Storage

Motivation

- So far, have assumed that there is no technology for transforming one type of good into another type of good
- In our dynamic context, this amounts to assuming goods are perishable
- Alternatively, can assume that goods are storable
- Here explore implications of that possibility

Storage

Technology

- The storage technology is a linear, constant returns to scale, intertemporal technology that transforms time t goods into time $t+1$ goods
  - For every unit of good stored, λ units are returned in the next period, where $\lambda \geq 0$
  - Let $K(t+1)$, called capital, be the amount of time t goods put into storage

Storage

Feasibility

Definition

A path of total consumption, $\{C(t)\}_{t=1}^{\infty}$*, is feasible for an economy with storage if, given* $K(1) \geq 0$*, there exist a non-negative sequence* $\{K(t)\}_{t=2}^{\infty}$ *that satisfies*

$$Y(t) + \lambda K(t) \geq C(t) + K(t+1)$$

for all $t \geq 1$

(Y(t) includes crop from land)

Storage

Budget Constraints

Let $k^h(t+1)$ denote the amount of time t goods put into storage by individual h, then we have the following budget constraints (assuming unanimity of expectations):

$$
\begin{aligned}
c_t^h(t) &= \omega_t^h(t) - l^h(t) - p(t)a^h(t) - k^h(t+1) \\
c_t^h(t+1) &= \omega_t^h(t+1) + r(t)l^h(t) \\
&+ d(t+1)a^h(t) + p^e(t+1)a^h(t) + \lambda k^h(t+1)
\end{aligned}
$$

$\ell^h(t) = \ldots$

The life-time budget constraint is:

$$
\begin{aligned}
c_t^h(t) + \frac{c_t^h(t+1)}{r(t)} &= \omega_t^h(t) + \frac{\omega_t^h(t+1)}{r(t)} \\
&\quad - a^h(t)\left(p(t) - \frac{d(t+1) + p^e(t+1)}{r(t)}\right) \\
&\quad - k^h(t+1)\left(1 - \frac{\lambda}{r(t)}\right)
\end{aligned}
$$

Thus, in equilibrium we must have

$$p(t) = \frac{d(t+1) + p^e(t+1)}{r(t)}$$

if $r(t) < \lambda$

$\rightarrow \ell_t^h(t) \rightarrow -\infty$

$k_t^h(t+1) \rightarrow \infty$

$\rightarrow$ not feasible

$$r(t) \geq \lambda$$

$$k^h(t+1)\left(1 - \frac{\lambda}{r(t)}\right) = 0$$

But if $r(t) > \lambda$

$\rightarrow$ no storage, can be an equilibrium

Storage
Market Clearing

Market clearing requires $\sum_{h=1}^{N(t)} l^h(t) = 0$, $\sum_{h=1}^{N(t)} a^h(t) = A$ and

$$\sum_{h=1}^{N(t)} c_t^h(t) + \sum_{h=1}^{N(t-1)} c_{t-1}^h(t) + \sum_{h=1}^{N(t)} k^h(t+1) =$$

$$= \sum_{h=1}^{N(t)} \omega_t^h(t) + \sum_{h=1}^{N(t-1)} \omega_{t-1}^h(t) + \sum_{h=1}^{N(t-1)} d(t)a^h(t-1) + \lambda \sum_{h=1}^{N(t-1)} k^h(t)$$

or

$$C(t) + K(t+1) = Y(t) + \lambda K(t)$$

Storage

Aggregate Savings

- As before, aggregate savings equals total value of all assets in economy
- Noting that sum of private loans equals zero, we have :

$$
\begin{aligned}
S_t(r(t)) &= \sum_{h=1}^{N(t)} s_t^h(t) = \sum_{h=1}^{N(t)} \left[l^h(t) + p(t)a^h(t) + k^h(t+1) \right] \\
&= \sum_{h=1}^{N(t)} \left[p(t)a^h(t) + k_t^h(t+1) \right] = p(t)A + K(t+1)
\end{aligned}
$$

Storage
Perfect Foresight

As before, perfect foresight implies

$$p^e(t+1) = p(t+1)$$

Definition

A perfect foresight competitive equilibrium for an economy with storage and land is a nonnegative sequence $\{p(t), r(t), K(t+1)\}_{t=1}^{\infty}$, *that for all* $t \geq 1$ *satisfies*

1. $S_t(r(t)) = p(t)A + K(t+1)$
2. $p(t) = \frac{d(t+1)+p(t+1)}{r(t)}$
3. $r(t) \geq \lambda$
4. $K(t+1)\left(1 - \frac{\lambda}{r(t)}\right) = 0$

given initial conditions $\{a^h(1), k^h(1), \omega_0^h(1)\}_{h=1}^{N(0)}$ *and sequences* $\{D(t), \{\omega_t^h\}_{h=1}^{N(t)}\}_{t=1}^{\infty}$

Finding an equilibrium with guess and verify

1. Choose a price at t and see whether the implied prices at $t+1$ obey the equilibrium conditions or
2. Guess that a stationary equilibrium exists

- A stationary equilibrium satisfies

$$S(r) = pA + K \tag{1}$$

$$p = \frac{d+p}{r} \tag{2}$$

$$r \geq \lambda$$

$$K\left(1 - \frac{\lambda}{r}\right) = 0$$

- Two possibilities:

1. Assume that $K = 0$: Solve for r and p using eq. (1) and (2). Check that $r \geq \lambda$.
2. Assume that $r = \lambda$: Solve for p using eq. (2) and check that (1) implies that $K \geq 0$, i.e., that $S(r) - pA \geq 0$.

Example

Consider a two-period OLG environment with storage but no land.
Let $u_t^h = \log c_t^h(t) + \beta \log c_t^h(t+1)$ *for all* h, t.
Also, let $\omega_t^h = [2, 1]$ *for all* h, t *and let* $N(t) = 1$ *for all* t.
Let $\beta = 1$. *The storage parameter is* $\lambda = 1$.
In each period, the government must spend $G = 3/40$ *on something useless. To finance this, it taxes the gross return on savings at the constant proportional rate* τ_k. *There is no government debt.*

Storage

Example

a. Define a feasible allocation in this environment.

b. Write down the budget constraint of the government.

c. Derive the savings function for this economy. Sketch a Laffer curve for this case. Indicate the two possible values of the tax rate that balance the government budget.

tax revenue as a function of the tax rate

d. Find the two competitive equilibria and compare.

Feasible allocation:

$$C(t) + K(t+1) + \underbrace{G}_{3/40} \leq \underbrace{Y(t)}_{3} + \underbrace{\lambda}_{1} K(t)$$

Government budget:

$$\tau_k K(t+1) = 3/40$$

$$\max \quad \log c_t^h(t) + \beta \log c_t^h(t+1)$$

$$c_t^h(t) = w_t^h(t) - \ell_t^h(t) - k^h(t+1)$$

$$c_t^h(t+1) = w_t^h(t+1) + (1-\tau_k)\left(r(t)\ell_t^h(t) + k^h(t+1)\right)$$

$$\rightarrow \ell_t^h(t) = \ldots$$

$$c_t^h(t) + \frac{c_t^h(t+1)}{(1-\tau_k)\,r(t)} = w_t^h(t) + \frac{w_t^h(t+1)}{(1-\tau_k)\,r(t)} - \underbrace{k^h(t+1)\left(1 - \frac{\lambda}{r(t)}\right)}_{=0}$$

Must have strictly positive storage, otherwise tax revenue 0

$$k^h(t+1) > 0 \rightarrow r(t) = \lambda \rightarrow r(t) = 1$$

$$c_t^h(t+1) = (1-\tau_k) r(t) \omega_t^h(t) + \omega_t^h(t+1) - (1-\tau_k) r(t) c_t^h(t)$$

FOC $c_t^h(t)$: $\frac{1}{c_t^h(t)} - \beta \frac{(1-\tau_k) r(t)}{c_t^h(t+1)} = 0 \rightarrow c_t^h(t+1) = (1-\tau_k) r(t) c_t^h(t)$

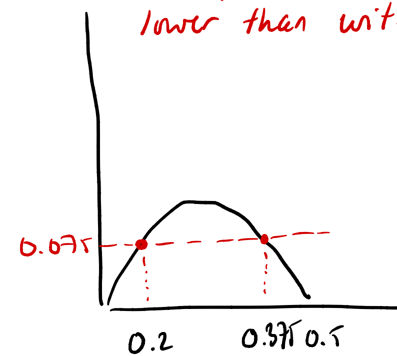
plugging into budget constraint:

$$2 c_t^h(t) = 2 + \frac{1}{(1-\tau_k) \underbrace{r(t)}_{=1}}$$

$$\rightarrow s_t = 1 - \frac{1}{2(1-\tau_k)}$$

$r(t) = 1$

utility higher in low tax equilibrium, but lower than with $\tau_k = 0$



$$K(t+1) = S(t) = 1 - \frac{1}{2(1-\tau_k)}$$

Laffer curve: $\tau_k K(t+1) = \tau_k - \frac{\tau_k}{2(1-\tau_k)} = 3/40$