Inequalities, Household Behavior and the Macroeconomy
(Consumption and income uncertainty)

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So far

- Introduced the basic consumption/ saving model
- Deterministic case and stochastic case with quadratic utility implies too much smoothing when no borrowing constraints.
- Saw that borrowing limits seem to play some role in actual behavior of households. Zeldes(1989)
- Discussed how to solve these models, relying on dynamic programming

So far

Some issues:

- Can we explain everything with borrowing limits, in a deterministic model?

  No! Allows only one of these two extreme situations:

  - Constraint not binding $\rightarrow$ Perfect smoothing
  - Constraint binding $\rightarrow$ Eat everything

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Some issues:

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  No! Allows only one of these two extreme situations:
  - Constraint not binding $\rightarrow$ Perfect smoothing
  - Constraint binding $\rightarrow$ Eat everything
- A puzzle in data:
  - In the aggregate, consumption closely follows income.
  - At the individual level, correlation between the two is weak.

Today

- (Practice) Gain some intuition on the consumption/saving model using our code. We experiment with parameter values a bit.
- (Theory) Why and how does uncertainty matter?
- (Empirics) How much uncertainty matters in real life consumption decisions? Carroll and Samwick (1997)

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2. Uncertainty, some theory

3. Precautionary savings in data

Some take-aways

- Under quadratic preferences, income uncertainty has no effect on consumption decisions (apart from effects of numerical errors and not allowing for negative consumption)
- Under CRRA utility,
  - uncertainty decreases consumption for young agents
  - and increases savings (decreases debt)

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Why do we care about uncertainty in income?

- Welfare is affected.
  - Income means fluctuations $->$ less smooth consumption $->$ lower average utility
- Saving decisions are affected.
  - Agents might attempt reducing these fluctuations $->$ save more to create a buffer

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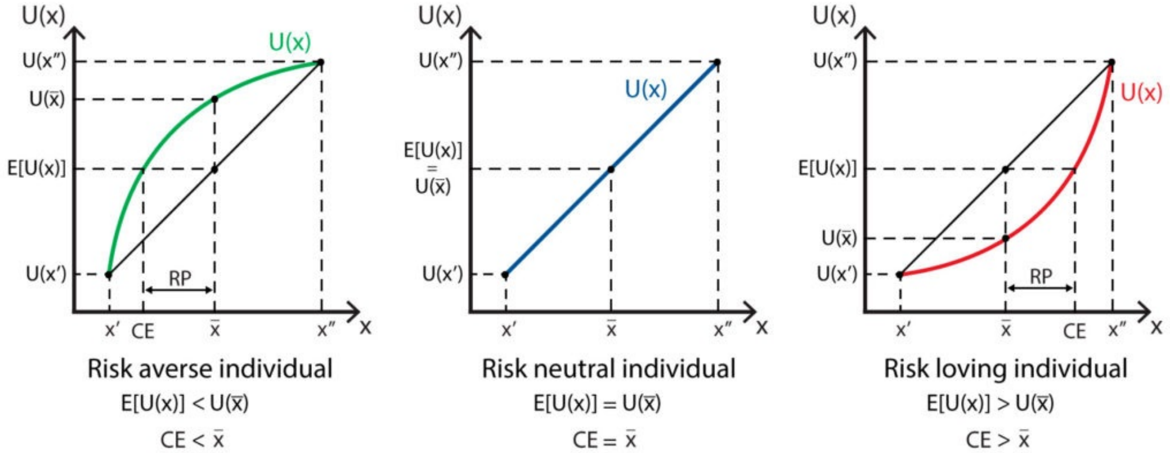
- Saving decisions are affected.
  - Agents might attempt reducing these fluctuations $->$ save more to create a buffer
  - How big effect? Prudence tells you!
    Linked to **third** derivative of the utility function

What is better: x' or x'' with probability $1/2$ each, or $\bar{x} = \mathbb{E}[x]$ for sure?



- CE: Certainty equivalent: $U(CE) = \mathbb{E}[U(x)]$
- (E)RP: Equivalent risk premium: $U(\bar{x} - RP) = \mathbb{E}[U(x)]$

Risk aversion

Even for a general x (i.e. not only two possible values),

$$\mathbb{E}\left[U(x)\right] < U(\mathbb{E}[x])$$

if U is strictly **concave** (due to the Jensen-inequality).

- Agent would be happier with the utility from the average outcome than the random utility from x.

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  losing RP amount from the mean outcome is equivalent to introducing uncertainty
- the size of RP depends on:
  - size of risk
  - how much the agent dislikes risk (shape of U)

Risk aversion - optional

Second-order Taylor approximation around $\mathbb{E}[x]$:

$$U(x) \approx U(\mathbb{E}[x]) + U'(\mathbb{E}[x])(x - \mathbb{E}[x]) + \frac{U''(\mathbb{E}[x])}{2}(x - \mathbb{E}[x])^2$$

implying

$$\mathbb{E}\big[U(x)\big] \approx U(\mathbb{E}[x]) + \frac{U''(\mathbb{E}[x])}{2} Var(x).$$

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Substituting into

$$U(\mathbb{E}[x] - RP) = \mathbb{E}\big[U(x)\big]$$

gives

$$RP \approx -\frac{U''(\mathbb{E}[x])}{U'(\mathbb{E}[x])} \frac{Var(x)}{2}.$$

Risk aversion

The risk-premium is approximately

$$RP \approx -\frac{U''(\mathbb{E}[x])}{U'(\mathbb{E}[x])}\frac{Var(x)}{2}.$$

- riskyness of x matters through $Var(x)$
- shape of U matters through

$$\text{Risk aversion} = -\frac{U''}{U'}$$

- This approximation is less precise for large risks!

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We know that

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Why?

- If the marginal utility on the RHS stays the same,
- then the LHS stays the same, so c_t is unaffected,
- hence saving today is unaffected.

Precautionary premium

(Equivalent) precautionary premium: Losing this amount from expected consumption leads to same decision as risky consumption:

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Precautionary premium - Intuition

- $$U'(C_t) = \beta(1+r)\mathbb{E}\Big[U'(C_{t+1})\Big]$$

- $$\mathbb{E}\Big[U'(C_{t+1})\Big] > U'\Big(\mathbb{E}[C_{t+1}]\Big)$$

  if $U''' > 0$. This means that consuming the average value for sure leads to too low marginal utility (i.e. C_{t+1} is too high, assuming C_t is unchanged). So without uncertainty, you should optimally consume less in the next period, i.e. save less and consume more now.
- By how much? PP tells you. Losing how much future consumption (relative to expected value) is equivalent to having uncertainty in consumption, in terms of ending up with the same expected marginal utility

$$\mathbb{E}\Big[U'(C_{t+1})\Big] = U'\Big(\mathbb{E}[C_{t+1}] - PP\Big)$$

- We are interested in keeping expected marginal utility constant, since that implies constant saving today (due to the Euler-equation).

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- As a response to risk, it makes sense to save more, if the unhappiness from risk is lower when having more money $->$ being able to consume more.
- That has to do with how the second derivative depends on your consumption level.
- Which is exactly what the third derivative tells you (i.e. change in the second derivative).

Linear utility

$$u(c) = a \cdot c$$

- Risk aversion $= 0$
- Prudence $= 0$
- Risk is irrelevant.

Quadratic utility

$$u(c) = -(c - \overline{c})^2$$

- Risk aversion $= \frac{1}{\overline{c} - c}$
- Prudence $= 0$
- Dislikes risk; especially when close to satiation point.
- But risk has no effect on actions.

CARA utility

$$u(c) = \frac{1 - e^{-ac}}{a}$$

- Risk aversion $= a$
  Hence the name: CARA = constant absolute risk aversion
- Prudence $= a$
- The harm and response linked to risk does not depend on consumption level.
- This sometimes makes analytical solutions possible, but
- is thought to be unrealistic (in reality poor people probably care more about identical risks than the rich).

CRRA utility

$$u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}$$

- Risk aversion $= \frac{\gamma}{c}$
- Prudence $= \frac{1+\gamma}{c}$
- The harm and response linked to a given risk decreases in the consumption level.

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- Risk aversion $= \frac{\gamma}{c}$
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- The harm and response linked to a given risk decreases in the consumption level.
- Name: CRRA = constant relative risk aversion. What's that?

$$\text{Relative Risk aversion} = -\frac{U''(c)}{U'(c)c}$$

  is the right measure when talking about relative risk (e.g. losing 10% of your wealth).
- For CRRA, relative risk aversion is constant (γ)

Elasticity of intertemporal substitution vs relative risk aversion - optional

For CRRA utility,

- Elasticity of intertemporal substitution is $\frac{1}{\gamma}$
- Relative risk aversion is γ, so the two things are reciprocals of each other.

This is a general phenomenon in our preference framework (additive over time, expected utility over different outcomes). Is this sensible?

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Yes! High γ means a very curved utility function, so

- you really want to smooth consumption across different histories, i.e. you don't like risk
- you really want to smooth consumption across time, i.e. you are insensitive to interest rate changes.

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No!

- Risk and time are very different concepts. Why should the same parameter determine behavior regarding both at once?
- E.g. maybe you enjoy going to a super fancy restaurant once a year. Based on this, can I tell if you are fine with risk or not? No.

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Precautionary savings in the data

How (empirically) important is precautionary savings?

Carroll and Samwick (1997) claim that 32-50% of wealth is attributable to the extra uncertainty that some consumers face compared to the lowest uncertainty group.

How do they get there?

1. Obtain the relationship between income uncertainty and net-worth using a life-cycle model with power utility
2. Using the functional form they obtained from their theory, they explore the relationship between income uncertainty and net-worth empirically
3. Using the empirical model estimated in point 2, they can answer the question: how much lower would the aggregate net-worth be if *everyone* faced the lowest income uncertainty observed among all socio-economic groups?

Precautionary savings in the data—Step 1

They consider the following life-cycle model

$$\max_{c_t, a_{t+1}} E_0 \sum_{t=0}^{T} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}$$

$$s.t. \quad a_t = (1+r)a_{t-1} + y_t - c_t$$

$$a_{t+1} \geq 0, \quad \log(y_t) = trend_t + \epsilon_t, \quad \epsilon_t \sim^{iid} \mathcal{N}(0, \sigma^2)$$

Note that in the paper, the notation is slightly different.

Precautionary savings in the data—Step 1

What is the right way to investigate the connection of savings and income uncertainty?

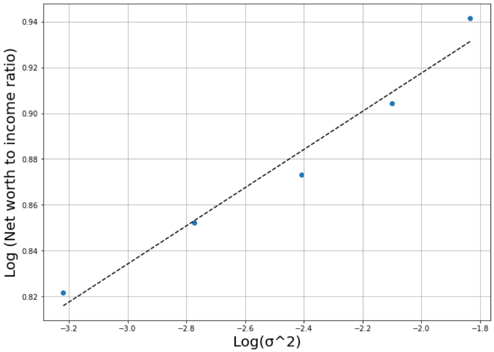
- In the data, we can work with linear relationships
- No theory to figure out if there is any linear connection between savings and income uncertainty

Solution:

1. Solve model numerically and simulate a population.
2. In simulated data, try which measures of income uncertainty and wealth show the most linear relationships.
    - Candidates for the former: (Log) variance of (log) income, and a proxy of EPP.
    - Wealth: (log) w^*, target wealth(-to-income ratio). Will learn more about this next lecture.
3. Use the analogous measures in actual data later on.

Precautionary savings in the data—Step 1

How much does net-worth in the model varies when we change σ^2?



The relationship is (almost) linear! We can use a linear model to study the relationship!
$\log(Nw_i/y_i) = \beta_0 + \beta_1 \log(\sigma_i^2) + \mu_i$, where μ_i is the error term.

Precautionary savings in the data—Step 2

Now we need data with net-worth and income uncertainty.

The answer is the PSID: the panel study of income dynamics

It is a longitudinal panel survey of American families: it measures economic and social factors over the life course of families over multiple generations. It started in 1968.

Carroll and Samwick (1997) measure the net-worth and (log) income uncertainty for each household in their sample

Precautionary savings in the data—Step 2

Using the relationship derived from the theory, they estimate

$$\log(Nw_i) = \beta_0 + \beta_1 \log(\sigma_i^2) + \beta_2 \log(\overline{y}_i) + \gamma' \mathbf{X_i} + \mu_i$$

where $\mathbf{X_i}$ is a vector of controls related to households i.

$\overline{y}_i$ and σ_i^2 are the mean and variance of the household income over the period they are observed
Since $\overline{y}_i$ and σ_i^2 are measured with error, they are instrumented with occupation and industry dummies.

Precautionary savings in the data—Step 3

The last step is quite intuitive: they construct a new measure of net-worth N_w^*

$$\log(Nw_i^*) = \log(Nw_i) - \hat{\beta}_1(\log(\sigma_i^2) - \log(\sigma_i^{2^*}))$$

which tells us how much the net-worth would change if uncertainty was changed to σ^{2^*}

They set σ^{2^*} such that it equals the average σ^2 of the socio-economic group with the lowest uncertainty and compute Nw_i^* for each household.

Then, they compute how lower is average Nw_i^* with respect to average Nw_i.

Their estimates range 32%-50%, depending on the specification.