# PS 1

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## Problem Set 1

1. Prove that $lim_{x \to -1} 2x + 1 = -1$.

Note: For any given $\epsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - f(x_0)| < \epsilon$ whenever $|x - x_0| < \delta$.

Draft:

$$
\begin{gathered}
|2x + 1 - (-1)| < \epsilon \\
|2x + 2| < \epsilon \\
|2||x - (-1)| < \epsilon \\
|x - (-1)| < \frac{\epsilon}{|2|} \\
\delta \leq \frac{\epsilon}{2}
\end{gathered}
$$

2. Determine all the numbers $c$ which satisfy the conclusions of the Mean Value Theorem for the following function and graph using R with the point/s identified. $f(x) = x^3 - 4x^2 - 2x - 5$ on $[-10, 10]$.
3. Find the point $c$ that satisfies the mean value theorem for integrals on the interval $[-1, 1]$. The function is $f(x) = 2e^x$.
4. Consider the function $f(x) = \cos(x/2)$. a Find the fourth Taylor polynomial for $f$ at $x = \pi$. b Use the fourth Taylor polynomial to approximate $\cos(\pi/2)$. c Use the fourth Taylor polynomial to bound the error.
5. If $fl(x)$ is the machine approximated number of a real number $x$ and $\epsilon$ is the corresponding relative error, then show that $fl(x) = (1 - \epsilon)x$.
6. For the following numbers $x$ and their corresponding approximations $x_A$, find the number of significant digits in $x_A$ with respect to $x$ and find the relative error.

a. $x = 451.01, x_A = 451.023$
b. $x = -0.04518, x_A = -0.045113$
c. $x = 23.4604, x_A = 23.4213$

Note:

Relative error

$E(x_A) = x - x_A$ $E_r(x_A) = \frac{E(x_A)}{x}$

Significant Digits

$|x - x_A| \leq \frac{1}{2}\beta^{s-r+1}$

Draft:

a. $x = 451.01, x_A = 451.023$

Relative Error:

$E(x_a) = 451.01 - 451.023 = -0.013$

$E_r(x_a) = \frac{-0.013}{451.01} = -0.00002882419$

Significant Digits:

$|x - x_A| = 0.013 < 0.5 = \frac{1}{2} \times 10^0$. But $10^2 < 451.01$. Therefore $s = 2$ and $r = 6$

$0.013 \le \frac{1}{2}\beta^{2-6+1} = \frac{1}{2} \times 10^{-3} = 0.0005$

b. $x = -0.04518, x_A = -0.045113$

Relative Error:

$E(x_a) = -0.04518 - (-0.045113) = -0.000067$

$E_r(x_a) = \frac{-0.000067}{-0.04518} = 0.00148295706$

Significant Digits:

$|x - x_A| = 0.000067 < 0.0005 = \frac{1}{2} \times 10^{-3}$. But $10^{-2} < 0.04518$. Therefore $s = -2$ and $r = 5$

$0.000067 \le \frac{1}{2}\beta^{-2-5+1} = \frac{1}{2} \times 10^{-6} = 0.0000005$

c. $x = 23.4604, x_A = 23.4213$

Relative Error:

$E(x_a) = 23.4604 - 23.4213 = 0.0391$

$E_r(x_a) = \frac{0.0391}{23.4604} = 0.00166663824$

Significant Digits:

$|x - x_A| = 0.0391 < 0.5 = \frac{1}{2} \times 10^0$. But $10^1 < 23.4213$. Therefore $s = 1$ and $r = 6$

$0.0391 \le \frac{1}{2}\beta^{1-6+1} = \frac{1}{2} \times 10^{-4} = 0.00005$

7. Find the condition number for the following functions

a. $f(x) = 2x^2$
b. $f(x) = 2\pi^x$
c. $f(x) = 2b^x$

8. Determine if the following series converges or diverges. If it converges determine its sum.

$$\sum_{n=1}^{\infty} \frac{1}{2^n}$$