PS 1

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Problem Set 1

1. Prove that $lim_{x \to -1} 2x + 1 = -1$.

Note: For any given $\epsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - f(x_0)| < \epsilon$ whenever $|x - x_0| < \delta$.

Draft:

$$|2x + 1 - (-1)| < \epsilon$$
$$|2x + 2| < \epsilon$$
$$|2||x - (-1)| < \epsilon$$
$$|x - (-1)| < \frac{\epsilon}{|2|}$$
$$\delta \leq \frac{\epsilon}{2}$$

2. Determine all the numbers c which satisfy the conclusions of the Mean Value Theorem for the following function and graph using R with the point/s identified. $f(x) = x^3 - 4x^2 - 2x - 5$ on $[-10, 10]$.

3. Find the point c that satisfies the mean value theorem for integrals on the interval $[-1, 1]$. The function is $f(x) = 2e^x$.

4. Consider the function $f(x) = \cos(x/2)$. a Find the fourth Taylor polynomial for f at $x = \pi$. b Use the fourth Taylor polynomial to approximate $\cos(\pi/2)$. c Use the fourth Taylor polynomial to bound the error.

5. If $fl(x)$ is the machine approximated number of a real number x and ϵ is the corresponding relative error, then show that $fl(x) = (1 - \epsilon)x$.

6. For the following numbers x and their corresponding approximations x_A, find the number of significant digits in x_A with respect to x and find the relative error.

a. $x = 451.01, x_A = 451.023$
b. $x = -0.04518, x_A = -0.045113$
c. $x = 23.4604, x_A = 23.4213$

7. Find the condition number for the following functions

a. $f(x) = 2x^2$
b. $f(x) = 2\pi^x$
c. $f(x) = 2b^x$

8. Determine if the following series converges or diverges. If it converges determine its sum.

$$\sum_{n=1}^{\infty} \frac{1}{2^n}$$