14.03/003 Microeconomic Theory and Public Policy
Fall 2022

Lecture 18. Uncertainty, Expected Utility Theory and the Market for Risk

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Motivation

- So far we only modeled choices without **uncertainty**: everything is known in advance. That's convenient, but not particularly plausible.
  - Income fluctuates (e.g., FTX)
  - Plans go awry (e.g., Covid)
  - Bad stuff happens (e.g., Hurricane Katrina)

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- If we want a realistic model of choice, we need to model how uncertainty affects choice and well-being. This model should help to explain:
  - How do people choose among 'bundles' that have uncertain payoffs?
  - Insurance: Why do people want to buy it?
  - How (and why) the **market** for risk operates? (Markets for risk include life insurance, auto insurance, gambling, futures markets, etc.)
- Central concepts for today: *Expected utility theory, insurance, risk pooling, risk spreading, and risk transfer*

Risk in Economic Decisions

People don't seem to want to play actuarially fair games. Such a game is one in which the cost of entry is equal to the expected payoff:

$$E(X) = P_{win} \cdot [\text{Payoff}|\text{Win}] + P_{lose} \cdot [\text{Payoff}|\text{Lose}] \,.$$

Example:

- Following a (fair) coin toss, you win \$ 10 M if tails and lose \$ 9.8 M if heads
- It's a good deal financially
- What would you be willing to pay to take this gamble?

Risk in Economic Decisions

- **A game with huge upside potential:**
  - Flip a coin repeatedly
  - Let n be the number of tosses until you get a head
  - You get 2^n dollars
  - The payoffs are:

$$X_{n=0} = 1, X_{n=1} = \$2, X_{n=2} = \$4, X_{n=3} = \$8, ...X_{n=n} = 2^n.$$

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  - **Why not?**

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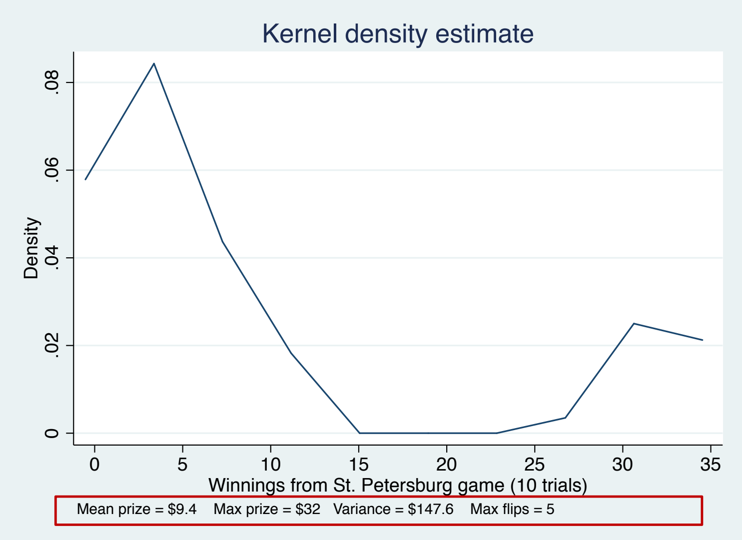
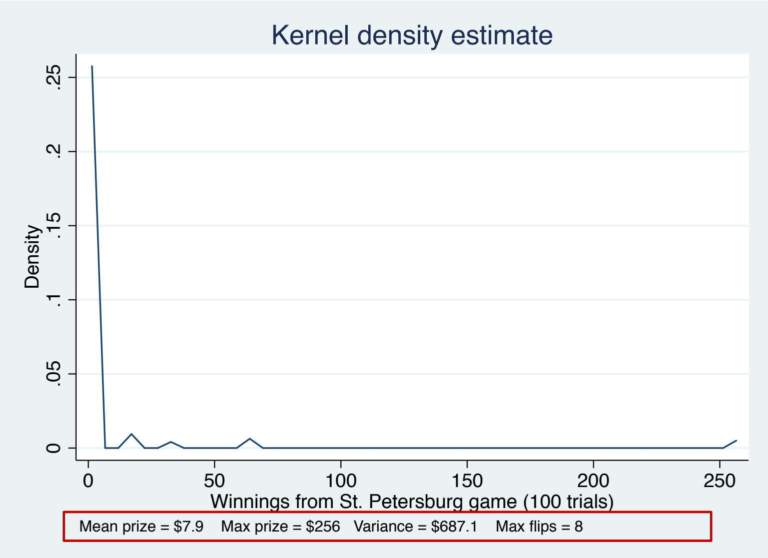
  - This example is called "The St. Petersburg Paradox"

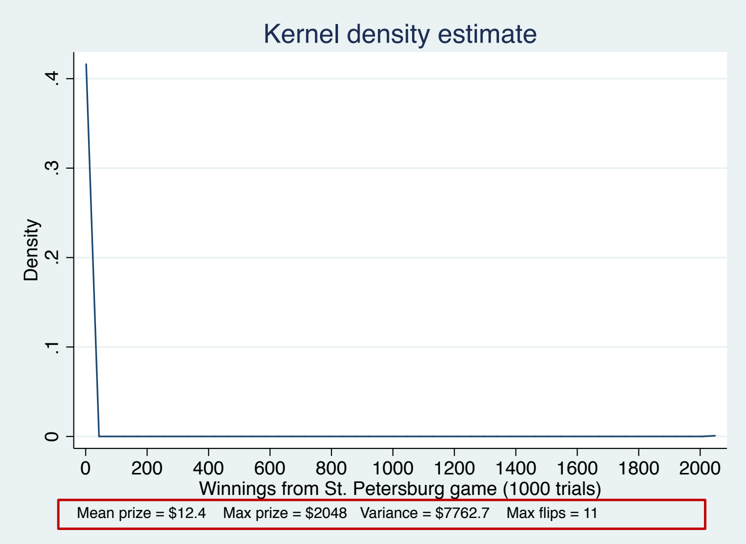
St. Petersburg Paradox: 10 coin flips (100 trials)



St. Petersburg Paradox: 100 coin flips (100 trials)



St. Petersburg Paradox: 1,000 coin flips (100 trials)



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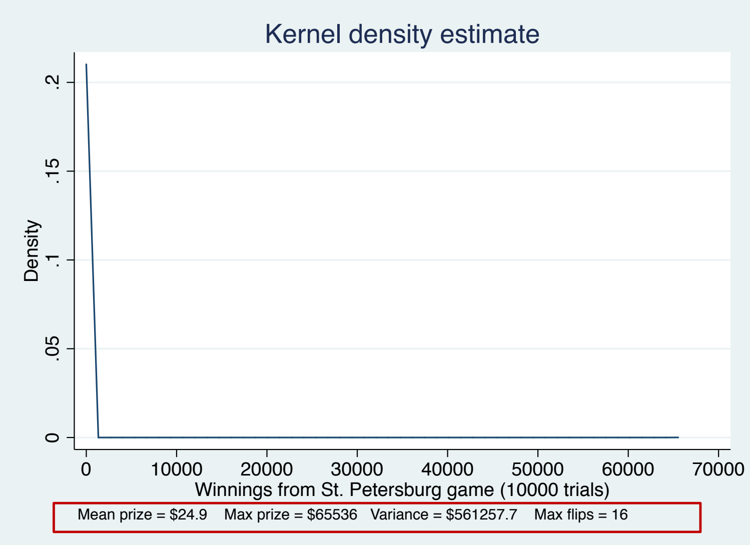

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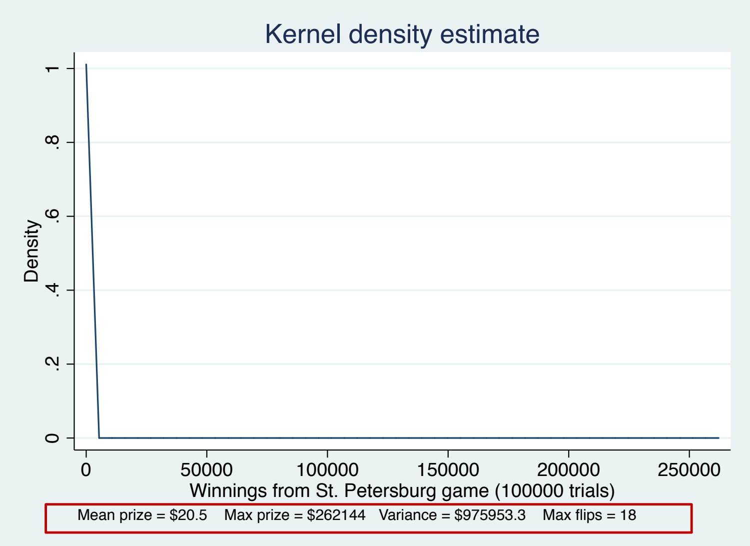

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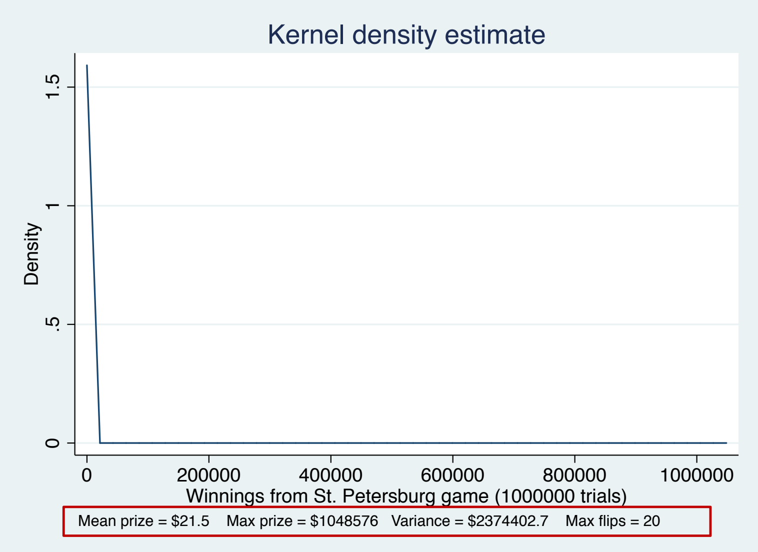

Expected Utility Theory

Expected Utility Theory

Preferences that satisfy **VNM Expected Utility theory** have the following property: We can assign a number u_n to each outcome $n = 1, ..., N$ in such a manner that for any two lotteries $L = (p_1, ..., p_N)$ and $L' = (p'_1, ...p'_N)$, we have $L \succsim L'$ if and only if:

$$U(L) = \sum_{n=1}^{N} u_n p_n \geq \sum_{n=1}^{N} u_n p'_n = U(L').$$

EU functions are linear in probabilities

- You should check for yourself that preferences that satisfy expected this implies that for any $\beta \in (0, 1)$, and lotteries L and L', we have that

$$U\left(\beta L + (1-\beta) L'\right) = \beta U(L) + (1-\beta) U(L').$$

- This equation says that for a person with VNM preferences, the utility of consuming two bundles L and L' with probabilities β and $(1-\beta)$, respectively, is equal to β times the utility of consuming bundle L plus $(1-\beta)$ times the utility of consuming bundle L'. Thus, the utility function is *linear* in probabilities.

Each bundle a convex combination of best and worst scenario

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Best Case

Each bundle a convex combination of best and worst scenario

Best Case $\longleftrightarrow$ **Worst Case**

The Expected Utility (EU) Representation

- A person who has VNM Expected Utility preferences (VNM EU, or just EU for short) over lotteries will act as if she is maximizing *expected utility*—a weighted average of utilities of each state, where weights equal probabilities.

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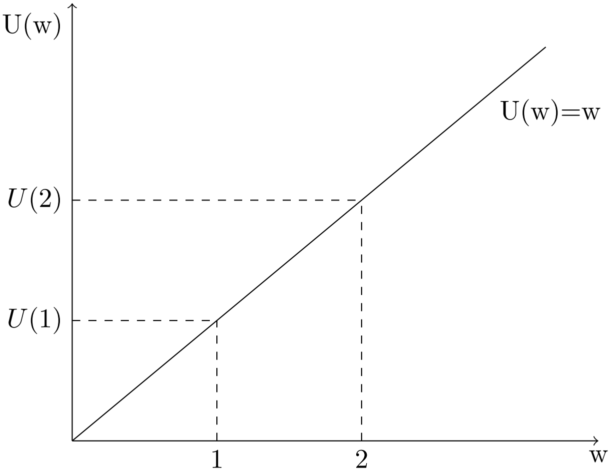
- Expected utility theory extends the model of consumer theory to choices over risky outcomes. Standard consumer theory continues to describe the utility of consumption of specific *bundles*
- Expected utility theory describes how a consumer might select among risky bundles

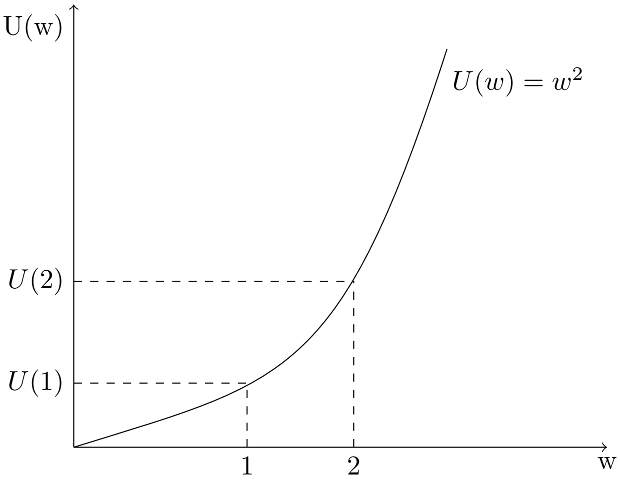
Linear Utility Function

Figure 1 : $u_1(w) = w$



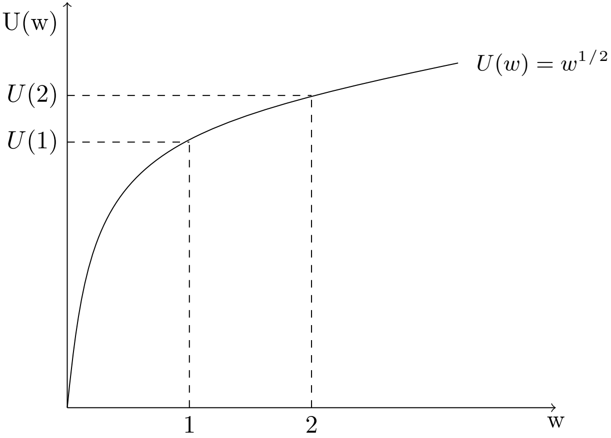
Convex Utility Function

Figure 2 : $u_2(w) = w^2$



Concave Utility Function

Figure 3 : $u_3(w) = \sqrt{w}$



Example of one gamble

- Consider a lottery with $50/50$ odds of either receiving \$2 or \$0. The expected \$ value of this lottery is $\$1$. How do consumers differ in risk preference?
- First notice that $u_1(1) = u_2(1) = u_3(1) = 1$. That is, they all value *one dollar with certainty* equally.
- Now consider the *Certainty Equivalent (CE)*for a lottery L that is a $50/50$ gamble over $\$2$ versus $\$0$. The CE is amount of cash that consumer would accept with certainty instead of facing lottery L

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- Step 1: What is the expected utility value?
  1. $u_1(L) = .5 \cdot u_1(0) + .5 \cdot u_1(2) = 0 + .5 \cdot 2 = 1$
  2. $u_2(L) = .5 \cdot u_2(0) + .5 \cdot u_2(2) = 0 + .5 \cdot 2^2 = 2$
  3. $u_3(L) = .5 \cdot u_3(0) + .5 \cdot u_3(2) = 0 + .5 \cdot 2^{.5} = .71$

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  3. $u_3(L) = .5 \cdot u_3(0) + .5 \cdot u_3(2) = 0 + .5 \cdot 2^{.5} = .71$
- Step 2: What is the "Certainty Equivalent" of lottery L for these three utility functions—that is, the cash value that the consumer would take in lieu of facing these lotteries?
  1. $CE_1(L) = U_1^{-1}(1) = \$1.00$
  2. $CE_2(L) = U_2^{-1}(2) = 2^{.5} = \$1.41$
  3. $CE_3(L) = U_3^{-1}(0.71) = 0.71^2 = \$0.51$

Example of one gamble

Depending on the utility function, a person would pay $\$1, \$1.41$, or $\$0.51$ dollars to participate in this lottery. Although the expected monetary value $E(V)$ of the lottery is $\$1.00$, the three utility functions value it differently:

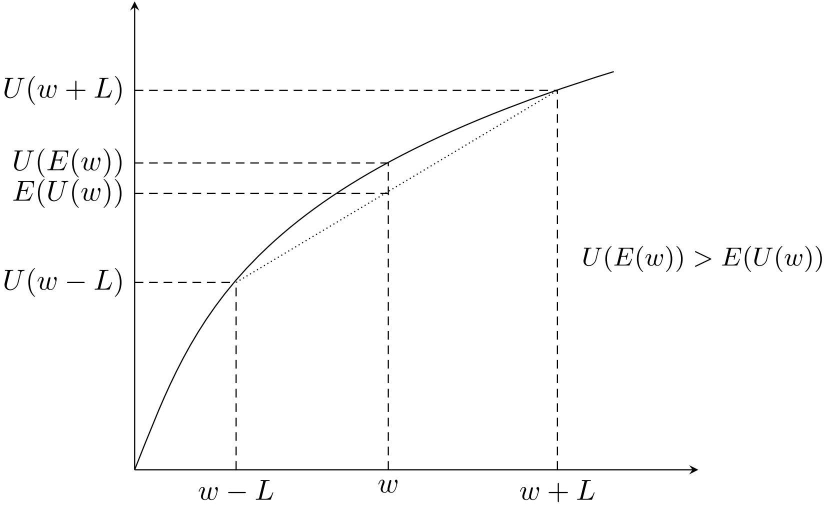
1. The person with U_1 is *risk neutral*: $CE = \$1.00 = E(Value) \Rightarrow$ Risk neutral
2. The person with U_2 is *risk loving*: $CE = \$1.41 > E(Value) \Rightarrow$ Risk loving
3. The person with U_3 is *risk averse*: $CE = \$0.50 < E(Value) \Rightarrow$ Risk averse

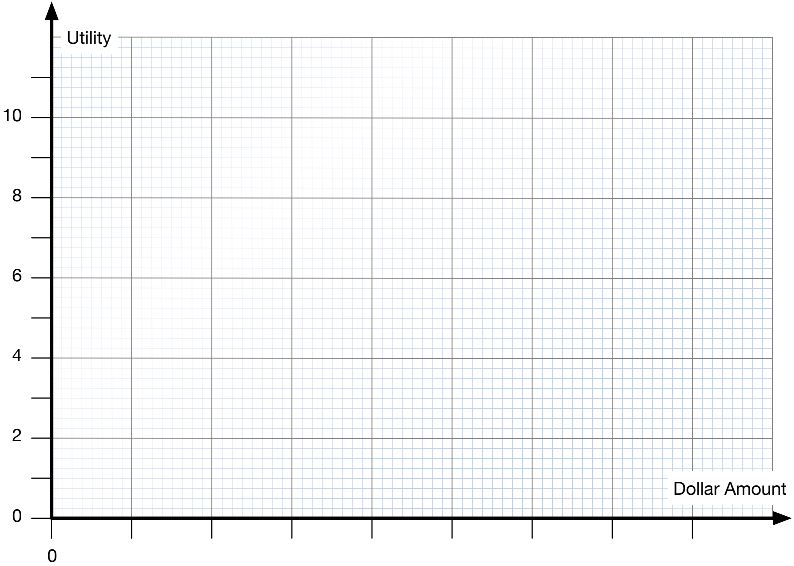
Example of one gamble

What gives rise to these inequalities is the shape of the utility function. Risk preference comes from the concavity/convexity of the utility function:

- Expected utility of wealth: $E(U(w)) = \sum_{i=1}^{N} p_i \cdot U(w_i)$
- Utility of expected wealth: $U(E(w)) = U\left(\sum_{i=1}^{N} p_i \cdot w_i\right)$
- Jensen's inequality:
  - $E(U(w)) = U(E(w)) \Rightarrow$ Risk neutral
  - $E(U(w)) > U(E(w)) \Rightarrow$ Risk loving
  - $E(U(w)) < U(E(w)) \Rightarrow$ Risk averse
- So, the core insight of expected utility theory is this: ***For a risk averse consumer facing an uncertain set of possible wealth levels, the expected utility of wealth is less than the utility of expected wealth***.

Example of one Gamble




Utility
10
8
6
4
2
0
0
Dollar Amount

Insurance, Risk Pooling, and the Law of Large Numbers

The Law of Large Numbers

Definition: The Law of Large Numbers: In repeated, independent trials with the same probability p of success in each trial, the chance that the percentage of successes differs from the probability p by more than a fixed positive amount $e > 0$ converges to zero as number of trials n goes to infinity for every positive e.

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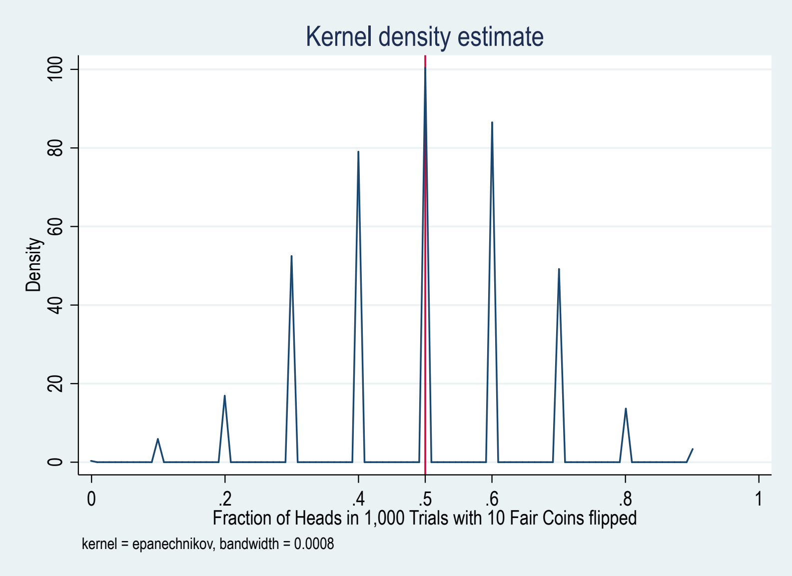
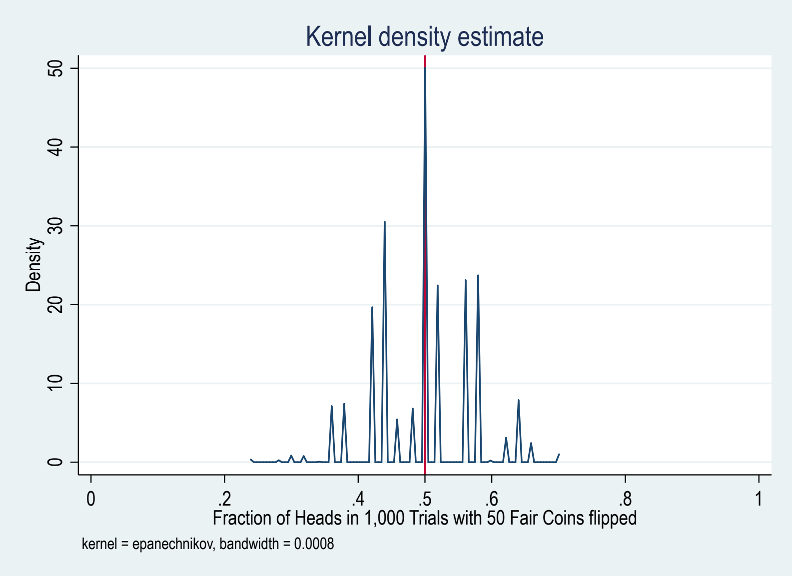
Example: If you flip a fair coin 100 times, the probability of getting heads more than $\geq 51\%$ of the time (that is, 51 or more times) is reasonably high. If you flip a fair coin 100,000 times, the probability of getting heads more than $\geq 51\%$ of the time (that is, 51000) is vanishingly small.

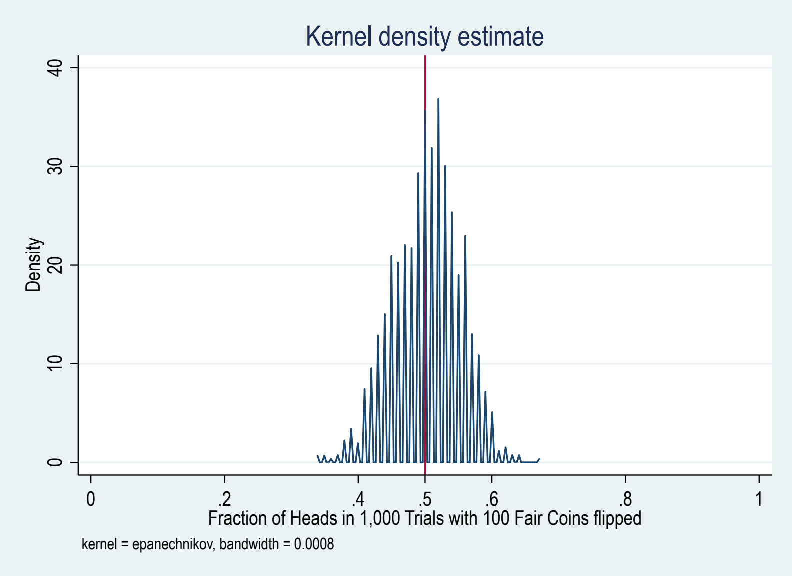
Fraction of flips that are 'heads' from 10 flips of a fair coin (1K trials)



Fraction of flips that are ‘heads’ from 50 flips of a fair coin (1K trials)



Fraction of flips that are ‘heads’ from 100 flips of a fair coin (1K trials)



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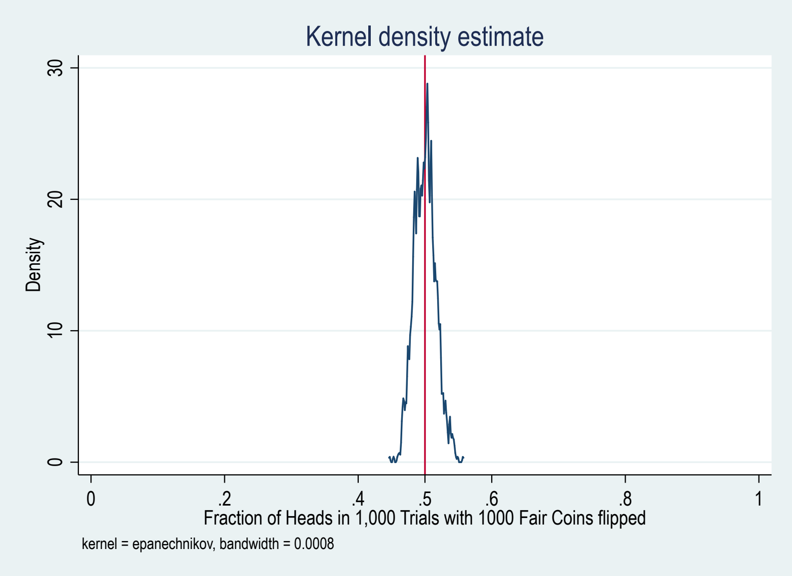

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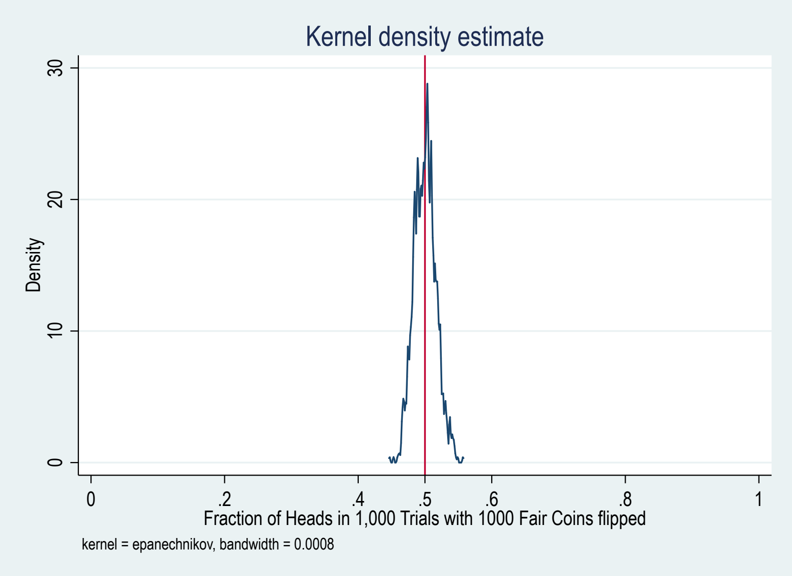

Risk Aversion and Insurance

- Consider insurance that is *actuarially fair*, meaning that the premium is equal to expected claims: Premium $= p \cdot A$ where p is the expected probability of a claim, and A is the amount that the insurance company will pay in the event of an accident
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- Consider a person with an initial endowment consisting of three things: A level of wealth w_0; a probability of an accident of p; and the amount of the loss, L (in dollars) should a loss occur:

$$\begin{aligned} \Pr(1-p) &: \quad U(\cdot) = U(w_o), \\ \Pr(p) &: \quad U(\cdot) = U(w_o - L) \end{aligned}$$

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- If insured, the endowment is (incorporating the premium pA, the claim paid A if a claim is made,and the loss L):

$$\begin{aligned} \Pr(1-p) &: U(\cdot) = U(w_o - pA), \\ \Pr(p) &: U(\cdot) = U(w_o - pA + A - L) \end{aligned}$$

Risk Aversion and Insurance

- Expected utility if uninsured is: $E(U|I=0) = (1-p)U(w_0) + pU(w_o - L)$.
- EU if insured is: $E(U|I=1) = (1-p)U(w_0 - pA) + pU(w_o - L + A - pA)$
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$$\begin{aligned}
\max_A E(U) &= (1-p)U(w_0 - pA) + pU(w_o - L + A - pA) \\
\frac{\partial E(U)}{\partial A} &= -p(1-p)U'(w_0 - pA) + p(1-p)U'(w_o - L + A - pA) = 0. \\
&\Rightarrow U'(w_0 - pA) = U'(w_o - L + A - pA), \\
&\Rightarrow A = L,
\end{aligned}$$

  which implies that wealth is $w_0 - L$ in both states of the world (insurance claim or no claim).
- A risk averse person will optimally buy *full insurance* if the insurance is actuarially fair.

Insurance as State Contingent Commodity

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- Consumers can use their endowment (equivalent to budget set) to shift wealth across states of the world via insurance, just like budget set can be used to shift consumption across goods X, Y.

Insurance as State Contingent Commodity

Example: Two states of world, good and bad, with $w_0 = 120$, $p = 0.75$, $L = 80$.

$$\begin{aligned} w_g &= 120 \\ w_b &= 120 - 80 \\ \Pr(g) &= p = 0.75 \\ \Pr(b) &= (1-p) = 0.25 \\ E(w) &= 0.75(120) + 0.25(40) = 100 \\ E(u(w)) &< u(E(w)) \text{ if consumer is risk averse.} \end{aligned}$$

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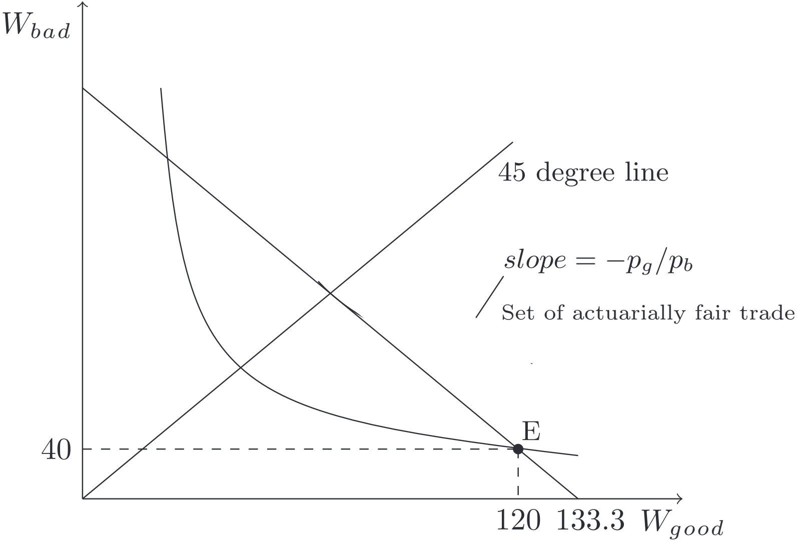
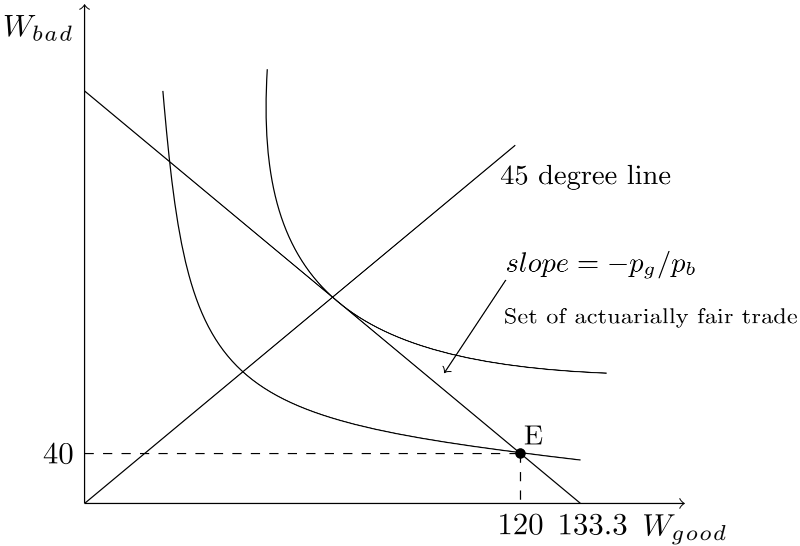


Insurance as State Contingent Commodity



Insurance as State Contingent Commodity

Let's say that this consumer can buy actuarially fair insurance. What will it sell for?

- If you want $\$1.00$ in Good state, this will sell of $\$0.75$ *prior to the state being revealed. The reason is that the good state will occur with* 75% *probability.*
- If you want $\$1.00$ in Bad state, this will sell for $\$0.25$ *prior to the state being revealed because the bad state will occur with* 25% *probability.*
- Note again that these prices reflect expected probabilities of making the claim. So, a *risk neutral* firm (say a central bank) could sell you insurance against bad states at a price of $\$0.25$ on the dollar and insurance again good states (assuming you wanted to buy it) at a price of $\$0.75$ on the dollar.
- The price ratio of payments in the Good state relative payments in the Bad state is therefore

$$\frac{P_g}{P_b} = \frac{p}{(1-p)} = 3.$$

Insurance as State Contingent Commodity

Recall that the utility of this lottery (the endowment) is:

$$u(L) = pu(w_g) + (1-p)u(w_b).$$

- Along an indifference curve

$$\begin{aligned} dU &= 0 = pu'(w_g)dw_g + (1-p)u'(w_b)dw_b, \\ \frac{dw_b}{dw_g} &= -\frac{pu'(w_g)}{(1-p)u'(w_b)} < 0. \end{aligned}$$

- Flat indifference curves would indicate risk neutrality—because for risk neutral consumers, expected utility is linear in expected wealth.
- Convex indifference curves mean that you must be compensated to bear risk.

Insurance as State Contingent Commodity

- In the figure, the movement from the lower (closer to the origin) to the upper indifference curve is the gain from shedding risk.
- Notice from the figure that, along the 45^0 line, $w_g = w_b$.
- But if $w_g = w_b$, this implies that

$$\frac{dw_b}{dw_g} = -\frac{pu'(w_g)}{(1-p)u'(w_b)} = -\frac{p}{(1-p)} = -\frac{P_g}{P_b}.$$

- Hence, the indifference curve will be tangent to the budget set at exactly the point where wealth is equated across states. [This is an alternative way of demonstrating the results above that a risk averse consumer will always fully insure if insurance is actuarially fair.]

Risk Pooling

Risk Pooling and the Market for Insurance

Risk pooling is main mechanism underlying most *private* insurance markets. Uses Law of Large Numbers to defray risk

- For any number of tosses n of a fair coin, expected fraction of heads H is $E(H) = \frac{0.5n}{n} = 0.5$. But the variance of this expectation (equal to $\frac{p(1-p)}{n}$) is declining in number of tosses:

$$\begin{aligned} V(1) &= 0.25 \\ V(2) &= 0.125 \\ V(10) &= 0.025 \\ V(1,000) &= 0.00025 \end{aligned}$$

- We cannot predict the **share** of heads in one coin toss with any precision, but we can predict the **share** of heads in $10,000$ coin tosses with confidence. It will be vanishingly close to 0.5.
- By *pooling* many independent risks, insurers can treat uncertain outcomes as *almost known*

Example: Insurance Market

- Let's say that each year, there is a $1/250$ chance that my house will burn down. If it does, I lose the entire $\$250,000$ house. The expected cost of a fire in my house each year is therefore about $\$1,000$.
- Given my risk aversion, it is costly in expected utility terms for me to bear this risk (i.e., much more costly than simply reducing my wealth by $\$1,000$).
- If $100,000$ owners of $\$250,000$ homes all put $\$1,000$ into the pool, this pool will collect $\$100$ million.
- In expectation, 400 of us will lose our houses $\left(\frac{100{,}000}{250} = 400\right)$.
- The pool will therefore pay out approximately $250,000 \cdot 400 = \$100$ million and approximately break even.
- Everyone who participated in this pool is better off to be relieved of the risk, though most will pay $\$1,000$ the insurance premium and not lose their house.

Example: Insurance Market

- However, there is still some risk that the pool will face a larger loss than the expected $1/250$ of the insured. How does the variance scale with the insurance pool?

$$\begin{aligned} V(Loss) &= \frac{P_{Loss} \cdot (1 - P_{Loss})}{100,000} = \frac{0.004 \cdot (1 - 0.004)}{100000} = 3.984 \times 10^{-8} \\ SD(Loss) &= \sqrt{3.984 \times 10^{-8}} = 0.0002 \end{aligned}$$

- Using the fact that the mean loss is approximately normally distributed when n is large, this implies that:

$$\Pr[Loss \in (0.004 \pm 1.96 \cdot 0.0002)] = 0.95$$

- So, there is a 95% chance that there will be somewhere between 361 and 439 losses, yielding a cost per policy holder in 95% of cases of $\$924.50$ to $\$1,075.50$.
- Most of the risk is defrayed in this pool of $100,000$ policies.
- And as $n \to \infty$, this risk entirely vanishes.

Risk Spreading

The Problem of Aggregate Risks

- This 'pooling' mechanism above *not* work when risks are *not independent.* Possible examples:
  - Earthquakes
  - Floods
  - Epidemics
- When a catastrophic event is likely to affect many people simultaneously, it is (to some extent) **non-diversifiable**. This is why many catastrophes such as floods, nuclear war, etc., are specifically not covered by insurance policies.

Question: But does this mean there is no way to insured against these correlated risks?

Risk Spreading

- Many risks cannot be covered by insurance companies, but the government can intercede by transferring money among parties
- Examples: victims compensation fund for World Trade Center, victims of floods and extreme weather, etc.
- Many of these insurance 'policies' are not even written until the disaster occurs—there was no market
- Why do we do this?

Risk Spreading

- For example, imagine 100 people, each with VNM utility function $u(w) = \ln(w)$ and wealth 500. Imagine that one of them experiences a loss of 200. His utility loss is

$$L = u(300) - u(500) = -0.511.$$

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- Now, instead consider if we took this loss and distributed it over the entire population:

$$L = 100 \cdot [\ln(498) - \ln(500)] = 100 \cdot [-0.004] = -0.401.$$

The aggregate loss (-0.401) is considerably smaller than the individual loss (-0.511). (This comes from the concavity of the utility function.)

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- Risk spreading may improve social welfare, even if it does not defray total risk faced by society
- **Q: Does risk spreading offer a Pareto improvement?**

Risk Transfer

Risk Transfer

- If utility cost of risk is declining in wealth, this means that *less wealthy people could pay more wealthy people to bear their risks* and both parties would be better off.$=$
- Again, take the case where $u(w) = \ln(w)$. Imagine that an individual faces a 50 percent chance of losing $\$100$. What would this person pay to eliminate this risk? It will depend on his or her initial wealth.

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- Assume that initial wealth is 200. Hence, expected utility is

$$u(L) = 0.5 \ln 200 + 0.5 \ln 100 = 4.952$$

  Expected wealth is $\$150$. The certainty equivalent of this lottery is $\exp[4.952] = \$141.5$. Hence, the "cost of the risk" to this consumer is $\$8.50$.

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- Now consider a person with the same utility function with wealth $1,000$. Expected utility is

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  Expected wealth is $\$950$. The certainty equivalent of this lottery is $\exp[6.855] = \$948.6$. Hence, the "cost of the risk" to this consumer is only $\$1.40$.

Risk Transfer

- The wealthy consumer could fully insure the poor consumer at psychic cost $\$1.40$ while the poor consumer would be "willing to pay" $\$8.50$ for defraying the risk. Any price that they can agree between $(\$1.40, \$8.50)$ represents a pure Pareto improvement.
- **Question:** Why does this form of risk transfer work?

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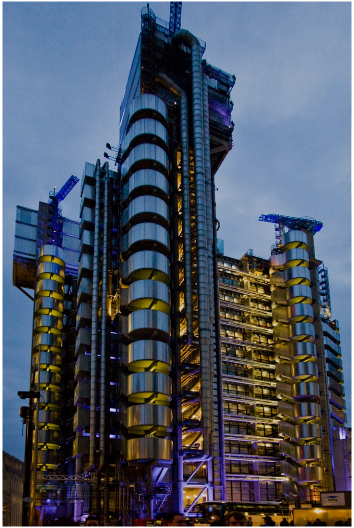
- **Question:** Why does this form of risk transfer work?
- Because the logarithmic (or more generally, concve) utility function exhibits declining absolute risk aversion—the wealthier someone is, the lower their psychic cost of bearing a fixed monetary amount of risk.
- This logic applies also to work relationships. We think that firms can bear risks more readily than workers.



LLOYD'S

LLOYD'S OF LONDON

Items insured by Lloyds of London: *German airship Hindenburg, 1937*

Items insured by Lloyds of London: *Keith Richards' hands*

Items insured by Lloyds of London: *Gene Simmons' tongue*