Heterogeneous Treatment Effects and Policy Assignment without Unconfoundedness

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Athey (The Impact of Machine Learning on Economics, forth.)
Athey, Tibshirani and Wager (Generalized Random Forests, AOS, 2019)
Athey and Wager (Efficient Policy Learning, 2016)

The potential outcomes framework

For a set of i.i.d. subjects $i = 1, ..., n$, we observe a tuple (X_i, Y_i, W_i), comprised of

- A **feature vector** $X_i \in \mathbb{R}^p$,
- A **response** $Y_i \in \mathbb{R}$, and
- A **treatment assignment** $W_i \in \{0, 1\}$.

Following the **potential outcomes** framework (Holland, 1986, Imbens and Rubin, 2015, Rosenbaum and Rubin, 1983, Rubin, 1974), we posit the existence of quantities $Y_i^{(0)}$ and $Y_i^{(1)}$.

- These correspond to the response we **would have measured** given that the i-th subject received treatment ($W_i = 1$) or no treatment ($W_i = 0$).

The potential outcomes framework

For a set of i.i.d. subjects $i = 1, \ldots, n$, we observe a tuple (X_i, Y_i, W_i), comprised of

- A **feature vector** $X_i \in \mathbb{R}^p$,
- A **response** $Y_i \in \mathbb{R}$, and
- A **treatment assignment** $W_i \in \{0, 1\}$.

Goal is to estimate the **conditional average treatment effect**

$$\tau(x) = \mathbb{E}\left[Y^{(1)} - Y^{(0)} \,\middle|\, X = x\right].$$

NB: In experiments, we only get to see $Y_i = Y_i^{(W_i)}$.

The potential outcomes framework

If we make no further assumptions, estimating $\tau(x)$ is not possible.

- Literature often assumes **unconfoundedness** (Rosenbaum and Rubin, 1983)

$$\{Y_i^{(0)}, Y_i^{(1)}\} \perp\!\!\!\perp W_i \mid X_i.$$

- When this assumption holds, methods based on matching or propensity score estimation are usually consistent.

ML Methods for Causal Inference: More general models

- Much recent literature bringing ML methods to causal inference focus on single binary treatment in environment with unconfoundedness
- Economic models often have more complex estimation approaches
- Athey, Tibshirani, and Wager (2016) tackle general GMM case:
  - Quantile regression
  - Instrumental Variables
  - Panel regression
  - Consumer choice
  - Euler equations
  - Survival analysis

Forests for GMM Parameter Heterogeneity

- Local GMM/ML uses kernel weighting to estimate personalized model for each individual, weighting nearby observations more.
  - Problem: curse of dimensionality
- We propose forest methods to determine what dimensions matter for "nearby" metric, reducing curse of dimensionality.
  - Estimate model for each point using "forest-based" weights: the fraction of trees in which an observation appears in the same leaf as the target
- We derive splitting rules optimized for objective
- Computational trick:
  - Use approximation to gradient to construct pseudo-outcomes
  - Then apply a splitting rule inspired by regression trees to these pseudo-outcomes

Solving estimating equations with random forests

We have $i = 1, \ldots, n$ i.i.d. samples, each of which has an **observable** quantity O_i, and a set of **auxiliary covariates** X_i.

Examples:

- Non-parametric regression: $O_i = \{Y_i\}$.
- Treatment effect estimation: $O_i = \{Y_i, W_i\}$.
- Instrumental variables regression: $O_i = \{Y_i, W_i, Z_i\}$.

Our **parameter of interest**, $\theta(x)$, is characterized by an estimating equation:

$$\mathbb{E}\left[\psi_{\theta(x),\,\nu(x)}(O_i) \,\middle|\, X_i = x\right] = 0 \quad \text{for all} \quad x \in \mathcal{X},$$

where $\nu(x)$ is an optional **nuisance parameter**.

The GMM Setup: Examples

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$$\mathbb{E}\left[\psi_{\theta(x),\,\nu(x)}(O_i) \,\middle|\, X_i = x\right] = 0 \quad \text{for all} \quad x \in \mathcal{X},$$

where $\nu(x)$ is an optional **nuisance parameter**.

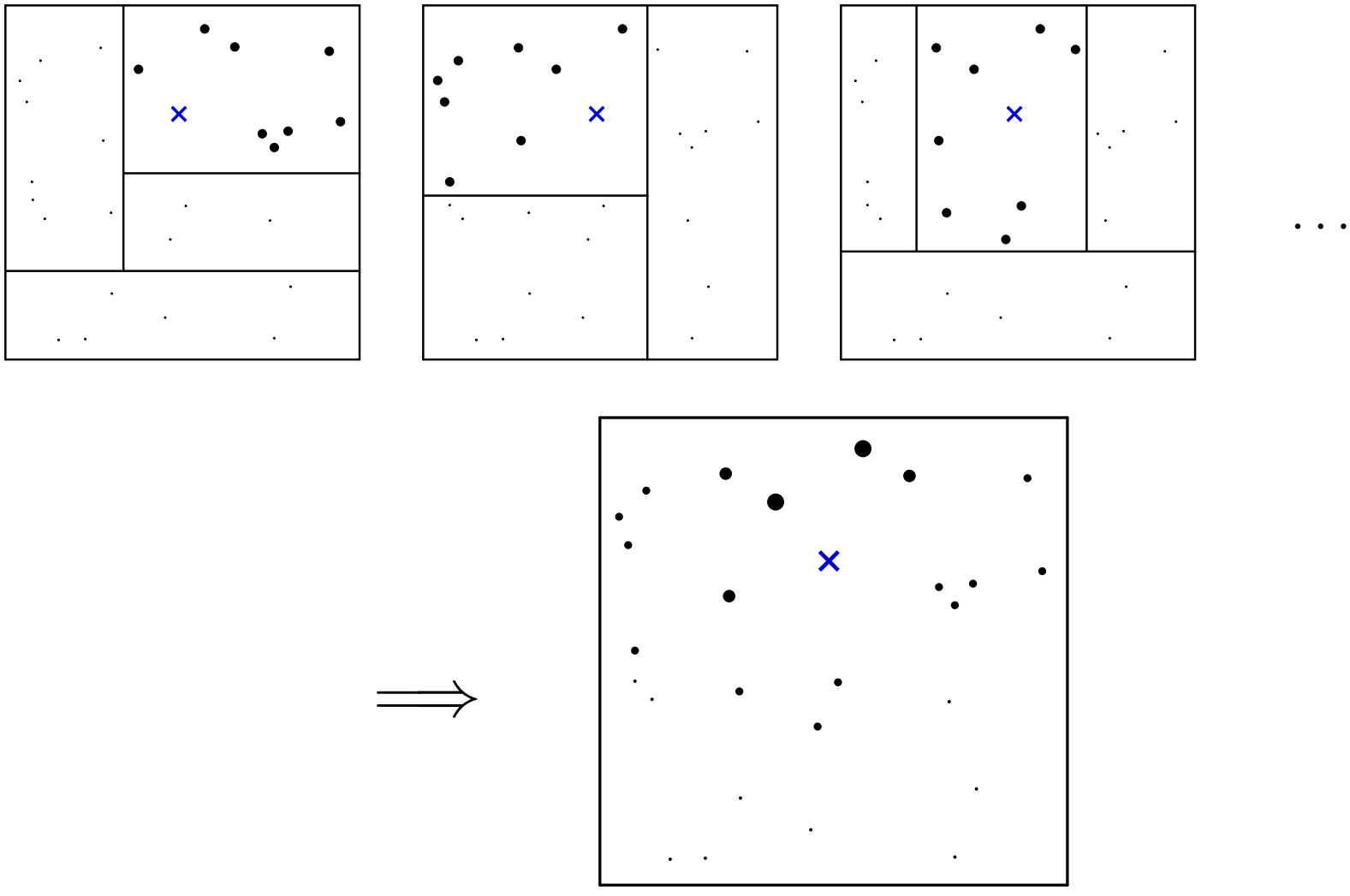
- **Quantile regression**, where $\theta(x) = F_x^{-1}(q)$ for $q \in (0,\, 1)$:

$$\psi_{\theta(x)}(Y_i) = q\,\mathbf{1}(\{Y_i > \theta(x)\}) - (1-q)\,\mathbf{1}(\{Y_i \leq \theta(x)\})$$

- **IV regression**, with treatment assignment W and instrument Z. We care about the treatment effect $\tau(x)$:

$$\psi_{\tau(x),\,\mu(x)} = \begin{pmatrix} Z_i(Y_i - W_i\,\tau(x) - \mu(x)) \\ Y_i - W_i\,\tau(x) - \mu(x) \end{pmatrix}.$$

The random forest kernel



Forests induce a kernel via **averaging tree-based neighborhoods**.
This idea was used by Meinshausen (2006) for quantile regression.

Solving estimating equations with random forests

We want to use an estimator of the form

$$\sum_{i=1}^{n} \alpha(x;\, X_i)\; \psi_{\hat{\theta}(x),\, \hat{\nu}(x)}\left(O_i\right) = 0,$$

where the weights $\alpha(x;\, X_i)$ are from a random forest.

Key Challenges:

- How do we grow trees that yield an **expressive** yet **stable** neighborhood function $\alpha(\cdot;\, X_i)$?
- We do not have access to **"prediction error"** for $\theta(x)$, so how should we **optimize splitting**?
- How should we account for **nuisance parameters**?
- Split evaluation rules need to be **computationally efficient**, as they will be run many times for each split in each tree.

Step #1: Conceptual motivation

Following CART (Breiman et al., 1984), we use **greedy splits**. Each split directly seeks to improve the fit as much as possible.

- For regression trees, in large samples, the **best split** is that which **increases the heterogeneity** of the predictions the most.
- The same fact also holds **locally** for estimating equations.

We split a parent node P into two children C_1 and C_2. In **large samples** and with **no computational constraints**, we would like to maximize

$$\Delta\left(C_1,\, C_2\right) = n_{C_1} n_{C_2} \left(\hat{\theta}_{C_1} - \hat{\theta}_{C_2}\right)^2,$$

where $\hat{\theta}_{C_1}$, $\hat{\theta}_{C_2}$ **solve the estimating equation in the children**.

Step #2: Practical realization

Computationally, solving the estimating equation in each possible child to get $\hat{\theta}_{C_1}$ and $\hat{\theta}_{C_2}$ can be **prohibitively expensive**.

To avoid this problem, we use a **gradient-based approximation**. The same idea underlies gradient boosting (Friedman, 2001).

$$\hat{\theta}_C \approx \tilde{\theta}_C := \hat{\theta}_P - \frac{1}{|\{i : X_i \in C\}|} \sum_{\{i:X_i \in C\}} \xi^\top A_P^{-1} \, \psi_{\hat{\theta}_P, \hat{\nu}_p}(O_i),$$

$$A_P = \frac{1}{|\{i : X_i \in P\}|} \sum_{\{i:X_i \in P\}} \nabla \psi_{\hat{\theta}_P, \hat{\nu}_P}(O_i),$$

where $\hat{\theta}_P$ and $\hat{\nu}_P$ are obtained by solving the estimating equation once in the parent node, and ξ is a vector that picks out the θ-coordinate from the (θ, ν) vector.

Step #2: Practical realization

In practice, this idea leads to a **split-relabel** algorithm:

1. **Relabel step:** Start by computing pseudo-outcomes

$$\tilde{\theta}_i = -\xi^\top A_P^{-1} \, \psi_{\hat{\theta}_P, \hat{\nu}_P}(O_i) \in \mathbb{R}.$$

2. **Split step:** Apply a CART-style regression split to the $\widetilde{Y}_i$.

This procedure has several advantages, including the following:

- **Computationally**, the most demanding part of growing a tree is in scanning over all possible splits. Here, we reduce to a regression split that can be efficiently implemented.
- **Statistically**, we only have to solve the estimating equation once. This reduces the risk of hitting a numerically unstable leaf—which can be a risk with methods like IV.
- From an **engineering** perspective, we can write a single, optimized split-step algorithm, and then use it everywhere.

Step #3: Variance correction

Conceptually, we saw that—in large samples—we want splits that maximize the heterogeneity of the $\hat{\theta}(X_i)$. In small samples, we need to account for **sampling variance**.

We need to penalize for the following two sources of variance.

- Our **plug-in estimates** for the heterogeneity of $\hat{\theta}(X_i)$ will be **overly optimistic** about the large-sample parameter heterogeneity. We need to correct for this kind of over-fitting.
- We **anticipate "honest" estimation**, and want to avoid leaves where the **estimating equation is unstable**. For example, with IV regression, we want to avoid leaves with an unusually weak 1st-stage coefficient.

This is a generalization of the analysis of Athey and Imbens (2016) for treatment effect estimation.

Generalized Random Forests

Our label-and-regress splitting rules can be used to grow an ensemble of trees that yield a forest kernel. We call the resulting procedure a **generalized random forest**.

- Regression forests are a special case of generalized random forests with a squared-error loss.

Available as an `R`-package, `grf`.

Asymptotic normality of generalized random forests

Theorem. (Athey, Tibshirani and Wager, 2016) Given regularity of both the estimating equation and the data-generating distribution, generalized random forests are **consistent** and **asymptotically normal**:

$$\frac{\hat{\theta}_n(x) - \theta(x)}{\sigma_n(x)} \Rightarrow \mathcal{N}(0, 1), \quad \sigma_n^2 \to 0.$$

Proof sketch.

- Influence functions: Hampel (1974); also parallels to use in Newey (1994).
- Influence function heuristic motivates approximating generalized random forests with a class of regression forests.
- Analyze the approximating regression forests using Wager and Athey (2018)
- Use coupling result to derive conclusions about generalized random forests.

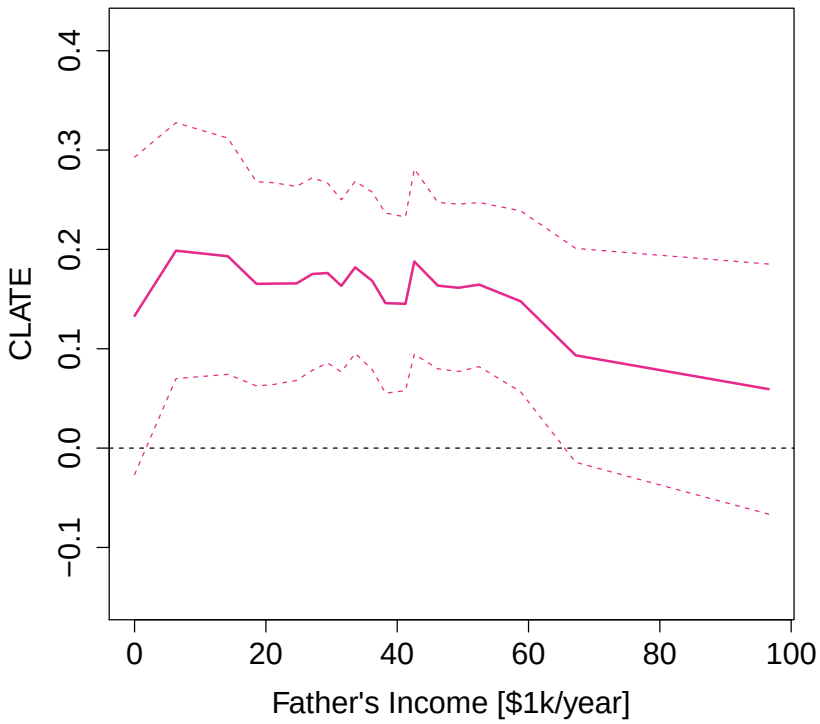
Empirical Application: Family Size

Angrist and Evans (1998) study the effect of family size on women's labor market outcomes. Understanding heterogeneity can guide policy.

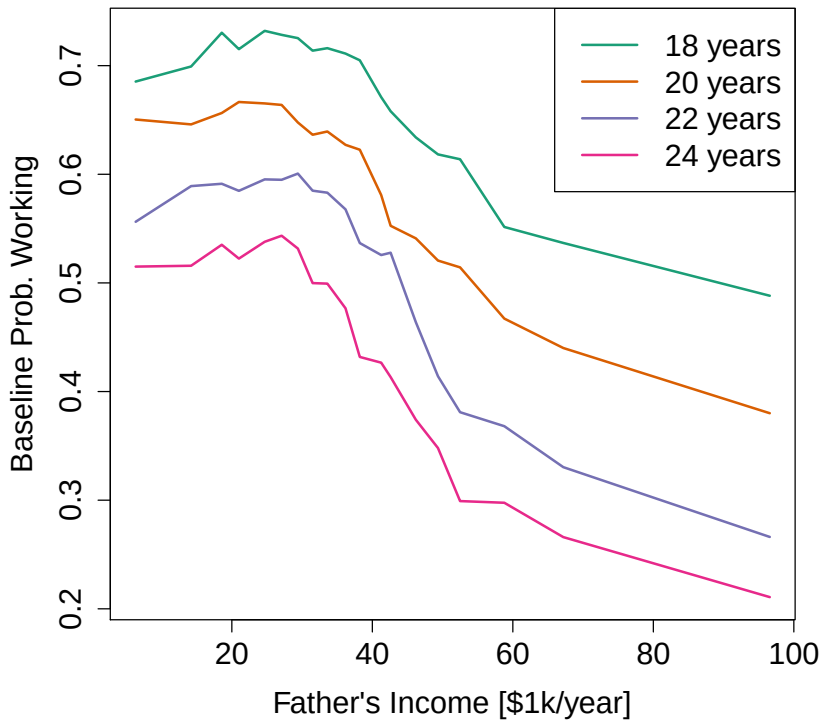
- Outcomes: participation, female income, hours worked, etc.
- Treatment: more than two kids
- Instrument: first two kids same sex
- First stage effect of same sex on more than two kids: .06
- Reduced form effect of same sex on probability of work, income: .008, $132
- LATE estimates of effect of kids on probability of work, income: .133, $2200

Treatment Effects: Magnitude of Decline

Effect on Participation



Baseline Probability of Working



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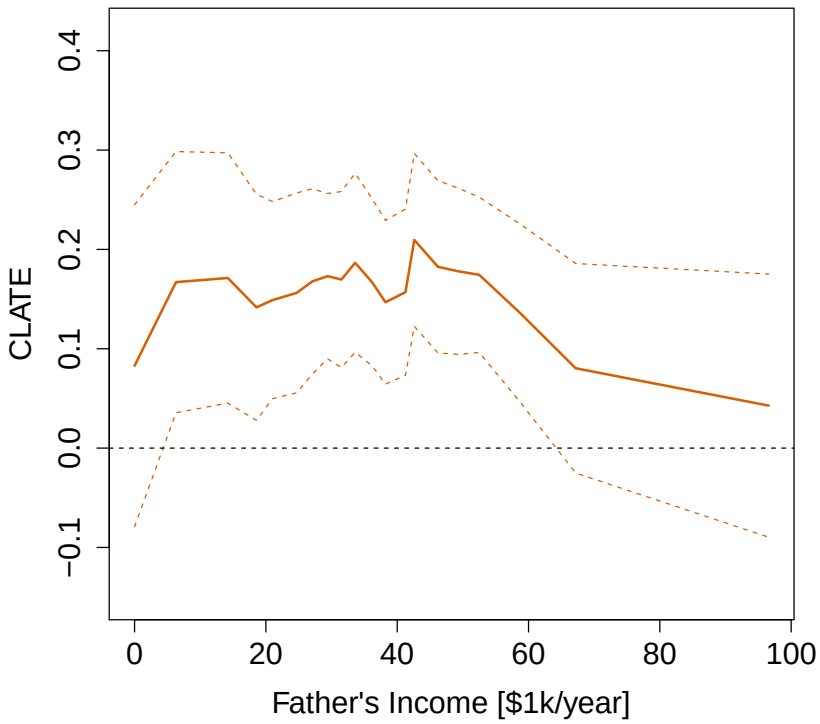

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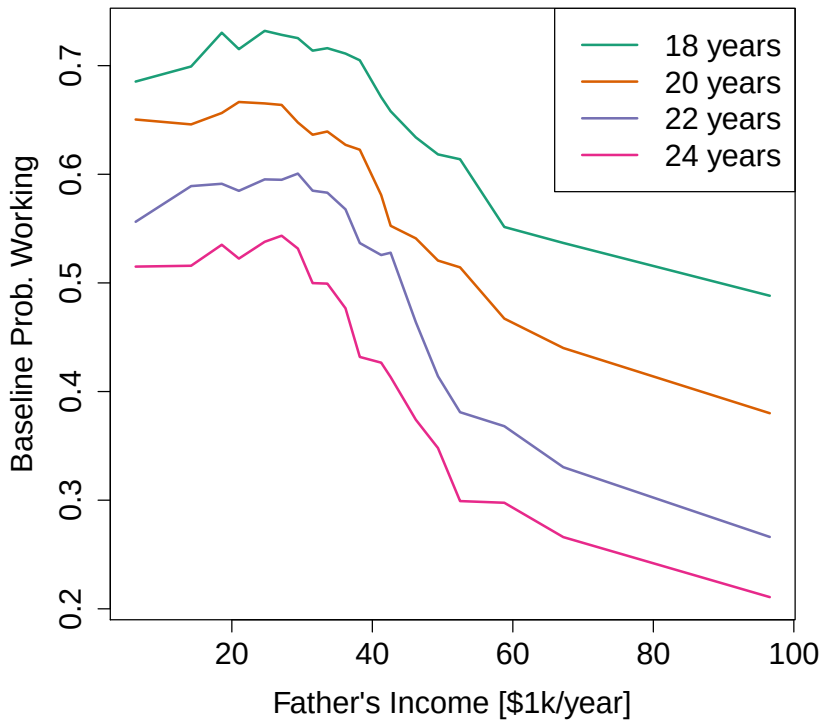

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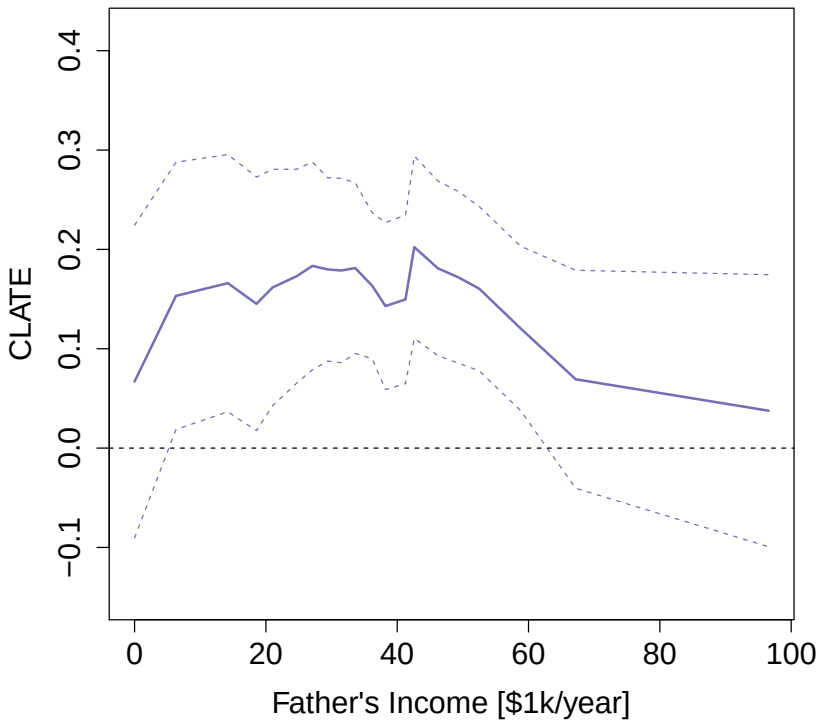

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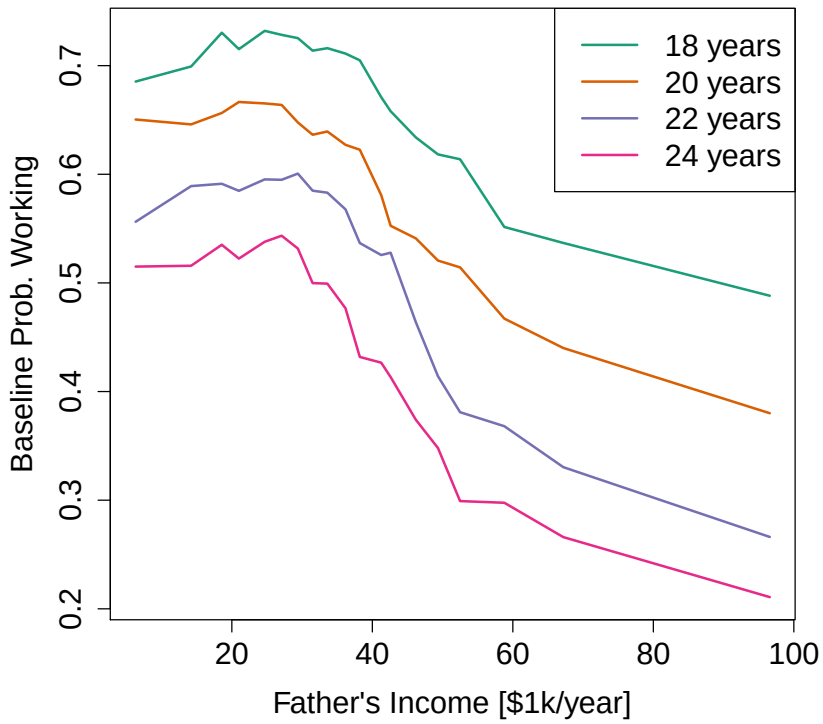

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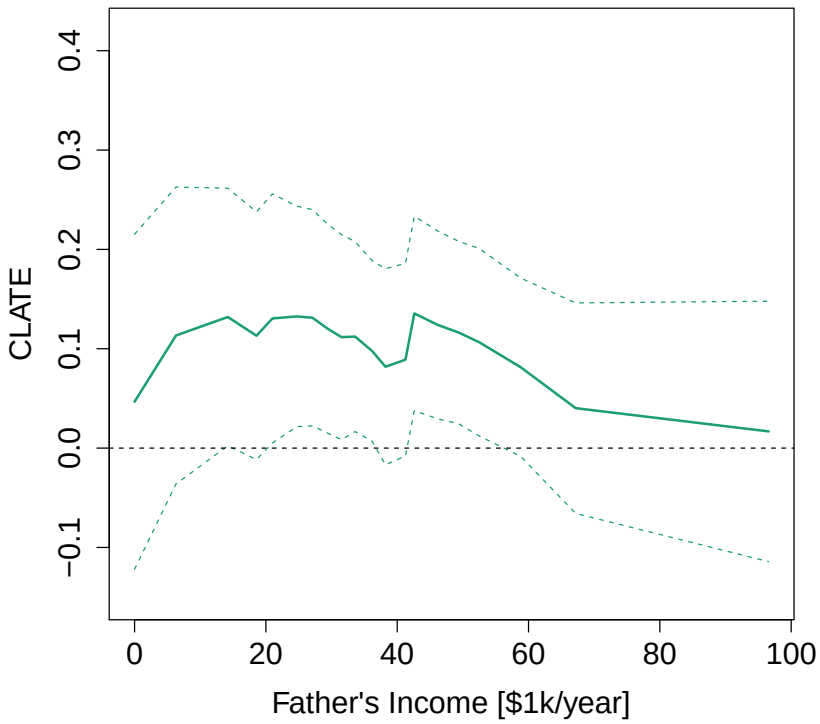

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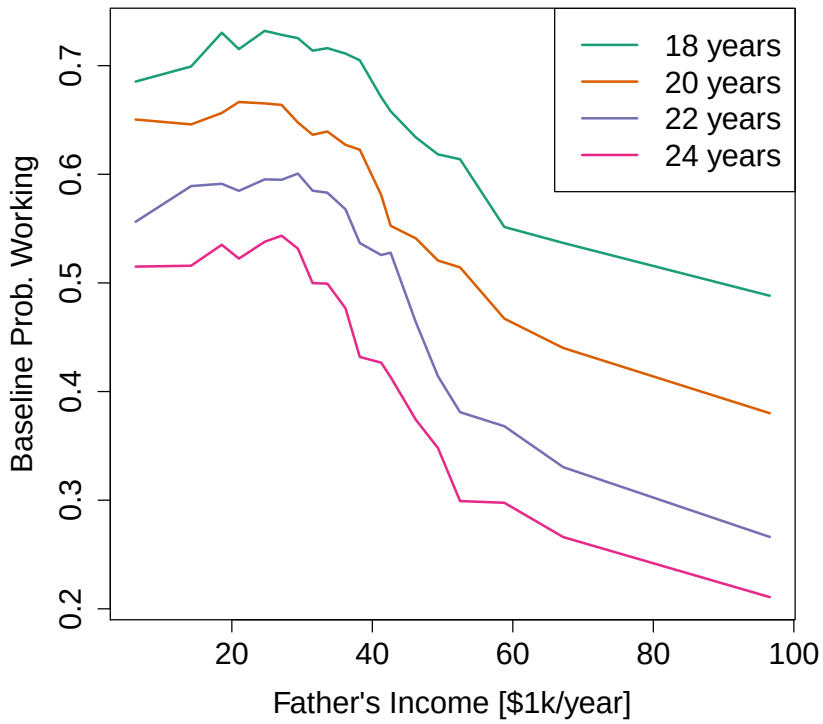

Using Heterogeneous Treatment Effect Estimates for Optimal Targeted Policies

- Problem: find a policy $\pi : \mathcal{X} \to \mathcal{W}$ to maximize $\mathbb{E}[Y_i(\pi(X_i))]$.
- First approach: non-parametric treatment assigment
  - Treat if $\hat{\tau}(X_i) > 0$
  - Hirano and Porter (2009) show efficient (under conditions)
- How to evaluate success?
- If $\hat{\tau}(X_i)$ OOB estimate from random forest, then the implied $\hat{\pi}(X_i)$ is independent of Y_i.
  - Define Group $G^w = \{i : \hat{\pi}(X_i) = w\}$, proportion in G^w is q^w.
  - Define $\hat{\gamma}^w$ as sample average treatment effect in G^w.
  - Improvement of $\hat{\pi}(\cdot)$ over treating no one: $q^1 \cdot \hat{\gamma}^1$
  - ... over random policy: $\frac{1}{2}(q^1\hat{\gamma}^1 - q^0\hat{\gamma}^0)$.
  - Standard errors straightforward

Policy Learning

The utilitarian **value** of a policy $\pi : \mathcal{X} \to \{0, 1\}$ is

$$V(\pi) = \mathbb{E}\left[Y_i(\pi(X_i))\right] = \mathbb{E}\left[Y_i(0)\right] + \mathbb{E}\left[\tau(X)\pi(X)\right].$$

In the abstract, we maximize utility by treating according to a **thresholding rule** $\tau(X_i) > c$.

But estimating the conditional average **treatment effect function** $\tau(\cdot)$ and learning a good **policy** $\pi(\cdot)$ are different problems.

- The correct **loss function** for policy learning is not mean-squared error on $\tau(\cdot)$.
- The $\tau(x)$ function may change with variables we cannot use for **targeting** (e.g., variables only measured after the fact).
- We may wish to impose other constraints on policy functions

Policy Learning

We seek to maximize the **utility** of the learned policy subject to **constraints** encoded via a class Π of allowed policies.

As in Manski (2004), we focus on **minimax regret** (Savage, 1951) relative to the policy class Π. We define utility regret as $R(\pi)$,

$$R(\pi) = \sup \left\{ V(\pi') : \pi' \in \Pi \right\} - V(\pi),$$

and seek a policy $\hat{\pi} \in \Pi$ satisfying a high-probability **regret bound**.

We can also write policy regret in terms of $\tau(x)$,

$$R(\pi) = \sup \left\{ \mathbb{E}\left[\tau(X)\pi'(X)\right] : \pi' \in \Pi \right\} - \mathbb{E}\left[\tau(X)\pi(X)\right],$$

meaning that baseline effects don't affect policy regret.

Imposing **structure** on Π is essential in many applications. In observational studies, we use many features with a non-parametric specification to make **unconfoundedness plausible**,

$$\{Y_i(0),\ Y_i(1)\} \perp\!\!\!\perp W_i \,\big|\, X_i.$$

Conversely, the policy $\pi(\cdot)$ must be **implementable in practice**. Features that should not be used in $\pi(\cdot)$ include:

- **Unreliably available features** (e.g., collected by specialist).
- **Gameable features** (e.g., self-reported preferences).
- **Legally protected classes** (e.g., religion, national origin).

Moreover, we may want Π to encode constraints on:

- **Total budget** or marginal **subgroup treatment rates** (e.g., Bhattacharya and Dupas, 2012).
- **Functional form** for easier implementation or audit.

We study policy learning in a way that is aware of such constraints.

From Evaluation to Learning

Recall that we want to pick a good **policy** $\pi : \mathcal{X} \to \{0, 1\}$ among a class Π of allowable interventions.

The **regret** from choosing π depends on the CATE function $\tau(x) = \mathbb{E}\left[Y_i(1) - Y_i(0) \,\middle|\, X_i = x\right]$:

$$R(\pi) = \sup\left\{\mathbb{E}\left[\tau(X)\pi'(X)\right] : \pi' \in \Pi\right\} - \mathbb{E}\left[\tau(X)\pi(X)\right].$$

Before discussing how to **learn** a policy, we review how how to estimate an **average effect**

$$\tau = \mathbb{E}\left[Y_i(1) - Y_i(0)\right].$$

We build on unifying results from Chernozhukov, Escanciano, Ichimura, Newey and Robins (CEINR, 2018).

From Evaluation to Learning

We have access to features X_i, an outcome Y_i, a treatment W_i, and an instrument Z_i. We assume that the exclusion restriction holds, such that potential outcomes only depend on W_i, and

$$m(x, w) = \mathbb{E}\left[Y_i(w) \,\middle|\, X_i = x\right], \quad \tau_m(x) = m(x, 1) - m(x, 0).$$

As in CEINR, suppose $\tau(x)$ can be represented via **weighting**:

$$\mathbb{E}\left[\tau_m(X) - g(X, Z)Y \,\middle|\, X = x\right] = 0 \text{ for all } x,\ m(\cdot).$$

CEINR then show that the **doubly robust** estimator is **efficient**,

$$\hat{\tau} = \frac{1}{n}\sum_{i=1}^{n} \widehat{\Gamma}_i, \quad \widehat{\Gamma}_i = \tau_{\hat{m}}(X_i) + \hat{g}(X_i, Z_i)\left(Y_i - \hat{m}(X_i, W_i)\right),$$

provided we use **cross-fitting** and have nuisance components that converge fast enough in L_2 (4th-root rates are sufficient).

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Example: Selection on observables, $\{Y_i(0), Y_i(1)\} \perp\!\!\!\perp W_i \,\big|\, X_i$. In this case, the **propensity score** can be used for weighting:

$$\tau(x) = \mathbb{E}\left[g(X_i, W_i)Y_i \,\middle|\, X_i = x\right], \; g(X_i, W_i) = \frac{(W_i - e(X_i))}{e(X_i)(1 - e(X_i)}.$$

The corresponding doubly robust estimator is **augmented IPW** (Robins, Rotnitzky, and Zhao, 1994).

From Evaluation to Learning

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Example: Endogenous treatment with instrument and **conditional homogeneity**, $\tau(x) = \text{Cov}\left[Y, Z \,\middle|\, X = x\right] / \text{Cov}\left[W, Z \,\middle|\, X = x\right]$. Now use the **compliance score** (Aronow and Carnegie, 2013),

$$g(X_i, Z_i) = \frac{1}{\Delta(X_i)}\frac{Z_i - z(X_i)}{z(X_i)(1 - z(X_i)}, \quad z(x) = \mathbb{P}\left[Z_i \,\middle|\, X_i = x\right],$$
$$\Delta(x) = \mathbb{P}\left[W \,\middle|\, Z = 1, X = x\right] - \mathbb{P}\left[W \,\middle|\, Z = 0, X = x\right],$$

to construct a doubly robust estimator.

From Evaluation to Learning

In many problems, the **doubly robust** estimator is **efficient**

$$\hat{\tau} = \frac{1}{n}\sum_{i=1}^{n} \widehat{\Gamma}_i, \quad \widehat{\Gamma}_i = \tau_{\hat{m}}(X_i) + \hat{g}(X_i, Z_i)\,(Y_i - \hat{m}(X_i, W_i))\,.$$

Our main result is that we can also use the same scores of **learning**

$$\hat{\pi} = \text{argmax}\left\{ \frac{1}{n}\sum_{i=1}^{n} (2\pi(X_i) - 1)\,\widehat{\Gamma}_i : \pi \in \Pi \right\}.$$

Regret bounds depend on n, Π, and the semiparametric efficient variance for policy evaluation.

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Regret bounds depend on n, Π, and the semiparametric efficient variance for policy evaluation.

NB: The policy $\pi^* = \text{argmax}\left\{\mathbb{E}\left[(2\pi(X_i) - 1)\tau(X_i)\right] : \pi \in \Pi\right\}$ gets **zero regret**. Our estimator effectively replaces $\tau(X_i)$ with $\widehat{\Gamma}_i$.

Back to the California GAIN Study

Each county enrolled participants with a **different covariate mix**, and randomized to treatment with **different probabilities**. Once we remove county information, this is not a **randomized study**, but Hotz et al. present evidence that **unconfoundedness** holds.

We set the **cost** C of treatment to match the **ATE**; thus, we need to find heterogeneity in order to get non-zero utility.

We estimate nuisance components with **forests**, and then optimize over the class Π of low-depth **trees**:

$$\hat{\pi} = \text{argmax} \left\{ \frac{1}{n} \sum_{i=1}^{n} (2\pi(X_i) - 1) \, \widehat{\Gamma}_i : \pi \in \Pi \right\},$$

$$\widehat{\Gamma}_i = \hat{\tau}^{(-i)}(X_i) - C + \frac{W_i - \hat{e}^{(-i)}(X_i)}{\hat{e}^{(-i)}(X_i) \left(1 - \hat{e}^{(-i)}(X_i)\right)} \times \left(Y_i - \hat{y}^{(-i)}(X_i) - (W_i - \hat{e}^{(-i)}(X_i)) \hat{\tau}^{(-i)}(X_i) \right).$$

Main Result

Goal is to show that we can use doubly robust scores of **learning**

$$\hat{\pi} = \text{argmax} \left\{ \frac{1}{n} \sum_{i=1}^{n} (2\pi(X_i) - 1) \widehat{\Gamma}_i : \pi \in \Pi \right\},$$
$$\widehat{\Gamma}_i = \tau_{\hat{m}}(X_i) + \hat{g}(X_i, Z_i) (Y_i - \hat{m}(X_i, W_i)).$$

Theorem. (Athey and Wager, 2018) Suppose $g(X, Z) \leq \eta^{-1}$, and that our nuisance estimates satisfy (we use **cross-fitting**)

$$\mathbb{E}\left[(\hat{m}(X, W) - m(X, W))^2\right] \mathbb{E}\left[(\hat{g}(X, Z) - g(X, Z))^2\right] = o_P\left(\frac{1}{n}\right).$$

Suppose moreover that Π has finite **VC-dimension**. Then,

$$R(\hat{\pi}) = \mathcal{O}_P\left(\sqrt{S \, \text{VC}(\Pi) / n}\right),$$

with $S = \mathbb{E}\left[(\tau_m(X_i) + g(X_i, Z_i)(Y_i - m(X_i, W_i)))^2\right]$.

Proof Ingredients

- ATE literature looks at efficient coupling of the **efficient score**: $|\widehat{A}(\pi) - \widetilde{A}(\pi)| = o_P(1/\sqrt{n})$, where

$$\widetilde{A}(\pi) = \sum_{i=1}^{n} (2\pi(X_i) - 1) \left(\tau_m(X_i) + g(X_i, Z_i)\left(Y_i - m(X_i, W_i)\right)\right).$$

- Here, need **uniform coupling**:
  $\sup\left\{\left|\widehat{A}(\pi) - \widetilde{A}(\pi)\right| : \pi \in \Pi\right\} = o_P(1/\sqrt{n})$.
  - Our uniform coupling result is specific to the **doubly robust** construction, and may not hold for other estimators that are efficient at a single π, e.g., empirical IPW (Hirano et al., 2003).
- Next: **concentration** of $\widetilde{A}(\pi)$ over the class $\pi \in \Pi$. Defining

$$S = \mathbb{E}\left[\left(\tau_m(X_i) + g(X_i, Z_i)\left(Y_i - m(X_i, W_i)\right)\right)^2\right],$$

  remix Dudley's classical **chaining argument** to verify that

$$\sup\left\{\left|\widetilde{A}(\pi) - A(\pi)\right| : \pi \in \Pi\right\} = \mathcal{O}_P\left(\sqrt{\frac{S\ VC(\Pi)}{n}}\right).$$

  Bound **Rademacher complexity** via chaining.

Lower bounds

Any statement about lower bounds depends on how **general** we want to be, and how **adaptive** we want to be to problem structure. We proved that $R(\hat{\pi}) = \mathcal{O}_P(\sqrt{S \operatorname{VC}(\Pi)/n})$, and argue that this is optimal. We first note, however:

- If treatment effects are **smaller** than $1/\sqrt{n}$, bound is loose.
- If treatment effects are **very large**, this bound is loose as finding the optimal rule is easy (Luedtke and Chambaz, 2017).
- VC-dimension may be a loose summary of the **complexity** of Π (Bartlett and Mendelson, 2006).

We show that our bound is **sharp** when our treatment effects scale as $1/\sqrt{n}$ and we summarize complexity via VC-dimension. Similar **local asymptotics** are also used by Hirano and Porter (2009).

NB: Our upper bound allows the data-generating distribution (and Π) to change with n, so changing $\tau(\cdot)$ with n is valid.

Lower bounds

In the **unconfoundedness** setting, define a sequence of problems

$$X_i \sim \text{Uniform}\left(\mathcal{X}_s\right), \quad W_i \,\Big|\, X_i \sim \text{Bernoulli}(e(X_i)),$$
$$Y_i \,\Big|\, X_i,\, W_i \sim \left(y(X_i) + (W_i - e(X_i))\,\frac{\tau(X_i)}{\sqrt{n}},\, \sigma^2(X_i)\right).$$

Theorem. (Athey and W., 2018) In this setting, there is a class Π with $\text{VC}(\Pi) = d$ whose **minimax regret** satisfies

$$\liminf_{n\to\infty}\left\{\sqrt{n}\,\inf_{\hat{\pi}_n}\left\{\sup_{|\tau(x)|\leq C}\left\{\mathbb{E}\left[R_n\left(\hat{\pi}_n\right)\right]\right\}\right\}\right\} \geq 0.33\sqrt{Sd},$$

where $S = \mathbb{E}\left[\sigma^2(X)/(e(X)(1-e(X)))\right]$.

Our method **achieves this bound** up to a universal constant. Other methods do not, e.g., for **IPW** with known propensity scores, Kitagawa & Tetenov (2018) prove a bound that depends on $\sup\{|Y_i|\} / \inf\{e(X_i), (1-e(X_i))\}$ instead of $\sqrt{S}$.

Conclusion

- Machine learning based methods very useful to analyze heterogeneous treatment effects and targeted policies
- Methods work in a variety of design settings (experiments, unconfounded, IV)
- Methods can give either simple or very complex policies, with statistical guarantees
- Online learning can help discover good policies
- Semi-parametric efficiency literature guides methods