From my Microphone to the Ether

An example-based approach to a bit of math

Marcus Müller

Software Defined Radio Academy 2016

Who am I?

- All-purpose SDR nut
- GNU Radio THE FREE & OPEN SOFTWARE RADIO ECOSYSTEM contributor and user
- …who was a bit overly present on the `discuss-gnuradio@gnu.org` mailing list
- Got hired by Ettus

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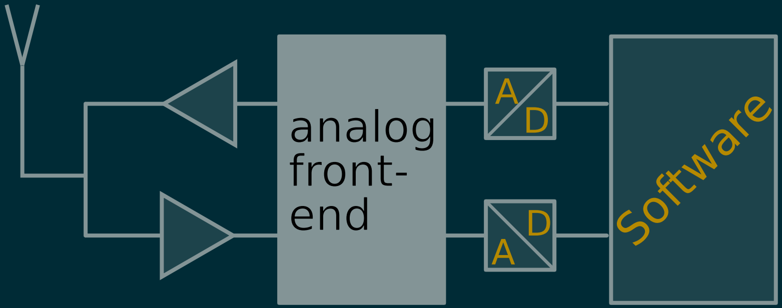
...and who is Ettus Research (A National Instruments Company)?

- Producer of the USRP series of SDR frontends
- `gr-uhd` integrates directly in GNU Radio
- `http://www.ettus.com`
- mostly directly mixing complex baseband receivers, but many can be used in Low-IF and direct sampling modes!

A short overview

What are we after?



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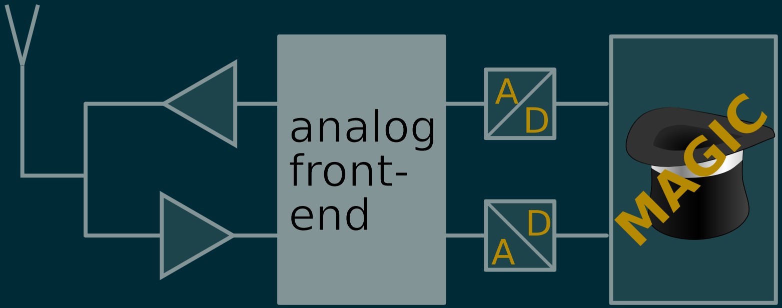

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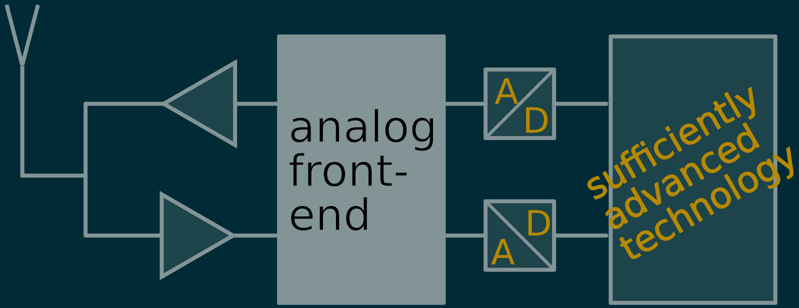

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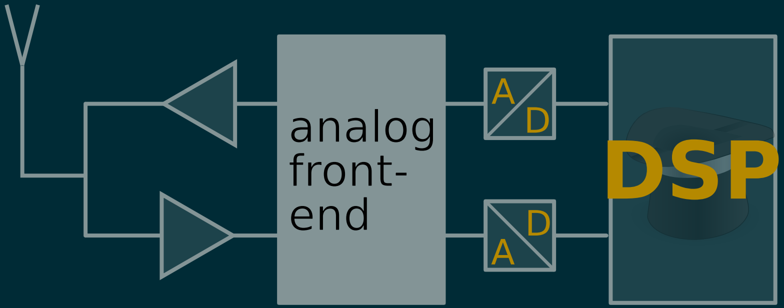


Understanding Signals in the Frequency Domain

Fourier states:

$\boxed{\text{Every sufficiently well-behaved[1] signal can be reproduced to an arbitrary amount of precision by combining harmonic functions}}$

Bonus: if they are periodic, it's only a discrete set of harmonics!

[1] i.e. the signals we care about

Understanding Signals in the Frequency Domain

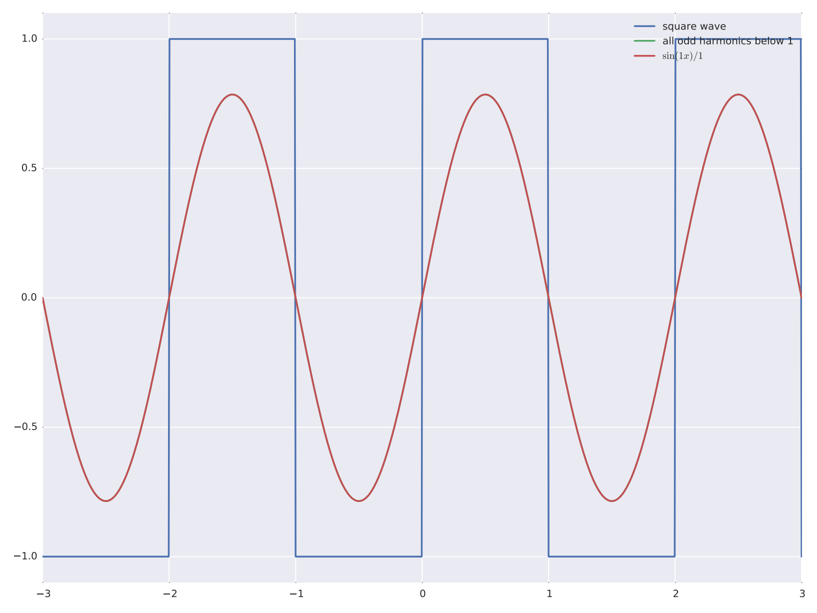
Fourier states:

Every sufficiently well-behaved[1] signal can be reproduced to an arbitrary amount of precision by combining harmonic functions

Bonus: if they are periodic, it's only a discrete set of harmonics!
Example: square wave

[1] i.e. the signals we care about

Square Wave reconstruction through harmonic functions



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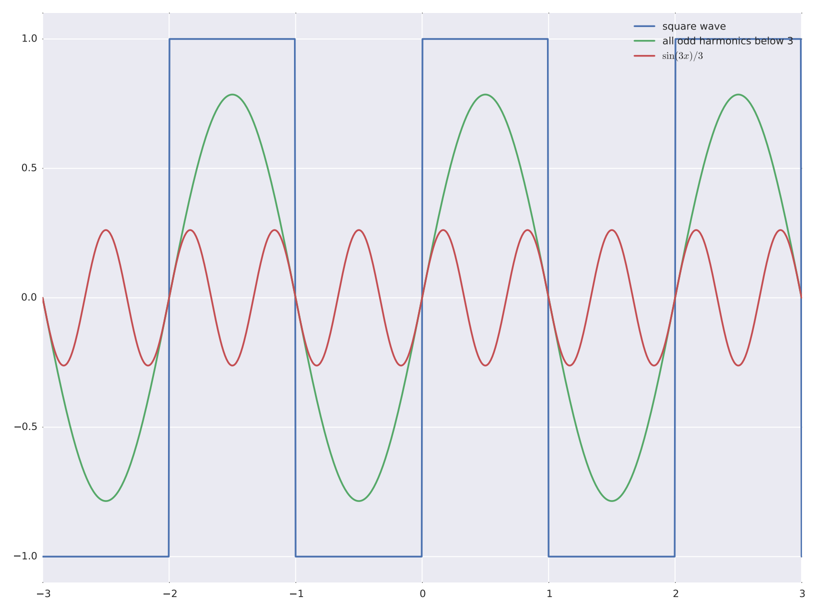

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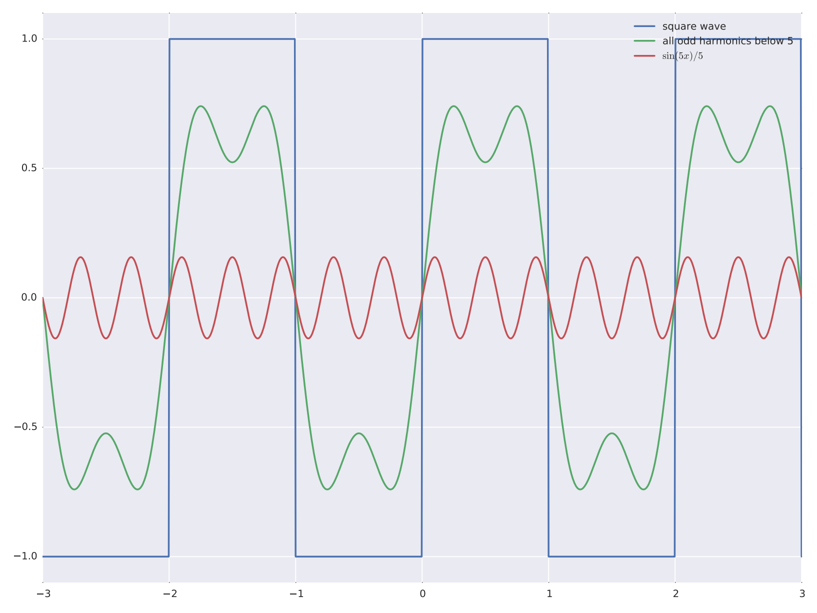

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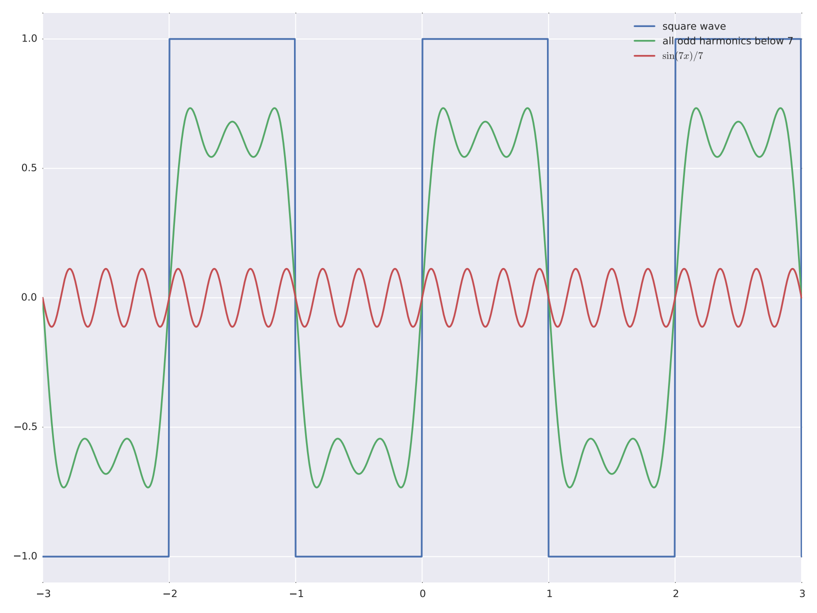

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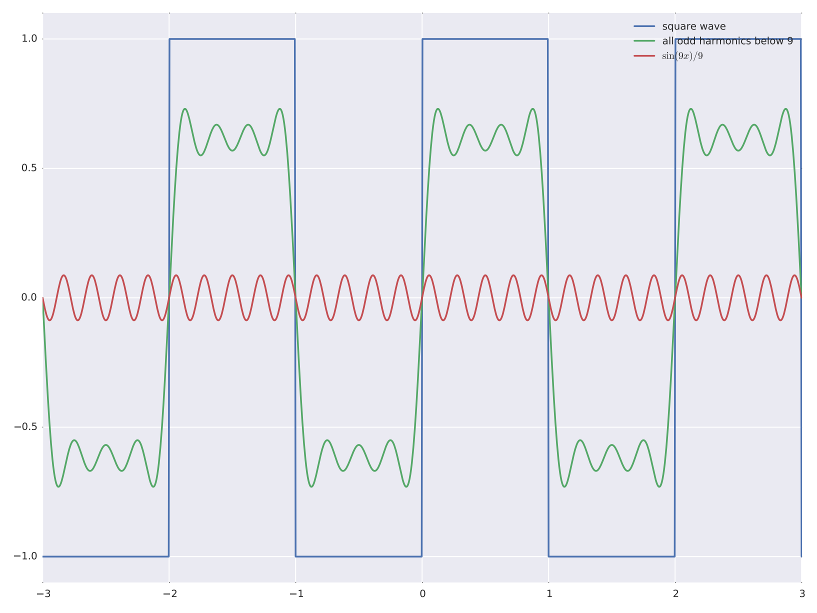

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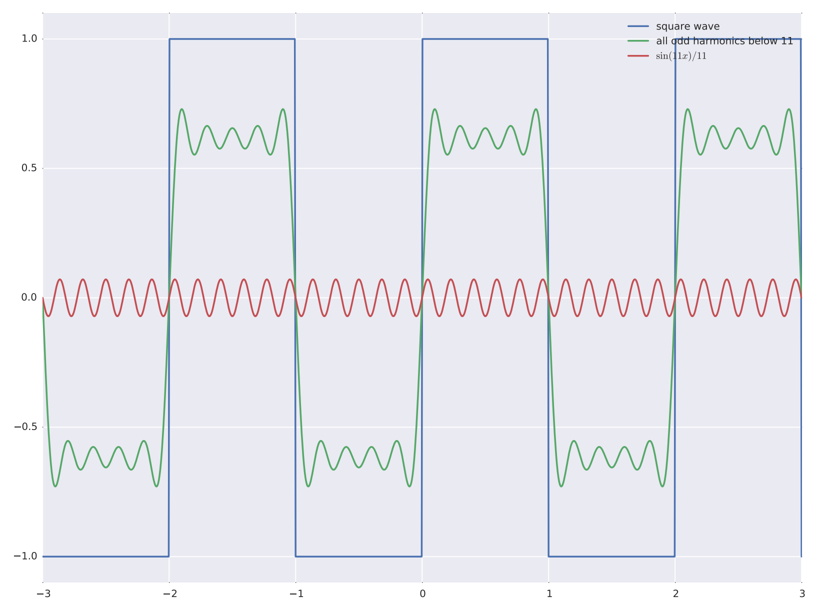

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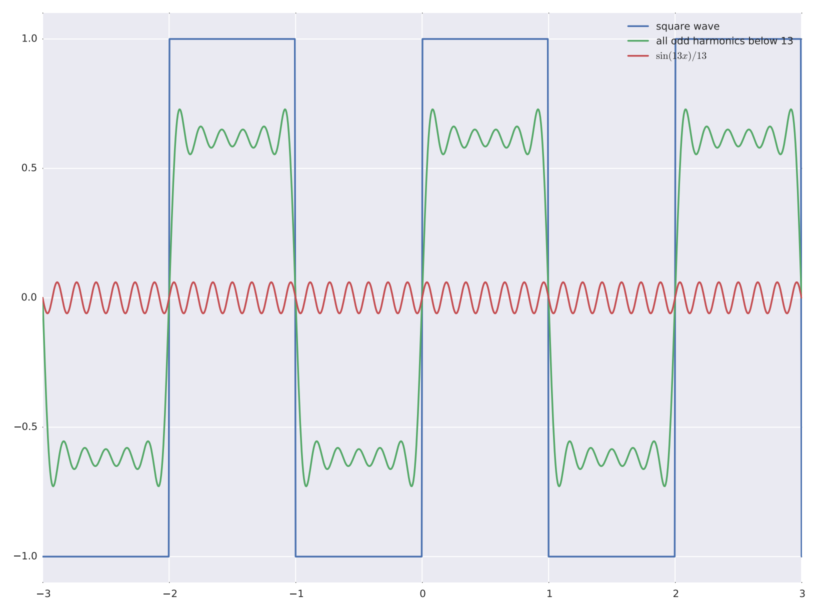

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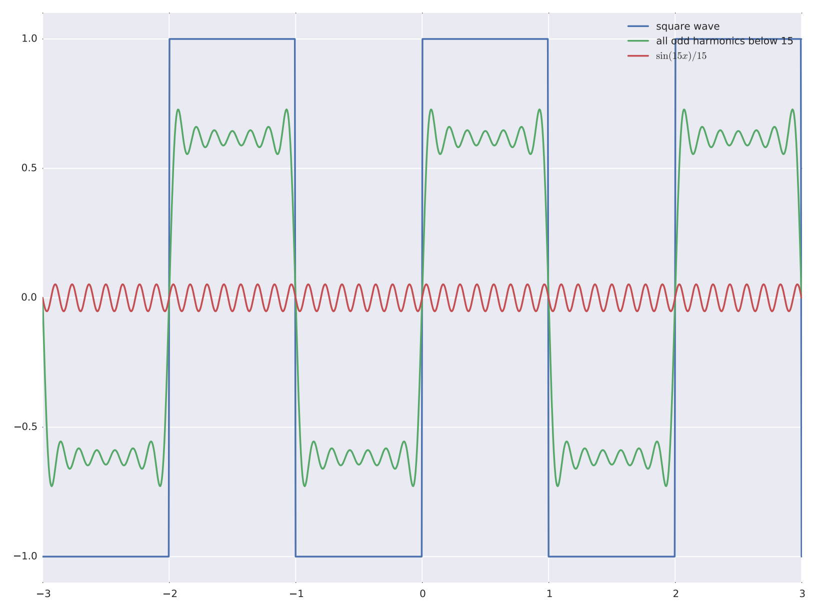

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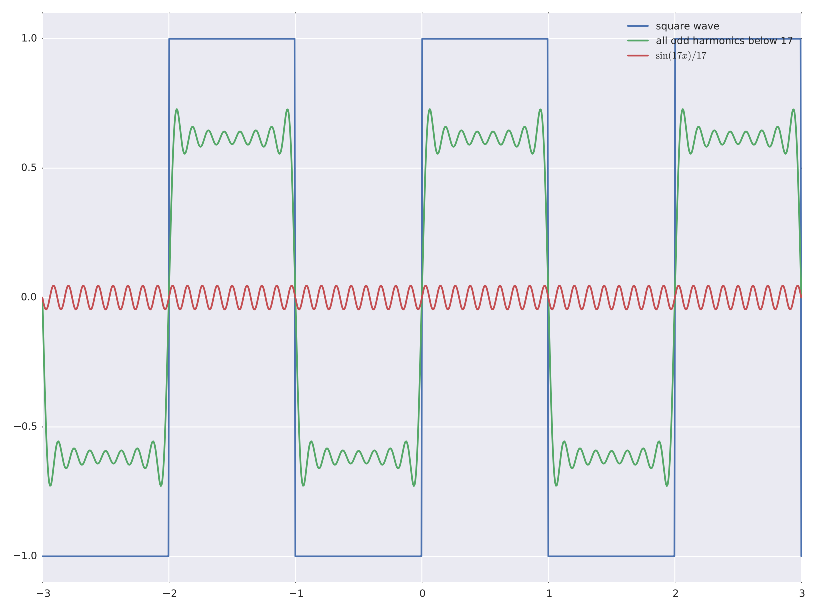

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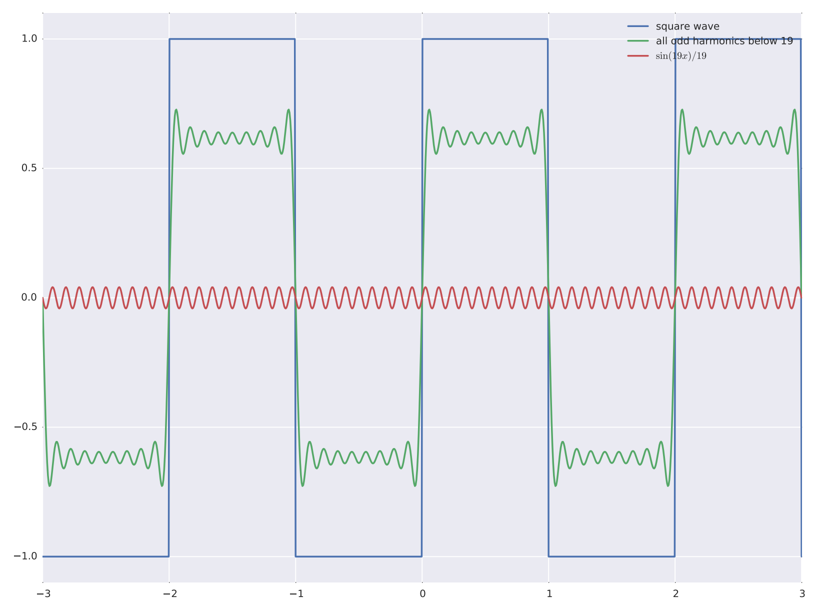

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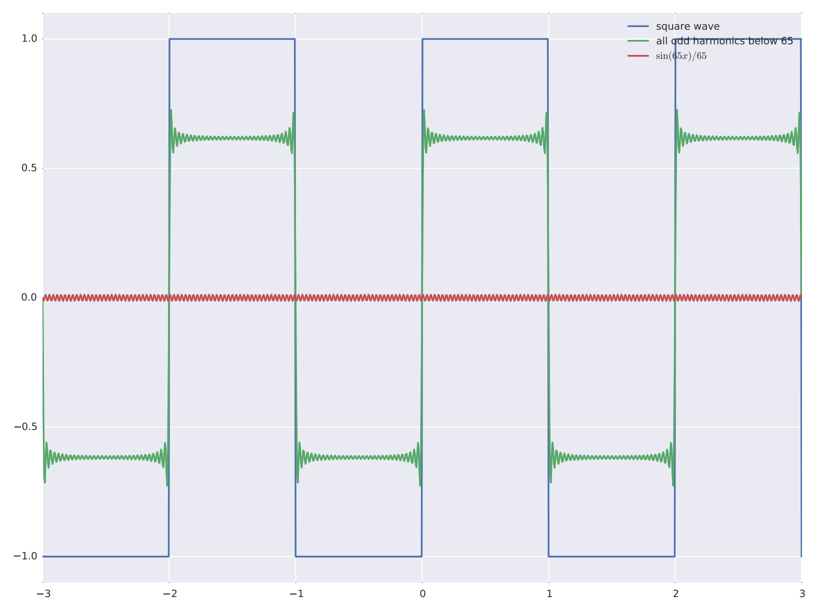

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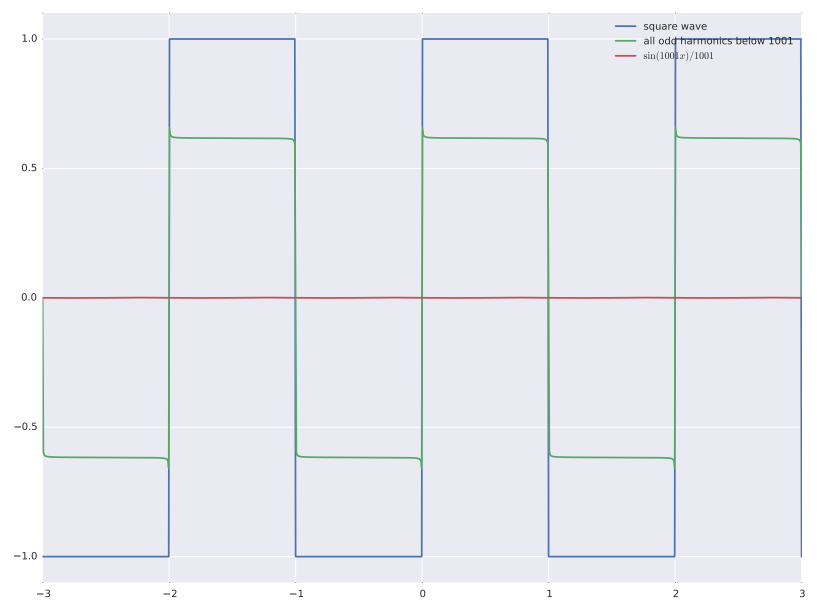
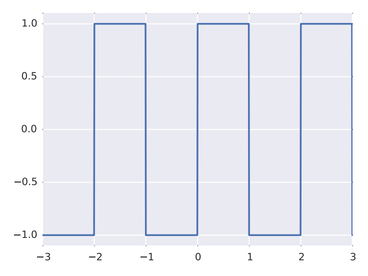


Taking a look at the spectrum

Intuitively, we know that all our sines are just single tones, and will leave a simple line in the spectrum



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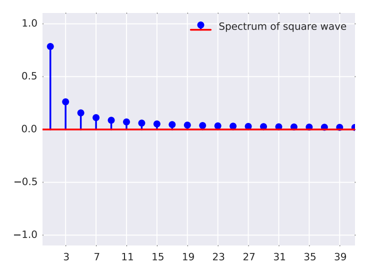

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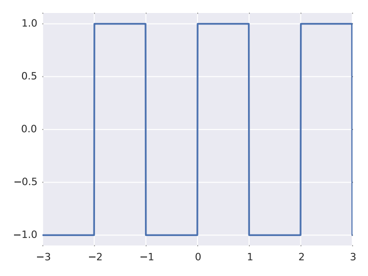

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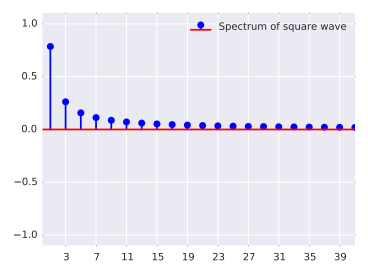

- The **Fourier Transform** actually does exactly that: Converting between time domain and frequency domain.
- Allows for negative frequencies and complex signals/spectra; a bit much math for 30 minutes

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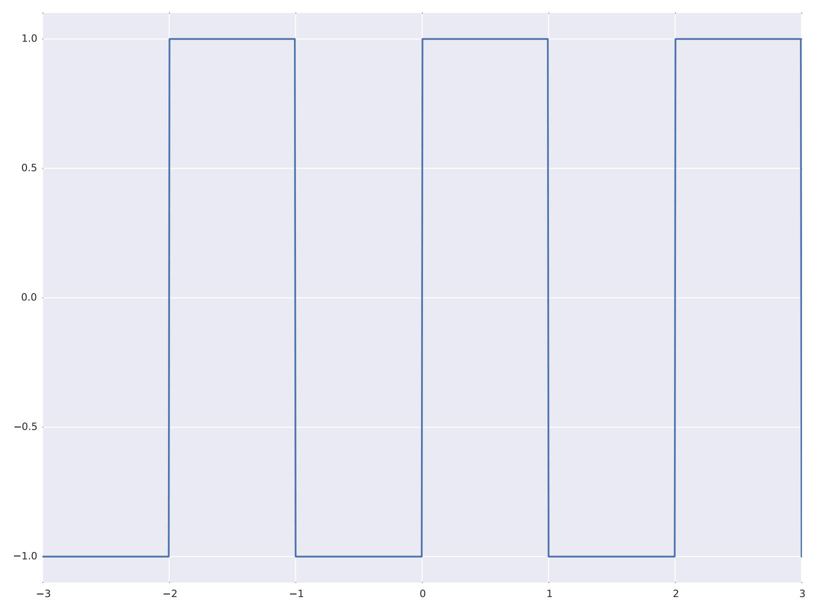

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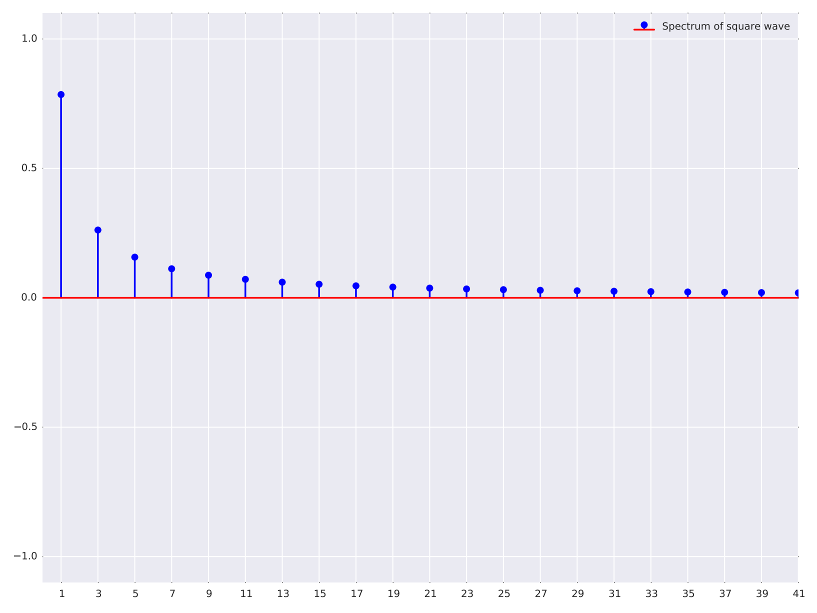

Why is this important?

- real-world systems always have a bandwidth-limiting, usually low-pass, behaviour
- that leaves us with only a limited amount of spectrum to reproduce the original signal

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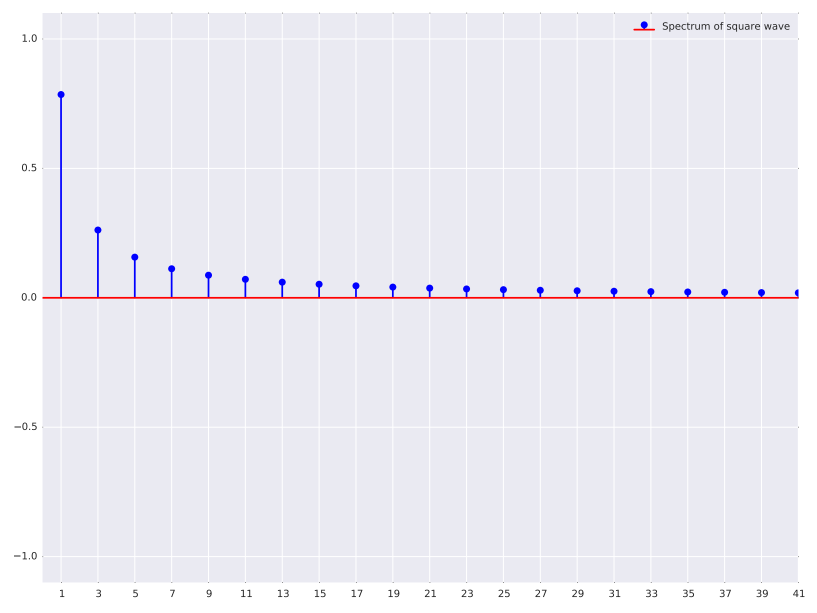

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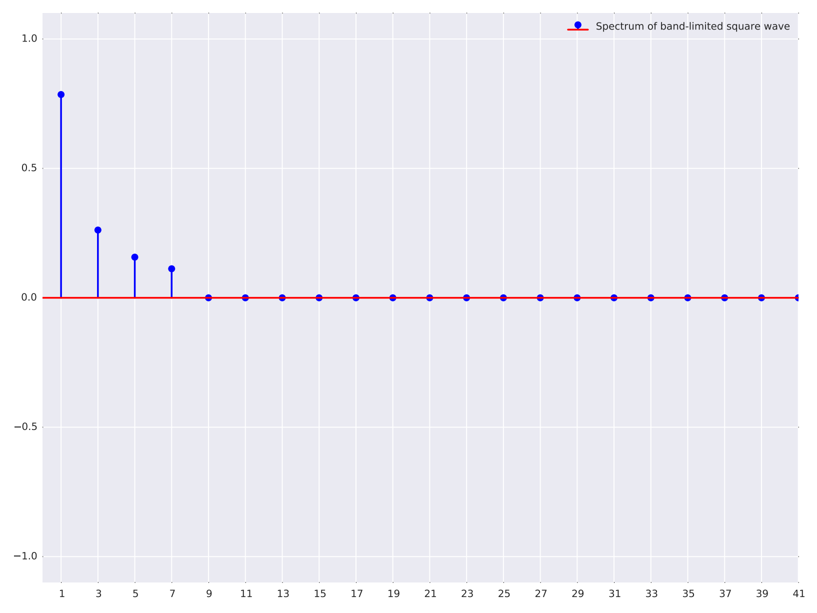

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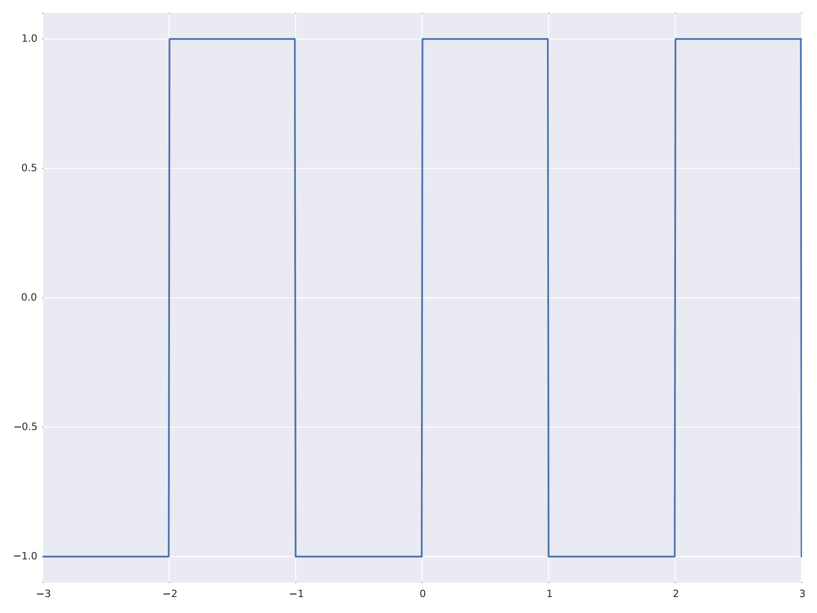

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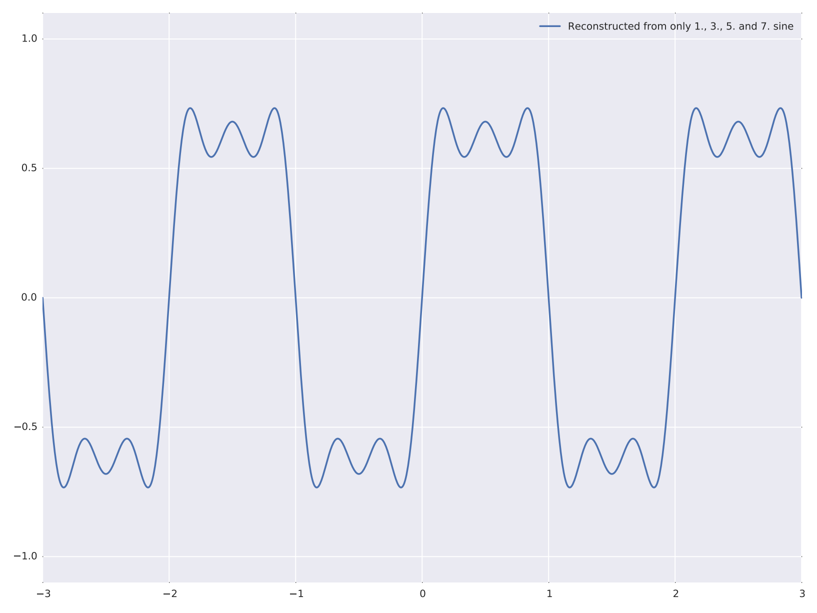
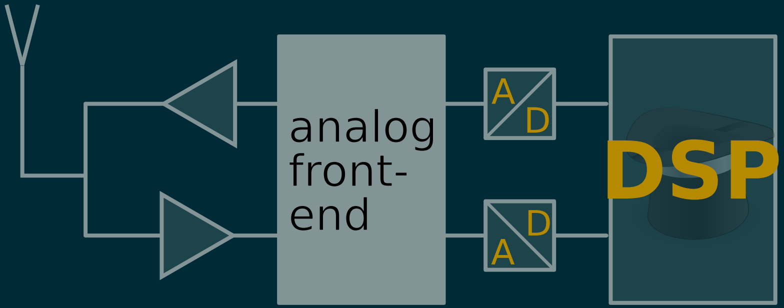


Remember this slide?



Digital Signal Processing

- works with a digital signal instead of analog things like voltages, currents
- typically uses software running on processors or specific hardware to implement all kinds of functionality
  - computers are cheap
  - good filters are expensive
  - software can much easier be written than implementing e.g. a cell phone in hardware alone

What *is* a Digital Signal?

formally

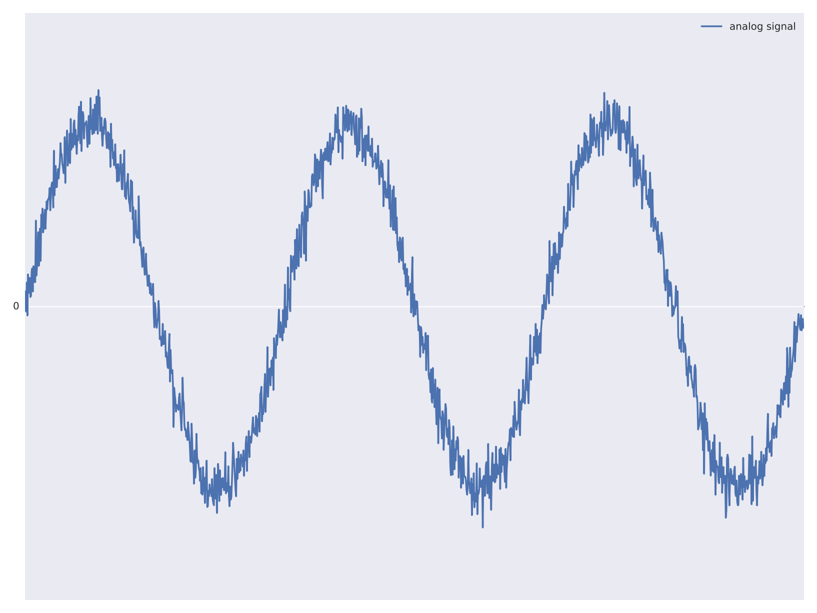
Definition:

A digital signal is a signal that

- only takes discrete values, and
- only exists for discrete times.

What *is* a Digital Signal?

visually



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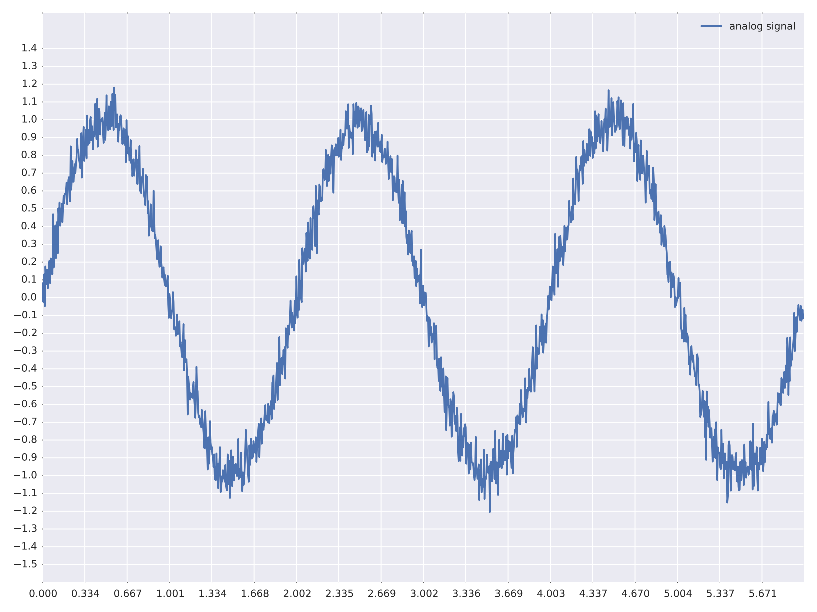

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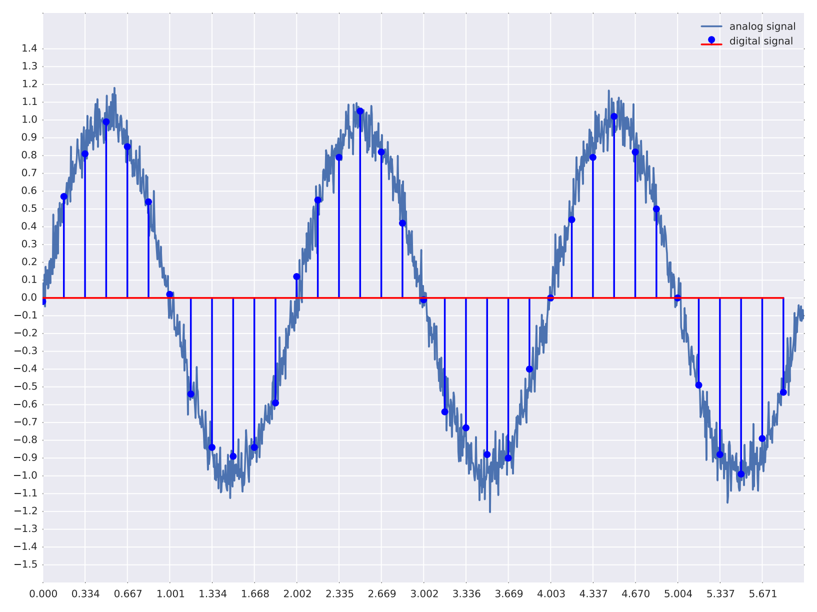

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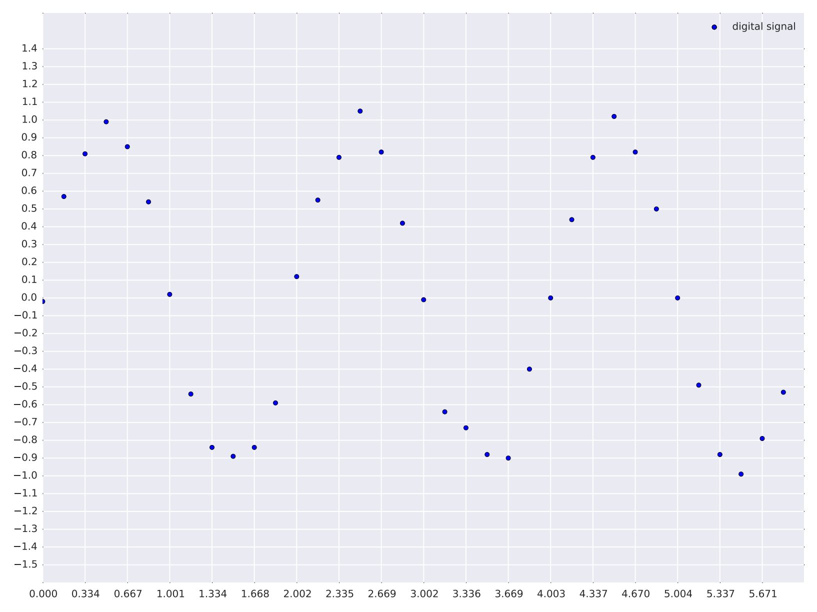
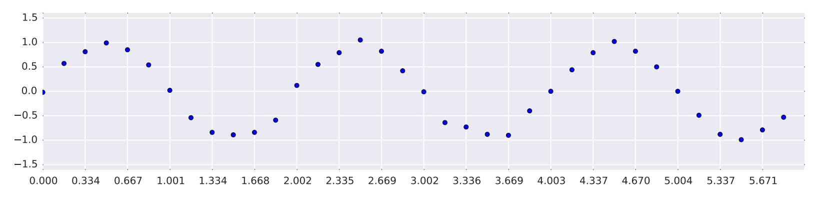


Digital Signals

are great!

- can just be represented as series of numbers



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  -0.02, 0.57, 0.81, 0.99, 0.85, 0.54, 0.02, -0.54, -0.84, -0.89, -0.84, -0.59, 0.12, 0.55, 0.79, 1.05, 0.82, 0.42, -0.01, -0.64, -0.73, -0.88, -0.90, -0.40, 0.00, 0.44, 0.79, 1.02, 0.82, 0.50, -0.00, -0.49, -0.88, -0.99, -0.79, -0.53

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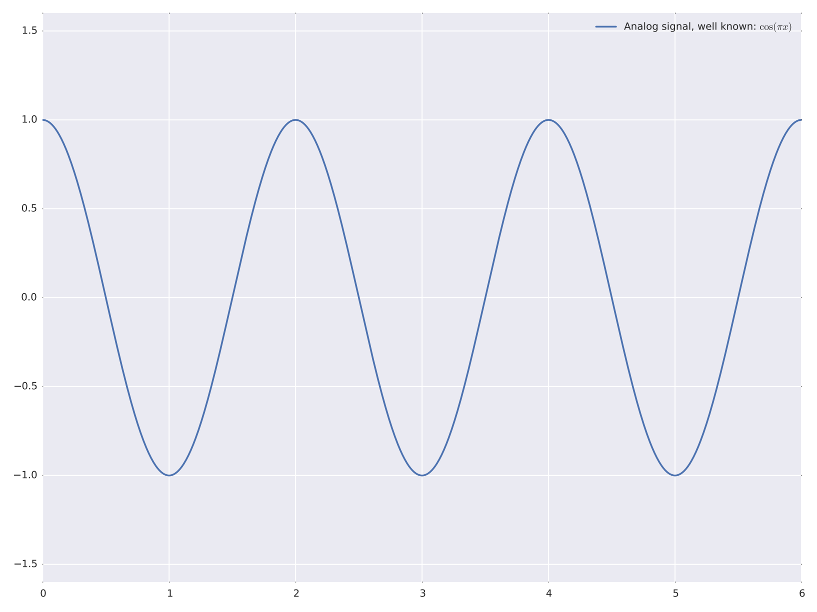
- which is actually something that computers can work with!

Sampling

Going from Analog to Digital in N discrete Steps



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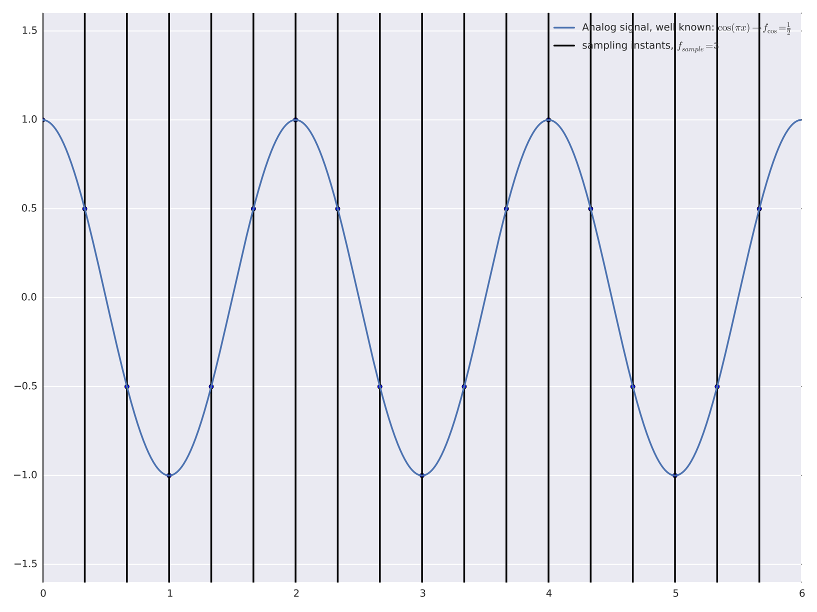
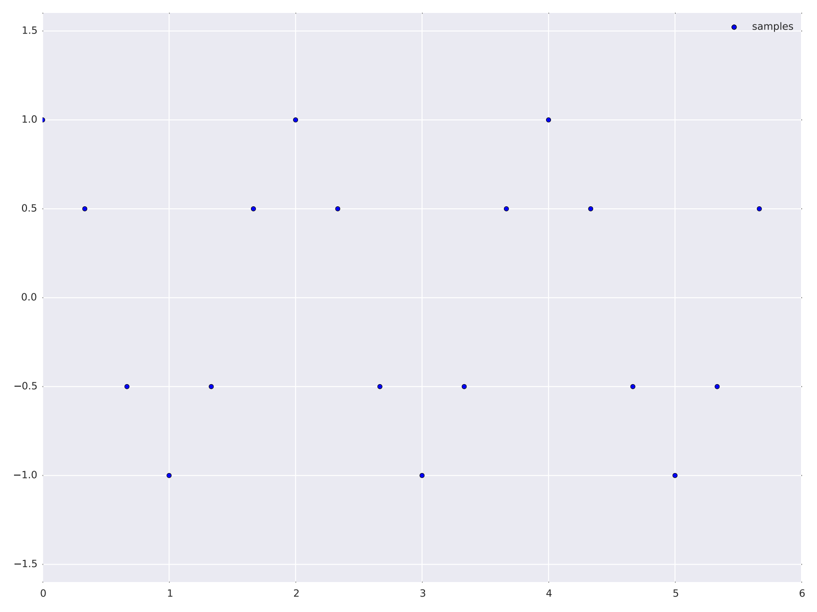


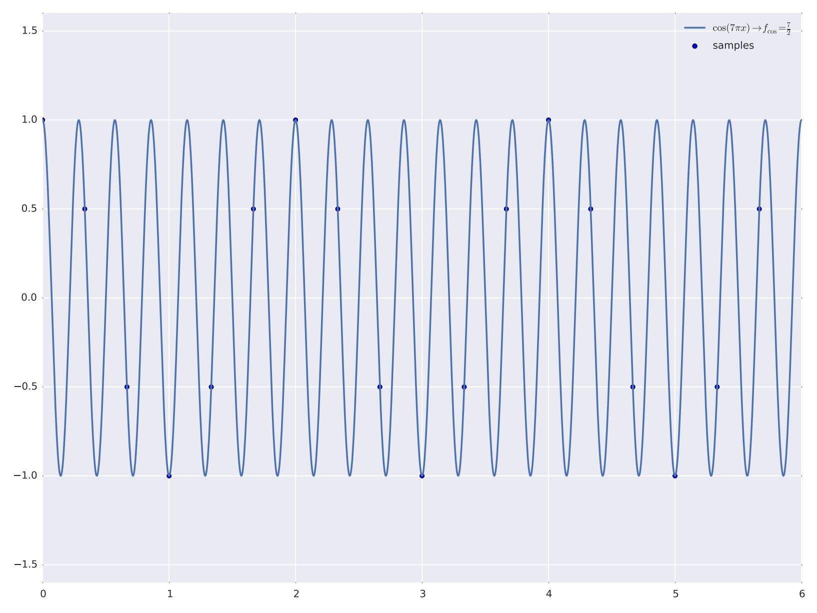
Sampling

Reconstructing is easy!



Sampling

Ouch.



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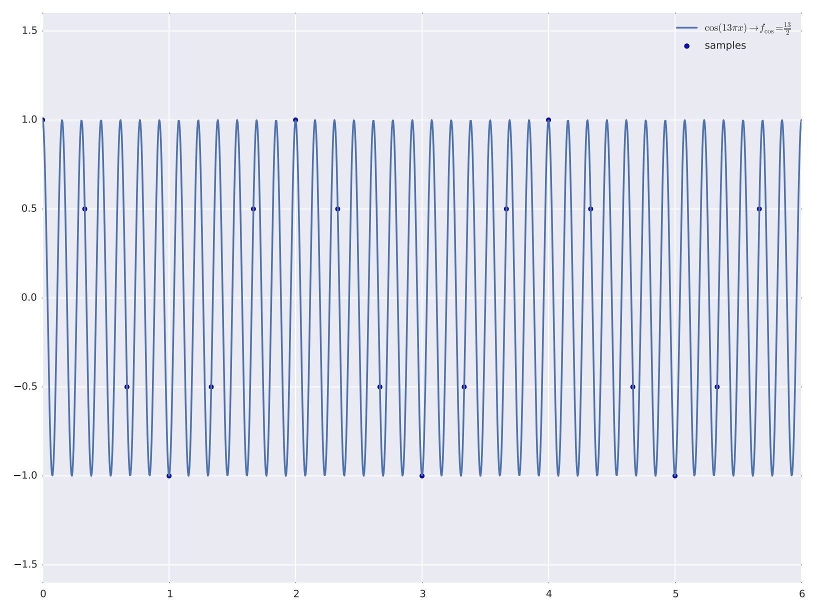

Sampling – Periodicity in Frequency Domain

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- or $f + f_{sample}$ ...
- or $f + 2f_{sample}$ ...
- or, in fact, $f + Nf_{sample}$ for any $N \in \mathbb{N}$.

Sampling – Periodicity in Frequency Domain

when considering a digital signal, its spectrum

- can only meaningfully be defined for a bandwidth of f_{sample},
- repeats every f_{sample}.

Sampling – Periodicity in Frequency Domain

Aliasing

Hence: Sampling analog signals demands:

$\boxed{\textbf{bandwidth limited sufficiently (filtered)!}}$

- frequencies at $f + Nf_{sample}$ ending up at f is called **aliasing**.
- Anti-Aliasing Filter: typically low pass filter

Works the other way around, too – **Images** every f_{sample} when DAC'ing $\rightarrow$ *Reconstruction* filter

Sampling – Periodicity in Frequency Domain

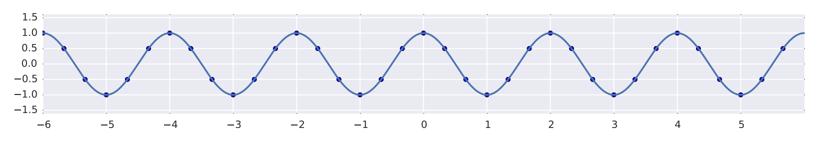
Undersampling

Alternatively *use* aliasing

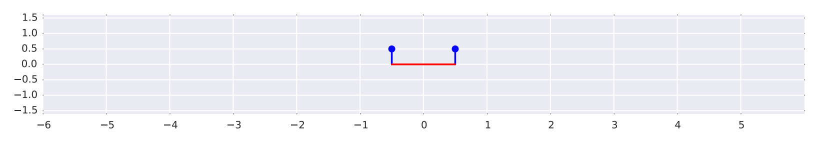
- alias a higher range of spectrum into baseband – **Undersampling**
- works well for relatively low frequencies (filters are easy/affordable/can be build adjustably)
- good high-frequency analog filters expensive/hard to make
- building a receiver working 0–25 MHz just as well as 5.000–5.025 GHz is physically hard
- ... and expensive: imagine the loads of filters!
- typical frequency agility seen with Ettus USRPs is achieved by first mixing to baseband, and then digitizing

Spectra of Digital Signals

- Example: impossible to tell whether your signal is $\cos(\pi x)$ or $\cos(-\pi x)$:



- spectrum of $\cos(\pi x)$ has a isolated value at both $\pm\frac{1}{2}$



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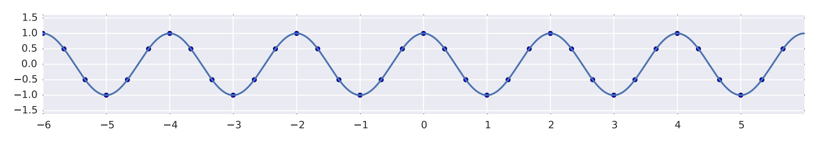
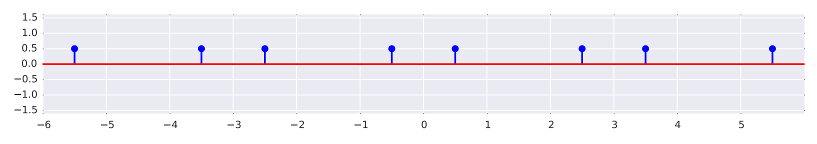


- spectrum of $\cos(\pi x)$ has a isolated value at both $\pm\frac{1}{2}$
- and because we've sampled it, it's f_{sample}-periodic:



Spectra of Digital Signals

Real Signals: Spectrum is hermitian symmetric

- spectra can be complex (otherwise, we couldn't represent phase of a signal)
- $\Re\{S(f)\} = \Re\{S(-f)\}$
- $\Im\{S(f)\} = -\Im\{S(-f)\}$
- Spectrum Analyzer only shows magnitude of spectrum: Can't tell sign of $\Im\{S\}$

Spectra of Digital Signals

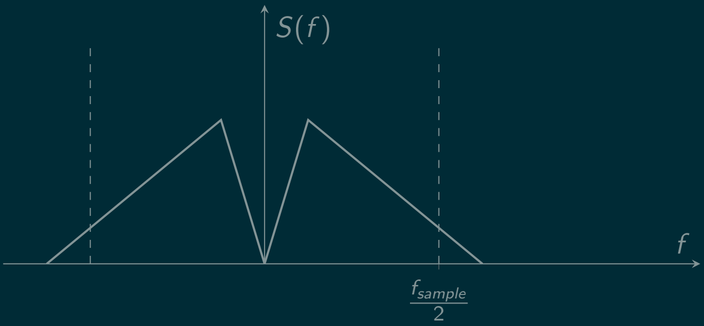
Real Signals: Spectrum is hermitian symmetric

- Positive half of spectrum fully defines negative half
- if symmetric and f_{sample} periodic...
- only half of f_{sample} "usable"

→ **Sampling Theorem** for real-valued signals

The Shannon-Nyquist Sampling Theorem

For real-valued sampling, the observed bandwidth of the analog signal must be limited to less than half the sampling rate.



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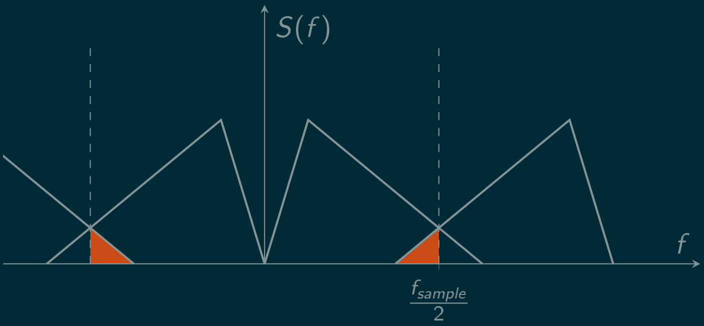

The Shannon-Nyquist Sampling Theorem

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Example:

- Sound cards sample with 44.1 kHz, 48 kHz or 96 kHz. Human perception reaches roughly from 10 Hz to 16 kHz.

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Example:

- Sound cards sample with 44.1 kHz, 48 kHz or 96 kHz. Human perception reaches roughly from 10 Hz to 16 kHz.
- Understanding voice possible using lower bandwidths – can find sampling rates of 16 kHz and below in standards.

The Shannon-Nyquist Sampling Theorem

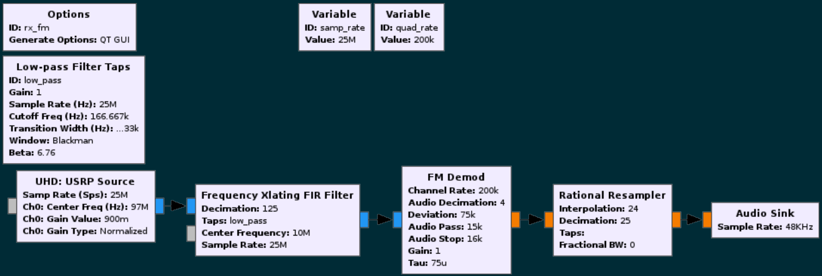
For real-valued sampling, the observed bandwidth of the analog signal must be limited to less than half the sampling rate.

Example:

- superheterodyne receiver:mix 1 MHz of signal from 465.5 – 4.665 MHz to 69.5 – 70.5 kHz ($f_c = 70$ MHz).
  - when considering 0 Hz – 70.5 MHz, one would need a sampling rate of at least 141 MHz
    - high, but far from impossible (USRPs currently do up to 200 MS/s)
    - totally unnecessary!
  - when undersampling, sampling rate of 2 MHz is sufficient
    - high requirement for quality of band-pass signal
    - good SAW filters exist for specific frequencies (reason Superhet is popular, even analog!)

Looking at a complete system

FM Receiver in GNU Radio



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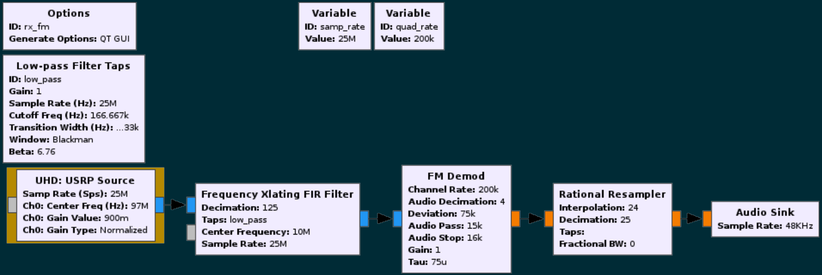
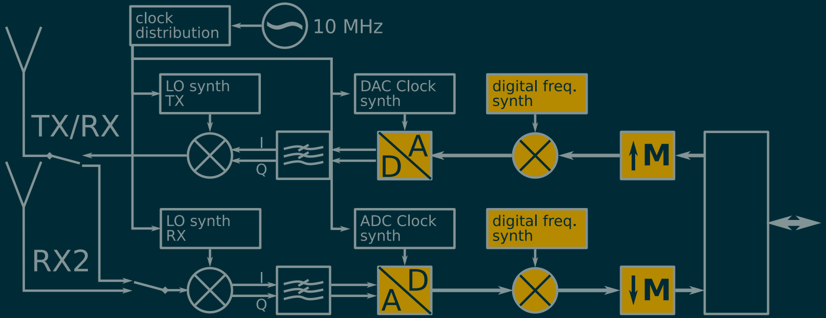


USRP Source

- interface to the USRP
- talks to the driver
- configures all analog and DSP aspects of the USRP
- receives samples

Looking at a complete system

DSP in the USRP



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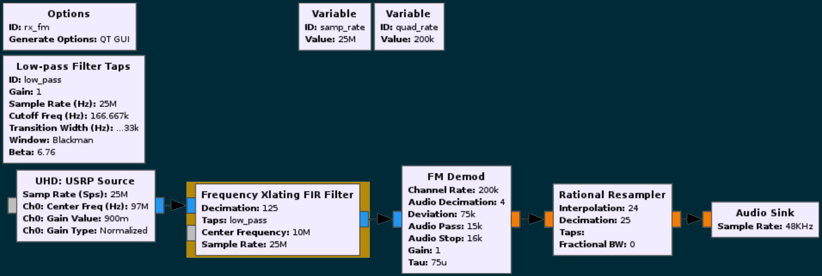
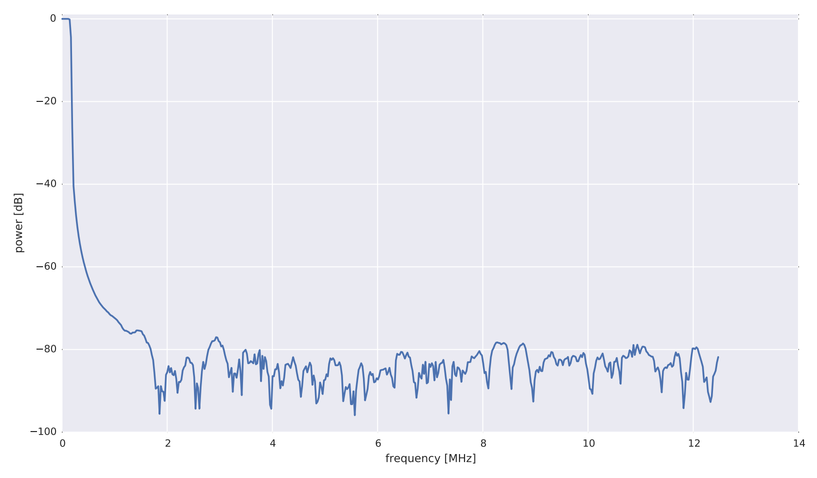


Frequency Translating FIR filter

- shifts the desired frequency to 0 Hz
- then applies filter
- and decimates on the go

Looking at a complete system

- 2045 tap monster of filter
- . . . runs in real time on old laptop at up to 10 MS/s
- attenuation far above necessity



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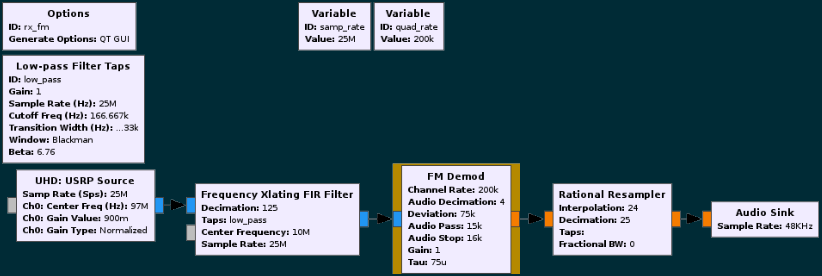

FM Demodulator

- Calculates instantaneous frequency of signal
- integrates and scales the result
- and decimates to an audio-typical rate on the go

Looking at a complete system

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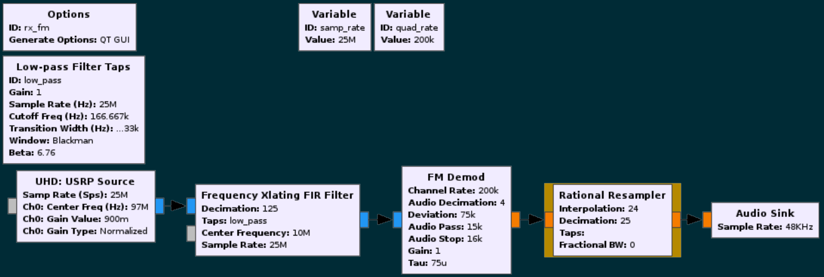
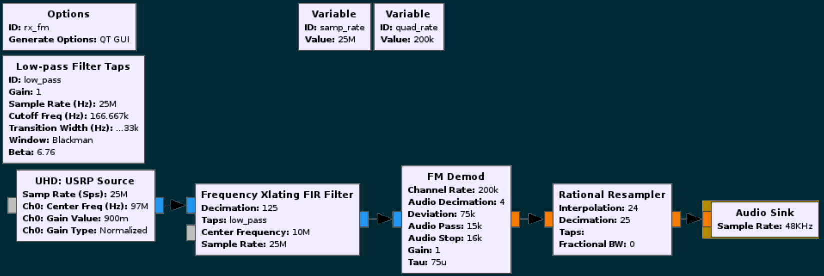


Rational Resampler

- No sound card can do 50 kS/s, but they do 48 kS/s
  - Interpolate to 48x input rate
  - suppress spectral images
  - filter and decimate by 50
- In fact, there's tricks to do the downsampling, filtering and upsampling without going to 50x input rate

Looking at a complete system

FM Receiver in GNU Radio



Audio Sink

- sends samples to the sound card, which Digital-to-Analog converts them

Conclusion

- any signal is representable in digital form …
- … as long as it's band-limited
- Aliasing can lead to out-of-band overlaying wanted signal
  - Anti-Alias Filtering necessary
  - Aliasing can be used for good
- DSP is a rich toolbox that allows construction of incredible filters at very low cost
- SDR hardware gives access to the raw digital signal – great flexibility
- Toolboxes like GNU Radio make it very easy to build extremely capable SDR applications