Day 1: Identification by Design

Peter Hull

Design-Based Regression Inference
Spring 2024

The Design of This Course

- This is a three-day intensive in design-based causal inference
  - Far from comprehensive: will focus on core concepts with regression/IV

  - Emphasis will be on practical lessons for applied work
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- 6-7 hours of lecture, two 30-minute coding demonstrations
  - Please ask questions in the Discord chat!
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  - Code assignments will be "take home," w/solutions the following class

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- Feedback/follow-up: *peter_hull@brown.edu*

Course Schedule

Monday 4/22	6:00-7:50pm	Lecture 1: Selection-on-Observables
	6:50-7:00pm	*Break*
	7:00-7:50pm	Lecture 2: Design vs. Outcome Models
	7:50-8:00pm	*Break*
	8:00-8:50pm	Lecture 3: Design-Based IV
	8:50-9:00pm	Application 1 Overview
Wednesday 4/24	6:00-6:30pm	Live-Coding Application 1
	6:30-6:40pm	*Break*
	6:40-7:40pm	Lecture 4: Negative Weights
	7:40-7:50pm	*Break*
	7:50-8:50pm	Lecture 5: Clustering
	8:50-9:00pm	Application 2 Overview
Friday 4/26	6:00-6:30pm	Live-Coding Application 2
	6:30-6:40pm	*Break*
	6:40-7:40pm	Lecture 6: Recentering
	7:40-7:50pm	*Break*
	7:50-9:00pm	Lecture 7: Nonlinear Models

The Design *in* This Course

- Design-based methods use knowledge on the assignment process of as-if-randomly assigned shocks to estimate causal effects
  - Mimic analysis of "true" experiments, w/known randomization protocol
  - Contrasts with identification strategies that model untreated potential outcomes (e.g. parallel trends) without appealing to randomization

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  - Clear(er) role of nonlinear/structural models as extrapolation devices
- We'll get into all of this over the next few days, building up slowly...

Outline

1. Selection on Observables
2. Design vs. Outcome Models
3. Design-Based IV

The Simplest Experimental Story

- Throughout today, we'll consider the goal of estimating parameter β in the constant-effects causal model

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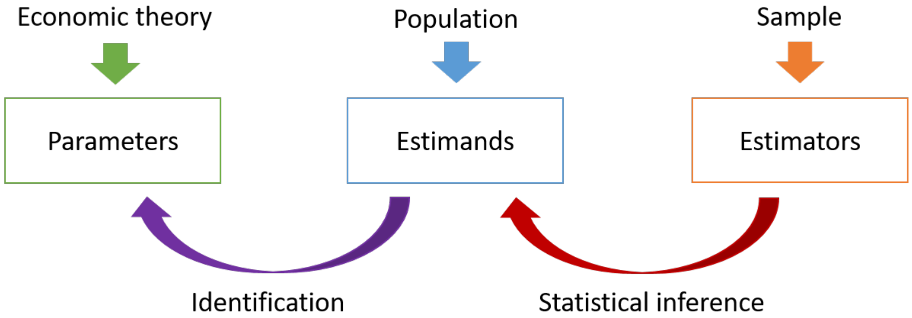
  $\Rightarrow$ We can estimate β by a sample (OLS) regression of y_i on x_i

Econometrics: The "Big Picture"



Always good to remember which part of the diagram you're working on!

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- Consider regressing y_i on x_i, controlling for strata FE, $w_{ik} = \mathbf{1}[w_i = k]$
  - By the Frisch-Waugh-Lovell theorem, equivalent to the regression of y_i on $\tilde{x}_i$: the residuals from regressing x_i on the FE

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  - Hence, the regression gives:

$$\frac{Cov(\tilde{x}_i, y_i)}{Var(\tilde{x}_i)} = \frac{Cov(\tilde{x}_i, \beta x_i + \varepsilon_i)}{Var(\tilde{x}_i)} = \beta + \frac{Cov(\tilde{x}_i, \varepsilon_i)}{Var(\tilde{x}_i)}$$

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  - Thus, the strata-controlled regression identifies the parameter β

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  1. Tell a clear *ex ante* story about where the $x_i \mid w_i$ variation comes from and why it is unlikely to be correlated with ε_i
  2. Use *ex post* balance tests to check that x_i is not correlated, conditional on w_i, with other observables that may proxy for ε_i

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- *Ex post* empirical validation:
  - Conditional on the selection controls, x_i appears uncorrelated with other baseline observables (demographics, etc)

Dale and Krueger Estimates (from MHE)

	No Selection Controls			Selection Controls		
	(1)	(2)	(3)	(4)	(5)	(6)
Private School	0.135	0.095	0.086	0.007	0.003	0.013
	(0.055)	(0.052)	(0.034)	(0.038)	(0.039)	(0.025)
Own SAT score/100		0.048	0.016		0.033	0.001
		(0.009)	(0.007)		(0.007)	(0.007)
Predicted log(Parental Income)			0.219			0.190
			(0.022)			(0.023)
Female			-0.403			-0.395
			(0.018)			(0.021)
Black			0.005			-0.040
			(0.041)			(0.042)
Hispanic			0.062			0.032
			(0.072)			(0.070)
Asian			0.170			0.145
			(0.074)			(0.068)
Other/Missing Race			-0.074			-0.079
			(0.157)			(0.156)
High School Top 10 Percent			0.095			0.082
			(0.027)			(0.028)
High School Rank Missing			0.019			0.015
			(0.033)			(0.037)
Athlete			0.123			0.115
			(0.025)			(0.027)
Selection Controls	N	N	N	Y	Y	Y

Notes: Columns (1)-(3) include no selection controls. Columns (4)-(6) include a dummy for each group formed by matching students according to schools at which they were accepted or rejected. Each model is estimated using only observations with Barron's matches for which different students attended both private and public schools. The sample size is 5,583. Standard errors are shown in parentheses.

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3. Design-Based IV

Why are Multi-Way FE Different?

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- We need a different justification for this sort of regression...

Two-Way FE as Outcome Modeling

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- Add in a model for untreated potential outcomes: $E[\varepsilon_{it} \mid w_{it}] = \alpha_i + \tau_t$
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- Logic clearly extends to more than two FEs, time-varying controls, unit-specific trends, or any other model for $E[\varepsilon \mid w]$

Example: Finkelstein (2007)

- Boiling multi-way FE regression specs down to simpler "diff-in-diff' comparisons can make the content of the outcome model clearer
  - Useful fact: in two periods, β in $y_{it} = \beta x_{it} + \alpha_i + \tau_t + \nu_{it}$ is given by the regression of $y_{i,Post} - y_{i,Pre}$ on $x_{i,Post} - x_{i,Pre}$ (and a constant)

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  - Event study version: $y_{it} = \alpha_i + \tau_t + \sum_s \beta_s (1 - x_{i,Pre}) \mathbf{1}[t = s] + \nu_{it}$; expect flat pre/post trends if the model is right...

Finkelstein Event Study

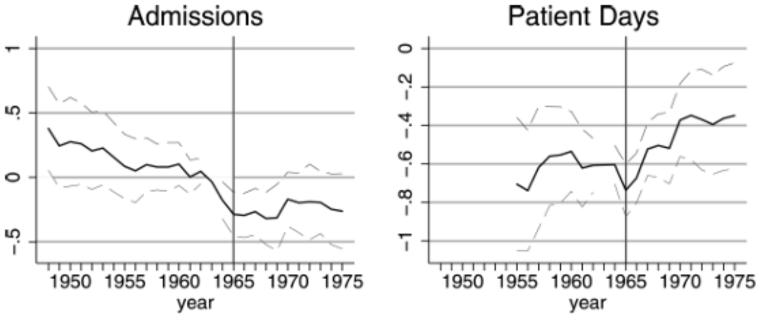

Figure II graphs the pattern of the λ_t coefficients from estimating (1) for the log of the dependent variable given above each graph. The scale of the graph is normalized so that in the reference year (1963) it is the average difference in the dependent variable between the south and west (where Medicare had a larger impact) relative to the north and northeast (where Medicare had a smaller impact). The dashed lines show the 95 percent confidence interval on each coefficient relative to the reference year (1963). Time varying state-level controls (X_{st}) in all analyses consist of eight indicator variables for the number of years before (or since) the implementation of Medicaid in state s (see text for more details).

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- Both strategies have *ex post* validations (balance tests / pre-trend checks), but the *ex ante* case for design is arguably easier to make
  - What ε_{it} model is best? E.g. does parallel trends hold in levels or logs?

Outline

1. Selection on Observables ✓
2. Design vs. Outcome Models ✓
3. Design-Based IV

The Simplest IV Story

- Again start w/constant fx model $y_i = \beta x_i + \varepsilon_i$, now $Cov(x_i, \varepsilon_i) \neq 0$
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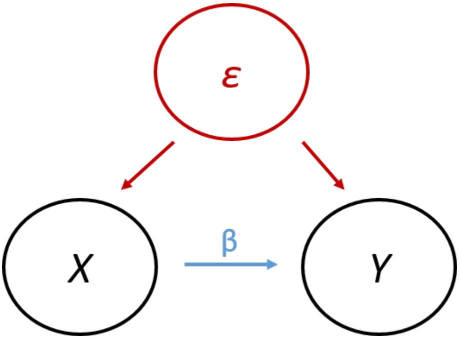
- Plugging in the model for $\varepsilon_i = y_i - \beta x_i$, we have IV identification:

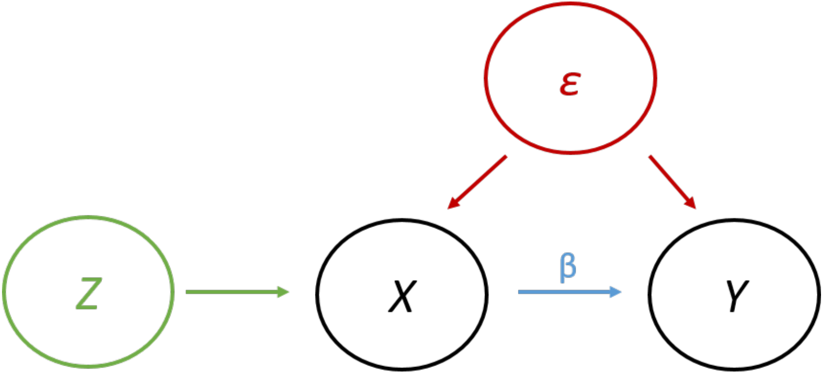
$$Cov(z_i, y_i - \beta x_i) = 0 \implies \frac{Cov(z_i, y_i)}{Cov(z_i, x_i)} = \beta$$

so long as $Cov(z_i, x_i) \neq 0$ ("relevance")

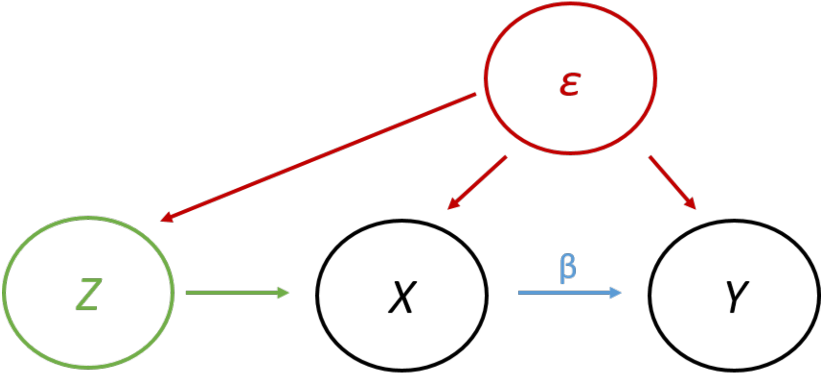
Regression "Endogeneity"



Instrument "Exogeneity" / "Validity"

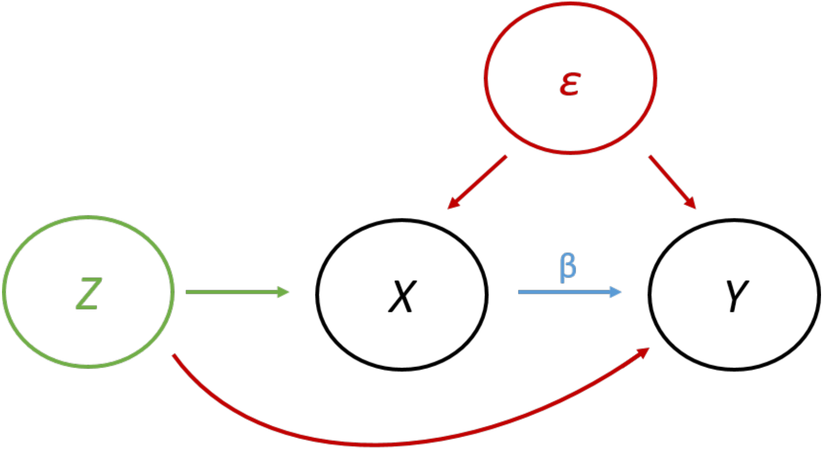


Threats to Validity: Instrument Assignment



We will later formalize this as a failure of instrument "independence"

Threats to Validity: Direct Effects



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Adding Controls and Instruments

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- RF&FS are the nuclei of IV; the design-based approach starts w/them

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- New twist: have to also argue exclusion in order to interpret RF/FS
  - Can both argue *ex ante* and sometimes test *ex post*: e.g. by looking at effects of z_i on other plausible treatment channels

Example: Abdulkadiroglu et al. (2016)

- AAHP are interested in the effect of "takeover" charter schools: ones which convert a low-performing traditional public school (TPS)
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  - Specifically, match each takeover school to a TPS using baseline test score performance and control for match cell fixed effects
- Exclusion: takeovers only affect later test scores via charter enrollment
  - Check whether there are takeover effects in the transition (pre-charter) year 0; develop a strategy to use these effects to relax exclusion

Abdulkadiroglu et al. Results

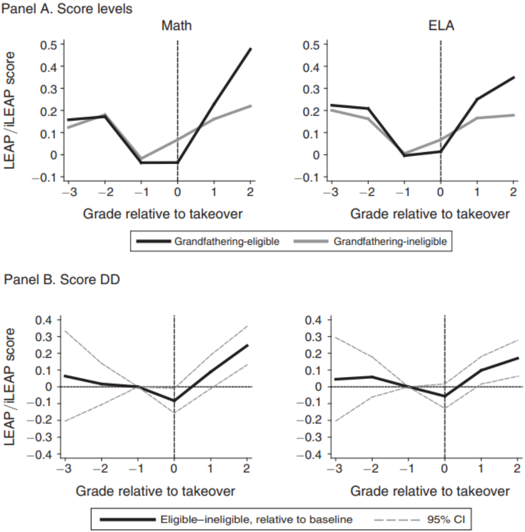

FIGURE 2. TEST SCORES IN THE RSD GRANDFATHERING SAMPLE

Notes: Panel A plots average LEAP/iLEAP math and ELA scores of students in the RSD legacy middle school matched sample. Panel B plots achievement growth relative to the baseline (−1) grade. Estimates in both panels control for matching cell fixed effects. Scores are standardized to those of students at direct-run schools in New Orleans RSD, by grade and year. Grade 0 is the last grade of legacy school enrollment.

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- Before then, you have the chance to play with a real-world application