Part B: Selection on Observables

B1: Regression Adjustment

Kirill Borusyak

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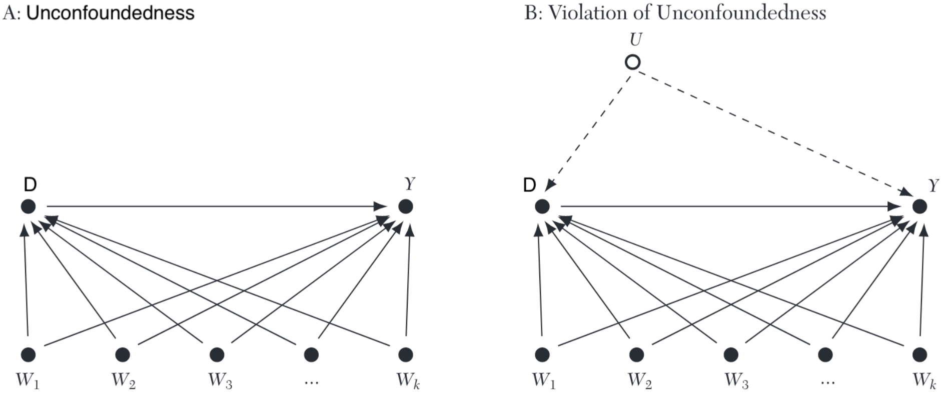
Outline

1. The concept of control variables
2. Regression adjustment
3. Application: National Support Work

What if treatment is not randomly assigned?

One basic approach is to control for observables



Identification assumptions

- Unconfoundedness $=$ Ignorability $=$ Conditional independence assumption (**CIA**) $=$ Selection on observables: $(Y_i(0), Y_i(1)) \perp D_i \mid X_i$
  - Sometimes viewed as a *definition* of a control variable
- **Overlap**: $0 < Pr(D_i = 1 \mid X_i) < 1$ on the support of X_i (testable)

Identification under CIA + overlap

- **Conditional average treatment effect** $CATE(x) \equiv \mathbb{E}\left[Y_i(1) - Y_i(0) \mid X_i = x\right]$ is identified by
$$\mathbb{E}\left[Y_i \mid D_i = 1, X_i = x\right] - \mathbb{E}\left[Y_i \mid D_i = 0, X_i = x\right]$$
- $ATE = \mathbb{E}\left[CATE(X_i)\right]$ is identified by
$$\mathbb{E}\left[\mathbb{E}\left[Y_i \mid D_i = 1, X_i\right] - \mathbb{E}\left[Y_i \mid D_i = 0, X_i\right]\right]$$
where the outer expectation is taken w.r.t. (observed) distribution of X_i
- $ATT = \mathbb{E}\left[CATE(X_i) \mid D_i = 1\right]$ is identified by
$$\mathbb{E}\left[\mathbb{E}\left[Y_i \mid D_i = 1, X_i = x\right] - \mathbb{E}\left[Y_i \mid D_i = 0, X_i = x\right] \mid D_i = 1\right]$$
where the outer expectation is taken w.r.t. (observed) distribution of $X_i \mid D_i = 1$
- Estimation is non-trivial — stay tuned

When may CIA hold?

1. RCT with unequal probabilities of treatment, e.g. stratified by age

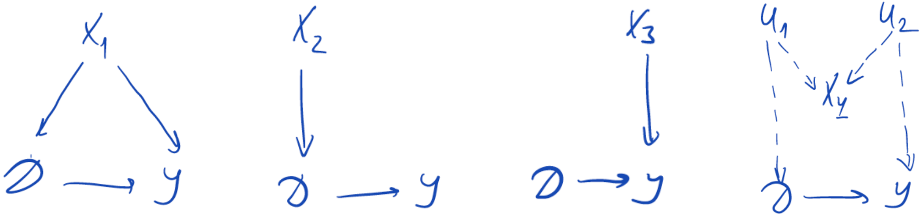
2. If you have a model of the factors driving D and believe none of them but X affect Y or are correlated with factors affecting Y

    - Understanding sources of variation in D is key for estimating the effects of D

3. Pragmatic approach: causal inference is always about comparing *some* treated and control outcomes $\Longrightarrow$ reduce bias by comparing units with similar X

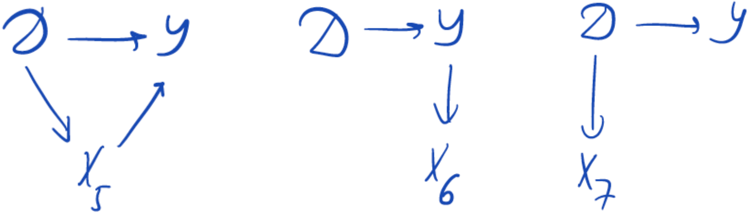
Good and bad controls

Which of these variables would you control for? Think identification and efficiency



Good and bad controls (2)

Which of these variables would you control for? Think identification and efficiency



Good and bad controls: Answer key

- X_1 Necessary for identification
- X_2 Bad for efficiency (but may be good for robustness)
- X_3 Good for efficiency
- X_4 "Collider," generates bias (but may serve as proxy for U_2 if U_2 affects D)
- X_5, X_6 Bad controls, generate bias
- X_7 Exercise for you

How many controls?

- Controlling for more pre-treatment variables relaxes CIA
  - But a clear narrative for a small set of controls is considered cleaner
- Weakens overlap but efficiency implications ambiguous
- Makes estimation trickier $\Longrightarrow$ machine learning solutions

Plan of attack

1. Regression adjustment
2. Matching
3. Propensity score-based methods
4. Doubly-robust methods
5. ML methods for high-dimensional covariates
6. Violations of CIA and coefficient stability

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Regression adjustment

- Under CIA, we have for $d = 0, 1$

$$\mathbb{E}\left[Y(d) \mid X\right] = \mathbb{E}\left[Y(d) \mid D = d, X\right] = \mathbb{E}\left[Y \mid D = d, X\right] \equiv h_d(X)$$

- If we estimate $h_0(\cdot)$ and $h_1(\cdot)$ by regression methods, we get

$$\widehat{ATE} = \frac{1}{N}\sum_i \left(\hat{h}_1(X_i) - \hat{h}_0(X_i)\right), \qquad \widehat{ATT} = \frac{1}{N_1}\sum_i \left(\hat{h}_1(X_i) - \hat{h}_0(X_i)\right) D_i$$

- Since average fitted values equal average outcomes, we also get the **imputation** representation:

$$\widehat{ATT} = \frac{1}{N_1}\sum_i \left(Y_i - \hat{h}_0(X_i)\right) D_i \qquad \text{and}$$

$$\widehat{ATE} = \frac{1}{N}\sum_i \left\{\left(Y_i - \hat{h}_0(X_i)\right) D_i + \left(\hat{h}_1(X_i) - Y_i\right)(1 - D_i)\right\}$$

Estimating $h_0(\cdot), h_1(\cdot)$

- Can use nonparametric regression, e.g. local linear regression
  - For each x, estimate $h_d(x)$ by an intercept from a regression of Y_i on $X_i - x$ keeping observations in the neighborhood of x (with $D_i = d$) only (assuming overlap)
- If $\mathbb{E}\left[Y(d) \mid X\right] = \gamma_d' X$ is linear in X (e.g. X is saturated): Oaxaca-Blinder estimator
  - Run linear regressions of Y on X within treated/control groups separately
  - Or a single fully-interacted regression

$$Y_i = \gamma_0' X_i + \tau' X_i D_i + \text{error}_i, \qquad \widehat{ATE} = \hat{\tau}' \bar{X}$$

  - Or its convenient reformulation

$$Y_i = \gamma_0' X_i + \beta D_i + \tau' \left(X_i - \bar{X}\right) D_i + \text{error}_i, \qquad \widehat{ATE} = \hat{\beta}$$

Regression adjustment with homogeneous effects

- If $\mathbb{E}\left[Y(0) \mid X\right] = \gamma' X$ is linear and causal effects are homogeneous,

$$Y_i = \beta D_i + Y_i(0) = \beta D_i + \gamma' X + u_i, \qquad \mathbb{E}\left[u_i \mid D_i, X_i\right] = 0$$

  - Regression that controls for X_i identifies the causal effect

- This still relies on overlap:
  - $\hat{\beta} = \left(\overline{Y_1} - \overline{Y_0}\right) - \hat{\gamma}' \left(\overline{X_1} - \overline{X_0}\right)$
    because $\overline{Y_1} = \hat{\beta} + \hat{\gamma}' \overline{X_1}$ and $\overline{Y_0} = \hat{\gamma}' \overline{X_0}$
  - If the distribution of X is the same among treated and controls, $\hat{\gamma}$ does not matter $\implies$ does not matter which nonlinear terms and interactions are included in X
  - If the distributions are very different, regression is doing a lot of extrapolation, and functional form becomes important

Uninteracted regression with heterogeneous effects

Mostly Harmless Econometrics advocates for $Y_i = \beta D_i + \gamma' X_i + \varepsilon_i$ regressions even when the effects are heterogeneous. Why?

- Assume the **propensity score** $p(X_i) \equiv \mathbb{E}[D_i \mid X_i] = Pr(D_i = 1 \mid X_i)$ is linear in X_i
  - Angrist (1998) focused on saturated controls X_i
- Then

$$\beta_{OLS} = \frac{\mathbb{E}[CATE(X_i) \cdot \omega(X_i)]}{\mathbb{E}[\omega(X_i)]}, \qquad \omega(X_i) = \mathrm{Var}[D_i \mid X_i] = p(X_i)(1 - p(X_i))$$

  - Groups with $p(X_i) \approx 1/2$ get the most weight (relative to their size)
  - Groups where overlap is limited ($p(X_i) \approx 0$ or $p(X_i) \approx 1$) get little weight
  - $\beta_{OLS} = ATE$ if $CATE(X_i)$ is constant, $\omega(X_i)$ is constant, or they are uncorrelated with each other

Variance weighting: Proof

- By linearity of the p-score, partialling out X_i from D_i yields residuals $\tilde{D}_i = D_i - \mathbb{E}\left[D_i \mid X_i\right]$
- By Frisch-Waugh-Lovell, $\beta_{OLS} = \mathbb{E}\left[\tilde{D}_i Y_i\right] / \mathbb{E}\left[\tilde{D}_i D_i\right]$ (where $\mathbb{E}\left[\tilde{D}_i D_i\right] = \mathrm{Var}\left[\tilde{D}_i\right]$)
- Using CIA and $\mathbb{E}\left[\tilde{D}_i \mid X_i\right] = 0$,

$$\begin{aligned}\mathbb{E}\left[\tilde{D}_i Y_i\right] &= \mathbb{E}\left[\mathbb{E}\left[\tilde{D}_i \left(Y_i(0) + \left(Y_i(1) - Y_i(0)\right) D_i\right) \mid X_i\right]\right] \\ &= \mathbb{E}\left[CATE(X_i) \cdot \mathbb{E}\left[\tilde{D}_i D_i \mid X_i\right]\right] = \mathbb{E}\left[CATE(X_i) \cdot \mathrm{Var}\left[D_i \mid X_i\right]\right]\end{aligned}$$

- Analogously, $\mathbb{E}\left[\tilde{D}_i D_i\right] = \mathbb{E}\left[\mathrm{Var}\left[D_i \mid X_i\right]\right]$

Multi-valued and multiple treatments

This is easy to extend to multi-valued (e.g. continuous) treatments, still assuming linear $\mathbb{E}[D \mid X]$ (see MHE p.58 for details)

$$\beta_{OLS} = \mathbb{E}\left[\int \frac{\partial \mathbb{E}\left[Y(\tilde{d}) \mid X\right]}{\partial \tilde{d}} \omega(\tilde{d}, X) d\tilde{d}\right] \Big/ \mathbb{E}\left[\int \omega(\tilde{d}, X) d\tilde{d}\right]$$

where

$$\begin{aligned}
\omega(\tilde{d}, x) &= \mathrm{Cov}\left[\mathbf{1}\left[D \geq \tilde{d}\right], D \mid X\right] \\
&= \left(\mathbb{E}\left[D \mid D \geq \tilde{d}, X\right] - \mathbb{E}\left[D \mid D < \tilde{d}, X\right]\right) P\left(D \geq \tilde{d} \mid X\right) P\left(D < \tilde{d} \mid X\right)
\end{aligned}$$

But extra care is needed with multiple treatments, even dummies of multi-valued treatments; see Goldsmith-Pinkham, Hull, and Kolesar (2022)

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Application: NSW

Lalonde (1986) studied National Support Work Demonstration (NSW)

- A government program in 1970s for groups with weak labor-force attachment (e.g. ex-convicts)
- Guaranteed a job for 9–18 months and paid for it. Expensive: $7–9k per person
- How did going through NSW in 1976–77 affect wage earnings in 1978?

NSW was designed as an RCT: random selection among qualified applicants

- So we know the ATE (=ATT) for the population of applicants
- But could one get the right answer by covariate adjustment?
- This setting has become the testing ground for CIA estimators

Application: NSW (2)

- Lalonde constructed several control groups:
  - Full CPS and PSID
  - Same but pre-screened: e.g. unemployed in 1976 and below poverty line in 1975
  - We focus on male workers
- And applied several estimators for ATT
  - Mostly as diff-in-diffs (1978 minus 1975) but we'll look at cross-sections
  - (Also Heckit-style selection corrections and more)

NSW: Covariate imbalance

Variable	NSW		Full Samples	
	Treated	Control	CPS-1	CPS-3
	(1)	(2)	(3)	(4)
Age	25.82	25.05	33.23	28.03
Years of schooling	10.35	10.09	12.03	10.24
Black	0.84	0.83	0.07	0.20
Hispanic	0.06	0.11	0.07	0.14
Dropout	0.71	0.83	0.30	0.60
Married	0.19	0.15	0.71	0.51
1974 earninigs	2,096	2,107	14,017	5,619
1975 earnings	1,532	1,267	13,651	2,466
Number of Obs.	185	260	15,992	429

(From MHE Table 3.3.2)

Reporting covariate imbalance

- It's good to report **normalized** (standardized) **differences** between treatment and control groups:
$$\text{Nor.Dif.} = \frac{\overline{X_1} - \overline{X_0}}{\sqrt{\left(\sigma_1^2 + \sigma_0^2\right)/2}}$$
where σ_1^2, σ_0^2 are SD of X_i in treated & control groups
- This is *not* the t-stat for $\mathbb{E}\left[X \mid D = 1\right] = \mathbb{E}\left[X \mid D = 0\right]$:
$$t = \frac{\overline{X_1} - \overline{X_0}}{\sqrt{\sigma_1^2/N_1 + \sigma_0^2/N_0}}$$
  - t-stat is larger in larger samples — but larger samples are not an indication of a more difficult causal inference problem

NSW: Regression adjustment estimates

Specification	Full Samples		
	NSW	CPS-1	CPS-3
	(1)	(2)	(3)
Raw Difference	1,794 (633)	-8,498 (712)	-635 (657)
Demographic controls	1,670 (639)	-3,437 (710)	771 (837)
1975 Earnings	1,750 (632)	-78 (537)	-91 (641)
Demographics, 1975 Earnings	1,636 (638)	623 (558)	1,010 (822)
Demographics, 1974 and 1975 Earnings	1,676 (639)	794 (548)	1,369 (809)

(From MHE Table 3.3.3. Demographics are age, years of schooling, dummies for Black, Hispanic, high school dropout, and married)

- Lalonde concludes it'd be hard for an analyst to pick a good estimate
- Could Oaxaca-Blinder do better?