Normal-Form Games with Pure Strategies

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JEB064 Game Theory and Applications

What are we up to?

Static games of complete information

- *game* = an n-person decision problem
- *static* = (once-and-for-all) decisions are made simultaneously and independently
- *simultaneity* = each player makes a decision without information about decisions of the others
- *independently* = no ability to agree on decisions
- *complete information* = the following components are *common knowledge*:
  - all the possible actions of all the players,
  - all the possible outcomes,
  - how each combination of actions of all players affects which outcome will materialize,
  - the preferences of each player over outcomes

Common knowledge

- An event E is common knowledge if (1) everyone knows E, (2) everyone knows that everyone knows E, and so on ad infinitum.
- Not only that everybody knows the components, but also everybody knows that everybody else knows the components.

Example: The Emperor's New Clothes

Normal-Form Games with Pure Strategies

Take a set of players, indexed $i \in N = \{1, 2, \ldots, n\}$.

- A *pure strategy* for player i, s_i, is a deterministic *plan* of his/her actions.
- S_i is the set of all pure strategies for player i.
- A *profile of pure strategies* $s = (s_1, s_2, \ldots, s_n)$, where $s_i \in S_i$ for all i, describes a pure strategies chosen by all n players in the game.
- pure = the plan is deterministic, not stochastic (no-one randomizes)

Normal-Form Games with Pure Strategies

A normal-form game is a triple of sets:

- A finite set of players, $N = \{1, 2, \ldots, n\}$.
- A collection of sets of pure strategies, $\{S_1, S_2, \ldots, S_n\}$.
- A set of payoff functions, $\{v_1, v_2, \ldots, v_n\}$, where $v_i : S_1 \times S_2 \times \cdots \times S_n \to \mathbb{R}$ for each $i \in N$.

Finite game

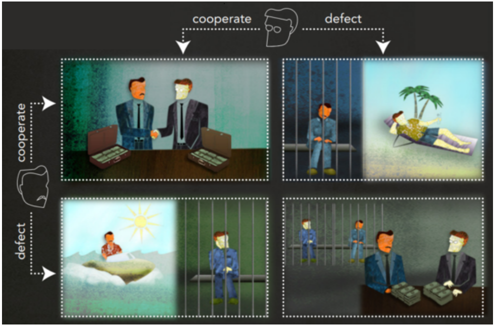
- N is finite and also each S_i is finite

Extra notation for the other players (opponents):

- $S_{-i} \equiv S_1 \times S_2 \times \ldots S_{i-1} \times S_{i+1} \times \ldots S_{n-1} \times S_n$
- the strategy profile of opponents: $s_{-i} \in S_{-i}$
- the strategy profile of all players: $s = (s_i, s_{-i}) = (s_1, s_2, \ldots, s_n)$

Example: The Prisoner's Dilemma

- 2 players are suspects in an armed robbery
- police has only evidence of a petty theft
- police is questioning each in a separate room
- confession/cooperation with police (C), silence (S)
- being a sole cooperator has a great advantage
- possible sentence: 1, 2, 4, 5 years



Example: The Prisoner's Dilemma

Formally:

- $N = \{1, 2\}, S_1 = S_2 = \{C, S\}$
- $v_1(S, S) = v_2(S, S) = -2, v_1(C, C) = v_2(C, C) = -4$
- $v_1(C, S) = v_2(S, C) = -1, v_1(S, C) = v_2(C, S) = -5$

How to represent payoffs? Bi-matrix (row player, column player)

	$s_2 = S$	$s_2 = C$
$s_1 = S$	$-2, -2$	$-5, -1$
$s_1 = C$	$-1, -5$	$-4, -4$

How to ‘solve’ a game?

We now proceed to solution concepts of games.

Solution concept = a method of analyzing games with the objective of restricting the set of all possible outcomes to those that are more ‘reasonable’ than others.

- a solution concept predicts not only the outcomes but also the players’ strategies
- prediction of a solution concept is called an *equilibrium*
- a solution depends on the assumptions about the players

Non-cooperative game theory derives solutions from these assumptions:

A1 rationality/consistency of preferences

A2 payoff-maximization/optimization

A3 common knowledge of both rationality and payoff-maximization

A4 correct beliefs in the equilibrium

Preview of solutions

- A1, A2: SDE
- A1, A2, A3: IDE
- A1, A2, A3, A4: NE

Strict dominance (SDE)

- Let $s_i \in S_i$ and $s_i' \in S_i, s_i' \neq s_i$ be possible strategies for player i.
- s_i' is **strictly dominated** by s_i if for *any possible combination* of the other players' strategies, $s_{-i} \in S_{-i}$, player i's payoff from s_i' is strictly less than that from s_i:

$$v_i(s_i, s_{-i}) > v_i(s_i', s_{-i}) \quad \forall s_{-i} \in S_{-i}$$

- s_i is a **strictly dominant** strategy for i if every other strategy of i, s_i', is strictly dominated by s_i:

$$v_i(s_i, s_{-i}) > v_i(s_i', s_{-i}) \quad \forall s_i' \in S_i, s_i' \neq s_i \quad \forall s_{-i} \in S_{-i}$$

- The strategy profile $s^D \in S$ is a **strictly dominant strategy equilibrium** (SDE) if $s_i^D \in S_i$ is a strictly dominant strategy for all $i \in N$.
- Uniqueness: If the game has a strictly dominant strategy equilibrium s^D, then it is a unique strictly dominant strategy equilibrium.
- Example: In Prisoners' Dilemma, $s = (C, C)$ is SDE.

Iterated-dominated equilibrium (IDE)

What if we eliminate strictly dominated strategies from the game?

- We obtain a reduced/restricted/smaller game.
- In the reduced game, we can again eliminate strictly dominated strategies.

= iterated elimination of strictly dominated strategies (IESDS).

- Any strategy profile s that survives IESDS is an **iterated-elimination equilibrium**.
- The iterated-elimination equilibrium (IDE) always exists. If SDE exists, then IDE = SDE.

Example

	L	C	R
U	$4, 3$	$5, 1$	$6, 2$
M	$2, 1$	$8, 4$	$3, 6$
D	$3, 0$	$9, 6$	$2, 8$

C is strictly dominated by R.

Iterated-dominated equilibrium (IDE)

A reduced game after the 1st iteration.

	L	R
U	$4, 3$	$6, 2$
M	$2, 1$	$3, 6$
D	$3, 0$	$2, 8$

- M is strictly dominated by U. D is strictly dominated by U.

A reduced game after the 2nd iteration.

	L	R
U	$4, 3$	$6, 2$

- R is strictly dominated by L. IESDS uniquely predicts (U, L).

Takeaways?

- Strict dominance requires only A1 and A2. (No need to know anything about the opponent.)
- Iterated strict dominance requires A1, A2 and A3. (Player must be sure that the opponent expects that all players eliminate strictly dominated strategies and that the opponent eliminates strictly dominated strategies himself/herself.)

Beliefs and best-responses

By IESDS, players eliminate some opponents' strategies, but cannot proceed further.

- But players can form specific beliefs (conjectures) about their opponents' strategies.
- A **belief** of player i is a possible profile of his opponents' strategies, $s_{-i} \in S_{-i}$.
- Rational players take these beliefs into account.

The strategy $s_i \in S_i$ is player i's **best response** to his opponent's strategies $s_{-i} \in S_{-i}$ if

$$v_i(s_i, s_{-i}) \geq v_i(s_i', s_{-i}) \quad \forall s_i' \in S_i.$$

- The **best-response correspondence** of player i selects for each $s_{-i} \in S_{-i}$ a subset $B_i(s_{-i}) \subseteq S_i$ where each strategy $s_i \in B_i(s_{-i})$ is a best response to s_{-i}.
- If the correspondence assigns only one element (i.e., if the best response strategy is unique), we speak of a **best-response function**.

Quiz

1. Suppose SDE exists. Are best responses to strategies s_{-i}^D unique or not?
2. A strictly dominated strategy $s_i \in S_i$ exists. Is it a best response to any $s_{-i} \in S_{-i}$?
3. Suppose IDE exists and is unique. Does SDE exist or not?
4. Suppose IDE exists and is unique. Are all strategies s_i^D best responses to s_{-i}^D or not?

Beliefs and best-responses

A rational player (recall A1, A2; but not necessarily A3, A4) who *believes* (for any reason) that his opponents are playing some $s_{-i} \in S_{-i}$ chooses a best response to s_{-i}.

- A1+A2 alone don't tell how the beliefs are formed.
- A3 helps only in a special case when IDE is unique.

What if we have multiple IDEs?

Nash equilibrium (NE)

We make a great leap by assuming that beliefs must be *correct* (A4).

- By rationality (A1+A2), each player is playing a best response to the strategies of all other players.
- With correct beliefs (A4), the outcome is 'self-enforcing' (= no profitable individual deviation).
- The solution outcome defined by A1–A4 is Nash equilibrium.

= mutual best responses

Nash equilibrium

- The pure-strategy profile $s^* = (s_1^*, s_2^*, \ldots, s_n^*) \in S$ is a **Nash equilibrium** if s_i^* is a best response to s_{-i}^* for all $i \in N$.

- Where do correct beliefs come from?
  - Deductive reasoning ('putting yourself in others shoes') due to common knowledge of rationality in A3 (and no limits to reasoning capacity) leads to correct beliefs at least if a unique NE exists.
  - Experience with past play of the opponents forms self-fulfilling beliefs.
  - Evolutionary/mass-action arguments: individuals with more successful strategies replicate faster. (In certain evolutionary games, evolutionarily stable outcomes simply correspond to Nash equilibria of baseline games.)

Nash equilibrium

Best-responses of row player and column player.

	L	C	R
U	7, 7	4, 2	1, 8
M	2, 4	5, 5	2, 3
D	8, 1	3, 2	0, 0

Nash equilibrium is a profile with mutual best responses.

Main issues with NE:

- non-existence: to be addressed by mixed strategies
- non-uniqueness (multiple Nash equilibria): to be addressed by selection criteria or perturbations to the game

Example: The Prisoners' Dilemma

	S	C
S	$-2, -2$	$-5, -1$
C	$-1, -5$	$-4, -4$

Solutions

- (C, C) is the (unique) SDE.
- (C, C) is a unique IDE.
- (C, C) is a unique NE.

Example: The Prisoners' Dilemma

Philosopher's perspective

- moral rationality: agents act in ways expected to bring about the 'greatest satisfaction' of their interests (this implies implicit cooperation).
- e.g., 'act a certain way only if you're willing to have everyone else act the same way too' (Kant's categorical imperative)

Economist's perspective

- Empirically, we observe that economic agents are willing to impose negative 'consumption' externalities on each other in the absence of devices that compensate production of such externalities (e.g., contracts) and in the absence of legal norms.
- On the other side, we observe that moral and social norms influence behavior.
- The norms can be represented in games either as (i) social preferences (altruism) or (ii) voluntary restrictions on strategy sets.
- The Prisoners' Dilemma is only a specific case of bilateral negative externalities.
- Willingness to impose negative externalities increases when views over legitimate and illegitimate harm differ.

Application: Grading

Absolute vs. relative grading

In many *competitive* economic environments, it is normal and 'desirable' to impose negative externalities on the opponents. The environment is even *designed* to exploit willingness of players to impose negative externalities on each other.

- 2 lazy students, $i = 1, 2$
- No learning: score $s_i = l \geq 0$
- Learning: score $s_i = h > 0$, cost $c > 0$
- Grade is an $(\alpha, 1 - \alpha)$ combination of *absolute* performance and *relative* performance (each multiplied by 2, w.l.o.g.), where $\alpha \in (0, 1)$:

$$g_i = 2\alpha s_i + 2(1-\alpha)\left[s_i - \tfrac{s_i + s_{-i}}{2}\right] = (1+\alpha)s_i - (1-\alpha)s_{-i}$$

- Learning increases the benchmark which decreases the opponent's score.
- Thanks to the relative performance component, *learning imposes a negative externality* on the opponent.

Absolute vs. relative grading

Row player's payoffs

	No learning	Learning
No learning	$2\alpha l$	$\alpha(l+h)-(h-l)$
Learning	$\alpha(l+h)+(h-l)-c$	$2\alpha h - c$

Best responses of Row player

- $(1+\alpha)(h-l)-c>0$: Learning strictly dominates No learning.
- $0>(1+\alpha)(h-l)-c$: No learning strictly dominates Learning.

What is efficient?

- $2\alpha(h-l)-c>0$: (Learning, Learning) is more efficient than (No learning, No learning).
- $0>2\alpha(h-l)-c$: (No learning, No learning) is more efficient than (Learning, Learning).

Absolute vs. relative grading

We exploit that

$$(1+\alpha)(h-l)-c > 2\alpha(h-l)-c.$$

Three environments exist:

- $0 > (1+\alpha)(h-l)-c > 2\alpha(h-l)-c$: (No learning, No learning) is NE and PE.
- $(1+\alpha)(h-l)-c > 0 > 2\alpha(h-l)-c$: (Learning, Learning) is NE but not PE.
- $(1+\alpha)(h-l)-c > 2\alpha(h-l)-c > 0$: (Learning, Learning) is NE and PE.

In the second environment, we observe a Prisoner's Dilemma in which lazy students compete over grades. The relative performance component motivates them to learn even if learning is inefficient for them. Designers of grading systems expect that the students are willing to impose negative externalities on each other (i.e., this is not considered 'immoral').

Application: Altruism

Example: The Prisoners' Dilemma

Altruism discourages production of negative externalities.

- Suppose robbers care about each other's sentences.
- Suppose each treats 1 year of his friend's sentence as $\frac{1}{2}$ year of his sentence.
- In other words, each robber is willing to prolong his sentence by 1 year if it saves 2 years of his friend's sentence.

	S	C
S	$-(2+\frac{2}{2}), -(2+\frac{2}{2})$	$-(5+\frac{1}{2}), -(1+\frac{5}{2})$
C	$-(1+\frac{5}{2}), -(5+\frac{1}{2})$	$-(4+\frac{4}{2}), -(4+\frac{4}{2})$

- Notice we had $-1 > -2$, but now $-3\frac{1}{2} = -(1+\frac{5}{2}) < -(2+\frac{2}{2}) = -3$.
- Also, we had $-4 > -5$ but now $-6 = -(4+\frac{4}{2}) < -(5+\frac{1}{2}) = -5\frac{1}{2}$.

Observing sentences is not enough to analyze the game!

The Samaritan's (Altruist's) Dilemma

When is altruism 'harmful'?

- donor, recipient
- donor's help/assistance is unconditional (i.e., simultaneous actions)
- the recipient shirks if he expects help
- the donor helps even if she expects that the recipient shirks[1]
- e.g., parents support their son (Help) or show 'tough love' (No help)
- a son works (Work) or loafs around (No work)

[1] Disincentive to exert costly effort after providing assistance (charity, aid, insurance, bailout) is often coined 'moral hazard'. This is not entirely correct. In microeconomics, moral hazard is reserved for disincentive to exert costly effort only under *private information* about effort.

The Samaritan's (Altruist's) Dilemma

Private payoffs

- son's no work: material values $(0, 0)$ for son and parents
- son's no work gives him leisure at value 1
- son's work: material values $(4, 5)$ for son and parents
- son's mental cost of work effort is 2 (plus he loses leisure)
- son's value of parental financial help is larger when he doesn't earn material values (3) than when he earns material values (1)
- son's payoffs (Column player):

	Work	No work
Help	$-2+4+1=3$	$1+3=4$
No help	$-2+4=2$	1

Private payoffs

- son's work generates parents' material value 5
- (selfish) parent's cost of financial help is 1
- (selfish) parents disregard son's leisure, mental cost of work effort, and his valuation of financial help

The game with selfish parents (Row player), with payoffs (u_p, u_s)

Selfish, Selfish	Work	No work
Help	$5 - 1$, 3	-1, 4
No help	$5, 2$	$0, 1$

Total *private* benefits in NE are 7.

The Samaritan's (Altruist's) Dilemma

Parental altruism

- (altruistic) parents reflect both their own a their son's private benefits, $u_a = u_p + 4u_s$
- i.e., they reflect son's leisure, cost of effort, material benefits and son's valuation of financial help
- (altruistic) parents' payoffs:

	Work	No work
Help	$4 + 4 * 3 = 16$	$-1 + 4 * 4 = 15$
No help	$5 + 4 * 2 = 13$	$0 + 4 * 1 = 4$

The game with altruistic parents, with payoffs (u_a, u_s)

Altruistic, Selfish	Work	No work
Help	$16, 3$	$15, 4$
No help	$13, 2$	$4, 1$

Total *private* benefits in NE decrease from $7 = 5 + 2$ to $3 = -1 + 4$.

Application: Battlefields/Advertising

Zero-sum games

- **Zero-sum game**: For any $(s_1, s_2) \in S_1 \times S_2$, total payoffs are constant

$$v_1(s_1, s_2) + v_2(s_1, s_2) = k.$$

- In zero-sum games, all outcomes are Pareto-efficient and therefore the game represents a *pure conflict* over the outcomes.
- The game is typically constructed such that a correct belief about an opponent's strategy that seeks to minimize your payoff allows you to avoid most adverse consequences.
- Then, a pure-strategy NE normally doesn't exist.
- Example: In Matching Pennies, each player has 1 coin and shows a head or tail. Player 1 wins if both coins are head or tail; Player 2 wins otherwise. There is no pure-strategy NE.
- This huge strategic uncertainty is also why it is funny to play zero-sum games.[2]
- Not all zero-sum games are like that.

[2] I suspect that fun also explains why computer scientists spend a lot of time analyzing these special games.

Battlefields/Advertising

- Attacker, Defender (Challenger, Incumbent)
- Battlefields $i = 1, 2$ (consumer groups)
- Attacker's strategy (a_1, a_2), where $a_i \in \mathbb{R}_+, a_1 + a_2 \le A$
- Defender's strategy (d_1, d_2), where $d_i \in \mathbb{R}_+, d_1 + d_2 \le D$
- Attacker's progress in Battlefield i
  - Symmetric power: $p_i = a_i - d_i$
  - Asymmetric power: $p_i = \max\{a_i - d_i, 0\}$
  - We can interpret that as whether big defense can or cannot cause extra harm to Attacker.
- Aggregation
  - Additive payoffs
  $$V_A = p_1 + p_2, V_D = -p_1 + (-p_2)$$
  - Attacker's best shot, Defender's weakest link,
  $$V_A = \max\{p_1, p_2\}, V_D = \min\{-p_1, -p_2\} = -\max\{p_1, p_2\}$$
  - Both aggregations are zero sum games, $V_A + V_D = 0$.

Battlefields/Advertising

We can narrow each strategy set to strategies in which *all* resources are spent:

$$d_1 + d_2 = D, a_1 + a_2 = A$$

- It never pays off strictly to not use all resources because payoffs are non-decreasing in the amounts of resources put into battlefields.
- Formally, any strategy that doesn't use all resources is a weakly dominated strategy. (More on that later.)

Game 1: Additive, Symmetric

- With the restricted set of strategy profiles, the payoff for each player is *constant* for any strategy profile. E.g., for the Attacker,

$$a_1 - d_1 + a_2 - d_2 = A - D.$$

- *Any strategy profile in which all resources are spent is a (pure-strategy) Nash equilibrium.*
- For each player, *each* unilateral deviation within the player's restricted strategy set leads to the *same outcome*.
- This is a game with zero strategic element.

Game 2: Additive, Asymmetric

The levels of progress are now *non-negative* (defense spending cannot be used for counter-attack):

- By additivity, Attacker doesn't care about allocation of her resources.
- By additivity, Defender doesn't care about allocation of her resources.
- By non-negativity, Attacker weakly prefers to concentrate on the weaker defense, where $d_i \leq d_{-i}$. (Weak defense attracts strong attack.)
- By non-negativity, Defender weakly prefers to concentrate on the stronger attack, where $a_j \geq a_{-j}$. (Strong attack attracts strong defense.)
- This is not mutually consistent.

A pure-strategy Nash equilibrium exists only if $D \geq 2A$.

Game 2: An example

Players

- d_1 policemen on East side of the street
- d_2 policemen on West side of the street, $d_1 + d_2 = D$
- a_1 activists run to the East side
- a_2 activists run to the West side, $a_1 + a_2 = A$

Arrests

- 1 policeman arrests only 1 activist
- East side arrests: $\min\{a_1, d_1\} = -\max\{-a_1, -d_1\}$
- West side arrests: $\min\{a_2, d_2\} = -\max\{-a_2, -d_2\}$

Activists payoff is convex on its resource line.

- $V_A = \max\{-a_1, -d_1\} + \max\{-a_2, -d_2\} = \max\{a_1 - d_1, 0\} + \max\{a_2 - d_2, 0\} - A$
- $V_A = p_1 + p_2 - A$, where $p_i = \max\{a_i - d_i, 0\}$

Police payoff is concave on its resource line.

- $V_D = \min\{a_1, d_1\} + \min\{a_1, d_1\} = -\max\{-a_1, -d_1\} - \max\{-a_2, -d_2\}$
- $V_D = -\max\{0, a_1 - d_1\} - \max\{0, a_2 - d_2\} + A = -p_1 - p_2 + A$

A pure-strategy Nash equilibrium exists only if $D \geq 2A$.

Game 3: Attacker's best shot/Defender's weakest link, Symmetric

- By best shot, Attacker *strictly* prefers to concentrate on the weaker defense. (Weak defense attracts strong attack.)
- By weakest link, Defender prefers to concentrate more on the stronger attack. (Strong attack attracts stronger defense.)
- This is not mutually consistent.
- Given Symmetry, Attacker is no longer indifferent over allocation of resources when $d_1 \geq A$ and $d_2 \geq A$.

Game 3: Attacker's best shot/Defender's weakest link, Symmetric

- Defender's (unconstrained) best response is to keep progress balanced.

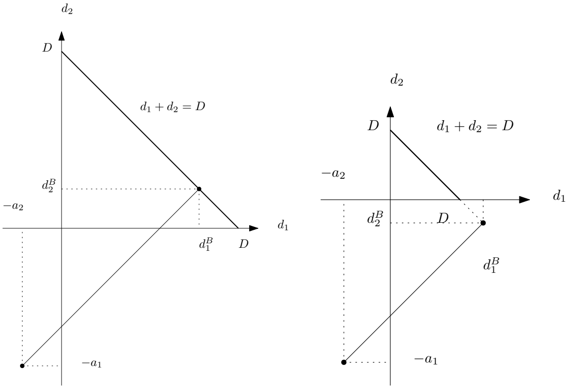
$$d_1 - a_1 = d_2 - a_2 = \frac{D - A}{2}.$$

- That is, $d_i^B = \frac{D-A}{2} + a_i$.
- But this is not feasible if either $d_i^B > D$ or $d_i^B < 0$!
- This occurs when A is large and (a_1, a_2) is highly asymmetric.
- Can we use infeasibility property to find a pure-strategy Nash equilibrium?

Game 3: Attacker's best shot/Defender's weakest link, Symmetric

When is the Defender's unconstrained best response in/feasible? With balance:

$$d_1^B + a_2 = d_2^B + a_1$$



(a) Feasible balance (b) Infeasible balance

Game 3: Attacker's best shot/Defender's weakest link, Symmetric

Suppose $D \geq A$.

- Defender has more resources and can (and will) always balance.
- However, for a balance to be NE, Attacker must not profitably deviate to an imbalance.
- But we know that Attacker profits from an imbalance.
- W.l.o.g., $a_1 \leq a_2$. Attacker redistributes a small $\Delta > 0$ from Battlefield 2 to Battlefield 1. (This is feasible since $a_2 \geq \frac{A}{2}$.)
- Attacker is strictly better off:

$$\max\{a_1 - d_1 + \Delta, a_2 - d_2 - \Delta\} = a_1 - d_1 + \Delta > a_1 - d_1 = \max\{a_1 - d_1, a_2 - d_2\}.$$

- Therefore, there is no NE in pure strategies. Defender balances any imbalance, but Attacker changes any balance into imbalance.

Game 3: Attacker's best shot/Defender's weakest link, Symmetric

Suppose $A > D$.

- Attacker has more resources.
- Suppose Attacker introduces an extreme imbalance, $(a_1, a_2) = (A, 0)$.
- Defender cannot balance an extreme imbalance due to low resources.
- Defender's best response to $(A, 0)$ is $(d_1, d_2) = (D, 0)$ such that $\min\{D - A, 0\} = D - A$.
- But then Attacker's best response to $(D, 0)$ is $(a_1, a_2) = (0, A)$ such that $\max\{-D, A\} = A$.
- Again, there is no NE in pure strategies. Defender tries to balance any imbalance by specializing into a single battlefield where both compete, but Attacker switches 'imperfect imbalance' into a 'perfect imbalance' by switching resources into the other battlefield where only Attacker competes.

Application: Auctions with complete information

Discrete sealed-bid auctions

- Two bidders bid only *integers* (discrete bids) in a sealed-bid auction.
- If both bidders submit an identical bid, the winner is determined by toss of a coin.
- Player 1 is **stronger** (more interested) than Player 2, $v_1 \gg v_2 \gg 0$ and valuations are integers.
- Either *first-price auction* or *second-price auction*.

- Which strategies are strictly dominated?
- Which strategies are weakly dominated?
- Find all Nash equilibria.

Each bid (except for zero) leads to either of three outcomes: win, tie, loss.

- win payoff: $\pi_i = v_i - b_i$
- tie payoff: $\pi_i = \frac{v_i - b_i}{2}$
- loss payoff: $\pi_i = 0$

b_i / b_{-i}	...	$v_i - 3$	$v_i - 2$	$v_i - 1$	v_i	$v_i + 1$	...
...	...	...	...	...	...	...	...
$v_i - 3$	...	$\frac{3}{2}$	0	0	0	0	...
$v_i - 2$	...	2	$\frac{2}{2}$	0	0	0	...
$v_i - 1$	...	1	1	$\frac{1}{2}$	0	0	...
v_i	...	0	0	0	0	0	...
$v_i + 1$	...	-1	-1	-1	-1	$-\frac{1}{2}$	...
...	...	...	...	...	...	...	...

Table: First-price auction: Row player's payoffs

Back to dominance . . .

- Let $s_i \in S_i$ and $s_i' \in S_i, s_i' \neq s_i$ be possible strategies for player i.
- s_i' is **strictly dominated** by s_i if for *any* possible profile of the other players' strategies, $s_{-i} \in S_{-i}$, player i's payoff from s_i' is strictly less than that from s_i:

$$v_i(s_i, s_{-i}) > v_i(s_i', s_{-i}) \quad \forall s_{-i} \in S_{-i}$$

- s_i' is **weakly dominated** by s_i if (i) for *any* possible profile of the other players' strategies, $s_{-i} \in S_{-i}$, player i's payoff from s_i' is weakly less than that from s_i,

$$v_i(s_i, s_{-i}) \geq v_i(s_i', s_{-i}) \quad \forall s_{-i} \in S_{-i},$$

  and (ii) there exists a profile of the other players' strategies, $s_{-i} \in S_{-i}$, such that player i's payoff from s_i' is strictly less than that from s_i,

$$\exists s_{-i} \in S_{-i} : v_i(s_i, s_{-i}) > v_i(s_i', s_{-i}).$$

First-price auction

Strict dominance

- There is no strictly dominated strategy because for each pair (b_i, b_i') such that $b_i' \neq b_i$, there exists a high b_{-i} such that $\pi_i(b_i, b_{-i}) = \pi_i(b_i', b_{-i}) = 0$.

Weak dominance

- Any excessive bid $b_i > v_i$ is weakly dominated by a *truthful* bid $b_i' = v_i$.
- Also, the truthful bid $b_i = v_i$ is weakly dominated by a bid $b_i' = v_i - 1$.
- Undominated strategies are $b_i \leq v_i - 1$.

Nash equilibria

NE are solved by necessary no-deviation conditions on the outcomes (*necessary equilibrium conditions*):

- Tie, Tie
  - If $b_1 = b_2 > v_2$ (overbidding), then the weaker player deviates to lose ($v_2 - b_2 < 0$).
  - If $b_1 = b_2 \leq v_2$, then the stronger player deviates to win ($\frac{v_1 - b_1}{2} < v_1 - b_1 - 1$.)
- Win, Loss
  - The weaker player cannot win. Either the weaker player overbids (then deviates to lose) or doesn't overbid (then the stronger player deviates to win).
  - The stronger player doesn't (strictly) deviate to loss, $b_1 \leq v_1$.
  - The weaker player doesn't (strictly) deviate to tie/win, $b_1 \geq v_2$.
  - The stronger player cannot decrease her bid and win; hence *a threat of the weak player's competing bid* must exist, $b_2 = b_1 - 1$.
- Nash equilibrium: Any $(b_1, b_1 - 1)$ such that $v_2 \leq b_1 \leq v_1$.
- Player 1 wins in each NE, but it is unclear on which NE the bidders coordinate.

Nash equilibria in weakly-undominated strategies

- We must disregard truthful $b_1 = v_1$ because v_1 is WD by $v_1 - 1$.
- We must disregard excessive $b_2 \geq v_2$ because it is WD by $v_2 - 1$.
- A unique NE profile in weakly-undominated strategies is $(b_1, b_2) = (v_2, v_2 - 1)$.
- The stronger player wins the item and grabs full surplus.

Second-price auction

- win payoff: $\pi_i = v_i - \min\{b_1, b_2\} = v_i - b_{-i}$
- tie payoff: $\pi_i = \frac{v_i - b_{-i}}{2} = \frac{v_i - b_i}{2}$
- loss payoff: $\pi_i = 0$

b_i / b_{-i}	...	$v_i - 3$	$v_i - 2$	$v_i - 1$	v_i	$v_i + 1$	$v_i + 2$	...
...	...	...	...	...	...	...	...	...
$v_i - 3$	...	$\frac{3}{2}$	0	0	0	0	0	...
$v_i - 2$	...	3	$\frac{2}{2}$	0	0	0	0	...
$v_i - 1$	...	3	2	$\frac{1}{2}$	0	0	0	...
v_i	...	3	2	1	0	0	0	...
$v_i + 1$	...	3	2	1	0	$-\frac{1}{2}$	0	...
$v_i + 2$	...	3	2	1	0	-1	$-\frac{2}{2}$	...
...	...	...	...	...	...	...	...	...

Table: Second-price auction: Row player's payoffs

Second-price auction

Strict dominance

- There is no strictly dominated strategy because for each pair (b_i, b_i') such that $b_i' \neq b_i$, we have $\pi_i(b_i, v_i) = \pi_i(b_i', v_i) = 0$.

Weak dominance

- The truthful strategy v_i weakly dominates any strategy $b_i > v_i$.
- The truthful strategy v_i weakly dominates any strategy $b_i < v_i$.

! In a *unique* profile of weakly undominated strategies, strategies are *truthful*.

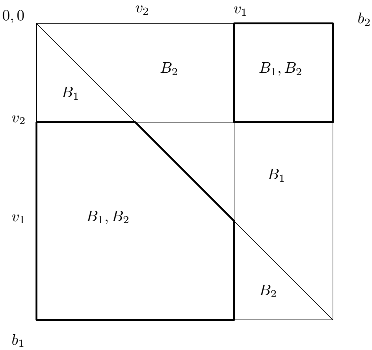
Nash equilibria

NE solved as mutual best responses. Best responses:

- If $b_{-i} < v_i$: Best response is $b_i > b_{-i}$ (win and pay low price).
- If $b_{-i} = v_i$: Best response is any $b_i \geq 0$.
- If $b_{-i} > v_i$: Best response is $b_i < b_{-i}$.

Second-price auction

- The weaker player wins above the diagonal.
- The stronger player wins below the diagonal.



Nash equilibria in weakly-undominated strategies

- We only check if $(b_1, b_2) = (v_1, v_2)$ is NE.
- More generally, if a unique profile of weakly-undominated strategies exists, it must be NE.

! Exactly like in first-price auction, if our solution is NE in weakly-undominated strategies, we predict that the stronger player wins the item and grabs full surplus.

Second-price auction with spiteful players

We observed that a NE exists where the weaker player wins!

- There is *double violation* of weak dominance.
- The strong player is loser as she plays a (weakly) dominated strategy of underbidding.
- The weak player is winner as she plays a (weakly) dominated strategy of overbidding.

See that the externalities of deviations are asymmetric:

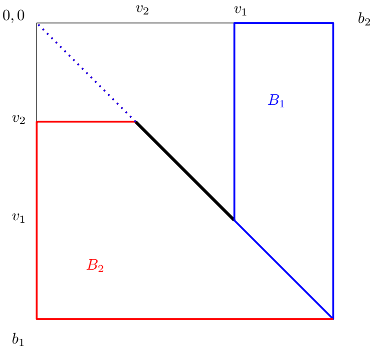
- A large deviation of the strong player (loser) harms the opponent.
- A large deviation of the weak player (winner) helps the opponent.

Suppose now that players have negative other-regarding preferences (spite).

- The strong player (loser) may consider the large deviation.
- Consider an ϵ-spite, $V_i = \pi_i - \epsilon\pi_{-i}$.
    - win payoff: $V_i = \pi_i$
    - loss payoff: $V_i = -\epsilon\pi_{-i}$

First-price auction with spiteful players

Without spite, win best responses are close to the diagonal, but loss best responses are intervals:



With spite, *all* best responses are close to the diagonal. NEs are identical.

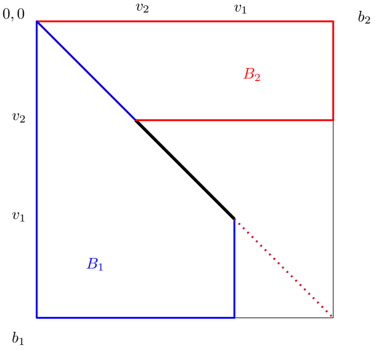
Second-price auction with spiteful players

Without spite, *all* best response are intervals. (See a few slides back.)

- In NE, either a strong player or a weaker player wins.

With spite, win best responses are intervals, but loss best responses are close to the diagonal.

- In NE, a strong player wins.



Summary: Nash equilibrium predictions

We can also map the structure of best responses (interval or close to the diagonal) to the environment (auction prices and existence of spite).

	Loss BR: diagonal	Loss BR: interval
Win BR: diagonal	First price, spite	First price, no spite
Win BR: interval	Second price, spite	Second price, no spite

	Loss BR: diagonal	Loss BR: interval
Win BR: diagonal	Stronger wins	Stronger wins
Win BR: interval	Stronger wins	Both may win

The object is efficiently allocated (stronger player wins) if (i) both players apply weak dominance and it is common knowledge that both players apply weak dominance, or (ii) the auction is first-price or (iii) the players are spiteful.

NE and IDE (Tadelis, p. 95, Exercise 5.3)

Consider a two-player game with 2 pure strategies for each player.

- If the game has a unique pure-strategy NE, (s_1^*, s_2^*), does any other IDE exist?
- Given NE, we have only 3 types of best responses at (s_1^*, s_2^*).

Strict, Strict	s_2^*		s_2'
s_1^*	s^*	$\leftarrow$	
	$\uparrow$		
s_1'			

Strict, Weak	s_2^*		s_2'
s_1^*	s^*	$=$	
	$\uparrow$		
s_1'			

Weak, Weak	s_2^*		s_2'
s_1^*	s^*	$=$	
	$\Vert$		
s_1'			

NE and IDE

- Consider Strict, Strict.

Strict, Strict	s_2^*		s_2'
s_1^*	s^*	$\leftarrow$	
	$\uparrow$		
s_1'			

- By uniqueness of NE, at least one remaining best response is unique/strict.
- We have 2 sub-types. In each we have a unique IDE (e.g., s_2' eliminated first).

	s_2^*		s_2'
s_1^*	s^*	$\leftarrow$	
	$\uparrow$		$\Vert$
s_1'		$\leftarrow$	

	s_2^*		s_2'
s_1^*	s^*	$\leftarrow$	
	$\uparrow$		$\uparrow$ or $\downarrow$
s_1'		$\leftarrow$ or $\rightarrow$	

NE and IDE

- Now consider Weak, Weak. (Weak means Indifference.)

Weak, Weak	s_2^*		s_2'
s_1^*	s^*	$=$	
	$\|$		
s_1'			

- By uniqueness of NE, both remaining best responses are strict.
- But this generates a NE at (s_1', s_2').
- This type of NE at s^* doesn't exist.

NE and IDE

- Now consider Strict, Weak

Strict, Weak	s_2^*		s_2'
s_1^*	s^*	$=$	
	$\uparrow$		
s_1'			

- By uniqueness of NE, both remaining best responses are strict ($\downarrow$, then $\leftarrow$).
- We don't have a unique IDE. All 4 profiles survive IESDS.

Strict, Weak	s_2^*		s_2'
s_1^*	s^*	$=$	
	$\uparrow$		$\downarrow$
s_1'		$\leftarrow$	

Application: Oligopoly competition

Oligopoly/duopoly theory

Motivation

- Each firm sells a quantity of goods at a certain price.
- Is the price set by the firm or is the quantity set by the firm? Or is there any other trading/price-making mechanism?
- If firms can announce prices and also plan quantities, does it make a difference how their strategy set is defined?

Two classic models

- Quantity (Cournot or Cournot-Nash) competition
- Price (Bertrand) competition

Are prices and quantities identical or not?

Cournot and Betrand duopoly

Assumptions

- 2 identical firms (duopoly): $i = 1, 2$
- a **constant** marginal cost $c > 0$; i's cost of production of $q_i \geq 0$ is cq_i
- a homogenous good
- market demand price is **linear**: $p(Q) = a - bQ$, where $Q \equiv q_1 + q_2 \leq \frac{a}{b}$
- for participation of firms, we obviously need $c < p(0) = a$
- monopolist's (total market) demand: $Q^m(p) = \frac{a-p}{b}$

Trading mechanisms

- Cournot duopoly: market 'clears' at a single price $p(Q)$ if $Q \leq \frac{a}{b}$ (otherwise clears at $p = 0$ and some supply is unsold); market clearing is not explicitly described
- Bertrand duopoly: by announcing a fixed price and nothing else, each firm commits to providing any positive quantity; consumers then trade *only* with the cheaper producer

Cournot duopoly

- strategy of Firm i is $q_i \in \mathbb{R}^+$ (continuous, not discrete!)
- payoff (profit) of Firm i is **continuous**

$$\pi_i(q_i, q_{-i}) = \begin{cases} q_i[a - bq_i - bq_{-i} - c], & \text{if } q_i + q_{-i} \leq \frac{a}{b} \\ -q_i c, & \text{otherwise.} \end{cases}$$

- best response is

$$B_i(q_{-i}) = \arg\max_{q_i \in \mathbb{R}^+} \pi_i(q_i, q_{-i})$$

By FOC, in the interior solution, $a - bq_i - bq_{-i} - c - bq_i = 0$.

$$B_i(q_{-i}) = \begin{cases} 0 & \text{if } a - bq_{-i} - c \leq 0 \\ \frac{a - bq_{-i} - c}{2b} \geq 0 & \text{otherwise} \end{cases}$$

- given symmetry of firms, best response functions are identical, $B_1(q) = B_2(q)$; thus, we will look for a symmetric equilibrium, $q_1 = q_2$
- insert symmetry $q \equiv q_1 = q_2$ into any best response function, $B_i(q) = q$

$$a - 2bq - c - bq = 0$$

- In Nash equilibrium, $Q^* = 2q^* = \frac{2}{3}\frac{a-c}{b}$ and $p^* = a - bQ^* > c$.

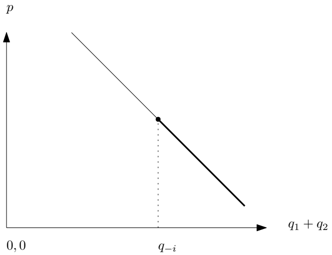
Bertrand duopoly

- strategy of Firm i is $p_i \in \mathbb{R}^+$
- total quantity sold at a market price $p \in [0, a]$ satisfies $Q^m(p) = \frac{a-p}{b}$
- payoff (profit) of Firm i is **discontinuous**

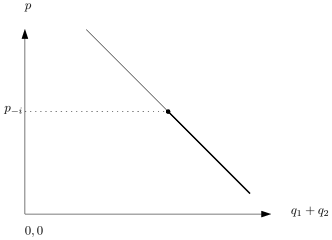
$$\pi_i(p_i, p_{-i}) = \begin{cases} 0 & \text{if } p_i > p_{-i} \\ (p_i - c)\frac{Q^m(p)}{2} = (p_i - c)\frac{a-p_i}{2b} & \text{if } p_i = p_{-i} \\ (p_i - c)Q^m(p) = (p_i - c)\frac{a-p_i}{b} & \text{if } p_i < p_{-i} \end{cases}$$

- Firm i selects an outcome: win, tie, or loss.
- Best response is (i) loss if $p_{-i} < c$ (set any p_i such that $p_i > p_{-i}$), and (ii) tie/loss if $p_{-i} = c$.
- Let $p^m = \frac{a-c}{2b}$ be the optimal monopolist's price. (iii) If $p_{-i} > c$ and $p_{-i} \leq p^m$, the best response is win with the maximal p_i such that $p_i < p_{-i}$. (iv) If $p_{-i} > c$ and $p_{-i} > p^m$, the best response is win with $p_i = p^m$.
- A unique profile with mutual best responses (NE) is $p_1 = p_2 = c$. Here, $Q^* = \frac{a-c}{b}$.
- Notice: If any player deviates to $p_i > c$, her payoff is *unchanged* (a weak NE), but the new profile is *not* a NE!

Look at implementable outcomes.

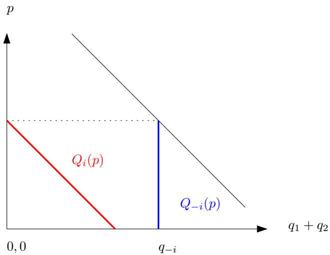


(a) Quantity-setter against Quantity-setter

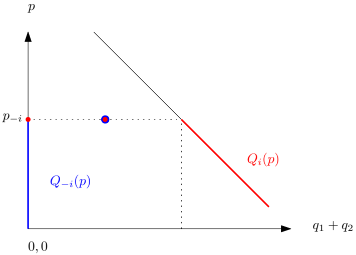


(b) Price-setter against Price-setter

In aggregate perspective, the implementable outcomes are identical.



(a) Quantity-setter against Quantity-setter



(b) Price-setter against Price-setter

However, the total sales are divided differently.

Key differences

Cournot (quantity) competition

- Each player is a monopolist on a *residual* market.
- Each player faces demand that is *continuous* in price.
- When competing, profits of the opponent are affected *continuously*: a zero effect on sales and a continuous effect on price.

Bertrand (price) competition

- Each player is a monopolist on a *full* market, but constrained by a price limit.
- Each player faces demand that is *discontinuous* in price.
- When competing, profits of the opponent are affected *discontinuously*: a large discontinuous effect on sales and a continuous effect on price.

Which model is more relevant?

Inflexible production and flexible prices

- Think of an input that must be invested into in advance (e.g., a factory is built, a professional is trained).
- Production has prohibitively large variable costs (e.g., no way to substitute for factory/professionals).
- The company has experienced sales team that easily adjusts prices to demand.

Flexible production and inflexible prices

- Think of production only through variable inputs (e.g., translators' hours).
- The company cannot negotiate the price (e.g., foreign owner requires a fixed price).

Are other models closer to Cournot or Bertrand?

Limited production capacity

- Idea: Capacity decision is a long-term decision, pricing decision is a short-term decision. Long-term decisions are less flexible.
- Consider *price* (Bertrand) competition within predetermined capacity *limits on production*.
- Assume zero cost to setting a capacity limit.
- We analyze predetermined capacity as a limit to the amount produced (*maximal supply*), not as the specific amount produced (*fixed supply*; solved in Belleflamme and Peitz, 2015).
- With a capacity limit and price competition, only part of the market can be served if a price decreases (not all consumers can buy cheaply = *rationing*).
- With rationing, the competitor with a high price is not afraid of losing all consumers.

Will capacity limits reduce price competition? What capacities will be set?

Price competition (short-term)

Can we have heterogeneous prices in NE?

- Firms with capacities (q_1, q_2) set prices $p_1 < p_2$.
- *Efficient rationing*: Suppose consumers with the higher WTP are served first (think of queues or secondary markets). Consumers with the lower WTP are *residual demand*.
- If $q_1 \geq Q^m(p_1)$, no rationing. Firm 2 has zero residual demand.
  - If $p_1 > c$, Firm 2 deviates to $p_2 < p_1$ (competes) to earn profits.
  - If $p_1 = c$, Firm 2 doesn't deviate (doesn't compete). Firm 1 increases profits by slightly increasing its price p_1 (competes less intensively).
- If $q_1 < Q^m(p_1)$, then the good of Firm 1 is rationed. Firm 1 increases profits by slightly increasing its price p_1 (competes less intensively).

Price are not heterogeneous in NE.

Price competition (short-term)

Which homogenous price p occurs in NE?

- Let $p^Q > c$ be the price for which all capacities are used, $Q^m(p^Q) = q_1 + q_2$.
- *Unused capacities*, $p > p^Q$: Any firm with unused capacity sets a slightly lower price to (massively) increase its sales (up to $\min\{q_i, Q^m(p)\}$) (steals full demand).
- *Excess demand*, $p < p^Q$: Each firm sets a higher price to sell its full capacity q_i at a higher price (no concern about being more expensive).
- *Used capacities*, $p = p^Q$. A lower price only lowers profits. A higher price makes Firm i more expensive; sales drop.
  - This drop of sales increases profits of Firm i if on a *residual* market, where residual demand is $Q^m(p^Q) - q_{-i}$, the monopolist's price satisfies $p^m_{res} \geq p^Q$.
  - If the monopolist's price satisfies $p^m_{res} \leq p^Q$, Firm i keeps price p^Q (no deviation).[3]
- If NE exists, then $p_1 = p_2 = p^Q$, where p^Q is above the monopolist's prices on the respective residual markets.

[3]Notice p^m_{res} equals to p^m when $q_{-i} = 0$, but typically differs when $q_{-i} > 0$.

Capacity competition (long-term)

- With low capacities, the price p^Q at which capacities are exactly used is large. Thus, the monopolist prices p^m_{res} on residual markets are likely smaller than p^Q, and the subsequent price competition clears the market at full capacities. (Firms are tempted to decrease prices but are at their capacity limits!)
- With high capacities, the price p^Q at which capacities are exactly used is small. Thus, the monopolist prices on residual markets p^m_{res} may be larger than p^Q, and in the subsequent price competition, NE in pure strategies doesn't exist.[4]
- If a unilateral deviation to a high capacity is unattractive, capacity competition is **Cournot competition**.

Price competition with limited production capacity limits is (almost) like Cournot competition.

[4] At this moment, we don't have tools to solve what is the expected equilibrium value of price competition with such high capacities.

Modern advanced oligopoly models

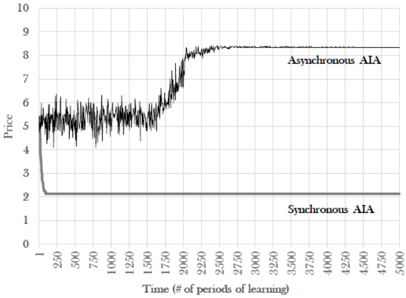
- dynamic games
- fluctuating demand
- production capacities fixed in the short run (i.e., adjustment costs)
- inventories
- posted prices fixed in the short run
- intermediation and platforms: firms may delegate price-setting to agents (e.g. to internet portals, auction houses, real-estate agents); however, reservation prices exist

$=$ capacity, inventory and price decisions are closely intertwined

Application: Algorithmic pricing (Asker et al., 2021)

Will AI algorithms promote or weaken market competitiveness?

- Bertrand duopoly, homogeneous good
- Monopoly price 10, competitive (Nash) price 2
- Asynchronous learning: AI learns only from own profit.
- Synchronous learning: AI learns from own profit and additional information.



Some agents are strategically naïve, some are strategically sophisticated.
Sophistication is measured by the depth of reasoning (the number of steps in iterative reasoning).

Definition: Level-k players

Level-k player makes at least k steps of iterative reasoning and believes that *all other players* are level-$k-1$ players.

- Level-0: no strategic thinking
- Level-1: believes that population consists of level-0 players
- Level-2: believes that population consists of level-1 players
- ...
- ...

Level-k players

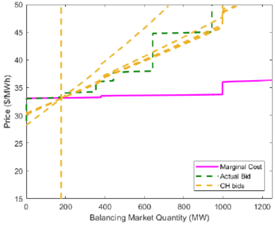
Example: Keynesian beauty contest

- Choose a number from $[0, 100]$ that is closest to p times the average of numbers of all players.
- A large number (continuum) of players.
- Typically, $p = \frac{1}{2}$. (Keynes described $p = 1$.)
- Level-0 selects randomly on $[0, 100]$ (uniformly).
- Level-1 expects 50, and selects 25.
- Level-2 expects 25, and selects 12.5.
- ...
- A unique Nash equilibrium is that all players set zero = as if all players make an infinite number of steps in iterative reasoning (level-∞ players).

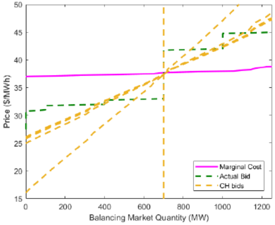
Texas electricity market (Hortaçsu et al., 2019)

- Data: bids and marginal costs in Texas electricity market
- Bids by some firms depart significantly from Nash equilibrium.
- Larger firms: more resources devoted to trading operations, including large trading floors, several employees with a PhD, and specially developed software applications to gather market information and compute bids
- Smaller firms: thinly staffed, with employees who appeared to specialize in operational/engineering details of the plant rather than in trading, less time to devote to balancing market bidding
- They identify and estimate heterogeneity in levels of strategic sophistication across power generation firms using the Cognitive Hierarchy model.
- Cognitive Hierarchy model: Defines a level-k player as a player that makes k steps of iterative reasoning and believes that all other players are distributed between level-0 and level $k-1$ players according to Poisson distribution.

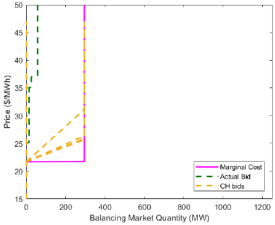
Texas electricity market (Hortaçsu et al., 2019)



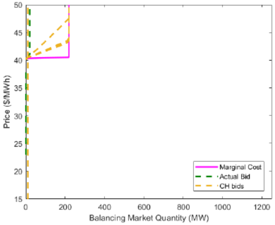
(a) Large Firm



(b) Medium Firm

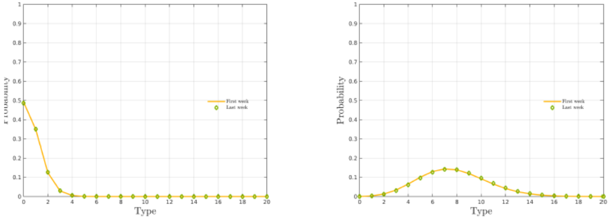


(c) Small Firm



(d) Very Small Firm

Texas electricity market (Hortaçsu et al., 2019)



(a) Smallest Firm (b) Largest Firm

FIGURE 8: Estimated Distributions of Types and Learning

What are the effects of mergers?

- cost synergies
- increased market power due to increased concentration
- ! but also more sophisticated bidding

Technology choice

Suppose each firm (non-cooperatively and simultaneously) decides whether to commit to price-setting or quantity-setting.

- For example, each firm invests either into *technology with flexible production* (price-setting) or *technology with inflexible production* (quantity-setting).
- 'Quantity' now means the specific amount produced (fixed supply), not the limit to the amount produced (maximal supply).
- (Quantity, Quantity) outcome yields Cournot competition with positive profits.
- (Price, Price) outcome yields Bertrand competition with zero profits.

Key question: Are firms better off with the ability to adjust production but not price, or with the ability to adjust price but not production?

- Does it pay off to keep (Quantity, Quantity) or change to (Price, Quantity)?
- Does it pay off to keep (Price, Quantity) or change to (Price, Price)?

Shall I commit to price or quantity?

If firms prefer price-setting (flexible production, inflexible price) in both cases, we have a Prisoners' Dilemma.

- NE = SDE is (Price, Price).
- Pareto-efficient is (Quantity, Quantity).

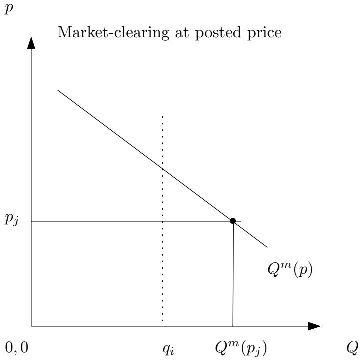
	Quantity		Price
Quantity	Cournot	$\rightarrow$	
	$\downarrow$		$\downarrow$
Price		$\rightarrow$	Bertrand

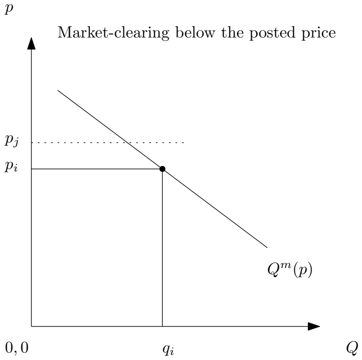
But is duopoly with this technology choice a Prisoners' Dilemma?

- We need to analyze (Price, Quantity) outcome.
- Like with Bertrand competition, we assume market clears without specifying the mechanism.

Shall I commit to price or quantity?

How does the market clear when it observes a quantity q_i and a price p_j?





i is Quantity-setter and j is Price-setter.

Shall I commit to price or quantity?

If $q_i \leq Q^m(p_j)$, market clears *at the posted price* p_j.

- Firm i sells q_i for price p_j, and Firm j sells *residual demand*.
- Market clearing works as if the market *slightly* prefers the good from i-firm which is already produced.
- Sales are $(q_i, Q^m(p_j) - q_i)$.

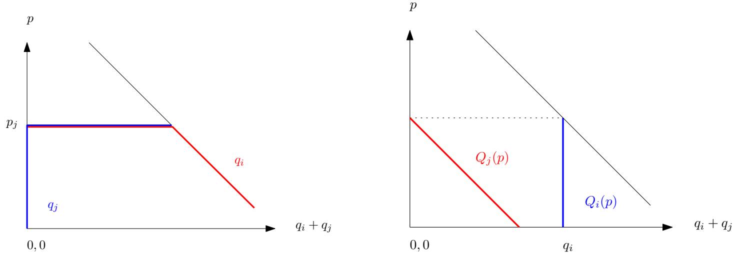
If $q_i > Q^m(p_j)$, market clears *below the posted price*, $p_i < p_j$.

- The market clears such that all promised quantity is sold, $Q^m(p_i) = q_i$.
- Sales are $(q_i, 0)$.

The quantity-setting firm *always* sells its production.

Shall I commit to price or quantity?

Implementable outcomes



(a) Quantity-setter against Price-setter (b) Price-setter against Quantity-setter

Best response of Quantity-setter

- Market price as a function of q_i is *flat* (clearing at p_j) and then *decreasing* (clearing below p_j).
- The quantity-setter optimizes production reflecting this market price function.
- If $p_j < c$, the quantity-setter sets $q_i = 0$. (lower corner)
- If $p_j = c$, the quantity-setter sets any quantity $q_i \in [0, Q^m(c)]$ (flat part).
- If $p_j > c$, the quantity-setter can *steal all consumers* by setting $q_i \geq Q^m(p_j)$.
- Then, the quantity-seller is a *monopolist on a full market* $Q^m(p_i)$, restricted by an *upper bound* on the price, $p_i \leq p_j$.
- If the bound p_j is above the monopoly price, $p_j \geq p^m$: The quantity-setter sets $q_i = Q^m(p^m) > Q^m(p_j)$ to reach the monopoly price. (decreasing part)
- If $p_j < p^m$: The quantity-setter sets $q_i = Q^m(p_j)$ to get as close to the monopoly price as possible. (almost upper corner)

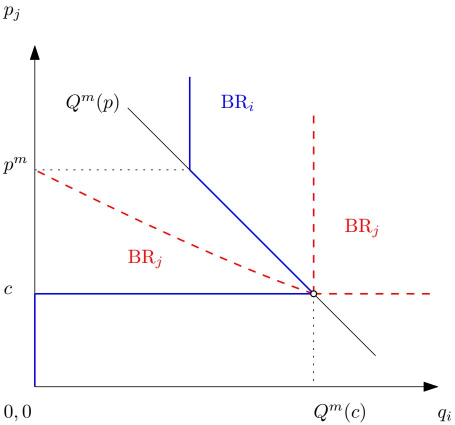
In any case, if $p_j > c$, then $q_i < Q^m(c)$.

Shall I commit to price or quantity?

Best response of Price-setter

- He knows that q_i is always sold.
- The price-setter is a *monopolist on a residual market* $Q^m(p_j) - q_i$.
- Sales are $Q^m(p_j) - q_i$.
- If q_i covers demand at zero profits, $q_i \geq Q^m(c)$: The price-setter sets any $p_j \geq c$ and sells nothing.
- If $q_i < Q^m(c)$: The price-setter is a monopolist on a residual market.
- The optimal price is $p_j = p^m$ if $q_i = 0$ (residual market is full market = monopoly).
- The optimal price is $p_j \in (c, p^m)$ if $q_i \in (0, Q^m(c))$ (residual market is a smaller market).
- In any case, the optimal monopolist's price is to largely 'undercut' the (implicit) price p_i (i.e., $q_i = Q^m(p_i)$), $p_j < p_i$, and generate positive sales.

Shall I commit to price or quantity?



Shall I commit to price or quantity?

- (Price, Quantity) outcome is Bertrand competition.
- The price-setter behaves as a (constrained) monopolist on the entire market and the quantity-setter as a monopolist of the residual market.
- We have two equilibria in this game: (Quantity, Quantity) is a strict Nash equilibrium, (Price, Price) is a weak Nash equilibrium.
- A unique weakly undominated strategy is Quantity.

	Quantity		Price
Quantity	Cournot	$\leftarrow$	Bertrand
	$\uparrow$		$\Vert$
Price	Bertrand	$=$	Bertrand

- Gaining price commitment here doesn't imply any advantage; it only starts an 'early race to the bottom' that can be avoided if both stay away from the race.
- Intuitively, each player prefers that price setting is left to the 'market mechanism' in later stage instead of competitive price setting in an early stage.

Application: Budgeting

Tragedy of commons

Commons

- any rival but non-excludable (open-access) resource
- environmental resources: common land, atmosphere, oceans, rivers, fish stocks
- tragedy of commons = the resource is overexploited (Pareto-inefficient outcome) if access is not regulated

Non-environmental examples

- budgetary commons: overspending when the authority on spending is decentralized (ministers in a cabinet, divisions in a company, departments at the university)
- 'pork-barrel' decisions in the legislatures/committees ("poslanecké pomníčky, malá domu, porcování medvěda")
  - in 2006, 20 bn CZK in proposals by MPs (finally 6.4 bn CZK) led to a vote of Finance Minister against his own budget proposal
- overeating when a bill in group dinners is shared
- addiction to coffee after free coffee machines are introduced in organizations

Rundale System and the tragedy of commons

Many of the stone walls in Ireland were built after the Great Famine of 1840. This replaced the previous open system of farming, known as the Rundale System.

Model of budgetary commons

socio-economic groups	$i \in \{1, \ldots, n\}$
tax paid by i	t_i
expenditures for a collective good of group i	x_i
balanced budget	$\sum_i t_i = \sum_i x_i$
fixed and equal cost-sharing (e.g., share of VAT base)	$t_i = \frac{1}{n} \sum_i x_i$
quasilinear utility function (natural log)	$u_i(x_i, t_i) = \log(x_i) - t_i$

- What is Pareto efficient? Imposing symmetry, $x = x_1 = \ldots = x_n$, hence $t_i = x$:

$$x^{SO} = \arg\max\{\log x - x\} = 1$$

- Nash equilibrium? Suppose each group i proposes x_i. Each group covers only n-th of the total cost, which is also n-th of the marginal cost of x_i.

$$x_i^* = \arg\max\left\{\log x_i - \frac{x_i + x_{-i}}{n}\right\} = n > 1$$

- The higher is fragmentation of decision-making (larger n), the larger is overspending.

Overspending ‘heuristic’

Mine or Ours? Neural Basis of the Exploitation of Common-Pool Resources

Mario Martinez-Saito, Sandra Andraszewicz, Vasily Klucharev, Jörg Rieskamp

Social Cognitive and Affective Neuroscience, nsac008,
https://doi.org/10.1093/scan/nsac008
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Abstract

Why do people often exhaust unregulated common (shared) natural resources but manage to preserve similar private resources? To answer this question, in this study we combine a neurobiological, economic, and cognitive modeling approach. Using functional magnetic resonance imaging on 50 participants, we show that a sharp decrease of common and private resources is associated with deactivation of the ventral striatum, a brain region involved in the valuation of outcomes. Across individuals, when facing a common resource, ventral striatal activity is anti-correlated with resource preservation (less harvesting), whereas with private resources the opposite pattern is observed. This indicates that neural value signals distinctly modulate behavior in response to the depletion of common versus private resources. Computational modeling suggested that over-harvesting of common resources was facilitated by the modulatory effect of social comparison on value signals. These results provide an explanation of people’s tendency to over-exploit unregulated common natural resources.

Keywords: Common Goods, Tragedy of the Commons, Social Competition, Ventral Striatum, Reinforcement Learning, Social Comparison

Model of budgetary commons

- Social optimum can be achieved even in a non-cooperative game.
- But the *cost-sharing rule* would have to force each group to pay the *full social marginal cost* of the group's marginal benefit.
- In our case of collective goods, the cost-sharing rule must impose

$$\frac{dt_i(x_1,\ldots,x_n)}{dx_i} = 1.$$

- A solution that balances the budget is simply to pay only for own benefits:

$$t_i(x_1,\ldots,x_n) = x_i$$

- Then, Nash equilibrium is $x_i^* = \arg\max\{\log x_i - x_i\} = 1 = x^{SO}$.
- But these properties are difficult to design in tax systems because (i) the structure of taxes is set independently on the structure of spending, and (ii) tax bases are shared.

Model of budgetary commons

Who overspends more, small or large divisions (parties, departments)?

- Consider a large Division A and a small Division B.
- Division A unites $m > n - m$ groups.
- Division B unites $n - m < m$ groups.
- Best responses:

$$x_A^* = \arg\max m \left\{ \log x_A - \frac{m x_A + (n-m) x_B}{n} \right\} = \frac{n}{m} \geq 1$$

$$x_B^* = \arg\max (n-m) \left\{ \log x_B - \frac{m x_A + (n-m) x_B}{n} \right\} = \frac{n}{n-m} \geq 1$$

- Since $m > n - m$, clearly $x_A^* < x_B^*$.
- The large division internalizes cost (tax) externalities more than the small division.

! Groups represented by the small division are better off (*benefit of smallness*):

$$u_B - u_A = \log x_B^* - t - \log x_A^* + t = \log x_B^* - \log x_A^* > 0$$

How to address overspending?

- Delegation/centralization
  - strong authority of a central agent (FM, PM); punishments for defecting spending ministers
  - large agenda-setting powers to the executive
  - the central agent is able to monitor and control the flow of expenditures during the year and to prevent spending departments from overspending their appropriations; tight limits on any changes in the budget law during the fiscal year; limits on the use of supplementary budgets
  - predicted for single-party governments
- Fiscal targets
  - binding budgetary agreements on a set of fiscal targets negotiated among all members of the executive; e.g., targets often derived from medium-term fiscal programs or coalition agreements among the ruling parties
  - FM has extra information, but no extra powers
  - more weight on the role of the legislature monitoring; more information rights and stronger amendment power
  - predicted for coalitional cabinets and minority governments
  - the key is credibility

Fiscal governance in the EU 15

Table 1
Electoral system, government constellation and type of fiscal governance, 1980–2000

	Electoral system	District magnitude	Average no. of parties in government	Change in coalition or ruling party	Mean ideological range	Ideological range small or large	Frequency of pre-electoral pacts	Predicted form of governance
Austria	2-tier PR	20/91	1.9	37.5	0.26	L 84-99, S 00-	0.71	C 84-99, D 00-
Belgium	PR	23	4.5	63.6	0.36	L	0.59	C
Denmark	2-tier PR	7/175	2.5	60.0	0.34	L	0.33	C
Finland	PR	13	3.9	66.7	0.41	L	0.14	C
France	Plurality	1	1.6	53.8	0.11	S	0.71	D
Germany	2-tier PR	1/603	1.9	30.0	0.04	S	0.93	D
United Kingdom	Plurality	1	1.0	20.0	0.00	S	0.14	D
Greece	Reinforced PR	6	1.0	42.8	0.02	S	.	D
Ireland	STV	4	1.8	77.8	0.20	L 85-97, S 98-	0.50	C 85-97, D 98-
Italy	2-tier PR	19/625	4.2	23.5	0.13	L 85-96, S 97-	0.31	C 85-96, D 97-
Luxembourg	PR	14	2.0	40.0	0.20	L	0.33	C
Netherlands	PR	150	2.4	71.4	0.30	L	0.38	C
Portugal	PR	12	1.7	18.2	0.14	S	0.78	D
Spain	PR	6	1.0	28.6	0.07	S	1.00	D
Sweden	2-tier PR	11/350	1.5	40.0	0.22	L	0.41	C

Example: Fiscal governance index by the World Bank

F Fiscal rules

- *Fiscal limits*: fiscal rules placing limits on Executive fiscal policy discretion; can the Executive Branch propose waiving or amending the limits
- *Medium Term Frameworks*: consistent medium-term fiscal framework (targets or ceilings for expenditures, deficits and revenues)
- *Borrowing limits*: limits on the borrowing activity of lower levels of government; explicit or implicit guarantee of the borrowing activity of lower levels of government; possibility to borrow against future appropriations for operating costs
- *Reserve Funds*: central reserve funds to meet unforeseen expenditures; regulations defining the permitted uses of the budget reserves; authorities for approving allocations from the reserves

T Transparent procedures

- comprehensive Budget Document; extra-budgetary funds

H Hierarchical procedures

- *Within the Executive*: fixed spending limits for initial Ministry spending plans; the last word; resolution of disputes between Ministries and the central budget authority; percentage of the initial executive branch budget decided by the President/Prime Minister/Principal Executive
- *Executive-Legislative relations*: consequences of rejection of the budget by Legislature; restrictions on amendments and their legal base
- *Cash management*: instruments to monitor budget execution; can the Central Budget authority withhold funds that are appropriated, but not available on a legal or entitlement basis; can it withhold funds for entitlement programs or other areas where legal obligations have been made on behalf of the state


WE'RE SUPPOSED TO HAVE OUR BUDGET APPROVED BY TUESDAY BUT THAT'S NOT WRITTEN IN STONE.
YES IT IS. I HAVE IT RIGHT HERE.
IT WAS KIND OF LUCKY BECAUSE THIS IS THE ONLY ONE I BROUGHT TO THE MEETING.
www.dilbert.com
scottadams@aol.com
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AND THEN WE'LL BUILD A SECOND PROTOTYPE AND. . .
IT'S NOT IN THE BUDGET
OH, SUDDENLY IT'S MY FAULT FOR CARING ABOUT THE BUDGET.
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What are cons of fiscal rules?

Delegation (monsters) brings power imbalance.

- overspending of winner
- underspending of losers
- ! Even symmetric groups may consider delegation too risky.

Fiscal limits (stones) may constrain the groups asymmetrically.

- even under uniform caps, asymmetric groups have asymmetric benefits
- for example, smaller groups are more constrained
- ! Asymmetrically constrained groups may veto fiscal limits unless compensation is provided.

Delegation (symmetric groups)

- Delegation reduces overspending but creates a single winner (e.g., Finance Minister) at the expense of multiple losers.
- Suppose that if a group spending x_i is critically small, $x_i < s < x^{SO} = 1$, there is a crisis. Therefore, FM provides $x_i \geq s$.
- Ex post allocation by the winner
  - $x = s < 1$ for all losers: underspending
  - $x = n > 1$ for own group: overspending
- Ex ante: each group gets power with probability $\frac{1}{n}$
  - expected utility of delegation:

$$\tfrac{1}{n}\log n + \tfrac{n-1}{n}\log s - \frac{n+(n-1)s}{n} = \tfrac{1}{n}(\log n - n) + \tfrac{n-1}{n}(\log s - s)$$

- What are ex ante preferences over delegation?

$$\tfrac{1}{n}(\log n - n) + \tfrac{n-1}{n}(\log s - s) > \log n - n$$

$$\tfrac{n-1}{n}(\log s - s) > \tfrac{n-1}{n}(\log n - n)$$

$$\log s - s > \log n - n$$

- Delegation is preferred (i) if it brings large total savings (large n), and (ii) if a loser can protect himself against large underspending (large s).

Spending caps

Symmetry

- Consider a non-discriminatory (uniform) cap X upon each item: $x \leq X$
- Cap effective/binding only if $X < n$. (Each minister sets $x = X$, and $t = X$.)
- Utility under effective cap: $u(X) = \log x - t = \log X - X$
- Identical preferences over cap: $\arg\max(\log X - X) = 1 = x^{SO}$
- All parties agree on the socially optimal cap.

Asymmetry

- Party A with m groups, Party B with $n - m < m$ groups
- Recall $1 < x_A^* = \frac{n}{m} < \frac{n}{n-m} = x_B^* < n$
- Cap ineffective: $X \geq x_B^*$
- Cap partially effective: $X \in [x_A^*, x_B^*]$
- Cap fully effective: $X < x_A^*$

Local preferences over the uniform spending cap (asymmetry)

- If X is partially effective, Party A wants a stricter cap, and B wants a looser cap:

$$X_A = \arg \max_{X \in [x_A^*, x_B^*]} \left\{ \log x_A^* - \frac{m x_A^* + (n-m)X}{n} \right\} = x_A^*$$

$$X_B = \arg \max_{X \in [x_A^*, x_B^*]} \left\{ \log X - \frac{m x_A^* + (n-m)X}{n} \right\} = x_B^*$$

- If X is fully effective, both parties want the socially optimal cap:

$$X = \arg \max_{X \in [0, x_A^*]} (\log X - X) = x^{SO}$$

- Party A has a single local maximum at x^{SO}.
- Party B has two local maxima at (x^{SO}, x_B^*) and prefers the socially optimal cap if:

$$\begin{aligned} -1 = \log 1 - 1 &\geq \log \frac{n}{n-m} - 2 \\ 1 &\geq \log \frac{n}{n-m} \\ e &\geq \frac{n}{n-m} \\ \frac{1}{2} < \frac{m}{n} &\leq \frac{e-1}{e} \doteq 0.63 \end{aligned}$$

- For consensus on the uniform cap $X = x^{SO}$, asymmetry must be small.

Application: Instrumental voting

Motivation

- For expressive voters, a vote is an expression of preferences, and 'sincere' vote gives an expressive benefit (consumption).
- For instrumental voters, a vote is an instrument to achieve a preferred voting outcome, not an expression of preferences.
- For instrumental voters, voting *sincerely* (i.e., to support the first alternative) may be suboptimal.
- In particular, rational instrumental voters avoid casting *wasted* votes.
- The instrumental effect of voting is especially relevant in small voting bodies (committees).

Manipulability is pervasive

Is there a voting system that would be immune to 'misrepresentation' of preferences?

- Each voter $i = 1, \ldots, n$ has strict preferences over subsets of alternatives from the set of alternatives K, denoted R_i.
- A *voting system* $G(\mathrm{R}; K)$ maps the profile of strict preferences $\mathrm{R} = (R_1, \ldots, R_n)$ into a set of winning alternatives (a voting outcome).
- A voting system $G(\mathrm{R}; K)$ is *manipulable* if there exists a preference profile $\mathrm{R} = (R_1, \ldots, R_n)$, and at least one person $i \in \{1, \ldots, n\}$ such that for some preference relation R_i', $G(R_1, \ldots, R_i', \ldots, R_n; K) \succ_i G(\mathrm{R}, K)$.
- Dictatorship is an extreme voting system where $\exists i$ such that for any R, $G(\mathrm{R}; K)$ is the preferred voting outcome (preferred set of winning alternatives) of i.
- Simply, in dictatorship, only one individual makes a decision independently upon preferences of the others.
- Think also of a 'random dictatorial' voting rule.

Manipulability is pervasive

Gibbard-Satterthwaite Theorem

If K contains more than two elements and G is non-manipulable, then G is dictatorship.

- Alternatively: If K contains more than two elements and G is not dictatorship, then G is manipulable.
- There is no voting rule (except for dictatorship) under which we could avoid the analysis of strategic voting in the presence of rational instrumental committee members!
- With restrictions on preference profiles (e.g., single-peakedness), manipulability is less pervasive. (Think of a voting rule that requests that each voter expresses her optimal policy and then sets the median optimal policy.)

Definitions

- The voting system/rule maps the voters' strategy profiles into *voting outcomes*, and payoffs are defined on the voting outcomes.
- Suppose a set of candidates (alternatives), $k = 1, \ldots, K$.
- A voting outcome is a set of winning candidates. (The composition of the losing candidates is irrelevant for payoffs.)
- We will analyze only *scoring voting rules*: Each voter i's strategy is a vector of points to all candidates, $(v_i^1, \ldots, v_i^K)$ (*ballot*).
- Points from ballots are summed into a vector of scores for candidates, $(v^1, \ldots, v^K)$.
- The candidates with the highest score v^k wins.
- With multiple winning candidates (tie), we typically think of a fair lottery.

How to get a best response for a given strategy profile?

- For each voter i, we always ask if she is *decisive/pivotal* or *indecisive*.
- That is, we ask if a change in her strategy can change her payoff or not (i.e., through the change in the set of winning candidates).
- If not for any change, then her best response is S_i. (This means that the *indecisive* voter can optimally submit any ballot!)
- If yes for some change, then the voter's best response contains only those strategies that deliver the best of achievable voting outcomes.

Serious and non-serious candidates for a profile s

- All candidates that do *not* gain the winning position or lose the winning position if some voter unilaterally deviates to some $s_i \in S_i$ are *non-serious candidates*.
- The other candidates are *serious*.
- Intuitively, strategic voters disregard non-serious candidates and optimally choose between serious candidates.

In any strategy profile s,

- either all candidates are non-serious and all voters are indecisive, or
- at least 2 candidates are serious and at least 1 voter is decisive.

Simultaneous voting

Voting games (especially when the committee is large) are special games:

- Many equilibria.
- ! In profiles where all candidates are non-serious and all voters are indecisive, no voter has any incentive to change his/her vote, whatever insincere ('perverse') the vote is.
- Therefore, any profile where *all candidates are non-serious and all voters are non-decisive* is a Nash equilibrium profile.

But some of these profiles are truly 'perverse'.

- Example: In plurality voting, three voters with preferences $A \succ_i B$ vote for B. This is a NE where none candidate is serious and none voter is decisive.
- Can we eliminate these equilibria by elimination of the *weakly dominated strategies*?

Weak dominance

- Let $s_i \in S_i$ and $s_i' \in S_i$ be possible strategies for player i.
- s_i' is **weakly dominated** by s_i if for *any possible combination* of the other players' strategies, $s_{-i} \in S_{-i}$, player i's payoff from s_i' is weakly less than that from s_i, and there exists a combination $s_{-i} \in S_{-i}$ such that the payoff from s_i' is strictly less than that from s_i,

$$v_i(s_i, s_{-i}) \geq v_i(s_i', s_{-i}) \quad \forall s_{-i} \in S_{-i}$$

$$\exists s_{-i} \in S_{-i} : v_i(s_i, s_{-i}) > v_i(s_i', s_{-i})$$

- s_i is a **weakly undominated** strategy for i if is not weakly dominated by any other strategy $s_i' \in S_i, s_i' \neq s_i$.
- The strategy profile $s^{WD} \in S$ is an equilibrium in **weakly undominated strategies** if for each $i \in N$, $s_i^{WD} \in S_i$ is a weakly undominated strategy .

Scoring voting rules

We will compare three *scoring voting rules*:

- plurality (1 positive vote): ballots $(1,0,0); (0,1,0); \dots; (0,0,0)$
- antiplurality (1 negative vote): ballots $(1,1,0); (1,0,1); \dots; (1,1,1) = (0,0,0)$
- approval voting (any number of positive votes)

Technical assumptions

- Preferences are *strict*.
- Abstention ballot $(0,0,\dots,0)$ is feasible.
- Ties are resolved fairly.
- When ranking lotteries (ties), voters are expected-utility maximizers. (If you prefer $A \succ_i B$, then you also strictly prefer A over a tie between A and B, and you strictly prefer the tie over B.)

Weak dominance can be applied when a change in ballot only increases the score of your best candidate (help the best) or decreases the score of your worst candidate (punish the worst).

- Condition 1 is satisfied as by additivity and separability of scores, the new outcome is at least as good as the old outcome.
- Condition 2 is satisfied as all players choose from an identical (and rich) set of ballots and therefore it is possible to construct the old outcome as the outcome where the voter is decisive.

Which ballots are weakly dominated?

- R1 help the best: $(1,0,0,0)$ weakly dominates $(0,0,0,0)$
- R2 punish the worst: $(0,0,0,0)$ weakly dominates $(0,0,0,1)$
  Index your worst alternative k. Let $v^{-k} = \max_{j \in K, j \neq k} v^j$. What is the effect of changing $(0,0,0,1)$ into $(0,0,0,0)$?
  - 0 $v^k < v^{-k}$: no effect, since $v^k - 1 < v^{-k}$
  - + $v^k = v^{-k}$: **improvement**, since $v^k - 1 < v^{-k}$
  - + $v^k = v^{-k} + 1$: **improvement**, since $v^k - 1 = v^{-k}$
  - 0 $v^k > v^{-k} + 1$: no effect, since $v^k - 1 > v^{-k}$

  Condition 2 is satisfied (a strategy profile with an improvement) exists. Here, suppose everybody else abstains. Then $v^k = 1$ and $v^{-k} = 0$, and an improvement exists such that $v^k = v^{-k} = 0$.
- R1+R2 $(1,0,0,0)$ weakly dominates $(0,0,0,1)$
  Condition 2 is satisfied under R1+R2 for any profile of the opponents' strategies where it is satisfied either under R1 or R2.

Scoring voting rules

R3 help the best: $(0,0,0,0) = (1,1,1,1)$ weakly dominates $(0,1,1,1)$

R4 punish the worst: $(1,1,1,0)$ weakly dominates $(1,1,1,1) = (0,0,0,0)$

R3+R4 $(1,1,1,0)$ weakly dominates $(0,1,1,1)$

If we add an identical vector of scores to the two ballots, then weak dominance preserves.

- Be careful to not make the affected candidates non-serious. (For instance, we cannot say $(1,21,21,0)$ weakly dominates $(0,21,21,1)$ if the voting body is small, because Condition 2 would not be satisfied.)

Plurality

Give always 0 to the worst candidate?

- Are all ballots with $(\dots, 1)$ weakly dominated?
- Yes. There is only one such ballot, $(0, 0, 0, 1)$, and it is weakly dominated by $(1, 0, 0, 0)$ (R1+R2).

Give always 1 to the best candidate?

- No, because the best candidate is not necessarily serious.
- The positive vote may be needed for another (serious) candidate.
- Suppose $(v^1, v^2, v^3) = (1, 2, 2)$.
- Voter who prefers Candidate 1 is strictly better off if she supports her 'second-best' candidate: $(v^1, v^2, v^3) = (0, 3, 2)$.

Note: Only in games with very few (2-4) voters and very few (2-3) alternatives, the 'top' ballot can be a unique weakly-undominated strategy.

Antiplurality

Give always 1 to the best candidate?

- Are all ballots with $(0, \dots)$ weakly dominated?
- Yes. There is only one such ballot, $(0, 1, 1)$, and it is weakly dominated by $(1, 1, 0)$ (R3+R4).

Give always 0 to the worst candidate?

- No, because the worst candidate is not necessarily serious.
- The negative vote may be needed for another (serious) candidate.
- Suppose $(v^1, v^2, v^3) = (5, 5, 2)$.
- Voter who prefers Candidate 1 is strictly better off if she punishes her 'second-worst' candidate: $(v^1, v^2, v^3) = (5, 4, 3)$.

The set of supported candidates is any subset of K. Thus, the voter is:

- not afraid to support her best candidate, and
- not afraid to not support her worst candidate.

The voter can combine 'help the best' with 'punish the worst'.

R5 $(1, y, w, z)$ weakly dominates $(0, y, w, z)$

R6 $(x, y, w, 0)$ weakly dominates $(x, y, w, 1)$

R5+R6 $(1, y, w, 0)$ weakly dominates (x, y, w, z), where $(x, y, w, z) \neq (1, y, w, 0)$

- R5 is a generalization of R1 and R3.
- R6 is a generalization of R2 and R4.
- R5+R6 is a generalization of R1+R2 and R3+R4.

Ballots permitted by weak dominance

Plurality

- 0 for the worst alternative
- 0 or 1 for the best alternative

Antiplurality

- 0 or 1 for the worst alternative
- 1 for the best alternative

Approval voting

- 0 for the worst alternative
- 1 for the best alternative

Coordination games

Stag Hunt game (Jean-Jacques Rousseau, 1775)

Two players choose to hunt a stag (S), or hunt a hare (H).

	Stag	Hare
Stag	5, 5	0, 3
Hare	3, 0	3, 3

- Multiple pure-strategy equilibria: (Stag, Stag) and (Hare, Hare).
- (Stag, Stag) Pareto-dominates (Hare, Hare).
- ! Unlike Prisoners' Dilemma, altruism doesn't help to coordinate on Pareto-dominating NE.

Multiplicity is interpreted as the existence of alternative *self-fulfilling* norms of behavior (no external enforcement, only enforced by the threat of miscoordination).

Team production with Leontieff technology

- Worker 1 independently produces x_1 left shoes.
- Worker 2 independently produces x_2 right shoes.
- The number of shoes is $y = \min\{x_1, x_2\}$.
- Each worker's cost of effort is $\frac{x^2}{2}$.

The social optimum is $x_1 = x_2 = 1$.

What best response of Worker 1?

- $B_1(x_2) = 1$ if $x_2 \geq 1$.
- $B_1(x_2) = x_2$ if $x_2 < 1$.

Nash equilibria are $(x_1, x_2) = (x, x)$, where $x \in [0, 1]$.

Bilateral meetings

- $2n$ players need n bilateral meetings in n meeting rooms.
- No pre-play communication; each player visits a single meeting room.
- At most 1 meeting can be held in each meeting room.
- When multiple pairs meet in a room, the room is used randomly.
- Each player has n strategies; $(2n)^n$ strategy profiles.
- Any profile in which each out of n pairs agrees on a meeting room is NE (irrespective of room occupancy); there are n^n such profiles.
- $n!$ profiles are efficient NE.

$n = 2$ 2 efficient NE, 2 inefficient NE, 12 non-NE
$n = 3$ 6 efficient NE, 19 inefficient NE, 191 non-NE
$n = 4$ 24 efficient NE, 232 inefficient NE, 65 280 non-NE

Coordination games

Coordination game

- a static game of complete information
- multiple Nash equilibria in pure strategies
- generally, when the strategies are *complementary* (players' best response functions are increasing)
- a coordination failure exists if a Pareto-dominated Nash equilibrium exists

Many examples

- adoption of standards (network externalities)
- location of firms (agglomeration externalities)
- search (market thickness)
- weakest-link (or weak-link) team production
- bank runs, speculative attacks, revolutions (regime changes)
- high-school applications (matching students to schools)

Private and public

Complementarities often exist between market and policy choices.

- policy is set by majority interests
- majority-driven policy motivates people to behave as in the majority
- majority-driven policy is then self-fulfilling

Example: Dog ownership in cities

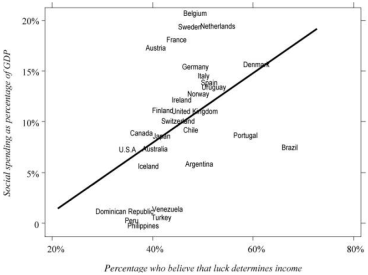
- with many dog owners, dog excrements are tolerated and the ownership fee (to cover a part of waste collection costs) is small

$\rightarrow$ many people become dog owners and tolerance is self-fulfilling

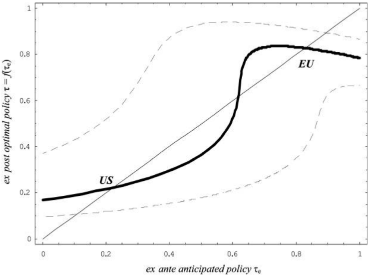
- with few dog owners, dog excrements are not tolerated and the ownership fee (to cover waste collection costs) is large

$\rightarrow$ people reconsider dog ownership and intolerance is self-fulfilling

Redistribution and entrepreneurship (Alesina and Angeletos, 2005)



World Values Survey



Multiple equilibria

- US: low taxes, many entrepreneurs, market income largely due to effort
- EU: high taxes, few entrepreneurs, market income largely due to luck

Selection criteria in coordination games

What about selecting a specific Nash equilibrium?

- Pareto dominance
- Risk dominance

Pareto dominance

- *Pareto dominance* is a reasonable criterion in 'small games' with pre-play communication.
- However, with a large number of players and no pre-play communication, multiple (equivalent) Pareto-dominant outcomes exist, and coordination is difficult.

Risk dominance with 2 NE (Harsanyi and Selten)

- For each NE strategy profile, multiply losses from the individual deviations.
- The profile with the higher product of losses *risk-dominates* the profile with the lower product of losses.

= Intuitively, the strategy profile has the largest 'basin of attraction'.

	High	Low
High	A, a	C, c
Low	B, b	D, d

- (High, High) risk-dominates (Low, Low) if $(A - B)(a - c) > (D - C)(d - b)$.

For example, (Hare, Hare) risk-dominates (Stag, Stag) since

$$(3 - 0)(3 - 0) > (5 - 3)(5 - 3).$$

Covid-19 shelter-in-place orders in Wisconsin

Enactment of the order

- The effect of order on the number of social contacts was **significant**.
- ! Measuring the effect of the order is not easy because of identification issues (states with initially lower levels of social distancing may impose more/less strict social distancing; both ways can be argued).

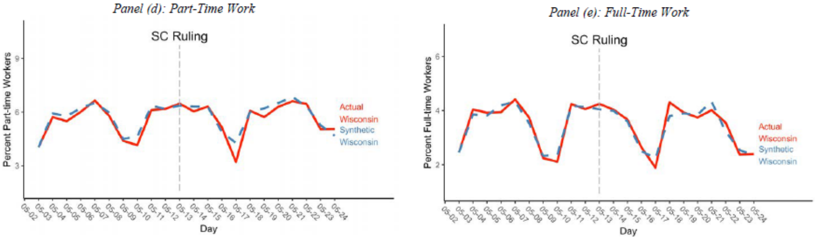
Lifting of the order (May 13, 2020)

- The Wisconsin Supreme Court abolished the state's "Safer at Home".
- The entire statewide order was overturned (with the exception of the school closures), making Wisconsin the only U.S. state without a single statewide protective measure in place. Non-essential businesses allowed to reopen without restriction, with many bars opening on the night of the decision.
- Wisconsin's Governor said that the ruling had "throw[n] the state into chaos".

Social distancing as a coordination game

The effect of lifting of order was **insignificant** (estimated by synthetic control design).

Figure 1. Synthetic Control Estimates of Effect of Wisconsin Supreme Court Abolition of SIPO on Social Distancing



2 possible channels

- a change in fundamental expectations (risk of Covid)
- a change in social/strategic expectations (coordination)

Social distancing as a coordination game

Should I stay (home), or should I go (out)? (The Clash, 1982).

- If (Out, Out) is unattractive, zero effect of lifting follows from strict dominance.
- But suppose (Out, Out) is sufficiently attractive.
  - (Out, Out) is a NE (offline meetings).
  - (Home, Home) is a NE (online meetings).
  - (Out, Home) and (Home, Out) represent miscoordination.

Social distancing as a coordination game

Initial game

- (Home, Home) and (Out, Out) are NEs.
- The agents initially coordinated on (Out, Out) (long-term 'social norm', perhaps Pareto dominance).

Enacted order

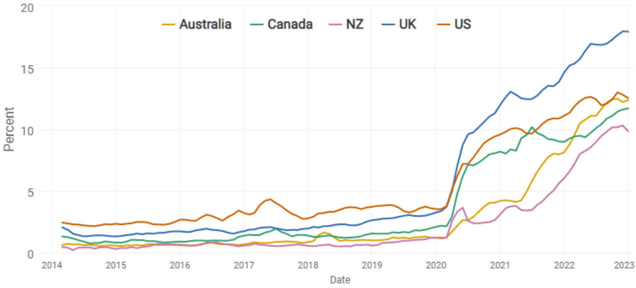
- (Home, Home) is the only NE.

Lifted order

- Both (Home, Home) and (Out, Out) are NEs, but now starting with (Home, Home).
- The agents likely coordinate on (Home, Home) (short-term 'social norm').

Notice that certain regulations are by definition *asymmetric*: In this case, they can restrict Out, but cannot restrict Home. Therefore, regulations can address inefficiency of (Out, Out) but not inefficiency of (Home, Home).

Figure 1: Share of new job vacancies advertising remote work rose dramatically



Note: This figure shows the share of vacancy postings that say the job allows one or more remote workdays per week. We compute these monthly, country-level shares as the weighted mean of the own-country occupation-level shares, with weights given by the U.S. vacancy distribution in 2019. Our occupation-level granularity is roughly equivalent to six-digit SOC codes. Figures depicts the 3-month moving average. Full screen. Access data.

Source: wfhmap.com

Miscoordination: a summary

To sum, when is it likely that players coordinate inefficiently?

- risk argument: high losses from individual deviations
- intensity of coordination problem: a large group size
- lack of communication: substantial communication costs
- efficiency argument: too small gains from an improvement
- path dependency: initial conditions matter