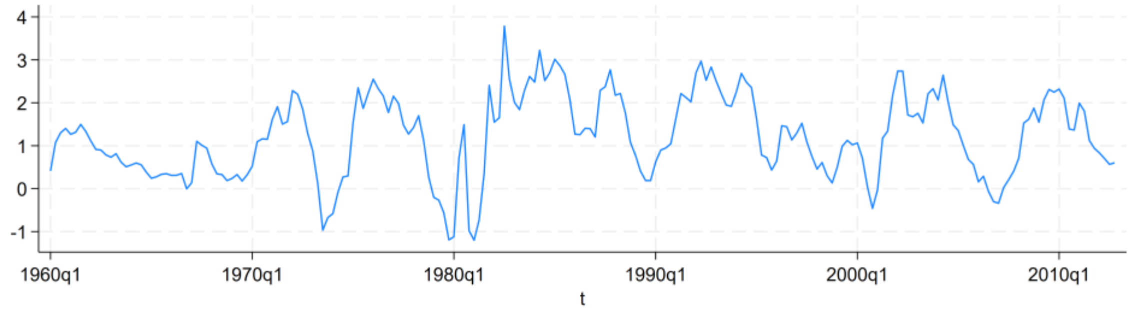
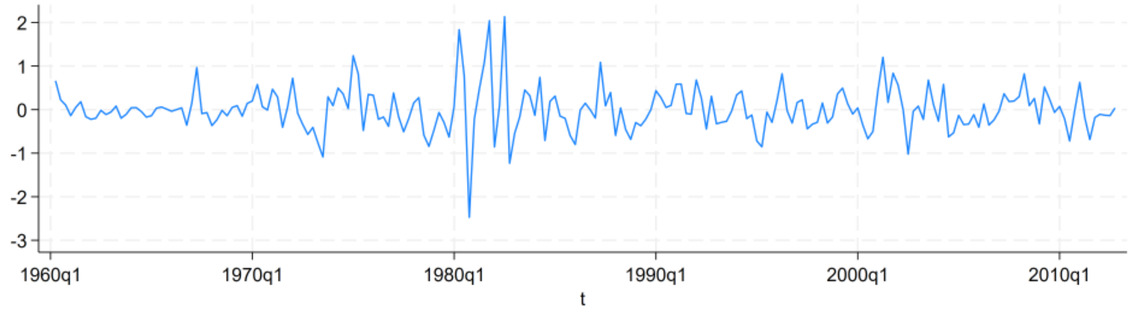
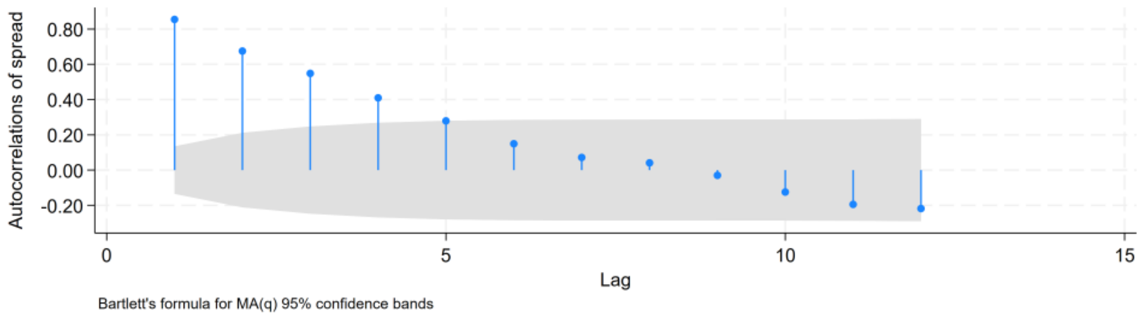
Time Series Analysis of US Interest Rates

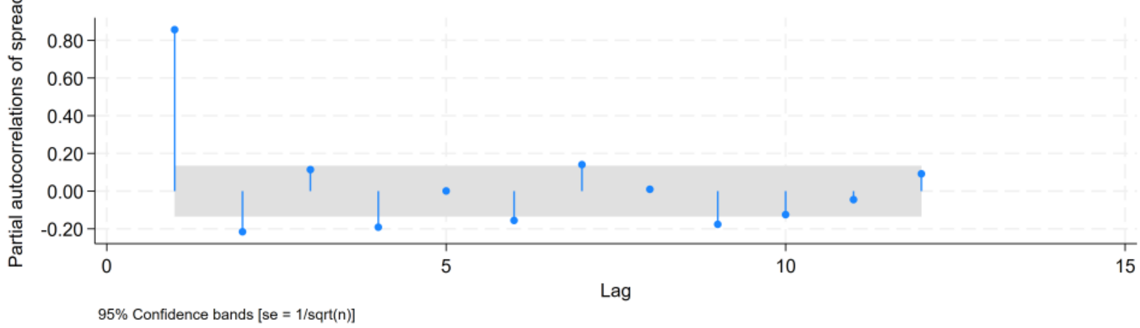
Question Go through the example of interest rate spreads Section 2.10 (pp. 88-96) and try to replicate the results.

Answer The spread over the period from 1960Q1 to 2012Q4 is presented below. Although there are a few instances in which the spread is negative, the difference between long- and short-term rates is generally positive (the sample mean is 1.21). In contrast, the first difference of the spread seems to be very erratic.







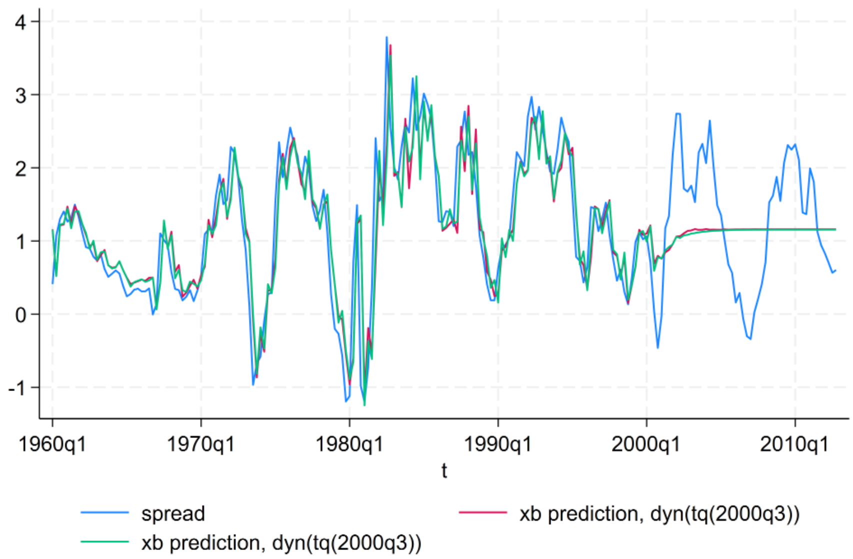


The ACF and PACF converge to zero relatively quickly. The ACF does not cut to zero and there appears to be an oscillating pattern in the PACF in that the first seven values alternate in sign.

	AR (7)	AR (6)	AR (2)	p = 1, 2, 7	ARMA (1,1)	ARMA (2,1)	p = 2; ma = (1, 7)
cons	1.209***	1.210***	1.204***	1.207***	1.204***	1.202***	1.204***
	(6.16)	(7.05)	(5.71)	(6.34)	(6.19)	(5.58)	(5.60)
L.ar	1.107***	1.086***	1.038***	1.028***	0.756***	0.407**	0.339**
	(15.18)	(17.45)	(18.32)	(15.79)	(14.80)	(3.03)	(3.21)
L2.ar	-0.451***	-0.427***	-0.213***	-0.195**		0.324**	0.399***
	(-4.16)	(-4.41)	(-3.82)	(-2.88)		(2.59)	(4.13)
L3.ar	0.404***	0.368***					
	(4.68)	(4.36)					
L4.ar	-0.305***	-0.255***					
	(-3.99)	(-3.67)					
L5.ar	0.227*	0.169*					
	(2.33)	(2.21)					
L6.ar	-0.298**	-0.152**					
	(-2.75)	(-2.72)					
L7.ar	0.135			-0.0297			
	(1.87)			(-0.92)			
L.ma					0.381***	0.706***	0.786***
					(5.83)	(6.36)	(10.67)
L7.ma							-0.136***
							(-3.49)
sigma	0.463***	0.468***	0.486***	0.485***	0.480***	0.474***	0.463***
	(24.01)	(24.36)	(37.42)	(36.90)	(30.91)	(28.17)	(28.15)
aic	286.1	287.9	295.2	296.6	290.7	287.0	279.9
bic	316.0	314.5	308.5	313.2	304.0	303.6	299.8

To ensure comparability, each equation was estimated over the 1961Q4 - 2012Q4 period.
t statistics in parentheses, * $p < 0.05$ ** $p < 0.01$, *** $p < 0.001$

The performance of the AR(7) and ARMA[2,(1,7)] models can be assessed by examining their bias and mean square prediction errors (2000Q3-2012Q4 out of sample forecast). The results is that the models are not substantially different from each other.



Question Conduct a Chow test for the spread series where you set the breakpoint at $t = 1981Q4$. What do you conclude? Will your conclusions change if you change the breakpoint?

Answer Let be the sum of the squared residuals from the AR(7) model estimated over the whole sample, period 1982Q1-2012Q4, and period 1982Q1-2012Q4 be SSR, SSR_1, and SSR_2, respectively. To test the restriction that all coefficients are equal before and a after $t = 1981Q4$, use an F-test and form:

$$F = \frac{(SSR - SSR_1 - SSR_2)/n}{(SSR_1 + SSR_2)/(T - 2n)}$$

where n = number of parameters estimated ($n = 7+1$) since an intercept is included and the number of degrees of freedom are $(n, T - 2n) = (8, 205 - 16)$. The resulting Chow Test Statistic

$$F = \frac{(43.976756 - 16.493662 - 22.716043)/8}{(493662 + 22.716043)/(205 - 16)} = 2.872288$$

corresponds to a p-value of approximately 0.004. We can therefore reject the null of no structural change in the coefficients. Changing the hypothetical breakpoint to 1991Q4 and using the same procedure leads to a statistic $F = 1.09$ with associated p-value 0.37, we thus cannot reject the hypothesis that there is no structural change in the coefficients.

Question Estimate an AR(1) process with intercept recursively for the sample sizes $n, n+1, \ldots, T-1, T$ where $n = 10$. Plot the estimation results for the intercept and the AR(1) coefficient with ± 2 standard deviation band (see p. 108). What do you conclude?

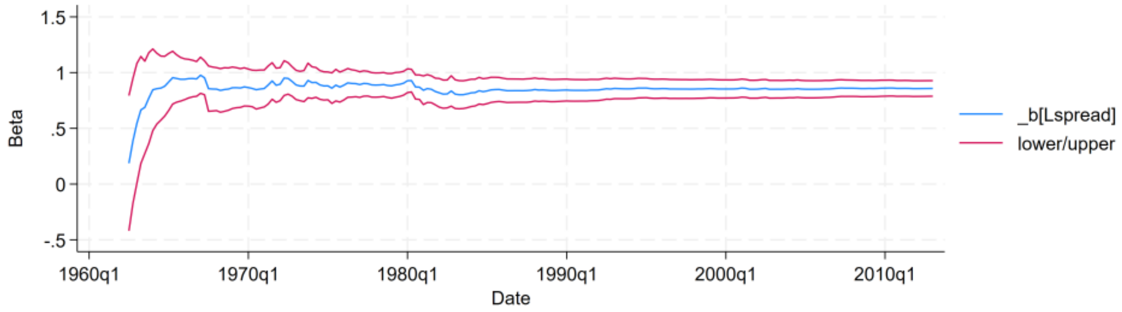
Answer The estimates of the AR(1) coefficient (along with their 2 standard deviation bands) resulting from a recursive estimation are shown below. The initial confidence intervals are quite wide since the first few estimations use a very small number of observations. The estimates all seem reasonable until about t = 100. A slight change in the AR(1) coefficient might indicate a potential structural break around 1981.

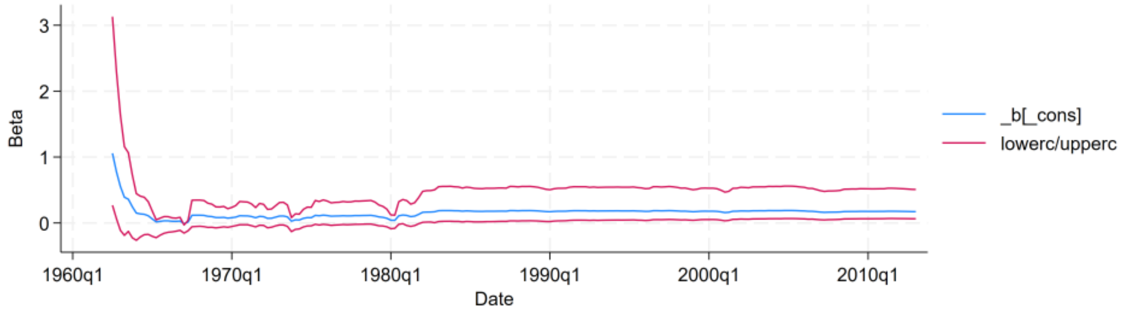
Question For the same sample sizes as in 2c, calculate the CUSUM test accompanied with ± 2 standard deviation band. What do you conclude?

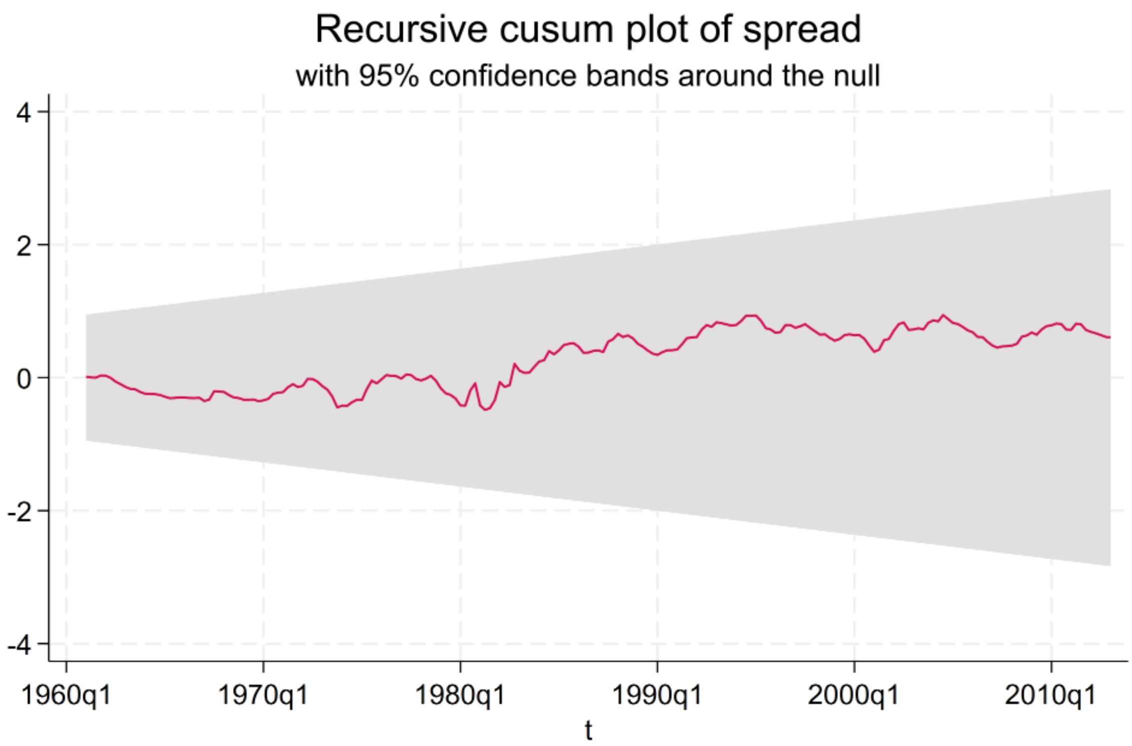
Answer The CUSUMs for $N = 10, \ldots, 205 - 1$

$$\sum_{i=n}^{N} \frac{e_i(1)}{\sigma_e},$$

where n denotes the date of the first forecast error constructed, T is the date of the last observation in the data set and σ_e is the estimated standard deviation of the forecast errors, are clearly within the confidence intervals $(0.948[(T-n)0.5+2(N-n)(T-n)-0.5])$. As such, the hypothesis of the AR(1) coefficient stability can not be rejected.


1.5
1
.5
0
-.5
Beta
1960q1
1970q1
1980q1
1990q1
2000q1
2010q1
Date
_b[Lspread]
lower/upper


3
2
1
0
Beta
1960q1
1970q1
1980q1
1990q1
2000q1
2010q1
Date
_b[_cons]
lowerc/upperc


Recursive cusum plot of spread
with 95% confidence bands around the null
4
2
0
-2
-4
1960q1
1970q1
1980q1
1990q1
2000q1
2010q1
t