Handout 11: Taxation, Student Loans, and Policy Design

1 Introduction

In this notes we explore tax policy design in two difference circumstances. The first is a usual income tax. We show how taxes distort labor choices, and who gains from taxation with equitable redistribution. In the second worked example we explore the effects of student loans on choices in the labor market. We discuss how student loans changes risk aversion, and thus changes the willingness of seniors to take a job offer or wait for a better one down the road. We compare different payment schemes (fixed vs income based) and the social welfare implications of it. The channel worked in this example has been the focus of recent research in economics. This example brings together two topics of the class: (i) choice with uncertainty and (ii) policy design to maximize social welfare.

The contrast between the examples about tax design is stark. In the first, taxation at the margin (income tax) distorts the decision, while a lump-sum tax causes no distortion. In the second, fixed student loan payments distort the labor decision, while an income based scheme (at the margin) removes the distortion. The overall message is that taxes have distortions, but the optimal tax design and its consequences depend on specific contexts.

2 Taxes and Redistribution

In this question we will explore how to design a tax and redistribution system. Individuals care about *after tax* income, which we'll call c, for consumption. (That is, we assume individuals consume all of their after tax income.). Assume an individual receives a hourly wage w. An individual's *pre*-tax income is given by $w \times (168 - n)$, where n is leisure (for the 168 hours of each week). Each individual's preferences are given by

$$U(c, n) = c - \frac{2}{3}(168 - n)^{\frac{3}{2}}$$

1. Solve for each individual's optimal leisure and consumption choice.

2. Now suppose the government imposes a "flat" income tax. This tax collects $\frac{1}{3}$ of each individual's earnings. *For the moment*, assume it takes this tax but doesn't do anything with it. Solve for each individual's new optimal leisure, labor, and consumption.

3. From now on, assume that there are two types of individuals in this society: rich and poor. The only difference between the two is that the rich have a higher wage $w_r = 9$ than the poor do ($w_p = 3$). $\frac{1}{4}$ of the population is rich, and the other $\frac{3}{4}$ are poor. Compute the total *per-capita* tax collected by the government in Item 2.

4. The government uses the tax revenue to give an equal lump-sum transfer payment T to each resident. Show that the lump sum transfer T will *not* affect the optimal labor supply of either type of individual.

5. Suppose the government sets T so that the government balances its budget. Show that this tax-plus-transfer policy is good for the poor and bad for the rich (relative to the no-tax regime from part 1).

6. Does the Rawlsian measure of social welfare increase? Does the utilitarian measure of social welfare increase?

Solution.

1. The leisure/consumption problem is given by

$$\max_{c,n} \ c - \frac{2}{3}(168-n)^{\frac{3}{2}}$$

subject to

$$c = w(168-n)$$

Therefore, we can rewrite the optimization problem as

$$\max_{n} \ w(168-n) - \frac{2}{3}(168-n)^{\frac{3}{2}}$$

Taking the FOC:

$$-w-(168-n)^{1/2} = 0 \Rightarrow n = 168-w^2$$

Therefore:

$$n = 168 - w^2 \text{ and } c = w^3$$

2. With the income tax, the labor/leisure problem becomes

$$\max_{n} \ \frac{2}{3}w(168-n) - \frac{2}{3}(168-n)^{\frac{3}{2}}$$

This is the same problem as in item 1, but now the total income - which equals consumption, is multiplied b 2/3 (1 −1/3), that is, what is left after the income tax. In this case, taking the FOC:

$$-\frac{2}{3}w-(168-n)^{1/2} = 0 \Rightarrow n = 168-w^2$$

Therefore:

$$n = 168 - \frac{4}{9}w^2 \text{ and } c = \frac{8}{27}w^3$$

Another way of solving this problem is realizing that we can use the solution of item 1 and replace w with $2/3w$, since the income tax in this case is equivalent to wage reduction.

3. Note that we can compute labor as (with taxes)

$$l = 168 - n \Rightarrow l = 168 - 168 + \frac{4}{9}w^2 = \frac{4}{9}w^2$$

For Rich consumers, we have that labor will be given by

$$l_{rich} = \frac{4}{9}w_r^2 = 36$$

Therefore, total income of rich consumers is:

$$I_{rich} = l_{rich} \times w_r = 36 \times 9 = 324$$

which the government keeps 1/3 off, that is, 108.

For Poor Consumers: we have that labor will be given by

$$l_{poor} = \frac{4}{9}w_p^2 = 4$$

Therefore, total income of rich consumers is:

$$I_{poor} = l_{poor} \times w_p = 4 \times 3 = 12$$

which the government keeps 1/3 off, that is, 4.

The per capita tax collection is thus:

$$T = \frac{1}{4} \times 108 + \frac{3}{4} \times 4 = 30$$

4. The leisure/consumption problem receiving a lump-sum transfer of T: With the income tax, the labor/leisure problem becomes

$$\max_n \; \underbrace{T + \frac{2}{3}w(168-n)}_{=c} - \frac{2}{3}(168-n)^{\frac{3}{2}}$$

This is the same problem as in item 2, but now the total income - which equals consumption, summed to T In this case, taking the FOC:

$$-\frac{2}{3}w - (168-n)^{1/2} = 0$$

which is the same as in Item 2. A lump-sum tax does not distort the choices at the margin (labor supply) as the income tax does. Importantly, this is a feature of these specific set of preferences and *not* a general conclusion.

5. If the government sets T to have a balanced budget (no deficits or surpluses), we will have that $T = 30$, the per capita tax revenue. Let's compute poor/rich utilities without any tax (item 1) and with the tax and redistribution:

   - *Poor*: without taxes, we can use the solution to item 1 to recover

   $$U_{poor}^{NoTax} = c - \frac{2}{3}(168-n)^{\frac{3}{2}} = w_p^3 - \frac{2}{3}(w_p^2)^{\frac{3}{2}} = \frac{1}{3}w_p^3 = 9$$

   with tax and redistribution (we are using here the fact that we can interpret the income tax as a change in wages (multiply it by 2/3) and that we can simply sum the tax rebate - as it does not change labor supply):

$$U_{poor}^{Tax} = T + \frac{1}{3}\frac{8}{27}w_p^3 = 30 + \frac{8}{3}$$

- *Rich*: without taxes, we can use the solution to item 1 to recover (analogous to the poor)

$$U_{rich}^{NoTax} = \frac{1}{3}w_r^3 = 243$$

with tax and redistribution :

$$U_{poor}^{Tax} = T + \frac{1}{3}\frac{8}{27}w_r^3 = 30 + 72 = 102$$

Therefore, the poor are much better off with income tax + redistribution and the rich are a worse off.

6. The Rawlsian measure of social welfare - the one where the planner/government evaluates welfare by the minimum of those in society - increases, since the poor are better off (not that it would not be enough in here to show that the poor receive more in transfers as they pay, as the income tax could create enough distortion as to generate a loss for the poor - imagine a tax close to 100%, for instance). The Utilitarian measure of social welfare - the one where the planner/government evaluates welfare by the summing the welfare of those in society [1] - decreases in this case. To see this, note that the change in the utilitarian social welfare change is given by

$$\frac{3}{4}\left[\frac{8}{3} + 30 - 9\right] + \frac{1}{4}[102 - 243] = -17.5 < 0$$

□

[1]That is, it would take \$1 of a poor person to given \$2 to Jeff Bezos.

3 Student Loans, Risk Aversion and Social Welfare

Suppose Oliver is receives a job offer at company A that pays \$85,000. Oliver believes that if he rejects the offer from company A, he will end up with a salary of:

- \$160,000 with probability 50%
- \$40,000 with probability 50%

Oliver values consumption with a utility of $U(c) = \sqrt{c}$.

1. Would Oliver find it optimal to reject or accept the offer from company A? Explain.

2. Suppose now that Dan is the same situation as Oliver. He has the same offer from company A, and if he rejects it he receives offers with the same probability as Oliver. The difference between the two is that in the first year after graduation Dan must pay \$27,400 on student loans that he took while an undergrad. Dan also has an utility over consumption of $U(c) = \sqrt{c}$. Would Dan find it optimal to reject or accept the offer from company A? (Assume his consumption is the salary minus the student loan payments. *If needed, use the following approximations:* $\sqrt{27400} \approx 165$, $\sqrt{132600} \approx 364$, $\sqrt{12600} \approx 112$, $\sqrt{57600} = 240$)

3. Suppose now that the student loan contract is instead income-based. In particular, it specifies that Dan must pay 19% of his income. Would Dan change his choice from item 5? Explain.

4. The government uses *realized* salary as a measure of welfare for each individual (and not individual consumption after the student loans). Is the Utilitarian measure of social welfare larger under the fixed student loan payments (as in iten 1 and 2) or an income-based system (as in item 3)?

Solution.

1. The expected utility of rejecting the offer is given by:

$$EU = \sqrt{160000} \times .5 + \sqrt{40000} \times .5 = 300$$

   Oliver's expected utility rejecting the offer , $EU = 300$, is higher than his utility taking the offer that pays \$ 85,000, since $\sqrt{85000} < \sqrt{90000} = 300$. Therefore, Oliver rejects the offer.

2. Dan's utility if he takes the offer from company A is:

$$EU_{take}^{Dan} = \sqrt{85000 - 27400} = \sqrt{57600} = 240$$

   Dan's utility if he rejects the offer from company A is:

$$EU_{reject}^{Dan} = \sqrt{160000 - 27400} \times .5 + \sqrt{40000 - 27400} \times .5 \approx 364 \times .5 + 112 \times .5 = 238$$

   Dan chooses the offer from company A.

3. Dan makes the same choice as Oliver. The expected utility of rejecting the offer is given by:

$$EU_{reject}^{Dan} = \sqrt{.81 \times 160000} \times .5 + \sqrt{.81 \times 40000} \times .5 = .9 \times 300$$

   and of taking it is:

$$EU_{take}^{Dan} = \sqrt{.81 \times 85000} = .9 \times \sqrt{85000}$$

   Therefore, as the .9 is just a constant, it does not affect the choice.

4. Utilitarian is larger under the income-based system, since the expected salary of those with student loans will be larger - given that they opt for the salary lottery, and thus receive \$ 100,000 on average.

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