Response Letter to Reviewers

Article reference: JAP24-AR-05791
A general spectral collocation method for computing the dispersion relations of guided acoustic waves in multilayer dissipative structures

We thank the associate editor for his interest in our paper and to the reviewers for their useful comments. Following the suggestions of the reviewers, we have revised and resubmitted our manuscript for consideration in Journal of Applied Physics. We have provided a point-by-point response to reviewer comments below and have submitted a revised manuscript that addresses their concerns. Reviewer comments are provided in black, which are followed by our responses in blue and associated changes are provided in red.

We sincerely appreciate the input from the reviewers, which we feel has improved the quality of the manuscript and we appreciate their efforts in providing helpful comments.

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1. Figure 4 shows the experimental f- Re(k) - Im (k) values. The authors are suggested to provide the procedure with one or two groups of the experimental data to show how to process the experimental data to get f- Re(k) - Im (k) values.

   - Some additional information concerning the processing of experimental data has been added to Appendix E of our manuscript, in which a figure has also been provided. We invite the reviewer to take note of this section and note that for further details, the original article on the subject presents the SLaTCoW method extensively, and has been cited in the manuscript.

2. Some wave modes are not generated. The authors may want to better explain this.

   - In the experimental setup, the structure is excited using a shaker having a metal plate attached to it, centered along the thickness of the poroelastic layer, on the edge of the sample. We would like to clarify to the reviewers that several excitations were tried out. Early on, the sample was positioned differently and the excitation located at the center of the aluminium plate. The head of the shaker was angled in order to excite some shear waves. This led to excite modes mostly located in the plate, even though we are more interested with what happens in the poroelastic layer. Multiple tests were done with regards to the excitation location, to try to find a suitable location. We acknowledge that a different excitation location and angle may have improved our experimental results, as the one chosen are not optimal. In addition, the modes mainly supported by the fluid phase should be better excited by a loudspeaker than by a shaker and therefore better measured by a microphone. However, we have chosen to focus on the modes mainly supported by the solid phase. In practice, it would be very difficult to retrieve the full dispersion relation experimentally because of the challenges faced with generating all types of waves supported by the poroelastic layer in our sample. As it is now explained on page 18 of the manuscript, the current location of the excitation limits the possible symmetries and the potential for shear waves excitation in our sample.

3. The authors may want to compare the dispersion curves with FEM results.

   - The present numerical method has been validated against analytical solutions, leading us to believe that additional validation methods would be extraneous, especially comparing with another numerical method like FEM, which would also introduce some error due to the discretization.

4. For multi-layered structures, global guided waves are present in the entire structure at low frequencies. As frequency goes higher, the guided wave mode gets closer to the wave mode only in the top solid plate (if using figure 2a as the example). I am wondering if the authors' model can capture this phenomenon.

   - The higher frequency behaviour of the structure can theoretically be captured by further refining the discretization, that is adding collocation points. However, this might require additional developments in practice and would require to test out some configurations in frequency ranges much higher than the one of interest for us.

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1. The semi-analytical finite element (SAFE) method is considered the most comprehensive and accurate method for analyzing the dispersion relationship of guided waves in structures. To better highlight the dispersion analysis method proposed in the article, please compare the results with the SAFE method.

   - It would have indeed been interesting to compare our SCM with some SAFE simulations, although this work does not provide an extensive review of the existing numerical methods to compute dispersion curves. Nevertheless, we do not agree with the statement that SAFE is the most accurate method for this kind of computation. There is already a large literature, including some books cited in our paper that discuss the fundamental aspects of SAFE and SCM, as well as the reference by A. Huber [2] that you provided in your next comment. The SCM is known as similar to SAFE due to the discretization in one dimension and a semi-analytical wave propagation in the other dimension that they both use. The key differences appear in the much faster computation time and higher accuracy achieved by the SCM. This is a point we were aware of from the outset of our work, in order to identify the most appropriate method for us.

2. In conclusion, "A spectral collocation method (SCM) is proposed to calculate dispersion relations for guided acoustic wave propagation in almost all multilayer dissipative structures." The results presented in the article are not sufficient to support it. Please provide more results to support the aforementioned conclusion. In addition, you can compared the calculated results by the Dispersion Calculator. For the the Dispersion Calculator, please refer to the following two references.

   [1]A. HUBER. The Dispersion Calculator - a free software for calculating dispersion curves of guided waves, e-Journal of Nondestructive Testing, 2024.

   [2]A. HUBER. Classification of solutions for guided waves in fluid-loaded viscoelastic composites with large numbers of layers, Journal of the Acoustical Society of America, 2023.

   - The results show some selected configurations, but we confirm that the method is developed for all configurations. We note that the discrete motion equations in fluid layers have been overlooked, and this neglect has been corrected in the manuscript. As of now, we indeed provide discretized motions equations for poroelastic, elastic and fluid layers as well as all types of coupling between those, as stated in the conclusion. The changes can be found in Section 3A of the manuscript.

   - We thank you for the proposition and the paper that you suggested was added as a citation and mentioned in our state of art. However, it does not appear to the authors as qualifying for a relevant comparison with our results. Our method was derived for general configurations where we are interested not only in elastic, but also in fluid and poroelastic layer which the Dispersion Calculator lacks as features. Since it provides tools for the computations for anisotropic viscoelastic elastic multilayers, and our method does not consider anisotropy, it leads to a poor comparison in our opinion.

3. For Abstract section, please add 3 5 keywodrs. The table caption should be placed directly above the table.

   - We thank the reviewer for the comment, which has been addressed in the manuscript. Changes were done accordingly.

4. Please give the definition of SLaTCoW when it first appears in in this paper.

   - We thank the reviewer for the comment, which has been addressed in the manuscript. Changes were done accordingly.

5. It is suggested that the legend should be added to the image to improve its readability. For example, Figure 3(a)-Figure 3(c), and Figure 3(e)-Figure 3(f) -. In addition, please check the legend located at the Figure 3(g) and Figure 3(d).

   - We are grateful for this advice and an effort has been made on this figure to make the legends and captions more legible.