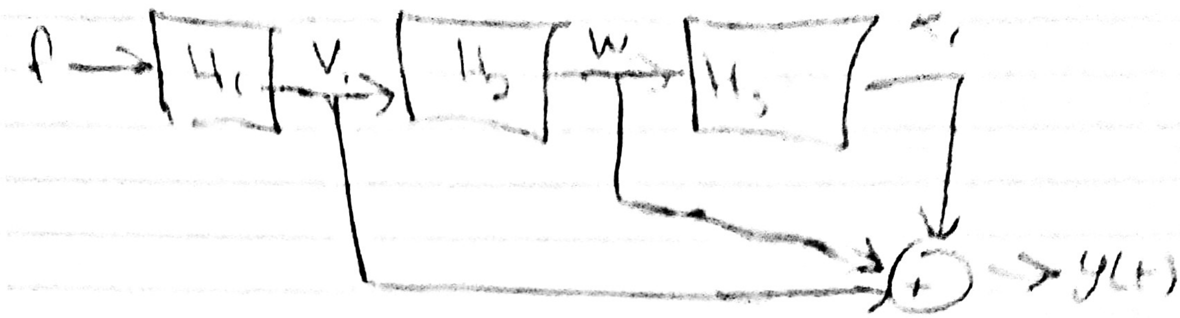
Miguel Flores
ECE450
Final Exam

Problem 1



$x_1 = \frac{1}{s+5} w \quad x_1(s+5) = w$

$\boxed{\dot{x}_1 = w - 5x_1} = x_3 + 4x_2 - 5x_1$

$w = \frac{s+4}{s^2+3s+9} v \quad w_0 = \frac{1}{s^2+3s+9} v$

$\dot{x}_3 = \ddot{w}_0 = -3\dot{w}_0 - 9w_0 + v$

$x_2 = w_0$

$x_3 = \dot{w}_0$

$\dot{x}_2 = \dot{w}_0 = -3\dot{w}_0 - 9w_0 + v = -3x_3 - 9x_2 + v$

$w = \dot{w}_0 + 4w_0 = x_3 + 4x_2$

$v = \frac{1}{s+20} f \quad \dot{v} = -20v + f$

$x_4 = v$

$\boxed{\dot{x}_4 = -20x_4 + f} \quad \dot{x}_3 = -3x_3 - 9x_2 + x_4 + f$

$\boxed{y = v + w + x_1 = x_4 + x_3 + 4x_2 + x_1}$

$$A = \begin{bmatrix} -5 & 4 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -9 & -3 & 1 \\ 0 & 0 & 0 & -20 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 4 & 1 & 1 \end{bmatrix} \qquad D = 0$$

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Problem 2

$$H(s) = \frac{5\times10^5}{s^2 + 200s + 10^4}$$

$\omega_{cg} \approx 700$ rad/s

The phase margin is at 180-165 for $G_c(s)=1$

$pm = 180 - 165 = 15°$

Design a phase lead..

So adding 40°, to that...

$\phi_m = 40°$

$$\alpha = \frac{1+\sin\phi_m}{1-\sin\phi_m} = 4.6$$

$-10\log(4.6) = -6.6\,dB \rightarrow \approx \omega_m = 900$ rad/s

$\omega_p = \omega_m\sqrt{\alpha} = 1930$

$\omega_z = \frac{\omega_m}{\sqrt{\alpha}} = 420$

$$G_c(s) = \frac{\omega_p}{\omega_z} \cdot \frac{s+\omega_z}{s+\omega_p} = \boxed{\frac{1930}{420} \cdot \frac{s+420}{s+1980}}$$

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$\boxed{\text{Problems 3}}$

Design a Butterworth w/: $n=2$
$\omega_p = 500$
$|H(\omega)| \geq .92 \quad \omega \geq \omega_p$

$n=2$ Butterworth:

$$H(s) = \frac{1}{s^2+1.41s+1}$$

Transforming low to high pass: $s \to \frac{1}{s}$

$$H(s) = \frac{s^2}{s^2+1.41s+1}$$

Solving for ω_s: $\frac{1}{1+(\omega_s)^{2n}} = |H_s|^2$

$\cancel{(\omega_s)^{2n} = \frac{1}{H_s^2} - 1 \quad \omega_s = 3.13}$ H_s is not known.

solving for ω_p: $\frac{1}{1+(\omega_p)^{2n}} = |H_p|^2$

$\omega_p = .652 \leftarrow$ to scale to 500, multiply by 766

Scaling $H_{Hp}(s) \to s = \frac{s}{766}$

$$\boxed{H(s) = \frac{s^2}{s^2+1080s+586756}}$$

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Problem 4

$$H(S) = \frac{7\times10^4}{S^2 + 800S + 7\times10^4}$$

$$H(S) = \frac{7\times10^4}{(S+100)(S+700)} = \frac{A}{S+100} + \frac{B}{S+700}$$

$$7\times10^4 = A(S+700) + B(S+100)$$
$$A = 117 \quad B = -117$$

$$H(S) = \frac{117}{S+100} + \frac{-117}{S+700}$$

Transforming to time domain...

$$H(t) = (117e^{-100t} - 117e^{-700t})u(t)$$

Transforming to discrete domain...

$$H[k] = (117e^{-100Tk} - 117e^{-700Tk})u[k]$$

Transforming to z domain...

$$H(z) = \frac{117z}{z - e^{-100T}} - \frac{117z}{z - e^{-700T}}$$

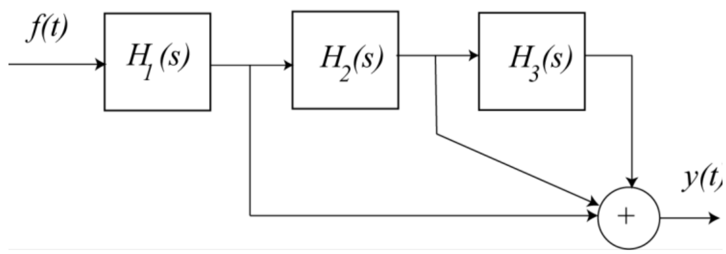
$$T = .0001$$

$$\boxed{H(z) = \frac{117z}{z - e^{-.01}} - \frac{117z}{z - e^{-.07}}}$$

Final Exam
ECE 450, Fall 2020
Two Hours
Open book

Printed Name: Miguel Mares

1. (25 points) Write the state space matrices **A, B, C** and **D,** for the system described by the block diagram. The input is *f(t)* and the output is *y(t).*



$$H_1(s) = \frac{1}{s+20}, \quad H_2(s) = \frac{s+4}{s^2+3s+9}, \quad H_3(s) = \frac{1}{s+5}$$

Hint: Choose the output of $H_3(s)$ to be x_1

A =

-5	4	1	0
0	2	0	0
0	-9	-3	1
0	0	0	-20

B =

0
0
0
1

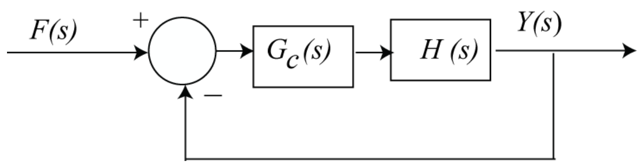
C =

1	4	1	1

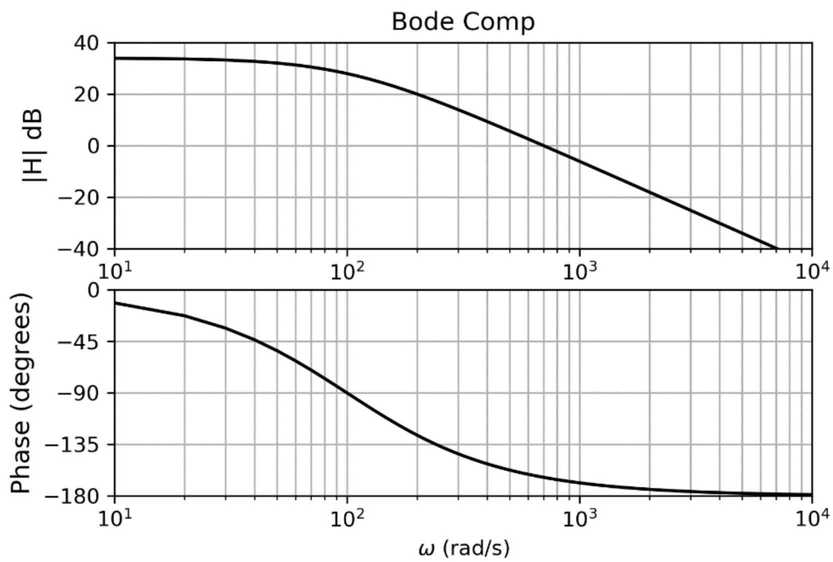
D = $\boxed{0}$

2. (25 points) A system *H(s)* is placed in a feedback loop as shown.

$$H(s) = \frac{5\times10^5}{s^2 + 200s + 10^4}$$



When $G_c(s) = 1$, the system has the following Bode plot.



Suggest a compensation network, $G_c(s)$, that results in a phase margin of about 50 degrees without altering the low frequency gain.

$$\boxed{G_c(s) = \frac{1930}{420} * \frac{s + 420}{s + 1930}}$$

3. (25 points) Design at 2-pole, high pass, Butterworth analog filter with the pass band starting at

$$\omega_p = 500\ rad/s,$$

and no more that 8 % attenuation in the pass band, i.e.,

$$|H(\omega)| \geq 0.92 \quad for\ \omega \geq \omega_p.$$

$$\boxed{H(s) = \frac{s^2}{s^2 + 1080s + 586756}}$$

4. 25 points) A system is described by the transfer function

$$H(s) = \frac{7\times 10^4}{s^2 + 800s + 7\times 10^4}.$$

Convert this to a digital system with a sampling time of T = 0.0001 sec. Write the Z domain transfer function.

$$\boxed{H(z) = \frac{117z}{z - e^{-.01}} - \frac{117z}{z - e^{-.07}}}$$