Handout 6: Market Equilibrium with Perfect Competition

1 Introduction

In this handout we will show how to go from the individual firm cost minimization problem to the individual firm supply and then to the aggregate market supply. We will assume that firms behave as price takers: for a given level of price, firms choose how much to produce without considering that their production can change equilibrium prices. This is a reasonable assumption in dispersed markets, where each firm individually is small and does not significantly affect market supply. Given the market supply that comes from the cost minimization problem, we will find the market equilibrium as we did in Handout 1, by equating quantity supplied and demanded.

The two examples covered here will bring together all that has been seen in the course so far. We start with the supply and demand framework (Handout 1). We then explore the problem of a consumer in Handout 3. The consumer choice problem (Handout 3) is what gives us the market demand. We then explore the problem of a firm (Handout 5). The cost minimization problem of a firm will give the market supply. All together, we *microfound* the decisions of buyers and sellers in the market to generate aggregate demand/supply functions and compute equilibrium prices/quantities. These are powerful tools, since we can analyze various shocks and policies (taxes, price floors, change on the price of substitutes, removal of entry barriers for firms etc..) quantitatively - since we know where the demand comes from.

2 Market Supply and Equilibrium

An economy is composed of $N = 4$ identical firms. In the short run, no other firms can enter this market. Each firm has a production function that combines capital K and labor L given by

$$F(K, L) = (K-1)^{\frac{1}{4}} L^{\frac{1}{4}} \tag{1}$$

and face input prices of $w = 1$ and $r = 1$. Demand is given by

$$Q_D = 12 - 2p$$

1. Find the cost function $C(Q)$ of each individual firm if they can choose both K and L.

2. Compute the short run market supply.

3. Compute the short run market equilibrium.

4. Compute the quantity produced and profit of each firm in the short run equilibrium.

5. Do you expect to see entry/exit in the long run? What will be the long run number of firms in this market?

Solution.

1. The firm cost minimization problem is given by:

$$\min_{L,K} \quad wL + rK \quad s.t. \quad F(L, K) = Q$$

In our case, replacing $w = r = 1$ and F

$$\min_{L,K} \quad L + K \quad s.t. \quad (K-1)^{\frac{1}{4}} L^{\frac{1}{4}} = Q$$

which states that the firm is minimizing the total cost of producing Q units by choosing the optimal combination of K and L. At the optimum, it must be that the marginal rate of transformation between capital and labor is equal to the absolute value of the ratio of wages and rental rate of capital. Intuitively, this means that the ratio at which the firm is willing to trade capital for labor ($MRTS$) is equal to the ratio the market is willing to trade one for the other (ratio of the prices of each input). This is the analogous of the firm for the condition that the marginal rate of substitution between two goods is equal to the ratio of prices for the consumer. Mathematically:

$$MRTS = -\frac{\frac{1}{4}L^{-3/4}(K-1)^{1/4}}{\frac{1}{4}L^{1/4}(K-1)^{-3/4}} = -\frac{w}{r} = -1$$
$$\Rightarrow K - 1 = L$$

We can then substitute this into our quantity constraint $F(L, K) = Q$

$$L^{\frac{1}{4}} L^{\frac{1}{4}} = Q \Rightarrow L^*(Q) = Q^2$$

Replacing back at the $MRTS = -1$ condition:

$$K^*(Q) = Q^2 + 1$$

Substituting these factor demands in the cost function gives the cost function:

$$C(Q) = wL^*(Q) + rK^*(Q) = 2Q^2 + 1$$

This is the cost curve for a firm to produce Q units. Note that the +1 is not a fixed cost in this production function - it comes from the fact that you need *at least* one unit of capital to produce. If the firm decides to shut down, however, it does not have to pay this unit of capital - so this is not characterized as a fixed cost. To compute the market supply, we will first compute the supply of each individual firm and them sum them horizontally (Handout 1). Given a price p, Firms maximize (economic) profits:

$$\max_Q pQ - C(Q) \Rightarrow p = MC(Q)$$

where the $MC(Q)$ marginal cost curve is the derivative of the total cost curve

$$MC(q) = TC'(q) = 4Q$$

So the profit maximizing condition of $p = MC(Q)$ give us:

$$p = 4Q \Rightarrow Q = \frac{p}{4}$$

Is this the end of it? No! $p = MC(Q)$ gives the maximum level of profits. However, it can be that this level is not enough to cover variable costs of production (recall that fixed costs do not influence short run decisions of firms, since they are fixed). In particular, the firm will shut down in the short run ($Q = 0$) if price is below the minimum average variable cost - producing in the short run is not profitable even without taking into account the fixed costs. To calculate minimum AVC, we can either use calculus to find the minimum or remember that this is the point at which MC=AVC.

$$MC(Q) = AVC(Q) = \frac{C(Q)}{Q} = 4Q = \frac{2Q^2+1}{Q} \Rightarrow Q_{AVC} = \frac{1}{\sqrt{2}}$$

This is the quantity where AVC attains its minimum. The minimum AVC (i.e. ATC at this Q_{AVC}) is the same as marginal cost at this $Q = \frac{1}{\sqrt{2}}$ (that is, $2^{-\frac{1}{2}}$) So quantity supplied is zero when

$$\frac{p}{4} < 2^{-\frac{1}{2}} \Rightarrow p < 2^{\frac{3}{2}}$$

Based on $MC = p$ and $Q = 0$ when $p < \min(AVC)$ get a supply function - quantity supplied at any price. Our firm-level supply function is therefore

$$Q(p) = \begin{cases} \frac{p}{4} & \text{if } p \geq 2^{\frac{3}{2}} \\ 0 & \text{otherwise} \end{cases}$$

Add up those supply curves across identical $N = 4$ firms in market, we get the market supply curve:

$$Q_S(p) = N \cdot Q_S(p) = \begin{cases} p & \text{if } p \geq 2^{-\frac{1}{2}} \\ 0 & \text{otherwise} \end{cases}$$

2. Market-clearing equilibrium price is (assuming $p_M > 2^{\frac{3}{2}}$) such that supply is equal to demand (Handout 1), that is

$$Q_S(p_M) = Q_D(p_M) \Rightarrow p_M = 12 - 2p_M \Rightarrow p_M = 4$$

So total quantity is $Q_M = 4$. Note that $p_M = 4 > 2^{\frac{3}{2}} \approx 2.83$, so our assumption that demand and supply intercept above $2^{\frac{3}{2}}$ is satisfied. This the market equilibrium.

3. The total quantity produced in the market is $Q_M = 4$, so each of the 4 firms is producing $Q = 1$ unit. The total profit of a firm is total revenue (price times quantity) minus costs:

$$\pi = 4 \times 1 - C(1) = 4 - 3 = 1 > 0$$

4. If there are positive profits, other firms are attracted to this market and enter. Here firms make profits of $\pi = 1 > 0$, so we have *entry* in the long run. In the long run, entry will drive profits

down until the marginal entrant has zero profits. The last firm to enter has to be making zero profits (i.e., be indifferent between entering or not). As all firms are identical in this example, all firms are making zero profits in the long run. In particular, each firm produces at the minimum of the ATC curve - and we have a perfectly elastic supply: prices in the long run will be given by the minimum ATC. If $p > \min(ATC)$, firms would enter and drive prices down. If $p < \min(ATC)$, firms would exit and drive prices up. In our problem, there are no fixed costs, so ATC = AVC. We know that the minimum ATC is $2^{-\frac{1}{2}}$. Therefore entry pushes $p \to 2^{-\frac{1}{2}}$. At this price, each firm produces

$$Q(2^{-\frac{1}{2}}) = \frac{1}{\sqrt{2}}$$

At this price, total demand is $Q_d(\sqrt{8}) = 12 - 2 \cdot 2^{\frac{3}{2}} = 12 - 2^{\frac{5}{2}}$. Since each firm produces $\frac{1}{\sqrt{2}}$ unit at this price, we must have that the number of firms is the total equilibrium quantity over how much each firm produces, that is

$$N^{LR} = \frac{12 - 2^{\frac{5}{2}}}{\frac{1}{\sqrt{2}}}$$

in the long run equilibrium. Note that the number of firms in here is not an integer. This is an *abstraction* of how the model applies to reality, just as in the consumer one could consume fractional amounts of goods. In reality, profits in the long run can still be > 0 for several reasons:

- barriers to entry
- firms differ
- input costs rise with output

Overall, however, we expect long run supply to be more elastic than short run, since markets have the extra dimension of entry and exit of firms.

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3 Consumer Choice, Cost Minimization and Market Equilibrium

Suppose there are 200 identical firms which use only labor L to produce output of good A with the following production function

$$F(L) = L^{\frac{1}{2}}$$

Suppose that wages are $w = 1$.

Additionally, suppose there are 1,000 consumers with utilities over goods A and B given by

$$U(A, B) = AB$$

where A and B are the quantities consumed of each good. The price of goods A and B are, respectively, given by P_A and P_B. Each consumer has 80 dollars in income to spend between A and B.

1. Find the market supply.

2. Find the market demand as a function of P_A and P_B.

3. Find the market equilibrium.

Solution.

1. To find the market supply, we must first find the supply of each individual firm. To produce Q units, each firm must use:

   $$L(Q) = Q^2$$

   units of labor. This comes from inverting the production function $F(L) = L^{\frac{1}{2}}$. Therefore, the total cost is given by

   $$C(Q) = wL(Q) = Q^2$$

   since $w = 1$ and the firms in this economy use only labor for production. The maximization problem of each firm of good A is given by

   $$\max_{Q} P_A Q - C(Q)$$

Taking the derivative with respect to Q and equating it to zero gives that prices are equal to marginal cost:

$$P_A = MC(Q) \Rightarrow P_A = 2Q$$

Therefore, the supply of each individual firm is given by: $Q = P_A/2$.Note that $P_A > \min(AVC)$ for sure in this economy, so we do not have to be concerned with firms shutdown in the short run. To see this, note that the $AVC = C(Q)/Q = Q$ is a straight line from the origin, so $\min(AVC) = 0$.

The market supply Q_S can be computed by summing the supply for each firm horizontally (i.e., in quantities) - see Handout 1. As we have 200 firms:

$$Q_S(P_A) = 200Q \Rightarrow Q_S(P_A) = 100P_A$$

2. To find the market demand, we must first find individual demands. The problem of each consumer is

$$\max_{A,B} AB \text{ s.t } P_A A + P_B B = 80$$

To find the optimal choice of A and B, we use two conditions: (i) the MRS= ratio of prices (the tangency of the indifference curve with the budget constraint) and (ii) the optimal bundle must be on the budget constraint itself. For more details, see the Handout 3. For each consumer:

$$MRS = -\frac{MU_A}{MU_B} = -\frac{B}{A}$$

Therefore,

$$MRS = -\frac{P_A}{P_B} \Rightarrow AP_A = BP_B$$

Replacing in the constraint that $P_A A + P_B B = 80$

$$2P_B B = 80 \Rightarrow B = \frac{40}{P_B}$$

Therefore

$$A = \frac{BP_B}{P_A} = \frac{40P_B}{P_B P_A} \Rightarrow A = \frac{40}{P_A}$$

The demand for A for each individual consumer is given by $\frac{40}{P_A}$. The total demand in the market

Q_D can be obtained by summing the quantity each consumer demands:

$$Q_D = 1000 \times \frac{40}{P_A} \Rightarrow Q_D = \frac{40,000}{P_A}$$

For these preferences, the demand is not a function of P_B. This is a characteristic of these specific preferences, and not true in general. In general, Q_D will be a function of both P_A and P_B.

3. In the equilibrium, P_A is such that $Q_S = Q_D$. Therefore:

$$100P_A = \frac{40,000}{P_A} \Rightarrow P_A = 20, Q_A = 2000$$

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