Inequalities, Household Behavior and the Macroeconomy *(Consumption - Budget contraints)*

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Last lecture

- Learnt how to set up and solve the Lagrangian for deterministic and stochastic optimal consumption/saving problems
- We derived the Euler-equation in each case
- Got explicit solutions for the deterministic case, and the stochastic case with quadratic utility
- Discussed Campbell &Mankiw (1989): they say PIH might hold for a part of the population if the rest is hand-to-mouth. Maybe hand-to-mouth consumers are borrowing constrained.

Today

Today we continue with exploring how borrowing limits change the picture

- Set up Lagrangian and discuss optimality conditions in the deterministic case
- Not the stochastic case, since we would get the same intuition through more complicated notation
- Zeldes(1989)
- How to solve these models on the computer?
- Code

The problem of borrowing constrained agents

We come back to the our previous problem with one small (but important!) difference: We maximize borrowing at b_t:

$$\max_{\{c_t, a_t\}_{t=0}^{T}} E_0 \sum_{t=0}^{T} \beta^t u(c_t)$$

$$\begin{aligned} s.t. \quad a_t &= (1+r)a_{t-1} + y_t - c_t \quad \forall t \in \{0, 1, \ldots, T\} \\ a_T &\geq 0 \\ a_t &\geq -b_t \quad \forall t \\ a_{-1} &\text{ given} \end{aligned}$$

Now, let's set up the Lagrangian for this new problem

The problem of borrowing constrained agents

The Lagrangian:

$$\mathcal{L} = \sum_{t=0}^{T} \beta^t \{ u(c_t) + \lambda_t [a_{t-1}(1+r) + y_t - c_t - a_t] + \mu_t[-a_t - b_t] \}$$

Now the first order conditions are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_t} = 0 &\implies u'(c_t) = \lambda_t \\
\frac{\partial \mathcal{L}}{\partial a_t} = 0 &\implies \lambda_{t+1}\beta(1+r) = \lambda_t - \mu_t \\
\mu_t \geq 0 & \\
\mu_t[a_t + b_t] = 0 &
\end{aligned}$$

The problem of borrowing constrained agents

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\end{aligned}$$

The last two lines are called dual feasibility and complementary slackness conditions. You get these applying the (Karush-)Kuhn-Tucker theorem

Euler-inequality

Instead of an Euler-equation, we get an Euler-inequality:

$$u'(c_t) - \mu_t = \beta(1 + r)u'(c_{t+1})$$

so

$$u'(c_t) \geq \beta(1 + r)u'(c_{t+1})$$

What does this mean?

- marginal utility might be lower in the next period than now
- intuitively this can happen if you want to transfer resources to the current period from the next one, but you can't
- why couldn't you? – borrowing limit!

Complementary slackness

We know that

$$\mu_t[a_t + b_t] = 0$$

Either:

1. $\mu_t = 0$. In this case, we fall back to the model without borrowing constraints, since the constraint on borrowing is at slack (and hence irrelevant). Hence the Euler-equation holds and

$$u'(c_t) = \beta(1 + r)u'(c_{t+1})$$

2. $\mu_t > 0$. Now we must have

$$a_t = -b_t,$$

so the borrowing constraint is binding and you eat everything you can:

$$c_t = a_{t-1}(1 + r) + y_t + b_t$$

Now the Euler-inequality is strict:

$$u'(c_t) > \beta(1 + r)u'(c_{t+1})$$

Testing the borrowing constraints hypothesis

Is it possible to test the hypothesis that consumers optimize subject to borrowing constraints?
Not obvious, since μ is not observable.

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Is it possible to test the hypothesis that consumers optimize subject to borrowing constraints? Not obvious, since μ is not observable.

Yes! Zeldes (1989) does it!

Note that Zeldes (1989) is on your reading list, so read and enjoy it!

Testing the borrowing constraints hypothesis

Zeldes (1989) develops a life-cycle model with borrowing constraints similar to ours

He makes slightly different assumptions and uses a different notiation:

- He has income uncertainty, we do not (nothing changes except plugging in some conditional expectation operators)
- He uses subscripts i for individual variables
- The utility function he considers is $u(c_{i,t}, \Theta_{i,t})$, where Θ_{it} is a preference shifter.
- The shape of the utility function he considers is $u(c_{i,t}, \Theta_{it}) = \frac{c_{i,t}^{1-\alpha}}{1-\alpha} \exp \Theta_{i,t}$
- He calls the discount factor $\frac{1}{1+\delta} = \beta$
- he has stochastic returns, while our r is constant
- The Lagrangian multiplier is denoted by $\lambda''_{i,t}$ and not μ_t like in the previous slides.

Testable version of our Euler ineqaulity

In a stochastic version of our model we would give

$$u'(c_t) - \mu_t = \beta \mathbb{E}_t \left[(1+r) u'(c_{t+1}) \right]$$

Rearrange:

$$1 = \frac{\beta \mathbb{E}_t \left[(1+r) u'(c_{t+1}) \right]}{u'(c_t)} \left(1 + \frac{\mu_t}{\beta \mathbb{E}_t \left[(1+r) u'(c_{t+1}) \right]} \right)$$

or

$$1 = \frac{\beta \mathbb{E}_t \left[(1+r) u'(c_{t+1}) \right]}{u'(c_t)} \left(1 + \lambda'_t \right)$$

by defining

$$\lambda'_t = \frac{\mu_t}{\beta \mathbb{E}_t \left[(1+r) u'(c_{t+1}) \right]}$$

Testable version of our Euler ineqaulity

So we have

$$1 = \frac{\beta \mathbb{E}_t \left[(1+r) u'(c_{t+1}) \right]}{u'(c_t)} \left(1 + \lambda_t' \right),$$

how to make it look like an estimable equation? Terms known at t can be taken inside the conditional expectation operator

$$1 = \mathbb{E}_t \left[\frac{\beta (1+r) u'(c_{t+1})}{u'(c_t)} \left(1 + \lambda_t' \right) \right]$$

But that means

$$\frac{\beta (1+r) u'(c_{t+1})}{u'(c_t)} \left(1 + \lambda_t' \right) = \mathbb{E}_t \left[\frac{\beta (1+r) u'(c_{t+1})}{u'(c_t)} \left(1 + \lambda_t' \right) \right] + e_{t+1} = 1 + e_{t+1}$$

where e_{t+1} is a forecast error, orthogonal to any information available at time t.

Testing the borrowing constraints hypothesis

Like us, Zeldes (1989) derived the Euler equation from the FOCs and obtained

$$\frac{c_{i,t+1}^{-\alpha} \exp\left(\Theta_{i,t+1} - \Theta_{i,t}\right)(1 + r_{i,t})(1 + \lambda'_{i,t})}{c_{i,t}^{-\alpha}(1 + \delta_i)} = 1 + e_{i,t+1}$$

- $\lambda'_{i,t}$ is a rescaled version of the original Lagrangian multiplier $\lambda''_{i,t}$
- $e_{i,t+1}$ is a measurement error uncorrelated with information available in t (note that in Zeldes it has a different meaning, but this is not crucial)

Taking the logs on both sides and rearranging he obtains:

$$\ln(c_{i,t+1}/c_{i,t}) = 1/\alpha[(\Theta_{i,t+1} - \Theta_{i,t}) + \ln(1 + r_{i,t}) - \ln(1 + e_{i,t+1}) + \ln(1 + \lambda'_{i,t+1}) - \ln(1 + \delta_{i,t})]$$

Testing the borrowing constraints hypothesis

The last equation of the previous can be written as follows:

$$\ln(c_{i,t+1}/c_{i,t}) = \mathbf{X}_{i,t}\beta + \gamma \ln(1 + r_{i,t}) + \underbrace{v_{i,t+1} + \ln\left(1 + \lambda_{i,t}\right)}_{=x_{i,t+1}} \quad (1)$$

Where:

$\mathbf{X}_{i,t}$ is an array containing a set of controls, among which age of the head, family fixed effects, time fixed effects, constant term...

- These controls contain individual-specific discount factors and preference shifters

$v_{i,t+1}$ is the rescaled error term (with mean zero)

$x_{i,t+1}$ is the excess growth with respect to the model with no borrowing constraints

$1 + \lambda_{i,t} = (1 + \lambda'_{i,t})^{\frac{1}{\alpha}}$

Testing the borrowing constraints hypothesis

To understand the role of borrowing constraints, Zeldes uses a straightforward strategy

He creates two samples using PSID data:

1. Households with a low ratio of wealth over income
2. Households with a high ratio of wealth over income

The idea is that, if borrowing constraints play some role, the Euler equation should be different in the two samples. Namely, it should be

- a (potentially strict) inequality in sample 1.
- an equality in sample 2.

Hence he performs three tests (i), (ii) and (iii) linked to the estimation of equation (1) in the two samples

Testing the borrowing constraints hypothesis—test (i)

Test: does the log of disposable income y_{it} enters significantly equation (1) while using samples 1 and 2 for the estimation?

The test is useful because:

- Current income y_{it} does not influence consumption growth if the PIH holds
- Current income y_{it} influences negatively consumption growth if borrowing constraints are binding (small current income $\Rightarrow$ low current consumption if cannot borrow $\Rightarrow$ high consumption growth)

Hence, the sign and significance of the estimated parameter liked to y_{it} is very informative!

If the PIH holds, then the parameter linked to y_{it} should be insignificant and small for both samples

If the borrowing constraints story holds, then the parameter linked to y_{it} should be insignificant in sample 2 + significant and negative in sample 1.

Testing the borrowing constraints hypothesis—test (ii)

- Sum of true residuals in equation (1) should be positive for constrained agents (since for them Lagrange multipliers are strictly positive)
- Challenge: this makes equation (1) impossible to estimate directly in constrained sample. If you do, your estimates will be biased.
- Solution: estimate (1) for non-constrained sample and use coefficients from there to obtain 'true' residuals for constrained solution.

So the test follows the steps below:

1. Estimate (1) for group 2 and store the parameter estimates
2. Using the parameter estimates, compute the residuals $\hat{x}_{t+1}$ in sample 1
3. Averaging $\hat{x}_{t+1}$ over the whole population 1, we have $\overline{\hat{x}_{t+1}} = E \ln(1 + \lambda_{it})$
4. If borrowing constraints are important for consumption behavior, then $\overline{\hat{x}_{t+1}}$ should be positive, statistically significant, and quantitatively large.
    - Why? If the borrowing constraint binds, the Lagrangian multiplier should be positive!

Testing the borrowing constraints hypothesis—test (iii)

Idea: When income is larger (all else equal), borrowing constraints should bite less.

Test: regress $\hat{x}_{t+1}$ on y_{it} for group 1 and test whether the sign is negative. Since the true error term $v_{i,t+1}$ should be uncorrelated with $y_{i,t}$, this tells us the sign of the correlation between the (rescaled) Lagrange multiplier and income.

Problem: this is an estimate of the total derivative of λ_{it} wrt y_{it}, not a partial derivative as it should be!

Testing the borrowing constraints hypothesis—Results

Test i: Euler Equation Estimates for Two Subsamples:
Dependent Variable: log $C_{i,t+1}/C_{it}$

Independent Variable	Group 1 (Low W/Y)	Group 2 (High W/Y)
Age of head	−.0084 (−.88)	−.0044 (−1.97)*
Growth in annual food needs t, $t+1$	.25 (8.26)*	.23 (3.97)*
Real after-tax Treasury bill rate t, $t+1$	.37 (.24)	.43 (.31)
Log of real disposable income t	−.071 (−4.40)*	−.039 (−1.49)
Degrees of freedom	9,362 (2,731 families)	4,477 (1,583 families)

Tests on the Group 1 Residuals Constructed Using Group 2 Parameter Estimates

Test ii: Estimate of Average Excess Consumption Growth for Group 1 Due to Binding Constraint

$\bar{\hat{x}}_{it}$	.017 (1.63)

Test iii: Regression of Estimate of Excess Consumption Growth for Group 1 on the Log of Real Disposable Income

y_{it}	−.024 (−1.31)
Degrees of freedom	4,267 (1,114 families)

NOTE.—Equations are estimated with instrumental variables and include time and family fixed effects. t-statistics

Testing the borrowing constraints hypothesis

Can we say that data supports the borrowing constraints theory? Generally yes!

Zeldes estimates are weakly significant in test (ii)

Another problem: In the data, only 10% of agents have zero or negative net worth, but the Campbell and Mankiw (1989) estimates of hand-to-mouth consumers is $>$30%

This last piece of evidence cannot rule out borrowing constraint theory, though:

- Many US households have positive net worth but also have a mortgage
- These households are constrained (rich hand-to-mouth) because household wealth is illiquid! You cannot sell your house when you become unemployed. Kaplan et al. (2014) study this and find that in the US around 20% of the population is a rich hand-to-mouth, while 10% is poor hand-to-mouth.

Dynamic Programming

How to solve models in practice?

- We have seen: solving a stochastic models involves finding an optimal policy for each time in the future, for all possible histories
- This is too complicated
- So we solve models backwards! When deciding, you have to think about how decisions today affect the future, but not the past. Hence starting from the last period instead of the first one is easier.

Bellman's principle of optimality (1957):
"An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision"

Deep property of dynamic optimization problems, not only relevant in economics.

Dynamic Programming

V_t is the value function (total discounted utility for the rest of the time horizon) for an agent of age t.

The problem of agent at period T is obvious

- consume everything
- value function is simply the utility of eating everything

$$V_T(x_T) = u_T(x_T)$$

The agent of age $T-1$ solves

$$V_{T-1}(x_{T-1}) = \max_{c_{T-1},a_{T-1}} E_{T-1}u(c_{T-1}) + \beta V_T(x_T)$$

$$\begin{aligned} s.t. \quad x_T &= (1+r)a_{T-1} + y_T \\ a_{T-1} &= x_{T-1} - c_{T-1} \\ a_{T-1} &\geq -b_{T-1} \end{aligned}$$

- x is cash-on-hand: income plus savings from the previous period. Only this matters!
- Next, we solve the problem at $T-2$, taken as given V_{T-1}, and so on...