Tutorial 9

1. Let A = {0, 2, 4, 6} and B = {1, 3, 5, 7}. Determine which of the following relations between A and B forms a function with domain A and codomains B. For those whose are functions, determine whether they are injective, surjective or bijective.
   i) {(6, 3), (2, 1), (0, 3), (4, 5)}
   ii) {(2, 3), (4, 7), (0, 1), (6, 5)}
   iii) {(2, 1), (4, 5), (6, 3)}
   iv) {(6, 1), (0, 3), (4, 1), (0, 7), (2, 5)}

2. Let A = {-1, 0, 1, 2} and $f: A \to Z$ be given by $f(x) = \left\lfloor \frac{x^2+1}{3} \right\rfloor$.
   i) Find the range of f.
   ii) Determine whether the function f is injective, surjective or bijective. Justify your answer.

3. Determine whether the function g: Z $\to$ Z given by $g(n) = \left\lfloor \frac{n}{2} \right\rfloor$ is injective, surjective or bijective. Justify your answer.

4. Given $f(x) = 2x - 1$, a function from X = {1, 2, 3} to Y = {1, 2, 3, 4, 5}. Find the domain and range of the function f. Hence determine whether the function is a bijective function and explain your answer.

5. Functions f: R $\to$ R and g: R $\to$ R are given by
   $f(x) = x^2$ and $g(x) = \begin{cases} 2x+1 & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$.
   Find formulae for $f \circ g$, $g \circ f$, and $g \circ g$.

6. Let f be the mod-10 function. Compute
   i) $f(417)$ ii) $f(38)$ iii) $f(253)$

7. Let the universal set $U = \{a, b, c, \ldots, y, z\}$ and the characteristic function for the specified subset to compute the following function values.
   i) $A = \{a, e, i, o, u\}$
      a) $f_A(i)$ b) $f_A(y)$ c) $f_A(o)$
   ii) $B = \{m, n, o, p, q, r, z\}$
      a) $f_B(a)$ b) $f_B(m)$ c) $f_B(s)$

8. Compute each of the following.
   i) $\lfloor 2.78 \rfloor, \lceil 2.78 \rceil$ ii) $\lfloor -2.78 \rfloor, \lceil -2.78 \rceil$ iii) $\lfloor 14 \rfloor, \lceil 14 \rceil$
   iv) $\lfloor -17.3 \rfloor, \lceil -17.3 \rceil$ v) $\lfloor 21.5 \rfloor, \lceil 21.5 \rceil$

9. Compute the function values indicated.
   i) $f(n) = 3n^2 - 1$
      a) $f(3)$ b) $f(17)$
      c) $f(5)$ d) $f(12)$
   ii) $f_2(n) = 2^n$
      a) $f_2(1)$ b) $f_2(3)$
      c) $f_2(5)$ d) $f_2(10)$

10. Let Q be the propositional function defined by $Q(x)$: $\exists y \in \mathbb{Z}^+$ such that $xy = 60$. Evaluate each of the following.
    i) $Q(3)$ ii) $Q(7)$
    iii) $Q(-6)$ iv) $Q(15)$

11. Let $A = \{1, 2, 3, 4, 5, 6\}$ and $p_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 1 & 2 & 6 & 5 \end{pmatrix}$, $p_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 5 & 4 & 6 \end{pmatrix}$, $p_3 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 2 & 5 & 4 & 1 \end{pmatrix}$. Compute the following.
    i) p_1^{-1} ii) $p_3 \circ p_1$
    iii) p_3^{-1} iv) $p_1^{-1} \circ p_2^{-1}$

12. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Compute the following products.
    i) $(3, 5, 7, 8) \circ (1, 3, 2)$ ii) $(2, 6) \circ (3, 5, 7, 8) \circ (2, 5, 3, 4)$

13. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Write $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 5 & 7 & 8 & 4 & 3 & 2 & 1 \end{pmatrix}$ as the product of disjoint cycles and product of transpositions.

14. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Write each permutation as a product of transpositions.
    i) $(2, 1, 4, 5, 8, 6)$ ii) $(3, 1, 6) \circ (4, 8, 2, 5)$

15. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Determine the given permutation is even or odd.
    i) $(6, 4, 2, 1, 5)$ ii) $(4, 8) \circ (3, 5, 2, 1) \circ (2, 4, 7, 1)$

16. Let $A = \{1, 2, 3, 4, 5\}$. Let $f = (5, 3, 2)$ and $g = (3, 4, 1)$ be permutations of A. Compute each of the following and write the results as the product of disjoint cycles.
    i) $f \circ g$ ii) $f^{-1} \circ g^{-1}$

17. Let $A = \{1, 2, 3, 4, 5, 6\}$ and $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 1 & 5 & 6 \end{pmatrix}$ be a permutation of A.
    i) Write p as a product of disjoint cycles.
    ii) Compute p^{-1}.
    iii) Compute p^2.