# Machine Learning and Causal Inference

MIXTAPE TRACK



Traditional strategy:  $Y_i = \delta D_i + X_i'\beta + \varepsilon_i$ , or

1. Regress  $Y_i$  on  $X_i$  and compute the residuals,

$$\tilde{Y}_{i} = Y_{i} - \hat{Y}_{i}^{OLS},$$

$$\hat{Y}_{i}^{OLS} = X'_{i} (X'X)^{-1} X'Y$$

2. Regress  $D_i$  on  $X_i$  and compute the residuals,

$$\begin{array}{rcl} \tilde{D}_{i} & = & D_{i} - \hat{D}_{i}^{OLS}, \\ \hat{D}_{i}^{OLS} & = & X_{i}' \left( X'X \right)^{-1} X'D \end{array}$$

3. Regress  $\tilde{Y}_i$  on  $\tilde{D}_i$ .

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When OLS might not be the right tool for the job:

- $\triangleright$  there are many variables in  $X_i$
- $\blacktriangleright$  the relationship between  $X_i$  and  $Y_i$  or  $D_i$  may not be linear

#### ML-augmented regression strategy:

1. Predict  $Y_i$  using  $X_i$  with ML and compute the residuals,

$$ilde{Y}_i = Y_i - \hat{Y}_i^{ML},$$
  
 $\hat{Y}_i^{ML} = \text{prediction generated by ML}$ 

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Two flavors of machine-assisted causal inference:

- 1. Post-double selection lasso (PDS lasso), introduced by Belloni, Chernozhukov, and Hansen
- 2. Double/De-biased machine learning (DML), introduced by Chernozhukov, Chetverikov, Demirer, Duflo, Hansen, Newey, and Robins

#### Machine-Assisted Causal Inference

▶ No identification ex machina! Still rely on

$$D_i \perp (Y_i(0), Y_i(1)) | X_i$$

What variables to include in  $X_i$ ? The omitted variables bias formula is our guide. Uncontrolled (bivariate) regression gives us:

$$\hat{\delta}^{\mathsf{bivariate}} o \delta + eta rac{\mathsf{Cov}\left(D_i, X_i
ight)}{\mathsf{Var}\left(D_i
ight)}$$

We need to control for variables that

- affect the outcome
- are correlated with treatment
- Beware of bad control: including post-treatment variables in X<sub>i</sub>

## PDS Lasso: Preliminaries

Begin with flexible version of our regression model:

$$Y_{i} = \tau D_{i} + g(X_{i}) + \varepsilon_{i}$$

Approximate the two CEFs,

$$m_D(X_i) \equiv E[D_i|X_i]$$
  
 $m_Y(X_i) \equiv E[Y_i|X_i] = \tau m_D(X_i) + g(X_i),$ 

With a sparse linear approximation:

$$m_Y(X_i) = X'_i \gamma_Y + r_i$$
  
 $m_D(X_i) = X'_i \gamma_D + s_i$ 

 $X_i$  should contain a **dictionary** of nonlinear transformations like powers and interactions

## PDS Lasso: The Recipe

#### PDS is implemented in three steps:

- 1. Lasso  $Y_i$  on  $X_i$ , collect retained features in  $X_i^Y$
- 2. Lasso  $D_i$  on  $X_i$ , collect retained features in  $X_i^D$
- 3. Regress  $Y_i$  on  $D_i$  and  $X_i^Y \cup X_i^D$

#### Caveats and considerations:

- Standardizing controls pre-lasso is important
- ▶ BCH have a formula for the penalty parameter, but cross-validation seems to work just fine
- ▶ Inference: just use robust SEs from last step!

Time for python!

## **DML**: Preliminaries

Stick with flexible version of our regression model:

$$Y_{i} = \tau D_{i} + g(X_{i}) + \varepsilon_{i}$$

1. Predict  $Y_i$  using  $X_i$  with ML and compute the residuals,

$$egin{array}{lll} & ilde{Y}_i & = & Y_i - \hat{Y}_i^{DML}, \ & \hat{Y}_i^{DML} & = & ext{prediction generated by ML} \end{array}$$

2. Predict  $D_i$  using  $X_i$  with ML and compute the residuals,

$$ilde{D}_i = D_i - \hat{D}_i^{DML},$$
  
 $\hat{D}_i^{DML} = \text{prediction generated by ML}$ 

3. Regress  $\tilde{Y}_i$  on  $\tilde{D}_i$ .

 $\hat{Y}_i^{DML}$  and  $\hat{D}_i^{DML}$  should be predictions generated by a machine learning model trained on a set of observations that *does not include i*. We accomplish this via *cross-fitting* 

## DML: Recipe

- 1. Divide the sample into K folds
- 2. For k = 1, ..., K
  - a Train a model to predict Y given X, leaving out observations i in fold k:  $\hat{Y}^{-k}(x)$
  - b Train a model to predict D given X, leaving out observations i in fold k:  $\hat{D}^{-k}(x)$
  - c Form residuals  $\tilde{Y}_{i} = Y_{i} \hat{Y}^{-k}(X_{i})$  and  $\tilde{D}_{i} = D_{i} \hat{D}^{-k}(X_{i})$
- 3. Regress  $\tilde{Y}_i$  on  $\tilde{D}_i$ .

#### Caveats and considerations:

- Cross-validation to choose tuning parameters
- ▶ Inference: use robust SEs from last step

Time for python!



## That's a wrap

#### What I hope you've gotten out of the last couple of days:

- Clarity on distinction between predictive and causal questions
- ► Foot in the door with python implementations of some common modern supervised machine learning methods
- ► Tools for using ML methods to control for high dimensional covariates in the service of causal inference

#### Preview for future workshop:

- Use ML to predict heterogeneous treatment effects (e.g., random causal forests)
- November 15: www.mixtapesessions.io/session/ml\_het\_effects\_nov15

# Thank you!