

# 14.03/003 Microeconomic Theory & Public Policy

---

## Lecture 11. General Equilibrium in a Pure Exchange Economy.

David Autor (Prof), MIT Economics and NBER

Jonathan Cohen (TA), MIT Economics

# Motivation for General Equilibrium

- ▶ So far we talked about one market at a time: labor, sugar, food, ...
- ▶ All changes in quantities or prices ultimately feed back into the demand and/or supply for other goods through several channels.
- ▶ We need a model that can accommodate the interactions of all markets simultaneously and allows us to determine the properties of the grand equilibrium.
- ▶ This is the goal of the General Equilibrium (GE) model.

# The Edgeworth Box

- ▶ To make the general equilibrium problem tractable, we want to reduce the dimensionality of the “all markets” problem to something manageable.
- ▶ The Edgeworth Box provides the tool we need.
- ▶ Focus here on two goods and two people.
- ▶ Edgeworth box depicts the gains in welfare that may accrue from pure exchange of goods.
- ▶ Intuitively demonstrates two fundamental results in economics: the First and Second Welfare Theorems.

# Before we dive in: What's this all about?

## 1 Where we started: **individual consumer choice**

- Preferences exogenous
- Budget set exogenous
- Prices exogenous
- Choices *endogenous*

# Before we dive in: What's this all about?

## 1 Where we started: **individual consumer choice**

- Preferences exogenous
- Budget set exogenous
- Prices exogenous
- Choices *endogenous*

## 2 Next: **equilibrium in a single market** (nutritive sweeteners, real estate brokers)

- Preferences exogenous
- Budget set exogenous
- Price and quantity in that single market is *endogenous*

# Before we dive in: What's this all about?

## 1 Where we started: **individual consumer choice**

- Preferences exogenous
- Budget set exogenous
- Prices exogenous
- Choices *endogenous*

## 2 Next: **equilibrium in a single market** (nutritive sweeteners, real estate brokers)

- Preferences exogenous
- Budget set exogenous
- Price and quantity in that single market is *endogenous*

## 3 Now: **general equilibrium**

- Preferences exogenous
- *Endowments* exogenous (an endowment is the 'stuff' you start with)
- Prices *endogenous*
- Budget set *endogenous*

# Edgeworth Notation

- ▶ There are two goods: call them food  $F$  and shelter  $S$ . There are two agents: call them  $A$  and  $B$ . The initial endowment is:

$$E_A = (E_A^F, E_A^S)$$

$$E_B = (E_B^F, E_B^S)$$

- ▶ The consumption of  $A$  and  $B$  are denoted as:

$$X_A = (X_A^F, X_A^S)$$

$$X_B = (X_B^F, X_B^S)$$

- ▶ *Without trade* between agents  $A$  and  $B$ , their consumption bundles will equal their endowments:

$$X_A = E_A$$

$$X_B = E_B$$

- ▶ *With trade*, many exchanges between  $A$  and  $B$  become feasible, but the following equalities must always hold:

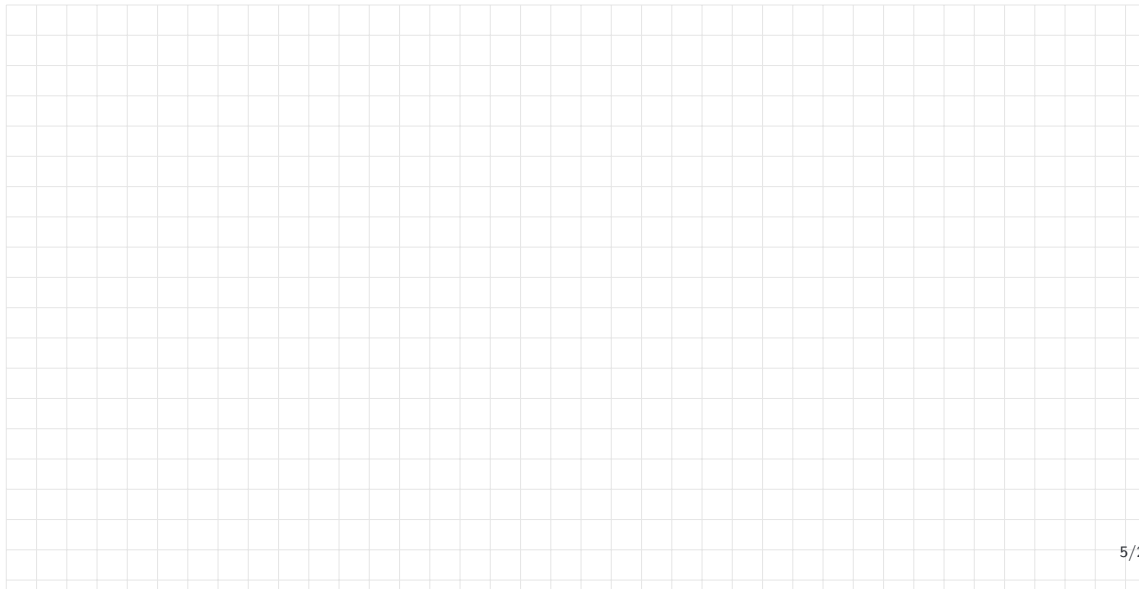
$$X_A^F + X_B^F = E_A^F + E_B^F$$

$$X_A^S + X_B^S = E_A^S + E_B^S$$

- ▶ Note: This is a model of *exchange without production*, but that's merely for simplicity. Adding production does not change the problem—though it makes it more fun

# The Edgeworth Box: Endowments and Preferences

## *A Graphical Example*





# Market Conditions for Trading

We assume that trade between  $A$  and  $B$  satisfies the following four conditions:

(C1) **No transaction costs**

(C2) **No market power**

(C3) **No externalities**

(C4) **Full information**

# What Constraints Must General Equilibrium Satisfy?

- 1 Preferences respected: No one prefers initial endowment to market equilibrium

# What Constraints Must General Equilibrium Satisfy?

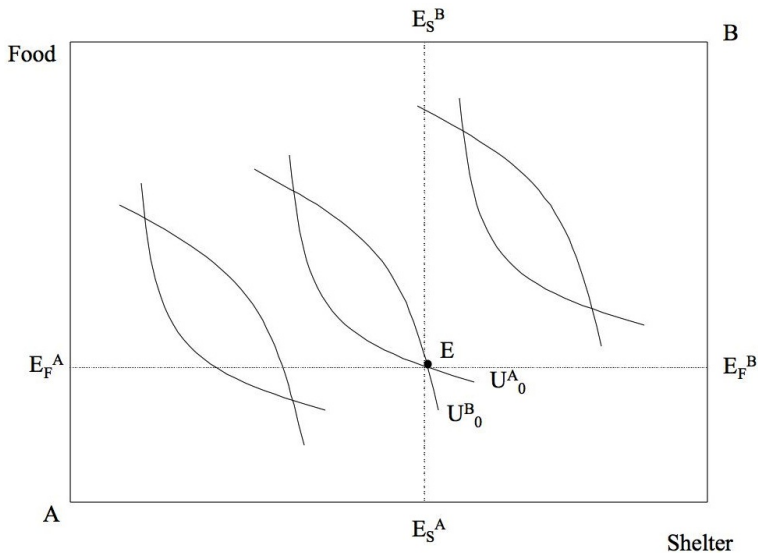
- 1 Preferences respected: No one prefers initial endowment to market equilibrium
- 2 All gains from trade exhausted: No Pareto-improving trades remain (allocative efficiency)

# What Constraints Must General Equilibrium Satisfy?

- 1 Preferences respected: No one prefers initial endowment to market equilibrium
  - 2 All gains from trade exhausted: No Pareto-improving trades remain (allocative efficiency)
  - 3 All markets clear: No excess demand or supply of any good
- Notice: there are no prices (yet) in this model. They are endogenous

# What happens when A and B trade

*Pareto improvements*



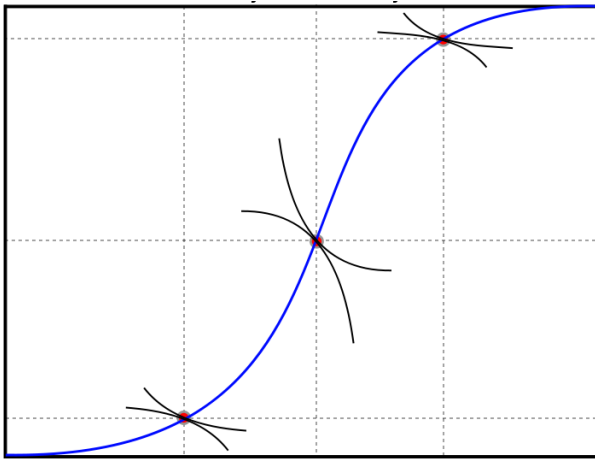
# Pareto Efficient Allocations

## *The Contract Curve*



# Pareto Efficient Allocations

## *The Contract Curve*



# How to get from $E$ to a point on the contract curve?

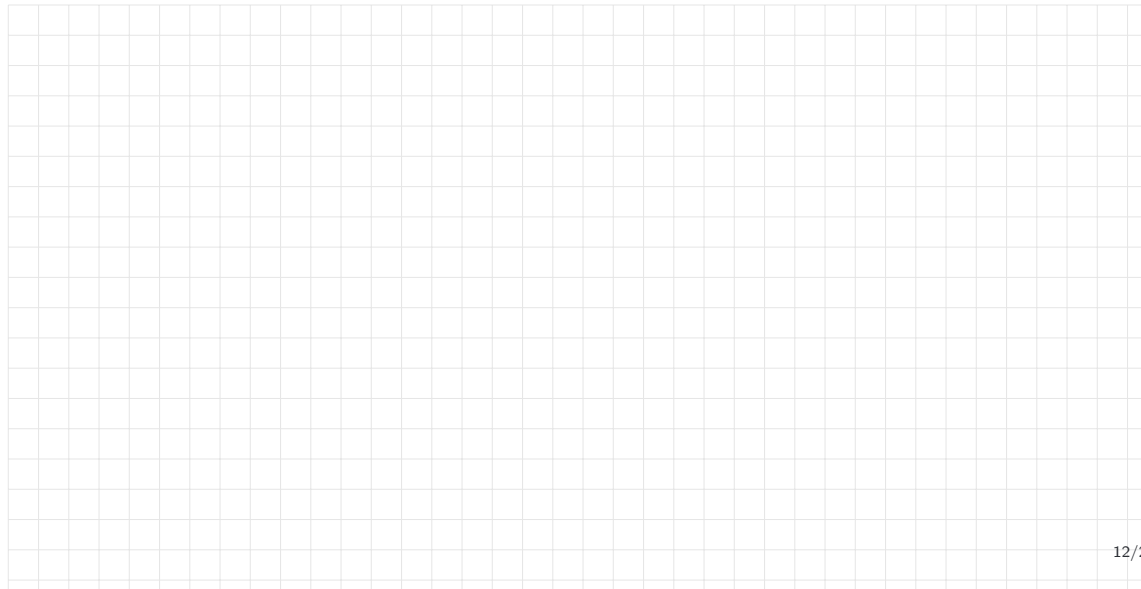
*Walrasian auctioneer*

- 1 In the initial endowment, *the market clears (that is, all goods consumed) but the allocation is not Pareto efficient.*
- 2 So, an auctioneer could announce some prices and then both parties could trade what they have for what they preferred at these prices.
- 3 Problem: *Choices would then be Pareto efficient but would not necessarily clear the market.*
- 4 It's possible there would be extra  $F$  and not enough  $S$  or vice versa.
- 5 So, must re-auction at new prices...



# Walrasian Auctioneer

*Prices with infeasible consumption plans*



# Walrasian Equilibrium

What should auctioneer do? Raise  $P_F/P_S$  (=decrease  $P_S/P_F$ ).

When the auctioneer gets the price ratio correct, the *market clears*.

- ▶ This is a market equilibrium or competitive equilibrium or Walrasian equilibrium.

In equilibrium:

- 1 Each consumer chooses his most preferred bundle given prices and his initial endowment.
- 2 All choices are compatible so that demand equals supply.
- 3 Pareto efficient consumption ('Allocative Efficiency'):  $\left(\frac{\partial U/\partial S}{\partial U/\partial F}\right)_A = \left(\frac{\partial U/\partial S}{\partial U/\partial F}\right)_B$

# Walrasian Equilibrium

## *Pareto efficiency*

How do we know **Pareto Efficiency** will be satisfied?

- ▶ Because both  $A, B$  face the same prices.
- ▶ Each person's optimal choice will therefore be the highest indifference curve that is tangent to her budget set given by the line with the slope  $P_S/P_F$  that intersects  $E$ .
- ▶ Because these choice sets (for  $A, B$ ) are separated by the price ratio, we know they will be tangent but not intersect.

## How do we reach the equilibrium?

- ▶ Leon Walras loosely proved that the market can reach this equilibrium without assistance from a central planner.
- ▶ This result—the existence of general equilibrium as a self-organizing outcome of the market—is fundamental.
- ▶ The description that Walras used was that the economy would reach equilibrium through a process of Tatonnement.

# First Welfare Theorem

*A free market, in equilibrium, is Pareto efficient*

- ▶ A competitive market exhausts all the gains from trade.
  - It is Pareto efficient.
- ▶ Note that the following conditions must be satisfied for this result to hold:
  - (C1) No externalities
  - (C2) Perfect competition
  - (C3) No transaction costs
  - (C4) Full information

It is not from the benevolence of the butcher, the brewer, or the baker, that we expect our dinner, but from their regard to their own interest. We address ourselves, not to their humanity but to their self-love, and never talk to them of our necessities but of their advantages.

– Adam Smith, *The Wealth of Nations*, 1776

Every individual necessarily labors to render the annual revenue of the society as great as he can.... He intends only his own gain, and he is in this, as in many other cases, led by an **invisible hand** to promote an end which was no part of his intention...

– Adam Smith, *The Wealth of Nations*, 1776

# First Welfare Theorem: Another Perspective

- ▶ We can think of the General Equilibrium problem as a utility maximization subject to:
  - 1 No actor is worse off in the market equilibrium than in the initial allocation.
  - 2 In equilibrium, no party can be made better off without making another party worse off.
  - 3 No more goods can be demanded/consumed than the economy is endowed with.
- ▶ The *First Welfare Theorem* says that the free market equilibrium is the solution to the above problem. Simply by allowing trade among small market actors, the market solution—that is, the price vector and resulting equilibrium choices—will satisfy the three constraints above.
- ▶ This is an important result. It implies that the decentralized market continually “solves” a complex problem that would be difficult for any individual (or large government agency) to solve by itself.



## Second Welfare Theorem

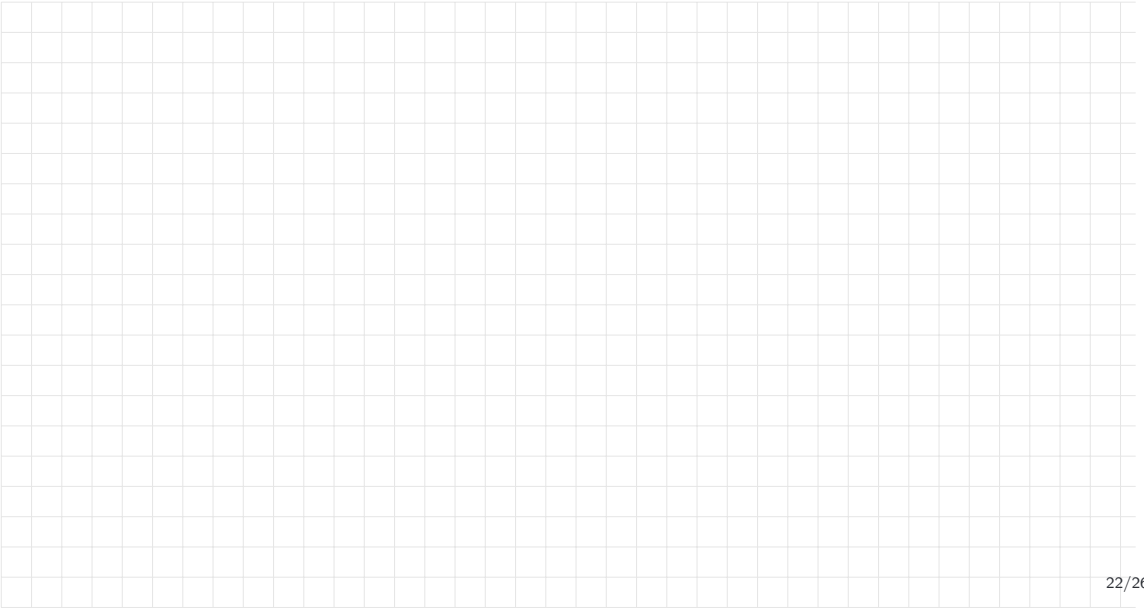
- ▶ **Question:** Does the First Welfare Theorem guarantee that the market allocation will be *fair* or *equitable*?
  - No!
  - There are many alternative Pareto efficient allocations of resources differing in allocations among parties
  - Pareto efficient allocations can be pretty ugly
- ▶ Is there a trade-off between efficiency and equity in equilibrium?
- ▶ Or: Given a Pareto efficient allocation of resources, will there exist prices and an initial endowment such that this allocation is *also* an equilibrium?
- ▶ The Second Welfare Theorem says that the answer is **yes**.
- ▶ In other words, there is **no** trade-off between equity and efficiency (*iff* lump-sum transfers are feasible)

## Second Welfare Theorem

*Providing that preferences are convex and conditions C1-C4 are satisfied, any Pareto efficient allocation can be supported as a market equilibrium.*

**Question:** If we don't like the distribution of wealth in the market equilibrium, how do we change it?

# Edgeworth and The Second Welfare Theorem



# Proof of Welfare Theorems and the Existence of General Equilibrium

- 1 Adam Smith intuited these theorems in 1776
- 2 Vilfredo Pareto and Enrico Barone proposed 1st and 2nd welfare theorems in 1930s
- 3 1st and 2nd welfare theorems proved mathematically by Oskar Lange and Maurice Allais in 1942 and 1943
- 4 Not until 1954 did Lionel McKenzie and (independently) Kenneth Arrow and Gerard Debreu prove the existence of general equilibrium in a market economy

# The Non-Obviousness of Welfare Economics

- ▶ Prior to Adam Smith—and long afterward—market behavior has been viewed with great suspicion
- ▶ In 1639 in Boston, the respected merchant Robert Keayne was charged with a crime: making over sixpence profit on the shilling, an outrageous gain
- ▶ The Boston court debated whether to ex-communicate him for his sin
- ▶ In view of his spotless past, the court instead fined him 200 pounds (a huge sum!)
- ▶ Keayne was so distraught that “with tears acknowledges his covetous and corrupt heart”

# The Non-Obviousness of Welfare Economics

The Minister of Boston used the example of Keayne's greed to denounce “**some false principles of trade**”

- ▶ That a man might sell as dear as he can, and buy as cheap as he can. [Arbitrage]
- ▶ If a man loses by casualty of sea, etc., in some of his commodities, he may raise the price of the rest. [Inward supply shifts raise prices]
- ▶ That he may sell as he bought, though he paid too dear. [Selling at the market price, independent of what you paid]

# The Non-Obviousness of Welfare Economics

- ▶ That free markets may tend to produce socially desirable outcomes is a fundamental, non-obvious insight of economics

# The Non-Obviousness of Welfare Economics

- ▶ That free markets may tend to produce socially desirable outcomes is a fundamental, non-obvious insight of economics
- ▶ 240 years after Smith wrote *Wealth of Nations*, it is not widely understood outside of the economics profession



# The Non-Obviousness of Welfare Economics

- ▶ That free markets may tend to produce socially desirable outcomes is a fundamental, non-obvious insight of economics
- ▶ 240 years after Smith wrote *Wealth of Nations*, it is not widely understood outside of the economics profession
- ▶ Nevertheless, it has had a profound effect on the organization of modern economies