AIP | Review of Scientific Instruments

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Citation: Review of Scientific Instruments **66**, 256 (1995); doi: 10.1063/1.1145268
View online: http://dx.doi.org/10.1063/1.1145268
View Table of Contents: http://scitation.aip.org/content/aip/journal/rsi/66/1?ver=pdfcov
Published by the AIP Publishing

Tuning a rotating wave resonator

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(Received 15 October 1993; accepted for publication 16 September 1994)

Creation of a rotating wave field in a high-Q resonator usually requires the resonator to be tuned to compensate for manufacturing errors. The tuning of a rotating wave resonator is more complicated than that of a common resonator. A theory of tuning rotating wave resonators and a procedure for efficiently carrying out this tuning is presented in this paper, along with the authors' experience in tuning a rotating TM_{110} mode in a 1.28 GHz microwave resonator. © *1995 American Institute of Physics.*

I. ROTATING WAVES

In resonators having cylindrical symmetry, all standing wave modes for which the azimuthal index is greater than zero are "doubly degenerate".[1] For example, Figs. 1(a) and 1(b) show two TM_{110} modes in a cylindrical microwave resonator which resonate at the same frequency and have the same arrangement of fields, except that one has its fields orientated 90° from the other. If we use the fact that these two modes can be independently excited and drive one with an oscillating source that is delayed in time by one quarter of the oscillation period (a 90° phase shift) as compared with the source driving the other, then we will create a rotating wave[2,3] in the resonator, as shown in Fig. 1(c). Whereas a simple standing wave has fixed nodal planes where the field is zero and regions between the nodal planes where the field is oscillating, a rotating wave field has a fixed field profile that rotates in time around the cylindrical axis of the resonator. Thus, it has a field profile of a traveling wave, however with the distinct resonant modes that characterize standing wave fields. Figure 2 shows a surface plot of the z-directed electric field of another rotating wave field (TM_{520}), this one having many more peaks and valleys than the lower order one of Fig. 1.

Rotating wave fields have a variety of potential applications in acoustics,[4] optics,[5] and electromagnetic[6] field areas. The language and examples in this paper are drawn from the microwave area but the concepts involved with tuning rotating wave modes apply to the acoustical and optical wave fields as well.

II. THEORY OF TUNING ROTATING MODES

In actual practice, most resonators do not have perfect cylindrical symmetry. This problem is particularly evident in low loss, high-Q resonators. Figure 3 shows a typical response spectrum of such a resonator, where because of manufacturing errors, the two standing wave modes which would be degenerate in an ideal cylindrical resonator, resonate at slightly different frequencies. If the two resonances are distinctly separated, then it is impossible to excite the two resonances with a single-frequency source and produce a rotating wave field. In this case it is necessary to tune the resonator to re-establish the degeneracy and allow the excitation of rotating waves.

One might expect to tune the resonator with two tuners, one to tune each mode. But, in reality, tuning rotating modes is not such an easy process. One complicating factor is that when the degeneracy is removed, the orientations of the modes are set, not by the exciting sources, but by the perturbations that are responsible for the frequency splitting. The effectiveness of the probes and tuners, both of which are at fixed azimuthal angles, changes with the orientation of the modes and changes as one tunes the resonator. This can be quite unsettling for a person who is used to simple resonances where these can be relied upon to be constant.

Tuning is normally accomplished by perturbing the oscillating fields inside a resonator, often by deforming the walls of the resonator or by pushing slugs of metal or dielectric into the resonator. Similarly, the detuning we would hope to correct occurs because of errors or deformations of the resonator walls modifying the fields from what they would be in a perfect cylindrically symmetric resonator. The frequency shift $\delta\omega$ from either phenomenon is given by[7]

$$\delta\omega=\omega\,\frac{\text{magnetic energy changed}-\text{electric energy changed}}{\text{total energy in the resonator}}. \tag{1}$$

To simplify the discussion and emphasize the concepts of the cylindrical geometry, in this section we will assume (1) that the resonator is a right circular cylinder, (2) that the deformations and detuning occur due to the distortions of the curved walls of this cylinder, (3) that the magnetic fields are only functions of r and ϕ, and (4) that there are only magnetic fields and no electric fields at these walls to be displaced. The more general case is a straightforward application of the principles to be laid out below. With these assumptions, the magnetic fields on the cylindrical walls of the resonator for the two nearly degenerate modes are given by

$$\begin{aligned} B_1&=B_0\sin m(\phi-\alpha),\\ B_2&=B_0\cos m(\phi-\alpha), \end{aligned} \tag{2}$$

where m is the azimuthal order of the resonant mode and α is an azimuthal offset or orientation angle of the modes. Equation (2) substituted into Eq. (1) yields for the two modes:

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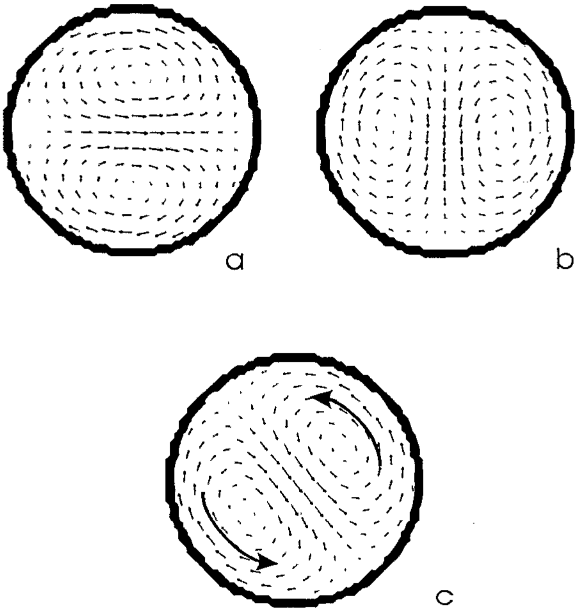

FIG. 1. Magnetic fields in a TM_{110} microwave resonator. The first two figures are the two degenerate standing wave modes, while the third is a composite of the two which is actually a rotating wave field.

$$\delta\omega_{1,2}=\omega\,\frac{\int\delta r(\phi)B_{1,2}^2 a\,d\phi}{\int B_{1,2}^2 dS}$$

$$=\frac{\omega a B_0^2}{\int B_{1,2}^2 dS}\int\delta r(\phi)\begin{pmatrix}\sin^2 m(\phi-\alpha)\\ \cos^2 m(\phi-\alpha)\end{pmatrix}d\phi, \tag{3}$$

where a is the radius of the resonator, $\delta r(\phi)$ is the distortion of the radius as a function of ϕ, the integrals in the numerators are carried out over the circumference of the resonator (ϕ=0 to 2π), and the integrals in the denominators are with respect to the circular cross section of the resonator ($dS=rdrd\phi$). Using the double angle formulas, Eq. (3) becomes

$$\delta\omega_{1,2}=A\int\delta r(\phi)d\phi\mp A\int\delta r(\phi)\cos 2m(\phi-\alpha)d\phi, \tag{4}$$

where $A\equiv\omega a B_0^2/2\int B^2 dS$, where we have used the fact that $\int B_1^2 dS=\int B_2^2 dS=\int B^2 dS$. Thus

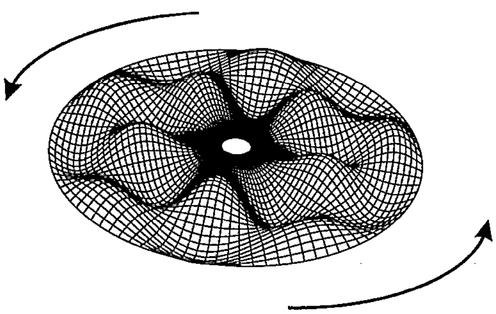

FIG. 2. A surface plot of the electric fields of a TM_{520} rotating mode in a cylindrical resonator. The height of the surface is proportional to E_z. The fields will rotate (in revolutions per second) about the center at one fifth the driving microwave frequency (in Hz, or GHz in this case).

$$\overline{\delta\omega}\equiv\frac{\delta\omega_1+\delta\omega_2}{2}=A\int\delta r(\phi)d\phi \tag{5}$$

and

$$\Delta\omega\equiv\omega_2-\omega_1=\delta\omega_2-\delta\omega_1$$

$$=2A\int\delta r(\phi)\cos 2m(\phi-\alpha)d\phi. \tag{6}$$

Therefore, the average frequency shift is proportional to the average of $\delta r(\phi)$, while the frequency separation between modes is proportional to the amplitude of the $2m$th cosine term in the Fourier series[8] expansion of $\delta r(\phi+\alpha)$. As shown in the appendix, the modes will align themselves at an orientation angle α which maximizes the frequency difference between the modes, which means maximizing the $2m$th cosine harmonic of $\delta r(\phi+\alpha)$.

The total $2m$th harmonic of $\delta r(\phi+\alpha)$ (both sine and cosine terms) can be expressed as

$$A\sin 2m\phi+B\cos 2m\phi=C\cos 2m(\phi-\theta), \tag{7}$$

where the right-hand side is an alternate expression written in terms of an amplitude C and a distortion offset or orientation angle θ, also perhaps called the quadrature angle of the distortion (A, B, and C are constants). Note that this alternate expression is written entirely in terms of cosines. Clearly, if α equals θ then the cosine harmonic of Eq. (6) becomes the entire total harmonic and is therefore maximized. Thus, the amplitude of the total $2m$th harmonic (i.e., C) of $\delta r(\phi)$ will indicate the frequency difference between the modes and the quadrature angle of the $2m$th harmonic (θ) of $\delta r(\phi)$ will equal the alignment angle α of the modes. Thus, we see that for tuning, we are interested, not in the details of how the tuning function $\delta r(\phi)$ varies with angle, but only in three numbers: The amplitude and phase (quadrature angle) of the total $2m$th harmonic of $\delta r(\phi)$ and the average value of $\delta r(\phi)$. (This dependence on the $2m$th harmonic instead of the mth harmonic is somewhat nonintuitive.) Since the normal excitation[9] of rotating waves requires the two resonant frequencies to be equal, the $2m$th harmonic of $\delta r(\phi)$ caused by manufacturing errors must be nulled out by the $2m$th harmonic of $\delta r(\phi)$ caused by the tuner. Figure 4 shows two resonators with $m=1$ and $m=2$ modes for

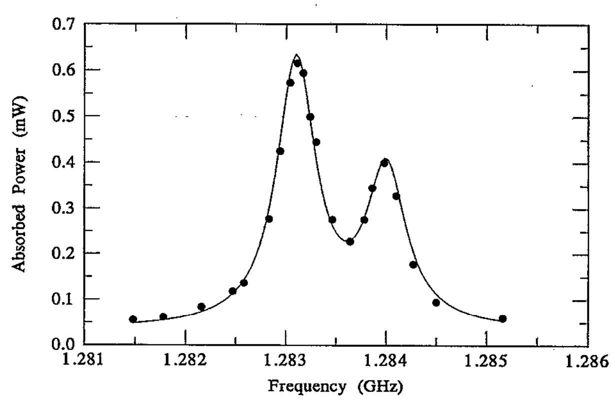

FIG. 3. The response spectra of the two nearly degenerate standing wave modes, split by manufacturing defects in the resonator.

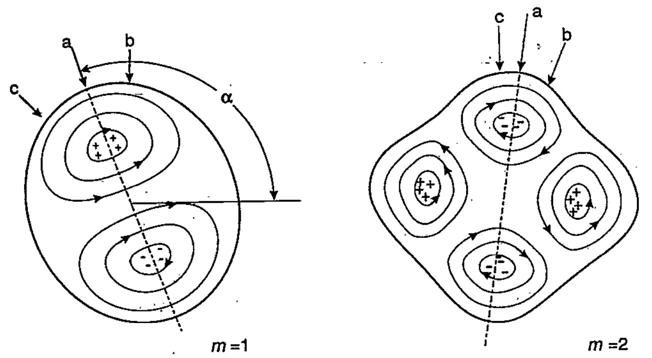

FIG. 4. Resonators with exaggerated $2m$th harmonic of wall distortion for an $m=1$ mode and for an $m=2$ mode, along with the fields of one of the two standing wave modes. The arrows show location of the tuners.

which we have eliminated all wall distortion except the $2m$th harmonics, which are exaggerated for clarity. We also show the fields of the lower frequency modes in each case. For $m=1$, the harmonic makes the wall elliptical with the long axis at an angle $\alpha=\theta$. For $m=2$, the harmonic makes the wall a four-lobed closed curve. A single tuner or compensating distortion of the wall would need to be placed at one of the ends of the major axis in the $m=1$ case or at one of the lobes in the $m=2$ case, shown by an arrow "a" in Fig. 4 in both cases. If one has fixed tuners that are not free to be applied at any arbitrary azimuthal angle, then two fixed tuners can be used at locations "b" and "c" separated by 45° in the $m=1$ case and by 22.5° in the $m=2$ case, to compensate for both the $2m$th cosine harmonic and $2m$th sine harmonic of the distortion.

As indicated in Fig. 4, when we only wish to eliminate the frequency difference and not correct the absolute, or average frequencies, and are free to locate the tuner at the optimal azimuthal angle at will, then we can tune for rotating waves with just one tuner. This might correspond to the case where we add an external clamp or otherwise externally deform or "dimple" the resonator and so can first measure the mode orientation α and then select the azimuthal angle to deform at. On the other hand, if we have fixed tuners that perhaps fit in holes made during manufacturing process, then we need to have two tuners to be able to generate an arbitrary composite amplitude and phase (or orientation) of a compensating $2m$th harmonic. In this case, we will generally locate these two tuners at azimuthal angles $\pi/2m \pm n\pi/m$ (where n is an integer) radians apart, so as to generate orthogonal $2m$th harmonic components (i.e., tune in quadrature) and so add up to a correct nulling amplitude of a $2m$th harmonic. The $\pm n\pi/m$ term in the azimuthal angle is based on the fact that the $2m$th harmonic repeats itself in this angle and so allows one the flexibility of having several equivalent places to put a tuner (or probe).

If we also wish to compensate for the average or absolute frequency error of the modes, we need an additional tuner or tuners. One possibility is to make this tuner deform the resonator in a azimuthally symmetric way to have no $2m$th harmonic and so only affect the average frequency. Another possibility is for this tuner to be similar to the first tuners, localized that is, and use it to nullout some of the $2m$th harmonic of the first tuners which are purposely set to have excessive $2m$th harmonic. Since the average tuning is all cumulative, this will allow the freedom to adjust the average frequency while keeping the $2m$th harmonic or frequency difference fixed. One might also employ totally separate opposing pairs of tuners, which null out each others' $2m$th harmonic, but since the averages add, these would give net average tuning.

Last, we need to discuss the direction of tuning. If one were to start with a tuner protruding somewhat into the fields, then moving the tuner more into, or alternately, out of the resonator would result in making the frequency larger or smaller, respectively. In this case we would have a bidirectional tuner capable of both positive and negative tuning. On the other hand, if we start with the tuners backed out of the resonator's fields and wish to push them in a minimal amount, then they will only engage the field if they are pushed in and are therefore only capable of positive tuning. In this case, if we need negative tuning, we will need another tuner that is located so that it produces negative tuning, i.e., by displacing electric fields instead of magnetic fields. In the case of needing negative $2m$th harmonic tuning, we may simply locate the other tuner $\pi/2m$ radians around the resonator from the first. Thus, if the tuners are not bidirectional and bidirectional tuning is needed, more tuners will be required.

III. HOW TO TUNE A ROTATING WAVE RESONATOR

Since we are usually not in a position to easily measure the distortion $\delta r(\phi)$, how do we adjust our tuners? The most obvious way is to play with the tuners (located as prescribed above) while monitoring the resonant frequencies and gradually converge on a tuned situation. This approach, although not elegant, should, with enough patience, work. One needs to use two probes located at different azimuthal angles to reliably sense both orthonormal modes. This is because the resonances may shift their alignment during tuning such that a node of one of the modes moves to the position of a probe, at which point that probe could not sense the mode.

A more straightforward method of tuning, suggested by Nezhevenko and Yakovlev, involves first aligning the modes and then tuning them. To do this, one uses two sets of tuners and probes, which we denote as major or minor, based on the azimuthal angle they are located at. This requires one to define or fix the azimuthal coordinate axes. We define the major probes and tuners as those that are located at $\phi=0$ or at increments of π/m, while minor probes and tuners are located $\pi/2m$ away from the possible major probe locations. This way, the major tuners are able to cancel the $2m$th cosine harmonic of the wall distortion, while the minor tuners are able to cancel the $2m$th sine harmonic. Figure 5 shows the setup we used in the experiments. Here probes p1–p4 are the major probes and tuners t1 and t2 are the major tuners. Probes p5 and p6 and tuner t3 are the minor probes and tuner shown and are set 45° to the major probes and tuner locations, because we were using an $m=1$ mode. Note however use of a mode with a larger m would result in a smaller angle between the major and minor tuners and probes. To align the

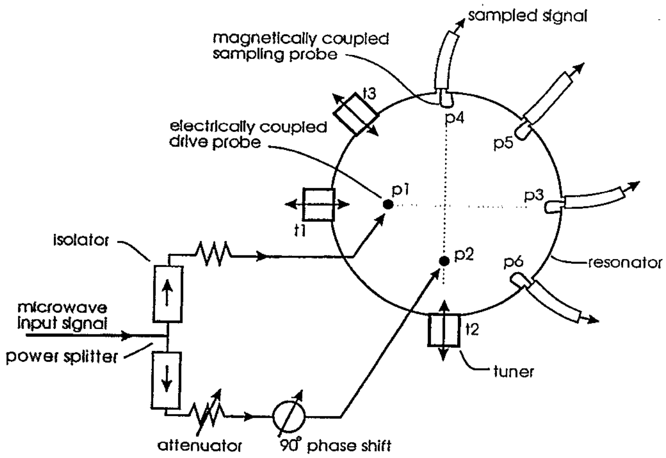

FIG. 5. Microwave electronics, probes, and tuners used in the experiment.

modes with the major probes, one adjusts the minor tuner t3 so as to make the cross coupling between the major probes p1 and p4 (set at azimuthal angles $\pi/2m$ apart) vanish, such that when a microwave signal is injected into probe p1 at a resonance (as indicated by the reflected power at p1), no signal will be detected on p4. When we eliminate the cross coupling, we are canceling the $2m$th sine harmonic of the manufacturing distortion, leaving only the $2m$th cosine harmonic, which means the quadrature angle θ of the $2m$th harmonic, will now be zero. We have seen in the previous section that the modes will self-align to make α in Eq. (2) equal to θ. Thus, α now will also equal zero, meaning that the modes will be aligned with the major tuners. In this orientation, one major tuner and the corresponding probe(s) lie in a node of one of the modes, while the other major tuner and probe(s) lie in a node of the other mode, which is the reason the cross coupling is zero. This is the maximum correction that the minor tuner can effect, and since the major tuners only add to and subtract from the $2m$th cosine harmonic, adjustment of the major tuners will have no effect on θ. It also means that adjustment of the major tuners to bring the two modes to the same frequency is very straightforward, since one mode is now tuned only by one of the major tuners and sensed only by the corresponding major probe(s), while the other mode is tuned only by the other major tuner and sensed by the corresponding other major probe(s). Thus, now we are set up to tune each mode independently in the standard fashion that simple resonators are tuned.

After the resonances of the modes are tuned to within a bandwidth of each other, more precise tuning can be accomplished by exciting a rotating wave using the microwave circuit shown in Fig. 5 to drive the two probes p1 and p2 with a 90° phase shift between them and then by adjusting the major tuners until there is a 90° phase shift in signals sampled from the cavity via separate sampling probes p3 and p4. This is important, because a resonator can have a large phase error (up to ±90°) between the driving signal and its resonant fields due to tuning errors and the production of good rotating waves depends on a proper 90° phase shift between the two modes, which in turn depends on having a known phase difference between the drive signals and the resonator fields. Also one should use this setup to check the amplitudes on both the sampling probes and adjust the two driving amplitudes to make them equal. This is also important for the creation of a good rotating wave field.

An alternative diagnostic method involving an additional field sampling probe p5 located $\pi/4m$ radians apart from the other probes (around the perimeter of the resonator), is useful in the event that one does not have an accurate way to measure the relative phase of the two modes. One proceeds as above in aligning the modes and tuning them. One knows that the modes are fine tuned when the signal strength from all three sampling probes p3–p5 are equal, since the third probe p5 will see the same fields as the first two only if the phasing between the modes is the desired 90° or $\pi/2$ radians. Still another fine tuning method which features a null measurement requires still another minor probe p6 in Fig. 5. By injecting a signal at resonance into one of the minor probes, e.g., p5, and adjusting the major tuners to cause a null measurement of the cross coupling to the other minor probe p6, we force cancellation of the $2m$th cosine harmonic of the distortion, which is responsible for the cross coupling between the minor probes. This is perhaps the most elegant and accurate of the three methods, but requires the most probes and also will not correct for phasing and amplitude errors in the drive of the two modes as the first and second methods will. The first two methods are also useful to set up the resonator for single point excitation for rotating waves as discussed in Refs. 4 and 9.

IV. EXPERIENCE WITH TUNING TM_{110} ROTATING WAVE FIELDS

To test the above theory, we used the setup shown in Fig. 5. The magnetic coupling loops of p3–p6 had about 240 mm^2 area and were set to weakly couple to the resonances and serve as sampling probes. The microwave resonator was a copper plated 65-mm-long by 280-mm-i.d. right circular cylinder made of stainless steel and equipped with ultrahigh-frequency metal vacuum flanges on all ports. The tuners consisted of 33-mm-diam copper slugs or pistons that could be made to protrude into the resonator by adjustment of nuts on threaded rods incorporated in the vacuum flanges and metal bellows. We started with the ends of the copper slugs even with the inside resonator walls and defined this to be the zero tuner position. With this setting the slugs could be withdrawn for a limited (−1.3 MHz) negative tuning range, and considerably more tuning in the positive, protruding direction (approximately 0.4 MHz per mm of travel in this direction). Electric probes p1 and p2 were used as driving probes and were adjusted for near unity coupling at a $Q_L = 2700$, however the actual coupling coefficients changed as the modes were realigned. Figure 3 shows the actual spectra we first observed from sampling probe p3 when probe p1 alone was driving the resonator. While monitoring the cross coupling on probe p4, we adjusted minor tuner t3 to eliminate the cross coupling and align the modes. Figure 6 shows the change in frequency of the resonant modes as a function of the motion of tuner t3. Near alignment, the second mode disappeared (its observed amplitude became zero) from probe p3 as the modes became aligned with the probes. At

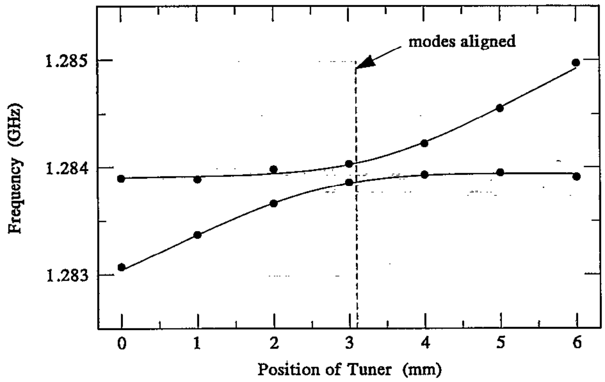

FIG. 6. Changes in the resonant frequencies of the two modes caused by moving the minor tuner, t3.

this point, probe p1 drove only the first mode, while probe p2 drove only the second mode. Similarly, probe p3 sensed only the first mode and probe p4 sensed only the second mode. After the modes were aligned with the probes, tuning using the major tuner t1 caused no additional mode rotation (no cross coupling was reintroduced because of its motion) and it was easy to adjust the frequencies of the two modes to be equal. Moving t1 by −1.3 mm brought the resonant frequencies of both modes to 1.2838 GHz. Some further fine tuning was required of tuner t1 and of the driving amplitudes to cause the amplitudes of the two modes to be equal and have a phase difference of 90°±5° (±5° was our phase measurement accuracy) when the resonator was subsequently driven as per Fig. 5. We also experimented with using the second major tuner t2 in Fig. 5 to effect changes in the average frequency of the resonator. After the above tuning, we adjusted t2 in unison with tuner t1 so that the two cancelled out each others' added $2m$th harmonic and affected only the average frequency of the two modes. Using these tuners, we were able to create a rotating wave field of frequencies anywhere from 1.2838 to 1.2868 GHz with 8 mm of travel of the two tuners. We also tried the alternate diagnostic method which used three sampling probes located 45° apart and found that indeed the amplitudes of the three probes were equal when the two standing wave modes were properly phased 90° apart in time. The third fine diagnostic method also proved to be straightforward in practice and produced a tuning which agreed with the first two methods. Generally, we found the methods proposed here of orienting the modes first, then tuning each separately, to be very much faster and more straightforward than our previous random tuning attempts.

ACKNOWLEDGMENTS

We are thankful for the financial support from the Department of Energy and the Virginia Center for Innovative Technology, as well as the help of coworkers Professor Murray Black, Dr. Terry Godlove, Dr. Frederick Mako, Kirk Evans, and Kevin Thomason. We also wish to acknowledge a very helpful discussion with O. A. Nezhevenko and V. P. Yakovlev from the Institute of Nuclear Physics, Novosibirsk, Russia.

APPENDIX: PROOF OF MAXIMUM FREQUENCY SEPARATION

Differentiating the first line of Eq. (3) with respect to α gives:

$$\frac{\partial(\delta\omega)}{\partial\alpha} = \mp \frac{2m\omega}{\int B^2 dS} \int \delta r(\phi) B_0 \sin m(\phi-\alpha) \times B_0 \cos m(\phi-\alpha) a\, d\phi, \tag{A1}$$

while the orthogonality relationship between normal modes is[10]

$$\int_V B_n B_{n'} dV = 0 \quad \text{for } n \neq n'. \tag{A2}$$

The integral on the left-hand side of Eq. (A2) can be separated into two integrals: One over the undistorted perfect cylinder and one over the distortion [the same volume as in Eq. (A1)]:

$$\int_{\text{cylinder}} B_n B_{n'} dV + \int_{\text{distortion}} B_n B_{n'} dV = 0. \tag{A3}$$

If we assume the fields in the undistorted part of the resonator to be approximately the same as those without the distortion [this is assumed in the derivation of Eq. (1)], then the first term will be zero, since it is the orthogonality relationship in the undistorted case. The second term is all that is left and equals zero on the other side of Eq. (A3). Examining Eq. (A1), we see that the second term in Eq. (A3) equals $\mp(1/2m\omega)(\int B^2 dS)\partial(\delta\omega)/\partial\alpha$, where we have assumed that $B_n = B_1$ and $B_{n'} = B_2$ in Eqs. (A2) and (A3). Since $\int B^2 dS$ will not be zero in an excited mode, this implies that $\partial(\delta\omega)/\partial\alpha$ is zero. This derivative will only be zero at the α's for which $\delta\omega$ is a maximum and for which it is minimum. In Eq.(4) the first term does not depend on α while the second term, which has the α dependence is of opposite sign for the two modes. This means that if α is such that $\delta\omega_1$ is greater than the first term in Eq. (4) and is therefore a maximum, then $\delta\omega_2$ will be less than the first term in Eq. (4) and be a minimum. Conversely, if $\delta\omega_1$ is a minimum, $\delta\omega_2$ will be a maximum. In both cases, α will be such as to maximize the absolute difference between the two modes.

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