Causality Search

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Chapter 1

Introduction

We consider the family of functions of type

$$f(x) = A * \sin(B * \sin(\Omega * x + \Phi) + C) \tag{1.1}$$

where $*$ is some kind of meaningfull product and A; B, C, Ω and Φ are real numbers, matrices or any objects for which $*$ makes sense. These are the parameters that we need to determine in the context of machine learning. This means that, given a set of data $\{(x_\nu, y_\nu) | 1 \leq \nu \leq h\}$, we try to find the "best" possible values for A; B, C, Ω and Φ such that

$$y_\nu \approx f(x_\nu) = A * \sin(B * \sin(\Omega * x_\nu + \Phi) + C) \text{ for } 1 \leq \nu \leq h \tag{1.2}$$

The function sin is the usual sine function. Of course, we need to explain how this function applies on our objects if they are not numbers. if $M = (m_{i,j})$ is a matrix, then $\sin(M)$ can be defined component wise, that is $\sin(M) = (\sin(m_{i,j}))$. Another choice, if M is a square matrix, is to set $\sin(M) = \sum_{k=0}^{\infty}(-1)^k \frac{M^{2k+1}}{(2k+1)!}$, the usual power series of sin.

At last, the variable x can be a number, a vector or even a matrix and the same choices apply to the variable y.

A lot of possibilities, indeed!

Now, a word about how the document is organized.

Each chapter, subchapter treats a specific situation (see above) for the function f. If the material related to the chapter, subchapter is not considered complete, the title of the chapter is preceded by "wip" work in progress.

Generally, one or several Jupyter notebooks illustrate some of the ideas developed. The name given for the notebooks relate to the chapter, subchapter in a very simple way : the title of the chapter ends with a string in parenthesis which defines the beginning of the notebook filename. Then, comes the date (which acts as versioning) followed by a small description or number.

Sometimes, a GeoGebra file is used, that follows the same naming procedure.

Chapter 2

wip : One dimensional case (causality_search_one_dim_case)

2.1 The simpler case $f(x) = b\sin(\omega x + \phi)$

2.1.1 Some simple cases

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Appendices

Appendix A

Homotopy method : presentation

We suppose we want to solve $y_1 = f(x)$, where $x \in D \subset \mathbb{R}^n$, D is an open domain and $y_1 \in \mathbb{R}^n$. We could start by making a "good guess" $x_0 \in D$ and consider solving

$$(y_1 - y_0)t + y_0 = f(x(t)) \text{ with } x(0) = x_0 \text{ and } t \in [0;1] \tag{A.1}$$

where $x : [0;1] \to \mathbb{R}^n$ and $x(0) = x_0$. If f is differentiable with $Df(x) \in \mathbb{R}^{n\times n}$ then, if we take the t derivative of equation (A.1) we get :

$$(y_1 - y_0) = Df(x(t)) \cdot \frac{d}{dt}x(t) \text{ for } t \in [0;1] \tag{A.2}$$

The next step we would like to undertake is multiply (A.2) by $[Df(x(t))]^{-1}$ (the inverse of the matrix $Df(x(t))$) :

$$[Df(x(t))]^{-1}(y_1 - y_0) = \frac{d}{dt}x(t) \text{ for } t \in [0;1] \tag{A.3}$$

(A.3) is a differential equation that could be solved numericaly.

In the different steps above, we ignored some hurdles that must be overcome in order to get this technique working. Lets list them here :

$$\begin{array}{ll} 1. & x_1 \in D \text{ or equivalently } y_1 \in f(D) = \{f(x)\,|x \in D\} \text{ and} \\ 2. & x_0 \text{ and } x_1 \text{ must be path-connected in } D \\ 3. & [Df(x(t))]^{-1} \text{ must exist for all } t \in [0;1] \end{array} \tag{A.4}$$

A.1 Homotopy method : solve $y = \sin(x)$

The only use of this simple example is to illustrate the working of homotopy.

Obviously $\sin(0) = 0$, so lets choose $x_0 = 0$ and $y_0 = 0$. Let $y_1 \in \mathbb{R}$, then following A, we get :

$$\begin{aligned}
(y-0)t+0 &= \sin(x(t)) \\
y &= \cos(x(t))\frac{d}{dt}x(t) \\
\frac{d}{dt}x(t) &= \frac{y}{\cos(x(t))} \qquad \text{(A.5)}
\end{aligned}$$

(A.5) is only possible if $x(t) \neq \frac{\pi}{2} + k\pi$ (for $k \in \mathbb{Z}$).

As $x_0 = 0$, this means that $x(t) \in]\frac{-\pi}{2}; \frac{\pi}{2}[$, the open connected component that contains x_0. This in turn implies $y \times t \in]-1; 1[$ for $t \in [0; 1]$, i.e. $|y| < 1$.