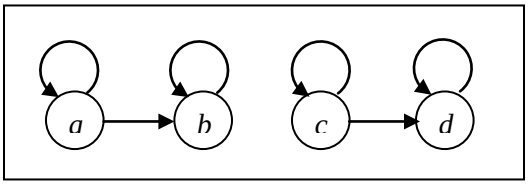
Tutorial 10

1. Determine whether the relation R is a partial order on the set A.
   i) $A = \mathrm{Z}$, and $a \; R \; b$ if and only if $a = 2b$.
   ii) $A = \mathrm{R}$, and $a \; R \; b$ if and only if $a \leq b$.
   iii) $\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$
   iv) $\mathbf{M}_R = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
   v)

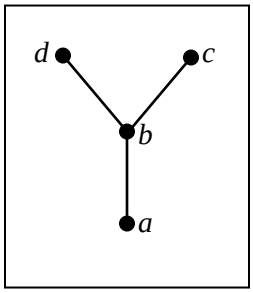


2. Find the lexicographic ordering of the following strings of lowercase English letters:
   i) quack, quick, quicksilver, quicksand, quacking
   ii) zoo, zero, zoom, zoology, zoological

3. Find the lexicographic ordering of the bit strings 0, 01, 11, 001, 010, 011, 0001, and 0101 based on the ordering $0 < 1$.

4. Write the dual of the following posets.
   i) $(\{0, 1, 2\}, \leq)$
   ii) $(\mathrm{Z}^+, |)$

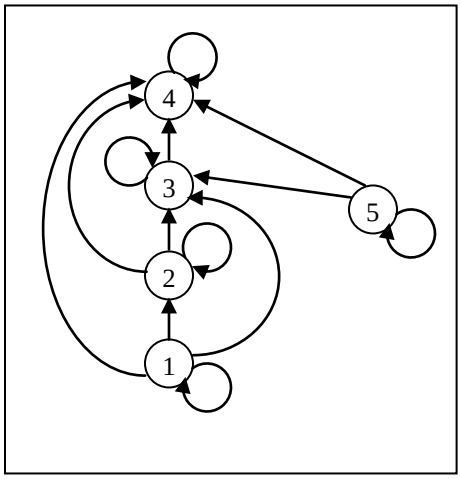
5. List all ordered pairs in the partial order whose Hasse diagram is shown as below.



6. Draw the Hasse diagram for each of the following posets.
   i) a is a divisor of b on the set $\{1, 2, 3, 5, 7, 11, 13\}$.
   ii) X is a subset of Y on the set of all subsets of $\{1, 2, 3\}$.

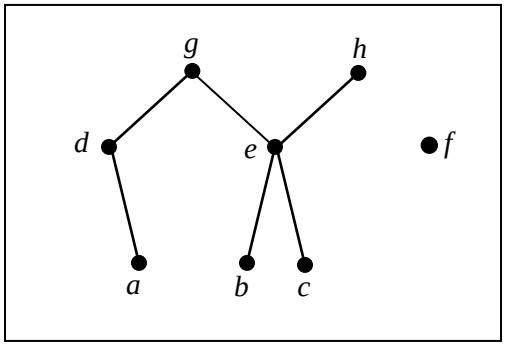
iii) $A = \{1, 2, 3, 4, 5\}$, $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

iv)



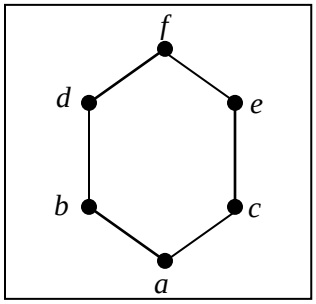
7. Consider the partial order of divisibility on the set A. Draw the Hasse diagram of the poset and determine which posets are linearly ordered.
   i) $A = \{1, 2, 3, 4, 5, 10, 15, 30\}$
   ii) $A = \{3, 6, 12, 36, 72\}$

8. Given the Hasse diagram of a partial order R on $A = \{a, b, c, d, e, f, g, h\}$. List the elements of R and write down the maximal and minimal elements of A.

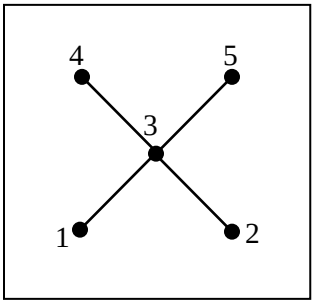


9. Determine the greatest and least elements, if exist, of the following posets.

i)

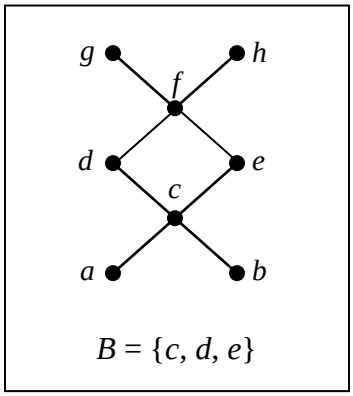


ii)

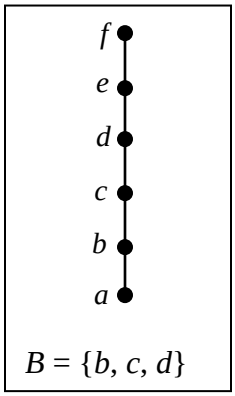


10. Consider the following posets whose Hasse diagrams are shown. Find, if they exist,
    i) maximal and minimal elements;
    ii) all upper bounds of B;
    iii) all lower bounds of B;
    iv) the least upper bound of B;
    v) the greatest lower bound of B.

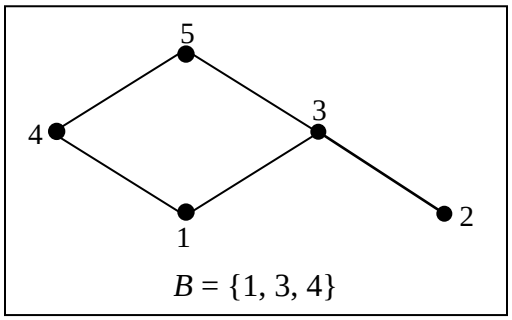
a)



b)



c)



11. Answer the following questions concerning the poset $(\{3, 5, 9, 15, 24, 45\}, |)$.
    i) Find the maximal and minimal elements.
    ii) Determine the greatest element and least element, if exist.
    iii) Find all upper bounds and least upper bounds of $\{3, 5\}$, if exist.
    iv) Find all lower bounds of $\{15, 45\}$. Hence determine the greatest lower bound of $\{15, 45\}$, if exist.

12. Let $A = \{1, 2, 3, 4, 5, 6\}$ and consider the partial order R on A as
    $R = \{(6, 6), (6, 5), (6, 4), (6, 3), (6, 2), (6, 1), (5, 5), (5, 3), (5, 2), (5, 1), (4, 4), (4, 3), (4, 2)(4, 1), (3, 3), (3, 2), (3, 1), (2, 2), (1, 1)\}$.
    i) Draw a Hasse diagram of the poset $[A, R]$.
    ii) Find the minimal and maximal elements of the poset $[A, R]$.
    iii) Find the least upper bound of $\{2, 5\}$, if it exists.
    iv) Find the greatest lower bound of $\{5, 4\}$, if exists.