# TTT 4280 - Øving 4 - Sammensatte feil

## Oppgave 1:

romets volum

$$\alpha_{absorbent} = \frac{24 \ln 10 V}{c \cdot S_{absorbent}} \left( \frac{1}{T_{60,med}} - \frac{1}{T_{60,uten}} \right)$$

Lydhastighet

absorbentprøvens areal i m²

Etterklangstiden

$$\left| \frac{\Delta T_{60,med}}{T_{60,med}} \right| \qquad \left| \frac{\Delta T_{60,uten}}{T_{60,uten}} \right|$$

$$\alpha(T_{med,0} + \Delta T_{med,0}, T_{uten,0} + \Delta T_{uten}) = \alpha(T_{med,0}, T_{uten,0}) + \Delta\alpha$$

$$\Delta\alpha_{T_{med,0},T_{uten,0}} = \frac{\partial \alpha}{\partial T_{med}} \Delta T_{med} + \frac{\partial \alpha}{\partial T_{uten}} \Delta T_{uten}$$

$$= \frac{24 \ln(10) V}{c \cdot S_{absorbent}} \cdot \left( -\frac{1}{T^2_{60,med}} \Delta T_{med} + \frac{1}{T^2_{60,uten}} \Delta T_{uten} \right)$$

Relativ std.feil

$$\left( \frac{\Delta\alpha}{\alpha} \right) = \frac{\left( \frac{\Delta T_{uten}}{T^2_{60,uten}} - \frac{\Delta T_{med}}{T^2_{60,med}} \right)}{\left( \frac{1}{T^2_{60,med}} - \frac{1}{T_{60,uten}} \right)} = \left| \frac{\Delta T_{med}}{T_{60,med}} \right| - \left| \frac{\Delta T_{uten}}{T^2_{60,uten}} \right|$$

For forventet std.avvik: $\sigma^2_\alpha \approx \left( \frac{\partial \alpha}{\partial T_{med}} \sigma_{med} \right) + \left( \frac{\partial \alpha}{\partial T_{uten}} \sigma_{uten} \right)^2$

$$\sigma_\alpha = \left[ \left[ \frac{24 \ln(10) V}{c \cdot S_{abs}} \right]^2 \cdot \left[ \frac{1}{T^4_{med}} \sigma^2_{med} + \frac{1}{T^4_{uten}} \sigma^2_{uten} \right] \right]^{1/2}$$

b) $V = 240\,m^3$

$S_{absorbent} = 10\,m^2$ $\qquad c = 343{,}4\,m/s$

$\overline{T}_{60,med} = 3{,}554$ $\qquad \overline{T}_{uten} = 4{,}154$

$\sigma^2_{med} = 0{,}1$ $\qquad \sigma^2_{uten} = 0{,}0281$

Stdavvik $S^2_{med} = 0{,}115$ $\qquad S^2_{uten} = 0{,}032$

Estimat for std. avviket: (normalfordelt og uavhengig)

$$\frac{\sigma^2_{med}}{T_{med}} = \frac{0{,}1}{3{,}554} = \underline{0{,}03} \qquad \frac{\sigma^2_{uten}}{T_{uten}} = \underline{0{,}007}$$

$$\sigma_\alpha^2 = \left(\frac{\partial \alpha}{\partial T_{med}} \cdot \sigma_{med}\right)^2 + \left(\frac{\partial \alpha}{\partial T_{uten}} \cdot \sigma_{uten}\right)^2$$

$$= \left(\frac{24 \cdot \ln(10) V}{c \cdot S_{abs.}}\right)^2 \left[\frac{1}{T^4_{med}} \cdot \sigma^2_{med} + \frac{1}{T^4_{uten}} \sigma^2_{uten}\right]$$

$$= (3{,}86)^2 \cdot \left[\frac{0{,}1}{3{,}554^4} + \frac{0{,}03}{4{,}154^4}\right] = 0{,}0107$$

$$\underline{\sigma_\alpha = 0{,}104}$$

c) 95% Konfidensintervall:

Må her bruke t-fordelingen:

$t_{0,025,7} = 2,365$

$$\hat{\alpha} = m_\alpha = \frac{24 \cdot \ln(10) \cdot 240}{343,4 \cdot 10} \left( \frac{1}{3,554} - \frac{1}{4,104} \right)$$

$$\hat{\alpha} = \underline{0,157}$$

$$x_{n+1} \in m_\alpha \pm t_p \frac{\hat{\sigma}_\alpha}{\sqrt{n}}$$

$$x_{n+1} \in 0,157 \pm 2,365 \cdot \frac{0,104}{\sqrt{8}} = \underline{\underline{[0,157 \pm 0,087]}}$$

## Oppgave 2 :

- $Y = kx \ldots$

Minste kvadratmetoden:

$$k = \frac{y - \bar{y}}{x - \bar{x}} \qquad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \qquad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$k = \frac{S_{xy}}{S_x^2} \Rightarrow \qquad S_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

$$S_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Regresjonslinje :

$$k = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n(n-1)S_x^2}$$

# Oppgave 3 :

$X = x_{sann} + \xi \qquad \xi = \mathcal{N}(0, \sigma_x)$

$Y = Y_{sann} + \eta \qquad \eta = \mathcal{N}(0, \sigma_x)$

a)

$d = Y - X = Y_{sann} + \eta - x_{sann} - \xi$

$= Y_{sann} - x_{sann} = d_{sann}$ hvis $\eta = \xi$

Dette er ikke sant da variansen for d ikke vil være 0

Siden $\eta \neq \xi$ for en måling, men over tid kan man estimere en verdi $d = d_{sann}$

b) Std. avviket for d:

$\sigma_d = \sqrt{1 \cdot \sigma_x^2 + 1 \cdot \sigma_x^2} = \sqrt{2}\, \sigma_x$

c) $d = \sqrt{(Y - X)^2} = Y - X$

$\cancel{\sigma_d^2 \approx \sigma_x^2 \left(\frac{\partial d}{\partial x}\right)^2 + \sigma_x^2 \left(\frac{\partial d}{\partial Y}\right)^2 = \sigma_x^2 \cdot \frac{1}{2} \cdot 2 \left(\frac{1}{\sqrt{(Y-X)^2}}\right)^2 + \sigma_x^2 \left(\frac{1}{\sqrt{(Y-X)^2}}\right)^2}$

$\cancel{= 0 \ (?)}$

c) $d = \sqrt{(y-x)^2} = y - x$

dermed blir $\sigma_d = \sqrt{2}\,\sigma_x$

d) Forventet verdi for $d^2 = (y_i - x_i)^2$

$d^2 = (y_{sann} + \beta - x_{sann} - \xi)^2$

$\Rightarrow$ får dermed ved mange målinger, $\alpha = N(0, \sqrt{2}\,\sigma_x)$

$d^2 = (d_{sann} + \alpha)^2$