2(a) Let $x \in \Delta_n$. For each coordinate i, $0 \leq x_i \leq 1$ and consequently $x_i^2 \leq x_i$. So

$$\|x\| = \sqrt{x_1^2 + \cdots + x_n^2} \leq \sqrt{x_1 + \cdots + x_n} = \sqrt{1} = 1.$$

Or via the triangle inequality: x has nonnegative coordinates summing to one, so $\|x\| \leq \sum_{i=1}^n |x_i| \|e_i\| = 1$.
2(b) Let $k \in \mathbb{N}$. Using properties (N3) and (N4) of norms and the fact that $A^k x$ and x lie in Δ_n and consequently have a length at most one by our answer above, we find

$$\begin{aligned}\|Av_k - v_k\| &= \left\| \tfrac{1}{k}(Ax + Ax^2 + \cdots + A^k x) - \tfrac{1}{k}(x + Ax + \cdots + A^{k-1}x) \right\| = \left\| \tfrac{1}{k}(A^k x - x) \right\| \\ &= \tfrac{1}{k}\|A^k x - x\| \leq \tfrac{1}{k}(\|A^k x\| + \|-x\|) = \tfrac{1}{k}(\|A^k x\| + \|x\|) \leq \tfrac{2}{k}.\end{aligned}$$

2(c) By 2(a), sequence $(v_k)_{k \in \mathbb{N}}$ in Δ_n is bounded, so it has a convergent subsequence (Thm. 9.2(d)). Δ_n is a polyhedron, hence closed (p. 66), so its limit lies in Δ_n (Thm. 9.4). Or use Thm. 13.4.
2(d) Denote our subsequence by $(v_{k(n)})_{n \in \mathbb{N}}$. It converges to v^* and the function $v \mapsto Av - v$ is linear, hence continuous (Ex. 8.3), so the sequence $(Av_{k(n)} - v_{k(n)})_{n \in \mathbb{N}}$ converges to $Av^* - v^*$ by Thm. 9.3. Since $\frac{2}{k} \to 0$ as $k \to \infty$, it also converges to $\mathbf{0}$ by 2(b). The limit of a (sub)sequence in a metric space is unique (Thm. 9.1), so $Av^* - v^* = \mathbf{0}$, i.e., $Av^* = v^*$.