Applied Econometric Time Series – Problem Set 4

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1. (a) X_t which is an I(1) process is said to be cointegrated if there exists a cointegrating vector β ($\neq 0$) such that the linear combination $X_t.\beta'$ is an I(0) process.

   r_{St} and r_{Lt} are both I(1) processes. However, the model uses error correction so that $\delta.r_{St}$ and $\delta.r_{Lt}$ are I(0) processes. All differences are stationary processes and ϵ_{St} and ϵ_{Lt} are also stationary since they are white noise processes. For the given model to be valid, the only remaining term of $\alpha.\beta'.X_{t-1}$ must also be stationary or an I(0) process. Thus, the two variables r_{St} and r_{Lt} must be cointegrated.

   The cointegrating vector is $\beta = \begin{bmatrix} 1 \\ -\beta \\ -\mu \end{bmatrix}$
   The long run equilibrium is the long run relationship between the two variables r_{St} and r_{Lt}. This is represented by:

$$r_{Lt} = \beta.r_{St} + \mu$$

   The intercept μ is included in the cointegrating relationship to allow for a non-zero intercept. It also represents the trend in the data i.e. a random walk process representing the stochastic trend in the variable.
   The restrictions $\alpha_S.\mu = 0$ and $\alpha_L.\mu = 0$ should be imposed if it is certain that the underlying series are random walks without a drift. The μ here represents the drift term, hence its effect scaled by the speed of adjustment parameter must be null for data which is a random walk without a drift.

   (b) If r_{lt} does not Granger cause r_{st}, then all the lags of that variable can not be used to predict of r_{st}. This can be formulated as the hypothesis $a_{1,12} = a_S = 0$ for this system.

   (c) Describe the adjustment mechanisms towards the long-run equilibrium.

   The long-run equilibrium of Δr_{Lt} and Δr_{St} through the error-correction mechanism is dependent of changes in long-term and short-term . We begin by assuming that we have $\alpha_S, \alpha_L \neq 0$. Since r_{Lt} and r_{St} is cointegrated, then any deviation is a stat process with mean zero so the system will return to equilibrium over time.

   We start by assuming no previous stochastic shocks, then we at time t have that:

$$\Delta r_{St} = \alpha_S(r_{Lt-1} - \beta r_{St-1} - \mu) \tag{1}$$
$$\Delta r_{Lt} = \alpha_L(r_{Lt-1} - \beta r_{St-1} - \mu) \tag{2}$$

   Given $r_{Lt-1} > \beta r_{St-1} + \mu$, it follows that $r_{Lt-1} - \beta r_{St-1} - \mu > 0$. This condition influences Δr_{St} proportionally to $\alpha_S(r_{Lt-1} - \beta r_{St-1} - \mu)$, necessitating $\alpha_S > 0$ for the short term rate increase needed for equilibrium. Conversely, as r_{Lt} exceeds r_{St}, the long term rate must decrease, hence Δr_{Lt} is affected by $\alpha_L(r_{Lt-1} - \beta r_{St-1} - \mu)$ and requires $\alpha_L < 0$ to ensure an equilibrium is eventually met.

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In period $t+1$ the error correction will come from both the first term parenthesis as described above, but also from the $\alpha_{1,11}\Delta r_{St} + \alpha_{1,12}\Delta r_{Lt}$ and $\alpha_{1,21}\Delta r_{St} + \alpha_{1,22}\Delta r_{Lt}$.

In the case of weakly exogenous r_{Lt}, the error correction will only come from:

$$\Delta r_{St} = \alpha_S(r_{Lt-1} - \beta r_{St-1} - \mu) \tag{3}$$
$$\Delta r_{Lt} = 0 \tag{4}$$

(d) This will be a spurious regression. The OLS coefficient estimate requires stationarity, but the variables are not. This will for example imply that the long run variances are not finite, which is a violation of OLS assumptions. The regression could yield significant results due to a drift in both varibles, without there being an actual relationship between the two.

2. (a) Deleting lags until the t-statistic on the last lag is significant at the 5% level we obtain lags $p = (7, 7, 5)$ and ADF test results $\hat{a}_1 = (-1.613, -0.785, -1.033)$ and $t = (-1.613, -0.785, -1.033)$ for tbill, r5 and r10 respectively. Since the p-values are insignificant the data are consistent with individual rates acting as I(1) process.

```
foreach var of varlist tbill r5 r10 {
forvalues i=10(-1)0 {dfuller `var', lags(`i') regress}}
```

		Coefficient	Std. err.	t	P>|t|
D.tbill	L1.	−.0276525	.0171431	−1.61	0.108
	L7D.	−.1984513	.0704491	−2.82	0.005
D.r5	L1.	−.0112213	.0142965	−0.78	0.433
	L7D.	−.1700385	.0709976	−2.39	0.018
D.r10	L1.	−.0131819	.0127584	−1.03	0.303
	L5D.	−.1460179	.0704762	−2.07	0.040

```
Augmented Dickey-Fuller test for unit root    Number of obs  = 204
H0: Random walk without drift, d = 0          Number of lags = 7
                                   MacKinnon approximate
Variable: tbill                  p-value for Z(t) = 0.4764.
Variable: r5                     p-value for Z(t) = 0.8235.
Variable: r10                    p-value for Z(t) = 0.7410.
```

(b) Given that each rate acts as a unit-root process, we can begin by estimating the long-run equilibrium relationship. Using tbill as the dependent variable, we find

```
regress tbill r5 r10
```

tbill	Coefficient	Std. err.	t
r5	2.743012	.1320313	20.78
r10	−1.905496	.1417471	−13.44
_cons	.3667713	.158628	2.31

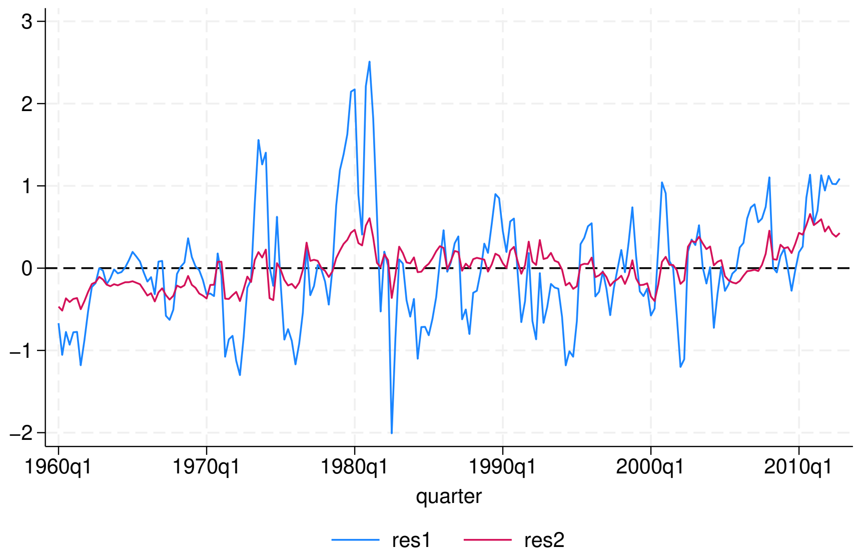
Next, we perform the no constant Engle–Granger as the residuals from regression with an intercept are mean zero. Using the same sequential rule, we find that it is appropriate to use eight lags in the augmented form of the test. Since the test statistic -4.081 is less than the 5% critical value of about -3.76 we conclude that the data are in line with the variables being cointegrated.

```
forvalues i=10(-1)0 {dfuller res1, lags(`i') noconstant regress}
H0: Random walk without drift, a = 0, d = 0
      D.res1 | Coefficient  Std. err.      t    P>|t|
-------------+----------------------------------------
res1     L1. |  -.275921    .0676191    -4.08   0.000
        L8D. |   .168825    .0711123     2.37   0.019
-------------+----------------------------------------
Test statistic Z(t)=-4.081
```

(c) Using r10 as the dependent variable the data do not support the three interest rates being cointegrated. Using 6 lags in the augmented form of the Engle–Granger test we find t-statistic of $-2.34 > -3.76$ and fail to reject the null.

```
regress r10 tbill r5, predict res2, res
         r10 | Coefficient  Std. err.      t
-------------+------------------------------
       tbill | -.2433523    .0181026    -13.44
          r5 |  1.163033    .0187604     61.99
       _cons |   .502806    .0456741     11.01

forvalues i=10(-1)0 {dfuller res2, lags(`i') noconstant regress}
H0: Random walk without drift, a = 0, d = 0
      D.res2 | Coefficient  Std. err.      t    P>|t|
-------------+----------------------------------------
res1     L1. |  -.1029057   .0439012    -2.34   0.020
        L6D. |  -.2116664   .0695921    -3.04   0.003
-------------+----------------------------------------
Test statistic Z(t)=-2.344
```



(d) i. We estimate the model using the Johansen procedure. use 7 lags (one less than the associated VAR) and include a constant in the cointegrating vector.

```
vecrank tbill r5 r10, trend(rconstant) lags(8) max
Trend: Restricted constant      Number of obs  = 204
Sample: 1962q1 thru 2012q4      Number of lags =   8
Max rank   Params      LL       Eigenvalue     max stat    trace stat
0          63      -59.047374            .      37.8328       45.5024
1          69      -40.130982      0.16927       6.8929        7.6697*
2          73      -36.684529      0.03322       0.7768        0.7768
```

To test the null hypothesis of no cointegration against the general alternative of cointegrating vectors compare the sum 37.8328 + 6.8929 to the 5% critical value of the trace statistic. Since 45.5024 exceeds the critical value of 34.91, reject the null and conclude that the data is in support of at least one cointegrating vector. To test the null of one cointegrating vector against the alternative of more than one cointegrating vector, compare the sample value of 7.6697 the 5% critical value of 19.96. Conclude that the data resembles one cointegrating vector. This result is reinforced by the max statistics. The null hypothe-sis of no cointegrating vectors (r = 0) against the specific alternative r = 1 is clearly rejected. The value 37.8328 exceeds the 5% critical value of 22.00. The test of the null hypothesis r = 1 against the specific alternative r = 2 cannot be rejected at the 5%, nor at the 10%, significance level. The value is 6.8929, whereas the critical values at the 5 and 10% significance levels are 15.67 and 13.75, respectively.

ii. We verify the cointegrating vector (assuming r5 and r10 typo switch).

```
qui: vec tbill r5  r10, trend(rconstant) lags(8) rank(1)
matrix betahat = e(beta)*1.99, matrix list betahat
betahat[1,4]   tbill          r5          r10        _cons
beta             1.99   -2.6646288    .87674226    .82675533
```

The result, normalized with respect to tbill $tbill - 1.34r_5 + .44r_{10} + .42 = 0$ is not the same as the one obtained in (b) $tbill - 2.74r_5 + 1.90r_{10} - .37 = 0$. Yet, the sum of the coefficients on the long term interest rates is similar, around -0.9.

(e)
```
reg r5 r10, predict res3, res
reg r10 r5, predict res4, res
for values i=10(-1)0 {dfuller res3, lags(`i') nocons regress}
forvalues i=10(-1)0 {dfuller res4, lags(`i') nocons regress}

L5D.res3 | P>|t| =  0.012
L5D.res4 | P>|t| =  0.009

H0: Random walk without drift, a = 0, d = 0
Z(t)_res3              -3.258
Z(t)_res4              -3.193
```

In a model with 2 variables and around 200 usable observations, we refer to the Engle-Granger 5% critical value -3.368 and the 1% value -3.95. We follow the sequential rule and augment the test with 5 lags of the residuals from both the regression of r5 on r10 and r10 on r5 and get t-statistics of -3.258 and -3.1938 respectively. We fail to reject the null in both cases and conclude that the data suggest no cointegration in the U.S. 5-year/10-year interest rate pair.