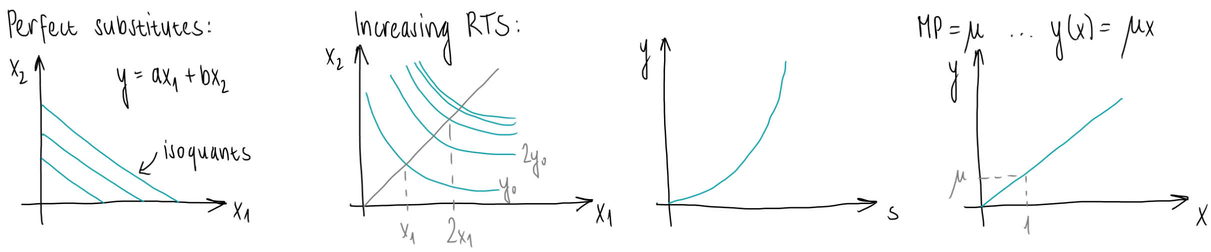
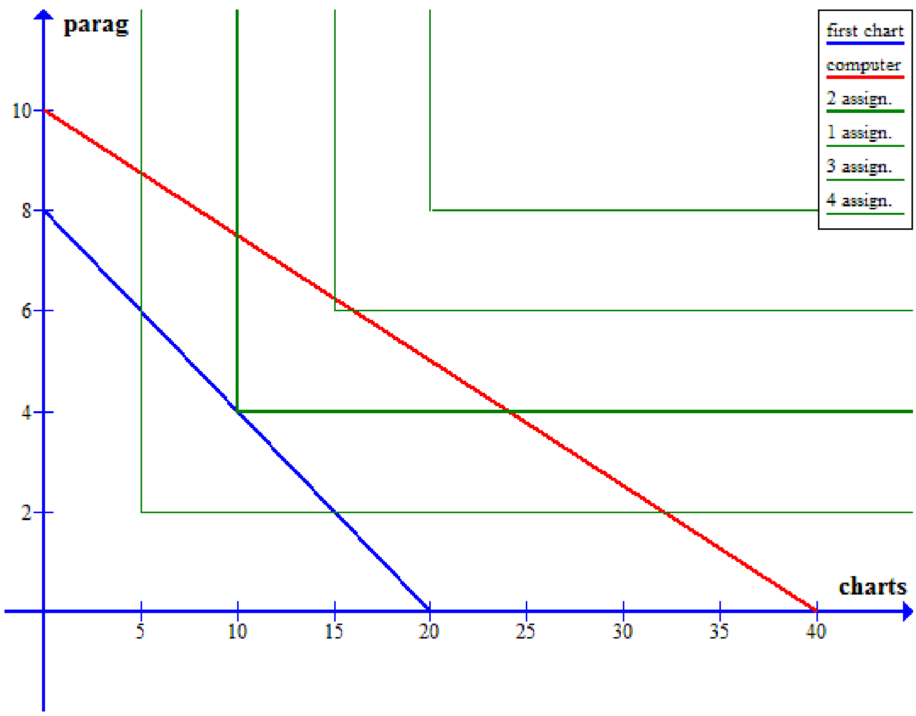
# Seminar 1 & 2 - Solution

1. Draw and explain the following: Perfect substitutes, increasing returns to scales, production function when the only input has a constant marginal product ($MP = \mu$, $\mu \in \mathbb{R}$).
   A:




2. You have 40 hours of time, you need to write paragraphs and draw charts. To write a paragraph, you need 5 hours and for one chart you need 2 hours.
   - Show in a diagram, how much of your duties you are able to complete within your time.
   - If you use a computer, you can depict one chart in 1 hour and one paragraph takes you only 4 hours. How will your digram change?
   - One assignment requires 2 paragraphs and 5 charts. What is your production function $f(c,p) =$? Does it make sense to plot it into your diagram? What is the connection between these two?

   A: Recall what you know from Micro I. You can think of hours as a time constraint (similar to budget constraint in Micro I with hours corresponding to prices). The first diagram will be a straight line between two points: vertical axis 8 units of $p$, horizontal axis 20 units of $c$.
   When using computer, the straight line will change its slope (from $-2/5$ to $-1/4$) and move to the right.
   $f(c,p) = \min\{\frac{c}{5}, \frac{p}{2}\}$, you can plot it in the same graph and see how many assignments are within your feasible production area. In the original setting (without computer), you would be able to produce 2 assignments in 40 hours.



3. Production function $f(x_1, x_2) = \frac{2}{5}\ln(x_1) + \frac{4}{5}\ln(x_2)$:
   - Calculate the technical rate of substitution ($TRS(x_1, x_2)$)
   - What is the value of TRS for $x_1 = 3$ and $x_2 = 4$
   - Derive the elasticity of substitution $\sigma$

   A: $TRS(x_1, x_2) = -\frac{x_2}{2x_1}$, $TRS(3, 4) = -\frac{2}{3}$, $\sigma = 1$

4. Derive for production functions $f_1(x_1, x_2) = x_1^{\frac{1}{7}} x_2^{\frac{3}{7}}$ and $f_2(x_1, x_2) = (\frac{1}{3}x_1^{\frac{2}{3}} + \frac{4}{3}x_2^{\frac{2}{3}})^{\frac{3}{2}}$ the elasticity of substitution $\sigma$.

   A: $\sigma_1 = 1$, $\sigma_2 = 3$

5. For production function $f(x_1, x_2) = \sqrt[3]{x_1} + 2\sqrt[3]{x_2}$ compute:
   - Marginal product of factor 1 and 2 ($MP_1$, $MP_2$) and technical rate of substitution (TRS)
   - Elasticity of returns to scale $e_t^y$
   - Returns to scale are increasing, decreasing or constant?

   A: $TRS = -\frac{x_1^{-\frac{2}{3}}}{2x_2^{-\frac{2}{3}}}$, $e_t^y = \frac{1}{3}$, decreasing

6. For production function $f(x_1, x_2) = \sqrt{5x_1 + 2x_2}$
   - Marginal product of factor 1 and 2 ($MP_1$, $MP_2$) and technical rate of substitution (TRS)
   - Elasticity of returns to scale $e_t^y$
   - Are they increasing, decreasing or constant?

   A: $TRS = -\frac{5}{2}$, $e_t^y = \frac{1}{2}$

7. For production function $f(x_1, x_2) = \sqrt{x_1}(\sqrt{x_1} - \sqrt[3]{x_2}) + 2$:
   - Derive the short-run production function when $x_2 = 8$
   - Calculate the elasticity of returns to factor 1 and find the amount of $x_1$ where the returns change from increasing to decreasing

   A: $f(x_1, x_2) = \sqrt{x_1}(\sqrt{x_1} - 2) + 2$, $x_1 = 4$

8. For production function $f(x_1, x_2) = (x_1^{\frac{1}{2}} - x_2^{-\frac{1}{2}})^2$:
   - Derive short-term production function when $x_1 = 4$
   - Calculate $MP_2$ and $AP_2$
   - Calculate the elasticity of returns to factor 2 and find the amount of $x_2$ where the returns change from increasing to decreasing

   A: $f(\bar{x}_1, x_2) = (2 - x_2^{-\frac{1}{2}})^2$, $x_2 = 1$

9. We have the production function $f(x_1, x_2) = x_1(\sqrt{x_1} + \sqrt{x_2})$.
   - Determine if the production function has increasing, constant or decreasing returns to scale.
   - Calculate returns to factor $x_1$ and $x_2$ and find out if they are increasing, constant or decreasing.

   A: $e_t^y = \frac{3}{2}$, $e_{x_1}^y$ - increasing or constant, $e_{x_2}^y$ - decreasing

10. For the short-term production function $f(x_1, \overline{x_2}) = 60x_1^2 - x_1^3$ where factor $x_2$ is fixed factor.
    - Calculate where is the maximum of marginal product.
    - Calculate where is the maximum of average product.
    - Calculate where the elasticity of returns to factor changes from increasing to decreasing.

    A: $x_1 = 20$; $x_1 = 30$; $x_1 = 30$

11. In the short run, the relation between number of hours worked and quantity produced looks like in the table:

| L | q |
|---|---|
| 0 | 0 |
| 20 | 30 |
| 40 | 100 |
| 60 | 170 |
| 80 | 210 |
| 100 | 200 |

- Draw a graph of what the production function looks like.
- Explain the concepts of average product of labor - $AP_L$ and marginal product of labor - $MP_L$ and what they correspond in the graph.
- Draw another graph below the production curve, illustrating the shapes of $AP_L$ and $MP_L$. Indicate the relations of $AP_L$ and $MP_L$ on the graph.

A: For the explanation, see the lecture notes and Figure 1 below. Note that this production function has also its maximum. At the maximum, marginal product will be equal to zero.

12. For the production function $f(x_1, x_2) = (\frac{1}{3}x_1^{\frac{2}{3}} + \frac{2}{3}x_2^{\frac{2}{3}})^{\frac{3}{2}}$ calculate:

- technical rate of substitution TRS
- change of the TRS with the change in ratio of inputs
- the elasticity of substitution $\sigma$

A: $TRS = -\frac{1}{2}(\frac{x_2}{x_1})^{1/3}$

$\frac{\partial TRS}{\partial \frac{x_2}{x_1}} = -\frac{1}{6}(\frac{x_2}{x_1})^{-2/3}$

$\sigma = 3$

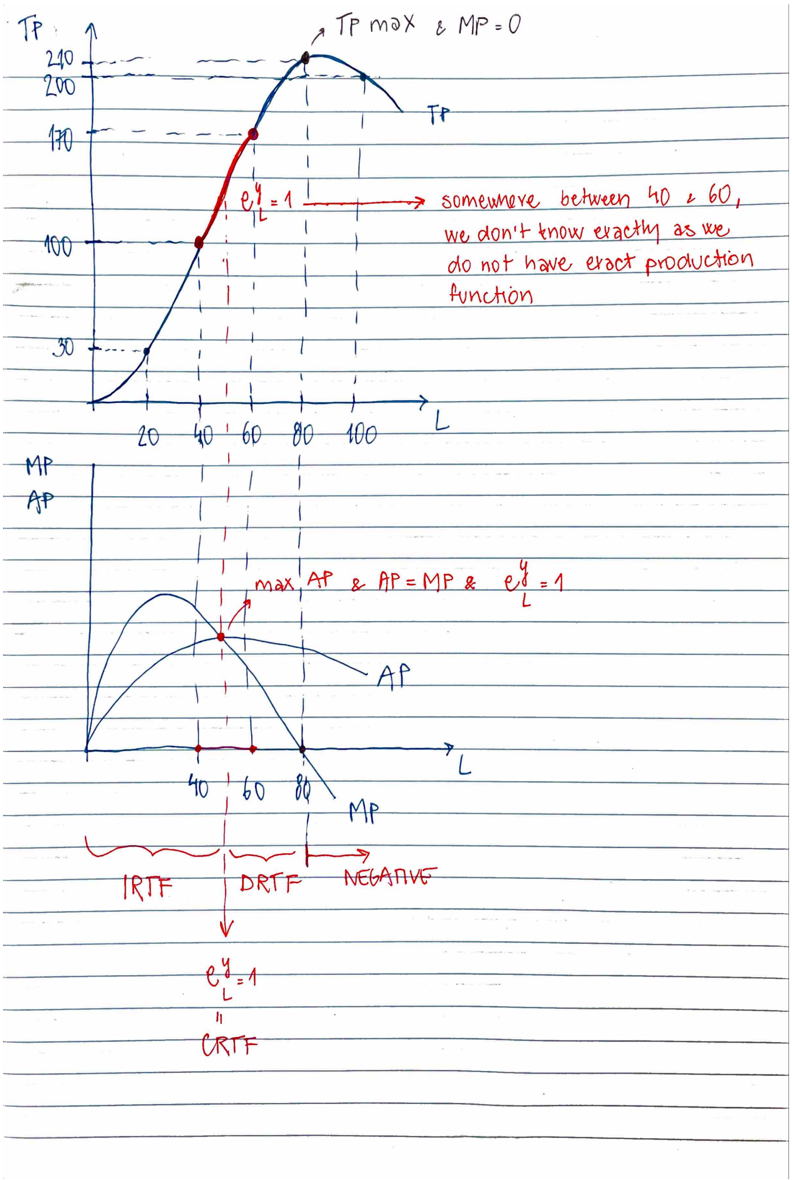



Figure 1: Diagram for task 11