BAMS1623 DISCRETE MATHEMATICS
ASSIGNMENT
Academic Year 2020/21
Session 202005
Submission date: 28.8.2020

Q1. a) Use mathematical induction to prove that $1 + 6 + 6^2 + \cdots + 6^{n-1} = \frac{6^n - 1}{5}$ for all integers $n \geq 1$. (13 marks)

b) (i) Let $a = 368$ and $b = 224$. Find the greatest common divisor of a and b and express it in the form of $ma + nb$, where $m, n \in \mathbb{Z}$. Hence, find the least common multiple of a and b. (10 marks)

(ii) Determine whether 368 and 224 are relatively prime. Justify your answer. (2 marks)

[Total: 25 marks]

Q2. a) The relation R on $A = \{2, 3, 4, 8, 9, 12\}$ is defined by $x \ R \ y$ if and only if x is a multiple of y.

(i) List the ordered pairs of relation R. (4 marks)

(ii) Determine the domain and range of the relation R. (4 marks)

(iii) Find the in-degree and out-degree of each vertex. (4 marks)

b) Let $A = \{1, 2, 3, 4, 5\}$ and R be the relation on A where $R = (1,1), (1,4), (2,3), (3,2), (3,5), (4,1), (5,5)\}$. Use Warshall's algorithm to compute the matrix of transitive closure of R. (13 marks)

[Total: 25 marks]

Q3. a) Given $f(x) = 5 + 2e^{-x}$ is a function from $\mathbb{R}$ to $\mathbb{R}$, where $\mathbb{R}$ is the set of real numbers.

(i) Find the domain and range of the function f. (3 marks)

(ii) Determine whether the function is everywhere defined, injective, surjective and/or bijective. Justify each of your answers. (10 marks)

Q3.(Continued)

b) Let $A = \{1, 2, 3, 4, 5, 6\}$, the permutation $\rho_1 = (1, 4, 3, 6)$ and $\rho_2 = (2, 5, 4)$

(i) Compute ρ_1^2 and ρ_2^{-1}. (6 marks)

(ii) Compute $\rho_2^{-1} \circ \rho_1^2$ and write the result as a product of disjoint cycles and as a product of transpositions. (5 marks)

(iii) Determine whether the permutation $\rho_2^{-1} \circ \rho_1^2$ is even or odd. (1 mark)

[Total: 25 marks]

Q4. a) Let $A = (p \wedge r) \vee (q \rightarrow r)$.

(i) Complete the truth table given below and determine whether A is a tautology, contradiction or contingency. (7 marks)

p	q	r	$p \wedge r$	$q \rightarrow r$	$(p \wedge r) \vee (q \rightarrow r)$
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

(ii) Write the Principal Disjunctive Normal Form (PDNF) and Principal Conjunctive Normal Form (PCNF) of A and $\sim A$ by using the truth table obtained in part a)(i). (6 marks)

b) Let $x \in \{2, 4\}$ and $y \in \{6, 8\}$, consider a predicate $P(x, y)$ is defined as $P(x, y)$: x is a factor of y. Rewrite the expression $\exists y[\forall x P(x, y)]$ by eliminating the quantifiers. Hence, determine the truth value of the statement. (6 marks)

c) Use the laws of algebra of propositions, show that $[(a \rightarrow b) \wedge (c \rightarrow b)]$ and $[(a \vee c) \rightarrow b]$ are logically equivalent. (6 marks)

[Total: 25 marks]