3rd Group Assignment

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5329: Inequality, Household Behavior, and the Macroeconomy

Instructions

- This assignment should be completed in groups of 2-3 students, and handed in through Canvas before 10 a.m. on the 8th of May.
- This assignment contributes 10% to your final grade.
- You can hand in either an ipynb or a jl, and a pdf file. For full marks, you are expected to carefully motivate your answers. Codes should be easy to follow and well-commented. The connection of verbal answers with corresponding codes (when relevant) should be made clear, especially if they are in separate files.

Tasks

1 Identification based on covariances

In the lecture, we saw that STY identified the parameters of their income process based on the relation of age and the cross-sectional variance of the log income residual. In this exercise, you need to explore an alternative method, relying on the covariance structure of log income residuals.

Similarly to STY, assume that

$$u_{i,h} = \alpha_i + \epsilon_{i,h} + z_{i,h} \text{ and } z_{i,h} = \rho z_{i,h-1} + \eta_{i,h}$$

$$\eta_{i,h} \sim \mathcal{N}(0, \sigma_\eta^2) \quad \epsilon_{i,h} \sim \mathcal{N}(0, \sigma_\epsilon^2) \quad \alpha_i \sim \mathcal{N}(0, \sigma_\alpha^2)$$

where u denotes the log income residual and h is age, expressed in years. Assume that $z_{i,22} = 0$, so the persistent income component of 22-year-old individuals is 0.

1. Express the following quantities as formulas containing the four parameters $(\sigma_\alpha, \sigma_\epsilon, \sigma_\eta, \rho)$: $Var(u_{i,60})$, $Cov(u_{i,60}, u_{i,58})$, $Cov(u_{i,60}, u_{i,56})$, $Cov(u_{i,60}, u_{i,30})$

2. Write a function in Julia, that takes a 4-element candidate vector of parameters and computes the corresponding moments from 1.1.

3. Compute these same moments in PSID. (Hint: It's simplest to create residuals as in 40_*Income_empirics.jl* up to line 69. Then proceed as in the following 10 lines, but use the 'age' variable instead of 'year' (and make other changes that you see necessary). If unstack complains about duplicate rows, you might need to run it with an extra keyword argument to drop all but the last one, like this:

   $unstack(psid\_residual, "id\_ind", "age", "income\_residual", combine = last))$

4. Find parameters that make the theoretical moments equal to the empirical ones. (Hint: relying on what you have from 1.2 and 1.3, you can write a function that to any parameter guess returns the vector of differences between the implied theoretical moments and the empirical ones. This new function you can pass to nlsolve. You can find some examples of using NLsolve in 7_*numerics.ipynb*.)

5. Compare your parameters to the ones obtained by STY. Compute the theoretical cross-sectional variances for each age group implied by your parameters, and compare the pattern with the empirical one (obtained in line 79 of 40_*Income_empirics.jl*). Discuss the differences.

2 Luxury bequests

When there was no bequest motive, the optimal decision of an agent in the final time-period was trivial. Not anymore. As in class, the oldest agents solve

$$\max_{a \geq 0} \frac{(x-a)^{1-\gamma}-1}{1-\gamma} + \Phi_1 \Big(1 + \frac{a[1+r(1-\tau_c)](1-\tau_b)}{\Phi_2}\Big)^{1-\gamma}$$

1. Derive the optimal saving level as a function of x (cash-on-hand).

2. Show that in this model leaving bequests is indeed a luxury good.

3. Based on the solution you derived, interpret the two Φ parameters.

4. Which quantities are used to discipline the two Φ parameters in De Nardi (2004)? How does her approach conform to your findings?

3 Capital tax and welfare

Consider the setting from 41_*Inequality_inheritence.jl* with bequest motive and ability transfer as your benchmark. As an alternative setting, consider an otherwise identical economy where capital income tax equals 0.

1. Compute the labor income tax level that balances the budget for your alternative model. (The balancing labor income tax for the benchmark setting should already be available from class.) Solve both models with the appropriate tax levels.

2. We want to figure out which model agents are happier in, depending on their wealth.

   (a) Write a function, that to any cash-on-hand level assigns the welfare gain of being in the alternative model instead of the benchmark one, for someone in the first period (age 22), with middle (so 0) z and α states. The welfare gain should be expressed as a percentage of consumption units.

   (b) Choose an appropriate grid for coh.

   (c) Plot your function over this grid and interpret the results.

3. Repeat the previous point with the lowest and highest values of α. Discuss the differences.

4. To quantify the average gain or loss from being in the alternative world, compute the expected (average) value of age 22 agents in each economy. This is the simplest to do via simulation. Using these average values, again compute the welfare difference between the two settings expressed in percentages of consumption.

5. Our model is simpler than that of De Nardi (2004). How do each of the following simplifications affect your findings, in your opinion? Only qualitative reasoning is expected here.

   (a) We don't close the capital market, so capital is not used in production.

   (b) Our agents do not expect to receive bequests (even though they do receive them).