Metodologia Ecológica

- Análise de Covariância – ANCOVA
- Partição da variação explicada por cada fator em ANOVA
- Fatores fixos e fatores aleatórios em ANOVA

TABLE 10.2 ANOVA table for one-way layout

Source	Degrees of freedom (df)	Sum of squares (*SS*)	Mean square (*MS*)	Expected mean square	F-ratio	*P*-value
Among groups	$a-1$	$\sum_{i=1}^{a}\sum_{j=1}^{n}(\bar{Y}_i-\bar{Y})^2$	$\frac{SS_{among\ groups}}{(a-1)}$	$\sigma^2+n\sigma_A^2$	$\frac{MS_{among\ groups}}{MS_{within\ groups}}$	Tail of the F-distribution with $(a-1)$ $a(n-1)$ degrees of freedom
Within groups (residual)	$a(n-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{n}(Y_{ij}-\bar{Y}_i)^2$	$\frac{SS_{within\ groups}}{a(n-1)}$	σ^2		
Total	$an-1$	$\sum_{i=1}^{a}\sum_{i=1}^{n}(Y_{ij}-\bar{Y})^2$	$\frac{SS_{total}}{(an-1)}$	σ_Y^2		

TABLE 10.3 One-way ANOVA table for the hypothetical data in Table 10.1

Source	Degrees of freedom (df)	Sum of squares (*SS*)	Mean square (*MS*)	F-ratio	*P*-value
Among groups	2	22.17	11.08	5.11	0.033
Within groups (residual)	9	19.50	2.17		
Total	11	41.67			

$$Var\,\mathrm{Pr}oporcional_A = \frac{\sigma_A^{\,2}}{\sigma_A^{\,2} + \sigma_e^{\,2}}$$

$$\sigma_e^{\,2} = QM_{dentro}$$

$$\sigma_e^{\,2} + n\sigma_A^{\,2} = QM_{entre}$$

$$\sigma_A^{\,2} = \frac{QM_{entre} - QM_{dentro}}{n}$$

$$\sigma_A^{\,2} = \frac{(QM_{entre} - QM_{dentro})(a-1)}{na}$$

TABLE 10.11 Variance components in two-way ANOVA models with fixed, mixed, or random factors

Component of variance	Fixed effects model (A fixed, B fixed)	Random effects model (A random, B random)	Mixed effects model (A fixed, B random)
A	$\frac{(MS_A - MS_{residual})(a-1)}{abn}$	$\frac{(MS_A - MS_{A\times B})}{bn}$	$\frac{(MS_A - MS_{A\times B})(a-1)}{bna}$
B	$\frac{(MS_B - MS_{residual})(b-1)}{abn}$	$\frac{(MS_B - MS_{A\times B})}{an}$	$\frac{(MS_B - MS_{A\times B})}{an}$
$A \times B$	$\frac{(MS_{A\times B} - MS_{residual})(a-1)(b-1)}{abn}$	$\frac{(MS_{A\times B} - MS_{residual})}{n}$	$\frac{(MS_{A\times B} - MS_{residual})}{n}$
Residual	$MS_{residual}$	$MS_{residual}$	$MS_{residual}$

TABLE 10.9 ANOVA for two-way ANOVA with random effects

Source	Degrees of freedom (df)	Sum of squares (SS)	Mean square (MS)	Expected mean square	F-ratio	P-value
Factor A	$a-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(\bar{Y}_i-\bar{Y})^2$	$\frac{SS_A}{(a-1)}$	$\sigma^2+n\sigma^2_{AB}+nb\sigma^2_A$	$\frac{MS_A}{MS_{AB}}$	Tail of the F-distribution with $(a-1)$, $(a-1)(b-1)$ degrees of freedom
Factor B	$b-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(\bar{Y}_j-\bar{Y})^2$	$\frac{SS_B}{(b-1)}$	$\sigma^2+n\sigma^2_{AB}+na\sigma^2_B$	$\frac{MS_B}{MS_{AB}}$	Tail of the F-distribution with $(b-1)$, $(a-1)(b-1)$ degrees of freedom
Interaction $(A\times B)$	$(a-1)(b-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(\bar{Y}_{ij}-\bar{Y}_i-\bar{Y}_j+\bar{Y})^2$	$\frac{SS_{AB}}{(a-1)(b-1)}$	$\sigma^2+n\sigma^2_{AB}$	$\frac{MS_{AB}}{MS_{within\ groups}}$	Tail of the F-distribution with $(a-1)(b-1)$, $ab(n-1)$ degrees of freedom
Within groups (residual)	$ab(n-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(Y_{ijk}-\bar{Y}_{ij})^2$	$\frac{SS_{within\ groups}}{ab(n-1)}$	σ^2		
Total	$abn-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(Y_{ijk}-\bar{Y})^2$	$\frac{SS_{total}}{(abn-1)}$	σ^2_Y		

TABLE 10.6 ANOVA table for two-way design

Source	Degrees of freedom (df)	Sum of squares (SS)	Mean square (MS)	Expected mean square	F-ratio	P-value
Factor A	$a-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(\bar{Y}_i-\bar{Y})^2$	$\frac{SS_A}{(a-1)}$	$\sigma^2+nb\sigma^2_A$	$\frac{MS_A}{MS_{within\ groups}}$	Tail of the F-distribution with $(a-1)$, $ab(n-1)$ degrees of freedom
Factor B	$b-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(\bar{Y}_j-\bar{Y})^2$	$\frac{SS_B}{(b-1)}$	$\sigma^2+na\sigma^2_B$	$\frac{MS_B}{MS_{within\ groups}}$	Tail of the F-distribution with $(b-1)$, $ab(n-1)$ degrees of freedom
Interaction $(A\times B)$	$(a-1)(b-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(\bar{Y}_{ij}-\bar{Y}_i-\bar{Y}_j+\bar{Y})^2$	$\frac{SS_{AB}}{(a-1)(b-1)}$	$\sigma^2+n\sigma^2_{AB}$	$\frac{MS_{AB}}{MS_{within\ groups}}$	Tail of the F-distribution with $(a-1)(b-1)$, $ab(n-1)$ degrees of freedom
Within groups (residual)	$ab(n-1)$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(Y_{ijk}-\bar{Y}_{ij})^2$	$\frac{SS_{within\ groups}}{ab(n-1)}$	σ^2		
Total	$abn-1$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(Y_{ijk}-\bar{Y})^2$	$\frac{SS_{total}}{(abn-1)}$	σ^2_Y		

ANCOVA

Modelo linear

Objetivos:

- Remover de *Y* a variação devido a uma covariável *X*
- Ajustar a média de *Y* para um valor padrão da variável *X* (desta forma “fixando” o valor de X)

Como fazer isso? Como medir a variação comum de *X* e *Y* e descontá-la de *Y*?

ANOVA de um fator:

$$Y_{ij} = \mu + A_i + \varepsilon_{ij}$$

Hipótese nula:

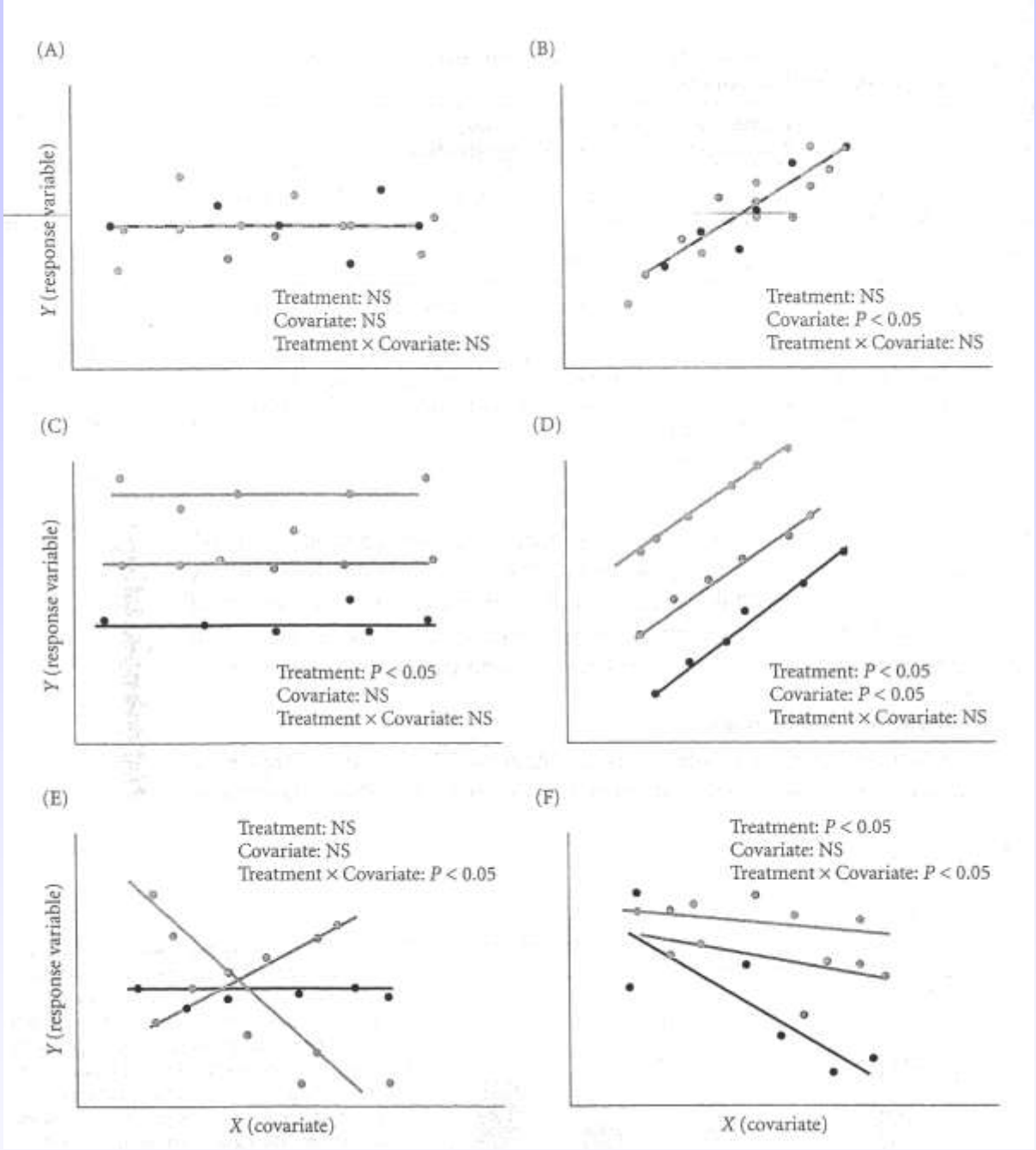
$$Y_{ij} = \mu + \varepsilon_{ij}$$

ANCOVA, inclinações diferentes:

$$Y_{ij} = \mu + A_i + B_i\left(X_{ij} - \bar{X}_{ij}\right) + \varepsilon_{ij}$$

ANCOVA, inclinação Comum:

$$Y_{ij} = \mu + A_i + B_C\left(X_{ij} - \bar{X}_{ij}\right) + \varepsilon_{ij}$$


(A)
Y (response variable)
Treatment: NS
Covariate: NS
Treatment × Covariate: NS
(B)
Treatment: NS
Covariate: P < 0.05
Treatment × Covariate: NS
(C)
Y (response variable)
Treatment: P < 0.05
Covariate: NS
Treatment × Covariate: NS
(D)
Treatment: P < 0.05
Covariate: P < 0.05
Treatment × Covariate: NS
(E)
Treatment: NS
Covariate: NS
Treatment × Covariate: P < 0.05
Y (response variable)
X (covariate)
(F)
Treatment: P < 0.05
Covariate: NS
Treatment × Covariate: P < 0.05
X (covariate)