project n°1
Study of a $\mathcal{PT}$-symetric system...

1 Introduction

The objective of the projects proposed here is to evaluate your ability to state and to solve problems by yourself, on a topic related to the lecture Acoustics 1. The expected work is theoretical in nature, on a somewhat framed but also open subject. You are expected to work independently for a limited period of 5 weeks (note however that you can ask me for help or advice all along the project).

What prevails in the work proposed here is the "obligation of means", not the "obligation of results" (as for a research work where it is not always easy to anticipate difficulties). The proposed evaluation format is a report and a 30-minutes individual interview (mainly for you to have a feedback on your work...). Beyond your knowledge of acoustics in general, you will be evaluated on skills that are those expected for a (young) researcher, e.g. :

- ⋆ ability to present concisely and clearly the approaches adopted and the results obtained,
- ⋆ ability to have a critical look at your work,
- ⋆ ability to suggest interesting future prospects,
- ⋆ etc ...

The proposed scheduled is the following :

- ⋆ **Tuesday November** 10^{th}. Presentation of each subjects. Then you'll have a few days to chose one of the proposed subjects, and inform me about your choice by E-mail.
- ⋆ **November** 10^{th} → **December 18**th. All along the duration of the project, you can contact me for advice or guidelines. Please contact me by E-mail beforehand so that we take an appointement for a skype/discord/zoom meeting.
- ⋆ **Friday December 18**th. Deadline for sending me your report. The report part can take the form that suits you, as long as it clearly and effectively summarizes your work. You should send me this report as a pdf file, and you are strongly encouraged to write it with latex (ideally, it would be good if it could have the form of a research article, e.g. using the AIP template).

2 Description of the project, guidelines

The objective of this project is to provide a theoretical study of an exotic wave scatterer based on $\mathcal{PT}$-symetry. This project is directly inspired from recent research works described in two papers, namely one paper published by Fleury et al. in Nature Communications and another one published by Auregan and Pagneux in PRL. The first task will therefore be to read these two papers. The project aims at studying and designing a two-port exhibiting unidirectional transparency from the use of a gain and a loss units (gain and loss should be balanced) that would be made with mass-spring oscillators driven by a feedback loop...It is proposed to study two types of gain and loss unit : a first type should provide a pressure drop driven by a velocity (as in the two papers mentioned above) while the second type would provide a velocity drop driven by a pressure.

2.1 A $\mathcal{PT}$-symetric system made with a feedback loop on a velocity sensor

Let's consider the device of Fig. 1 : we consider a duct inside which, at position x_0, there is a mass-spring oscillator. The wavelength is assumed to be large compared with the radius of the duct, so that we assume that there are only plane waves at both side of the mass-spring oscillator. The system of Fig. 1 is supposed to provide gain or loss, which is done by means of a feedback loop, i.e. we assume that there is a velocity sensor on the membrane, and that there is an actuator connected to this velocity sensor which provides an external force to the mass-spring oscillator, this force being proportional to the measured velocity : $\tilde{F} = \alpha_g \tilde{V}$. The mass-pring oscillator is therefore submitted both to a restoring force proportional to the pressure drop $\tilde{p}(x_0)^+ - \tilde{p}(x_0)^-$ at both sides of the duct and to an external force provided through the feedback-loop.

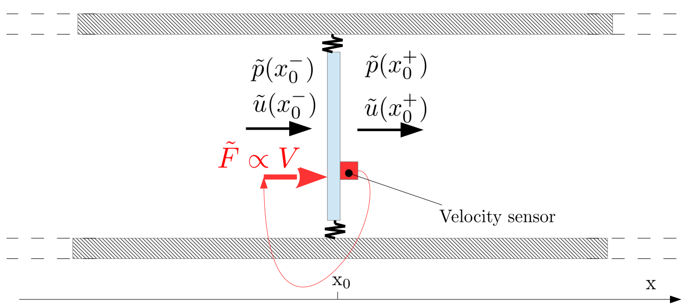

FIGURE 1 – First type of gain/loss unit

- ⋆ Let's assume that the mass-spring oscillator has a mass M, a compliance C (the compliance is the inverse of the stiffness) and a mechanical resistance R. Let's assume that the feedback loop is equiped with an amplifier (or attenuator if needed) enabling to control the parameter α_g. Derive the expression of the transfer matrix relating the pressures $\tilde{p}(x_0^\pm)$ and volume velocities $\tilde{u}(x_0^\pm)$ at both sides of the oscillator.
- ⋆ Now we consider the loss unit, which is supposed to be identical to the gain unit, except for the parameter α_ℓ of the feedback loop. How should be fixed α_ℓ so that the loss and the gain unit be balanced ?
- ⋆ Now let's consider the complete two-port, which is represented in Fig. 2. The gain unit (red fonts) is separated from the loss unit (blue fonts) with a duct of length D. Following the approach described in the two papers mentioned above, derive the expression of the scattering matrix od the complete two-port.
- ⋆ The gain and loss units could be made in practice by means of small moving coil loudspeakers. Typical values of the mechanical parameters of such loudspeakers are $M \approx 1-2$ grams, $C \approx 500\ \mu m/N$ and $R \approx 1 N.m^{-1}.s$. Let's assume that the radius of the duct is 2 cm. Write a program enabling to calculate the scatering matrix and its eigen values as a function of the various control parameters, i.e. the frequency, the distance D and the feedback loop parameters α_ℓ andα_g. The objective would be to find some possible configurations where unidirectional transparency is observed.

2.2 A $\mathcal{PT}$-symetric system made with a feedback loop on a pressure sensor

Now we consider a gain/loss unit which will be made accordingly with the sketch of Fig. 3 : it would therefore provide a velocity jump proportional to a measured pressure.

You are asked to treat this new system similarly to what you have done for the first system.

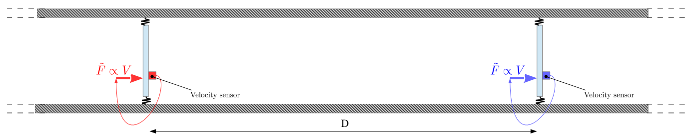

FIGURE 2 – The complete $\mathcal{PT}$-system.

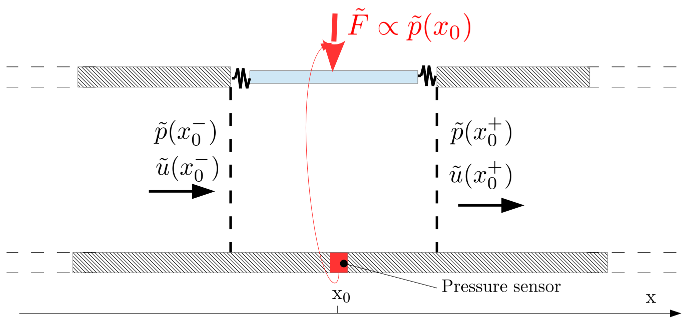

FIGURE 3 – Second type of gain/loss unit

2.3 Should you have further time at disposal…

Several additional studies could be performed to go further, like for instance :

⋆ including in the model a more accurate description of the moving coil electrodynamic loudspeaker. Should you plan to investigate this, ask me for help and for a short introduction to electroacoustics…

⋆ investigating the impact of viscothermal losses along the duct : this can be made by replacing the wave number $k = \omega/c_0$ by a complex wave number $k = \omega/c_0\,(1 + (1-i)\alpha)$ with

$$\alpha = \alpha(\omega) = \frac{\sqrt{2\kappa\omega}}{R}\left(\sqrt{\mathrm{Pr}} + \gamma - 1\right)$$

⋆ investigating the potentiality of the two systems described above for achieving CPA-lasing (see the paper by Auregan et al.).