Neoclassical Growth

Motivation

- So far, have assumed individuals are endowed with goods
- Haven't been concerned with how these goods were generated or how individuals came into possession of them
- Now, assume individuals endowed with labor (or time) rather than goods
- Introduce production technology

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Environment

- People endowed with labor, not goods
- Goods produced by combining labor (L) and forgone consumption, which call capital (K):

$$Y(t) = F(K(t), L(t))$$

- Labor endowments denoted by:

$$\triangle_t^h = [\triangle_t^h(t), \triangle_t^h(t+1)]$$

delta

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Environment

- For now, assume people don't care about leisure:

$$u_t^h = \log c_t^h(t) + \beta \log c_t^h(t+1)$$

- People spend all available time working, so total amount of labor at date t is:

$$L(t) = \sum_{h=1}^{N(t)} \triangle_t^h(t) + \sum_{h=1}^{N(t-1)} \triangle_{t-1}^h(t)$$

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Feasibility

- A feasible allocation is defined as in the economy with storage
- An allocation is feasible if there is a sequence $K(t)$ such that $K(t) \geq 0$ and

$$Y(t) \geq C(t) + K(t+1) \tag{1}$$

- This looks like economy with full depreciation, but can add any rate of depreciation if want

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Firm Profit Maximization

- In competitive equilibrium, factors of production paid their marginal products

$$w(t) = MPL = \frac{\partial F(.)}{\partial L(t)} \tag{2}$$

$$R(t) = MPK = \frac{\partial F(.)}{\partial K(t)} \tag{3}$$

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Production Function

θ = capital share

- Cobb Douglas production function commonly assumed

$$Y(t) = K(t)^{\theta} L(t)^{1-\theta}$$

- With this production function, factor payments exhaust output profit max

$$K(t)^{\theta} L(t)^{1-\theta} - w(t)L(t) - R(t)K(t)$$

$$K(t): \underbrace{\theta K(t)^{\theta-1} L(t)^{1-\theta}}_{MPK = \theta \frac{Y(t)}{K(t)}} - R(t) = 0$$

$$L(t): \underbrace{(1-\theta) K(t)^{\theta} L(t)^{-\theta}}_{MPL = (1-\theta)\frac{Y(t)}{L(t)}} - w(t) = 0$$

profits:

$$\rightarrow \underbrace{K(t)^{\theta} L(t)^{1-\theta}}_{Y(t)} - (1-\theta)\frac{Y(t)}{\cancel{L(t)}} \cdot \cancel{L(t)} - \theta \frac{Y(t)}{\cancel{K(t)}} \cdot \cancel{K(t)} = 0$$

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Budget Constraints

Budget constraints individual h faces:

$$c_t^h(t) = w(t)\triangle_t^h(t) - l^h(t) - k^h(t+1) \tag{4}$$

$$c_t^h(t+1) = \underbrace{w(t+1)\triangle_t^h(t+1)}_{\text{earnings}} + r(t)l^h(t) + R(t+1)k^h(t+1) \tag{5}$$

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Indifference Between Holding Two Assets

If $R(t+1) > r(t)$

want to borrow as much as possible, but no one will lend

$\rightarrow$ not an eq.

In equilibrium we must have:

$$R(t+1) = r(t) \quad (6)$$

If $R(t+1) < r(t) \leftrightarrow$ no capital

$MPK \rightarrow \infty \rightarrow K(t+1) > 0$

- Want to investigate long run behavior of economy under competitive equilibrium
- To do so, make some further simplifying assumptions
  - Population grows at rate n: $N(t+1) = (1+n)N(t)$ (normalize $N(0) = 1$)
  - Assume all young people endowed with one unit of labor, and all old people endowed with no labor

$$\max \quad \log c_t^h(t) + \beta \log c_t^h(t+1)$$

$$\text{s.t.} \quad c_t^h(t) + \frac{c_t^h(t+1)}{r(t)} = w(t)\Delta_t^h(t)$$

$$\rightarrow c_t^h(t+1) = r(t)w(t)\Delta_t^h(t) - r(t)c_t^h(t)$$

$$c_t^h(t): \quad \frac{1}{c_t^h(t)} - \frac{\beta r(t)}{c_t^h(t+1)} = 0 \rightarrow c_t^h(t+1) = \beta r(t) c_t^h(t)$$

$$c_t^h(t) + \beta c_t^h(t) = w(t)\Delta_t^h(t)$$

$$\rightarrow c_t^h(t) = \frac{w(t)\Delta_t^h(t)}{1+\beta}$$

$$\rightarrow S_t = \frac{\beta}{1+\beta} w(t) \underbrace{\Delta_t^h(t)}_{=1}$$

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Optimality Conditions

- Consumption satisfies:

$$\frac{c_t^h(t+1)}{c_t^h(t)} = \beta r(t) \tag{7}$$

- Savings satisfies:

$$s_t^h(t) = \frac{\beta}{1+\beta} w(t) \tag{8}$$

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Aggregate Savings

$N(t) = L(t)$

$k(t) = \frac{K(t)}{N(t)}$

$N(t) = \frac{N(t+1)}{1+n}$

- Thus, the aggregate savings function is:

$$\frac{K(t+1)}{N(t)} = \frac{\beta}{1+\beta} w(t) N(t) = \frac{\beta}{1+\beta} \underbrace{(1-\theta) \frac{K(t)^{\theta} N(t)^{1-\theta}}{N(t)}} \tag{9}$$

$$k(t+1) = \frac{\beta}{1+\beta} \frac{(1-\theta)}{(1+n)} k(t)^{\theta}$$

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Aggregate Savings in Per Capita Terms

- Divide through by $N(t)$

$$k(t+1) = \frac{\beta(1-\theta)}{(1+\beta)(1+n)} k(t)^{\theta} \tag{10}$$

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What happens in the long run?

- For a given $k(0)$, this gives us $k(1)$, $k(2)$, etc.
- What happens in the long run? Does $k(t)$ tend to a limit that is independent of $k(0)$?
  - Yes, as long as $k(0) > 0$
  - This limit solves

k^* steady state

$$k^* = \frac{\beta(1-\theta)}{(1+\beta)(1+n)} (k^*)^\theta \tag{11}$$

  - and thus

$$k^* = \left(\frac{\beta(1-\theta)}{(1+\beta)(1+n)} \right)^{\frac{1}{1-\theta}} \tag{12}$$

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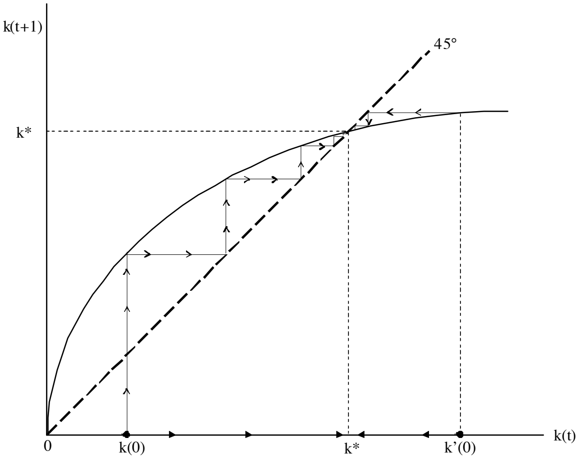
What happens in the long run?



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What happens in the long run?

- Looks a lot like the Solow model
  - Difference is that savings rate endogenous, but still constant

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Compare with Social Planner

- Compare equilibrium with choice of social planner wishing to maximize weighted average of all generations' utilities
- Social planner maximizes

$$\sum_{t=0}^{\infty} \beta_S^t U(t)$$

- β_S reflects how social planner values utilities of different generations

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Compare with Social Planner

- Plugging in utility function, this implies that social planner maximizes:

$$\sum_{t=0}^{\infty} \beta_S^t (\log c_t(t) + \beta \log c_t(t+1))$$

- subject to resource constraints

for each t

$$F(K(t), N(t)) = K(t+1) + N(t)c_t(t) + N(t-1)c_{t-1}(t) \quad \Big| : N(t)$$

Planner's Lagrangian:

$$\mathcal{L} = \sum_t \beta_s^t \left(\log c_t^h(t) + \beta \log c_t^h(t+1)\right) + \sum_t \lambda_t \left[f(k(t)) - (1+n)k(t+1) - c_t(t) - \frac{c_{t-1}(t)}{1+n} \right]$$

$$c_t^h(t): \quad \frac{\beta_s^t}{c_t^h(t)} - \lambda_t = 0$$

$$c_t^h(t+1): \quad \frac{\beta_s^t \beta}{c_t^h(t+1)} - \frac{\lambda_{t+1}}{1+n} = 0$$

$$\frac{c_t^h(t+1)}{\beta c_t^h(t)} = \frac{\lambda_t}{\lambda_{t+1}} (1+n)$$

$$k(t+1): -\lambda_t (1+n) + \lambda_{t+1} f'(k(t+1)) = 0 \rightarrow \frac{\lambda_t}{\lambda_{t+1}} = \frac{f'(k(t+1))}{1+n} = \frac{R(t+1)}{1+n}$$

$$= \frac{r(t)}{1+n}$$

$$\rightarrow c_t^h(t+1) = \beta r(t) c_t^h(t)$$

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Compare with Social Planner

- Divide through by $N(t)$ and utilize population growth equation

$$f(k(t)) = (1+n)k(t+1) + c_t(t) + \frac{c_{t-1}(t)}{1+n}$$

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Compare with Social Planner

- Combining optimality conditions w.r.t. $c_t(t)$, $c_t(t+1)$ and $k(t+1)$ gives

$$\frac{c_t(t+1)}{c_t(t)} = \beta f'(k(t+1)) = \beta MPK = \beta R(t+1) = \beta r(t)$$

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Compare with Social Planner

- Allocate consumption of given individual in exactly same way as individual him/herself would
- Allocations across generations depend on (potentially different) weights planner attaches to different generations

Neoclassical Growth

Overaccumulation?

- Resource constraint in steady state is:

$$f(k^*) - (1+n)k^* = c^*_{young} + \frac{c^*_{old}}{1+n} \equiv c^*$$

- Therefore

$$\frac{\partial c^*}{\partial k^*} = f'(k^*) - (1+n)$$

- k_{gold} defined as

$$f'(k_{gold}) = (1+n)$$

Neoclassical Growth

Overaccumulation?

- If $k^* > k_{gold}$, then $\frac{\partial c^*}{\partial k^*} < 0$
  - Reducing savings can increase total consumption for everybody
  - Economy referred to as dynamically inefficient
  - Involves overaccumulation of capital
  - Another way of expressing dynamic inefficiency is $r^* < 1 + n$

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Adding Technological Change

- Now add exogenous technological change
- General production function now given by:

$$Y(t) = F(K(t), \gamma(t)L(t))$$

  - $\gamma(t)$ is labor-augmenting productivity at t
  - $\gamma(t+1) = (1+g)\gamma(t)$
- With Cobb-Douglas this takes form:

$$Y(t) = K(t)^{\theta}[\gamma(t)L(t)]^{1-\theta}$$

- Aggregate savings function is now:

$$\underbrace{K(t+1)}_{\gamma(t)N(t)} = \frac{\beta}{1+\beta} w(t)N(t) = \frac{\beta}{1+\beta}(1-\theta)K_t^{\theta}\underbrace{[\gamma(t)N(t)]^{1-\theta}}_{\gamma(t)N(t)} \tag{13}$$

$$\gamma(t+1)N(t+1) = (1+g)(1+n)\gamma(t)N(t)$$

Neoclassical Growth

Aggregate Savings in Intensive Form

$\tilde{k}(t)$

- Define $k(t) = \frac{K(t)}{\gamma(t)L(t)} \left(= \frac{K(t)}{\gamma(t)N(t)}\right)$
- Then the aggregate savings function in intensive form is:

$$k(t+1) = \frac{\beta(1-\theta)}{(1+\beta)(1+n)(1+g)} k(t)^{\theta} \qquad (14)$$

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What happens in the long run?

- Now capital stock in efficient labor is constant

$$k^* = \left(\frac{\beta(1-\theta)}{(1+\beta)(1+n)(1+g)} \right)^{\frac{1}{1-\theta}} \tag{15}$$

- So, capital stock per capita grows at rate g

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Overaccumulation?

Define: $c(t) = \dfrac{N(t)c_t(t) + N(t-1)c_{t-1}(t)}{\gamma(t)N(t)}$

- Resource constraint of social planner is now

$$F(K(t), \gamma(t)N(t)) = K(t+1) + N(t)c_t(t) + N(t-1)c_{t-1}(t)$$

- Now, divide through by $\gamma(t)N(t)$:

$$f(k(t)) = (1+n)(1+g)k(t+1) + c(t)$$

$$f(k^*) = k^*(1+n)(1+g) + c^*$$

$$\rightarrow \frac{\partial c^*}{\partial k^*} = \underbrace{f'(k^*)}_{r} - (1+g)(1+n)$$

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Overaccumulation?

- Dynamic inefficiency empirically much more likely than when $g = 0$
- Social security way of dealing with overaccumulation

Why are some countries rich while others are poor?

Development Accounting: Hall and Jones (1999)

- View the world through the lens of the Cobb Douglas production function
- Output in country i produced according to

$$Y_i = K_i^{\theta}(A_i H_i)^{1-\theta} \tag{16}$$

where K_i denotes stock of physical capital, H_i human capital-augmented labor and A_i labor-augmenting measure of productivity (TFP)

total factor productivity

Why are some countries rich while others are poor?

Development Accounting: Hall and Jones (1999)

- Assume labor L_i homogeneous within a country and each unit of labor trained with E_i years of schooling
- Human capital-augmented labor given by

$$H_i = e^{\phi(E_i)} L_i \tag{17}$$

  - $\phi(E)$ reflects efficiency of a unit of labor with E years of schooling relative to one with no schooling
  - Derivative $\phi'(E)$ is return to schooling estimated in a Mincerian wage regression
  - If $\phi(E) = 0$ for all E this is a standard production function with undifferentiated labor

Why are some countries rich while others are poor?

Development Accounting: Hall and Jones (1999)

i = country

- Convenient to rewrite production function in terms of output per worker, $y = Y/L$:

$\theta \sim 1/3$

$$y_i = \left(\frac{K_i}{Y_i}\right)^{\theta/(1-\theta)} h_i A_i \qquad (18)$$

where $h = H/L$

A also called Solow residual

Why are some countries rich while others are poor?

Development Accounting: Hall and Jones (1999)

- This equation allows us to decompose differences in output per worker across countries into differences in the capital-output ratio, educational attainment, and productivity
- With data on output, capital, and schooling, and knowledge of θ and $\phi(.)$, can calculate level of productivity as residual

Why are some countries rich while others are poor?

Development Accounting: Hall and Jones (1999)

		Contribution from		
Country	Y/L	$(K/Y)^{\alpha/(1-\alpha)}$	H/L	A
United States	1.000	1.000	1.000	1.000
Canada	0.941	1.002	0.908	1.034
Italy	0.834	1.063	0.650	1.207
West Germany	0.818	1.118	0.802	0.912
France	0.818	1.091	0.666	1.126
United Kingdom	0.727	0.891	0.808	1.011
Hong Kong	0.608	0.741	0.735	1.115
Singapore	0.606	1.031	0.545	1.078
Japan	0.587	1.119	0.797	0.658
Mexico	0.433	0.868	0.538	0.926
Argentina	0.418	0.953	0.676	0.648
U.S.S.R.	0.417	1.231	0.724	0.468
India	0.086	0.709	0.454	0.267
China	0.060	0.891	0.632	0.106
Kenya	0.056	0.747	0.457	0.165
Zaire	0.033	0.499	0.408	0.160
Average, 127 Countries:	0.296	0.853	0.565	0.516
Standard Deviation:	0.268	0.234	0.168	0.325
Correlation w/ Y/L (logs)	1.000	0.624	0.798	0.889
Correlation w/ A (logs)	0.889	0.248	0.522	1.000

Why are some countries rich while others are poor?

Development Accounting: Caselli (2005)

- Similar decomposition, but construct different metric
- Take per capita production function

$$y = Ak^{\alpha}h^{1-\alpha} \equiv Ay_{KH}$$

refer to y_{KH} as factor only model

Why are some countries rich while others are poor?

Development Accounting: Caselli (2005)

- How successful is factor only model in explaining cross-country income differences?
  - Compare observed variation in y_{KH} to observed variation in y
  - In other words, suppose all countries had same efficiency A, what would world income distribution look like compared to actual one?

Variance decomposition

$$var[log(y)] = var[log(y_{KH})] + var[log(A)] + 2cov[log(A), log(y_{KH})]$$

If all countries have same TFP, we would have

$$var[log(A)] = cov[log(A), log(y_{KH})] = 0$$

Why are some countries rich while others are poor?

Development Accounting: Caselli (2005)

- Thus, the first measure of success of factor-only model is:

$$success_1 = \frac{var[log(y_{KH})]}{var[log(y)]}$$

- A second measure of success of factor-only model is:

$$success_2 = \frac{y_{KH}^{90}/y_{KH}^{10}}{y^{90}/y^{10}}$$

where 90 and 10 refer to 90th and 10th percentiles

- Results for full sample
    - $success_1$: 0.39
    - $success_2$: 0.34

- Results for sub-samples

Sub-sample	Obs.	var[log(y)]	var[log(y_{KH})]	$success_1$
Above the median	47	0.172	0.107	0.620
Below the median	47	0.624	0.254	0.407
OECD	24	0.083	0.050	0.606
Non-OECD	70	1.047	0.373	0.356
Africa	27	0.937	0.286	0.305
Americas	25	0.383	0.179	0.468
Asia and Oceania	25	0.673	0.292	0.434
Europe	17	0.136	0.032	0.233
All	94	1.297	0.500	0.385

- Seminal papers conclude that accumulated factors can't account for majority of cross-country income differences
- Calls for theory of TFP?
- Subsequent papers try to unpack black box of TFP
  - Health
  - Better measures of human capital

Why are some countries rich while others are poor?

Development Accounting: Weil (2007)

- Adding health to the production function
- Output in country i produced according to

$$Y_i = A_i K_i^{\theta} H_i^{1-\theta}, \;\; H_i = h_i v_i L_i \tag{19}$$

where h_i denotes per worker human capital in form of education, v_i per worker human capital in form of health and L_i number of workers

- How to measure health?
  - Adult height
  - Adult survival rate
  - Age at onset of menstruation

Why are some countries rich while others are poor?

Development Accounting: Weil (2007)

- Results
  - Eliminating health differences across countries would reduce variance of log GDP per worker by 9.9%
  - And would reduce ratio of GDP per worker at 90th percentile to GDP per worker at 10th percentile from 20.5 to 17.9

Why are some countries rich while others are poor?

Development Accounting: Lagakos et al. (2018)

- Quality of education not just quantity?
  - Return to education not the same across countries
  - Return to education not the same for everyone in a country
- Human capital accumulation after finishing schooling?

Why are some countries rich while others are poor?

Development Accounting: Lagakos et al. (2018)

- Study US immigrants to understand differences in life cycle human capital accumulation across countries
    - Advantage that all workers observed in common labor market

Why are some countries rich while others are poor?

Development Accounting: Lagakos et al. (2018)

- Insight from data
  - Returns to experience lower among immigrants from poor countries than from rich countries
- Potential mechanisms
  - Workers in poor countries accumulate less human capital
  - Immigrants from poor countries less strongly selected on learning ability than counterparts in rich countries
  - Immigrants from poor countries lose larger fraction of their skills after migrating

Why are some countries rich while others are poor?

Development Accounting: Lagakos et al. (2018)

- Insight from data
  - Returns to experience lower among immigrants from poor countries than from rich countries
- Potential mechanisms
  - **Workers in poor countries accumulate less human capital**
  - Immigrants from poor countries less strongly selected on learning ability than counterparts in rich countries
  - Immigrants from poor countries lose larger fraction of their skills after migrating

Why are some countries rich while others are poor?

Development Accounting: Lagakos et al. (2018)

- Use estimated returns to experience among US immigrants to construct stocks of human capital from experience in each source country
- Compute success measure from Caselli (2005) with these returns/stocks
  - Differences in accumulated factors K and H account for 60% rather than 40%

Why are some countries rich while others are poor?

Fundamental Causes of Growth

- Why do countries end up with different technology and accumulation choices?
  - Luck
  - Geography
  - Institutions
  - Culture