Microeconomics II - Home Assignment 1

October 2023

Home assignments are supposed to be handed out via SIS UK in .pdf format until 29.10.2023 23:59. Late submission means 0 points. File naming convention: HA1_Surname_Firstname.pdf. If you have any questions regarding the assignment, do not hesitate to email me at diana.kmetkova@fsv.cuni.cz.

1. (1p.) Suppose a firm produces widgets using labour (L) and capital (K) as inputs. The production function is given as:

$$Y = f(L, K) = 3L^{\frac{1}{2}}K^{\frac{1}{2}}$$

   (a) Calculate the marginal product of labour (MP_L) and the marginal product of capital (MP_K) when the firm employs 16 units of labour and 25 units of capital.

   (b) Explain how changes in labour and capital affect the output.

2. (1p.) For the following production function $f(x_1, x_2) = \sqrt[3]{9x_1 + 12x_2}$, calculate:

   (a) marginal product of factors 1 and 2 (MP_1, MP_2).

   (b) technical rate of substitution (TRS).

   (c) elasticity of returns to scale e_y^s. Are they increasing, decreasing or constant?

3. (1p.) Consider a firm with the following production function:

$$y = f(x_1, x_2) = (x_1 + 4)^{1/2} + (x_2 + 4)^{1/2} - 4.$$

   (a) Derive the profit function $\pi(w_1, w_2, p)$.

   (b) Using the profit function, derive factor demand functions $x_1(w_1, w_2, p)$ and $x_2(w_1, w_2, p)$.

   (c) Using factor demand functions, express the cost function $C(w_1, w_2, p)$.

4. (1.5p.) A firm produces bicycles using labour (L) and steel (S) as inputs. The firm's production function is given as: $Y = f(L, S) = 2L^{\frac{1}{2}}S^{\frac{1}{2}}$.

   (a) Calculate the slope of an isoquant for a general bundle of inputs (L, S) and the elasticity of returns to scale.

   (b) Derive conditional demands $L(w_L, w_S, Y)$ and $S(w_L, w_S, Y)$.

   (c) Using conditional demands, express the cost function $C(w_L, w_S, Y)$.

   (d) Calculate the cost-minimizing combination of labour and steel required to produce 72 bicycles while minimizing the cost of production. The cost of labour is \$16 per hour, and the cost of steel is \$9 per unit.

5. (0.5p.) Imagine Martha, the owner of a bakery called Martha's Delights, who is now concerned about her flour and sugar supplies. She adjusts her ingredient usage based on their prices. According to data from her kitchen staff, when the price of sugar was \$6 per pound and the price of flour was \$3 per pound, they used 10 pounds of sugar and 20 pounds of flour for each batch of cupcakes. When the price of sugar increased to \$9 per pound while flour remained at \$3 per pound, they used 25 pounds of flour and 5 pounds of sugar for each batch. Lastly, when the price of flour jumped to \$5 per pound, and the price of sugar decreased to \$5 per pound, they used 15 pounds of sugar and 20 pounds of flour for each batch.

(a) How much money did Martha spend per batch of cupcakes when the prices were (p_{f1},p_{s1})=(3,6)? How much would it be if the prices were (p_{f2},p_{s2})=(3, 9) and (p_{f3},p_{s3})=(5, 5)?

(b) Create a graph to illustrate these three ingredient combinations and corresponding isocost lines.

Solution

1. To calculate the marginal product of labour (MP_L) and the marginal product of capital (MP_K) when the firm employs 16 units of labour and 25 units of capital, we first need to find the partial derivatives of the production function $Y = 3L^{\frac{1}{2}}K^{\frac{1}{2}}$.

$$MP_L = \frac{\partial Y}{\partial L} = \frac{3}{2}L^{-\frac{1}{2}}K^{\frac{1}{2}}$$
$$MP_K = \frac{\partial Y}{\partial K} = \frac{3}{2}L^{\frac{1}{2}}K^{-\frac{1}{2}}$$

When the firm employs 16 units of labour and 25 units of capital:

$$MP_L = \frac{3}{2} \times 16^{-\frac{1}{2}} \times 25^{\frac{1}{2}} = 1.5 \times 0.25 \times 5 = 1.875$$
$$MP_K = \frac{3}{2} \times 16^{\frac{1}{2}} \times 25^{-\frac{1}{2}} = 1.5 \times 4 \times 0.2 = 1.2$$

Changes in labour and capital:

- MP_L is positive, so increasing L would result in an increase in Y. At the same time, if more labour is hired while keeping capital constant, MP_L would decrease so this positive effect gets smaller and smaller.
- MP_K is positive, so increasing K would result in an increase in Y. At the same time, if more capital is hired while keeping labour constant, MP_K would decrease so this positive effect gets smaller and smaller.

2. (1p.) For the following production function $f(x_1, x_2) = \sqrt[3]{9x_1 + 12x_2}$, calculate:

(a)

$$MP_1 = \frac{\partial f(x_1, x_2)}{\partial x_1} = \frac{1}{3}(9x_1 + 12x_2)^{-2/3} \times 9 = 3(9x_1 + 12x_2)^{-2/3}$$
$$MP_2 = \frac{\partial f(x_1, x_2)}{\partial x_2} = \frac{1}{3}(9x_1 + 12x_2)^{-2/3} \times 12 = 4(9x_1 + 12x_2)^{-2/3}$$

(b) technical rate of substitution (TRS):

$$TRS = -\frac{MP_1}{MP_2} = -\frac{3(9x_1 + 12x_2)^{-2/3}}{4(9x_1 + 12x_2)^{-2/3}} = -\frac{3}{4}$$

(c) elasticity of returns to scale e_y^s. Are they increasing, decreasing or constant?

$$e_y^s = \frac{\partial f(sx)}{\partial s}\frac{s}{f(sx)}$$

$$\frac{\partial f(sx)}{\partial s} = \frac{1}{3}(9sx_1 + 12sx_2)^{-2/3}(9x_1 + 12x_2) = \frac{1}{3}s^{-2/3}(9x_1 + 12x_2)^{1/3}$$

$$e_y^s = \frac{1}{3}s^{-2/3}(9x_1 + 12x_2)^{1/3}\frac{s}{(9sx_1 + 12sx_2)^{1/3}} = \frac{1}{3} < 1$$

Decreasing returns to scale.

3. Consider a firm with the following production function:

$$y = f(x_1, x_2) = (x_1 + 2)^{1/2} + (x_2 - 1)^{1/2} - 4.$$

   (a) The profit function π is the difference between total revenues and total costs.

$$\pi(y) = TR(y) - TC(y)$$

   We will express it as a function of prices of inputs and output.

$$\pi(w_1, w_2, p) = p[(x_1 + 4)^{1/2} + (x_2 + 4)^{1/2} - 4] - w_1 x_1 - w_2 x_2$$

   (b) To derive the factor demand functions, we will take the first derivative of the profit function with respect to x_1 and x_2 and set it equal to 0.

$$\frac{\partial \pi}{\partial x_1} = \frac{1}{2} p(x_1 + 4)^{-1/2} - w_1 = 0 \rightarrow x_1(w_1, w_2, p) = (\frac{p}{2w_1})^2 - 4$$

$$\frac{\partial \pi}{\partial x_2} = \frac{1}{2} p(x_2 + 4)^{-1/2} - w_2 = 0 \rightarrow x_2(w_1, w_2, p) = (\frac{p}{2w_2})^2 - 4$$

   (c) Using factor demand functions, we will express the cost function $C(w_1, w_2, p)$.

$$C(w_1, w_2, p) = w_1 x_1(w_1, w_2, p) + w_2 x_2(w_1, w_2, p) = w_1[(\frac{p}{2w_1})^2 - 4] + w_2[(\frac{p}{2w_2})^2 - 4]$$

$$C(w_1, w_2, p) = -4(w_1 + w_2) + w_1(\frac{p}{2w_1})^2 + w_2(\frac{p}{2w_2})^2 = -4(w_1 + w_2) + \frac{p^2}{4}(\frac{1}{w_1} + \frac{1}{w_2})$$

$$C(w_1, w_2, p) = -4(w_1 + w_2) + \frac{p^2}{4w_1 w_2}(w_2 + w_1) = (w_1 + w_2)[\frac{p^2}{4w_1 w_2} - 4]$$

4. (a) The slope of an isoquant can be calculated using technical rate of substitution between inputs L, S.

$$MP_L = \frac{\partial f(L,S)}{\partial L} = \frac{S^{1/2}}{L^{1/2}}$$

$$MP_S = \frac{\partial f(L,S)}{\partial L} = \frac{L^{1/2}}{S^{1/2}}$$

$$TRS = -\frac{MP_1}{MP_2} = -\frac{(\frac{S}{L})^{1/2}}{(\frac{L}{S})^{1/2}} = -\frac{S}{L}$$

   The slope of an isoquant is therefore $-\frac{S}{L}$.

$$e_y^s = \frac{\partial f(sL, sS)}{\partial s} \frac{s}{f(sL, sS)}$$

$$f(sL, sS) = 2(sL)^{1/2}(sS)^{1/2} = 2s(LS)^{1/2}$$

$$\frac{\partial f(sL, sS)}{\partial s} = 2(LS)^{1/2}$$

$$e_y^s = 2(LS)^{1/2} \frac{s}{2s(LS)^{1/2}} = 1$$

   Constant returns to scale.

(b) Derive conditional demands $L(w_L, w_S, Y)$ and $S(w_L, w_S, Y)$. In the optimum:

$$\frac{w_L}{w_S} = \frac{MP_L}{MP_S} = \frac{\frac{S^{1/2}}{L^{1/2}}}{\frac{L^{1/2}}{S^{1/2}}} = \frac{S}{L}$$

$$S = \frac{w_L}{w_S} L$$

We will plug in S into the production function:

$$Y = 2(LS)^{1/2} = 2L^{1/2}(\frac{w_L}{w_S}L)^{1/2} = 2L(\frac{w_L}{w_S})^{1/2}$$

We can express the conditional demand for L and S:

$$L^\star = \frac{Y}{2}(\frac{w_S}{w_L})^{1/2}$$

$$S^\star = \frac{Y}{2}(\frac{w_L}{w_S})^{1/2}$$

(c) Using conditional demands, express the cost function $C(w_L, w_S, Y)$.

$$C(w_L, w_S, Y) = w_L L^\star + w_S S^\star = Y(w_L w_S)^{1/2}$$

(d) Calculate the cost-minimizing combination of labour and steel required to produce 72 bicycles while minimizing the cost of production. The cost of labour is \$16 per hour, and the cost of steel is \$9 per unit.

After having derived conditional demands for L and S, we will plug in w_L and w_S:

$$\frac{w_L}{w_S} = \frac{MP_L}{MP_S} = \frac{S}{L}$$

$$S = \frac{w_L}{w_S} L = \frac{16}{9} L$$

We will plug in the conditional demand of S into the production function:

$$Y = 2(LS)^{1/2} = 2L^{1/2}(\frac{16}{9}L)^{1/2} = 2L\frac{4}{3} = \frac{8}{3}L$$

We want to produce 72 bicycles, hence:

$$72 = \frac{8}{3}L \to L = \frac{3}{8}72 = 27$$

$$S = \frac{16}{9}27 = 48$$

5. (a)

$$TC_1 = 3 * 20 + 6 * 10 = 120$$

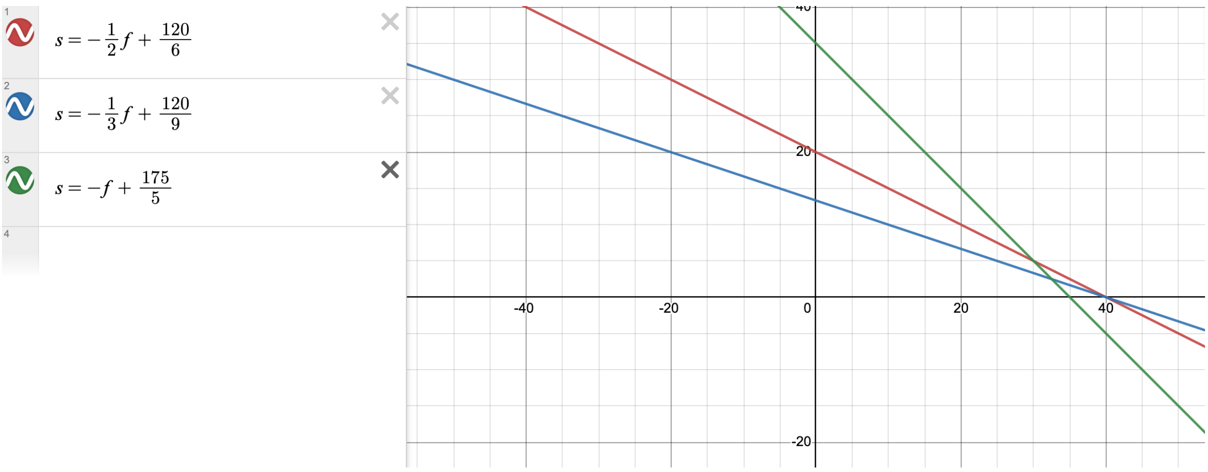
$$TC_2 = 3 * 25 + 9 * 5 = 120$$

$$TC_3 = 5 * 20 + 5 * 15 = 175$$

(b) Graphical solution to express isocost lines:

$$S = \frac{TC}{p_S} - \frac{p_F}{p_S} F$$

Hence:


1
$s = -\frac{1}{2}f + \frac{120}{6}$
2
$s = -\frac{1}{3}f + \frac{120}{9}$
3
$s = -f + \frac{175}{5}$
4
-40
-20
0
20
40
20
-20