Coupled 1D Richards and 2D Boussinesq Model

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About This File

SECTION 1

Introduction to the Model

Welcome to this manual. The objective of this file is to provide a brief introduction to the *Coupled 1D Richards and 2D Boussinesq Model* first developed in (Brandhorst et al., 2021).

This file is organized to present the rationale behind all modeling aspects, as well as examples of how to apply the model to real-world case studies. If you have any questions, please contact us at marcusnobrega.engcivil@gmail.com.

The model is developed in Matlab and was adapted to have most of its input data entered from Microsoft Excel spreadsheets. The model allows a variety of simulations with different boundary conditions, such surface irrigation/drainage and groundwater Dirichlet and Neumann boundary conditions.

The model is designed to perform either CPU or GPU computations, which is an important feature when high resolution modeling is required. Parameters can be spatially assigned, therefore states are simulated per each cell of the groundwater domain and per each vertical cell of the unsaturated flow zones of the rectangular grid of the domain.

1.1 Downloading

The model files and input data used in the paper are found on Github and the link is shown (here). Files are organized in folders and can be easily accessed in Github.

1.2 Applications

A few examples of the application of this modeling approach can be:

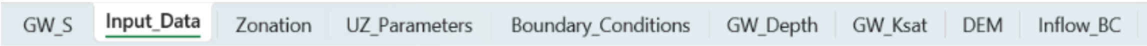


Figure 1: Sheets in the Input Excel File. Each of this sheets are presented following this file.

- Estimate hydrological processes in unconfined aquifers under discrete or continuous simulation
- Estimate groundwater recharge for a given time series of climatological forcing
- Estimate residence times and storage discharge functions
- Simulate effects of wells and rivers in unconfined aquifers.

1.3 Computational Requirements and Software Packages

The model requires a Matlab version 2021 or superior and at least 8 GB of ram memory.

1.4 Additional Software Required

Users need to have access to additional software for data preparation and post-processing activities:

- Microsoft Excel, this spreadsheet application will help to organize the input data required by the model, such as rainfall data, and land use and land cover characteristics, coordinates of points of interest, models performance parameters, among others.
- Geoprocessing tool such as QGIS or ArcGIS to allow georeferenced visualization of model outputs.

1.5 Input Data Files

The current version of the model has all data entry derived from an Excel Spreadsheet, named "General_Input.

In this file, 7 sheets are presented and detailed as shown in Fig. 1.

1.6 Input_Data

The main sheet is the Input_Data, which contains time-stepping information, UZ model discretization, GW model discretization, and the label (i.e., name) of the simulation. The time-stepping control data is presented in Fig. 2. The unsaturated zone parameters are shown in Fig. 3. The GW discretization parameters are shown in Fig. 4 and the setup name is shown in Fig. 5.

1.7 Zonation

Running a fully 1D-2D GW model would require 1D unsaturated flow models per each 2D GW cell, potentially making the model slow. To deal with that, zones are defined to reduce the number of 1D UZ models. These zones are defined by entering the relative coordinates of each UZ

Time-Stepping

Running Control

t0	0	sec
tfinal	518400	sec
tsave	3600	sec
dtUZ	600	sec
dtGW	600	sec
AdaptiveUZ	1	
AdaptiveGW	0	

Figure 2: Time-step running control, where users can define the initial and end of the simulation, as well as the time-step configurations used in the GW and UZ models.

Unsaturated Zone

Grid Discretization

H	1	m
nz_top	5	nodes
dz_top	0.05	m
nz_down	5	nodes
dz_down	0.05	m
dz	0.01	m
UZ nodes	100	

Figure 3: Unsaturated zone discretization. The groundwater thickness is specified by H. At the top of the domain, nz_top nodes are discretizing the dz_top discrete length at the top. Similarly, nz_down nodes divide the discrete length dz_down at the bottom of the aquifer. In the between the bottom and top, the domain is discretized with a resolution of dz.

Groundwater Model

Grid Discretization

ny	60	
nx	24	
dx	11.5	m
dy	30	m

Figure 4: Discretization of the 2D GW domain. Notice that y represent the rows of the matrices, whereas x are the collumns. Ny defines the number of rows and nx the number of rows. Lengths dx and dy are the width and length of the 2D domain, defined by the DEM, followed explained.

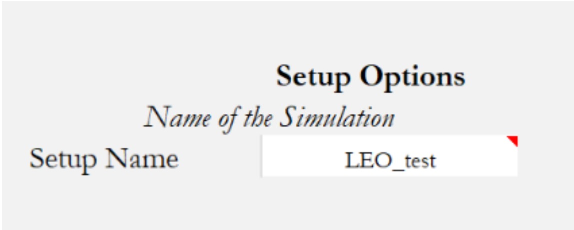
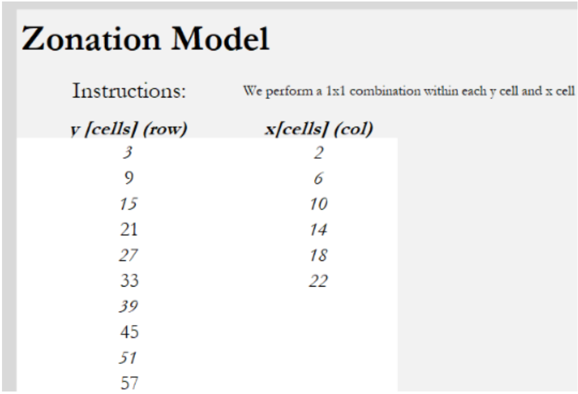


Figure 5: Simulation setup name.



y [cells] (row)	x[cells] (col)
3	2
9	6
15	10
21	14
27	18
33	22
39	
45	
51	
57	

Figure 6: Coordinates entered for each UZ model.

model in the sheet *Zonation*. Fig. 6 shows an example of data used to define the zones. (**add zonation figure**). The zonation code is simply an algorithm that calculates the closest distance from each GW cell from the correspondent UZ model as follows:

```
m = 1;
for i = 1:size(Surf_Elevation,1)
    for j = 1:size(Surf_Elevation,2)
        y_cell = i;
        x_cell = j;
        for k = 1:length(yi) % For the number of UZM
            uz_ycell = yi(k);
            uz_xcell = xi(k);
            D(1,k) = sqrt((uz_xcell - x_cell)^2 + (uz_ycell - ...
                y_cell)^2); % Distance in cells
        end
        % Find the closest
        idx = find(D ≤ min(D),1,'last'); % Last value with the ...
            smallest distance
        % Fill ZoneMatrix with corresponding UZM
        ZonationMap(i,j) = idx;
    end
end
```

An example of a zonation map is shown in Fig. 7

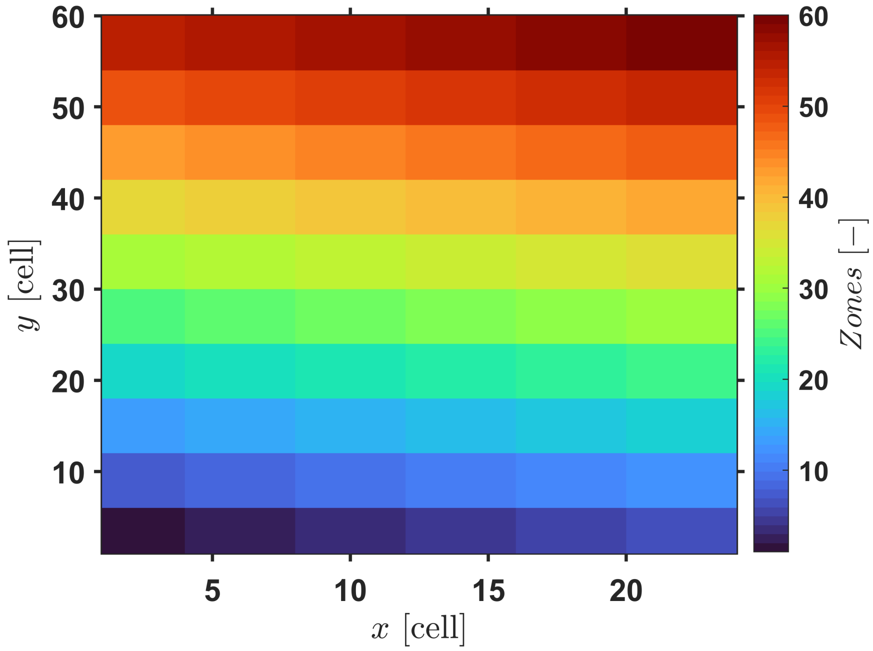


Figure 7: Example of a zonation map with 60 UZ models.

1.8 GW Boundary Conditions

For a given set of cells $\mathscr{S}$ at the groundwater domain $\mathscr{D}$, we can set Dirchlet's (i.e., known heads over time) and Neumann (i.e., known flow rates), such that:

$$h_i(t) = f(t),\ \forall i \in \mathscr{S}_1 \in \mathscr{D}\ \textit{Dirchlet Boundary Condition} \tag{1}$$

$$q_i(t) = g(t),\ \forall i \in \mathscr{S}_2 \in \mathscr{D}\ \textit{Neumann Boundary Condition} \tag{2}$$

The values of $f(t)$ and $g(t)$ are entered in the table shown in Fig. 8. The values associated with this table are spatially assigned by the GW boundary condition matrix, entered in Fig. 9.

For example, assuming you have only one cell with a Dirichlet boundary condition of atmospheric pressure. A way to simulate it is, given the ground elevation of this cell indexed at position o, g_e^o, assume a Dirichlet value D, such that:

$$D = g_e^o \tag{3}$$

or, if the DEM is given,

$$D = \mathrm{DEM}^o - H \tag{4}$$

in case the aquifer has a constant thickness.

Boundary Conditions at the Perimeter

Dirichlet Index	Dirichlet Value [head in m]	Neumann B.C	Neumann B.C [flux in m/s]
1	-4.106158457	2	0

Figure 8: GW Boundary conditions. For each different Dirichlet or Neumann boundary condition, users can specify the values. In the example shown in this figure, only a single condition is entered for the B.C., however, more than one can be entered. Note that the head entered for the Dirichlet is the total head ($z+P/\gamma$), not the head pressure (P/γ).

	x[cells]				
y [cells]	2	2	2	2	1
	0	0	0	0	1
	0	0	0	0	1
	2	2	2	2	1

Figure 9: Matrix indicating which boundary condition is entered for each cell of the GW domain. Values equal 0 means that no boundary condition is specified.

1.9 UZ Parameters and Initial Conditions

The current version of the model, as shown previously, allow to enter a space-variant discretization in the z direction to accurately represent the unsaturated flow. Given the distance of each UZ node z_{UZ}, taken from the bedrock elevation, we can assign UZ parameters for each node to represent the Van-Genuchten parameters.

An example of the data entering is shown in Fig. 10.

Initial Conditions and Parameters for the UZ model

Node	Pressure [cm]	Ksat [m/s]	Porosity [-]	α [-]	n [-]	Sres	aKR	Ssat [-]	Storage [-]
1	**50**	2.2E-05	0.37	1.85	1.75	0.01	0.5	1	0.01
2	**50**	2.2E-05	0.37	1.85	1.75	0.01	0.5	1	0.01
3	**50**	2.2E-05	0.37	1.85	1.75	0.01	0.5	1	0.01
4	**50**	2.2E-05	0.37	1.85	1.75	0.01	0.5	1	0.01
5	**50**	2.2E-05	0.37	1.85	1.75	0.01	0.5	1	0.01

Figure 10: UZ model initial conditions and parameters, where the input is discretized (from the bedrock to the surface). Nodes can have specified head pressure that can be converted into soil moisture using the V-G retention curve. The parameters following the table are: (Head Pressure), (Porosity), (α) V-G parameter, n V-G parameter, (S_{res} Residual Saturation), (aKR - V-G Parameter), S_{sat} - Maximum Saturation, (Storage)

1.10 GW Intial Water Depth

The groundwater initial water depth (i.e., the depth taken from the bedrock to the water table) can be assigned spatially in the sheet "GW_Depth", as shown in Fig. 11.

Initial GW Depth [m]

Instructions: Paste the initial GW table data [m] where y is the vertical axis and x is the horizontal axis

	x[cells]							
y [cells]	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Figure 11: Initial GW depth. Please notice that the y-axis is oriented downwards, and the x-axis is oriented rightwards.

1.11 GW Saturated Hydraulic Conductivity

Similarly to all other spatially-varied input, the K_{sat} is entered in matrix-wise fashion as following Fig. 12.

GW Ksat [m/s]

Instructions: Paste the initial GW saturated hydraulic conductivity data [m/s] where y is the vertical axis and x is the horizontal axis

	x[cells]										
y [cells]	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022
	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022
	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022
	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022
	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022
	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022	0.000022

Figure 12: Spatial input of GW K_{sat}. Sometimes, to accurately estimate the outlet flow, one might increase the K_{sat} at the outlet cells.

1.12 Surface DEM

An approximation from the surface DEM is done to determine the bedrock elevation model, such that:

$$\text{BEM} = \text{DEM} - H \tag{5}$$

where H is the aquifer thickness entered in Fig. 3

The surface DEM is entered following Fig. 13

Surface Elevation [m]

Instructions: Paste the DEM data where y is the vertical axis in x is the horizontal axis

	x[cells]					
y [cells]	9.959890366	9.950086594	9.940842628	9.93876267	9.93971348	9.93953896
	9.886141777	9.854210854	9.833338737	9.832221031	9.833305359	9.833601952
	9.789600372	9.751794815	9.726305008	9.723222733	9.724627495	9.725279808
	9.684222221	9.645529747	9.618868828	9.616868019	9.618397713	9.618764877

Figure 13: Surface Digital Elevation Model.

1.13 Surface Boundary Condition

Irrigation/Rainfall and Evapotranspiration fluxes can be assigned at the surface of UZ model. This data is shown in Fig. 14.

Hydrologic Input

Time (min)	*Rainfall/Irrigation* (mm/hr)	*ETR* (mm/h)
0	3	0
60	3	0
120	3	0
180	3	0
240	3	0
300	3	0
360	3	0
420	0	0
480	0	0
540	0	0

Figure 14: Hydrologic Input for each time. Note that the time-steps not necessarily have to be equal.

1.14 GW Specific Yield or Drainable Porosity

Simiarly to the other spatial parameters, the specific yield can be spatially assigned for each GW cell, following Fig. 15.

Specific Yield [-]

Instructions: Paste the initial Specific Yield data [-] where y is the vertical axis and x is the horizontal axis

	x[cells]							
y [cells]	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

Figure 15: Spatial specific yield for the GW model.

SECTION

2D Boussinesq Model Background

Although this is the manual of the coupled model, both UZ and GW model can work independent. The input for the GW model is basically the recharge rate at each particular GW cell. The 2D Boussinesq equation can be written as (Bear and Verruijt, 2012):

$$\frac{\partial}{\partial x}\left(K_x(h-\eta)\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y(h-\eta)\frac{\partial h}{\partial y}\right)+N=S\frac{\partial h}{\partial t} \tag{6}$$

where K_x is the saturated hydraulic conductivity in the x direction, K_y is the saturated hydraulic conductivity the y (vertical) direction, h is the hydraulic head, η is the bedrock elevation from reference datum, S is the specific yield, and $K_x = K_y$ in the current version of the model.

To ensure no pressure heads at the outlet, we set K at the outlet cells as a very high number (e.g., 2-3 orders of magnitude of the soil matrix K) and assume the outlet cell as a Dirichlet cell with $D = z_0$, where z_0 is the bottom elevation of the cell.

2.1 Input Data

The input data used for the GW model is the same presented in Sec. 1. Only data for the GW model is required.

2.2 Example 1 - 100 m Aquifer

In this series of examples taken from Troch et al. (2003), we use the 2D boussinesq model to simulate a variety of different groundwater flow. A schematic of the hillslope is presented in Fig. 16.

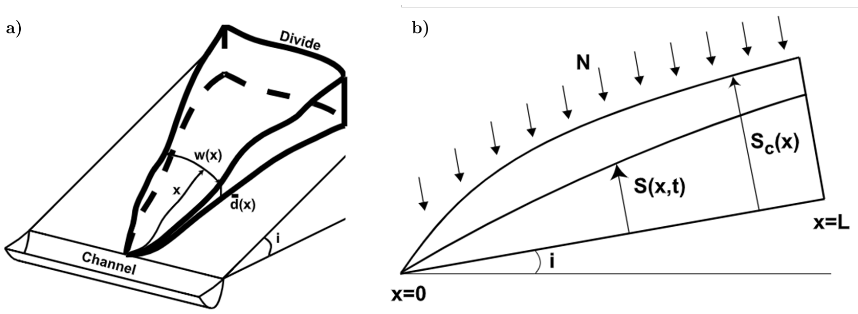


Figure 16: Hillslope 1D representation, where (a) represents a profile of a hilsslope with known width function $\omega(x)$, with average aquifer depth $\bar{d}$, and constant slope i, and (b) is a 1D lateral view of the hillslope, where $S(x,t)$ is the width-averaged storage at position x and time t and S_c is the storage capacity at x.

All examples have an aquifer length of $L = 100$ m, aquifer tickness of $H = 2$ m, $K = 1$ m$\cdot$h^{-1}, drainable porosity $S_y = 0.3$ and recharge rate of $N = 10$ mm$\cdot$day^{-1}.

The DEM elevation of the exercises proposed here are calculated as (Troch et al., 2003):

$$z(x,y) = E + H(x/L)^n + \omega y^2 \tag{7}$$

where y is the distance (perpendicular to x) from the slope center. E defines the reference datum for elevation, H represents the elevation at the top of the hillslope with respect to this reference datum, and L defines the length of the hillslope.

To generate a variety of different DEMs, we use the function using two files: the Generating_DEM.m script and hillslope_DEM.m function.

```
% Generating DEM
% Developer: Marcus Nobrega, Ph.D.
% Goal: Develop matrices to represent model inputs and B.C. for ...
    the 2D
% Boussinesq Model

%% Width Function
width_function = @(x)(50); % User defined width function in terms ...
    of x

%% Input Data
L = 100;                            % Length of the hillslope [m]
angle = 5;                          % Hillslope angle in percentage
E = 0;                              % Datum reference [m]
n = 1;                              % Profile curvature parameter
omega = 0;                          % Plan Curvature Parameter
dx = 2;                             % Grid discretization [m]
dy = 10;                            % Grid discretization [m]
ksat = 1/3600;                      % Ksat [m/s]
sy = 0.3;                           % Specific yield
Aquif_Tickness = 2;                 % Aquifer tickness [m]
gw_IC = 0.2*Aquif_Tickness;         % GW Initial Depth [m]
```

```matlab

Angle = rad2deg(atan(5/100));         % Angle of the hillslope in deg
H = tan(pi/180*Angle)*100 + E;        % Maximum elevation [m]


%% Call DEM Function
[hillslope.DEM,hillslope.Area,hillslope.Ksat,hillslope.Sy, ...
    hillslope.GW_IC, ...
    hillslope.per,hillslope.dir_val,neu_val, ...
    hillslope.lsurf] = hillslope_DEM(E,H,L,1,omega,dx ...
    ,dy,width_function, ...
    ksat,sy,gw_IC,Aquif_Tickness);

clearvars -except hillslope
```

```matlab
function [Z,Area,Ksat,Sy,GW_IC,per,dir_val,neu_val,Surf_DEM] = ...
    hillslope_DEM(E,H,L,n,omega,dx,dy,Width_Function,ksat,sy,gw_IC, ...
    ...
Aquif_Tickness)
% Generates a surface DEM following:
% Z(x,y) = E + H.*(x/L).^n + omega*y.^2;
% E: datum reference elevation
% H: Elevation at the top of the hillslope with respect to the ...
    reference
% datum
% L: Length of the hillslope
% n: profile curvature parameter
% omega: plan curvature parameter
% dx: x discretization [m]
% dy: y discretization [m]
% Width_Function: hillslope width function [m]
% ksat: sat. hyd. conductivity
% sy: specific yield
% gw_IC: gw initial water depth
% dir_index: index to represent dirichlet cells at the outlet
% neu_index: index to represesent neuman cells at the borders
% Aquif_Tickness: Soil depth [m]

nx = L/dx;
if floor(nx)≠ceil(nx)
    error('Please, enter a mesh for x that produces integer ...
        finite elements.')
end

max_width = 0;
% Maximum Width
for i = 1:nx
    max_width = max(max_width,Width_Function((i-1)*dx));
end

```

```
close all;
ny = max_width/dy;
if floor(ny)≠ceil(ny)
    error('Please, enter a mesh for y that produces integer ...
        finite elements.')
end

idx = zeros(ny,nx);
for i = 1:nx
    width_i = Width_Function(i*dx); % [m]
    ny_i = ceil(width_i/dy);
    idx((ny/2 - ny_i/2+1):(ny/2 + ny_i/2),i) = 1;
end
idx = logical(idx);
[X,Y] = meshgrid(dx:dx:L,(-max_width/2 + dy/2):dy:(max_width/2 - ...
    dy/2));
X(idx ≠ 1) = nan;
Y(idx ≠ 1) = nan;
idx = double(idx);

Z = idx*E + idx*H.*((X)/L).^n + omega*Y.^2;
Ksat = ksat*idx;
Sy = sy*idx;
GW_IC = gw_IC*idx;

% Plotting
set(gcf, 'Units', 'normalized', 'OuterPosition', [.25, .25, .5, .5]);
surf(X,Y,Z);
xlabel('x [m]','Interpreter','latex','FontSize',12)
ylabel('y [m]','Interpreter','latex','FontSize',12)
zlabel('z [m]','Interpreter','latex','FontSize',12)
colormap('turbo');
box on;
grid on
view(0,90);
clb = colorbar;
ylabel(clb,'Elevation [m]','Interpreter','latex','FontSize',14);

shading interp;
set(gca,'FontName','Garamond','FontSize',12)
exportgraphics(gcf,'DEM.png','Resolution',600)
Area = (dx*dy)*numel(X); % m2

% Boundary Conditions
per = double(bwperim(¬isnan(Z)));
n_dir = length(¬isnan(per(:,1))); % Number of dirichlet cells
per(isnan(Z)) = nan;
per(per == 1) = n_dir + 1; % Neumann B.C
per(¬isnan(per(:,1))) = 1:1:n_dir; % Dirchlet B.C

% Boundary Values
```

```
pressure_head = 0;
dir_val = Z(¬isnan(per(:,1))) + pressure_head; % Total Head
neu_val = 0; % m/s

% Surf_DEM = DEM + Aquif_Tickness
Surf_DEM = Z + Aquif_Tickness ;
end
```

2.3 Case 1 - Uniform Hillslopes with $i = 5\%$

Two hillslopes of $w(x) = w = 50$ m, with $n = 1$, and $\omega = 0$ m are tested under drainage (Initial GW depth of 20% of the aquifer thickness) and recharge (Recharge rate of 10 mm/day with initial GW depth of 0).

The DEM of this simulation is shown in Fig. 17.

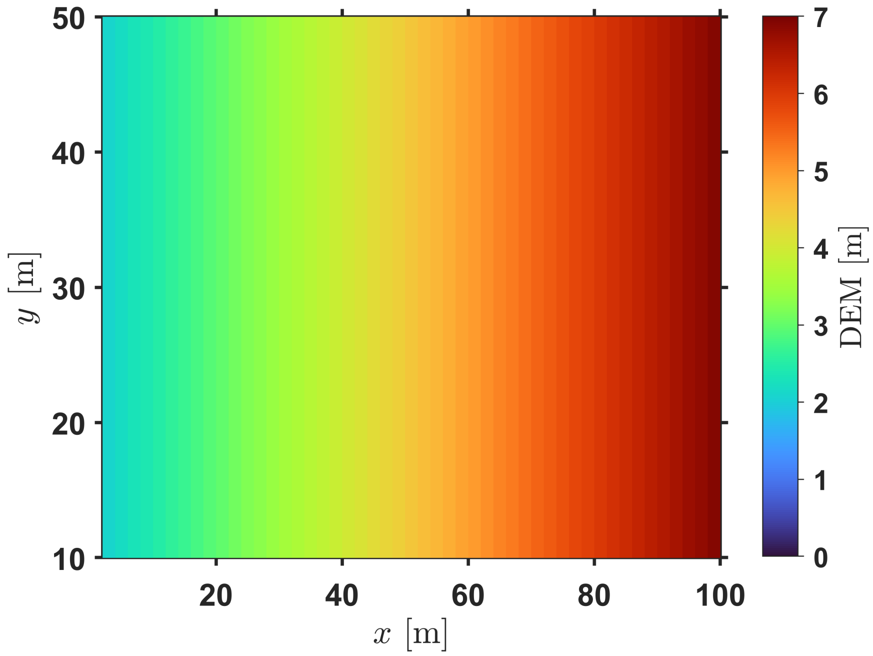


Figure 17: Hillslope with 5% inclination, $L = 100$ m, and $w(x) = w = 50$ m.

The results reported used for comparison are shown in Fig. 18.

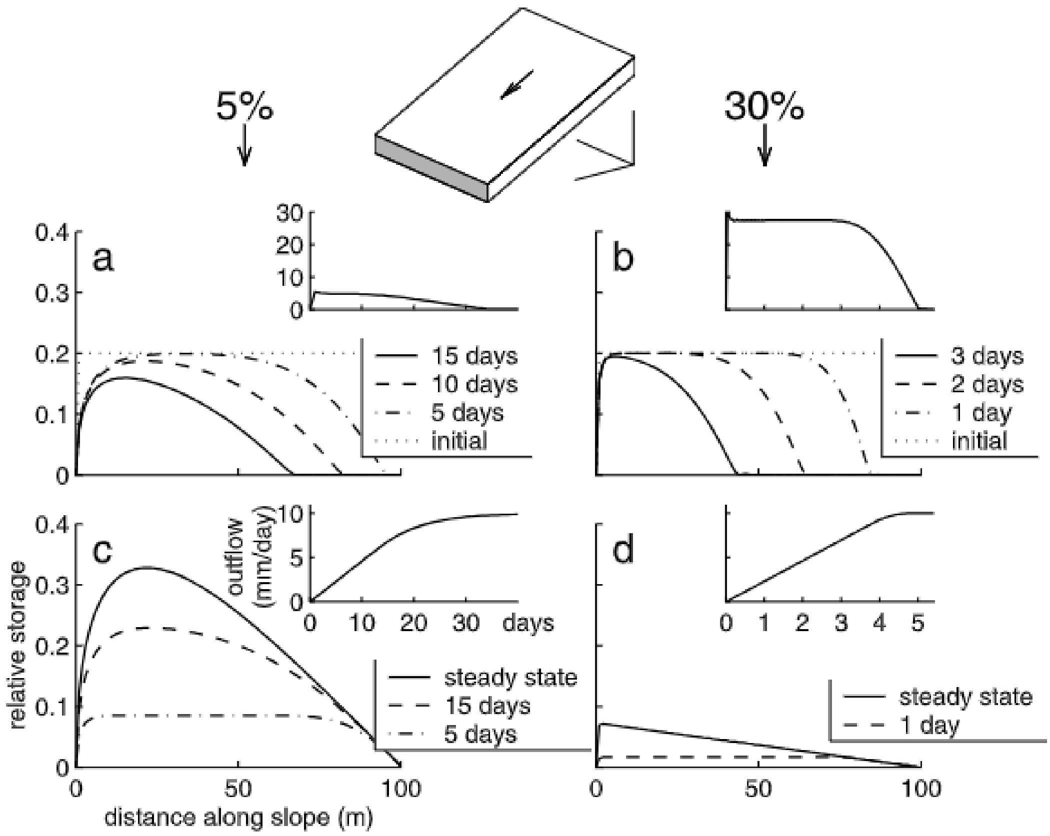


Figure 4. Relative storage profiles along the hillslope and normalized subsurface flow rates (mm/day) at the outlet for the uniform hillslope at (left) 5% and (right) 30% slope angle. (a and b) Drainage scenario results; (c and d) recharge scenario results.

Figure 18: Reported results from hsB model from Troch et al. (2003).

2.3.1 Drainage of 20% of the storage

The profiles of relative storage (i.e., ratio between storage and total storage) are shown for different durations at different locations ($x = 2,\ ,25,\ 50$ m, $y = 2,\ 3,\ 5$ m) in Fig. 19.

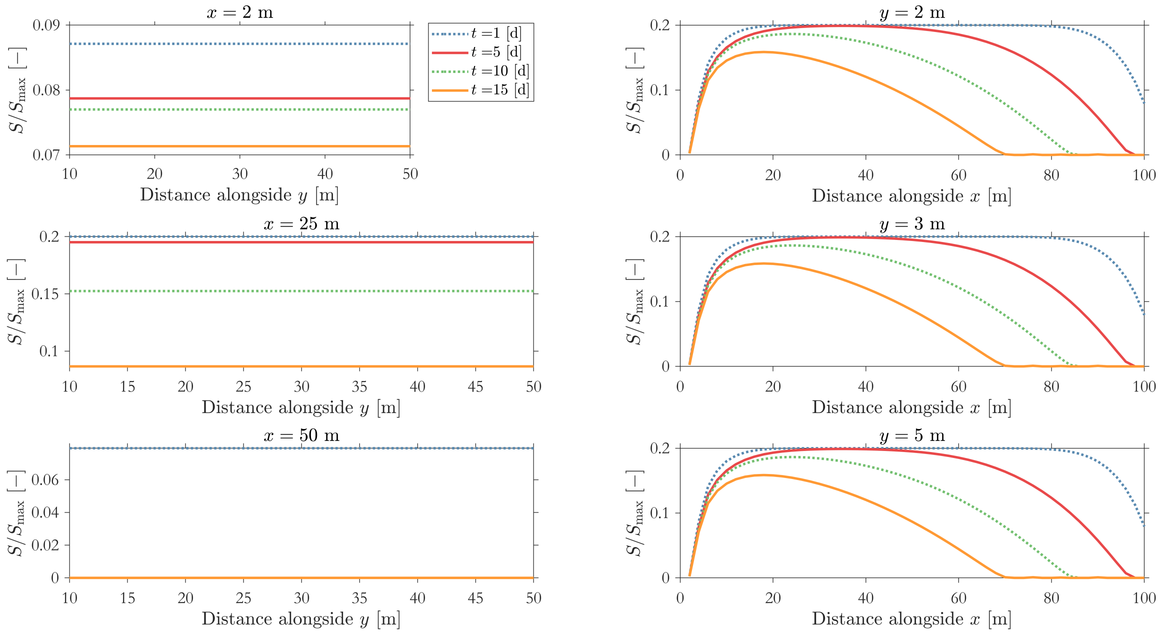


Figure 19: Temporal propagation of the storage for different durations in different locations for a drainage scenario, with initial storage of 20% of the soil matrix.

The normalized hydrograph with a day resolution is presented in Fig. 20

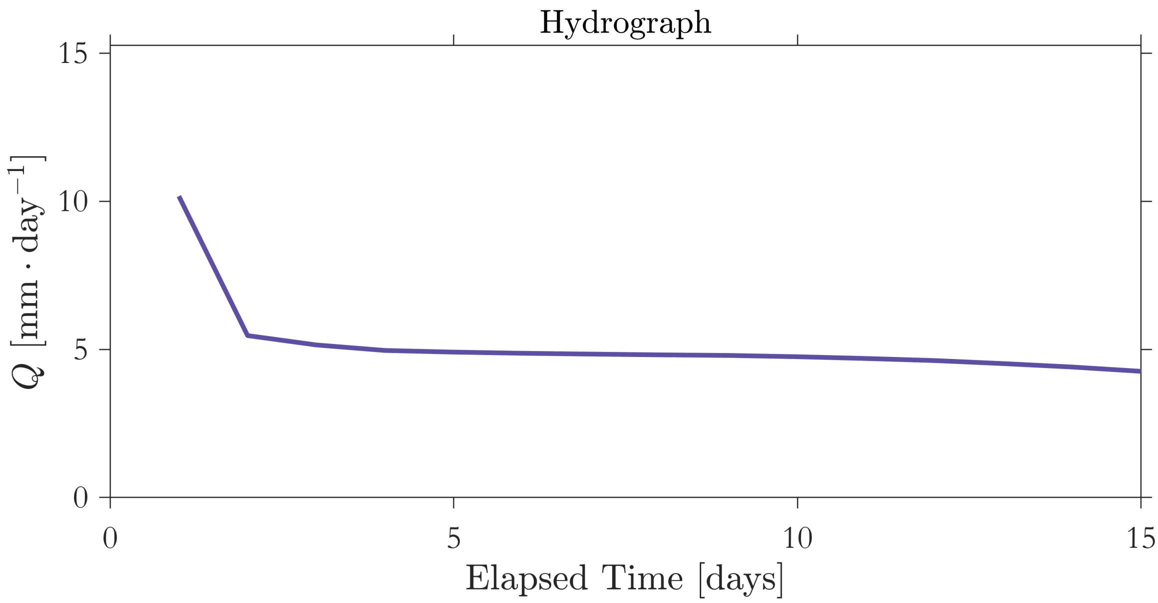


Figure 20: Normalized hydrograph for the 5% slope draining 20% of the storage. Note that this figure has results retrieved every day.

The evolution of the GW depths is shown in Fig. 21.

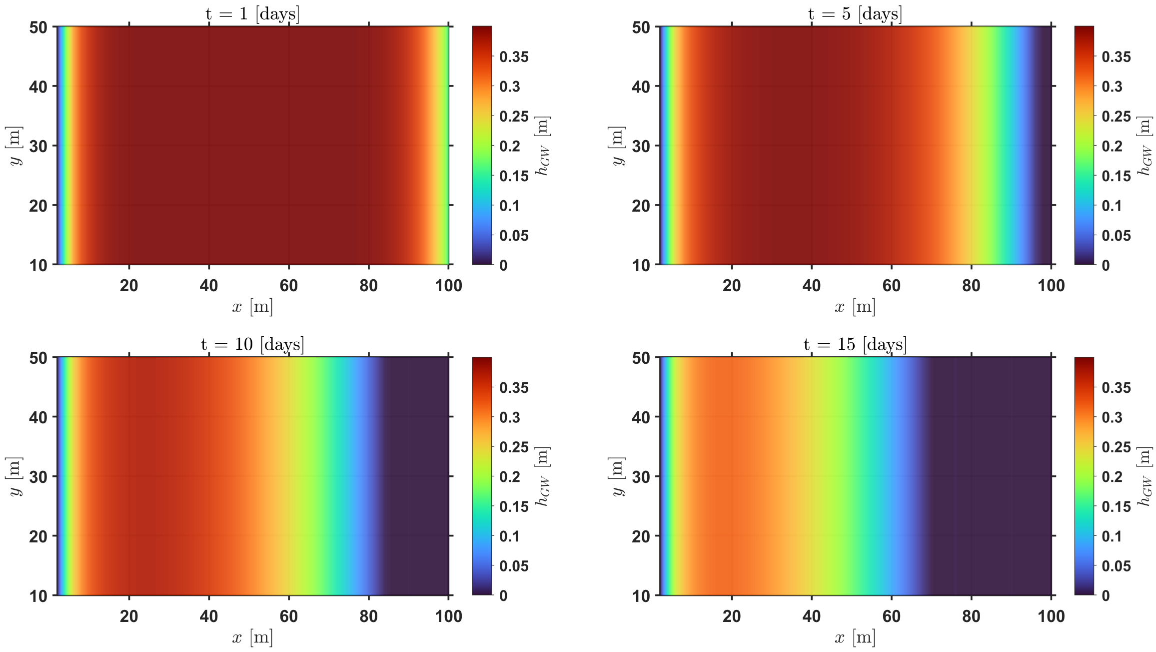


Figure 21: Evolutin of GW depth for the 5% slope draining 20% of the storage

2.3.2 Recharge of 10 mm/day

The soil moisture profiles are shown in Fig. 22.

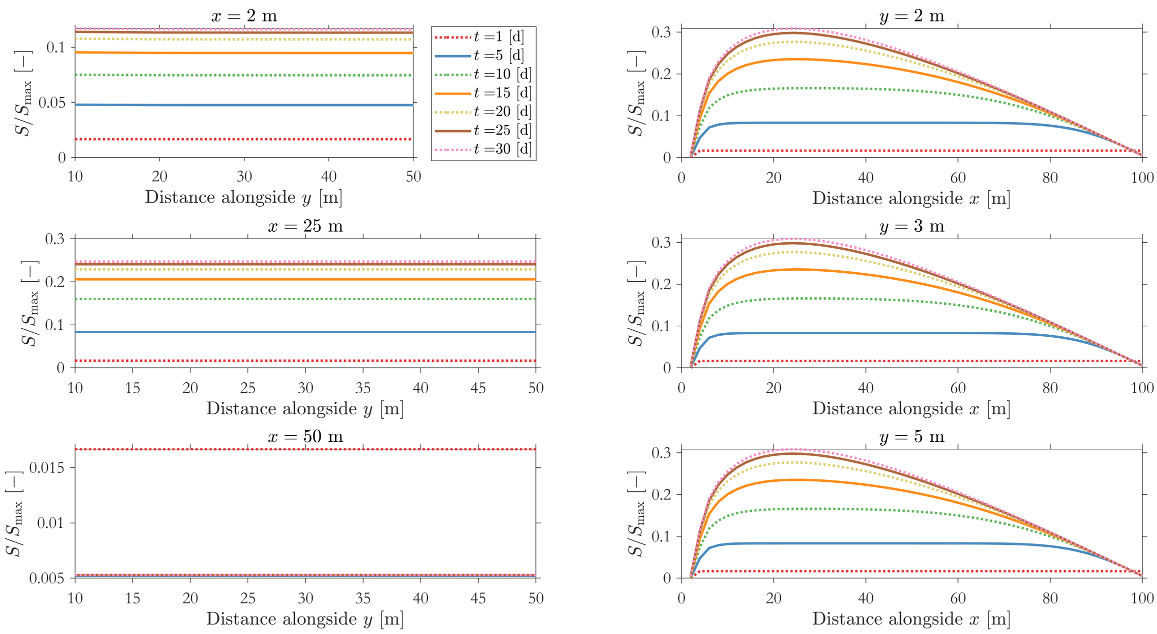


Figure 22: Soil moisture profiles for a recharge scenario of 10 mm/day.

The hydrograph, normalized by the hillslope area, is shown in Fig. 23.

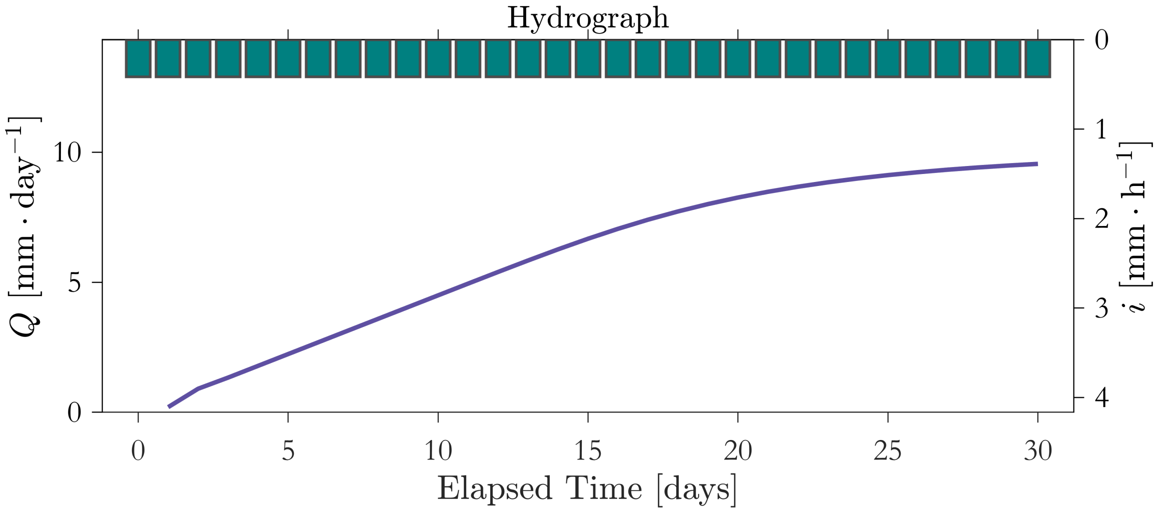


Figure 23: Normalized hydrograph of the 5% hillslope with 10 mm/h recharge.

2.4 Case 2 - Uniform Hillslopes with $i = 30\%$

2.4.1 Drainage of 20% of the storage

The soil moisture profiles are shown in Fig. 24.

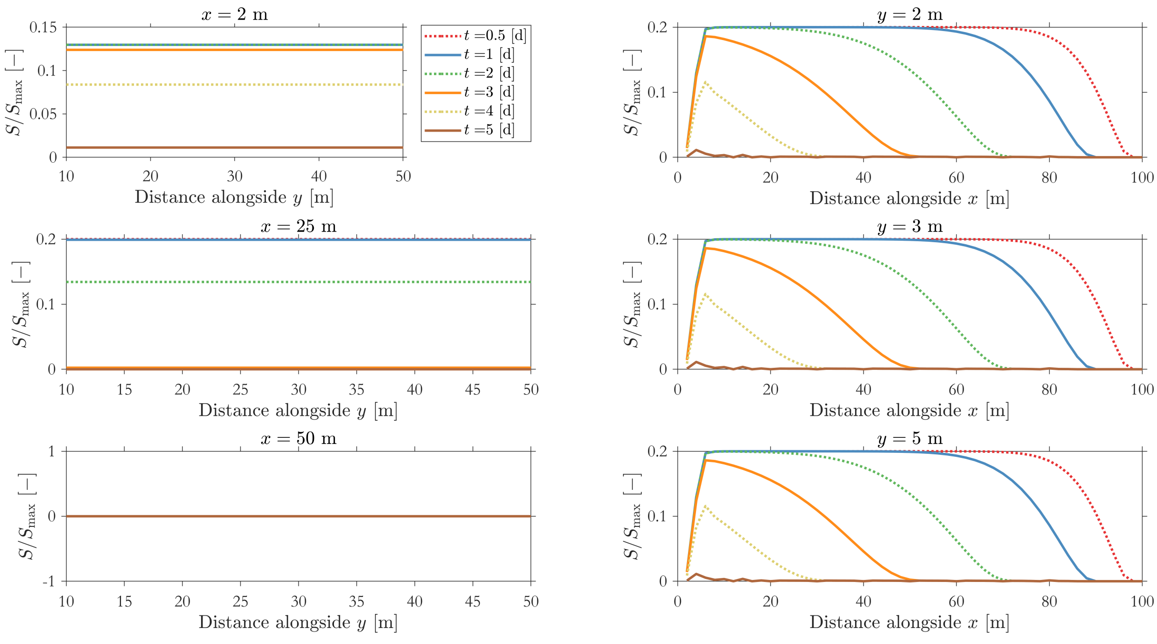


Figure 24: Storage evolution of the 30% hillslope draining 20% of the storage.

The evolution of the GW depths is shown in Fig. 25.

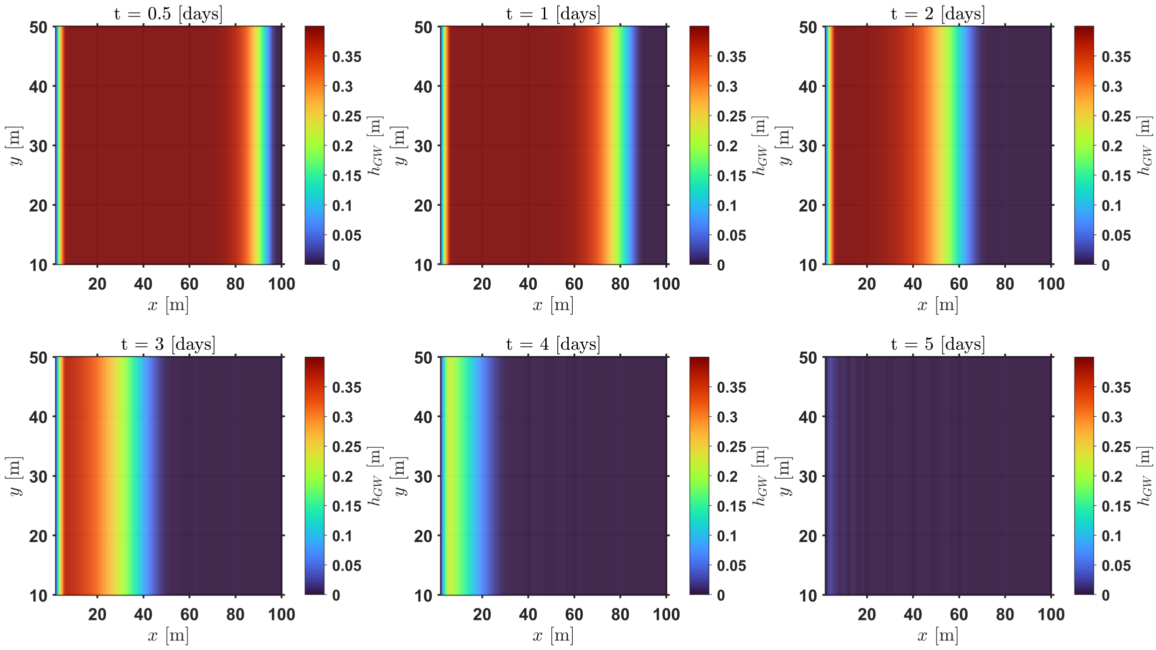


Figure 25: Depth evolution of the 30% hillslope draining 20% of the storage.

The hydrograph, normalized by the hillslope area, is shown in Fig. 26.

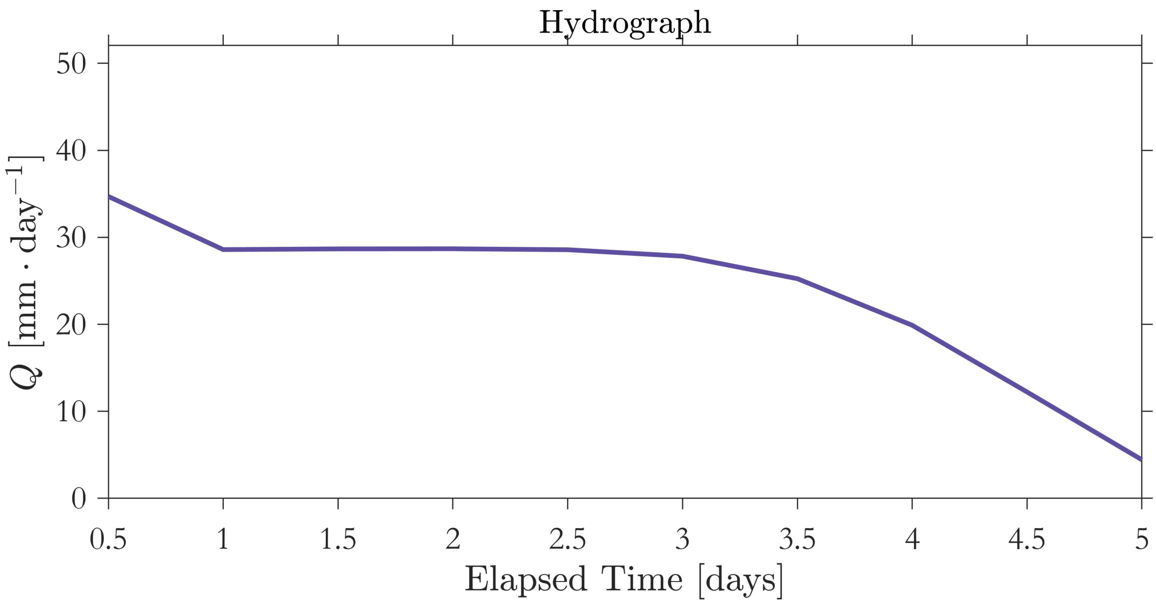


Figure 26: Normalized Hydrograph of the 30% hillslope draining 20% of the storage.

2.4.2 Recharge of 10 mm/day

The soil moisture profiles are shown in Fig. 27.

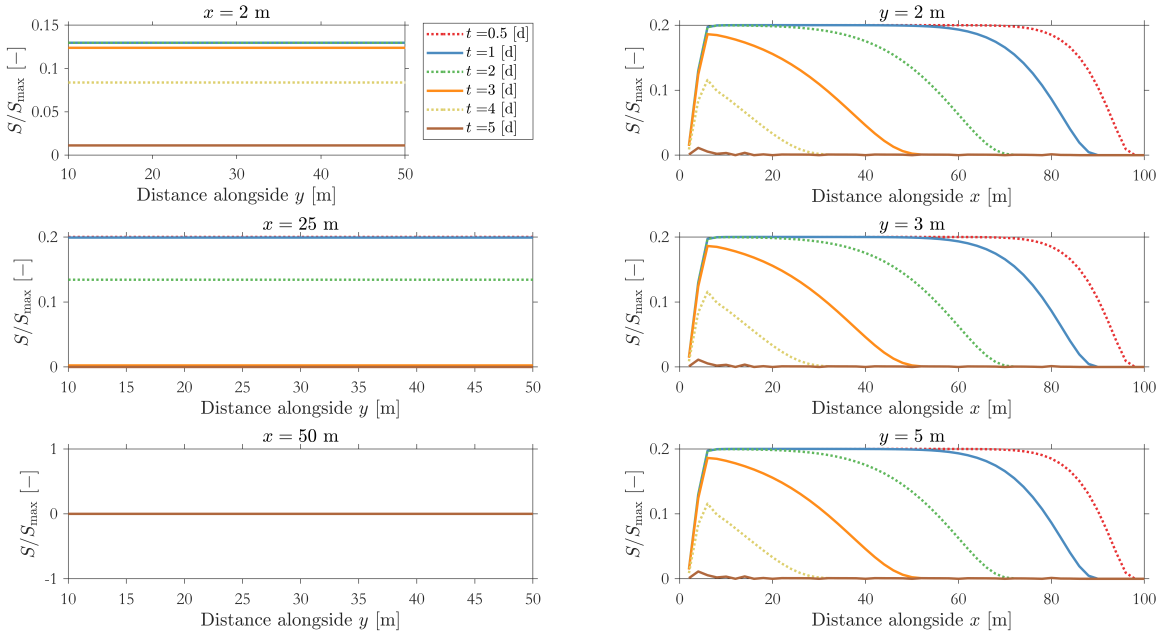


Figure 27: Storage evolution of the 30% hillslope draining 20% of the storage.

The evolution of the GW depths is shown in Fig. 25.

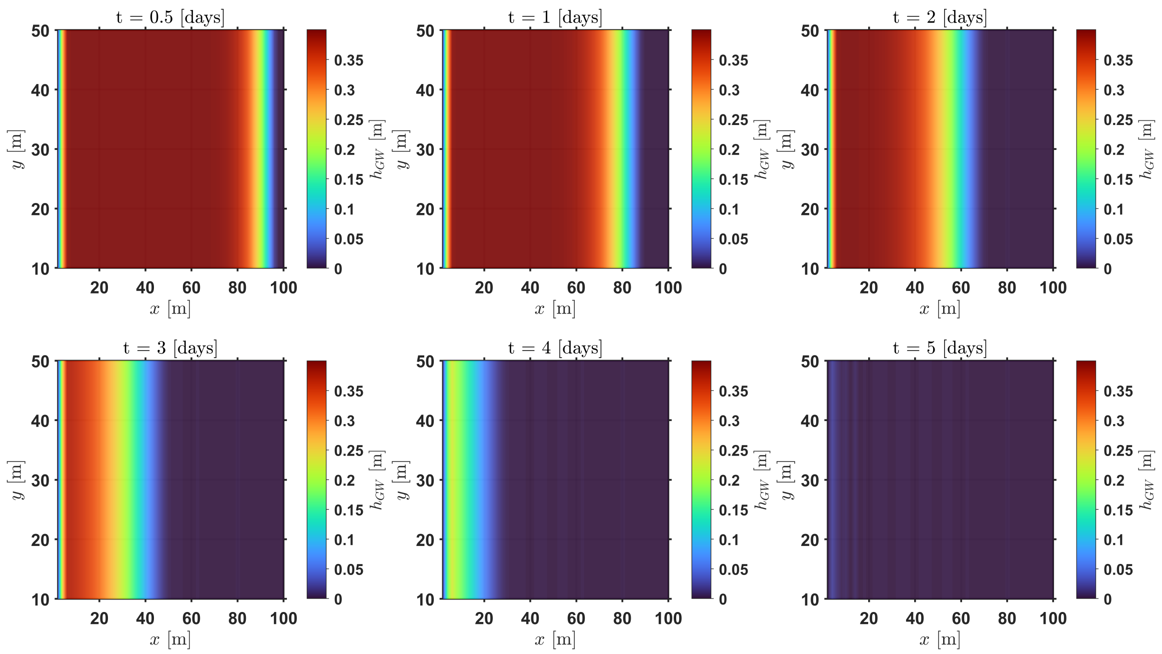


Figure 28: Depth evolution of the 30% hillslope recharging 10 mm/day.

The hydrograph, normalized by the hillslope area, is shown in Fig. 29.

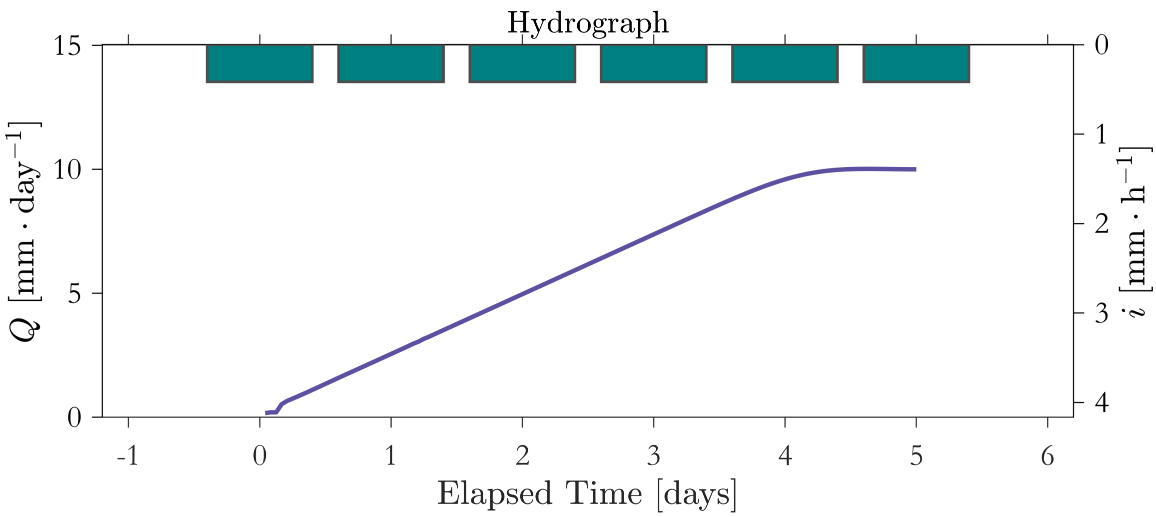


Figure 29: Normalized Hydrograph of the 30% hillslope recharging 10 mm/day.

2.5 Case 3 - Comparing the coupled model with hsB Model

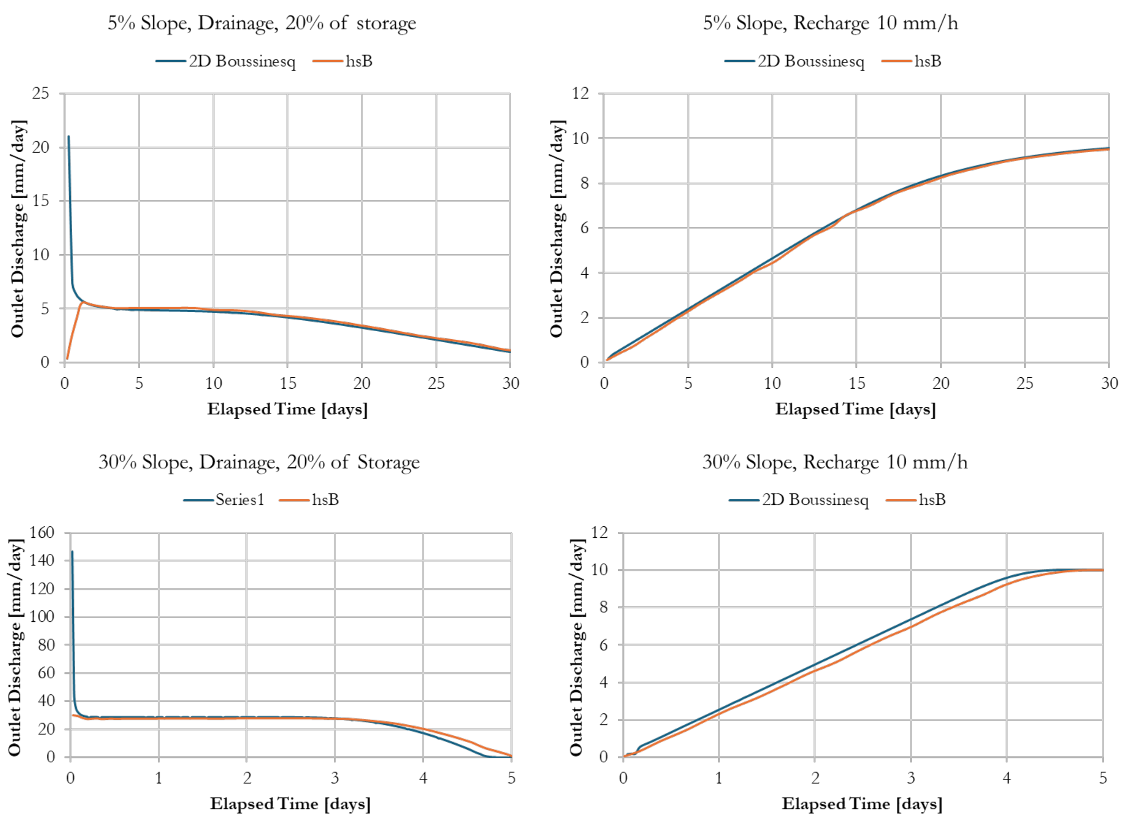


Figure 30: Comparison of the 2D Boussinesq model with the Hillslope-Storage-Boussinesq model. The differences between both models are due to the initial water depth at the outlet node. Time-step was tunned to avoid numerical instabilities, especially for the 30% hillslope. For the 2D Boussinesq Model, results were retrieved each 30 min.

2.6 Case 3 - Convergent Hillslopes

2.7 Case 4 - Divergent Hillslopes

3

SECTION

References

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