1 Budgeting

1.1 Voting on total spending

In a coalitional government, there are two parties, A and B. Party A represents m groups, and Party B represents $n-m$ groups, where $m \geq n-m$. Groups are indexed $i = 1,\ldots,n$. The government prepares a budget which consists of two items (x_A, x_B), where $x_i \geq 0$ is expenditure for collective goods for a group i. Each group has utility $u_i(x_i,t) = \log(x_i) - t$, where log is for natural logarithm and t is a tax paid by the group. (All groups pay an identical tax.)

All groups in the economy are represented by Parties A and B, and budget is balanced, $mx_A + (n-m)x_B = nt$.

We suppose that the coalitional government first agrees on the total spending $X = \sum_i x_i$. Then, the total spending X is allocated by a predetermined division rule. Agreement over total spending is achieved when Party A's optimal total spending X_A equals to Party B's optimal total spending X_B, i.e., when $X_A = X_B$.

- *Equal shares*: Suppose that for any X, each party receives half of the total tax revenues to cover spending for all their groups. Will the parties agree? If so, what is total spending?
- *Equal shares*: For each party j, when is better if the government first agrees on total spending and then allocates it with equal-shares rule and when is it better if both parties independently determine their optimal spending levels and taxes are set residually?
- *Proportional shares*: Now, suppose that Party A gets share $\frac{m}{n}$ and Party B gets share $\frac{n-m}{n}$ of the tax revenues to cover spending for their groups. Will the parties agree? If so, what is total spending?
- *Proportional shares*: For each party j, when is better if the government first agrees on total spending and then allocates it with proportional-shares rule and when is it better if both parties independently determine their optimal spending levels and taxes are set residually?

Sample solution We begin with equal shares:

- Party A maximizes $\log(\frac{X}{2m}) - \frac{X}{n}$, where the optimum is $X_A = n$. Also $X_B = n$. Therefore, there is agreement.

  In this budget, the levels of group spending are different, $x_A = \frac{n}{2m} < 1 < \frac{n}{2(n-m)} = x_B$. Precisely, see that the levels are exactly $\frac{1}{2}$ of the levels in the decentralized budgeting when the levels are set independently and non-cooperatively.

- Party A is better off with a vote on total spending if

  $$\log\frac{n}{2m} - 1 > \log\frac{n}{m} - 2,$$

  which is equivalent to $1 > \log 2 \doteq 0.69$. This is true always.

- Party B is better off with a vote on total spending if

$$\log \frac{n}{2(n-m)} - 1 > \log \frac{n}{n-m} - 2,$$

which is again equivalent to $1 > \log 2 \doteq 0.69$. Hence, also Party B is always better off.

Proportional shares:

- Party A maximizes $\log(\frac{mX}{nm}) - \frac{X}{n} = \log(\frac{X}{n}) - \frac{X}{n}$. Party B maximizes $\log(\frac{(n-m)X}{n(n-m)}) - \frac{X}{n} = \log(\frac{X}{n}) - \frac{X}{n}$. Hence, each party maximizes $\log(\frac{X}{n}) - \frac{X}{n}$, where the optimum is $X_j = n$. Therefore, there is agreement.

  This budget has the same size like with equal shares. However, in contrast to the equal shares, in this budget all groups receive the same level of spending, $x_A = x_B = 1$.

- Larger Party A is better off with a vote on total spending if

$$\log 1 - 1 > \log \frac{n}{m} - 2,$$

which is equivalent to $\frac{m}{n} > \frac{1}{e} \doteq 0.36$. This is true since $m \geq \frac{1}{2}$.

- Smaller Party B is better off with a vote on total spending if

$$\log 1 - 1 > \log \frac{n}{n-m} - 2,$$

which is equivalent to $\frac{n-m}{n} > \frac{1}{e} \doteq 0.36$. This is true only if the asymmetry between the parties is small enough. For a greater asymmetry, Party B is worse off, because it has to reduce its initially great spending $x_B = \frac{n}{n-m} \gg 1$ down to $x_B = 1$.

  Intuitively, the common vote increases efficiency of the budget, but also fully equalizes payoffs; Party B loses its benefit of smallness. The problem is that (i) all surplus from the regime change goes to the larger Party A, and moreover (ii) the regime change distributes some initial value from Party B to Party A. To get approval of Party B, we rather need that some surplus from the regime changes goes to the smaller Party B.

To sum up: The budgetary regime with proportional shares is more fair than the budget with equal shares, but fails to be approved. In contrast, the budgetary regime with equal shares is less fair but is approved. The takeaway is that efficiency-enhancing reforms of public expenditures often fail to be approved when the reform aims seek to achieve too many objectives. Here, one objective is to cut budgets (reduce overspending and generate a more efficient outcome) and the other objective is fairness (balance public spending across more and less 'privileged' groups).

2 Contests

2.1 Public procurement as a two-level Tullock contest

Suppose there is a public contract to develop a new IT system that is organized in the following way:

Two companies $i = 1, 2$ are invited. Each has to create a testing IT platform. The quality of the testing platform is $x \geq 0$ and the cost of developing a platform of quality x is linear to both companies, $c(x) = x$. Invited companies are not compensated for the costs of development. Based on the qualities of the platforms, there is an *independent committee* that awards the contract. For computational convenience, suppose the committee's decision-making can be approximated by the Tullock lottery. (This could be modeled by adding additive noise into their observations of the quality.) In other words, the probability that an invited Company i wins after (x_1, x_2) platforms are developed is $\frac{x_i}{x_1+x_2}$. A winning company gains R, which are net profits associated with the contract. (These profits obviously do not include sunk costs of the development, which is x_i for a Company i.)

This is not the end of the story. There are in total four relevant companies in the market, $j = A, B, C, D$. There is a policy-maker who *arbitrarily* defines criteria for an invitation. These entry criteria determine which pair of companies will be subsequently invited. The policy-maker knows that companies profit from being invited, hence wants to use his power strategically to his benefit. Suppose that the policy-maker can organize a *contest for invitation*. In the contest for invitation, each company $j = A, B, C, D$ pays a bribe y_j to the policy-maker. The policy-maker allocates the two "entry ticket" (i.e., the policy-maker specifies the entry criteria such that the other companies are excluded) to companies using *Tullock lottery over pairs*.

To understand Tullock lottery with pairs, notice that we have 6 pairs that may be invited. The set of pairs is $S = \{(A,B); (A,C); (A,D); (B,C); (B,D); (C,D)\}$. The probability that a pair $(k,l) \in S$ is invited is

$$\frac{y_k + y_l}{\sum_{(m,n)\in S}(y_m + y_n)} = \frac{y_k + y_l}{3\sum_{j\in\{A,B,C,D\}} y_j}.$$

Therefore, the probability that, w.l.o.g., a company A is invited, is

$$\frac{(y_A + y_B) + (y_A + y_C) + (y_A + y_D)}{3\sum_{j\in\{A,B,C,D\}} y_j} = \frac{3y_A + y_B + y_C + y_D}{3\sum_{j\in\{A,B,C,D\}} y_j} = \frac{3y_A + y_{-A}}{3y_A + 3y_{-A}},$$

where $y_{-A} := y_B + y_C + y_D$.

We have a two-level contest: There is a *contest for invitation* that proceed into a *contest for profits*.

1. Derive the level of the equilibrium bribes y^* and the expected payoff of each company associated with the two-level contest.

2. How is the prize R split into profits of the policy-maker, development costs, and profits of the companies?

Sample solution We proceed backwards and solve the contest for profits first.

1. There is a common prize R. (Bribes are sunk costs.) From the class, we know that for any pair of invited companies, their equilibrium payments for Tullock contest are

$$x^* = \frac{R}{4},$$

and the net payoff of the invited companies is the expected payoff of the Tullock contest for $n = 2$, i.e., $\frac{R}{2} - x^* = \frac{R}{4}$. This expected payoff constitutes a prize in the contest for invitation.

2. Second, solve the contest for invitations. There are two common prizes $\frac{R}{4}$. In this contest, the expected payoff for A is

$$\pi_A = \frac{3y_A + y_{-A}}{3y_A + 3y_{-A}} \frac{R}{4} - y_A.$$

The F.O.C. writes:

$$\frac{\partial \pi_A(y_A, y_{-A})}{\partial y_A} = \frac{3(3y_A + 3y_{-A} - 3y_A - y_{-A})}{9(y_A + y_{-A})^2} \frac{R}{4} - 1 = \frac{2y_{-A}}{3(y_A + y_{-A})^2} \frac{R}{4} - 1 = 0.$$

By symmetry, $y_A = y$ and $y_{-A} = 3y$, hence:

$$\frac{6y}{48y^2} \frac{R}{4} = 1$$

$$y^* = \frac{R}{32}.$$

Ex ante, the distribution of the prize R is as follows:

- The policy-maker's profits (rents) are $4y^* = \frac{R}{8}$ (12.5%).
- Development costs are $2x^* = \frac{R}{2}$ (50%).
- Companies gain the complementary $\frac{3R}{8}$ (37.5%) in total profits in expectation.

Ex post, we may distinguish between companies in the following way:

- Each of two non-invited company has a loss $-y^* = -\frac{R}{32}$ (-3.13%).
- The invited but non-winning company has a loss $-y^* - x^* = -\frac{9R}{32}$ (-28.13%).
- The winning company has profits $R - y^* - x^* = \frac{23R}{32}$ (71.88%).

Indeed, the companies' total payoffs are $\frac{(-2-9+23)R}{32} = \frac{12R}{32} = \frac{3R}{8}$.

2.2 How to derive the degree of discrimination in a contest?

Two large construction firms compete over public tenders by influencing local politicians. They also build properties for property developers. We observe their returns on capital (calculated for projects for property developers), which indicates the differences in their construction costs. These differences then also indicate the differences in their valuations of the public tenders; we let R_1 and R_2 be these valuations.

We also observe the shares of tenders awarded to each of firms, p_1 and p_2. Since the firms are large, we suppose they are risk-neutral.

- What is the degree of discrimination v in the bilateral contest of the two construction firms over the public tenders?

Sample solution First, we derive how the ratio of the award shares $\frac{p_1}{p_2}$ depends on the ratio of the equilibrium investments $\frac{x_1}{x_2}$. Using $p_i = \frac{x_i^v}{x_1^v + x_2^v}$ for both players, we have

$$x_1^v + x_2^v = \frac{x_1^v}{p_1} = \frac{x_2^v}{p_2} = x_1^v + x_2^v.$$

As a consequence,

$$\left(\frac{x_1}{x_2}\right)^v = \frac{p_1}{p_2}. \tag{1}$$

Second, we derive conditions that characterize the equilibrium ratio of the investments in the contest $\frac{x_1}{x_2}$. We start with best response of Player 1. His payoff is

$$\pi_1 = \frac{x_1^v}{x_1^v + x_2^v} R_1 - x_1,$$

and the F.O.C. that characterizes his best response is

$$\frac{\partial \pi_1}{\partial x_1} = \frac{v x_1^{v-1} x_2^v}{(x_1^v + x_2^v)^2} R_1 - 1 = 0.$$

By symmetry, we have in equilibrium a well-known condition that the investments are proportional to values,

$$\frac{x_1}{x_2} = \frac{R_1}{R_2}. \tag{2}$$

We insert (2) into (1) and obtain

$$\left(\frac{R_1}{R_2}\right)^v = \frac{p_1}{p_2}.$$

After rearranging,

$$v = \frac{\ln \frac{p_1}{p_2}}{\ln \frac{R_1}{R_2}} = \frac{\ln p_1 - \ln p_2}{\ln R_1 - \ln R_2}.$$

2.3 Which contest is more intensive under asymmetry?

Derive total expenditures in a contest for two players with valuations $R_1 = \sigma(R_1+R_2)$ and $R_2 = (1-\sigma)(R_1+R_2)$ for any $\sigma \in [0,1]$ for (i) Tullock lottery and (ii) all-pay auction. For all-pay auction, use that for $R_1 < R_2$, the equilibrium strategies are characterized by distribution functions $F_1(x) = \frac{R_2-R_1}{R_2} + \frac{x}{R_2}$ and $F_2(x) = \frac{x}{R_1}$ on the support $x \in [0,R_1]$. For which σ does all-pay auction raise more expenditures than Tullock lottery?

Sample solution W.l.o.g., let $\sigma < \frac{1}{2}$.

- APA: The total expected lobbying expenditures are

$$E(x_1)+E(x_2) = \int_0^{R_1} \frac{x}{R_2}\,dx + \int_0^{R_1} \frac{x}{R_1}\,dx = \frac{R_1^2}{2R_2} + \frac{R_1}{2} = \frac{\sigma(R_1+R_2)}{2(1-\sigma)}. \tag{3}$$

- Tullock lottery: From the class, we know that the expected lobbying expenditures are

$$E(x_1)+E(x_2) = \sigma(1-\sigma)(R_1+R_2). \tag{4}$$

APA yields more total expenditures than Tullock lottery if $\frac{1}{2(1-\sigma)} > (1-\sigma)$, which is equivalent to $\sigma > \frac{\sqrt{2}-1}{\sqrt{2}} \doteq 0.29$. Recall that this condition is valid for our assumption $\sigma < \frac{1}{2}$.

For $\sigma > \frac{1}{2}$, we use that for APA, $E(x_1)+E(x_2) = \frac{(1-\sigma)R}{2\sigma}$. Therein, APA yields more total expenditures than Tullock lottery if $\sigma < \frac{1}{\sqrt{2}}$. To sum up, APA yields more expenditures if

$$\frac{\sqrt{2}-1}{\sqrt{2}} < \sigma < \frac{1}{\sqrt{2}}. \tag{5}$$

2.4 A U.S. President linked only to domestic lobbyists

A newly elected U.S. President considers to change trade policy in favor of domestic firms. Suppose that the change increases profits of the domestic firms by $R_1 > 0$ at the expense of a profit losses $R_2 > 0$ of foreign firms (and importers). The change decreases overall welfare, $R_1 < R_2$.

The two companies engage in a lobbying contest over the trade policy. The domestic company gains political influence through payments to domestic lobbyists, x_1, and the foreign company gains political influence through payments to foreign lobbyists, x_2. The President is 'linked' only to the domestic lobbyists, and therefore can appropriate only part of payments given to these lobbyists. In other words, the policy-maker maximizes x_1 (or its expected value).

1. Does the President prefer Tullock contest or an all-pay auction? (The answer may depend on the asymmetry of prizes which you may denote by $\sigma := \frac{R_1}{R_1+R_2}$.) For all-pay auction, use that for $R_1 < R_2$, the equilibrium mixed strategies are characterized by the distribution functions $F_1(x) = \frac{R_2-R_1}{R_2} + \frac{x}{R_2}$ and $F_2(x) = \frac{x}{R_1}$ on the support $x \in [0, R_1]$.
2. How does the President's preferences over the two contests change if the President links to *both* domestic and foreign lobbyists?

Sample solution First, we calculate the lobbying effort of the domestic company:

- Tullock lottery (T): From the lecture, the lobbying effort of the domestic company is

$$x_1^T = \frac{R_1 R_2}{(R_1+R_2)^2} R_1 = \sigma(1-\sigma)R_1.$$

- All-pay auction (A): The expected lobbying effort of the domestic company is

$$E(x_1^A) = \int_0^{R_1} \frac{x}{R_2}\, dx = \frac{R_1^2}{2R_2} = \frac{\sigma}{2(1-\sigma)} R_1.$$

By comparing x_1^T with $E(x_1^A)$, we observe that Tullock lottery is preferred when $\sigma < \frac{\sqrt{2}-1}{\sqrt{2}} \doteq 0.29$. This is exactly the same cutoff level as if total expenditures matter. A bit surprisingly, the President's preference over Tullock contest vs. all-pay auction *don't* change even if the President gains benefits only from the effort raised by one of the competitors.

2.5 Asymmetric valuation and asymmetric profits

Two players with asymmetric valuations of the prize, w.l.o.g, $R_1 > R_2 > 0$, are engaged in a slightly discriminative contest, $1 \leq v \leq 2$.

In the equilibrium, is the expected payoff of the player with a higher valuation (Player 1) always larger than the the expected payoff of the player with a lower valuation (Player 2)? (Hint: You may try to express π_1 as a function of π_2; or vice versa.)

Sample solution For convenience, we will again introduce $\sigma := \frac{R_1}{R_1+R_2}$, where $\sigma > \frac{1}{2}$. We will start with profit of Player 2. To convert x_2 into x_1 (such that we can obtain profit of Player 1), we will multiply profit of Player 2 by $\frac{R_1}{R_2} = \frac{\sigma}{1-\sigma}$, and use that $\frac{x_1}{x_2} = \frac{R_1}{R_2}$:

$$\frac{\sigma}{1-\sigma}\pi_2 = p_2 R_1 - x_2 \frac{R_1}{R_2} = p_2 R_1 - x_1.$$

Next, we will use that

$$p_2 = \frac{x_2^v}{x_1^v + x_2^v} = \frac{(1-\sigma)^v}{\sigma^v + (1-\sigma)^v} = p_1 \frac{(1-\sigma)^v}{\sigma^v}.$$

Hence,

$$\frac{\sigma}{1-\sigma}\pi_2 = \frac{(1-\sigma)^v}{\sigma^v} p_1 R_1 - x_1 = \frac{(1-\sigma)^v}{\sigma^v} - \frac{\sigma^v}{\sigma^v} p_1 R_1 + p_1 R_1 - x_1 = \frac{(1-\sigma)^v - \sigma^v}{\sigma^v} p_1 R_1 + \pi_1.$$

Finally, using $\sigma > \frac{1}{2} > 1-\sigma$ and $\sigma^v > (1-\sigma)^v$,

$$\pi_1 = \frac{\sigma}{1-\sigma}\pi_2 + \frac{\sigma^v - (1-\sigma)^v}{\sigma^v} p_1 R_1 > \frac{\sigma}{1-\sigma}\pi_2 > \pi_2.$$

2.6 To subsidize or tax sports?

A sport association considers optimal funding of its two sport clubs, $i = 1,2$, where the clubs compete over a prize $E+R > 0$ in a club competition. The prize has two components: (i) A trophy $R \geq 0$ provided by the association and (ii) external benefits from victory, $E \geq 0$ (e.g., sponsorship thanks to media coverage). Suppose the competition has a form of Tullock lottery, where the win probability of a prize depends on the level of (training) activity in each club, denoted $x_i \geq 0$. The federation can observe the amount of activity and can subsidize or tax it (e.g., taxation through membership fee, participation fees in trophy competition, or by setting fees for infrastructure owned by the federation). For each club, the marginal cost of activity x_i is $1+t$, where $t > 0$ is for the case when each activity is taxed by the amount t and $t < 0$ is for the case when the sport is subsidized by the amount $-t$.

The association pays the prize out of two sources, the initial budget $B > 0$ and the net payments from clubs, $t(x_1+x_2)$, where the net payments are positive when the activity is taxed and negative when the activity is subsidized. It means the following:

1. The association announces (t,R) to the clubs.
2. The clubs engage in competitive sport activities, (x_1,x_2), with the aim to win the prize $E+R$.
3. The prize is allocated by Tullock lottery.

We assume that the association is *ex ante budgetary neutral*. It means two things: (i) If the association announces (t,R), it is committed to pay R even if any of the clubs deviates from the equilibrium (expected) amount of activity. (ii) In equilibrium, the association has exactly zero funds, so $R = B+t(x_1+x_2)$. This means that the association adjusts the value of the prize to the *expected* amount of total subsidies (or total tax revenues).[1] Simply, if the association subsidizes clubs, it must correspondingly reduce the prize. And if it taxes clubs, it uses all tax revenues to increase the initial prize.

Finally, we make a technical assumption that the initial budget plus external benefits are large enough, $E+B > 2$.

- Suppose that the association is interested only in maximizing total sports activity, x_1+x_2. What is the optimal t and what is the level of prize it will give to the clubs?

Sample solution We will first solve the Tullock contest of clubs which expect the marginal cost $1+t$ and the announced (i.e., exogenous) total prize R. Denoting total spending $X \equiv x_1+x_2$, the F.O.C. of Club 1 writes:

$$\frac{x_2}{(x_1+x_2)^2}(E+R) = \frac{x_2}{X^2}(E+R) = 1+t.$$

[1] Notice that the ex post budgetary neutral association adjusts the value of the prize to the real amount of total subsidies (or total tax revenues). This means that when some of the clubs deviates, the prize also changes, and the deviating club takes this effect into account.

By summing both F.O.Cs, total activities satisfy

$$\frac{X}{X^2}(E+R) = \frac{E+R}{X} = 2(1+t),$$

and therefore

$$X = \frac{E+R}{2(1+t)}.$$

This shows you the key tradeoff of the association. It can stimulate total activity X either by (i) reducing t or (ii) increasing R. But this is inconsistent: A lower t means a lower R and vice versa. So which channel does the association choose—will it focus on reducing t or rather on increasing R?

To address the question, see that the association is ex ante budgetary neutral, so in the equilibrium, $R = B + tX$. Therefore,

$$X = \frac{E+B+tX}{2(1+t)}.$$

By rearranging,

$$X = \frac{E+B}{2+t}.$$

Since X is decreasing in t, the association is *decreasing* the total sports activities if it taxes the activities (and uses the tax revenues to boost the prize) and is *increasing* the total sport activities if it subsidizes the investments out of the initial prize. If only total activities matter for the association, it pays off to subsidize activities as much as possible.

What is the maximal subsidy (i.e., a minimal t)? This subsidy must satisfy $t > -1$, because with $t = -1$, we would violate that the activity cost, $(1+t)x_i$, is positive (in a contest, this is necessary for getting a finite optimal x_i). Also, it must satisfy that the trophy is non-negative, $R \geq 0$. Depending on t, the prize R is

$$R = B + tX = B + t\frac{E+B}{2+t} = \frac{2(1+t)B + tE}{2+t}.$$

The numerator is non-negative, $2(1+t)B + tE \geq 0$, if and only if $t \geq \frac{-2}{B+E}$, where we use $t \geq \frac{-2}{B+E} > -1$ as $B+E > 2$. Therefore, the association optimally provides a subsidy $t^* = \frac{-2}{B+E}$, pays zero trophy, and the clubs compete only over the external benefits E.

To summarize, the association solves the tradeoff by focusing on *subsidizing activity instead of on increasing the prize* for the winning club. The optimum is at the corner; it pays off to provide a maximal feasible subsidy (a direct incentive) at the expense of the prize (an indirect incentive).

2.7 Proposals of lobbies: Tullock's lottery, concave policy cost

In the class, we did analyze lobbies which first committed to policy proposals and then engaged in a competition over the right to set the proposed policy. The competition is modeled as a perfectly discriminating contest (all-pay auction). But what if the contest is less discriminating?

Use the setting exactly as in the class, but assume that Tullock's lottery determines the winner in Stage 2. What are the equilibrium proposals?

Sample solution In Tullock's lottery, a player A invests $x_A = \frac{R_A^2 R_B}{(R_A+R_B)^2}$, wins with probability $p_A = \frac{x_A}{x_A+x_B} = \frac{R_A}{R_A+R_B}$, and therefore the expected payoff is

$$\pi_A = p_A R_A - x_A = \frac{R_A^3}{(R_A+R_B)^2}.$$

What is the effect of convergence, i.e., an increase in a, on the expected payoff? We know that $\frac{\partial R_A}{\partial a} = -2a$ and $\frac{\partial R_B}{\partial a} = -4+2a$. We insert into the marginal payoff to obtain

$$\frac{\partial \pi_A}{\partial a} = \frac{R_A^2}{(R_A+R_B)^3}\left[-6a(R_A+R_B)+8R_A\right].$$

Divergence is clearly not in the equilibrium, since the bracket is for $a=0$ positive. We will find whether a symmetric equilibrium $(a,b)=(a,2-a)$ with incomplete convergence exists, using that in the equilibrium with incomplete convergence, the marginal effect of a convergence is zero. Using symmetry, we derive values, $R_A(a,2-a) = R_B(a,2-a) = 4(1-a)$. Then, we inserting into the marginal payoff and setting equal zero yields

$$(a^*,b^*) = \left(\frac{2}{3},\frac{4}{3}\right).$$

Unlike for APA, we observe *incomplete convergence* for Tullock's lottery.

2.8 Proposals of lobbies: Tullock's lottery, linear policy cost

We have two lobbies competing over a policy $x \in R$. Lobby A's utility is $u_A(x) = -|x|$ (bliss point is at $x = 0$), and Lobby B's utility is $u_B(x) = -|2-x|$ (bliss point is at $x = 2$). In Stage 1, Lobby A commits to implementing policy $a \in [0,1]$ if it wins a lobbying contest; Lobby B commits to implementing policy $b \in [1,2]$ if it wins a lobbying contest.

In Stage 2, the two lobbies are engaged in a contest which has structure of Tullock's lottery (imperfect discrimination). Find the equilibrium platforms (a^*, b^*) that the two competing lobbies commit to implement.

Sample solution The key point is that for linear preferences, the contest prizes are identical for any (a,b). Namely, given $0 \le a \le 1 \le b \le 2$, the prize is

$$R := R_A = R_B = b - a.$$

For Tullock contest over a symmetric prize R, we know that the equilibrium profit is $\frac{R}{4} = \frac{b-a}{4}$. Therefore, $U_A(a,b) = \frac{b-a}{4} + u_A(b)$. For A, a deviation from her bliss point decreases her expected payoff,

$$\frac{\partial U_A(a,b)}{\partial a} = -\frac{1}{4} < 0.$$

Hence, best response of A is $a(b) = 0$. Similarly, the best response of B is $b(a) = 2$. Mutual best responses are $(a^*, b^*) = (0,2)$. The lobbies *diverge completely*; convergence doesn't pay off since it doesn't create any relative advantage in the subsequent contest.

Notice that complete divergence is primarily because the disutility is linear; imperfection in the discrimination is neither necessary nor sufficient for complete divergence.