Seminar 13 (Bertrand model) - Solution

Homogeneous products:

- Assume three firms: $MC_1 = 200, MC_2 = 200, MC_3 = 205$. The market demand function is $D(p) = 300 - p$.
  - How much will be the market price?
    A: $p = 200$. Firms 1 and 2 push the market price towards their marginal costs.
  - How much will be output of Firm 1?
    A: $y_1 = 50$. Total output ($y = 100$) is divided equally between Firms 1 and 2. Firm 3 is not able to produce anything at $p = 200$, so its output is zero.
- Assume three firms: $MC_1 = 200, MC_2 = 203, MC_3 = 205$. The market demand function is $D(p) = 300 - p$.
  - How much will be the market price?
    A: $p = 202$ (or just below 203 if you assume non-integer prices). Firm 1 has no incentive to push price down towards its marginal cost, so it will undercut marginal costs of Firm 2 by setting price below $MC_2 = 203$. Thus, Firm 2 will leave the market and Firm 1 remains the only producer.
  - How much will be the output of Firm 1?
    A: $y_1 = 98$. Firm 1 takes the whole market.
    This is the profit-maximizing optimum. If you consider $p = 202, y_1 = 98$, profit is $\pi_1 = 202 * 98 - 200 * 98 > 0$. On the other hand, profit is zero for $p = 200, y_1 = 100$ as price equals marginal costs.
    Note: This is a little controversial exercise, as we have just one firm operating in the market. The Bertrand's optimum for identical products when $p = MC$ assumes there are at least two firms with the lowest marginal costs.
- Assume two firms that compete on prices. The market demand function: $D(p) = 92 - p$; marginal cost functions: $MC_1(y_1) = 3y_1$, $MC_2(y_2) = 5y_2$. How much are the optimal outputs? How much is the market price?
  A: $y_1 = 20$, $y_2 = 12$, $p = 60$

Differentiated products:

1. Consider two gas stations in the duopoly market where gas stations compete with each other with price of gas. Gas stations have following cost functions $TC_1(y_1) = cy_1 + 4,000$ and $TC_2(y_2) = cy_2 + 4,000$. Monthly demand functions are $D_1(p_1, p_2) = 1,000 - 30p_1 + 10p_2$ and $D_2(p_1, p_2) = 1,000 - 30p_2 + 10p_1$.
   - Due to high price of oil on financial markets were costs unit costs of gasoline in September $c^0 = 20$. Calculate Bertrand equilibrium (p_1^{b0}, p_2^{b0}) and corresponding outputs and profits.
     A: $p_1^{b0} = p_2^{b0} = 32$; $y_1^{b0} = y_2^{b0} = 360$; $\Pi_1^{b0} = \Pi_2^{b0} = 320$
   - Decrease of oil price in October led to drop in unit costs of gasoline $c^1 = 10$. Because of low profits, firms decided to collude, therefore maximize joint profit $\Pi_1 + \Pi_2$ (no wealth redistribution) and choose corresponding prices p_1^k, p_2^k. Calculate these prices, outputs and profits of both firms.
     A: $p_1^k = p_2^k = 30$; $y_1^k = y_2^k = 400$; $\Pi_1^k = \Pi_2^k = 4000$
   - Both firms were punished for collusion by fine of 100 and restrictions for further collusion. Therefore, firms came back to Bertrand equilibrium in November, however with the lower marginal costs $c^1 = 10$. Calculate equilibrium price p_1^{b1}, p_2^{b1}, outputs and profits.
     A: $p_1^{b1} = p_2^{b1} = 26$; $y_1^{b1} = y_2^{b1} = 480$; $\Pi_1^{b1} = \Pi_1^{b2} = 3680$.
   - Compare Π^k and Π^{b1} and decide if the collusion was profitable in spite of the fine 100.
     A: The collusion is still profitable despite the fine: $\Pi^k > \Pi^{b1}$ $(3900 > 3680)$.

2. Levi's and Diesel are producing jeans. Each of them produce slightly different types of jeans - Levi's produces skinny jeans whereas Diesel focuses more on straight jeans. The market demand for Levi's jeans is $y_1 = 80 - p_1 + \frac{1}{2}p_2$, while the market demand for Diesel's jeans is $y_2 = 160 - p_2 + \frac{1}{2}p_1$. Levi's cost function is $TC_1(y_1) = 80y_1$ while Diesel's cost function is $TC_2(y_2) = 160y_2$.

   - Calculate the Bertrand equilibrium in this market. Indicate each firm's price, output level, and profits.
     A: $p_1 = 128$, $p_2 = 192$, $y_1 = 48$, $y_2 = 32$, $\Pi_1 = 2304$, $\Pi_2 = 1024$
   - Find prices and output levels that would maximize joint profits, and calculate the maximum joint profits.
     A: $p_1 = 146,7$, $p_2 = 213.3$, $y_1 = 40$, $y_2 = 20$, $\Pi_1 = 2666.7$, $\Pi_2 = 1066.7$