Infinitely Lived Assets

Motivation

- Introduce infinitely lived assets
- Some assets do not have a finite maturity date - for example land
- Pricing such assets is more difficult

Infinitely Lived Assets

Introducing Land

- Let A units of land give a total crop of $D(t)$ units of time t good in each period t
  - land - equity
  - crops - dividends
- Let $d(t)$ denote the amount of crop per unit of land

$$d(t) = \frac{D(t)}{A}$$

  - Sequence $\{d(t)\}_{t=1}^{\infty}$ known in advance
- Assume initial old own some land, collect crop at no cost, and then sell land to young at price $p(t)$ per unit of land

Infinitely Lived Assets

Expectations of Prices

- Need to know what individual expects price of land to be next period
- Initially, consider expectations as given arbitrarily
- Resulting equilibrium called *temporary equilibrium*
- Later, consider equilibrium concept that requires people hold correct expectations
- This is called *perfect foresight equilibrium*

Infinitely Lived Assets

Temporary Equilibrium with Land

- For simplicity, assume only assets are land and private borrowing and lending
- Budget constraints are:

$$c_t^h(t) = \omega_t^h(t) - l^h(t) - p(t)a^h(t) \tag{1}$$

$$c_t^h(t+1) = \omega_t^h(t+1) + r(t)l^h(t) \tag{2}$$
$$+ d(t+1)a^h(t) + p^{h,e}(t+1)a^h(t)$$

$l^h(t) = \ldots$

where $a^h(t)$ is quantity of land individual h at t chooses to purchase at price $p(t)$, when expecting price of land in next period to be $p^{h,e}(t+1)$

Infinitely Lived Assets

Temporary Equilibrium with Land

- Expected wealth – life-time budget constraint is:

$$\begin{aligned} c_t^h(t) + \frac{c_t^h(t+1)}{r(t)} &= \omega_t^h(t) + \frac{\omega_t^h(t+1)}{r(t)} \\ &\quad - a^h(t)\left(p(t) - \frac{d(t+1) + p^{h,e}(t+1)}{r(t)}\right) \end{aligned} \tag{3}$$

Thus, household $h's$ demand for land equals

$$a^h(t) = \begin{cases} 0 & \text{if } p(t) > \frac{d(t+1)+p^{h,e}(t+1)}{r(t)} \\ \infty & \text{if } p(t) < \frac{d(t+1)+p^{h,e}(t+1)}{r(t)} \\ ? & \text{if } p(t) = \frac{d(t+1)+p^{h,e}(t+1)}{r(t)} \end{cases}$$

Infinitely Lived Assets

Temporary Equilibrium with Land

Proposition

If there is unanimity of expectations such that

$$p^{h,e}(t+1) = p^{e}(t+1)$$

for all h of generation t, and if some land exist at t, then in any equilibrium

$$p(t) = \frac{d(t+1) + p^{e}(t+1)}{r(t)}$$

- Proposition states that the present value of the expected value of land in the next period, including crops, equals today's price

Infinitely Lived Assets

Temporary Equilibrium with Land

Definition

Given $\{u_t^h(.,.), \omega_t^h, A, D(t), D(t+1), p^e(t+1)\}$*, a time t temporary equilibrium is a pair of prices* $[r(t), p(t)]$ *and a set of allocations* $\{c_t^h, l^h(t), a^h(t)\}_{h=1}^{N(t)}$ *such that the equilibrium conditions (i)*

$$p(t) = \frac{d(t+1) + p^e(t+1)}{r(t)}$$

and (ii)

$$S_t(r(t)) = p(t)A(t)$$

are fulfilled.

Infinitely Lived Assets

Temporary Equilibrium with Land

To show that 2nd condition is equilibrium condition:

Market clearing in goods market implies

$$\sum_{h=1}^{N(t)} c_t^h(t) + \sum_{h=1}^{N(t-1)} c_{t-1}^h(t) = \sum_{h=1}^{N(t)} \omega_t^h(t) + \sum_{h=1}^{N(t-1)} \omega_{t-1}^h(t) + D(t)$$

Total consumption of old equals

$$\sum_{h=1}^{N(t-1)} c_{t-1}^h(t) = \sum_{h=1}^{N(t-1)} \omega_{t-1}^h(t) + D(t) + p(t)A$$

thus total consumption of young equals

$$\sum_{h=1}^{N(t)} c_t^h(t) = \sum_{h=1}^{N(t)} \omega_t^h(t) - p(t)A$$

thus aggregate savings equal

$$S_t\left(r(t)\right) = \sum_{h=1}^{N(t)} s_t^h(r(t)) = \sum_{h=1}^{N(t)} \left(\omega_t^h(t) - c_t^h(t)\right) = p(t)A$$

Infinitely Lived Assets

Temporary Equilibrium with Land

Prices defining a temporary equilibrium can be found as before by solving for $r(t)$:

$$S_t\left(r(t)\right) = \frac{d(t+1) + p^e(t+1)}{r(t)} A$$

If $S_t(.)$ is increasing in $r(t)$ there is at least one $r(t)$ that solves this equation

Alternatively, solving for $p(t)$:

$$S_t\left(\frac{d(t+1) + p^e(t+1)}{p(t)}\right) = p(t)A$$

a *price function* can be defined:

$$p(t) = f_t(p^e(t+1), d(t+1), A)$$

Infinitely Lived Assets

Perfect Foresight Equilibrium with Land

Definition

A perfect foresight competitive equilibrium with land is a sequence of prices $p(t)$ and $r(t)$ and other endogenous variables such that the time t values are a temporary equilibrium for

$$p^e(t+1) = p(t+1)$$

Infinitely Lived Assets

Perfect Foresight Equilibrium with Land

The price of land that is expected to hold in the next period is always the price that does hold.

From now on, refer to perfect foresight equilibrium simply as an *equilibrium*

Infinitely Lived Assets
Finding Equilibrium

Finding an equilibrium with guess and verify

1. Choose a price at t and see whether the implied prices at $t+1$ obey the equilibrium conditions or
2. Guess that a stationary equilibrium exist
3. Economy imposes some natural restrictions on price sequence, such as ruling out negative prices or explosive price sequences

Definition

Given an economy with a stationary environment in the sense that the environmental variables are the same in each period, the stationary equilibrium is a perfect foresight equilibrium in which consumptions are the same for every generation.

Example

Consider a two-period OLG environment with land.
Let $u_t^h = \log c_t^h(t) + \beta \log c_t^h(t+1)$ *for all* h, t.
Also, let $\omega_t^h = [2, 1]$ *for all* h, t *and let* $N(t) = 1$, $A = 1$ *and* $d(t) = 1$ *for all* t.
Let $\beta = 1$.

Solve for the price of land in the stationary equilibrium.

$$s_t^h = \frac{\omega_t^h(t)}{2} - \frac{\omega_t^h(t+1)}{2r(t)}$$ endowments [2, 1]

$$\rightarrow 1 - \frac{1}{2r(t)}$$

$$S(t) = p(t)A \qquad r(t) = \frac{d(t+1) + p(t+1)}{p(t)}$$

$$\rightarrow 1 - \frac{1}{2}\frac{p(t)}{1+p(t+1)} = p(t)$$

$$p(t+1) = p(t) = p$$

$$\rightarrow 2p^2 + p - 2 = 0$$

Remember: $ax^2+bx+c=0$ $\quad x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$p \sim 0.78$ (other root negative)

What if crop is zero?

$d(t)=0 \rightarrow p(t+1) = \frac{1}{2}\dfrac{p(t)}{1-p(t)}$

$\rightarrow p=0$ and $p=1/2$

Consistent with equilibrium not to value something that's fundamentally worthless

But model economy crying out for asset that makes inter-generational trade possible