Inequality, Household Behavior, & the Macroeconomy
Problem Set 4

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1 Taxation

6. The case with a benefit financed by higher labor income tax yields higher average welfare than the benchmark case. The difference in consumption equivalents is approximately 15.21% (extra consumption relative to the optimal path).

   Labor income tax is distortive due to heterogeneity in the y process, which disincentivizes more productive individuals. General equilibrium implies that the total labor supply of wage earners equals the labor input of the corporate sector, which produces with a Cobb-Douglas technology and maximizes profits. Consequently, the wage is proportional to the total labor supply.

   The channel that offsets the reduced efficiency is the distributional impact of benefits financed by taxation. Concave preferences make the overall welfare difference from the benchmark to the alternative setting positive.

7. Average welfare in both the capital income (-5.126) and wealth tax (-5.042) settings is greater than in the first labor tax settings ($-5.99, -5.58$). The reason for this difference could be that entrepreneurs are more productive, and the former settings imply redistribution to workers, which in turn makes the overall effect positive.

8. The wealth tax setting leads to higher average welfare, but the difference is not too large (around 3.4% extra consumption). While less pronounced, the results are in line with Guvenen et al. (2023), who use a model similar (but more elaborate) to Cagetti and De Nardi (2006). Wealth taxes reduce the after-tax returns of high-productivity entrepreneurs less than those of low-productivity ones, creating a behavioral savings response that shifts resources toward the more productive individuals. Key differences in our setting include the entrepreneurial and corporate sectors being more detached (more efficient entrepreneurs do not increase wages).

   When are capital income and wealth taxes equivalent? As long as everyone earns the same return. This is not true in our model. Returns, defined as capital income divided by beginning-of-period wealth, vary across agents in the two models: workers have an implied $r \approx 6\%$ (with no variation), while the median entrepreneur returns are an order of magnitude greater and vary according to the θ process.

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Taxation in Cagetti & De Nardi (2006)

Before starting this exercise, make sure that you downloaded the updated version of 50_cagettidenardi.jl, 50_cagettidenardi_solve.jl and you looked at the extra slides on market clearing from the same class. Consider a modification of Cagetti & De Nardi (2006) with a richer tax system, as follows:

- We keep the labor income tax from the original version of the model, but will denote it with τl instead of τ .
- Everyone with beginning-of-period assets a pays an $\tau w \cdot a$ amount of wealth tax.
- A capital income tax with rate τk is applied on capital income. The latter is defined as
  - $r \cdot a$ for non-entrepreneurs, and
  - $\theta k\nu - \delta k - r(k - a)$ for entrepreneurs
- Everyone receives b amount of benefits.
- As in the original model, retired people receive p amount of pension, where p is a fixed fraction of w. This is in addition to b.
- The government balances the budget: Total revenues from the labor income, capital income and wealth taxes should exactly cover expenses on benefits and pension.

Make a copy of 50_cagettidenardi_solve.jl and rename it. Next, in this file:

1.

Modify function solve in a way that it correctly solves for the optimal policy and value functions of all agents, given this richer tax system.

- In addition to r and τl, also make τk, τw and b (all Reals) function arguments to the function solve. We don't put these in EconPars as independent parameters, since we want to solve for the levels balancing the budget (or in the case of r, clear the capital market).
- Next, amend the budget constraints appropriately for all possible states (old/young, entrepreneur or not). Therefore update all lines computing cash-on-hand (4 such places)

In [ ]:
```
#function solve(ep::EconPars, np::NumPars, r::Real, τ_l::Real, τ_w::Real, τ_k::Real, b::Real; conv_tol=0.0001, max
# 1: retired people
# coh = (1 - τ_k) * (r * assetgrid[i]) + (1 - τ_w) * assetgrid[i] + b + p
# 2: old entrepreneur
# coh = (1 - τ_k) * (ϑs[ϑi] * k^ν - δ * k - r*(k - assetgrid[i])) + (1 - τ_w) * assetgrid[i] + b
# 3) young
# coh if enterprise
# ccoh_e = (1 - τ_k) * (ϑs[ϑi] * k^ν - δ * k - r*(k - assetgrid[i])) + (1 - τ_w) * assetgrid[i] + b
# coh if being a wage earner
# coh_w = (1 - τ_k) * (r * assetgrid[i]) + (1 - τ_w) * assetgrid[i] + (1 - τ_l) * w * ys[yi] + b
```

2.

Modify GE_difference such that input x is interpreted as a 5-element vector with elements r, τl, τk, τw and b (instead of just the first two elements). Within the function, compute

- the total amount of benefits paid out
- total tax revenues from capital and wealth taxes

and take into account the corresponding terms in the line defining budget_balance_relative. It should still give the budget surplus per person, divided by w.

The output of the GE_difference should be left unchanged: the 2-element vector it returns will help us pin down r and one free taxation parameter at a time.

We will work with four setups. The parameters from 50_cagettidenardi.jl serve as a benchmark. In the other settings, the government finances b = 0.05 (approximately 1% of average labor earnings) via different taxes. More concretely:

- Benchmark: r and τl as defined in calib in 50_cagettidenardi.jl. (These

parameters clear the capital market and balance the government budget). The new taxes and b should all be 0s Benefit financed by a higher labor income tax: b = 0.05, while the new taxes are still 0. r and τl are re-estimated to clear the capital market and balance the government budget.

- Benefit financed by capital income tax: b = 0.05, and τl is as in the benchmark.

$\tau w = 0$, while the r and τk are re-estimated to clear the capital market and balance the government budget.

- Benefit financed by wealth income tax: b = 0.05, and τl is as in the benchmark.

τk = 0, while the r and τw are re-estimated to clear the capital market and balance the government budget

In [ ]:
```
# ci = similar(ks)
# for t in 1:size(ci,2)
#     for n in 1:size(ci,1)
#         if ϑis[n,t] == 0 # if not entrepreneur
#             ci[n,t] = r * as[n,t]
#         else
#             ci[n,t] = ϑs[ϑis[n,t]] * ks[n,t]^ν - δ * ks[n,t] - r * (ks[n,t] - as[n,t])
#         end
#     end
# end
# k_total = sum(ci) # total capital income
# w_total = sum(as) # total wealth
# benefits_total = N_sample * b
# budget_balance_relative = (earnings_total*τ_l + k_total*τ_k + w_total*τ_w - pension_total - benefits_total) / (N_
```

3.

Calibrate the appropriate parameters in all three new settings. Some hints:

- Use nested_find_zero as in line 21 of 50_cagettidenardi.jl.
- To do so, you always need bracketing intervals for relevant parameters.

For me, (0.06001, 0.07) worked for r, (0.08, 0.15) worked for τl, (0.0, 0.1) worked for τk, (0.0, 0.01) worked for τw.

- We always calibrate two parameters at once (r and a tax) since we have two conditions (capital market and government budget). To write

GE_difference as a function of only two elements of x, while keeping the rest (and the other inputs fixed), one can use an anonymous function such as y− > GE_dif f erence([42, y[1], 21, y[2], 6], otherinputs) as an input of nested_find_zero. Here, y is interpreted as a length two vector, whose elements are used in the positions of your choice of x, while for the other elements of x you can give values you would like.

- On my laptop, calibration took around 10 minutes, for each setting

In [ ]:
```
include("50_cagettidenardi_solveA4.jl"); np = NumPars(); ep = EconPars(); b = 0.05; calib = [0.06602,0.10471]  # r
(olds, yis, θtrans_shocks) = simulate_shocks(ep::EconPars, np::NumPars, 10000, 1000) # simulate shocks that are us
println(GE_difference([calib[1], calib[2], 0, 0, 0],ep,np,olds,yis,θtrans_shocks))
```

```
[0.003418750020585559, 0.0006791427051933911]
```

In [ ]:
```
# nested_find_zero(y -> GE_difference([y[1], y[2], 0, 0, b],ep,np,olds, yis, ϑtrans_shocks),  (0.06001,0.07), (0.0
laborcalib = [0.064436674591436 61, 0.115446544400454 63] # r, τ_l
println(GE_difference([laborcalib[1], laborcalib[2], 0, 0, b], ep, np, olds, yis, θtrans_shocks))
```

```
didn't converge in 150 iterations
[-0.004124389459341904, 0.0007200008815168021]
[-0.004124389459341904, 0.0007200008815168021]
```

In [ ]:
```
# nested_find_zero(y -> GE_difference([y[1], calib[2], y[2], 0.0, 0.05 ],ep, np, olds, yis, ϑtrans_shocks),  (0.06
capitalcalib = [0.06290095399839181,  0.050396721692878246] # r, τ_k
println(GE_difference([capitalcalib[1], calib[2], capitalcalib[2], 0, b], ep, np, olds, yis, θtrans_shocks))
```

```
[-0.0033225644905510165, 0.0005121416797894637]
```

In [ ]:
```
# nested_find_zero(y -> GE_difference([y[1], calib[2], 0, y[2], b],ep,np,olds, yis, ϑtrans_shocks),  (0.06001,0.07
wealthcalib = [0.0627620293814927, 0.008021976857079024] # r, τ_w
println(GE_difference([wealthcalib[1], calib[2], 0, wealthcalib[2], b], ep, np, olds, yis, θtrans_shocks))
```

```
[-0.0036088733195061007, 0.0012467546438097014]
```

4.

After having the right calibrated parameters, solve and simulate all settings.

In [ ]:
```
nobenefit = solve(ep,np,calib[1],calib[2],0,0,0); (olds1, yis1, θis1, ks1, cs1, as1) = simulate(ep, np, nobenefit)
laborbenefit = solve(ep,np,laborcalib[1],laborcalib[2],0,0,b); (olds2, yis2, θis2, ks2, cs2, as2) = simulate(ep, n
capitalbenefit = solve(ep,np,capitalcalib[1],calib[2],capitalcalib[2],0,b); (olds3, yis3, θis3, ks3, cs3, as3) = s
wealthbenefit = solve(ep,np,wealthcalib[1],calib[2],0,wealthcalib[2],b); (olds4, yis4, θis4, ks4, cs4, as4) = simu
```

```
didn't converge in 500 iterations
```

5.

Write a function that takes a Solution structure and matrices containing simulated olds, θis, yis and as as inputs, and gives a correct matrix of simulated value functions as an output, with the same dimensions as the other simulated matrices. (It might be helpful to take a look at how consumption is simulated within simulate_rest). Simulate values in each setting, using the corresponding inputs. Compute the average of simulated values in each setting separately. For any calculations based on simulations, use only simulated figures from the last period.

In [ ]:
```
function simulate_vs(sol::Solution, olds, θis, yis, as)
    N = size(olds,1)
    T = size(olds,2)
    vs = fill(0.0, N, T)
        for t in 2:T
            for n in 1:N
                if olds[n,t]
                    if θis[n,t] == 0
                        vs[n,t] = sol.vfR(as[n,t])
                    else
                        vs[n,t] = sol.vfO[θis[n,t]](as[n,t])
                    end
                else
                    vs[n,t] = sol.vfY[θis[n,t],yis[n,t]](as[n,t])
                end
            end
    end
    return (vs)
end
```

```
simulate_vs (generic function with 1 method)
```

In [ ]:
```
nobenefitvs = simulate_vs(nobenefit, olds1, θis1, yis1, as1); println(mean(nobenefitvs[:,end]))
laborbenefitvs = simulate_vs(laborbenefit, olds2, θis2, yis2, as2); println(mean(laborbenefitvs[:,end]))
capitalbenefitvs = simulate_vs(capitalbenefit, olds3, θis3, yis3, as3); println(mean(capitalbenefitvs[:,end]))
wealthbenefitvs = simulate_vs(wealthbenefit, olds4, θis4, yis4, as4); println(mean(wealthbenefitvs[:,end]))
```

```
-5.990402091843159
-5.580969318303463
-5.126135236742837
-5.041841113782776
```

6.

Compare the benchmark with the case with a benefit financed by higher labor income tax.

- Which one gives higher average welfare?

How big is the difference expressed in consumption equivalents?

- In class we have seen that a labor income tax is distortive when leisure

provides utility. Is there a reason for it to be distortive in this model? If yes, what should be the effect on welfare?

- Are there other channels at work that create a welfare difference between

these two settings?

In [ ]:
```
println(mean(nobenefitvs[:,end]) < mean(laborbenefitvs[:,end]))
println((mean(laborbenefitvs[:,end])/mean(nobenefitvs[:,end]))^(1/(1-1.5))-1)
```

```
true
0.15210663787748802
```

7.

How does welfare in the capital income and wealth tax settings relate to the first two settings? What could be the reason for this difference?

In [ ]:
```
println(mean(nobenefitvs[:,end]) < mean(laborbenefitvs[:,end]) < mean(capitalbenefitvs[:,end]) < mean(wealthbenefi
```

```
true
```

8.

Now let's compare the welfare effects across the settings with capital income tax and wealth tax.

- Which one leads to higher average welfare? Is the difference large?
- Are the results you find in line with Guvenen at al (2023)?
    - If yes, what common features of the two models drive this finding?

- If not, what differences between the two models might be responsible?

In either case, you might find it helpful to think through how returns vary across agents in the two models, and how the entrepreneurial sector is linked to wage earners. In the model, you can define return as capital income divided by beginning-of-period wealth. (No need to know the technical details of Guvenen et al, what I said in class is sufficient to answer this question.)

In [ ]:
```
println((mean(wealthbenefitvs[:,end])/mean(capitalbenefitvs[:,end]))^(1/(1-1.5))-1)
```

```
0.03371735612577709
```

In [ ]:
```
returns3 = similar(ks3);  δ= 0.06; v = 0.82;
    for t in 1:size(ks3, 2)
        for n in 1: size(ks3, 1)
            if ks3[n,t] == 0
                returns3[n,t] = capitalcalib[1]
            else
                returns3[n,t] = (θis3[n,t] * ks3[n,t]^v - δ * ks3[n,t] - capitalcalib[1] * (ks3[n,t] - as3[n,t]))/
            end
        end
    end
sqrt(var(returns3[:,end]))
```

```
0.3440864829555074
```

In [ ]:
```
returns3_w = returns3[ks3[:,end].==0,end]; println(median(returns3_w));
returns3_e = returns3[ks3[:,end].>0,end]; println(median(returns3_e))
sqrt(var(returns3_e))
```

```
0.06290095399839181
1.7354531279326837
1.2336540119728099
```

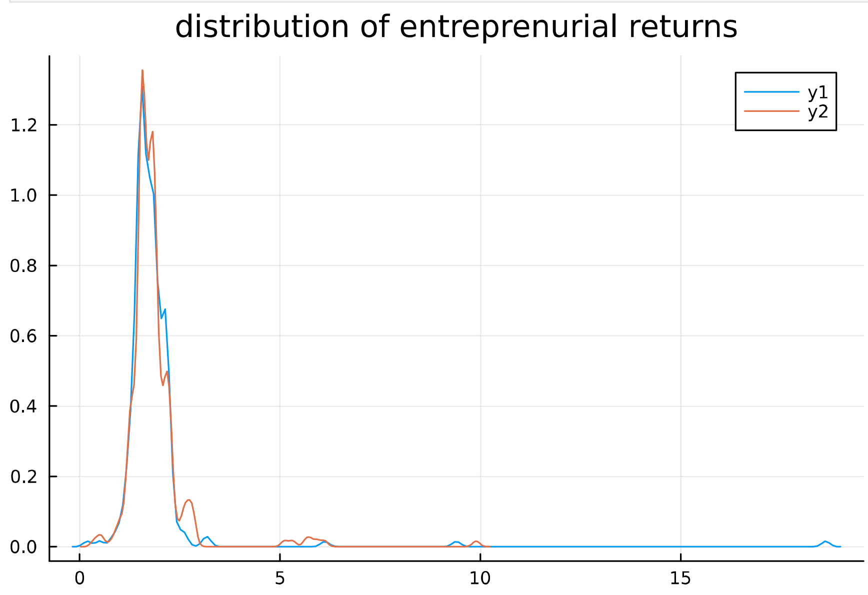
In [ ]:
```
returns4 = similar(ks4);  δ= 0.06; v = 0.82;
    for t in 1:size(ks4, 2)
        for n in 1: size(ks4, 1)
            if ks4[n,t] == 0
                returns4[n,t] = wealthcalib[1]
            else
                returns4[n,t] = (θis4[n,t] * ks4[n,t]^v - δ * ks4[n,t] - wealthcalib[1] * (ks4[n,t] - as4[n,t]))/a
            end
        end
    end
sqrt(var(returns4[:,end]))
```

```
0.0283
```

In [ ]:
```
returns4_w = returns4[ks4[:,end].==0,end]; println(median(returns4_w))
returns4_e = returns4[ks4[:,end].>0,end]; println(median(returns4_e))
sqrt(var(returns4_e))
```

```
0.0627620293814927
1.7421481575469908
0.8315515336973741
```

In [ ]:
```
using Plots
density(returns3_e, title = "distribution of entreprenurial returns")
density!(returns4_e)
```



2 MPC

2.1

Plot the optimal consumption function from model 'nolinks' of age 22 people in middle (6th) α and z states. Explain the intuition behind the shape of the consumption policy.

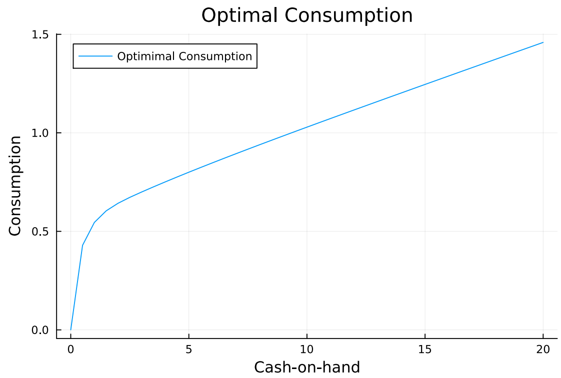


Figure 1: Optimal Consumption (nolinks) for age 22 agents with middle α and z states.

In Figure 1 we see that optimal consumption follows cash-on-hand exactly until approximately 0.5. This "45-degree-line" shows that the very poor are hand-to-mouth consumers. Once cash on hand is sufficiently high, agents no longer eat their entire income; instead they begin to save, primarily for precautionary reasons. Agents may also save in order to smooth consumption around retirement, but this motive is less salient for the youngest agents.

2.2

Keep age fixed, but vary α and z, and compare with your result from the previous point. How do these income states affect the level and steepness of the consumption policy functions? Is there a difference between how α and z affects the shape of the consumption policy? (Compare what happens with the lowest α and middle z state against the lowest z with middle α level.)

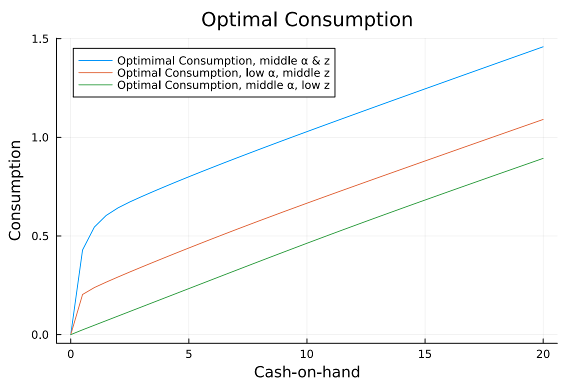


Figure 2: Optimal Consumption with varying α and z states.

Compared to the middle-z middle-α agents, agents with low z consume less and save more for every value of cash-on-hand, including at very low levels of cash-on-hand, yielding a linear optimal consumption path. Since z represents persistent income shocks, those with low z values (negative persistent income shocks, e.g. being laid off) have an incentive to save even at low values of cash-on-hand as they do not expect income to return to its original level and still want to engage in precautionary savings (compared to higher-z agents, who are hand-to-mouth consumers up to a certain level of cash-on-hand).

Compared to the middle-z middle-α agents, agents with low α also consume less of any amount of cash-on-hand. Agents with lower alpha values have lower levels of income, all else equal, and thus follow a similar (nonlinear) consumption path to the middle-z middle-α agents, just at a lower level of consumption.

2.3

Go back to middle α and z and vary age. Try $t = 1, 11, 21, 41, 61$. How does the level and steepness of optimal consumption change with age? What is the intuition? (Comparing with the analytically tractable deterministic case might help figure this out.)

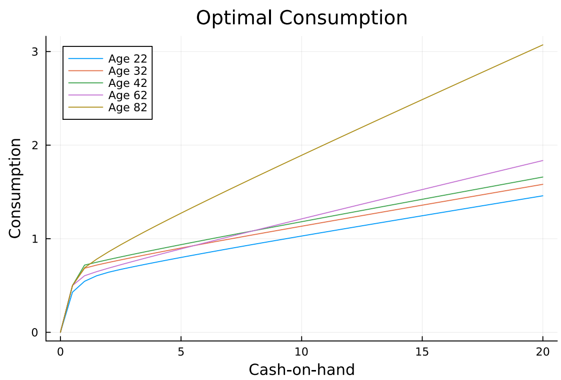


Figure 3: Optimal consumption (nolinks, mid α and z) for different ages.

In Figure 3 we see that the level and steepness of the optimal consumption path each increase in age. Intuitively, this is due to the absence of bequest motives. In the no-link environment, precautionary savings is the primary savings motive. As agents age and face an increasing likelihood of death, they need less savings to protect against the possibility of living for a long time, and thus their optimal consumption is higher for most levels of cash-on-hand.

2.4

Repeat the previous point with 'bequest'. How do your results differ?

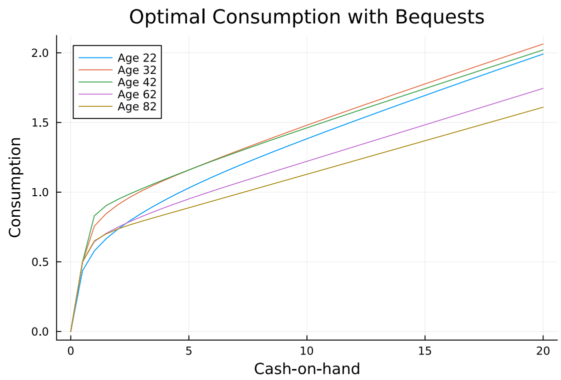


Figure 4: Optimal consumption (bequests, mid α and z) for different ages.

In the bequest world, we now see that the oldest agents in fact consume the *least* for higher levels of cash-on-hand. This is due to the desire to leave bequests for their offspring, which offsets the desire to consume everything at the end of one's life.

2.5 & 2.6

Try to find the intuition for your findings! What drives the high/low values for different age groups?

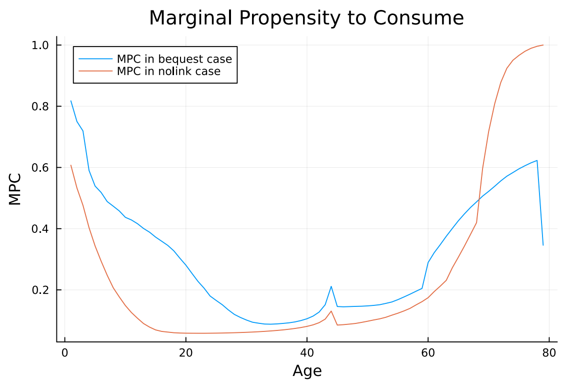


Figure 5: Age average MPC

The red curve in figure 5 shows the average MPC (per age) for agents of generation 20 in our simulated economy with no intergenerational links/no bequest motives. We observe that the marginal propensity to consume is high when agents are young, low in the periods leading up to retirement (note retirement occurs at around age index 43), before the MPC increases sharply in the periods before death. The blue curve illustrating the MPCs for the model allowing for bequests shows a slower decrease in MPC in the beginning and a less significant increase in the years preceding death. Additionally, the line shows a significant drop in the final period(s) before death.

Sources for High/Low MPCs for Age Groups
From the figure above, it appears that the magnitude of the MPC is (at least partially) determined by

- **Bequest Motives**: Allowing for bequests affects MPC. Older agents without a bequest motive have no reason to not consume the extra cash as they do not derive utility from doing so. Allowing for bequests thus decreases the MPC an agent's later years.

- **Available Cash on Hand**: Younger agents in the *bequest* case who may already have received bequests in the past have already built up precautionary savings and, thus have little reason not to spend the extra cash on hand. This is not the case in the *nolink* case, where (accidental) bequests received are smaller.

Differences
Differences between the two models arise mostly in the early and late stages of an agent's lifecycle. In absence of bequest motives, an aging agent in the *nolink* case has little reason to build up savings and, thus will consume a large share (or everything in the last periods) of the extra cash on hand value provided. An agent in the *bequest* case, however, derives utility from leaving bequests, thus consuming less of the extra cash-on-hand, to leave a larger estate to their heirs.
In earlier periods, we observe that agents in the *nolink*-case have lower MPCs as their *bequest*-case counterparts. This can be explained as, in absence of intentional bequests, agents (on average) hold less wealth in younger years and, thus have a motive to build up savings. In contrast, allowing for bequests, some agents who have already received their bequests already have more savings built up, thus having the possibility and desire to spend a larger share of the extra cash-on-hand.