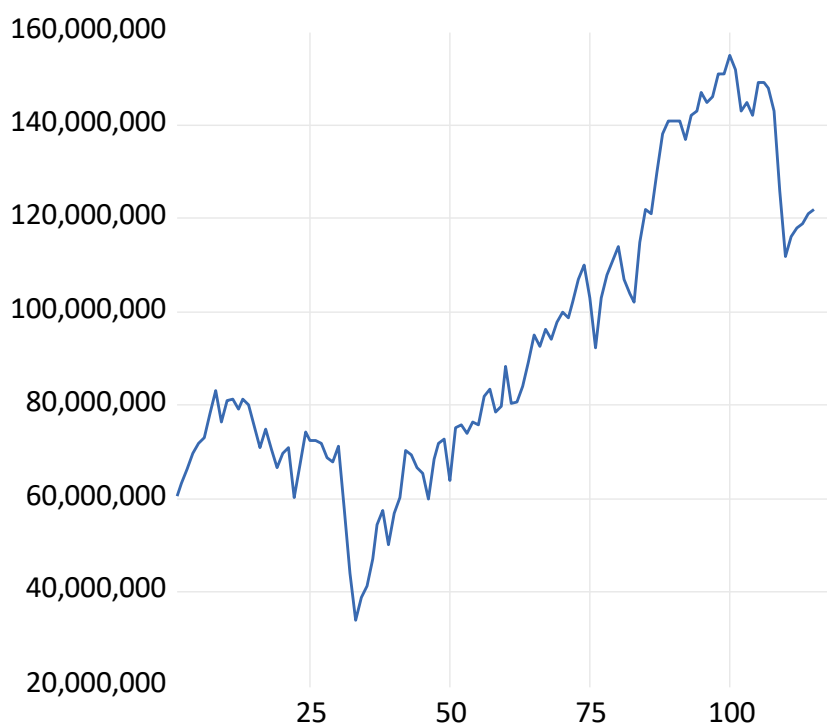


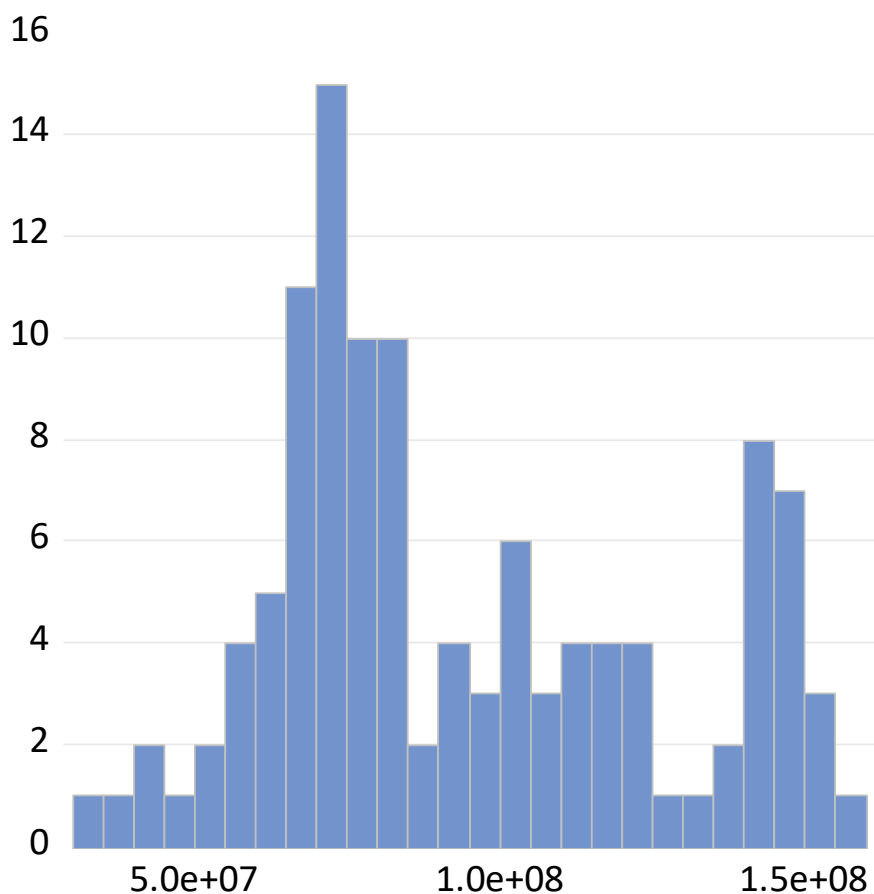
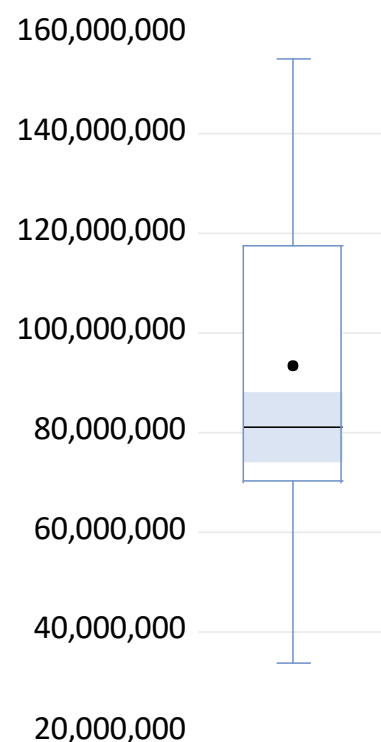
Data

TOTAL FORESTRY STATISTICS, U.S. GEOLOGICAL SURVEY. [All values in metric tons], annual from 1900 to 2014

Production



Production



Series: PRODUCTION

Sample 1 118

Observations 115

Mean	93499130
Median	81400000
Maximum	1.55e+08
Minimum	33900000
Std. Dev.	31417376
Skewness	0.457300
Kurtosis	2.097688
Jarque-Bera	7.909415
Probability	0.019164

Dependent Variable: PRODUCTION

Method: Least Squares

Date: 03/01/23 Time: 13:02

Sample (adjusted): 1 115

Included observations: 115 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	48339325	3165251.	15.27188	0.0000
@TREND	792277.3	47985.90	16.51063	0.0000

R-squared	0.706951	Mean dependent var	93499130
Adjusted R-squared	0.704357	S.D. dependent var	31417376
S.E. of regression	17082577	Akaike info criterion	36.16225
Sum squared resid	3.30E+16	Schwarz criterion	36.20999
Log likelihood	-2077.330	Hannan-Quinn criter.	36.18163
F-statistic	272.6008	Durbin-Watson stat	0.107119
Prob(F-statistic)	0.000000		

Breusch-Godfrey Serial Correlation LM Test:

Null hypothesis: No serial correlation at up to 10 lags

F-statistic	93.09727	Prob. F(10,103)	0.0000
Obs*R-squared	103.5442	Prob. Chi-Square(10)	0.0000

Test Equation:

Dependent Variable: RESID

Method: Least Squares

Date: 03/01/23 Time: 13:10

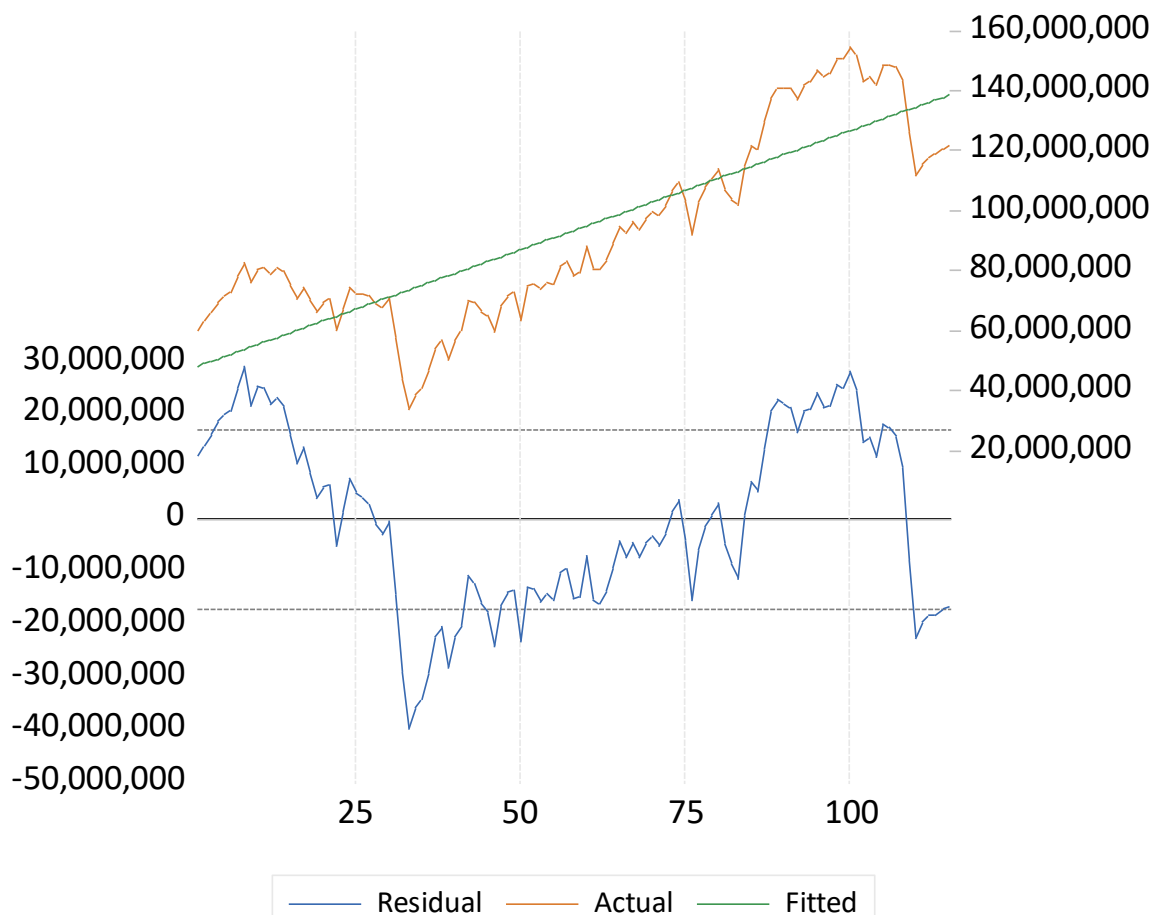
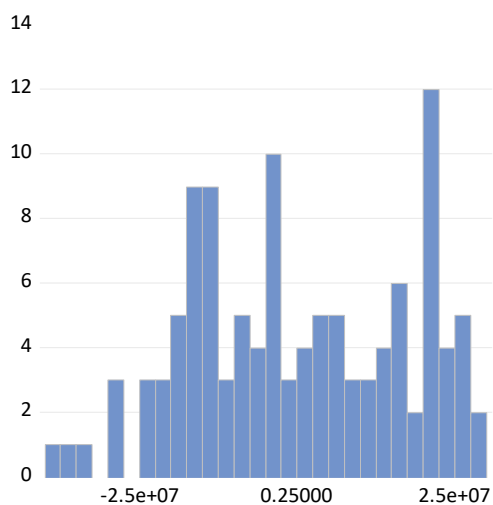
Sample: 1 115

Included observations: 115

Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	178635.3	1050519.	0.170045	0.8653
@TREND	-4814.162	16046.79	-0.300008	0.7648
RESID(-1)	1.115993	0.098617	11.31642	0.0000
RESID(-2)	-0.242851	0.147357	-1.648048	0.1024
RESID(-3)	0.095898	0.146754	0.653459	0.5149
RESID(-4)	-0.038736	0.146778	-0.263906	0.7924
RESID(-5)	0.034905	0.146970	0.237500	0.8127
RESID(-6)	0.057546	0.149739	0.384309	0.7015
RESID(-7)	0.085544	0.149959	0.570446	0.5696
RESID(-8)	-0.287926	0.150711	-1.910455	0.0589
RESID(-9)	0.114149	0.152424	0.748890	0.4556
RESID(-10)	-0.007647	0.106861	-0.071564	0.9431

R-squared	0.900384	Mean dependent var	-7.48E-09
Adjusted R-squared	0.889746	S.D. dependent var	17007488
S.E. of regression	5647264.	Akaike info criterion	34.02973
Sum squared resid	3.28E+15	Schwarz criterion	34.31616
Log likelihood	-1944.710	Hannan-Quinn criter.	34.14599
F-statistic	84.63388	Durbin-Watson stat	1.953568
Prob(F-statistic)	0.000000		



Dependent Variable: PRODUCTION

Method: Least Squares

Date: 03/01/23 Time: 13:04

Sample (adjusted): 1 115

Included observations: 115 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	67581937	4025147.	16.78993	0.0000
@TREND	-229454.3	163172.4	-1.406208	0.1624
@TREND^2	8962.558	1385.137	6.470521	0.0000

R-squared	0.786690	Mean dependent var	93499130
Adjusted R-squared	0.782881	S.D. dependent var	31417376
S.E. of regression	14639255	Akaike info criterion	35.86205
Sum squared resid	2.40E+16	Schwarz criterion	35.93366
Log likelihood	-2059.068	Hannan-Quinn criter.	35.89112
F-statistic	206.5287	Durbin-Watson stat	0.149193
Prob(F-statistic)	0.000000		

Breusch-Godfrey Serial Correlation LM Test:

Null hypothesis: No serial correlation at up to 10 lags

F-statistic	64.99266	Prob. F(10,102)	0.0000
Obs*R-squared	99.40007	Prob. Chi-Square(10)	0.0000

Test Equation:

Dependent Variable: RESID

Method: Least Squares

Date: 03/01/23 Time: 13:12

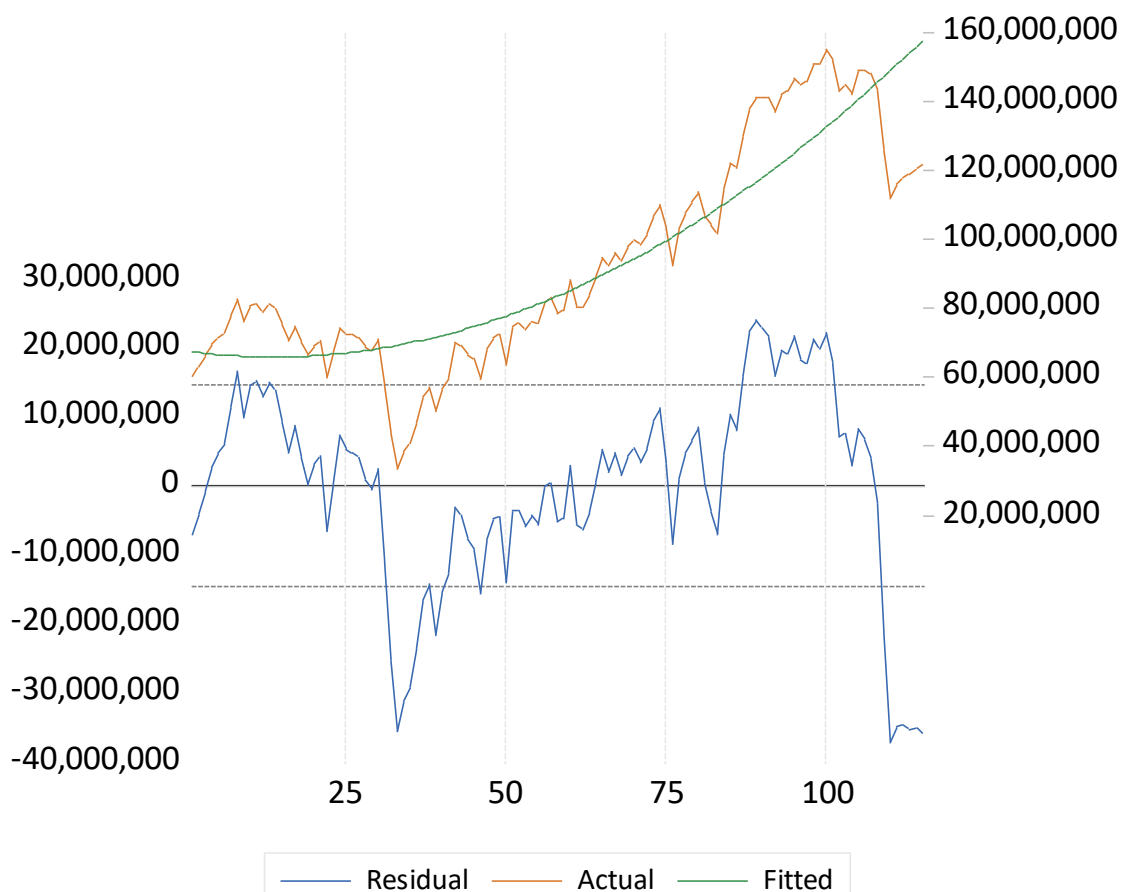
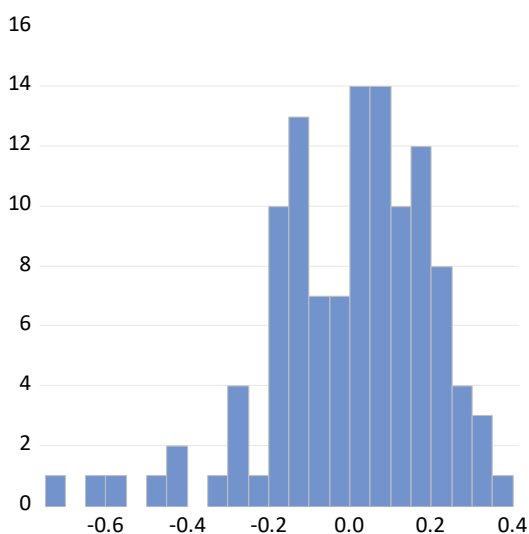
Sample: 1 115

Included observations: 115

Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-583462.9	1647215.	-0.354212	0.7239
@TREND	41892.37	73274.61	0.571717	0.5688
@TREND^2	-463.5714	665.0100	-0.697089	0.4873
RESID(-1)	1.105811	0.098924	11.17837	0.0000
RESID(-2)	-0.241159	0.147557	-1.634345	0.1053
RESID(-3)	0.094575	0.148207	0.638127	0.5248
RESID(-4)	-0.037059	0.148247	-0.249980	0.8031
RESID(-5)	0.038824	0.148499	0.261446	0.7943
RESID(-6)	0.032648	0.151573	0.215398	0.8299
RESID(-7)	0.074283	0.151855	0.489171	0.6258
RESID(-8)	-0.213286	0.152416	-1.399364	0.1647
RESID(-9)	0.021547	0.152750	0.141060	0.8881
RESID(-10)	0.050256	0.111072	0.452463	0.6519

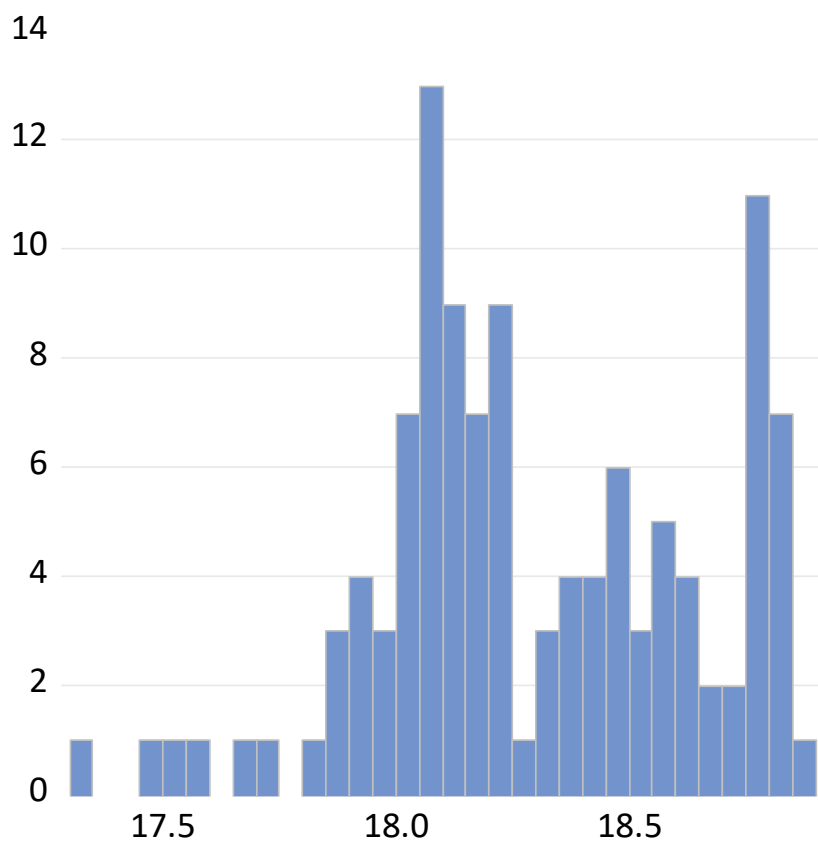
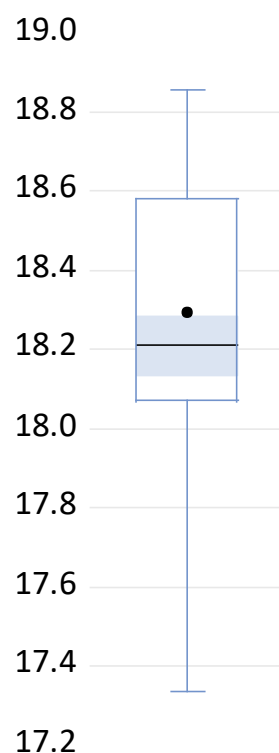
R-squared	0.864348	Mean dependent var	-1.81E-09
Adjusted R-squared	0.848389	S.D. dependent var	14510272
S.E. of regression	5649894.	Akaike info criterion	34.03830
Sum squared resid	3.26E+15	Schwarz criterion	34.34860
Log likelihood	-1944.202	Hannan-Quinn criter.	34.16425
F-statistic	54.16055	Durbin-Watson stat	1.979798
Prob(F-statistic)	0.000000		



LOG(PRODUCTION)



LOG(PRODUCTION)



Series: LOG(PRODUCTION)

Sample 1 118

Observations 115

Mean 18.29672

Median 18.21489

Maximum 18.85894

Minimum 17.33893

Std. Dev. 0.342029

Skewness -0.144876

Kurtosis 2.541114

Jarque-Bera 1.411302

Probability 0.493787

Dependent Variable: LOG(PRODUCTION)

Method: Least Squares

Date: 03/01/23 Time: 13:14

Sample (adjusted): 1 115

Included observations: 115 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	17.82293	0.037305	477.7564	0.0000
@TREND	0.008312	0.000566	14.69691	0.0000

R-squared	0.656534	Mean dependent var	18.29672
Adjusted R-squared	0.653494	S.D. dependent var	0.342029
S.E. of regression	0.201334	Akaike info criterion	-0.350461
Sum squared resid	4.580514	Schwarz criterion	-0.302723
Log likelihood	22.15152	Hannan-Quinn criter.	-0.331085
F-statistic	215.9991	Durbin-Watson stat	0.139894
Prob(F-statistic)	0.000000		

Breusch-Godfrey Serial Correlation LM Test:

Null hypothesis: No serial correlation at up to 10 lags

F-statistic	74.27011	Prob. F(10,103)	0.0000
Obs*R-squared	100.9939	Prob. Chi-Square(10)	0.0000

Test Equation:

Dependent Variable: RESID

Method: Least Squares

Date: 03/01/23 Time: 13:13

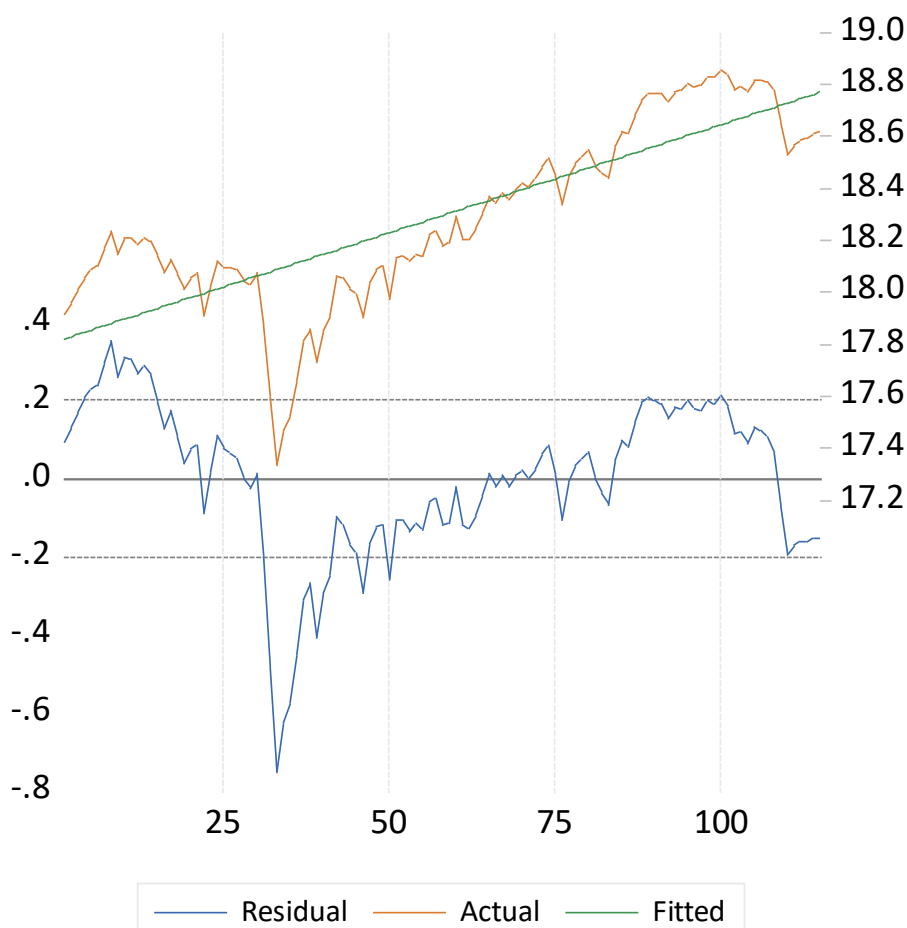
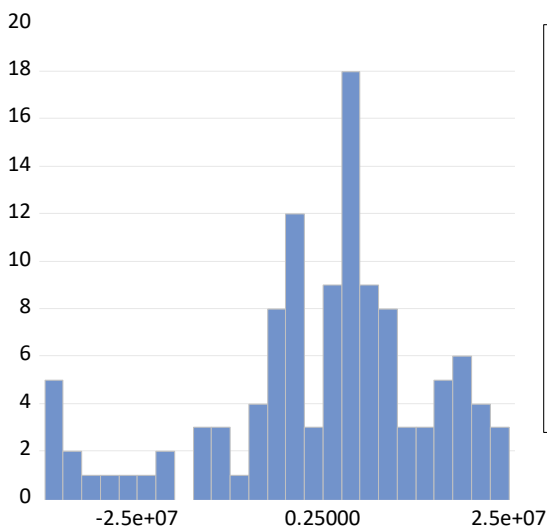
Sample: 1 115

Included observations: 115

Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.001633	0.013659	0.119547	0.9051
@TREND	-4.30E-05	0.000208	-0.206982	0.8364
RESID(-1)	1.120479	0.098340	11.39391	0.0000
RESID(-2)	-0.208247	0.147487	-1.411963	0.1610
RESID(-3)	-0.009531	0.146249	-0.065172	0.9482
RESID(-4)	0.060101	0.146253	0.410941	0.6820
RESID(-5)	-0.109439	0.145209	-0.753666	0.4528
RESID(-6)	0.200573	0.146335	1.370644	0.1735
RESID(-7)	-0.015133	0.147515	-0.102584	0.9185
RESID(-8)	-0.288828	0.147818	-1.953941	0.0534
RESID(-9)	0.109781	0.149374	0.734943	0.4640
RESID(-10)	0.064136	0.100941	0.635385	0.5266

R-squared	0.878208	Mean dependent var	4.62E-15
Adjusted R-squared	0.865201	S.D. dependent var	0.200449
S.E. of regression	0.073595	Akaike info criterion	-2.281985
Sum squared resid	0.557872	Schwarz criterion	-1.995557
Log likelihood	143.2141	Hannan-Quinn criter.	-2.165726
F-statistic	67.51828	Durbin-Watson stat	1.981908
Prob(F-statistic)	0.000000		



Dependent Variable: LOG(PRODUCTION)

Method: Least Squares

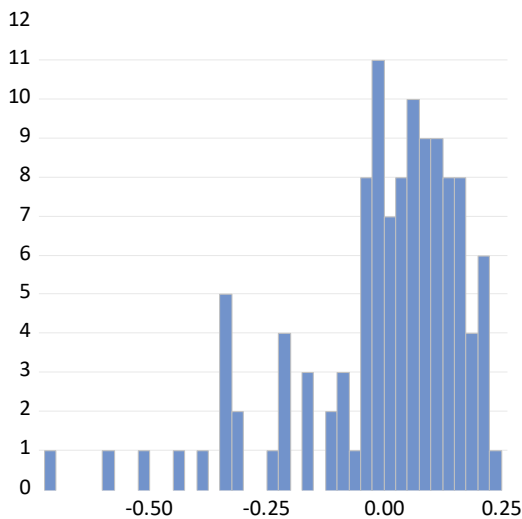
Date: 03/01/23 Time: 13:15

Sample (adjusted): 1 115

Included observations: 115 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	18.01583	0.049832	361.5284	0.0000
@TREND	-0.001930	0.002020	-0.955496	0.3414
@TREND^2	8.98E-05	1.71E-05	5.239201	0.0000

R-squared	0.724142	Mean dependent var	18.29672
Adjusted R-squared	0.719216	S.D. dependent var	0.342029
S.E. of regression	0.181238	Akaike info criterion	-0.552272
Sum squared resid	3.678884	Schwarz criterion	-0.480665
Log likelihood	34.75562	Hannan-Quinn criter.	-0.523207
F-statistic	147.0030	Durbin-Watson stat	0.175567
Prob(F-statistic)	0.000000		



Breusch-Godfrey Serial Correlation LM Test:

Null hypothesis: No serial correlation at up to 10 lags

F-statistic	56.14266	Prob. F(10,102)	0.0000
Obs*R-squared	97.31907	Prob. Chi-Square(10)	0.0000

Test Equation:

Dependent Variable: RESID

Method: Least Squares

Date: 03/01/23 Time: 13:15

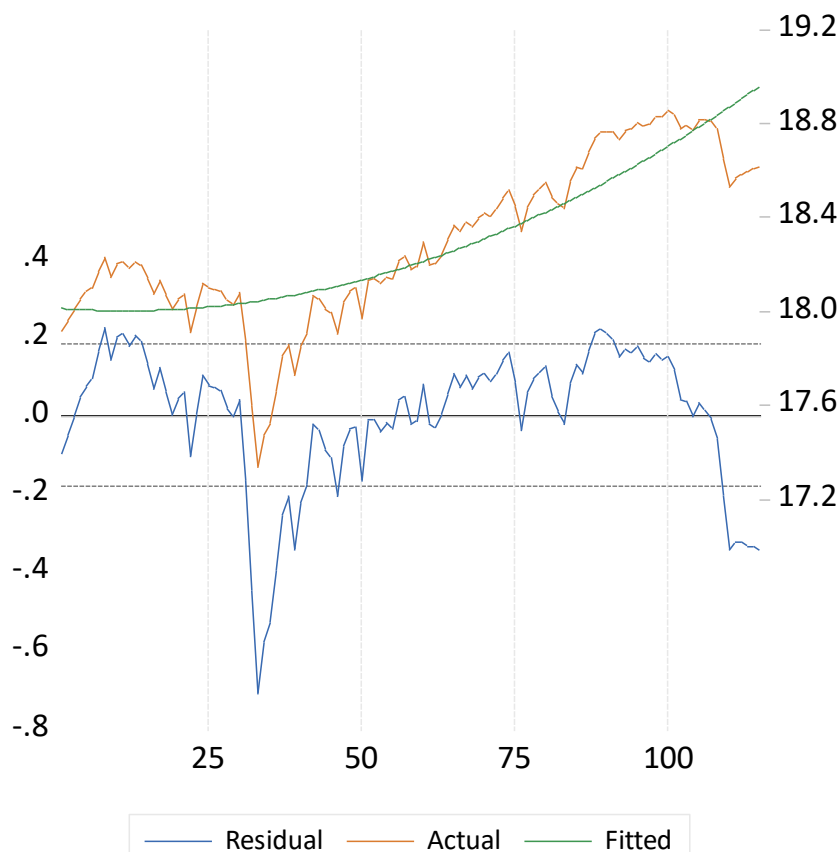
Sample: 1 115

Included observations: 115

Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.007857	0.020936	-0.375291	0.7082
@TREND	0.000551	0.000882	0.624783	0.5335
@TREND^2	-5.97E-06	7.72E-06	-0.773065	0.4413
RESID(-1)	1.099922	0.098461	11.17110	0.0000
RESID(-2)	-0.202623	0.146797	-1.380294	0.1705
RESID(-3)	-0.003032	0.146480	-0.020697	0.9835
RESID(-4)	0.051468	0.146481	0.351360	0.7260
RESID(-5)	-0.096264	0.145693	-0.660731	0.5103
RESID(-6)	0.174071	0.146895	1.184997	0.2388
RESID(-7)	-0.015813	0.147832	-0.106966	0.9150
RESID(-8)	-0.227884	0.148058	-1.539156	0.1269
RESID(-9)	0.034992	0.148626	0.235434	0.8143
RESID(-10)	0.112218	0.103324	1.086075	0.2800

R-squared	0.846253	Mean dependent var	-3.09E-15
Adjusted R-squared	0.828165	S.D. dependent var	0.179641
S.E. of regression	0.074467	Akaike info criterion	-2.250804
Sum squared resid	0.565618	Schwarz criterion	-1.940507
Log likelihood	142.4212	Hannan-Quinn criter.	-2.124856
F-statistic	46.78555	Durbin-Watson stat	1.984513
Prob(F-statistic)	0.000000		



Estimation

As autocorrelation was present in all the models, the HAC standard errors need to be considered for inference

Dependent Variable: PRODUCTION

Method: Least Squares

Date: 03/01/23 Time: 13:11

Sample (adjusted): 1 115

Included observations: 115 after adjustments

HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	48339325	7433188.	6.503175	0.0000
@TREND	792277.3	109352.0	7.245203	0.0000
R-squared	0.706951	Mean dependent var	93499130	
Adjusted R-squared	0.704357	S.D. dependent var	31417376	
S.E. of regression	17082577	Akaike info criterion	36.16225	
Sum squared resid	3.30E+16	Schwarz criterion	36.20999	
Log likelihood	-2077.330	Hannan-Quinn criter.	36.18163	
F-statistic	272.6008	Durbin-Watson stat	0.107119	
Prob(F-statistic)	0.000000	Wald F-statistic	52.49297	
Prob(Wald F-statistic)	0.000000			

Dependent Variable: LOG(PRODUCTION)

Method: Least Squares

Date: 03/01/23 Time: 13:13

Sample (adjusted): 1 115

Included observations: 115 after adjustments

HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	17.82293	0.093700	190.2128	0.0000
@TREND	0.008312	0.001198	6.938777	0.0000
R-squared	0.656534	Mean dependent var	18.29672	
Adjusted R-squared	0.653494	S.D. dependent var	0.342029	
S.E. of regression	0.201334	Akaike info criterion	-0.350461	
Sum squared resid	4.580514	Schwarz criterion	-0.302723	
Log likelihood	22.15152	Hannan-Quinn criter.	-0.331085	
F-statistic	215.9991	Durbin-Watson stat	0.139894	
Prob(F-statistic)	0.000000	Wald F-statistic	48.14662	
Prob(Wald F-statistic)	0.000000			

Estimating the four given models and evaluating their performance led us to conclude that the quadratic trend is significant in both specifications. Graphs of the residuals as well as the goodness of fit and information criteria noticeably improved by its addition.

Dependent Variable: PRODUCTION

Method: Least Squares

Date: 03/01/23 Time: 13:12

Sample (adjusted): 1 115

Included observations: 115 after adjustments

HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	67581937	6199084.	10.90192	0.0000
@TREND	-229454.3	315225.5	-0.727905	0.4682
@TREND^2	8962.558	3313.599	2.704781	0.0079
R-squared	0.786690	Mean dependent var	93499130	
Adjusted R-squared	0.782881	S.D. dependent var	31417376	
S.E. of regression	14639255	Akaike info criterion	35.86205	
Sum squared resid	2.40E+16	Schwarz criterion	35.93366	
Log likelihood	-2059.068	Hannan-Quinn criter.	35.89112	
F-statistic	206.5287	Durbin-Watson stat	0.149193	
Prob(F-statistic)	0.000000	Wald F-statistic	37.30606	
Prob(Wald F-statistic)	0.000000			

Dependent Variable: LOG(PRODUCTION)

Method: Least Squares

Date: 03/01/23 Time: 13:16

Sample (adjusted): 1 115

Included observations: 115 after adjustments

HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	18.01583	0.080462	223.9057	0.0000
@TREND	-0.001930	0.003534	-0.546112	0.5861
@TREND^2	8.98E-05	3.52E-05	2.555074	0.0120
R-squared	0.724142	Mean dependent var	18.29672	
Adjusted R-squared	0.719216	S.D. dependent var	0.342029	
S.E. of regression	0.181238	Akaike info criterion	-0.552272	
Sum squared resid	3.678884	Schwarz criterion	-0.480665	
Log likelihood	34.75562	Hannan-Quinn criter.	-0.523207	
F-statistic	147.0030	Durbin-Watson stat	0.175567	
Prob(F-statistic)	0.000000	Wald F-statistic	30.61205	
Prob(Wald F-statistic)	0.000000			

Since the dependent variable is different among the two transformations, we cannot directly compare the level and log quadratic models. It is therefore not clear which one is better. Visualizing the production series and its log transformation, the latter exhibits distribution closer to the Gaussian, therefore we favor the log model.

Forecast

When comparing the RMSEs on the last 10 forecasts, we can see that the quadratic models performed better as expected by their in-sample superiority. However, our preferred log model did not do as well in the out of sample forecast as the linear one. It is worth noting that the graph of our series shows that there was a drop in forestry production during the last 10 recorded years. Perhaps the log model would otherwise do better.

Dependent Variable: PRODUCTION

Method: Least Squares

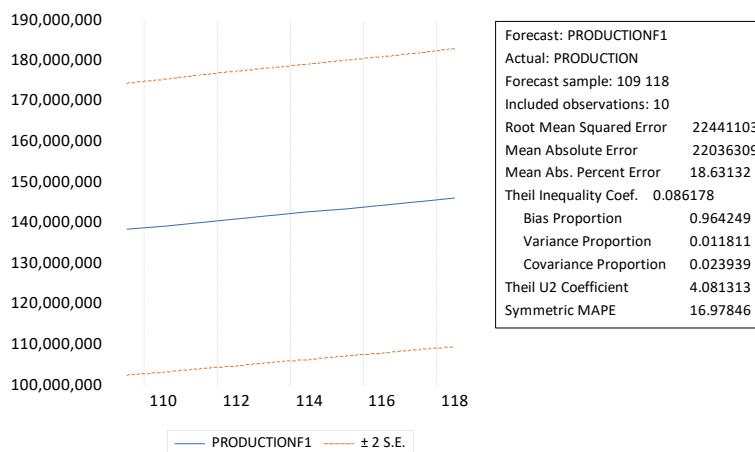
Date: 03/01/23 Time: 14:09

Sample: 1 108

Included observations: 108

HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	45927404	8078966.	5.684812	0.0000
@TREND	858124.0	117865.5	7.280534	0.0000
R-squared	0.719016	Mean dependent var	91837037	
Adjusted R-squared	0.716365	S.D. dependent var	31696790	
S.E. of regression	16880879	Akaike info criterion	36.13961	
Sum squared resid	3.02E+16	Schwarz criterion	36.18928	
Log likelihood	-1949.539	Hannan-Quinn criter.	36.15975	
F-statistic	271.2456	Durbin-Watson stat	0.098800	
Prob(F-statistic)	0.000000	Wald F-statistic	53.00618	
Prob(Wald F-statistic)	0.000000			



Dependent Variable: PRODUCTION

Method: Least Squares

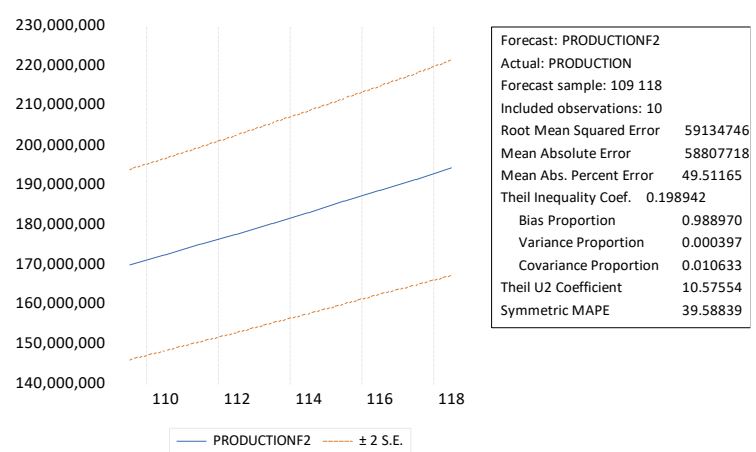
Date: 03/01/23 Time: 14:07

Sample: 1 108

Included observations: 108

HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	75653799	4920373.	15.37562	0.0000
@TREND	-824502.2	214843.6	-3.837686	0.0002
@TREND^2	15725.48	2160.143	7.279831	0.0000
R-squared	0.906711	Mean dependent var	91837037	
Adjusted R-squared	0.904934	S.D. dependent var	31696790	
S.E. of regression	9772981.	Akaike info criterion	35.05553	
Sum squared resid	1.00E+16	Schwarz criterion	35.13003	
Log likelihood	-1889.998	Hannan-Quinn criter.	35.08573	
F-statistic	510.2691	Durbin-Watson stat	0.300904	
Prob(F-statistic)	0.000000	Wald F-statistic	96.77499	
Prob(Wald F-statistic)	0.000000			



Dependent Variable: LOG(PRODUCTION)

Method: Least Squares

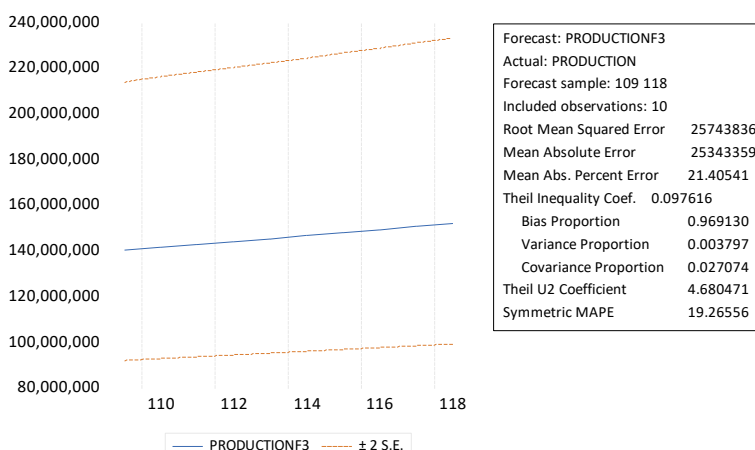
Date: 03/01/23 Time: 14:15

Sample: 1 108

Included observations: 108

HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	17.80176	0.099888	178.2181	0.0000
@TREND	0.008890	0.001297	6.852790	0.0000
R-squared	0.655079	Mean dependent var	18.27737	
Adjusted R-squared	0.651825	S.D. dependent var	0.344020	
S.E. of regression	0.202994	Akaike info criterion	-0.332939	
Sum squared resid	4.367880	Schwarz criterion	-0.283270	
Log likelihood	19.97873	Hannan-Quinn criter.	-0.312800	
F-statistic	201.3166	Durbin-Watson stat	0.138718	
Prob(F-statistic)	0.000000	Wald F-statistic	46.96073	
Prob(Wald F-statistic)	0.000000			



Dependent Variable: LOG(PRODUCTION)

Method: Least Squares

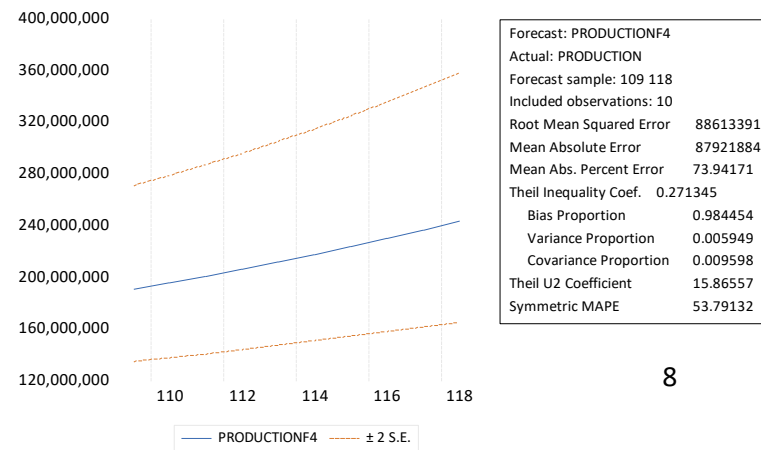
Date: 03/01/23 Time: 14:12

Sample: 1 108

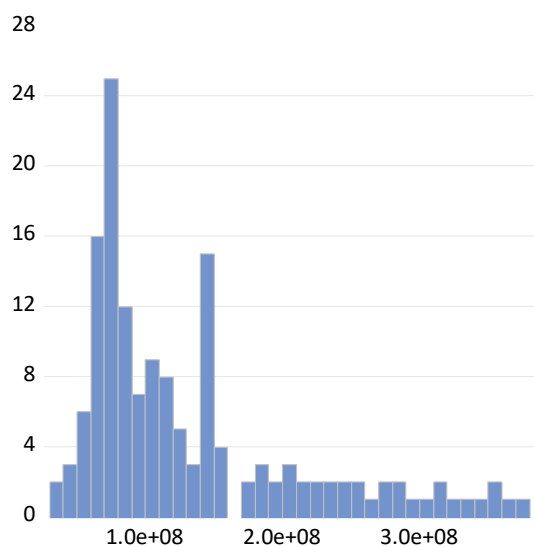
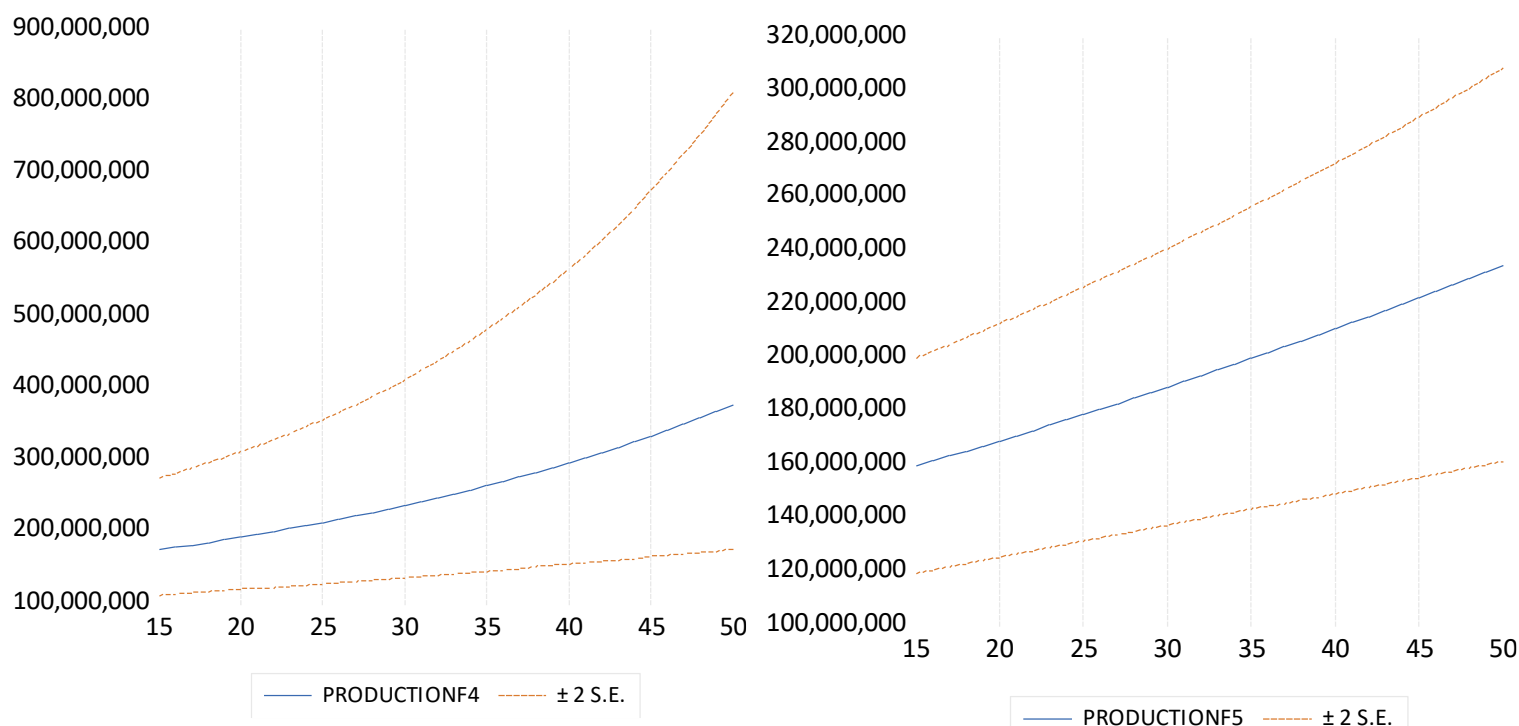
Included observations: 108

HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 5.0000)

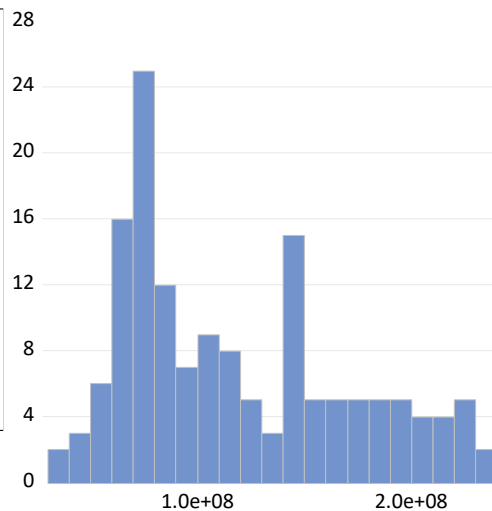
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	18.09165	0.068579	263.8083	0.0000
@TREND	-0.007519	0.003041	-2.472956	0.0150
@TREND^2	0.000153	3.04E-05	5.050595	0.0000
R-squared	0.806613	Mean dependent var	18.27737	
Adjusted R-squared	0.802930	S.D. dependent var	0.344020	
S.E. of regression	0.152719	Akaike info criterion	-0.893046	
Sum squared resid	2.448935	Schwarz criterion	-0.818542	
Log likelihood	51.22446	Hannan-Quinn criter.	-0.862837	
F-statistic	218.9769	Durbin-Watson stat	0.249396	
Prob(F-statistic)	0.000000	Wald F-statistic	55.41371	
Prob(Wald F-statistic)	0.000000			



The 2050 forecasts are presented for both quadratic trend model. PRODUCTIONF4, PRODUCTIONF5 represents the log and the level model, respectively. Both forecasts are for the forestry series in levels. It is visible that the production is forecasted to be higher by the log specification.



Series: PRODUCTIONF4	
Sample 1900 2050	
Observations 151	
Mean	1.33e+08
Median	1.03e+08
Maximum	3.77e+08
Minimum	33900000
Std. Dev.	81313483
Skewness	1.297302
Kurtosis	3.811389
Jarque-Bera	46.49743
Probability	0.000000



Series: PRODUCTIONF5	
Sample 1900 2050	
Observations 151	
Mean	1.18e+08
Median	1.03e+08
Maximum	2.35e+08
Minimum	33900000
Std. Dev.	52641160
Skewness	0.588622
Kurtosis	2.186711
Jarque-Bera	12.88121
Probability	0.001595