Inequalities, Household Behavior and the Macroeconomy
(Income and consumption inequality over the life-cycle)

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Last two lectures

- Found that uncertainty increases savings – especially for the poor
- In data, people with higher income uncertainty indeed seem to hold more wealth
- Precautionary saving to create a buffer against risk $\Rightarrow$ wealth inequality

Today and next time

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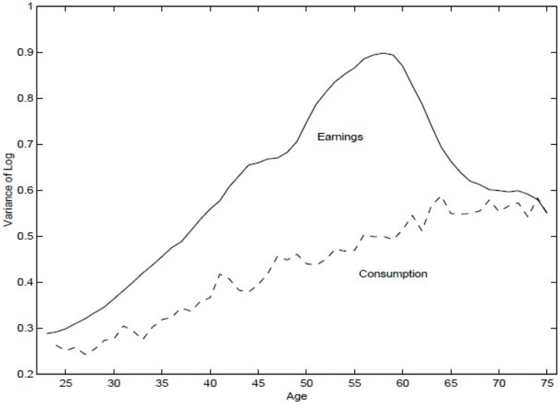
- Next time: 3 (+ inheritence)

Consumption and earnings inequality over age

Variance of Earnings and Consumption over age, US; CEX, PSID



Facts:

- Inequalities in earnings and consumption rise with age
- The rise in consumption inequality is less steep

Why is that?

Roadmap for Today

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- Compare resulting consumption patterns with data
  - Let's say we get a good match. How to understand why? – We turn off some features of the model and see how the match gets ruined!

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- Quantify welfare effects of income risk

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A model of log income

We model **log** income. Why?

- Distribution of log income is more well-behaved
- Easier to produce high earnings inequality, as the right tail is 'brought closer' by log
- Model should work well both for people with low or high earnings:
  - +200 SEK might mean a lot to the poor, means nothing to the rich
  - +10% has a meaning for everybody

$\Rightarrow$ better to model multiplicative changes

  - multiplicative changes $\Leftrightarrow$ additive changes in logs

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Source of confusion: Two conceptual ways of grouping components:

We are interested in this split:

1. Fixed/determinate characteristics
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a is not the same as 1:

- Business cycles are not idiosyncratic, but random
- Abilities are idiosyncratic, but partially predetermined

General procedure

$$\log(y_{i,a,c,t}) = age_a + year_t + cohort_c + education_i + gender_i + race_i + \cdots + u_{i,t}$$

- $y_{i,a,c,t}$ is the income of household i of age a and cohort c in year t.
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- $year_t$ matters due to business cycle
- $cohort_c$ matters due to time of entering labor market

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- $u_{i,t}$ is error term $\rightarrow$ contains everything random and idiosyncratic

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Two problems:

- Multicollinearity in age, year, cohort. Current year and age determine when you were born!
- Who knows if a linear model is sensible

Procedure in STY

Formally, they model earnings as follows:

$$\log(y_{i,a,c,t}) = age_a + cohort_c + u_{i,t}$$

$$\text{where } u_{i,h} = \alpha_i + \epsilon_{i,h} + z_{i,h} \text{ and } z_{i,h} = \rho z_{i,h-1} + \eta_{i,h}$$

$$\eta_{i,h} \sim \mathcal{N}(0, \sigma^2_\eta) \quad \epsilon_{i,h} \sim \mathcal{N}(0, \sigma^2_\epsilon) \quad \alpha_i \sim \mathcal{N}(0, \sigma^2_\alpha)$$

Household fixed effects are represented by α_i, while "luck" is represented by the iid earnings shocks $\epsilon_{i,h}$ and the persistent earnings shock $z_{i,h}$.

- α_i picks up everything indexed by i in previous slide and unobservables too.

Identifying parameters

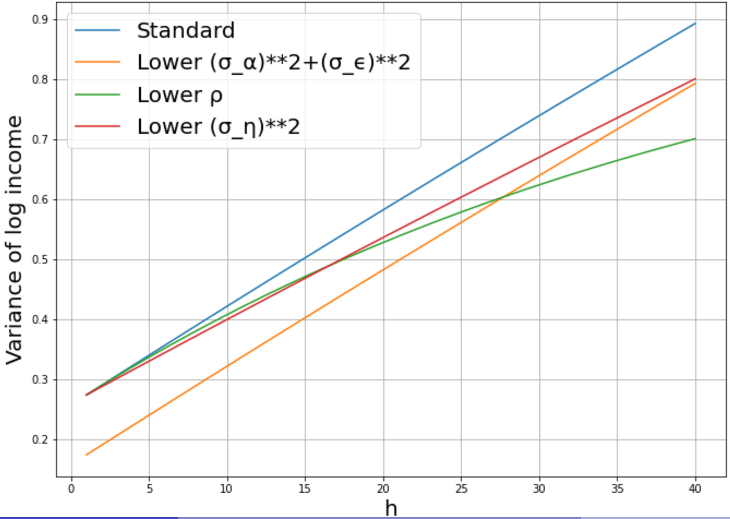
Can the earnings process described above reproduce the increase in cross-sectional inequality over the life cycle?

Yes! If we set $z_{i,0} = 0$ for everyone, the cross-sectional variance is

$$Var(u_{i,h}) = \sigma_\alpha^2 + \sigma_\epsilon^2 + \sigma_\eta^2 \sum_{j=0}^{h-1} \rho^{2j}$$

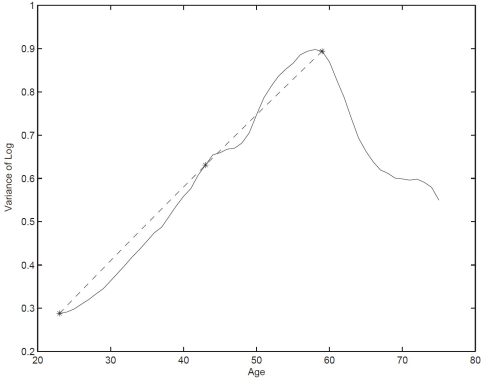
$\sigma_\alpha^2 + \sigma_\epsilon^2$ is informative about the variance in $h = 0$, while σ_η and ρ tell us how fast the variance increases with age and with which curvature (linear, concave or flat)!

Identifying parameters



Identifying parameters

STY set $\sigma_\alpha^2 + \sigma_\epsilon^2$, σ_η^2 and ρ to match the cross sectional earnings variance in the data at selected ages "*", as shown below



They get $\sigma_\alpha^2 + \sigma_\epsilon^2 = 0.27$, $\sigma_\eta^2 = 0.0166$ and $\rho = 0.9989$

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Problem - (Bellman equation form)

Consider a slightly more simplified model than the one we saw above with our usual notation

$$V(\alpha, t, z_t, x_t) = \max_{a_t \geq 0} \frac{(x_t - a_t)^{1-\gamma}}{1-\gamma} + \xi_t \beta E_t V(\alpha, t+1, z_{t+1}, x_{t+1})$$

$$s.t. \quad x_{t+1} = (1+r)a_t + y_{t+1}$$

$$y_t = \begin{cases} \exp\left(\alpha + gt + k_t + z_t + \epsilon_t\right) & \text{if } t \leq 65 \\ B \cdot \bar{Y} & \text{if } t > 65 \end{cases}$$

$$\text{where } z_t = \rho z_{t-1} + \eta_t$$

$$\eta \sim \mathcal{N}(\sigma_\eta^2) \quad \epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

ξ_t is conditional probability of survival, g is growth rate, k is age fixed-effect, B is replacament ratio, $\bar{Y}$ is average income of working age population.

Differences from STY

Our setup is simpler. They have:

- GE
- process for labor supply, not earnings directly
- more sophisticated social security: pension depends on average earnings over working life
- taxes, balance government budget
- allow for borrowing

Solving the model

In 40_*lifecycle_solve.jl*

- Lifecycle with borrowing limit, like 30_*ageprofile.jl*
- Endogenous grid point method, like 31_*infinite_horizon.jl*
- Load realistic age profile of earnings and survival probabilities
- Future earnings depend on α and z. These become state variables! (α is constant over life, $z_{i,t}$ helps predict $z_{i,t+1}$)
- Continuous random variables as shocks (α, ϵ, η), need to discretize them!

Parametrization—Discretizing the earnings process

As you know, we need to make continuous variables discrete on the computer

For ϵ: we consider some values $(\epsilon_1, \ldots, \epsilon_{N_\epsilon})$ that stay the same during the household life and draw from them using appropriate probabilities (no need to know details)

Parametrization—Discretizing the earnings process

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For z_t this is trickier! The idea is to pick a grid $\mathbf{z} = [z_1, z_2, ..., z_{N_z}]$ and a transition matrix Π with dimension $N_z \times N_z$ to approximate the continuous stochastic process z_t

Π gives the probabilities of ending up in each gridpoint of $\mathbf{z}$ tomorrow, given current state $i \in \mathbf{z}$.

If $N_z = 2$, then $\Pi = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} =$

$$\begin{bmatrix} \text{Probability of } z_1 \text{ tomorrow if today } z_1 & \text{Probability of } z_2 \text{ tomorrow if today } z_1 \\ \text{Probability of } z_1 \text{ tomorrow if today } z_2 & \text{Probability of } z_2 \text{ tomorrow if today } z_2 \end{bmatrix}$$

Parametrization—Discretizing the earnings process

The question is how to choose the grid $\mathbf{z}$ and transition matrix Π

Rouwenhorst (95) came up with a technique (no need for you to know the details...) to pick an optimal grid $\mathbf{z}$ and matrix Π for a given ρ and σ_η of the original process

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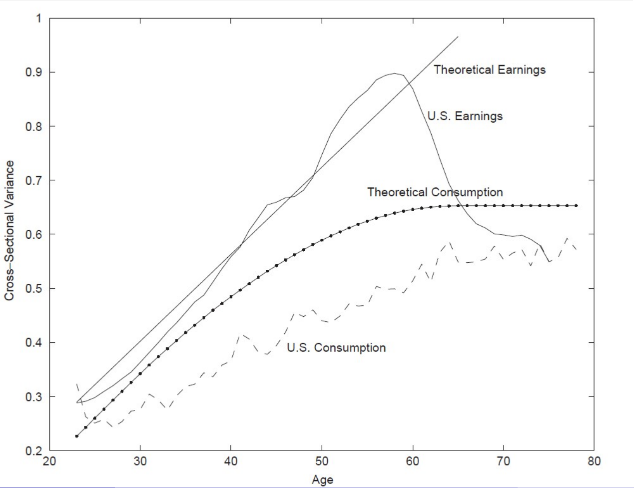
The role of Incomplete Markets

The question is *how much* a model with partial insurance can get close the data

STY, after calibrating earnings as we have seen above, got the following results



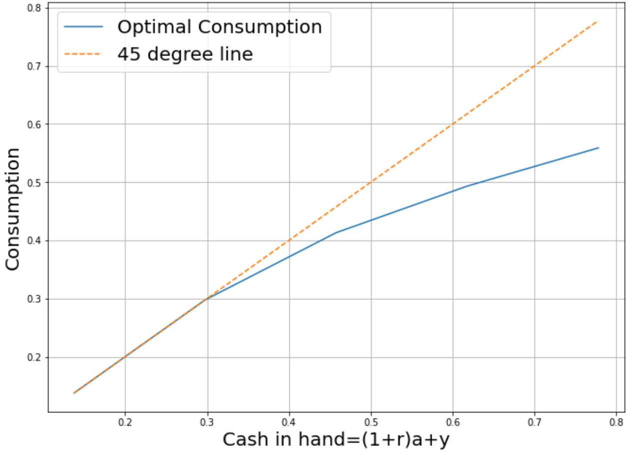
The role of Incomplete Markets

By simulating the model "switching off" one part of the model at the time, they can understand which ingredients were crucial to getting this result

They learned that:

1. Tighter borrowing constraints ($a_h \geq 0$) increase consumption inequality among younger individuals, but the effect is quantitatively small.
2. Risk aversion γ increases precautionary savings substantially, but only if β stays the same...
3. Precautionary savings play an important role in explaining the lower rise of consumption than earnings inequality over age. If the model featured quadratic utilities, the rise in consumption inequality would be very similar to the rise in earnings inequality
4. Without social security, consumption inequality would be too high

The role of Incomplete Markets



Excess savings from precautionary savings of non-borrowing constrained individuals (aka the rich) decrease their consumption: this generates less consumption inequality than in autarky

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Interpreting if value function is higher in a model than another

- Recently, we have mostly seen V in the Bellman equation
- But it is still true that

$$V_0(state_0) = \mathbb{E}_0 \sum_{t=0}^{\infty \text{ or } T} \beta^t u(c_t)$$

given constraints in your model

$\rightarrow$ V expresses total discounted utility

- Higher V is better!
- Given your preferences (u and β), would you be happier in setting A or B?
  1. solve both models
  2. compare corresponding value functions
- Comparison is not ok, if your preferences are not the same across settings

Quantifying welfare differences

- So let's say $V_0^A(state_0) > V_0^B(state_0)$. Then being born in setting A is better. By how much?

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- Nice answer when $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$.
- Imagine you are in setting B, but someone increases your consumption by $d\%$ every period? What would be your value?

$$\mathbb{E}_0 \sum_{t=0}^{\infty \text{ or } T} \beta^t \frac{((1+d)\cdot c_t)^{1-\gamma}}{1-\gamma} = (1+d)^{1-\gamma} \mathbb{E}_0 \sum_{t=0}^{\infty \text{ or } T} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} = (1+d)^{1-\gamma} V_0^B$$

Quantifying welfare differences

- So let's say $V_0^A(state_0) > V_0^B(state_0)$. Then being born in setting A is better. By how much?
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- which d would make you indifferent between model A and B?

$$(1+d)^{1-\gamma} V_0^B = V_0^A \quad \Rightarrow \quad 1+d = \left(\frac{V_0^A}{V_0^B}\right)^{\frac{1}{1-\gamma}}$$

d has a meaningful interpretation (extra consumption in percents, relative to optimal path)