Part C: Panel Data Methods

C3: Staggered-Adoption Difference-in-Differences

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ARE 213 Applied Econometrics

UC Berkeley, Fall 2023

C3 outline

1. Staggered adoption: Setting and estimands
2. Traditional estimators
3. What to do instead
4. Extensions

Staggered adoption/rollout setting

	$i = Z$	$i = A$	$i = B$	$i = C$	$i = D$
$t = 1$					
$t = 2$					
$t = 3$					
$t = 4$					
$t = 5$					
$t = 6$					

- Assume binary treatment; i gets treated at $t = E_i$ and stays treated forever: $D_{it} = \mathbf{1}[t \geq E_i]$
  - "Cohort" $=$ units with the same E_i
- May or may not have never-treated units ($E_D = \infty$), always-treated units ($E_Z = 1$)
  - Come back to always-treated units later

Why staggered adoption?

- A fact of life
  - Unilateral divorce laws adopted in different years across states
- Researcher's choice to have more comparable units
  - A panel of mothers, where $E_i =$ year of birth of first child
  - May intentionally drop women without kids, as they are not expected to be on parallel trends

Notation and assumptions

- Fixed sample $\Omega = \{it\}$ with untreated obs Ω_0 and treated obs Ω_1
- Causal structure: $Y_{it}(D_{it})$ (no spillovers, no anticipation effects)
- Causal effects: $\tau_{it} = \mathbb{E}\left[Y_{it}(1) - Y_{it}(0)\right]$
- Parallel trends: $\mathbb{E}\left[Y_{it}(0)\right] = \alpha_i + \beta_t$
- Linear target estimand: $\sum_{it\in\Omega_1} w_{it}\tau_{it}$ for w_{it} chosen by researcher...

Some estimands of interest

- ATT: $\frac{1}{|\Omega_1|}\sum_{it\in\Omega_1}\tau_{it}$ where $\Omega_1 = \{it: \; D_{it} = 1\}$

	$i = A$	$i = B$	$i = C$	$i = D$
$t = 1$				
$t = 2$				
$t = 3$				
$t = 4$				
$t = 5$				
$t = 6$				

Some estimands of interest

- ATT

- ATT $h \geq 0$ periods since treatment (typically fewer units for longer horizons)

	$i = A$	$i = B$	$i = C$	$i = D$
$t = 1$				
$t = 2$	$h = 0$			
$t = 3$	$h = 1$	$h = 0$		
$t = 4$	$h = 2$	$h = 1$		
$t = 5$	$h = 3$	$h = 2$	$h = 0$	
$t = 6$	$h = 4$	$h = 3$	$h = 1$	

Some estimands of interest

- ATT
- ATT $h \geq 0$ periods since treatment
- ATT $h \geq 0$ periods since treatment on a balanced set of units for different h

	$i = A$	$i = B$	$i = C$	$i = D$
$t = 1$				
$t = 2$	$h = 0$			
$t = 3$	$h = 1$	$h = 0$		
$t = 4$	$h = 2$	$h = 1$		
$t = 5$	$h = 3$	$h = 2$	$h = 0$	
$t = 6$	$h = 4$	$h = 3$	$h = 1$	

Some estimands of interest

- ATT
- ATT $h \geq 0$ periods since treatment
- ATT $h \geq 0$ periods since treatment on a balanced set of units for different h
- Difference between ATT between subgroups
- Size-weighted ATT; etc.

Outline

Conventional practice

By analogy with non-staggered DiD, common to estimate:

- Static TWFE specification — to get a single summary statistic of treatment effects:

$$Y_{it} = \tilde{\alpha}_i + \tilde{\beta}_t + \tau_{\text{static}} D_{it} + \varepsilon_{it}$$

- Event study (dynamic) specification:

$$Y_{it} = \tilde{\alpha}_i + \tilde{\beta}_t + \sum_{\substack{h=-a \\ h \neq -1}}^{b-1} \tau_h \mathbf{1}\left[t = E_i + h\right] + \tau_{b+} \mathbf{1}\left[t \geq E_i + b\right] + \varepsilon_{it}$$

  - "Fully-dynamic" if $h = -1$ is the only omitted term
  - Some dummies are often binned on the left and/or on the right

Static TWFE specification

- Does the static TWFE specification estimate the ATT?

$$Y_{it} = \tilde{\alpha}_i + \tilde{\beta}_t + \tau_{\text{static}} D_{it} + \varepsilon_{it}$$

- de Chaisemartin and D'Haultfœuille (AER 2020), Borusyak, Jaravel, Spiess (2023) (BJS):
    - Yes if the effects are homogeneous across units and periods. Not otherwise!
    - Under PTA, estimand $\tau_{\text{static}} = \sum_{it \in \Omega_1} w_{it}^{\text{static}} \tau_{it}$ for some weights w_{it}^{static} that add up to one
    - But $w_{it}^{\text{static}} \neq \frac{1}{|\Omega_1|}$ and some can be negative due to "forbidden comparisons"...

Forbidden comparisons

$$Y_{it} = \tilde{\alpha}_i + \tilde{\beta}_t + \tau_{\text{static}} D_{it} + \varepsilon_{it}$$

$\mathbb{E}[Y_{it}]$	$i = A$	$i = B$
$t = 1$	α_A	α_B
$t = 2$	$\alpha_A + \beta_2 + \tau_{A2}$	$\alpha_B + \beta_2$
$t = 3$	$\alpha_A + \beta_3 + \tau_{A3}$	$\alpha_B + \beta_3 + \tau_{B3}$

Here $\hat{\tau}_{\text{static}} = \left(Y_{A2} - Y_{B2} \right) - \frac{1}{2}\left(Y_{A1} - Y_{B1} \right) - \frac{1}{2}\left(Y_{A3} - Y_{B3} \right)$

- Treated observations for early adopters are used as controls for treated observations of late adopters
- Long-term effects for early adopters can get a negative weight

Mechanics of negative weights

- By Frisch-Waugh-Lovell, $\hat{\tau}_{\text{static}}$ can be obtained from

$$Y_{it} = \tau_{\text{static}} D_{it}^{\perp} + \text{error}$$

where $D_{it}^{\perp}$ are residuals from regressing $D_{it} = a_i + b_t + \text{error}$.

- Thus, $\hat{\tau}_{\text{static}} = \frac{\sum_{it} D_{it}^{\perp} Y_{it}}{\sum_{js}\left(D_{js}^{\perp}\right)^2}$. Weights $\frac{D_{it}^{\perp}}{\sum_{js}\left(D_{js}^{\perp}\right)^2}$ are easy to compute
- But they can be negative for some treated observations: where $\hat{a}_i$ is high (early adopters) and $\hat{b}_t$ is high (late periods if few never-treated units)
- Angrist (1998) result does not apply!

Characterizing negative weights

- You can compute w_{it}^{static} (by observation or group totals) and total negative weights
- Goodman-Bacon (2021) provides a decomposition of τ_{static} as convex weighted average of several types of comparisons (package *bacondecomp*):
  - Treated vs. never treated *(good)*
  - Early adopters vs. late adopters *(good)*
  - Late adopters vs. early adopters *(forbidden)*
  - Treated during the sample vs. always-treated *(forbidden)*
- *Note*: only useful if you plan to use the static TWFE specification, and you don't

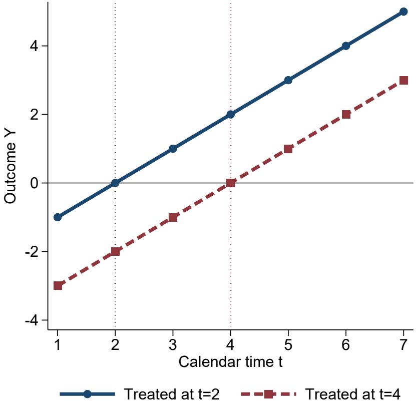
Under-identification of the fully-dynamic specification

- "Fully-dynamic" specification:

$$Y_{it} = \tilde{\alpha}_i + \tilde{\beta}_t + \sum_{h \neq -1} \tau_h \mathbf{1}\left[t = E_i + h\right] + \varepsilon_{it}$$

- **Proposition** (BJS): Without never-treated units, the path $\{\tau_h\}_{h \neq -1}$ is not point identified in the fully-dynamic specification. Adding any linear trend to this path, $\{\tau_h + \kappa(h+1)\}$, fits the data equally well

Under-identification of the fully-dynamic specification



(from BJS)

- Diff-in-diff doesn't work without some assumption of no anticipation effects!

Spurious identification of very long-run effects

- "Semi-dynamic" specification can be estimated:

$$Y_{it} = \tilde{\alpha}_i + \tilde{\beta}_t + \sum_{h \geq 0} \tau_h \mathbf{1}\left[t = E_i + h\right] + \varepsilon_{it}$$

- **Proposition** (BJS): Without never-treated units and with heterogeneous effects, long-run effects ($h \geq \max_i E_i - \min_i E_i$) are not identified by PTA, while the semi-dynamic specification produces some (spurious) estimates

Spurious identification of very long-run effects

$$Y_{it} = \tilde{\alpha}_i + \tilde{\beta}_t + \sum_{h=0}^{1} \tau_h \mathbf{1}\left[t = E_i + h\right] + \varepsilon_{it}$$

$\mathbb{E}\left[Y_{it}\right]$	$i = A$	$i = B$
$t = 1$	α_A	α_B
$t = 2$	$\alpha_A + \beta_2 + \tau_0$	$\alpha_B + \beta_2$
$t = 3$	$\alpha_A + \beta_3 + \tau_1$	$\alpha_B + \beta_3 + \tau_0$

- Here $\hat{\tau}_1 = \left(Y_{A3} - Y_{B3} \right) + \left(Y_{A2} - Y_{B2} \right) - 2\left(Y_{A1} - Y_{B1}\right)$

Spurious identification of very long-run effects

$$Y_{it} = \tilde{\alpha}_i + \tilde{\beta}_t + \sum_{h=0}^{1} \tau_h \mathbf{1}\left[t = E_i + h\right] + \varepsilon_{it}$$

$\mathbb{E}\left[Y_{it}\right]$	$i = A$	$i = B$
$t = 1$	α_A	α_B
$t = 2$	$\alpha_A + \beta_2 + \tau_{A2}$	$\alpha_B + \beta_2$
$t = 3$	$\alpha_A + \beta_3 + \tau_{A3}$	$\alpha_B + \beta_3 + \tau_{B3}$

- Here $\hat{\tau}_1 = \left(Y_{A3} - Y_{B3} \right) + \left(Y_{A2} - Y_{B2} \right) - 2 \left(Y_{A1} - Y_{B1} \right)$
- Estimand $\tau_1 = \tau_{A3} + \tau_{A2} - \tau_{B3}$ inevitably involves extrapolation that is invalid with heterogeneous effects

Cross-horizon contamination

Sun and Abraham (2021):

- Similar problems occur even for short-run effects in dynamic specification
  - Estimand τ_h is not an average of horizon-h effects
  - Contaminated by heterogeneity of effects at other horizons
- And pre-trend coefficients are contaminated by treatment effect heterogeneity
  - Can be significant even if PTA holds!
- *Note*: these problems tend to be small in practice

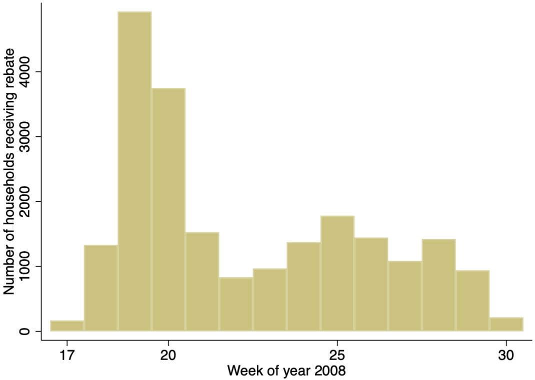
Pre-testing problems

Roth (AER:I 2022):

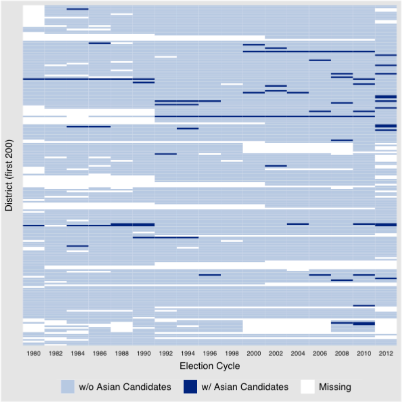
- Estimators of causal effects and pre-trends are correlated, e.g. by using the same reference period
  - In the non-staggered case, $(\bar{Y}_{\text{treated},E+h} - \bar{Y}_{\text{control},E+h}) - (\bar{Y}_{\text{treated},E-1} - \bar{Y}_{\text{control},E-1})$ and $(\bar{Y}_{\text{treated},E-\ell} - \bar{Y}_{\text{control},E-\ell}) - (\bar{Y}_{\text{treated},E-1} - \bar{Y}_{\text{control},E-1})$
- Suppose you only report the results if some pre-trend test doesn't reject ("pre-testing")
- If PTA holds, you are distorting inference for causal effects
- If PTA doesn't hold, you are changing the bias; under some conditions increase it

Outline

Plot treatment timing

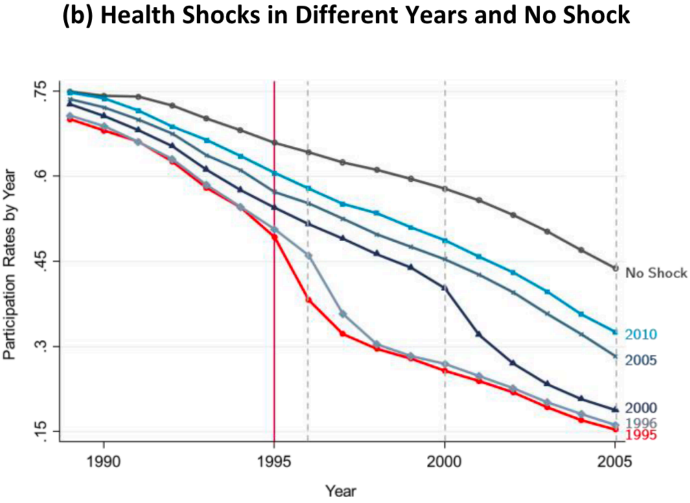


(From BJS)



(from Chiu, Lan, Liu, Xu 2023; package *panelView* in Stata & R)

Plot raw outcome data by cohort



(from Fadlon and Nielsen 2015)

Estimation robust to heterogeneous effects

- The problems arise from conventional specifications being too restrictive
- They are not fundamental to staggered adoption DiD
  - Under PTA, there are many valid 2x2 contrasts
- How to combine them?
  - Manual averaging approaches
  - Imputation approaches
  - Regression implementations of both approaches

Manual averaging estimators

de Chaisemartin and D'Haultfœuille (AER 2020) for $h = 0$:

	$i = A$	$i = B$	$i = C$	$i = D$
$t = 1$				
$t = 2$	$h = 0$			
$t = 3$		$h = 0$		
$t = 4$				
$t = 5$			$h = 0$	
$t = 6$				

	$i = A$	$i = B$	$i = C$	$i = D$
$t = 1$				
$t = 2$	$h = 0$			
$t = 3$		$h = 0$		
$t = 4$				
$t = 5$			$h = 0$	
$t = 6$				

- For each cohort $E_i = e$: form the clean control group; compute **cohort-average treatment effect** $(CATT_{e,e+0})$ by comparing $Y_{ie} - Y_{i,e-1}$
- Average across cohorts weighting by cohort size. Get SE by bootstrap
- *Note:* PTA is not fully exploited by comparing to $e - 1$ only, without earlier periods

Manual averaging estimators (2)

de Chaisemartin and D'Haultfœuille (2023 WP) for $h \geq 0$:

	$i = A$	$i = B$	$i = C$	$i = D$
$t = 1$				
$t = 2$				
$t = 3$	$h = 1$			
$t = 4$		$h = 1$		
$t = 5$				
$t = 6$			$h = 1$	

	$i = A$	$i = B$	$i = C$	$i = D$
$t = 1$				
$t = 2$				
$t = 3$	$h = 1$			
$t = 4$		$h = 1$		
$t = 5$				
$t = 6$			$h = 1$	

- For each cohort $E_i = e$: form the control group as cohorts not treated by $e + h$; compute $CATT_{e,e+h}$ by comparing $Y_{i,e+h} - Y_{i,e-1}$
- Sun and Abraham (2021): same but use never-treated controls only
  - If no never-treated, use latest-treated cohort instead

Manual averaging: Pre-trend tests

de Chaisemartin and D'Haultfœuille (2023 WP) pre-trend tests:

	$i = A$	$i = B$	$i = C$	$i = D$
$t = 1$				
$t = 2$			$\ell = 3$	
$t = 3$				
$t = 4$				
$t = 5$				
$t = 6$				

- For cohort e and lead $\ell > 1$, measure $Y_{i,e-\ell} - Y_{i,e-1}$
- Compare to the control group of units not treated by e
- Average across cohorts; test it's zero

Imputation estimators

- PTA requires $\mathbb{E}\left[Y_{it}(0)\right] = \alpha_i + \beta_t$
- No anticipation effects means $Y_{it} = Y_{it}(0)$ for untreated observations ($it \in \Omega_0$)
- Imputation approach:
  1. Estimate α_i and β_t from untreated observations
     - ★ Need pre-treatment obs for every unit to get $\hat{\alpha}_i$
     - ★ Need untreated obs in every period to get $\hat{\beta}_t$
  2. For each treated observation $it \in \Omega_1$, compute $\hat{\tau}_{it} = Y_{it} - \hat{\alpha}_i - \hat{\beta}_t$
     - ★ Each $\hat{\tau}_{it}$ is very noisy!
  3. Estimate $\tau_w = \sum_{it \in \Omega_1} w_{it} \tau_{it}$ by $\hat{\tau}_w = \sum_{it \in \Omega_1} w_{it} \hat{\tau}_{it}$
     - ★ Averaging across many units makes $\hat{\tau}_w$ consistent

Efficient imputation

How to estimate α_i and β_t?

- $\hat{\tau}_w$ is unbiased for any unbiased $\hat{\alpha}_i, \hat{\beta}_t$

Proposition (BJS) If $Y_{it}(0) = \alpha_i + \beta_t + \varepsilon_{it}$ for *spherical* ε_{it} (i.e., homoskedastic and serially uncorrelated), estimating $\hat{\alpha}_i, \hat{\beta}_t$ by OLS in the untreated sample yields most efficient $\hat{\tau}_w$ for any τ_w

- This imputation estimator can be obtained by OLS from a very flexible regression

$$Y_{it} = \alpha_i + \beta_t + \tau_{it} D_{it} + \varepsilon_{it}$$

  where each treated observation gets its own coef

- By Gauss-Markov, OLS is efficient for the vector of τ_{it} and for any linear combination τ_w

Comparison to manual averaging

- **Proposition** (BJS): Any unbiased estimator for τ_w under arbitrary heterogeneity of treatment effects can be represented as an imputation estimator for some unbiased $\hat{\alpha}_i, \hat{\beta}_t$
- de Chaisemartin and D'Haultfœuille (2023) and Sun and Abraham (2021) are also imputation estimators that use less information to estimate $\hat{\alpha}_i, \hat{\beta}_t$
- Exception (Harmon 2022): If ε_{it} is a random walk, de Chaisemartin and D'Haultfœuille's estimator is efficient for $h = 0$
  - Outcomes at $E_i - 1$ contain all useful information; previous periods only add noise

BJS: Asymptotic standard errors

- Represent $\hat{\tau}_w = \sum_{it} v_{it} Y_{it}$: $v_{it} = w_{it}$ for $it \in \Omega_1$; v_{it} can be computed for Ω_0
- True variance: $\mathrm{Var}\left[\hat{\tau}_w\right] = \mathrm{Var}\left[\sum_{it} v_{it}\varepsilon_{it}\right] = \mathbb{E}\left[\sum_i \left(\sum_t v_{it}\varepsilon_{it}\right)^2\right]$
- Plug-in estimator: $\hat{\sigma}_w^2 = \sum_i \left(\sum_t v_{it}\hat{\varepsilon}_{it}\right)^2$ where $\hat{\varepsilon}_{it} = Y_{it} - \hat{\alpha}_i - \hat{\beta}_t$ for untreated obs.
- Key challenge: $Y_{it} - \hat{\alpha}_i - \hat{\beta}_t - \hat{\tau}_{it} = 0$ for treated obs. by construction
  - Impossible to separate variation in τ_{it} from ε_{it}:
    $\hat{\tau}_{it} = \tau_{it} + \varepsilon_{it} +$ noise from estimating FEs
- Obtain conservative SE by attributing some τ_{it} variation to ε_{it}

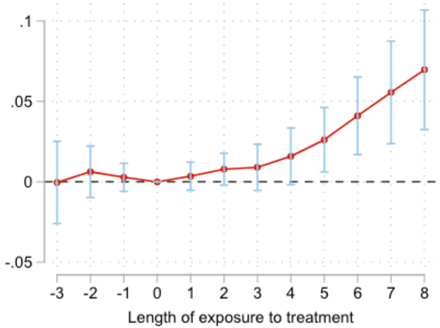
BJS: Asymptotic standard errors (2)

- E.g. $\tilde{\varepsilon}_{it} = \hat{\tau}_{it} - \hat{\tau}_{E_i t}$ where $\hat{\tau}_{et} =$ Avg of $\hat{\tau}_{jt}$ in the cohort $E_j = e$
- If cohorts are large, $\tilde{\varepsilon}_{it} \approx \varepsilon_{it} + (\tau_{it} - \bar{\tau}_{E_i t})$
  - SE are conservative when there is variation in τ_{it} within cohorts; otherwise asymptotically exact
- If cohorts are small, can replace $\hat{\tau}_{E_i t}$ with averages that pool multiple cohorts or use leave-out estimation; see BJS
  - SE of other estimators assume random sample of units $\Longrightarrow$ require large cohorts

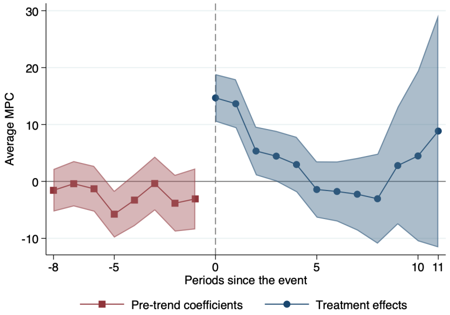
BJS: Pre-trend testing

- To test for pre-trends, always use only untreated observations
- Null hypothesis: $Y_{it} = \alpha_i + \beta_t + \varepsilon_{it}$
- Choose a richer alternative model: $Y_{it} = \alpha_i + \beta_t + \eta' W_{it} + \varepsilon_{it}$
  - E.g. anticipation effects: W_{it} are $\mathbf{1}[t = E_i - 1], \ldots, \mathbf{1}[t = E_i - L]$
  - Non-parallel linear trends: W_{it} are cohort dummies $\times t$
  - Structural break: W_{it} are cohort dummies $\times$ post financial crisis
- Use F-test for $\eta = 0$. (Don't include too many covariates to avoid low power)
- *Note:* not all violations affect causal estimates much
  - But Rambachan-Roth approach is not available for imputation yet
- *Bonus:* pre-trend coefs are not correlated w/ causal effects under spherical errors

dCDH and BJS event study graphs



(from de Chaisemartin, D'Haultfœuille, 2023 WP)

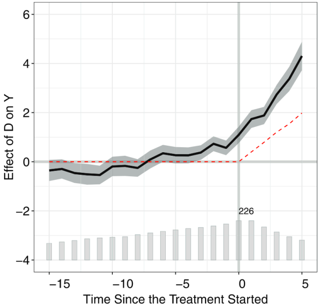


(BJS Figure 2a, using event_plot Stata command)

- Note different reference groups and different behavior of SE

Liu, Wang, Xu (AJPS 2022) pre-trend test

- Liu et al. also derive an imputation approach: "FE counterfactual estimator"
- Both for $h \geq 0$ and $h < 0$, plot averages of $Y_{it} - \hat{\alpha}_i - \hat{\beta}_t$



- Like with dCDH graphs, same interpretation on the left and right
- But no single reference period because imputation-based
- Unlike BJS, no explicit alternative hypothesis
- See Gardner (2021) and Tharkal, To (2021) for a similar approach

Flexible regression estimator

- Can the convenience of OLS be preserved without negative weights and other problems?
- Wooldridge (2021): the problem with old-school estimators is restrictive specifications $\Longrightarrow$ let's keep running regressions but more flexibly

$$Y_{it} = \alpha_i + \beta_t + \sum_e \sum_{s \geq e} \tau_{es} \mathbf{1}\left[E_i = e\right] \times \mathbf{1}\left[t = s\right] + \text{error}$$

- τ_{et} estimates CATT for cohort e in period t; then aggregate estimates as required
- For CATT-type estimands, equivalent to BJS in complete panels

Local projection DiD

- Dube, Girardi, Jorda, Taylor (2023) propose to estimate, for each h,

$$Y_{i,t+h} - Y_{i,t-1} = \beta_{ht} + \tau_h \mathbf{1}\left[t = E_i\right] + \text{error}$$

  on the subsample where $E_i = t$ or $E_i > t + h$

  - Combines the local projections estimator for time series of Jorda (2005) with "stacking" approach of Cengiz, Dube, Lindner, Zipperer (2019, Appendix D)

- τ_h estimates a convex weighted average of treatment effects
  - Can be reweighted to get dCDH
  - Instead of subtracting $Y_{i,t-1}$ can subtract average of several pre-period outcomes $\Longrightarrow$ closer to imputation
  - Can stack the data for different h to get joint confidence intervals

Does heterogeneity-robust estimation matter?

BJS application:

- Yes, relative to specifications that restrict dynamics
- But the semi-dynamic specification would also be fine

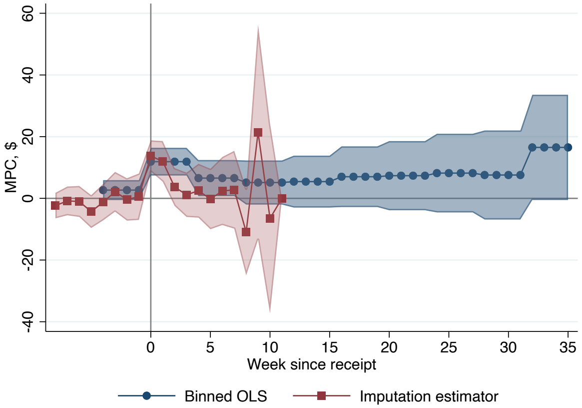
Setting: marginal propensity to spend (MPC) from the 2008 Economic Stimulus Payments (tax rebates)

- Staggered disbursement of rebates. Weekly spending data from Nielsen

Broda and Parker (2014) find a very large MPC from a monthly-binned specification:

$$Y_{it} = \alpha_i + \beta_t + \sum_{m=-1}^{\infty} \tau_m \mathbf{1}\left[t - E_i \in \{4m-3, \ldots, 4m\}\right] + \text{error}_{it}$$

BJS results

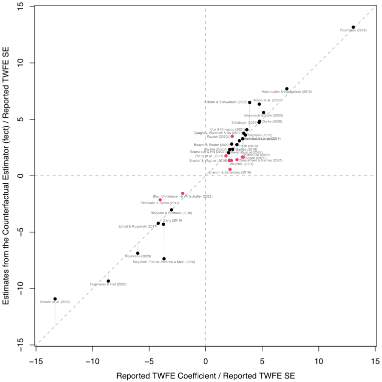


- Because the effects are fast decaying, binned specification overstates them in the first month
- Binned specification also extrapolates them to all future months

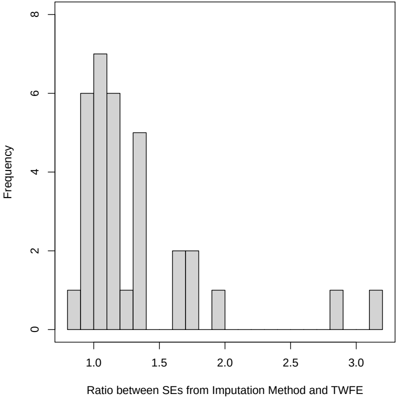
Chiu, Lan, Liu, Xu (2023) are less convinced

Chiu et al. reanalyze main significant TWFE coefficients in 37 political science papers — but also find no cost of being robust

FIGURE 4. TWFE VS. IMPUTATION METHOD



(From Chiu et al.)



(Computation by Yixing Xu)

Outline

Extensions

Feature	Baseline	Extensions
Reason for PTA:	None	Linking to selection models
Model of $Y(0)$:	TWFE (PTA)	Covariates
Model of $Y(1)$:	Arbitrary heterogeneity	*Ex ante* restrictions
Treatment:	Binary, absorbing	Continuous treatments Treatment reversals
Data structure:	Panel	Two-dimensional cross-sections Triple-differences

Link to selection models

New literature tries to reconcile PTA with self-selection into treatment

- de Chaisemartin, D'Haultfœuille (2022), Ghanem, Sant'Anna, Wuthrich (2023), Marx, Tamer, Tang (2023)
- Themes:
  - Is self-selection based on time-invariant or time-varying unobservables?
  - Is self-selection based on gains from treatment or on untreated potential outcomes ("Ashenfelter's dip")
  - Are agents forward-looking?
  - Can they learn about future potential outcomes from observing earlier outcomes?
  - Are $Y_{it}(0)$ a random walk, up to period FEs?

Covariates

Generalize the two approaches from canonical DiD:

- Imputation approach extends directly:
  - Estimate $Y_{it}(0) = \alpha_i + \beta_t + \gamma' X_{it}$ from untreated observations in the first step
  - Use $\hat{\tau}_{it} = Y_{it} - \hat{Y}_{it}(0)$ for treated observations

Covariates (2)

Generalize the two approaches from canonical DiD:

- Callaway and Sant'Anna (2021) apply the doubly-robust approach to canonical DiD (Sant'Anna and Zhao 2020) to dCDH
  - Impose $\mathbb{E}\left[Y_{it}(0) - Y_{i,t-1}(0) \mid X_i, E_i = e\right] = \mathbb{E}\left[Y_{it}(0) - Y_{i,t-1}(0) \mid X_i, E_i > s\right]$ for all $s \geq t \geq e$
  - For $t \geq e$ define $S_{ite} = \mathbf{1}\left[E_i = e \text{ or } E_i > t\right]$ and

$$p_{e,t}(X_i) = Pr(E_i = e \mid X_i; S_{ite} = 1); \quad m_{e,t}(X_i) = \mathbb{E}\left[Y_{it} - Y_{i,e-1} \mid X_i; E_i > t\right]$$

  - Estimate $p_{e,t}(X_i)$ and $m_{e,t}(X_i)$ to get $CATE_{et}$ by AIPW (or just regression adjustment or IPW), then average across cohorts
  - *Optional:* use Sun and Abraham (2021) instead of dCDCH
  - *Note:* good software packages even for the case without covariates

Restrictions on treatment effects

So far we have considered estimators robust to arbitrary effect heterogeneity

- This is a strong demand on the estimator $\Longrightarrow$ can't get all estimands
- Model is very asymmetric: strong assumptions on $Y_{it}(0)$ and none on $Y_{it}(1)$
- With so much heterogeneity possible, are ATTs informative about future policy?

  "Anyone who makes a living out of data analysis probably believes that heterogeneity is limited enough that the well-understood past can be informative about the future" (Angrist and Pischke 2010)

Restrictions on treatment effects

Can we get more power under extra restrictions on τ_{it} and $Y_{it}(1)$?

- Impose a linear treatment effect model $\tau_{it} = \Gamma_{it}'\theta$: e.g.
  - $\tau_{it} = \bar{\tau}$ is homogeneous across i and t $\implies$ *static TWFE specification*
  - $\tau_{it} \equiv \tau_{t-E_i}$ is homogeneous across i for any given horizon $\implies$ *semi-dynamic*
  - $\tau_{it} = 0$ when $t > E_i + K$ for some K
  - $\tau_{it+1} = \tau_{it}$ when $t > E_i + K$
- **Proposition** (BJS): Under PTA, spherical errors, and model of τ_{it}, the efficient unbiased estimator of τ_w is:
  1. Run OLS on the full sample for the "true" model: $Y_{it} = \alpha_i + \beta_t + D_{it}\Gamma_{it}'\theta + \varepsilon_{it}$
  2. Compute $\hat{\tau}_{it} = \Gamma_{it}'\hat{\theta}$ and $\hat{\tau}_w = \sum_{it\in\Omega_1} w_{it}\hat{\tau}_{it}$

Continuous treatments

Imputation works with untreated periods for every unit, untreated units for every period

- In BJS, the tax rebate amount is a continuous treatment but that's no problem
- Can identify e.g. the average MPC as % of household-specific rebate

Otherwise have to restrict effect heterogeneity in some way

- Explicit model of heterogeneity $\Longrightarrow$ imputation with restrictions
- Explicit contrasts (justified by stronger PTA): de Chaisemartin, D'Haultfœuille, Pasquier, Vazquez-Bare 2023
  - Consider D_{it} continuously distributed in every period (e.g. tariffs)
  - Impose $\mathbb{E}\left[\Delta Y_{i2}(d) \mid D_{i1}=d, D_{i2}\right]=\mathbb{E}\left[\Delta Y_{i2}(d) \mid D_{i1}=d\right] \Longrightarrow$ compare stayers and switchers with the same initial treatment
  - Can't match on D_{i1} exactly $\Longrightarrow$ estimate $\mathbb{E}\left[\Delta Y_{i2}(d) \mid D_{i1}=D_{i2}=d\right]$ nonparametrically for stayers and impute the counterfactual for switchers

Treatment reversals

What if treatment is not an absorbing state?

- E.g. Adda (2016): Economic effects of epidemics (e.g., flu); D_{it} = dummy for school holidays
- Key assumption: **no carryover effects**
  - If lagged treatment can have effects far into the future, there is no control group
- Imputation approach extends immediately
  - Estimate α_i, β_t from untreated observations both before and after treatment
- dCDH (2020) assume PTA on $Y_{it}(1)$ and use groups with $D_{i,t-1} = D_{it} = 1$ as controls for "leavers" who switch from $D_{i,t-1} = 1$, $D_{it} = 0$
- The no-carryover effects assumption should be tested

Two-dimensional cross-sections

- Suppose we have data by region i and age group g in a single period
- $D_{ig} = \mathbf{1}\left[g < E_i\right]$, e.g. a policy applies to 18y.o. but was rolled out in a staggered way
  - Impose $Y_{ig}(0) = \alpha_i + \beta_g$; use older groups as controls
- dCDH and imputation extend by redefining variables
  - E.g. $D_{ib} = \mathbf{1}\left[b \geq B_i\right]$ where b is birth year and B_i is the earliest birth year eligible

Triple-differences

- Manual averaging can be manually extended

- Imputation extends immediately: estimate

$$Y_{igt} = \alpha_{it} + \beta_{gt} + \gamma_{ig} + \text{error}$$

on all untreated observations