

Lecture Note 8b: — Taxation Versus Lump Sum Transfers in the Edgeworth Box

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Setting

Consider an economy with two goods, X and Y , and two consumers A and B . These consumers have identical preferences:

$$U_A(X, Y) = X_A Y_A, \quad U_B(X, Y) = X_B Y_B$$

Initial allocations are $E_A(X, Y) = (1, 2)$ and $E_B(X, Y) = (1, 0)$. Hence, the total endowment of the economy is $E(X, Y) = (2, 2)$ and A is wealthier than B . With these specific preferences, the contract curve of the economy runs through the center of the Edgeworth box (that is, it a line connecting the southwest and northeast vertices). You can confirm for yourself that from any initial endowment, A and B will trade at a price ratio $p = p_X/p_Y = 1$ until they reach the contract curve. (Of course, if they are initially on the contract curve, then there will be no further trade). With these initial allocations, the equilibrium of this economy will have the following consumption bundles: $d_A(X_A, Y_A) = (1.5, 1.5)$ and $d_B(X_B, Y_B) = (0.5, 0.5)$.

Aside: Deriving the equilibrium in this economy

How did we derive that? Here's one way. First, solve for the Marshallian demands of both consumers, which are:

$$d_A^X = \frac{I_A}{2P_X}$$

$$d_A^Y = \frac{I_A}{2P_Y}$$

$$d_B^X = \frac{I_B}{2P_X}$$

$$d_B^Y = \frac{I_B}{2P_Y}$$

Notice that I_A and I_B are determined by the initial endowments and the price ratio. Since it is the price *ratio* (not the level of both prices) that determines consumption choices, we can normalize $P_X = 1$ while using P_Y as our price variable (we could instead have normalized $P_Y = 1$ and used P_X as our price variable). Applying this normalization, we have:

$$I_A = 1 + 2P_Y, \quad I_B = 1 + 0P_Y.$$

We can substitute these income levels back into the demand functions to obtain:

$$d_A^X = \frac{1 + 2P_Y}{2}, \quad d_A^Y = \frac{1 + P_Y}{2P_Y}.$$

$$d_B^X = \frac{1}{2}, \quad d_B^Y = \frac{1}{2P_Y}$$

Finally, we can find the price P_Y that clears the market, respecting the economy's endowment of each good ($2X$ and $2Y$):

$$d_A^X + d_B^X = 2 = \frac{1}{2} + \frac{1 + 2P_Y}{2} \Rightarrow P_Y = 1.$$

By Walras law, this same P_Y should clear the market for good X . Does it?

$$d_A^Y + d_B^Y = 2 = \frac{1 + 2P_Y}{2P_Y} + \frac{1}{2P_Y} \Rightarrow P_Y = 1.$$

Yes it does.

It's also useful to note the properties of the MRS with these utility functions.

$$MRS_{xy} = \frac{\partial U / \partial X}{\partial U / \partial Y} = \frac{-Y}{-X} = \frac{Y}{X},$$

which is true for both consumers since they have identical utility functions. This MRS is compatible with many price ratios; it simply says that the more X that the consumer has relative to Y the more she is willing to pay for Y at the margin. However, if consumers A and B are trading with one another, and they are on the contract curve (and not at a corner where either A or B possesses the entire endowment, it must be that

$$MRS_{xy}^A = MRS_{xy}^B.$$

This will only be true if $\frac{Y_A}{X_A} = \frac{Y_B}{X_B}$. Thus, the contract curve in this particular case will have the property that if A is consuming (X_A, Y_A) then B will be consuming $(X_B, Y_B) = (\alpha X_A, \alpha Y_A)$, where α is a positive constant on the open interval between 0 and infinity. (Note again that this reasoning does not apply at the corners since the MRS of this particular utility function is undefined at $X = Y = 0$.) The contract curve will therefore be a line that extends from the southwest to northeast corners of the Edgeworth box with slope equal to the ratio of Y to X in the economy. In our example here, the economy is endowed with 2 units each of X and Y , which implies that the slope of the contract curve is 1.

Redistribution

A hypothetical government in this two-person economy might think that the final allocation is undesirable (inequitable) since A is consuming three times as much of each good as B . Let's say that this government decides that a preferable final consumption bundle would be: $d'_A(X_A, Y_A) = (1.25, 1.25)$ and $d'_B(X_B, Y_B) = (0.75, 0.75)$. Notice that this bundle is on the contract curve, hence, by the Second Welfare Theorem, it can be supported as a competitive equilibrium. However, this bundle is not attainable as a market equilibrium from the initial endowment. We know this because agent A would be worse off under the government's preferred allocation than under his initial endowment: $U_A(1, 2) = 2$, $U_A(1.25, 1.25) = 1.56$. Thus, A would never voluntarily move from E_A to d'_A . How could the government attain its preferred outcome? Let's consider two options.

Altering prices

One option is for the government to announce a price ratio *other than* the market price ratio to change the final consumption equilibrium. If, for example, the government were to announce that $P_Y = 1/3$, the new price ratio would trace a ray that extends through both the initial endowment point and the point on the contract curve that corresponds to the government's preferred outcomes. You can see this as follows: with prices $P_X = 1$ and $P_Y = 1/3$, A 's bundle would have value $I'_A = 1 + \frac{2}{3} = \frac{5}{3}$, which would provide the purchasing power required to buy 1.25 each of good X and Y . Similarly, B 's bundle would be worth 1, which would allow B to purchase 0.75 each of goods X and Y . This price ratio would therefore be compatible with A and B each consuming the government's preferred bundles d'_A and d'_B .

But would A and B choose these bundles? We can answer this question by calculating their demands at these prices:

$$d_A^{X'} = \frac{1 + 2/3}{2} = 0.83$$

$$d_A^{Y'} = \frac{1 + 2/3}{2/3} = 2.5$$

$$d_B^{X'} = \frac{1}{2}$$

$$d_B^{Y'} = \frac{3}{2}.$$

The answer is **no**, A and B would not choose the government's preferred bundles, even though both would be feasible given the government's mandated price ratio and A 's and B 's endowments. With the price of Y depressed below its equilibrium level, there would be excess demand

for Y ($d_A^{Y'} + d_B^{Y'} = 3.25$) and insufficient demand for X ($d_A^{X'} + d_B^{X'} = 1.23$). In fact, there is no price ratio other than $p = \frac{P_Y}{P_X} = 1$ that would *clear the market* given consumers' preferences and the economy's total endowment of X and Y . At any announced price ratio, consumers A and B will have Marshallian demands that are privately optimal given their preferences; that is, if these demands are feasible, we would have $\frac{\partial U/\partial Y}{\partial U/\partial X} = \frac{P_Y}{P_X}$ for both A and B . But if $p \neq 1$, these demands will not be feasible because there will be excess supply of one good and excess demand for the other.

One could concoct more elaborate policies, of course. For example, the government could charge *different* prices (or price ratios) to A and B . Given the preferences specified above and the economy's total endowment of X and Y , however, there is exactly one price ratio that extends from the initial endowment through the contract curve *and* crosses that curve at a slope that is tangent to the indifference curves of both consumers. The only price ratio that will put both A and B on the contract curve is $p = 1$. (Of course, there are an infinite number of rays that extend from the endowment to the contract curve and an infinite number of rays that cross the contract curve at a slope tangent to both A 's and B 's indifference curves at the point of intersection. But there is only one ray that satisfies both criteria.)

What this example indicates is that manipulating prices will not simultaneously solve the efficiency and equality problems (the equality problem being the government's objective of generating a more equal distribution of consumption). The market will solve the efficiency problem *on its own* through the emergence of prices that allow consumers to move from E_A, E_B onto the contract curve. Thus, the price ratio *is* the market's solution to the efficiency problem. If the price ratio is instead used by the government to solve a different problem, it will not likely *also* solve the efficiency problem simultaneously (unless the government desires the market outcome).

Lump sum redistribution

Does this mean that the government is out of options? Nope. The Second Welfare Theorem says that any Pareto efficient allocation—any allocation on the contract curve—is supportable as a market equilibrium. The question is how we get from E_A, E_B to that equilibrium. If manipulating the price ratio is not a feasible solution, then there's another free parameter that we can work, which is the endowment itself. (We assume that the government cannot directly manipulate preferences, though some would disagree.) The government could potentially perform lump sum transfers to redistribute goods from A to B or vice versa. This lump sum redistribution is different from directly altering the price ratio because after redistribution has

occurred, the price ratio would still be allowed to adjust freely to find the market equilibrium.

How would the government accomplish its objective through lump sum transfers? One way is for the government simply to move the endowment to the desired location: $E'_A(X_A, Y_A) = (1.25, 1.25)$ and $E'_B(X_B, Y_B) = (0.75, 0.75)$. Because this location is on the contract curve, A and B would remain at this location if placed there. But in fact, there are many allocations that would serve this purpose. Given that the price ratio will always be $p = 1$ in this economy given A 's and B 's preferences and the economy's total endowment of X and Y , any endowment that has the following form will lead to the same consumption choices:

$$E_A^X + E_A^Y = 2.5,$$

(which further implies that $E_B^X + E_B^Y = 1.5$ due to the adding up constraint). Restating, any point on a line that is perpendicular to the contract curve at the government's desired location d'_A, d'_B will yield an equilibrium consumption set that is equal to d'_A, d'_B . (Note that this linear/perpendicular solution is *not* a general result for all preferences. It derives specifically from the preferences of A and B and the endowment of this economy, all specified above.)

What's the lesson here?

The point of this analysis is a simple but general one. Prices serve a fundamental function in a competitive economy, which is to move the economy from the initial endowment to a Pareto efficient allocation that clears the market (no excess demand or supply) and leaves no consumer worse off than she was at the initial endowment. (Note: competitive means that *all* of the strong preconditions for the First Welfare Theorem are met). Altering prices in a competitive economy either by fiat—i.e., announcing that the new price ratio is P_Z —or by taxing one price and subsidizing another will thwart the allocative function of market prices. This will generally lead to a set of undesirable distortions, manifest in the presence of excess demands and supplies of various goods. At regulated (non-market) prices, consumers will not be able to obtain all of the goods that they're seeking at regulated prices; simultaneously, other goods will go unconsumed.

In broadest terms, altering prices (typically through taxation) is a distortionary mechanism for achieving redistribution because it generates Pareto inefficient consumption. A potentially more efficient way to achieve similar objectives is to perform lump sum redistribution while still allowing prices to clear the market. Lump sum transfers (or lump sum taxation), unlike direct adjustments to prices, do not distort consumer decisions at the margin. Once transfers are complete, the market will generate equilibrium prices that accurately reflect the opportunity

costs of goods that consumers wish to trade and thus move consumption to Pareto efficiency.