4th Group Assignment

Course director: Zoltán Rácz
5329: Inequality, Household Behavior, and the Macroeconomy

Instructions

- This assignment should be completed in groups of 2-3 students, and handed in through Canvas before 10 a.m. on the 20th of May.
- This assignment contributes 10% to your final grade.
- You can hand in either an ipynb or a jl, and a pdf file. For full marks, you are expected to carefully motivate your answers. Codes should be easy to follow and well-commented. The connection of verbal answers with corresponding codes (when relevant) should be made clear, especially if they are in separate files.

Tasks

1 Taxation in Cagetti & De Nardi (2006)

Before starting this exercise, make sure that you downloaded the updated version of 50_cagettidenardi.jl, 50_cagettidenardi_solve.jl and you looked at the extra slides on market clearing from the same class.

Consider a modification of Cagetti & De Nardi (2006) with a richer tax system, as follows:

- We keep the labor income tax from the original version of the model, but will denote it with τ_l instead of τ.
- Everyone with beginning-of-period assets a pays an $\tau_w \cdot a$ amount of wealth tax.
- A capital income tax with rate τ_k is applied on capital income. The latter is defined as
  - $r \cdot a$ for non-entrepreneurs, and

– $\theta k^{\nu} - \delta k - r(k-a)$ for entrepreneurs.

- Everyone receives b amount of benefits.
- As in the original model, retired people receive p amount of pension, where p is a fixed fraction of w. This is in addition to b.
- The government balances the budget: Total revenues from the labor income, capital income and wealth taxes should exactly cover expenses on benefits and pension.

Make a copy of 50_cagettidenardi_solve.jl and rename it. Next, in this file:

1. Modify function *solve* in a way that it correctly solves for the optimal policy and value functions of all agents, given this richer tax system.
   (a) In addition to r and τ_l, also make τ_k, τ_w and b (all *Real*s) function arguments to the function *solve*. We don't put these in EconPars as independent parameters, since we want to solve for the levels balancing the budget (or in the case of r, clear the capital market).
   (b) Next, amend the budget constraints appropriately for all possible states (old/young, entrepreneur or not). Therefore update all lines computing cash-on-hand (4 such places). [1]
2. Modify GE_difference such that input x is interpreted as a 5-element vector with elements r, τ_l, τ_k, τ_w and b (instead of just the first two elements). Within the function, compute
   - the total amount of benefits paid out
   - total tax revenues from capital and wealth taxes

   and take into account the corresponding terms in the line defining budget_balance_relative. It should still give the budget surplus per person, divided by w. The output of the GE_difference should be left unchanged: the 2-element vector it returns will help us pin down r and one free taxation parameter at a time.

We will work with four setups. The parameters from 50_cagettidenardi.jl serve as a benchmark. In the other settings, the government finances $b = 0.05$ (approximately 1% of average labor earnings) via different taxes. More concretely:

- Benchmark: r and τ_l as defined in *calib* in 50_cagettidenardi.jl. (These parameters clear the capital market and balance the government budget). The new taxes and b should all be 0s.

[1] Unlike in Assignment 2, no need to worry about first-order conditions, since we use value function iteration to solve the model. Also, no need to update the simulation function this time (since our state variable is a, not *coh*).

- Benefit financed by a higher labor income tax: $b = 0.05$, while the new taxes are still 0. r and τ_l are re-estimated to clear the capital market and balance the government budget.
- Benefit financed by capital income tax: $b = 0.05$, and τ_l is as in the benchmark. $\tau_w = 0$, while the r and τ_k are re-estimated to clear the capital market and balance the government budget.
- Benefit financed by wealth income tax: $b = 0.05$, and τ_l is as in the benchmark. $\tau_k = 0$, while the r and τ_w are re-estimated to clear the capital market and balance the government budget.

3. Calibrate the appropriate parameters in all three new settings. Some hints:
   (a) Use nested_find_zero as in line 21 of 50_cagettidenardi.jl.
   (b) To do so, you always need bracketing intervals for relevant parameters. For me, $(0.06001, 0.07)$ worked for r, $(0.08, 0.15)$ worked for τ_l, $(0.0, 0.1)$ worked for τ_k, $(0.0, 0.01)$ worked for τ_w.
   (c) We always calibrate two parameters at once (r and a tax) since we have two conditions (capital market and government budget). To write GE_difference as a function of only two elements of x, while keeping the rest (and the other inputs fixed), one can use an anonymous function such as
   $$y- > GE\_difference([42, y[1], 21, y[2], 6], otherinputs)$$
   as an input of nested_find_zero. Here, y is interpreted as a length two vector, whose elements are used in the positions of your choice of x, while for the other elements of x you can give values you would like.
   (d) On my laptop, calibration took around 10 minutes, for each setting.
4. After having the right calibrated parameters, solve and simulate all settings.
5. Write a function, that takes a Solution structure and matrices containing simulated $olds$, $\theta$is, yis and as as inputs, and gives a correct matrix of simulated value functions as an output, with the same dimensions as the other simulated matrices. (It might be helpful to take a look at how consumption is simulated within simulate_rest). Simulate values in each setting, using the corresponding inputs. Compute the average of simulated values in each setting separately.[2]
6. Compare the benchmark with the case with a benefit financed by higher labor income tax.

[2]For any calculations based on simulations, use only simulated figures from the last period.

- Which one gives higher average welfare? How big is the difference expressed in consumption equivalents?
- In class we have seen that a labor income tax is distortive when leisure provides utility. Is there a reason for it to be distortive in this model? If yes, what should be the effect on welfare?
- Are there other channels at work that create a welfare difference between these two settings?

7. How does welfare in the capital income and wealth tax settings relate to the first two settings? What could be the reason for this difference?

8. Now let's compare the welfare effects across the settings with capital income tax and wealth tax.

   - Which one leads to higher average welfare? Is the difference large?
   - Are the results you find in line with Guvenen at al (2023)?
     - If yes, what common features of the two models drive this finding?
     - If not, what differences between the two models might be responsible?

   In either case, you might find it helpful to think through how returns[3] vary across agents in the two models, and how the entrepreneurial sector is linked to wage earners. (No need to know the technical details of Guvenen et al, what I said in class is sufficient to answer this question.)

2 MPCs in a lifecycle model

In this task you'll learn about the determinants of MPCs in a reasonable calibrated lifecycle consumption saving model. Make available the following files from lecture 41: 41_Inequality_inheritence.jl, 41_inheritance_solve.jl, age_fe.csv and mortality.csv.

Copy the necessary lines from 41_Inequality_inheritence.jl to solve the models 'nolinks' and 'bequest' with the same parameters as in the original file. Create a grid of cash-on-hand from 0 to 20.

1. Plot the optimal consumption function from model 'nolinks' of age 22 people in middle (6th) α and z states. (If you forgot what is in the Solution structure, see the description in 41_inheritance_solve.jl, around line 117) Explain the intuition behind the shape of the consumption policy.

[3] In the model, you can define return as capital income divided by beginning-of-period wealth.

2. Keep age fixed, but vary α and z, and compare with your result from the previous point. How do these income states affect the level and steepness of the consumption policy functions? Is there a difference between how α and z affects the shape of the consumption policy? (Compare what happens with the lowest α and middle z state against the lowest z with middle α level.)

3. Go back to middle α and z and vary age. Try $t = \{1, 11, 21, 41, 61\}$. How does the level and steepness of optimal consumption change with age? What is the intuition? (Comparing with the analytically tractable deterministic case might help figure this out.)

4. Repeat the previous point with 'bequest'. (No need to redo point 2. in 'bequest'.) How do your results differ?

5. Compute the average MPCs by age group in 'nolinks'. One method is the following one:

   (a) Run the simulate_shocks function with appropriate inputs

   (b) Run simulate_decisions with appropriate inputs, including outputs from the previous point

   (c) Choose an appropriate small number. Let's call it dif

   (d) Create a three-dimensional array for alternative consumption values, with identical dimensions as the simulated consumption array from the previous point. (Looking inside function simulate_decisions might help.) Use the consumption policy from the Solution of 'nolink' and use the correct simulated $\alpha$is, zis and cohs from the simulation (for each n, m and t), but **add dif to coh** in each case. (If you set $dif = 0$ and replicate the simulated consumption array exactly, you did it right). With a small, positive dif this is the alternative consumption when getting a little bit of extra income.

   (e) Take the difference of the alternative and original simulated consumption arrays and divide the difference with dif. Now you have an array of MPCs.

   (f) Finally, take means by age group (the method $mean(matrix, dims = 1)$ does the job, when applied to the simulated matrix corresponding to the last generation of simulated individuals). Plot your results!

   Try to find the intuition for your findings! What drives the high/low values for different age groups?

6. Repeat the previous point with 'bequest'. How do your results differ?