Handout 7: Monopoly

1 Introduction

In this handout we will explore a market under the control of a monopolist. In Handout 7, we discussed how to get from the problem of the firm to the supply in a competitive market. Now, we will do the same, but in a situation where firms do not take the price as given. In particular, we assume that there is only one firm in the market, which internalizes that it can *increase* market price if it reduces quantity. We first do an example where the monopolist cannot discriminate prices and compute equilibrium outcomes and surpluses of consumers and producers. Second, we discuss price discrimination and how the equilibrium price is related to the elasticity of demand.

2 Monopoly

Consider a monopolist that faces the market demand $Q = (40 - P)/3$. The cost of Q to produce the good is given by is given by $C(Q) = Q^2$. The monopolist can charge only one price (no price discrimination).

1. Find the marginal revenue and the marginal cost as a function of Q. Draw them along with the demand curve.

2. Find the profit-maximizing price and quantity. What is the marginal cost at the profit-maximizing quantity?

3. How much profits does the monopolist make?

4. Calculate the producer surplus, consumer surplus and total surplus in the market. Illustrate them in a graph.

5. What is the price elasticity of demand at the equilibrium point? What is the markup of the monopolist, defined by $(P - MC)/P$ (this is known as the Lerner index)?

Solution.

1. The Marginal cost (MC) is given by

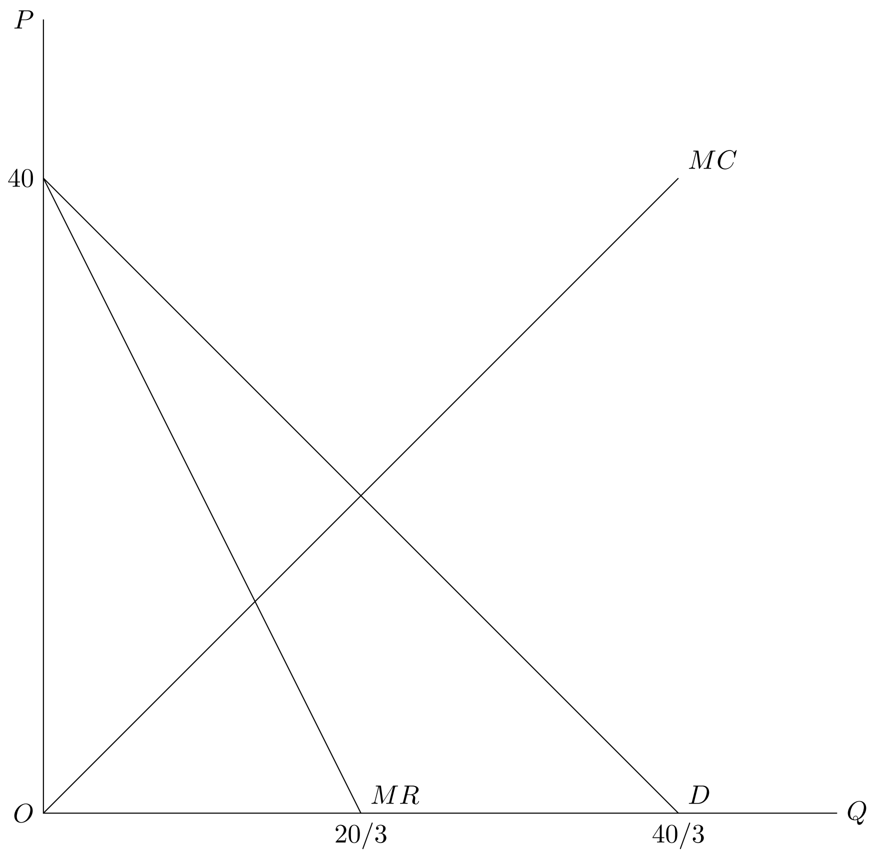
$$MC = \frac{dC(Q)}{dQ} \Rightarrow MC = 2Q$$

and marginal revenue is

$$MR = \frac{d(P(Q)Q)}{dQ} = \frac{d(40Q - 3Q^2)}{dQ} \Rightarrow MR = 40 - 6Q$$

We first invert the market demand to get $P = 40 - 3Q$, multiply this by Q, and finally differentiate w.r.t. Q. We show all curves in Figure 1.

Figure 1: Demand, MR and MC



2. Setting $MR = MC$ (the profit maximizing condition of the firm) implies profit-maximizing quantity

$$40 - 6Q^* = 2Q^* \Rightarrow Q^* = 5$$

To find the price, we replace this quantity in the demand to recover: $P^* = 40 - 3Q = 25$. Finally, the marginal cost at this quantity equals $MC(5) = 10$.

3. The profit level of the monopolist is given by total revenue (price times quantity) minus total

cost, that is

$$\pi = P^*Q^* - C(Q^*) = 25 \times 5 - 25 \Rightarrow \pi = 100$$

4. The consumer surplus (CS) is the are above the equilibrium price and below the demand. Recall that the equilibrium quantity is where $MC = MR$, but the equilibrium price is that on the demand at the equilibrium quantity, as shown in Figure 2. The consumer surplus is the are of the blue triangle:

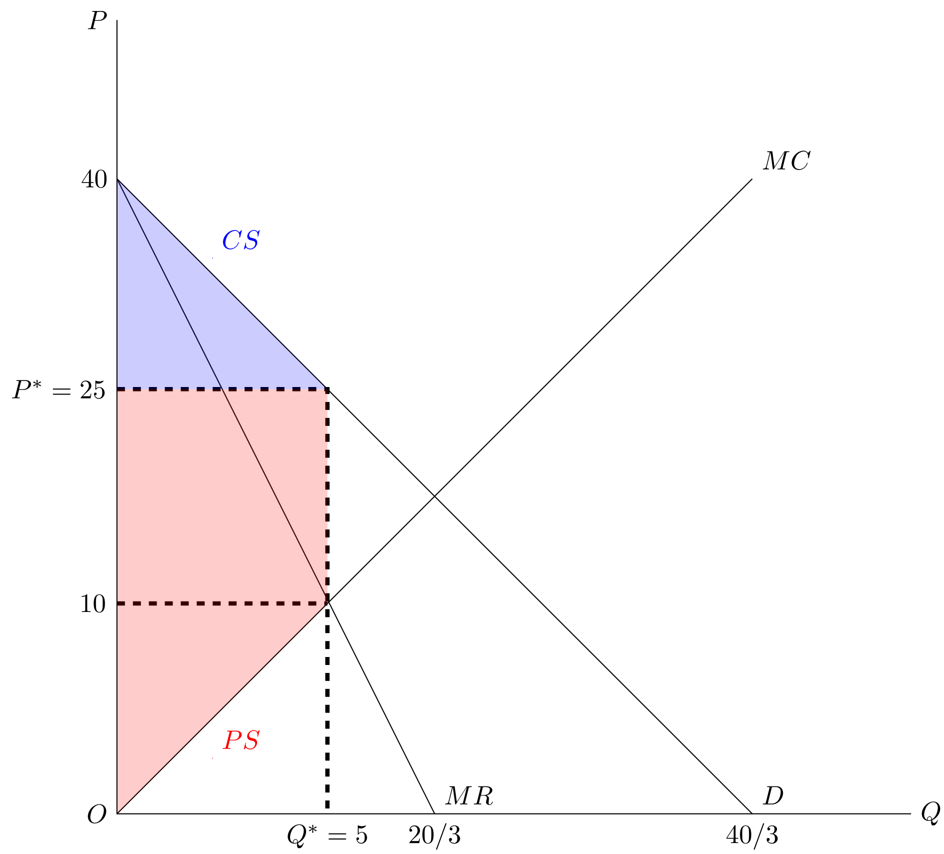
$$CS = \frac{(40-25)\times 5}{2} = \frac{75}{2}$$

The producer surplus is the area of the trapeze, which can be computed as

$$PS = \frac{(25+(25-10))\times 5}{2} = 100$$

The total surplus is the sum of CS and PS, thus: $TS = 275/2$.

Figure 2: Consumer, Producer and Total Surplus



5. The elasticity of demand at any point is given by

$$\epsilon_D = \frac{dQ}{dP}\frac{P}{Q} = -\frac{P}{40-P}$$

Therefore, at the equilibrium price, the elasticity is $-25/15 = -5/3$. Remembering the formula for markup: $\frac{P-MC}{P} = -\frac{1}{\epsilon_D}$, we get the markup 3/5. If you don't remember the formula for markup, here is a derivation. Consider the maximization problem of any monopolist that faces inverse demand $P(Q)$ and has a marginal cost c:

$$\max_Q \quad (P(Q)-c)Q$$

where $P(Q)$ is the inverse demand function. Taking the FOC:

$$P(Q) + Q\frac{dP(Q)}{dQ} = c$$

Therefore:

$$1 + \frac{dP}{dQ}\frac{Q}{P} = \frac{c}{P}$$

Note that

$$\frac{dP(Q)}{dQ}\frac{Q}{P} = \frac{1}{\epsilon_D}$$

since the derivative of the inverse of a function is the inverse of the derivative. Therefore:

$$\frac{P-c}{P} = -\frac{1}{\epsilon_D}$$

□

3 Price Discrimination

A price discriminating monopolist sells a good for two types of consumers: young and old. Old consumers have an aggregate demand given by:

$$Q_D^{OLD} = 110 - \frac{p}{4}$$

While young consumers have an aggregate demand given by

$$Q_D^{Young} = 100 - \frac{p}{2}$$

The cost of production is $C(Q) = Q^2$.

1. Find the equilibrium price, quantity and aggregate demand elasticity if the monopolist does not distinguish between Old and Young Consumers.

2. Find the equilibrium prices, quantities and aggregate demand elasticities of Young and Old if the monopolist can price discriminate between Old and Young consumers.

3. Are Young consumers better or worse off with price discrimination?

Solution.

1. We must first compute the aggegate demand curve. For $100 - p/2 \geq 0$, both Young and Old consume positive amounts of the good - and we sum their demands. For $100 - p/2 \leq 0$, only Old consumers demand the good. For $110 - p/4 \leq 0$, no consumer demands the good. Together:

$$Q_D = \begin{cases} 210 - \frac{3}{4}p, & \text{if } p \leq 200 \\ 110 - p/4, & \text{if } 200 < p \leq 440 \\ 0, & \text{if } p > 440 \end{cases}$$

To find out what the price maximizing profit for the monopolist is, we must separate the three cases above. **Case 1:** Sell to both Young and Old. $p \leq 200$. If $p < 200$, the inverse demand equation is given by

$$p = 280 - \frac{4}{3}Q$$

Therefore, for the monopolist to maximize profits with $MR = MC$

$$MR = \frac{d(\left[(280 - \frac{4}{3}Q)Q\right]}{dQ} = 280 - \frac{8}{3}Q = 2Q = MC \Rightarrow 280 - \frac{8}{3}Q = 2Q$$

Which implies $Q_1 = 60$ (Q of the first case). Replacing at the demand, we get $P_1 = 280 - \frac{4}{3}Q = 200$, which satisfies the condition that $p \leq 200$ we started with. Therefore, profits in the first case are

$$\pi_1 = 200 \times 60 - 60^2 = 8400$$

Note that as $\pi_1 > 0$, we do not have to consider $p > 440$ as a case (the third case), since it would yield zero profits. We have only to additionally consider case 2, where $200 < p \leq 440$.

Case 2: Sell to Old Only. $200 < p \leq 440$. IThe inverse demand equation is given by

$$p = 440 - 4Q$$

Therefore, for the monopolist to maximize profits with $MR = MC$

$$MR = \frac{d([(440 - 4Q)Q]}{dQ} = 440 - 8Q = 2Q = MC \Rightarrow 440 - 8D = 2Q$$

Which implies $Q_2 = 44$ (Q of the second case). Replacing at the demand, we get $P_2 = 264$, which satisfies the condition that $200 < p \leq 440$ we started with. Therefore, profits in the second case are

$$\pi_2 = 264 \times 44 - 44^2 = 9680$$

Conclusion. The monopolist is better off in case 2, where it excludes young people from the market to focus solely on the old. Old consumers are willing to pay more, and thus the monopolist can decrease quantities (and costs) and increase prices to maximize its profits. Therefore, the equilibrium price is $p = 264$, the equilibrium quantity is $Q = 44$ and the elasticity of demand at this point is

$$\varepsilon_D = -\frac{1}{4}\frac{264}{44} = -\frac{3}{2}$$

2. The Monopolist can now offer different prices for young and old consumers. The profit maximization problem can be written as (already writing prices as their inverted demand counterparts):

$$\max_{Q_Y, Q_O} \; (200 - 2Q_Y)Q_Y + (440 - 4Q_O)Q_O - (Q_Y + Q_O)^2$$

Note that we *cannot* solve the problem of the monopolist for young/old consumers separately. The cost function $C(Q) = Q^2$ is convex and, thus, the quantities produced in one market affect the marginal cost of production in the other.

Taking the FOC with respet to Q_Y and Q_O

$$\begin{aligned} 200 - 4Q_Y &= 2(Q_Y + Q_O) \\ 440 - 8Q_O &= 2(Q_Y + Q_O) \end{aligned} \tag{1}$$

Subtracting one from the other:

$$Q_Y = 2Q_O - 60$$

Replacing it back:

$$440 - 10Q_O = 2(2Q_O - 60) \Rightarrow 14Q_O = 560$$

Which yields: $Q_Y = 20, Q_O = 40$. Prices come from plugging into the inverse demand curves, and yield

$$P_Y = 200 - 2Q_Y \Rightarrow P_Y = 160$$

$$P_O = 440 - 4Q_O \Rightarrow P_O = 280$$

And the demand elasticities are

$$\varepsilon_{D,Y} = -\frac{1}{2} \times \frac{160}{20} = -4$$

$$\varepsilon_{D,O} = -\frac{1}{4} \times \frac{280}{40} = -\frac{7}{4}$$

There are two noteworthy things in this example:

(a) Even though in Item 1 the monopolist was only producing for the old, the quantities were larger there than they are here. Here, the convexity of the cost implies that the production for the young will have an impact in price/quantity decisions for the old (and vice -versa).

(b) The old consumers, which are more inelastic (i.e., elasticity) close to zero, are the ones charged the higher price. This is consistent with our derivation of the Lerner Index in Item 5 of the first section of this handout. As will be seen later on, demand elasticity and market structure together are what generally determines equilibrium prices.

3. Young consumers are better off, because without price discrimination they would be excluded from the market. This means that price discrimination can increase total welfare not only by

increasing the ability of the monopolist to extract rents from consumers, but also by allowing more consumers to be in the market place.

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