Counterfactuals via Deep IV

Matt Taddy (Chicago + MSR)   Greg Lewis (MSR)
Jason Hartford (UBC) Kevin Leyton-Brown (UBC)

Endogenous Errors

$$y = g(p, \boldsymbol{x}) + e \text{ and } \mathbb{E}[\, p\, e\,] \neq 0$$

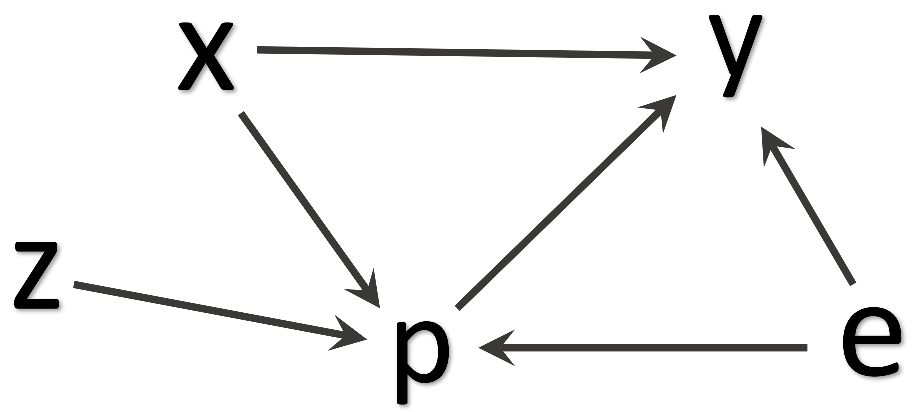
If you estimate this using naïve ML, you'll get

$$E[y|p, \boldsymbol{x}] = E_{e|p}[g(p, \boldsymbol{x}) + e] = g(p, \boldsymbol{x}) + E[e|p, \boldsymbol{x}]$$

This works for prediction. It doesn't work for counterfactual inference:

What happens if I change p independent of e ?

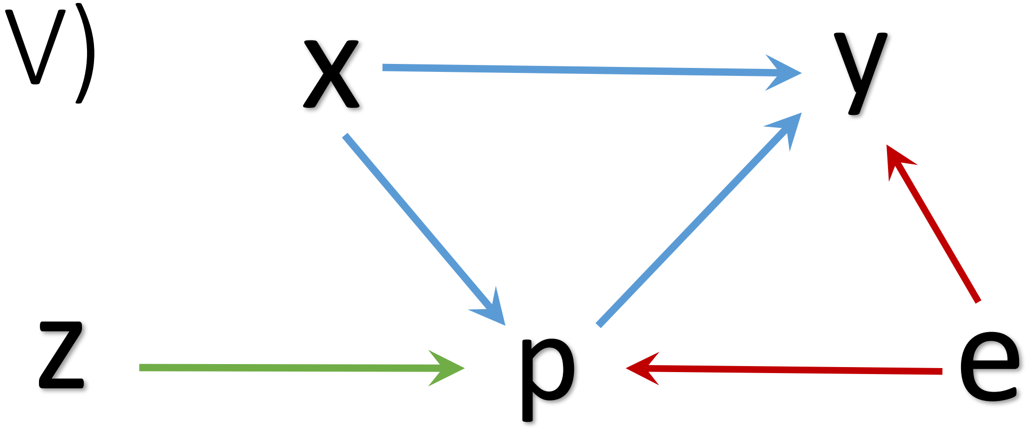
Instrumental Variables (IV)



In IV we have a special $z \perp e$ that influences policy p but not response y.

- Supplier costs that move price independent of demand (e.g., fish, oil)
- Any source of treatment randomization (intent to treat, AB tests, lottery)

Instrumental Variables (IV)



The *exclusion structure* implies

$$\mathbb{E}[y|x,z] = \int g(p,x)dF(p|x,z)$$

You can observe and estimate $\hat{\mathbb{E}}[y|x,z]$ and $\hat{F}(p|x,z)$

$\Rightarrow$ to solve for *structural* $g(p,x)$ we have an inverse problem.

cf Newey+Powell 2003

$$\min_{g \in G} \sum \left(y_i - \int g(p, x_i) dF(p|x_i, z_i) \right)^2$$

2SLS: $p = \beta z + v$ and $g(p) = \tau p$ so that $\int g(p) dF(p|z) = \tau \mathbb{E}[p|z]$

So you first regress p on z then regress y on $\hat{p}$ to recover $\hat{\tau}$.

$$\min_{g \in G} \sum \left(y_i - \int g(p, x_i) dF(p|x_i, z_i) \right)^2$$

Or nonparametric sieves where $g(p, x_i) \approx \sum_k \gamma_k \varphi_k(p, x_i)$ and

$$\mathbb{E}_F[\varphi_k(p, x_i)] \approx \sum_j \alpha_{kj} \beta_j(x_i, z_i) \quad \text{(Newey+Powell)}$$

or

$$\mathbb{E}_F[\, y_i - \sum_k \gamma_k \varphi_k(p, x_i)\,] \approx \sum_j \alpha_j \beta_j(x_i, z_i) \quad \text{(BCK, Chen+Pouzo)}$$

Also Darolles et al (2011) and Hall+Horowitz (2005) for kernel methods.

But this requires careful crafting and will not scale with $\dim(x)$

$$\min_{g \in G} \sum \left(y_i - \int g(p, x_i) dF(p|x_i, z_i) \right)^2$$

Instead, we propose to target the integral loss function directly

For discrete (or discretized) treatment

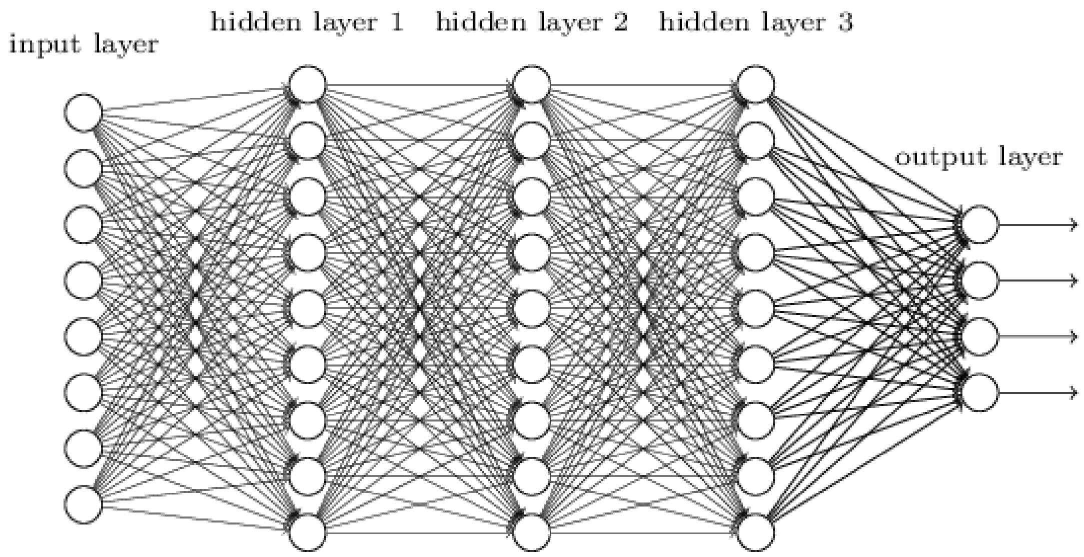
- Fit distributions $\hat{F}(p|x_i, z_i)$ with probability masses $\hat{f}(p_b|x_i, z_i)$
- Train $\hat{g}$ to minimize $\left[\mathrm{y_i} - \sum_b g(\hat{p}_b, x_i) \hat{f}(p_b|x_i, z_i) \right]^2$

And you've turned IV into two *generic* machine learning tasks

Deep Neural Networks

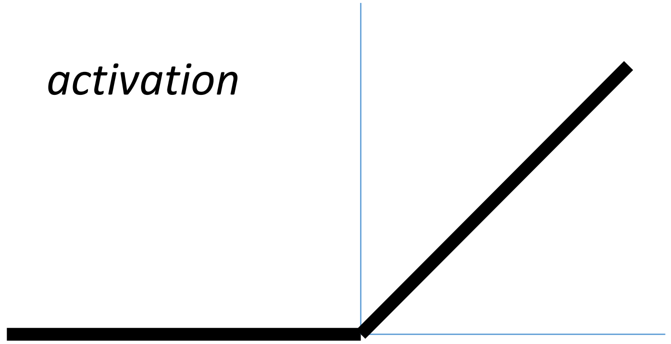
Massive number of parameters, mapping output of each layer to each node activation in the next

$$\boldsymbol{z}_i^L \rightarrow h_k(\langle W_k^{L+1}, \boldsymbol{z}_i^L \rangle)$$



Regularize

- deviance penalties $\lambda ||W||$
- dropout training (zeros in grad)



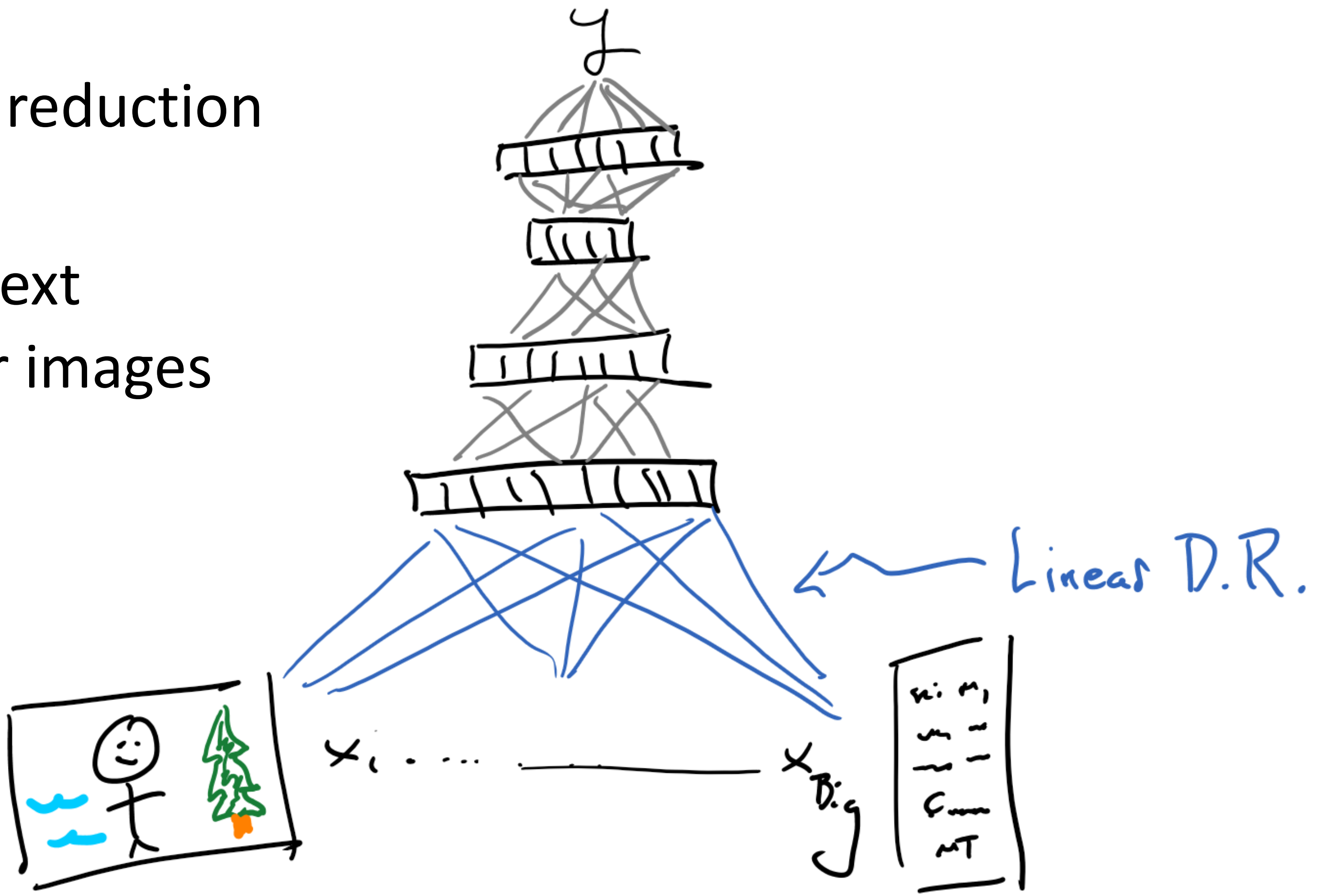
Deep nets are not really sieves

1^{st} layer is a big dimension reduction

e.g.,

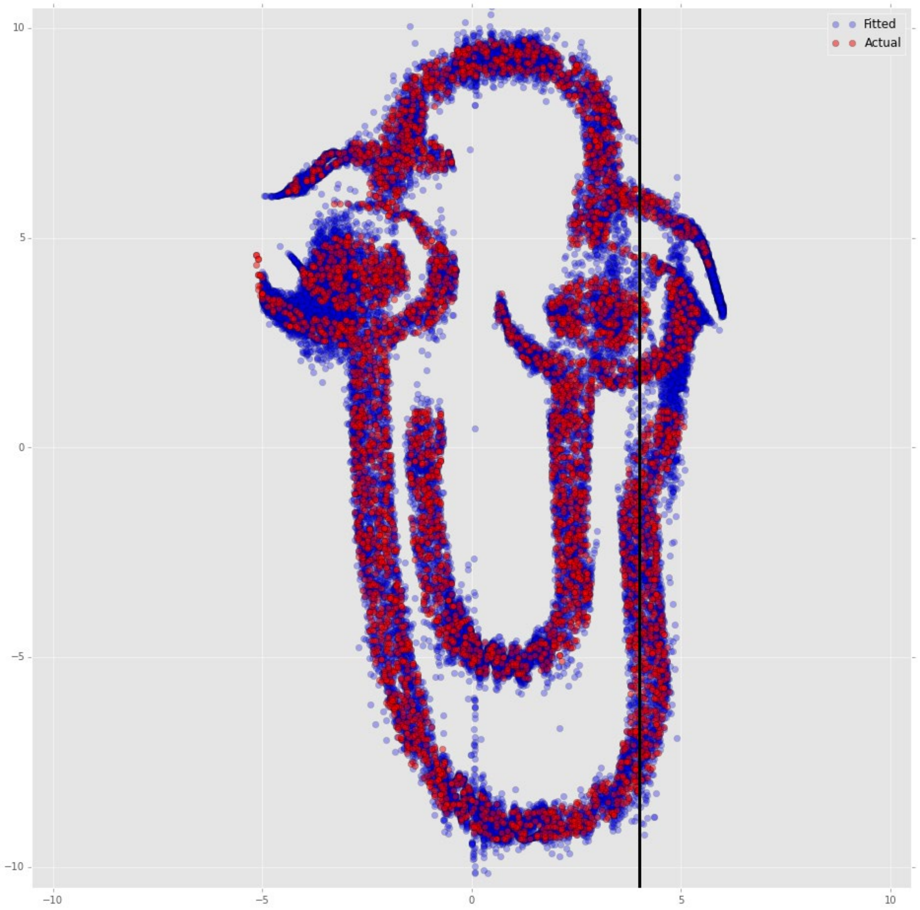
- word embedding for text
- matrix convolution for images

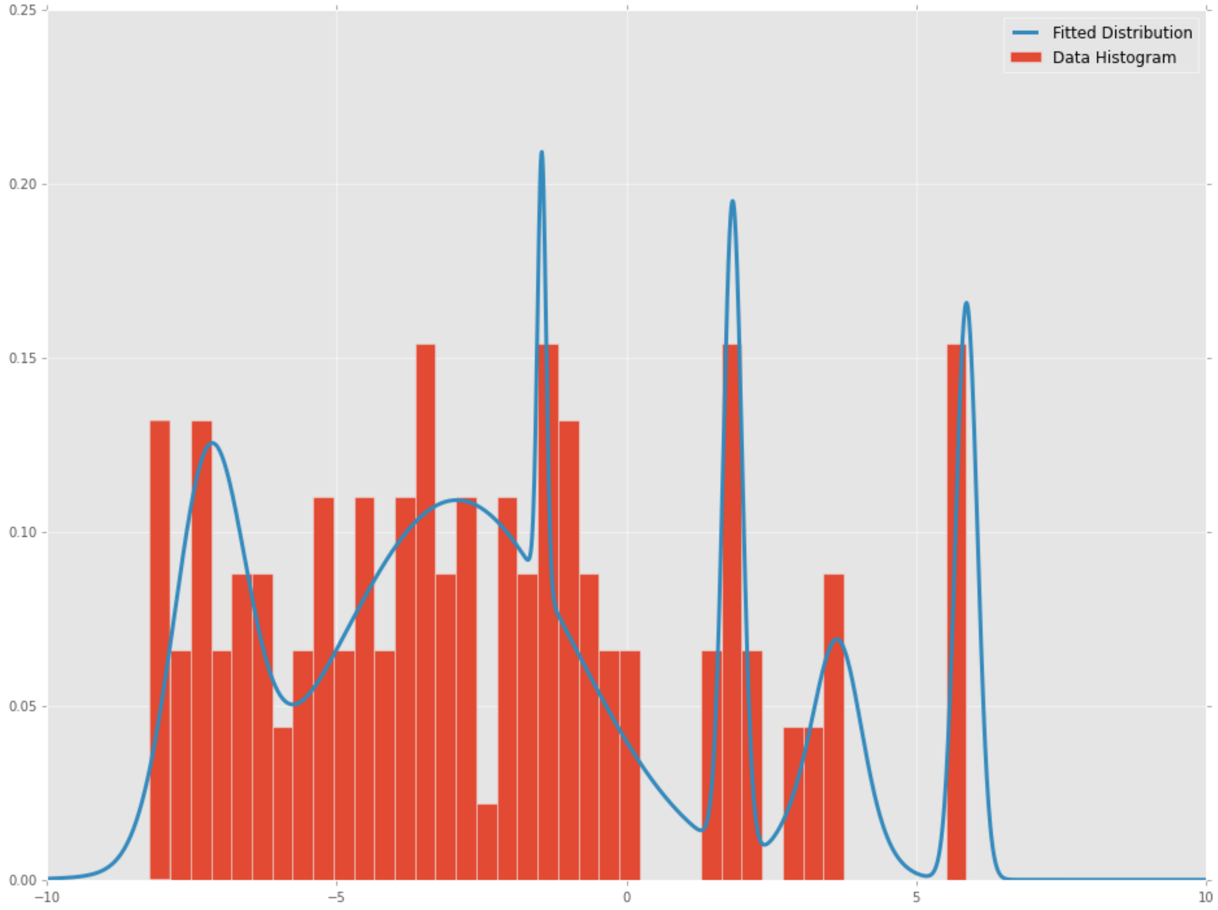
We need to study these…



e.g., first-stage learning for $\mathrm{F}(p|x_i, z_i)$

Bishop 96: Final layer of network parametrizes a mixture of Gaussians





Stage 2: Integral Loss

The second stage involves an integral loss function

If p is not discrete or can take many values, not easy!

Brute force just samples from $\hat{\mathrm{F}}(p|x_i, z_i)$ and you take gradients on

$$\frac{1}{N}\sum_i \left(y_i - \frac{1}{B}\sum_b g(p_b\,, x_i; \theta)\right)^2, \quad p_b \sim \hat{\mathrm{F}}(p|x_i, z_i)$$

This is what economists usually do, but this is super inefficient

Stochastic Gradient Descent

You have loss $L(\boldsymbol{D},\ \theta)$ where $\boldsymbol{D} = [\boldsymbol{d}_1\ \ldots \boldsymbol{d}_N]$

In the usual GD, you iteratively descend

$$\theta_t = \theta_{t-1} - \boldsymbol{C}_t \nabla L(\boldsymbol{D}, \theta_{t-1})$$

In SGD, you instead follow *noisy* but *unbiased* sample gradients

$$\theta_t = \theta_{t-1} - \boldsymbol{C}_t \nabla L(\{\boldsymbol{d}_{t_b}\}_{b=1}^{B}, \theta_{t-1})$$

SGD for integral loss functions

Our one-observation stochastic gradient is

$$\nabla L(d_i,\theta) \ = -2\left(y_i \ - \ \int g_\theta(p,x_i)d\hat{F}(p|x_i,z_i)\right) \ \int g_\theta{}'(p,x_i)d\hat{F}(p|x_i,z_i)$$

Do SGD by pairing each observation with *two independent* treatment draws

$$\nabla\hat{L}(d_i,\theta) = -2(y_i - g_\theta(\dot{p}\ ,x_i))\, g_\theta'(\ddot{p}\ ,x_i), \quad \dot{p},\ddot{p}\sim\hat{\mathrm{F}}(p|x_i,z_i)$$

So long as the draws are independent, $\mathbb{E}\nabla\hat{L}(d_i,\theta) = \mathbb{E}\nabla L(d_i,\theta) = L(\boldsymbol{D},\theta)$

Aside: we can use SGD more in econ …

There are a ton of setups where we use simulation to solve

$$\min_{\beta} \sum \left(y_i - \int g(x_i; \theta) d\mathrm{P}(\theta | \beta) \right)^2$$

Random coefficient models, simulate ML or simulate MM…

Monte Carlo SGD is a perfect fit here

Validation and model tuning

We can do OOS *causal validation*

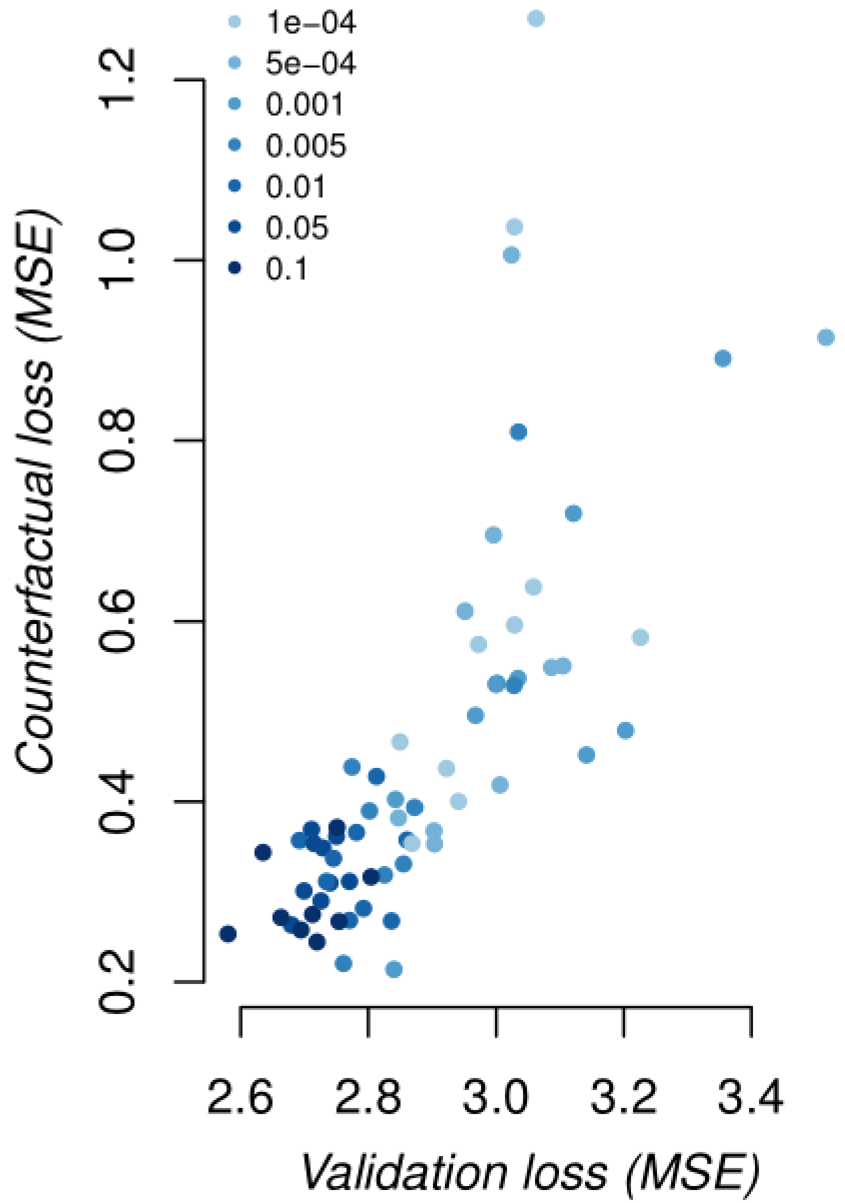
Leave-out deviance on first stage

$$\sum_{i \in LO} -\log \hat{f}(p|x_i, z_i)$$

Leave-out loss on second

$$\sum_{i \in LO} \left(y_i - \int g_\theta(p, x_i) d\hat{F}(p|x_i, z_i)\right)^2$$

You want to minimize both of these (in order).

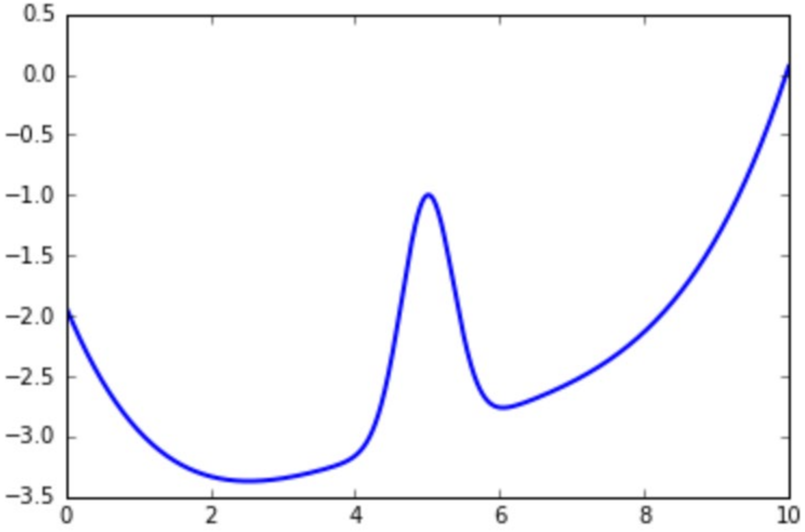


heterogeneous price effects

$$y = 100 + s\psi_t + (\psi_t - 2)p + e,$$

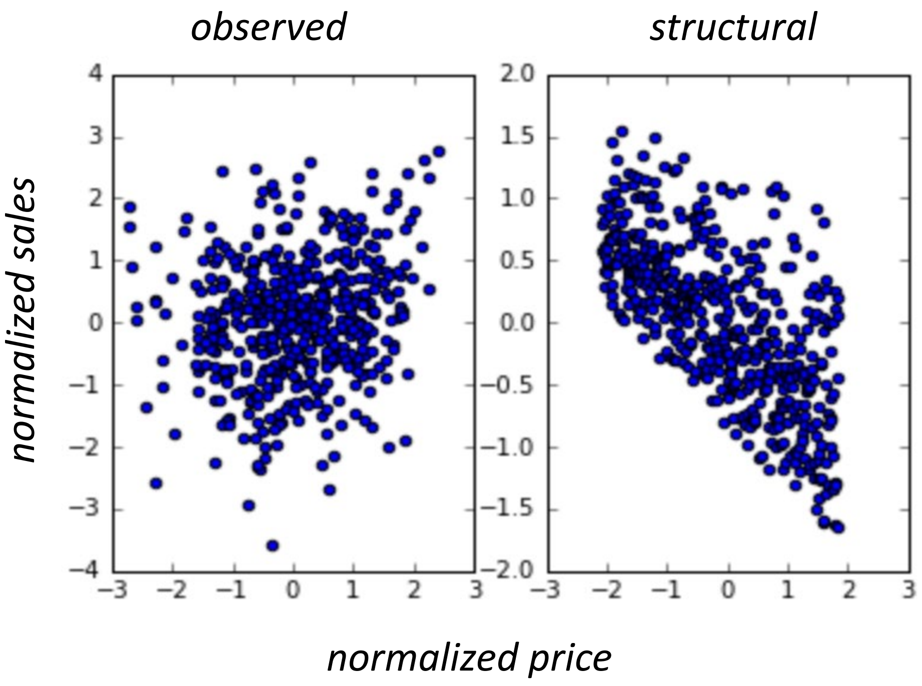
$$p = 25 + (z+3)\psi_t + v$$

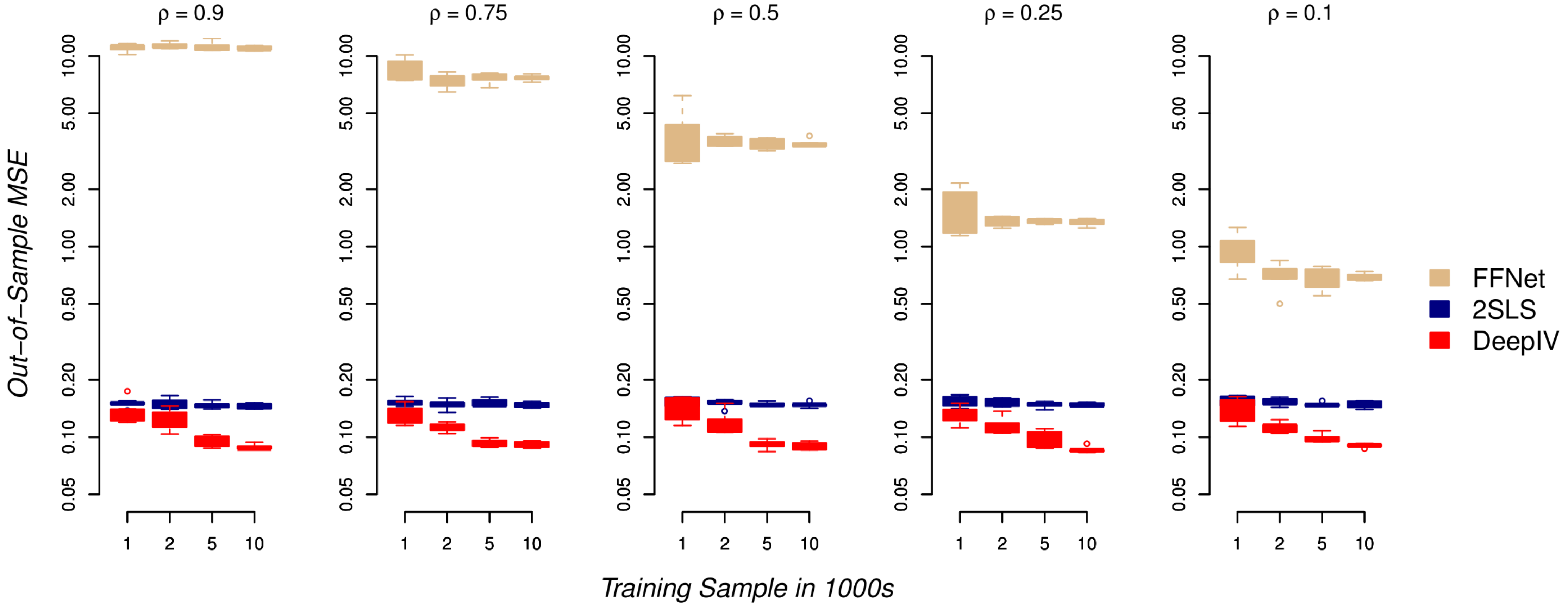
$$z,\ v \sim \mathrm{N}(0,1) \ \text{ and } \ e \sim \mathrm{N}(\rho v, 1-\rho^2),$$



'time' dependent prices, sensitivity, utility

Customer 'type' 1-7 impacts demand





ρ = 0.9
ρ = 0.75
ρ = 0.5
ρ = 0.25
ρ = 0.1
Out-of-Sample MSE
0.05
0.10
0.20
0.50
1.00
2.00
5.00
10.00
1
2
5
10
Training Sample in 1000s
FFNet
2SLS
DeepIV

Inference? Good question

Data split! Get top node values and averages on left-out data:

$$\eta_{ik} = \eta_k(x_i, p_i) \quad \text{and} \quad \bar{\eta}_{ik} = \mathbb{E}_{\hat{F}(p|x_i,z_i)} \eta_k(x_i, p)$$

Stack as instruments $\overline{\mathrm{H}} = [\bar{\eta}_1 \cdots \bar{\eta}_L]'$ and treatments $\mathrm{H} = [\eta_1 \cdots \eta_L]'$

Post-net 2SLS coefficients are $\hat{\beta} = (\overline{H}' H)^{-1} \overline{H}' y$ with variance V_β and

$$\mathrm{var}[\hat{g}(x,p)] = \boldsymbol{\eta}'(x,p)\, V_\beta\, \boldsymbol{\eta}(x,p)$$

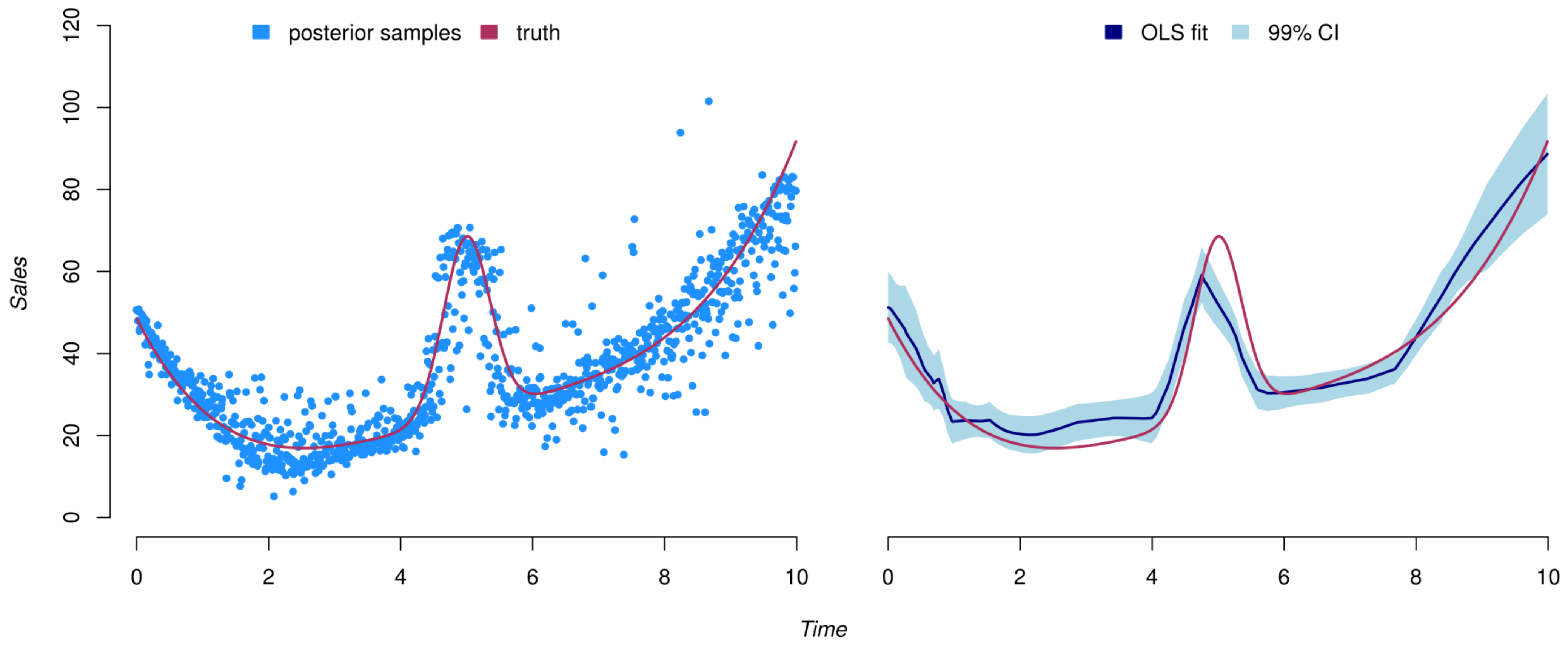


Figure 3: Bayesian (left) and Frequentist (right) inference for a central slice of the counterfactual function, taken at the average price and in our 4^{th} customer category. Since the price effect for a given customer at a specific time is constant in (27), the curves here are a rescaling of the customer *price sensitivity* function.

Ads Application

Taken from Goldman and Rao (2014)

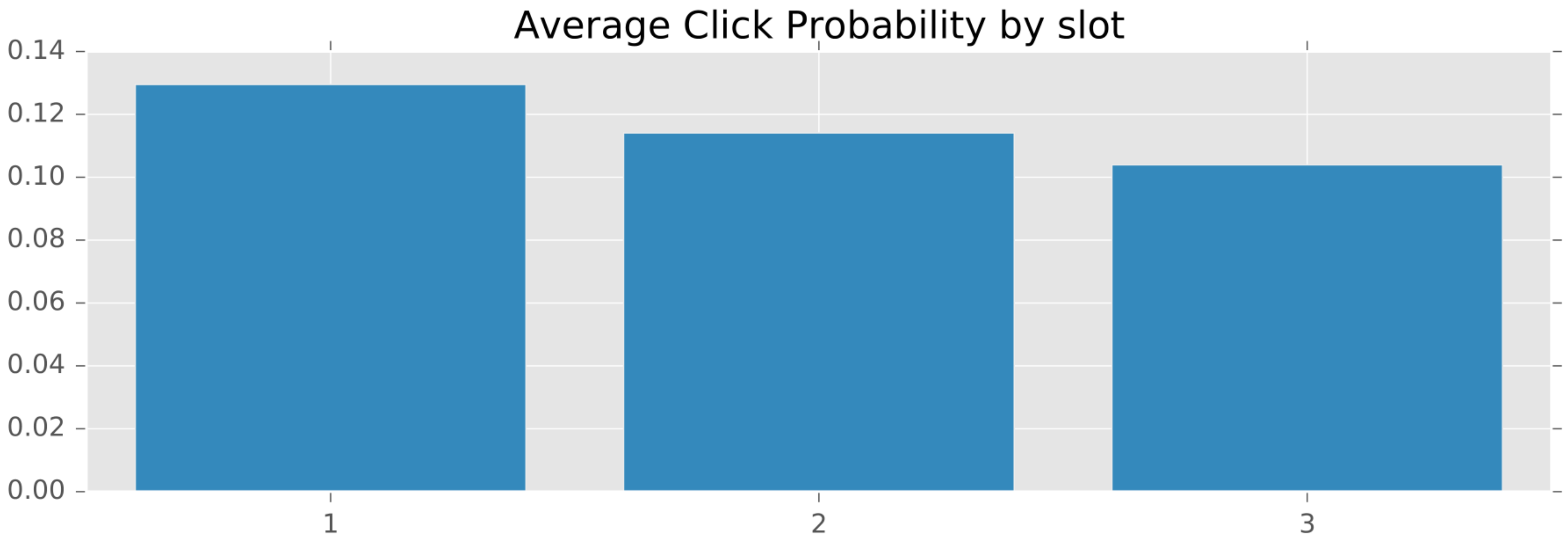
We have 74 mil click-rates over 4 hour increments for 10k search terms

Treatment: ad position 1-3

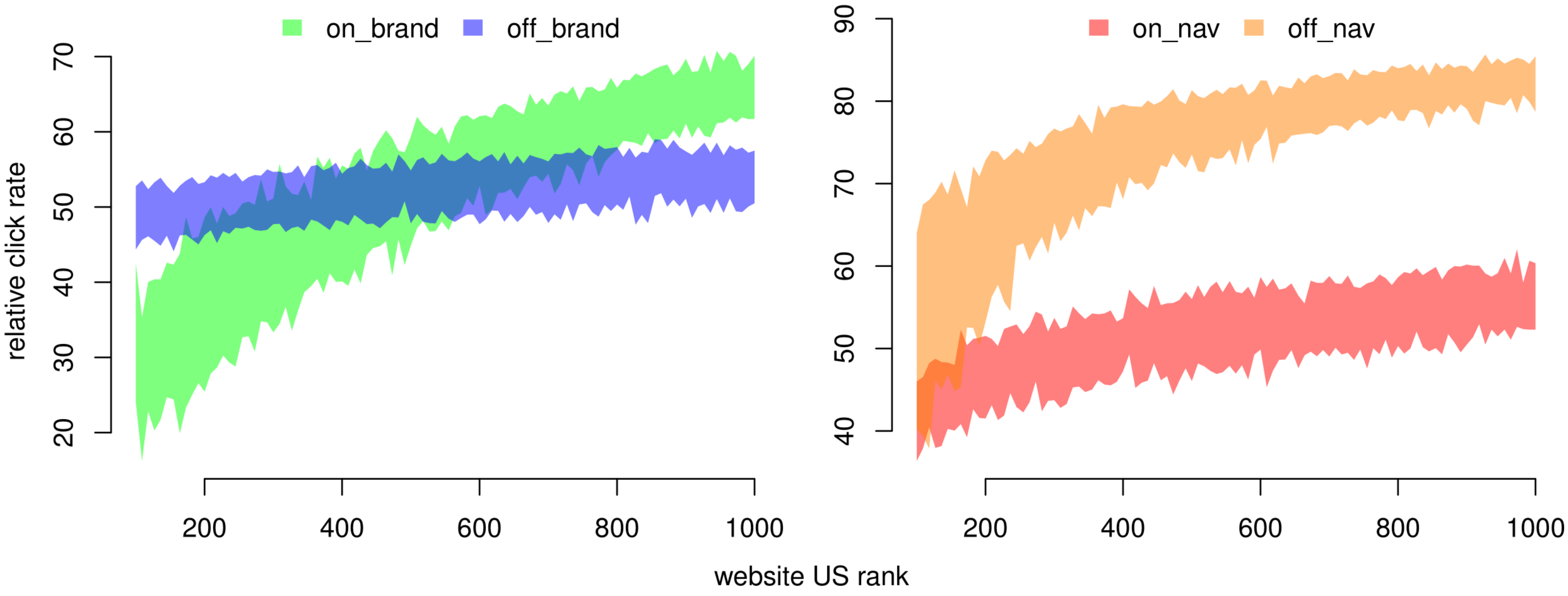
Instrument: background AB testing (bench of ~ 100 tests)

Covariates: advertiser id and ad properties, search text, time period

Average Treatment Effects



These compare to observed click probabilities of 0.33, 0.1, and 0.05.



Heterogeneity across advertiser and search