Chapter 2 Logic

2.1 Introduction
2.2 Compound Statements
2.3 Conditional Statement
2.4 Biconditional Statement
2.5 Tautology, Contradiction and Contingency
2.6 Logical Equivalence
2.7 Logic Diagram
2.8 Normal Forms
2.9 Predicates and Quantifiers

2.1 Introduction

- Logic is the discipline that deals with the method of reasoning.
- Logic provides rules and techniques for determining whether a given argument is valid.
- A statement or proposition is a declarative sentence that is either true or false, but not both.

2.1 Introduction (cont)

- The truthfulness or falsity of a statement is called its truth value.
- Use “0” or “F” for false statement; “1” or “T” for true statement.

E.g.1

Which of the following are statements? For each statement, state the truth value if it is known.

i. $2 + 3 = 5$

ii. Sarah is a doctor.

iii. Do you like Mathematics?

iv. Open the door.

2.2 Compound Statements

- Simple statements may be combined by connectives like “and”, “or”, and “not” to form compound statements.
- The truth value of a compound statement depends on the truth value of the compound statements and on the types of connectives used.

2.2 Compound Statements (cont)

- A table giving the truth values of a compound statement in terms of the truth values of its component parts, is called a truth table.
- As each component statement has two possible truth values, if there are n component statements, the number of rows in its truth table is 2^n.

2.2.1 Conjunction

- If p and q are statements, the conjunction of p and q is the compound statement "p and q", denoted by $p \wedge q$.
- The compound statement $p \wedge q$ is true when both p and q are true; otherwise, it is false.

2.2.1 Conjunction (cont)

p	q	$p \wedge q$
T	T	
T	F	
F	T	
F	F	

2.2.2 Disjunction

- If p and q are statements, the disjunction of p and q is the compound "p or q", denoted by $p \vee q$.
- The compound statement $p \vee q$ is true at least one of p or q is true; it is false when both p and q are false.

2.2.2 Disjunction (cont)

p	q	$p \vee q$
T	T	
T	F	
F	T	
F	F	

2.2.3 Negation

- If p is a statement, the negation of p is the statement "not p", denoted by $\sim p$.
- The statement $\sim p$ is true if p is false; and $\sim p$ is false if p is true.
-

p	$\sim p$
T	F
F	T

2.3 Conditional Statement

- The statement "if p then q", denoted by $p \to q$, is called a conditional statement or implication.
- The statement p is called the antecedent or hypothesis, and the statement q is called the consequent or conclusion.

2.3 Conditional Statement (cont)

- The conditional statement $p \rightarrow q$ requires q to be true if p is true, but impose nothing on q when p is false.
- $p \rightarrow q$ is false only when p is true and q is false.
-

p	q	$p \rightarrow q$
T	T	
T	F	
F	T	
F	F	

2.3 Conditional Statement (cont)

- Each of the following expressions is an equivalent form of the conditional statement $p \to q$:
  - p implies q
  - If p then q
  - q, if p
  - p only if q
  - p is sufficient for q
  - q is necessary for p
- $p \to q \equiv {\sim}p \vee q$

E.g.2

Rewrite the following statement in if-then form.

Either you go to work on time or you are fired.

E.g.2

Rewrite the following statement in if-then form.

Either you go to work on time or you are fired.

Let $\sim p$: You go to work on time.

q: You are fired.

2.3.1 The Negation of a Conditional Statement

- $\sim(p \to q) \equiv p \wedge \sim q$

E.g.3

Write the negation for the statement “If my car is in the repair shop, then I cannot go to the class”.

E.g.3

Write the negation for the statement “If my car is in the repair shop, then I cannot go to the class”.

Let p: My car is in the repair shop.

q: I cannot go to the class.

2.3.2 The Contrapositive, Converse and Inverse of a Conditional Statement

- The contrapositive of $p \to q$ is $\sim q \to \sim p$.
- The converse of $p \to q$ is $q \to p$.
- The inverse of $p \to q$ is $\sim p \to \sim q$.

E.g.4

Write the contrapositive, converse and inverse of the following statement.

If Howard can swim across the lake, then Howard can swim to the island.

E.g.4

Write the contrapositive, converse and inverse of the following statement.

If Howard can swim across the lake, then Howard can swim to the island.

Let p: Howard can swim across the lake.

q: Howard can swim to the island.

Given $p \to q$.

Contrapositive $\equiv$

Converse $\equiv$

Inverse $\equiv$

E.g.5

Consider the following conditional statement:

If the flood destroys my house or the fire destroys my house, then my insurance company will pay me.

Identify the converse and contrapositive of the above statement.

E.g.5 (cont)

i. If my insurance company pays me, then the flood destroys my house or the fire destroys my house.

ii. If my insurance company pays me, then the flood destroys my house and the fire destroys my house.

iii. If my insurance company does not pay me, then the flood does not destroy my house or the fire does not destroy my house.

iv. If my insurance company does not pay me, then the flood does not destroy my house and the fire does not destroy my house.

Let p: The flood destroys my house.

q: The fire destroys my house.

r: My insurance company will pay me.

Given $(p \vee q) \rightarrow r$.

i.

ii.

iii.

iv.

2.4 Biconditional Statement

- If $p \rightarrow q$ and $q \rightarrow p$, denoted by $p \leftrightarrow q$, is called an equivalent or biconditional statement.
- $p \leftrightarrow q$ is true when p and q have the same truth value.
-

p	q	$p \leftrightarrow q$
T	T	
T	F	
F	T	
F	F	

2.4 Biconditional Statement (cont)

- Read as "p if and only if q" or "p is necessary and sufficient condition for q".
- $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

2.5 Tautology, Contradiction and Contingency

- A tautology is a proposition that is always true whatever the truth values of the component statements. The last column of the truth table of a tautology consists of T only.
- A contradiction or an absurdity is a proposition which is always false whatever the truth values of the component statements. The last column of the truth table of a contradiction consists of F only.

2.5 Tautology, Contradiction and Contingency (cont)

- If a proposition P is a tautology, then $\sim P$ is a contradiction.
- A proposition that can be either true or false, depending on the truth values of its component statements, is called a contingency.

E.g.6

Use truth table to show that

i. $p \vee \sim p$ is a tautology;

ii. $p \wedge \sim p$ is a contradiction;

p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$

E.g.6

Use truth table to show that

i. $p \vee \sim p$ is a tautology;

ii. $p \wedge \sim p$ is a contradiction;

p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$
T	F		
F	T		

E.g.6 (cont)

iii. $\sim(p \wedge \sim q)$ is a contingency;

p	q			

E.g.6 (cont)

iv. $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(p \to q) \wedge (q \to r)$	$(p \to q) \wedge (q \to r) \to (p \to r)$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

2.6 Logical Equivalence

- Two propositions are considered to be logically equivalent if they have the same truth value for any combination of truth values of their component statements. The last columns of their truth tables are identical.
- Write as $P \Leftrightarrow Q$ or $P \equiv Q$ if P and Q are logical equivalent.

E.g.7

Determine whether the propositions are logically equivalent.

i. $p \rightarrow q$

ii. $(\sim p \vee q)$

iii. $\sim q \rightarrow \sim p$

p	q					
T	T					
T	F					
F	T					
F	F					

E.g.8

By constructing a truth table, show that

$$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r).$$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

E.g.9

Determine whether $\sim(p \wedge q) \Leftrightarrow \sim p \wedge \sim q$.

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$
T	T					
T	F					
F	T					
F	F					

/

2.6.1 Laws of Algebra of Propositions

Given any statement variables p, q, and r, a tautology t, and a contradiction c, the following logical equivalences hold:

- Idempotent Law
  - $p \vee p \equiv p$
  - $p \wedge p \equiv p$
- Commutative Law
  - $p \vee q \equiv q \vee p$
  - $p \wedge q \equiv q \wedge p$

2.6.1 Laws of Algebra of Propositions (cont)

- Associative Law
  - $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  - $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
- Distributive Law
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- Identity Law
  - $p \vee c \equiv p$, where c is a contradiction
  - $p \wedge t \equiv p$, where t is a tautology

2.6.1 Laws of Algebra of Propositions (cont)

- Complement Law
  - $p \vee \sim p \equiv t$
  - $p \wedge \sim p \equiv c$
  - $\sim t \equiv c$
  - $\sim c \equiv t$
  - $\sim(\sim p) \equiv p$
- De Morgan's Law
  - $\sim(p \vee q) \equiv \sim p \wedge \sim q$
  - $\sim(p \wedge q) \equiv \sim p \vee \sim q$

2.6.1 Laws of Algebra of Propositions (cont)

- Contrapositive Law
  - $p \rightarrow q \equiv \sim q \rightarrow \sim p$
- Absorption Law
  - $p \vee (p \wedge q) \equiv p$
  - $p \wedge (p \vee q) \equiv p$

E.g.10

Use the laws above to prove the following logical equivalences:

i. $\sim(p \to q) \equiv p \wedge \sim q$

$$\sim(p \to q) \equiv$$

$$\equiv$$

$$\equiv$$

E.g.10 (cont)

ii. $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

$\sim(p \leftrightarrow q) \equiv$

$\equiv$

$\equiv$

$\equiv$

E.g.10 (cont)

iii. $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

$p \leftrightarrow q$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

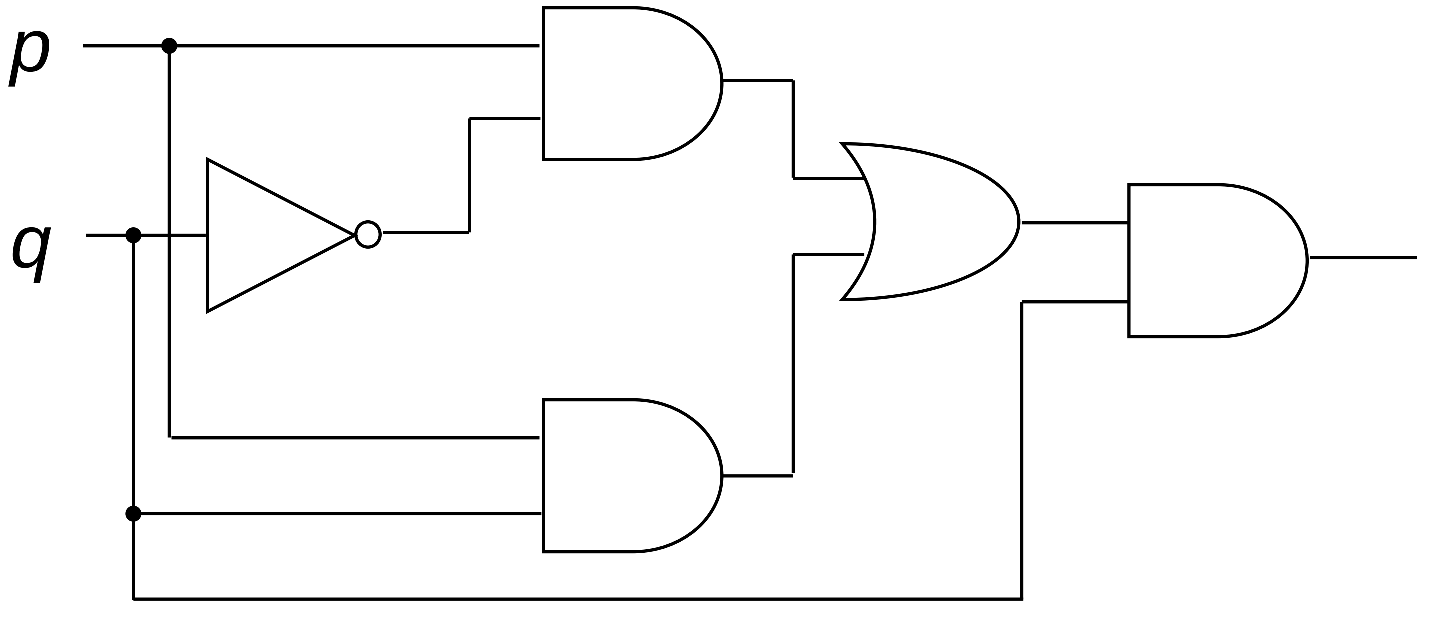
$\equiv$

2.7 Logic Diagram (cont)

- An input output table for a logic diagram is similar to a truth table, where signal 0 represents a False statement and signal 1 represents a True statement.

E.g.11

Simplify the following circuit.



$$[(p \wedge \sim q) \vee (p \wedge q)] \wedge q$$

E.g.11

$[(p \wedge \sim q) \vee (p \wedge q)] \wedge q$

2.8 Normal Forms

- Often we wish to compare two logical expressions whether they are logical equivalent.
- Comparison would be convenient if both expressions can be converted into some standard form.

2.8 Normal Forms (cont)

- For example, we may express a logical expression into a normal form, which is
  - the disjunctive of several terms, each of which is the conjunction of some simple statements and the negations of some simple statements, such as $(p \wedge q) \vee (p \wedge {\sim}q) \vee ({\sim}p \wedge q \wedge r)$; or
  - the conjunction of several terms, each of which is the disjunction of some simple statements and the negations of some simple statements, such as $(p \vee q) \wedge (p \vee {\sim}q) \wedge ({\sim}p \vee q \vee r)$.

2.8 Normal Forms (cont)

- For convenience, we use the word "product" for conjunction, and "sum" for disjunction.
- An elementary product is the conjunction of some simple statement variables and the negations of some simple statement variables, such as p, $p \wedge q$, $\sim q \wedge p \wedge \sim p$, $q \wedge \sim p$.
- An elementary sum is the disjunction of some simple statement variables and the negations of some simple statement variables, such as p, $\sim q$, $p \vee q$, $p \vee \sim q \vee r$, $\sim p \vee \sim r$.

2.8 Normal Forms (cont)

- A part of an elementary sum or product, which is itself an elementary sum or product, is called a factor of the original elementary sum or product.
- A Disjunctive Normal Form of a given expression is an expression which is equivalent to the logical expression and which consists of a sum of elementary products.

2.8 Normal Forms (cont)

- A Conjunctive Normal Form of a given expression is an expression which is equivalent to the logical expression and which consists of a product of elementary sums.
- A given expression is identically false, i.e. a contradiction, if every elementary product in its disjunctive normal form is identically false.
- A given expression is identically true, i.e. a tautology, if every elementary sum in its conjunctive normal form is identically true.

2.8 Normal Forms (cont)

- The following rules apply to elementary sums and products:
  - 1) A necessary and sufficient condition for an elementary product to be identically false is that it contains at least one pair of factors in which one is the negation of the other. For example:

$$p \wedge q \wedge {\sim}p \equiv c$$

$${\sim}p \wedge q \wedge r \wedge {\sim}r \equiv c$$

2.8 Normal Forms (cont)

- 2) A necessary and sufficient condition for an elementary sum to be identically true is that it contains at least one pair of factors in which one is the negation of the other.

$$p \vee q \vee \sim p \equiv t$$

$$\sim p \vee q \vee r \vee \sim r \equiv t$$

2.8 Normal Forms (cont)

- Procedure to obtain a disjunctive normal form or conjunctive normal form:

1. Change $\rightarrow$ and $\leftrightarrow$ using equivalent expressions in terms of $\wedge$, $\vee$ and ~.
2. For ~ which appears in front of a compound statement enclosed in parentheses, use De Morgan's laws to bring in the ~.

2.8 Normal Forms (cont)

iii. If the expression is still not in disjunctive normal form / conjunctive normal form because of some parts are products of sums/ sums of products, use distributive laws repeatedly to convert into sums of products/ products of sums.

E.g.12

Express the following statements in disjunctive normal form and conjunctive normal form.

i. $p \wedge (p \rightarrow q) \equiv$

$\equiv$

ii. $\sim(p \vee q) \leftrightarrow (p \wedge q)$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

ii. $\sim(p \vee q) \leftrightarrow (p \wedge q)$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

$\equiv$

E.g.13

Obtain a conjunctive normal form of $q \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$ and show that this expression is a tautology.

$$q \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$$

$$\equiv$$

2.8.1 Principal Disjunctive Normal Form (PDNF)

- The disjunctive normal form and the conjunctive normal form of a given expression are not unique, i.e. there may be several normal forms that are equivalent to each other.
- For example,

$p \vee (q \wedge r) \equiv$

2.8.1 Principal Disjunctive Normal Form (PDNF)(cont)

- Comparison would be easier if the expressions can be converted into some standard form that does not have many variations, preferably that is unique for each expression.

2.8.1 Principal Disjunctive Normal Form (PDNF)(cont)

- Minterm or Boolean conjunction of several simple statement variables p, q, r,… is a product of p, q, r, … or their negations, formed in such a way that each variable appears exactly once either as itself or its negation. For example,

  $$p \wedge q,\ p \wedge {\sim}q,\ {\sim}p \wedge q,\ {\sim}p \wedge {\sim}q$$

- Any expression that obtained by commuting the factors in the expressions above is not included in the list, as it would be equivalent to one of the minterms above.

2.8.1 Principal Disjunctive Normal Form (PDNF)(cont)

- The truth table for these minterms is as below:

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge \sim q$	$\sim p \wedge q$	$\sim p \wedge \sim q$

2.8.1 Principal Disjunctive Normal Form (PDNF)(cont)

- From the truth table,
  - each minterm has the truth value 1(T) for exactly one combination of the truth values of *p* and *q*.
  - no two minterms are equivalent.
- Principal disjunctive normal form (PDNF) or sum-of-product canonical form refers to an equivalent expression consisting of disjunctions of minterms only.

2.8.1 Principal Disjunctive Normal Form (PDNF)(cont)

- To obtain PDNF,

1) For every truth value 1(T) in the truth table of the given expression, select the minterm which also has the truth value 1(T) for the same combination of the truth values of *p* and *q*.
2) Take the disjunction of these minterms.

Then the disjunction will be equivalent to the given expression.

E.g.14

Construct truth table for $p \to q$, $p \vee q$, and $\sim(p \wedge q)$ and obtain the principal disjunctive normal forms of these expressions.

p	q	$p \wedge q$	$p \wedge \sim q$	$\sim p \wedge q$	$\sim p \wedge \sim q$	$p \rightarrow q$	$p \vee q$	$\sim(p \wedge q)$
T	T							
T	F							
F	T							
F	F							

$p \rightarrow q \equiv$

$p \vee q \equiv$

$\sim(p \wedge q) \equiv$

Notes:

- The number of minterms appearing in the normal form is the same as the number of entries with the truth value 1 in the truth table of the given expression.
- Every expression which is not a contradiction has an equivalent principal disjunctive normal form. Such a normal form is unique, except for the rearrangements of the factors in the disjunctions as well as in each of the minterms.

Notes: (cont)

- A certain order in which the variables appear in the minterms as well as a definite order in which the minterms appear in the disjunction may impose. Then, the given two equivalent expressions must have identical principal disjunctive normal forms.
- Then for 3 variables *p*, *q*, and *r*, the 8 minterms are

Notes: (cont)

- To obtain PDNF without constructing the truth table, first obtain a disjunctive normal form as before. Any elementary product which is a contradiction is dropped.
- Minterms are obtained in the disjunctions by introducing the missing factors. For example, if p is missing in a product, introduce $p \vee {\sim}p$, and then expand the term by the distributive law.
- Repeated minterms in the disjunctions are deleted.

E.g.15

Obtain the principal disjunctive normal forms of

i. $\sim p \vee q$

E.g.15 (cont)

ii. $(p \wedge q) \vee (\sim p \wedge r) \vee (q \wedge r)$

$\equiv$

E.g.16

Show that the following are equivalent expressions:

i. $p \vee (p \wedge q) \Leftrightarrow p$

Since the PDNF of LHS $\equiv$ PDNF of RHS, thus $p \vee (p \wedge q) \Leftrightarrow p$.

E.g.16 (cont)

ii. $p \vee (\sim p \wedge q) \Leftrightarrow p \vee q$

Since the PDNF of LHS $\equiv$ PDNF of RHS, thus $p \vee (\sim p \wedge q) \Leftrightarrow p \vee q$.

E.g.17

Obtain the principal disjunctive normal form of $p \to [(p \to q) \wedge \sim(\sim q \vee \sim p)]$.

$$p \to [(p \to q) \wedge \sim(\sim q \vee \sim p)]$$

$$\equiv \sim p \vee [(\sim p \vee q) \wedge (q \wedge p)]$$

2.8.2 Principal Conjunctive Normal Form (PCNF)

- For a given number of variables, a maxterm consists of disjunctions in which each variable or its negation, but not both, appears only once.
- The maxterms are the duals of minterms.

2.8.2 Principal Conjunctive Normal Form (PCNF) (cont)

- Each of the maxterms has the truth value 0 (F) for exactly one combination of the truth values of the variables. Different maxterms have the truth value 0 for different combinations of the truth values of the variables.

2.8.2 Principal Conjunctive Normal Form (PCNF) (cont)

- For a given expression, an equivalent expression consisting of conjunctions of the maxterms only is called its principal conjunctive normal form, or the product-of-sums canonical form.
- Every logical expression that is not a tautology has an equivalent principal conjunctive normal form, which is unique except for the rearrangement of the factors in the maxterms as well as in their conjunctions.

2.8.2 Principal Conjunctive Normal Form (PCNF) (cont)

- If the principal disjunctive (conjunctive) normal form of a given expression *A* is known, then the principal disjunctive (conjunctive) normal form of ~*A* will consist of the disjunctive (conjunctive) of the remaining minterms (maxterms) which do not appear in the principal disjunctive (conjunctive) normal form of *A*.

2.8.2 Principal Conjunctive Normal Form (PCNF) (cont)

- From $A \equiv {\sim}{\sim}A$, the principal disjunctive (conjunctive) normal form of A can be obtained by repeated applications of De Morgan's laws to the principal conjunctive (disjunctive) normal form of ${\sim}A$.

E.g.18

Let A represent $(\sim p \rightarrow r) \wedge (q \leftrightarrow p)$. Obtain the principal conjunctive normal form of A, and of $\sim A$. Deduce the principal disjunctive normal form of A.

$$A \equiv (\sim p \to r) \wedge (q \leftrightarrow p)$$

$$\equiv (p \vee r) \wedge [(q \to p) \wedge (p \to q)]$$

$$\equiv$$

$$\equiv$$

$$\equiv$$

$$\equiv$$

PCNF of A

$\equiv$

PCNF of $\sim A$

$\equiv$

PDNF of $A \equiv \sim$(PCNF of $\sim A$)

$\equiv$

PDNF of $\sim A \equiv \sim$(PCNF of A)

$\equiv$

Notes:

- The principal conjunctive normal form of a given expression can be written based on its truth table.
- The maxterms included correspond to the truth value 0 (F) in the truth table of that expression.
- The maxterms are written down by including the variable of its truth value is 0 (F) and its negation if the value is 1 (T).
- This is opposite to writing the minterms in principal disjunctive normal form.

E.g.19

Construct a truth table for the expression *A* in the previous example, and write its principal disjunctive normal form and the principal conjunctive normal form.

p	q	r	$\sim p$	$\sim p \to r$	$q \leftrightarrow p$	$(\sim p \to r) \wedge (q \leftrightarrow p)$
T	T	T	F	T	T	
T	T	F	F	T	T	
T	F	T	F	T	F	
T	F	F	F	T	F	
F	T	T	T	T	F	
F	T	F	T	F	F	
F	F	T	T	T	T	
F	F	F	T	F	T	

PDNF of A

$\equiv$

PCNF of A

$\equiv$

2.9 Predicates and Quantifiers

- Propositional logic applies to simple declarative statements where the basic propositions are either true or false.
- Statements involving one or more variables may be true for some values but not for others.

2.9 Predicates and Quantifiers (cont)

- A predicate is a sentence containing variables which is either true or false depending on the values assigned to the variables.
  - For example, x is an integer satisfying $x = x^2$ is a predicate since it is true for $x = 0$ or 1, and false for all other values of x.

2.9 Predicates and Quantifiers (cont)

- Logical operations can now be applied to predicates. In general, the truth of a compound predicate ultimately depends on the values assigned to the variables involved.

2.9 Predicates and Quantifiers (cont)

- The quantifiers, which refer to quantities, are added to predicate to produce either a true proposition or a false proposition.
  - All triangles have the sum of their angles equal to 180°.
  - All cats have tails.
  - There is an integer x satisfying $x^2 = 2$.
  - There is a prime number that is not odd.

2.9 Predicates and Quantifiers (cont)

- The universal quantifier, $\forall$, denotes “for all”, also can be stated as “for each”, “for every”, “for any”.
- The existential quantifier, $\exists$, denotes “there exists”, also can be read as “there is some”, “there is at least one”.

2.9 Predicates and Quantifiers (cont)

- For a general predicate $P(x)$.
  - not $(\exists x\, P(x)) \equiv \forall x$ (not $P(x)$);
  - not $(\forall x\, P(x)) \equiv \exists x$ (not $P(x)$).

2.9 Predicates and Quantifiers (cont)

- Let $Q(x)$ be a predicate and D the domain of x.
  - A Universal Statement , "$\forall x \in D, Q(x)$", is defined to be true if, and only if, $Q(x)$ is true for every x in D, and false if, and only if, $Q(x)$ is false for at least one x in D. A value for x for which $Q(x)$ is false is called a counterexample to the universal statement.

2.9 Predicates and Quantifiers (cont)

- An Existential Statement, "$\exists x \in D$, $Q(x)$", is defined to be true if, and only if, $Q(x)$ is true for at least one x in D, and false if, and only if, $Q(x)$ is false for every x in D.

E.g.20

Let P(x) be the following predicates. Express the proposition $\exists x$ P(x) in words and determine the truth value.

i. x is an integer and $x^2 = 16$

E.g.20 (cont)

ii. x is a real number and $x^2 + 1 = 0$

$\exists x$ P(x):

False statement,

E.g.21

Assume that x and y are real numbers and let $P(x, y)$ denote the predicate $x + y = 0$. Express each of the following propositions in words and determine the truth value.

i. $\forall x\ (\exists y\ P(x, y))$

ii. $\exists y\,(\forall x\, P(x, y))$