

Covariance of 58 and 60

$$y_{60} = \alpha + \underbrace{\varepsilon_{60}}_{\text{iid}} + \underbrace{\eta_{60}}_{\text{iid (presumably)}} + p\eta_{59} + p^2\eta_{58} + p^3\eta_{57} + \dots + p^{37}\eta_{23}$$

$$y_{58} = \alpha + \underbrace{\varepsilon_{58}}_{\text{iid}} + \dots + \eta_{58} + p\eta_{57} + \dots + p^{35}\eta_{23}$$

$$\alpha^2$$

$$p^2\eta_{58} + p^4\eta_{57} + \dots + p^{72}\eta_{23}$$

$$\text{Cov}(y_{60}, y_{58}) = \sigma_\alpha^2 + \underbrace{\sigma_\eta^2 (p^2 + p^4 + \dots + p^{72})}_{p^2 \frac{1 - p^{72}}{1 - p^2}}$$

$$p^2 \frac{1 - p^{72}}{1 - p^2}$$

This is another geometric sum.

$$a \frac{1 - r^{N+1}}{1 - r}$$

$$a = \sigma_\eta^2, \quad r = p^2, \quad N = 35$$

$$1. \max \frac{(x-a)^{1-\gamma} - 1}{1-\gamma} + \phi_1 \left(1 + \frac{a [1 + r(1-\tau_c)] (1-\tau_b)}{\phi_2}\right)^{1-\gamma}$$

such that  $a \geq 0$

Define  $A := [1 + r(1-\tau_c)](1-\tau_b)$

Interior maximum: negative from moving  
to other side

$$(x-a)^{-\gamma} = (\gamma-1) \frac{\phi_1}{\phi_2} A \left(1 + \frac{A}{\phi_2} a\right)^{-\gamma}$$

$$(x-a) = [(\gamma-1)A]^{-\frac{1}{\gamma}} \left[\frac{\phi_1}{\phi_2}\right]^{-\frac{1}{\gamma}} \left(1 + \frac{A}{\phi_2} a\right)$$

$$\text{Let } B := [(\gamma-1)A]^{-\frac{1}{\gamma}} \text{ and } \psi := \left(\frac{\phi_1}{\phi_2}\right)^{-\frac{1}{\gamma}}$$

$$x-a = B\psi \left(1 + \frac{A}{\phi_2} a\right)$$

$$a = \frac{x - B\psi}{1 + B\psi \frac{A}{\phi_2}}$$

$$\text{General maximum: } a = \frac{\max\{0, x - B\psi\}}{1 + B\psi \frac{A}{\phi_2}}$$



Q2

$$2. \quad \underbrace{\frac{a}{x}}_{\text{total resources}} = \underbrace{\frac{a}{x - B\Psi}}_{\text{bequest}} \cdot \frac{x - B\Psi}{x} = \underbrace{\frac{1}{1 + B\Psi \frac{A}{\phi_2}}}_{*} \cdot \underbrace{\left(1 - \frac{B\Psi}{x}\right)}_{\cdot}$$

\* From the found value of  $a$ , which gives a constant, asymptotic share strength of the bequest motive. The value asymptotically approaches this constant.

• luxury. If not luxury, then this would be a constant share of resources.

— we consume or eat in last period. Bequest luxury to consumption

Share of  $\frac{a}{x}$  should be constant if not a luxury.

$\phi_1$  : bequest motive  $\phi_2$  : luxury.

$$3. \quad \Psi = \left(\frac{\phi_1}{\phi_2}\right)^{-\frac{1}{\gamma}} \quad \text{So the relative } \phi_1 \text{ to } \phi_2 \text{ gives the measure of luxury, and } \frac{\phi_1}{\phi_2} \cdot \frac{B\Psi A}{\phi_2} \text{ gives the bequest motive.}$$

It is not exactly as put in the paper. They have these as  $\phi_1$  and  $\phi_2$  being luxury vs. bequest. Of course once we know about these factors  $\phi_1$  and  $\phi_2$  identify these concepts.

4. De Nardi :

$\phi_1$  : transfer wealth share (60% in US)

$\phi_2$  : average bequest left by singles (\$10,000).