Tutorial 3

1. Which of the following are statements?
    i) Is 2 a positive number?
    ii) $x^2 + x + 1 = 0$
    iii) Study logic.
    iv) There will be snow in January in Malaysia.
    v) If stock prices fall, then I will lose money.

2. Give the negation of each of the following statements.
    i) $\sim p \vee \sim q$
    ii) $p \vee (q \wedge \sim r)$
    iii) It will rain tomorrow or it will snow tomorrow.
    iv) 2 is even or -3 is negative.

3. In each of the following, form the conjunction and the disjunction of p and q.
    i) p: I will drive my car. q: I will be late.
    ii) p: NUM > 10 q: NUM $\leq$ 15

4. Find the truth value of each proposition if p and r are true and q is false.
    i) $\sim p \wedge (q \vee r)$ ii) $p \wedge (\sim(q \vee \sim r))$
    iii) $(r \wedge \sim q) \vee (p \vee r)$ iv) $(q \wedge r) \wedge (p \vee \sim r)$

5. Let p: $9 > 6$, q: $7 < 8$, r: $5 \leq 5$, and s: $10 \geq 7$. Write the following statements in terms of p, q, r, s, and logical connectives.
    i) If $9 > 6$, then $5 \leq 5$.   ii) Either $7 \geq 8$ or $10 < 7$.
    iii) It is not true that $7 < 8$ or $5 \leq 5$.

6. Rewrite each statement below in "if – then" form.
    i) I am on time for lecture if I catch the 7 am bus.
    ii) David studies hard or he fails the examination.
    iii) The program is readable only if it is well structured.
    iv) This door will not open unless a security code is entered.
    v) $2x - 5 = 11$ implies $x = 8$.
    vi) Having two 45⁰ angles is a sufficient condition for this triangle to be a right triangle.
    vii) Solving all tutorial's questions is a necessary condition for Alan to pass this subject.
    viii) To be a citizen in this country, it is sufficient that you were born in this country.
    ix) It is necessary to have a valid password to log on to the server.

7. Let p, q, and r be statements variables. Assuming that p is true, q is false, and r is true, find the truth value of each statement forms.
    i) $p \wedge q \rightarrow r$ ii) $p \vee q \rightarrow \sim r$
    iii) $p \wedge (q \rightarrow r)$ iv) $p \leftrightarrow q \vee r$

v) $(p \rightarrow q) \vee (q \rightarrow r)$

8. State the converse, inverse, and contrapositive of each of the following implications.
   i) If $2 + 2 = 4$, then I am not the Queen of England
   ii) If I am not President of the United States, then I will walk to work.
   iii) If I am late, then I did not take the train to work.
   iv) If I have time and I am not too tired, then I will go to the store.
   v) If I have enough money, then I will buy a car and I will buy a house.

9. Construct truth tables to determine whether the given statement is a tautology, a contradiction, or a contingency.

   i) $\sim p \wedge \sim q$ ii) $\sim(\sim p \leftrightarrow q)$
   iii) $p \rightarrow (\sim p \vee q)$ iv) $(p \wedge \sim q) \rightarrow (\sim p \vee q)$
   v) $p \rightarrow (q \rightarrow p)$ vi) $q \rightarrow (q \rightarrow p)$
   vii) $(q \wedge p) \vee (q \wedge \sim p)$ viii) $(p \wedge q) \rightarrow p$
   ix) $p \rightarrow (q \wedge p)$ x) $(\sim p \wedge (q \vee \sim r))$
   xi) $(p \vee \sim q) \rightarrow (p \vee \sim r)$ xii) $((\sim p \wedge q) \wedge (q \vee r)) \wedge \sim q$

10. Prove by constructing truth table for the following logical equivalences.
    i) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
    ii) $(p \wedge q) \rightarrow r \equiv (p \rightarrow r) \vee (q \rightarrow r)$