Tutorial 8

1. Let A = {1, 2, 3, 4} and a relation R on A is R = {(1, 2), (1, 3), (2, 3), (3, 4)}. Find the reflexive closure and symmetric closure of R.

2. i) Let A = {1, 2, 3} and let R = {(1, 1),(1, 2), (2, 3), (1, 3), (3, 1), (3, 2)}. Compute the matrix $\mathbf{M}_{R^\infty}$ of the transitive closure R by using the formula

$$\mathbf{M}_{R^\infty} = \mathbf{M}_R \vee (\mathbf{M}_R)^2_\odot \vee (\mathbf{M}_R)^3_\odot.$$

   ii) List the relation R^∞ whose matrix was computed in part (i).

3. Let A = {1, 2, 3, 4}. For the relation R whose matrix is given, find the matrix of the transitive closure by using Warshall's algorithm.

   i) $\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ ii) $\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

   iii) $\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ iv) $\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

4. Let A = {1, 2, 3, 4, 5} and let R and S be the equivalence relations on A whose matrices are given. Compute the matrix of the smallest equivalence relation containing R and S, and list the elements of this relation.

$$\mathbf{M}_R = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}, \mathbf{M}_S = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

5. Compute A/R, A/S, and the partition of A that corresponds to the equivalence relation found in Question 4.

6. Let A = {1, 2, 3, 4} and let R and S be relations on A described by

$$\mathbf{M}_R = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \mathbf{M}_S = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

   Use Warshall's algorithm to compute the transitive closure of $R \cup S$.