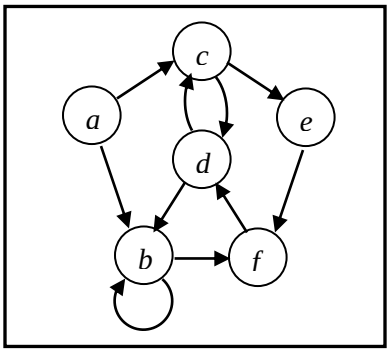
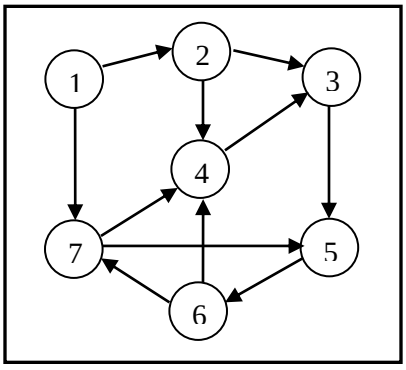
Tutorial 6

1. Let R be the relation whose digraph is given as follow:



   i) List all paths of length 1.
   ii) List all paths of length 3 starting from vertex a.
   iii) Find a cycle starting at vertex d.
   iv) Draw a digraph of R^2.
   v) Find $\mathbf{M}_{R^\infty}$ and R^∞.

2. Let R be the relation whose digraph is given as below. If π_1: 2, 3, 5, 6, 7 and π_2: 7, 5, 6, 4, find the composition $\pi_2 \circ \pi_1$.



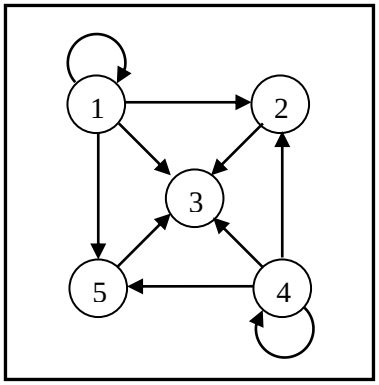
3. Determine whether the given relation on $A = \{1, 2, 3, 4\}$ is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive. Explain your answers.
   i) $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4)\}$
   ii) $R = \{(1, 3), (1, 1), (3, 1), (1, 2), (3, 3), (4, 4)\}$
   iii) $R = \varnothing$
   iv) $R = \{(1, 2), (1, 3), (3, 1), (1, 1), (3, 3), (3, 2), (1, 4), (4, 2), (3, 4)\}$

4. Let $A = \{w, x, y, z\}$. Determine whether the relation R whose matrix $\mathbf{M}_R$ given below is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive. Explain your answers.

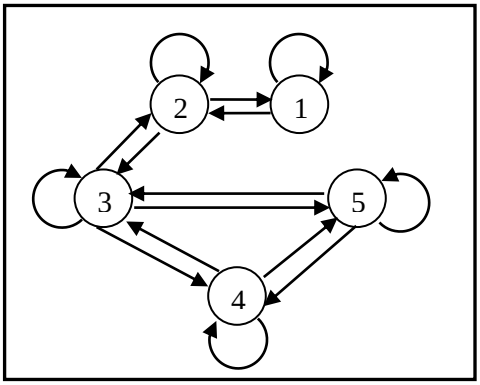
i) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$ ii) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

5. Let $A = \{1, 2, 3, 4, 5\}$. Determine whether the relation R whose digraph is given below is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive. Explain your answers.

i)



ii)



6. Let R be the following symmetric relation on the set $A = \{1, 2, 3, 4, 5\}$ where $R = \{(1, 2), (2, 1), (3, 4), (4, 3), (3, 5), (5, 3), (4, 5), (5, 4), (5, 5)\}$. Draw the graph of R.

7. Consider the graph of a symmetric relation R on $A = \{1, 2, 3, 4, 5, 6, 7\}$ is shown as follows. Determine R.