TFY4280 - Øving 3 - sammensatte feil

Oppgave 1 :

a) $\left|\frac{\Delta \tau}{\tau_0}\right|_{maks} = 0{,}01$ $\left|\frac{\Delta d}{d_0}\right|_{maks} = 0{,}02$

$f = \frac{c\tau}{d}$ $\left|\frac{\Delta f}{f}\right|_{maks} = ?$

Taylor-rekkeutvikling:
med hensyn på τ

$$f(\tau_0 + \Delta\tau) = f(\tau_0) + \frac{df}{d\tau}\bigg|_{\tau_0} \Delta\tau + \frac{1}{2}\frac{d^2f}{d\tau^2}\bigg|_{\tau_0}(\Delta\tau)^2 \ldots$$

$$= f(\tau_0) + \Delta\tau$$

$$\Rightarrow \left|\frac{\Delta f}{f(\tau_0)}\right| \approx \left|\frac{\frac{c\Delta\tau}{d}}{\frac{c}{d}\tau_0}\right| = \left|\frac{\Delta\tau}{\tau_0}\right|$$

her er usikkerheten både i τ og d:

$$f(\tau, d) \Rightarrow f(\tau_0 + \Delta\tau, d_0 + \Delta d)$$

$$= f(\tau_0, d_0) + \frac{\partial f}{\partial \tau}\bigg|_{\tau_0, d_0}\Delta\tau + \frac{\partial f}{\partial d}\bigg|_{\tau_0, d_0}\Delta d$$

$$+ \frac{1}{2}\frac{\partial^2 f}{\partial \tau^2}\bigg|_{\tau_0, d_0}(\Delta\tau)^2 + \frac{1}{2}\frac{\partial^2 f}{\partial d^2}\bigg|_{\tau_0, d_0}(\Delta d)^2 + \frac{\partial^2 f}{\partial\tau\partial d}\bigg|_{\tau_0, d_0}\Delta d\Delta\tau + \ldots$$

$$= f(\tau_0, d_0) + \Delta f$$

Ved små feil kan man gjøre antagelsen:

$$\Delta f|_{d_0,\tau_0} \approx \frac{\partial f}{\partial \tau}\Big|_{\tau_0,d_0} \Delta\tau + \frac{\partial f}{\partial d}\Big|_{\tau_0,d_0} \Delta d$$

Antar at feilene er avhengige?

$$|\Delta f|_{\tau_0,d_0}| \approx \frac{c}{d_0}|\Delta\tau| + -\frac{c\tau_0}{d_0^2}|\Delta d|$$

$$f(\tau,d) = \frac{c\tau}{d}$$

$$\Rightarrow \left|\frac{\Delta f}{f}\right| = \left|\frac{\frac{c}{d}\left(|\Delta\tau| - \frac{\tau}{d}|\Delta d|\right)}{f(\tau,d)}\right|$$

$$= \left|\frac{\frac{c}{d}\tau\left(\left|\frac{\Delta\tau}{\tau}\right| - \left|\frac{\Delta d}{d}\right|\right)}{f(\tau,d)}\right| = \left|\left|\frac{\Delta\tau}{\tau}\right| - \frac{|\Delta d|}{d}\right|$$

$$= |0{,}01 - 0{,}02| = \underline{\underline{0{,}01}}$$

Hvis vi antar at feilene er uavhengige:

$$|\Delta f|_{\tau_0,d_0}| \approx \left|\frac{\partial f}{\partial \tau}\Big|_{\tau_0,d_0}\Delta\tau\right| + \left|\frac{\partial f}{\partial d}\Big|_{\tau_0,d_0}\Delta d\right|$$

$$\left|\frac{\Delta f}{f}\right| = \frac{\frac{c}{d}\Delta\tau + \frac{\tau}{d}|\Delta d|}{f(\tau,d)} = \underline{\underline{0{,}03}}$$

b) Den maksimale relative feilen avhenger ikke av verdien på c, τ eller d, men på $\left|\frac{\Delta \tau}{\tau}\right|$ og $\left|\frac{\Delta d}{d}\right|$

c) Monte-Carlo Simulering:

-> repeter forsøk mange ganger

-> se matlab-fil

$$\Rightarrow \sigma_f^2 = \sigma_\tau^2\left(\frac{\partial f}{\partial \tau}\right)^2 + \sigma_d^2\left(\frac{\partial f}{\partial d}\right)^2 + \dots$$

$$f(\tau, d) = \frac{c\tau}{d}$$

$$\sigma_f^2 \approx \sigma_\tau^2\left(\frac{c}{d}\right)^2 + \sigma_d^2\left(-\frac{c\tau}{d^2}\right)^2 + \dots$$

$$\approx \left(\frac{c\tau}{d}\right)^2\left(\frac{\sigma_\tau^2}{\tau^2} + \frac{\sigma_d^2}{d^2}\right) = f^2\left[\frac{\sigma_\tau^2}{\tau^2} + \frac{\sigma_d^2}{d^2}\right]$$

$$\frac{\sigma_f^2}{f^2} = \frac{\sigma_\tau^2}{\tau^2} + \frac{\sigma_d^2}{d^2}$$

Smaller?

Oppgave 2 :

$$\theta = \cos^{-1} f = \cos^{-1} \frac{c\tau}{d}$$

a) $\left|\frac{\Delta\theta}{\theta}\right|_{maks} \Rightarrow \quad \theta(f_a + \Delta f) = \theta(f_a) + \Delta f$

Dermed blir $\Delta\theta \approx \frac{d\theta}{df} \Delta f$

$$\approx \frac{d \cos^{-1} f}{df} \Delta f = -\frac{1}{\sqrt{1-f^2}} \Delta f$$

$$\Rightarrow \frac{\Delta\theta}{\theta} = -\frac{1}{\sqrt{1-f^2}} \frac{\Delta f}{\cos^{-1} f}$$

b) Ut fra uttrykket over så er $\frac{\Delta\theta}{\theta}$ avhengig av f og dermed av c, τ og d

Den er også avhengig av $\theta = \cos^{-1} f$

Oppgave 3 :

- $\frac{\sigma_\tau}{\tau} = 0{,}01$ $\qquad \frac{\sigma_d}{d} = 0{,}02$

$\Rightarrow$ relative std. avvik

Bruker de første leddene i taylor rekken:

$\sigma_f^2 = \sigma_\tau^2 \left(\frac{\delta f}{\delta \tau}\right)^2 + \sigma_d^2 \left(\frac{\delta f}{\delta d}\right)^2 + \dots$

- $\frac{\sigma_f^2}{f^2} \approx \frac{\sigma_\tau^2}{\tau^2} + \frac{\sigma_d^2}{d^2} = 5 \cdot 10^{-4}$ (se 2c)

$\Rightarrow$ samme for $\frac{\sigma_\theta^2}{\theta}$?

dermed blir

$\sigma_\theta^2 = \sigma_f^2 \left(\frac{d\theta}{df}\right)^2 = \frac{\sigma_f^2}{\quad} \cdot \frac{1}{1-f^2} = \frac{\sigma_f^2}{f^2} \cdot \frac{1}{\frac{1}{f^2}-1}$

- $\frac{\sigma_\theta^2}{\theta} = \frac{\sigma_f^2}{f^2} \cdot \frac{1}{\frac{1}{f^2}-1} \cdot \frac{1}{\cos^{-1} f}$ (?)

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