$f: A \subset \mathbb{R} \to \mathbb{R} \qquad f \sim g \Leftrightarrow f(1) = g(1)$

RIF

$f \sim f \Rightarrow f(1) = f(1) \quad \forall f \in A$

SIM $\forall f, g \in A$

$f \sim g \Rightarrow g \sim f$ per la prop. COMM

$f(1) = g(1) \Rightarrow g(1) = f(1)$ è VERA

TRANS $\forall f, g, z \in A$

$f \sim g \wedge g \sim z \Rightarrow f \sim z$

$f(1) = g(1) \wedge g(1) = z(1)$

$f(1) \cancel{-g(1) + g(1)} - z(1) = 0$

$f(1) = z(1)$

$\mathbb{Q} \qquad x \rho y \Leftrightarrow \frac{x-y}{2} \in \mathbb{Z} \qquad \{0, \pm 1, \pm 2, \pm 3\}$

RIFF

$x \rho x \Rightarrow \frac{x-x}{2} \in \mathbb{Z}$

$\frac{0}{2} \in \mathbb{Z}$

SIMM

$x \rho y \Rightarrow y \rho x$

$\frac{x-y}{2} \Rightarrow \frac{y-x}{2}$

$\frac{x-y}{2} = a \in \mathbb{Z}$

$\frac{-(x-y)}{2} = -a \in \mathbb{Z}$

TRANS

$x \rho y \wedge y \rho z \Rightarrow x \rho z$

$\frac{x-y}{2} = 0 \wedge \frac{y-z}{2} = 0 \qquad \Downarrow \frac{x-z}{2}$

$\cancel{2} \cdot \frac{x-y}{\cancel{2}} = -\frac{y-z}{\cancel{2}} \cdot \cancel{2}$

$x - y = -y + z$

$x \cancel{-y+y} - z = 0$

$\frac{x-z}{2} = \frac{0}{2} \Rightarrow \frac{x-z}{2} = 0$

OPPURE

$\frac{x-y}{2} + \frac{y-z}{2} = 0$

$\frac{x \cancel{-y+y} - z}{2} = 0$

$\frac{x-z}{2} = 0$

INS QUOZ.

$[a] = \{x \in \mathbb{Q} : x \rho a\} = \{\frac{x-a}{2} = b, b \in \mathbb{Z}\} = \{x = 2b + a, b \in \mathbb{Z}\}$

$\mathbb{Q}/\rho = \{\{2b + a : b \in \mathbb{Z}\} : a \in \mathbb{Q}\}$

$x \rho y \Leftrightarrow x \cdot y > 0 \quad \mathbb{Z} \setminus \{0\}$

$$[a] = \left\{ x \cdot a > 0 \;,\; a \in \mathbb{Z} \setminus \{0\} \right\} = \left\{ \underbrace{x \cdot a}_{b} = b > 0 \;,\; b \in \mathbb{Z} \setminus \{0\} \right\}$$

$$= \left\{ x = \frac{b}{a} \;,\; a, b \in \mathbb{Z} \setminus \{0\} \right\}$$

$x \cdot a > 0$

$b > 0$

$$\mathbb{Z} \setminus \{0\} / \rho = \left\{ \left\{ \frac{b}{a} \;;\; b > 0 \right\} : a \in \mathbb{Z} \setminus \{0\} \right\}$$