MICROECONOMICS II

Topic 3 - Cost minimization

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BRAILLE DOTS ON DRIVE-UP ATMS, PUZZLE

Why do the keypad buttons on drive-up cash machines have Braille dots?

Milk and soft drink containers, Puzzle

Why is milk sold in rectangular containers and soft drinks in round ones?

Cost minimization

Profit-maximizing firm behavior

- Minimize costs of producing any given output level, find optimal structure of inputs
- Choose the most profitable output level when costs are minimized (differs across market structures)

Problem: $min(w_1x_1 + w_2x_2)$, such that $f(x_1, x_2) = y$

Include all costs, concept of opportunity cost

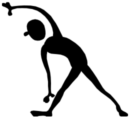
Solution: cost function $c(w_1, w_2, y)$

Cost minimization

- Substitute the constraint into the objective function. Useful for specific production function, not in general case.
- Method of Lagrange multipliers
  - $L = w_1x_1 + w_2x_2 - \lambda(f(x_1, x_2) - y)$
  - First order conditions: $\frac{w_1}{w_2} = \frac{MP_1}{MP_2}$
  - Technical rate of substitution equals factor price ratio.
- Alternative way
  - Change in inputs that keeps output constant: $MP_1 \vartriangle x_1 + MP_2 \vartriangle x_2 = 0$
  - Cost minimum: $w_1 \vartriangle x_1 + w_2 \vartriangle x_2 \geq 0$ and $-w_1 \vartriangle x_1 - w_2 \vartriangle x_2 \geq 0$
  - $w_1 \vartriangle x_1 + w_2 \vartriangle x_2 = 0$, thus $\dfrac{\vartriangle x_2}{\vartriangle x_1} = -\dfrac{w_1}{w_2} = -\dfrac{MP_1}{MP_2}$

Economic efficiency (minimized costs) $\Rightarrow$ technological efficiency $\Rightarrow$ output efficiency

Cost minimization



Exercise

- Cobb-Douglas production function, general case: $f(x_1, x_2) = x_1^a x_2^b$
- Cobb-Douglas production function, specific case: $f(x_1, x_2) = 3x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}; y = 48; w_1 = 4; w_2 = 64$

Find the amount of both factors demanded by the firm and minimized cost.

Cost minimization

Graphical solution

- Isoquant of production
- Isocost line: $x_2 = \frac{C}{w_2} - \frac{w_1}{w_2} x_1$

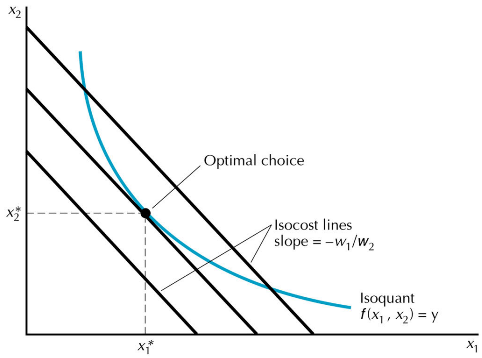
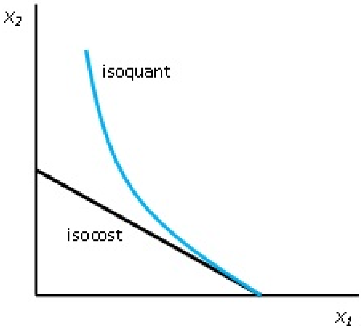


FIGURE 20.1 Cost minimization

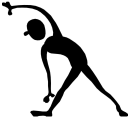
Corner solution

Tangency condition does not hold. Too costly to use some inputs.

- For $x_j^* = 0$: $\frac{w_j}{MP_j} \geq \frac{w_i}{MP_i} = MC$; $\frac{MP_i}{MP_j} \geq \frac{w_i}{w_j}$
- Complementarity conditions: $\frac{\delta L(x^*)}{\delta x_j} \geq 0 \wedge x_j^* \geq 0 \wedge \frac{\delta L(x^*)}{\delta x_j} \cdot x_j^* = 0$



Corner solution



Exercise

Production function $f(x) = \sqrt{x_1 + 5} + 2\sqrt{x_2}$

$y^0 = 50; w_1 = 12; w_2 = 2$

Find the amount of both factors demanded by the firm and minimized cost.

Conditional factor demands and cost function

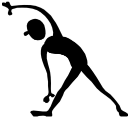
Conditional factor demand functions (derived factor demands)

- $x_1(w_1, w_2, y); x_2(w_1, w_2, y)$
- Cost minimizing choices for given input prices and output level

Cost function

- Minimized costs as a function of input prices and required output
- $c = \Sigma w_i x_i(w_1, ..., w_n, y) = c(w, y)$

Conditional factor demands and cost function



Exercise

Production function $f(x) = 3x_1^{\frac{1}{3}}x_2^{\frac{2}{3}}$

Output y^0, input prices $w_1; w_2$

Find conditional factor demands and the cost function.

Properties of the cost function

- Increasing in y: $LMC = \frac{\triangle\ c}{\triangle\ y} = \frac{w_i}{MP_i}$
- Non-decreasing in w
- Linearly homogenous in w: $c(kw, y) = kc(w, y)$
- Continuous in w and y
- Concave in w_i
- Shephard's lemma: $\frac{\delta c(w,y)}{\delta w_i} = x_i(w, y)$
  - Useful for estimate of change in costs when there is a small change in input price.
  - Derive conditional factor demands from the cost function.

Cost function

Examples of technologies

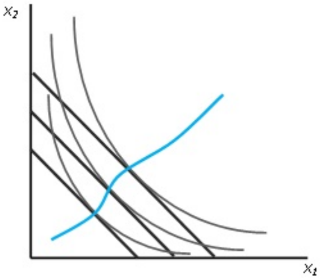
- Perfect complements
  - $f(x_1, x_2) = min\{x_1, x_2\}$
  - $c(w_1, w_2, y) = y(w_1 + w_2)$
- Perfect substitutes
  - $f(x_1, x_2) = x_1 + x_2$
  - $c(w_1, w_2, y) = min\{w_1 y, w_2 y\}$

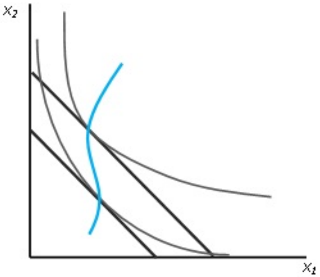
Expansion path

Effect of change in output on the optimal choice of inputs, input prices being constant.

Types of inputs

- Normal input: EP positive slope, conditional factor demand increasing in output.
- Inferior input: EP negative slope, conditional factor demand decreasing in output.
- At least one input normal, no input inferior at all output levels.



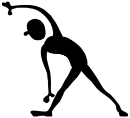


Revealed cost minimization

Weak axiom of cost minimization (WACM)

- We observe choices at two points of time t and s. Prices differ, output is the same.
- $w_1^t x_1^t + w_2^t x_2^t \leq w_1^t x_1^s + w_2^t x_2^s$
- $w_1^s x_1^s + w_2^s x_2^s \leq w_1^s x_1^t + w_2^s x_2^t$
- $\triangle x_1 \triangle w_1 + \triangle x_2 \triangle w_2 \leq 0$
- Comparative statics results

Opportunity cost



Exercise

- You won a ticket to see an Eric Clapton concert. You cannot resell it.
- The only other activity you consider is a Bob Dylan concert. The ticket costs $40 and you are willing to pay $50 to see him perform.
- No other cost of seeing either performer.

What is your opportunity cost of attending the Clapton concert?

Short-run and long-run costs

Short-run costs

- Minimum costs if all variable factors are adjusted
- $c_s(y, \bar{x_2}) = min(w_1 x_1 + w_2 \bar{x_2})$, such that $f(x_1, \bar{x_2}) = y$
- Factor demands: $x_1 = x_1^s(w_1, w_2, \bar{x_2}, y)$ and $x_2 = \bar{x_2}$

Long-run costs

- Minimum costs if all factors are adjusted
- $c(y) = min w_1 x_1 + w_2 x_2$, such that $f(x_1, x_2) = y$
- Factor demands: $x_1 = x_1(w_1, w_2, y)$ and $x_2 = x_1(w_1, w_2, y)$

Fixed costs: independent on output. Not in the long-run.
Quasi-fixed costs: independent on output, paid only if output is positive. Exist in the long-run.
Sunk costs: cannot be recovered

SUNK COST, APPLICATION

Why do film studios make movies that they know will lose money?

SUNK COST, APPLICATION

DellaVigna and Malmendier, 2006. Paying Not to Go to the Gym.

Sunk cost, application

Sunk cost fallacy: A way to motivate people to use bed nets?

Debate about pricing of bed nets - for free or for positive price?

- Downward sloping demand: more people can use them if for free.
- Selection effect: positive price selects out people who do not value the net.
- Sunk cost effect: positive price can induce people to use the nets.

Sunk cost, application

Cohen and Dupas, 2010. Free distribution or cost sharing? Evidence from a randomized malaria prevention experiment.

- Sample: 20 prenatal clinics in Kenya
- Randomized the price of nets (0-$0.6 (90% subsidy)).
- Identify women who use the net by surprise visits at home a few months later.
- The link between usage and price: combination of the selection and sunk cost effects.
  - Randomized two-stage pricing design. A sub-sample of women who decided to buy the net for positive price were surprised with a lottery for additional discount.
- Results
  - Demand drops significantly with price.
  - Usage intensity not increasing in price.