Full Name: Lee Kai Yang
Student ID: 19WMR11671
Programme: RSFIS3

Q1 (a) $5, -11, -27, -43, \ldots\ldots$

(ii) $T_n = a + (n-1)d$

When $n=1$, $T_1 = 5$, $a = 5$

$d = -11 - 5$

$= -16$

$T_n = 5 + (n-1)(-16)$

$= 5 - 16(n-1)$

} explicit

(i) For $n > 1$, $T_n = T_{n-1} - 16$

$n \geq 2$, $T_1 = 5$

} recursive

(b) $1, -0.2, \cancel{0.04}, -0.008, \ldots\ldots$ (0.04)

(ii) $\cancel{T_n = ar^{n-1}}$

$\cancel{\text{When } n=1, T_1 = 1, a = 1}$

$\cancel{r = \frac{-0.008}{-0.04}}$

$\cancel{r = 0.2}$

$\cancel{T_n = r}$

$\cancel{T_n = (0.2)^{n-1}, n > 1}$

$\cancel{T_n}$

$\cancel{T_n = ar^{n-1}}$

$\cancel{r = \frac{-0.04}{-0.2}}$

$\cancel{r = 0.2}$

$\cancel{T_n = 1(0.2)^{n-1}}$

(i) $\cancel{T_n = (0.2)^{n-1}, n > 1}$

$\cancel{n \geq}$

$T_n = -0.2\,(T_{n-1}), \; n > 1$

$n \geq 2$, $T_1 = 1$

} recursive

$T_n = (-0.2)^n$ — explicit

$\cancel{T_n = (0.2)^n}$

Q2 $n(U) = 200$

$n(B) = 110$

$n(G) = 120$

$\cancel{n(E) = 140}$

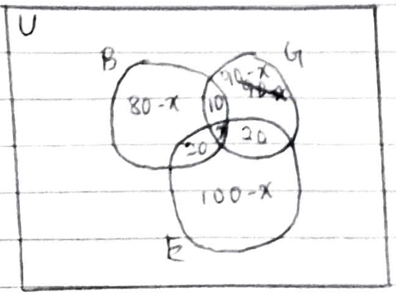
$n(E) = 140$

$n(B \cap E) = 20$

$n(G \cap E) = 20$

$n(G \cap B) = 10$

$x = n(B \cap G \cap E)$

(a) 


$\cancel{80 - x + 10 + 20 + x + 90 - x + 10 + 20 + x + 100 - x + 20 + 120 + x} = 110 + \cancel{120}$

(b) $(80 - x) + (90 - x) + (100 - x) + 20 + 20 + 10 + x = 200$

$270 - 3x + 50 + x = 200$

$320 - 2x = 200$

$x = \frac{-120}{-2}$

$x = 60$

Full Name : Lee Kai Yang
Student ID : 19WMR11671
Programme : RSF1S3

Q2 (c) $n(U) - n(B \cap E) - n(G \cap E) - n(G \cap B) - n(B \cap G \cap E)$

$= 200 - 20 - 20 - 10 - 60$

$= 90$

(d) $n(B \cap E) + n(G \cap E) + n(G \cap B) + n(B \cap G \cap E)$

$= 20 + 20 + 10 + 60$

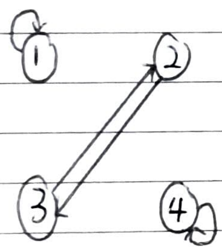
$= 110$

Q3 $A = \{1, 2, 3, 4\}$

$xRy \Leftrightarrow 5 \mid (3 + 2xy)$ $\nearrow$ $\frac{3+2xy}{5}$

(a) $R = \{(1,1), (2,3), (3,2), (4,4)\}$

(b)




$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c) $Dom(R) = \{1, 2, 3, 4\}$

$Ran(R) = \{1, 2, 3, 4\}$

(d)

| Vertex | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| in-degree | 1 | 1 | 1 | 1 |
| out-degree | 1 | 1 | 1 | 1 |

(e) R is not reflexive since $3 \not R 3$.

R is not irreflexive since $1R1$.

R is symmetric.

R is not asymmetric since $1R1$.

R is not ~~any~~ antisymmetric since $2R3$ and $3R2$ but $2 \neq 3$.

R is not transitive since $2R3$ and $3R2$ but $2 \not R 2$.

Full Name: Lee Kai Yang
Student ID: 19WMR11671
Programme: RSF1S3

(f) R is not an equivalence relation on A because R is ~~not reflexive, R is not~~ symmetric but not reflexive and not transitive.

Q4 $A = \{p, q, r, s\}$

(a) ~~$A \cup B = M_A \cup M_B$~~

~~A~~

$$M_{A \cup B} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

(b) $$M_{A \cap B} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad M_{\overline{A \cap B}} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

(c) ~~$M_{B \circ A} = [0 \times 1 + 1 \times 1 + 1 \times 0 + 0 \times 0 \quad 0 \times 0$~~

$$M_{B \circ A} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 2 \end{bmatrix}$$

Q5 (a) p is true, q is false, r is false

(i) $(p \to q) \vee r$
$= (\sim p \vee q) \vee r$
$= (\sim T \vee F) \vee F$
$= F$

(ii) $(q \wedge r) \leftrightarrow p$
$= [(q \wedge r) \to p] \wedge [p \to (q \wedge r)]$
$= [\sim(q \wedge r) \vee p] \wedge [\sim p \vee (q \wedge r)]$
$= [\sim(F \wedge F) \vee T] \wedge [\sim T \vee (F \wedge F)]$
$= [T \vee T] \wedge [F \vee F]$
$= F$

(iii) $p \wedge (\sim q \vee r)$
$= \cancel{p}\ T \wedge (\sim F \vee F)$
$= T \wedge T$
$= T$

(iv) $(q \wedge r) \to \sim p$
$= \sim(q \wedge r) \vee \sim p$
~~$= \sim F$~~
$= \sim(F \wedge F) \vee \sim T$
$= T \vee F$
$= T$

Full Name : Lee Kai Yang
Student ID : 19 WMR 11671
Programme : RSF1 S3

Q5 (b) (i) $p \rightarrow q$

(ii) $\cancel{p \leftrightarrow q}$ $p \rightarrow q$

(iii) ~~if~~ $\cancel{\sim p \rightarrow}$ $\cancel{\sim p \rightarrow \sim q}$ $\sim q \rightarrow \sim p$

(c) ~~(i)~~ $p \rightarrow q$

contrapositive $= \sim q \rightarrow \sim p$
$= q \vee \sim p$

converse $= q \rightarrow p$
$= \sim q \vee p$

inverse $= \sim p \rightarrow \sim q$
$= p \vee \sim q$

~~(ii)~~ $\cancel{p \rightarrow q}$

~~contrapositive~~ $\cancel{= \sim q \rightarrow \sim p}$
$\cancel{= q \vee \sim p}$

~~converse~~ $\cancel{= q \rightarrow p}$
$\cancel{= \sim q \vee p}$

~~inverse~~ $\cancel{= \sim p \rightarrow \sim q}$
$\cancel{= p \vee \sim q}$

~~(iii)~~ $\cancel{\sim q \rightarrow \sim p}$

~~contrapositive~~

Full Name : Lee Kai Yang
Student ID : 19WMR11671
Programme : RSF1S3

Q6 (a) $A = \sim p \leftrightarrow (q \vee p)$
$B = q \wedge (p \rightarrow q)$

(i)

| $p$ | $q$ | $\sim p$ | $\sim q$ | $q \vee p$ | $\sim p \leftrightarrow (q \vee p)$ | $p \rightarrow q$ | $q \wedge (p \rightarrow q)$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | F 0 | T 1 | 0 |
| 0 | 1 | 1 | 0 | 1 | T 1 | T 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | F 0 | F 0 | 0 |
| 1 | 1 | 0 | 0 | 1 | F 0 | T 1 | 1 |

(ii) contingency

(iii) ~~$A = \sim p \leftrightarrow (q \vee p)$~~
~~$= [\sim p \rightarrow (q \vee p)] \wedge [(q \vee p) \rightarrow \sim p]$~~
~~$= [p \vee (q \vee p)] \wedge [\sim q \wedge \sim p \rightarrow$~~
~~$= [p \vee (q \vee p)] \wedge [\sim q \wedge \sim p \vee \sim p]$~~

PDNF A $= \sim p \wedge q$
$= \bar{p}q$

~~PCNF A $= (p \vee q) \wedge (p \vee \sim q) \wedge ($~~
PCNF A $= (p \vee q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)$
$= (p+q)(\bar{p}+q)(\bar{p}+\bar{q})$

PDNF $\sim$A $= (\sim p \wedge \sim q) \vee (p \wedge \sim q) \vee (p \wedge q)$
$= \bar{p}\bar{q} + p\bar{q} + pq$

PCNF $\sim$A $= p \vee \sim q$
$= p + \bar{q}$

(b) $\exists r (\forall s\, Q(r,s))$

There exists ~~an integer~~ a number r for every number s such that r|s.

When $r = 1$, $s = 4$, $r|s$ is true.
When $r = 1$, $s = 5$, $r|s$ is true.
When $r = 2$, $s = 4$, $r|s$ is true.
When $r = 2$, $s = 5$, $r|s$ is false.

$\therefore$ Truth value of this expression is false.