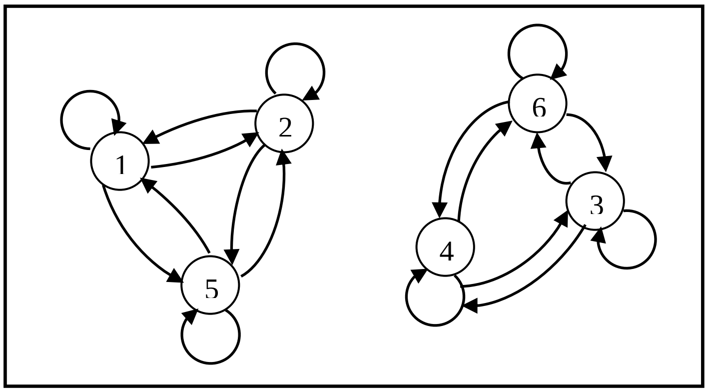
Tutorial 7

1. Let $A = \{a, b, c\}$. Determine whether the relation R whose matrix $\mathbf{M}_R$ given is an equivalence relation. If yes, find A/R.

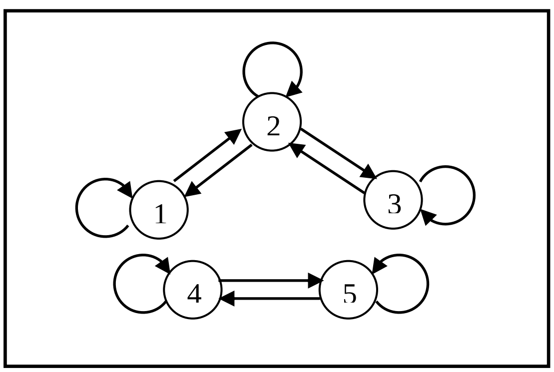
   i) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ ii) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

2. Determine whether the relation R whose digraph is given as below is an equivalence relation. If yes, find A/R.

   i)



   ii)



3. Determine whether the following relation R on the set A is an equivalence relation. If yes, find A/R.

   i) $A = \{a, b, c, d\}$, $R = \{(a, a), (a, b), (b, a), (b, b), (c, c), (c, d), (d, c), (d, d)\}$

   ii) $A = \{1, 2, 3, 4\}$, $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 1), (3, 3), (1, 3), (4, 1), (4, 4)\}$

   iii) $A = \{2, 3, 5, 6, 8\}$, $x\ R\ y$ if and only if $3|(x - y)$.

   iv) $A = \{1, 2, 3, 4, 5\}$, $x\ R\ y \Leftrightarrow x \equiv y \pmod 2$.

4. If $\{\{a, c, e\}, \{b, d, f\}\}$ is a partition of the set $A = \{a, b, c, d, e, f\}$, determine the corresponding equivalence relation R.

5. The following arrays describe a relation R on a set $A = \{1, 2, 3, 4\}$:
VERT = [1, 2, 6, 4]
TAIL = [1, 2, 2, 4, 4, 3, 4, 1]
HEAD = [2, 2, 3, 3, 4, 4, 1, 3]
NEXT = [8, 3, 0, 5, 7, 0, 0, 0]
Compute both the digraph of R and the matrix $\mathbf{M}_R$.

6. Let $A = B = \{1, 2, 3\}$ and let $R = \{(1, 1), (1, 2), (2, 3), (3, 1)\}$ and let $S = \{(2, 1), (3,1), (3, 2), (3, 3)\}$. Let R and S be the relations from A to B. Compute

i) $\overline{R}$ ii) $R \cap S$
iii) $R \cup S$ iv) S^{-1}

7. Let $A = \{2, 4, 5, 7\}$ and let R and S be the relations on A described by $x\,R\,y$ if and only if $x + y$ is even and $\mathbf{M}_S = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. List the ordered pairs belonging to the following relations.
i) S^{-1} ii) $S^{-1} \cap R$ iii) $(S^{-1} \circ R)^{-1}$

8. Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3\}$. The matrices $\mathbf{M}_R$ and $\mathbf{M}_S$ of the relation R and S be the relations from A to B are given by $\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $\mathbf{M}_S = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Compute

i) $\mathbf{M}_{R \cup S}$ ii) $\mathbf{M}_{R \cap S}$
iii) $\mathbf{M}_{R^{-1}}$ iv) $\mathbf{M}_{\bar{S}}$

9. Let $A = \{a, b, c, d, e\}$ and let the equivalence relations R and S on A be given by

$\mathbf{M}_R = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$, $\mathbf{M}_S = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

i) Compute
a) $\mathbf{M}_{R \circ R}$ b) $\mathbf{M}_{S \circ R}$
c) $\mathbf{M}_{R \circ S}$ d) $\mathbf{M}_{S \circ S}$

ii) Compute the partition of A corresponding to $R \cap S$.

10. Given $A = \{w, x, y, z\}$, $B = \{1, 2, 3, 4\}$ and $C = \{a, b, c, d\}$. Let R be a relation from A to B and S be a relation from B to C defined as follow:

$R = \{(w, 2), (x, 3), (y, 4), (z, 1), (z, 2), (y, 3)\}$

$S = \{(1, a), (1, c), (2, c), (2, d), (3, a), (4, b), (4, d)\}$

i) Find $\mathbf{M}_{S \circ R}$.

ii) Verify that $\mathbf{M}_{R^{-1}} = (\mathbf{M}_R)^T$.