2nd Group Assignment

Course director: Zoltán Rácz
5329: Inequality, Household Behavior, and the Macroeconomy

Instructions

- This assignment should be completed in groups of 2-3 students, and handed in through Canvas before 10 a.m. on the 23rd of April.
- This assignment contributes 10% to your final grade.
- You can hand in either an ipynb or a jl, and a pdf file. For full marks, you are expected to carefully motivate your answers. Codes should be easy to follow and well-commented. The connection of verbal answers with corresponding codes (when relevant) should be made clear, especially if they are in separate files.

Tasks

1 Risk aversion and precautionary premium

In the lecture, we saw that in the case of the CRRA utility function, risk aversion is lower than prudence. In this exercise, you need to understand the economic intuition of this statement, through an example.

Utility comes from consumption now and in the next period. Income now is 1, but income in the next period (y) is uncertain, with mean 1, i.e. $\mathbb{E}[y] = 1$. This is because your salary depends on the performance of your firm. You can save, but the interest rate is 0. Per-period utility function is of the CRRA kind, and there is no impatience. Therefore, you need to maximize

$$u(c_0) + \mathbb{E}[u(c_1)]$$

where $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$, subject to the constraints

$$c_0 = 1 - s$$
$$c_1 = y + s$$

1. Suppose you figure out that your optimal saving level is s^*, therefore you expect $\mathbb{E}[u(y+s^*)]$ utility in the next period. However, your boss offers you an insurance policy: instead of your risky future income (y), you can get $1-\tau$ for sure (τ can be interpreted as the price of this insurance). From now on suppose that
$$u(1-\tau+s^*) > \mathbb{E}[u(y+s^*)]$$
holds. Should you take this insurance?

2. What is the relation between τ and the risk premium?

3. If you take the insurance, how should you change your optimal saving level relative to s^*? Increase, decrease, or it is impossible to tell without knowing more?

2 Consumption and saving over the life-cycle

Consider the file $30\_ageprofile.jl$. It contains the same code as $21\_borrowinglimits.jl$, but now we introduce a semi-realistic age-profile for income, i.e. for individual i when of age t, income equals $f_t \cdot y_{it}$ where f_t is a deterministic age-dependent component and y_{it} is the random component. From now on we interpret timing in the following way: First period corresponds to age 25 individuals, while $T = 60$ corresponds to 84-year-old individuals. As in the data, the age-profile of income grows for the young, flattens out around age 50 and drops at retirement (age 65), after which it stays constant. If you want to see where the code changed due to the income profile, you should simply ctrl/command+f for fs. I also included the *solve_simul* function from Monday's class in the script.

Throughout this exercise, we consider the following three parameter combinations:

- Model A: $EconPars(ys = [1.0], pys = [1.0], bl = 7.0, \beta = 0.96)$
- Model B: $EconPars(ys = [1.0], pys = [1.0], bl = 0.0, \beta = 0.96)$
- Model C: $EconPars(ys = [0.8, 1.2], pys = [0.5, 0.5], bl = 0.0, \beta = 0.96)$

and I recommended solving the models with $NumPars(max\_coh = 10.0, N\_coh = 500)$.

1. After loading $30\_ageprofile.jl$ with *include*, simulate the average paths of income, consumption and wealth over age in the three economies. (You can use directly the *solve_simul* function for this.) Explain the economic intuition behind the patterns you observe. In particular, discuss how the introduction of borrowing constraints and uncertainty affect the results. In addition, explain how the new income profile changes findings relative to the flat income profile used in class.

2. Simulate all three models, and for each, perform the following exercise: for each simulated individual, compute their correlation between consumption and income changes over their working age (i.e. ignoring everything after period 40). How do the distributions of the obtained correlations compare across the three models? Discuss the economic intuition.

3. Comparing the three models regarding

   - the relation between average income and consumption paths over age;
   - and the individual-specific correlations you computed in point 2,

   which of the three models seems to fit best the stylized facts in the data?

Note: Since the codes used in Task 2 and 3 are similar enough to mix up, but work differently, I highly recommend restarting your Julia between producing results for the two tasks.

3 Wealth tax and inequality

Consider the following modification of the infinite-horizon model considered during the Wednesday lecture:

$$\mathbb{E}\Big[\sum_{t=0}^{\infty} \beta^t u(c_{i,t})\Big]$$

subject to

$$\begin{aligned} a_{i,t} &= (1+r)(1-\tau)a_{i,t-1} + b + y_{i,t} - c_{i,t} \\ a_{i,t} &\geq 0 \end{aligned}$$

where τ is a tax rate on wealth and b is a benefit. The idea is that the state collects tax from taxing savings that it distributes back to the population. This of course makes sense only if the budget is balanced, i.e.

$$\sum_{i=1}^{N} \tau(1+r)a_{i,t-1} = Nb$$

or

$$\tau(1+r)\sum_{i=1}^{N}\frac{a_{i,t-1}}{N} = b \tag{1}$$

where we are summing over individuals at a given time. Your first task is to write a code finding the b for a given τ, which balances the budget (this is not obvious, since decisions, and hence the distribution of as depend on b). Then you need to answer some questions on economics.

1. Write a code that (keeping the defaults for all other economic parameters) computes the budget-balancing level of benefit for any given tax rate τ. You might want to make a copy of 31_$infinite_horizon.jl$ and amend it at the right places[1], following the steps below:

   (a) Add a new field τ in EconPars. Give any default (for example 0 makes sense).

   (b) Make b (a *Real*) a function argument to the function *solve*, since we want to be able to solve the model for any level of b. We don't put b in EconPars as it is not an independent parameter of the model. Next, we amend *solve* to give the correct solution of the model with τ (included in *ep*) and b given as a function argument. In particular, τ and b affect the budget constraint. Therefore update all lines relying on that (i.e. lines where cash-on-hand in the next period is computed). In addition, τ should pop up somewhere else in the first-order condition. Not to miss it, you might want to derive the first-order conditions of the model on paper.

   (c) Add b as a function input to the *simulate* function and update all the lines with b and τ where it is necessary.

   (d) Write a function *budget_balance* that takes inputs τ and b. Inside the function, the following things could happen:

   - Define an EconPars structure such that the given τ overwrites the default, but keep defaults for the other parameters
   - Define a default NumPars structure.
   - Solve the model. Simulate it for 200 periods and 10000 individuals. Compute average wealth across all individuals in the last 100 simulated time periods.
   - Return the per-capita budget deficit based on equation (1).

   (e) Write a function taking τ as input and returning

$$find\_zero(b- > budget\_balance(\tau, b), (m, M), atol = 0.001),$$

[1] Among other things, τ and b affect the natural borrowing limit. Since we will assume no borrowing in this exercise and hence the natural borrowing limit is irrelevant, feel free to ignore this issue.

where m and M are sensible bounds for looking for the right b value. (Setting atol makes the algorithm stop without striving for great accuracy. Due to sampling errors, convergence would be tricky with a much lower tolerance.)

2. Choose a range/vector of various candidates for τ between 0 and 1. (Computing results for many τs might take long, but use at least 10 candidates so that you can see patterns. Using more points close to 0 might make sense if more action seems to be going on there, based on your finding below.) Compute the corresponding b for each of these τs and comment on your findings.

3. For the τ candidates in 2, compute average wealth, the Gini index of wealth, and the share of wealth held by the richest 1% in the implied steady-state distribution. (To fight sampling errors, you might want to compute these statistics over all individuals AND over many time periods after the burn-in period.) Discuss.

4. For the τ candidates in 2, compute the value of holding 1 unit of cash-on-hand. Comment on your results.