Tutorial 1

1. Enumerate the elements of the following sets:
$A = \{x| \ x \in Z \text{ and } 10 \le x \le 17\}$
$B = \{x| \ x \in Z \text{ and } x^2 < 24\}$
$C = \{x| \ x \in Z \text{ and } 6x^2 + x - 1 = 0\}$
$D = \{x| \ x \in R \text{ and } 6x^2 + x - 1 = 0\}$

2. Given the universal set is $\{p, q, r, s, t, u, v, w\}$. Let $A = \{p, q, r, s\}$, $B = \{r, t, v\}$, and $C = \{p, s, t, u\}$. Find the elements of the following sets:
   i) $B \cap C$
   ii) $A \cup C$
   iii) $\overline{C}$
   iv) $A \cap B \cap C$
   v) $(A \cup B) \cap (A \cap C)$
   vi) $\overline{(A \cup B)}$
   vii) $B - C$
   viii) $B \oplus C$

3. Let $U = \{x: x \in Z, 0 \le x \le 10\}$, $A = \{x: x \in U, x^2 - 11x + 10 = 0\}$, $B = \{x: x \in U, x \text{ is an odd integer}\}$, and $C = \{x \in U, x \text{ is an even integer}\}$.
   i)   Find   1) $B \cap C$;   2) $\overline{(A \oplus C)}$.
   ii) List the power set of $(B - A)$.

4. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{x: x \in U, x \text{ is an odd number}\}$, $B = \{x: x \in U, x \text{ is a multiple of 3}\}$, and $C = \{x: x \in U, x \text{ is a factor of 18}\}$. Find
   i) $A \cup B$;
   ii) $A - B$;
   iii) $\overline{A \cap C}$;
   iv) $A \oplus C$.

5. Consider the following subsets of a standard English language dictionary:
$A = \{x: x \text{ is a word that appears before } dog\}$
$B = \{x: x \text{ is a word that appears after } cat\}$
$C = \{x: x \text{ is word containing two identical consecutive letters}\}$
   i)   Decide which of the following statements are true:
        a) $C \subseteq A \cup B$
        b) $aardvark \in \overline{B} \cap C$
        c) $moose \in B \oplus C$
        d) $A \cap B = \varnothing$
   ii) Describe in words the elements of the following sets:
        a) $A \cap B \cap C$
        b) $(A \cup B) \cap \overline{C}$

6. Consider the following subsets of Z:
$A = \{3n: n \in Z \text{ and } n \ge 4\}$
$B = \{2n: n \in Z\}$
$C = \{n: n \in Z \text{ and } n^2 \le 100\}$
   Using set operations, express each of the following in terms of A, B, and C:
   i)   The set of odd integers.
   ii) $\{-10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10\}$
   iii) $\{6n: n \in Z \text{ and } n \ge 2\}$
   iv) $\{-9, -7, -5, -3, -1, 1, 3, 5, 7, 9\}$

7. An operator $*$ is defined on sets by $A * B = \overline{(A \cap B)}$.

    i) Draw a Venn diagram representation of $A * B$.

    ii) Make a sequence of Venn diagrams to prove the following:

        a) $A * A = \overline{A}$

        b) $(A * A) * (B * B) = A \cup B$

        c) $(A * B) * (A * B) = A \cap B$

8. i) Find $\wp(A)$ when $A = \{1, 2, 3\}$.

    ii) Show by example that $\wp(A) \cup \wp(B)$ need not equal $\wp(A \cup B)$.

9. First-year students studying computing at a university can study a number of optional units. Last year 25 students chose to study accounting option, 27 chose the business option and 12 chose the tourism option. There are 20 students who took both the accounting and the business options, 5 who opted for accounting and tourism, and 3 who studied business and tourism. No students took all three options.

    i) How many students are taking at least one of the three options?

    ii) How many of the students involved took only tourism?

10. The records of 200 students at a college show the following courses taken:

    104 students took Latin

    103 students took Greek

    35 students took Sanskrit

    46 students took Latin and Greek

    24 students took Greek and Sanskrit

    9 students took all three languages

    28 students have taken none of these languages

    i) How many students took only Greek?

    ii) How many students took only Latin and Sanskrit?