Homework 08

MO412 - Network Science

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(Star Network) Consider a star network, where a single node is connected to $N-1$ nodes of degree one. Assume that N is much larger than 1.

(a) **What is the degree distribution p_k of this network?**

$$p_k = \begin{cases} \frac{N-1}{N}, & \text{if } k = 1 \\ \frac{1}{N}, & \text{if } k = N-1 \\ 0, & \text{other cases} \end{cases}$$

(b) **What is the probability q_k that moving along a randomly chosen link we find at its end a node with degree k?**
Remember that

$$\boxed{q_k = \frac{kp_k}{\langle k \rangle}}$$

So we need to obtain $\langle k \rangle$.

$$\langle k \rangle = \frac{2\langle L \rangle}{N}$$

$$\langle L \rangle = N - 1$$

$$\langle k \rangle = \frac{2(N-1)}{N}$$

Finally

$$q_k = \frac{kp_k}{\frac{2(N-1)}{N}}$$

We need to evaluate the three cases for p_k:

- Case: when $k = 1$, $p_k = \frac{N-1}{N}$

$$q_k = \frac{kp_k}{\frac{2(N-1)}{N}}$$

$$q_k = \frac{1\frac{N-1}{N}}{\frac{2(N-1)}{N}}$$

$$\boxed{q_k = \frac{1}{2}}$$

- Case: when $k = N-1$, $p_k = \frac{1}{N}$

$$q_k = \frac{kp_k}{\frac{2(N-1)}{N}}$$

$$q_k = \frac{(N-1)\frac{1}{N}}{\frac{2(N-1)}{N}}$$

$$\boxed{q_k = \frac{1}{2}}$$

- Case: Other cases $p_k = 0$

$$q_k = \frac{kp_k}{\frac{2(N-1)}{N}}$$

$$q_k = \frac{k * 0}{\frac{2(N-1)}{N}}$$

$$\boxed{q_k = 0}$$

(c) **Calculate the degree correlation coefficient r for this network.**

$$r = \sum_{jk} \frac{jk(e_{jk} - q_j q_k)}{\sigma^2}$$

$$\sigma^2 = \sum_k k^2 q_k - \left[\sum_k k q_k\right]^2$$

We will solve σ^2 first:
When is $k = 1$ and $k = N-1$, $q_k = 0.5$ as we saw in the previous exercise

$$\sigma^2 = \left[1^2 * 0.5 + (N-1)^2 * 0.5\right] - \left[1 * 0.5 + (N-1)0.5\right]^2$$

When k takes other values $q_k = 0$, for example $k = 4$

$$k^2 q_k = 4^2 * 0$$

$$k q_k = 4 * 0$$

these multiplications are equal to zero.

$$\sigma^2 = \left[1^2 * 0.5 + (N-1)^2 * 0.5\right] - \left[1 * 0.5 + (N-1)0.5\right]^2$$
$$\sigma^2 = \left[0.5 + (N^2 - 2N + 1) * 0.5\right] - \left[0.5 + 0.5N - 0.5\right]^2$$
$$\sigma^2 = \left[0.5 + 0.5N^2 - N + 0.5\right] - 0.25N^2$$
$$\sigma^2 = 0.5N^2 - N + 1 - 0.25N^2$$
$$\sigma^2 = 0.25N^2 - N + 1$$
$$\sigma^2 = (0.5N - 1)^2$$

Now we are going to solve r

$$r = \sum_{jk} \frac{jk(e_{jk} - q_j q_k)}{\sigma^2}$$

When is $k = 1$ and $k = N - 1$, $q_k = 0.5$, we can take $j = 1$ and $k = N - 1$ or vice versa. In other cases the multiplication will be zero.

$$r = \frac{1 * (N-1)(1 - 0.5 * 0.5)}{(0.5N - 1)^2}$$

$$\boxed{r = \frac{(N-1)(0.75)}{(0.5N - 1)^2}}$$

(d) **Is this network assortative or disassortative? How can we tell?**

This network is assortative. As long as N is bigger r tends to zero, but it is greater than zero.