Majorana fermions - concepts and applications

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Introduction

In the centenary of quantum mechanics and in the age of digital life, in these disruptive times, a transformation similar to the passage from the steam engine to the industrial revolution takes place, that is, the change from classical computing (Boolean algebra, bits) to quantum computing (quantum algebra, qubits).In this scene one of the main characters is an entity proposed by Ettore Majorana in 1937, the commonly called Majorana Fermions.

1 Concepts

A Majorana fermion is a particle that has the strange property of constituting its own antiparticle. In formal second quantization language of Quantum Mechanics, the operator that creates a Majorana fermion is identical to the operator that annihilates that fermion. In the concept of Etore Majorana, such particles would be another type of elementary particle existing in nature.

However, to date, there is no proof of the existence of such particles, although it is speculated that neutrinos may be Majorana fermions. Recently in Condensed Matter Physics Majorana fermions have been studied. However, in this context Majorana fermions do not constitute any elementary particles like those predicted by Etore Majorana. What we have in are just electrons and "holes" whose combinations of states (which we call excitations or quasi-particles) can have properties similar to those of Majorana fermions.

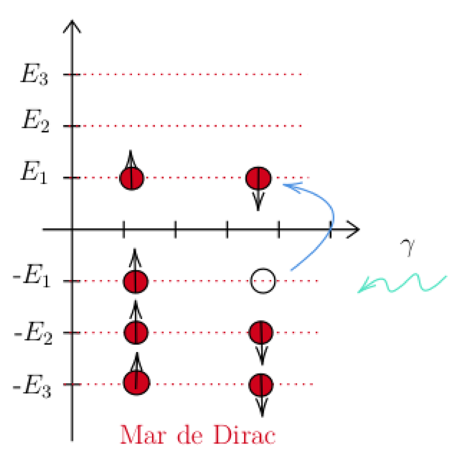

Figura 1: Mar dr Dirac.

Dirac's hole theory. When a photon excites the Dirac Sea, an electron is promoted leaving a hole behind, which behaves like a particle with same properties as electron, but with positive charge.

2 Mathematical Section

Dirac sought to find a covariant relativistic equation for the Schrödinger wave equation,

$$i\frac{\partial\psi}{\partial t} = H\psi, \tag{1}$$

$$E^2 = p^2c^2 + m^2c^4, \tag{2}$$

Dirac proposed the following Hamiltonian:

$$H = \vec{\alpha}\cdot\vec{p} + \beta m, \tag{3}$$

with α and β

$$i\frac{\partial\Psi}{\partial t} = [\vec{\alpha}\cdot(-i\nabla) + \beta m]\Psi. \tag{4}$$

Using the dispersion relation, we find

$$H^2\Psi = [p^2 + m^2]\Psi. \tag{5}$$

$$(\vec{\alpha}\cdot\vec{p} + \beta m)(\vec{\alpha}\cdot\vec{p} + \beta m)\Psi = [p^2 + m^2]\Psi. \tag{6}$$

FM is the only fermionic particle that is its own antiparticle. Their dynamic correspondence is determined by the Majorana equation, where the spinorial field is by definition equal to its charge-conjugate field.

Generally speaking, spinors ψ and the basis of Clifford's algebra are complex objects as shown in Dirac's γ matrices. However, it is possible to propose real spinors that satisfy the DE using purely imaginary objects in the choice of matrices γ.

$$(i\gamma^\mu\partial_\mu - m)\Psi = 0. \tag{7}$$

$$(\psi_R)^{(c)} = \psi_L, \tag{8}$$

$$\psi^{(C)} = C\psi^*. \tag{9}$$

where C is a 4×4 unit matrix that satisfies $C^\dagger\gamma^\mu C = -(\gamma^\mu)^*$ with ψ being the Majorana spinor, and L e R, representing the helicity of spinors.

3 Conclusions and Future Prospects

Based on Dirac's theory, Ettore Majorana proposes the existence of fermions in which the particle is equivalent to the antiparticle. With this premise, we observe that the charge conjugation of the Dirac theory needs to be reformulated so that the spin field is equal to the conjugate charge field.

In fact, this characteristic of Majorana fermions generates a series of applications in several branches of physics, for example, in High Energy Physics (in the study of neutrinos); as well as in Condensed Matter Physics (in the study of quantum wires).

We hope as future work perspectives to develop applied studies on the importance and description of Majorana fermions for neutrinos as well as for quantum wires, among others.

4 References

[1] https://repositorio.ufc.br/handle/riufc/63106

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