

Advanced Panel Data: Interpreting FE?

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Causal FE

Recall the FE Assumptions

$$y_{it} = x'_{it}\beta + \eta_i + \varepsilon_{it}$$

- η_i is a **fixed effect**.
- To estimate everything consistently, we need $E[\varepsilon_{it}|x_{it}, \eta_i] = 0$
- Mostly this is not true. Instead usually treat η_i as a **control variable** or **nuisance parameter**.
 - A nuisance parameter is one that we estimate but don't care about interpreting.
 - If we only care about β then η_i is a nuisance parameter.
- With a control or nuisance parameter we only require that $E[\varepsilon_{it}|\eta_i] = E[\varepsilon_{it}|x_{it}, \eta_i]$ **conditional mean independence**.
- Once we condition on η_i it is as if ε_{it} and x_{it} are uncorrelated.

- We can get away with conditional mean independence if we don't care about η_i .
- But suppose that we care about $\hat{\eta}_i$?
 - Teacher FE
 - Physician/Hospital FE
 - Location/County FE
- Suppose move someone from the 10th percentile to the 90th percentile
 - This about the variance/second moment of η_i .

- Now we have to really believe $E[\varepsilon_{it}|x_{it}, \eta_i] = 0$
- We should worry about the conventional **omitted variable bias** problem.
- Suppose there exists a variable w_{it} so that:

$$y_{it} = x'_{it}\beta + w'_{it}\gamma + \eta_i + \varepsilon_{it}$$

- Recall the conditions for OVB
 - w_{it} is correlated with x_{it}
 - w_{it} is a determinant of y_{it}
- New one: w_{it} is correlated with η_i
 - This is easy to satisfy!
 - w_{it} needs to be uncorrelated with anything about the individual i .

Example: Test Scores

- Students s , Teachers i
- Want to measure effect of Teachers on Test Scores

$$y_{is} = x'_{is}\beta + \eta_i + \varepsilon_{is}$$

- We observe some features of students but not all of them (parent's education, household income, language spoken at home).
- We also observe some school specific variables w_{is} but not all of them (district spending per pupil, % free lunch, etc.).
- But we don't observe other things (jackhammering outside the classroom, which students have disruptive home lives, etc.).
 - If the mean of those things varies across teachers \rightarrow we are screwed!
 - Can't get an accurate estimate of η_i .

Example: Test Scores

We need a better design:

- We probably need random assignment of students to teachers.
- Ideally we would be able to control for student and school unobservables.
- Might want to see many students match with many teachers.

Example: Test Scores

Even random assignment is not enough:

$$y_{is} = \eta_i + \varepsilon_{is}$$

For each teacher the mean test score is:

$$\bar{y}_i = \eta_i + \frac{1}{S_i} \sum_{s=1}^{S_i} \varepsilon_{is}$$

- But $S_i \rightarrow \infty$ doesn't hold. So relative variance of η_i and ε_{is} matters.
- People with large \bar{y}_i may mostly be lucky (small S_i).
- May want to apply **shrinkage** estimator again.

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Example: Test Scores

$$y_{ist} = x_{ist}\beta + \underbrace{\eta_i + \lambda_{it} + \varepsilon_{ist}}_{u_{ist}}$$

- Now residual u_{ist} contains a teacher “quality” η_i that we are interested in
- λ_{it} a “classroom” or teacher-year specific deviation.
- Goal: think about a forecast of $y_{i,t+1}|\mathbf{y}_t$

How ?

Estimate \hat{u}_{ist} and construct the following:

- Within classroom student variance: $\hat{\sigma}_s^2 = \text{Var}(u_{ist} - \bar{u}_{it})$
- Auto-Covariance within teacher across years/classrooms: $\hat{\sigma}_\eta^2 = \text{Var}(\bar{u}_{it}, \bar{u}_{i,t+k})$
- Classroom Variance: $\hat{\sigma}_\lambda^2 = \text{Var}(u_{ist}) - \hat{\sigma}_\eta^2 - \hat{\sigma}_s^2$.
- Weight each classroom by its inverse variance (n_{it} is students per classroom):

$$\bar{u}_i = \sum_t w_{it} \bar{u}_{it}, \quad w_{it} = \frac{h_{it}}{\sum_t h_{it}}$$
$$h_{it} = \frac{1}{\text{Var}(\bar{u}_{it}|\eta_i)} = \frac{1}{\sigma_\lambda^2 + \frac{\sigma_s^2}{n_{it}}}$$

- But \bar{u}_i is still not the best **forecast** estimate of η_i .

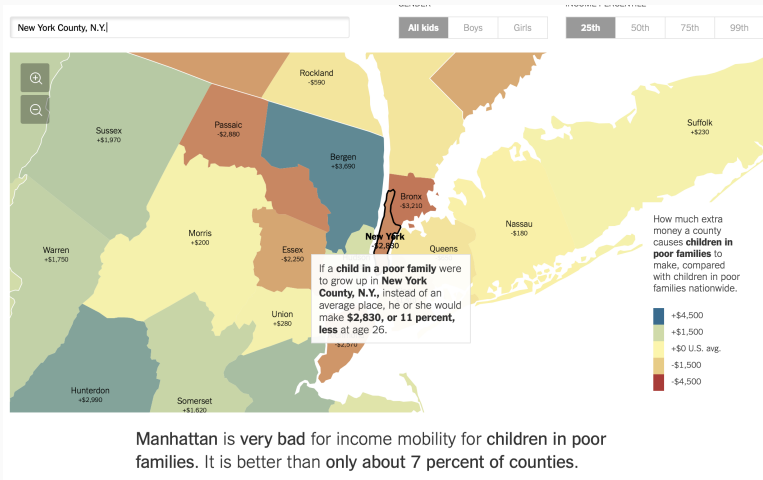
How ?

We want to further shrink our estimate \bar{u}_i to account for the variance in our estimate:

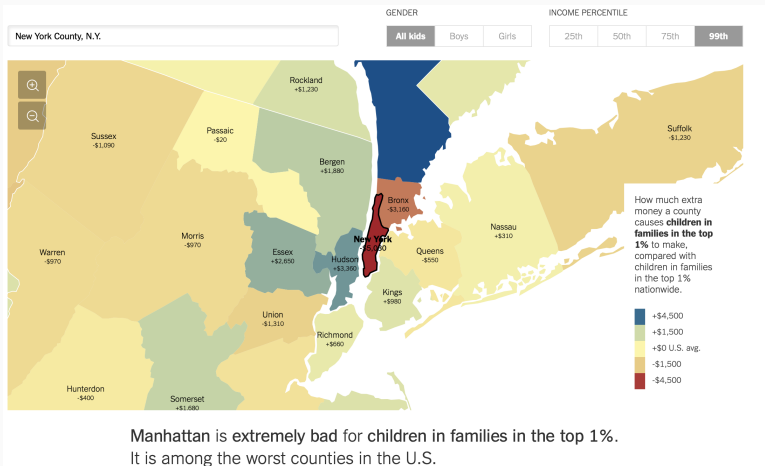
$$\eta_i = \bar{u}_i \times \frac{\sigma_\eta^2}{\text{Var}(\bar{u}_i)}$$

- Where $\text{Var}(\bar{u}_i) = \sigma_\eta^2 + (\sum_t h_{it})^{-1}$
- Idea: fraction of teacher variance explained by FE η_i as opposed to λ_{it} and student factors.
- Shrinkage factor always ≤ 1 and smaller for less precise estimates.
- Hope: if teachers teach many students with similar scores, we have a precise estimate of η_i .

Best Counties? (Chetty Hendren 2016 AER)

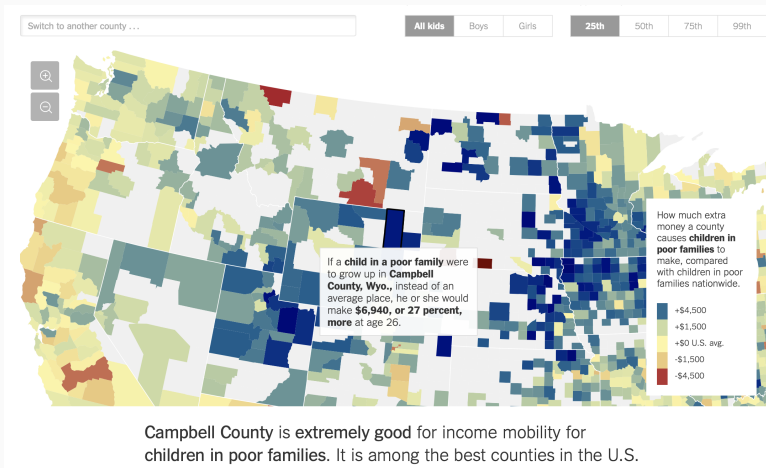


Best Counties?



Best Counties?

50,000 residents (and very few people move here).



Time to move to Campbell County, WY?

Less than 10% of Campbell County residents graduated college (among worst in US):



Quick Stats (2020)

- > Grades: **9-12**
- > Students: **1,103 students**
- > Student:Teacher Ratio: **16:1**
- > Minority Enrollment: **18%**
- > Graduation Rate: **86%** (Btm 50% in WY)
- > Overall School Rank: **Bottom 50%**
- > Math Proficiency: **35%** (Btm 50%)
- > Reading Proficiency : **30%** (Btm 50%)
- > Diversity Score: **0.31** (Btm 50%)

What's going on?

Is Campbell county the land of opportunity?

- Or is it a small place that nobody moves to with very imprecisely measured FE γ_i ?
- Helped that people found coal in their backyards (literally)
- If you moved there today would you still expect to do well?

This problem is endemic:

- Our “best” and “worst” teachers tend to be those who teach the fewest students.
- Presumably FE are estimated with precision that is not equal.

Healthcare Exceptionalism: Static Reallocation

Is **quality** (or productivity) correlated with **market share**?

$$\ln(N_h) = \beta_0^s + \beta_1^s q_h + \gamma_M^s + \varepsilon_h^s$$

- N_h measures market size for hospital h
- γ_M^s are market FE
- q_h is measure of hospital quality
- Goal: Is $\beta_1^s > 0$ or not. $\beta_1^s < 0$ is usually only Soviet countries or 1970's steel.
- $\beta_1^s > 0$ means allocation towards productive firms (or just returns to scale?)

Healthcare Exceptionalism: Dynamic Reallocation

$$\Delta_h = \beta_0^d + \beta_1^d q_h + \gamma_M^d + \varepsilon_h^d$$

$$\Delta_h = \frac{N_{h,2010} - N_{h,2008}}{\frac{1}{2} (N_{h,2010} + N_{h,2008})}$$

- $\beta_1^d > 0$ means growth towards productive firms (not just returns to scale)
- Same idea but now we capture **dynamics**.
- Patients may still be attracted to **unobservables correlated with quality**.

TABLE 1—STATIC AND DYNAMIC ALLOCATION METRICS ACROSS CONDITIONS

Condition:	AMI (1)	Heart failure (2)	Pneumonia (3)	Hip/knee (4)
<i>Panel A. Composition of all Medicare discharges in 2008</i>				
Number of patients in 2008	263,485	545,363	475,756	350,536
Share through emergency dept.	0.71	0.76	0.76	0.02
Share of all Medicare discharges	0.03	0.06	0.05	0.04
Share of Medicare hospital spending	0.04	0.05	0.04	0.05
Number of hospitals in 2008	4,257	4,547	4,607	3,297
<i>Panel B. Static allocation: patients in 2008</i>				
Patients (index events)	190,189	308,122	354,319	267,557
Average number of patients per hospital	65.8	76.6	81.9	101.7
SD of patients per hospital	67.6	78.2	70.8	118.0
Hospitals	2,890	4,023	4,325	2,632
Average number of hospitals per market	9.4	13.1	14.1	8.6
<i>Panel C. Dynamic allocation: growth in patients from 2008 to 2010</i>				
Average growth rate across hospitals	−0.17	−0.10	−0.13	−0.03
SD across hospitals	0.42	0.38	0.36	0.46
Hospitals	2,890	4,023	4,325	2,632

Notes: Panel A is calculated on a 100 percent sample of age 65 and older fee-for-service Medicare patients in 2008 and counts all patients with the condition, not just the index events that are the subject of the remainder of this study and panels B and C. The sample in panels B and C is all hospitals that had at least 1 index admission in 2008 for the condition shown in the column heading and had a valid risk-adjusted survival rate for that condition (risk-adjusted readmission for hip/knee replacement). There are 306 hospital markets, called Hospital Referral Regions (HRRs). Growth is calculated based on the formula in equation (3) that restricts values to between −2 and 2.

TABLE 2—SUMMARY STATISTICS ON QUALITY METRICS ACROSS CONDITIONS

Condition:	AMI (1)	Heart failure (2)	Pneumonia (3)	Hip/knee (4)
<i>Panel A. Risk-adjusted survival rates (30 days): patients in 2006–2008</i>				
Average 30-day survival rate	0.82	0.89	0.88	
SD of risk-adjusted measure	(0.03)	(0.02)	(0.02)	
Hospitals in risk-adjusted measure	2,890	4,023	4,325	
<i>Panel B. Risk-adjusted readmission rates (30 days): patients in 2006–2008</i>				
Average 30-day readmission rate	0.21	0.21	0.16	0.06
SD of risk-adjusted measure	(0.03)	(0.02)	(0.02)	(0.02)
Hospitals in risk-adjusted measure	2,322	3,904	4,264	2,632
<i>Panel C. Processes of care: shares of patients receiving appropriate treatments in 2006–2008</i>				
Average score	0.93	0.83	0.88	
SD	(0.05)	(0.14)	(0.07)	
Hospitals	2,398	3,666	3,920	
Average number of processes reported	4.40	3.30	6.22	
<i>Panel D. Patient survey: survey covers all patients in 2008 (not limited to particular condition)</i>				
Average overall rating (1–3, higher is better)	2.53	2.53	2.53	2.53
SD	(0.14)	(0.14)	(0.14)	(0.14)
Hospitals	3,498	3,598	3,610	3,061

Notes: Sample restrictions are specific to the condition and quality metric; see text for more details of the metric definitions and sample restrictions. Summary statistics are reported across hospitals. In panels A and B, the standard deviations are of the risk-adjusted measures and are empirical-Bayes-adjusted to account for measurement error (see online Appendix Section C.3.1). In panel D, the number of hospitals differs across conditions even though the patient survey metric is not condition-specific because we calculate the ratings on the subset of hospitals that reported at least one patient with the condition in 2008.

TABLE 3—CORRELATION OF QUALITY METRICS WITHIN CONDITION

Metric	AMI				HF			
	Risk-adj survival (1)	Risk-adj readm (2)	Process of care Z (3)	Patient survey Z (4)	Risk-adj survival (5)	Risk-adj readm (6)	Process of care Z (7)	Patient survey Z (8)
Risk-adjusted survival	1.00 [2,890]				1.00 [4,023]			
Risk-adjusted readmission	0.03 [2,322]	1.00 [2,322]			0.35 [3,904]	1.00 [3,904]		
Process of care Z-score	0.24 [2,346]	−0.25 [2,214]	1.00 [2,398]		0.17 [3,607]	−0.15 [3,578]	1.00 [3,666]	
Patient survey Z-score	−0.06 [2,799]	−0.26 [2,293]	0.18 [2,370]	1.00 [3,498]	−0.18 [3,447]	−0.36 [3,398]	0.01 [3,392]	1.00 [3,598]
Metric	Pneumonia				Hip/knee replacement			
	Risk-adj survival	Risk-adj readm	Process of care Z	Patient survey Z	Risk-adj survival	Risk-adj readm	Process of care Z	Patient survey Z
Risk-adjusted survival	1.00 [4,325]							
Risk-adjusted readmission	0.08 [4,264]	1.00 [4,264]				1.00 [2,632]		
Process of care Z-score	0.08 [3,871]	−0.18 [3,847]	1.00 [3,920]					
Patient survey Z-score	−0.03 [3,527]	−0.36 [3,503]	0.18 [3,512]	1.00 [3,610]		−0.23 [2,542]		1.00 [3,061]

Notes: Hospitals used to calculate correlation in brackets. All quality metrics are condition-specific except the patient survey, which is only available as an all-patient average. Correlations involving risk-adjusted survival and

TABLE 4—ALLOCATION ACROSS CONDITIONS

Measure/condition	Static allocation				Dynamic allocation			
	AMI (1)	HF (2)	Pneu (3)	Hip/knee (4)	AMI (5)	HF (6)	Pneu (7)	Hip/knee (8)
<i>Risk-adjusted survival</i>								
Coef. on survival rate	17.496 (0.995)	15.360 (1.320)	5.140 (0.777)		1.533 (0.379)	0.774 (0.501)	1.220 (0.354)	
Hospitals	2,890	4,023	4,325		2,890	4,023	4,325	
<i>Risk-adjusted readmission</i>								
Coef. on readmission rate	−9.162 (1.621)	−10.346 (1.782)	0.499 (1.575)	−21.037 (2.027)	−1.428 (0.611)	−2.300 (0.651)	−1.138 (0.679)	−1.112 (0.836)
Hospitals	2,322	3,904	4,264	2,632	2,322	3,904	4,264	2,632
<i>Process of care Z-score</i>								
Coef. on process Z-score	0.319 (0.026)	0.332 (0.016)	0.211 (0.015)		0.048 (0.010)	0.043 (0.009)	0.026 (0.009)	
Hospitals	2,398	3,666	3,920		2,398	3,666	3,920	
<i>Patient survey Z-score</i>								
Coef. on survey Z-score	−0.321 (0.052)	−0.252 (0.038)	−0.210 (0.030)	0.057 (0.051)	−0.065 (0.015)	−0.003 (0.011)	0.007 (0.011)	0.037 (0.022)
Hospitals	3,498	3,598	3,610	3,061	3,498	3,598	3,610	3,061

Notes: The static allocation results are estimated using equation (1), a hospital-level regression of log-patients in 2008 on market fixed effects and the quality measure named in the row. The dynamic allocation results are estimated using equation (2), which is an identical regression except for the dependent variable, which is now growth in patients from 2008 to 2010. Growth is defined as in equation (3). Standard errors are bootstrapped with 300 replications and are clustered at the market level. Risk-adjusted survival and readmission are reported in percentage points (e.g., a value of 0.1 is 10 percentage points); process of care and patient survey metrics are reported in standard deviation units (e.g., a value of 1 is one standard deviation).

Healthcare Exceptionalism: Production Function

Hospital Production Function:

$$y_p^s = a_h + \sum_k \lambda_k r_{pk} + \mu x_p + \xi_p$$

- a_h is **hospital productivity** (a FE) and variable of interest
- y_p is a patient outcome (survival-days, etc.)
- x_p are (log) hospital inputs
- r_{pk} are patient risk factors.
- This has interpretation as a **production function**. Why?

TABLE 7—ALLOCATION OF AMI WITH RESPECT TO AMI PRODUCTIVITY AND ITS COMPONENTS

Measure	Static allocation for AMI				Dynamic allocation for AMI			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Productivity (fed \$)	17.637 (1.118)				1.491 (0.420)			
Productivity (resources)		17.540 (1.013)				1.471 (0.386)		
Risk-adjusted ln(fed \$)			1.447 (0.169)				0.246 (0.064)	
Risk-adjusted ln(resources)				0.620 (0.406)				0.468 (0.162)
Risk-adjusted survival			17.940 (1.192)	19.789 (1.297)			1.479 (0.446)	1.559 (0.441)
Hospitals	2,890	2,890	2,890	2,890	2,890	2,890	2,890	2,890

Notes: This table extends the analysis of Table 4 but is limited to static and dynamic allocation for AMI. It shows how allocation is related to AMI productivity or its two components (risk-adjusted survival and risk-adjusted log inputs). Productivity is defined as risk- and inputs-adjusted survival; see Section IIIC and equation (9). We consider two input measures, “federal expenditures” and “resources,” also defined in the text. Standard errors are bootstrapped with 300 replications and are clustered at the market level. The standard deviation of productivity is 0.03 (Fed \$ or Resources), of risk-adjusted log-inputs is 0.22 (Fed \$) and 0.07 (Resources), and of risk-adjusted survival is 0.04—this number differs from that of Table 2 because it comes from estimating the joint distribution of survival and inputs, not survival alone.

Healthcare Exceptionalism: EB Adjustment

Table A2 - Sensitivity of Allocation Results to Empirical Bayes Adjustment

Condition	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Static Allocation				Dynamic Allocation			
	AMI	HF	Pneu	Hip/Knee	AMI	HF	Pneu	Hip/Knee
<i>Panel A - Risk-Adjusted Survival</i>								
Baseline (EB-Adjusted)	17.496	15.360	5.140		1.533	0.774	1.220	
	(0.995)	(1.320)	(0.777)		(0.379)	(0.501)	(0.354)	
Raw (No EB Adjustment)	6.833	3.761	1.957		0.645	0.084	0.340	
	(0.342)	(0.425)	(0.403)		(0.175)	(0.199)	(0.175)	
Hospitals	2,890	4,023	4,325		2,890	4,023	4,325	
Raw SD / Corrected SD	1.597	1.788	1.547		1.597	1.788	1.547	
<i>Panel B - Risk-Adjusted Readmission</i>								
Baseline (EB-Adjusted)	-9.162	-10.346	0.499	-21.037	-1.428	-2.300	-1.138	-1.112
	(1.621)	(1.782)	(1.575)	(2.027)	(0.611)	(0.651)	(0.679)	(0.836)
Raw (No EB Adjustment)	-1.699	-1.043	0.755	-6.492	-0.307	-0.217	-0.189	-0.431
	(0.395)	(0.346)	(0.427)	(0.727)	(0.197)	(0.162)	(0.195)	(0.382)
Hospitals	2,322	3,904	4,264	2,632	2,322	3,904	4,264	2,632
Raw SD / Corrected SD	1.870	2.132	1.864	1.794	1.870	2.132	1.864	1.794

This table shows the sensitivity of the allocation results of Table 4 to the empirical Bayes adjustment procedure. In each panel, we first repeat the baseline allocation results in which the quality metric is empirical-Bayes-adjusted. We then show the same allocation models using the raw quality metric without empirical Bayes adjustment. Lastly, we show the ratio of the raw standard deviation of the quality measure to its standard deviation after correcting for measurement error (see Appendix Section C.3.1). Standard errors are bootstrapped with 300 replications and are clustered at the market level.

Thanks!
