Application of Machine Learning to the Simulated Method of Moments in Financial Agent-Based Models Estimation

Bachelor's Thesis

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1. Introduction
2. Simulated Method of Moments Extension
3. Results
4. Conclusion

1 Introduction

2 Simulated Method of Moments Extension

3 Results

4 Conclusion

Motivation

- Financial agent-based models as an alternative to traditional capital asset pricing models (CAPMs)
- More realistic assumptions
- Stylised facts of financial data
- Complexity of agent-based models and their estimation

Simulated Method of Moments (SMM)

- Extension of the generalised method of moments (GMM)
- Necessary choice of a set of moments
  - e.g. mean, variance, kurtosis, auto-correlation, ...
- Minimisation of the distance between their sample counterparts computed using empirical data and simulated data

$$h(\theta) = m^{emp} - m^{sim}(\theta)$$

$$J(\theta) = h(\theta)' W h(\theta)$$

$$\hat{\theta} = \arg\min_{\theta \in \Theta} J(\theta)$$

where m^{emp} is a vector of empirical moments, m^{sim} is a vector of simulated moments, θ is a vector of estimated parameters, W is a positive semi-definite (weighting) matrix, Θ is a parameter search space

- Criticism for arbitrariness of the moment set selection

Objectives

- Offer a more consistent approach to the moment set selection when using the SMM
- Improve estimation performance of the estimator by selecting a more optimal moment set
- Enable various implementations of the SMM

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Machine Learning Extension

- Intuitively follows stepwise regression
- Decisions based on adjusted RMSE averaged over parameters

$$RMSE(\hat{\theta}_i) = \sqrt{\frac{(\hat{\theta}_i - \theta_i)^2 + \hat{\sigma}_i^2}{|\theta_i|}}$$

Forward stepwise moment selection (FSMS)

- Iterative expansion of an initially empty set of moments
- Addition of one moment in each selection round

Backward stepwise moment elimination (BSME)

- Iterative restriction of an initially full set of moments
- Removal of one moment in each elimination round

FSMS: Example 1

Initialisation

Available set: $\{m_{mean}, m_{variance}, m_{kurtosis}\}$

Base set: $\{\}$

FSMS: Example 2

First selection round

Available set: $\{m_{mean}, m_{variance}, m_{kurtosis}\}$

Tried sets: $\{m_{mean}\}, \{m_{variance}\}, \{m_{kurtosis}\}$

Base set: $\{\}$

FSMS: Example 3

After first selection round

Available set: $\{m_{mean}, m_{variance}, m_{kurtosis}\} \Rightarrow \{m_{mean}, m_{kurtosis}\}$

Base set: $\{\} \Rightarrow \{m_{variance}\}$

FSMS: Example 4

Second selection round

Available set: $\{m_{mean}, m_{kurtosis}\}$

Tried sets: $\{m_{mean}, m_{variance}\}, \{m_{variance}, m_{kurtosis}\}$

Base set: $\{m_{variance}\}$

FSMS: Example 5

After second selection round

$$\text{Available set:} \quad \{m_{mean}, m_{kurtosis}\} \Rightarrow \{m_{kurtosis}\}$$

$$\text{Base set:} \quad \{m_{variance}\} \Rightarrow \{m_{mean}, m_{variance}\}$$

FSMS: Example 6

Third selection round

Available set: $\{m_{kurtosis}\}$

Tried sets: $\{m_{mean}, m_{variance}, m_{kurtosis}\}$

Base set: $\{m_{mean}, m_{variance}\}$

FSMS: Example 7

After third selection round

Available set: $\{m_{kurtosis}\} \Rightarrow \{\}$

Base set: $\{m_{mean}, m_{variance}\} \Rightarrow \{m_{mean}, m_{variance}, m_{kurtosis}\}$

Moment Set

- Constructed theoretically to reflect desired model behaviour
- Franke and Westerhoff (2012) benchmark set of 9 moments
  - r_t: auto-correlation at lag 1; $|r_t|$: mean, Hill estimator, auto-correlation at lags $\{1, 5, 10, 25, 50, 100\}$
- Chen and Lux (2018) benchmark sets of 4 moments
  - r_t: variance, excess kurtosis, auto-correlation at lag 1; r_t^2: auto-correlation at lag 1
- Chen and Lux (2018) benchmark sets of 15 moments
  - r_t: variance, excess kurtosis, auto-correlation at lag 1; $|r_t|$: auto-correlation at lags $\{1, 5, 10, 15, 20, 25\}$; r_t^2: auto-correlation at lags $\{1, 5, 10, 15, 20, 25\}$
- Full set of 19 moments provided to the algorithms

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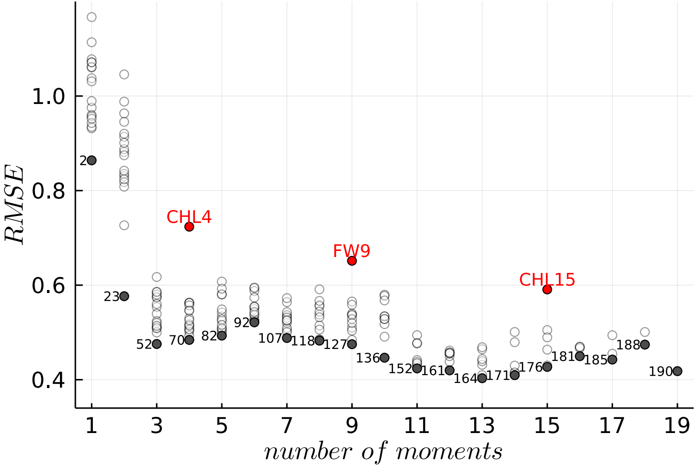
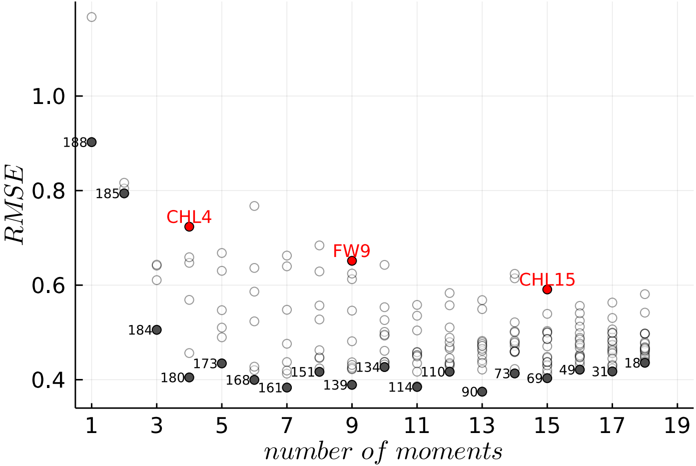
Candidate Models

- Random walk with a drift
  - 1 estimated parameter
- Random walk with a structural break
  - 2 estimated parameters
- Cox–Ingersoll–Ross (1985) model
  - 3 estimated parameters
- Franke and Westerhoff (2011) model
  - three parameter sets of an increasing size
  - 7 estimated parameters in the largest set

Franke and Westerhoff (2011)—Parameter Set 3: FSMS



Franke and Westerhoff (2011)—Parameter Set 3: BSME



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Summary

- Problem of finding a more consistent approach to moment set selection for the SMM
- Design of two machine learning algorithms allowing automatic selection of the moment set
- Both algorithms provide a severe performance boost over benchmark sets from the literature for all candidate models

Thank you for your attention.

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Dennis Kristensen and Yongseok Shin. Estimation of Dynamic Models With Nonparametric Simulated Maximum Likelihood. *Journal of Econometrics*, 167:76–94, 2012.

Francesco Lamperti. An Information Theoretic Criterion for Empirical Validation of Simulation Models. *Econometrics and Statistics*, 5:83–106, 2018.

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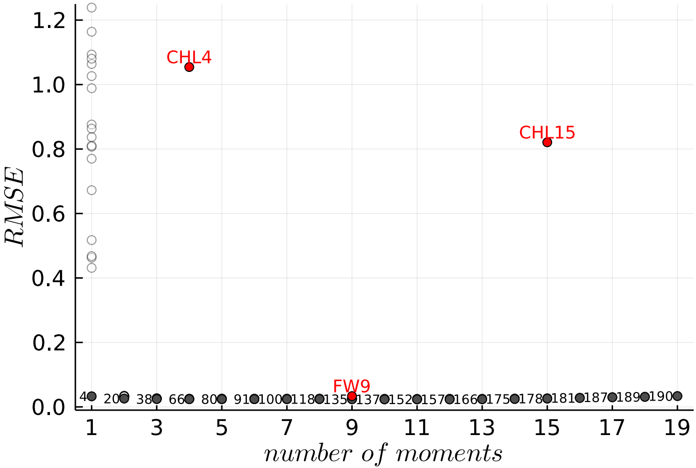
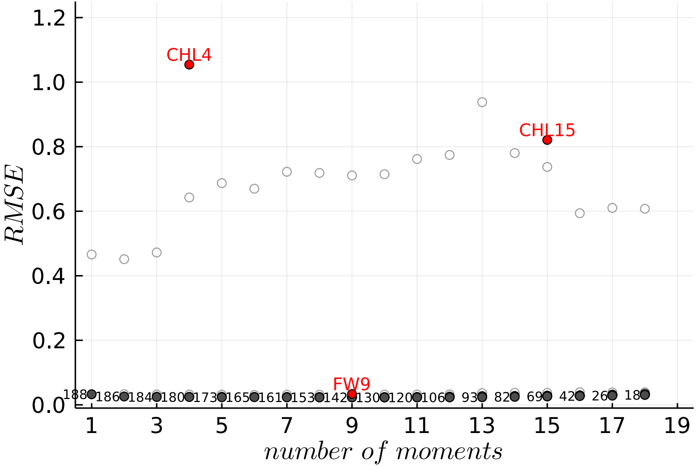
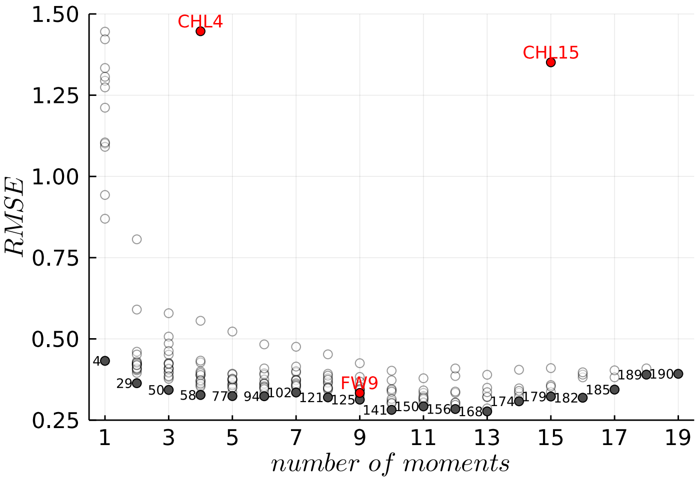
Random Walk With a Drift: FSMS



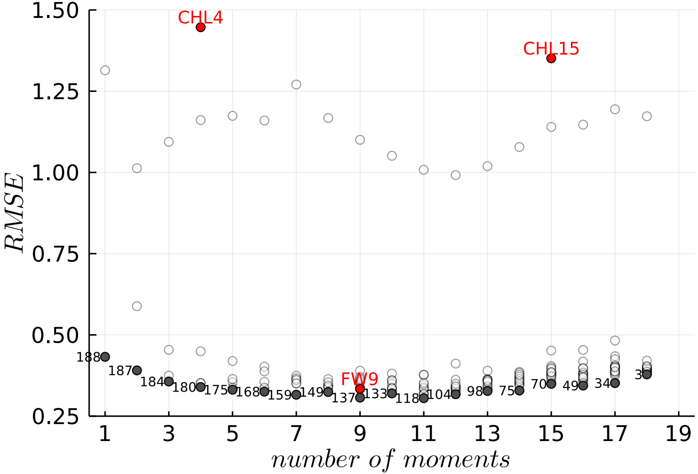
Random Walk With a Drift: BSME



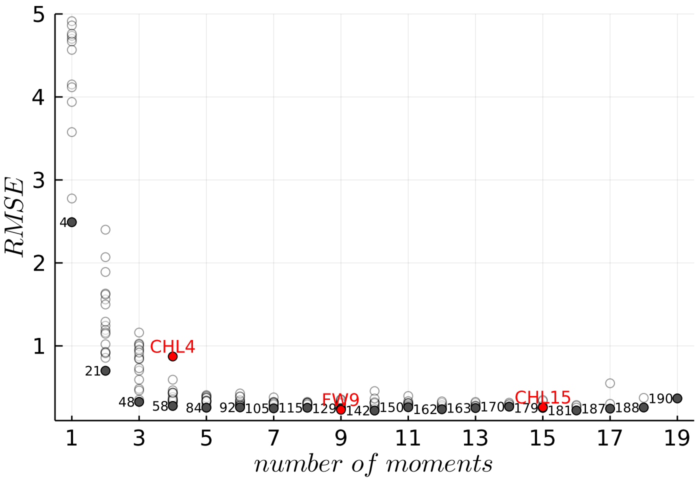
Random Walk With a Structural Break: FSMS



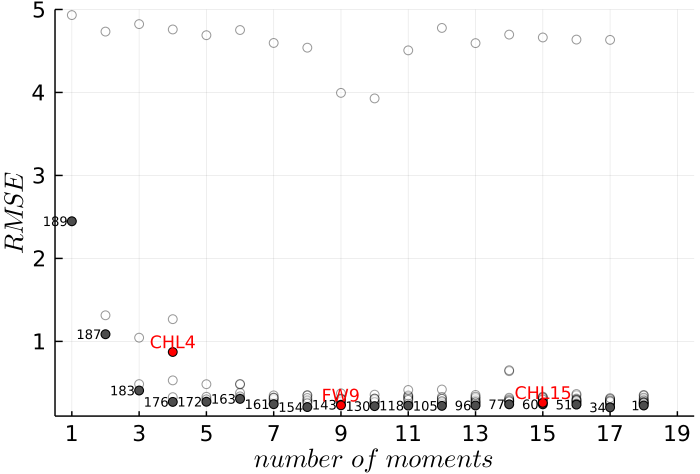
Random Walk With a Structural Break: BSME



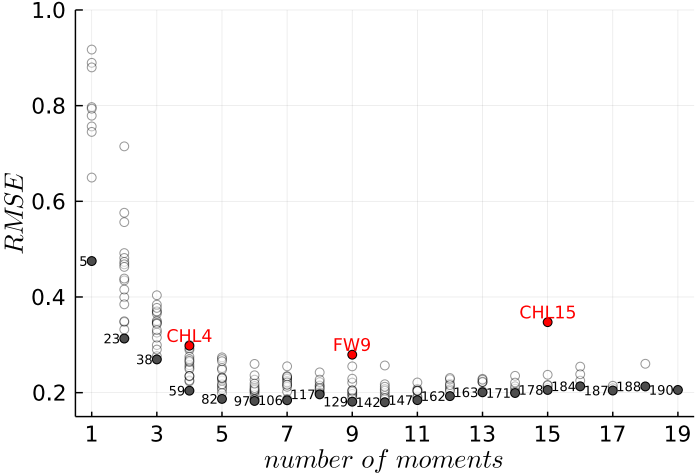
Cox–Ingersoll–Ross (1985) Model: FSMS



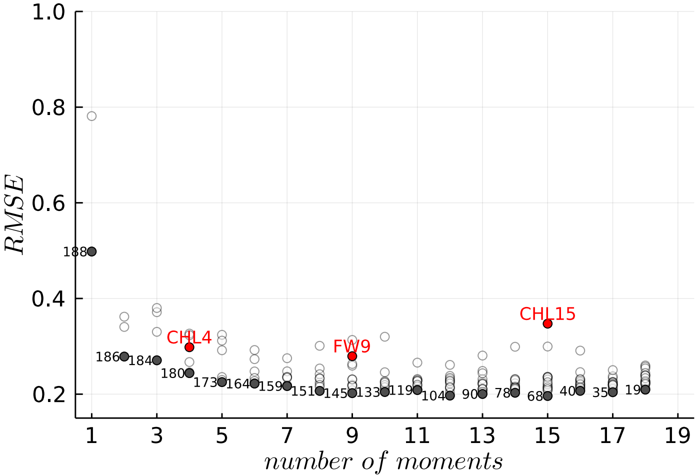
Cox–Ingersoll–Ross (1985) Model: BSME



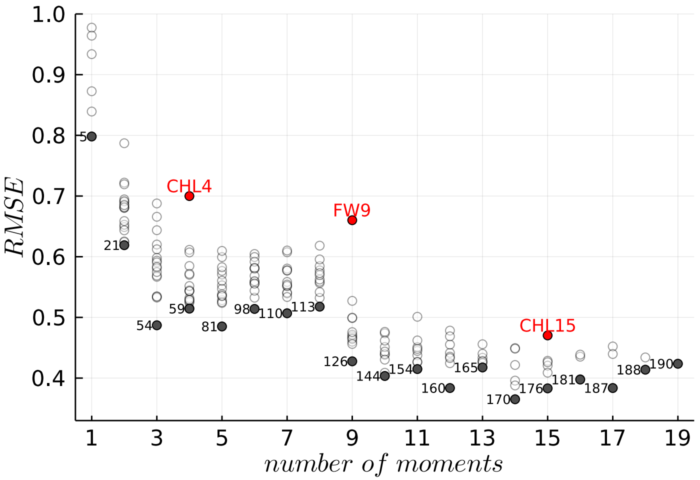
Franke and Westerhoff (2011) Model—Parameter Set 1: FSMS



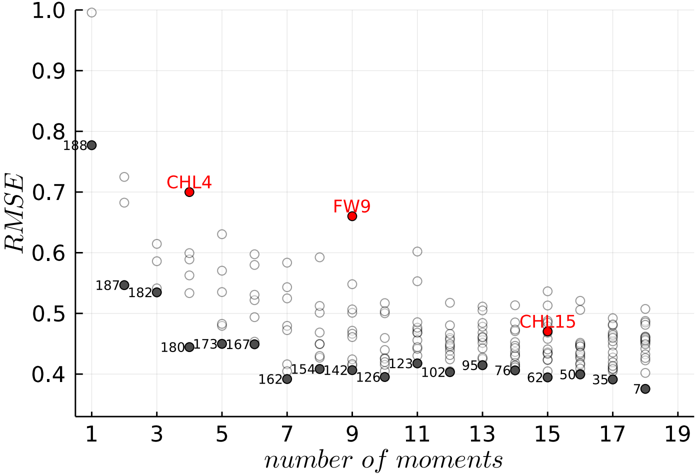
Franke and Westerhoff (2011) Model—Parameter Set 1: BSME



Franke and Westerhoff (2011) Model—Parameter Set 2: FSMS



Franke and Westerhoff (2011) Model—Parameter Set 2: BSME



Benchmark Moment Sets

Moment function	*FW9*	*CHL4*	*CHL15*
$m_1 = E(r_t^2)$	-	✓	✓
$m_2 = E(r_t^4)$	-	✓	✓
$m_3 = E(r_t r_{t-1})$	✓	✓	✓
$m_4 = E(\|r_t\|)$	✓	-	-
$m_5 = Hill(\|r_t\|, 5)$	✓	-	-
$m_6 = E(\|r_t\|\|r_{t-1}\|)$	✓	-	✓
$m_7 = E(\|r_t\|\|r_{t-5}\|)$	✓	-	✓
$m_8 = E(\|r_t\|\|r_{t-10}\|)$	✓	-	✓
$m_9 = E(\|r_t\|\|r_{t-15}\|)$	-	-	✓
$m_{10} = E(\|r_t\|\|r_{t-20}\|)$	-	-	✓
$m_{11} = E(\|r_t\|\|r_{t-25}\|)$	✓	-	✓
$m_{12} = E(\|r_t\|\|r_{t-50}\|)$	✓	-	-
$m_{13} = E(\|r_t\|\|r_{t-100}\|)$	✓	-	-
$m_{14} = E(r_t^2 r_{t-1}^2)$	-	✓	✓
$m_{15} = E(r_t^2 r_{t-5}^2)$	-	-	✓
$m_{16} = E(r_t^2 r_{t-10}^2)$	-	-	✓
$m_{17} = E(r_t^2 r_{t-15}^2)$	-	-	✓
$m_{18} = E(r_t^2 r_{t-20}^2)$	-	-	✓
$m_{19} = E(r_t^2 r_{t-25}^2)$	-	-	✓

Random Walk With a Drift

$$x_t = x_{t-1} + d + \varepsilon_t$$

$$\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$$

- Baseline model to test elementary functionality
- 1 estimated parameter [d]

Random Walk With a Structural Break

$$x_t = x_{t-1} + d_t + \varepsilon_t$$

$$\varepsilon_t \sim \mathcal{N}(0, \sigma_t^2)$$

$$d_t, \sigma_t = \begin{cases} d_1, \sigma_1 & \text{if } t \leq \tau \\ d_2, \sigma_2 & \text{if } t > \tau \end{cases}$$

- Extension of the random walk with a drift
- Selection following Lamperti (2018) and Platt (2020, 2021)
- Parametrization from Platt (2021)
- 2 estimated parameters $[d_1, d_2]$

Cox–Ingersoll–Ross (1985) Model

$$dx_t = \beta(\alpha - x_t)dt + \sigma\sqrt{x_t}dW_t$$

- Model of short-term interest rates
- Replicates stylised facts from financial markets
- Selection following Kristensen and Shin (2012)
- Parametrization from Kristensen and Shin (2012)
- 3 estimated parameters $[\alpha, \beta, \sigma]$

Franke and Westerhoff (2011) model

$$p_{t+1} = p_t + \frac{\mu}{2}\left[(1+x_t)\phi(p^* - p_t) + (1-x_t)\chi(p_t - p_{t-1}) + \varepsilon_t\right]$$

$$\varepsilon_t \sim \mathcal{N}(0, \sigma_t^2) \quad \sigma_t^2 = \frac{(1+x_t)^2\sigma_f^2 + (1-x_t)^2\sigma_c^2}{2}$$

$$x_{t+1} = x_t + (1-x_t)min\{1, \nu exp(s_t)\} - (1+x_t)min\{1, \nu exp(-s_t)\}$$

$$s_t = \alpha_0 + \alpha_x x_t + \alpha_d(p_t - p^*)^2$$

- Selection following Barde (2016) and Platt (2021)
- Parametrization from Franke and Westerhoff (2016)
- Three sets of parameters, the largest one with 7 parameters