Tutorial 4

1. Express the following using $\wedge$, $\vee$, and ~ only. Simplify your expressions.

   i) $(p \to q) \vee \sim p$ ii) $(p \to q) \vee p$

   iii) $(p \leftrightarrow \sim q) \vee q$ iv) $p \wedge \sim(q \to p)$

2. Write negations for each of the following statements.

   i) If P is a square, then it is a rectangle.

   ii) If the sun is shining, then I shall play tennis or swimming this afternoon.

   iii) If I am free and I am not tired, then I will go to the supermarket.

   iv) If $x = 17$ or $x^3 = 8$, then x is prime.

3. Obtain the principal disjunctive normal form and the principal conjunctive normal form of each of the following expressions. Hence deduce the principal disjunctive normal form and the principal conjunctive normal form of their negation statements.

   i) $\sim(p \vee q)$ ii) $\sim(p \wedge q)$

   iii) $\sim(p \to q)$ iv) $\sim(p \leftrightarrow q)$

4. Construct a truth table for the following expressions. Based on the truth table, write the principal disjunctive normal form of A, the principal disjunctive normal form of $\sim A$, the principal conjunctive normal form of A, and the principal conjunctive normal form of $\sim A$.

   i) $A \equiv (p \to q) \wedge (\sim p \vee r)$ ii) $A \equiv (\sim p \vee \sim q) \to (p \leftrightarrow \sim q)$

5. Without constructing truth tables, obtain the principal disjunctive normal form of A, the principal disjunctive normal form of $\sim A$, the principal conjunctive normal form of A, and the principal conjunctive normal form of $\sim A$, if the normal form exist, for each expression A below:

   i) $q \wedge (p \vee \sim q)$ ii) $p \to (p \wedge (q \to p))$

   iii) $(q \to p) \wedge (\sim p \wedge q)$

6. Let P(x): x is even; and Q(x): x is a prime number; and R(x, y): $x + y$ is even.
   The variables of x and y represent integers.
   Write each of the following in terms of P(x), Q(x), R(x, y), logical connectives and quantifiers. Determine the truth value of each statement.

   i) Every integer is an odd integer.

   ii) The sum of any two integers is an even number.

   iii) There are no even prime numbers.

   iv) Every integer is even or a prime.

7. Show the scope of each quantifier in the following expression $\forall y[\mathrm{P}(x, y) \to \exists x \mathrm{Q}(x, y)]$.
   Determine whether the given expression is a statement or not. If not, state the reason.

8. Let the universe of discourse be {1, 2} and the predicates P(x): $x = x^2$ and Q(y): y is a prime number.
   i) State if the expression $\forall x\ [P(x) \rightarrow \exists y\ Q(y)]$ is a statement. State its truth value if yes, and state the reason if no.
   ii) Rewrite the expression in (i) by eliminating the symbol, $\rightarrow$ and quantifiers.

9. Let P(x) be "x is short", and Q(x) be the "x is thin", where x is a person. Express the proposition "Everyone is short and thin" in symbolic form. Then write its negation.

10. Let $P = \{2, 3, 4\}$ and $Q = \{7, 8, 9\}$.
    Consider the quantified statement $\forall x \in P, \exists y \in Q \ni (x + y < 14)$.
    i) Eliminate the quantifiers in the above statement.
    ii) State the truth value for the above statement.