Tutorial 5

1. In the following, find x and y so that the statement is true.
   i) $(x, 3) = (4, 3)$ ii) $(a, 3y) = (a, 9)$
   iii) $(4x, 6) = (16, y)$ iv) $(2x - 3, 3y - 1) = (5, 5)$

2. Let $A = \{a, b\}$ and $B = \{4, 5, 6\}$.
   i) List the elements in
   1) $A \times B$ 2) $B \times A$
   ii) List all partitions of B.

3. Let the universal set, $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and S, T be the subsets of U defined as $S = \{x | \ x \in U$ and 3 divides $x\}$, $T = \{x | \ x \in U$ and 5 divides $x\}$. List the elements in $S \times T$.

4. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A_1 = \{1, 2, 3, 4\}$, $A_2 = \{5, 6, 7\}$, $A_3 = \{4, 5, 7, 9\}$, $A_4 = \{4, 8, 10\}$, $A_5 = \{8, 9, 10\}$, $A_6 = \{1, 2, 3, 6, 8, 10\}$.
   List the possible partitions of A.

5. Let $A = Z^+$, the positive integers, and R be the relation defined by $a \ R \ b$ if and only if $2a \leq b + 1$. Which of the following ordered pairs belong to R?
   i) $(2, 2)$ ii) $(3, 2)$ iii) $(6, 15)$
   iv) $(1, 1)$ v) $(15, 6)$ vi) $(n, n), n > 1$

6. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{3, 4\}$ and define a binary relation R from A to B as follows:
   For $(x, y) \in A \times B$, $(x, y) \in R \Leftrightarrow x \geq y$.
   Write R as a set of ordered pairs.

7. For each of the following relation on N, list the ordered pairs that belong to the relation.
   $R = \{(x, y): 2x + y = 9\}$
   $S = \{(x, y): x + y < 7\}$
   $T = \{(x, y): y = x^2\}$

8. Let $A = \{1, 3, 5, 7\}$ and R be the relation on A whose matrix is given below.

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

   i) Write R as a set of ordered pairs.
   ii) Draw the digraph of R.
   iii) Find the domain and range of R.
   iv) Give the in-degree and out degree of each vertex.

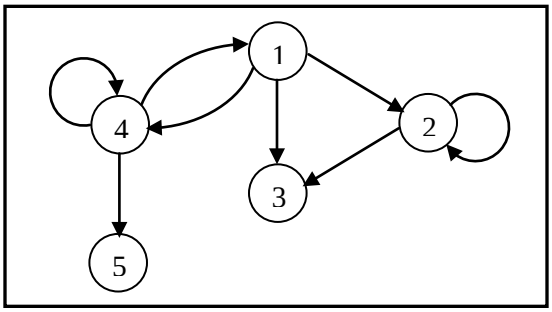
9. Let R be the relation on {1, 2, 3, 4} given by $u\ R\ v$ if and only if $u + 2v$ is odd. Represent R in each of the following ways:
   i) as a set of ordered pairs; ii) in graphical form;
   iii) in matrix form.
   Give the in-degree and out-degree of each vertex.

10. Find the domain, range, matrix, and, when $A = B$, the digraph of the relation R.
   i) $A = \{1, 2, 3, 4, 8\} = B$; $a\ R\ b$ if and only if $a = b$.
   ii) $A = \{1, 2, 3, 4, 6\} = B$; $a\ R\ b$ if and only if a is a multiple of b.
   iii) $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 4, 6, 8\}$; $a\ R\ b$ if and only if $b < a$.

11. Let A = R, set of real numbers. Consider the following relation R on A: $a\ R\ b$ if and only if $a^2 + b^2 = 25$. Find Dom (R) and Ran (R).

12. Let $A = \{1, 2, 3, 4, 6\}$ and R be the relation defined as $a\ R\ b$ if and only if a is a multiple of b. Find each of the following.
   i) $R(3)$ ii) $R(6)$ iii) $R(\{2, 4, 6\})$

13. Refer to the following digraph of a relation R.



   i) Give the in-degree and the out-degree of each vertex.
   ii) List the ordered pairs belonging to the relation R.
   iii) Represent R in the matrix form.

14. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{2, 3, 4, 6\}$, and $R = \{(1, 2), (1, 4), (2, 3), (2, 5), (3, 6), (4, 7)\}$. Compute the restriction of R to B.