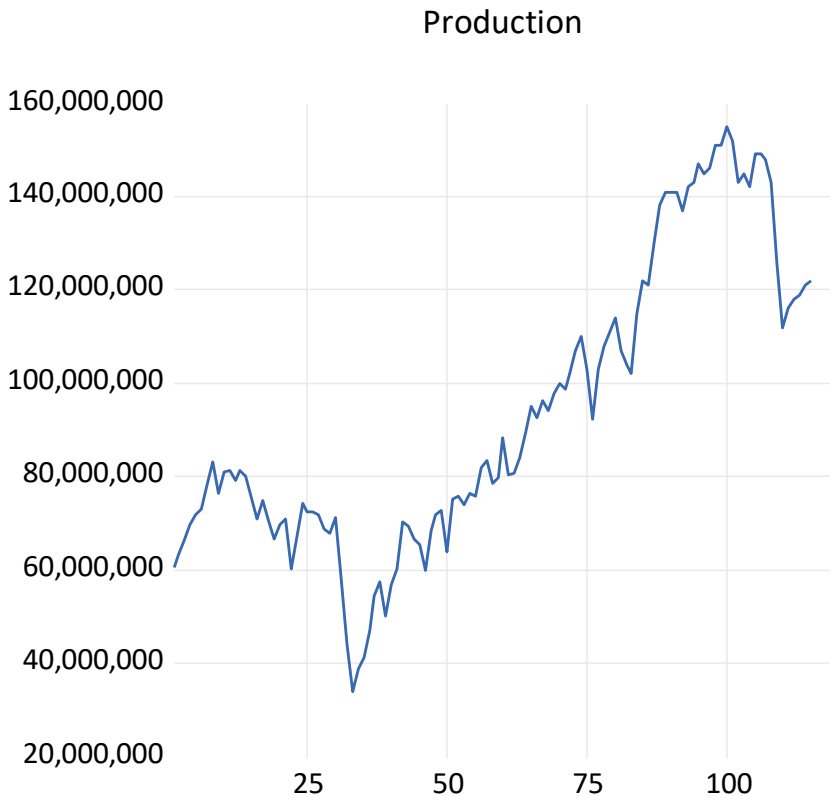
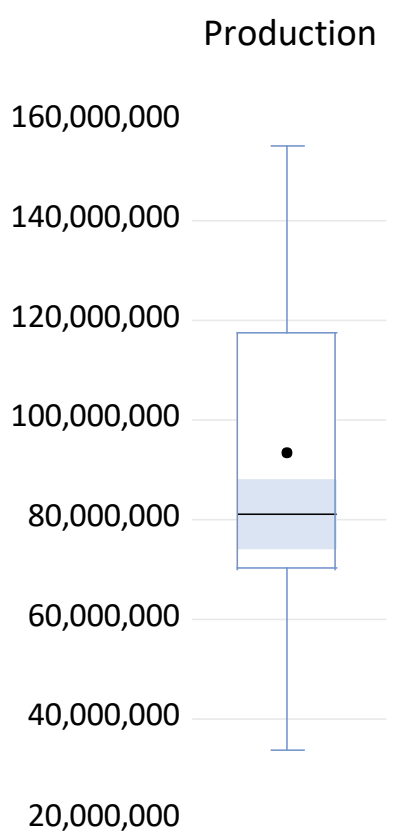
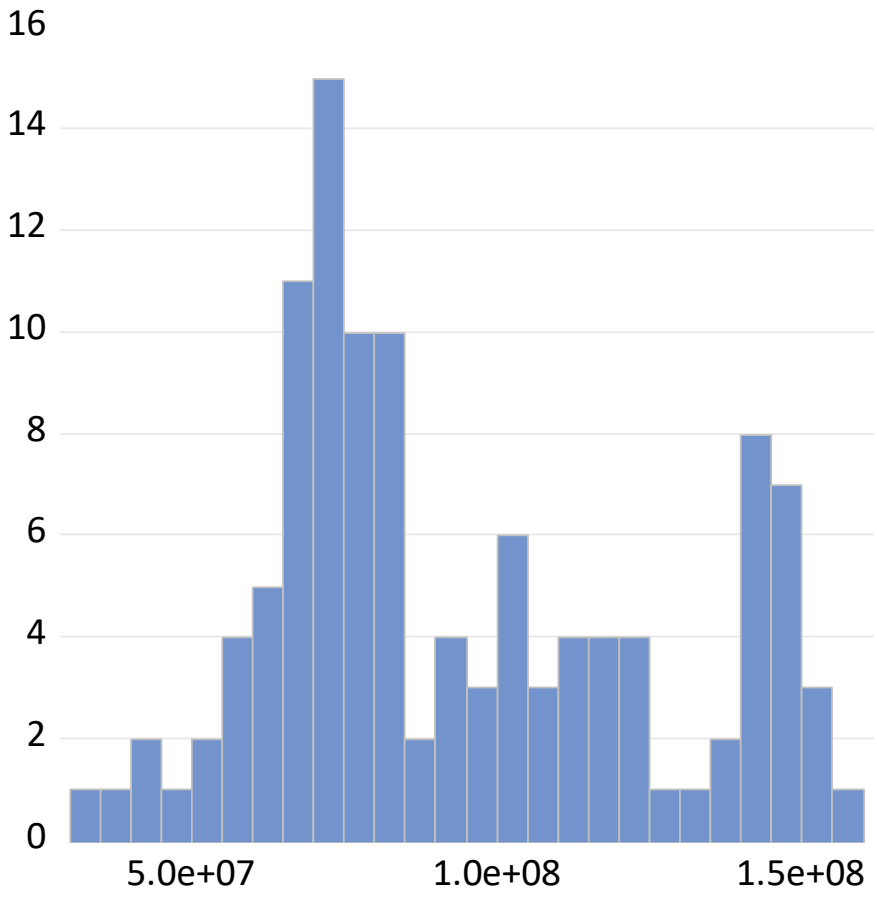
PART 1

Data

TOTAL FORESTRY STATISTICS, U.S. GEOLOGICAL SURVEY. [All values in metric tons], annual from 1900 to 2014







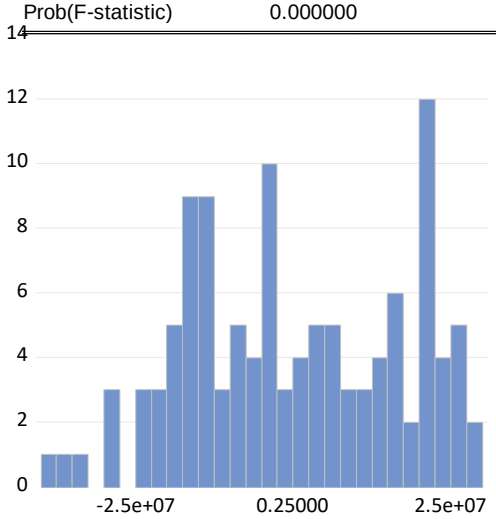
Series: PRODUCTION	
Sample 1 118	
Observations 115	
Mean	93499130
Median	81400000
Maximum	1.55e+08
Minimum	33900000
Std. Dev.	31417376
Skewness	0.457300
Kurtosis	2.097688
Jarque-Bera	7.909415
Probability	0.019164

Estimation

Dependent Variable: PRODUCTION
Method: Least Squares
Date: 03/01/23 Time: 13:02
Sample (adjusted): 1 115
Included observations: 115 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	48339325	3165251.	15.27188	0.0000
@TREND	792277.3	47985.90	16.51063	0.0000

R-squared	0.706951	Mean dependent var	93499130
Adjusted R-squared	0.704357	S.D. dependent var	31417376
S.E. of regression	17082577	Akaike info criterion	36.16225
Sum squared resid	3.30E+16	Schwarz criterion	36.20999
Log likelihood	-2077.330	Hannan-Quinn criter.	36.18163
F-statistic	272.6008	Durbin-Watson stat	0.107119
Prob(F-statistic)	0.000000		



Series: Residuals
Sample 1 115
Observations 115

Mean	-7.48e-09
Median	-1344676.
Maximum	29114734
Minimum	-39792198
Std. Dev.	17007488
Skewness	-0.065956
Kurtosis	1.987706
Jarque-Bera	4.993584
Probability	0.082349

Breusch-Godfrey Serial Correlation LM Test:
Null hypothesis: No serial correlation at up to 10 lags

F-statistic	93.09727	Prob. F(10,103)	0.0000
Obs*R-squared	103.5442	Prob. Chi-Square(10)	0.0000

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 03/01/23 Time: 13:10
Sample: 1 115
Included observations: 115
Presample missing value lagged residuals set to zero.

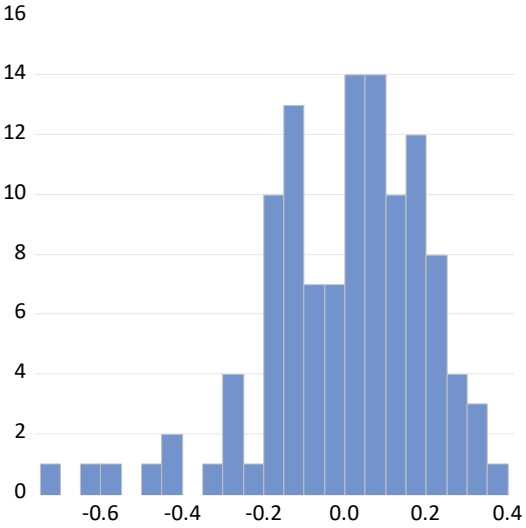
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	178635.3	1050519.	0.170045	0.8653
@TREND	-4814.162	16046.79	-0.300008	0.7648
RESID(-1)	1.115993	0.098617	11.31642	0.0000
RESID(-2)	-0.242851	0.147357	-1.648048	0.1024
RESID(-3)	0.095898	0.146754	0.653459	0.5149
RESID(-4)	-0.038736	0.146778	-0.263906	0.7924
RESID(-5)	0.034905	0.146970	0.237500	0.8127
RESID(-6)	0.057546	0.149739	0.384309	0.7015
RESID(-7)	0.085544	0.149959	0.570446	0.5696
RESID(-8)	-0.287926	0.150711	-1.910455	0.0589
RESID(-9)	0.114149	0.152424	0.748890	0.4556
RESID(-10)	-0.007647	0.106861	-0.071564	0.9431

R-squared	0.900384	Mean dependent var	-7.48E-09
Adjusted R-squared	0.889746	S.D. dependent var	17007488
S.E. of regression	5647264.	Akaike info criterion	34.02973
Sum squared resid	3.28E+15	Schwarz criterion	34.31616
Log likelihood	-1944.710	Hannan-Quinn criter.	34.14599
F-statistic	84.63388	Durbin-Watson stat	1.953568
Prob(F-statistic)	0.000000		

Dependent Variable: PRODUCTION
Method: Least Squares
Date: 03/01/23 Time: 13:04
Sample (adjusted): 1 115
Included observations: 115 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	67581937	4025147.	16.78993	0.0000
@TREND	-229454.3	163172.4	-1.406208	0.1624
@TREND^2	8962.558	1385.137	6.470521	0.0000

R-squared	0.786690	Mean dependent var	93499130
Adjusted R-squared	0.782881	S.D. dependent var	31417376
S.E. of regression	14639255	Akaike info criterion	35.86205
Sum squared resid	2.40E+16	Schwarz criterion	35.93366
Log likelihood	-2059.068	Hannan-Quinn criter.	35.89112
F-statistic	206.5287	Durbin-Watson stat	0.149193
Prob(F-statistic)	0.000000		



Series: Residuals
Sample 1 115
Observations 115

Mean	4.62e-15
Median	0.018874
Maximum	0.353234
Minimum	-0.749991
Std. Dev.	0.200449
Skewness	-1.023169
Kurtosis	4.630822
Jarque-Bera	32.80895
Probability	0.000000

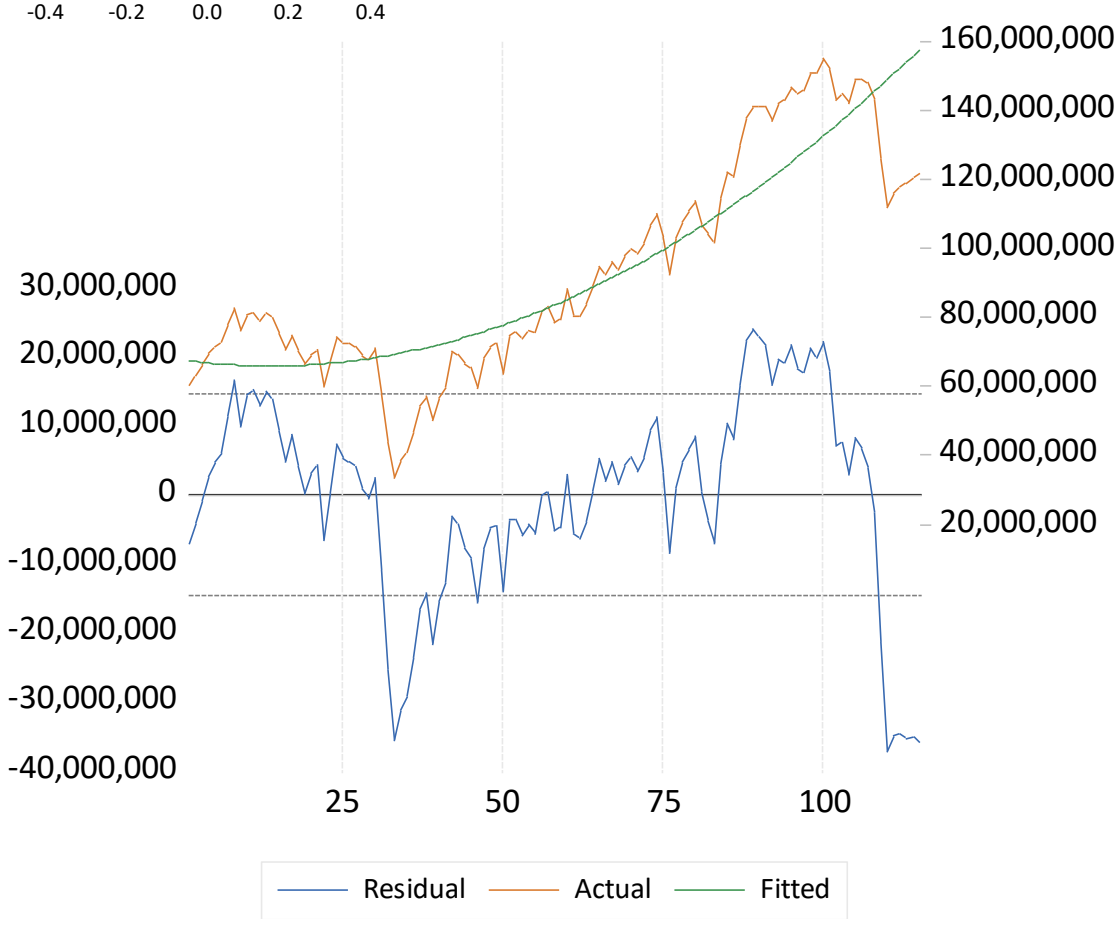
Breusch-Godfrey Serial Correlation LM Test:
Null hypothesis: No serial correlation at up to 10 lags

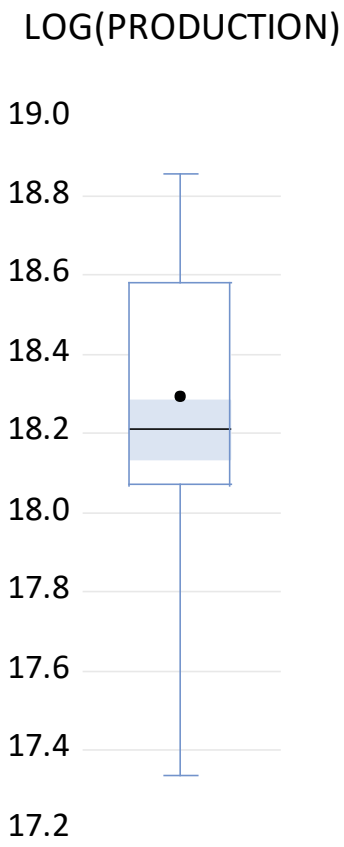
F-statistic	64.99266	Prob. F(10,102)	0.0000
Obs*R-squared	99.40007	Prob. Chi-Square(10)	0.0000

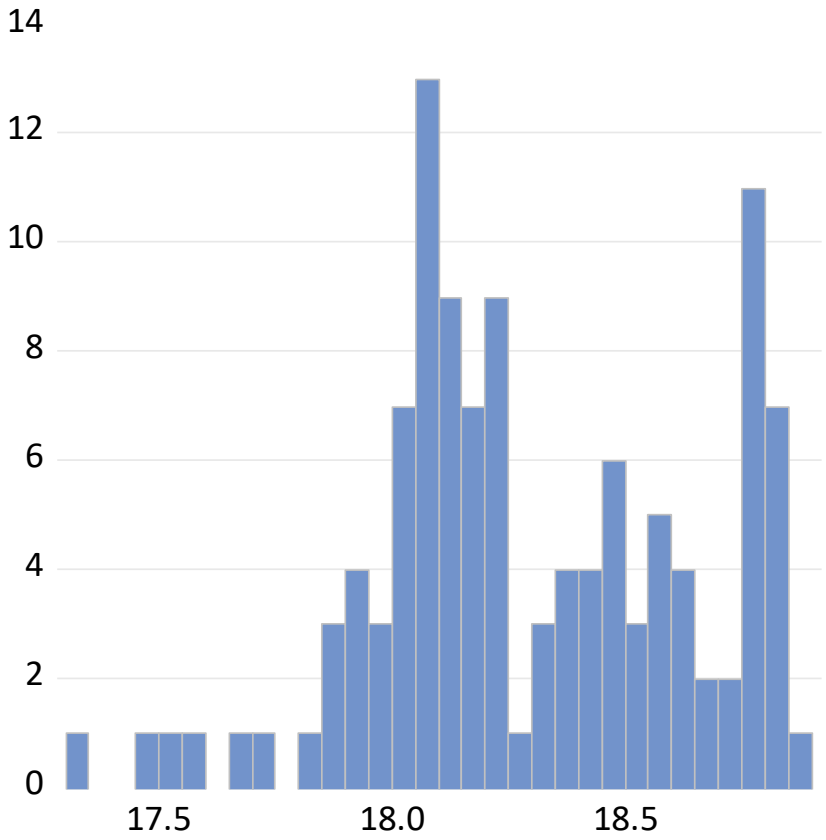
Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 03/01/23 Time: 13:12
Sample: 1 115
Included observations: 115
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-583462.9	1647215.	-0.354212	0.7239
@TREND	41892.37	73274.61	0.571717	0.5688
@TREND^2	-463.5714	665.0100	-0.697089	0.4873
RESID(-1)	1.105811	0.098924	11.17837	0.0000
RESID(-2)	-0.241159	0.147557	-1.634345	0.1053
RESID(-3)	0.094575	0.148207	0.638127	0.5248
RESID(-4)	-0.037059	0.148247	-0.249980	0.8031
RESID(-5)	0.038824	0.148499	0.261446	0.7943
RESID(-6)	0.032648	0.151573	0.215398	0.8299
RESID(-7)	0.074283	0.151855	0.489171	0.6258
RESID(-8)	-0.213286	0.152416	-1.399364	0.1647
RESID(-9)	0.021547	0.152750	0.141060	0.8881
RESID(-10)	0.050256	0.111072	0.452463	0.6519

R-squared	0.864348	Mean dependent var	-1.81E-09
Adjusted R-squared	0.848389	S.D. dependent var	14510272
S.E. of regression	5649894.	Akaike info criterion	34.03830
Sum squared resid	3.26E+15	Schwarz criterion	34.34860
Log likelihood	-1944.202	Hannan-Quinn criter.	34.16425
F-statistic	54.16055	Durbin-Watson stat	1.979798
Prob(F-statistic)	0.000000		





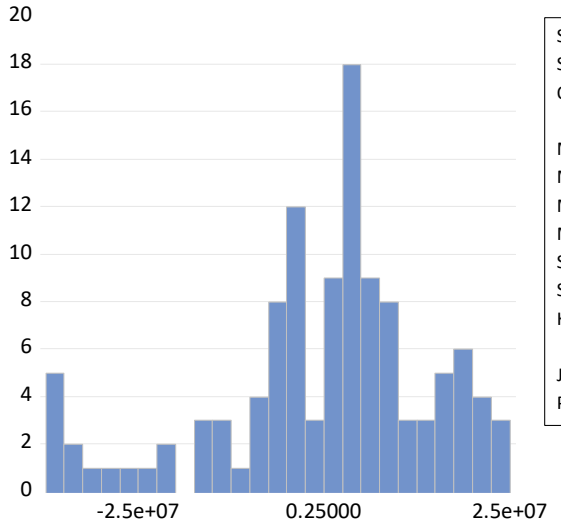


Series: LOG(PRODUCTION)	
Sample 1 118	
Observations 115	
Mean	18.29672
Median	18.21489
Maximum	18.85894
Minimum	17.33893
Std. Dev.	0.342029
Skewness	-0.144876
Kurtosis	2.541114
Jarque-Bera	1.411302
Probability	0.493787

Dependent Variable: LOG(PRODUCTION)
Method: Least Squares
Date: 03/01/23 Time: 13:14
Sample (adjusted): 1 115
Included observations: 115 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	17.82293	0.037305	477.7564	0.0000
@TREND	0.008312	0.000566	14.69691	0.0000

R-squared	0.656534	Mean dependent var	18.29672
Adjusted R-squared	0.653494	S.D. dependent var	0.342029
S.E. of regression	0.201334	Akaike info criterion	-0.350461
Sum squared resid	4.580514	Schwarz criterion	-0.302723
Log likelihood	22.15152	Hannan-Quinn criter.	-0.331085
F-statistic	215.9991	Durbin-Watson stat	0.139894
Prob(F-statistic)	0.000000		



Series: Residuals
Sample 1 115
Observations 115

Mean	-1.81e-09
Median	2725763.
Maximum	24203994
Minimum	-37055567
Std. Dev.	14510272
Skewness	-0.812321
Kurtosis	3.445996
Jarque-Bera	13.60053
Probability	0.001113

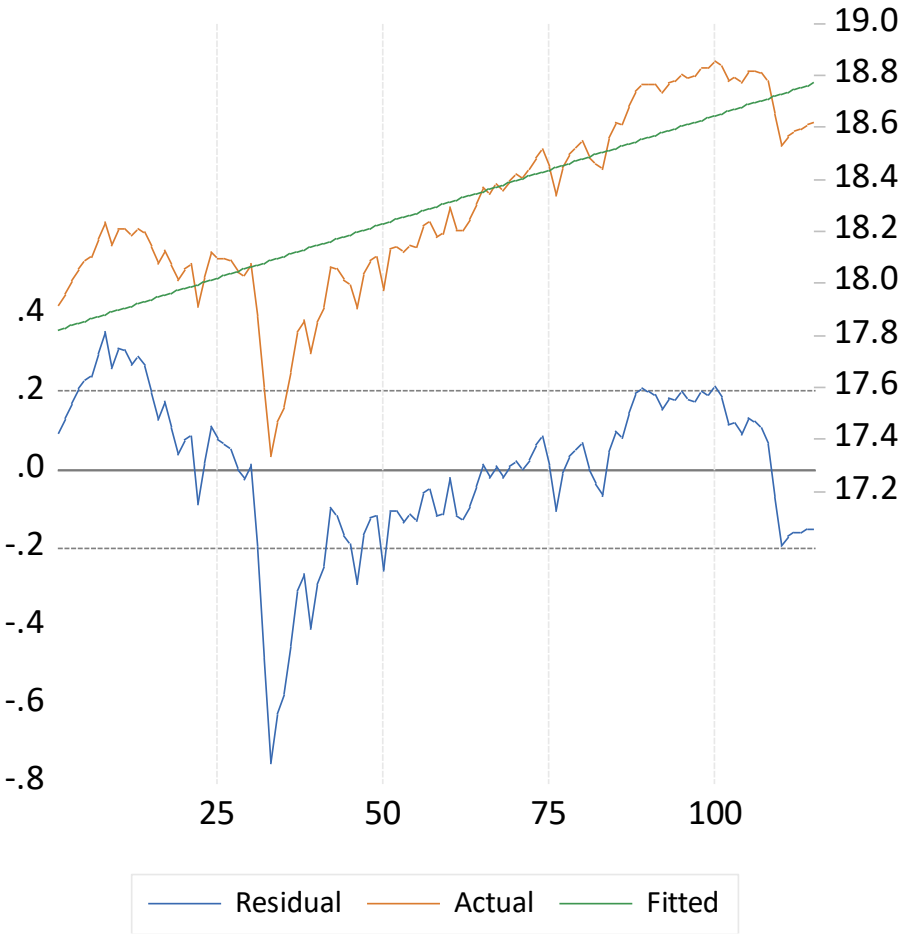
Breusch-Godfrey Serial Correlation LM Test:
Null hypothesis: No serial correlation at up to 10 lags

F-statistic	74.27011	Prob. F(10,103)	0.0000
Obs*R-squared	100.9939	Prob. Chi-Square(10)	0.0000

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 03/01/23 Time: 13:13
Sample: 1 115
Included observations: 115
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.001633	0.013659	0.119547	0.9051
@TREND	-4.30E-05	0.000208	-0.206982	0.8364
RESID(-1)	1.120479	0.098340	11.39391	0.0000
RESID(-2)	-0.208247	0.147487	-1.411963	0.1610
RESID(-3)	-0.009531	0.146249	-0.065172	0.9482
RESID(-4)	0.060101	0.146253	0.410941	0.6820
RESID(-5)	-0.109439	0.145209	-0.753666	0.4528
RESID(-6)	0.200573	0.146335	1.370644	0.1735
RESID(-7)	-0.015133	0.147515	-0.102584	0.9185
RESID(-8)	-0.288828	0.147818	-1.953941	0.0534
RESID(-9)	0.109781	0.149374	0.734943	0.4640
RESID(-10)	0.064136	0.100941	0.635385	0.5266

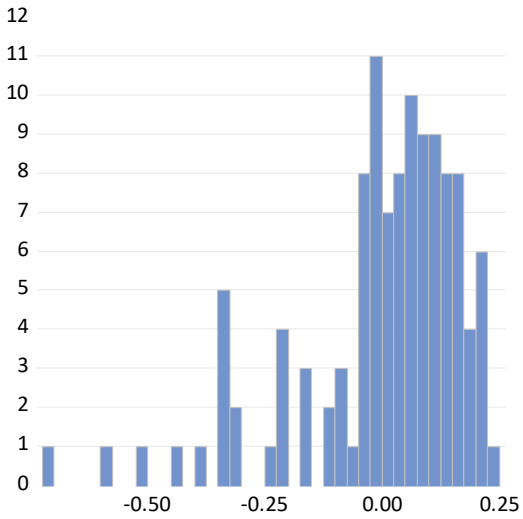
R-squared	0.878208	Mean dependent var	4.62E-15
Adjusted R-squared	0.865201	S.D. dependent var	0.200449
S.E. of regression	0.073595	Akaike info criterion	-2.281985
Sum squared resid	0.557872	Schwarz criterion	-1.995557
Log likelihood	143.2141	Hannan-Quinn criter.	-2.165726
F-statistic	67.51828	Durbin-Watson stat	1.981908
Prob(F-statistic)	0.000000		



Dependent Variable: LOG(PRODUCTION)
Method: Least Squares
Date: 03/01/23 Time: 13:15
Sample (adjusted): 1 115
Included observations: 115 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	18.01583	0.049832	361.5284	0.0000
@TREND	-0.001930	0.002020	-0.955496	0.3414
@TREND^2	8.98E-05	1.71E-05	5.239201	0.0000

R-squared	0.724142	Mean dependent var	18.29672
Adjusted R-squared	0.719216	S.D. dependent var	0.342029
S.E. of regression	0.181238	Akaike info criterion	-0.552272
Sum squared resid	3.678884	Schwarz criterion	-0.480665
Log likelihood	34.75562	Hannan-Quinn criter.	-0.523207
F-statistic	147.0030	Durbin-Watson stat	0.175567
Prob(F-statistic)	0.000000		



Series: Residuals
Sample 1 115
Observations 115

Mean	-3.09e-15
Median	0.046687
Maximum	0.227632
Minimum	-0.707136
Std. Dev.	0.179641
Skewness	-1.509789
Kurtosis	5.343114
Jarque-Bera	69.99683
Probability	0.000000

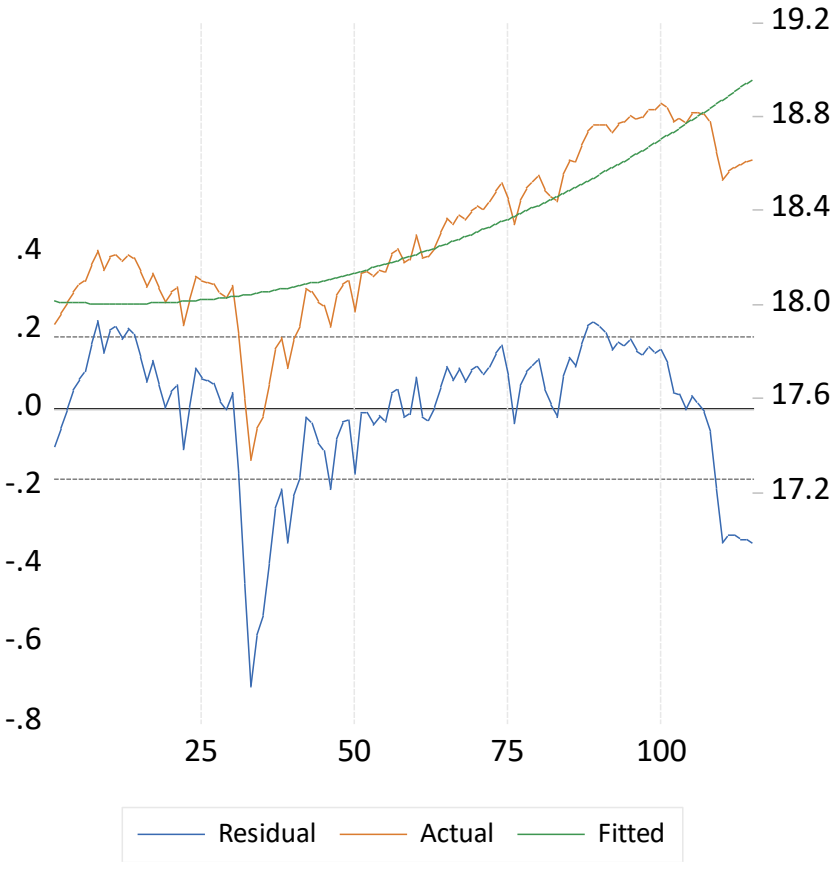
Breusch-Godfrey Serial Correlation LM Test:
Null hypothesis: No serial correlation at up to 10 lags

F-statistic	56.14266	Prob. F(10,102)	0.0000
Obs*R-squared	97.31907	Prob. Chi-Square(10)	0.0000

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 03/01/23 Time: 13:15
Sample: 1 115
Included observations: 115
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.007857	0.020936	-0.375291	0.7082
@TREND	0.000551	0.000882	0.624783	0.5335
@TREND^2	-5.97E-06	7.72E-06	-0.773065	0.4413
RESID(-1)	1.099922	0.098461	11.17110	0.0000
RESID(-2)	-0.202623	0.146797	-1.380294	0.1705
RESID(-3)	-0.003032	0.146480	-0.020697	0.9835
RESID(-4)	0.051468	0.146481	0.351360	0.7260
RESID(-5)	-0.096264	0.145693	-0.660731	0.5103
RESID(-6)	0.174071	0.146895	1.184997	0.2388
RESID(-7)	-0.015813	0.147832	-0.106966	0.9150
RESID(-8)	-0.227884	0.148058	-1.539156	0.1269
RESID(-9)	0.034992	0.148626	0.235434	0.8143
RESID(-10)	0.112218	0.103324	1.086075	0.2800

R-squared	0.846253	Mean dependent var	-3.09E-15
Adjusted R-squared	0.828165	S.D. dependent var	0.179641
S.E. of regression	0.074467	Akaike info criterion	-2.250804
Sum squared resid	0.565618	Schwarz criterion	-1.940507
Log likelihood	142.4212	Hannan-Quinn criter.	-2.124856
F-statistic	46.78555	Durbin-Watson stat	1.984513
Prob(F-statistic)	0.000000		



As autocorrelation was present in all the models, the HAC standard errors need to be considered for inference

Dependent Variable: PRODUCTION
Method: Least Squares
Date: 03/01/23 Time: 13:11
Sample (adjusted): 1 115
Included observations: 115 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	48339325	7433188.	6.503175	0.0000
@TREND	792277.3	109352.0	7.245203	0.0000

R-squared	0.706951	Mean dependent var	93499130
Adjusted R-squared	0.704357	S.D. dependent var	31417376
S.E. of regression	17082577	Akaike info criterion	36.16225
Sum squared resid	3.30E+16	Schwarz criterion	36.20999
Log likelihood	-2077.330	Hannan-Quinn criter.	36.18163
F-statistic	272.6008	Durbin-Watson stat	0.107119
Prob(F-statistic)	0.000000	Wald F-statistic	52.49297
Prob(Wald F-statistic)	0.000000		

Dependent Variable: LOG(PRODUCTION)
Method: Least Squares
Date: 03/01/23 Time: 13:13
Sample (adjusted): 1 115
Included observations: 115 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	17.82293	0.093700	190.2128	0.0000
@TREND	0.008312	0.001198	6.938777	0.0000

R-squared	0.656534	Mean dependent var	18.29672
Adjusted R-squared	0.653494	S.D. dependent var	0.342029
S.E. of regression	0.201334	Akaike info criterion	-0.350461
Sum squared resid	4.580514	Schwarz criterion	-0.302723
Log likelihood	22.15152	Hannan-Quinn criter.	-0.331085
F-statistic	215.9991	Durbin-Watson stat	0.139894
Prob(F-statistic)	0.000000	Wald F-statistic	48.14662
Prob(Wald F-statistic)	0.000000		

Estimating the four given models and evaluating their performance led us to conclude that the quadratic trend is significant in both specifications. Graphs of the residuals as well as the goodness of fit and information criteria noticeably improved by its addition.

Dependent Variable: PRODUCTION
Method: Least Squares
Date: 03/01/23 Time: 13:12
Sample (adjusted): 1 115
Included observations: 115 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	67581937	6199084.	10.90192	0.0000
@TREND	-229454.3	315225.5	-0.727905	0.4682
@TREND^2	8962.558	3313.599	2.704781	0.0079

R-squared	0.786690	Mean dependent var	93499130
Adjusted R-squared	0.782881	S.D. dependent var	31417376
S.E. of regression	14639255	Akaike info criterion	35.86205
Sum squared resid	2.40E+16	Schwarz criterion	35.93366
Log likelihood	-2059.068	Hannan-Quinn criter.	35.89112
F-statistic	206.5287	Durbin-Watson stat	0.149193
Prob(F-statistic)	0.000000	Wald F-statistic	37.30606
Prob(Wald F-statistic)	0.000000		

Dependent Variable: LOG(PRODUCTION)
Method: Least Squares
Date: 03/01/23 Time: 13:16
Sample (adjusted): 1 115
Included observations: 115 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	18.01583	0.080462	223.9057	0.0000
@TREND	-0.001930	0.003534	-0.546112	0.5861
@TREND^2	8.98E-05	3.52E-05	2.555074	0.0120

R-squared	0.724142	Mean dependent var	18.29672
Adjusted R-squared	0.719216	S.D. dependent var	0.342029
S.E. of regression	0.181238	Akaike info criterion	-0.552272
Sum squared resid	3.678884	Schwarz criterion	-0.480665
Log likelihood	34.75562	Hannan-Quinn criter.	-0.523207
F-statistic	147.0030	Durbin-Watson stat	0.175567
Prob(F-statistic)	0.000000	Wald F-statistic	30.61205
Prob(Wald F-statistic)	0.000000		

Since the dependent variable is different among the two transformations, we cannot directly compare the level and log quadratic models. It is therefore not clear which one is better. Visualizing the production series and its log transformation, the latter exhibits distribution closer to the Gaussian, therefore we favor the log model.

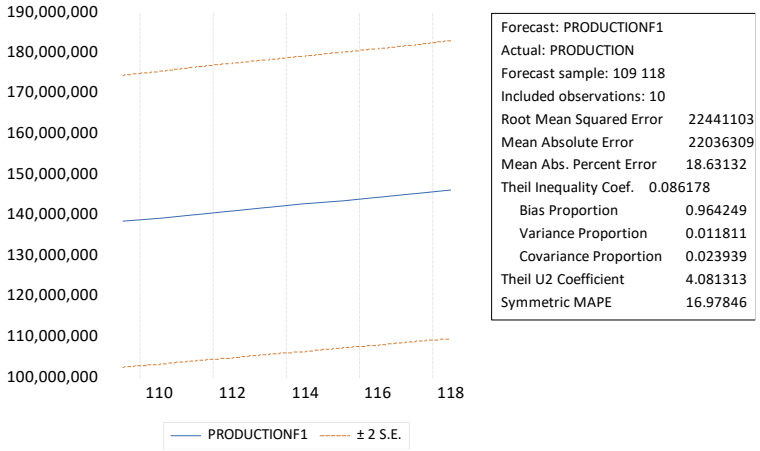
Forecast

When comparing the RMSEs on the last 10 forecasts, we can see that the quadratic models performed better as expected by their in-sample superiority. However, our preferred log model did not do as well in the out of sample forecast as the linear one. It is worth noting that the graph of our series shows that there was a drop in forestry production during the last 10 recorded years. Perhaps the log model would otherwise do better.

Dependent Variable: PRODUCTION
Method: Least Squares
Date: 03/01/23 Time: 14:09
Sample: 1 108
Included observations: 108
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	45927404	8078966.	5.684812	0.0000
@TREND	858124.0	117865.5	7.280534	0.0000

R-squared	0.719016	Mean dependent var	91837037
Adjusted R-squared	0.716365	S.D. dependent var	31696790
S.E. of regression	16880879	Akaike info criterion	36.13961
Sum squared resid	3.02E+16	Schwarz criterion	36.18928
Log likelihood	-1949.539	Hannan-Quinn criter.	36.15975
F-statistic	271.2456	Durbin-Watson stat	0.098800
Prob(F-statistic)	0.000000	Wald F-statistic	53.00618
Prob(Wald F-statistic)	0.000000		



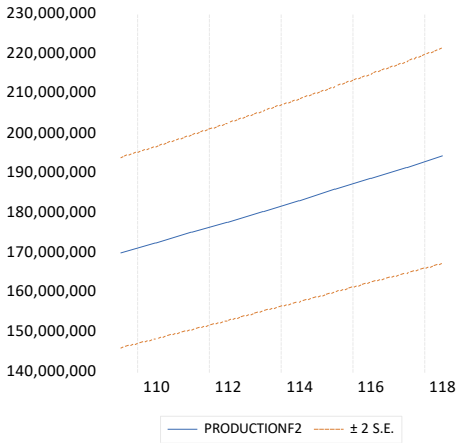
Forecast: PRODUCTIONF1
Actual: PRODUCTION
Forecast sample: 109 118
Included observations: 10

Root Mean Squared Error	22441103
Mean Absolute Error	22036309
Mean Abs. Percent Error	18.63132
Theil Inequality Coef.	0.086178
Bias Proportion	0.964249
Variance Proportion	0.011811
Covariance Proportion	0.023939
Theil U2 Coefficient	4.081313
Symmetric MAPE	16.97846

Dependent Variable: PRODUCTION
Method: Least Squares
Date: 03/01/23 Time: 14:07
Sample: 1 108
Included observations: 108
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	75653799	4920373.	15.37562	0.0000
@TREND	-824502.2	214843.6	-3.837686	0.0002
@TREND^2	15725.48	2160.143	7.279831	0.0000

R-squared	0.906711	Mean dependent var	91837037
Adjusted R-squared	0.904934	S.D. dependent var	31696790
S.E. of regression	9772981.	Akaike info criterion	35.05553
Sum squared resid	1.00E+16	Schwarz criterion	35.13003
Log likelihood	-1889.998	Hannan-Quinn criter.	35.08573
F-statistic	510.2691	Durbin-Watson stat	0.300904
Prob(F-statistic)	0.000000	Wald F-statistic	96.77499
Prob(Wald F-statistic)	0.000000		



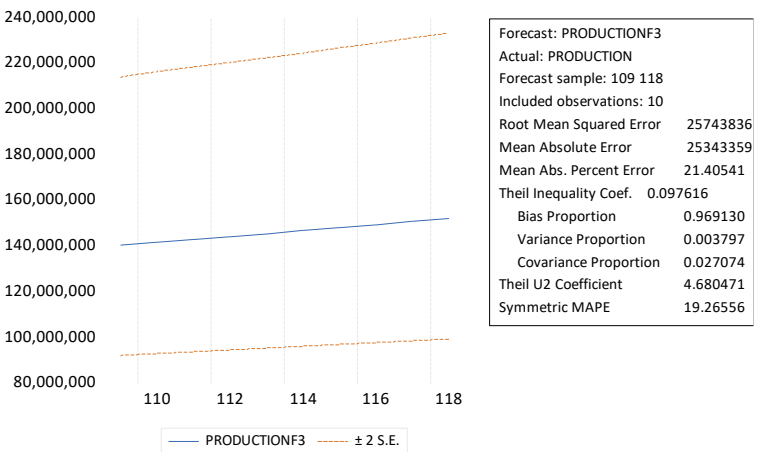
Forecast: PRODUCTIONF2
Actual: PRODUCTION
Forecast sample: 109 118
Included observations: 10

Root Mean Squared Error	59134746
Mean Absolute Error	58807718
Mean Abs. Percent Error	49.51165
Theil Inequality Coef.	0.198942
Bias Proportion	0.988970
Variance Proportion	0.000397
Covariance Proportion	0.010633
Theil U2 Coefficient	10.57554
Symmetric MAPE	39.58839

Dependent Variable: LOG(PRODUCTION)
Method: Least Squares
Date: 03/01/23 Time: 14:15
Sample: 1 108
Included observations: 108
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	17.80176	0.099888	178.2181	0.0000
@TREND	0.008890	0.001297	6.852790	0.0000

R-squared	0.655079	Mean dependent var	18.27737
Adjusted R-squared	0.651825	S.D. dependent var	0.344020
S.E. of regression	0.202994	Akaike info criterion	-0.332939
Sum squared resid	4.367880	Schwarz criterion	-0.283270
Log likelihood	19.97873	Hannan-Quinn criter.	-0.312800
F-statistic	201.3166	Durbin-Watson stat	0.138718
Prob(F-statistic)	0.000000	Wald F-statistic	46.96073
Prob(Wald F-statistic)	0.000000		



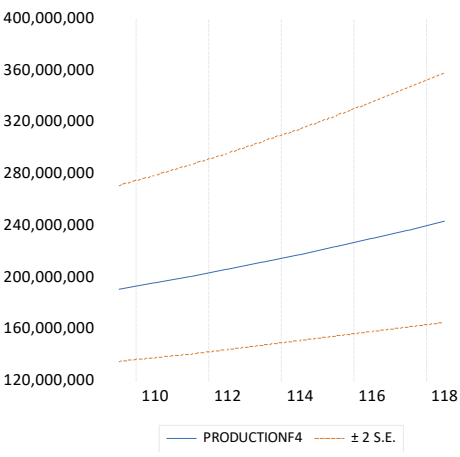
Forecast: PRODUCTIONF3
Actual: PRODUCTION
Forecast sample: 109 118
Included observations: 10

Root Mean Squared Error	25743836
Mean Absolute Error	25343359
Mean Abs. Percent Error	21.40541
Theil Inequality Coef.	0.097616
Bias Proportion	0.969130
Variance Proportion	0.003797
Covariance Proportion	0.027074
Theil U2 Coefficient	4.680471
Symmetric MAPE	19.26556

Dependent Variable: LOG(PRODUCTION)
Method: Least Squares
Date: 03/01/23 Time: 14:12
Sample: 1 108
Included observations: 108
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	18.09165	0.068579	263.8083	0.0000
@TREND	-0.007519	0.003041	-2.472956	0.0150
@TREND^2	0.000153	3.04E-05	5.050595	0.0000

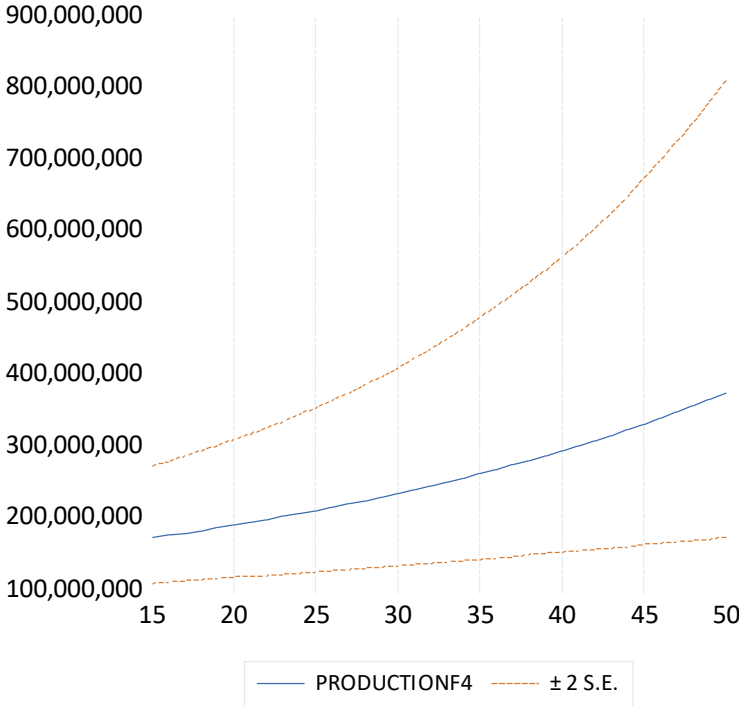
R-squared	0.806613	Mean dependent var	18.27737
Adjusted R-squared	0.802930	S.D. dependent var	0.344020
S.E. of regression	0.152719	Akaike info criterion	-0.893046
Sum squared resid	2.448935	Schwarz criterion	-0.818542
Log likelihood	51.22446	Hannan-Quinn criter.	-0.862837
F-statistic	218.9769	Durbin-Watson stat	0.249396
Prob(F-statistic)	0.000000	Wald F-statistic	55.41371
Prob(Wald F-statistic)	0.000000		

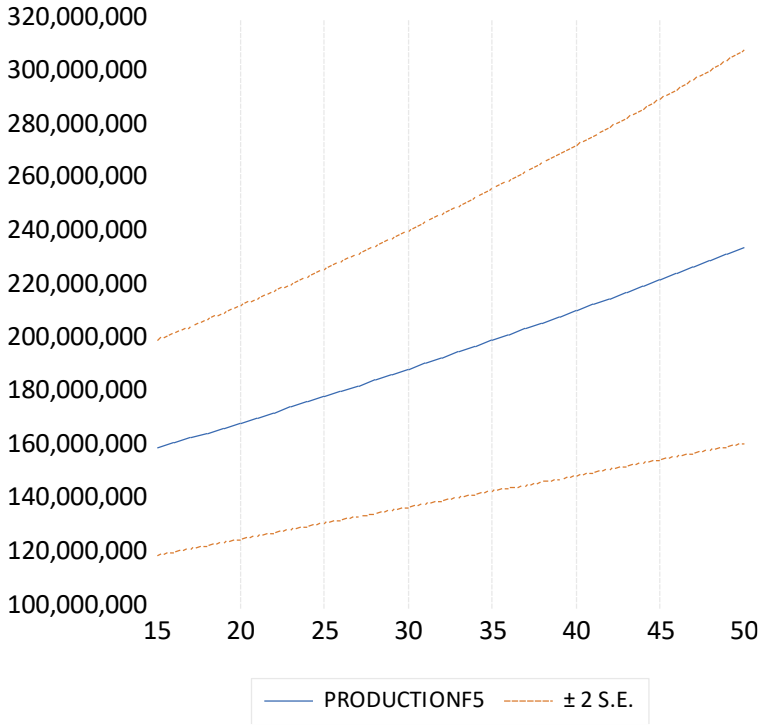


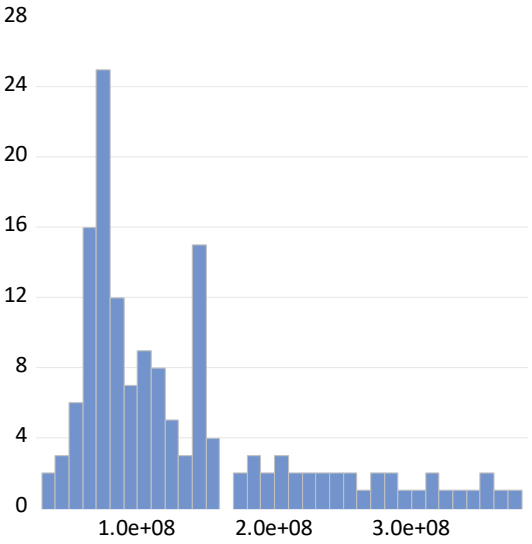
Forecast: PRODUCTIONF4
Actual: PRODUCTION
Forecast sample: 109 118
Included observations: 10

Root Mean Squared Error	88613391
Mean Absolute Error	87921884
Mean Abs. Percent Error	73.94171
Theil Inequality Coef.	0.271345
Bias Proportion	0.984454
Variance Proportion	0.005949
Covariance Proportion	0.009598
Theil U2 Coefficient	15.86557
Symmetric MAPE	53.79132

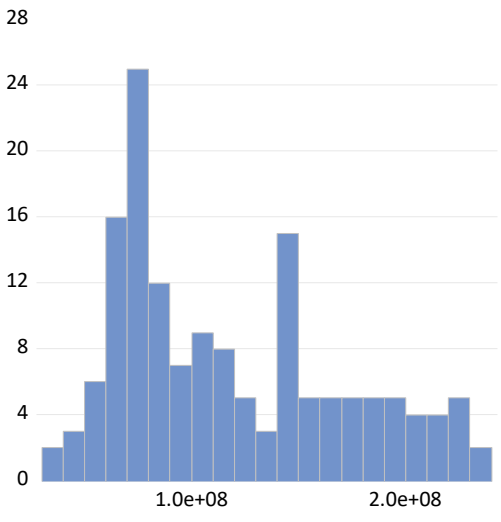
The 2050 forecasts are presented for both quadratic trend model. Productionf4, productionf5 represents the log and the level model, respectively. both forecasts are for the forestry series in levels. It is visible that the production is forecasted to be higher by the log specification.







Series: PRODUCTIONF4	
Sample 1900 2050	
Observations 151	
Mean	1.33e+08
Median	1.03e+08
Maximum	3.77e+08
Minimum	33900000
Std. Dev.	81313483
Skewness	1.297302
Kurtosis	3.811389
Jarque-Bera	46.49743
Probability	0.000000



Series: PRODUCTIONF5	
Sample 1900 2050	
Observations 151	
Mean	1.18e+08
Median	1.03e+08
Maximum	2.35e+08
Minimum	33900000
Std. Dev.	52641160
Skewness	0.588622
Kurtosis	2.186711
Jarque-Bera	12.88121
Probability	0.001595

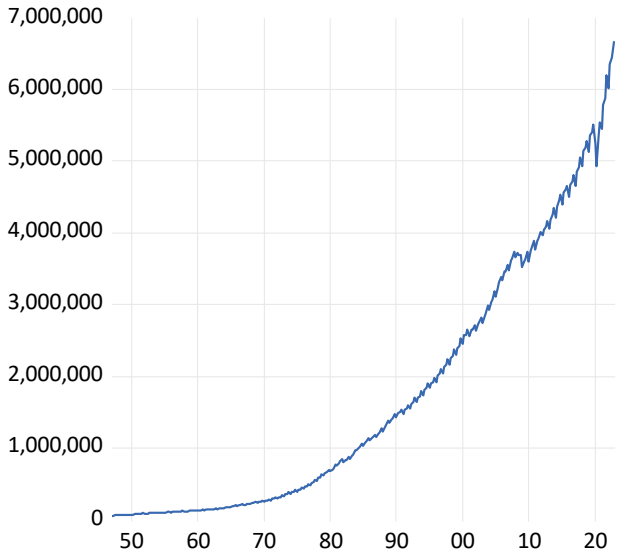
Part 2

Data

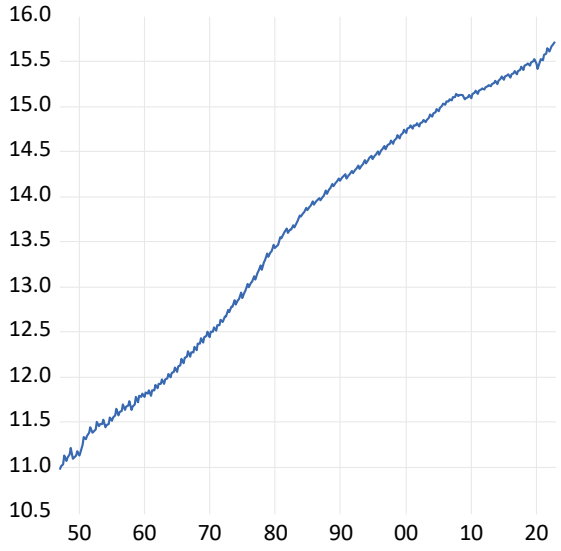
U.S. Bureau of Economic Analysis, Gross Domestic Product [NA000334Q], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/NA000334Q, March 4, 2023.

For the purpose of this exercise, we wanted to us quarterly seasonally unadjusted data. This requirement led to use the GDP (Millions of Dollars) instead of real GDP (Billions of Chained 2012 Dollars) in order to achieve sufficient number of observations as the not seasonally adjusted series latter series starts from the year 2002 while the former from 1947.

US



LOG(US)

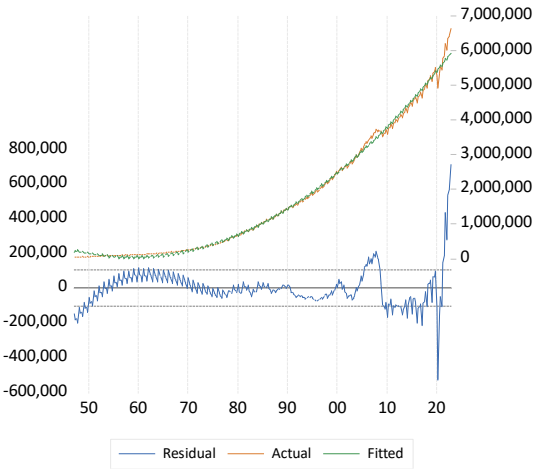


For comparison, let us estimate both the level and log models first.

Dependent Variable: US
Method: Least Squares
Date: 03/04/23 Time: 06:43
Sample: 1947Q1 2022Q4
Included observations: 304

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	248503.6	18772.15	13.23789	0.0000
@TREND	-8342.466	286.2139	-29.14767	0.0000
@TREND^2	89.18569	0.914418	97.53279	0.0000

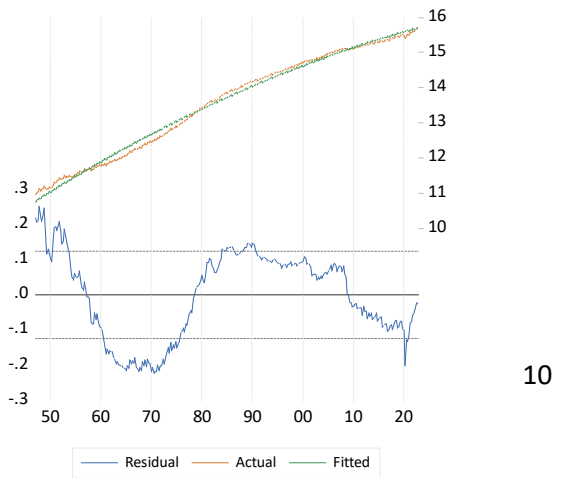
R-squared	0.996119	Mean dependent var	1718474.
Adjusted R-squared	0.996093	S.D. dependent var	1756988.
S.E. of regression	109819.8	Akaike info criterion	26.06089
Sum squared resid	3.63E+12	Schwarz criterion	26.09757
Log likelihood	-3958.255	Hannan-Quinn criter.	26.07556
F-statistic	38627.85	Durbin-Watson stat	0.467371
Prob(F-statistic)	0.000000		



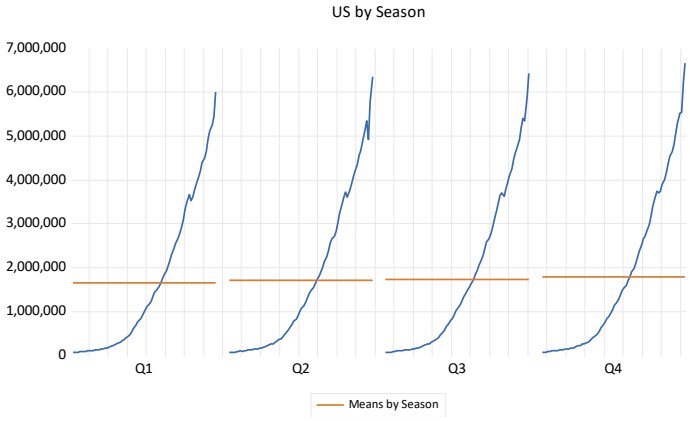
Dependent Variable: LOG(US)
Method: Least Squares
Date: 03/04/23 Time: 06:42
Sample: 1947Q1 2022Q4
Included observations: 304

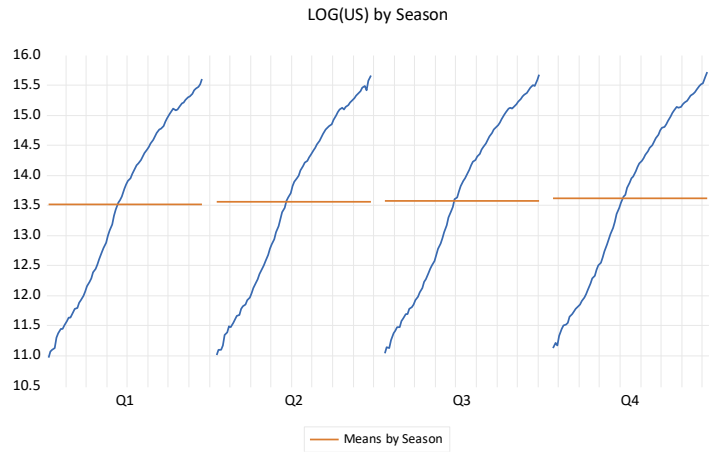
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	10.78205	0.021338	505.2867	0.0000
@TREND	0.022626	0.000325	69.54629	0.0000
@TREND^2	-2.10E-05	1.04E-06	-20.17854	0.0000

R-squared	0.992565	Mean dependent var	13.56701
Adjusted R-squared	0.992515	S.D. dependent var	1.442936
S.E. of regression	0.124833	Akaike info criterion	-1.313858
Sum squared resid	4.690579	Schwarz criterion	-1.277177
Log likelihood	202.7064	Hannan-Quinn criter.	-1.299184
F-statistic	20091.19	Durbin-Watson stat	0.092600
Prob(F-statistic)	0.000000		



Seasonality





Dependent Variable: US
Method: Least Squares
Date: 03/04/23 Time: 06:50
Sample: 1947Q1 2022Q4
Included observations: 304

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	206412.8	20964.91	9.845636	0.0000
@TREND	-8346.602	277.0544	-30.12622	0.0000
@TREND^2	89.18572	0.885150	100.7578	0.0000
@QUARTER=2	41137.33	17245.05	2.385457	0.0177
@QUARTER=3	46877.17	17245.47	2.718231	0.0069
@QUARTER=4	82851.60	17246.17	4.804058	0.0000

R-squared	0.996400	Mean dependent var	1718474.
Adjusted R-squared	0.996339	S.D. dependent var	1756988.
S.E. of regression	106304.8	Akaike info criterion	26.00555
Sum squared resid	3.37E+12	Schwarz criterion	26.07891
Log likelihood	-3946.843	Hannan-Quinn criter.	26.03489
F-statistic	16494.47	Durbin-Watson stat	0.325337
Prob(F-statistic)	0.000000		

Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	7.744806	(3, 298)	0.0001
Chi-square	23.23442	3	0.0000

Null Hypothesis: C(4)=C(5)=C(6)=0
Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(4)	41137.33	17245.05
C(5)	46877.17	17245.47
C(6)	82851.60	17246.17

Restrictions are linear in coefficients.

Dependent Variable: LOG(US)
Method: Least Squares
Date: 03/04/23 Time: 06:46
Sample: 1947Q1 2022Q4
Included observations: 304

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	10.75785	0.024472	439.5988	0.0000
@TREND	0.022624	0.000323	69.95614	0.0000
@TREND^2	-2.10E-05	1.03E-06	-20.29979	0.0000
@QUARTER=2	0.024839	0.020130	1.233962	0.2182
@QUARTER=3	0.021839	0.020130	1.084892	0.2788
@QUARTER=4	0.051620	0.020131	2.564191	0.0108

R-squared	0.992727	Mean dependent var	13.56701
Adjusted R-squared	0.992605	S.D. dependent var	1.442936
S.E. of regression	0.124088	Akaike info criterion	-1.316117
Sum squared resid	4.588531	Schwarz criterion	-1.242755
Log likelihood	206.0498	Hannan-Quinn criter.	-1.286770
F-statistic	8134.651	Durbin-Watson stat	0.020526
Prob(F-statistic)	0.000000		

Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	2.209153	(3, 298)	0.0871
Chi-square	6.627459	3	0.0848

Null Hypothesis: C(4)=C(5)=C(6)=0
Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(4)	0.024839	0.020130
C(5)	0.021839	0.020130
C(6)	0.051620	0.020131

Restrictions are linear in coefficients.

The null hypothesis of joint insignificance of the seasonal deterministic dummies was rejected in the level model as the p-value for both the F-statistic and the chi-square test were less than 0.05, indicating strong evidence against the null. In the log model, the null hypothesis is the same, but the p-values for both the F-statistic and chi-square test are greater than 0.05, indicating that there is not enough evidence to reject the null hypothesis at the 95% confidence level.

Autocorrelation

From now, we will consider only the level model.

Breusch-Godfrey Serial Correlation LM Test:
Null hypothesis: No serial correlation at up to 2 lags

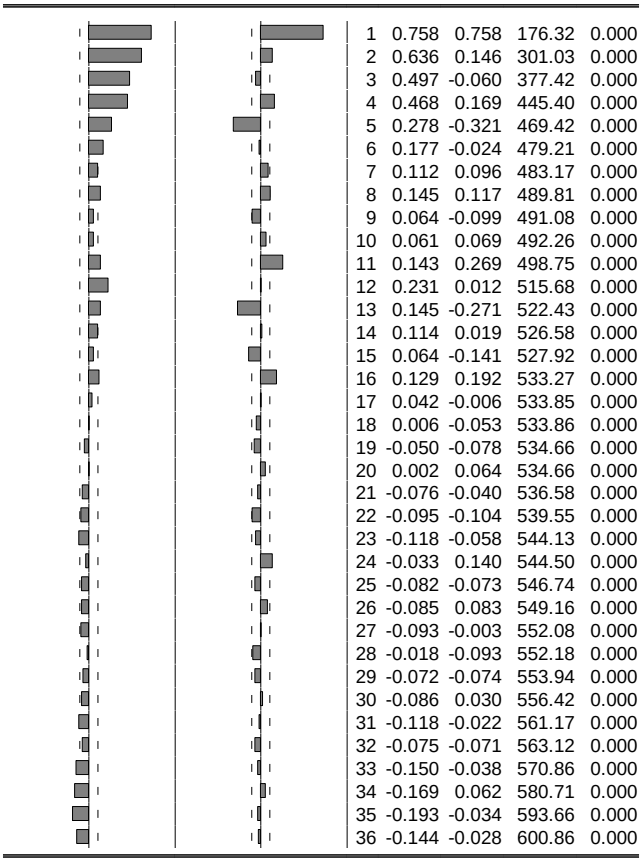
F-statistic	348.3484	Prob. F(2,296)	0.0000
Obs*R-squared	213.3540	Prob. Chi-Square(2)	0.0000

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 03/05/23 Time: 06:40
Sample: 1947Q1 2022Q4
Included observations: 304
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	14272.77	11502.41	1.240851	0.2156
@TREND	-222.6802	152.4411	-1.460762	0.1451
@TREND^2	0.919159	0.488382	1.882050	0.0608
@QUARTER=2	-6147.672	9469.972	-0.649175	0.5167
@QUARTER=3	-8739.146	9454.577	-0.924330	0.3561
@QUARTER=4	-8796.809	9455.032	-0.930384	0.3529
RESID(-1)	0.700326	0.057431	12.19422	0.0000
RESID(-2)	0.268896	0.061512	4.371424	0.0000

R-squared	0.701822	Mean dependent var	-2.23E-10
Adjusted R-squared	0.694771	S.D. dependent var	105424.0
S.E. of regression	58244.18	Akaike info criterion	24.80864
Sum squared resid	1.00E+12	Schwarz criterion	24.90646
Log likelihood	-3762.913	Hannan-Quinn criter.	24.84777
F-statistic	99.52810	Durbin-Watson stat	1.952836
Prob(F-statistic)	0.000000		

Date: 03/05/23 Time: 06:37
Sample: 1947Q1 2022Q4
Included observations: 304



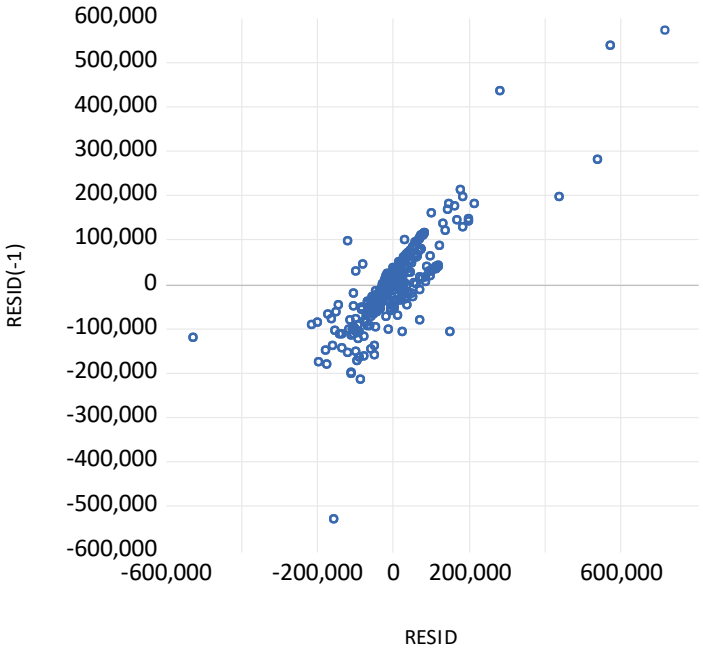
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.758	0.758	176.32	0.000
		2	0.636	0.146	301.03	0.000
		3	0.497	-0.060	377.42	0.000
		4	0.468	0.169	445.40	0.000
		5	0.278	-0.321	469.42	0.000
		6	0.177	-0.024	479.21	0.000
		7	0.112	0.096	483.17	0.000
		8	0.145	0.117	489.81	0.000
		9	0.064	-0.099	491.08	0.000
		10	0.061	0.069	492.26	0.000
		11	0.143	0.269	498.75	0.000
		12	0.231	0.012	515.68	0.000
		13	0.145	-0.271	522.43	0.000
		14	0.114	0.019	526.58	0.000
		15	0.064	-0.141	527.92	0.000
		16	0.129	0.192	533.27	0.000
		17	0.042	-0.006	533.85	0.000
		18	0.006	-0.053	533.86	0.000
		19	-0.050	-0.078	534.66	0.000
		20	0.002	0.064	534.66	0.000
		21	-0.076	-0.040	536.58	0.000
		22	-0.095	-0.104	539.55	0.000
		23	-0.118	-0.058	544.13	0.000
		24	-0.033	0.140	544.50	0.000
		25	-0.082	-0.073	546.74	0.000
		26	-0.085	0.083	549.16	0.000
		27	-0.093	-0.003	552.08	0.000
		28	-0.018	-0.093	552.18	0.000
		29	-0.072	-0.074	553.94	0.000
		30	-0.086	0.030	556.42	0.000
		31	-0.118	-0.022	561.17	0.000
		32	-0.075	-0.071	563.12	0.000
		33	-0.150	-0.038	570.86	0.000
		34	-0.169	0.062	580.71	0.000
		35	-0.193	-0.034	593.66	0.000
		36	-0.144	-0.028	600.86	0.000

The LM and BP tests reject the null of no autocorrelation, we therefore add the HAC standard errors to our equation.

Dependent Variable: US
Method: Least Squares
Date: 03/05/23 Time: 06:44
Sample: 1947Q1 2022Q4
Included observations: 304
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 6.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	206412.8	40557.35	5.089407	0.0000
@TREND	-8346.602	749.2437	-11.14004	0.0000
@TREND^2	89.18572	2.801583	31.83404	0.0000
@QUARTER=2	41137.33	10139.05	4.057318	0.0001
@QUARTER=3	46877.17	11037.28	4.247167	0.0000
@QUARTER=4	82851.60	14500.84	5.713574	0.0000

R-squared	0.996400	Mean dependent var	1718474.
Adjusted R-squared	0.996339	S.D. dependent var	1756988.
S.E. of regression	106304.8	Akaike info criterion	26.00555
Sum squared resid	3.37E+12	Schwarz criterion	26.07891
Log likelihood	-3946.843	Hannan-Quinn criter.	26.03489
F-statistic	16494.47	Durbin-Watson stat	0.325337
Prob(F-statistic)	0.000000	Wald F-statistic	2004.769
Prob(Wald F-statistic)	0.000000		



Correction

Using ARDL EViews method with Schwarz criterion model selection, we arrived at the following specification

Dependent Variable: US
Method: ARDL
Date: 03/05/23 Time: 06:47
Sample (adjusted): 1950Q1 2022Q4
Included observations: 292 after adjustments
Maximum dependent lags: 12 (Automatic selection)
Model selection method: Schwarz criterion (SIC)
Dynamic regressors (0 lag, automatic): @TREND @TREND^2 @EXPAND(@QUARTER, @DROP(1))
Fixed regressors: C
Number of models evaluated: 12
Selected Model: ARDL(12, 0, 0, 0, 0, 0)
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 6.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
US(-1)	0.721938	0.099007	7.291818	0.0000
US(-2)	0.219332	0.144848	1.514219	0.1311
US(-3)	0.038894	0.101731	0.382322	0.7025
US(-4)	0.122306	0.256304	0.477191	0.6336
US(-5)	-0.176803	0.215452	-0.820617	0.4126
US(-6)	-0.203408	0.072026	-2.824096	0.0051
US(-7)	0.160318	0.074034	2.165457	0.0312
US(-8)	0.070218	0.215310	0.326125	0.7446
US(-9)	-0.230807	0.144169	-1.600948	0.1105
US(-10)	0.001070	0.105432	0.010145	0.9919
US(-11)	-0.394109	0.181523	-2.171120	0.0308
US(-12)	0.621368	0.294938	2.106776	0.0360
@TREND	-397.0712	604.3644	-0.657006	0.5117
@TREND^2	5.760244	6.056564	0.951075	0.3424
@QUARTER=2	33011.77	17458.07	1.890918	0.0597
@QUARTER=3	29146.22	12661.54	2.301948	0.0221
@QUARTER=4	25188.92	11693.30	2.154132	0.0321
C	-6390.324	22618.11	-0.282531	0.7777

R-squared	0.999295	Mean dependent var	1786368.
Adjusted R-squared	0.999251	S.D. dependent var	1759866.
S.E. of regression	48149.52	Akaike info criterion	24.46167
Sum squared resid	6.35E+11	Schwarz criterion	24.68832
Log likelihood	-3553.404	Hannan-Quinn criter.	24.55246
F-statistic	22851.41	Durbin-Watson stat	1.771457
Prob(F-statistic)	0.000000		

*Note: p-values and any subsequent tests do not account for model selection.

Breusch-Godfrey Serial Correlation LM Test:
Null hypothesis: No serial correlation at up to 2 lags

F-statistic	20.65944	Prob. F(2,272)	0.0000
Obs*R-squared	38.50746	Prob. Chi-Square(2)	0.0000

Test Equation:
Dependent Variable: RESID
Method: ARDL
Date: 03/05/23 Time: 07:01
Sample: 1950Q1 2022Q4
Included observations: 292
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
US(-1)	-1.023862	0.173884	-5.888179	0.0000
US(-2)	0.609707	0.235927	2.584305	0.0103
US(-3)	0.334795	0.147816	2.264940	0.0243
US(-4)	-0.086888	0.089034	-0.975892	0.3300
US(-5)	0.338056	0.092028	3.673403	0.0003
US(-6)	-0.191151	0.102386	-1.866971	0.0630
US(-7)	-0.303186	0.098682	-3.072350	0.0023
US(-8)	-0.011710	0.094274	-0.124214	0.9012
US(-9)	0.311108	0.087398	3.559680	0.0004
US(-10)	-0.319925	0.110632	-2.891800	0.0041
US(-11)	0.110794	0.140587	0.788083	0.4313
US(-12)	0.120945	0.109107	1.108498	0.2686
@TREND	-1012.259	502.7887	-2.013289	0.0451
@TREND^2	11.02484	4.719278	2.336128	0.0202
@QUARTER=2	-37739.09	12584.91	-2.998757	0.0030
@QUARTER=3	-13461.55	11264.39	-1.195053	0.2331
@QUARTER=4	-7694.301	10831.10	-0.710390	0.4781

The autocorrelation tests show that even after including 12 lags, the model still exhibits some autocorrelation. To tackle this issue, we return to the log specification.

Date: 03/05/23 Time: 06:51
Sample (adjusted): 1950Q1 2022Q4
Q-statistic probabilities adjusted for 12 dynamic regressors

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.113	0.113	3.7775	0.052
		2	0.040	0.028	4.2532	0.119
		3	0.041	0.034	4.7539	0.191
		4	-0.042	-0.052	5.2698	0.261
		5	0.103	0.112	8.4127	0.135
		6	0.041	0.019	8.9236	0.178
		7	0.038	0.030	9.3676	0.227
		8	-0.095	-0.119	12.121	0.146
		9	0.051	0.087	12.924	0.166
		10	-0.006	-0.029	12.936	0.227
		11	0.024	0.034	13.110	0.286
		12	0.036	0.001	13.509	0.333
		13	0.022	0.052	13.663	0.398
		14	-0.029	-0.058	13.917	0.456
		15	-0.042	-0.023	14.472	0.490
		16	0.050	0.038	15.262	0.506
		17	0.020	0.033	15.392	0.567
		18	-0.010	-0.044	15.423	0.633
		19	-0.017	-0.007	15.517	0.689
		20	0.030	0.049	15.802	0.729
		21	-0.015	-0.019	15.874	0.777
		22	-0.098	-0.125	18.936	0.649
		23	-0.086	-0.069	21.285	0.564
		24	0.030	0.086	21.572	0.605
		25	-0.005	-0.016	21.580	0.660
		26	-0.034	-0.053	21.948	0.692
		27	-0.046	-0.030	22.633	0.705
		28	0.055	0.128	23.628	0.701
		29	-0.021	-0.065	23.769	0.740
		30	0.045	0.032	24.432	0.752
		31	-0.003	-0.018	24.436	0.792
		32	0.025	0.094	24.643	0.820
		33	-0.050	-0.132	25.479	0.822
		34	0.023	0.075	25.652	0.848
		35	0.020	0.003	25.781	0.872
		36	-0.001	0.045	25.782	0.896

*Probabilities may not be valid for this equation specification.

C	50781.08	20726.00	2.450115	0.0149
RESID(-1)	1.111878	0.185780	5.984933	0.0000
RESID(-2)	0.151113	0.192771	0.783899	0.4338

R-squared	0.131875	Mean dependent var	-8.79E-10
Adjusted R-squared	0.071234	S.D. dependent var	46721.93
S.E. of regression	45027.10	Akaike info criterion	24.33395
Sum squared resid	5.51E+11	Schwarz criterion	24.58578
Log likelihood	-3532.757	Hannan-Quinn criter.	24.43483
F-statistic	2.174678	Durbin-Watson stat	1.999970
Prob(F-statistic)	0.003640		

Dependent Variable: LOG(US)
Method: ARDL
Date: 03/05/23 Time: 06:54
Sample (adjusted): 1949Q2 2022Q4
Included observations: 295 after adjustments
Maximum dependent lags: 12 (Automatic selection)
Model selection method: Schwarz criterion (SIC)
Dynamic regressors (0 lag, automatic): @TREND @TREND^2 @EXPAND(@QUARTER, @DROP(1))
Fixed regressors: C
Number of models evaluated: 12
Selected Model: ARDL(9, 0, 0, 0, 0, 0)
Note: final equation sample is larger than selection sample
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 6.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
LOG(US(-1))	1.008117	0.111590	9.034100	0.0000
LOG(US(-2))	0.114393	0.104629	1.093319	0.2752
LOG(US(-3))	-0.217638	0.082528	-2.637139	0.0088
LOG(US(-4))	0.334381	0.167310	1.998571	0.0466
LOG(US(-5))	-0.401562	0.173603	-2.313111	0.0214
LOG(US(-6))	0.012223	0.097179	0.125778	0.9000
LOG(US(-7))	0.082418	0.103046	0.799815	0.4245
LOG(US(-8))	0.252321	0.121136	2.082960	0.0382
LOG(US(-9))	-0.198402	0.075561	-2.625723	0.0091
@TREND	0.000359	0.000225	1.597019	0.1114
@TREND^2	-5.16E-07	2.98E-07	-1.731090	0.0845
@QUARTER=2	0.027568	0.008801	3.132438	0.0019
@QUARTER=3	0.026262	0.006844	3.837215	0.0002
@QUARTER=4	0.028966	0.008487	3.413143	0.0007
C	0.139570	0.084780	1.646250	0.1008

R-squared	0.999897	Mean dependent var	13.64262
Adjusted R-squared	0.999892	S.D. dependent var	1.397104
S.E. of regression	0.014506	Akaike info criterion	-5.579042
Sum squared resid	0.058916	Schwarz criterion	-5.391569
Log likelihood	837.9087	Hannan-Quinn criter.	-5.503973
F-statistic	194785.0	Durbin-Watson stat	2.042829
Prob(F-statistic)	0.000000		

*Note: p-values and any subsequent tests do not account for model selection.

Breusch-Godfrey Serial Correlation LM Test:
Null hypothesis: No serial correlation at up to 2 lags

F-statistic	1.451617	Prob. F(2,278)	0.2360
Obs*R-squared	3.048929	Prob. Chi-Square(2)	0.2177

Test Equation:
Dependent Variable: RESID
Method: ARDL
Date: 03/05/23 Time: 06:59
Sample: 1949Q2 2022Q4
Included observations: 295
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LOG(US(-1))	0.250270	0.191217	1.308828	0.1917
LOG(US(-2))	-0.360736	0.234729	-1.536814	0.1255
LOG(US(-3))	0.094179	0.201623	0.467107	0.6408
LOG(US(-4))	0.058751	0.087266	0.673239	0.5014
LOG(US(-5))	-0.140028	0.113345	-1.235416	0.2177
LOG(US(-6))	0.191204	0.139177	1.373816	0.1706
LOG(US(-7))	-0.082488	0.126294	-0.653143	0.5142
LOG(US(-8))	-0.009115	0.080575	-0.113125	0.9100
LOG(US(-9))	0.000450	0.064062	0.007028	0.9944
@TREND	-7.10E-05	0.000225	-0.315414	0.7527
@TREND^2	1.12E-07	3.14E-07	0.355591	0.7224
@QUARTER=2	0.009673	0.008985	1.076502	0.2826
@QUARTER=3	-0.002461	0.007617	-0.323112	0.7469
@QUARTER=4	0.001435	0.006484	0.221269	0.8250

Applying the same method led us to include 9 lags of the log(GDP). The LM and BP test results now indicate that the model residuals are not autocorrelated.

Date: 03/05/23 Time: 06:58
Sample (adjusted): 1949Q2 2022Q4
Q-statistic probabilities adjusted for 9 dynamic regressors

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.029	-0.029	0.2498	0.617
		2	0.019	0.018	0.3538	0.838
		3	0.003	0.004	0.3567	0.949
		4	-0.018	-0.018	0.4556	0.978
		5	0.026	0.024	0.6531	0.985
		6	0.062	0.064	1.8008	0.937
		7	0.057	0.060	2.7939	0.903
		8	-0.103	-0.104	6.0627	0.640
		9	-0.024	-0.033	6.2414	0.716
		10	0.060	0.066	7.3573	0.691
		11	0.022	0.028	7.5058	0.757
		12	-0.009	-0.023	7.5335	0.820
		13	-0.006	-0.013	7.5449	0.872
		14	-0.037	-0.023	7.9817	0.890
		15	0.010	0.024	8.0150	0.923
		16	0.101	0.089	11.207	0.797
		17	0.047	0.037	11.907	0.806
		18	0.064	0.072	13.184	0.781
		19	-0.017	-0.003	13.277	0.824
		20	0.051	0.050	14.091	0.826
		21	-0.028	-0.032	14.341	0.855
		22	0.003	-0.020	14.344	0.889
		23	-0.048	-0.067	15.072	0.892
		24	0.055	0.069	16.053	0.886
		25	0.043	0.057	16.661	0.894
		26	0.014	0.014	16.723	0.917
		27	-0.024	-0.042	16.905	0.933
		28	0.028	0.037	17.154	0.945
		29	-0.015	-0.004	17.231	0.958
		30	0.010	0.007	17.268	0.969
		31	-0.004	-0.032	17.274	0.978
		32	0.051	0.055	18.148	0.976
		33	-0.054	-0.042	19.117	0.974
		34	0.004	-0.012	19.123	0.981
		35	-0.017	-0.047	19.223	0.986
		36	0.005	-0.003	19.230	0.990

*Probabilities may not be valid for this equation specification.

C	-0.030880	0.097514	-0.316669	0.7517
RESID(-1)	-0.275539	0.201766	-1.365637	0.1732
RESID(-2)	0.126413	0.201408	0.627648	0.5307

R-squared	0.010335	Mean dependent var	1.04E-16
Adjusted R-squared	-0.046624	S.D. dependent var	0.014156
S.E. of regression	0.014482	Akaike info criterion	-5.575872
Sum squared resid	0.058307	Schwarz criterion	-5.363402
Log likelihood	839.4411	Hannan-Quinn criter.	-5.490794
F-statistic	0.181452	Durbin-Watson stat	1.992008
Prob(F-statistic)	0.999845		

Misspecification tests

The null hypotheses of homoskedasticity and normality were both rejected. HAC standard errors are thus justified. Further, the squared fitted values significantly contribute to the model's explanatory power.

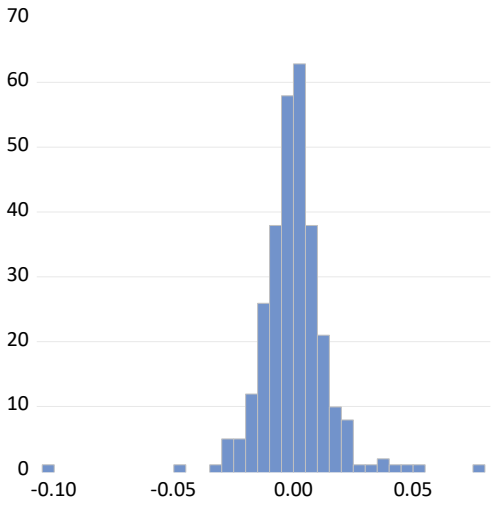
Heteroskedasticity Test: Breusch-Pagan-Godfrey
Null hypothesis: Homoskedasticity

F-statistic	2.755327	Prob. F(14,280)	0.0008
Obs*R-squared	35.72005	Prob. Chi-Square(14)	0.0011
Scaled explained SS	219.6156	Prob. Chi-Square(14)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 03/05/23 Time: 08:07
Sample: 1949Q2 2022Q4
Included observations: 295
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 6.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.003892	0.005212	0.746688	0.4559
LOG(US(-1))	-0.008470	0.007492	-1.130522	0.2592
LOG(US(-2))	0.007216	0.005645	1.278394	0.2022
LOG(US(-3))	0.003578	0.004620	0.774342	0.4394
LOG(US(-4))	-0.002131	0.004503	-0.473226	0.6364
LOG(US(-5))	0.005660	0.005874	0.963657	0.3360
LOG(US(-6))	-0.011126	0.007665	-1.451657	0.1477
LOG(US(-7))	0.003939	0.003343	1.178238	0.2397
LOG(US(-8))	0.003332	0.004991	0.667706	0.5049
LOG(US(-9))	-0.002289	0.003066	-0.746772	0.4558
@TREND	-1.30E-06	8.74E-06	-0.148726	0.8819
@TREND^2	2.06E-08	6.99E-09	2.945878	0.0035
@QUARTER=2	-3.77E-05	0.000185	-0.203459	0.8389
@QUARTER=3	-0.000436	0.000257	-1.696859	0.0908
@QUARTER=4	-1.84E-05	0.000206	-0.089412	0.9288

R-squared	0.121085	Mean dependent var	0.000200
Adjusted R-squared	0.077139	S.D. dependent var	0.000739
S.E. of regression	0.000710	Akaike info criterion	-11.61304
Sum squared resid	0.000141	Schwarz criterion	-11.42556
Log likelihood	1727.923	Hannan-Quinn criter.	-11.53797
F-statistic	2.755327	Durbin-Watson stat	1.339992
Prob(F-statistic)	0.000764		



Series: Residuals	
Sample 1949Q2 2022Q4	
Observations 295	
Mean	4.38e-15
Median	8.33e-05
Maximum	0.075483
Minimum	-0.101109
Std. Dev.	0.014156
Skewness	-0.462411
Kurtosis	14.64926
Jarque-Bera	1678.556
Probability	0.000000

Ramsey RESET Test
Equation: EQ01
Omitted Variables: Squares of fitted values
Specification: LOG(US) LOG(US(-1)) LOG(US(-2)) LOG(US(-3)) LOG(US(-4)) LOG(US(-5)) LOG(US(-6)) LOG(US(-7)) LOG(US(-8)) LOG(US(-9)) @TREND @TREND^2 @QUARTER=2 @QUARTER=3 @QUARTER=4 C

	Value	df	Probability
t-statistic	3.656647	279	0.0003
F-statistic	13.37107	(1, 279)	0.0003
Likelihood ratio	13.80954	1	0.0002

F-test summary:

	Sum of Sq.	df	Mean Squares
Test SSR	0.002694	1	0.002694
Restricted SSR	0.058916	280	0.000210
Unrestricted SSR	0.056222	279	0.000202

LR test summary:

	Value
Restricted LogL	837.9087
Unrestricted LogL	844.8135

Unrestricted Test Equation:
Dependent Variable: LOG(US)
Method: Least Squares
Date: 03/05/23 Time: 08:08
Sample: 1949Q2 2022Q4
Included observations: 295
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 6.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LOG(US(-1))	1.299510	0.094920	13.69062	0.0000
LOG(US(-2))	0.145207	0.107407	1.351929	0.1775
LOG(US(-3))	-0.274353	0.071906	-3.815419	0.0002
LOG(US(-4))	0.436388	0.138577	3.149053	0.0018
LOG(US(-5))	-0.525037	0.137758	-3.811289	0.0002
LOG(US(-6))	0.024603	0.094878	0.259310	0.7956
LOG(US(-7))	0.110719	0.096635	1.145749	0.2529
LOG(US(-8))	0.310413	0.120771	2.570267	0.0107
LOG(US(-9))	-0.206669	0.069234	-2.985084	0.0031
@TREND	-0.000644	0.000418	-1.541345	0.1244
@TREND^2	2.19E-06	1.08E-06	2.027277	0.0436
@QUARTER=2	0.038941	0.010618	3.667511	0.0003
@QUARTER=3	0.036221	0.009078	3.990175	0.0001
@QUARTER=4	0.041423	0.011242	3.684759	0.0003
C	-2.083984	0.859129	-2.425693	0.0159
FITTED^2	-0.012059	0.004757	-2.535113	0.0118

R-squared	0.999902	Mean dependent var	13.64262
Adjusted R-squared	0.999897	S.D. dependent var	1.397104
S.E. of regression	0.014195	Akaike info criterion	-5.619074
Sum squared resid	0.056222	Schwarz criterion	-5.419103
Log likelihood	844.8135	Hannan-Quinn criter.	-5.539001
F-statistic	189832.5	Durbin-Watson stat	1.986182
Prob(F-statistic)	0.000000	Wald F-statistic	292394.0
Prob(Wald F-statistic)	0.000000		

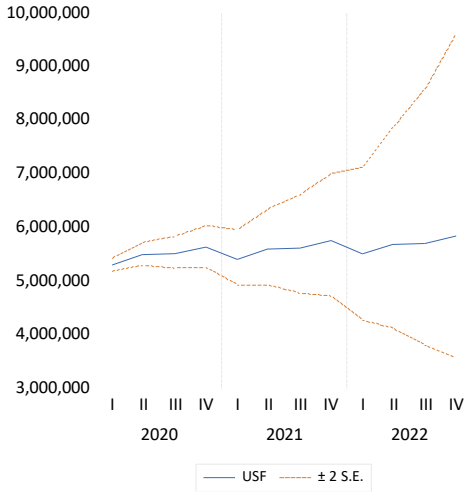
Forecast

Finally, we estimate the model without the last 12 observations and provide a forecast to test its out of sample performance. The resulting RMSE is 463 356.6 $

Dependent Variable: LOG(US)
Method: Least Squares
Date: 03/05/23 Time: 08:18
Sample (adjusted): 1949Q2 2019Q4
Included observations: 283 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 6.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
@TREND	6.65E-05	5.55E-05	1.197781	0.2321
@TREND^2	-2.89E-07	1.42E-07	-2.029510	0.0434
LOG(US(-1))	1.205725	0.053066	22.72105	0.0000
LOG(US(-2))	-0.108931	0.090006	-1.210262	0.2272
LOG(US(-3))	-0.261323	0.104281	-2.505945	0.0128
LOG(US(-4))	0.569696	0.085366	6.673567	0.0000
LOG(US(-5))	-0.718828	0.073373	-9.796864	0.0000
LOG(US(-6))	0.195451	0.102882	1.899768	0.0585
LOG(US(-7))	0.128470	0.109627	1.171880	0.2423
LOG(US(-8))	0.153998	0.082729	1.861470	0.0638
LOG(US(-9))	-0.164633	0.055650	-2.958379	0.0034
@QUARTER=2	0.020451	0.005191	3.940006	0.0001
@QUARTER=3	0.018950	0.004458	4.250483	0.0000
@QUARTER=4	0.018799	0.004549	4.132343	0.0000

R-squared	0.999935	Mean dependent var	13.56076
Adjusted R-squared	0.999932	S.D. dependent var	1.367225
S.E. of regression	0.011313	Akaike info criterion	-6.077554
Sum squared resid	0.034427	Schwarz criterion	-5.897214
Log likelihood	873.9739	Hannan-Quinn criter.	-6.005244
Durbin-Watson stat	2.078043		



Forecast: USF	
Actual: US	
Forecast sample: 2020Q1 2022Q4	
Included observations: 12	
Root Mean Squared Error	463356.6
Mean Absolute Error	378697.6
Mean Abs. Percent Error	6.299744
Theil Inequality Coef.	0.040513
Bias Proportion	0.245820
Variance Proportion	0.631300
Covariance Proportion	0.122880
Theil U2 Coefficient	1.669551
Symmetric MAPE	6.479467