

Tutorial 2

1. Give the set corresponding to the sequence.
 - i) 1, 2, 1, 2, 1, 2, 1, 2, 1
 - ii) $aabbccdde...zz$
2. Write out the first four terms (begin with $n = 1$) of the sequence whose general term is given as follows.
 - i) $a_n = 5^n$
 - ii) $c_1 = 2.5, c_n = c_{n-1} + 1.5$
3. Write a formula for the n th term of the sequence. Identify your formula as a recursive or explicit.
 - i) 1, 3, 5, 7, ...
 - ii) 1, -1, 1, -1, 1, -1, ...
4. Prove the following statements by induction:
 - i) $1 + 5 + 9 + \dots + (4n - 3) = n(2n - 1)$ for all $n \geq 1$.
 - ii) $\frac{1}{1(3)} + \frac{1}{3(5)} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$ for all $n \geq 1$.
 - iii) $(n^3 - n)$ is divisible by 3 for all $n \geq 1$.
5. A sequence of integers x_1, x_2, \dots, x_n is defined by

$$x_1 = 1 \text{ and } x_{k+1} = \frac{x_k}{x_k + 2} \text{ for } k \geq 1.$$
 - i) Calculate x_2, x_3 , and x_4 .
 - ii) Prove by induction that $x_n = \frac{1}{2^n - 1}$ for all $n \geq 1$.
6. For the following pairs of integers m and n ,
 - i) find the greatest common divisor of the m and n . Hence deduce the least common multiple of m and n .
 - ii) rewrite m and n in the form of $am + bn, a, b \in \mathbb{Z}$.
 - a) $m = 27, n = 72$
 - b) $m = 3510, n = 672$
7. Compute $A \vee B, A \wedge B$, and $A \ominus B$ for the given matrices A and B .
 - i) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}$
 - ii) $A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$
 - iii) $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$