Part C: Panel Data Methods

C2: Canonical Difference-in-Differences

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ARE 213 Applied Econometrics

UC Berkeley, Fall 2023

C2 outline

1. $2x2$ DiD
2. DiD with multiple units and periods
3. Extensions

Difference-in-differences: Idea

- We are interested in causal effects of some binary treatment
- But treatment is not randomly assigned across unit
- "Quasi-experimental" contrasts are easier to find in panel data
- Card and Krueger (1994): effect of minimum wages on employment
    - On April 1, 1992, NJ raised the min.wage from \$4.25 to \$5.05
    - Min.wage in PA stayed at \$4.25
    - Measure average employment at fast food restaurants before (Feb 1992) and after (Nov 1992)

Card and Krueger (1994)

	Variable	PA (i)	NJ (ii)	Difference, NJ-PA (iii)
1.	FTE employment before,	23.33	20.44	-2.89
	all available observations	(1.35)	(0.51)	(1.44)
2.	FTE employment after,	21.17	21.03	-0.14
	all available observations	(0.94)	(0.52)	(1.07)
3.	Change in mean FTE	-2.16	0.59	2.76
	employment	(1.25)	(0.54)	(1.36)

(Card-Krueger Table 3, reproduced from MHE Table 5.2.1)

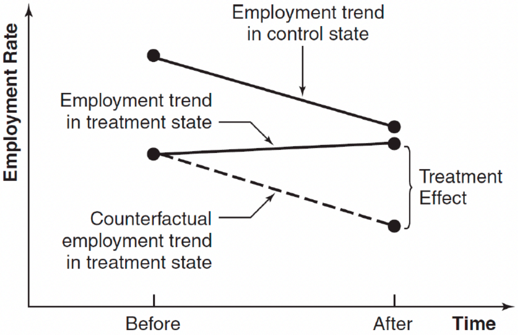
Is this causal?

- Define potential outcomes $Y_{it}(d)$ for $d = 0, 1$ (low/high min.wage), with $\tau_{it} = Y_{it}(1) - Y_{it}(0)$ and $Y_{it} = Y_{it}(D_{it})$

D_{it}	$i = PA$	$i = NJ$
$t = 1$	0	0
$t = 2$	0	1

- Estimand: $ATT = \tau_{NJ,2}$
- Assumptions for this potential outcomes formulation:
  - No spillovers
  - No anticipation effects: Y_{i1} does not depend on D_{i2}
  - No lagged effects: Y_{i2} does not depend on D_{i1}
- Parallel trends...

Parallel trends (PTA)



(MHE Figure 5.2.1, corrected by Peter Hull)

Parallel trends: $\mathbb{E}\left[Y_{NJ,2}(0) - Y_{NJ,1}(0)\right] = \mathbb{E}\left[Y_{PA,2}(0) - Y_{PA,1}(0)\right]$

- Equivalently, $\mathbb{E}\left[Y_{it}(0)\right] = \alpha_i + \beta_t$ for $i = NJ, PA$ and $t = 1, 2$
- *Note*: notation for a fixed sample, $\mathbb{E}\left[Y_{it}(0)\right]$ depends on i, t

2x2 DiD estimator

- $\hat{\tau} = \left(\boxed{Y_{NJ,2}} - Y_{NJ,1} \right) - \left(Y_{PA,2} - Y_{PA,1} \right)$ is unbiased for ATT

- $\hat{\tau}$ can be obtained as OLS from the TWFE specification

$$Y_{it} = \tilde{\alpha}_i + \tilde{\beta}_t + \tau D_{it} + \varepsilon_{it}$$

  - Proof: with 2 periods, the first-differenced equation $Y_{i2} - Y_{i1} = \beta + \tau \left(D_{i2} - D_{i1} \right) + \left(\varepsilon_{i2} - \varepsilon_{i1} \right)$ gives the same $\hat{\tau}$

Problems with 2x2 designs (1)

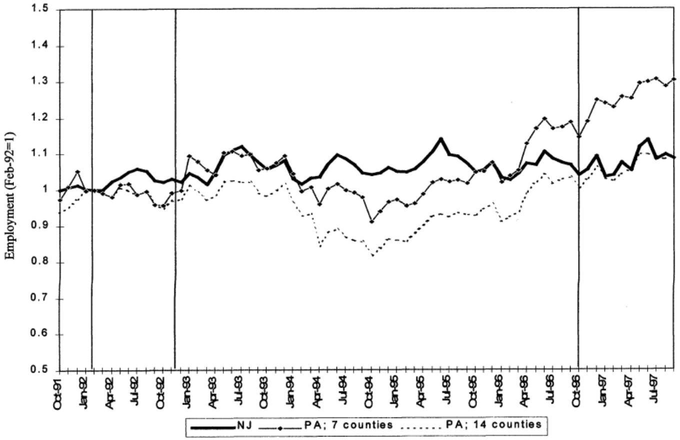


FIGURE 2. EMPLOYMENT IN NEW JERSEY AND PENNSYLVANIA FAST-FOOD RESTAURANTS, OCTOBER 1991 TO SEPTEMBER 1997

(Card-Krueger 2000 for an extended time period)

Are these fluctuations are captured by SE in firm-level analysis of Card-Krueger (1994)?

Problems with 2x2 designs (2)

1. Effectively 4 observations: can't separate τ from other shocks in NJ relative to PA

Variable	PA (i)	NJ (ii)	Difference, NJ-PA (iii)
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These SE are not clustered, and cannot be!

Problems with 2x2 designs (2)

1. Effectively 4 observations: can't separate τ from other shocks in NJ relative to PA
2. Can't test parallel trends

Solution: have more (pre-)periods and more units

	$i = A$	$i = B$	$i = C$	$i = D$
$t = 1$				
$t = 2$				
$t = 3$				
$t = 4$				
$t = E = 5$				
$t = 6$				

Outline

Causal structure with multiple periods

This setting accommodates two scenarios:

1. Event happens at $t = 5$ only but can have persistent effects
   - D_{it} = having ever been exposed to the treatment/event
2. Policy switches on at $t = 5$ and stays on
   - Two types of effects at $t = 6$: contemporaneous effects of D_{i6}, delayed effects of D_{i5}
   - Potential outcomes $Y_{i6}(d_{i5}, d_{i6})$ would acknowledge this
   - But can't distinguish them here $\implies$ simplify notation to $Y_{i6}(d_{i6})$

PTA with multiple units and periods

- PTA at the unit level (in a fixed sample):

$$\begin{aligned} & \mathbb{E}\left[Y_{it}(0)\right] = \alpha_i + \beta_t \qquad \forall i, t \\ \iff & \mathbb{E}\left[Y_{it}(0) - Y_{i,t-1}(0)\right] = \mathbb{E}\left[Y_{jt}(0) - Y_{j,t-1}(0)\right] \qquad \forall i, j, t \end{aligned}$$

- Or impose PTA at the treatment/control group level (in a random sample):

$$\mathbb{E}\left[Y_{it}(0) - Y_{i,t-1}(0) \mid Tr_i = 1\right] = \mathbb{E}\left[Y_{it}(0) - Y_{i,t-1}(0) \mid Tr_i = 0\right], \qquad \forall t$$

where $Tr_i = \mathbf{1}\left[\text{treament group}\right]$

Why might PTA hold?

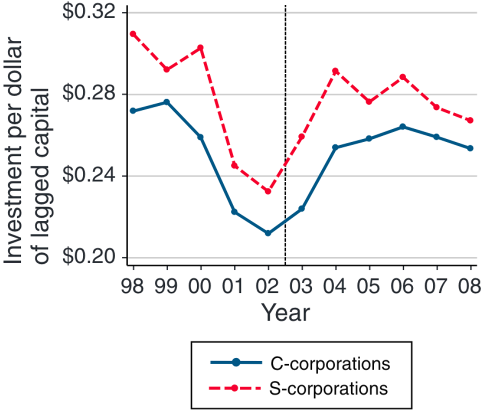
- **Design-based approach:** group assignment is random
  - Balance on levels in addition to trends $\Longrightarrow$ have better tools than DiD! (Roth and Sant'Anna, JPE:Micro 2023)
  - Justifies group-level PTA
- **Model-based approach:** stability of the environment, i.e. unobserved factors
  - Better with large shocks and in the short-run
  - Individual-level PTA is more appropriate
  - Levels vs logs matters (Roth and Sant'Anna, Ecma 2023)
- It's not clear how seriously researchers take PTA: research design or data mining?

Example: Yagan (2015, AER)

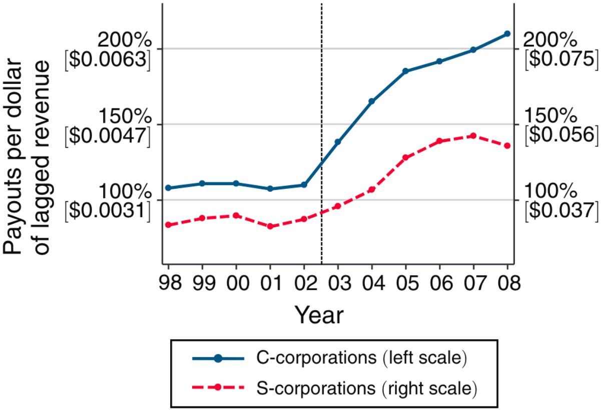
- Studies the effects of the 2003 dividend tax cut from 38.6% to 15%
- Treatment group: C-corporations; control group: S-corporations
- S-corporations don't pay dividend tax; otherwise similar taxation
- *"The identifying assumption is not random assignment of C- versus S-status; it is that C- and S-corporation outcomes would have trended similarly in the absence of the tax cut"*
- Justifications:
  1. *"C- and S-corporations of the same ages operate in the same narrow industries and at the same scale"*
  2. *"Contemporaneous stimulative tax provisions applied almost identically"*
  3. *"Key outcomes empirically trended similarly for C- and S-corporations before 2003"*

Yagan (2015): Plotting raw data

Panel A. Investment



Panel D. Total payouts to shareholders



“Static” and “dynamic” ATT estimation

Assume a balanced panel. Static TWFE specification: $Y_{it} = \tilde{\alpha}_i + \tilde{\beta}_t + \tau D_{it} + \varepsilon_{it}$

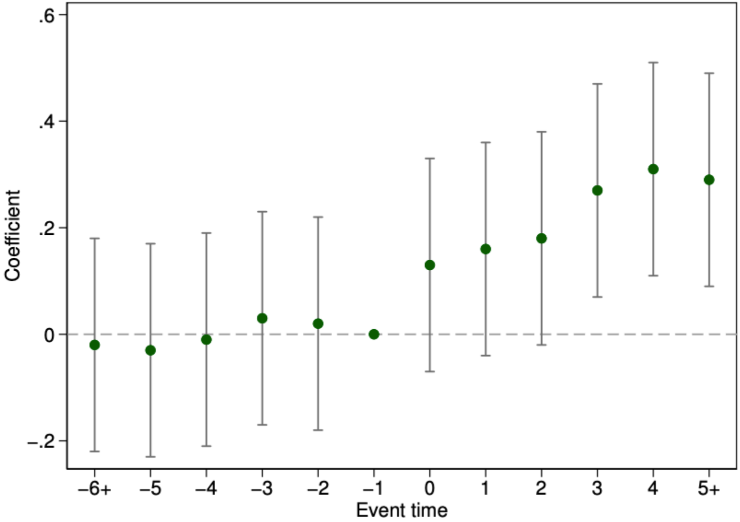
- $\hat{\tau} = \left(\bar{Y}_{\text{treated,post}} - \bar{Y}_{\text{treated,pre}}\right) - \left(\bar{Y}_{\text{control,post}} - \bar{Y}_{\text{control,pre}}\right)$
- Under PTA, $\mathbb{E}\left[\hat{\tau}\right] = ATT \equiv$ avg τ_{it} across all treated units and “post” periods
- Doesn't assume static effects but not very useful if effects exhibit strong dynamics

“**Event study**” dynamic specification:

$$Y_{it} = \tilde{\alpha}_i + \tilde{\beta}_t + \sum_{h=-(E-1)}^{T-E} \tau_h \mathbf{1}\left[t - E = h\right] Tr_i + \varepsilon_{it}, \qquad \tau_{-1} = 0$$

- $\hat{\tau}_h = \left(\bar{Y}_{\text{treated},E+h} - \bar{Y}_{\text{treated},E-1}\right) - \left(\bar{Y}_{\text{control},E+h} - \bar{Y}_{\text{control},E-1}\right)$
- For $h \geq 0$, unbiased for ATT h periods after the event
- For $h < -1$, unbiased for zero under PTA: yields a “pre-trends” test

Exemplary event study plot



(Example from Freyaldenhoven et al. 2021, Fig. 1)

Discussion of conventional event studies

- Event study regression conflates estimation and testing
  - If PTA holds, why not use all pre-periods for more efficient estimation:

$$Y_{it} = \tilde{\alpha}_i + \tilde{\beta}_t + \sum_{h \geq 0} \tau_h \mathbf{1}\left[t - E = h\right] Tr_i + \varepsilon_{it}$$

- Pre-trend test is at best suggestive:
  - We imposed parallel trends in $Y_{it}(0)$ both pre- and post-treatment
  - Parallel trends post-treatment is necessary and untestable

Discussion of conventional event studies (2)

- Pre-trend tests are not always informative:
  - Mann and Pozzoli (2023) study the effects of low-skill immigration into Danish municipalities on industrial robot adoption by firms in 1995-2019
  - How informative are their regressions of pre-period (1993-95) changes in robot adoption and other variables on the 1995-2019 change in low-skill immigration?

Table 3: Pre-Sample Trends and Long-Run Changes in Automation

	Δ robot users (broad)	Δ imports	Δ exports	Δ capital stock	Δ low skill share
	(1)	(2)	(3)	(4)	(5)
Δ Non-West Img IV (in sample)	-0.849	0.301	-0.260	-0.362	-0.018
	(0.674)	(0.289)	(0.503)	(0.247)	(0.029)
N	97	97	97	97	97

Discussion of conventional event studies (3)

- Pre-trend tests can have low power against substantially different alternatives (Roth AER:I 2022)
  - He and Wang (2017 AEJ:Applied) study the effects of college-graduated bureaucrats placed to Chinese villages
  - Outcome = % of subsidized population (a measure of poverty)

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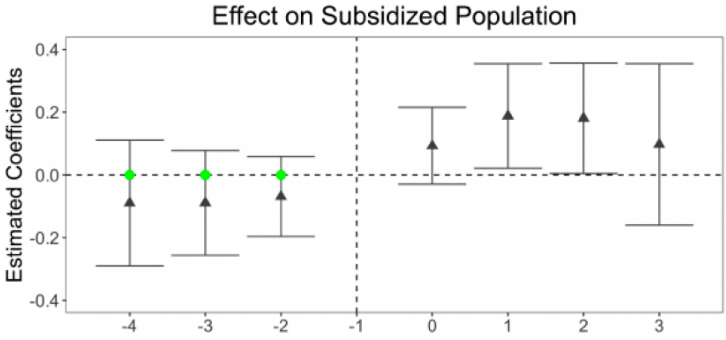
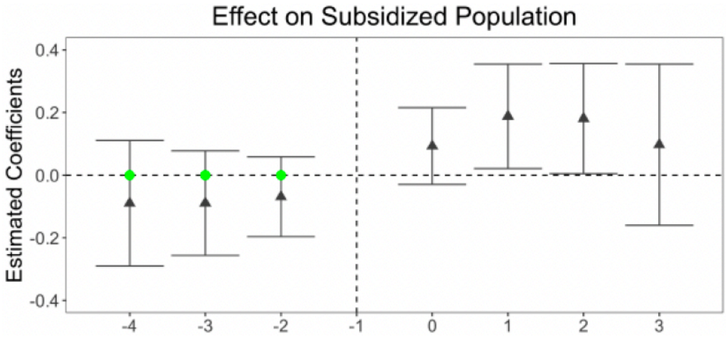


(From Jonathan Roth's Mixtape Session slides)

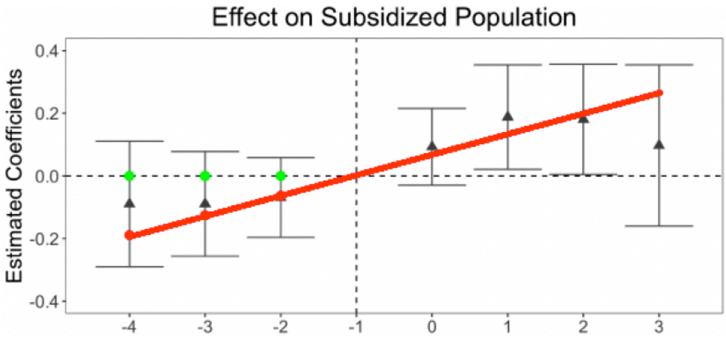
Discussion of conventional event studies (3)



(From Jonathan Roth's Mixtape Session slides)

- *"The estimated coefficients on the leads of treatment ... are statistically indifferent from 0. ... We conclude that the pretreatment trends in the outcomes in both groups of villages are similar, and villages without CGVOs [treatment] can serve as a suitable control group for villages with CGVOs in the treatment period."*

Discussion of conventional event studies (3)



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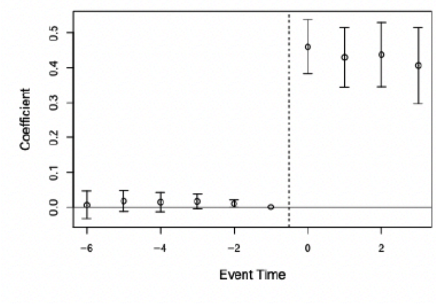
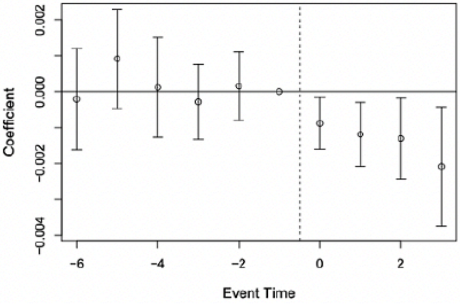
The "Straight line test"

- The more nonlinearity is needed to kill the effects, the more robust the results are



vs.



- See Stata package *pretrends* accompanying Roth (2022)

Checking robustness to PTA violations

Rambachan-Roth (forthcoming): partial identification approach

- Let $\delta_t = \mathbb{E}\left[Y_{it}(0) - Y_{i,t-1}(0) \mid Tr_i = 1\right] - \mathbb{E}\left[Y_{it}(0) - Y_{i,t-1}(0) \mid Tr_i = 0\right]$
- Estimable for $t < E$ but not $t \geq E$; PTA requires $\delta_t = 0$
- Weaker assumptions for $t \geq E$ as robustness checks:
  1. Differential trends are not too large: $|\delta_t| \leq M \cdot \max_{s<E} |\delta_s|$
  2. Differential trends are smooth: $|\delta_t - \delta_{t-1}| \leq M$
     - ⋆ *Note:* for $M = 0$ this is not PTA but a linear differential trend
- When pre-trends are noisily estimated, conf.interval will be wider
- Need to pick M or compute the largest M that doesn't kill your findings

Outline

Extensions

1. Continuous treatment intensity
2. Including covariates
3. Spillovers
4. Triple-diffs

Continuous treatment intensity: Setting

- Suppose $D_{it} = \mathbf{1}\left[t \geq E\right] \times Tr_i$ where Tr_i is treatment dosage which is not binary
  - Enikolopov et al. (2011): effect of region i's exposure to independent TV channel (NTV) on election outcomes in Russia
  - In 1995, $D_{it} \equiv 0$; in 1999, $D_{it} = Tr_i =$ % of regional population with NTV access
- Can still impose PTA on $Y_{it}(0)$
- Estimands of interest:

$$\mathbb{E}\left[Y_{it}(d) - Y_{it}(d') \mid Tr_i = d\right] \quad \text{or} \quad \mathbb{E}\left[\frac{\partial Y_{it}(d)}{\partial d} \mid Tr_i = d\right]$$

Continuous treatment intensity: Event study

- Can still run the event study regression:

$$Y_{it} = \tilde{\alpha}_i + \tilde{\beta}_t + \sum_{h \neq -1} \tau_h \mathbf{1}\left[t = E + h\right] \times Tr_i + \text{error}_{it}$$

- Or, equivalently, get $\hat{\tau}_h$ from h-specific regression

$$Y_{i,E+h} - Y_{i,E-1} = \tilde{\beta}_h + \tau_h Tr_i + \text{error}_{it}$$

- But with heterogeneous effects and without randomization of Tr_i, don't get a causal estimand
  - PTA on $Y_{it}(0)$ does not allow comparing units with different dosages
  - OLS subtracts outcomes of units with low $Tr_i > 0 \implies$ puts "**negative weights**" on their effects

Continuous treatment intensity: What to do?

- If $Pr(Tr_i = 0) = 0$, PTA on $Y_{it}(0)$ is entirely unhelpful
- If $Pr(Tr_i = 0) > 0$, PTA allows comparisons relative to the never-treated group
  - Yields $ATT(d) = \mathbb{E}\left[Y_{it}(d) - Y_{it}(0) \mid Tr_i = d\right]$; linked to **imputation** strategies
  - Still cannot identify $\mathbb{E}\left[Y_{it}(d) - Y_{it}(d') \mid Tr_i = d\right]$ for $d' \neq 0$
- Can get further with randomization, restrictions on heterogeneous effects, or stronger PTAs
  - See Callaway, Goodman-Bacon, Sant'Anna (2021), expect more work

DiD with covariates

Two ways of thinking about covariates:

- *Borusyak, Jaravel, Spiess (2023)*: relax the model of $Y_{it}(0)$

$$\mathbb{E}\left[Y_{it}(0)\right] = \alpha_i + \beta_t + \gamma' X_{it}$$

e.g. add unit-specific linear trends $\gamma_i \cdot t$, time-interacted baseline characteristics $\gamma_t' X_i$

$\Rightarrow$ Estimate γ from untreated observations only, then imputation

- *Sant'Anna and Zhao (2020)*: impose PTA conditional on baseline characteristics

$$\mathbb{E}\left[\Delta Y_{it}(0) \mid Tr_i = 1, X_i\right] = \mathbb{E}\left[\Delta Y_{it}(0) \mid Tr_i = 0, X_i\right]$$

$\Rightarrow$ AIPW (or other covariate adjustment) for ΔY_{it}

DiD with covariates: What NOT to do

- With heterogeneous effects, do not just add covariates to TWFE regressions, e.g.

$$Y_{it} = \tilde{\alpha}_i + \tilde{\beta}_t + \tau D_{it} + \gamma' X_{it} + \text{error}$$

  - $\hat{\gamma}$ will use treated observations too, misattributing some treatment effects

- *Example:* Wolfers (AER 2006) studies the effect of unilateral divorce laws on divorce rates

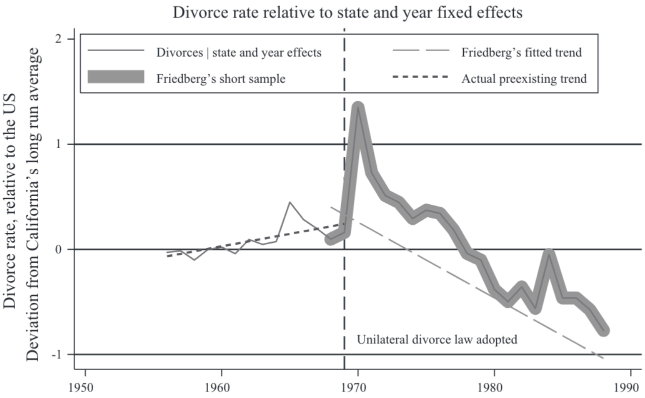
  - Reanalyzes Friedberg (1998) finding of a positive effect

Results without and with state-specific trends

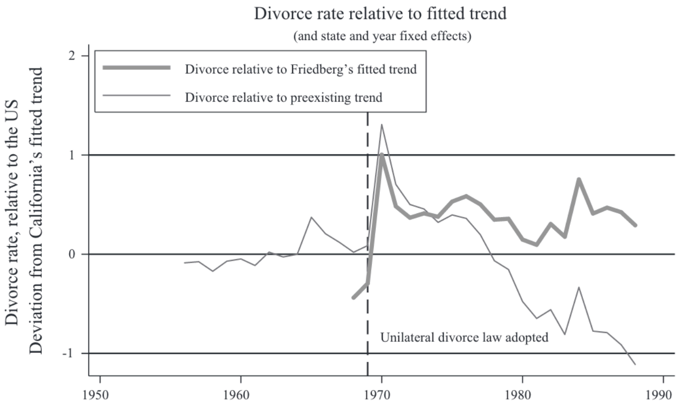
	(1) Basic specification	(2) State-specific trends linear
	Panel A. Friedberg (1998)	
Unilateral	0.004	0.447
	(0.056)	(0.050)
Year effects	$F = 89.0$	$F = 95.3$
State effects	$F = 217.3$	$F = 196.2$
State trend, linear	No	$F = 24.7$
State trend, quadratic	No	No
Adjusted R^2	0.946	0.976

Which results should we trust?

Example of California



Example of California (2)



- With appropriate estimators, state-specific trends don't change estimates (de Chaisemartin and D'Haultfœuille, Econometrics J. forthcoming)

Spillovers

- Consider a 2-period DiD with spillovers: $Y_{i2} = Y_{i2}(\boldsymbol{D}_2) \equiv Y_{i2}(\mathbf{0}) + \tau_{i2}(\mathbf{0}, \boldsymbol{D}_2)$
- Under PTA, get *relative* effect $\mathbb{E}\left[\tau_{i2}(\mathbf{0}, \boldsymbol{D}_2) \mid D_{i2} = 1\right] - \mathbb{E}\left[\tau_{i2}(\mathbf{0}, \boldsymbol{D}_2) \mid D_{i2} = 0\right]$
- Identifies the *aggregate* effect if and only if $\mathbb{E}\left[\tau_{i2}(\mathbf{0}, \boldsymbol{D}_2) \mid D_{i2} = 0\right] = 0$
- What to do if the control group can be affected?
- Local spillovers are easier:
  - e.g., "donut approach" in spatial settings
- To identify spillovers that affect everyone:
  - Two-tier designs: e.g. use variation in % of treated across regions, on top of treated-control comparisons within regions
  - Theoretical models: e.g. how import competition affect equilibrium wages
  - Common sense: e.g. if Y_{it} = net internal migration across regions i

Triple-differences

- Suppose for each state $i = NJ, PA$ and period $t = 1, 2$ we observe groups $g = L, H$: low- and high-wage occupations
- Min.wage should affect group L only: $D_{igt} = \mathbf{1}[i = NJ] \times \mathbf{1}[g = L] \times \mathbf{1}[t = 2]$
- Strategy 1:
  - Run **placebo** DiD: $Y_{iHt} = \alpha_{iH} + \beta_{Ht} + \tau_H \mathbf{1}[i = NJ] \times \mathbf{1}[t = 2] + \varepsilon_{iHt}$
  - Test $\tau_H = 0$; then use DiD to learn the ATT for group L
- Strategy 2 (**triple-differences**):
  - Allow non-parallel trends between NJ and PA, but the same for $g = L, H$

$$\mathbb{E}[\Delta Y_{NJ,L}(0) - \Delta Y_{PA,L}(0)] = \mathbb{E}[\Delta Y_{NJ,H}(0) - \Delta Y_{PA,H}(0)]$$

  - Equivalent to $\mathbb{E}[\Delta Y_{ig}(0)] = \alpha_i + \beta_g$ and $\mathbb{E}[Y_{igt}(0)] = \alpha_{it} + \beta_{gt} + \gamma_{ig}$
  - Can estimate $ATT = \tau_{NJ,L,2}$ from $Y_{igt} = \tilde{\alpha}_{it} + \tilde{\beta}_{gt} + \tilde{\gamma}_{ig} + \tau D_{igt} + \text{error}_{igt}$