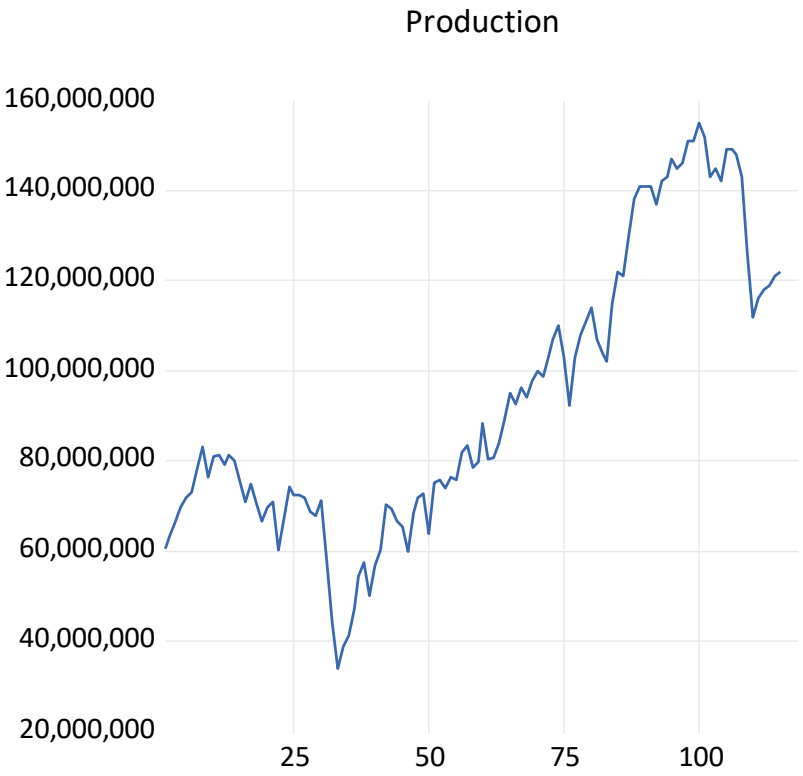
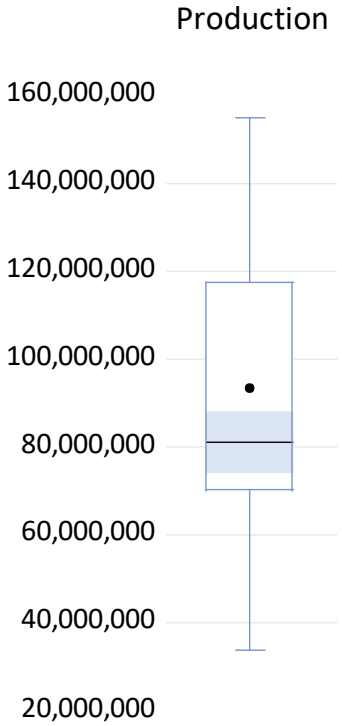
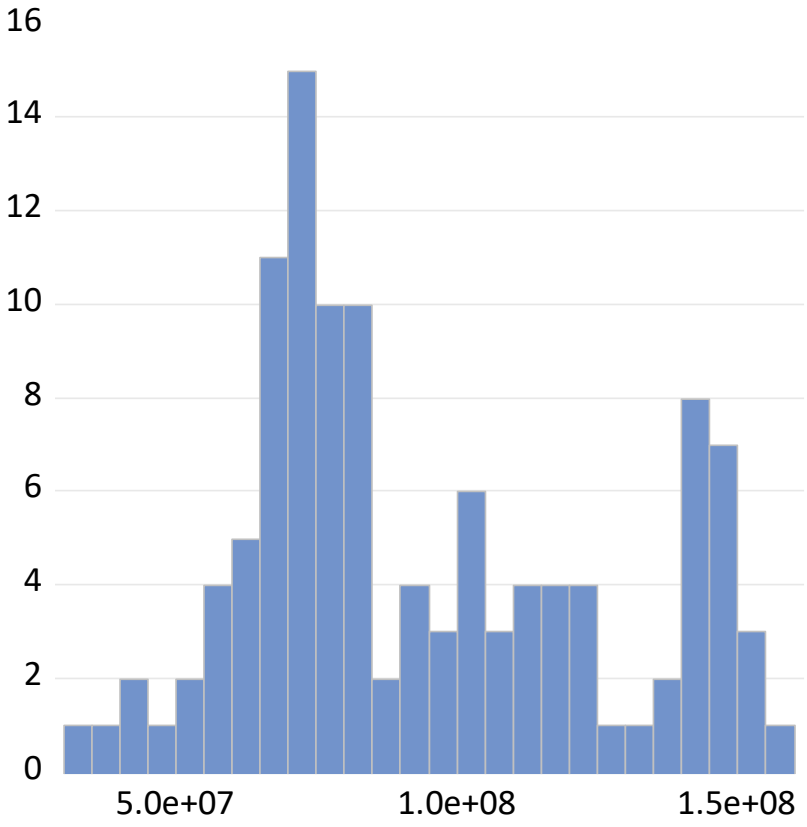
PART 1

Data

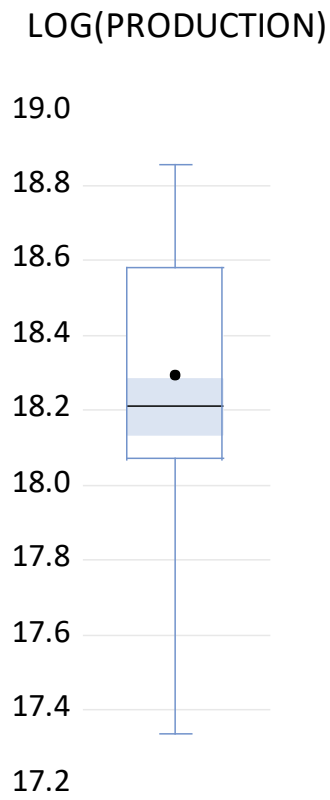
TOTAL FORESTRY STATISTICS, U.S. GEOLOGICAL SURVEY. [All values in metric tons], annual from 1900 to 2014

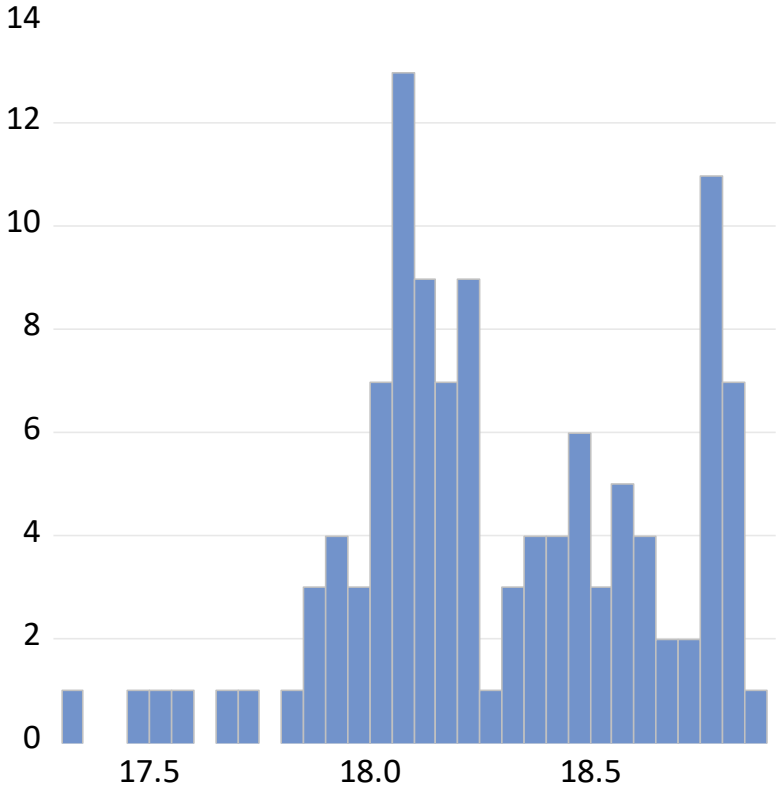






Series: PRODUCTION	
Sample 1 118	
Observations 115	
Mean	93499130
Median	81400000
Maximum	1.55e+08
Minimum	33900000
Std. Dev.	31417376
Skewness	0.457300
Kurtosis	2.097688
Jarque-Bera	7.909415
Probability	0.019164



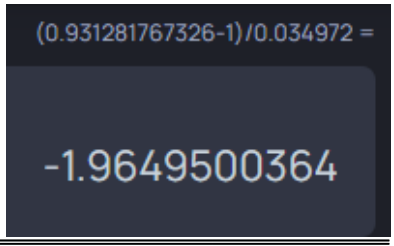


Series: LOG(PRODUCTION)
Sample 1 118
Observations 115

Mean	18.29672
Median	18.21489
Maximum	18.85894
Minimum	17.33893
Std. Dev.	0.342029
Skewness	-0.144876
Kurtosis	2.541114
Jarque-Bera	1.411302
Probability	0.493787

- The Jarque-Bera statistic and its corresponding p-value indicate that the log series is more likely to be normally distributed than the level series, therefore we favor the transformation.
- There is a visual trend in both series, namely a steady increase from the 1930s to the 2000s.

Dependent Variable: LOG(PRODUCTION)
Method: Least Squares
Date: 03/10/23 Time: 13:33
Sample (adjusted): 2 115
Included observations: 114 after adjustments



Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.231589	0.623085	1.976600	0.0506
@TREND	0.000551	0.000362	1.521378	0.1310
LOG(PRODUCTION(-1))	0.931282	0.034972	26.62901	0.0000

R-squared	0.953107	Mean dependent var	18.30002
Adjusted R-squared	0.952262	S.D. dependent var	0.341688
S.E. of regression	0.074655	Akaike info criterion	-2.325904
Sum squared resid	0.618651	Schwarz criterion	-2.253899
Log likelihood	135.5765	Hannan-Quinn criter.	-2.296681
F-statistic	1128.047	Durbin-Watson stat	1.641250
Prob(F-statistic)	0.000000		

Dependent Variable: DLOG(PRODUCTION)
Method: Least Squares
Date: 03/10/23 Time: 13:32
Sample (adjusted): 2 115
Included observations: 114 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.231589	0.623085	1.976600	0.0506
@TREND	0.000551	0.000362	1.521378	0.1310
LOG(PRODUCTION(-1))	-0.068718	0.034972	-1.964925	0.0519

R-squared	0.033732	Mean dependent var	0.006138
Adjusted R-squared	0.016322	S.D. dependent var	0.075272
S.E. of regression	0.074655	Akaike info criterion	-2.325904
Sum squared resid	0.618651	Schwarz criterion	-2.253899
Log likelihood	135.5765	Hannan-Quinn criter.	-2.296681
F-statistic	1.937496	Durbin-Watson stat	1.641250
Prob(F-statistic)	0.148905		

- Manually computed and directly estimated t-statistics are both equal.

- The 5 % critical value for Dickey-Fuller test with a trend and a sample size 114 is about -3.45.
  - Thus the null hypothesis of a unit root is not rejected since -1.964925 > -3.45.
    - Therefore the trend is stochastic.

Breusch-Godfrey Serial Correlation LM Test:
Null hypothesis: No serial correlation at up to 2 lags

F-statistic	2.247876	Prob. F(2,109)	0.1105
Obs*R-squared	4.515726	Prob. Chi-Square(2)	0.1046

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 03/10/23 Time: 14:00
Sample: 2 115
Included observations: 114
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.338102	0.714607	0.473130	0.6371
@TREND	0.000159	0.000396	0.400198	0.6898
LOG(PRODUCTION(-1))	-0.018981	0.040112	-0.473198	0.6370
RESID(-1)	0.207505	0.101000	2.054502	0.0423
RESID(-2)	-0.060786	0.102875	-0.590874	0.5558

R-squared	0.039612	Mean dependent var	5.26E-16
Adjusted R-squared	0.004368	S.D. dependent var	0.073992
S.E. of regression	0.073830	Akaike info criterion	-2.331234
Sum squared resid	0.594145	Schwarz criterion	-2.211225
Log likelihood	137.8803	Hannan-Quinn criter.	-2.282529
F-statistic	1.123938	Durbin-Watson stat	1.998874
Prob(F-statistic)	0.349046		

Date: 03/10/23 Time: 14:02
Sample (adjusted): 2 115
Q-statistic probabilities adjusted for 1 dynamic regressor

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.178	0.178	3.6969	0.055
		2	-0.045	-0.079	3.9351	0.140
		3	-0.057	-0.036	4.3290	0.228
		4	-0.016	-0.002	4.3584	0.360
		5	-0.056	-0.061	4.7454	0.448
		6	0.108	0.132	6.1719	0.404
		7	0.099	0.050	7.3857	0.390
		8	-0.159	-0.192	10.533	0.230
		9	-0.166	-0.088	13.995	0.122
		10	-0.032	0.004	14.126	0.167
		11	0.101	0.105	15.442	0.163
		12	-0.061	-0.122	15.932	0.194
		13	0.189	0.209	20.585	0.082
		14	0.035	-0.033	20.747	0.108
		15	-0.079	-0.028	21.587	0.119
		16	-0.072	-0.026	22.279	0.134
		17	0.186	0.156	27.002	0.058
		18	0.020	-0.064	27.057	0.078
		19	-0.078	-0.067	27.903	0.085
		20	-0.061	-0.073	28.423	0.100
		21	-0.039	0.037	28.642	0.123
		22	-0.019	0.045	28.692	0.154
		23	-0.037	-0.072	28.887	0.184
		24	-0.053	-0.177	29.298	0.209
		25	-0.145	-0.019	32.401	0.147
		26	-0.043	0.023	32.674	0.172
		27	-0.028	-0.054	32.795	0.204
		28	0.033	-0.049	32.964	0.237
		29	-0.081	-0.058	33.987	0.240
		30	0.061	0.067	34.575	0.258
		31	-0.022	-0.042	34.650	0.298
		32	-0.057	-0.059	35.178	0.320
		33	0.010	0.031	35.194	0.365
		34	-0.016	-0.087	35.234	0.410
		35	0.001	0.014	35.234	0.457
		36	0.029	0.050	35.379	0.498

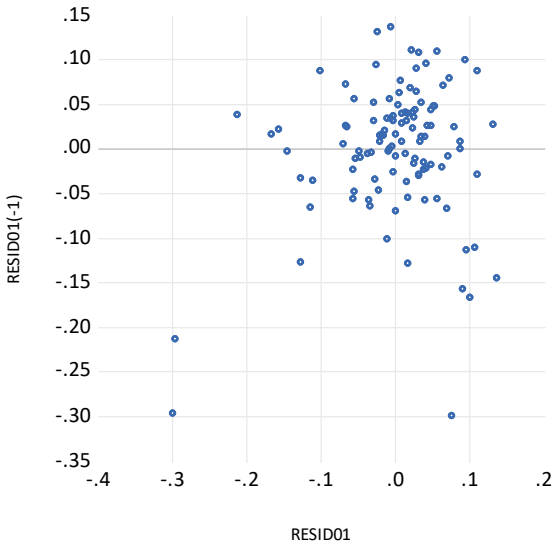
*Probabilities may not be valid for this equation specification.

Dependent Variable: DLOG(PRODUCTION)
Method: ARDL
Date: 03/10/23 Time: 14:17
Sample (adjusted): 3 115
Included observations: 113 after adjustments
Maximum dependent lags: 12 (Automatic selection)
Model selection method: Schwarz criterion (SIC)
Dynamic regressors (0 lag, automatic): @TREND LOG(PRODUCTION)
Fixed regressors: C
Number of models evaluated: 12
Selected Model: ARDL(1, 0, 0)
Note: final equation sample is larger than selection sample

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
DLOG(PRODUCTION(-1))	0.109796	0.096416	1.138774	0.2573
@TREND	-0.000514	0.000374	-1.374789	0.1720
LOG(PRODUCTION)	0.060464	0.036160	1.672130	0.0974
C	-1.071752	0.644142	-1.663844	0.0990

R-squared	0.046946	Mean dependent var	0.005793
Adjusted R-squared	0.020716	S.D. dependent var	0.075517
S.E. of regression	0.074730	Akaike info criterion	-2.315102
Sum squared resid	0.608726	Schwarz criterion	-2.218557
Log likelihood	134.8032	Hannan-Quinn criter.	-2.275925
F-statistic	1.789742	Durbin-Watson stat	1.850450
Prob(F-statistic)	0.153388		

*Note: p-values and any subsequent tests do not account for model selection.



- The selected ARDL model based on the Schwarz information criterion includes 1 lag of the differenced log transformed production series.

Dependent Variable: DLOG(PRODUCTION)
Method: Least Squares
Date: 03/10/23 Time: 14:23
Sample (adjusted): 3 115
Included observations: 113 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.493528	0.628104	2.377836	0.0192
@TREND	0.000703	0.000366	1.919223	0.0576
LOG(PRODUCTION(-1))	-0.083601	0.035269	-2.370368	0.0195
DLOG(PRODUCTION(-1))	0.192574	0.093993	2.048803	0.0429

R-squared	0.070417	Mean dependent var	0.005793
Adjusted R-squared	0.044832	S.D. dependent var	0.075517
S.E. of regression	0.073805	Akaike info criterion	-2.340036
Sum squared resid	0.593735	Schwarz criterion	-2.243491
Log likelihood	136.2120	Hannan-Quinn criter.	-2.300859
F-statistic	2.752274	Durbin-Watson stat	1.982426
Prob(F-statistic)	0.046111		

- The null hypothesis of a unit root is still not rejected as -2.370368 > -3.45.
  - EViews t-test corresponds to the one above.

Null Hypothesis: LOG(PRODUCTION) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on SIC, maxlag=12)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.370368	0.3929
Test critical values:	1% level	-4.041280	
	5% level	-3.450073	
	10% level	-3.150336	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LOG(PRODUCTION))
Method: Least Squares
Date: 03/10/23 Time: 14:10
Sample (adjusted): 3 115
Included observations: 113 after adjustments

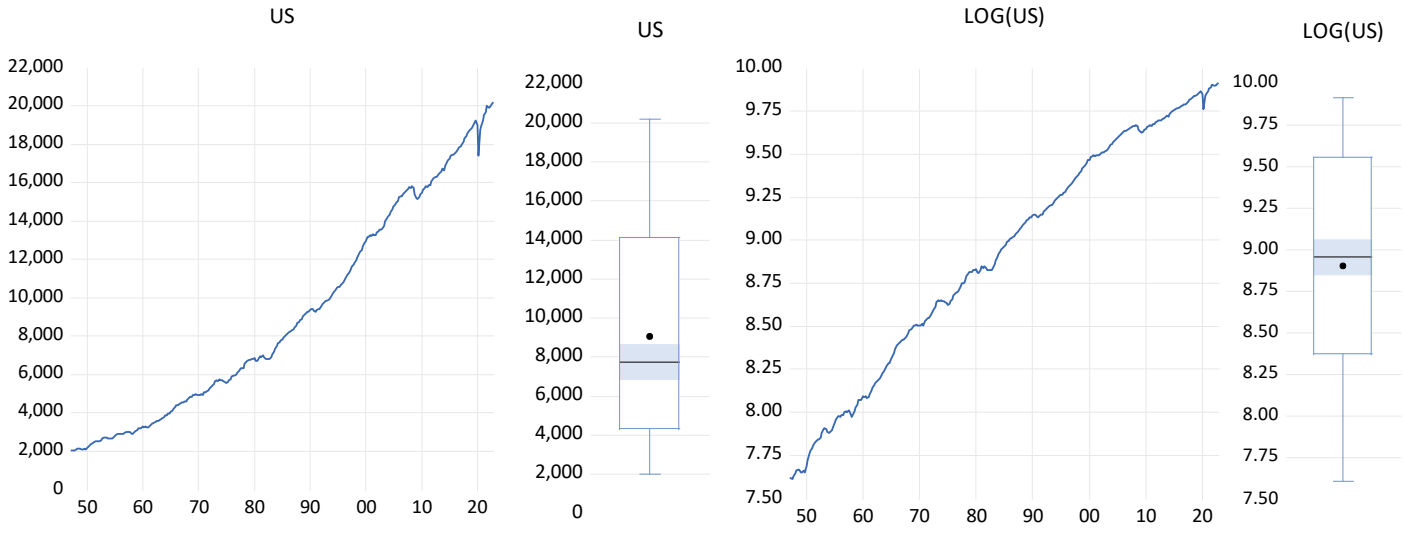
Variable	Coefficient	Std. Error	t-Statistic	Prob.
LOG(PRODUCTION(-1))	-0.083601	0.035269	-2.370368	0.0195
D(LOG(PRODUCTION(-1)))	0.192574	0.093993	2.048803	0.0429
C	1.493528	0.628104	2.377836	0.0192
@TREND("1")	0.000703	0.000366	1.919223	0.0576

R-squared	0.070417	Mean dependent var	0.005793
Adjusted R-squared	0.044832	S.D. dependent var	0.075517
S.E. of regression	0.073805	Akaike info criterion	-2.340036
Sum squared resid	0.593735	Schwarz criterion	-2.243491
Log likelihood	136.2120	Hannan-Quinn criter.	-2.300859
F-statistic	2.752274	Durbin-Watson stat	1.982426
Prob(F-statistic)	0.046111		

PART 2

Data

US real GDP . [Billions of Chained 2012 Dollars, Seasonally Adjusted], Quarterly, 1947 Q1 - 2022 Q1

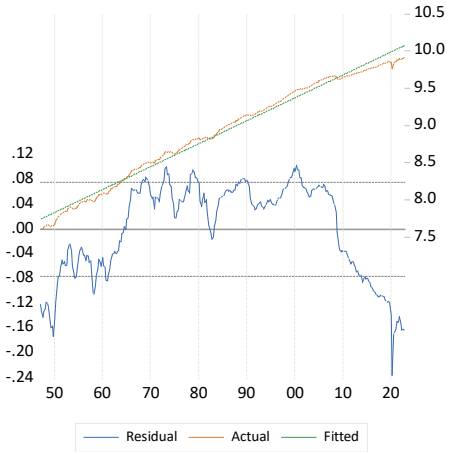


The logged GDP plot series increased steadily during the whole period, the regressions below provide evidence of a possible determinist nature of the time trend.

Dependent Variable: LOG(US)
Method: Least Squares
Date: 03/13/23 Time: 09:07
Sample: 1947Q1 2022Q4
Included observations: 304

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	7.738875	0.008679	891.6547	0.0000
@TREND	0.007711	4.96E-05	155.5523	0.0000

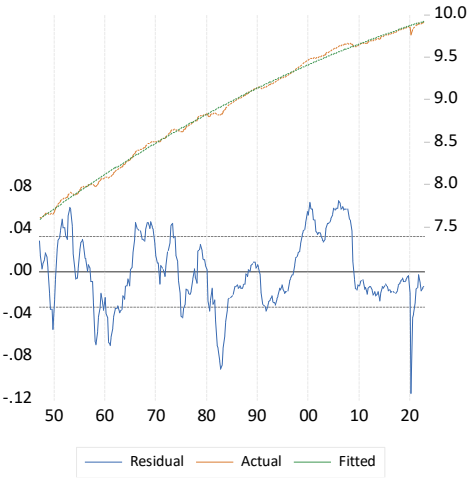
R-squared	0.987673	Mean dependent var	8.907110
Adjusted R-squared	0.987632	S.D. dependent var	0.682036
S.E. of regression	0.075851	Akaike info criterion	-2.313548
Sum squared resid	1.737496	Schwarz criterion	-2.289093
Log likelihood	353.6592	Hannan-Quinn criter.	-2.303765
F-statistic	24196.53	Durbin-Watson stat	0.022869
Prob(F-statistic)	0.000000		



Dependent Variable: LOG(US)
Method: Least Squares
Date: 03/13/23 Time: 09:09
Sample: 1947Q1 2022Q4
Included observations: 304

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	7.588431	0.005694	1332.769	0.0000
@TREND	0.010700	8.68E-05	123.2574	0.0000
@TREND^2	-9.86E-06	2.77E-07	-35.56709	0.0000

R-squared	0.997631	Mean dependent var	8.907110
Adjusted R-squared	0.997615	S.D. dependent var	0.682036
S.E. of regression	0.033309	Akaike info criterion	-3.956150
Sum squared resid	0.333959	Schwarz criterion	-3.919468
Log likelihood	604.3347	Hannan-Quinn criter.	-3.941476
F-statistic	63367.94	Durbin-Watson stat	0.116592
Prob(F-statistic)	0.000000		

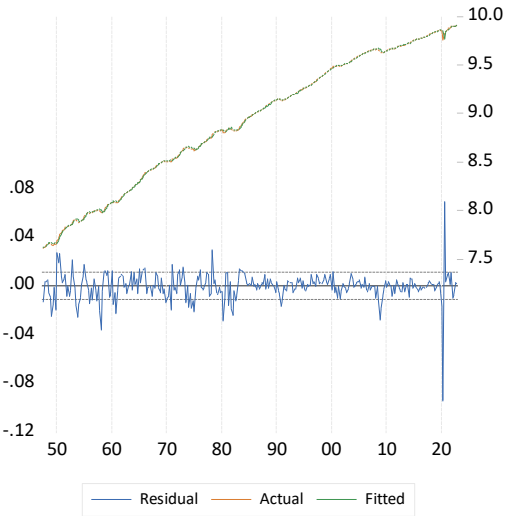


Another way of explaining the series is by including its lagged value in the regression, doing so results in a higher adjusted R-squared, which indicates to a stochastic nature of the time trend.

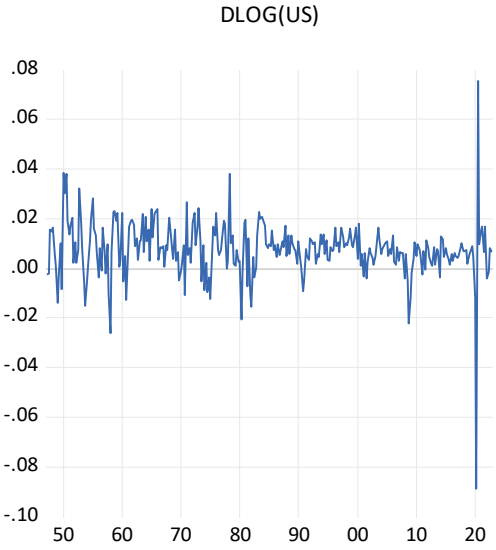
Dependent Variable: LOG(US)
Method: Least Squares
Date: 03/13/23 Time: 09:16
Sample (adjusted): 1947Q2 2022Q4
Included observations: 303 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.029598	0.008579	3.450259	0.0006
LOG(US(-1))	0.997526	0.000961	1038.355	0.0000

R-squared	0.999721	Mean dependent var	8.911364
Adjusted R-squared	0.999720	S.D. dependent var	0.679111
S.E. of regression	0.011364	Akaike info criterion	-6.110123
Sum squared resid	0.038872	Schwarz criterion	-6.085610
Log likelihood	927.6836	Hannan-Quinn criter.	-6.100316
F-statistic	1078181.	Durbin-Watson stat	1.799961
Prob(F-statistic)	0.000000		



Indeed, plot of the differenced shows that the transformed series may be stationary.



However, the t-test corresponding an inclusion of a time trend in the regression of the differenced series results in a statistical significance of the coefficient.

Dependent Variable: DLOG(US)
Method: Least Squares
Date: 03/13/23 Time: 09:21
Sample (adjusted): 1947Q2 2022Q4
Included observations: 303 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.010398	0.001310	7.938445	0.0000
@TREND	-1.86E-05	7.47E-06	-2.488027	0.0134

R-squared	0.020151	Mean dependent var	0.007574
Adjusted R-squared	0.016896	S.D. dependent var	0.011470
S.E. of regression	0.011372	Akaike info criterion	-6.108693
Sum squared resid	0.038928	Schwarz criterion	-6.084180
Log likelihood	927.4670	Hannan-Quinn criter.	-6.098886
F-statistic	6.190277	Durbin-Watson stat	1.801846
Prob(F-statistic)	0.013386		

Therefore, the ADF test of unit root was performed twice: 1) with a constant only, and 2) with a constant and a linear trend. In both cases the null of a unit root was not rejected.

Null Hypothesis: LOG(US) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=15)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.906392	0.9528
Test critical values:	1% level	-3.988635	
	5% level	-3.424726	
	10% level	-3.135436	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LOG(US))
Method: Least Squares
Date: 03/13/23 Time: 08:46
Sample (adjusted): 1947Q2 2022Q4
Included observations: 303 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LOG(US(-1))	-0.007883	0.008697	-0.906392	0.3655
C	0.071336	0.067244	1.060858	0.2896
@TREND("1947Q1")	4.23E-05	6.76E-05	0.625819	0.5319

R-squared	0.022827	Mean dependent var	0.007574
Adjusted R-squared	0.016313	S.D. dependent var	0.011470
S.E. of regression	0.011376	Akaike info criterion	-6.104827
Sum squared resid	0.038822	Schwarz criterion	-6.068057
Log likelihood	927.8813	Hannan-Quinn criter.	-6.090117
F-statistic	3.504077	Durbin-Watson stat	1.792579
Prob(F-statistic)	0.031311		

Null Hypothesis: LOG(US) has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=15)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.574860	0.0993
Test critical values:	1% level	-3.451775	
	5% level	-2.870868	
	10% level	-2.571811	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LOG(US))
Method: Least Squares
Date: 03/13/23 Time: 08:57
Sample (adjusted): 1947Q2 2022Q4
Included observations: 303 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LOG(US(-1))	-0.002474	0.000961	-2.574860	0.0105
C	0.029598	0.008579	3.450259	0.0006

R-squared	0.021552	Mean dependent var	0.007574
Adjusted R-squared	0.018301	S.D. dependent var	0.011470
S.E. of regression	0.011364	Akaike info criterion	-6.110123
Sum squared resid	0.038872	Schwarz criterion	-6.085610
Log likelihood	927.6836	Hannan-Quinn criter.	-6.100316
F-statistic	6.629904	Durbin-Watson stat	1.799961
Prob(F-statistic)	0.010506		

The trend stationary and a difference stationary specification estimation without the last 20 observations resulted as follows

Dependent Variable: LOG(US)
Method: Least Squares
Date: 03/13/23 Time: 09:38
Sample: 1947Q1 2017Q4
Included observations: 284

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	7.716663	0.007677	1005.168	0.0000
@TREND	0.007939	4.69E-05	169.1145	0.0000

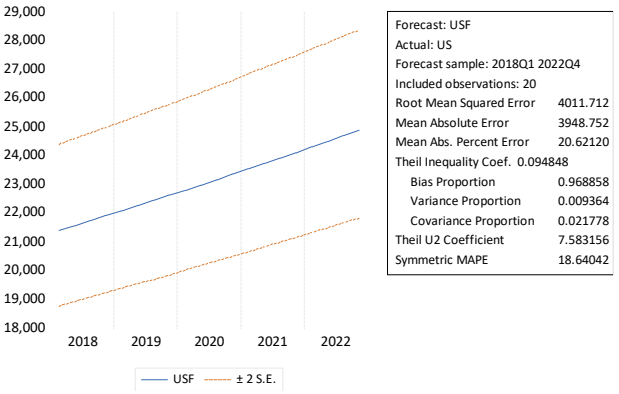
R-squared	0.990236	Mean dependent var	8.840023
Adjusted R-squared	0.990201	S.D. dependent var	0.655215
S.E. of regression	0.064858	Akaike info criterion	-2.626205
Sum squared resid	1.186263	Schwarz criterion	-2.600508
Log likelihood	374.9211	Hannan-Quinn criter.	-2.615902
F-statistic	28599.70	Durbin-Watson stat	0.021035
Prob(F-statistic)	0.000000		

Dependent Variable: DLOG(US)
Method: Least Squares
Date: 03/13/23 Time: 09:39
Sample (adjusted): 1947Q2 2017Q4
Included observations: 283 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.007764	0.000559	13.88754	0.0000

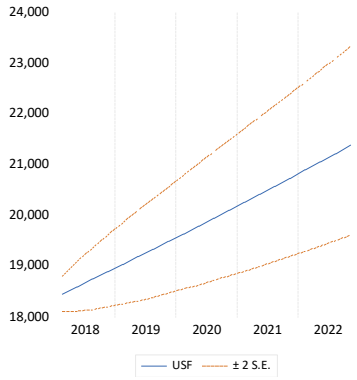
R-squared	0.000000	Mean dependent var	0.007764
Adjusted R-squared	0.000000	S.D. dependent var	0.009405
S.E. of regression	0.009405	Akaike info criterion	-6.491624
Sum squared resid	0.024944	Schwarz criterion	-6.478742
Log likelihood	919.5648	Hannan-Quinn criter.	-6.486459
Durbin-Watson stat	1.273531		

Afterwards forecast were generated, the RMSE was more than four times lower in the differenced specification. The TS forecast confidence interval is around 3000 $ for all the observations, while the DS one is around 1000 $ for the first and approximately 2000 $ for the last observation.



Forecast: USF
Actual: US
Forecast sample: 2018Q1 2022Q4
Included observations: 20

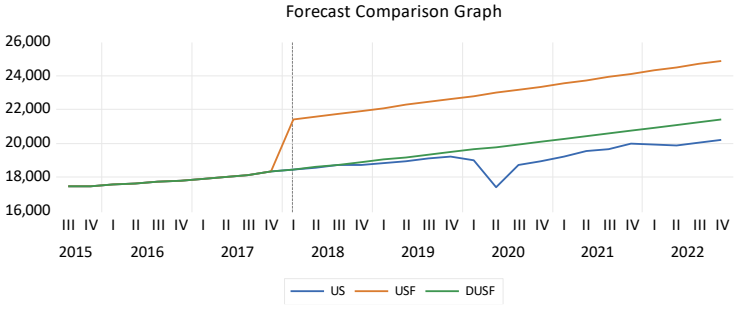
Root Mean Squared Error	4011.712
Mean Absolute Error	3948.752
Mean Abs. Percent Error	20.62120
Theil Inequality Coef.	0.094848
Bias Proportion	0.968858
Variance Proportion	0.009364
Covariance Proportion	0.021778
Theil U2 Coefficient	7.583156
Symmetric MAPE	18.64042



Forecast: USF
Actual: US
Forecast sample: 2018Q1 2022Q4
Included observations: 20

Root Mean Squared Error	936.0452
Mean Absolute Error	729.5647
Mean Abs. Percent Error	3.817688
Theil Inequality Coef.	0.023956
Bias Proportion	0.607483
Variance Proportion	0.055653
Covariance Proportion	0.336864
Theil U2 Coefficient	1.794146
Symmetric MAPE	3.697961

The Diebold Mariano test rejected the null of both forecast having the same accuracy, as expected since the DS forecast significantly outperformed the TS one.



Forecast Evaluation
Date: 03/13/23 Time: 10:36
Sample: 2018Q1 2022Q4
Included observations: 20
Evaluation sample: 2018Q1 2022Q4
Number of forecasts: 2

Combination tests
Null hypothesis: Forecast i includes all information contained in others

Forecast	F-stat	F-prob
USF	28.43577	0.0000
DUSF	13.82788	0.0016

Diebold-Mariano test (HLN adjusted)
Null hypothesis: Both forecasts have the same accuracy

Accuracy	Statistic	<> prob	> prob	< prob
Abs Error	83.83697	0.0000	1.0000	0.0000
Sq Error	14.17144	0.0000	1.0000	0.0000

Evaluation statistics

Forecast	RMSE	MAE	MAPE	SMAPE	Theil U1	Theil U2
USF	4011.712	3948.752	20.62120	18.64042	0.094848	7.583156
DUSF	936.0452	729.5647	3.817688	3.697961	0.023956	1.794146

Finally, after generating the differenced series, the automatic model selection based on the BIC (Schwarz) information criterion was applied to two cases, once with only a constant and once with an inclusion of a time trend. In both of them the best ARMA(p,q) model was identified as ARMA(0,0).

Automatic ARIMA Forecasting Selected dependent variable: DLOGUS Date: 03/14/23 Time: 11:12 Sample: 1947Q1 2022Q4 Included observations: 303 Forecast length: 0 Model maximums: (4,4)0(0,0) Regressors: C	Automatic ARIMA Forecasting Selected dependent variable: DLOGUS Date: 03/14/23 Time: 11:20 Sample: 1947Q1 2022Q4 Included observations: 303 Forecast length: 0 Model maximums: (4,4)0(0,0) Regressors: C @TREND
Number of estimated ARMA models: 25 Number of non-converged estimations: 0 Selected ARMA model: (0,0)(0,0) SIC value: -6.06382269557	Number of estimated ARMA models: 25 Number of non-converged estimations: 0 Selected ARMA model: (0,0)(0,0) SIC value: -6.06532257605

The resulting model is given below

Dependent Variable: DLOGUS
Method: Least Squares
Date: 03/14/23 Time: 11:23
Sample (adjusted): 1947Q2 2022Q4
Included observations: 303 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.010398	0.001310	7.938445	0.0000
@TREND	-1.86E-05	7.47E-06	-2.488027	0.0134

R-squared	0.020151	Mean dependent var	0.007574
Adjusted R-squared	0.016896	S.D. dependent var	0.011470
S.E. of regression	0.011372	Akaike info criterion	-6.108693
Sum squared resid	0.038928	Schwarz criterion	-6.084180
Log likelihood	927.4670	Hannan-Quinn criter.	-6.098886
F-statistic	6.190277	Durbin-Watson stat	1.801846
Prob(F-statistic)	0.013386		