Handout 9: Choice under Uncertainty

1 Introduction

Even though economic decisions are made under uncertain and risky scenarios, we have so far in this class focused on consumers and firms making decisions without any risk. In this handout, we will relax this assumption and use a *expected utility theory* framework to understand how agents make decisions under uncertainty. We start the handout by quickly reviewing expected utility and contrasting it with expected value. We then discuss a few short questions to fix the key concepts.

Our main application in this handout is the market for health insurance. Agents must decide to buy insurance exactly because their are exposed to risk, but their willingness to pay for it depends on their preferences and price of insurance. We will show that even though two agents are subject to the same risk level and have the same risk of falling ill, poorer agents will be more risk averse. Intuitively, poorer agents do not have wealth to sustain high levels of consumption with bad draws from the lottery and thus must insure themselves against it. Still in the example, we will discuss the price of insurance and how to guarantee all agents sign up for healthcare by using a penalty (as is the case of the Affordable Care Act). Finally, we show one situation where the expected utility theory can fail, and how researchers have developed new theories of how agents make choices under uncertainty.

2 Review of Expected Utility

Suppose that you have a coin. At each flip, the coin lands on heads with a probability p and tails with a probability $1 - p$ (not a balanced coin). If the coin lands on heads, you get a prize of \$ H, while if it lands on tails you get \$ T. The definition of expeted value is:

Expected Value: the sum of the probability of each outcome times the value of that outcome.

In our coin example:

$$EV = Prob(Heads) * Value(Heads) + Prob(Tails) * Value(Tails) = pH + (1 - p)T$$

When facing uncertainty, agents can be risk: (i) averse, (ii) neutral or (iii) loving. To assess how individuals make choices under uncertainty, we use *expected utility theory*. The expected utility over a uncertain outcome, as the coin flip, is given by:

$$EU = Prob(Heads) * u(Heads) + Prob(Tails) * u(Tails) = pu(H) + (1-p)u(T)$$

Expected utility EU is different from EV due to the fact that the utility function is not necessarily linear. In fact, we capture risk aversion through the *concavity* of the utility function. To understand why, consider that we have two cases:

1. $p = .5$, $H = T = 1$, and $u(C) = \sqrt{C}$. The expected value in this case is $EV = 1$, and the expected utility is:
$$EU_1 = .5 \times 1 + .5 \times 1 = 1$$

2. $p = 3/4$, $H = 0, T = 4$. The expected value in this case is still $EV = 1$, but the expected utility is:
$$EU_2 = .75 \times 0 + .25 \times \sqrt{4} = .5$$

Even though both cases have the same expected value, case 2 is riskier: you can either win 0 or 4, while case 1 is such that you win 1 for sure. With the concavity in the utility, the agent discounts large gains by more than in a linear case. If given a choice, a risk averse agent would choose case 1, since $EU_1 > EU_2$.

Now consider a case 3: $p = .5$, $H = T = .16$. The expected utility is: $EU_3 = .4$. Even though case 3 has no risk, this risk averse agent still prefers case 2 over case 3, because case 2 has enough extra return (a higher expected value) to compensate this agent for incurring in the risk.

Caveat. Finally, there is an important distinction from consumer choice to expected utility. Without uncertainty, any increasing transformation of a utility function delivers the same choices over bundles. With uncertainty, any increasing *linear* transformation of a utility function delivers the same choices over lotteries. The statement changes to include the term linear, since non-linear transformation change the risk aversion profile of the agent.

3 Short Questions

1. Consider a gamble that pays \$10 with probability 50% and \$0 with probability 50%. No expected-utility maximizing individual will strictly prefer this gamble over a certain payment of

5.

2. Consider a gamble with which you gain \$50 with probability 20%, you gain \$130 with probability 30%, or you lose \$95 with probability 50%. A risk-neutral individual would take this gamble.

3. (Extremely Hard) Suppose that from any initial wealth, an expected utility maximizer would turn down a 50/50 chance to loses \$1000 or gains \$1050. Then this person must also always turn down a 50/50 bet of losing \$20, 000 and gaining any sum.

Solution.

1. *False.* The expected value of the gamble is \$5. So the agent has to choose between the gamble with an expected value of \$5 and \$5 for sure. If the agent is risk averse, he prefers the \$5 dollars for sure - since he would require a larger return to incur some risk. A risk-neutral individual does not care about risk, only expected value - and thus would be indifferent. A risk-loving individual prefers the gamble, since it attaches value to risk. This means that The risk-loving individuals, or individuals with convex utility curves will prefer this gamble. Therefore, as risk-neutral/averse/loving individuals can be expected utility maximiziers, the statement is false.

2. *True.* The expected value of the gamble is strictly positive:

$$EV = .2 \times 50 + .3 \times 130 - .5 \times 95 = 1.5 > 0$$

so a risk-neutral individual would take this gamble.

3. *True.* This a conclusion of Rabin (2000) paper. The idea here is not to actually show it, but just to get you thinking about the surprising result and some limitations of expected utility.

□

4 Insurance

John and Jane are deciding to buy or not insurance from a given company. Each has utility function

$$u(x) = 2\ln x$$

where x is measured in dollars. Each of them falls ill with probability 0.5, in which case he incurs medical bills of \$24. If they have insurance, the company covers all of the medical costs. John has an initial wealth of 25, while Jane has a initial wealth of 28.

1. What is John's willingness to pay for the policy (the maximum premium he is willing to pay)? (The premium is the amount you pay for the insurance company regardless if you use or not your insurance).

2. What is Jane's willingness to pay for the policy (the maximum premium she is willing to pay)? Who has a higher willingness to pay, and why? (*Use* $\sqrt{7} \approx 2.75$ *as an approximation.*)

3. Suppose the insurance company sets a price of \$22, but the government wants everyone to purchase health insurance. It decides to institute a penalty of p dollars for anyone who does not have insurance, with the penalty waived for anyone who falls ill. What is the minimum penalty it must set to induce both John and Jane to purchase insurance?

Solution.

1. John's expected utility without insurance is the probability he falls ill times the utility of $25 - 24 = 1$ dollar, since he must pay his medical bills, summed to the probability he does not fall ill and still has 25 dollars, that is

$$EU = (0.5)2\ln 1 + (0.5)2\ln 25 = \ln 25.$$

For John, a expected utility of $\ln 25$ is equivalent to a certain wealth of x^*, where x^* is given by

$$\ln 25 = 2\ln x^* \Rightarrow x^* = 5.$$

Thus willingness to pay for insurance is thus

$$WTP = 25 - 5 = \$20.$$

John is indifferent between paying 20 dollars for insurance or play the illness lottery.

2. Jane's expected utility without insurance is

$$EU = (0.5)2\ln 4 + (0.5)2\ln 28 = \ln 112.$$

The certain wealth level x^* that is equivalent for this level of utility is

$$\ln 112 = 2\ln x^* \Rightarrow x^* = \sqrt{112} = 4\sqrt{7} \approx 11.$$

Thus willingness to pay for insurance is

$$WTP \approx 28 - 11 \approx \$17.$$

John's willingness to pay is higher. The benefit of insurance is greater to John because both individuals are risk averse and John is poorer, so the $24 medical bills would give him a greater utility loss (due to the concavity of utility functions).

3. John will purchase if the expected utility with insurance is greater than the expected utility without

$$\begin{aligned}
2\ln 3 &\geq (0.5)2\ln 1 + (0.5)2\ln(25-p) \\
\ln 9 &\geq \ln 1(25-p) \\
p &\geq 16
\end{aligned}$$

Similarly for Jane:

$$\begin{aligned}
\ln 36 &\geq (0.5)2\ln 4 + (0.5)2\ln(28-p) \\
36 &\geq 4(28-p) \\
p &\geq 19
\end{aligned}$$

So the penalty must be at least $19.

□

5 Allais' Experiment - Kahneman and Tversky (1979)

The idea is to show that expected utility is useful, but there are some pitfalls. This example is part of Kahneman and Tversky's research, which is basis for the bestseller *Thinking Fast and Slow.*. They and other researchers developed other theories of how individuals make decisions under uncertainty, but expected utility theory is still used today.

Problem 1. Imagine you are choosing between two lotteries. The first pays \$55, 000 with probability .33, \$48, 000 with probability .66, and 0 with probability .01. The second pays \$48, 000 for sure.

Ask students here what they would choose - without the math.

Problem 2 Imagine you are choosing between other two lotteries. The first pays \$55, 000 with probability .33 and nothing with probability .67. The second pays \$48, 000 with probability .34 and nothing with probability .66.

Without looking below and doing the math, which lotteries would you take?

A typical response to these two problems is to choose the certain payoff in the first case and take the first lottery in the second case. This pattern of choices, however, is not rational in a expected utility framework.

To see that: Normalizing $u(0) = 0$, the first choice implies that

$$u(48) > .33u(55) + .66u(48) \Leftrightarrow .34u(48) > .33u(55),$$

while the second choice implies the reverse inequality. Kahneman and Tversky explain the experimental result as a certainty effect: people tend to overvalue a sure thing. Their paper has many other examples that also lead experimental subjects to violate.