + l \cdot \sin(\Theta_{sum}) \cdot \Theta_{next} &= c_x + l \cdot \cos(\Theta_{sum}) + l \cdot \sin(\Theta_{sum}) \cdot \Theta_{curr}
\end{aligned}$$

Eq2

$$\begin{aligned}
p_{y_{next}} &= c_y + l \cdot \sin(\Theta_{sum} + \Delta\Theta) \Longrightarrow \\
p_{y_{next}} &= c_y + l \cdot \sin(\Theta_{sum}) + l \cdot \cos(\Theta_{sum}) \cdot \Delta\Theta \Longrightarrow \\
p_{y_{next}} &= c_y + l \cdot \sin(\Theta_{sum}) + l \cdot \cos(\Theta_{sum}) \cdot \Theta_{next} - l \cdot \cos(\Theta_{sum}) \cdot \Theta_{curr} \Longrightarrow \\
p_{y_{next}} - l \cdot \cos(\Theta_{sum}) \cdot \Theta_{next} &= c_y + l \cdot \sin(\Theta_{sum}) - l \cdot \cos(\Theta_{sum}) \cdot \Theta_{curr}
\end{aligned}$$

Eq2

$$\begin{aligned}
F_{xnext} \cdot \sin(\Theta_{sum}) - F_{ynext} \cdot \cos(\Theta_{sum}) &= \frac{\tau_{next}}{l} \Longrightarrow \\
F_{xnext} \cdot l \cdot \sin(\Theta_{sum}) - F_{ynext} \cdot l \cdot \cos(\Theta_{sum}) - \tau_{next} &= 0
\end{aligned}$$

# 2 Spring pendlum

## 2.1 Enery loss

# 3 Lose pendlum

Object variables: $p_1 = \{x_1, y_1\}, p_2 = \{x_2, y_2\}$
Forces: $F_1 = \{F_x, F_y\}$
Constants: $l$
Model:

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 = l^2$$

$$-vy \cdot F_x + vx \cdot F_y = 0$$

$$\{vx, vy\} = \{x_1 - x_2, y_1 - y_2\}$$

Implementation:

$$\begin{aligned}
(x_1 - x_2)^2 + (y_1 - y_2)^2 &= l^2 \Rightarrow \\
(x_{1_{curr}} - x_{2_{curr}}) \cdot (x_{1_{next}} - x_{2_{next}}) + (y_{1_{curr}} - y_{2_{curr}}) \cdot (y_{1_{next}} - y_{2_{next}}) &= l \left| x_{1_{curr}} - x_{2_{curr}}, y_{1_{curr}} - \right. \\
vx \cdot x_1 - vx \cdot x_2 + vy \cdot y_1 - vy \cdot y_2 &= l \cdot \sqrt{(vx \cdot vx + vy \cdot vy)}
\end{aligned}$$

# 4 One side fixed connection

Object variables: $p_1 = \{x_1, y_1\}, p_2 = \{x_2, y_2\}, \theta_1$
Forces: $F = \{F_1, F_1\}, \tau$
Constants: $l, \theta_f$
Model:

$$p_2 = p_1 + l \cdot e^{i \cdot \Theta}$$

$$\tau = l \cdot \{-\sin(\theta_1 + \theta_f) \cdot F_x, \cos(\theta_1 + \theta_f) \cdot F_y\}$$

Help:

$$\begin{aligned}
\cos(\Theta + \Delta\Theta) &\approx \cos(\Theta) - \sin(\Theta) \cdot \Delta\Theta \\
\sin(\Theta + \Delta\Theta) &\approx \sin(\Theta) + \cos(\Theta) \cdot \Delta\Theta \\
\Delta\Theta &= \Theta_{next} - \Theta_{curr}
\end{aligned}$$

Implementation:
eq1:

$$
\begin{aligned}
p_2 &= p_1 + l \cdot e^{i \cdot \Theta} \Rightarrow \\
x_{2_{next}} &= x_{1_{next}} + l \cdot \cos\left(\theta_{1_{curr}} + \theta_f + \Delta\theta\right) \Rightarrow \\
x_{2_{next}} - x_{1_{next}} &= l \cdot \cos\left(\theta_{1_{curr}} + \theta_f\right) - l \cdot \sin\left(\theta_{1_{curr}} + \theta_f\right) \cdot \left(\Theta_{1_{next}} - \Theta\right. \\
x_{2_{next}} - x_{1_{next}} + l \cdot \sin\left(\theta_{1_{curr}} + \theta_f\right) \cdot \Theta_{1_{next}} &= l \cdot \cos\left(\theta_{1_{curr}} + \theta_f\right) + l \cdot \sin\left(\theta_{1_{curr}} + \theta_f\right) \cdot \Theta_{1_{curr}}
\end{aligned}
$$

eq2:

$$
\begin{aligned}
p_2 &= p_1 + l \cdot e^{i \cdot \Theta} \Rightarrow \\
y_{2_{next}} &= y_{1_{next}} + l \cdot \sin\left(\theta_{1_{curr}} + \theta_f + \Delta\theta\right) \Rightarrow \\
y_{2_{next}} - y_{1_{next}} &= l \cdot \sin\left(\theta_{1_{curr}} + \theta_f\right) + l \cdot \cos\left(\theta_{1_{curr}} + \theta_f\right) \cdot \left(\Theta_{1_{next}} - \Theta\right. \\
y_{2_{next}} - y_{1_{next}} - l \cdot \cos\left(\theta_{1_{curr}} + \theta_f\right) \cdot \Theta_{1_{next}} &= l \cdot \sin\left(\theta_{1_{curr}} + \theta_f\right) - l \cdot \cos\left(\theta_{1_{curr}} + \theta_f\right) \cdot \Theta_{1_{curr}}
\end{aligned}
$$

eq3:

$$\tau_{next} = l \cdot \left\{-\sin\left(\theta_{1_{curr}} + \theta_f\right) \cdot F_{x_{nect}}, \cos\left(\theta_{1_{curr}} + \theta_f\right) \cdot F_{y_{next}}\right\}$$

# 5 Fixed connection

Object variables: $p_1 = \{x_1, y_1\}, p_2 = \{x_2, y_2\}, \theta_1, \theta_2$
Forces: $F_1 = \{F_{1x}, F_{1y}\}, F_2 = \{F_2, F_2\}, \tau_1, \tau_2$
Constants: $l, \theta_{f1}, \theta_{f2}$
Model:

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 = l^2$$

$$-vy_{norm} \cdot F_{1x} + vx_{norm} \cdot F_{1y} = \frac{\tau_2}{l}$$

$$-vy_{norm} \cdot F_{2x} + vx_{norm} \cdot F_{2y} = \frac{\tau_1}{l}$$

$$vx_{norm} \cdot F_{1x} + vy_{norm} \cdot F_{1y} + vx_{norm} \cdot F_{2x} + vy_{norm} \cdot F_{2y} = 0$$

$$\tan\left(\theta_1 + \theta_{f1}\right) = \frac{x_1 - x_2}{y_1 - y_2}$$

$$\theta_1 + \theta_{f1} = \theta_2 + \theta_{f2}$$

$$\{vy_{norm}, vy_{norm}\} = \{x_1 - x_2, y_1 - y_2\} / abs(vx, vy)$$

Implementation:

$$\begin{aligned} vx^* &= x_1 - x_2 \\ vy^* &= y_1 - y_2 \\ scale &= \frac{l}{vx^{*2} + vy^{*2}} \\ vx &= vx^* \cdot scale \\ vy &= vy^* \cdot scale \end{aligned}$$

eq1:

$$\begin{aligned} (x_1 - x_2)^2 + (y_1 - y_2)^2 &= l^2 \Rightarrow \\ (x_{1_{curr}} - x_{2_{curr}}) \cdot (x_{1_{next}} - x_{2_{next}}) + (y_{1_{curr}} - y_{2_{curr}}) \cdot (y_{1_{next}} - y_{2_{next}}) &= l \left| x_{1_{curr}} - x_{2_{curr}}, y_{1_{curr}} - \right. \\ vx \cdot x_1 - vx \cdot x_2 + vy \cdot y_1 - vy \cdot y_2 &= l^2 \end{aligned}$$

eq2:

$$\begin{aligned} \cos(\Theta + \Delta\Theta) &\approx \cos(\Theta) - \sin(\Theta) \cdot \Delta\Theta \\ \sin(\Theta + \Delta\Theta) &\approx \sin(\Theta) + \cos(\Theta) \cdot \Delta\Theta \\ \Delta\Theta &= \Theta_{next} - \Theta_{curr} \end{aligned}$$

$$\begin{aligned} l \cdot cos(\theta_1 + \theta_{f1}) &= x_1 - x_2 \Rightarrow \\ l \cdot cos(\theta_1 + \theta_{f1} + \Delta\theta_1) &= \Delta x_1 - \Delta x_2 \Rightarrow \\ l \cdot \cos(\theta_1 + \theta_{f1}) - l \cdot \sin(\theta_1 + \theta_{f1}) \cdot (\Theta_{1next} - \theta_{1curr}) &= x_{1next} - x_{1curr} - x_{2next} + x_{2curr} \Rightarrow \\ -l \cdot \sin(\theta_1 + \theta_{f1}) \cdot \Theta_{1next} + x_{1next} - x_{2next} &= -l \cdot \cos(\theta_1 + \theta_{f1}) - l \cdot \sin(\theta_1 + \theta_{f1}) \cdot \theta_{1cur} \end{aligned}$$

eq3:

$$\begin{aligned} \theta_1 + \theta_{f1} &= \theta_2 + \theta_{f2} \Rightarrow \\ \theta_1 - \theta_2 &= -\theta_{f1} + \theta_{f2} \end{aligned}$$

eq4:

$$
\begin{aligned}
-vy_{norm} \cdot F_{1x} + vx_{norm} \cdot F_{1y} &= \frac{\tau_2}{l} \Rightarrow \\
-vy \cdot F_{1x} + vx \cdot F_{1y} \quad -\tau_2 &= 0
\end{aligned}
$$

eq5:

$$
\begin{aligned}
-vy_{norm} \cdot F_{2x} + vx_{norm} \cdot F_{2y} &= \frac{\tau_1}{l} \\
-vy \cdot F_{2x} + vx \cdot F_{2y} &= \tau_1
\end{aligned}
$$

eq6:

$$
x_{norm} \cdot F_{1x} + y_{norm} \cdot F_{1y} + x_{norm} \cdot F_{2x} + y_{norm} \cdot F_{2y} = 0
$$

# 6 Friction

## 6.1 Point connected to line

$$
p_1 + l_1 \cdot e^{j(\theta_{f1}+\theta_1)} = p_2 + l_2 \cdot e^{j(\theta_{f2}+\theta_2)}
$$

trig

$$
\begin{aligned}
\cos(\Theta + \Delta\Theta) &\approx \cos(\Theta) - \sin(\Theta) \cdot \Delta\Theta \\
\sin(\Theta + \Delta\Theta) &\approx \sin(\Theta) + \cos(\Theta) \cdot \Delta\Theta \\
\Delta\Theta &= \Theta_{next} - \Theta_{curr}
\end{aligned}
$$

Eq1

$$
\begin{aligned}
p_1 + l_1 \cdot e^{j(\theta_{f1}+\theta_1)} &= p_2 + l_2 \cdot e^{j(\theta_{f2}+\theta_2)} \Rightarrow \\
x_{1_{next}} + l_1 \cdot \cos(\theta_{f1} + \theta_{1_{curr}} + \Delta\theta_1) &= x_{2_{nect}} + l_2 \cdot \cos(\theta_{f2} + \theta_{2_{curr}} + \\
x_{1_{next}} + l_1 \cdot \cos(\theta_{f1} + \theta_{1_{curr}}) - l_1 \cdot \sin(\theta_{f1} + \theta_{1_{curr}}) \cdot (\theta_{1_{next}} - \theta_{1_{curr}}) &= x_{2_{next}} + l_2 \cdot \cos(\theta_{f2} + \theta_{2_{curr}}) \\
x_{1_{next}} - l_1 \cdot \sin(\theta_{f1} + \theta_{1_{curr}}) \cdot \theta_{1_{next}} - x_{2_{next}} + l_2 \cdot \sin(\theta_{f2} + \theta_{2_{curr}}) \cdot \theta_{2_{next}} &= -l_1 \cdot \cos(\theta_{f1} + \theta_{1_{curr}}) - l_1 \cdot \mathrm{si}
\end{aligned}
$$

Eq2

$$
\begin{aligned}
p_1 + l_1 \cdot e^{j(\theta_{f1}+\theta_1)} &= p_2 + l_2 \cdot e^{j(\theta_{f2}+\theta_2)} \Rightarrow \\
y_{1_{next}} + l_1 \cdot \sin(\theta_{f1} + \theta_{1_{curr}} + \Delta\theta_1) &= y_{2_{nect}} + l_2 \cdot \sin(\theta_{f2} + \theta_{2_{curr}} + \\
y_{1_{next}} + l_1 \cdot \sin(\theta_{f1} + \theta_{1_{curr}}) + l_1 \cdot \cos(\theta_{f1} + \theta_{1_{curr}}) \cdot (\theta_{1_{next}} - \theta_{1_{curr}}) &= y_{2_{next}} + l_2 \cdot \sin(\theta_{f2} + \theta_{2_{curr}}) \\
y_{1_{next}} + l_1 \cdot \cos(\theta_{f1} + \theta_{1_{curr}}) \cdot \theta_{1_{next}} - y_{2_{next}} - l_2 \cdot \cos(\theta_{f2} + \theta_{2_{curr}}) \cdot \theta_{2_{next}} &= -l_1 \cdot \sin(\theta_{f1} + \theta_{1_{curr}}) + l_1 \cdot \mathrm{co}
\end{aligned}
$$

Eq3

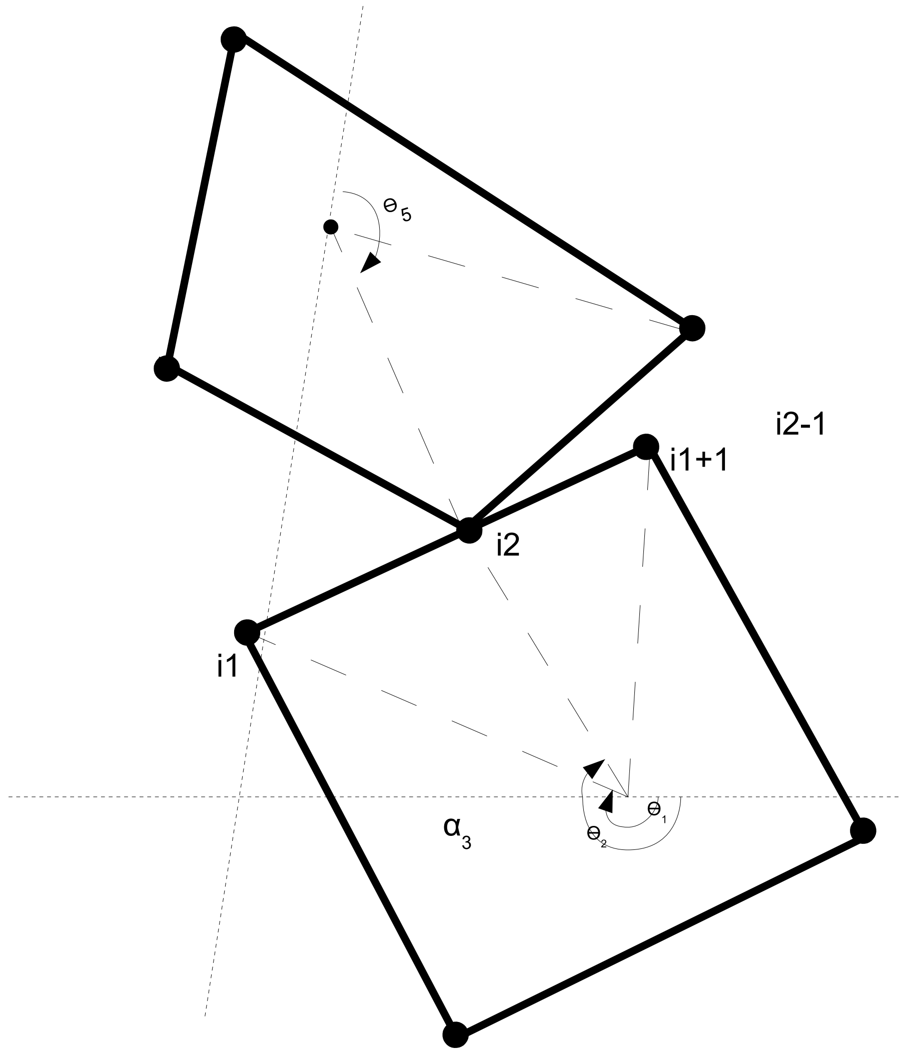
$$\begin{aligned} -vy \cdot F_{1x} + vx \cdot F_{1y} &= \frac{\tau_1}{l_1} \Rightarrow \\ -l_1 \cdot \sin\left(\theta_{f1} + \theta_{1_{curr}}\right) \cdot F_{1x} + l_1 \cdot \cos\left(\theta_{f1} + \theta_{1_{curr}}\right) \cdot F_{1y} - \tau_1 &= 0 \end{aligned}$$

Eq4

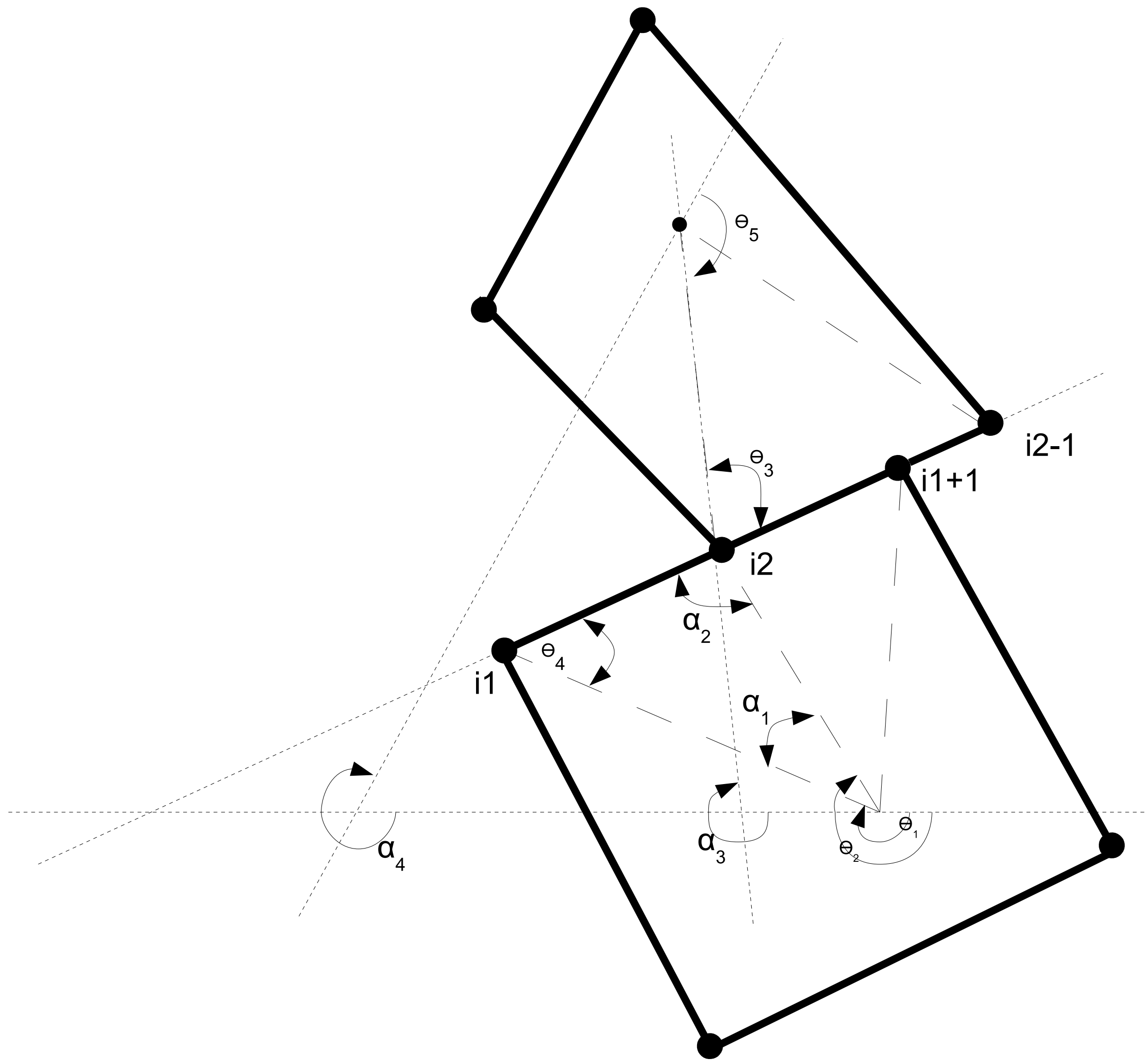
$$\begin{aligned} -vy \cdot F_{1x} + vx \cdot F_{1y} &= - \frac{\tau_2}{l} \Rightarrow \\ -l_2 \cdot \sin\left(\theta_{f2} + \theta_{2_{curr}}\right) \cdot F_{1x} + l_2 \cdot \cos\left(\theta_{f1} + \theta_{2_{curr}}\right) \cdot F_{1y} + \tau_1 &= 0 \end{aligned}$$

# 7 Collision

before:




after:




Know angles: $\Theta_n$
Unknown angles: $\alpha_n$

$$\begin{aligned}
\alpha_1 &= \theta_2 - \theta_1 \\
\alpha_2 &= \pi - \theta_4 - \alpha_1 \\
\alpha_3 &= \theta_2 + \alpha_2 - \theta_3 \\
\alpha_4 &= \pi - \theta_5 + \alpha_3
\end{aligned}$$