

Lecture Note 4 – Demand Functions: Income Effects, Substitution Effects, and Labor Supply

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1 Demand Functions

Now, let's use the Indirect Utility function and the Expenditure function to get Demand functions. Up to now, we have been solving for:

- Utility as a function of prices and budget
- Expenditure as a function of prices and utility

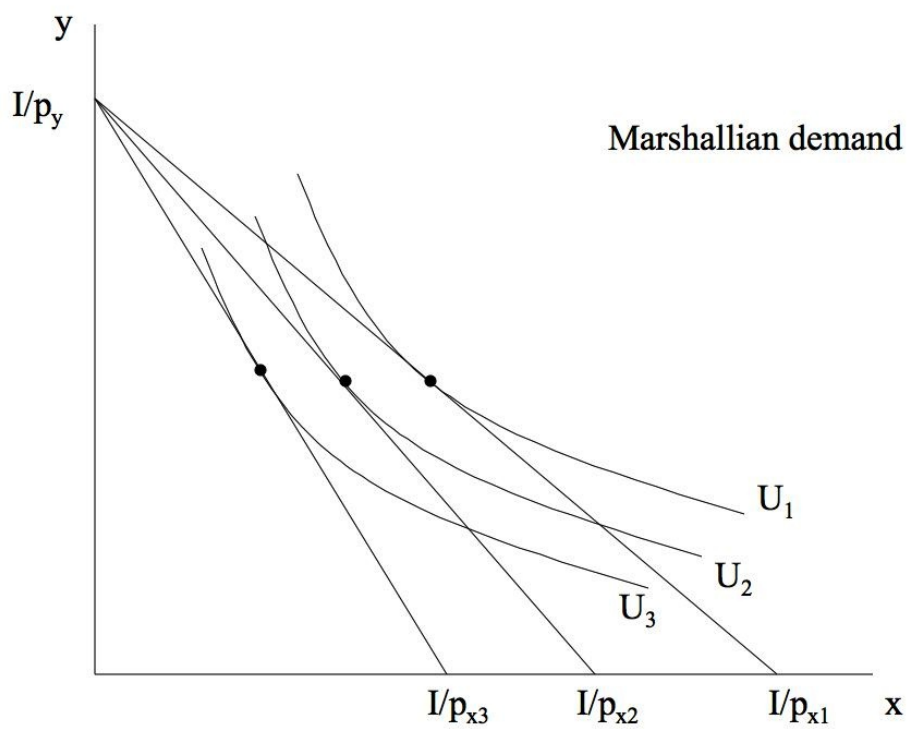
Implicitly we have already found demand schedules—a demand schedule is immediately implied by an individual utility function. For any utility function, we can solve for the quantity demanded of each good as a function of its price, holding the price of all other goods constant *and* holding *either* income *or* utility constant.

2 Two types of demand curves: Marshallian (uncompensated) and Hicksian (compensated)

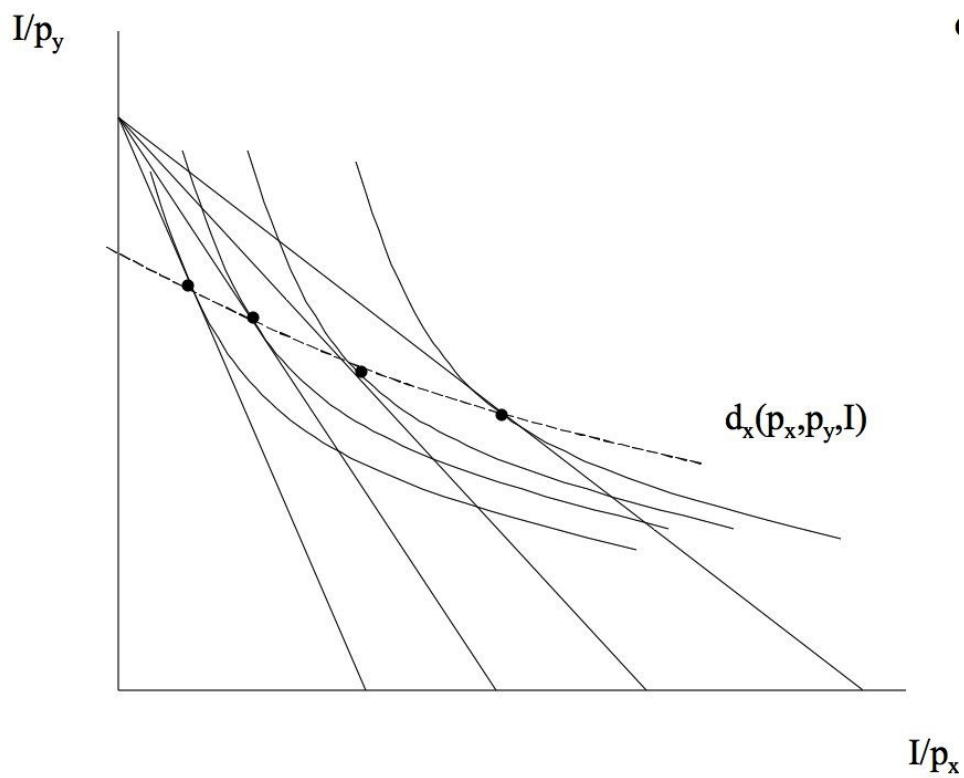
2.1 Uncompensated ('Marshallian') demand—Holding *income* constant

Alfred Marshall was the first economist to draw supply and demand curves. The 'Marshallian cross' is the staple tool of blackboard economics — *the most famous diagram in economics*. *Marshallian demand curves* correspond to conventional market-level or consumer-level demand curves. They answer the question:

- Holding *income and all other prices constant*, how does the quantity of good X demanded change with p_x ? We notate this demand function as $d_x(p_x, p_y, \bar{I})$. Marshallian demand curves implicitly *combine* income and substitution effects. The income effect is the change in consumption that arises if the consumer's income falls but if prices stay the same. The substitution effect is the change in consumption that arises if the prices change but the agent is given enough income to maintain the same utility they had at the initial prices. They are *net* demands that sum over these two conceptually distinct behavioral responses to price changes.



- A Marshallian demand curve for x as a function of p_x



- So a demand function is a set of tangency points between indifference curves and budget set holding I and p_y (all other prices) constant.

Mathematics of uncompensated (‘Marshallian’) demand—Holding *income* constant

- In our previous example where:

$$U(x, y) = x^{.5}y^{.5}$$

we derived:

$$\begin{aligned}x(p_x, p_y, I) &= 0.5 \frac{I}{p_x} \\ y(p_x, p_y, I) &= 0.5 \frac{I}{p_y}\end{aligned}$$

- In general we will write these demand functions (for individuals) as:

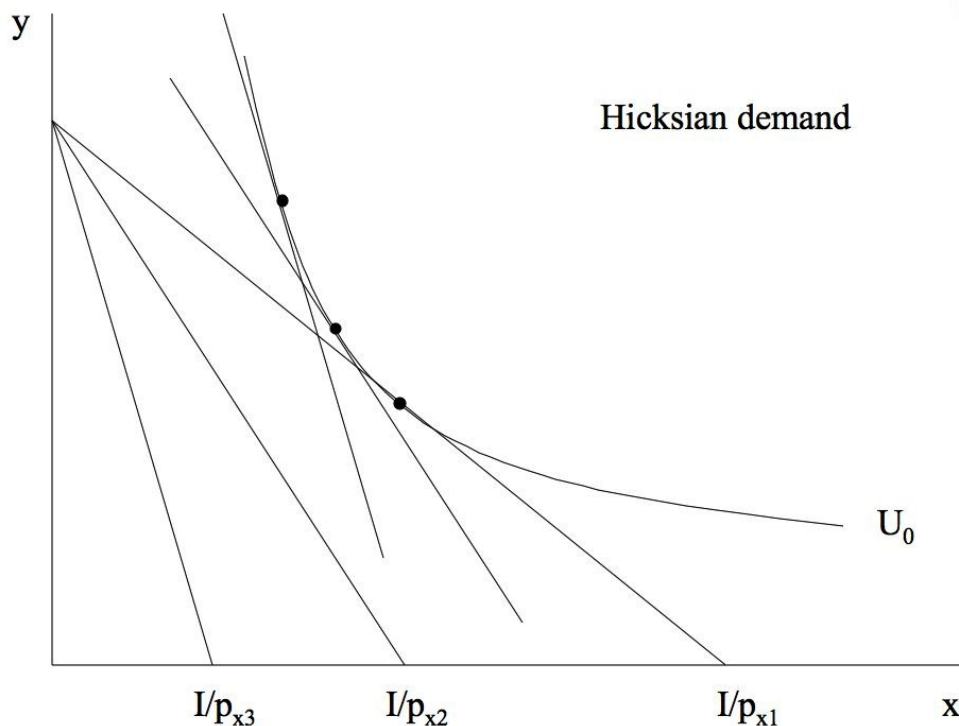
$$\begin{aligned}x_1^* &= d_1(p_1, p_2, \dots, p_n, I) \\ x_2^* &= d_2(p_1, p_2, \dots, p_n, I) \\ &\dots \\ x_n^* &= d_n(p_1, p_2, \dots, p_n, I)\end{aligned}$$

- We call this “Marshallian” demand after Alfred Marshall (who first drew demand curves). You are also welcome to call it uncompensated demand.

2.2 Compensated (‘Hicksian’) demand—Holding *utility* constant

- One can also conceive of a demand curve that is composed solely of substitution effects. This is called Hicksian demand (after the economist J. R. Hicks) and it answers the question: Holding *consumer utility constant*, how does the quantity of good X demanded change with p_x ? We notate this demand function as $h_x(p_x, p_y, \bar{U})$. The presence of \bar{U} as a parameter in the Hicksian demand function indicates that this function holds consumer utility constant—on the same indifference curve—as prices change. Hicksian demand is also called *compensated* demand. This name follows from the fact that to keep the consumer on the same indifference curve as prices vary, one would have to

adjust the consumer's income, i.e., compensate them. For the analogous reason, the Marshallian demand is called *uncompensated* demand.



Mathematics of compensated (‘Hicksian’) demand—Holding *utility* constant

- We can write this mathematically using the dual problem to utility maximization, which is expenditure minimization. Using the utility function from earlier examples, we had previously derived that:

$$\begin{aligned} x(p_x, p_y, U) &= \left(\frac{p_y}{p_x} \right)^{.5} U_p \\ y(p_x, p_y, U) &= \left(\frac{p_x}{p_y} \right)^{.5} U_p \end{aligned}$$

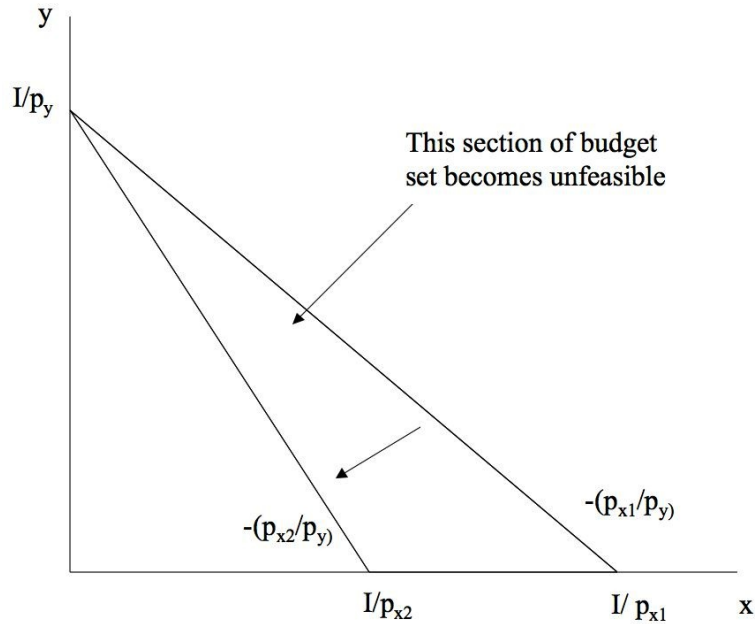
- These are Hicksian (compensated) demand function because they take prices and utility as arguments rather than prices and income.

- We will denote the Hicksian demand functions (of an individual) as:

$$\begin{aligned}x_{1,c}^* &= h_1(p_1, p_2, \dots, p_n, U) \\x_{2,c}^* &= h_2(p_1, p_2, \dots, p_n, U) \\&\dots \\x_{n,c}^* &= h_n(p_1, p_2, \dots, p_n, U)\end{aligned}$$

3 The effect of price changes on Marshallian (uncompensated) demand

- A simple change in the consumer's budget (i.e., an increase or decrease of I) involves a parallel shift of the feasible consumption set inward or outward from the origin. The economics of this are straightforward. Since this shift preserves the price ratio $\left(\frac{p_x}{p_y}\right)$, it typically has no effect on the consumer's marginal rate of substitution (MRS), $\left(\frac{U_x}{U_y}\right)$, unless the chosen bundle is either initially or ultimately at a corner solution.
- By contrast, a rise in the price of one good holding constant both income and the prices of other goods has economically more complex effects:
 1. It shifts the budget set inward toward the origin for the good whose price has risen. In other words, the consumer is now effectively poorer. This component is the 'income effect.'
 2. It changes the slope of the budget set so that the consumer faces a different set of market trade-offs. This component is the 'substitution' effect.
- Although both shifts occur simultaneously, they are conceptually distinct and have potentially different implications for consumer behavior.



Impact of an Increase in P_x on the Budget Set

3.1 Substitution effect

- In a two good economy, what happens to consumption of X_1 if

$$\frac{p_1}{p_2} \uparrow$$

while utility is held constant? Formally, we are asking for the sign of this derivative:

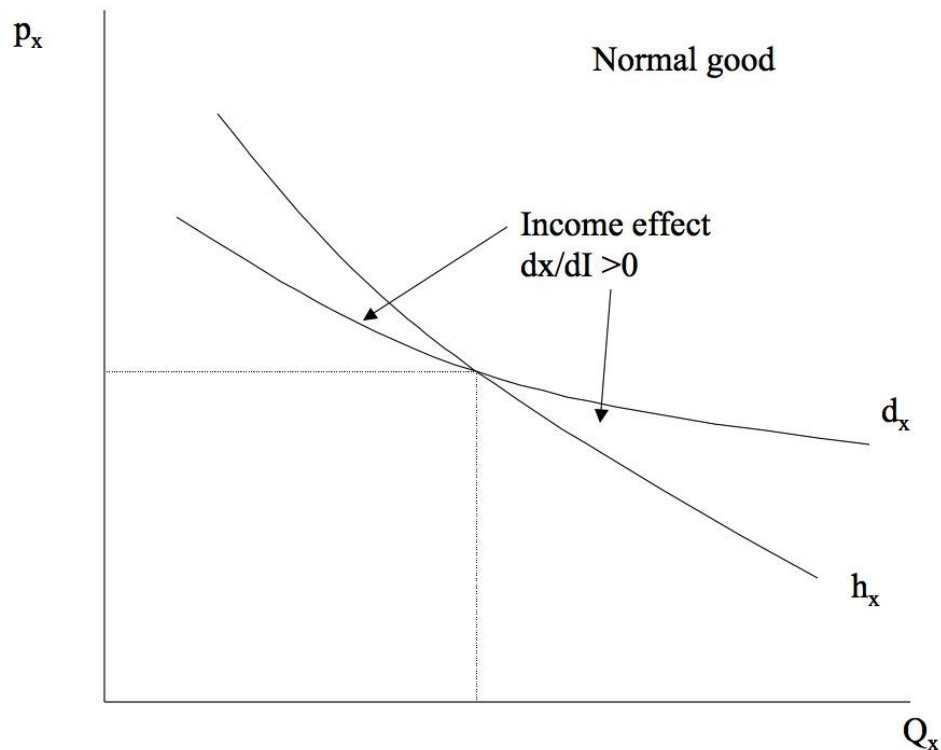
$$\text{Sign} \left\langle \frac{\partial X_1}{\partial p_1} \Big|_{U=U_0} \right\rangle.$$

- Provided that the axiom of diminishing MRS applies, we'll have $\frac{\partial X_1}{\partial p_1} \Big|_{U=U_0} < 0$.
- In words, holding utility constant, the substitution effect is *always* negative.

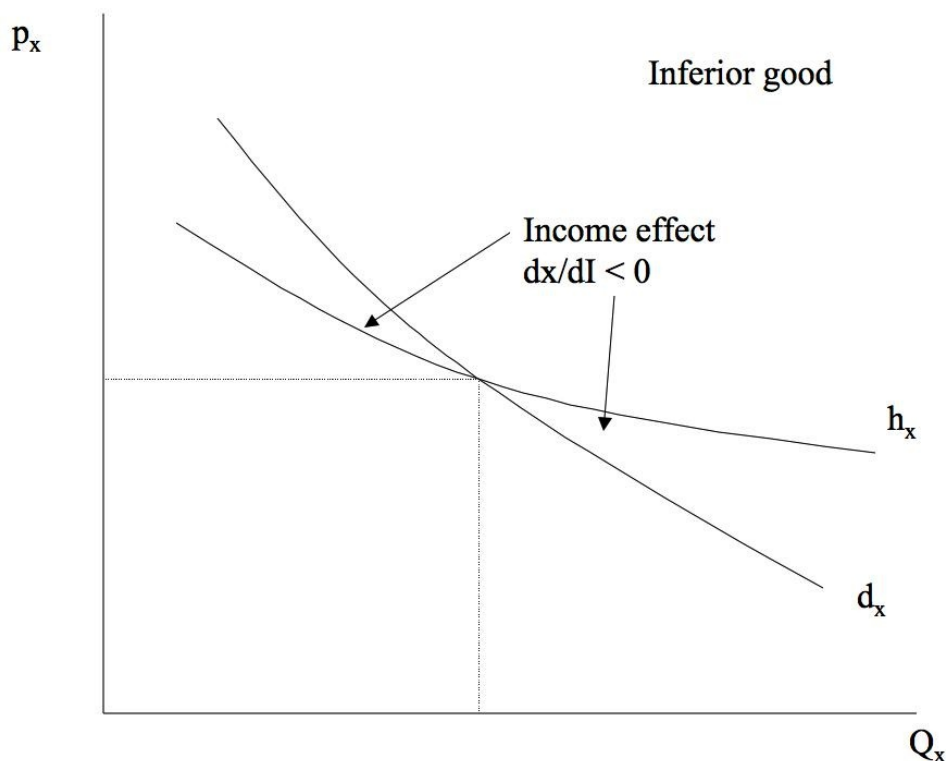
3.2 Income effect

- Consider now the effect of a change in income on consumption of good X . The income effect is defined as $\partial X / \partial I$, the change in the consumer's quantity demanded of X for a rise in income I .
- The income effect can be either negative or positive, depending on the specific good and the consumer's income level.

- If $\partial X/\partial I > 0$ at given prices and income, good X is said to be a “normal” good. If $\partial X/\partial I < 0$ at given prices and income, good X is said to be an “inferior” good. Inferior goods can be further subdivided into two groups, as we’ll see below.
- What is the impact of an inward shift in the budget set in a 2-good economy (X_1, X_2) ?
 1. Total consumption? [Falls]
 2. Utility? [Falls]
 3. Consumption of X_1 ? [Answer depends on normal, inferior]
 4. Consumption of X_2 ? [Answer depends on normal, inferior]



Comparison of Compensated (h_x) and Uncompensated (d_x) Demand Curves for a Normal Good



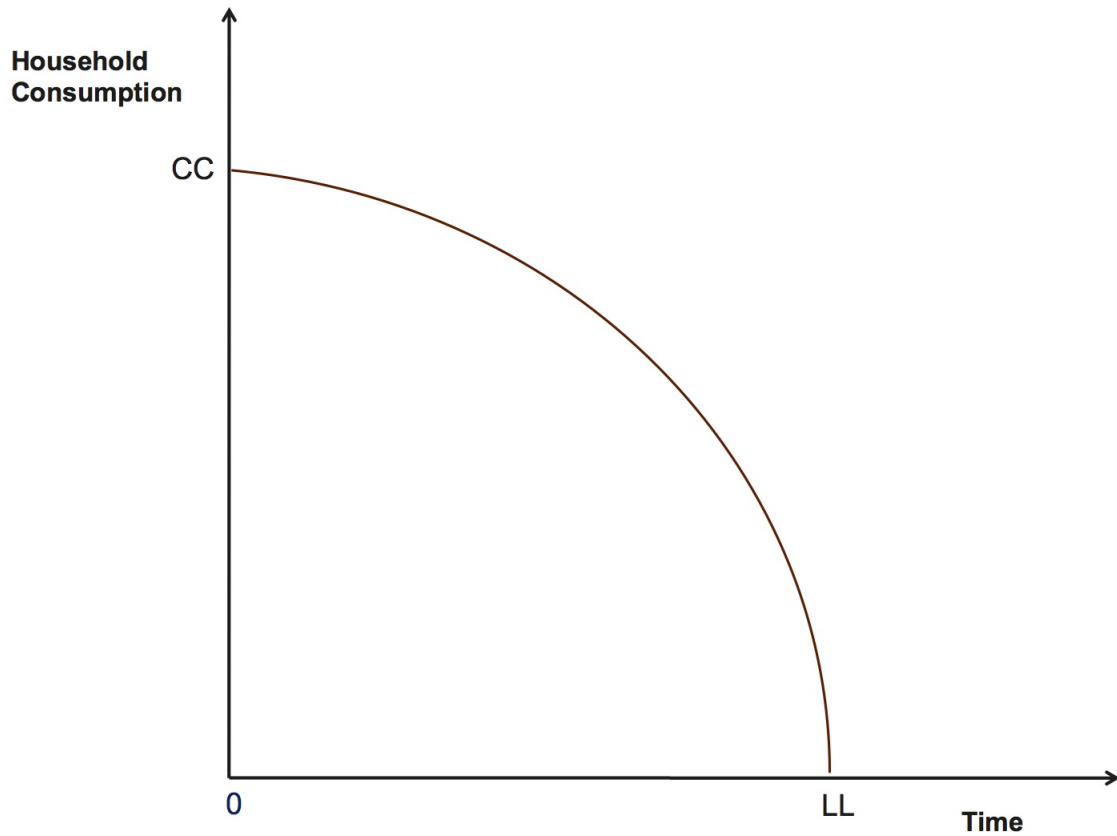
Comparison of Compensated (h_x) and Uncompensated (d_x) Demand Curves for an Inferior Good

4 Income, substitution, and labor supply

Although we typically think of demand functions as describing goods consumption, the same reasoning applies to labor supply functions. Seeing this same logic through the labor supply lens will deepen your understanding of the material. [Don't worry: we'll return to demand for goods in the next lecture when we study Giffen goods.]

Consider the figure below, which depicts the Production Possibility Frontier of a subsistence household—that is, a household that works only for itself. For simplicity, let's assume there's only one household member, Zara. She can choose any bundle on the frontier of this budget set (or in the interior—but she would not, by non-satiation). If Zara works 24 hours per day, she maximizes consumption (point CC). If she sleeps or relaxes 24/7 (*chillaxes* as they say in Singapore) her consumption is zero (point LL). The curvature of the PPF comes from diminishing returns to home production (technically, this is a diminishing marginal rate of technical substitution between production and leisure/sleep). The implicit assumption is that the first hours of home production in the day are highly productive (taking a shower, brewing coffee, making a kale smoothie). Zara's marginal productivity of home production

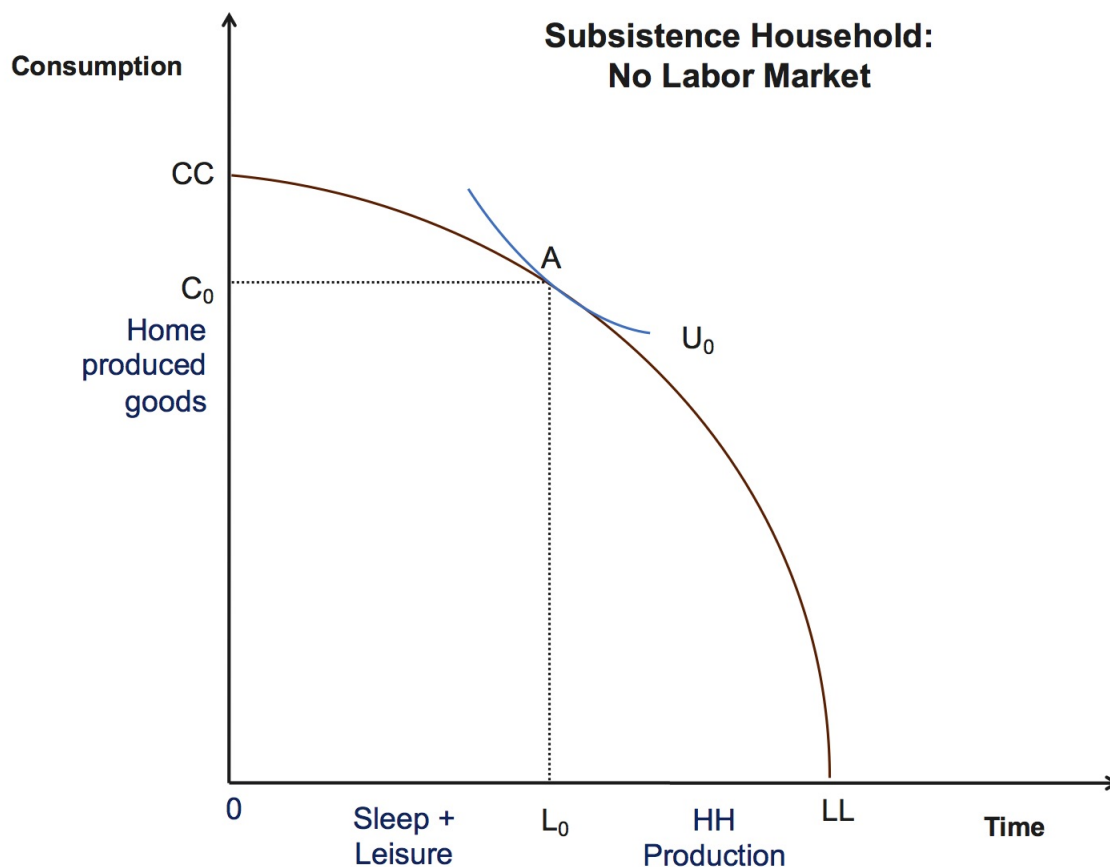
falls as Zara does all of the essential stuff and/or becomes increasingly exhausted.¹



Production Possibility Frontier of a Subsistence Household

Let's say Zara chooses point A on this PPF. Notice that A corresponds to the point where the PPF is tangent to the highest attainable indifference curve. Zara devotes time $LL - L_0$ to household production and consumes goods C_0 . She spends hours $L_0 - 0$ on leisure and sleep (the complement of the time spent on household production).

¹This graphical depiction of the household time's allocation trade-offs among sleep/leisure, home production, market production, and consumption, was introduced in a 1977 *Journal of Political Economy* paper by Reuben Gronau, "Leisure, Home Production, and Work—The Theory of the Allocation of Time Revisited." It's a remarkably compact graphical representation of a sophisticated problem.



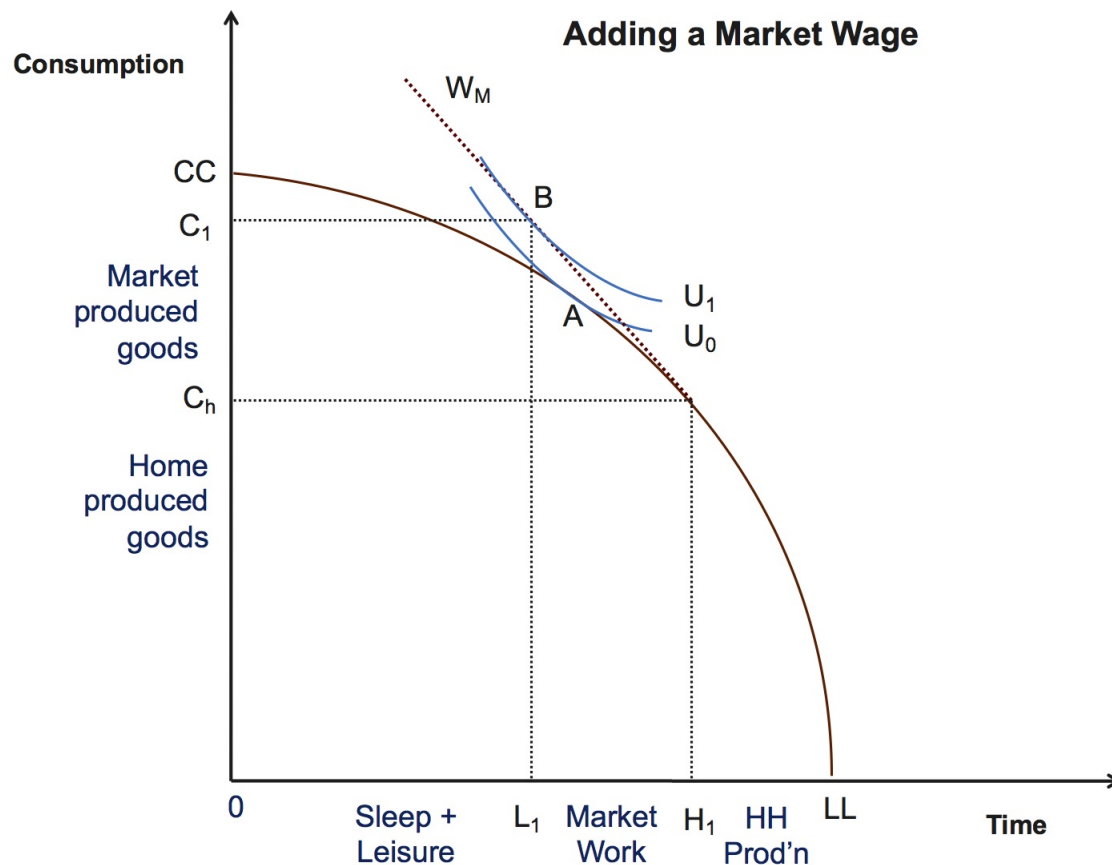
Consumption-Leisure Bundle of Subsistence Household

Now let's introduce a market wage, W_m . This will change the equilibrium in a subtle and interesting way. This wage W_m enters the figure as a slope that measures the additional consumption attainable per hour of labor supply. W_m connects to the PPF at the point of tangency between the wage slope and the PPF's frontier. It thus *expands* the PPF by allowing Time-Consumption bundles that were previously infeasible (beyond the frontier).

How does the market wage affect home production, labor supply, and consumption? A first thing to notice is that it will never make sense for Zara to spend more than $LL - H_1$ hours on home production. Why? Because to the left of H_1 , she gets more marginal consumption per hour by supplying labor to the market at wage W_m than she does through home production. This is because the wage-augmented PPF is steeper to the left of H_1 than is the subsistence PPF.

How much does Zara work? Zara's new utility maximizing bundle is at point B . Her utility has risen to U_1 , her home production has *fallen* to $LL - H_1$ and her market work has risen from zero to $H_1 - L_1$. (Market work has risen by more than work overall because home production has fallen.) Zara's hours of sleep and leisure have declined ($L_1 - 0$ is shorter

than $L_0 - 0$). Consumption has risen to C_1 , with the length $0 - C_h$ corresponding to home-produced goods and the length $C_1 - C_h$ corresponding to market-produced goods. (The figure implicitly assumes that home- and market-produced goods are perfect substitutes; that is, one unit of either is exactly as good as one unit of the other. If this were not the case, the figure would need another dimension). In net, Zara is consuming more, working more, chilling less, and happier for it (see her higher indifference curve).



Adding a Market Wage

Let's now consider the impact of the Earned Income Tax Credit. For simplicity, assume initially that the EITC is exclusively an *hourly wage subsidy* that does not have the complex phase-in and phase-out properties of the actual EITC; it just raises the hourly wage from W_m to $W_m + W_E$ as depicted below.



Adding an EITC Subsidy to the Market Wage

What effect does this have on home production, market work, consumption, and well-being? You can see that home production unambiguously falls (from $LL - H_1$ to $LL - H_2$). This outcome is immediate because the point of tangency between the wage locus and the PPF has moved southeast (down and to the right). Similarly, utility must rise: as long as Zara is participating in the subsidy (that is, her chosen location on the PPF is along the segment containing $W_m + W_E$), she must be better off; otherwise she would not participate. Graphically, utility has risen from point B on indifference curve U_1 to point C on indifference curve U_2 . Notice also that consumption has risen to C_2 , which exceeds consumption at the previous location B . Although consumption of home produced goods falls, consumption of market produced goods rises by more, leading to a net increase in consumption.

The most interesting observation here is how the wage subsidy affects labor supply. Intuitively, you might think that Zara's labor supply would necessarily increase since her wage has risen—and in the picture, it does. But the rise is modest (from $H_1 - L_1$ to $H_2 - L_2$). This modest increase reflects the net impact of two countervailing effects. Holding income constant, the rise in the wage would certainly cause Zara to work more; this is a pure *substi-*

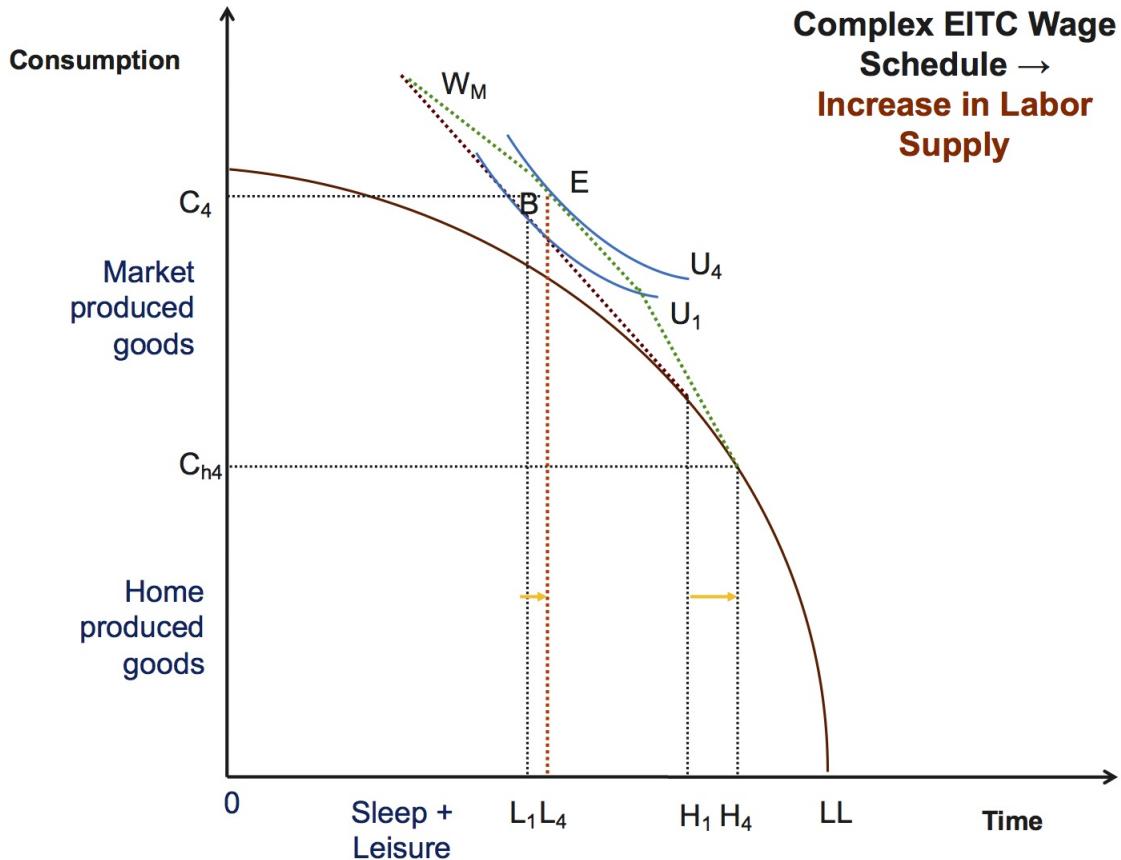
tution effect. You can visualize this by rotating the ray W_m through point B until it parallels the ray $W_m + W_E$. The indifference curve tangent to that rotated ray would clearly lie north-west of U_1 , meaning more hours of labor supply, more consumption, and higher utility. To reiterate, this notional change would correspond to the pure *substitution effect* of a higher wage rate offered at the initial income level.

But that's *not* precisely what's happening here. If Zara made *no adjustment* in her hours at all, her income/consumption would still rise when the EITC was implemented. Prior to the EITC policy, she was working hours $H_1 - L_1$ in the figure above. When the EITC is enacted, her income rises by the amount $(H_1 - L_1) \times W_E$ with no change in labor supply. Thus, the wage subsidy has two components from Zara's perspective: (1) a transfer of income equal to $(H_1 - L_1) \times W_E$; and (2) an increase of W_E in the hourly wage for each additional hour worked. All else equal, this income transfer is likely to *reduce* labor supply because leisure is, for most people, a normal good—the wealthier you get, the more of it you consume (holding the wage constant). Because of these countervailing substitution and income effects, a wage subsidy may either increase or reduce labor supply among those already working. Notice, however, that for a person who is not initially working, the wage subsidy has no income effect but does generate a potential substitution effect. A wage subsidy will therefore normally cause some non-workers to enter the labor market. This observation is quite important for the Eissa-Liebman paper, which we'll be discussing next.

The next figure depicts a case where the wage subsidy is substantially larger, reflected in a steeper slope for $W_m + W_{EE}$ as compared to $W_m + W_E$ in the figure above. Relative to the case with no EITC subsidy (wage is W_m), home production falls from the $LL - H_1$ to $LL - H_3$ while market labor supply shifts from $H_1 - L_1$ to $H_3 - L_3$. As drawn, the inward shift from L_1 to L_3 is slightly larger than the outward shift from H_1 to H_3 , meaning that market labor supply has fallen. Thus, the income effect of this large wage subsidy dominates the substitution effect for Zara. The subsidy raises her welfare, increases her consumption, increases her leisure and reduces her time spent on both home production and market work. She's definitely more chillaxed than she used to be.

Summing up: in the phase-in range, the EITC has both income and substitution effects (except in the case of individuals who were not initially working—for them, there is no income effect). We generally expect the EITC to increase labor supply in this region, though it's possible for the income effect to dominate, lowering labor supply. In the plateau range, the EITC has only an income effect; it raises total earnings but does not raise earnings at the margin, so we expect it to (modestly) decrease labor supply in this range. In the phase-out range, where the EITC reduces the marginal wage below W_m , both the income and substitution effects work in the direction of lowering labor supply. We expect the EITC to reduce labor supply in this range.

How does Zara respond to the EITC? Her hours of home production fall from $LL - H_1$ to $LL - H_4$ while her hours of market work rise from $H_1 - L_1$ to $H_4 - L_4$ (both shift right, but home production shifts rightward by more). The EITC permits Zara to choose modestly higher consumption, modestly higher labor market hours, modestly higher leisure, and modestly lower household production. She is, of course, better off.



Adding the Full EITC Budget Set to the Market Wage

As this figure underscores, the complexity of the EITC budget set implies that *many*

things can happen. Because the EITC reduces the marginal wage during the phase-out range, it provides an incentive to reduce labor supply; many people will shift labor supply until they are at the kink where the plateau meets the phase-out region. Others whose initial market labor supply was quite low or zero will face a strong substitution effect but have little or no income effect, which should increase labor supply. For a person like Zara whose hours were already in the plateau range, there is an income effect but no substitution effect. *Total* hours worked (home + market) are likely to decline due to the income effect. But market hours may rise (as in this figure) because home production hours fall by more than total labor supply (meaning market hours have risen). In all cases, individuals receiving the EITC are made better off: whether they increase or decrease total work hours, market hours, and/or leisure, those who receive the subsidy must necessarily obtain a bundle of consumption, work, and leisure that is preferred to the initial starting point.

5 Bringing the theory to the data: the Tax Reform Act of 1986 (TRA86)

The 1996 paper by Eissa and Liebman in the *Quarterly Journal of Economics* provides a credible test of whether the EITC affects labor supply in practice. The Tax Reform and Fiscal Responsibility Act of 1996 (TRA86) yielded a substantial expansion in the generosity of the EITC. As Eissa/Liebman note, at every level of earnings, the EITC amount after the expansion was at least as large as it was before. We would therefore expect it to increase the fraction of potentially eligible workers participating in the labor force (due to the substitution effect). For eligible workers already participating in the labor force, however, we would generally expect the EITC expansion to modestly reduce hours (due to the income effect). We'll discuss the results of the Eissa/Liebman study in class.