14.03 Micro Theory & Public Policy

Lecture 3. Axioms of Consumer Preference and the Theory of Choice

David Autor (Prof), MIT Economics and NBER

Jonathan Cohen (TA), MIT Economics

Cardinal and ordinal utility

A consumer's utility from consumption of a given bundle "A" is determined by a personal *utility function*.

Cardinal utility function

– $U(A)$ is a cardinal number: $U : consumption\ bundle \longrightarrow reals$ measured in "utils"

Ordinal utility function

– U provides a "ranking" or "preference ordering" over bundles.

$$U : (A, B) \longrightarrow \begin{cases} A \ ^P \ B \\ B \ ^P \ A \\ A \ ^I \ B \end{cases}$$

Cardinal vs. ordinal utility functions

- Problems with cardinal utility functions
  1. Difficult to find the appropriate measurement index (metric)
  2. Invite you to make interpersonal comparisons of utility, which is problematic. Want to focus on *intrapersonal* choices
- Using unit-free *ordinal* utility functions avoids these problems
- Significant progress on positive and normative questions is still possible

The axioms of consumer preference theory

The axioms of consumer preference theory were developed for three purposes:

1. Portray rational behavior
2. Mathematical representation of utility functions
3. Derive "well-behaved" demand curves

Axiom 1: Completeness

Axiom 1: Preferences are complete ("completeness")

- For any two bundles A and B, a consumer can establish a preference ordering.

1. $A \ ^P \ B$
2. $B \ ^P \ A$
3. $A \ ^I \ B$

Axiom 2: Transitivity

Axiom 2: Preferences are transitive ("transitivity")

- For any consumer if $A\ ^{P}\ B$ and $B\ ^{P}\ C$ then it must be that $A\ ^{P}\ C$.

- Consumers are consistent in their preferences

Axiom 3: Continuity

Axiom 3: Preferences are continuous ("continuity")

- If $A \; ^P \; B$ and C lies within an ε radius of B then $A \; ^P \; C$.

- We need continuity to derive well-behaved demand curves.

Axioms: Completeness, transitivity, and continuity

- *Axiom 1: Preferences are complete ("completeness")*
- *Axiom 2: Preferences are transitive ("transitivity")*
- *Axiom 3: Preferences are continuous ("continuity")*

Theorem

If Axioms 1–3 are obeyed, then we can define a cardinal utility function that represents the individual's preference.

Note: this theorem should be interpreted as an "as if" statement.

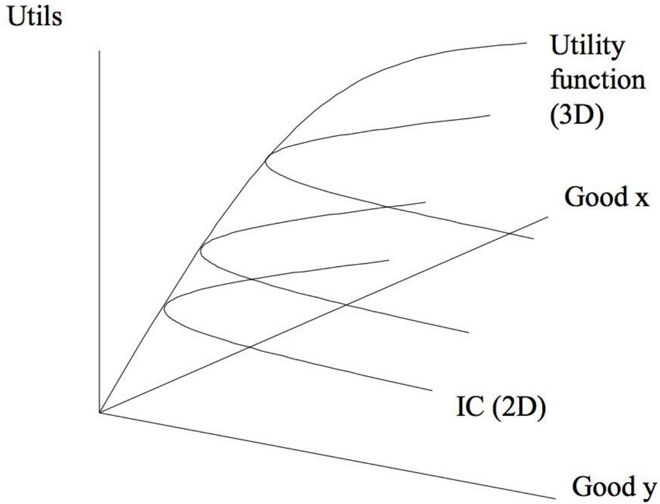
Indifference curves

- The indifference curve $IC(\overline{U})$ is the set of consumption bundles that generate utility level $\overline{U}$ for a utility function U
- An *Indifference Curve Map* is a sequence of indifference curves defined over every utility level:

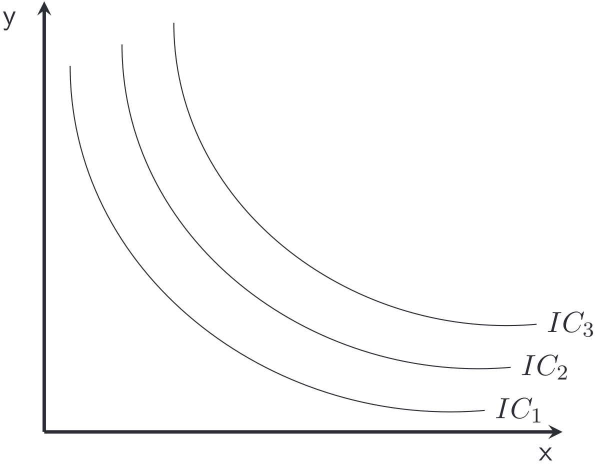
  $$\{IC(0), IC(\varepsilon), IC(2\varepsilon), ...\}$$

  with a small positive value for ε

Indifference curves



Indifference curves



$$\left.\begin{array}{l} IC_3 \longrightarrow \text{ Utility level } U_3 \\ IC_2 \longrightarrow \text{ Utility level } U_2 \\ IC_1 \longrightarrow \text{ Utility level } U_1 \end{array}\right\} U_3 > U_2 > U_1$$

Axiom 4: Non-satiation (never get enough)

We usually use two additional axioms

- Introduced to reflect observed behavior and to simplify
- But, they are not *necessary* for a theory of rational choice

Axiom 4: Non-Satiation

- Given two bundles A and B of goods X and Y, if $X_A = X_B$ and $Y_A > Y_B$ then $A \ ^P \ B$, regardless of the levels of X_A, X_B, Y_A, Y_B

- Implications:
  1. The consumer always places positive value on more consumption
  2. Indifference curve map stretches out endlessly

Axiom 5: Diminishing marginal rate of substitution

- “The more of something you have, the less you value it”
  - Captures, what we believe, is a fundamental feature of human preferences
  - Role in consumer theory:
    - Makes the mathematics of consumer theory much simpler
    - Avoids consumers spending all their money on one good

- Need to define **Marginal Rate of Substitution** first

Marginal rate of substitution

Definition (Marginal rate of substitution)

MRS measures willingness to trade one bundle for another.

- *Example:*
  - Bundle $A =$ (7 hours of sleep, 80 points on the problem set)
  - Bundle $B =$ (6 hours of sleep, 90 points on the problem set)
  - If indifferent, a student is willing to give up 1 more hour of sleep for 10 more points on the problem set

$$MRS \text{ (hours of sleep for points)} = |-10| = 10$$

- MRS is measured along an indifference curve and *may* vary along the same indifference curve
  - MRS is defined relative to some bundle (starting point)

Marginal rate of substitution

- By definition, utility is constant along an indifference curve:

$$\bar{U} = U(x, y)$$

Marginal rate of substitution

– By definition, utility is constant along an indifference curve:

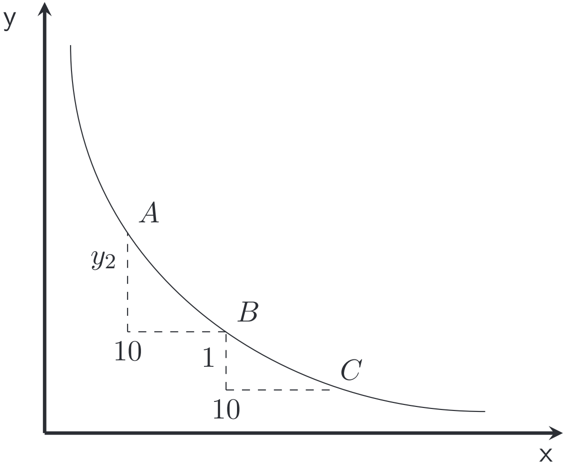
$$\begin{aligned}
\bar{U} &= U(x, y) \\
0 &= \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy \\
0 &= MU_x dx + MU_y dy \\
-\tfrac{dy}{dx} &= \frac{MU_x}{MU_y} = \text{MRS of x for y}
\end{aligned}$$

– MRS of x for y is the marginal utility of x divided by the marginal utility of y (holding total utility constant), which is equal to $-dy/dx$.

– "How much y do you need to compensate for a unit loss in x?"

Marginal rate of substitution

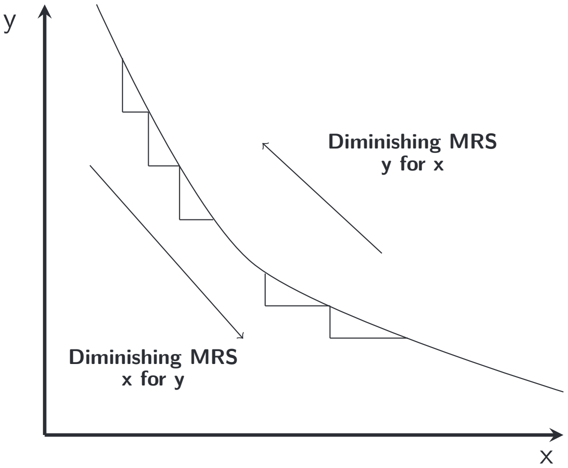
- MRS must always be evaluated at some particular point



Axiom 5: Diminishing marginal rate of substitution

Axiom 5: The MRS of x for y decreases as x increases

- The ratio MU_x/MU_y is decreasing in x



Convexity and MRS

- Diminishing MRS implies that consumers prefer diversity in consumption
- A convex utility function exhibits diminishing MRS

Definition

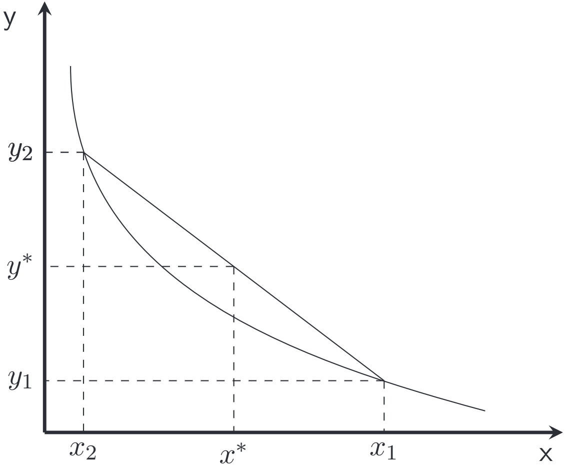
A function $U(x, y)$ is convex if for any arguments (x_1, y_1) and (x_2, y_2) where $(x_1, y_1) \neq (x_2, y_2)$:

$$U(\alpha x_1 + (1-\alpha)x_2, \alpha y_1 + (1-\alpha)y_2) \geq \alpha U(x_1, y_1) + (1-\alpha)U(x_2, y_2),$$

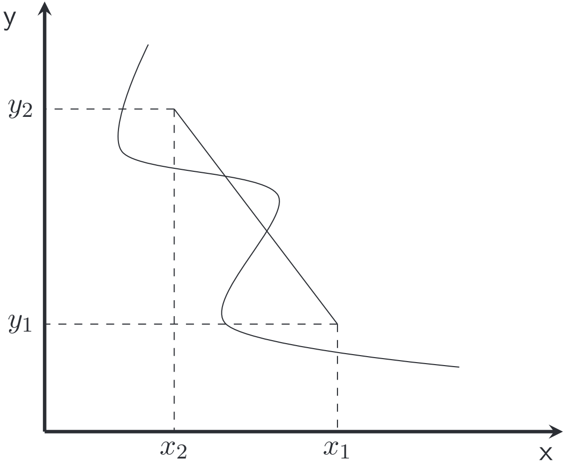
where $\alpha \in (0, 1)$.

Example of convex utility function

A utility function $U(\cdot)$ exhibits diminishing MRS iff the indifference curves of $U(\cdot)$ are convex.

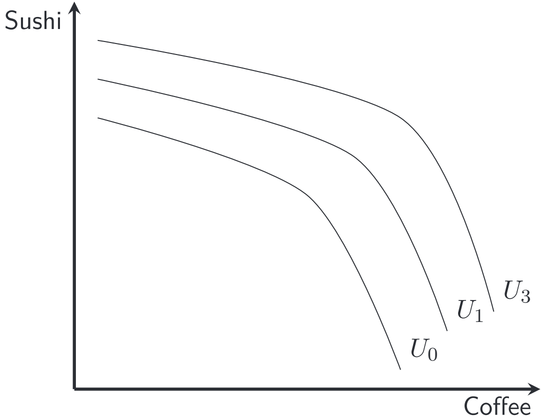


Example of non-convex utility function



Example of concave utility function

- Suppose you love coffee and sushi, but dislike consuming them together



- If your indifference curves were concave as above, you should not diversify consumption

Back to Indifference Curves

- Properties of Indifference Curve Map:
  - Every consumption bundle lies on some indifference curve

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  - Every consumption bundle lies on some indifference curve
    (Axiom 1: Completeness)
  - Indifference curves are smooth
    (Axiom 3: Continuity)
  - Indifference curves are convex
    (Axiom 5: Diminishing MRS)

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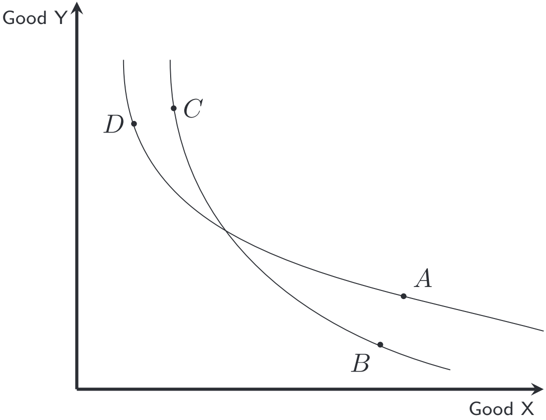
- Properties of Indifference Curve Map:
  - Every consumption bundle lies on some indifference curve (Axiom 1: Completeness)
  - Indifference curves are smooth (Axiom 3: Continuity)
  - Indifference curves are convex (Axiom 5: Diminishing MRS)
  - Indifference curves cannot intersect ...

Non-crossing of indifference curves

- Proof: say two indifference curves intersect:



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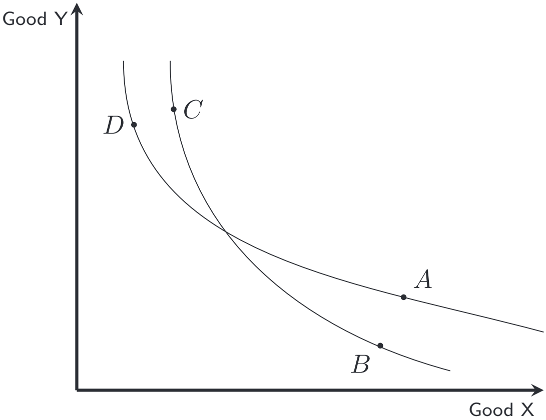

- According to these indifference curves, (i) $A \ {}^{P}\ B$ (by non-satiation), (ii) $B \ {}^{I}\ C$, (iii) $C \ {}^{P}\ D$ (by non-satiation), (iv) $D \ {}^{I}\ A$
- By transitivity, $A \ {}^{P}\ D$ and $A \ {}^{I}\ D$, which is a contradiction

Cardinal vs ordinal utility

- Utility function $U(x, y) = f(x, y)$ is cardinal
  - It reads off "utils" as a function of consumption
  - But, choices are inherently ordinal
- *However, we do care that the MRS along an indifference curve is well defined*
  - Important to know how people trade off among goods
- In consumer theory, we choose to use ordinal not cardinal utility functions

(Positive) Monotone transformation

- Q: How do we preserve properties of utility that we care about and believe in without imposing cardinal properties?
  - Utility function is only defined up to a "*positive monotone transformation*"

- If utility function $\tilde{g}()$ is a monotone transformation of utility function $g\,()$, they are identical for purposes of consumer theory

Definition (Monotone Transformation)

Let I be an interval on the real line $(\mathbb{R})$ then: $g : I \longrightarrow \mathbb{R}$ is a monotone transformation if g is a strictly increasing function on I.

- If $g(x)$ is differentiable and $g'(x) > 0\ \forall x$, then $g(x)$ is monotone.
- Note that not all monotone functions have $g'(x) > 0\ \forall x$, e.g., $x = y^3$.