The Single-Person Decision Problem

Martin Gregor
martin.gregor AT fsv.cuni.cz

JEB064 Game Theory and Applications

Decision problem

- Actions: the alternatives from which the decision-maker chooses
- Outcomes: consequences from the actions
- Preferences: preferences over outcomes

Decision under certainty

Outcomes

- a *finite* set of outcomes X

Preferences over (certain) outcomes

- $\succsim$ is a weak preference relation of a decision maker on X
- $x \succsim y$: for $x \in X$ and $y \in X$, 'x is at least as good as y'
- $\succsim$ is a binary relation[1] that induces:
  - strict preference relation $\succ$

$$x \succ y \Leftrightarrow x \succsim y \text{ but not } y \succsim x$$

  - indifference preference relation $\sim$

$$x \sim y \Leftrightarrow x \succsim y \text{ and } y \succsim x$$

[1] A binary relation on a set X is a collection of ordered pairs of elements of X.

Consistency/Rationality

$\succsim$ is consistent/rational if it possesses the following two properties:

1. completeness: for all $x \in X$ and $y \in X$

$$x \succsim y \text{ or } y \succsim x$$

2. transitivity: for all $x \in X$, $y \in X$ and $z \in X$

$$x \succsim y \text{ and } y \succsim z \Rightarrow x \succsim z$$

Consistency implies existence of a (payoff) function (over outcomes) $u : X \to \mathbb{R}$ such that $u(x) \geq u(y)$ if and only if $x \succsim y$.

Let $f : A \to X$ be mapping from actions to outcomes. Then, a payoff function over actions, $v : A \to \mathbb{R}$, is $v(a) \equiv u(f(a))$.

Homo oeconomicus under certainty

- consistency of preferences ($\succsim$)
  - preferences may be incomplete when the outcomes have very little in common
  - e.g., is it worse to have military coup in Barma or lose 100 CZK?
  - ! preferences are not necessary selfish
- payoff-maximization
  - A player facing a decision problem with a payoff function $v(\cdot)$ over actions is payoff-maximizing if he chooses an action $a \in A$ that maximizes his payoff. That is, $a^* \in A$ is chosen only if $v(a^*) \geq v(a)$ for all $a \in A$.

Is there a difference if preferences are also over actions, not only over outcomes?

Rational decision under certainty

How to interpret payoff-maximization?

- Knowledge of (i) all possible actions, A, (ii) all possible outcomes, X, (iii) exactly how each action affects which outcome will materialize (knowledge of mapping from A to X).
- No cost of optimization or constraint to optimization.
- Alternatively, some learning process that approximates the optimal solution.

Of course, not all of us are cool-blooded like Sherlock Holmes.

- Errors in preferences
- Errors in judgment
- Errors in behavior

Errors in preferences

Suppose your preferences are not transitive. Take outcomes that generate a cycle:

$$x \succ y \succ z \succ x$$

You are vulnerable to a 'money pump' by an arbitrageur (Dutch books).
Suppose your preferences over money are separable from preferences over x, y, z.

- You will pay $p_{zx} > 0$ for changing x to z.
- You will pay $p_{yz} > 0$ for changing z to y.
- You will pay $p_{xy} > 0$ for changing y to x.
- In total, you will pay $p_{zx} + p_{yz} + p_{xy} > 0$ for nothing.

You can protect against money-pumping by self-control but it still remains unclear how you should optimally impose self-control in the absence of 'consistent objectives'.

Errors in judgment

- Conjuction fallacy
  - Linda problem: Linda is a student deeply concerned with discrimination and social justice.
  - Is Linda more likely a bank teller or a bank teller and an active feminist?
  - But every feminist bank teller is a bank teller.
  - Conjuction fallacy is when specific conditions are seen as more probable than general conditions.
  - Why? Linda problem violates conversational maxims in that people assume that the question obeys the *maxim of relevance*.
- Anchoring bias
  - Survey respondents observed a roulette wheel that had been fixed to stop at either 65 or 10.
  - They were asked to estimate the percentage of UN countries located in Africa.
  - The respondents were influenced by irrelevant numbers.
  - Anchoring is a cognitive bias where an individual depends too heavily on an initial piece of information offered (anchor) to make subsequent judgments.
  - Why? We typically assume that the *initial information provided is relevant*.
  - Anchoring can be used in marketing and pricing.

See also: Wikipedia's list of cognitive errors

Framing (Tadelis 2013, p. 32)

Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people.

- Program A will save 200 lives.
- Program B will save no-one with probability $\frac{2}{3}$ and 600 lives with probability $\frac{1}{3}$.
- Which program do you prefer?

Now consider another choice.

- Under Program C, 400 people die.
- Under Program D, no-one dies with probability $\frac{1}{3}$ and 600 people die with probability $\frac{2}{3}$.
- Which program do you prefer?

Most physicians preferred A over B and D over C (Kahneman and Tversky, 1981).

Decision under uncertainty (Mas-Colell, Whinston, Green 1995)

Uncertain outcomes (lotteries)

- outcomes in X indexed $n = 1, \dots, N$
- objectively known probability of outcome n: $p_n \in [0, 1]$
- a simple lottery L:

  A list $L = (p_1, \dots, p_N)$ such that $p_n \geq 0$ for all n and $\sum_n p_n = 1$.
- the set of all simple lotteries is $\mathcal{L}$ (simplex with $N - 1$ dimensions)

Preferences

- $\succsim$ is a weak preference relation of a decision maker on $\mathcal{L}$
- consistency/rationality (completeness, transitivity) is assumed

In contrast to decision under certainty, we also assume continuity.

- Continuity: For all $L' \in \mathcal{L}$, the sets $\{L : L \succsim L'\}$ and $\{L : L' \succsim L\}$ are closed sets.

Lexicographic preferences

- Lexicographic preferences mean that individuals disregard 'second-order' considerations.
- For instance: survive first, follow Twitter second.

Example

- Parents: 1 EUR $\succ$ 0 EUR $\succ$ 'loss of a child'.
- By continuity of preferences, there exists $\alpha > 0$ such that $(1-\alpha, 0, \alpha) \succ (0, 1, 0)$.
- But a typical parent tells you that $(0, 1, 0) \succ (1-\alpha, 0, \alpha)$ for any $\alpha \in [0, 1]$.
- However, when a benefit increases far above 1 EUR, $(1-\alpha, 0, \alpha) \succ (0, 1, 0)$ for some $\alpha > 0$.
- Parents travel by car with kids instead of staying at home all day long.
- Parents don't maximize survival of kids at 'any cost'. Only avoid any risk when the benefit is very small.

Consistency and continuity (of preferences over lotteries) implies existence of a function $U : \mathcal{L} \to \mathbb{R}$ such that

$$L \succsim L' \Leftrightarrow U(L) \geq U(L').$$

To compare:

- For the existence of a payoff function under certainty (a finite set of outcomes X), we impose consistency.
- For the existence of a payoff function under uncertainty (an infinite set of lotteries $\mathcal{L}$), we impose consistency and continuity.

In addition, consider the following property of preferences:

- independence: For any $L, L', L'' \in \mathcal{L}$ and $\alpha \in (0,1)$:

$$L \succsim L' \Leftrightarrow \alpha L + (1-\alpha)L'' \succsim \alpha L' + (1-\alpha)L''$$

- intuition: If a third lottery may occur instead of the two lotteries, its possible occurrence doesn't affect the preference ordering of the two lotteries.

! Lottery L'' occurs *instead* of lottery L or L'. The consumer choosing between L and L' doesn't consume L *with* L'' or L' with L'', but only L'' *instead* of L or L'.

- This puzzled even Paul Samuelson (Moscati, 2016):

> I am simply confused... the assumption would impose an arbitrary "straight-jacket" on individual preferences over risky alternatives.... The postulate was a "gratuitously-arbitrary special-implausible hypothesis."

- After discussions with Friedman, Savage and Baumol:

> ... now I must eat my words. As you know I hate to change my mind, but I hate worse to hold wrong views, and so I have no choice.

Definition: A compound lottery is $(L_1, \dots, L_k; \alpha_1, \dots, \alpha_k)$.

- For K simple lotteries $L_k = (p_1^k, \dots, p_N^k)$ and probabilities $\alpha_k \geq 0$ with $\sum_k \alpha_k = 1$, the compound lottery yields a simple lottery L_k with probability α_k.
- A compound lottery generates a corresponding reduced (simple) lottery with

$$p_n = \alpha_1 p_n^1 + \cdots + \alpha_K p_n^K.$$

- That is, we obtain it by vector addition $L = \alpha_1 L_1 + \cdots + \alpha_K L_K$.
- For preferences over uncertain outcomes, it is irrelevant if a lottery is simple or compound, as long as the reduced lottery is identical to the simple lottery.

Now look at *linear* U functions.

- Utility function $U : \mathcal{L} \to \mathbb{R}$ is *linear* if and only if

$$U\left(\sum_{k=1}^{K} \alpha_k L_k\right) = \sum_{k=1}^{K} \alpha_k U(L_k)$$

  for any K lotteries $L_k \in \mathcal{L}$, and probabilities $(\alpha_1, \ldots, \alpha_K) \geq 0, \sum_k \alpha_k = 1$.
- Utility function $U : \mathcal{L} \to \mathbb{R}$ has an *expected utility form* if and only if there exists a vector $(u_1, \ldots, u_N) \in \mathbb{R}^N$ such that for every $L \in \mathcal{L}$

$$U(L) = p_1 u_1 + \cdots + p_N u_N.$$

- u_n is called Bernoulli utility of the outcome n.

The two properties are equivalent.

Expected utility theorem

Consider a rational and continuous preference relation $\succsim$ on $\mathcal{L}$ that satisfies independence. Then, $\succsim$ is represented by a utility function U with the expected utility form (i.e., by a linear utility function U).

Moving from preferences over outcomes to preferences over actions is simple.

- Let $g : A \to \mathcal{L}$ be mapping from actions to lotteries. Then, the expected payoff function over actions, $V : A \to \mathbb{R}$, is $V(a) \equiv U(g(a))$.

Rational decision under *uncertainty*

- consistency, continuity and independence of preferences over lotteries
- payoff-maximization: A player facing a decision problem with an expected payoff function $V(\cdot)$ over actions is payoff-maximizing if he chooses an action $a \in A$ that maximizes his *expected* payoff/utility. That is, $a^* \in A$ is chosen only if $V(a^*) \geq V(a)$ for all $a \in A$.

Application: Retrieving payoffs from choices

How to construct payoff (utility) functions under certainty?

- Suppose a homo economicus chooses $x \succ y \succ z$.
- If we analyze only decisions under certainty, we may impose any $u(\cdot)$ such that $u(x) > u(y) > u(z)$.
- These payoffs however do *not* represent decisions under uncertainty.

If we want to analyze decisions under uncertainty, we need preference over lotteries.

Application: Retrieving payoffs from choices

How to construct payoff (utility) functions under uncertainty?

- We use that preferences are invariant to adding a constant $c \in R$ and multiplying Bernoulli utilities by any $\beta > 0$.
- Formally, $\sum_i p_i u_i \geq \sum_i q_i u_i$ is equivalent to $\sum_i p_i(c + \beta u_i) \geq \sum_i q_i(c + \beta u_i)$.
- Therefore, we may impose $u_y = 0$.
- We will ask homo economicus to report $\alpha \in (0, 1)$ such that he/she is indifferent between a lottery $(\alpha, 0, 1 - \alpha)$ over (x, y, z) and a (degenerate) lottery $(0, 1, 0)$:

$$(\alpha, 0, 1 - \alpha) \sim (0, 1, 0)$$

- Then, (Bernoulli) payoffs must satisfy the constraint:

$$\alpha u_x + (1 - \alpha) u_z = u_y = 0$$

- A solution $u_x - u_z = 1$ is $(u_x, u_y, u_z) = (1 - \alpha, 0, -\alpha)$.
- By adding a constant and multiplying by a positive $\beta > 0$, we have other solutions, such as $(u_x, u_y, u_z) = (1, \alpha, 0)$.

With Bernoulli payoffs, we can predict any choice of homo economicus under uncertainty. Of course, our predictions hold only to the extent that consistency, independence and payoff-maximization hold.

What if independence is violated?

Such agent is potentially vulnerable to 'money pumping' (a collection of bets/trades that leaves one party strictly better off and the other party strictly worse off).

- 3 simple assets: $1BTC \sim 1500ETH$, $1BTC \succ 15AMZN$
- In extreme, suppose independence of preference $1BTC \succ 15AMZN$ is violated for some α such that complex assets

$$\alpha 15AMZN + (1-\alpha)1500ETH \succ \alpha 1BTC + (1-\alpha)1500ETH \sim 1BTC$$

- Suppose the agent initially owns the asset $1BTC$.
- A trader offers a complex asset = a compound lottery over the other assets $\alpha 15AMZN + (1-\alpha)15000ETH$.
- The individual pays a fee.
- After realization of uncertainty in the compound lottery, the trader offers the asset $1BTC$.
- The trader charges zero fee if the agent holds $1500ETH$, but charges a positive fee of the agent holds $15AMZN$.
- The individual ends with the initial asset $1BTC$ but loses money.

Violations of Expected Utility Theory 1 (Allais Paradox)

- Take certain outcomes: nothing, 100 million EUR, 500 million EUR
- Do you prefer lottery A: $(0, 1, 0)$ or lottery B: $(0.01, 0.89, 0.1)$?
- Do you prefer lottery C: $(0.89, 0.11, 0)$ or lottery D: $(0.9, 0, 0.1)$?

Most people say $A \succ B$ and $D \succ C$. But this is inconsistent with EUT!

Violations of Expected Utility Theory 1 (Allais Paradox)

How to visualize the preferences over lotteries A, B, C, and D?

- Expected utility function is **linear** in probabilities, $U(\mathrm{p}) = p_1 u_1 + \cdots + p_N u_N$.
- In a lottery, probabilities satisfy a **linear** constraint $\sum_n p_n = 1$ and $\mathrm{p} \in \mathbb{R}^N_+$.
- Given linearities, *level curves* (in 2D) are linear.
- Formally, the set of lotteries is a N-dimensional simplex

$$\Delta^N \equiv \{p \in \mathbb{R}^N_+ : p_1 + \cdots + p_N = 1\}.$$

- We can map the simplex Δ^3 into a triangle in 2D space. Consider a space defined by (p_1, p_3). The simplex maps into a set $\{(p_1, p_3) \in \mathbb{R}^2_+ : p_1 + p_3 \leq 1\}$.
- Utility function in the 2D space is $U_{13}(p_1, p_3) \equiv U(p_1, 1 - p_1 - p_3, p_3)$.
- A level curve of U_{13} is a set of pairs $\{(p_1, p_3) \in \mathbb{R}^2_+, p_1 + p_3 \leq 1 : U_{13}(p_1, p_3) = k\}$.

$$p_1 u_1 + (1 - p_1 - p_3) u_2 + p_3 u_3 = k.$$

$$p_3 = \frac{k - u_2}{u_3 - u_2} + \frac{u_2 - u_1}{u_3 - u_2} p_1$$

Of course, k must be in a range of U and $u_2 \neq u_3$.

Violations of Expected Utility Theory 1 (Allais Paradox)

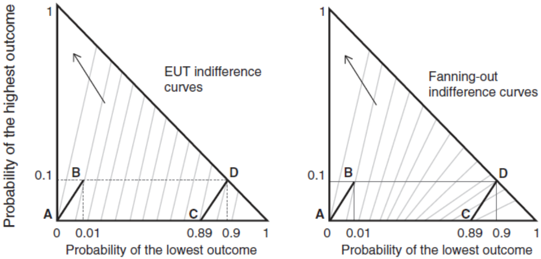

FIGURE 1. ILLUSTRATION OF THE ALLAIS PARADOX IN THE PROBABILITY TRIANGLE

- Blavatskyy, Ortmann and Panchenko (2022): Allias Paradox is observed in experiments with high hypothetical payoffs, the medium outcome being close to the highest outcome and when lotteries are presented in reduced (not compound) form.

Violations of Expected Utility Theory 2 (Ambiguity Aversion)

2 outcomes: good and bad

- $p \in (0,1)$ is probability of the good outcome
- *unambiguous* (simple) lottery: p is known
- *ambiguous* lottery: p is unknown and we have *no argument* why it should be of specific value

How to perceive an ambiguous lottery?

- a compound lottery where each unambiguous (simple) lottery has *equal* probability

$=$ principle of insufficient reason

- a reduced (simple) lottery of this compound lottery is a lottery with $p = \frac{1}{2}$

What is experimentally observed?

- Decision-makers prefer an unambiguous lottery with $p = \frac{1}{2}$ to the ambiguous lottery.

$=$ ambiguity aversion

But does the experiment reveal ambiguity aversion or something else?

Rule rationality (Aumann, 2019)

Key ideas:

- Act-rationality: Agents (consciously or sub-consciously) optimize over actions.
- Rule-rationality: Agents (evolutionarily) optimize over rules/heuristics.

Why heuristics?

- Limited cognitive abilities = a limited number of heuristics
- Behavior is driven by limited genes (nature) and memes (culture).
- Also, heuristics may serve as commitment devices (more on that later).

What are implications?

- Heuristics are optimal ‘on average’.
- In unusual situations, heuristics fail (even dramatically).
- Certain industries (e.g., advertising) tend to exploit failures.
- But if failures become ‘usual situations’, the heuristics will adapt.

Example: Overeating

- heuristic ‘eat when you have appetite’
- unusual situation: sedentary nature of modern life

Ellsberg experiment

- White bag ($\color{red}{\bullet}\ \circ\ \circ$): 1 red ball and 2 white balls
- Black bag ($\color{red}{\bullet}\ \bullet\ \bullet$): 1 red ball and 2 black balls
- Nature has chosen a white bag or a black bag, with **unknown** probability.
- You don't observe color of the bag.
- A fair draw is made from the chosen bag.
- You may bet on a red ball (red bet) or a white ball (white bet).
- If you win a bet, you get 1. If you lose, you get 0.
- By principle of insufficient reason, we can set the probabilities of bags to $(\frac{1}{2}, \frac{1}{2})$.
- Red and white bets are different compound lotteries over different simple lotteries, but they give an **identical** reduced lottery $L = (\frac{2}{3}, \frac{1}{3})$ over payoffs $(0, 1)$.

= as if we bet on 2 out of 6 equally likely balls ($\color{red}{\bullet}\ \color{red}{\bullet}\ \circ\ \circ\ \bullet\bullet$).

- The expected payoff in each bet is $\frac{1}{3}$.
- Ambiguity-averse decision makers prefer **red** bet.

Ellsberg experiment revisited

- We suppose that the agent expects that a small probability $\epsilon > 0$ exists that **another draw** will be possible.
- We also suppose that the agent is risk-averse. (This was irrelevant in Ellsberg experiment because only two outcomes exist for 1 draw.)

Either of 2 settings exists:

- double draw **without** a revision: the future draw must be identical like the initial draw
- double draw **with** a revision: the future draw can be different from the initial draw

Double draw without a revision of the initial bet

- Red bet: $L_r = (\frac{4}{9}, \frac{4}{9}, \frac{1}{9})$ over $(0, 1, 2)$ outcomes

$$\Pr(0 \text{ red}) = \frac{1}{2}\left(\frac{2}{3}\frac{2}{3}\right) + \frac{1}{2}\left(\frac{2}{3}\frac{2}{3}\right) = \frac{4}{9}$$

$$\Pr(1 \text{ red}) = \frac{1}{2}\left(\frac{2}{3}\frac{1}{3} + \frac{1}{3}\frac{2}{3}\right) + \frac{1}{2}\left(\frac{2}{3}\frac{1}{3} + \frac{1}{3}\frac{2}{3}\right) = \frac{4}{9}$$

$$\Pr(2 \text{ red}) = \frac{1}{2}\left(\frac{1}{3}\frac{1}{3}\right) + \frac{1}{2}\left(\frac{1}{3}\frac{1}{3}\right) = \frac{1}{9}$$

- White bet: $L_w = (\frac{5}{9}, \frac{2}{9}, \frac{2}{9})$ over $(0, 1, 2)$ outcomes

$$\Pr(0 \text{ white}) = \frac{1}{2}1 + \frac{1}{2}\left(\frac{1}{3}\frac{1}{3}\right) = \frac{5}{9}$$

$$\Pr(1 \text{ white}) = \frac{1}{2}0 + \frac{1}{2}\left(\frac{2}{3}\frac{1}{3} + \frac{1}{3}\frac{2}{3}\right) = \frac{2}{9}$$

$$\Pr(2 \text{ white}) = \frac{1}{2}0 + \frac{1}{2}\left(\frac{2}{3}\frac{2}{3}\right) = \frac{2}{9}$$

- The expected number of balls in both lotteries is **identical**:

$$\frac{4}{9}1 + \frac{1}{9}2 = \frac{6}{9} = \frac{2}{9}1 + \frac{2}{9}2$$

- But white bet is **more risky**. A risk-averse expected-utility maximizer prefers the red bet.

Double draw with a revision of the initial bet

- If the initial bet is red, revise bet (strictly) only if you observe the white ball.
  - red ball in Draw 1: beliefs about bags don't change, keep red bet

$$\Pr(\text{red}, \text{red}) = \tfrac{1}{2}\left(\tfrac{1}{3}\tfrac{1}{3}\right) + \tfrac{1}{2}\left(\tfrac{1}{3}\tfrac{1}{3}\right) = \tfrac{1}{9}$$

$$\Pr(\text{red}, \text{white}) = \tfrac{1}{2}0 + \tfrac{1}{2}\left(\tfrac{1}{3}\tfrac{2}{3}\right) = \tfrac{1}{9}$$

$$\Pr(\text{red}, \text{black}) = \tfrac{1}{2}\left(\tfrac{1}{3}\tfrac{2}{3}\right) + \tfrac{1}{2}0 = \tfrac{1}{9}$$

  - black ball in Draw 1: bag is black, keep red bet

$$\Pr(\text{black}, \text{red}) = \tfrac{1}{2}\left(\tfrac{2}{3}\tfrac{1}{3}\right) + \tfrac{1}{2}0 = \tfrac{1}{9}$$

  - white ball in Draw 1: bag is white, choose white bet

$$\Pr(\text{white}, \text{white}) = \tfrac{1}{2}0 + \tfrac{1}{2}\left(\tfrac{2}{3}\tfrac{2}{3}\right) = \tfrac{2}{9}$$

- Initial red bet: $L_r' = (\frac{3}{9}, \frac{5}{9}, \frac{1}{9})$ over $(0, 1, 2)$

$$\Pr(1 \text{ ball}) = 3\frac{1}{9} + \frac{2}{9} = \frac{5}{9}$$

$$\Pr(2 \text{ balls}) = \frac{1}{9}$$

- The expected number of balls is $\frac{7}{9}$.

Double draw with a revision of the initial bet

- If the initial bet is white, revise bet (strictly) only if you observe the black ball.
  - red ball in Draw 1: beliefs about bags don't change, keep white bet

$$\Pr(\text{red}, \text{white}) = \tfrac{1}{2}0 + \tfrac{1}{2}\left(\tfrac{1}{3}\tfrac{2}{3}\right) = \tfrac{1}{9}$$

  - white ball in Draw 1: bag is white, keep white bet

$$\Pr(\text{white}, \text{red}) = \tfrac{1}{2}0 + \tfrac{1}{2}\left(\tfrac{2}{3}\tfrac{1}{3}\right) = \tfrac{1}{9}$$

$$\Pr(\text{white}, \text{white}) = \tfrac{1}{2}0 + \tfrac{1}{2}\left(\tfrac{2}{3}\tfrac{2}{3}\right) = \tfrac{2}{9}$$

  - black ball in Draw 1: bag is black, choose red bet

$$\Pr(\text{black}, \text{red}) = \tfrac{1}{2}\left(\tfrac{2}{3}\tfrac{1}{3}\right) + \tfrac{1}{2}0 = \tfrac{1}{9}$$

- Initial white bet: $L'_w = \left(\frac{4}{9}, \frac{3}{9}, \frac{2}{9}\right)$ over $(0, 1, 2)$

$$\Pr(1 \text{ ball}) = 3\tfrac{1}{9} = \tfrac{3}{9}$$

$$\Pr(2 \text{ balls}) = \tfrac{2}{9}$$

- The expected number of balls is **identical**, $\frac{7}{9}$. But white bet is **more risky**.
- A risk-averse expected-utility maximizer prefers the red bet.

Double draw with a revision of the initial bet

The lotteries L'_r and L'_w don't change if the (indifferent) indecision-maker switches from her initial bet to an alternative bet.[2]

- Initial red bet
  - After a red ball (no learning), suppose the decision-maker switches to the white bet.
  - Now, (red, red) outcome is a 1-ball outcome, and (red, white) outcome is a 2-ball outcome.
  - Previously, (red, red) outcome was a 2-ball outcome, and (red, white) outcome was a 1-ball outcome.
  - This change is irrelevant because, conditional on the initial red ball, both outcomes are equally likely:
$$\Pr(\text{red, red}) = \tfrac{1}{9} = \Pr(\text{red, white})$$
- Initial white bet
  - After a red ball (no learning), suppose the decision-maker switches to the red bet.
  - By the same argument, the change is irrelevant because, conditional on the initial white ball, the following two outcomes are equally likely:
$$\Pr(\text{red, red}) = \tfrac{1}{9} = \Pr(\text{red, white})$$

[2] Also, the way we treat indifferences doesn't matter.

Ellsberg experiment revisited

- Any ϵ-uncertainty over the future *unrevised* draw coupled with risk aversion implies that the red bet is optimal.
- Any ϵ-uncertainty over the future *revised* draw coupled with risk aversion implies that the red bet is optimal.
- The decision to choose a red bet in Ellsberge experiment is not necessarily due to ambiguity aversion, but due to (i) the ϵ-uncertainty over the future draw, or (ii) the use of an optimal heuristic that accounts for the existence of both static and dynamic decision environments.

Key lessons

- Homo economicus is an agent with consistent, continuous and independent preferences that chooses his/her best outcome in a decision problem.
- Consistent, continuous and independent preferences over lotteries are represented by an expected utility function over Bernoulli utilities (in fact, by a class of EU functions that differ only in a constant and in a positive multiplier).
- Equivalently, homo economicus is an expected-utility maximizer.
    - Homo economicus can be altruistic.
    - Homo economicus can be moral (norms constitute part of outcomes, and preferences are over both consumption and norms).
- Homo sapiens sapiens is often a different animal:
    - no consistency because of incomplete preferences
    - no consistency because of intransitive preferences
    - no continuity because of lexicographic preferences
    - no independence of preferences
    - no payoff-maximization because of cognitive biases and imperfect heuristics
- When payoff-maximization is violated, it is often hard to identify which element of homo economicus is violated.
- With cognitive limits, heuristics-optimization replaces payoff-maximization.