Mock Exam

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5329: Inequality, Household Behavior, and the Macroeconomy

Instructions

- Please answer all the questions below for full points.
- About mathematical derivations: do not skip steps, show all work/derivations, and explain (the answer is meaningless if I do not understand how you got it).
- You may use calculators permitted by the school (but they are unlikely to be of any use). Other aids are not permitted.

Problem 1. Consider a household that maximizes expected lifetime utility defined by:

$$E_0 \sum_{t=0}^{T} \beta^t u(c_t)$$

Assume $u(c_t) = \log(c_t)$. The budget constraint of the household is given by:

$$a_t = (1+r)a_{t-1} - c_t + y_t \qquad \forall t$$

where y_t is stochastic labor income following $y_t = h_t + \epsilon_t$, with h_t denoting a deterministic trend and $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$ being a random fluctuation around this trend. Further assume $\beta(1+r) = 1$. Households cannot die in debt: $a_T \geq 0$.

1. Write down the problem above recursively, using the Bellman equation. Is it correct to write the value function with two only state variables [1], t and x_t, where $x_t = a_{t-1}(1+r) + y_t$? Motivate your answer (Max 5 lines for the latter.)

2. Write down the Lagrangian of the problem and derive the Euler-equation. Provide an economic intuition of your result. (Max 5 lines for the latter.)

3. Consider the Euler-equation relating optimal consumption in period t to the same object in time $t+1$. If you eliminated the randomness in c_{t+1} and replaced it with its expectation, how would current optimal consumption change? Would it be higher or lower? Argue formally and also provide economic intuition. (Max 10 lines for the latter.)

   You can invoke either version of Jensen's inequality without proof. If X is any random variable, f is a strictly concave, g is a strictly convex function, then

$$\mathbb{E}\big[f(x)\big] < f\big(\mathbb{E}[x]\big) \quad \text{and} \quad \mathbb{E}\big[g(x)\big] > g\big(\mathbb{E}[x]\big).$$

From now on, consider the following four variants of the model above:

(a) $u(c_t) = -(c_t - c^*)^2$.

(b) $u(c_t) = -(c_t - c^*)^2$ and $\sigma_\epsilon^2 = 0$.

(c) $u(c_t) = -(c_t - c^*)^2$ and $a_t \geq 0$ for all t.

(d) $u(c_t) = -(c_t - c^*)^2$, $\sigma_\epsilon^2 = 0$ and $a_t \geq 0$ for all t.

Note that c^* is a bliss point that is never reached.

[1] In other words, to write the value function as a function of only two arguments?

4. Rank the maximum lifetime utility attainable by individuals given models (a), (b), (c), and (d). Motivate your answer. It is possible that some pairs of these models cannot be ordered without more information. (Max 20 lines.)

5. Consider the three pictures below, where you see the average life-cycle (labor) income, assets, and savings of individuals obtained using the four model variants presented above. Each one of these pictures contains the outcomes of one or two models among $(a) - (d)$.[2] There are two models which result in the same figure (forgetting about numerical errors).

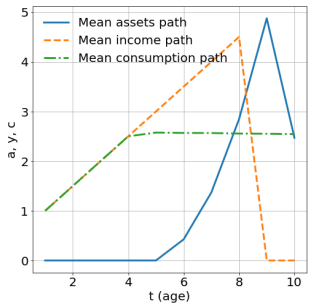

Figure 1:

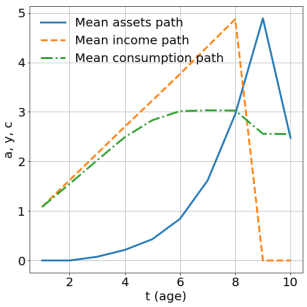

Figure 2:

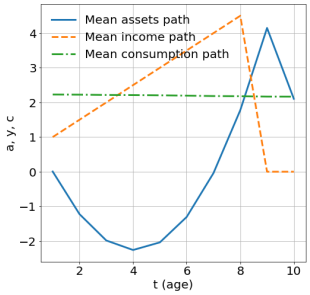

Figure 3:

Use three arrows to match the models (a), (b), (c) and (d) with the right Figure:

(a)

Figure 1

(b)

Figure 2

(c)

Figure 3

(d)

Motivate your answer. (Max 20 lines.)

[2]Under the assumption that income is 0 after retirement (age 9) and $a_{-1} = 0$.

Problem 2. Consider the following figure by Storesletten, Telmer, and Yaron (2004).

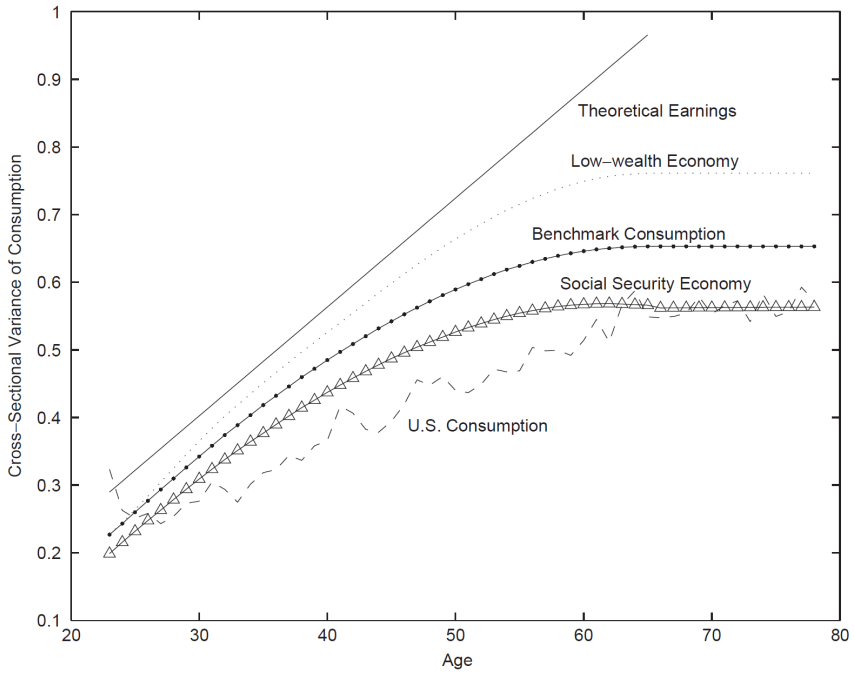

Fig. 5. The dashed line represents the empirical cross-sectional variance of consumption. The solid-dotted line represents the theoretical variance for the benchmark economy, whereas the dotted and triangle-marked lines represent the low wealth and social security economies, respectively, discussed in Section 4 of the text. All the economies have an identical pattern of earnings inequality, given by the solid line (which closely matches its empirical counterpart).

1. Explain why in every considered setting, consumption variance is always lower than the income variance of the corresponding age group. One explanation is expected, which applies to all settings at once. (Max 5 lines.)

2. Explain why the lines corresponding to the considered settings are in this particular order for each age group. Why are the differences bigger for older agents? (Max 20 lines.)

3. In all the settings shown on this figure, STY allowed for some borrowing. However, they also checked (not on this figure) what happens when borrowing

is ruled out completely, i.e. $a_t \geq 0$ holds in every period. Do you think they found higher or lower consumption variance relative to the benchmark? Was this effect more substantial for the young or the old? (Max 10 lines.)

Some reminders: In their paper,

- The Benchmark setting did not include social security.
- Social Security Economy differed from the benchmark by including social security.
- Low-Wealth Economy is like the benchmark setup, but with lower β, inducing less savings.

Problem 3. Please comment on the table below, taken from Cagetti and De Nardi (2006). Make sure you explain why results differ (or are similar) between the different lines and what we can learn from it. (Max 30 lines)

TABLE 7
THE ROLE OF BORROWING CONSTRAINTS AND VOLUNTARY BEQUESTS

	Capital-Output Ratio	Interest Rate	Wealth Gini	Entrepreneurs	Percentage Wealth in the Top			
					1%	5%	20%	40%
U.S. data	3.0	. . .	.8	7.55%	30	54	81	94
Baseline with entrepreneurs	3.0	6.5%	.8	7.50%	31	60	83	94
More stringent borrowing constraints: $f = .85$	2.7	7.5%	.7	6.90%	24	49	75	91
No altruism: $\eta = 0$, only involuntary bequests	2.5	9.3%	.7	7.55%	21	45	73	90
$\eta = 0$, recalibrated $\beta = .88$	3.0	6.4%	.8	7.9%	28	57	81	94

Some reminders: In their model,

- f denotes the fraction of entrepreneurial capital k that the entrepreneur can choose to run away with, instead of putting the capital into productive use. Baseline value was 0.75.
- β is the discount factor, as in all the other models we discussed. Baseline estimate was 0.865.
- η expresses how much an old agent cares about the value of their child. $\eta = 0$ means ignoring the child's utility, while $\eta = 1$ corresponds to perfect altruism. In their baseline model the authors assumed $\eta = 1$.

Problem 4. Assume that you are creating a model of the Swedish economy involving taxes. You make sure that the government budget is balanced and parameters are calibrated in a way that the model mimics reality in many desirable aspects. In particular, the discount factor β is chosen to match the capital/income ratio of the Swedish economy. Your final aim is to resolve the model with an alternative tax system and to figure out the welfare effects of moving into this alternative tax system by comparing value functions across the two models.

You get the following advice from your hairdresser: *When solving the model with the new taxes,*

(a) you shouldn't only change the taxes, but you have to recalibrate β as well,

(b) since otherwise in the new setting the capital/income ratio will be different than in the data,

(c) which would make your results meaningless.

Are each of the statements (a)-(c) correct or not? Motivate your answer! (Max 20 lines.)

Problem 5. Consider a household maximizing

$$\begin{aligned} &\max \sum_{t=0}^{\infty} \beta^t u\left(c_t, l_t\right) \\ &\text{s.t} \quad l_t = 1 - n_t \\ &c_t\left(1+\tau^c\right) + a_t + T = \left(1-\tau^n\right) w_t n_t + \left(1+r\right) a_{t-1} \quad \forall t \\ &a_{-1} \text{ is given} \end{aligned}$$

where l is leasure, n is labor supply, τ_n is a labor income tax and τ_c is consumption tax. u in concave in both arguments. There is no uncertainty.

1. Set up and solve the Lagrangian, and derive the intratemporal first-order condition, i.e. the FOC relating optimal leisure choice to the current marginal utility from consumption.
2. Explain the intuition behind this equation. (Max 10 lines.)
3. How does the presence of the two distortive taxes affect optimal labor and consumption choices, relative to a setting where the same amount of tax revenue is collected via a lump-sum tax instead? Argue formally and also explain the intuition. (Max 10 lines for the latter.)

Discussing the other first-order condition (i.e. the Euler equation) is not asked for in this exercise!