5303 Advanced Macroeconomics - Group 3
Assignment 5

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1. (a) Assuming agents live for two periods as we have discussed in class, this statement is true. There is no reason for the young and the old of each period to engage in trade. They have the same good during the period their lives overlap, so there are no taste differences that might lead to trade. The only trades that are of interest are trading for consumption in different time periods. Suppose that some old person at time t attempts to make a trade with a young person. There are two possibilities at time t: (1) the young person could give the old person some of the good for a promise of future repayment; or (2) the old person could give the young person some good for a promise of future repayment. But in the next period, the old person is dead. The young person could not collect a debt and the now dead old person could not enjoy a repayment. Therefore, no intergenerational trade takes place. Note that if agents lived more than two periods, trade could occur between all generations except for the one which will die after the current period.

   (b) The pay-as-you-go pension system requires population growth in order to uphold the size of transfers to the old. In this sense, a fully-funded system would be better suited to current demographics. However, outright replacing the PAYG system with a fully-funded one would severely hurt those who are currently retired or soon to retire (i.e. the current old) as they do not have the time to build up savings in a fully-funded system.

   (c) Studying Scandinavia leads one to conclude that the conclusions drawn in Prescott (2004) are incomplete. While Scandinavian countries do work much more than Prescott's model predicts, Rogerson (2007) shows that these differences are explained when different types of government spending are added to the model. Since Scandinavian governments spend more on family services, the incentives that workers face change, and they choose to work more. Thus Prescott's model is not false, it is merely incomplete.

   (d) True. By considering the utility of their children and adjusting the bequests, parents can address the issue of dynamic inefficiency, resembling a social security scheme, but dynamic inefficiency returns once the bequest reaches zero, thus altruism does not eliminate dynamic inefficiency, but positive bequests ensure dynamic efficiency. In other words, if the bequest motive is operative (positive), the economy achieves the a modified golden rule and is thus dynamically efficient. The market equilibrium aligns with the planner's solution and Ricardian equivalence holds across generations due to the present value budget constraint. If the bequest motive is not operative, altruism has no impact on the economy's dynamic efficiency, which may still be inefficient.

2.

FIRM $R_t = MPK = \theta K_t^{\theta-1} N_t^{1-\theta}$

$w_t = MPL = (1-\theta) K_t^{\theta} N_t^{-\theta}$

AGENTS $\max u$ s.t. $c_t + \frac{c_{t+1}}{r_t} = w_t(1-\tau) + \frac{T}{r_t}$

FOC

$$\frac{1}{c_t} + \frac{\beta r}{c_{t+1}} = 0$$

$$c_{t+1} = \beta r c_t$$

$$\to \quad c_t = \frac{1}{1+\beta}\left(w_t(1-\tau) + T r_t^{-1}\right)$$

$$s_t = \frac{1}{1+\beta}\left[\beta w_t(1-\tau) - T r_t^{-1}\right)$$

GOVERNMENT

$$n N \tau w_t - N T = 0$$

$$T = n \tau w_t$$

Equilibrium:

$$S(r_t) = s_t N_t = K_{t+1}$$

$$\to \quad \frac{K_{t+1}}{N_{t+1}} \equiv k_{t+1} = \frac{1}{n(1+\beta)}\left[w_t\left(\beta(1-\tau) - n\tau r_t^{-1}\right)\right]$$

steady state $k_t = k_{t+1} = k > 0$

(a) $\tau = 0$:

$$k = \frac{\beta w_t}{n(1+\beta)} = \frac{\beta(1-\theta)}{n(1+\beta)} k^{\theta}$$

$$\to \quad k = \left[\frac{\beta(1-\theta)}{n(1+\beta)}\right]^{\frac{1}{1-\theta}}$$

(b) $\tau \in (0,1)$:

$$= \frac{\beta(1-\theta)}{n(1+\beta)} k^{\theta} - \frac{\tau(1-\theta)}{\theta(1+\beta)} k \quad / : k$$

$$\frac{\beta(1-\theta)(1-\tau)}{n(1+\beta)} k^{\theta-1} = 1 + \frac{\tau(1-\theta)}{\theta(1+\beta)}$$

$$\to \quad k = \Big[\underbrace{\frac{n(1+\beta)}{\beta(1-\theta)}}_{A>0} \frac{1}{(1-\tau)} + \underbrace{\frac{n}{\theta\beta}}_{B>0} \frac{\tau}{(1-\tau)}\Big]^{-\frac{1}{1-\theta}}$$

$$\frac{\partial k}{\partial \tau} = \underbrace{-1 \underbrace{\frac{1}{1-\theta}}_{>0} \underbrace{k^{\frac{\theta-2}{1-\theta}}}_{>0} \Big[A \underbrace{\frac{1}{(1-\tau)^2}}_{>0} + B \underbrace{\frac{1}{(1-\tau)^2}}_{>0}\Big]}_{<0 \ldots}$$

decreasing in τ ✗

(c) AGENTS $\max u$ s.t. $c_t + \frac{c_{t+1}}{r_t} = w_t(1-\tau) + \frac{R_{t+1} T}{r_t}$

$$\iff \quad c_t + c_{t+1} r_t^{-1} = w_t$$

since $\underset{\text{no arbitrage}}{R_{t+1} = r_t}$ & $\underset{\text{tax scheme}}{\tau w_t = T_t}$

by (a) $\to$ $c_t = \frac{w_t}{1+\beta}$

$$\to \quad s_t = w_t\left(1 - \tau - \frac{1}{1+\beta}\right)$$

$$K_{t+1} = N_t(s_t + T_t) = N_t w_t\left(1 - \tau - \frac{1}{1+\beta} + \tau\right)$$

$$= \frac{\beta}{1+\beta} N_t w_t$$

which corresponds to K_{t+1} in (a),

$\to$ raising τ a bit from 0 doesn't affect K_{t+1} since lifetime budget constraint of agents equals w_t which is set by the firm based on K_t independent of whether it is provided by them or the government, however $\tau > \frac{\beta}{1+\beta}$ implies $s_t < 0$ thus $\tau \in [0, \frac{\beta}{1+\beta})$ needs to hold

Definition *A perfect foresight competitive equilibrium for an economy with labor endowments and a production function of $\gamma(t)F(L(t), K(t))$ is a sequence of $K(t)$, $r(t)$, wage(t), and rental(t) for $t \geq 1$ such that, given an initial $K(1) > 0$,*

$$S_t(r(t)) = K(t+1),$$

$$r(t) = \text{rental}(t+1),$$

$$\text{wage}(t) = \frac{\partial[\gamma(t)F(L(t), K(t))]}{\partial L(t)},$$

and $$\text{rental}(t) = \frac{\partial[\gamma(t)F(L(t), K(t))]}{\partial K(t)},$$

hold for all $t \geq 1$.

Question 3. Consider a two period overlapping generations economy with land. The number of people born in each period is $N(t) = 100$ for all t. Preferences are given by

$$u_t^h = c_t^h(t)c_t^h(t+1).$$

Everyone has endowment profile $\omega_t^h = (30, 24)$. The southern part of the country has 100 units of land each yielding a crop $d^S(t) = 1$ in every period. The northern part also has 100 units of land but it is barren $d^N(t) = 0$ and unpopulated. Ownership of southern land is in period 0 equally distributed among members of generation -1.

1. What allocations are feasible in this economy?
2. Solve for the stationary competitive equilibirum.
3. Now suppose that people leanr in period 0 that there will be golbabl warming so that from period 2 on each unit of northern land will aslo yield a crop.
   (a) What is the new stationary equilibrium?
   (b) What is the period 0 price of southern land $p^s(0)$.

Solution. a) Feasibility requires that the consumption of both young and old is less than total endowments and crop yield, implying

$$C(t) = \sum c_{t-1}^h(t) + \sum c_t^h(t) \leq Y(t) + D(t) = \sum \omega_{t-1}^h(t) + \sum \omega_t^h(t) + Ad(t).$$

Subsituting values we find consumption can be no greater than

$$C(t) \leq 100(30 + 24) + 100 = 5500.$$

b) Each individual h of generation t faces budget constraints

$$c_t^h(t) \leq \omega_t^h(t) - l^h(t) - p(t)a^h(t),$$

$$c_t^h(t) \leq \omega_t^h(t+1) + r(t)l^h(t) + d(t+1)a^h(t) + p(t+1)a^h(t).$$

The individual chooses consumption, land purchases and lending as to maximize utility, the maximization problem is

$$\begin{aligned}
\max \quad & c_t^h(t)c_t^h(t+1) \\
\text{s.t} \quad & c_t^h(t) \leq w_t^h(t) - l^h(t) - p(t)a^h(t) \\
& c^h(t+1) \leq w_t^h(t+1) + r(t)l^h(t) + d(t+1)a^h(t) + p(t+1)a^h(t).
\end{aligned}$$

The lagrangian is

$$L = c_t^h(t)c_t^h(t+1) - \lambda_t(c_t^h(t) - w_t^h(t) + l^h(t) + p(t)a^h(t)) - \lambda_{t+1}(c^h(t+1) - w_t^h(t+1) - r(t)l^h(t) - d(t+1)a^h(t) - p(t+1)a^h(t)).$$

The first order conditions are

$$c_t^h(t+1) - \mu_t = 0,$$

$$c_t^h(t) - \mu_{t+1} = 0,$$

$$-\mu_t + \mu_{t+1}r(t) = 0,$$

$$-\mu_t p(t) + \mu_{t+1}(d(t+1) + p(t+1)) = 0.$$

Solve the first and second equation for the multipliers and substitute into the thrid and fourth, this gives

$$c_t^h(t+1) = r(t)c_t^h(t),$$

$$r(t) = \frac{d(t+1) + p(t+1)}{p(t)}.$$

The expressions are then substituted into the life time budget constraint. Using the no-arbitrage condition we find

$$c_t^h(t) = \frac{1}{2}(\omega_t^h(t) + \frac{\omega_t^h(t+1)}{r(t)}).$$

Savings is the difference between endowment and consumption, equal to

$$s_t^h = \frac{w_t^h(t)}{2} - \frac{1}{2}\frac{\omega_t^h(t+1)}{r(t)}.$$

The equilibrium savings condition with land purchases and interest rate condition is

$$S(r(t)) = p(t)A,$$

$$r(t) = \frac{d(t+1) + p(t+1)}{p(t)}.$$

We can deduce an expression for $p(t)$ by stationarity. Since $p(t) = p(t+1)$ we have,

$$r = \frac{d+p}{p},$$

$$p = \frac{d}{r-1}.$$

Substituting values into the savings condition and simplifying yields

$$15r^2 - 28r + 12 = 0.$$

The roots for the second degree polynomial are $r = \frac{2}{3}$ and $r = \frac{6}{5}$. Using the price and interest rate equation we find that they correspond to prices $p = -3$ and $p = 5$ respectively. Prices cant be negative so we discard the root $r = \frac{2}{3}$. The equilibrium savings and consumption can now be calculated,

$$c_t^h(t) = \frac{1}{2}(30 + \frac{24 \cdot 5}{6}) = 25,$$

$$c_t^h(t+1) = \frac{6}{5} \cdot 25 = 30,$$

$$s_t^h(t) = \frac{1}{2}(30 - \frac{24 \cdot 5}{6} = 5.$$

c) I) With northern land yielding crop we now have equilibrium savings

$$100(\frac{30}{2} - \frac{24}{2r} = \frac{d}{r-1}A,$$

$$15r^2 - 29r + 12 = 0.$$

Only the root $\frac{4}{3}$ solves the polynomial and has a positive associated price of land $p = 3$. The new consumptions and savings, using the same equations as before, is

$$c_t^h(t) = 24,$$

$$c_t^h(t+1) = 32,$$

$$s_t^h(t) = 6.$$

II) The price of southern and northern land must follow a no-arbitrage condition, we get the two equations

$$r = \frac{3}{p^N},$$

$$r = \frac{4}{p^S}.$$

Setting them equal we find

$$p^N = \frac{3}{4}p^S.$$

This is then used in the savings market clearing expression

$$S(r) = 100p^N + 100p^S.$$

Plugging in individual savings expression and substituting the p^N we get

$$100(\frac{30}{2} - \frac{24}{2\frac{4}{p^S}}) = 100p^N + 100p^S,$$

$$p^s = \frac{60}{19}.$$

This means northern prices are

$$p^N = \frac{45}{19}.$$

Problem Set #5

(4) $N(t)=1$, $u_t^h = \ln c_t^h(t) + \beta \ln c_t^h(t+1)$, $\Delta_t^h = [1/2, 1/2]$, $Y(t) = K(t)^{\theta} L(t)^{1-\theta}$,
$C(t) + K(t+1) = Y(t) \quad \forall t$

(a) Find expression for steady state eq. capital stock & i.r.
What are numbers when $\beta = 1$ and $\theta = 1/3$?

Consumer's problem

$$\max \; u_t^h \quad \text{s.t.} \quad c_t^h(t) = w(t)\Delta_t^h(t) - k^h(t+1) - l^h(t)$$
$$c_t^h(t+1) = w(t+1)\Delta_t^h(t+1) + r(t)\, l^h(t) + R(t+1)\, k^h(t+1)$$

combining budget constraints yields lifetime BC:

$$\underbrace{c_t^h(t+1) = w(t+1)\Delta_t^h(t+1) + r(t)\left[w(t)\Delta_t^h(t) - k^h(t+1) - c_t^h(t)\right] + R(t+1)k^h(t+1)}_{1}$$

Substitute into maximization problem:

$$\max_{\{c_t^h(t),\, k^h(t+1)\}} \ln c_t^h(t) + \beta \ln(1)$$

FOCs:

(2) $c_t^h(t)$: $\dfrac{1}{c_t^h(t)} - \dfrac{\beta r(t)}{c_t^h(t+1)} = 0 \quad \Rightarrow c_t^h(t+1) = \beta r(t) c_t^h(t)$

(3) $k^h(t+1)$: $\dfrac{\beta}{c_t^h(t+1)}\left[-r(t) + R(t+1)\right] = 0$

since $\beta > 0$, we must have $r(t) = R(t+1)$

Substituting the above into the lifetime BC:

$$\beta r(t) c_t^h(t) = w(t+1)\Delta_t^h(t+1) + r(t)\left[w(t)\Delta_t^h(t) - k^h(t+1) - c_t^h(t)\right] + r(t)k^h(t+1)$$
$$\Leftrightarrow$$
$$\beta r(t) c_t^h(t) + c_t^h(t) r(t) = w(t+1)\Delta_t^h(t+1) + r(t) w(t)\Delta_t^h(t)$$
$$\Leftrightarrow$$
$$(1+\beta) r(t) c_t^h(t) = w(t+1)\Delta_t^h(t+1) + r(t) w(t)\Delta_t^h(t)$$
$$\Leftrightarrow$$
$$c_t^h(t) = \frac{w(t+1)\Delta_t^h(t+1) + r(t) w(t)\Delta_t^h(t)}{(1+\beta) r(t)} = \frac{w(t+1)\Delta_t^h(t+1)}{(1+\beta) r(t)} + \frac{w(t)\Delta_t^h(t)}{1+\beta}$$

Savings = endowment - consumption

$$s_t^h(t) = w(t)\Delta_t^h(t) - \frac{w(t+1)\Delta_t^h(t+1)}{(1+\beta) r(t)} - \frac{w(t)\Delta_t^h(t)}{1+\beta}$$

$$s_t^h(t) = \frac{\beta}{1+\beta} w(t)\Delta_t^h(t) - \frac{w(t+1)\Delta_t^h(t+1)}{(1+\beta) r(t)} = S_t(t) \quad \text{since } N(t) = 1 \;\; \forall t$$

Firm's Problem

$$\max_{\{K(t),\,L(t)\}} \quad K(t)^\theta L(t)^{1-\theta} - R(t)K(t) - w(t)L(t)$$

FOCs:

$$K(t):\ \theta K(t)^{\theta-1} L(t)^{1-\theta} - R(t) = 0$$

$$L(t):\ (1-\theta) K(t)^\theta L(t)^{-\theta} - w(t) = 0$$

So $$R(t) = \theta \left(\frac{K(t)}{L(t)}\right)^{\theta-1}$$

$$w(t) = (1-\theta)\left(\frac{K(t)}{L(t)}\right)^\theta$$

Since we have two agents alive with combined labor endowment $\frac{1}{2}+\frac{1}{2}=1$, $L(t)=1 \quad \forall t$. Thus:

$$R(t) = \theta\, K(t)^{\theta-1} \quad \text{and } R(t+1) = \theta K(t+1)^{\theta-1} = r(t)$$

$$w(t) = (1-\theta) K(t)^\theta \quad \text{and } w(t+1) = (1-\theta) K(t+1)^\theta$$

Substituting into aggregate savings:

$$S_t(r(t)) = \frac{\beta}{1+\beta}(1-\theta)K(t)^\theta \Delta_t^h(t) - \frac{(1-\theta)K(t+1)^\theta \Delta_t^h(t+1)}{(1+\beta)\,\theta K(t+1)^{\theta-1}}$$

$$= \frac{\beta}{2(1+\beta)}(1-\theta)K(t)^\theta - \frac{(1-\theta)}{2(1+\beta)\theta} K(t+1)$$

$$= \frac{(1-\theta)}{\theta 2(1+\beta)}\left(\theta\beta K(t)^\theta - K(t+1)\right)$$

By p. 238 in MW we know $S_t(r(t)) = K(t+1)$

Since we're looking at a stationary equilibrium, let $K(t) = K \in \mathbb{R} \ \forall t$.

Then $$K = \frac{(1-\theta)}{\theta 2(1+\beta)}\left(\theta\beta K^\theta - K\right)$$

$$\Rightarrow \frac{\beta(1-\theta)K^\theta}{2(1+\beta)} - \frac{(1-\theta)+\theta 2(1+\beta)}{\theta 2(1+\beta)} K = 0$$

$$\frac{\theta\beta(1-\theta)K^\theta}{(1-\theta)+\theta 2(1+\beta)} = K$$

$$\frac{\beta(1-\theta)\theta}{(1-\theta)+\theta 2(1+\beta)} = K^{1-\theta}$$

$$K = \left(\frac{\theta\beta(1-\theta)}{(1-\theta)+2\theta(1+\beta)}\right)^{\frac{1}{1-\theta}}$$

Plugging in $\theta = \frac{1}{3}$, $\beta = 1$, we get $K^* = \frac{1}{27}$, $r = \frac{1}{3}\left(\frac{1}{27}\right)^{\frac{1}{3}-1} = 3$.

(a) Checking feasibility: $\forall t$:

$$\star\ C(t) + K(t+1) = Y(t) = K^{1/3} L^{2/3}$$

$$w(t) = (1-\theta) K^{\theta} = \frac{2}{3} K^{1/3} = \frac{2}{9}$$

$$C(t) = c_t^n(t) + c_t^n(t+1) = c_t^n(t) + \beta r(t) c_t^n(t) = 4 c_t^n(t)$$
$$= 4\left(\frac{w(t+1)\Delta_t^n(t+1)}{(1+\beta) r(t)} + \frac{w(t)\Delta_t^n(t)}{1+\beta}\right) = 4\left(\frac{\frac{2}{9}\cdot\frac{1}{2}}{2\cdot 3} + \frac{\frac{2}{9}\cdot\frac{1}{2}}{2}\right) = \frac{8}{27}$$

$$C(t) + K(t+1) = \frac{1}{3} = K^{1/3}$$
$$8/27 + 1/27 = \frac{1}{3} = (1/27)^{1/3}$$

So the allocation is feasible

(b) Why is the interest rate so high in this economy?

Since agents in this economy want to smooth consumption, and have a balanced labor endowment of $[1/2, 1/2]$, agents have a low incentive to save. A high interest rate is necessary to incentivize saving.

(c) Golden rule level of capital

We know $C(t) = Y(t) - K(t+1) \quad \forall t \quad \Rightarrow C(t) = K(t)^{\theta} L(t)^{1-\theta} - K(t+1)$

To maximize consumption, we check where $\partial C / \partial K = 0$:

$\partial C/\partial K = \theta K^{\theta-1} L^{1-\theta} - 1 \Rightarrow \theta K^{\theta-1} = 1$

$\Rightarrow \frac{1}{3} K^{-2/3} = 1 \Rightarrow K^{-2/3} = 3 \Rightarrow K_{gold} = (\frac{1}{3})^{3/2} \approx 0.1924$

Since $K_{gold} > K^*$, stationary capital stock is below the golden rule level, so we conclude the economy is dynamically efficient.