

## 14.750X: VOTING SYSTEMS REVIEW

### Practice Problem 1

Consider the following model. We are making a decision about the amount of public good  $b$ . Any individual  $i$  is characterized by her productivity  $y_i$  and her utility  $u_i$ :

$$u_i = y_i b - \frac{1}{2} b^2$$

There are three groups of individuals. In Group A, individuals' productivity  $y_i$  is uniformly distributed on  $[0, 1]$ . In Group B, individuals' productivity  $y_i$  is uniformly distributed on  $[1, 2]$ . In Group C, individuals' productivity  $y_i$  is uniformly distributed on  $[2, 3]$ . Groups A, B, and C have equal size, so that overall productivity is uniformly distributed on  $[0, 3]$ .

- (1) Are the preferences single-peaked with respect to the public good  $b$ ?
- (2) What is the preferred level of public good for individual  $i$  with productivity  $y_i$ ?
- (3) There are two candidates in the election, who care only about winning the election, and the candidates can commit in advance to the policy they will implement if elected.
  - (a) First suppose that only Group B and Group C individuals have the right to vote. What amount of public good will be chosen in equilibrium, and why?
  - (b) What if all groups (A, B, and C) had the right to vote? What amount of public good will be chosen in equilibrium?
- (4) Groups B and C have the right to vote, and they are deciding whether to expand the franchise (give Group A the right to vote, so everyone can vote). They will first vote on whether to expand the franchise, and then everyone who is eligible will vote in the election that determines the amount of the public good. As before, there will be two candidates in the election, who care only about winning, and the candidates can commit in advance to the policy they will implement if elected (the candidates announce their policies after the vote on the franchise).
  - (a) Do Groups B and C expand the franchise in equilibrium?
  - (b) What if only Group B got to decide whether to expand the franchise to A. Would Group B expand the franchise?
- (5) Now suppose that Groups B and C find it unpleasant to live in a society that does not let everyone vote. Their utility function is now:

$$u_i = \begin{cases} y_i b - \frac{1}{2} b^2 & \text{if Group A can vote} \\ y_i b - \frac{1}{2} b^2 - \delta & \text{if Group A can't vote} \end{cases}$$

For what values of  $\delta$  would they expand the franchise?