14.03/14.003: Microeconomic Theory and Public Policy 10/21/22

Recitation 6: General Equilibrium and Non-Compliance in Randomized Control Trials

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The Edgeworth Box

The Edgeworth Box is a way of representing an economy with two goods and two agents. It shows how trade can make these two agents better off, and how prices will facilitate the right amount of trade, bringing us to an efficient equilibrium.

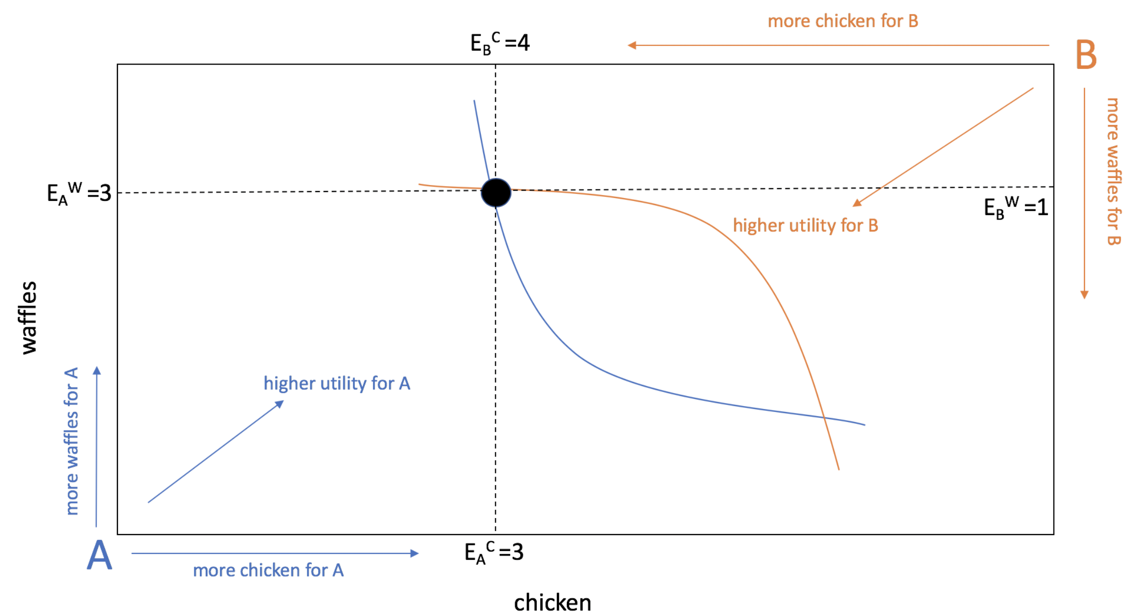
Setup

We have two agents, A and B. We also have two goods in the economy, c and w (chicken and waffles). Each agent has preferences (a utility function) and an initial endowment of each good. To be more concrete:

- Agent A's utility is $u_A(c, w) = \frac{1}{2}\ln c + \frac{1}{2}\ln w$
- Agent B's utility is $u_B(c, w) = \frac{1}{4}\ln c + \frac{3}{4}\ln w$
- Agent A starts with endowment $(E_A^c, E_A^w) = (3, 3)$
- Agent B starts with endowment $(E_B^c, E_B^w) = (4, 1)$

The figure below shows the setup of an Edgeworth Box in this context.

Figure 1: Endowments in the Edgeworth Box



Without any trade, A's utility is

$$u_A(3,4) = \frac{1}{2}\ln 3 + \frac{1}{2}\ln 3 \approx 1.01$$

and B's utility is

$$u_B(4,1) = \frac{1}{4}\ln 4 + \frac{3}{4}\ln 1 \approx 0.35.$$

We assume that A and B can trade under the following assumptions:

1. No transaction costs
2. No market power (agents take prices as given)
3. No externalities
4. Full information about the goods
5. Property rights (all goods are owned by somebody).

Question: without doing any math, which good do you expect will be more expensive and why?

Finding Demands as a Function of Prices (Partial Equilibrium)

As a first step, we solve for A and B's demand as a function of prices p^c and p^w. This is very similar to the problems we solved in the consumer theory section - for the time being, we still treat prices and given. The only wrinkle is that the budget set is given by the consumer's endowment. Agent A solves:

$$\max_{c_A, w_A} \left\{ \frac{1}{2} \ln c + \frac{1}{2} \ln w \right\} \quad \text{subject to} \quad p^c c_A + p^w w_A \leq 3p^c + 3p^w$$

and Agent B solves:

$$\max_{c_B, w_B} \left\{ \frac{1}{4} \ln c + \frac{3}{4} \ln w \right\} \quad \text{subject to} \quad p^c c_B + p^w w_B \leq 4p^c + 1p^w.$$

Setting up the two agents' Lagrangians gives us:

$$\mathcal{L}_A(p^c, p^w, \lambda_A) = \frac{1}{2} \ln c + \frac{1}{2} \ln w - \lambda_A(p^c c_A + p^w w_A - 3p^c - 3p^w)$$
$$\mathcal{L}_B(p^c, p^w, \lambda_B) = \frac{1}{4} \ln c + \frac{3}{4} \ln w - \lambda_B(p^c c_B + p^w w_B - 4p^c - 1p^w).$$

Solving both of these constrained optimization problems gives us each agent's demands as a function of prices:

$$c_A^* = \frac{3p^c + 3p^w}{2p^c} \qquad w_A^* = \frac{3p^c + 3p^w}{2p^w}$$
$$c_B^* = \frac{4p^c + p^w}{4p^c} \qquad w_A^* = \frac{3(4p^c + p^w)}{4p^w}.$$

Finding the Prices that "Clear" the Market (General Equilibrium)

Next, we want to solve for the price ratio p^c/p^w that clears the market.[1] We use the fact that markets must clear: the total demand for c must equal the total supply of c. Same for w. For chicken, this means that:

$$\begin{aligned} \underbrace{c_A^* + c_B^*}_{demand} &= \underbrace{3 + 4}_{supply} \\ \frac{3p^c + 3p^w}{2p^c} + \frac{4p^c + p^w}{4p^c} &= 7 \\ \frac{3}{2} + \frac{3}{2}\left(\frac{p^w}{p^c}\right) + 1 + \frac{1}{4}\left(\frac{p^w}{p^c}\right) &= 7 \\ \frac{7}{4}\left(\frac{p^w}{p^c}\right) &= \frac{9}{2} \\ \frac{p^c}{p^w} &= \frac{7}{18}. \end{aligned}$$

[1]Why are we solving for the ratio p^c/p^w rather than p^c and p^w separately?

For waffles, this means that:

$$\begin{aligned}
\underbrace{w_A^* + w_B^*}_{demand} &= \underbrace{3+1}_{supply} \\
\frac{3p^c + 3p^w}{2p^w} + \frac{3(4p^c + p^w)}{4p^w} &= 4 \\
\frac{3}{2}\left(\frac{p^c}{p^w}\right) + \frac{3}{2} + 3\left(\frac{p^c}{p^w}\right) + \frac{3}{4} &= 4 \\
\frac{9}{2}\left(\frac{p^c}{p^w}\right) &= \frac{7}{4} \\
\frac{p^c}{p^w} &= \frac{7}{18}.
\end{aligned}$$

Note that we only need to solve one of these equations, because we really only have one variable: p^c/p^w. Plugging this price ratio into our demands, we get:

$$\begin{aligned}
c_A^* &= \frac{3p^c + 3p^w}{2p^c} = \frac{3}{2} + \frac{3}{2}\left(\frac{p^w}{p^c}\right) = \frac{3}{2} + \frac{3}{2}\left(\frac{18}{7}\right) = \frac{75}{14} \\
w_A^* &= \frac{3p^c + 3p^w}{2p^w} = \frac{3}{2}\left(\frac{p^c}{p^w}\right) + \frac{3}{2} = \frac{3}{2}\left(\frac{7}{18}\right) + \frac{3}{2} = \frac{25}{12} \\
c_B^* &= \frac{4p^c + p^w}{4p^c} = 1 + \frac{1}{4}\left(\frac{p^w}{p^c}\right) = 1 + \frac{1}{4}\left(\frac{18}{7}\right) = \frac{23}{14} \\
w_B^* &= \frac{3(4p^c + p^w)}{4p^w} = 3\left(\frac{p^c}{p^w}\right) + \frac{3}{4} = 4\left(\frac{7}{18}\right) + \frac{3}{4} = \frac{23}{12}
\end{aligned}$$

Plugging these into the utility functions, we find that A's utility is

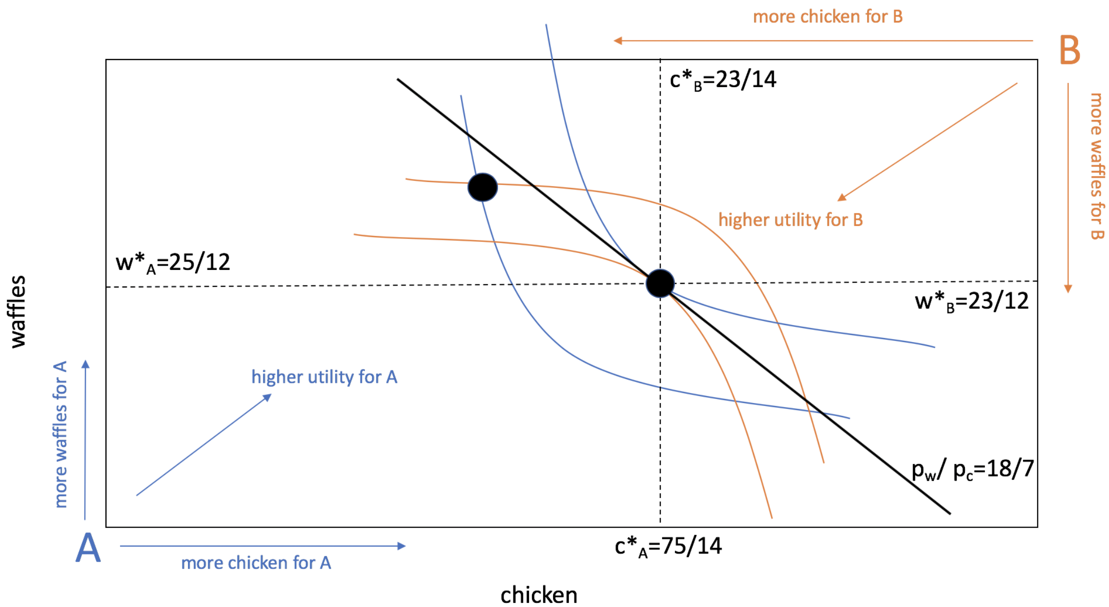
$$u_A(3,4) = \frac{1}{2}\ln\frac{75}{14} + \frac{1}{2}\ln\frac{25}{12} \approx 1.21$$

and B's utility is

$$u_B(4,1) = \frac{1}{4}\ln\frac{23}{14} + \frac{3}{4}\ln\frac{23}{12} \approx 0.61.$$

So both agents are better off after trade. The figure below shows the equilibrium in the Edgeworth Box.

Figure 2: Equilibrium in the Edgeworth Box



Properties of the Equilibrium and the First Welfare Theorem

An equilibrium is a price ratio p^c/p^w and a set of consumptions (c_A^*, w_A^*), (c_B^*, w_B^*). At the equilibrium we solved above, all of the following are true:

1. The market clears (demand = supply for both goods)
2. Each consumer is optimizing given his endowments and prices
3. Consumption is Pareto Efficient. Nobody can be made better off without making the other person worse off.

The **First Welfare Theorem** states that given our initial endowments $(E_A^c, E_A^w) = (3, 3)$ and $(E_B^c, E_B^w) = (4, 1)$ we will arrive at our equilibrium without the help of a central planner. In other words, the free market will get us to this equilibrium on its own. In fact, given any set of initial endowments (E_A^c, E_A^w) and (E_B^c, E_B^w) we will arrive at an equilibrium with these properties.

Different Endowments → Different Equilibrium and the Second Welfare Theorem

Suppose that we changed the initial endowments to $(E_A^c, E_A^w) = (3, 1)$ and $(E_B^c, E_B^w) = (4, 3)$. Preferences and total endowments $(E\prime_A + E_B)$ are unchanged. If we re-solve our demands, we find:

$$c_A^* = \frac{3p^c + p^w}{2p^c} \quad w_A^* = \frac{3p^c + p^w}{2p^w}$$

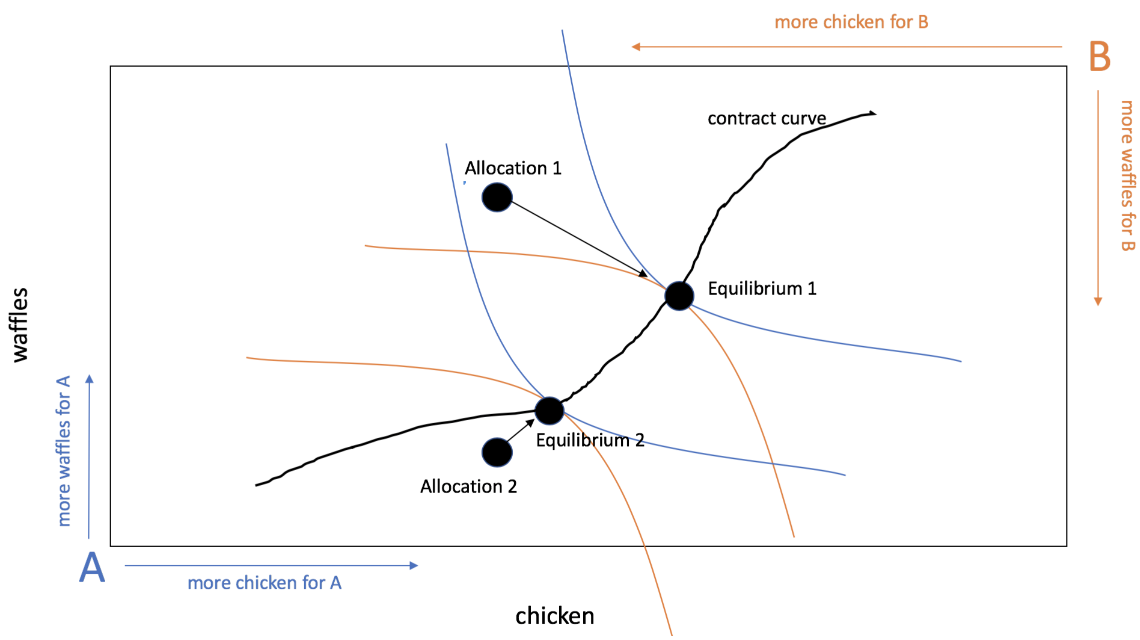
$$c_B^* = \frac{4p^c + 3p^w}{4p^c} \quad w_A^* = \frac{3(4p^c + 3p^w)}{4p^w}.$$

If we use market clearing, we find that the price ratio is $p^c/p^w = \frac{5}{18}$. This implies that:

$$c_A^* = \frac{33}{10}$$
$$w_A^* = \frac{11}{12}$$
$$c_B^* = \frac{37}{10}$$
$$w_B^* = \frac{37}{12}.$$

In other words, different endowments give us a different equilibrium (different consumption bundles and a different price ratio). The Second Welfare Theorem states that any Pareto efficient allocation can be supported as a market equilibrium. In other words, we can end up at any point on the contract curve. The key is that we need to start with the right endowments. The figure below shows how the two different endowments get us to two different efficient equilibria in the Edgeworth Box.

Figure 3: The Second Welfare Theorem in the Edgeworth Box



Non-Compliance in Randomized Control Trials

Setup

Suppose we want to assess the impact of a new blood pressure drug, so we run a randomized control trial. The problem is that we have some non-compliance - some of the people who we put in the treatment group don't actually take the drug (they forget, they don't like the side effects, whatever). Moreover, it's also possible that some people in the control group sneakily get ahold of the drug. Let:

- Y_i be the outcome of interest (in this case blood pressure - lower is better)
- $Z_i \in \{0, 1\}$ be the assignment to treatment or control (aka the instrument)
- $D_i \in \{0, 1\}$ be whether the subject actually took the drug (aka the treatment)

If we compare treatment and control groups in this setting, we won't get an accurate estimate of the ATT. In other words,

$$\tilde{T} = E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0] \neq ATT.$$

Why? And which way would you expect it to be biased (recall that the drug is supposed to lower Y)?

Compliance and Non-Compliance

Some subjects are good subjects - they comply with the group they are assigned. For compliers,

- **Compliers:** If $Z_i = 0$, then $D_i = 0$. If $Z_i = 1$, then $D_i = 1$.

There are three types of non-compliers. Always-takers will take the drug no matter which group they are assigned. Thus,

- **Always-takers:** If $Z_i = 0$, then $D_i = 1$. If $Z_i = 1$, then $D_i = 1$.

Never-takers will not take the drug no matter which group they are assigned. Thus,

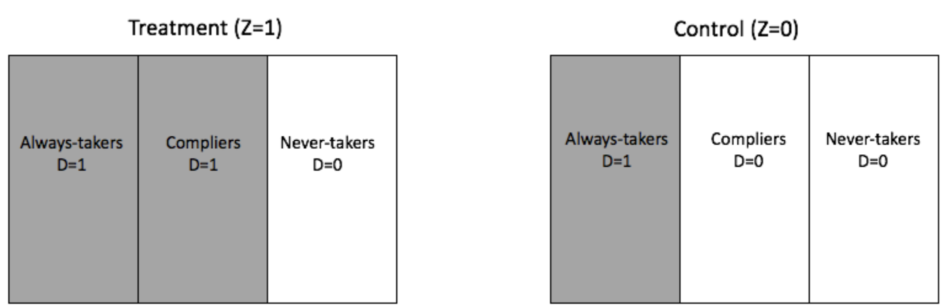
- **Never-takers:** If $Z_i = 0$, then $D_i = 0$. If $Z_i = 1$, then $D_i = 0$.

Defiers will always do the opposite of the group they were assigned. Thus,

- **Defiers:** If $Z_i = 0$, then $D_i = 1$. If $Z_i = 1$, then $D_i = 0$.

We will assume that we have no defiers in our study. Why is this a reasonable assumption? Figure 4 shows why $\tilde{T} = E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0]$ is not an unbiased estimate of the ATT. Note that the treatment group is "contaminated" the never-takers and the control group is "contaminated" by the always-takers.

Figure 4: Non-Compliance in RCTs



Recovering an Average Treatment Effect

There are compliers and non-compliers. Can we figure out what fraction of our subjects are compliers? Yes, because compliers are the ones who flip from $D_i = 0$ to $D_i = 1$ when Z_i goes from 0 to 1. So the fraction of subjects that are compliers is

$$E[D_i|Z_i = 1] - E[D_i|Z_i = 0].$$

For concreteness, say that the fraction of compliers equals $\frac{1}{3}$. Let's re-write $\tilde{T}$ as a weighted average of the effect on compliers and on non-compliers:

$$\begin{aligned}\tilde{T} &= E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0]\\ &= \frac{1}{3}\left[E[Y_i^C|Z_i = 1] - E[Y_i^C|Z_i = 0]\right] + \frac{2}{3}\left[E[Y_i^{NC}|Z_i = 1] - E[Y_i^{NC}|Z_i = 0]\right].\end{aligned}$$

Now, notice that $E[Y_i^C|Z_i = 1] - E[Y_i^C|Z_i = 0]$ is a true ATT for the compliers,[2] because we can re-write this as:

$$ATT_C = E[Y_i^C|Z_i = 1] - E[Y_i^C|Z_i = 0] = E[Y_i^C|Z_i = 1, D_i = 1] - E[Y_i^C|Z_i = 0, D_i = 0].$$

Next, think about the term $E[Y_i^{NC}|Z_i = 1] - E[Y_i^{NC}|Z_i = 0]$ from the perspective of both the always-takers and never-takers. Why does it make sense to assume it is zero for both types? Now, let's do some re-writing:

$$\begin{aligned}E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0] &= \frac{1}{3}ATT_C + \frac{2}{3}\cdot 0\\ ATT_C &= \frac{E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0]}{\frac{1}{3}}\\ ATT_C &= \frac{E[Y_i|Z_i = 1] - E[Y_i|Z_i = 0]}{E[D_i|Z_i = 1] - E[D_i|Z_i = 0]}.\end{aligned}$$

So the ATT for the compliers is the ratio of effect of being put in the treatment group on cholesterol, divided by the effect of being put in the treatment group on actually taking the drug. Perhaps a more intuitive way to think about this is to realize:

$$\text{Effect of treatment group on cholesterol} = (\text{Effect of treatment group on taking drug}) \times (\text{Effect of drug})$$

Therefore, to isolate the effect of the drug, we need to re-arrange to get:

$$\text{Effect of drug} = \frac{\text{Effect of treatment group on cholesterol}}{\text{Effect of treatment group on taking drug}}.$$

[2] This is sometimes called the Local Average Treatment Effect (LATE), because it is only the ATT for the compliers. The ATT could be different from the always-takers or never-takers, but we can't measure it (because they either always or never take the drug)