Acoustics I project: rotating and resonant waves in a cylindrical cavity

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This is an article describing a project done during the Winter semester of the Master 1 in Wave Physics. This article deals with rotating waves in a cylindrical cavity: a cylinder with a point source was studied and the way to generate rotating wave inside the resonator was studied analytically and numerically.

I. INTRODUCTION

II. INVESTIGATING THE NODAL LINES OF THE CYLINDRICAL CAVITY

The considered system is a cylinder having a loudspeaker at (r_0, θ_0) on its upper wall. This point source imposes a driving force q_0 oscillating at a ω frequency. The wave equation to solve in the cavity is inhomogenenous,

$$\frac{\partial \tilde{p}^2}{\partial r^2} + \frac{1}{r}\frac{\partial \tilde{p}}{\partial r} + \frac{1}{r^2}\frac{\partial \tilde{p}^2}{\partial \theta^2} + k^2 \tilde{p} = -i\omega q_0 \delta(r - r_0), \quad (1)$$

with $k = \omega / c_0$ and δ is the Dirac-Delta distribution.

$$\tilde{p}(r, \theta) = \sum_{m,n} \tilde{A}_{mn} \Psi_{mn} \quad (2)$$

Plugging the solution of the pressure field

The pressure field form is the following

$$\tilde{p}(r, \theta) = \sum_{m,n} \tilde{A}_{mn} J_m(k_{mn} r) \cos(m\theta) \quad (3)$$

We have

$$\sum_n \left[k^2 - k_{mn}^2\right] \frac{\tilde{A}_{mn}}{2 - \delta_m^0} J_m(k_{mn} r) = -i\omega q_0 \delta(r - r_0) \quad (4)$$

The inner product to imply orthogonality on the Bessel function $J_m(k_{mn} r)$

$$\int_0^R r J_m(k_{mn} r) J_m(k_{mn'} r) dr = \begin{cases} \frac{k_{mn}^2 R^2 - m^2}{2k_{mn}^2} J_m^2(k_{mn} R), & n = n' \\ 0, & n \neq n' \end{cases} \quad (5)$$

$$\tilde{A}_{mn} = \begin{cases} \frac{2 - \delta_m^0}{k_{mn}^2 - k^2} i\omega q_0 \Psi_{m'}(r_0), & m \neq m' \end{cases} \quad (6)$$

[1] P. H. Ceperley, Rotating waves, (1992).
[2] A. O. Santillán and K. Volke-Sepúlveda, A demonstration of rotating sound waves in free space and the transfer of their angular momentum to matter, (2009).