# Recitation 3 Practice Questions

## 1 Rational Choice

In the first period, $p_x = 2$ and $p_y = 1$ and Jon buys 11 units of $x$ and 8 units of $y$. In the second period, $p_x = 1$ and $p_y = 2$ and Jon buys 10 units of $x$ and 10 units of $y$. Prove that Jon's choices do not satisfy all 5 axioms of consumer theory in the specified ways. Be sure to cite specific axioms in your answer.

*Hint: For some of your responses, you may find it useful to prove by contradiction. In order to prove the statement "A is true implies B is true" by contradiction, you assume that B is false and deduce that A must also be false. This relies on the fact that a logical statement and its contrapositive are equivalent. In the context of this question, a proof by contradiction would assume that Jon's choice* ***do*** *satisfy all 5 axioms and conclude that he could not have made the choices that he did.*

1. (4 points) Prove using a graphical argument based on budget sets and/or indifference curves.
2. Prove using an algebraic argument based on conditions from Jon's utility maximization problem.
3. Prove using a revealed preference argument by comparing multiple bundles sequentially.
4. Consumer with preferences that satisfy all 5 axioms of consumer theory is indifferent between two bundles of $X$ and $Y$: $(3,2)$ and $(1,5)$. Can we infer that she prefers the bundle $(2,4)$ to either of the first two?

## 2 Regression Discontinuity

The basic idea of a regression discontinuity design (RDD) is to compare units just above a treatment cutoff and just below a treatment cutoff. David's lecture notes go through the math details, but the intuition is that these two types of units are virtually identical apart from their treatment exposure. In other words, the RDD is like an RCT for units just around the cutoff. The following questions will test your understanding of this.

1. High school students in the US take the PSAT only one time. Some scholarships have a minimum PSAT score cutoff. Suppose no students scoring below the threshold get a scholarship while all students scoring above the threshold get a scholarship? Do you think students just below the threshold are comparable to ones just above the threshold? Can you use this variation to evaluate the causal effect of the scholarship on student outcomes?
2. What if students who scored above the threshold additionally needed to fill out a scholarship application form. Everyone who meets the threshold and fills out the application gets the scholarship, but only $x$% who scored high enough did so?

3. What if students could retake the test multiple times and use their highest score for the scholarship?

## 3 Indirect utility function and expenditure function

Let $U = [\alpha x^r + (1-\alpha)y^r]^{\frac{1}{r}}$ be the utility function (it is called constant elasticity of substitution (CES) utility function), where $x$ and $y$ are two goods, and $\alpha \in (0,1)$ and $r > 0$ are parameters. Denote $p_x$ and $p_y$ as respectively the prices of the two goods $x$ and $y$, and $I$ as the income of the consumer. This question has challenging algebra, so be careful!

1. Derive the Marshallian demand functions $d_x(p_x, p_y, I)$, $d_y(p_x, p_y, I)$, and the indirect utility function $V\left(p_x, p_y, I\right)$.
2. Apply Roy's Identity to find Marshallian demand functions.
3. Derive the Hicksian demand functions $h_x(p_x, p_y, U_0)$, $h_y(p_x, p_y, U_0)$, and the expenditure function $E\left(p_x, p_y, U_0\right)$.
4. Apply Shephard's lemma to find Hicksian demand functions.
5. Find $V\left(p_x, p_y, E\left(p_x, p_y, U_0\right)\right)$ and $E\left(p_x, p_y, V\left(p_x, p_y, I\right)\right)$ and explain.
6. Suppose now that the utility function is given by $\widehat{U} = g([\alpha x^r + (1-\alpha)y^r]^{\frac{1}{r}})$, where $g(.)$ is a strictly increasing function. Find indirect utility and expenditure functions $\widehat{V}\left(p_x, p_y, I\right)$, $\widehat{E}\left(p_x, p_y, \widehat{U}_0\right)$. How are they related to $V\left(p_x, p_y, I\right)$ and $E\left(p_x, p_y, U_0\right)$ found in 2? Why?