School of Business and Economics
Maastricht University
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EBC2089: Forecasting for Economics and Business

Final case

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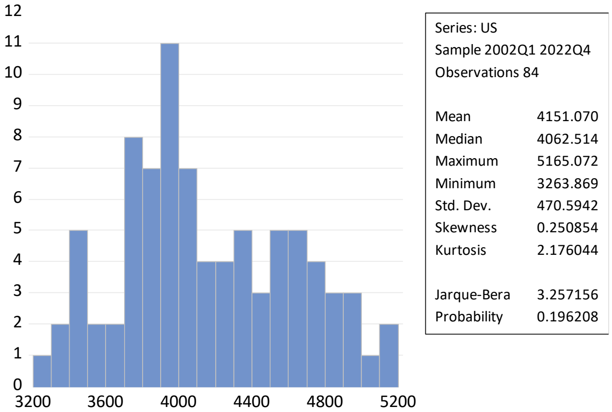
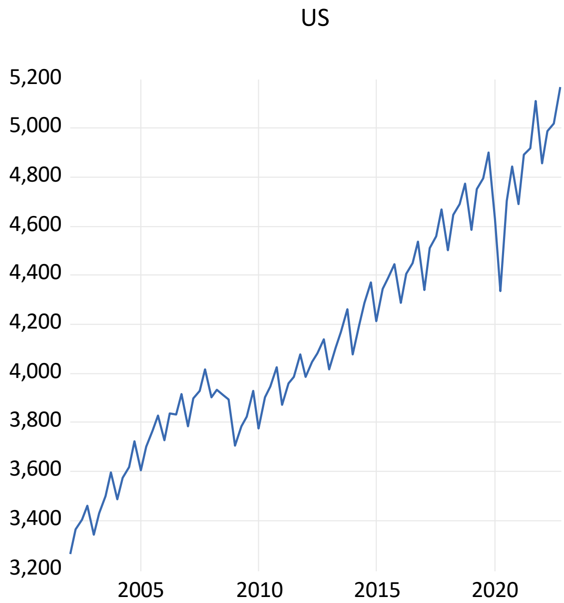
Data and motivation

US Real Gross Domestic Product

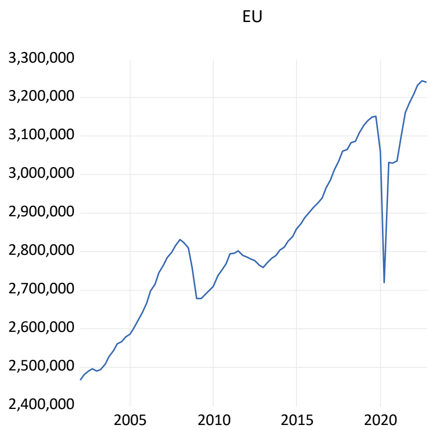
Billions of Chained 2012 Dollars, Not Seasonally Adjusted

https://fred.stlouisfed.org/series/ND000334Q





Because of its size and interconnectedness, developments in the US economy are bound to have important effects around the world. The US has the world’s single largest economy, accounting for almost a quarter of global GDP (at market exchange rates), one-fifth of global FDI, and more than a third of stock market capitalisation. It is the most important export destination for one-fifth of countries around the world. The US dollar is the most widely used currency in global trade and financial transactions, and changes in US monetary policy and investor sentiment play a major role in driving global financing conditions (World Bank 2016).



Real GDP is a measure of a country's economic output adjusted for inflation. It is calculated by taking the nominal GDP, which is the total value of goods and services produced in a country and adjusting it for inflation to determine the real value of the economic output.

Real Gross Domestic Product for European Union

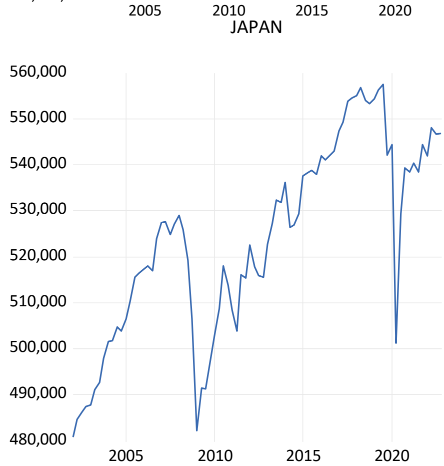
Millions of Chained 2010 Euros, Seasonally Adjusted

https://fred.stlouisfed.org/series/CLVMEURSCAB1GQEU272020

Real Gross Domestic Product for Japan

Billions of Chained 2015 Yen, Seasonally Adjusted

https://fred.stlouisfed.org/series/JPNRGDPEXP

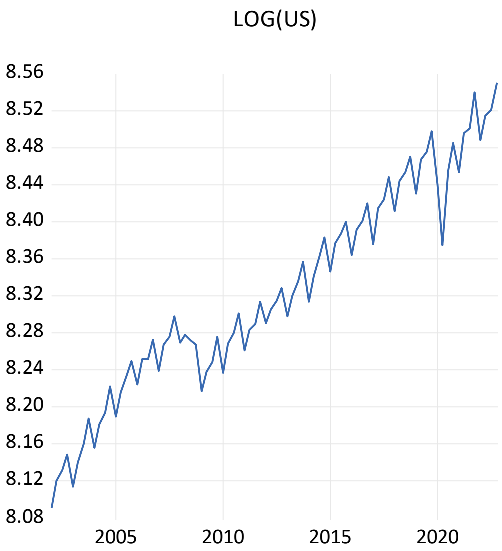


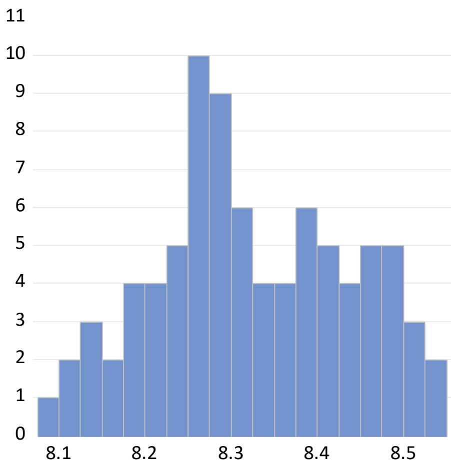
Importance of the real US GDP indicator motivated our choice of the dependent variable. The two explanatory series for real GDP for EU and Japan were chosen based on on the longstanding cooperation, shared values and principles such as democracy, the rule of law, human rights, good governance, multilateralism and open market economies shared among them and the US.

We envision a positive correlation between the evolvement of these series over time.

Univariate analysis

Taking the logs transforms the data from levels to growth rates and stabilizes the variance.





Series: LOG(US)	
Sample 2002Q1 2022Q4	
Observations 84	
Mean	8.324806
Median	8.309549
Maximum	8.549674
Minimum	8.090669
Std. Dev.	0.112990
Skewness	0.058913
Kurtosis	2.168593
Jarque-Bera	2.467923
Probability	0.291137

The ADF test resulted in not rejecting the null of unit root in the log series, therefore we proceeded with taking the first difference as a remedy for the stochastic trend. The differenced series now exhibits stationarity.

Null Hypothesis: LOG(US) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 4 (Automatic - based on SIC, maxlag=11)

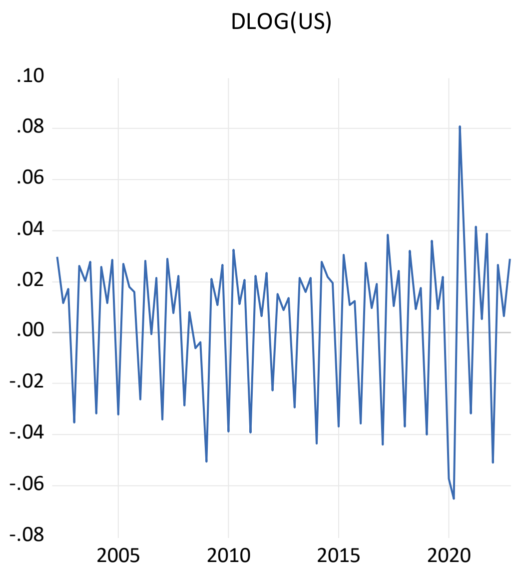
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.875045	0.1762
Test critical values:	1% level	-4.078420	
	5% level	-3.467703	
	10% level	-3.160627	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LOG(US))
Method: Least Squares
Date: 03/20/23 Time: 14:21
Sample (adjusted): 2003Q2 2022Q4
Included observations: 79 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LOG(US(-1))	-0.335920	0.116840	-2.875045	0.0053
D(LOG(US(-1)))	-0.139959	0.139899	-1.000426	0.3205
D(LOG(US(-2)))	-0.134491	0.126037	-1.067077	0.2895
D(LOG(US(-3)))	-0.218816	0.114562	-1.910016	0.0601
D(LOG(US(-4)))	0.449227	0.105680	4.250808	0.0001
C	2.740118	0.949931	2.884543	0.0052
@TREND("2002Q1")	0.001457	0.000520	2.802440	0.0065

R-squared	0.608585	Mean dependent var	0.005518
Adjusted R-squared	0.575967	S.D. dependent var	0.028819
S.E. of regression	0.018767	Akaike info criterion	-5.029051
Sum squared resid	0.025357	Schwarz criterion	-4.819100
Log likelihood	205.6475	Hannan-Quinn criter.	-4.944938
F-statistic	18.65802	Durbin-Watson stat	2.104270
Prob(F-statistic)	0.000000		



The joint F-test indicates strong significance of the seasonal components

Dependent Variable: DLOG(US)
Method: Least Squares
Date: 03/21/23 Time: 14:13
Sample (adjusted): 2002Q2 2022Q4
Included observations: 83 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
@QUARTER=1	-0.037330	0.003347	-11.15379	0.0000
@QUARTER=2	0.022892	0.003266	7.008989	0.0000
@QUARTER=3	0.013289	0.003266	4.068774	0.0001
@QUARTER=4	0.021228	0.003266	6.499312	0.0000

R-squared	0.736665	Mean dependent var	0.005530
Adjusted R-squared	0.726665	S.D. dependent var	0.028629
S.E. of regression	0.014967	Akaike info criterion	-5.518891
Sum squared resid	0.017698	Schwarz criterion	-5.402320
Log likelihood	233.0340	Hannan-Quinn criter.	-5.472059
Durbin-Watson stat	2.386542		

Wald Test:
Equation: Untitled

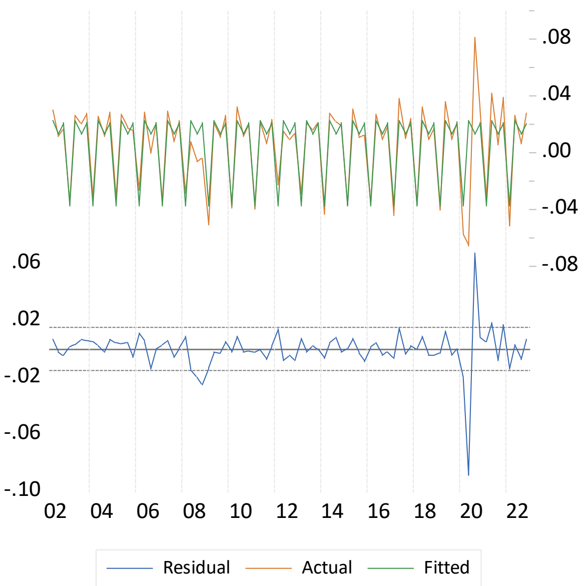
Test Statistic	Value	df	Probability
F-statistic	58.08222	(4, 79)	0.0000
Chi-square	232.3289	4	0.0000

Null Hypothesis: C(1)=C(2)=C(3)=C(4)=0
Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(1)	-0.037330	0.003347
C(2)	0.022892	0.003266
C(3)	0.013289	0.003266
C(4)	0.021228	0.003266

Restrictions are linear in coefficients.

Further improvement was made after inspecting the residual plot and incorporating dummies for the financial crisis of 2008 and the Covid pandemic in 2020.



Dependent Variable: DLOG(US)
Method: Least Squares
Date: 03/20/23 Time: 14:57
Sample (adjusted): 2002Q2 2022Q4
Included observations: 83 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.036272	0.001468	-24.70346	0.0000
Q2Y08	-0.020174	0.006566	-3.072339	0.0030
Q3Y08	-0.016942	0.006566	-2.580084	0.0119
Q4Y08	-0.026391	0.006558	-4.024065	0.0001
Q1Y20	-0.021145	0.006566	-3.220177	0.0019
Q2Y20	-0.093602	0.006566	-14.25448	0.0000
Q3Y20	0.070108	0.006566	10.67663	0.0000
@QUARTER=2	0.064583	0.002077	31.10163	0.0000
@QUARTER=3	0.047030	0.002077	22.64857	0.0000
@QUARTER=4	0.058757	0.002050	28.65648	0.0000

R-squared	0.955506	Mean dependent var	0.005530
Adjusted R-squared	0.950020	S.D. dependent var	0.028629
S.E. of regression	0.006400	Akaike info criterion	-7.152385
Sum squared resid	0.002990	Schwarz criterion	-6.860958
Log likelihood	306.8240	Hannan-Quinn criter.	-7.035306
F-statistic	174.1854	Durbin-Watson stat	2.404380
Prob(F-statistic)	0.000000		

Using Schwarz information criterium, the best ARMA(p,q) model was evaluated as ARMA(0,0).

Automatic ARIMA Forecasting
Selected dependent variable: DLOGUS
Date: 03/20/23 Time: 15:05
Sample: 2002Q1 2022Q4
Included observations: 83
Forecast length: 0
Model maximums: (4,4)0(0,0)
Regressors: C @EXPAND(@QUARTER, @DROP(1))
Q2Y08 Q3Y08 Q4Y08 Q1Y20 Q2Y20 Q3Y20

Number of estimated ARMA models: 25
Number of non-converged estimations: 0
Selected ARMA model: (0,0)(0,0)
SIC value: -6.80771890355

Autocorrelation Q statistics p-values are at the edge of the 5% significance level for the ARMA(0,0) model.Adding the AR(1) makes some of the year dummies become insignificant. However, we get the ACF and PACF to lie inside the bands and the p-values increase considerably.

Dependent Variable: DLOG(US)
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 03/21/23 Time: 14:40
Sample: 2002Q2 2022Q4
Included observations: 83
Convergence achieved after 13 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.036362	0.001288	-28.23158	0.0000
Q2Y08	-0.018436	0.176734	-0.104313	0.9172
Q3Y08	-0.016494	0.049530	-0.333013	0.7401
Q4Y08	-0.029760	0.011008	-2.703392	0.0086
Q1Y20	-0.021164	0.339955	-0.062254	0.9505
Q2Y20	-0.093898	0.092451	-1.015654	0.3132
Q3Y20	0.071899	0.019049	3.774463	0.0003
@QUARTER=2	0.064620	0.002068	31.25512	0.0000
@QUARTER=3	0.047008	0.001946	24.15913	0.0000
@QUARTER=4	0.059012	0.002094	28.18062	0.0000
AR(1)	-0.233981	0.100356	-2.331499	0.0226
SIGMASQ	3.42E-05	5.73E-06	5.972963	0.0000

R-squared	0.957726	Mean dependent var	0.005530
Adjusted R-squared	0.951177	S.D. dependent var	0.028629
S.E. of regression	0.006326	Akaike info criterion	-7.154698
Sum squared resid	0.002841	Schwarz criterion	-6.804986
Log likelihood	308.9200	Hannan-Quinn criter.	-7.014204
F-statistic	146.2293	Durbin-Watson stat	1.981343
Prob(F-statistic)	0.000000		

Inverted AR Roots -.23

Date: 03/21/23 Time: 14:32
Sample (adjusted): 2002Q2 2022Q4
Q-statistic probabilities adjusted for 9 dynamic regressors

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.209	-0.209	3.7713	0.052
		2	0.092	0.050	4.5053	0.105
		3	-0.194	-0.173	7.8279	0.050
		4	0.147	0.078	9.7462	0.045
		5	-0.166	-0.115	12.224	0.032
		6	0.068	-0.021	12.647	0.049
		7	0.023	0.083	12.697	0.080
		8	0.063	0.027	13.072	0.109
		9	-0.083	-0.041	13.726	0.132
		10	0.183	0.176	16.966	0.075
		11	-0.074	-0.009	17.509	0.094
		12	-0.073	-0.119	18.031	0.115
		13	0.073	0.141	18.572	0.137
		14	0.084	0.065	19.288	0.154
		15	-0.001	0.024	19.289	0.201
		16	-0.002	0.057	19.289	0.254
		17	-0.051	-0.101	19.569	0.297
		18	-0.019	-0.040	19.607	0.355
		19	0.066	0.144	20.087	0.389
		20	-0.031	-0.093	20.194	0.446
		21	-0.100	-0.162	21.331	0.439
		22	-0.156	-0.149	24.157	0.339
		23	0.123	-0.007	25.950	0.303
		24	-0.103	-0.122	27.217	0.294
		25	0.085	0.050	28.105	0.303
		26	0.082	0.139	28.932	0.314
		27	0.012	-0.047	28.950	0.363
		28	-0.104	-0.033	30.325	0.348
		29	0.047	0.036	30.619	0.384
		30	0.023	0.062	30.689	0.431
		31	-0.101	-0.017	32.076	0.413
		32	-0.058	-0.036	32.549	0.440
		33	0.080	-0.044	33.446	0.446
		34	-0.102	-0.110	34.953	0.423
		35	-0.048	-0.015	35.291	0.454
		36	-0.114	-0.194	37.247	0.411

*Probabilities may not be valid for this equation specification.

Date: 03/21/23 Time: 14:40
Sample (adjusted): 2002Q2 2022Q4
Q-statistic probabilities adjusted for 1 ARMA term and 9 dynamic regressors

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.004	0.004	0.0015	
		2	-0.010	-0.010	0.0102	0.920
		3	-0.139	-0.138	1.7024	0.427
		4	0.094	0.097	2.4943	0.476
		5	-0.158	-0.167	4.7412	0.315
		6	0.056	0.050	5.0335	0.412
		7	0.068	0.090	5.4663	0.486
		8	0.048	-0.009	5.6864	0.577
		9	-0.069	-0.021	6.1352	0.632
		10	0.157	0.157	8.5148	0.483
		11	-0.051	-0.065	8.7736	0.554
		12	-0.075	-0.065	9.3380	0.591
		13	0.097	0.171	10.280	0.591
		14	0.092	0.006	11.145	0.599
		15	0.008	0.043	11.151	0.674
		16	-0.004	0.041	11.153	0.742
		17	-0.050	-0.120	11.424	0.783
		18	-0.025	0.022	11.494	0.830
		19	0.065	0.125	11.966	0.849
		20	-0.032	-0.146	12.082	0.882
		21	-0.158	-0.166	14.931	0.780
		22	-0.162	-0.112	17.951	0.652
		23	0.086	-0.003	18.823	0.656
		24	-0.076	-0.108	19.508	0.671
		25	0.083	0.108	20.337	0.677
		26	0.126	0.101	22.315	0.617
		27	0.006	-0.080	22.319	0.671
		28	-0.115	0.004	24.019	0.629
		29	0.041	0.042	24.244	0.669
		30	0.015	0.023	24.276	0.715
		31	-0.120	-0.039	26.244	0.663
		32	-0.072	-0.047	26.952	0.675
		33	0.052	-0.076	27.327	0.702
		34	-0.102	-0.080	28.835	0.675
		35	-0.105	-0.032	30.444	0.643
		36	-0.115	-0.209	32.413	0.594

*Probabilities may not be valid for this equation specification.

Since the automatic selection chose no p=q=0, we do not think that we have ARMA common factors. Further, we will maintain the model chosen based on the Scharz criterion.

The LM test does not reject the null of no serial correlation under the ARMA(0,0) specification. Moreover, both the normality and homoskedasticity tests resulted in not rejecting the null hypotheses, providing evidence of no major misspecification issues. Finally, the residual plot reveals no patterns. Instead, the residuals look like white noise, as they should.

Breusch-Godfrey Serial Correlation LM Test:
Null hypothesis: No serial correlation at up to 4 lags

F-statistic	1.604787	Prob. F(4,69)	0.1829
Obs*R-squared	7.064379	Prob. Chi-Square(4)	0.1325

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 03/21/23 Time: 14:55
Sample: 2002Q2 2022Q4
Included observations: 83
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000205	0.001450	-0.141162	0.8882
Q2Y08	0.000788	0.006559	0.120087	0.9048
Q3Y08	0.000227	0.006557	0.034678	0.9724
Q4Y08	0.001343	0.006534	0.205556	0.8377
Q1Y20	0.001747	0.006547	0.266931	0.7903
Q2Y20	-0.001061	0.006542	-0.162110	0.8717
Q3Y20	0.000143	0.006467	0.022063	0.9825
@QUARTER=2	0.000248	0.002052	0.120642	0.9043
@QUARTER=3	0.000117	0.002046	0.057315	0.9545
@QUARTER=4	0.000170	0.002024	0.084051	0.9333
RESID(-1)	-0.179193	0.121687	-1.472577	0.1454
RESID(-2)	0.016289	0.122696	0.132756	0.8948
RESID(-3)	-0.164420	0.124791	-1.317568	0.1920
RESID(-4)	0.096796	0.128389	0.753928	0.4535

R-squared	0.085113	Mean dependent var	2.20E-18
Adjusted R-squared	-0.087257	S.D. dependent var	0.006039
S.E. of regression	0.006297	Akaike info criterion	-7.144954
Sum squared resid	0.002736	Schwarz criterion	-6.736956
Log likelihood	310.5156	Hannan-Quinn criter.	-6.981043
F-statistic	0.493781	Durbin-Watson stat	1.960185
Prob(F-statistic)	0.920875		

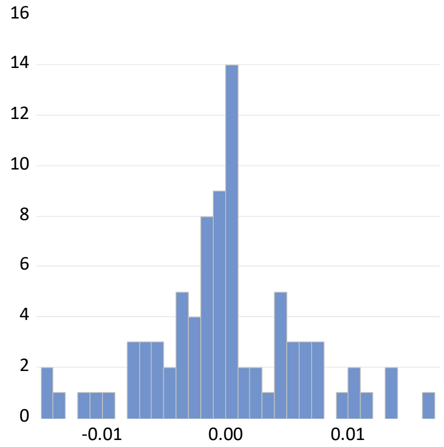
Heteroskedasticity Test: White
Null hypothesis: Homoskedasticity

F-statistic	0.494528	Prob. F(9,73)	0.8737
Obs*R-squared	4.769645	Prob. Chi-Square(9)	0.8539
Scaled explained SS	4.554014	Prob. Chi-Square(9)	0.8713

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 03/21/23 Time: 14:56
Sample: 2002Q2 2022Q4
Included observations: 83
Collinear test regressors dropped from specification

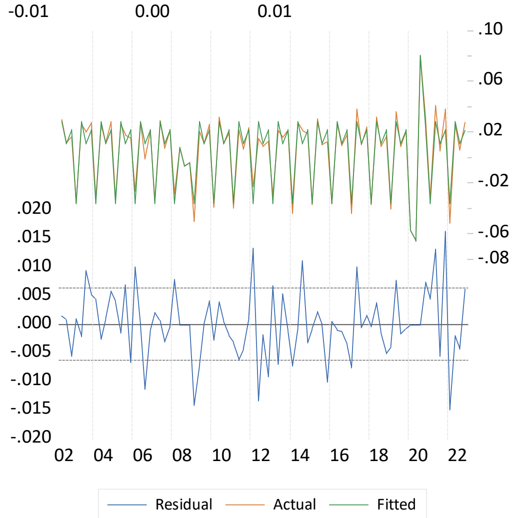
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	5.34E-05	1.34E-05	3.975359	0.0002
Q2Y08^2	-3.73E-05	6.01E-05	-0.620179	0.5371
Q3Y08^2	-2.68E-05	6.01E-05	-0.445080	0.6576
Q4Y08^2	-3.79E-05	6.00E-05	-0.631228	0.5299
Q1Y20^2	-5.34E-05	6.01E-05	-0.888917	0.3770
Q2Y20^2	-3.73E-05	6.01E-05	-0.620179	0.5371
Q3Y20^2	-2.68E-05	6.01E-05	-0.445080	0.6576
(@QUARTER=2)^2	-1.62E-05	1.90E-05	-0.849824	0.3982
(@QUARTER=3)^2	-2.67E-05	1.90E-05	-1.403538	0.1647
(@QUARTER=4)^2	-1.55E-05	1.88E-05	-0.827794	0.4105

R-squared	0.057466	Mean dependent var	3.60E-05
Adjusted R-squared	-0.058737	S.D. dependent var	5.69E-05
S.E. of regression	5.86E-05	Akaike info criterion	-16.53915
Sum squared resid	2.51E-07	Schwarz criterion	-16.24772
Log likelihood	696.3746	Hannan-Quinn criter.	-16.42207
F-statistic	0.494528	Durbin-Watson stat	1.647894
Prob(F-statistic)	0.873676		



Series: Residuals
Sample 2002Q2 2022Q4
Observations 83

Mean	2.20e-18
Median	-9.51e-05
Maximum	0.016324
Minimum	-0.014927
Std. Dev.	0.006039
Skewness	0.103106
Kurtosis	3.468589
Jarque-Bera	0.906426
Probability	0.635583



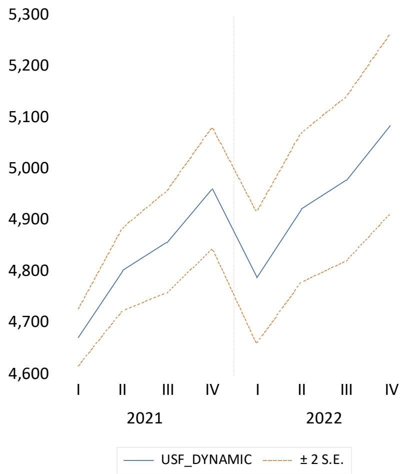
As we include the 2020 dummies in our model, the forecast was performed on 2021Q1 - 2022Q4.

- RMSE_dynamic = 77 443 830 000 $
- RMSE_static = 52 109 100 000 $

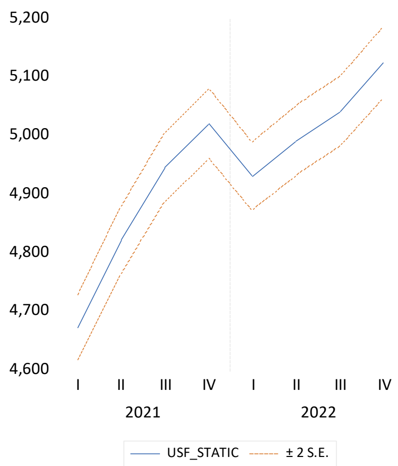
Dependent Variable: DLOG(US)
Method: Least Squares
Date: 03/21/23 Time: 15:42
Sample (adjusted): 2002Q2 2020Q4
Included observations: 75 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.035652	0.001399	-25.47928	0.0000
Q2Y08	-0.019506	0.005937	-3.285694	0.0016
Q3Y08	-0.017521	0.005937	-2.951414	0.0044
Q4Y08	-0.025129	0.005927	-4.239550	0.0001
Q1Y20	-0.021766	0.005937	-3.666397	0.0005
Q2Y20	-0.092933	0.005937	-15.65444	0.0000
Q3Y20	0.069529	0.005937	11.71205	0.0000
@QUARTER=2	0.063293	0.001979	31.98509	0.0000
@QUARTER=3	0.046989	0.001979	23.74547	0.0000
@QUARTER=4	0.056875	0.001951	29.14917	0.0000

R-squared	0.963451	Mean dependent var	0.005263
Adjusted R-squared	0.958390	S.D. dependent var	0.028283
S.E. of regression	0.005769	Akaike info criterion	-7.348976
Sum squared resid	0.002163	Schwarz criterion	-7.039978
Log likelihood	285.5866	Hannan-Quinn criter.	-7.225597
F-statistic	190.3794	Durbin-Watson stat	2.197996
Prob(F-statistic)	0.000000		



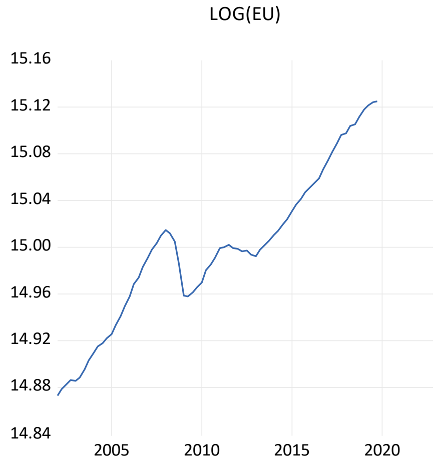
Forecast: USF_DYNAMIC	
Actual: US	
Forecast sample: 2021Q1 2022Q4	
Included observations: 8	
Root Mean Squared Error	77.44383
Mean Absolute Error	68.64554
Mean Abs. Percent Error	1.373794
Theil Inequality Coef.	0.007867
Bias Proportion	0.785689
Variance Proportion	0.059801
Covariance Proportion	0.154510
Theil U2 Coefficient	0.514019
Symmetric MAPE	1.385774



Forecast: USF_STATIC	
Actual: US	
Forecast sample: 2021Q1 2022Q4	
Included observations: 8	
Root Mean Squared Error	52.10910
Mean Absolute Error	43.84350
Mean Abs. Percent Error	0.882316
Theil Inequality Coef.	0.005262
Bias Proportion	0.033920
Variance Proportion	0.029238
Covariance Proportion	0.936842
Theil U2 Coefficient	0.343204
Symmetric MAPE	0.883975

Conditional regression models

Our explanatory variables are the log real GDP series of the EU and Japan. Based on the ADF test, both series follow a stochastic trend. Therefore, we apply first differencing to achieve stationarity.



Null Hypothesis: LOG(EU) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on SIC, maxlag=11)

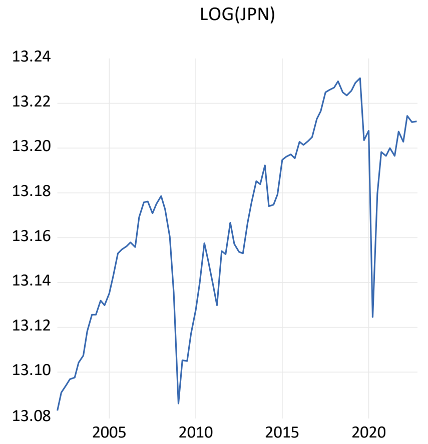
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.574426	0.2930
Test critical values:	1% level	-4.094550	
	5% level	-3.475305	
	10% level	-3.165046	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LOG(EU))
Method: Least Squares
Date: 03/21/23 Time: 16:05
Sample (adjusted): 2002Q3 2019Q4
Included observations: 70 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LOG(EU(-1))	-0.061309	0.023814	-2.574426	0.0123
D(LOG(EU(-1)))	0.698329	0.088724	7.870808	0.0000
C	0.913915	0.354567	2.577554	0.0122
@TREND("2002Q1")	0.000181	7.45E-05	2.433342	0.0177

R-squared	0.492581	Mean dependent var	0.003514
Adjusted R-squared	0.469516	S.D. dependent var	0.005776
S.E. of regression	0.004207	Akaike info criterion	-8.048845
Sum squared resid	0.001168	Schwarz criterion	-7.920360
Log likelihood	285.7096	Hannan-Quinn criter.	-7.997809
F-statistic	21.35665	Durbin-Watson stat	1.997671
Prob(F-statistic)	0.000000		



Null Hypothesis: LOG(JPN) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=11)

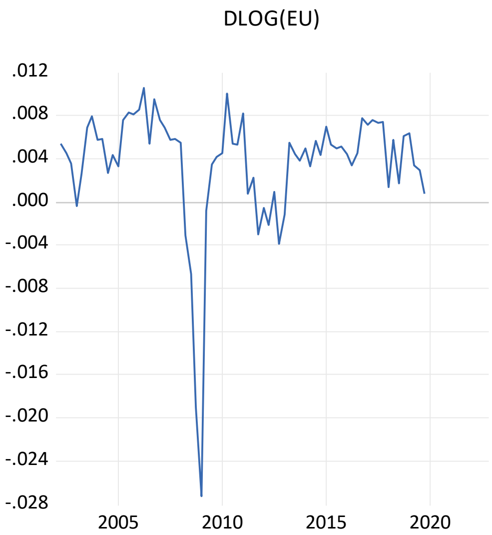
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.158021	0.1002
Test critical values:	1% level	-4.072415	
	5% level	-3.464865	
	10% level	-3.158974	

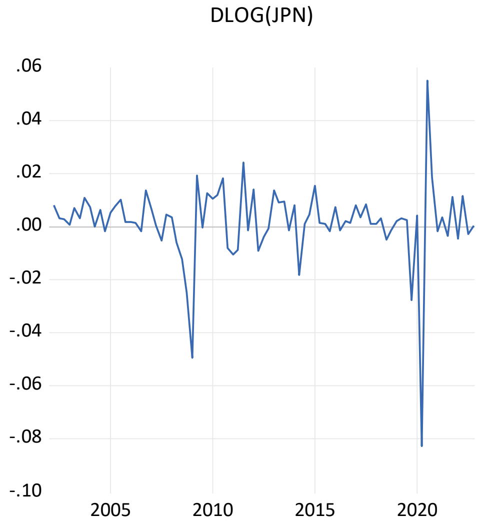
*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LOG(JPN))
Method: Least Squares
Date: 03/21/23 Time: 16:06
Sample (adjusted): 2002Q2 2022Q4
Included observations: 83 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LOG(JPN(-1))	-0.213011	0.067451	-3.158021	0.0022
C	2.795842	0.884344	3.161486	0.0022
@TREND("2002Q1")	0.000251	0.000112	2.233496	0.0283

R-squared	0.113864	Mean dependent var	0.001550
Adjusted R-squared	0.091711	S.D. dependent var	0.015097
S.E. of regression	0.014388	Akaike info criterion	-5.609421
Sum squared resid	0.016560	Schwarz criterion	-5.521993
Log likelihood	235.7910	Hannan-Quinn criter.	-5.574297
F-statistic	5.139794	Durbin-Watson stat	2.125217
Prob(F-statistic)	0.007944		





dlog(jpn) dlog(eu) q2y08 q3y08 q4y08 q1y20 q2y20 q3y20 dlog(us(-1)) dlog(us(-2)) dlog(us(-3)) dlog(us(-4)) dlog(eu(-1)) dlog(eu(-2)) dlog(eu(-3)) dlog(eu(-4)) dlog(jpn(-1)) dlog(jpn(-2)) dlog(jpn(-3)) dlog(jpn(-4))

- Using the search regressors above, the automatic variable selection based on both the stepwise forwards (t-stat = 2) and AutoGets (SIC) methods chose the following model

Dependent Variable: DLOG(US)
Method: Variable Selection
Date: 03/21/23 Time: 16:26
Sample (adjusted): 2003Q2 2019Q4
Included observations: 67 after adjustments
Number of always included regressors: 4
Number of search regressors: 20
Selection method: Stepwise forwards
Stopping criterion: t-stat forwards/backwards = 2/2

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
C	-0.029155	0.003884	-7.507179	0.0000
DLOG(EU)	0.696122	0.114413	6.084300	0.0000
Q2Y08	-0.016899	0.005599	-3.018118	0.0038
DLOG(JPN(-3))	-0.235518	0.068867	-3.419882	0.0011
DLOG(US(-4))	0.232050	0.102862	2.255938	0.0278
@QUARTER=2	0.048311	0.006530	7.398145	0.0000
@QUARTER=3	0.034489	0.005093	6.772262	0.0000
@QUARTER=4	0.042582	0.005882	7.239730	0.0000

R-squared	0.959779	Mean dependent var	0.005726
Adjusted R-squared	0.955007	S.D. dependent var	0.025084
S.E. of regression	0.005321	Akaike info criterion	-7.522777
Sum squared resid	0.001670	Schwarz criterion	-7.259530
Log likelihood	260.0130	Hannan-Quinn criter.	-7.418609
F-statistic	201.1261	Durbin-Watson stat	2.081863
Prob(F-statistic)	0.000000		

Selection Summary

Number of selected regressors: 4
Added DLOG(EU)
Added Q2Y08
Added DLOG(JPN(-3))
Added DLOG(US(-4))

*Note: p-values and subsequent tests do not account for variable selection.

Dependent Variable: DLOG(US)
Method: Variable Selection
Date: 03/21/23 Time: 16:51
Sample (adjusted): 2003Q2 2019Q4
Included observations: 67 after adjustments
Number of always included regressors: 4
Number of search regressors: 20
Selection method: AutoGets
Number of blocks: 1

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
C	-0.029155	0.003884	-7.507179	0.0000
DLOG(EU)	0.696122	0.114413	6.084300	0.0000
Q2Y08	-0.016899	0.005599	-3.018118	0.0038
DLOG(US(-4))	0.232050	0.102862	2.255938	0.0278
DLOG(JPN(-3))	-0.235518	0.068867	-3.419882	0.0011
@QUARTER=2	0.048311	0.006530	7.398145	0.0000
@QUARTER=3	0.034489	0.005093	6.772262	0.0000
@QUARTER=4	0.042582	0.005882	7.239730	0.0000

R-squared	0.959779	Mean dependent var	0.005726
Adjusted R-squared	0.955007	S.D. dependent var	0.025084
S.E. of regression	0.005321	Akaike info criterion	-7.522777
Sum squared resid	0.001670	Schwarz criterion	-7.259530
Log likelihood	260.0130	Hannan-Quinn criter.	-7.418609
F-statistic	201.1261	Durbin-Watson stat	2.081863
Prob(F-statistic)	0.000000		

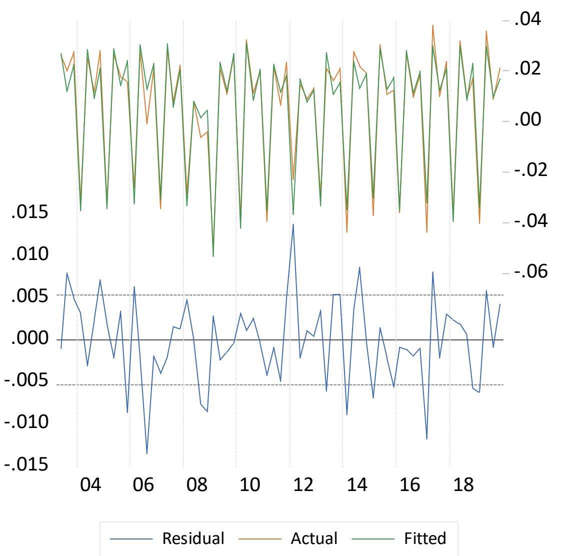
Selection Summary

Number of selected regressors: 4
Number of models compared: 5

*Note: p-values and subsequent tests do not account for variable selection.

For comparison, the model on the right was obtained using the ARDL method.

Based on the performance metrics, we favor the VARSEL selection, with the following residuals



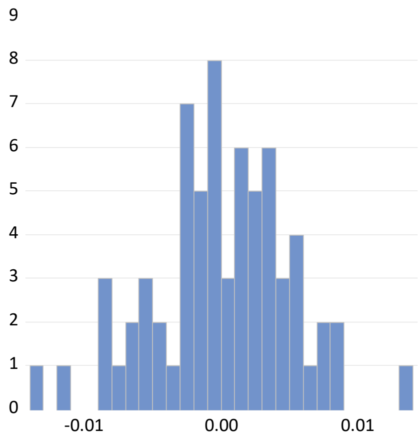
Dependent Variable: DLOG(US)
Method: ARDL
Date: 03/27/23 Time: 08:53
Sample (adjusted): 2002Q3 2019Q4
Included observations: 70 after adjustments
Maximum dependent lags: 4 (Automatic selection)
Model selection method: Schwarz criterion (SIC)
Dynamic regressors (4 lags, automatic): DLOG(JPN) DLOG(EU)
Fixed regressors: @EXPAND(@QUARTER, @DROP(1)) Q2Y08 C
Number of models evaluated: 100
Selected Model: ARDL(1, 0, 0)
Note: final equation sample is larger than selection sample

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
DLOG(US(-1))	-0.051437	0.120655	-0.426316	0.6714
DLOG(JPN)	0.085795	0.083401	1.028705	0.3076
DLOG(EU)	0.567841	0.176782	3.212092	0.0021
@QUARTER=2	0.059218	0.007129	8.306303	0.0000
@QUARTER=3	0.045632	0.002084	21.89881	0.0000
@QUARTER=4	0.054533	0.002248	24.25852	0.0000
Q2Y08	-0.013882	0.006138	-2.261613	0.0272
C	-0.036405	0.002532	-14.37758	0.0000

R-squared	0.953797	Mean dependent var	0.005385
Adjusted R-squared	0.948581	S.D. dependent var	0.025069
S.E. of regression	0.005685	Akaike info criterion	-7.394897
Sum squared resid	0.002004	Schwarz criterion	-7.137926
Log likelihood	266.8214	Hannan-Quinn criter.	-7.292825
F-statistic	182.8444	Durbin-Watson stat	2.186567
Prob(F-statistic)	0.000000		

*Note: p-values and any subsequent tests do not account for model selection.

Misspecification tests conducted on the residuals indicate that the model is specified correctly, namely we do not reject the null of normality, heteroskedasticity and no serial corellation.



Series: Residuals	
Sample 2003Q2 2019Q4	
Observations 67	
Mean	9.45e-19
Median	-0.000157
Maximum	0.013749
Minimum	-0.013379
Std. Dev.	0.005031
Skewness	-0.192250
Kurtosis	3.327373
Jarque-Bera	0.711914
Probability	0.700503

Heteroskedasticity Test: White
Null hypothesis: Homoskedasticity

F-statistic	0.836978	Prob. F(22,44)	0.6671
Obs*R-squared	19.76664	Prob. Chi-Square(22)	0.5976
Scaled explained SS	17.83706	Prob. Chi-Square(22)	0.7156

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 03/21/23 Time: 17:05
Sample: 2003Q2 2019Q4
Included observations: 67
Collinear test regressors dropped from specification

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000177	0.000156	-1.131655	0.2639
DLOG(EU)^2	0.125009	0.114165	1.094982	0.2795
DLOG(EU)*Q2Y08	-0.000889	0.018887	-0.047069	0.9627
DLOG(EU)*DLOG(US(-4))	0.709120	0.336377	2.108108	0.0408
DLOG(EU)*DLOG(JPN(-3))	0.016198	0.220927	0.073317	0.9419
DLOG(EU)*(@QUARTER=2)	-0.041696	0.019260	-2.164935	0.0359
DLOG(EU)*(@QUARTER=3)	-0.031589	0.014388	-2.195464	0.0334
DLOG(EU)*(@QUARTER=4)	-0.039986	0.017765	-2.250835	0.0294
DLOG(EU)	0.024520	0.011055	2.217926	0.0318
DLOG(US(-4))^2	-0.081413	0.113219	-0.719077	0.4759
DLOG(US(-4))*DLOG(JPN(-3))	-0.068072	0.089932	-0.756930	0.4531
DLOG(US(-4))*(@QUARTER=2)	0.011302	0.013896	0.813337	0.4204
DLOG(US(-4))*(@QUARTER=3)	0.010729	0.010636	1.008754	0.3186
DLOG(US(-4))*(@QUARTER=4)	0.010705	0.013001	0.823382	0.4147
DLOG(US(-4))	-0.009156	0.008321	-1.100270	0.2772
DLOG(JPN(-3))^2	0.042932	0.061388	0.699359	0.4880
DLOG(JPN(-3))*(@QUARTER=2)	0.005927	0.005780	1.025324	0.3108
DLOG(JPN(-3))*(@QUARTER=3)	0.004975	0.004494	1.106973	0.2743
DLOG(JPN(-3))*(@QUARTER=4)	0.006104	0.005482	1.113486	0.2715
DLOG(JPN(-3))	-0.005377	0.004123	-1.303953	0.1990
(@QUARTER=2)^2	0.000181	0.000119	1.522715	0.1350
(@QUARTER=3)^2	0.000193	0.000152	1.267294	0.2117
(@QUARTER=4)^2	0.000204	0.000138	1.479222	0.1462

R-squared	0.295024	Mean dependent var	2.49E-05
Adjusted R-squared	-0.057463	S.D. dependent var	3.83E-05
S.E. of regression	3.94E-05	Akaike info criterion	-17.17933
Sum squared resid	6.83E-08	Schwarz criterion	-16.42249
Log likelihood	598.5075	Hannan-Quinn criter.	-16.87985
F-statistic	0.836978	Durbin-Watson stat	1.964683
Prob(F-statistic)	0.667101		

Date: 03/21/23 Time: 17:00
Sample (adjusted): 2003Q2 2019Q4
Q-statistic probabilities adjusted for 7 dynamic regressors

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	-0.047	-0.047	0.1536	0.695
		2	-0.112	-0.115	1.0476	0.592
		3	0.094	0.084	1.6828	0.641
		4	-0.045	-0.050	1.8299	0.767
		5	-0.069	-0.055	2.1895	0.822
		6	-0.017	-0.042	2.2126	0.899
		7	0.052	0.045	2.4196	0.933
		8	0.080	0.088	2.9176	0.939
		9	-0.061	-0.046	3.2111	0.955
		10	0.065	0.066	3.5493	0.965
		11	-0.043	-0.065	3.7022	0.978
		12	0.010	0.046	3.7107	0.988
		13	-0.211	-0.238	7.5385	0.872
		14	-0.118	-0.124	8.7438	0.847
		15	-0.027	-0.116	8.8108	0.887
		16	-0.019	-0.021	8.8445	0.920
		17	-0.009	-0.030	8.8525	0.945
		18	0.057	0.016	9.1578	0.956
		19	-0.009	-0.019	9.1661	0.971
		20	-0.218	-0.247	13.832	0.839
		21	-0.054	-0.047	14.123	0.864
		22	0.009	-0.086	14.130	0.897
		23	-0.014	0.049	14.152	0.922
		24	-0.032	-0.098	14.261	0.941
		25	-0.007	-0.046	14.266	0.957
		26	0.024	-0.103	14.332	0.968
		27	-0.017	-0.084	14.364	0.977
		28	-0.049	-0.117	14.648	0.982

*Probabilities may not be valid for this equation specification.

Breusch-Godfrey Serial Correlation LM Test:
Null hypothesis: No serial correlation at up to 4 lags

F-statistic	0.392063	Prob. F(4,55)	0.8134
Obs*R-squared	1.857451	Prob. Chi-Square(4)	0.7620

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 03/21/23 Time: 17:01
Sample: 2003Q2 2019Q4
Included observations: 67
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.001281	0.005512	0.232487	0.8170
DLOG(EU)	0.008527	0.120113	0.070988	0.9437
Q2Y08	0.000125	0.005919	0.021054	0.9833
DLOG(US(-4))	0.035396	0.147639	0.239750	0.8114
DLOG(JPN(-3))	0.008798	0.072657	0.121084	0.9041
@QUARTER=2	-0.002345	0.009253	-0.253425	0.8009
@QUARTER=3	-0.001664	0.007087	-0.234853	0.8152
@QUARTER=4	-0.002037	0.008338	-0.244267	0.8079
RESID(-1)	-0.034496	0.144369	-0.238943	0.8120
RESID(-2)	-0.125180	0.137484	-0.910505	0.3665
RESID(-3)	0.082196	0.138077	0.595289	0.5541
RESID(-4)	-0.097515	0.203179	-0.479946	0.6332

R-squared	0.027723	Mean dependent var	9.45E-19
Adjusted R-squared	-0.166732	S.D. dependent var	0.005031
S.E. of regression	0.005434	Akaike info criterion	-7.431489
Sum squared resid	0.001624	Schwarz criterion	-7.036618
Log likelihood	260.9549	Hannan-Quinn criter.	-7.275237
F-statistic	0.142568	Durbin-Watson stat	1.977847
Prob(F-statistic)	0.999360		

VAR

Based on the lag order selection criteria, the optimal VAR between our stationary series was estimated as VAR(4). The VAR exclusion tests further supports including the 4 lags.

VAR Lag Order Selection Criteria
Endogenous variables: DLOG(US) DLOG(EU) DLOG(JPN)
Exogenous variables: C
Date: 03/31/23 Time: 10:31
Sample: 2002Q1 2022Q4
Included observations: 75

Lag	LogL	LR	FPE	AIC	SC	HQ
0	597.6556	NA	2.60e-11	-15.85748	-15.76478	-15.82047
1	619.2685	40.92032	1.86e-11	-16.19383	-15.82303	-16.04577
2	630.3136	20.02856	1.76e-11	-16.24836	-15.59947	-15.98927
3	696.6291	114.9468	3.84e-12	-17.77678	-16.84978	-17.40664
4	718.6976	36.48662*	2.73e-12*	-18.12527*	-16.92018*	-17.64409*
5	723.8640	8.128481	3.05e-12	-18.02304	-16.53985	-17.43082
6	729.6465	8.635275	3.37e-12	-17.93724	-16.17595	-17.23398
7	734.7769	7.250909	3.81e-12	-17.83405	-15.79466	-17.01974
8	742.5455	10.35818	4.04e-12	-17.80121	-15.48373	-16.87587

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

VAR Lag Exclusion Wald Tests
Date: 03/31/23 Time: 10:33
Sample (adjusted): 2003Q2 2022Q4
Included observations: 79 after adjustments

Chi-squared test statistics for lag exclusion:
Numbers in [ ] are p-values

	DLOG(US)	DLOG(EU)	DLOG(JPN)	Joint
Lag 1	4.040263	2.069661	3.385236	28.52742
	[0.2571]	[0.5581]	[0.3360]	[0.0008]
Lag 2	21.05844	13.00404	10.35518	33.84741
	[0.0001]	[0.0046]	[0.0158]	[0.0001]
Lag 3	10.29443	1.249912	7.211884	34.07459
	[0.0162]	[0.7411]	[0.0654]	[0.0001]
Lag 4	11.29880	0.319172	1.011191	54.03423
	[0.0102]	[0.9564]	[0.7985]	[0.0000]
df	3	3	3	9

VAR Granger Causality/Block Exogeneity Wald Tests
Date: 03/31/23 Time: 10:40
Sample: 2002Q1 2022Q4
Included observations: 79

Results of the exogeneity tests indicate the following, highly significant, Granger causalities among the series:

Dependent variable: DLOG(US)

Excluded	Chi-sq	df	Prob.
DLOG(EU)	19.16999	4	0.0007
DLOG(JPN)	22.87020	4	0.0001
All	64.46497	8	0.0000

dlog(eu) -> dlog(us)
dlog(jpn) -> dlog(us)

Dependent variable: DLOG(EU)

Excluded	Chi-sq	df	Prob.
DLOG(US)	1.488650	4	0.8287
DLOG(JPN)	15.60966	4	0.0036
All	18.10397	8	0.0205

dlog(jpn) -> dlog(eu)

Dependent variable: DLOG(JPN)

Excluded	Chi-sq	df	Prob.
DLOG(US)	2.150748	4	0.7081
DLOG(EU)	12.75702	4	0.0125
All	18.75155	8	0.0162

dlog(eu) -> dlog(jpn)

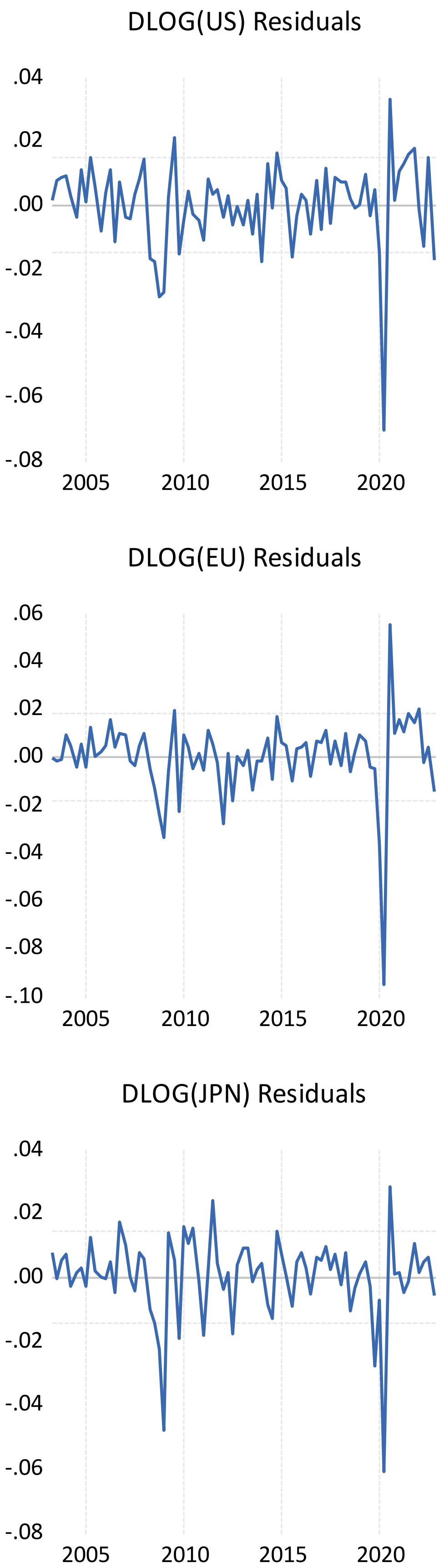
The estimation output with the respective residual graphs follows.

Vector Autoregression Estimates
Date: 03/31/23 Time: 10:26
Sample (adjusted): 2003Q2 2022Q4
Included observations: 79 after adjustments
Standard errors in ( ) & t-statistics in [ ]

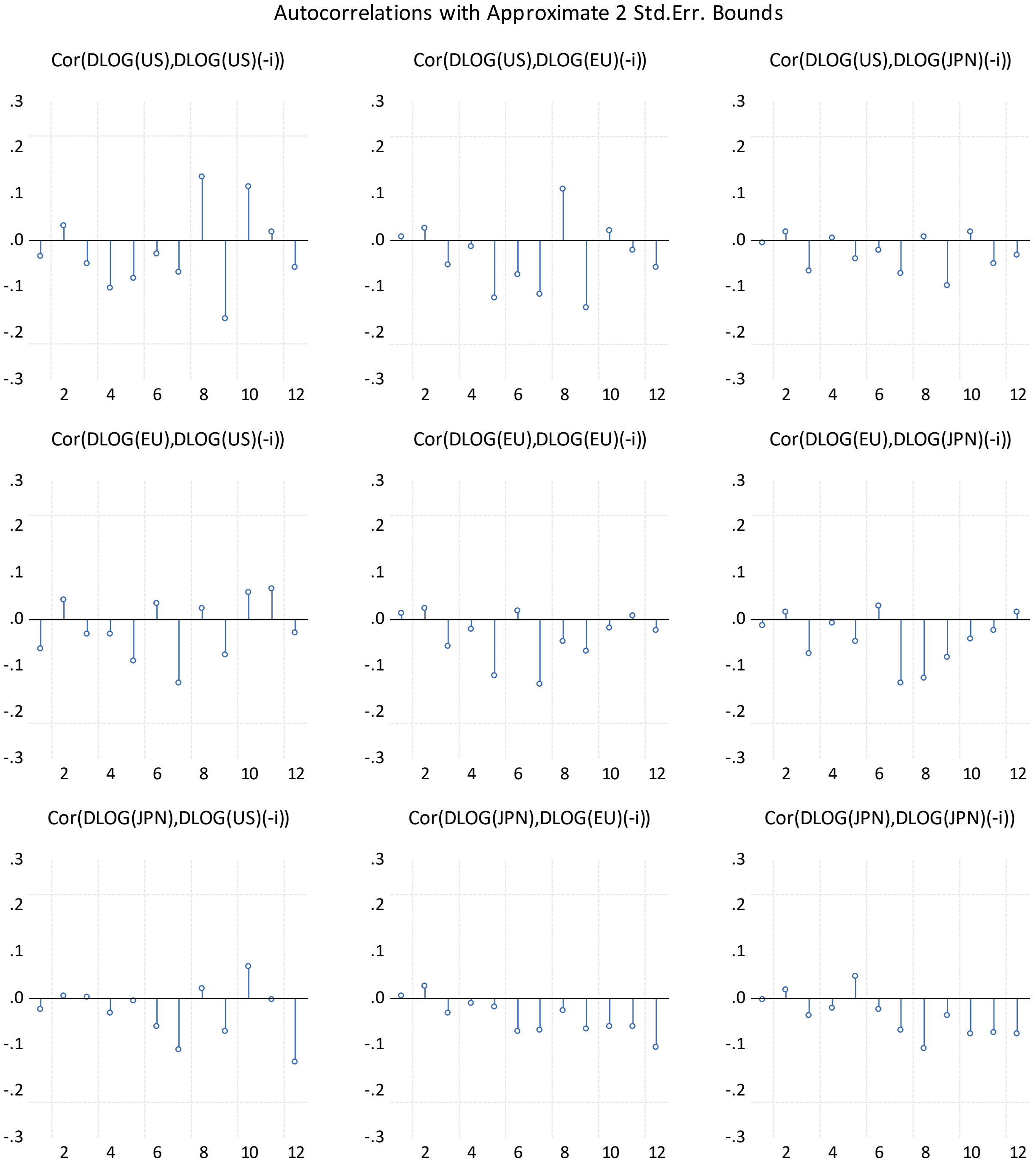
	DLOG(US)	DLOG(EU)	DLOG(JPN)
DLOG(US(-1))	-0.334327	-0.029274	0.199187
	(0.18698)	(0.23086)	(0.18138)
	[-1.78802]	[-0.12680]	[1.09821]
DLOG(US(-2))	-0.398496	-0.149588	0.122060
	(0.18806)	(0.23219)	(0.18242)
	[-2.11897]	[-0.64424]	[0.66911]
DLOG(US(-3))	-0.395458	-0.130896	0.170814
	(0.18628)	(0.22999)	(0.18069)
	[-2.12293]	[-0.56913]	[0.94533]
DLOG(US(-4))	0.624467	-0.067226	0.162933
	(0.19261)	(0.23781)	(0.18683)
	[3.24215]	[-0.28269]	[0.87208]
DLOG(EU(-1))	0.209816	-0.056581	0.118755
	(0.17961)	(0.22175)	(0.17422)
	[1.16820]	[-0.25515]	[0.68164]
DLOG(EU(-2))	-0.359133	-0.491162	-0.519470
	(0.18610)	(0.22978)	(0.18052)
	[-1.92975]	[-2.13756]	[-2.87758]
DLOG(EU(-3))	0.280484	0.144601	0.042493
	(0.18262)	(0.22548)	(0.17715)
	[1.53587]	[0.64131]	[0.23988]
DLOG(EU(-4))	-0.450102	0.125054	-0.103577
	(0.18687)	(0.23072)	(0.18127)
	[-2.40865]	[0.54201]	[-0.57141]
DLOG(JPN(-1))	-0.055689	-0.138817	-0.300551
	(0.20212)	(0.24955)	(0.19606)
	[-0.27553]	[-0.55627]	[-1.53297]
DLOG(JPN(-2))	0.832099	0.842226	0.488861
	(0.19128)	(0.23617)	(0.18555)
	[4.35011]	[3.56618]	[2.63471]
DLOG(JPN(-3))	-0.203745	-0.153802	-0.470911
	(0.22027)	(0.27196)	(0.21366)
	[-0.92499]	[-0.56554]	[-2.20400]
DLOG(JPN(-4))	-0.077158	-0.052514	-0.079115
	(0.20987)	(0.25911)	(0.20357)
	[-0.36765]	[-0.20267]	[-0.38864]
C	0.007575	0.005341	0.000199
	(0.00283)	(0.00350)	(0.00275)
	[2.67250]	[1.52604]	[0.07240]
R-squared	0.779196	0.271714	0.277931
Adj. R-squared	0.739050	0.139298	0.146646
Sum sq. resids	0.014304	0.021806	0.013459
S.E. equation	0.014722	0.018177	0.014280
F-statistic	19.40900	2.051975	2.117002
Log likelihood	228.2610	211.6076	230.6662
Akaike AIC	-5.449646	-5.028040	-5.510537
Schwarz SC	-5.059736	-4.638131	-5.120628
Mean dependent	0.005518	0.003332	0.001444
S.D. dependent	0.028819	0.019592	0.015459

Determinant resid covariance (dof adj.)	1.50E-12
Determinant resid covariance	8.77E-13
Log likelihood	760.3395
Akaike information criterion	-18.26176
Schwarz criterion	-17.09203
Number of coefficients	39

VAR Residuals



With autocorrelations well inside the standard error bounds, the lag length is likely to be correctly specified.



For comparison, the model reestimation includes observations up to 2020Q4 and period 2021Q1 - 2022Q4 is forecasted again.

- RMSE_dynamic = 839 648 800 000 $
- RMSE_static = 251 947 200 000 $

Forecast Evaluation
Date: 03/31/23 Time: 11:05
Sample: 2021Q1 2022Q4
Included observations: 8
Evaluation sample: 2021Q1 2022Q4
Number of forecasts: 1

Evaluation statistics

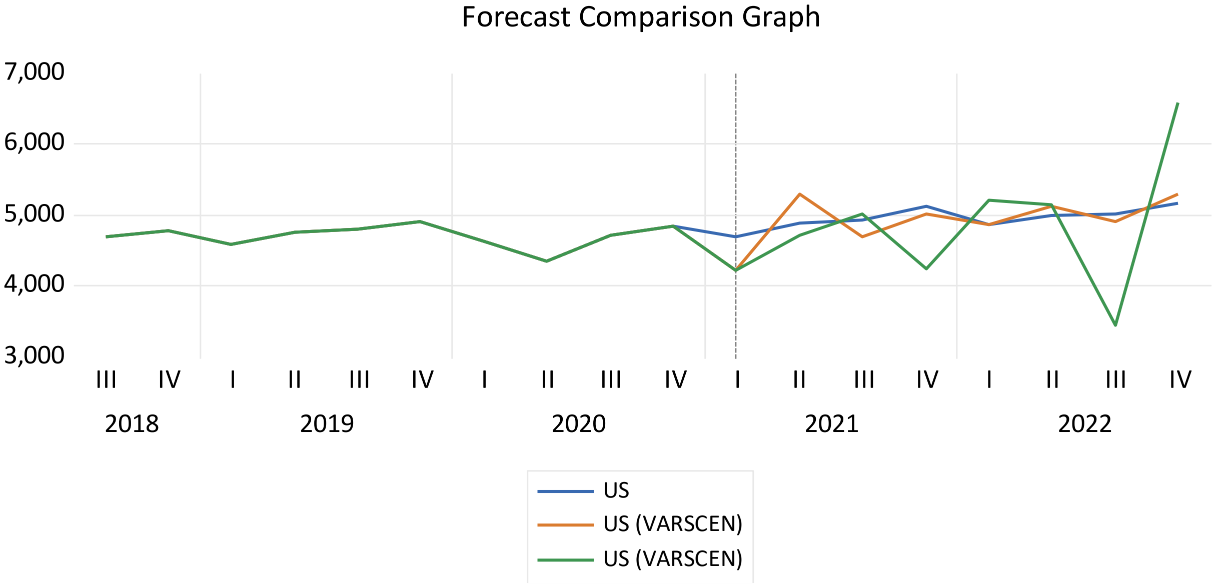
Forecast	RMSE	MAE	MAPE	SMAPE	Theil U1	Theil U2
US_FVD	839.6488	639.9528	12.74530	13.30848	0.085261	5.381669

Forecast Evaluation
Date: 03/31/23 Time: 11:03
Sample: 2021Q1 2022Q4
Included observations: 8
Evaluation sample: 2021Q1 2022Q4
Number of forecasts: 1

Evaluation statistics

Forecast	RMSE	MAE	MAPE	SMAPE	Theil U1	Theil U2
US_FVS	251.9472	199.9395	4.095955	4.136174	0.025476	1.262187

The graph depicts the dynamic forecast in green and the static one in red.



Forecast comparison

In terms of included evaluation statistics, the univariate model is almost five times better than the VAR approach for both the static and dynamic forecasting.

Static

Forecast Evaluation
Date: 03/31/23 Time: 11:13
Sample: 2021Q1 2022Q4
Included observations: 8
Evaluation sample: 2021Q1 2022Q4
Number of forecasts: 2

Combination tests
Null hypothesis: Forecast i includes all information contained in others

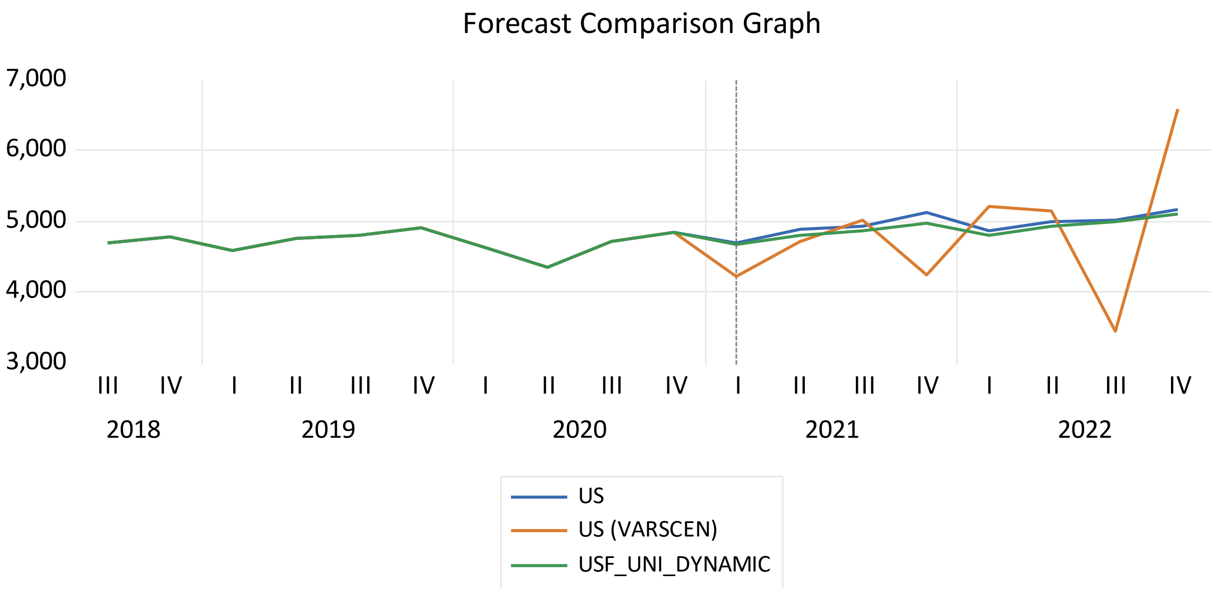
Forecast	F-stat	F-prob
USF_UNI_STATIC	0.763700	0.4158
US_FVS	0.797087	0.4063

Diebold-Mariano test (HLN adjusted)
Null hypothesis: Both forecasts have the same accuracy

Accuracy	Statistic	<> prob	> prob	< prob
Abs Error	-2.533817	0.0390	0.0195	0.9805
Sq Error	-1.944014	0.0930	0.0465	0.9535

Evaluation statistics

Forecast	RMSE	MAE	MAPE	SMAPE	Theil U1	Theil U2
USF_UNI_STATIC	52.10910	43.84350	0.882316	0.883975	0.005262	0.343204
US_FVS	251.9472	199.9395	4.095955	4.136174	0.025476	1.262187



Dynamic

Forecast Evaluation
Date: 03/31/23 Time: 11:15
Sample: 2021Q1 2022Q4
Included observations: 8
Evaluation sample: 2021Q1 2022Q4
Number of forecasts: 2

Combination tests
Null hypothesis: Forecast i includes all information contained in others

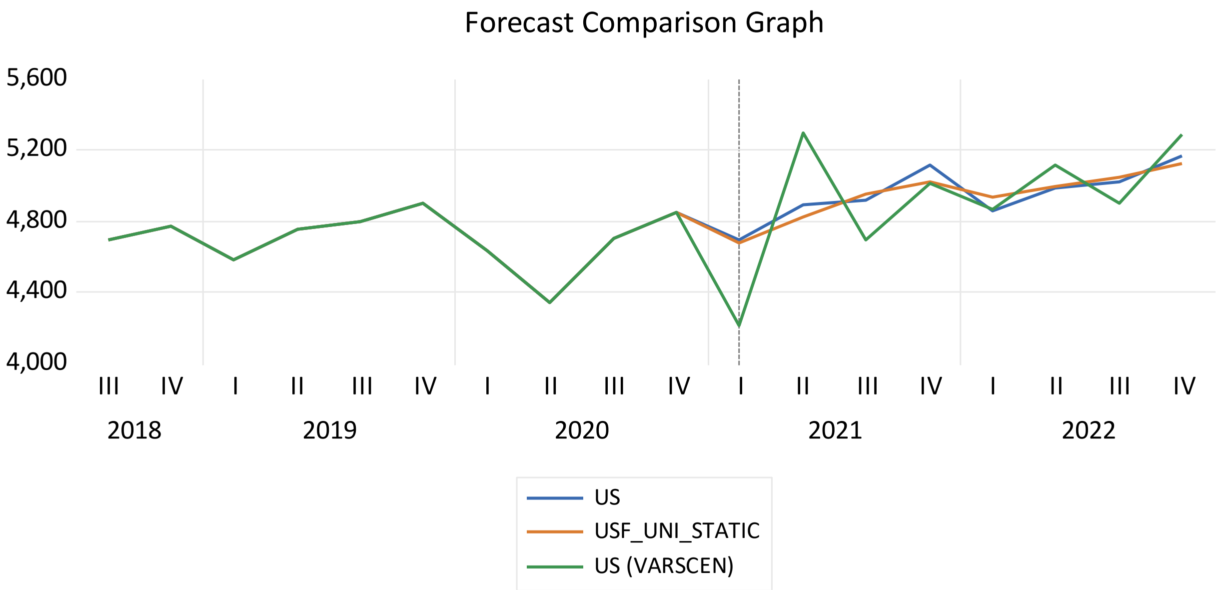
Forecast	F-stat	F-prob
US_FVD	0.227683	0.6501
USF_UNI_DYNAMIC	0.131516	0.7293

Diebold-Mariano test (HLN adjusted)
Null hypothesis: Both forecasts have the same accuracy

Accuracy	Statistic	<> prob	> prob	< prob
Abs Error	2.783488	0.0272	0.9864	0.0136
Sq Error	2.018649	0.0833	0.9584	0.0416

Evaluation statistics

Forecast	RMSE	MAE	MAPE	SMAPE	Theil U1	Theil U2
US_FVD	839.6488	639.9528	12.74530	13.30848	0.085261	5.381669
USF_UNI_DYNAMIC	77.44383	68.64554	1.373794	1.385774	0.007867	0.514019



The null of the univariate and VAR forecasts having the same accuracy was rejected for both the static and dynamic forecast

- on the significance level of 5% for absolute error
- and 10% for squared error.

Average of the univariate model and the VAR static and dynamic forecasts performance was better than the former but worse than the latter, namely

- RMSE_static = 122 865 400 000$,
- RMSE_dynamic = 426 425 600 000 $.

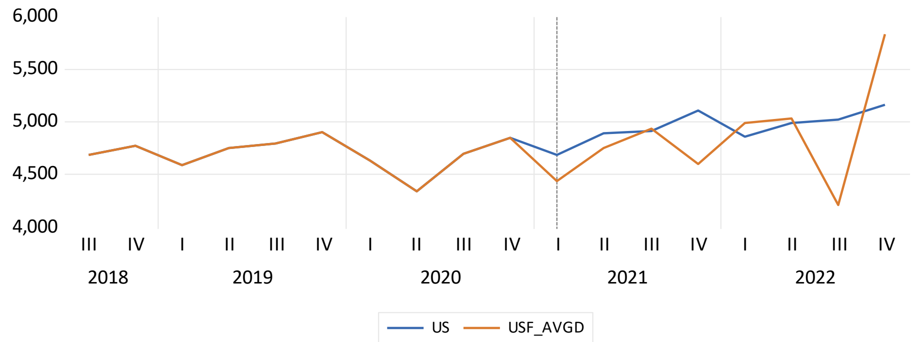
Realizations of the series(blue) together with the forecast (red) are given below.

Forecast Evaluation
Date: 03/31/23 Time: 11:34
Sample: 2021Q1 2022Q4
Included observations: 8
Evaluation sample: 2021Q1 2022Q4
Number of forecasts: 1

Evaluation statistics

Forecast	RMSE	MAE	MAPE	SMAPE	Theil U1	Theil U2
USF_AVGS	122.8654	101.6665	2.082937	2.097039	0.012418	0.577189

Forecast Comparison Graph

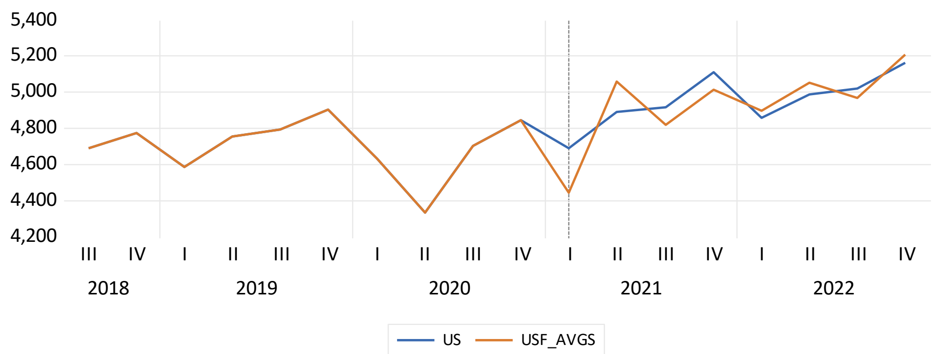


Forecast Evaluation
Date: 03/31/23 Time: 11:33
Sample: 2021Q1 2022Q4
Included observations: 8
Evaluation sample: 2021Q1 2022Q4
Number of forecasts: 1

Evaluation statistics

Forecast	RMSE	MAE	MAPE	SMAPE	Theil U1	Theil U2
USF_AVGD	426.4256	321.6357	6.404765	6.566305	0.043389	2.734775

Forecast Comparison Graph



Conclusion

After a brief introduction, this paper investigated the out-of-sample forecast performances of three approaches using a statistical and econometric package EViews. The real GDP of the United States was chosen as the dependent variable, with the regression framework extended by the real GDP of the EU and Japan. The studied period is from 2002Q1 to 2022Q4.

First, the univariate time series model, including trends, seasonality outliers, and cycles, was identified. Using natural logarithms, the series was transformed from levels to growth rates. Stationarity was obtained by first differencing, seasonal components were incorporated, and the optimal ARMA model was determined. The misspecification test concluded the preparation, and the static and dynamic forecasts for the 2021 and 2022 (the last eight quarters) were obtained.

Second, additional variables were considered based on economic theory and intuition. Unit root tests determined their order of integration. The conditional model in which the dependent variable was regressed on the contemporaneous explanatory variables, necessary deterministic elements, and the history was estimated.

Third, optimal VAR was established, Granger causalities were tested, and the last eight observations were forecasted after re-estimation. Both static and dynamic forecasting models were compared with the univariate ones based on forecast evaluation metrics as well as with the Diebold-Mariano tests. The average forecast of the univariate and the VAR models was obtained, and its results were analyzed.