project n°2

Investigations on the image source method…

1 Introduction

The objective of the projects proposed here is to evaluate your ability to state and to solve problems by yourself, on a topic related to the lecture Acoustics 1. The expected work is theoretical in nature, on a somewhat framed but also open subject. You are expected to work independently for a limited period of 5 weeks (note however that you can ask me for help or advice all along the project).

What prevails in the work proposed here is the ”obligation of means”, not the ”obligation of results” (as for a research work where it is not always easy to anticipate difficulties). The proposed evaluation format is a report and a 30-minutes individual interview (mainly for you to have a feedback on your work…). Beyond your knowledge of acoustics in general, you will be evaluated on skills that are those expected for a (young) researcher, e.g. :

- ⋆ ability to present concisely and clearly the approaches adopted and the results obtained,
- ⋆ ability to have a critical look at your work,
- ⋆ ability to suggest interesting future prospects,
- ⋆ etc …

The proposed scheduled is the following :

- ⋆ **Tuesday November** 10^{th}. Presentation of each subjects. Then you'll have a few days to chose one of the proposed subjects, and inform me about your choice by E-mail.
- ⋆ **November** 10^{th} → **December 18**th. All along the duration of the project, you can contact me for advice or guidelines. Please contact me by E-mail beforehand so that we take an appointement for a skype/discord/zoom meeting.
- ⋆ **Friday December 18**th. Deadline for sending me your report. The report part can take the form that suits you, as long as it clearly and effectively summarizes your work. You should send me this report as a pdf file, and you are strongly encouraged to write it with latex (ideally, it would be good if it could have the form of a research article, e.g. using the AIP template).

2 Description of the project, guidelines

The objective of this project is to investigate the image source method with applicability to guided waves (first) and next to room acoustics (in the low frequency range where the Sabine formula does not work…).

2.1 Preliminary work : propagative and evanescent waves in an infinite 2D duct…

NB : the part to be treated in this paragraph actually corresponds to a copy/paste of some part of the Matlab/python simulation tutorial labs proposed to the IMA/IMDEA students.

If one considers a string with radius a pulsating with a velocity V_0 at angular frequency ω, the complex amplitude of the acoustic pressure radiated at a distance r from the cylinder axis can be written as :

$$\tilde{p}(r) = \frac{i\rho_0 V_0 \omega}{H_1^{(2)}(ka)} H_0^{(2)}(kr)$$

where ρ_0 is the density of the fluid, $H_n^{(2)}$ is the Hankel function of the second kind and order n, and $k = \omega/c_0$ where c_0 is the adiabatic sound speed.

Now, we consider the *two-dimensional* problem presented in Fig. 1. A point source is placed in the middle of two rigid walls. Because the problem is two-dimensional, the point source is here equivalent to an infinite pulsating string in 3D-space. The distance between the two walls is 2d. We want to calculate the acoustic field $\tilde{p}(x, y)$ "far" from the source, e.g. at $x = 20d$, as a function of the frequency. To that purpose, we want to use the method of the image sources.

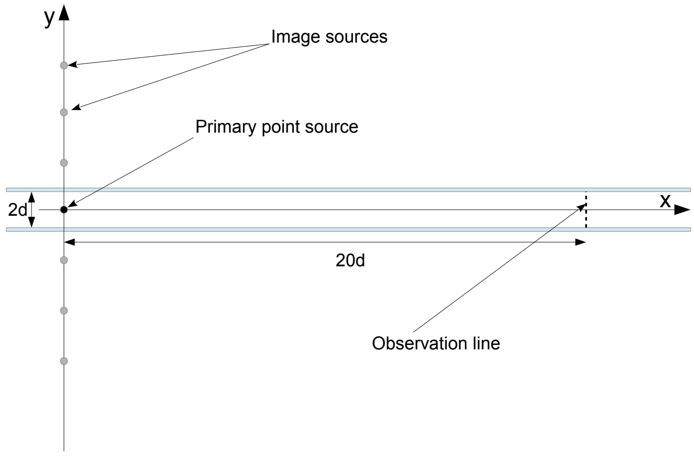

FIGURE 1 – Sketch of the problem considered

a.- Calculate the positions of the image sources, and show that the the acoustic pressure radiated at position (x, y) can be written as :

$$\tilde{p}(x, y) = \frac{i\rho_0 V_0 \omega}{H_1^{(2)}(ka)} \sum_{n=-\infty}^{n=-\infty} H_0^{(2)}(kr_n)$$

with

$$r_n = \sqrt{x^2 + (y - 2nd)^2}$$

and where V_0 and a respectively stand for the oscillating velocity and the radius of the point source.

b.- Download and execute the program main_source_image.m (and related sub-programs). Launch the program with different values of the frequency of the source (which should be chosen appropriately), and draw your own conclusions regarding the acoustic field generated along the line $(x = 20d, y)$. Are the obtained results consistent with what one should expect regarding higher order (i.e. non plane-wave) modes ?

c.- Create a new program in order to plot the pressure $\tilde{p}$ at position $(x = 20d, y = 0)$ as a function of frequency. Repeat the calculations to plot $\tilde{p}$ at position $(x = 20d, y = d/2)$. Are the obtained results consistent with what one should expect regarding higher order (i.e. non plane-wave) modes ?

d.- Create a new program to calculate the acoustic field along the line $(x = 20d, y)$ if the primary source is placed at positions $(x_0 = 0, y_0)$ where y_0 is an arbitrary position between 0 and d (to that purpose you need to calculate the new positions of the image sources). Launch your program with different values of the frequency of the source positioned (for instance) at $(x_0 = 0, y_0 = d/2)$, and draw your own conclusions regarding the acoustic field generated along the line $(x = 20d, y)$. Are the obtained results consistent with what one should expect regarding higher order (i.e. non plane-wave) modes ?

2.2 Image source method and room acoustics...

Now, the device we are considering is the one schematically presented in Fig. 2 : it actually corresponds to the problem treated at the end of lecture 4, where the method of modal analysis is employed to calculate the frequency response of a 3D cavity at some position (x, y, z), when a point source is placed at some position (x_0, y_0, z_0).

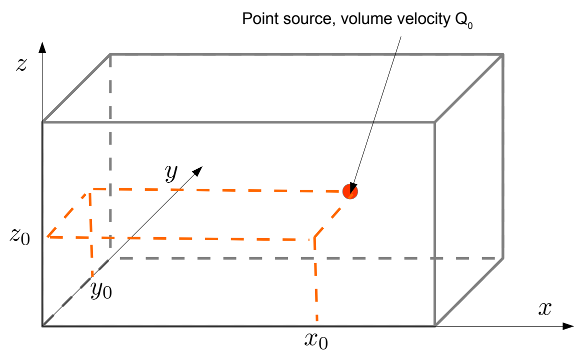

FIGURE 2 – Schematic drawing of the problem considered.

2.2.1 Acoustic field in a perfect parallelepiped...

First you can start with the problem treated in lecture 4. This problem can be treated using modal analysis, as done in lecture 4, and next you are asked to treat it (actually a simpler 2D problem using the image source method...). Let's assume, for instance [1], that the cavity has the following dimensions :

$$L_x = 6.8m, \quad L_y = 4.35m, \quad L_z = 3.47m.$$

Let's assume that the microphone is placed close to a corner, say for instance $(x, y, z) = (0.1, 0.1, 0.1)$.

- ⋆ Should a point source be placed at some position (x_0, y_0, z_0), write the general solution for the pressure measured at position (x, y, z) in terms of a modal expansion.
- ⋆ Now assume that the point source is placed at position :

$$x_0 = L_x - 0.01, \quad y_0 = L_y/2 \quad z_0 = L_z/2.$$

1. a random choice...

Write a program enabling to calculate the acoustic pressure at position (x, y, z) as a function of the frequency of the point source, and plot the predicted sound pressure level (dB SPL) as a function of the frequency, say within the frequency range from 20 Hz up to 200 Hz (or another frequency range if you think it is better).

⋆ Now assume that the point source is placed at position :

$$x_0 = L_x - 0.01, \quad y_0 = L_y/2 \quad z_0 = L_z/(2.05).$$

Calculate the new frequency response of the system, and discuss the differences with the previous case considered. . .

2.2.2 Acoustic field in an imperfect parallelepiped.

Now let's consider the 3D cavity of Fig. 3. We assume that the wall at position $x = 0$ is not perfectly perpendicular to the other ones. This means that the boundary condition at position $x = 0$ should now be written as

$$\partial_n \left. \tilde{p} \right|_{x=0} \approx 0$$

with $\mathbf{n} = -\cos\theta \mathbf{e_x} + \sin\theta \mathbf{e_z}$, with $\theta \ll 1$ the small angle defined in Fig. 3.

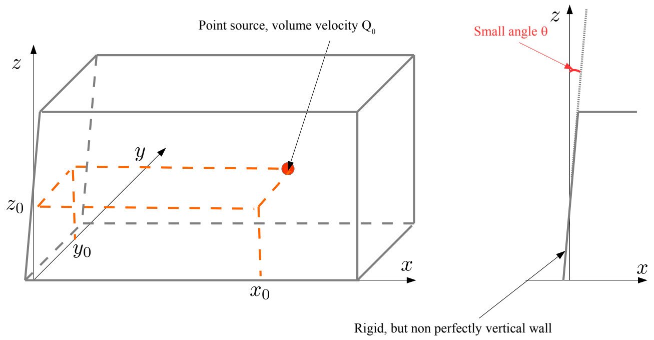

FIGURE 3 – Now the 3D cavity has one wall (at $x = 0$) which is rigid but which is *not* perfectly orthogonal to the other walls.

⇒ For this new problem, we see that a difficulty arises, which is the fact that the eigenfunctions used to treat the previous problem of of a perfect parallelepiped no longer correspond to this new problem.

⇒ My proposal here is to treat this new problem by means of an image source method, rather than a modal expansion. You are asked to investigate this approach. As you may expect, one of the key question will be how much reflexion orders should we consider so as to get a sufficiently accurate result. Another question will be that of determining the positions of the image sources.

⇒ I am not a specialist of such methods, but I found for you three references which might be useful (see the bibliography file in the UMTICE space)

⇒ It is a possibility that the 3D case be too complex, so my proposal is that you start by considering the 2D case where the point source is a pulsating string. . . You could treat both a perfectly rectangular cavity and a cavity with a (slightly) tilted wall. This way, the solution obtained by the image source method could be compared to the modal solution for the perfectly rectangular case, which would lead to an evaluation of its domain of validity within a given frequency range (and for a given number of reflections considered). Next, the case of a tilted wall could be treated by means of the image source method, and the impact of the tilted wall on the frequency response could be analyzed...

2.3 Should you have further time at disposal...

Another way to treat the problem of Fig. 3 would be to make use of the integral formulation of acoustics, where the Green function would be chosen as a modal expansion using the modes of a perfectly parallelepipedic cavity. The integral formulation would allow to account for the fact that the wall is not perfectly vertical. Green function and the integral formulation are out of the scope of the lecture Acoustics 1, but should you have time at disposal (and I don't think so...), you could try to investigate this other approach [2]

2. In this case, you can obviously ask me for help...