02/11/2021

DIMOSTRARE che per ogni $n \geq 1$ si ha

$$\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - \frac{1}{n}$$

PASSO BASE $P(n) = \sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - \frac{1}{n}$

$n = 1$

$$P(1) = \text{"} \sum_{i=1}^{1} 1 \leq 2 - \frac{1}{1} \text{"} \quad \text{VERO}$$

PASSO INDUTTIVO supponiamo $P(n)$ sia vero

$$P(n+1) = \text{"} \sum_{i=1}^{n+1} \frac{1}{i^2} \leq 2 - \frac{1}{n+1} \text{"}$$

$$\sum_{i=1}^{n+1} \frac{1}{i^2} = \underbrace{\sum_{i=1}^{n} \frac{1}{i^2}}_{\leq\, 2 - \frac{1}{n}} + \frac{1}{(n+1)^2} \leq \underbrace{2 - \frac{1}{n} + \frac{1}{(n+1)}}$$

$+$

$$2 - \frac{1}{n} + \frac{1}{(n+1)^2} = 2 - \frac{(n+1)^2 - n}{n(n+1)^2} = 2 - \frac{n^2+2n+1-n}{〃} = \boxed{2 - \frac{n^2+n+1}{n(n+1)^2}}$$

$$\cancel{2} - \frac{n^2+n+1}{n(n+1)^2} \overset{?}{\leq} \cancel{2} - \frac{1}{n+1}$$

$$\frac{n^2+n+1}{n(n+1)^2} \geq - \frac{1}{n+1}$$

$$\frac{n^2+n+1}{n(n+1)^2} \geq \frac{(n+1)\, n}{(n+1)^2 \cdot n}$$

$$\frac{n^2+n+1}{n\cancel{(n+1)^2}} \geq \frac{n^2+n}{\cancel{(n+1)^2} \cdot n}$$

VERO $2 - \frac{n^2+n+1}{n(n+1)^2} \leq 2 - \frac{1}{n+1}$

DIM. che per ogni $n \geq 2$

si HA

$$\sum_{k=2}^{n}\left(1-\frac{2}{k^2}\right)=\frac{1+n}{2n}$$

PASSO BASE

$$P(2) = \text{"}\sum_{k=2}^{2}\left(1-\frac{2}{k^2}\right) = \frac{3}{4}\text{"}$$

$$1-\frac{2}{4} = \frac{3}{4} \quad \text{NON È VERIFICATO}$$

DIM. che per ogni $n \geq$

si HA

$$\sum_{k=2}^{n}\left(1-\frac{1}{k^2}\right)=\frac{1+n}{2n}$$

PASSO BASE

$$P(2) = \text{"}\sum_{k=2}^{2}\left(1-\frac{1}{k^2}\right) = \frac{3}{4}\text{"}$$

$$1-\frac{1}{4} = \frac{3}{4} \quad \text{VERIFICATO}$$

PASSO IND

$P(n)$ V

$P(n+1)$?

$$P(n+1) = \sum_{k=2}^{n+1}\left(1-\frac{1}{k^2}\right) = \frac{n+2}{2(n+1)}$$

$$\sum_{k=2}^{n+1}\left(1-\frac{1}{k^2}\right) = \underbrace{\sum_{k=2}^{n}\left(1-\frac{1}{k^2}\right)} + \left(1-\frac{1}{(n+1)^2}\right)$$

IPOTESI INDUTTIVA

$$= \frac{n+1}{2n} + 1 - \frac{1}{(n+1)^2} =$$

$$= \frac{(n+1)(n+1)^2 + 2n(n+1)^2 - 2n}{2n(n+1)^2} = \frac{(n+1)^3 + 2n(n+1)^2 - 2n}{2(n+1)^2}$$

$$= \frac{n^3+3n^2+3n+1+2n(n^2+2n+1)-2n}{2n(n+1)^2} =$$

$$= \frac{n^3+3n^2+3n+1+2n^3+4n^2+2n-2n}{2n(n+1)^2}$$

$$= \frac{3n^3+7n^2+3n+1}{2n(n+1)^2} = \frac{n+2}{2(n+1)}$$

$$\frac{3n^3+7n^2+3n+1}{n} = n+2$$

$$\frac{3n^3+7n^2+3n+1}{n} = \frac{(n+2)n}{n}$$

$$3n^3+7n^2+3n+1 = n^2+2n$$ NON è VERIFICATO

$\alpha > -1$ FISSATO

$$\forall n \geq 1 \Rightarrow (1+\alpha)^n \geq 1+\alpha n$$

PASSO BASE

$n=1$ $(1+\alpha)^1 \geq 1+\alpha \cdot 1$ VERIFICATO

$(1+\alpha)^n \geq 1+\alpha n$ VERO

$(1+\alpha)^{n+1} \geq 1+\alpha \cdot (n+1)$?

$3x \geq 3y$
$-3x \leq -3y$

$$(1+\alpha)^{n+1} = \underbrace{(1+\alpha)^n}_{\substack{\geq \\ 1+\alpha n}} \cdot (1+\alpha) \geq \boxed{(1+\alpha n)\cdot(1+\alpha)}$$

IPOTESI INDUT.

perche ho moltiplicato $(1+\alpha)$ POSITIVO ed $(1+\alpha)>0$.

$$= 1+\alpha+\alpha n+\alpha^2 n = 1+\alpha(n+1)+\alpha^2 n \geq \boxed{1+\alpha(n+1)}$$

perche $\alpha^2 n \geq 0$