Lecture 2: Time Series

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January 30, 2024

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Packages for Today

Let's load some packages so that I can make some better looking plots:

```r
#always
library(fixest)
library(tidyverse)
# for loading data
library(tidyquant)
# for cleaning up time series
library(timetk)
library(broom)
library(sweep)
library(forecast)
```

What is Time Series?

Big Picture

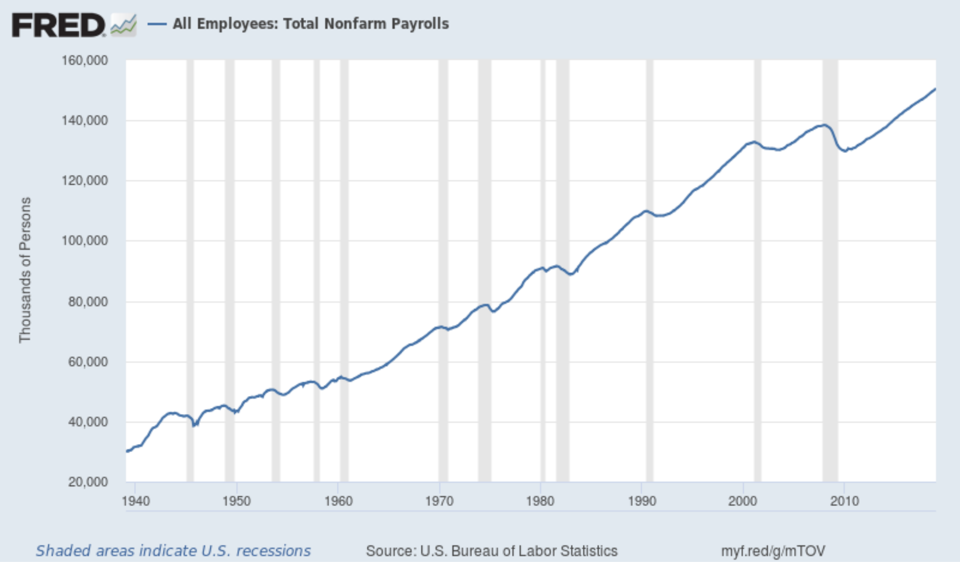
So far you have mostly studied cross sectional econometrics (subscript i):

- Individual observations are independent of one another (mostly).
- Large number of individuals allows us to do inference (LLN, CLT).

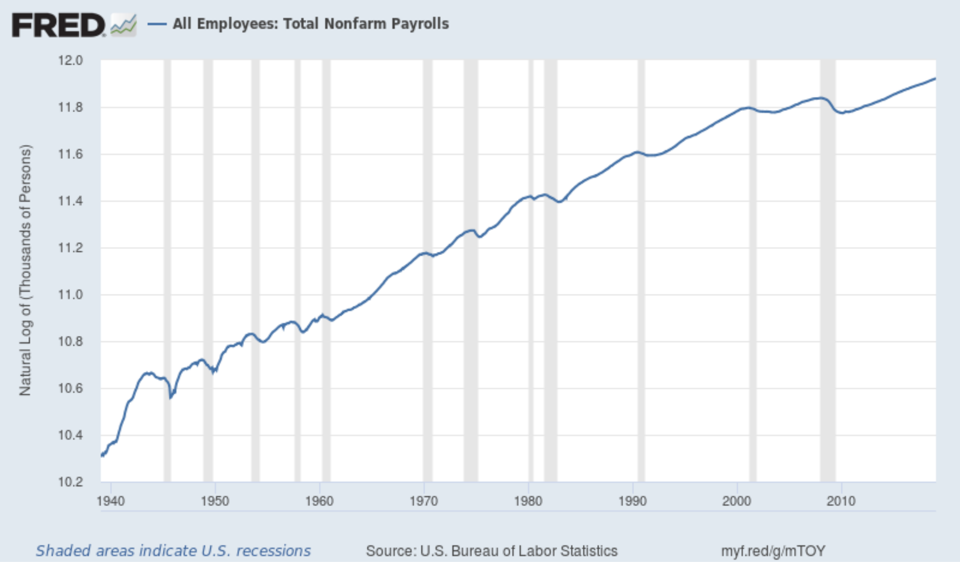
But suppose we observe a single object for may periods (subscript t):

- Now we worry that y_t is autocorrelated with y_{t-1}
- This means that independence is not going to hold.
- This presents a number of challenges addressed in time series econometrics.

Employment


FRED
— All Employees: Total Nonfarm Payrolls
Thousands of Persons
160,000
140,000
120,000
100,000
80,000
60,000
40,000
20,000
1940
1950
1960
1970
1980
1990
2000
2010
Shaded areas indicate U.S. recessions
Source: U.S. Bureau of Labor Statistics
myf.red/g/mTOV

Employment


FRED
— All Employees: Total Nonfarm Payrolls
Natural Log of (Thousands of Persons)
12.0
11.8
11.6
11.4
11.2
11.0
10.8
10.6
10.4
10.2
1940
1950
1960
1970
1980
1990
2000
2010
Shaded areas indicate U.S. recessions
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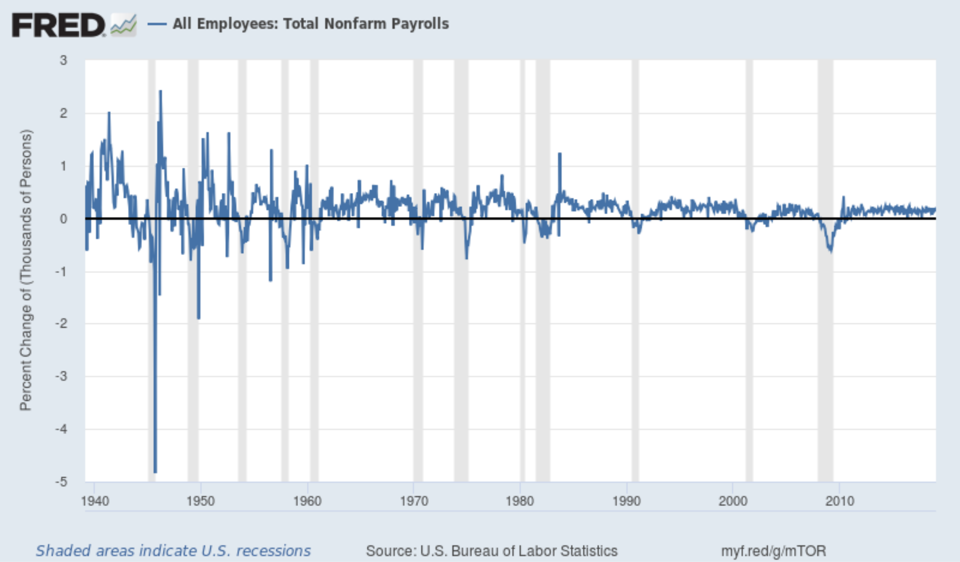
Employment



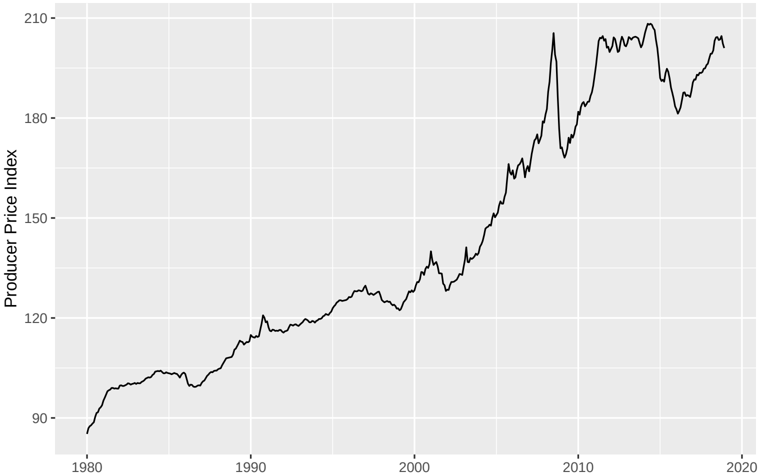
FRED
— All Employees: Total Nonfarm Payrolls
Percent Change of (Thousands of Persons)
3
2
1
0
-1
-2
-3
-4
-5
1940
1950
1960
1970
1980
1990
2000
2010
Shaded areas indicate U.S. recessions
Source: U.S. Bureau of Labor Statistics
myf.red/g/mTOR

Do it ourselves

```r
# Load and plot the PPI Data
ppi<-tq_get("PPIACO", get  = "economic.data",
         from='1980-01-1',to='2018-12-31')
tail(ppi)
```

Do it ourselves

```
ggplot(ppi, aes(date, price)) + geom_line() +
    scale_x_date() + ylab("Producer Price Index") + xlab("")
```



Theory of Time Series

Notation

We observe a sample $\{y_1, y_2, \ldots, y_{t-1}, y_t, y_{t+1}\}$.

- We call y_{t-1} the first lag of y_t.
- We call $\Delta y_t = y_t - y_{t-1}$ the first difference
- We might also want $\Delta \ln y_t = \ln y_t - \ln y_{t-1}$
- We can approximate percentage change as $100 \cdot \Delta \ln y_t$

Autocovariance, Serial Correlation

Measure the correlation of a series with its own lagged values

- First autocovariance of y_t is $Cov(y_t, y_{t-1}) = \gamma(1)$.
- The jth autocovariance of y_t is $Cov(y_t, y_{t-j}) = \gamma(j)$.

Questions

1. How do we represent $Var(y_t)$?
2. Can we show that $\gamma(k) = \gamma(-k)$? (even function)
3. Can we show that $\gamma(0) \geq |\gamma(k)|$ for any k?
4. Does this imply that $|\gamma(k)| \geq |\gamma(k-1)|$?

Autocorrelation

We can also compute the autocorrelaton coefficient j:

$$Corr(y_t, y_{t-j}) = \frac{Cov(y_t, y_{t-j})}{Var(y_t)} = \frac{\gamma(j)}{\gamma(0)} = \rho(j)$$

With sample analogue

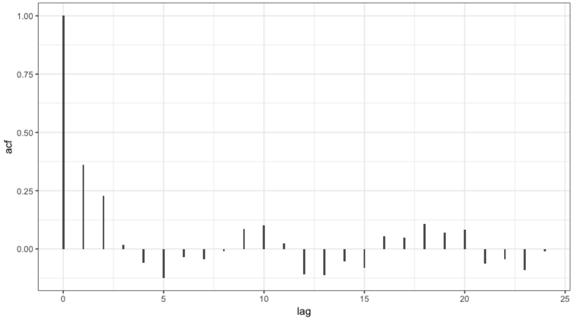
$$\widehat{Corr(y_t, y_{t-j})} = \frac{\widehat{\gamma}(j)}{\widehat{\gamma}(0)} = \widehat{\rho}(j)$$

Which we can estimate via:

$$\widehat{\rho}(j) = \frac{1}{T} \sum_{t=j+1}^{T} (y_t - \overline{y})(y_{t-j} - \overline{y})$$

- Most software uses $\frac{1}{T}$ instead of d.o.f corrected $\frac{1}{T-j}$
- Some software uses mean of $\{y_{j+1}, y_T\}$ and $\{y_1, y_{T-j}\}$ instead of grand mean

ACF plots




```
gdp<-tq_get("A191RL1Q225SBEA", get  = "economic.data",
            from='1950-01-1',to='2018-12-31')
result<-tidy(acf(gdp$price))
ggplot(result, aes(x=lag, y=acf)) +
          geom_bar(stat='identity', width=0.1) + theme_bw()
```

Stationarity

Conceptually stationarity is one of the most important issues with time series:

- Basic idea: the future needs to look like the past (at least probabilistically)
- I cheated on previous slides and assumed stationarity. Why?
- Simplified: $Cov(y_t, y_{t-k})$ is allowed to depend on k but not on t.
  - Relationship between y_t and its lags is constant across time
- Formally we need the joint distribution $f(y_{s+1}, y_{s+2}, \ldots, y_{s+T})$ to be invariant to s.
- Weaker form: Covariance Stationary

Hand Waving Technical Stuff

We probably want something like an LLN or CLT:

- Independence is violated between (y_t, y_{t-k})
- Idea: consider a large value H and assume stationarity:
  - The block $(y_t, y_{t-1}, y_{t-2}, \ldots, y_{t-k})$ and $(y_{t+H}, y_{t-1+H}, y_{t-2+H}, \ldots, y_{t-k+H})$ are as if they are independent for some large enough choice of H.
  - How is H determined? The mixing rate of the time series?
  - In practice? Looking at the ACF function/plot

Hand Waving Technical Stuff

- Soemtimes people will talking about mixing properties or the mixing rate
- This tells us how far apart in time two observations are before we can treat them as if they are "independent".
- Another property is ergodicity

$$\sum_{k=0}^{\infty} |\gamma(k)| = \gamma(0)\tau < \infty$$

- τ is the correlation time
- We could look at the variance of $\overline{X}_t$ to derive this but
- It is as if we have $\frac{n}{1+2\tau}$ effective independent observations

$AR(1)$ Regression

Consider the first-order autoregression for a forecast:

$$y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t$$

- No causal interpretation of (β_0, β_1).
- $\beta_1 = 0$ means that y_{t-1} is not informative about y_t.
- We can run this regression using OLS

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- We can run this regression using OLS

$AR(1)$ Example

```
> ar(gdp$price,order=1)

Call:
ar(x = gdp$price, order.max = 1)

Coefficients:
     1
0.3816

Order selected 1  sigma^2 estimated as  12.65
```

Wold Decomposition

Start with the AR(1) where ε_t is I.I.D with some variance σ^2:

$$y_t = \beta_0 + \beta_1 y_{t-1} + \varepsilon_t$$

$$y_{t-1} = \beta_0 + \beta_1 y_{t-2} + \varepsilon_{t-1}$$

$$y_{t-2} = \beta_0 + \beta_1 y_{t-3} + \varepsilon_{t-2}$$

Can we re-write the sequence as function of ϵ_t's only?

$$y_t = \underbrace{\beta_0 + \beta_1\beta_0 + \beta_1^2\beta_0}_{\widetilde{\beta}_0} + \beta_1\varepsilon_{t-1} + \beta_1^2\varepsilon_{t-2} + \varepsilon_t \dots$$

$$y_t = \widetilde{\beta}_0 + \sum_{k=1}^{t} \beta^k \varepsilon_{t-k}$$

Wold Decomposition

Our $AR(1)$ can be written as a $MA(\infty)$ moving average process:

$$y_t = \widetilde{\beta}_0 + \sum_{k=1}^{\infty} \beta^k \varepsilon_{t-k}$$

- We call this an $MA(\infty)$ process because it represents a β_1 weighted moving average of all past realizations of ε_t
- Wold's Theorem tells us we can write any stationary time series as the sum of a deterministc and stochastic component.

Wold Decomposition

Consider the Wold Representation of the $AR(1)$

$$y_t = \widetilde{\beta}_0 + \sum_{k=1}^{\infty} \beta_1^k \varepsilon_{t-k}$$

Assume that $\varepsilon \sim N(0, \sigma^2)$ and IID

$$E[y_t] = \widetilde{\beta}_0$$

$$V[y_t] = \sum_{k=1}^{\infty} \beta_0^k Var(\varepsilon_{t-k}) \rightarrow \frac{1}{1-\beta_1}\sigma^2$$

- Here stationarity requires $\beta_1 \in (0, 1)$.
- Note that as $\beta_1 \rightarrow 1$ implies that the series no longer converges
- This is what is known as a unit root

Other Autoregressive Processes

We could also construct an $AR(2)$

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \varepsilon_t$$

Or an $AR(p)$:

$$y_t = \beta_0 + \sum_{k=1}^{p} \beta_k y_{t-k} + \varepsilon_t$$

Or an $ARMA(p, q)$ which adds moving average terms:

$$y_t = \beta_0 + \sum_{k=1}^{p} \beta_k y_{t-k} + \sum_{k=1}^{q} \theta_k \varepsilon_{t-k}$$

What About Lag Selection

Think about the $AR(p)$ model, which order lag do we choose?

$$y_t = \beta_0 + \sum_{k=1}^{p} \beta_k y_{t-k} + \varepsilon_t$$

- More lags $\longrightarrow$ Better Fit
- Potential for overfitting
- Bias vs. Variance tradeoff

Information Criteria

$$AIC(p) = \ln\left(\frac{SSR(p)}{T}\right) + (p+1)\frac{2}{T}$$
$$BIC(p) = \ln\left(\frac{SSR(p)}{T}\right) + (p+1)\frac{\ln T}{T}$$

The penalty is smaller for AIC than for BIC

- AIC estimates more lags (bigger p) than BIC
- AIC tends to overestimate p

There are other information criteria and ways to calculate.

$AR(p)$ Example: Auto-selecting

```
> ar(gdp$price)

Call:
ar(x = gdp$price)

Coefficients:
      1        2        3
 0.3461   0.1505  -0.0880

Order selected 3  sigma^2 estimated as  12.46
```

Autoregressive Distributed Lag Models

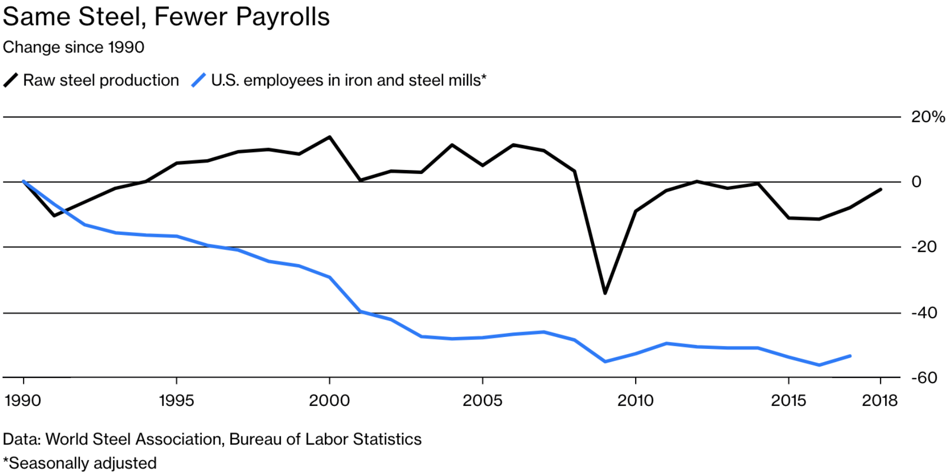
$ADL(p,r)$ models add the covariate X (and its lags). Usually contemporaneous X_t is excluded:

$$y_t = \beta_0 + \sum_{k=1}^{p} \beta_k y_{t-k} + \sum_{k=1}^{r} \theta_k X_{t-k} + \varepsilon_t$$

An important issue is Granger Causality

- This has nothing to do with actual causality
- Include $p > r$ lags of y_t. Does $(x_t, x_{t-1}, \ldots, x_{t-p})$ have any predictive value?
- Joint F-test of all coefficients on x_t lags

Steel Production and Employment



$ADL(3,3)$ Example

```
dt<-read.csv("steel.csv")
dt2<-ts(dt)
>summary(dynlm(output~L(output,1:3)+L(hours,1:3), data=dt2))

Residuals:
    Min      1Q  Median      3Q     Max
-34.162  -4.769   0.439   6.952  13.480

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)     196.56293   55.49461   3.542  0.00205 **
L(output, 1:3)1  -0.01371    0.29492  -0.046  0.96339
L(output, 1:3)2   0.01829    0.31067   0.059  0.95363
L(output, 1:3)3  -0.17356    0.21767  -0.797  0.43459
L(hours, 1:3)1    0.46844    0.92788   0.505  0.61918
L(hours, 1:3)2   -0.90532    1.33926  -0.676  0.50679
L(hours, 1:3)3   -0.20820    0.84409  -0.247  0.80769
---
Signif. codes:  0 ?***? 0.001 ?**? 0.01 ?*? 0.05 ?.? 0.1 ? ? 1

Residual standard error: 10.44 on 20 degrees of freedom
Multiple R-squared:  0.5985,        Adjusted R-squared:  0.478
F-statistic: 4.969 on 6 and 20 DF,  p-value: 0.002905
```

Granger Test

```
>grangertest(output~ hours, order=3,data=dt)
Granger causality test

Model 1: output ~ Lags(output, 1:3) + Lags(hours, 1:3)
Model 2: output ~ Lags(output, 1:3)
  Res.Df Df      F  Pr(>F)
1     20
2     23 -3 3.8094 0.02612 *
---
Signif. codes:  0 ?***? 0.001 ?**? 0.01 ?*? 0.05 ?.? 0.1 ? ? 1
```

Significant! Hours predict output.

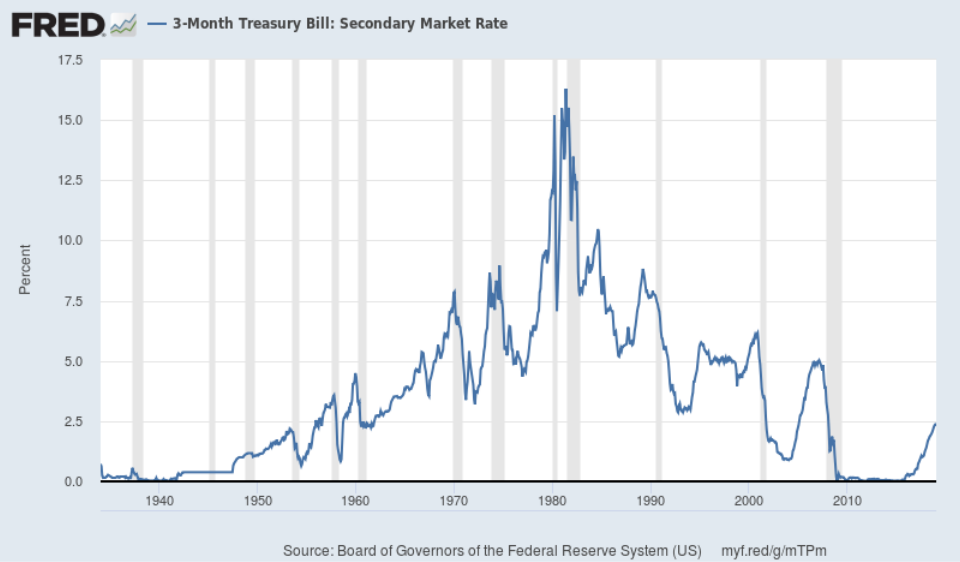
Granger Test: Other Direction

```
> grangertest(hours ~ output, order=3,data=dt)
Granger causality test

Model 1: hours ~ Lags(hours, 1:3) + Lags(output, 1:3)
Model 2: hours ~ Lags(hours, 1:3)
  Res.Df Df      F Pr(>F)
1     20
2     23 -3 1.8272 0.1747
```

Not significant! Output does not predict hours.

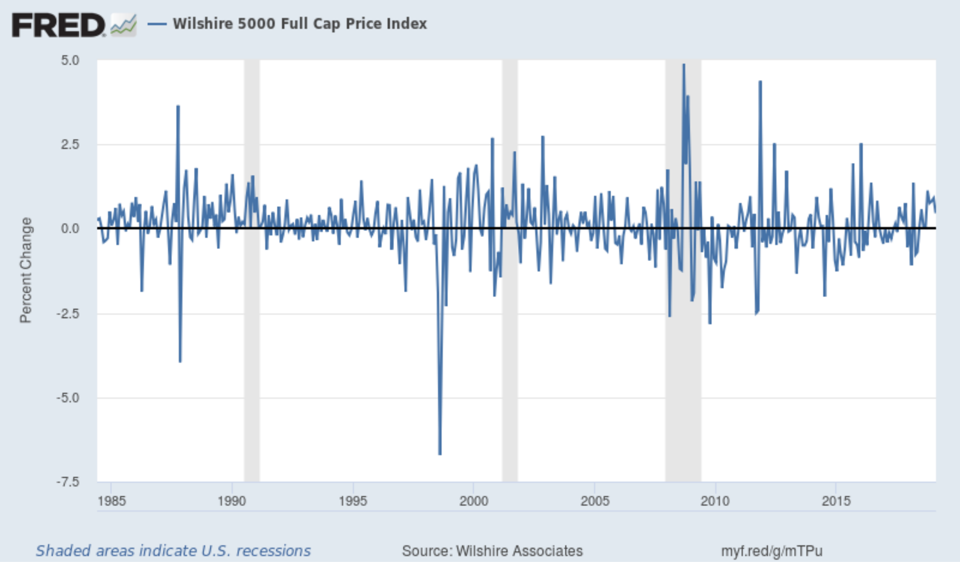
Trends

Which Series has a Trend?



Which Series has a Trend?

Which Series has a Trend?



Two Kinds of Trends

- Deterministic Trends: $y_t = a \cdot t + \epsilon_t$ or $y_t = a \cdot t + b \cdot t^2 + \epsilon_t$
- Stochastic Trend: random and time varying trend (see how this works later)
- Random Walk: $Y_t = Y_{t-1} + \varepsilon_t$

What is a random walk

$$Y_t = Y_{t-1} + \varepsilon_t, \quad E[\varepsilon_t] = 0, V[\varepsilon_t] = \sigma^2$$

- Best guess of tomorrow is today
- $E[y_{t+h}|y_t] = y_t$ for any t and h
- If Y_0 then $V(y_t) = t\sigma^2$

Generate Random Random Walks

```r
tibble(x = 1:1000, y = cumsum(rnorm(1000, mean = 0))) %>%
  ggplot(aes(x=x,y=y))+
  geom_point()+
  geom_line()
```

Adding Drift

We an easily add a drift term β_0

$$Y_t = Y_{t-1} + \beta_0 + \varepsilon_t$$

- $E[y_{t+h}|y_t] = y_t + h \cdot \beta_0$ for any t and h
- If Y_0 then $V(y_t) = t\sigma^2$

Log stock prices are roughly RWD

(stock returns are random but positive on average)

Where are we heading?

Suppose we have a stochastic (random walk) trend:

- We no longer satisfy stationarity
- We can run OLS but we can't trust the results (not even a little bit)
  - Recall $AR(1)$ has non-convergent series!
  - Coefficients are biased towards zero
  - Not asymptotically normal
- We are going to want to transform things to return to stationary case
- Easy for RW trend because Δy_t is stationary!

$$y_t = y_{t-1} + \varepsilon_t$$

$$\Delta y_t = \varepsilon_t$$

A Simple Example: $AR(1)$

We can think about RWD as a special case of $AR(1)$ with $\beta_1 = 1$

$$
\begin{aligned}
Y_t &= \beta_0 + \beta_1 Y_{t-1} + \varepsilon_t \quad AR(1) \\
Y_t &= \beta_0 + Y_{t-1} + \varepsilon_t \quad \text{RWD} \\
\Delta Y_t &= \beta_0 + \varepsilon_t
\end{aligned}
$$

We call the β_1 case unit root because $1 - \beta_1 z = 0$ has root $z = \frac{1}{\beta_1}$ so that β_1 when $z = 1$.

Harder Example: $AR(2)$

This case is more complicated

$$
\begin{aligned}
Y_t &= \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \varepsilon_t \\
&= \beta_0 + (\beta_1 + \beta_2) Y_{t-1} - \beta_2 Y_{t-1} + \beta_2 Y_{t-2} + \varepsilon_t \\
&= \beta_0 + (\beta_1 + \beta_2) Y_{t-1} - \beta_2 (Y_{t-1} - Y_{t-2}) + \varepsilon_t
\end{aligned}
$$

Now difference Y_{t-1}:

$$
Y_t - Y_{t-1} = \beta_0 + \underbrace{(\beta_1 + \beta_2 - 1)}_{\delta} Y_{t-1} - \beta_2 \underbrace{(Y_{t-1} - Y_{t-2})}_{\Delta Y_t} + \varepsilon_t
$$

$$
\Delta Y_t = \beta_0 + \delta Y_{t-1} - \beta_2 \Delta Y_t \varepsilon_t
$$

A Harder Example: $AR(2)$

What is a unit root now?

$$\Delta Y_t = \beta_0 + \delta Y_{t-1} - \beta_2 \Delta Y_t \varepsilon_t$$

- $1 - \beta_1 z - \beta_2 z^2 = 0$ a unit root implies that $\beta_1 + \beta_2 = 1$
- If there is a unit root then $\delta = 0$
  - We can use this to construct a test for a unit root
- If $AR(2)$ has a unit root, then write as an $AR(1)$ in first differences

$$\Delta Y_t = \beta_0 - \beta_2 \Delta Y_t \varepsilon_t$$

The General Case $AR(p)$

What is a unit root now?

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \cdot + \beta_p Y_{t-p} + \varepsilon_t$$

$$\Delta Y_t = \beta_0 + \Delta Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \gamma_2 \Delta Y_{t-2} + \cdots + \gamma_p \Delta Y_{t-p} + \varepsilon_t$$

With coefficients:

$$\begin{aligned}
\delta &= \beta_1 + \beta_2 + \cdots + \beta_p - 1 \\
\gamma_1 &= -(\beta_2 + \cdots + \beta_p) \\
\gamma_2 &= -(\beta_3 + \cdots + \beta_p) \\
\gamma_{p-1} &= -\beta_p
\end{aligned}$$

Detecting Trends

- Plot the Data: are there persistent long run movements?
- Run the Dickey-Fuller Test for unit roots

Dickey Fuller Test for $AR(1)$:

$$Y_t = \beta_0 + \beta_1 Y_{t_1} + \varepsilon_t$$

$$\Delta Y_t = \beta_0 + \delta Y_{t-1} + \mu \cdot t + \varepsilon_t$$

- $H_0 : \delta = 0$ vs $H_1 : \delta < 0$ (one sided test)
- The usual critical values for t-stats don't work (because at $\delta = 0$ things are non-normal).
- Software usually has adjusted critical values

Dickey Fuller Test

Which test do we want?

$$\Delta Y_t = \beta_0 + \delta Y_{t-1} + \mu \cdot t + \varepsilon_t$$

- Can include the trend $\mu \cdot t$ or not
- Leads to different critical values
- Depends on whether y_t is stationary around a trend or not
- Need to choose number of lags first

Dickey Fuller Test: Example

```
# convert to time-series
gdp2<-ts(gdp$price)
> tidy(dynlm(d(gdp2)~L(gdp2,1)+L(d(gdp2),2:4)))
# A tibble: 5 x 5
  term               estimate std.error statistic  p.value
  <chr>                 <dbl>     <dbl>     <dbl>    <dbl>
1 (Intercept)          2.19      0.280       7.80 1.40e-13
2 L(gdp2, 1)          -0.693     0.0593    -11.7  9.98e-26
3 L(d(gdp2), 2:4)2     0.128     0.0553      2.32 2.12e- 2
4 L(d(gdp2), 2:4)3     0.0786    0.0609      1.29 1.98e- 1
5 L(d(gdp2), 2:4)4     0.0683    0.0547      1.25 2.12e- 1
```

Dickey Fuller Test: Example

```
adf.test(gdp2, k=3)

    Augmented Dickey-Fuller Test

data:  gdp2
Dickey-Fuller = -8.1364, Lag order = 3, p-value = 0.01
alternative hypothesis: stationary
```

Spurious Regression/Correlation

Imagine we have two series each with a trend

$$y_t = a_0 + a_1 t + \varepsilon_t$$

$$x_t = b_0 + b_1 t + \mu_t$$

- Both are related to t but neither has anything to do with each other.
- Regression of x_t on y_t can produce very high R^2

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t$$

$$y_t = \beta_0 + \beta_1(b_0 + b_1 \cdot t + \mu_t) + \varepsilon_t$$

$$y_t = \underbrace{(\beta_0 + \beta_1 b_0)}_{\widetilde{\beta}} + \underbrace{\beta_1 b_1}_{\widetilde{\beta}} \cdot t + \underbrace{(\beta_1 \mu_t + \varepsilon_t)}_{\widetilde{\varepsilon}_t}$$

Spurious Regression/Correlation

- This is a huge mistake and people make it all of the time
- http://www.tylervigen.com/spurious-correlations
- This problem is insidious: it seems obvious and then you do it

Applications of Time Series

Moving Average Models

We might want a trend but one that isn't a straight line.

Enter the simple q Moving average (SMA):

$$Y_t = \frac{Y_{t-1} + Y_{t-2} + \cdot + Y_{t-m}}{m}$$

- The average age of the data is around $\frac{m+1}{2}$ periods.
- We are always behind what is happening at time t
- As we include more lags, we use more data, but we get further behind today.
- Gets plotted a lot on stock market prices, etc.

Moving Average: S&P 500 w/ MA(60)

Simple Exponential Smoothing (SES)

We might want to weight older observations less and more recent observations more. Think about $L_t = E[Y_{t+1}|Y_t]$ our forecast of Y_{t+1}:

$$L_t = \alpha Y_t + (1-\alpha)L_{t-1}$$
$$E[Y_{t+1}|Y_t]] = \alpha Y_t + (1-\alpha)\hat{Y}_t$$

Notice that $\varepsilon_t \equiv Y_t - E[Y_t|Y_{t_1}]$ so that

$$E[Y_{t+1}|Y_t]] = \alpha E[Y_t|Y_{t-1}] + \alpha\varepsilon_t$$

Rewriting as a moving average

$$E[Y_{y+1}|Y_t] = \alpha[Y_t + (1-\alpha)Y_{t-1} + (1-\alpha)^2 Y_{t-2} + (1-\alpha)^3 Y_{t-3} + \ldots]$$

Decomposing Trends and Seasonality

Given some time series data how should we start?

- Plot the series
- Try and `decompose` the series
  - Extract trends
  - Look for seasonality
  - Remainder should be random

Loading Alcohol Data :

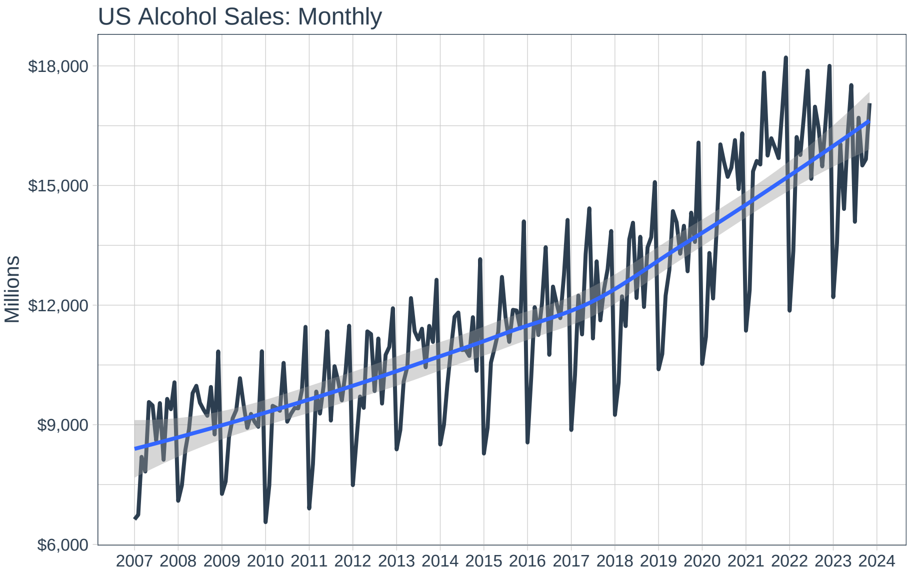
https://fred.stlouisfed.org/series/S4248SM144NCEN

```
alcohol_sales_tbl <- tq_get("S4248SM144NCEN",
                            get  = "economic.data",
                            from = "2007-01-01",to    = "2023-12-31")
                           # A tibble: 120 x 2
   date        price # note expenditure not prices!
   <date>      <int>
 1 2007-01-01  6627
 2 2007-02-01  6743
 3 2007-03-01  8195
 4 2007-04-01  7828
 5 2007-05-01  9570
 6 2007-06-01  9484
 7 2007-07-01  8608
```

Plotting Alcohol Data

```r
alcohol_sales_tbl %>%
    ggplot(aes(x = date, y = price)) +
    geom_line(size = 1, color = palette_light()[[1]]) +
    geom_smooth(method = "loess") +
    labs(title = "US Alcohol Sales: Monthly", x = "", y = "Millions") +
    scale_y_continuous(labels = scales::dollar) +
    scale_x_date(date_breaks = "1 year", date_labels = "%Y") +
    theme_tq()
```

Alcohol Example



Rearranging Alcohol Data

Notice the strong seasonal pattern (December and June)

```
> alcohol_sales_ts <- tk_ts(alcohol_sales_tbl, start = 2007, freq = 12, silent = TRUE)
       Jan   Feb   Mar   Apr   May   Jun   Jul   Aug   Sep   Oct   Nov   Dec
2007  6627  6743  8195  7828  9570  9484  8608  9543  8123  9649  9390 10065
2008  7093  7483  8365  8895  9794  9977  9553  9375  9225  9948  8758 10839
2009  7266  7578  8688  9162  9369 10167  9507  8923  9272  9075  8949 10843
2010  6558  7481  9475  9424  9351 10552  9077  9273  9420  9413  9866 11455
2011  6901  8014  9832  9281  9967 11344  9106 10469 10085  9612 10328 11483
2012  7486  8641  9709  9423 11342 11274  9845 11163  9532 10754 10953 11922
2013  8383  8870 10085 10462 12177 11342 11139 11409 10442 11479 11077 12636
2014  8506  9003  9991 10903 11709 11815 10875 10884 10725 11697 10353 13153
2015  8279  8926 10557 10933 11330 12708 11700 11079 11882 11865 11420 14100
2016  8556 10199 11949 11253 12046 13453 10755 12465 12038 11674 12761 14137
2017  8870 10251 12241 11266 13275 14428 11165 13098 11619 12386 12904 13859
2018  9248 10056 12221 11474 13650 14067 12178 13714 11954 13450 13706 15086
2019 10391 10776 12238 12879 14358 14076 13290 13990 12849 14318 13584 16076
2020 10524 11206 13308 12167 13925 16032 15598 15217 15449 16139 14911 16309
2021 11360 12380 15354 15617 15527 17832 15751 16185 15944 15687 16909 18211
2022 11862 13358 16216 15766 16755 17882 15168 16977 16430 15480 16718 18001
2023 12201 13552 16041 14412 16225 17519 14091 16699 15503 15660 17065
```

Rearranging Alcohol Data

Apply Error Trend Seasonal Decomposition (ETS) to data. These are not really interpretable on their own:

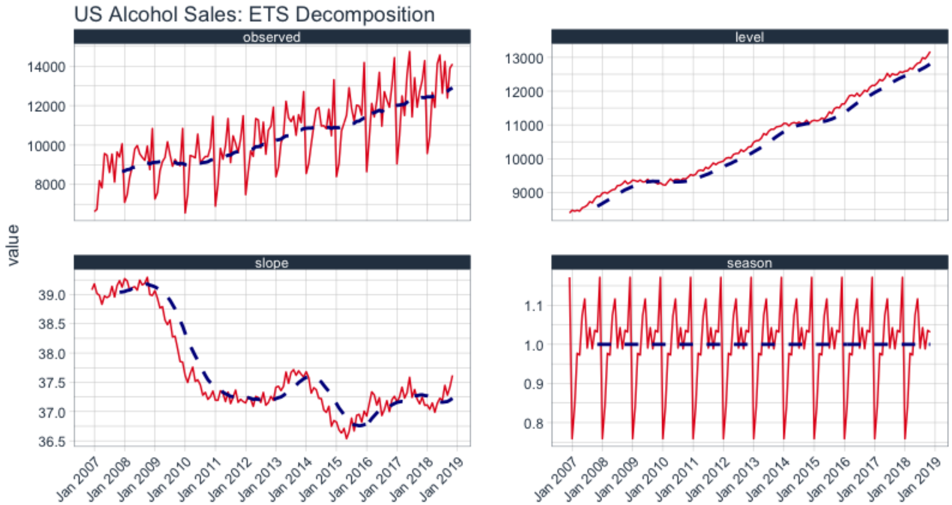
```
> fit_ets <- alcohol_sales_ts %>%
+     ets()
ETS(M,Ad,M)

Call:
 ets(y = .)

  Smoothing parameters:
    alpha = 0.0783
    beta  = 0.0772
    gamma = 0.0053
    phi   = 0.9511

  Initial states:
    l = 8387.3061
    b = 39.5634
    s = 1.1755 1.0241 1.041 0.9894 1.0455 0.9968
           1.1104 1.0675 0.9733 0.9743 0.8323 0.7699

  sigma:  0.0451

     AIC     AICc      BIC
2058.006 2064.778 2108.181
```

Rearranging Alcohol Data

Run the equivalent of `decompose` on the data:

```
> decomp_fit_ets <- sw_tidy_decomp(fit_ets)
> decomp_fit_ets
# A tibble: 121 x 5
   index          observed level slope season
   <S3: yearmon>     <dbl> <dbl> <dbl>  <dbl>
 1 Dec 2006             NA 8387.  39.6  1.18
 2 Jan 2007           6627 8439.  51.7  0.770
 3 Feb 2007           6743 8458.  19.4  0.832
 4 Mar 2007           8195 8471.  13.4  0.974
 5 Apr 2007           7828 8450. -21.3  0.973
 6 May 2007           9570 8471.  21.1  1.07
 7 Jun 2007           9484 8495.  23.9  1.11
 8 Jul 2007           8608 8527.  31.8  0.997
 9 Aug 2007           9543 8602.  74.2  1.05
10 Sep 2007           8123 8636.  34.9  0.989
```

ETS/decompose Example



ARIMA Models

Consider Auto-Regressive Integrated Moving Average $ARIMA(p, d, q)$

- Autoregressive p terms like $AR(p)$: lags of y_{t-p}
- Integrated d Differenced out unit roots
- Moving Average q include lags of forecast errors ϵ_{t-h}

ARIMA Models

Denote by (p, d, q)

- $(0, 0, 0) + c$ constant model
- $(0, 1, 0)$ RW
- $(0, 1, 0) + c$ RW w/ drift
- $(1, 0, 0)$ $y_t \sim y_{t-1}$
- $(1, 1, 0)$ $\Delta y_t \sim \Delta y_{t-1}$
- $(2, 1, 0)$ $\Delta y_t \sim \Delta y_{t-1} + \Delta y_{t-2}$
- $(0, 1, 1)$ SES model
- $(0, 1, 1) + c$ SES with constant trend

More Serious: X-13 ARIMA

Lots of government economic series are seasonally adjusted

- The Census uses X-13 software to seasonally adjust most series
- Also popular is Bank of Spain (SEATS) adjustment
- available in R package `seasonal`
- https://github.com/christophsax/seasonal/wiki/Examples-of-X-13ARIMA-SEATS-in-R

Next time: Panel Data

- Linear Model
- Serial Correlation
- Fixed Effects, Random Effects
- Dynamic Panel: Arellano Bond, etc.

Thanks!