
Seminar 12 - Solution

1. Consider duopoly with a homogenous product. Duopoly firms have following cost functions: $TC_1 = 10y_1 + 2$ and $TC_2 = y_2^2 + 6y_2$. Market demand function is $D(p) = 12 - \frac{p}{2}$.
 - Calculate inverse demand function of the market and reaction curve of the second producer $R_2(y_1)$.
A: $p = 24 - 2y_1 - 2y_2$, $R_2(y_1) = \frac{9-y_1}{3}$
 - Suppose, that the first producer is Stackelberg leader and the second producer is follower. Calculate equilibrium outputs (y_1^{SL}, y_2^{SF}) , equilibrium price and profits of both producers.
A: $y_1^{SL} = 3$, $y_2^{SF} = 2$, $p = 14$, $\Pi_1 = 10$, $\Pi_2 = 12$
 - Suppose that the second producer offers collusion to the first producer which would maximize joint profit of both producers. First producer would not have to use fixed costs $FC_1 = 2$ for getting leader position. On the other hand, he would have to use fixed costs $FC_1 = 1$ for monitoring the cooperation. The second producer would have to use only $FC_2 = 1$ to monitor the first producer. Calculate optimal outputs (y_1^k, y_2^k) in this case, equilibrium price and profits of both producers. Decide if the collusion is profitable for both producers and if the first producer has the incentive to accept the offer for collusion.
A: $y_1^k = 1.5$, $y_2^k = 2$, $\Pi_1 = 9.5$, $\Pi_2 = 17$. The leader has no incentive to accept the offer for collusion.
2. Consider a duopoly with homogenous product and demand curve $D(p) = 140 - p$. Cost functions of firms are $TC_1 = y_1^2 + 35y_1 + 58$ and $TC_2 = 3.5y_2^2 + 60.5$.
 - Find Cournot equilibrium (y_1^c, y_2^c) , equilibrium price p^c and output Y^c of the industry.
A: $y_1^c = 23$, $y_2^c = 13$, $p^c = 104$, $Y^c = 36$
 - Suppose that firms have a possibility to collude. Calculate output y_1^k and y_2^k of firms in collusion, price p^k and output of the whole industry.
A: $y_1^k = 20.78 \doteq 21$, $y_2^k = 10.94 \doteq 11$, $p^k = 108.3 \doteq 108$
 - Compare outputs of the industry Y^c and Y^k and prices p^c and p^k from the consumers' perspective
A: Cournot equilibrium has higher output at lower price which is better for consumers.
 - Calculate profits of both firms with and without collusion and decide if firms will be willing to cooperate. If not, is there a possibility for redistribution of profits?
A: $\Pi_1^c \doteq 1000$, $\Pi_2^c \doteq 700$, $\Pi_1^k \doteq 1034$, $\Pi_2^k \doteq 704$. Both firms are willing to cooperate.