26/10/2021

(ES) DIMOSTRARE per induzione

che $\boxed{\forall\, n \geq 0}$ si ha $n < 10^n$ $\qquad P(n) = \text{"}n < 10^n\text{"}$

1) passo BASE

$P(0) = \text{"}0 < 10^0\text{"}$

$= \text{"}0 < 1\text{"}$

VERO

2) PASSO INDUTTIVO

supporre che $p(n)$ è vero e dimostrare

che $p(n+1)$ è VERO

$p(n) = \text{"}\underline{n < 10^n}\text{"}$ VERA

$p(n+1) = \text{"}\underline{n+1 < 10^{n+1}}\text{"}$? VERO

SOSTITUITO

$n + 1 < \underline{10}^n + 1 < 10^n + 9 \leq 10^n + 9 \cdot 10^n = 10^{n+1}$

IPOTESI INDUTTIVA

$10^{n+1} = 10^n \cdot 10$

$= 10^n + 9 \cdot 10^n$

$10x = \underbrace{x + x + \ldots + x}_{10}$

$=$

$x + 9x$

$10 \cdot (\underbrace{10^n}_{x}) = \underbrace{10^n}_{x} + 9 \cdot (\underbrace{10^n}_{x})$

DIM

$\forall\, n \geq 1$ si ha

$$\sum_{i=1}^{n} (2i - 1) = n^2$$

$p(n) = \text{"}\sum_{i=1}^{n} (2i-1) = n^2\text{"}$

1) P.B. $n = 1$

$p(1) = \text{"}\sum_{i=1}^{1} (2i-1) = 1^2\text{"} = \text{"}(2 \cdot 1) - 1 = 1^2\text{"}$

$\text{"}1 = 1\text{"}$ VERO

2) P.I. $n+1$

$p(n) = \text{"}\sum_{i=1}^{n} (2i-1) = n^2\text{"}$ VERO

$p(n+1) = \text{"}\sum_{i=1}^{n+1} (2i-1) = (n+1)^2\text{"}$? VERA

$$\boxed{\sum_{i=1}^{n+1} (2i-1)} = \underbrace{\sum_{i=1}^{n} (2i-1)}_{} + \underbrace{\sum_{i=n+1}^{n+1} (2i-1)}_{}$$

IPOTESI INDUTTIVA

$= n^2 + 2(n+1) - 1 =$

$= n^2 + 2n + 2 - 1 = n^2 + 2n + 1 = \boxed{(n+1)^2}$

DETERMINARE

$$\sum_{i=2}^{n} \frac{i^2-3i+1}{6}$$

$$\sum_{i=1}^{n} 1 = n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} \frac{i^2-3i+1}{6} = \frac{i^2}{6} - \frac{1}{2}i + \frac{1}{6}$$

$$= \sum_{i=1}^{n} \frac{i^2}{6} - \sum_{i=1}^{n} \frac{1}{2}i + \sum_{i=1}^{n} \frac{1}{6}$$

$$= \frac{1}{6}\underbrace{\sum_{i=1}^{n} i^2}_{} - \frac{1}{2}\underbrace{\sum_{i=1}^{n} i}_{} + \frac{1}{6}\underbrace{\sum_{i=1}^{n} 1}_{}$$

$$= \frac{1}{6}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{1}{2}\cdot\frac{n(n+1)}{2} + \frac{1}{6}\cdot n =$$

$$= \frac{n(n+1)(2n+1)}{36} - \frac{n(n+1)}{4} + \frac{n}{6} =$$

$$= \frac{n(n+1)(2n+1) - 9n(n+1) + 6n}{36} =$$

$$= \frac{(n^2+n)(2n+1) - 9n^2 - 9n + 6n}{36}$$

$$= \frac{2n^3 + n^2 + 2n^2 + n - 9n^2 - 9n + 6n}{36} =$$

$$= \frac{2n^3 - 6n^2 - 2n}{36} =$$ DIVIDO 2

$$= \frac{n^3 - 3n^2 - n}{18} = \sum_{i=1}^{n} \frac{i^2-3i+1}{6}$$

$$\sum_{i=2}^{n} \frac{i^2-3i+1}{6} = \sum_{i=1}^{n} \frac{i^2-3i+1}{6} - \underbrace{\frac{1-3+1}{6}}_{} =$$

TOGLI $i=1$

$$= \frac{n^3-3n^2-n}{18} + \frac{1}{6}$$

DIM $n \geq 2$

$$\sum_{i=2}^{n} \frac{i^2-3i-1}{6} = \frac{n^3-3n^2-n+3}{18} =$$

$$P(n) = \text{“} \sum_{i=2}^{n} \frac{i^2-3i+1}{6} = \frac{n^3-3n^2-n+3}{18} \text{”}$$

P.B. $n=2$

$$P(2) = \sum_{i=2}^{2} \frac{i^2-3i+1}{6} = \frac{2^3-3(2)^2-2+3}{18}$$

$$= \frac{4-3\cdot 2+1}{6} = \frac{8-12-2+3}{18}$$

$$= -\frac{1}{6} = -\frac{3}{18}$$ VERO

P.I. $P(n+1)$

$$P(n+1) = \underbrace{\sum_{i=2}^{n+1} \frac{i^2-3i+1}{6}}_{} = \underbrace{\frac{(n+1)^3-3(n+1)^2-(n+1)+3}{18}}_{}$$

$$\sum_{i=2}^{n+1} \frac{i^2-3i+1}{6} = \underbrace{\sum_{i=2}^{n} \frac{i^2-3i+1}{6}}_{} + \sum_{i=n+1}^{n+1} \frac{i^2-3i+1}{6}$$

$$= \frac{n^3-3n^2-n+3}{18} + \frac{(n+1)^2-3(n+1)+1}{6} =$$

$$= \frac{n^3-3n^2-n+3+3(n+1)^2-9(n+1)+3}{18} =$$

$$= n^3 \cancel{-3n^2} - n \cancel{+3} \cancel{+3n^2} + 6n \cancel{+3} \cancel{-9}n - 3 \cancel{+3}$$

$$= \frac{n^3-6n}{18} \stackrel{?}{=} \frac{(n+1)^3-3(n+1)^2-(n+1)+3}{18}$$

$$= \frac{n^3 \cancel{+3n^2} + 3n \cancel{+1} \cancel{-3n^2} - 6n \cancel{-3} - n \cancel{-1} \cancel{+3}}{18}$$

$$= \frac{n^3-6n}{18} = \frac{n^3-6n}{18}$$

DIM. per IND

che dato $\alpha \in (0,1)$ si ha $\forall n \geq 1 \Rightarrow (1-\alpha)^n < \frac{1}{1+n\alpha}$

$$P(n) = (1-\alpha)^n < \frac{1}{1+n\alpha}$$

PASSO BASE $n=1$

$$P(1) = (1-\alpha)^1 < \frac{1}{1+\alpha}$$

$$1-\alpha < \frac{1}{1+\alpha}$$

$$\frac{(1-\alpha)(1+\alpha)}{1+\alpha} < \frac{1}{1+\alpha}$$

$$\frac{1-\alpha^2-1}{1+\alpha} < 0 \Rightarrow \frac{\overbrace{-\alpha^2}^{<0}}{\underbrace{1+\alpha}_{>0}} < 0$$

È MINORE di 0

PASSO INDUT.

$$P(n) = (1-\alpha)^n < \frac{1}{1+n\alpha}$$

$$P(n+1) = (1-\alpha)^{n+1} < \frac{1}{1+\alpha(n+1)}$$

essendo che $\alpha \in (0,1)$ È VERO

$$(1-\alpha)^{n+1} = \underbrace{(1-\alpha)^n}_{\wedge\ \frac{1}{1+\alpha n}} \cdot (1-\alpha) < \frac{1-\alpha}{1+\alpha n} < \frac{1}{1+\alpha(n+1)}$$

VERO

PRODOTTO

$2\cdot 3 < 5 \cdot 3$

$2\cdot(-3) > 5\cdot(-3$

$$\frac{1+\alpha}{1+\alpha n} < \frac{1}{1+\alpha(n+1)} \quad ?$$

$$\frac{(1-\alpha)(1+\alpha(n+1)) - 1\cdot(1+\alpha n)}{(1+\alpha n)(1+\alpha(n+1))} < 0$$

$$\frac{1+\alpha(n+1)-\alpha-\alpha^2(n+1)-1-\alpha n}{} < 0$$

$$\frac{\cancel{1}+\cancel{\alpha n}\cancel{+\alpha}\cancel{-\alpha}-\alpha^2(n+1)\cancel{-1}\cancel{-\alpha n}}{} < 0$$

$$\frac{\underbrace{-\alpha^2(n+1)}_{<0}}{\underbrace{(1+\alpha n)(1+\alpha(n+1))}_{>0}} < 0 \quad \text{VERO}$$

ES da fare

$\alpha > -1 \quad \forall n \geq 1 \Rightarrow (1+\alpha)^n \geq (1+n\alpha)$

per INDUZIONE