Assignment 3

Inequality

Question 1

1 Identification based on covariances

In the lecture, we saw that STY identified the parameters of their income process based on the relation of age and the cross-sectional variance of the log income residual. In this exercise, you need to explore an alternative method, relying on the covariance structure of log income residuals.

Similarly to STY, assume that

$$u_{i,h} = \alpha_i + \epsilon_{i,h} + z_{i,h} \text{ and } z_{i,h} = \rho z_{i,h-1} + \eta_{i,h}$$

$$\eta_{i,h} \sim \mathcal{N}(0, \sigma_\eta^2) \quad \epsilon_{i,h} \sim \mathcal{N}(0, \sigma_\epsilon^2) \quad \alpha_i \sim \mathcal{N}(0, \sigma_\alpha^2)$$

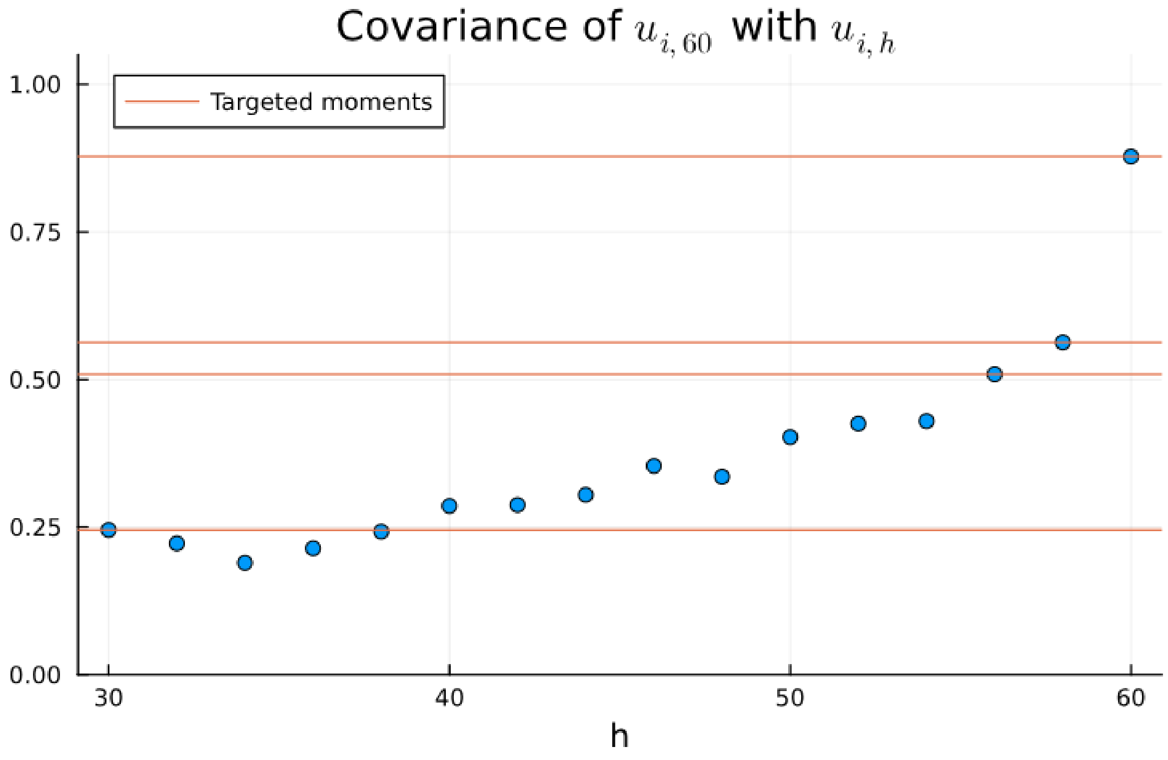
where u denotes the log income residual and h is age, expressed in years. Assume that $z_{i,22} = 0$, so the persistent income component of 22-year-old individuals is 0.

1.1 Express the following quantities as formulas containing the four parameters ($\sigma\alpha$, $\sigma\epsilon$, $\sigma\eta$, ρ):

1.2

```
function theory_covs(x)
    # elements of x: σ_α^2, σ_ϵ^2, σ_η^2, ρ
    ρ = x[4]
    V = x[1] + x[2] + x[3] * (ρ^76-1)/(ρ^2-1)
    Cov2 = x[1] + x[3] * ρ^2 * (ρ^72-1)/(ρ^2-1)
    Cov4 = x[1] + x[3] * ρ^4 * (ρ^68-1)/(ρ^2-1)
    Cov30 = x[1] + x[3] * ρ^30 * (ρ^16-1)/(ρ^2-1)
    return [V,Cov2,Cov4,Cov30]
end | theory_covs (generic function with 1 method)
```

1.3 Find the corresponding moments from the candidate parameters in PSID

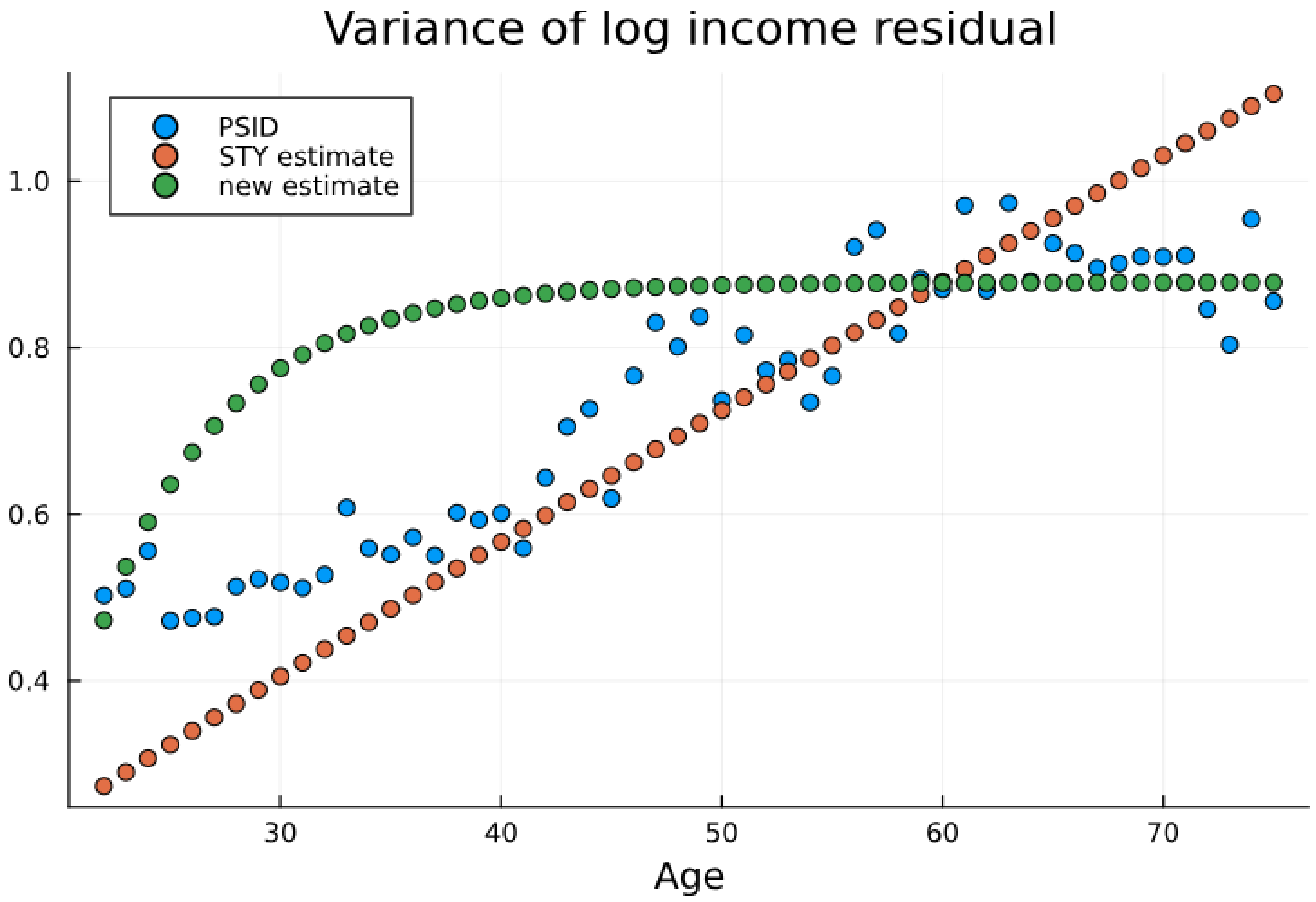


1.4 Find parameters that make the theoretical moments equal to the empirical ones.

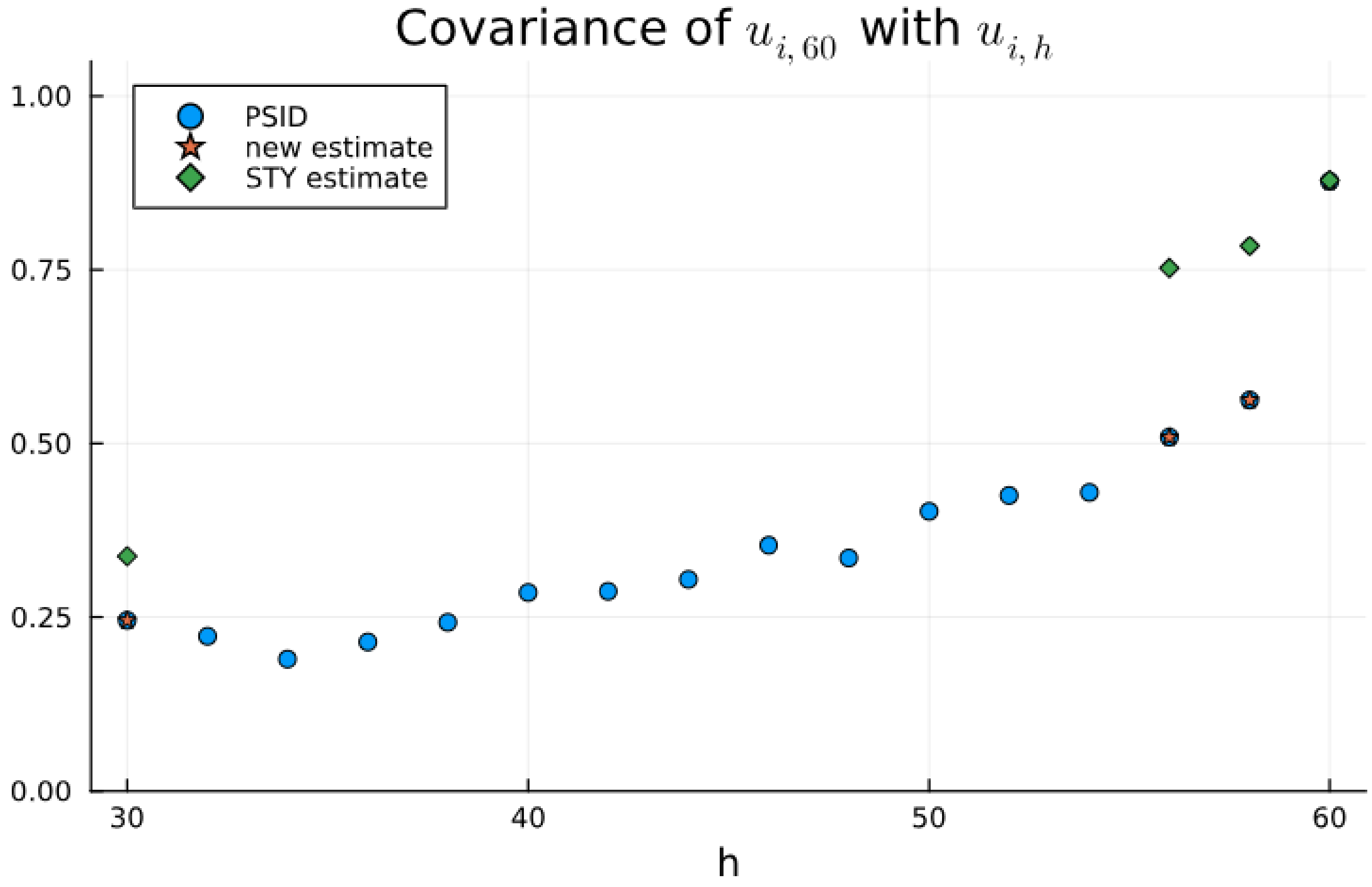
- [σα, σϵ, ση, ρ] = [0.2221, 0.2507, 0.0639, 0.9179]

```
function theory_covs(x)
    # elements of x: σ_α^2, σ_ϵ^2, σ_η^2, ρ
    ρ = x[4]
    V = x[1] + x[2] + x[3] * (ρ^76-1)/(ρ^2-1)
    Cov2 = x[1] + x[3] * ρ^2 * (ρ^72-1)/(ρ^2-1)
    Cov4 = x[1] + x[3] * ρ^4 * (ρ^68-1)/(ρ^2-1)
    Cov30 = x[1] + x[3] * ρ^30 * (ρ^16-1)/(ρ^2-1)
    return [V,Cov2,Cov4,Cov30]
end

using NLsolve

function diff_theory_data(x)
    theory_covs(x) .- data_moments
end

sol = nlsolve(diff_theory_data, [0.05, 0.05, 0.05, 0.8])
```

1.5 Compare your parameters to the ones obtained by STY.



1.5 Comparing on the moments we targeted.



Question 3

3 Capital tax and welfare

Consider the setting from 41_*Inequality_inheritence.jl* with bequest motive and ability transfer as your benchmark. As an alternative setting, consider an otherwise identical economy where capital income tax equals 0.

3.1 Compute the labor income tax level that balances the budget for your alternative model.

```
# setting in class
ep_both = EconPars(β = 0.9313, ρ_α = 0.6, ϕ_1 = -55.0) | EconPars{Float64}
#τ_both = find_τ_l(ep_both, np, N = 10000, M = 20)
τ_both = 0.1430

sol_both = solve(ep_both,np, τ_both) | Solution{Interpolations.Extrapolation{Float64, 1, Interpolations.GriddedInterpolation{Float64, 1, V…


# setting without capital income
ep_no_tauc = EconPars(β = 0.9313, ρ_α = 0.6, ϕ_1 = -55.0, τ_c = 0.0)
#τ_no_tauc = find_τ_l(ep_no_tauc, np, N = 10000, M = 20)
τ_no_tauc = 0.2074 # need higher labor income tax to compensate for no capital income tax

sol_no_tauc = solve(ep_no_tauc,np, τ_no_tauc) | Solution{Interpolations.Extrapolation{Float64, 1, Interpolations.GriddedInterpolation{Float64, 1, V…
```

3.2(a) Write a function, that to any cash-on-hand level assigns the welfare gain of being in the alternative model instead of the benchmark one

```
gr = 0.01:0.01:10.0 | 0.01:0.01:10.0

# middle alpha, out of 11, the 6th element is the middle one
plot(gr,x-> (sol_no_tauc.vf[6,1,6](x)/sol_both.vf[6,1,6](x))^(1/(1-1.5)))
 | Plot{Plots.GRBackend() n=1}
# lowest alpha
plot(gr,x-> (sol_no_tauc.vf[1,1,6](x)/sol_both.vf[1,1,6](x))^(1/(1-1.5))) | Plot{Plots.GRBackend() n=1}

# highest alpha
plot(gr,x-> (sol_no_tauc.vf[11,1,6](x)/sol_both.vf[11,1,6](x))^(1/(1-1.5))) | Plot{Plots.GRBackend() n=1}
```

Comparing welfare from the lecture notes.

Quantifying welfare differences

- So let's say $V_0^A(state_0) > V_0^B(state_0)$. Then being born in setting A is better. By how much?
- Nice answer when $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$.
- Imagine you are in setting B, but someone increases your consumption by $d\%$ every period? What would be your value?

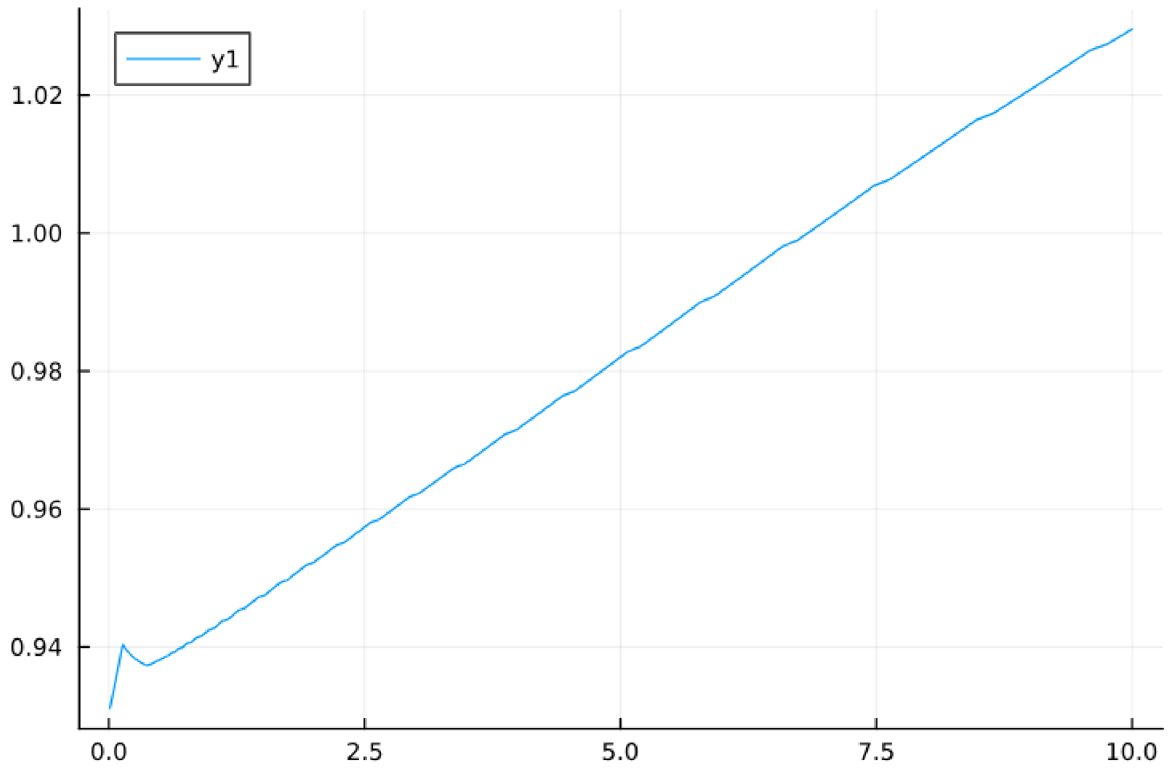
$$\mathbb{E}_0 \sum_{t=0}^{\infty \text{ or } T} \beta^t \frac{((1+d)\cdot c_t)^{1-\gamma}}{1-\gamma} = (1+d)^{1-\gamma} \mathbb{E}_0 \sum_{t=0}^{\infty \text{ or } T} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} = (1+d)^{1-\gamma} V_0^B$$

- which d would make you indifferent between model A and B?

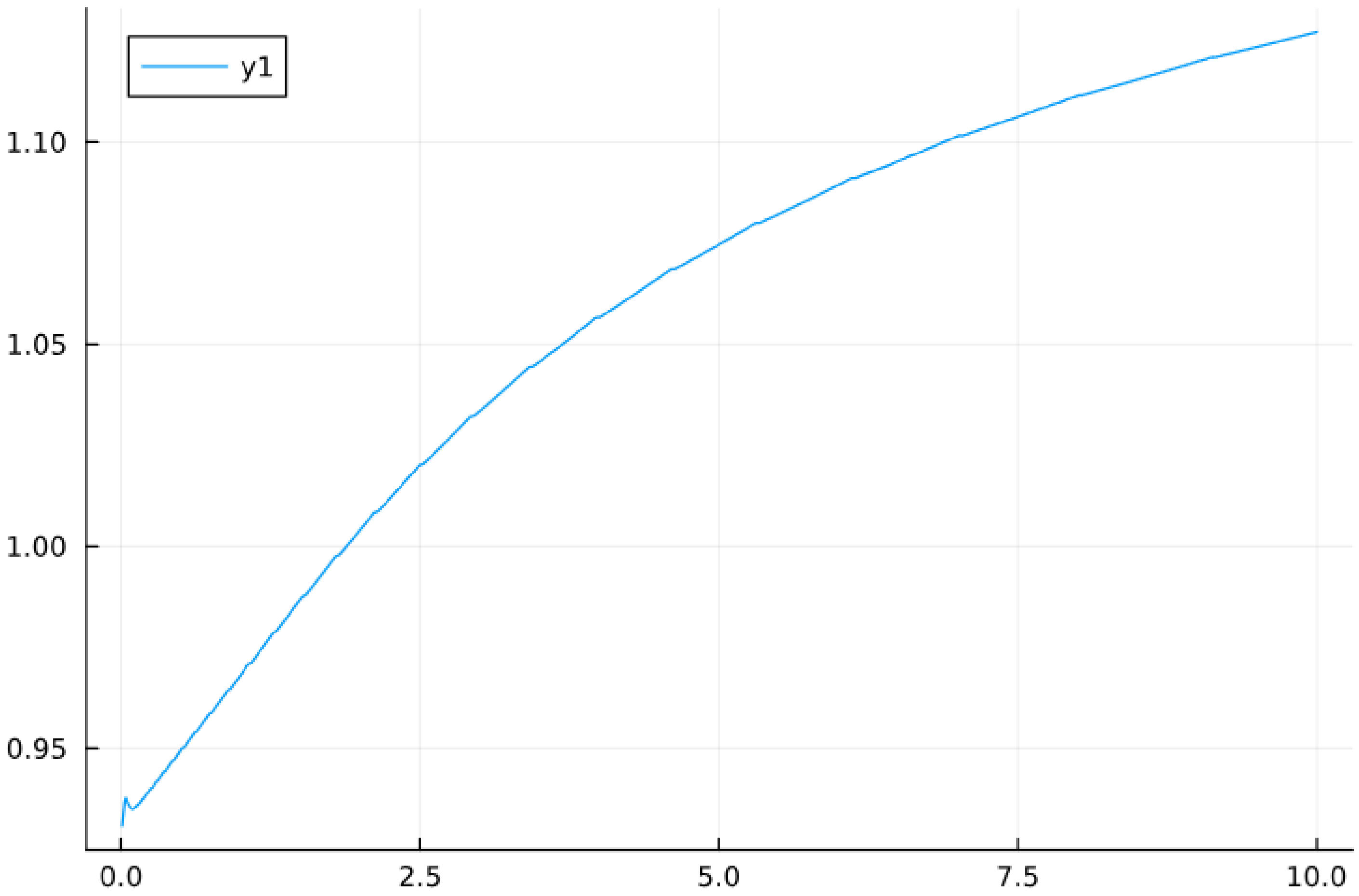
$$(1+d)^{1-\gamma} V_0^B = V_0^A \quad \Rightarrow \quad 1+d = \left(\frac{V_0^A}{V_0^B}\right)^{\frac{1}{1-\gamma}}$$

d has a meaningful interpretation (extra consumption in percents, relative to optimal path)

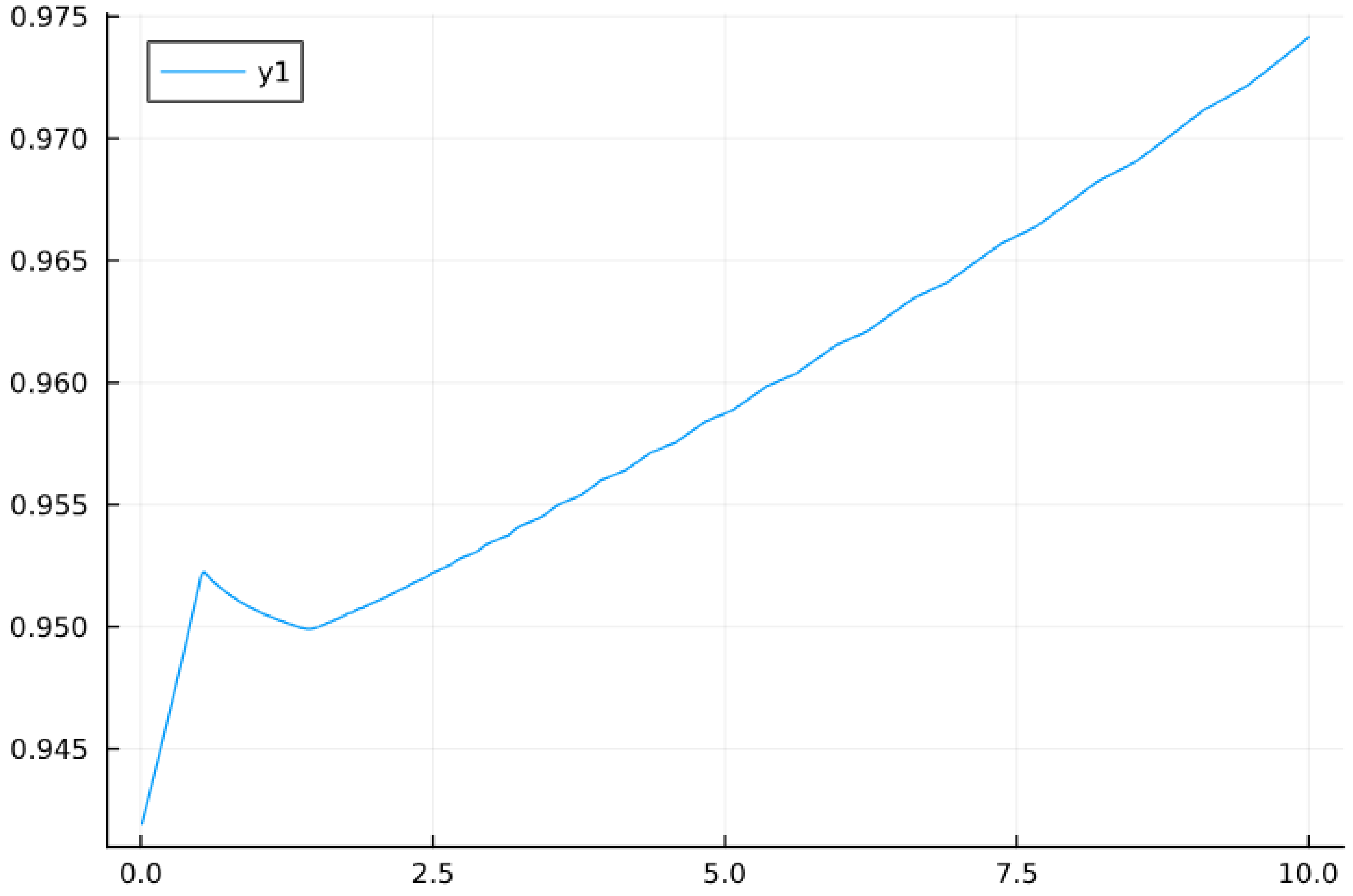
3.2 Middle Alpha



3.3 Low Alpha



3.3 High Alpha



3.4 To quantify the average gain or loss from being in the alternative world, compute the expected (average) value of age 22 agents in each economy.

```
# we can use middle z value, since in simulations age 22 agents start with z=0
v2 = mean(sol_no_tauc.vf[αis[n,end],1,6](incomes[n,1,end]) for n in 1:10000)  -40.326426492814946

v = mean(sol_both.vf[αis[n,end],1,6](incomes[n,1,end]) for n in 1:10000)
 -39.06252050268808
# without being born, (and thus knowing your initial state), one would prefer to be born in the economy with positive capital income tax
(v2/v)^(1/(1-1.5))  0.9382985548754622
```

3.5 How do each of the following simplifications affect your findings, in your opinion?

(a) We don't close the capital market, so capital is not used in production.

(b) Our agents do not expect to receive bequests (even though they do receive them).