Instrumental Variables

Professor Susan Athey

Machine Learning and Causal Inference

What if unconfoundedness fails?

- Alternate assumption: there exists an instrumental variable Z_i that is correlated with W_i ("relevance") and where:

$$(Y_i(0), Y_i(1)) \perp Z_i | X_i$$

Treatment W_i	Instrument Z_i	Outcome Y_i
Military service	Draft Lottery Number	Earnings
Price	Fuel cost	Sales
Having 3 or more kids	First 2 kids same sex	Mom's wages
Education	Quarter of birth	Wage
Taking a drug	Assigned to treatment group	Health
Seeing an ad	Assigned to group of users advertiser bids on in experiment	Purchases at advertiser's web site

Instrumental Variables: Binary Experiment Case

	Assigned to Treatment	**Not Assigned to Treatment**
Compliers	Treated	Not treated
Always-Takers	Treated	Treated
Never-Takers	Not treated	Not treated
Defiers	Not treated	Treated

Different Estimands

- Why not look at who was actually treated?
  - Those who complied or defied were probably not random
- Intention-to-treat (ITT)
  - Compare average outcomes of those assigned to treatment with those assigned to control
  - This may be interesting object if compliance will be similar when you actually implement the treatment, e.g. recommend patients for a drug
- Local Average Treatment Effect (effect of treatment on compliers)
  - Calculated as ITT/Pr(treat|assigned treatment)=ITT/Pr(W_i=1|Z_i=1)
  - This clearly works if you can’t get the treatment without being assigned to treatment group (no always-takers, no defiers)
  - This also works as long as there are no defiers
  - LATE is always larger than ITT

Local Average Treatment Effects

- Special case: W_i,Z_i both binary
- Relevance: Z_i is correlated with W_i
- Exclusion: $(Y_i(0), Y_i(1)) \perp Z_i$
- Monotonicity: No defiers
- Then the LATE is:

$$\frac{\mathbb{E}[Y_i|Z_i = 1] - \mathbb{E}[Y_i|Z_i = 0]}{\mathbb{E}[W_i|Z_i = 1] - \mathbb{E}[W_i|Z_i = 0]}$$

Local Average Treatment Effects: Including Covariates

- Special case: W_i,Z_i both binary
- Relevance: Z_i is correlated with W_i
- Exclusion: $(Y_i(0), Y_i(1)) \perp Z_i | X_i$
- Monotonicity: No defiers
- Then the LATE conditional on $X_i = x$ is:

$$\frac{\mathbb{E}[Y_i | X_i = x, Z_i = 1] - \mathbb{E}[Y_i | X_i = x, Z_i = 1]}{\mathbb{E}[W_i | X_i = x, Z_i = 1] - \mathbb{E}[W_i | X_i = x, Z_i = 1]}$$

IV Approaches: Including Covariates

- Two-stage least squares approach

$$Y_i = \beta_0 + \beta_1 W_i + \beta_2' X_i + \varepsilon_i$$
$$W_i = \gamma_0 + \gamma_1 Z_i + \gamma_2' X_i + \varepsilon_i$$

- Chernozhukov et al:
  - Use LASSO to select which X's to include and partial them out
  - If there are many instruments, use LASSO to construct the optimal instrument, which is the predicted value of W_i
    - Formally, estimate first stage using Post-LASSO
    - In second stage, run 2SLS using predicted value of treatment as instrument
    - Theorem: if model is sparse and instruments are strong, estimator is semi-parametrically efficient
- Note: doesn't consider observable or unobservable heterogeneity of treatment effects

IV Approaches: Including Covariates

- Two-stage least squares approach

$$Y_i = \beta_0 + \beta_1 W_i + \beta_2' X_i + \varepsilon_i$$
$$W_i = \gamma_0 + \gamma_1 Z_i + \gamma_2' X_i + \varepsilon_i$$

- Chernozhukov et al example:
  - Angrist and Krueger quarter of birth paper
  - Instruments: quarter of birth, and interactions with controls
  - Using few instruments gives large standard errors

Estimator	Instruments	Schooling Coef	Rob Std Error
2SLS (3 IVs)	3	.10	.020
2SLS (All IVs)	1530	.10	.042
2SLS (LASSO IVs)	12	.10	.014