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Four different cases – the VAR(1) model

(1) $X_t = A_1 X_{t-1} + e_t$; $X_t = \begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix}$, $A_1 = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, $e_t = \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix}$

The error correction form of (1) is given by:

(2) $\Delta x_t = \Pi X_{t-1} + e_t$; $\Pi = A_1 - I_2$

The four different cases are now related to the roots of the characteristic polynomial, $C(z)$, for A_1, i.e. the roots of:

$C(z) = \det(I_2 - A_1 z) = 0$

I. $|z_1| > 1$, $|z_2| > 1$

"Ch. 5"

This implies that $X_t \sim I(0)$ (a stationary system)

Example: $A_1 = \begin{pmatrix} 0.5 & 0.2 \\ 0.6 & 0.4 \end{pmatrix}$, $z_1 = 10$, $z_2 = 1.25$. Moreover,

$\Pi = \begin{pmatrix} -0.5 & 0.2 \\ 0.6 & -0.6 \end{pmatrix}$; $\text{rank}(\Pi) = 2$ (full rank)

II. $z_1 = z_2 = 1$

"Ch. 6"

This implies that $X_t \sim I(1)$ (a non-stationary system)

Example: $A_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2$, $z_1 = 1$, $z_2 = 1$. Moreover,

$\Pi = I_2 - I_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ which implies in (2) that

$\Delta X_t = e_t \iff \begin{pmatrix} x_{1t} - x_{1,t-1} \\ x_{2t} - x_{2,t-1} \end{pmatrix} = \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \iff$

$\left.\begin{aligned} x_{1t} &= x_{1,t-1} + e_{1t} \\ x_{2t} &= x_{2,t-1} + e_{2t} \end{aligned}\right\}$ two indep. R.W.'s.

Finally, $\text{rank}(\Pi) = 0$ (minimum rank)

We conclude, $X_t \sim I(1)$ but <u>not</u> a cointegrated system.

(III.) $z_1 = 1$, $|z_2| > 1$

This implies that $X_t \sim I(1)$ (a non-stationary system)

Example: $A_1 = \begin{pmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{pmatrix}$, $z_1 = 1$, $z_2 = 1.4286$. Moreover,

$$\Pi = \begin{array}{c} \begin{array}{cc} c_1 & c_2 \end{array} \\ \begin{pmatrix} -0.2 & 0.1 \\ 0.2 & -0.1 \end{pmatrix} \end{array}$$; Since $c_2 = -0.5c_1$ we will have reduced rank, i.e. $\text{rank}(\Pi) = 1$

"Ch. 6"

Remark: Since we have a reduced rank case, X_{1t} and X_{2t} will have a common stochastic trend (up to a scalar).

We conclude, $X_t \sim I(1)$ and is a cointegrated system.

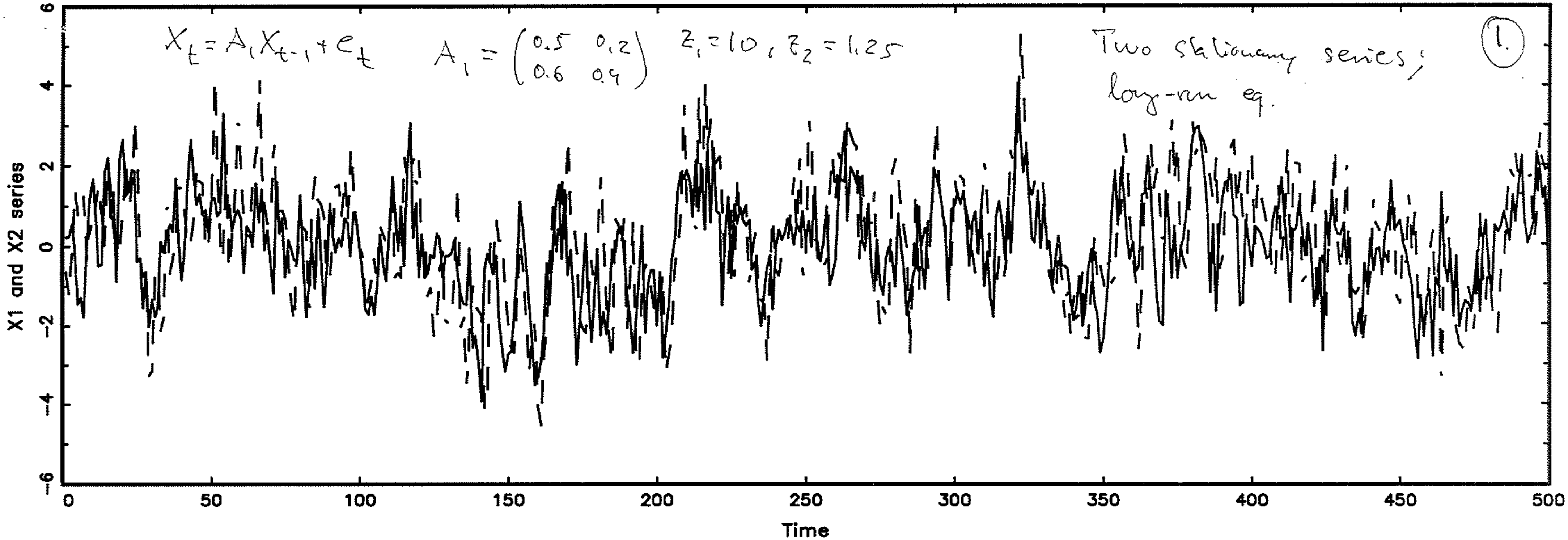
(IV.) $|z_1| < 1$ $|z_2| < 1$ (or if any root is ~~less~~ less than one in absolute value)

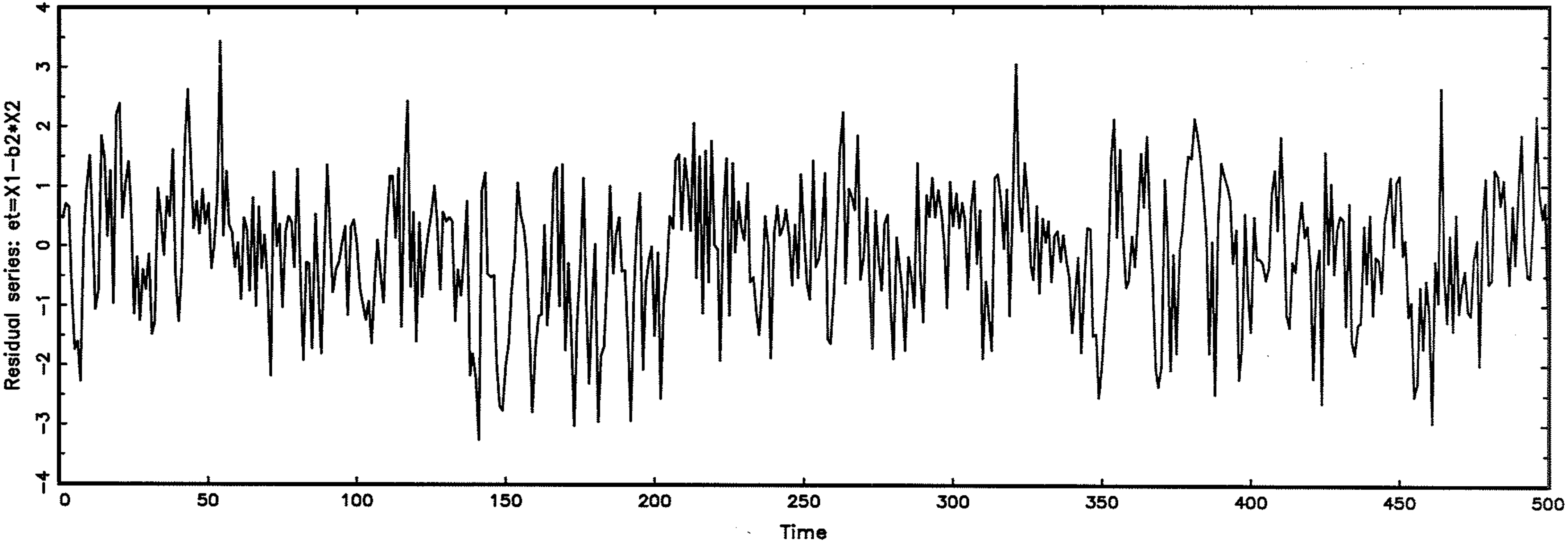
This case implies an explosive system (exponential trend), and is not further studied in this course.

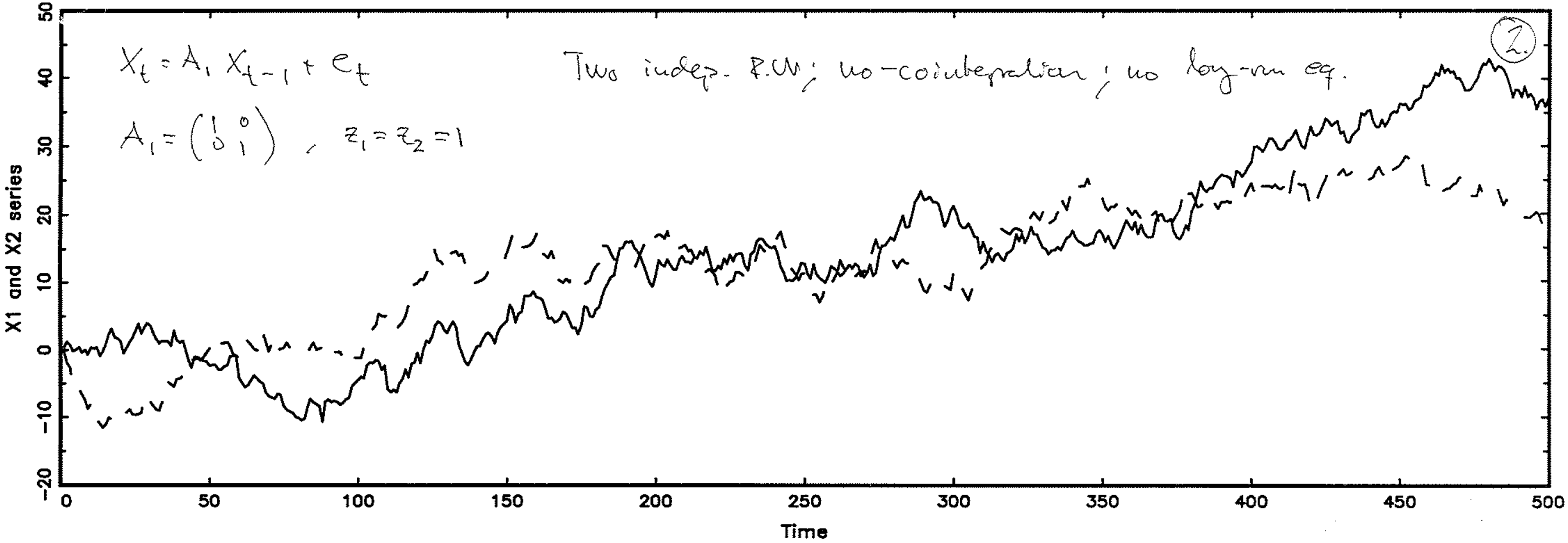
(1.)

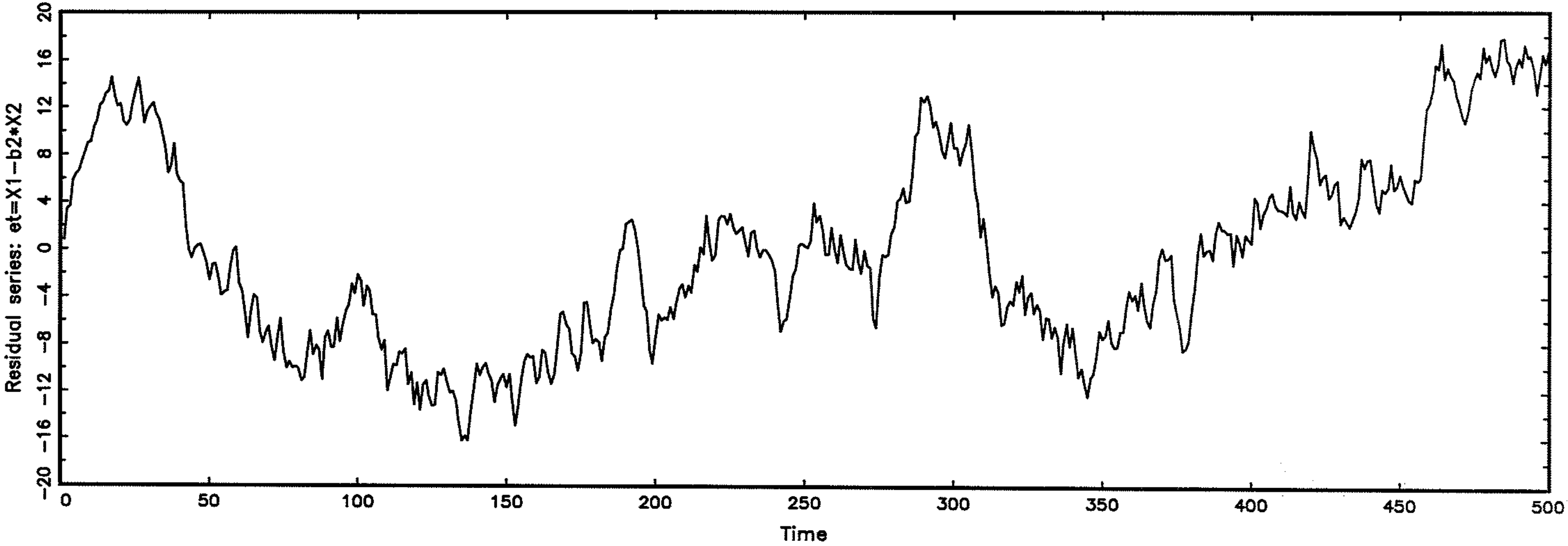
$X_t = A_1 X_{t-1} + e_t$ $A_1 = \begin{pmatrix} 0.5 & 0.2 \\ 0.6 & 0.4 \end{pmatrix}$ $z_1 = 10,\ z_2 = 1.25$

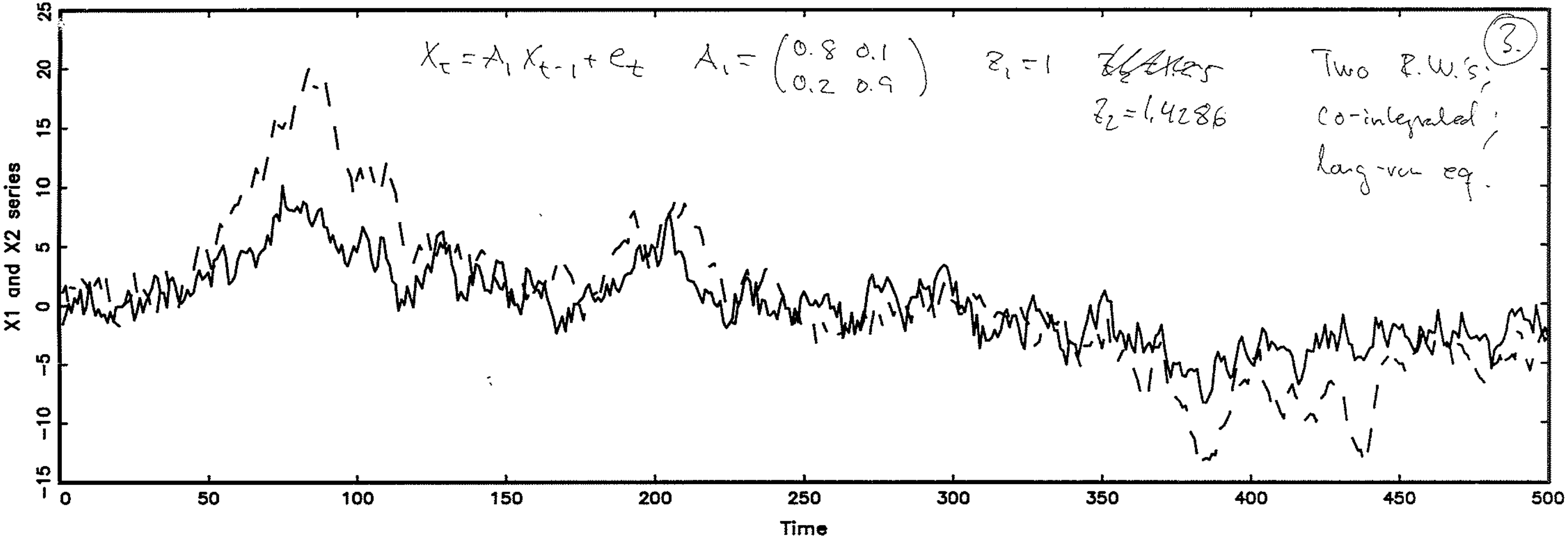
Two stationary series; long-run eq.

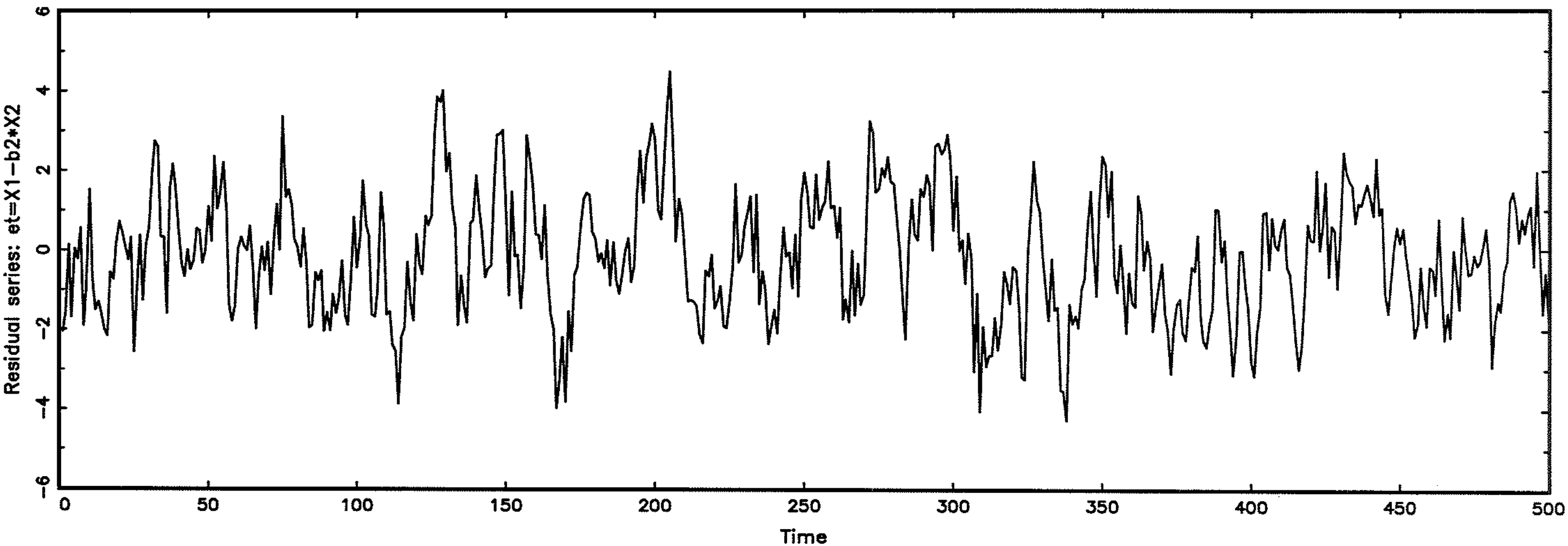





$X_t = A_1 X_{t-1} + e_t$
$A_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $z_1 = z_2 = 1$
Two indep. R.W.; no-cointegration; no long-run eq.
2.
X1 and X2 series
Time


Residual series: et=X1−b2*X2
Time


X1 and X2 series
Time
$X_t = A_1 X_{t-1} + e_t$ $A_1 = \begin{pmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{pmatrix}$
$z_1 = 1$
$z_2 = 1.4286$
Two R.W.'s;
co-integrated;
long-run eq.
3.


Residual series: et=X1-b2*X2
Time