Part 8: Program Evaluation (d): Difference in Difference

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Applied Econometrics

Motivation: Recap Matching

Matching estimators had some advantages:

- Limited assumptions on functional forms
- We could do nearest neighbor matching and use kernels to compute treatment effects

Matching estimators had some drawbacks:

- Treated patients were "matched" to control patients based only on observable characteristics
  - Ignored selection on unobservables.
- Relied on cross sectional variation to construct a control group.

Motivation

IV estimators resolve some of those issues but

- Good IV are in short supply!

Often (in this course at least) we have access to panel data.

- What if we could use panel data to control for unobserved heterogeneity within a treated individual/group?

Difference in Difference estimators are like the opposite of matching

- Strong assumptions on functional form
- but... allow for unobservable heterogeneity in outcomes.

A Famous Example: Card and Krueger (AER 1994)

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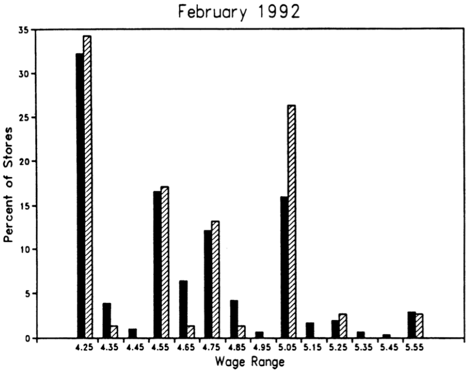
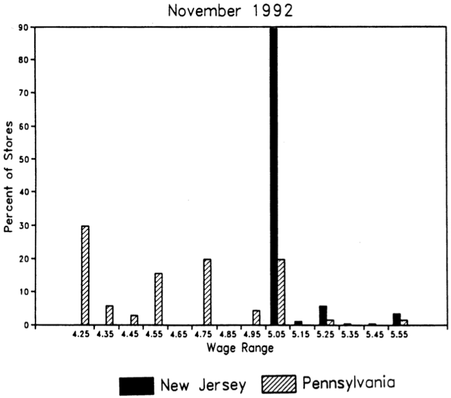
- On April 1, 1992 NJ raises its minimum wage from \$4.25 $\longrightarrow$ \$5.05 per hour.
- Question: Econ 101 predicts this will reduce demand for low wage workers
  - Focus on fast food restaurants (since they pay min wage)
  - Focus on starting wage (avoid tenure, high turnover)
- Survey 410 restaurants in NJ (treated group) and eastern PA (control group).
- Idea: Compare change in wages in *NJ* to *PA*: $\Delta_{DD} = \Delta_{NJ} - \Delta_{PA}$
  - Wave 1: February 15-March 4, 1992
  - Wave 2: November 5 - December 31, 1992

Balance Table: Covariates

Variable	Stores in: NJ	Stores in: PA	t^{a}
1. *Distribution of Store Types (percentages):*			
a. Burger King	41.1	44.3	−0.5
b. KFC	20.5	15.2	1.2
c. Roy Rogers	24.8	21.5	0.6
d. Wendy's	13.6	19.0	−1.1
e. Company-owned	34.1	35.4	−0.2
2. *Means in Wave 1:*			
a. FTE employment	20.4 (0.51)	23.3 (1.35)	−2.0
b. Percentage full-time employees	32.8 (1.3)	35.0 (2.7)	−0.7
c. Starting wage	4.61 (0.02)	4.63 (0.04)	−0.4
d. Wage = $4.25 (percentage)	30.5 (2.5)	32.9 (5.3)	−0.4
e. Price of full meal	3.35 (0.04)	3.04 (0.07)	4.0
f. Hours open (weekday)	14.4 (0.2)	14.5 (0.3)	−0.3
g. Recruiting bonus	23.6 (2.3)	29.1 (5.1)	−1.0
3. *Means in Wave 2:*			
a. FTE employment	21.0 (0.52)	21.2 (0.94)	−0.2
b. Percentage full-time employees	35.9 (1.4)	30.4 (2.8)	1.8
c. Starting wage	5.08 (0.01)	4.62 (0.04)	10.8
d. Wage = $4.25 (percentage)	0.0	25.3 (4.9)	—
e. Wage = $5.05 (percentage)	85.2 (2.0)	1.3 (1.3)	36.1
f. Price of full meal	3.41 (0.04)	3.03 (0.07)	5.0
g. Hours open (weekday)	14.4 (0.2)	14.7 (0.3)	−0.8
h. Recruiting bonus	20.3 (2.3)	23.4 (4.9)	−0.6

Distribution of Wages





Differences in Wages : 2 x 2 Table

TABLE 3—AVERAGE EMPLOYMENT PER STORE BEFORE AND AFTER THE RISE IN NEW JERSEY MINIMUM WAGE

Variable	Stores by state			Stores in New Jersey[a]			Differences within NJ[b]	
	PA (i)	NJ (ii)	Difference, NJ − PA (iii)	Wage = \$4.25 (iv)	Wage = \$4.26–\$4.99 (v)	Wage ≥ \$5.00 (vi)	Low–high (vii)	Midrange–high (viii)
1. FTE employment before, all available observations	23.33 (1.35)	20.44 (0.51)	−2.89 (1.44)	19.56 (0.77)	20.08 (0.84)	22.25 (1.14)	−2.69 (1.37)	−2.17 (1.41)
2. FTE employment after, all available observations	21.17 (0.94)	21.03 (0.52)	−0.14 (1.07)	20.88 (1.01)	20.96 (0.76)	20.21 (1.03)	0.67 (1.44)	0.75 (1.27)
3. Change in mean FTE employment	−2.16 (1.25)	0.59 (0.54)	2.76 (1.36)	1.32 (0.95)	0.87 (0.84)	−2.04 (1.14)	3.36 (1.48)	2.91 (1.41)
4. Change in mean FTE employment, balanced sample of stores[c]	−2.28 (1.25)	0.47 (0.48)	2.75 (1.34)	1.21 (0.82)	0.71 (0.69)	−2.16 (1.01)	3.36 (1.30)	2.87 (1.22)
5. Change in mean FTE employment, setting FTE at temporarily closed stores to 0[d]	−2.28 (1.25)	0.23 (0.49)	2.51 (1.35)	0.90 (0.87)	0.49 (0.69)	−2.39 (1.02)	3.29 (1.34)	2.88 (1.23)

Notes: Standard errors are shown in parentheses. The sample consists of all stores with available data on employment. FTE (full-time-equivalent) employment counts each part-time worker as half a full-time worker. Employment at six closed stores is set to zero. Employment at four temporarily closed stores is treated as missing.

[a]Stores in New Jersey were classified by whether starting wage in wave 1 equals \$4.25 per hour ($N = 101$), is between \$4.26 and \$4.99 per hour ($N = 140$), or is \$5.00 per hour or higher ($N = 73$).

[b]Difference in employment between low-wage (\$4.25 per hour) and high-wage (≥ \$5.00 per hour) stores; and difference in employment between midrange (\$4.26–\$4.99 per hour) and high-wage stores.

[c]Subset of stores with available employment data in wave 1 and wave 2.

[d]In this row only, wave-2 employment at four temporarily closed stores is set to 0. Employment changes are based on the subset of stores with available employment data in wave 1 and wave 2.

Outcome Equation

- Differences lack any covariates (different fast food chains).
- Also $\Delta_{PA} < 0$ and $\Delta_{NJ} > 0$ (!)
- Recall i denotes stores, $t \in 1, 2$. Run the following regression:

$$Y_{it} = \beta X_{it} + \alpha \cdot [i \in \text{NJ}] + \gamma \cdot \text{After}_t + \delta \cdot NJ_i \times After_t + u_i$$

$$Y_{it} = \beta X_{it} + \alpha \cdot [\text{wage gap}_i] + \gamma \cdot \text{After}_t + \delta \cdot \text{wage gap}_i \times After_t + u_i$$

- α is mean difference between NJ and PA
- γ is mean difference between period 1 and 2
- δ is the parameter of interest, the difference in difference
- $\text{wage gap}_i = [\text{min wage}_{i,2} - w_{i1}]_+ = \max\{0, \text{min wage}_{i,2} - w_{i1}\}$.
  (How much do you need to raise $t = 1$ wages to achieve minimum wage in $t = 2$?)

Differences in Wages

TABLE 4—REDUCED-FORM MODELS FOR CHANGE IN EMPLOYMENT

	Model				
Independent variable	(i)	(ii)	(iii)	(iv)	(v)
1. New Jersey dummy	2.33 (1.19)	2.30 (1.20)	—	—	—
2. Initial wage gap[a]	—	—	15.65 (6.08)	14.92 (6.21)	11.91 (7.39)
3. Controls for chain and ownership[b]	no	yes	no	yes	yes
4. Controls for region[c]	no	no	no	no	yes
5. Standard error of regression	8.79	8.78	8.76	8.76	8.75
6. Probability value for controls[d]	—	0.34	—	0.44	0.40

Notes: Standard errors are given in parentheses. The sample consists of 357 stores with available data on employment and starting wages in waves 1 and 2. The dependent variable in all models is change in FTE employment. The mean and standard deviation of the dependent variable are −0.237 and 8.825, respectively. All models include an unrestricted constant (not reported).

[a]Proportional increase in starting wage necessary to raise starting wage to new minimum rate. For stores in Pennsylvania the wage gap is 0.

[b]Three dummy variables for chain type and whether or not the store is company-owned are included.

[c]Dummy variables for two regions of New Jersey and two regions of eastern Pennsylvania are included.

[d]Probability value of joint F test for exclusion of all control variables.

A More General Method

Difference in Difference: General Approach

$$\text{Potential outcome in period 1} = Y_{i1}(0)$$

$$\text{Potential outcome in period 2} = \left\{ \begin{array}{ll} Y_{i2}(1) & \text{if } T_{i2} = 1 \\ Y_{i2}(0) & \text{if } T_{i2} = 0 \end{array} \right\}$$

	Treatment	Control
Before	$Y_{i1}(0)$	$Y_{i1}(0)$
After	$Y_{i2}(1)$	$Y_{i2}(0)$

We can write the outcome as:

$$Y_{it} = T_{it} Y_{it}(1) + (1 - T_{it})\, Y_{it}(0) = T_{it}\,(Y_{it}(1) - Y_{it}(0)) + Y_{it}(0)$$

Difference in Difference: General Approach

Consider the first difference $\Delta Y_{it} = Y_{i2} - Y_{i1}$:

$$\Delta Y_{it} = T_{i2}\left(Y_{i2}(1) - Y_{i2}(0)\right) + Y_{i2}(0) - Y_{i1}(0)$$

For treated group (first difference):

$$E[\Delta Y_{it} | T_{i2} = 1] = \overbrace{E\left[Y_{i2}(1) - Y_{i2}(0) | T_{i2} = 1\right]}^{ATT} + \overbrace{E\left[Y_{i2}(0) - Y_{i1}(0) | T_{i2} = 1\right]}^{\gamma(1)}$$

For control group (second difference):

$$E[\Delta Y_{it} | T_{i2} = 0] = \overbrace{E\left[Y_{i2}(0) - Y_{i1}(0) | T_{i2} = 0\right]}^{\gamma(0)}$$

The DiD (difference in difference) estimator

$$\Delta_{DD} = E[\Delta Y_{it} | T_{i2} = 1] - E[\Delta Y_{it} | T_{i2} = 0]$$

Difference in Difference: Parallel Trends

- If $\gamma(1) = \gamma(0)$ then the DiD estimator cancels and we are left with the $\Delta_{DD} = \text{ATT}$.
- This is the parallel trends assumption

$$E\left[Y_{i2}(0) - Y_{i1}(0) | T_{i2} = 0\right] = E\left[Y_{i2}(0) - Y_{i1}(0) | T_{i2} = 1\right]$$

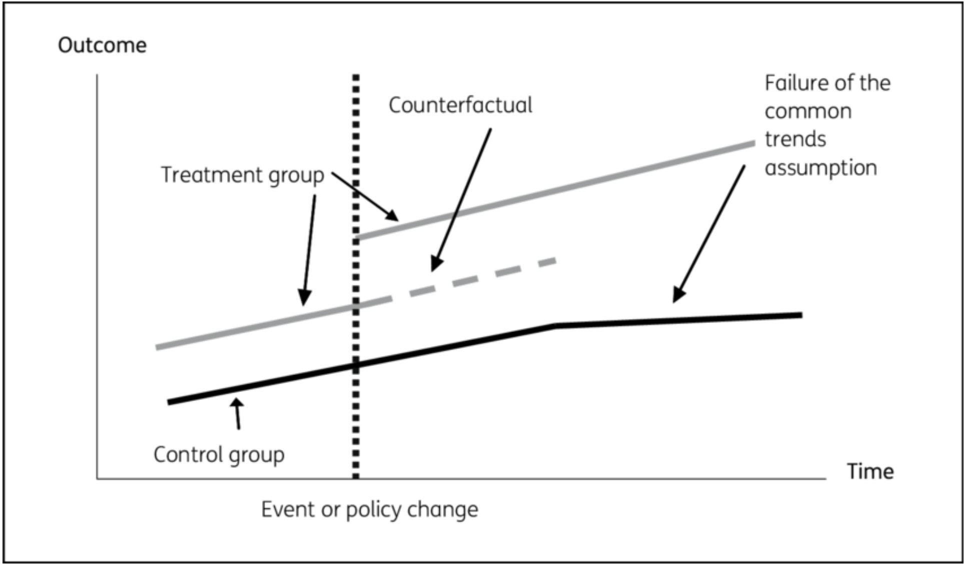
- Absent the treatment effect, both treatment and control would evolve identically over time.
- But, treatment and control groups can start from very different places...

$$E\left[Y_{it}(0) | T_{i2} = 1\right] \neq E\left[Y_{it}(0) | T_{i2} = 0\right], t = 1, 2$$

- And have selection on treatment effects...

$$E\left[Y_{i2}(1) - Y_{i2}(0) | D_{i2} = 1\right] \neq E\left[Y_{i2}(1) - Y_{i2}(0) | D_{i2} = 0\right]$$

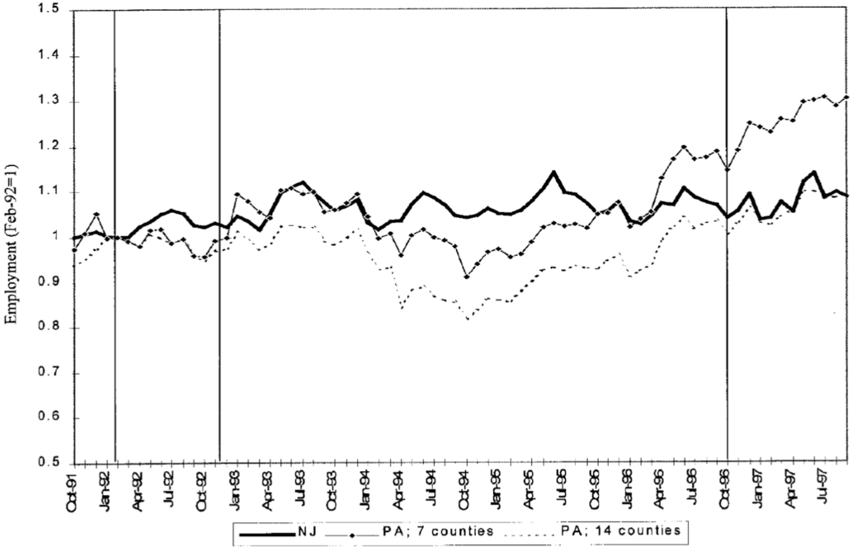
Parallel Trends



Difference in Differences: Limitations

1. Functional form restrictions
    - Parallel trends assumes that absent treatment that we add $\gamma_2 - \gamma_1$ to each unit
    - Because this is additive it is not invariant to transformations $f(Y_{it})$ (ie: taking logs)
2. Parallel Trend Assumption is not testable
    - Best we can hope is that it looks similar in the pre-period
3. Compositional Effects: the treatment may affect who is in each group
    - Restaurants could close in NJ and open nearby in PA to avoid minimum wage.
    - A good job training program may lead to migration, etc.
    - One approach: redefine the population so that it doesn't endogenously respond to treatment
        - Recover something, but probably not ATT anymore...

Checking Pre-Trend: Card Krueger (2000)



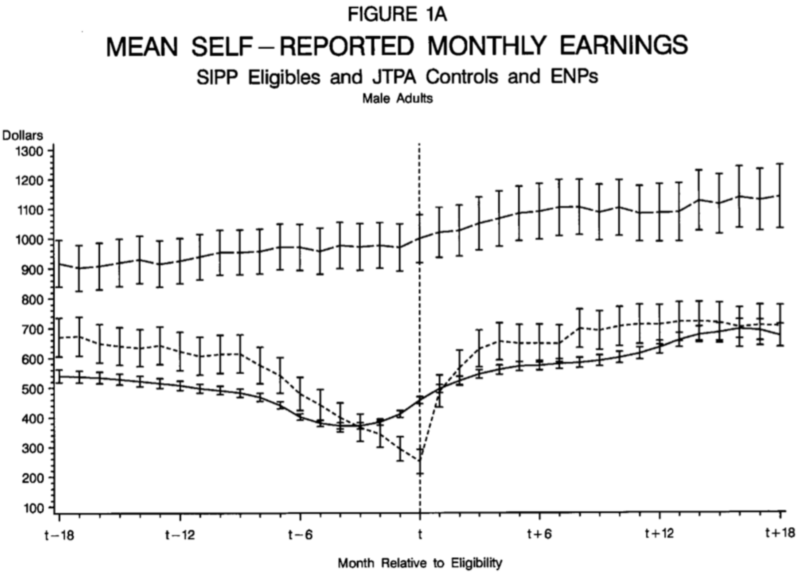
Difference in Differences

Just like in Card and Kruger, we can write as a regression equation:

$$Y_{it} = \alpha_i + \gamma_t + \beta X_{it} + \delta_i T_{it} + u_{it}$$

- Suppose we wish to evaluate a training program for those with low earnings. Let the threshold for eligibility be B.
- We have a panel of individuals and those with low earnings qualify for training, forming the treatment group.
- Those with higher earnings form the control group.
- Now the low earning group is low for two reasons
  1. They have low permanent earnings (α_i is low) - this is accounted for by diff in diffs.
  2. They have a negative transitory shock (u_{i1} is low) - this is not accounted for by diff in diffs.

The “Ashenfelter Dip” (Heckman and Smith 2000)



Difference in Differences

- $\#2$ above violates the assumption $E[Y_{i2}(0) - Y_{i1}(0)|T] = E[Y_{i2}(0) - Y_{i1}(0)]$.
- To see why note that those participating into the program are such that $Y_{i0}(0) < B$. Assume for simplicity that the shocks u are *iid*. Hence $u_{i1} < B - \alpha_i - \gamma_1$. This implies:

$$E[Y_{i2}(0) - Y_{i1}(0)|T = 1] = \gamma_2 = \gamma_1 - E[u_{i1}|u_{i1} < B - \alpha_i - \gamma_1]$$

For the control group:

$$E[Y_{i2}(0) - Y_{i1}(0)|T = 1] = \gamma_2 = \gamma_1 - E[u_{i1}|u_{i1} > B - \alpha_i - \gamma_1]$$

$$E[Y_{i2}(0) - Y_{i1}(0)|T = 1] - E[Y_{i2}(0) - Y_{i1}(0)|T = 0] =$$

$$E[u_{i1}|u_{i1} > B - \alpha_i - \gamma_1] - E[u_{i1}|u_{i1} < B - \alpha_i - \gamma_1] > 0$$

- This is effectively regression to the mean: those unlucky enough to have a bad shock recover and show greater growth relative to those with a good shock. The nature of the bias depends on the stochastic properties of the shocks and how individuals select into training.

Difference in Differences

- The assumption on growth of the non-treatment outcome being independent of assignment to treatment may be violated, but it may still be true conditional on X.
- Consider the assumption

$$E[Y_{i2}(0) - Y_{i1}(0)|X, T] = E[Y_{i2}(0) - Y_{i1}(0)|X]$$

- This is just matching assumption on a redefined variable, namely the growth in the outcomes. In its simplest form the approach is implemented by running the regression

$$Y_{it} = \alpha_i + \gamma_t + \delta_i T_{it} + \beta_t' X_i + u_{it}$$

  which allows for differential trends in the non-treatment growth depending on X_i. More generally one can implement propensity score matching on the growth of outcome variable when panel data is available.

Variants

Difference in Difference in Difference

The triple difference is also a thing:

- Suppose that we have: before/after, treated-state/untreated-state, treated-group (men)/ untreated-group women.
- We can compute two D-i-D here: $\Delta_{DDD} = \Delta_{DD,state} - \Delta_{DD,gender}$
- Literally difference, the difference in differences estimators.
- As a regression: interpret the triple-interaction term (make sure to control for ALL double interactions).

Difference in Differences with Repeated Cross Sections

- Suppose we do not have available panel data but just a random sample from the relevant population in a pre-treatment and a post-treatment period. We can still use difference in differences.
- First consider a simple case where $E[Y_{i2}(0) - Y_{i1}(0)|T] = E[Y_{i2}(0) - Y_{i1}(0)]$.
- We need to modify slightly the assumption to

$$\begin{aligned} E[Y_{i2}(0)|_{\text{Group receiving training}}] - E[Y_{i1}(0)|_{\text{Group receiving training in the next period}}] \\ = E[Y_{i2}(0) - Y_{i1}(0)] \end{aligned}$$

  which requires, in addition to the original independence assumption that conditioned on particular individuals that population we will be sampling from does not change composition.
- We can then obtain immediately an estimator for ATT as

$$\begin{aligned} E[\beta_i|T_{i2} = 1] = E[Y_{i2}|_{\text{Group receiving training}}] - E[Y_{i1}|_{\text{Group receiving training next period}}] \\ - \{E[Y_{i2}|_{\text{Non-trainees}}] - E[Y_{i1}|_{\text{Group not receiving training next period}}]\} \end{aligned}$$

Difference in Differences with Repeated Cross Sections

- More generally we need an assumption of conditional independence of the form

$$
\begin{aligned}
& E[Y_{i2}(0)|X, _{\text{Group receiving training}}] - E[Y_{i1}(0)|X, _{\text{Group receiving training next period}}] \\
&= E[Y_{i2}(0)|X] - E[Y_{i1}(0)|X]
\end{aligned}
$$

- Under this assumption (and some auxiliary parametric assumptions) we can obtain an estimate of the effect of treatment on the treated by the regression

$$Y_{it} = \alpha_g + \gamma_t + \gamma T_{it} + \beta' X_{it} + u_{it}$$

Difference in Differences with Repeated Cross Sections

- More generally we can first run the regression

$$Y_{it} = \alpha_g + \gamma_t + \delta(X_{it})T_{it} + \beta' X_{it} + u_{it}$$

  where α_g is a dummy for the treatment of comparison group, and $\delta(X_{it})$ can be parameterized as $\delta(X_{it}) = \delta' X_{it}$. The ATT can then be estimated as the average of $\delta' X_{it}$ over the (empirical) distribution of X.
- A non parametric alternative is offered by Blundell, Dias, Meghir and van Reenen (2004).

Difference in Differences and Selection on Unobservables

- Suppose we relax the assumption of *no selection* on unobservables.
- Instead we can start by assuming that

$$E[Y_{i2}(0)|X,Z] - E[Y_{i1}(0)|X,Z] = E[Y_{i2}(0)|X] - E[Y_{i1}(0)|X]$$

  where Z is an instrument which determines training eligibility say but does not determine outcomes in the non-training state. Take Z as binary (1,0).
- Non-Compliance: not all members of the eligible group ($Z = 1$) will take up training and some of those ineligible ($Z = 0$) may obtain training by other means.
- A difference in differences approach based on grouping by Z will estimate the impact of being allocated to the eligible group, but not the impact of training itself.

- Now suppose we still wish to estimate the impact of training on those being trained (rather than just the effect of being eligible)
- This becomes an IV problem and following up from the discussion of LATE we need stronger assumptions
  - Independence: for $Z = a, \{Y_{i2}(0) - Y_{i1}(0), Y_{i2}(1) - Y_{i1}(1), T(Z = a)\}$ is independent of Z.
  - Monotonicity $T_i(1) \geq T_i(0) \forall i$
- In this case LATE is defined by

$$\frac{E(\Delta Y_{it}|Z_{it} = 1) - E(\Delta Y_{it}|Z_{it} = 0)}{Pr(T_{it} = 1|Z_{it} = 1) - Pr(T_{it} = 1|Z_{it} = 0)}$$

assuming that the probability of training in the first period is zero.

Changes in Changes: Dealing w Nonlinearity

- Athey and Imbens (2006) develop a model robust to nonlinearity complaints
- Combines nonparametrics with DiD.
- Works with quantile treatment effects and limits selection on unobservables
- Assume that your relative location in distribution is invariant to difference.

Next time

What if we can combine the benefits of matching with DiD approaches?