# Seminar 8 + 9 - Exercises

Review questions

- If $S_1(p) = p - 10$ and $S_2(p) = p - 5$, then at what price does the industry supply curve have a kink in?
  A: when $p = 10$
- In the short run the demand for cigarettes is totally inelastic. In the long run, suppose that it is perfectly elastic. What is the impact of a cigarette tax on the price consumers pay in the short run and in the long run?
  A: In the short run, tax increases price for consumers; in the long run, tax increases costs for producers.
- True or false? In long–run industry equilibrium no firm will be losing money. Why?
  A: True, firms with negative profit would exit the market (the least profit in long run is 0, as there are no fixed costs).
- A New York City cab operator appears to be making positive profits in the long run after carefully accounting for the operating and labor costs. Does this violate the competitive model? Why or why not?
  A: In LR, $\Pi = 0$ for perfect competition, so it violates the competitive model.

1. Assume a firm operating in the competitive market where all firms have same cost functions: $LTC = y^3 - 3y^2 + 5y$:
   - Calculate $LAC(y)$ and $LMC(y)$. Find minimum of $LAC(y)$.
     A: $LAC(y) = y^2 - 3y + 5$, $LMC(y) = 3y^2 - 6y + 5$, $\min LAC(y) = \frac{3}{2}$
   - What will be the optimal output $y^*$ and price $p^*$ in the long–run? What will be expected profit $\Pi^*$? A: $y^* = \frac{3}{2}$, $p^* = \frac{11}{4}$, $\Pi = 0$
   - Suppose long–run market demand $D_Q(p) = 101 - 4p$, what will be the equilibrium output of the industry $Q^*$ and number of firms $n^*$?
     A: $Q^* = 90$, $n^* = 60$
   - Suppose increase in the long–run market demand to $D_Q^1(p) = 152 - 4p$ and simultaneously rise of input prices to their double value. What are new optimal values $y^{*1}$, $p^{*1}$, $Q^{*1}$, $n^{*1}$
     A: $y^{*1} = \frac{3}{2}$, $p^{*1} = \frac{11}{2}$, $Q^{*1} = 130$, $n^{*1} = 86$
2. Joshua is producing special green beverage. It takes $0.5l$ of one secret ingredient and $0.5l$ of the second secret ingredient to produce $1l$ of the green beverage. Total costs of producing $y$ liters of beverage are described by the cost function $c(p_0 y) = \dfrac{y^2}{2} + p_0 y$, where $p_0$ is price of $1l$ of both ingredients.
   - Express the marginal cost of producing beverage as a function of $p_0$ and $y$.
     A: $MC = y + p_0$
   - Suppose that Joshua can buy 25 litres of each ingredient for £5 a liter and must pay £15 for every additional liter. What are $MC$ when $y \leq 50$ and $y > 50$?
     A: $MC = y + 5$ when $y \leq 50$ and $MC = y + 15$ when $y > 50$
   - Derive the supply of the firm and plot it into a graph.
     A: $S(p) = p - 5$ when $y \leq 50$ and $S(p) = p - 15$ when $y > 50$. See seminar slides for graph.
   - What is the firm's output when demand is horizontal line at a price of £30? A: $y = 25$
3. A firm in a perfectly competitive market has short-run cost function: $STC = y^3 - \frac{15}{2}y^2 + 20y + 5$
   - Calculate minimal $y_0$ and $p_0$ when a firm starts producing positive output.
     A: $y_0 = \frac{15}{4}$, $p_0 = 5.94$

- Suppose that market price is $p = 10$. What is the profit of the firm? What is the output of the whole industry when 150 firms operate on the market in this moment?
  $\Pi = 11.2$, $Q = 631.5$
- What will be the long-run inverse supply $S_Q^{-1}(y)$ of the industry considering that every firm in the market has the same cost function and no input is fixed.
  $S_Q^{-1}(y) = 5.94$

4. Firm in a perfectly competitive market has the equilibrium output equal to 20. Long run costs for given level of production are $LTC(20) = 700$.

   - What is the price of the output?
     A: $p = 35$
   - Draw the situation into a graph.

5. Assume that the market demand curve is $Q_D = 300 - 2p^2$ and the market supply curve is $Q_S = 20 + 4p^2$.

   - Find the equilibrium price and quantity.
     A: $p^* = 6.83$, $Q^* = 206.7$
   - Suppose the government imposes on each firm a \$5 tax on each unit sold. How much of the \$5 tax is shifted to the customer in the short-run? How much in the long run?
     A: $p^*_{SR} = 9.75$, $p^*_{LR} = 11.83$
   - Calculate tax revenue in the short–run.
     A: Firm has to pay $tax = 550$ to the government, but it obtains only 320 from consumers.
   - Draw the situation of both long and short run into one graph.
     See seminar slides.

6. Assume that a firm in a perfect competitive market has cost function: $TC(q) = 3q^3 - 6q^2 + 10q + 4$

   - Find the minimum price for which the firm is willing to operate.
     A: $p_0 = 7$, $q_0 = 1$
   - What is the profit of the firm in this case?
     A: $\pi(p_0) = -4 = -FC$
   - Depict the situation graphically, depict MR, AR, MC, AC and AVC, and show the profit/loss of the firm.
     A: $MR = AR = p$; $MC = 9q^2 - 12q + 10$; $AC = 3q^2 - 6q + 10 + \frac{4}{q}$; $AVC = 3q^2 - 6q + 10$

7. Assume that there is 1000 identical firms operating in a perfectly competitive market. Each of them has the same short-run total cost function: $STC(q) = q + q^2 + FC$ and faces market demand $P = 10 - \frac{Q}{1000}$. Calculate the producer and consumer surplus of the market.

   - A: $CS = 4500$; $PS = 9000$

8. Assume that a firm in a perfect competitive market has short-term cost function: $STC(y) = y^2 + 30y + 400$

   - Calculate the output $y_0$ and price $p_0$ at the shut-down point.
     A: $y_0 = 0$; $p_0 = 30$
   - What is the short-term supply of the firm?
     A: $S(p) = \frac{p-30}{2}$ for $p \geq 30$
   - Find the point where costs equal revenues.
     A: $\pi = 0$ for $y = 20$ and $p = 70$

   Now assume that 100 such firms operate in the short-term perfect competitive market and that all has the same cost functions as above.

   - Calculate the short-term industry supply.
     A: $S(p) = 50(p - 30)$ for $p \geq 30$

- What is the total amount of goods supplied for $p^* = 80$?
  A: $Q_S(80) = 2500$

Now consider a demand $Q_D^* = Q_D(p) = 2500 - 30p$ on the same market.

- Calculate the equilibrium price.
  A: $p^* = 50$
- Calculate the equilibrium quantity of the industry.
  A: $Q_S^* = Q_D^* = 1000$

9. Assume a firm's production function $f(z) = (3 \cdot z_1^{\frac{1}{3}} z_2^{\frac{2}{3}})^{\frac{1}{2}}$, with the $y_0$ and input prices $w_1$ and $w_2$.

- Find the conditional demand functions $z_1(w, y)$, $z_2(w, y)$ and derive the cost function $C(w, y)$.
  A: $\frac{w_1}{w_2} = \frac{MP_1}{MP_2} \Rightarrow z_1 = \frac{y^2}{3}\left(\frac{w_2}{2w_1}\right)^{\frac{2}{3}}$; $z_2 = \frac{y^2}{3}\left(\frac{2w_1}{w_2}\right)^{\frac{1}{3}}$;
  $$C(w, y) = w_1\frac{y^2}{3}\left(\frac{w_2}{2w_1}\right)^{\frac{2}{3}} + w_2\frac{y^2}{3}\left(\frac{2w_1}{w_2}\right)^{\frac{1}{3}} = \frac{y^2}{3} w_1^{\frac{1}{3}} w_2^{\frac{2}{3}} (2^{-\frac{2}{3}} + 2^{\frac{1}{3}}) = \frac{y^2}{3} w_1^{\frac{1}{3}} w_2^{\frac{2}{3}} \frac{3}{2^{\frac{2}{3}}} = \frac{y^2 w_1^{\frac{1}{3}} w_2^{\frac{2}{3}}}{2^{\frac{2}{3}}}$$
- Find the supply function $y(p, w)$.
  A: $S(p) = \frac{p}{2^{\frac{1}{3}} w_1^{\frac{1}{3}} w_2^{\frac{2}{3}}}$
- Derive the Marshallian demand curves $z_1(p, w)$, $z_2(p, w)$.
  A: $z_1 = \left(\frac{p}{2^{\frac{1}{3}} w_1^{\frac{1}{3}} w_2^{\frac{2}{3}}}\right)^2 \frac{1}{3}\left(\frac{w_2}{2w_1}\right)^{\frac{2}{3}} = \frac{p^2}{3 \cdot 2^{\frac{4}{3}} w_1^{\frac{4}{3}} w_2^{\frac{2}{3}}}$;
  $$z_2 = \frac{1}{3}\left(\frac{p}{2^{\frac{1}{3}} w_1^{\frac{1}{3}} w_2^{\frac{2}{3}}}\right)^2 \left(\frac{2w_1}{w_2}\right)^{\frac{1}{3}} = \frac{p^2}{3 \cdot 2^{\frac{1}{3}} w_1^{\frac{1}{3}} w_2^{\frac{5}{3}}}$$
- Derive the profit function of the firm $\pi(p, w)$.
  A: $\pi = \frac{1}{2}\frac{p^2}{2^{\frac{1}{3}} w_1^{\frac{1}{3}} w_2^{\frac{2}{3}}}$
- Verify the Hotelling's Lemma holds for these functions.
  $$\frac{\partial \pi}{\partial p} = \frac{p}{2^{\frac{1}{3}} w_1^{\frac{1}{3}} w_2^{\frac{2}{3}}} = S(p)$$
  $$\frac{\partial \pi}{\partial w_1} = -\frac{p^2}{3 \cdot 2^{\frac{4}{3}} w_1^{\frac{4}{3}} w_2^{\frac{2}{3}}} = -z_1(w, p)$$
  $$\frac{\partial \pi}{\partial w_2} = -\frac{p^2}{3 \cdot 2^{\frac{1}{3}} w_1^{\frac{1}{3}} w_2^{\frac{5}{3}}} = -z_2(w, p)$$