Problem Set 6

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Lecture 12 Generalized methods of moments

Exercise 1

(Ch. 13, ex. 13.6) Derive the constrained GMM estimator (13.16).

Exercise 2

(Ch. 13, ex. 13.12) In the linear model $Y = X'\beta + e$ with $\mathbb{E}[Xe] = 0$, the GMM criterion function for β is

$$J(\beta) = \frac{1}{n}(\mathbf{Y} - \mathbf{X}\beta)'\mathbf{X}\hat{\mathbf{\Omega}}^{-1}\mathbf{X}'(\mathbf{Y} - \mathbf{X}\beta) \tag{*}$$

where $\hat{\mathbf{\Omega}} = n^{-1}\sum_i X_i X_i' \hat{e}_i^2,\ \hat{e}_i^2 = y_i - X_i'\beta$ are the OLS residuals, and $\hat{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$ is the least squares estimator. The GMM estimator for β subject to the restriction $r(\beta) = 0$ is

$$\tilde{\beta} = \arg\min_{r(\beta)=0} J_n(\beta).$$

The GMM test statistic (the distance statistic) of the hypothesis $r(\beta) = 0$ is

$$D = J(\tilde{\beta}) = \min_{r(\beta)=0} J(\beta). \tag{+}$$

(a) Show that you can rewrite $J(\beta)$ in (*) as

$$J(\beta) = n(\hat{\beta} - \beta)'\hat{\mathbf{V}}_\beta^{-1}(\hat{\beta} - \beta)$$

and thus $\tilde{\beta}$ is the same as the minimum distance estimator.

(b) Show that under linear hypotheses the distance statistic D in (+) equals the Wald statistic.

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Exercise 3

(Ch. 13, ex. 13.27) Continuation of Exercise 12.22, based on the empirical work reported in Acemoglu, Johnson, and Robinson (2001).

(a) Re-estimate the model in part (j) by efficient GMM. se the 2SLS estimates as the first step for the weight matrix and then calculate the GMM estimator using the weight matrix without further iteration. Report the estimates.

(b) Calculate and report the J statistic for overidentification.

(c) Compare the GMM and 2SLS estimates. Discuss your findings.

Lecture 13 Time series

Exercise 4

(Ch. 14, ex. 14.8) Suppose $y_t = y_{t-1} + e_t$ with e_t iid $N(0,1)$ and $y_0 = 0$. Find $\operatorname{Var}[y_t]$. Is y_t stationary?

Exercise 5

(Ch. 14, ex. 14.9) Take the AR(1) model with no intercept $y_t = \alpha_1 y_{t-1} + e_t$.

(a) Find the impulse response function $b_j = \frac{\partial}{\partial e_t} y_{t+j}$.

(b) Let $\hat{\alpha}_1$ be the least squares estimator of α_1. Find an estimator for b_j.

(c) Let $s(\hat{\alpha}_1)$ be s standard error for $\hat{\alpha}_1$. Use the delta method to find a 95% asymptotic confidence interval for b_j.

Exercise 6

(Ch. 14, ex. 14.18) Take the quarterly series *pnfix* (nonresidential real private fixed investment) from FRED-Q.

(a) Transform the seres into quarterly growth rates.

(b) Estimate an AR(4) model. Report using heteroskedastic-consistent standard errors.

(c) Repeat using Newey-West standard errors, using $M = 5$.

(d) Comment on the magnitude and interpretation of the coefficients.

(e) Calculate (numerically) the impulse response function for $j = 1, ..., 10$.