5330 Advanced Microeconomic Theory

Lecture: The Market

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Introduction

This lecture is mainly based on

- Geoffrey Jehle and Philip Reny (2011), *Advanced Microeconomic Theory*, chapter 4, and any good mathematics textbook.

Perfect Competition

- In perfectly competitive markets, buyers and sellers are sufficiently large in number to ensure that no single one of them, alone, has the power to determine market price.
- The demand side of a market is made up of all potential buyers of the good, each with their own preferences, consumption set and income.
- We let $\mathcal{I} \equiv \{1, \ldots, I\}$ index the set of individual buyers and $q^i(p, \mathbf{p}, y^i)$ be i's nonnegative demand for good q as a function of its own price p, income y^i and prices $\mathbf{p}$ for the remaining goods in i's budget.
- Market demand for q is simply the sum of all buyers' individual demand:

$$q^d(p) \equiv \sum_{i \in \mathcal{I}} q^i(p, \mathbf{p}, y^i).$$

- The supply side of the market is made up of all potential sellers of q.
- Earlier, we defined the short run as that period of time in which at least one input is fixed to the firm. Then the number of potential suppliers is fixed, finite and limited to those firms that "already exist."
- If we let $\mathcal{J} \equiv \{1, \ldots, J\}$ index those firms, the **short-run market supply function** is the sum of individual firm short-run supply functions $q^j(p, \mathbf{w})$:

$$q^s(p) \equiv \sum_{j \in \mathcal{J}} q^j(p, \mathbf{w}).$$

- Market demand and market supply together determine the price and total quantity traded.
- We say that a competitive market is in **short-run equilibrium** at price p^* when $q^d(p^*) = q^s(p^*)$.
- Geometrically, this corresponds to the familiar intersection of market supply and market demand curves drawn in the (p, q) plane.

Example: SR Equilibrium with Cobb-Douglas technology

- Consider a competitive industry composed of J identical firms.
- Firms produce output according to the Cobb-Douglas technology $q = x^{\alpha} k^{1-\alpha}$, where x is some variable input such as labor, k is some input such as plant size, which is fixed in the short run, and $0 < \alpha < 1$.
- Then, at prices p, w_x and w_k, the individual firm's profit maximization problem is

$$\max_{x,q \geq 0} \; pq - w_x x - w_k k \quad \text{s.t.} \quad x^{\alpha} k^{1-\alpha} \geq q.$$

- Assuming an interior solution, the constraint holds with equality, so the problem reduces to

$$\max_{x \geq 0} \; p\, x^{\alpha} k^{1-\alpha} - w_x x - w_k k.$$

- $$\max_{x \geq 0} \; p\, x^{\alpha} k^{1-\alpha} - w_x x - w_k k.$$

  The first-order condition requires that

  $$p\,\alpha x^{\alpha-1} k^{1-\alpha} - w_x = 0.$$

  Solving for x gives

  $$\begin{aligned}
  \left(\frac{x}{k}\right)^{\alpha-1} &= \frac{w_x}{\alpha p} \\
  \frac{x}{k} &= \left(\frac{w_x}{\alpha p}\right)^{1/(\alpha-1)} \\
  x &= p^{1/(1-\alpha)} \alpha^{1/(1-\alpha)} w_x^{1/(\alpha-1)} k.
  \end{aligned}$$

- $$\max_{x \geq 0} \; p\, x^{\alpha} k^{1-\alpha} - w_x x - w_k k.$$

$$x = p^{1/(1-\alpha)} \alpha^{1/(1-\alpha)} w_x^{1/(\alpha-1)} k.$$

Substituting for x and simplifying gives the short-run profit function

$$\pi^j = p \left(p^{1/(1-\alpha)} \alpha^{1/(1-\alpha)} w_x^{1/(\alpha-1)} k \right)^{\alpha} k^{1-\alpha}$$

$$-w_x \left(p^{1/(1-\alpha)} \alpha^{1/(1-\alpha)} w_x^{1/(\alpha-1)} k \right) - w_k k$$

$$\pi^j = p^{1/(1-\alpha)} \alpha^{\alpha/(1-\alpha)} w_x^{\alpha/(\alpha-1)} k$$

$$-p^{1/(1-\alpha)} \alpha^{(1-\alpha)/(1-\alpha)} \alpha^{\alpha/(1-\alpha)} w_x^{\alpha/(\alpha-1)} k - w_k k$$

$$\pi^j = p^{1/(1-\alpha)} w_x^{\alpha/(\alpha-1)} \alpha^{\alpha/(1-\alpha)} (1-\alpha) k - w_k k$$

- $$\pi^j = p^{1/(1-\alpha)} w_x^{\alpha/(\alpha-1)} \alpha^{\alpha/(1-\alpha)} (1-\alpha)k - w_k k$$

  Notice that because $\alpha < 1$, short-run profits are well-defined even though the production function exhibits (long-run) constant returns to scale.
- By Hotelling's lemma, short-run supply can be found by differentiating

$$q^j = \frac{\partial \pi^j}{\partial p} = \frac{1}{1-\alpha} p^{1/(1-\alpha)-(1-\alpha)/(1-\alpha)} w_x^{\alpha/(\alpha-1)} \alpha^{\alpha/(1-\alpha)} (1-\alpha)k$$

$$= p^{\alpha/(1-\alpha)} w_x^{\alpha/(\alpha-1)} \alpha^{\alpha/(1-\alpha)} k$$

Note that this supply function is upward-sloping in p.

- If $\alpha = 1/2$, $w_x = 4$ and $w_k = 1$, then supposing each firm operates a plant of size $k = 1$, firm supply reduces to

$$q^j = p^{\alpha/(1-\alpha)} w_x^{\alpha/(\alpha-1)} \alpha^{\alpha/(1-\alpha)} k = p\, 4^{-1} 2^{-1} = \frac{p}{8}.$$

  The market supply function with $J = 48$ firms will be

$$q^s(p) \equiv \sum_{j \in \mathcal{J}} q^j(p, \mathbf{w}) = 48 \frac{p}{8} = 6p.$$

- Let market demand be given by

$$q^d = \frac{294}{p}.$$

- Then solving for the short-run equilibrium yields:

$$q^s = 6p = q^d = \frac{294}{p}$$

$$p^* = \left(\frac{294}{6}\right)^{1/2} = 7$$

$$q^* = 6p^* = 6 \cdot 7 = 42$$

$$q^j = \frac{q^*}{48} = \frac{42}{48} = \frac{7}{8}$$

$$\pi^j = p^{1/(1-\alpha)} w_x^{\alpha/(\alpha-1)} \alpha^{\alpha/(1-\alpha)} (1-\alpha)k - w_k k$$

$$\pi^j = 7^2 4^{-1} (1/2)^1 (1/2) 1 - 1 \cdot 1 = 2.0625$$

- Firm supply $q^j = p/8$ implies that $p = 8q^j$ and since price equals short-run marginal cost for a competitive firm, it follows that

$$smc = 8q^j.$$

- Since firm profits equal price minus short-run average cost times output, $\pi^j = (p^* - sac(q^j))q^j$, we can solve for short-run average cost

$$sac(q^j) = p^* - \frac{\pi^j}{q^j} = 7 - \frac{2.0625}{7/8} = 4.64$$

- Note that

$$p^* - sac(q^j) = \frac{\pi^j}{q^j}.$$

Price minus short-run average cost equals short-run profits per unit.

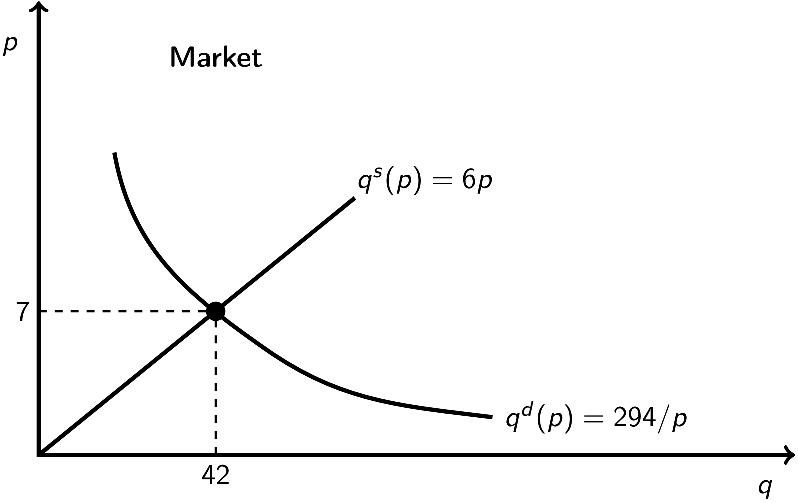

Figure: Short-run equilibrium in a single market.

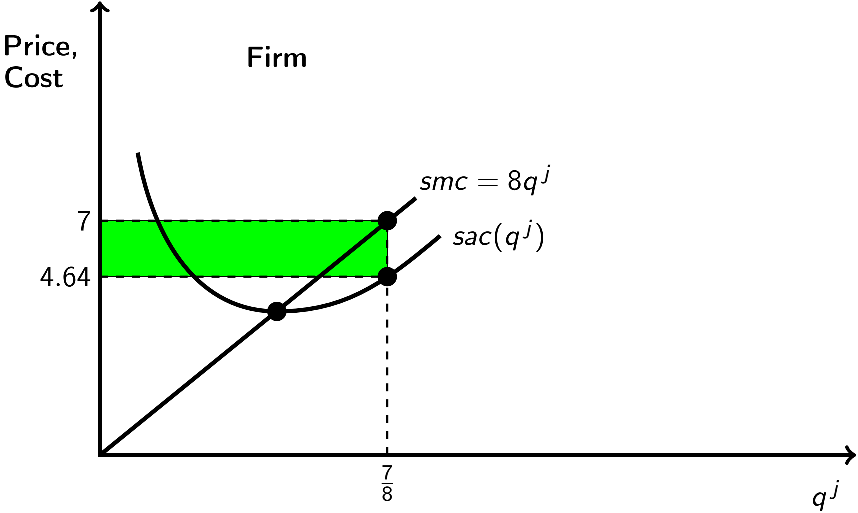

Figure: Short-run equilibrium in a single market.

The Long-Run

- In the long run, no inputs are fixed for the firm.
- Incumbent firms – those already producing – are free to choose optimal levels of all inputs, including, for example, the size of their plants.
- They are also free to leave the industry entirely and new firms may decide to begin producing the good in question.
- Thus, in the long run, there are possibilities of **entry** and **exit** of firms.

- There are two conditions that characterize long-run equilibrium in a competitive market:

$$q^d(\hat{p}) = \sum_{j=1}^{\hat{J}} q^j(\hat{p}),$$

$$\pi^j(\hat{p}) = 0, \quad j = 1, \ldots, \hat{J}$$

- The first condition simply says that the market must clear.
- The second says long-run profits for all firms in the industry must be zero so that no firm wishes to enter or exit the industry.

- In the long run, the 2 conditions (market-clearing and zero-profit) determine price $\hat{p}$ and number of firms $\hat{J}$.

Example: LR Equilibrium with Cobb-Douglas technology

- Consider a competitive industry composed of J identical firms.
- Firms produce output according to the Cobb-Douglas technology $q = x^{\alpha} k^{1-\alpha}$, where x is some variable input such as labor, k is some input such as plant size, which is fixed in the short run, and $0 < \alpha < 1$.
- In the short-run analysis, we assumed that $J = 48$ and $k = 1$.
- Now, we solve for a long-run equilibrium where each firm chooses k and the number of firms J adjusts (up or down) so that profits for all firms equal zero.

- From before, when $\alpha = 1/2$, $w_x = 4$ and $w_k = 1$, the short-run profit is

$$\pi^j = p^{1/(1-\alpha)} w_x^{\alpha/(\alpha-1)} \alpha^{\alpha/(1-\alpha)} (1-\alpha)k - w_k k$$
$$= p^2 4^{-1} (1/2)^1 (1/2) k - k = \frac{p^2 k}{16} - k$$

and the short-run supply function is

$$q^j = \frac{\partial \pi^j}{\partial p} = \frac{pk}{8}.$$

- With market demand

$$q^d = \frac{294}{p},$$

48 firms in the industry and $k = 1$, we obtained a short-run equilibrium of $p^* = 7$, giving firm profits of $\pi^j = 2.0625 > 0$.

- In the long run,

$$\pi^j = \frac{p^2 k}{16} - k = k\left(\frac{p^2}{16} - 1\right) = 0$$

for all $k > 0$ if and only if $\hat{p} = 4$.

- The market-clearing condition is

$$q^d(\hat{p}) = \frac{294}{p} = \frac{294}{4} = \sum_{j=1}^{\hat{J}} q^j(\hat{p}) = \hat{J}\,\frac{pk}{8} = \hat{J}\,\frac{4\hat{k}}{8}$$

or

$$147 = \hat{J}\,\hat{k}.$$

- $$147 = \hat{J}\hat{k}$$

  Because at $\hat{p} = 4$ firm profits are zero regardless of plant size $\hat{k}$, long-run equilibrium is consistent with a wide range of market structures.
- Long-run equilibrium may involve a single firm operating a plant of size $\hat{k} = 147$, two firms each with plants $\hat{k} = 147/2$, three firms with plants $\hat{k} = 147/3$, all the way up to any number J of firms, each with a plant size $147/J$.
- This indeterminacy in the long-run equilibrium number of firms is a phenomenon common to all constant-returns industries.

- $$147 = \hat{J}\hat{k}$$

  If we fix $\hat{k} = 1$, then going from the short-run equilibrium to the long-run equilibrium, price drops from $p^* = 7$ to $\hat{p} = 4$ and the number of firms increases from $J = 48$ to $\hat{J} = 147$.

Imperfect Competition

- Perfect competition occupies one polar extreme on a spectrum of possible market structures ranging from the "more" to the "less" competitive.
- **Pure monopoly**, the least competitive market structure imaginable, is at the opposite extreme.
- In pure monopoly, there is a single seller of a product for which there are no close substitutes in consumption, and entry into the market is completely blocked by technological, financial, or legal impediments.

- The monopolist takes the market demand function as given and chooses price and quantity to maximize profits.
- Price and quantity are related by the market demand function, so the firm may choose either one, with demand determining the other.
- For convenience, we will suppose the firm chooses quantity.

- As a function of q, profit is the difference between revenue and cost,

$$\Pi(q) = r(q) - c(q).$$

If $q^* > 0$ maximizes profit, it satisfies the first-order condition

$$\Pi'(q^*) = r'(q^*) - c'(q^*) = 0,$$

which means that marginal revenue equals marginal cost:

$$mr(q^*) \equiv r'(q^*) = mc(q^*) \equiv c'(q^*).$$

Equilbrium price will be $p^* = p(q^*)$, where $p(q)$ is the inverse demand function.

- Given that revenue is $r(q) = p(q)q$, we can differentiate to obtain marginal revenue:

$$mr(q) = r'(q) = p(q) + q\frac{dp(q)}{dq}$$

$$= p(q)\left[1 + \frac{q}{p(q)}\frac{dp(q)}{dq}\right]$$

$$= p(q)\left[1 - \frac{1}{|\epsilon(q)|}\right]$$

where

$$\epsilon(q) \equiv \frac{dq}{dp}\frac{p}{q} = \frac{dq}{q}\Big/\frac{dp}{p}$$

is the elasticity of demand at output q and the absolute value $|\epsilon(q)| = -(dq/dp)(p/q) > 0$ whenever market demand is negatively sloped.

- Consequently, q^* will satisfy

$$mr(q^*) = p(q^*)\left[1 - \frac{1}{|\epsilon(q^*)|}\right] = mc(q^*) > 0$$

because marginal cost is positive. Price is also positive, so we must have $|\epsilon(q^*)| > 1$. Thus, the monopolist never chooses an output in the inelastic range of market demand.

$$\frac{p(q^*)}{p(q^*)} - \frac{1}{|\epsilon(q^*)|} = \frac{mc(q^*)}{p(q^*)}$$

$$\frac{p(q^*) - mc(q^*)}{p(q^*)} = \frac{1}{|\epsilon(q^*)|}$$

Price will exceed marginal cost by a greater amount the more market demand is inelastic.

Example: Linear Demand

- Consider the linear demand curve

$$q = a - bp.$$

Then the inverse demand curve is

$$p = \frac{a}{b} - \frac{1}{b}q$$

and the elasticity of demand is

$$\epsilon(q) \equiv \frac{dq}{dp}\frac{p}{q} = -b\frac{p}{q} = -b\left[\frac{a}{bq} - \frac{1}{b}\right] = 1 - \frac{a}{q}$$

- $$q = a - bp$$
  $$\epsilon(q) = 1 - \frac{a}{q}$$

  When $q = a$ and $p = 0$, the elasticity of demand is zero.

  When $q = 0$ and $p = a/b$, the elasticity of demand is (negative) infinity.

  When is the elasticity of demand equal to -1?
  $\epsilon(q) = 1 - (a/q) = -1$ implies that $q = a/2$, which is halfway down the demand curve.

- Consider the linear demand curve and inverse demand curve

$$q = a - bp \qquad p = \frac{a}{b} - \frac{1}{b}q$$

- Marginal revenue is

$$mr(q) = p(q) + q\frac{dp(q)}{dq} = \frac{a}{b} - \frac{1}{b}q + q\left(-\frac{1}{b}\right) = \frac{a}{b} - \frac{2}{b}q$$

- Thus, marginal revenue is linear, has the same vertical intercept a/b and twice the slope of the inverse demand curve ($-2/b$ versus $-1/b$).

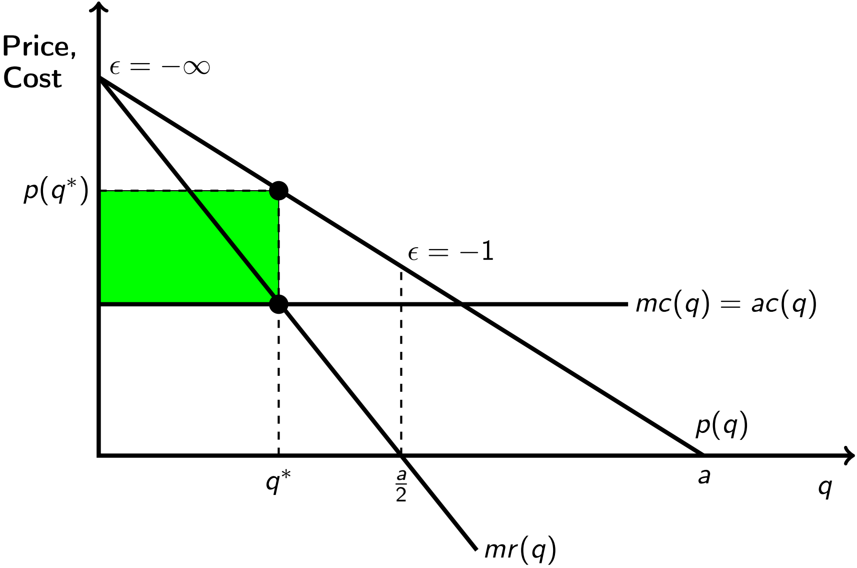

Figure: Equilibrium in a pure monopoly.

Between Pure Competition and Pure Monopoly

- Pure competition and pure monopoly are opposing extreme forms of market structure.
- Nonetheless, they share one important feature: Neither the pure competitor nor the pure monopolist needs to pay any attention to the actions of other firms in formulating its own profit-maximizing plans.
- Many markets display a blend of monopoly and competition simultaneously.
- The smaller the number of firms in the industry, the easier entry, and the closer the substitute goods available to consumers, the more interdependent firms become.

- Because firms are aware of their interdependence, and because the actions of one firm may reduce the profits of others, won't they simply work together or collude to extract as much total profit as they can from the market and then divide it between themselves?
- Putting the legality of such collusion aside, there is something tempting in the idea of a **collusive equilibrium** such as this.
- Let us consider a simple market consisting of J firms, each producing output q^j. We will suppose each firm's profit is adversely affected by an increase in the output of any other firm, so that

$$\Pi^j = \Pi^j(q^1, \ldots, q^j, \ldots, q^J) \quad \text{and} \quad \frac{\partial \Pi^j}{\partial q^k} < 0, \quad j \neq k.$$

- $$\Pi^j = \Pi^j(q^1, \ldots, q^j, \ldots, q^J) \quad \text{and} \quad \frac{\partial \Pi^j}{\partial q^k} < 0, \quad j \neq k.$$

  Now suppose firms cooperate to maximize joint profits. If $\bar{\mathbf{q}}$ maximizes

  $$\sum_{j=1}^{J} \Pi^j,$$

  it must satisfy the first-order condition

  $$\frac{\partial \Pi^k(\bar{\mathbf{q}})}{\partial q^k} + \sum_{j \neq k} \frac{\partial \Pi^j(\bar{\mathbf{q}})}{\partial q^k} = 0, \quad k = 1, \ldots, J,$$

  which implies that

  $$\frac{\partial \Pi^k(\bar{\mathbf{q}})}{\partial q^k} > 0, \quad k = 1, \ldots, J.$$

- $$\frac{\partial \Pi^k(\bar{\mathbf{q}})}{\partial q^k} > 0, \quad k = 1, \ldots, J.$$

  Each firm can increase its own profit by increasing output away from its assignment under $\bar{\mathbf{q}}$, provided, of course, that everyone else continues to produce their assignment under $\bar{\mathbf{q}}$!
- If even one firm succumbs to this temptation, $\bar{\mathbf{q}}$ will not be the output vector that prevails in the market.
- Virtually all collusive solutions give rise to incentives such as these for the agents involved to cheat on the collusive agreement they fashion.

- Any appeal there may be in the idea of a collusive outcome as the likely "equilibrium" in a market context is therefore considerably reduced.
- It is perhaps more appropriate to think of self-interested firms as essentially *noncooperative*.
- The most common concept of noncooperative equilibrium is due to John Nash (1951). In a **Nash equilibrium**, every agent must be doing the very best he or she can, given the actions of all the other agents.
- If $\mathbf{q}^*$ is to be a Nash equilibrium, each firm's output must maximize its own profit given the other firms' output choices. Thus, $\mathbf{q}^*$ must satisfy the first-order conditions:

$$\frac{\partial \Pi^k(\mathbf{q}^*)}{\partial q^k} = 0, \quad k = 1, \ldots, J.$$

Cournot Oligopoly

- The following oligopoly model dates from 1838 and is due to the French economist Auguste Cournot.
- Here we consider a simple example of **Cournot oligopoly** in the market for some homogeneous good.
- We will suppose that there are J identical firms, that entry by additional firms is effectively blocked, and that each firm has identical costs,

$$C(q^j) = c\,q^j, \quad c \geq 0 \quad \text{and} \quad j = 1, \ldots, J.$$

- Firms sell output on a common market, so market price depends on the total output sold by all firms in the market.
- Let inverse market demand be the linear form,

$$p = a - b \sum_{j=1}^{J} q^j$$

where $a > 0$, $b > 0$ and we will require $a > c$.

- Then profit for firm j is

$$\Pi^j(q^1, \ldots, q^J) = \left(a - b \sum_{k=1}^{J} q^k \right) q^j - c\, q^j.$$

- We seek a vector of outputs $(\bar{q}^1, \ldots, \bar{q}^J)$ such that each firm's output choice is profit-maximizing given the output choices of the other firms. Such a vector of outputs is called a **Cournot-Nash equilibrium**.
- Thus, given

$$\Pi^j(q^1, \ldots, q^J) = \left(a - b \sum_{k=1}^{J} q^k \right) q^j - c\, q^j,$$

$$\frac{\partial \Pi^j(\bar{q}^1, \ldots, \bar{q}^J)}{\partial q^j} = \left(a - b \sum_{k=1}^{J} \bar{q}^k \right) - b\, \bar{q}^j - c = 0$$

or

$$b\, \bar{q}^j = a - c - b \sum_{k=1}^{J} \bar{q}^k.$$

- $$b\,\bar{q}^{\,j} = a - c - b\sum_{k=1}^{J}\bar{q}^{\,k}.$$

  Noting that the right-hand side is independent of which firm j we are considering, we conclude that all firms must produce the same amount of output in equilibrium.
- By letting $\bar{q}$ denote this common equilibrium output, this equation reduces to

$$b\,\bar{q} = a - c - bJ\bar{q}$$

$$b(J+1)\bar{q} = a - c$$

$$\bar{q} = \frac{a-c}{b(J+1)}.$$

- $$\bar{q}^{j} = \frac{a-c}{b(J+1)}, \quad j = 1, \ldots, J,$$

$$\sum_{k=1}^{J} \bar{q}^{k} = \frac{J(a-c)}{b(J+1)},$$

$$\bar{p} = a - b\sum_{j=1}^{J} \bar{q}^{j} = a - \frac{J(a-c)}{J+1},$$

$$\begin{aligned}\bar{\Pi}^{j} &= \left(a - b\sum_{k=1}^{J} \bar{q}^{k}\right) \bar{q}^{j} - c\,\bar{q}^{j} \\ &= \left(a - c - \frac{J(a-c)}{J+1}\right) \frac{a-c}{b(J+1)} \\ &= \frac{(a-c)^2}{b(J+1)^2}\end{aligned}$$

- We can calculate the deviation of price from marginal cost:

$$\bar{p} = a - \frac{J(a-c)}{J+1}$$

$$\bar{p} - c = (a-c) - \frac{J(a-c)}{J+1} = \frac{a-c}{J+1} > 0.$$

  The equilibrium price will typically exceed the marginal cost of each identical firm.

- When $J = 1$ and that single firm is a pure monopolist, the deviation of price from marginal cost is greatest. At the other extreme

$$\lim_{J \to \infty} (\bar{p} - c) = 0.$$

  Perfect competition can be viewed as a limiting case of imperfect competition, as the number of firms becomes large.

Bertrand Oligopoly

- Almost 50 years after Cournot another French economist, Joseph Bertrand (1883), offered a different view of firm rivalry under imperfect competition.
- Bertrand argued it is much more natural to think of firms competing in their choice of price, rather than quantity.
- This small difference is enough to completely change the character of market equilibrium.

- Suppose there are just two firms. In a simple **Bertrand duopoly**, two firms produce a homogeneous good, each has identical marginal costs $c > 0$, and no fixed costs.
- Suppose that market demand is linear in total output Q, so

$$Q = \alpha - \beta p,$$

where p is market price.

- Firms simultaneously declare the prices they will charge and they stand ready to supply all that is demanded of them at their price.
- Consumers buy from the cheapest source.
- Thus, the firm with the lowest price will serve the entire market at the price it has declared, whereas the firm with the highest price, if prices differ, gets no customers at all.
- If both firms declare the same price, then they share the market demand equally, and each serves half.

- $Q = \alpha - \beta p$ and $Q = 0$ implies that $p = \alpha/\beta$. Restricting attention to prices satisfying $\alpha/\beta > p^i \geq c$, firm 1's profit will be

$$\Pi^1(p^1, p^2) = \begin{cases} (p^1 - c)(\alpha - \beta p^1), & c < p^1 < p^2, \\ 0.5(p^1 - c)(\alpha - \beta p^1), & c < p^1 = p^2 \\ 0, & \text{otherwise.} \end{cases}$$

- Claim: In the unique Nash equilibrium, both firms charge price equal to marginal cost $p^i = c$, and both earn zero profits.
- Proof: If $p^i = c$ for $i = 1, 2$, then each firm serves half the market and earns zero profits. By increasing its price, a firm ceases to obtain any demand at all. Consequently, it is not possible to earn more than zero profits, given what the other firm is doing.

- Next, we argue that there are no other Nash equilibria.
- Because any equilibrium must involve $p^i \geq c$, it suffices to show that there are no equilibria in which $p^i > c$ for some i. So let (p_1, p_2) be an equilibrium.
- If $p_1 > c$, then $p_2 \in (c, p_1]$ because some such choice earns firm 2 strictly positive profits, whereas all other choices earn firm 2 zero profits.
- Moreover, $p_2 \neq p_1$ because if firm 2 can earn positive profits by choosing $p_2 = p_1$ and splitting the market, it can earn even higher profits by choosing p_2 just slightly below p_1 and supplying the entire market at virtually the same price.

- Therefore

$$p_1 > c \Longrightarrow p_2 > c \quad \text{and} \quad p_2 < p_1$$

and

$$p_2 > c \Longrightarrow p_1 > c \quad \text{and} \quad p_1 < p_2$$

Consequently, if one firm's price is above marginal cost, both prices must be above marginal cost and each firm must be strictly undercutting the other, which is impossible.

□

- In the Bertrand model, price is driven to marginal cost by competition among just *two* firms. This is striking, and it contrasts starkly with what occurs in the Cournot model, where the difference between price and marginal cost declines only as the number of firms in the market increases.

Monopolistic Competition

- Firms in both Cournot and Bertrand oligopolies sell a homogeneous product.
- In **monopolistic competition**, a "relatively large" group of firms sell *differentiated* products that buyers view as close, though not perfect, substitutes for one another.
- Each firm therefore enjoys a limited degree of monopoly power in the market for its particular product variant, though the markets for different variants are closely related.
- Firms produce their products with the same technology.
- In a monopolistically competitive group, entry occurs when a new firm introduces a previously nonexistent variant of the product.

- Assume a potentially infinite number of possible product variants $j = 1, 2, \ldots$
- The demand for product j depends on its own price and the price of other variants:

$$q^j = q^j(\mathbf{p}), \qquad \text{where } \frac{\partial q^j}{\partial p^j} < 0 \text{ and } \frac{\partial q^j}{\partial p^k} > 0 \text{ for } k \neq j,$$

  and $\mathbf{p} = (p^1, \ldots, p^j, \ldots)$.
- Also, we assume there is always some price $\tilde{p}^j > 0$ at which demand for j is zero, regardless of the prices of the other products.

- Then one firm's profit depends on the prices of all variants:

$$\Pi^j(\mathbf{p}) = q^j(\mathbf{p})p^j - c^j(q^j(\mathbf{p})).$$

- In a short run equilibrium, a fixed finite number of active firms choose price to maximize profit, given the prices chosen by the others.
- Let $j = 1, \ldots, J$ be the active firms in the short run.
- For simplicity, set the price "charged" by each inactive firm k to $\tilde{p}^k$ to ensure that each of them produce no output. To ease notation, we will drop explicit mention of inactive firms for the time being.
- Now suppose $\bar{\mathbf{p}} = (\bar{p}^1, \ldots, \bar{p}^J)$ is a Nash equilibrium in the short run.

- If $0 < \bar{p}^j < \tilde{p}^j$ and firm j produces a positive output, then $\bar{\mathbf{p}}$ must satisfy the first-order conditions for an interior maximum:

$$\Pi^j(\bar{\mathbf{p}}) = q^j(\bar{\mathbf{p}})\bar{p}^j - c^j(q^j(\bar{\mathbf{p}}))$$

$$\frac{\partial \Pi^j(\bar{\mathbf{p}})}{\partial p^j} = q^j(\bar{\mathbf{p}}) + \bar{p}^j \frac{\partial q^j(\bar{\mathbf{p}})}{\partial p^j} - \frac{dc^j(q^j(\bar{\mathbf{p}}))}{dq^j} \frac{\partial q^j(\bar{\mathbf{p}})}{\partial p^j} = 0$$

$$\frac{\partial \Pi^j(\bar{\mathbf{p}})}{\partial p^j} = \frac{\partial q^j(\bar{\mathbf{p}})}{\partial p^j} \left[q^j(\bar{\mathbf{p}}) \frac{\partial p^j}{\partial q^j} + \bar{p}^j - \frac{dc^j(q^j(\bar{\mathbf{p}}))}{dq^j} \right] = 0$$

Now remember from before that when revenue is $r(q) = q\,p(q)$, then marginal revenue is $mr(q) = q\,dp(q)/dq + p(q)$.

$$\frac{\partial \Pi^j(\bar{\mathbf{p}})}{\partial p^j} = \frac{\partial q^j(\bar{\mathbf{p}})}{\partial p^j} \left[mr^j(q^j(\bar{\mathbf{p}})) - mc^j(q^j(\bar{\mathbf{p}})) \right] = 0.$$

- $$\frac{\partial \Pi^j(\bar{\mathbf{p}})}{\partial p^j} = \frac{\partial q^j(\bar{\mathbf{p}})}{\partial p^j}\left[mr^j(q^j(\bar{\mathbf{p}})) - mc^j(q^j(\bar{\mathbf{p}}))\right] = 0.$$

  Because $\partial q^j/\partial p^j < 0$, this reduces to the familiar requirement that price and output be chosen to equate marginal revenue and marginal cost:

  $$mr^j(q^j(\bar{\mathbf{p}})) = mc^j(q^j(\bar{\mathbf{p}})).$$

- As usual, the monopolistic competitor may have positive, negative or zero short-run profit.

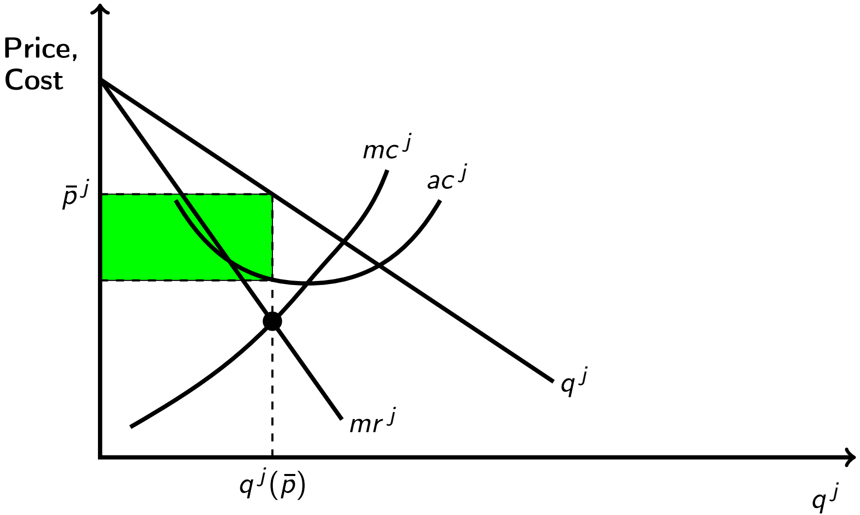

Figure: Short-run equilibrium in monopolistic competition.

- In the long-run, firms will exit the industry if their profits are negative.
- Positive profits for any single firm will induce the entry of firms producing close substitutes.
- Long-run equilibrium requires there to be no incentive for entry or exit.
- Suppose $\mathbf{p}^*$ is a Nash equilibrium vector of long-run prices. Then the following two conditions must hold for all active firms:

$$\frac{\partial \Pi^j(\mathbf{p}^*)}{\partial p^j} = \frac{\partial q^j(\mathbf{p}^*)}{\partial p^j}\left[mr^j(q^j(\mathbf{p}^*)) - mc^j(q^j(\mathbf{p}^*))\right] = 0$$

$$\Pi^j(\mathbf{p}^*) = 0$$

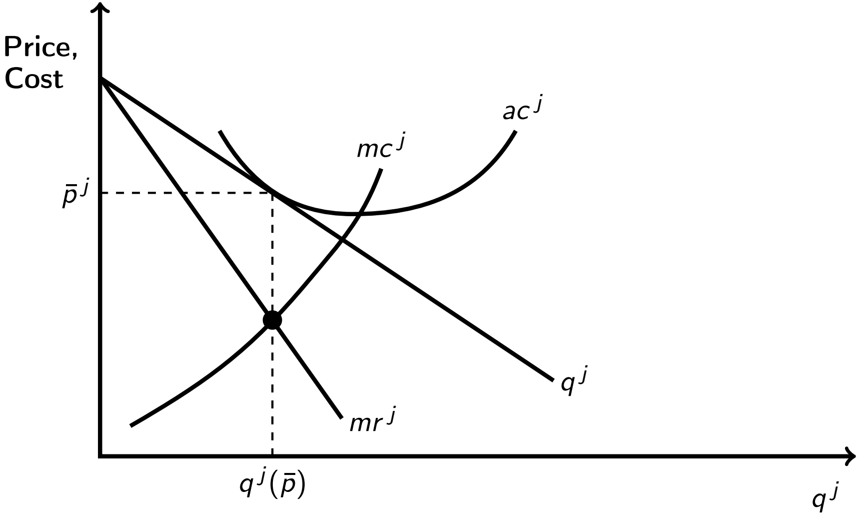

Figure: Long-run equilibrium in monopolistic competition.

Price and Individual Welfare

- How does a change in the price of a good affects a person's welfare?
- Let us suppose the price of every other good except good q remains fixed throughout our discussion. This is the essence of the partial equilibrium approach.
- So if the price of good q is p, and the vector of all other prices is $\mathbf{p}$, then instead of writing the consumer's indirect utility function as $v(p, \mathbf{p}, y)$, we shall simply write it as $v(p, y)$.

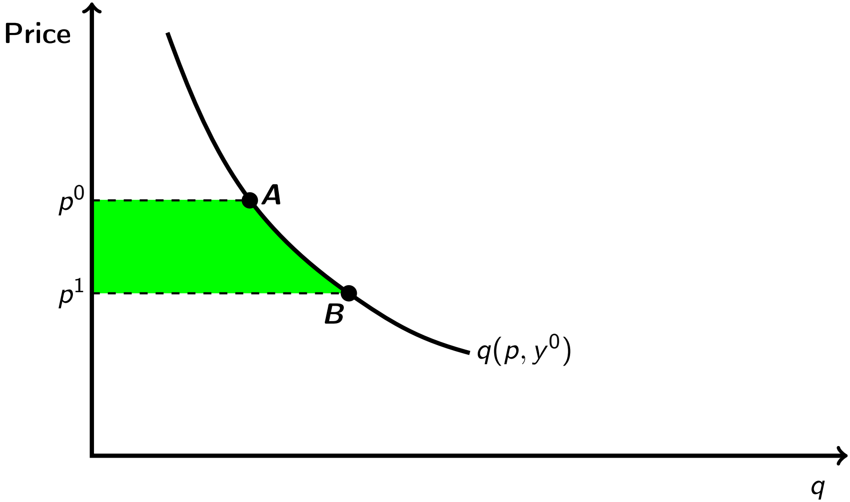

Figure: How does a price decrease affect a consumer's welfare? "Increase in consumer surplus" is the bachelor-level textbook answer. It's not correct.

- It will be convenient to introduce a **composite commodity** m as the amount of income spent on all goods other than q.
- If $\mathbf{x}(p, \mathbf{p}, y)$ denotes demand for the vector of all other goods, then the demand for the composite commodity is

$$m(p, \mathbf{p}, y) \equiv \mathbf{p} \cdot \mathbf{x}(p, \mathbf{p}, y),$$

  which we will denote simply as $m(p, y)$.
- Then the consumer's utility function over the two goods q and m is

$$\bar{u}(q, m) \equiv \max_{\mathbf{x}} u(q, \mathbf{x}) \quad \text{subject to} \quad \mathbf{p} \cdot \mathbf{x} \leq m.$$

- We can use $\bar{u}$ to analyze the consumer's problem as if there were only two goods, q and m.

- By solving

$$\max_{q,m} \ \bar{u}(q,m) \quad \text{s.t.} \quad pq + m \leq y,$$

we obtain the consumer's demands for $q(p,y)$ and $m(p,y)$, and the maximized value of $\bar{u}$ is $v(p,y)$.

- Consider now the following situation in which a typical practicing economist might find himself.
- The local government is considering plans to modernize the community's water-treatment facility. The planned renovations will improve the facility's efficiency and will result in a decrease in the price of water.
- The cost of the improvements will be offset by a one-time "water tax."
- The question is: Should the improvement be undertaken? Would consumers be willing to pay the additional tax in order to obtain the reduction in the price of water?

- Consider a particular consumer whose income is y^0. Suppose that the initial price of water is p^0 and that it will fall to p^1 as a result of the improvement project.
- By letting v denote the consumer's indirect utility function, $v(p^0, y^0)$ denotes his utility before the price fall and $v(p^1, y^0)$ his utility after.
- Now the amount of income the consumer is willing to give up for the price decrease will be just enough so that at the lower price and income levels he would be just as well off as at the initial higher price and income levels.
- Letting the **compensating variation** CV denote this change in the consumer's income, we have

$$v(p^1, y^0 + CV) = v(p^0, y^0).$$

Note that $CV < 0$ given $p^1 < p^0$ (and $CV > 0$ if $p^1 > p^0$).

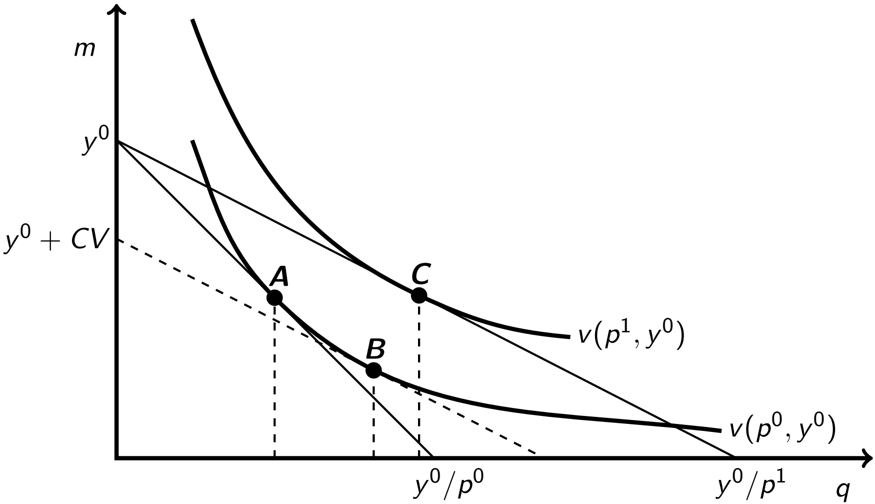

Figure: Price and individual welfare

- **THEOREM (Relations between Indirect Utility and Expenditure Functions)**: Let $v(\mathbf{p}, y)$ and $e(\mathbf{p}, u)$ be the indirect utility function and expenditure function for some consumer whose utility function is continuous and strictly increasing. Then for all $\mathbf{p} \gg \mathbf{0}$, $y \geq 0$ and $u \in \mathcal{U}$,

  1. $e(\mathbf{p}, v(\mathbf{p}, y)) = y$.
  2. $v(\mathbf{p}, e(\mathbf{p}, u)) = u$.

- $$v(p^1, y^0 + CV) = v(p^0, y^0)$$

  Let $v^0 \equiv v(p^0, y^0)$ stand for the consumer's base utility level facing base prices and income.

- Using the first identity $e(\mathbf{p}, v(\mathbf{p}, y)) = y$ relating expenditure and indirect utility functions, we obtain

  $$e(p^1, v(p^0, y^0)) = e(p^1, v(p^1, y^0 + CV)) = y^0 + CV$$

  and

  $$e(p^0, v(p^0, y^0)) = y^0.$$

- Rearranging yields

  $$CV = e(p^1, v^0) - e(p^0, v^0).$$

- Definition: A function $P(x)$ is called a **primitive** of a function $f(x)$ if $P'(x) = f(x)$ for all x (for which the function is defined).
- Definition: The **integral** of a continuous function $f(x)$ from $x = a$ to $x = b$ is denoted by

$$\int_a^b f(x)\, dx$$

and equals

$$P(b) - P(a) \equiv P(x)\Big|_a^b$$

where $P(x)$ is any primitive of $f(x)$.
- Shephard's lemma:

$$\frac{\partial e(\mathbf{p}^0, u^0)}{\partial p_i} = x_i^h(\mathbf{p}^0, u^0), \quad i = 1, \ldots, n.$$

- Using the definition of an integral and Shepard's lemma, we obtain

$$
\begin{aligned}
CV &= e(p^1, v^0) - e(p^0, v^0) \\
&= \int_{p^0}^{p^1} \frac{\partial e(p, v^0)}{\partial p} dp \\
&= \int_{p^0}^{p^1} q^h(p, v^0) dp \\
&= -\int_{p^1}^{p^0} q^h(p, v^0) dp < 0
\end{aligned}
$$

CV is therefore equal to the (negative of the) yellow shaded area to the left of the Hicksian demand curve $q^h(p, v^0)$ between p^1 and p^0.

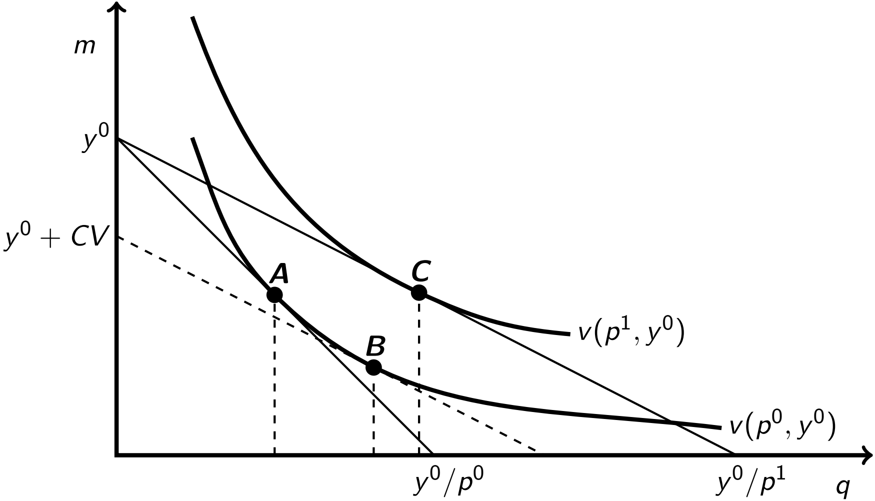

Figure: Price and individual welfare

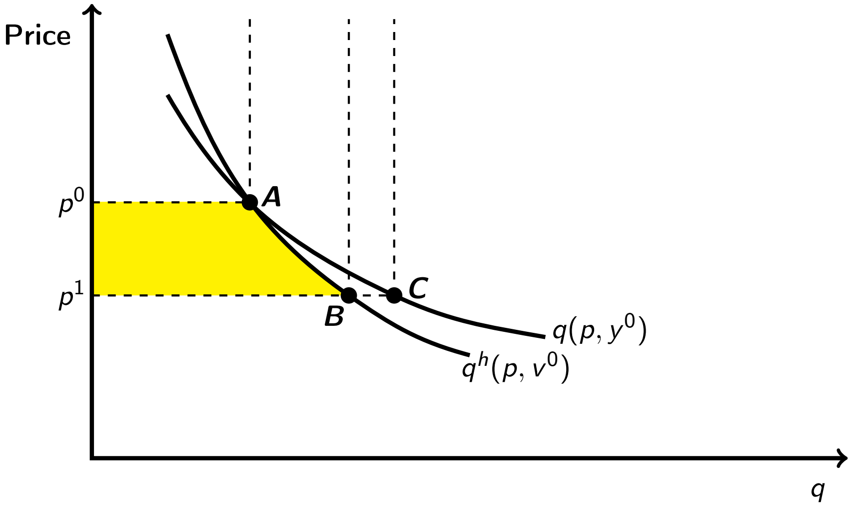

Figure: Prices, welfare and consumer demand.

- If price rises ($p > p^0$), a positive income adjustment will be necessary to restore the original utility level ($CV > 0$), and if price declines ($p < p^0$), a negative income adjustment will restore the original utility level ($CV < 0$).
- If our economist knows the consumer's Hicksian demand, then he can directly calculate CV. However, suppose our economist only has access to the consumer's Marshallian demand curve $q(p, y^0)$ for this one good corresponding to one fixed level of income y^0.
- The Hicksian and Marshallian demand curves will generally diverge because of the income effect of a price change.

- We would like to relate John Hicks's idea of compensating variation to the notion of **consumer surplus** because the latter is easily measured directly from Marshallian demand.
- Recall that at the price-income pair (p^0, y^0), consumer surplus $CS(p^0, y^0)$ is simply the area to the left of the Marshallian demand curve and above the price p^0.
- Consequently, the gain in consumer surplus due to the price fall from p^0 to p^1 is

$$\Delta CS \equiv CS(p^1, y^0) - CS(p^0, y^0) = \int_{p^1}^{p^0} q(p, y^0)dp > 0.$$

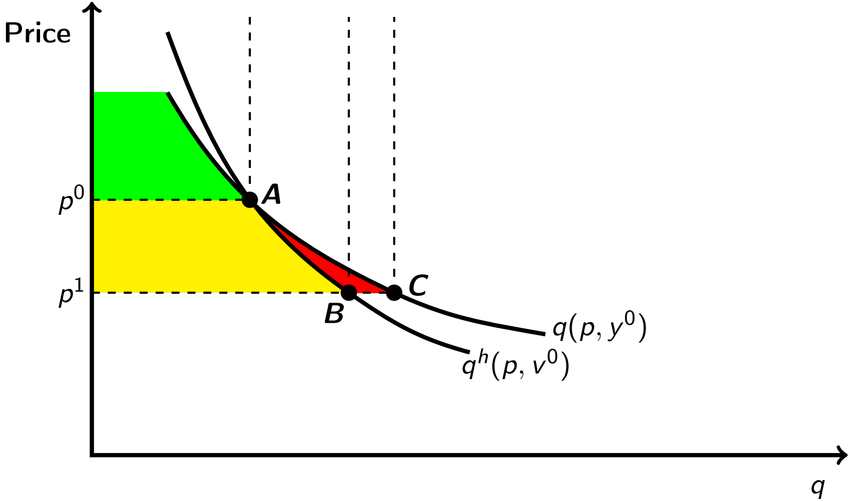

Figure: Consumer surplus CS when price is p^0 (green) and increase in consumer surplus ΔCS when price decreases from p^0 to p^1 (yellow + red).

- As you can see, ΔCS will always diverge from $-CV$ whenever demand depends in any way on the consumer's income (red area), due to the income effect of a price change.
- Because we want to know $-CV$ but can only calculate ΔCS, a natural question immediately arises. How good an approximation of $-CV$ is ΔCS?
- Robert Willig (1976, "Consumer's Surplus without apology", *American Economic Review*) studied this question and reports some useful results. For small price changes, the size of the error one makes when using ΔCS instead of $-CV$ is usually so small that one can "without apology" simply ignore it.

- When $\underline{\eta}$ and $\bar{\eta}$ are the smallest and largest values of the income elasticity of demand

$$\eta \equiv \frac{\partial q(p,y)}{\partial y} \frac{y}{q(p,y)}$$

in the region under consideration, then Willig (1976) shows that

$$\frac{1}{2}\underline{\eta}\frac{\Delta CS}{y^0} \leq \frac{\Delta CS - (-CV)}{\Delta CS} \leq \frac{1}{2}\bar{\eta}\frac{\Delta CS}{y^0}$$

- For example, if the income elasticity varies from $\underline{\eta} = 0.8$ to $\bar{\eta} = 0.9$, and consumer surplus as a fraction of base income is $\Delta CS/y^0 = 0.01$, then the error is

$$\frac{1}{2}(0.8)(0.01) = 0.004 \leq \frac{\Delta CS - (-CV)}{\Delta CS} \leq \frac{1}{2}(0.9)(0.01) = 0.0045$$

- Actually, instead of using consumer surplus "without apology," there is an even better approach:

$$\frac{1}{2}(0.8)(0.01) = 0.004 \leq \frac{\Delta CS - (-CV)}{\Delta CS} \leq \frac{1}{2}(0.9)(0.01) = 0.0045$$

$$0.004\,\Delta CS \approx \Delta CS - (-CV)$$

$$-CV \approx (1 - 0.004)\Delta CS$$

- If you have information about the size of the income effects, why not use that information to construct a better estimate!

- The concept of consumer's surplus originated in 1844 (Jules Dupuit) and has been controversial ever since.
- Alfred Marshall, who popularized the tool, stipulated that for it to be validly used the marginal utility of money must be constant.
- John Hicks (1956) stated: "In order that the Marshallian measure of consumer's surplus should be a good measure, one thing alone is needful – that the income effect should be small".
- Paul Samuelson (1947, *Foundations of Economic Analysis*) concluded that consumer's surplus is a worse than useless concept (because it confuses) and I.M.D. Little agreed, calling it no more than a "theoretical toy."

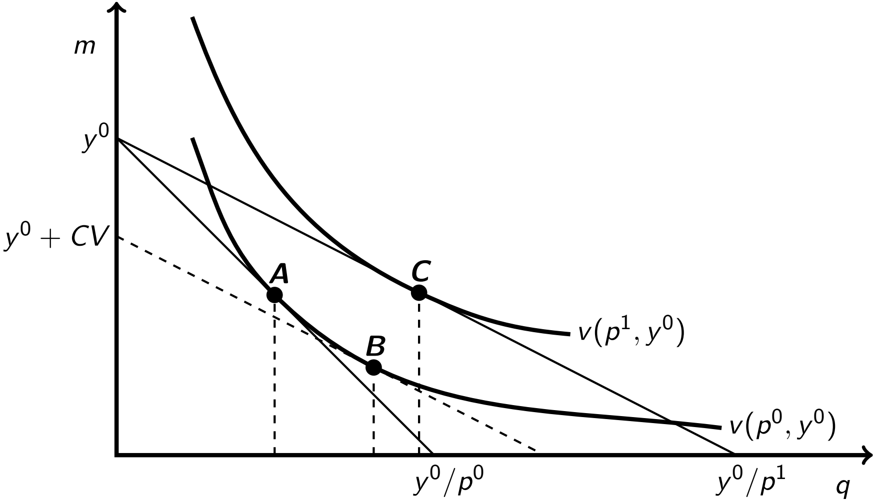

Figure: Textbook diagram misleads because roughly half of income is spent on good q (water) at point A.

Efficiency of the Competitive Outcome

- In the example just considered, it seemed clear that the water project should be implemented if after taking account of both the costs (taxes) and benefits (from lower price of water), everyone could be made better off.
- In general, when it is possible to make someone better off and no one worse off, we say that a **Pareto improvement** can be made.
- If there is no way at all to make a Pareto improvement, then we say that the current situation is **Pareto efficient**. That is, a situation is Pareto efficient if there is no way to make someone better off without making someone else worse off.

- The idea of Pareto efficiency is pervasive in economics and it is often used as one means to evaluate the performance of an economic system.
- The basic idea is that if an economic system is to be considered as functioning well, then given the distribution of resources it determines, it should not be possible to redistribute them in a way that results in a Pareto improvement.
- We now ask the question: Which, if any, of the three types of market competition – perfectly competitive, monopoly or Cournot oligopoly – functions well in the sense that they yield a Pareto efficient outcome?

- Note that the difference between the three forms of competition is simply the prices and quantities they determine.
- For example, were a perfectly competitive industry taken over by a monopolist, the price would rise from the perfectly competitive equilibrium price to the monopolist's profit-maximizing price and the quantity of the good produced and consumed would fall.
- Consequently, we might just as well ask: Which price-quantity pairs on the market demand curve yield Pareto efficient outcomes?

- To simplify the discussion, we shall suppose from now on that there is just one producer and one consumer. (The arguments generalize.)
- The following figure depicts the consumer's (and therefore the market) Marshallian demand $q(p, y^0)$, his Hicksian-compensated demand $q^h(p, v^0)$ where $v^0 = v(p^0, y^0)$, and the firm's marginal cost curve $mc(q)$.
- Consider now the price-quantity pair (p^0, q^0) on the consumer's demand curve. We will argue that this market outcome is not Pareto efficient.

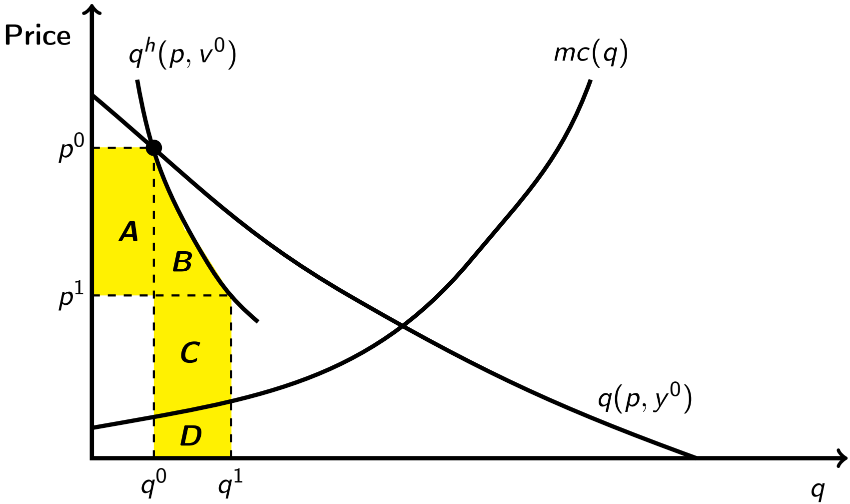

Figure: Inefficiency of monopoly equilibrium.

- To do so, we need to demonstrate that we can redistribute resources in a way that makes someone better off and no one worse off.
- So, consider reducing the price of q from p^0 to p^1. What would the consumer be willing to pay for this reduction? $-CV = A + B$.
- Let us then reduce the price to p^1 and take $A + B$ units of income away from the consumer. Consequently, he is just as well off as he was before, and he now demands q^1 units of the good according to his Hicksian-compensated demand.
- In order to fulfill the additional demand for q, let us insist that the firm produce just enough additional output to meet it.

- The price-quantity change will have an effect on the profits earned by the firm. If $c(q)$ denotes the cost of producing q units of output, then the change in the firm's profits will be

$$
\begin{aligned}
[p^1 q^1 - c(q^1)] - [p^0 q^0 - c(q^0)] &= [p^1 q^1 - p^0 q^0] - [c(q^1) - c(q^0)] \\
&= [p^1 q^1 - p^0 q^0] - \int_{q^0}^{q^1} mc(q)\, dq \\
&= [C + D - A] - D \\
&= C - A
\end{aligned}
$$

- Consequently, if after making these changes, we give the firm A dollars out of the $A + B$ collected from the consumer, the firms will have come out strictly ahead by C dollars.
- We can then give the B dollars we have left over so that in the end, both the consumer and the firm are strictly better off.
- Thus, beginning from the market outcome (p^0, q^0), we have been able to make both the consumer and the firm strictly better off simply by redistributing the available resources. Consequently, the initial situation was not Pareto efficient.

- A similar argument applies to price-quantity pairs on the consumer's Marshallian demand curve lying below the competitive point.
- Hence, the only price-quantity pair that can possibly result in a Pareto efficient outcome is the perfectly competitive one – and indeed it does (shown in the next lecture).
- Neither the monopoly outcome nor the Cournot-oligopoly outcome (for any finite n) is Pareto efficient.

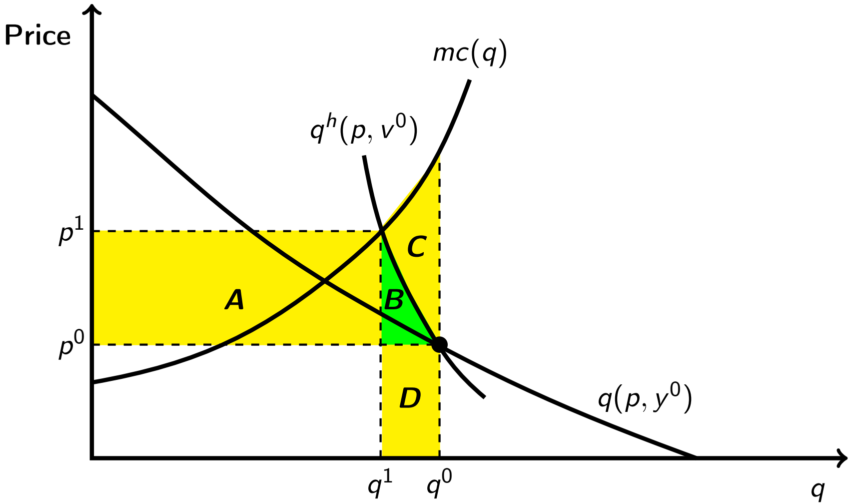

Figure: Inefficiency of low-price, high-quantity outcome.

- Consider raising the price of q from p^0 to p^1. What does the consumer need to be paid to compensate for the price increase? $CV = A + B$.
- Let us then increase the price to p^1 and give $A + B$ units of income to the consumer. Consequently, he is just as well off as he was before, and he now demands q^1 units of the good according to his Hicksian-compensated demand.
- To satisfy this reduced demand for q, let us insist that the firm reduces its production from q^0 to q^1 (and the firm is forced to pay the consumer $A + B$).

- The price-quantity change will have an effect on the profits earned by the firm. The change in the firm's profits will be

$$\begin{aligned}
&[p^1 q^1 - c(q^1) - (A + B)] - [p^0 q^0 - c(q^0)] \\
&= [p^1 q^1 - p^0 q^0] - [c(q^1) - c(q^0)] - [A + B] \\
&= [p^1 q^1 - p^0 q^0] + \int_{q^1}^{q^0} mc(q)\, dq - [A + B] \\
&= [A - D] + [D + B + C] - [A + B] \\
&= C
\end{aligned}$$

- Thus, beginning from the market outcome (p^0, q^0), we have been able to make the firm strictly better off without hurting the consumer by redistributing the available resources. Consequently, the initial situation was not Pareto efficient.

- To see that the only price-quantity pair that can possibly result in a Pareto efficient outcome is the perfectly competitive one, we try the same argument starting from the perfectly competitive outcome.

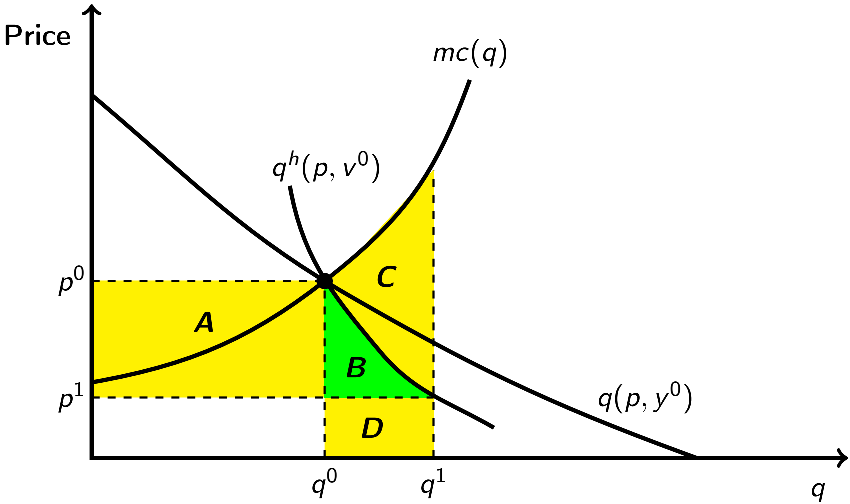

Figure: Trying to show that the perfectly competitive outcome is not efficient.

- Consider lowering the price of q from p^0 to p^1. What is the consumer will to pay for this price decrease? $-CV = A + B$.
- Let us then decrease the price to p^1 and take $A + B$ units of income from the consumer. Consequently, he is just as well off as he was before, and he now demands q^1 units of the good according to his Hicksian-compensated demand.
- To satisfy this increased demand for q, let us insist that the firm raises its production from q^0 to q^1 (and the consumer is forced to pay the firm $A + B$).

- The price-quantity change will have an effect on the profits earned by the firm. The change in the firm's profits will be

$$
\begin{aligned}
& [p^1 q^1 - c(q^1) + (A + B)] - [p^0 q^0 - c(q^0)] \\
& = [p^1 q^1 - p^0 q^0] - [c(q^1) - c(q^0)] + [A + B] \\
& = [p^1 q^1 - p^0 q^0] - \int_{q^0}^{q^1} mc(q)\, dq + [A + B] \\
& = [D - A] - [B + C + D] + [A + B] \\
& = -C
\end{aligned}
$$

- Thus, beginning from the perfectly competitive market outcome (p^0, q^0), we have made the firm strictly worse off while not helping the consumer by redistributing the available resources.
- Congratulations! We have tried to show that the perfectly competitive outcome is not Pareto efficient, and have not succeeded.

Efficiency and Total Surplus Maximization

- **Consumer surplus** is simply the area under the demand curve and above the market price. Consumer surplus is close to being a dollar measure of the gains going to the consumer as a result of purchasing the good. Consumer surplus overstates slightly the dollar benefits to the consumer whenever income effects are present and the good is normal.
- **Producer surplus** is simply the firm's revenue over and above its variable cost. This is an exact measure of the dollar value to the producer of selling the good.

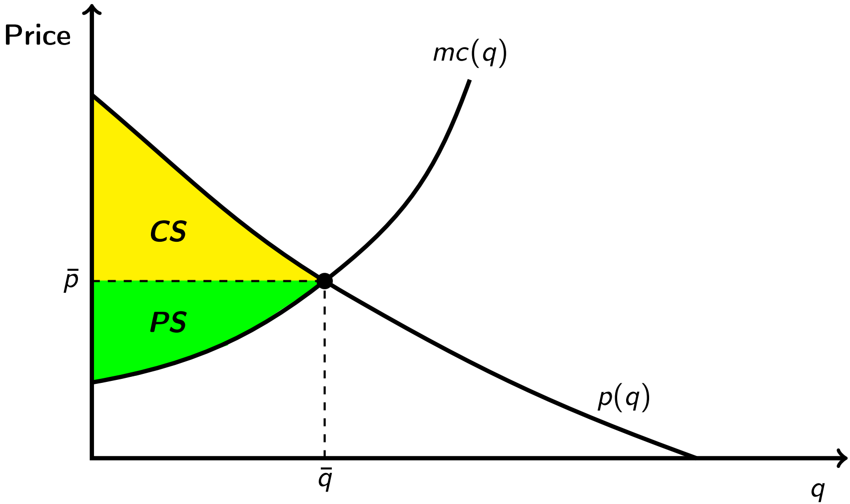

Figure: Consumer plus producer surplus at the competitive market equilibrium.

- Now it would seem that in order to obtain an efficient outcome, the **total surplus** – the sum of consumer surplus and producer surplus – must be maximized.
- Otherwise, both the producer and consumer could be made better off by redistributing resources to increase the total surplus, and then dividing the larger surplus among them so that each obtains strictly more surplus than before.
- We will now show that this is true, for the case of a single consumer and a single producer, with downward-sloping demand and upward-sloping marginal cost.

- At an arbitrary price-quantity pair (p, q) on the (inverse) demand curve $p(q)$, the sum of consumer and producer surplus is

$$CS(q) + PS(q) = \left[\int_0^q p(\omega)\,d\omega - p(q)q\right] + \left[p(q)q - \int_0^q mc(\omega)\,d\omega\right]$$

$$= \int_0^q [p(\omega) - mc(\omega)]\,d\omega.$$

Choosing q to maximize this expression (using Leibniz Rule) leads to the first-order condition

$$CS'(q) + PS'(q) = \frac{dq}{dq}\,[p(q) - mc(q)] = 0 \quad \text{or} \quad p(q) = mc(q),$$

which occurs precisely at the perfectly competitive equilibrium quantity.

- $$\frac{d}{dq}\int_0^q \left[p(\omega) - mc(\omega)\right] d\omega = \frac{dq}{dq}\left[p(q) - mc(q)\right]$$

- **Theorem (Leibniz Rule)**: If $g_y \equiv \partial g / \partial y$ is continuous and h_1 and h_2 are differentiable, then

$$\frac{d}{dy}\int_{h_1(y)}^{h_2(y)} g(x, y)\, dx = \int_{h_1(y)}^{h_2(y)} \frac{\partial g(x, y)}{\partial y}\, dx$$

$$+h_2'(y) g(h_2(y), y) - h_1'(y) g(h_1(y), y)$$

- Whenever price and marginal cost differ, the total surplus can be increased and a Pareto-improvement can be implemented, as we showed earlier.
- When markets are imperfectly competitive, the market equilibrium generally involves prices that exceed marginal cost.
- However, "price equals marginal cost" is a necessary condition for a maximum of consumer and producer surplus.
- It should therefore come as no surprise that the equilibrium outcomes in most imperfectly competitive markets are *not* Pareto efficient.

Example: Cournot oligopoly

- Let us consider the performance of the previously studied Cournot oligopoly.
- Let market (inverse) demand be $p = a - bq$ for total market output q.
- Firms are identical, with marginal cost $c > 0$.
- When each firm produces the same output q/J, total surplus is illustrated in the following figure:

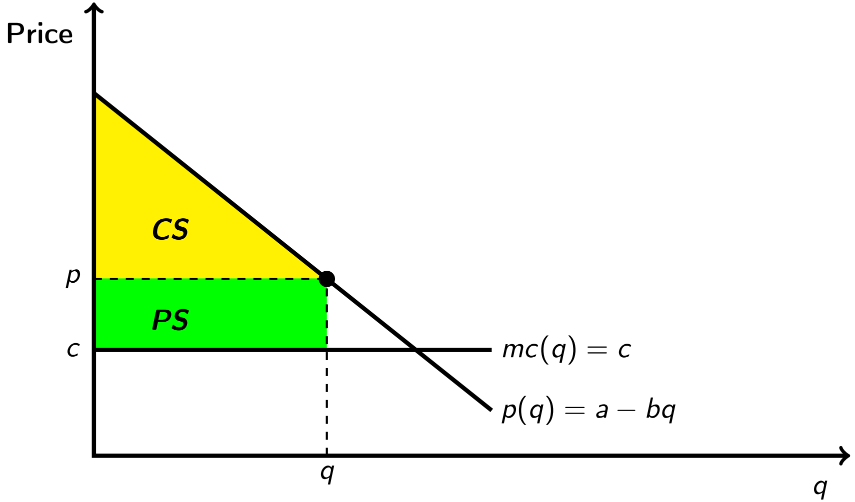

Figure: Total surplus when there is Cournot oligopoly with linear demand and constant marginal cost.

- When each firm produces the same output q/J, total surplus as a function of total output will be

$$
\begin{aligned}
W(q) &= \int_0^q (a - b\,\omega) d\omega - J \int_0^{q/J} c\, d\omega \\
&= \left(a\,\omega - \frac{b}{2}\,\omega^2 \right)\Bigg|_0^q - J\left(c\,\omega \Bigg|_0^{q/J} \right) \\
&= a\,q - \frac{b}{2}\,q^2 - Jc\frac{q}{J} \\
&= a\,q - \frac{b}{2}\,q^2 - cq
\end{aligned}
$$

Maximizing total surplus yields

$$
W'(q) = a - c - bq = 0 \qquad q^* = \frac{a-c}{b} \qquad W''(q) = -b < 0
$$

- Thus the maximum total surplus in this market will be

$$
\begin{aligned}
W(q^*) &= a\,q^* - \frac{b}{2}\,(q^*)^2 - cq^* \\
&= (a-c)\frac{a-c}{b} - \frac{b}{2}\left(\frac{a-c}{b}\right)^2 \\
&= \frac{(a-c)^2}{b} - \frac{(a-c)^2}{2b} \\
&= \frac{(a-c)^2}{2b}
\end{aligned}
$$

- In the Cournot-Nash equilibrium with J symmetric firms, we have seen that total market output will be

$$\bar{q} = \frac{J(a-c)}{(J+1)b}.$$

Clearly

$$\bar{q} = \frac{J(a-c)}{(J+1)b} < q^* = \frac{a-c}{b},$$

so the Cournot oligopoly produces too little output from a social point of view.

- Total surplus in the Cournot equilibrium will be

$$\begin{aligned}
W(\bar{q}) &= a\bar{q} - \frac{b}{2}\bar{q}^2 - c\bar{q} \\
&= (a-c)\frac{J(a-c)}{(J+1)b} - \frac{b}{2}\left(\frac{J(a-c)}{(J+1)b}\right)^2 \\
&= \frac{(a-c)^2}{2b}\frac{2J}{J+1} - \frac{(a-c)^2}{2b}\frac{J^2}{(J+1)^2} \\
&= \frac{(a-c)^2}{2b}\frac{J}{J+1}\left(\frac{2(J+1)}{J+1} - \frac{J}{J+1}\right) \\
&= \frac{(a-c)^2}{2b}\frac{J(J+2)}{(J+1)^2}
\end{aligned}$$

- Therefore, the **deadweight loss** with Cournot oligopoly is

$$
\begin{aligned}
W(q^*) - W(\bar{q}) &= \frac{(a-c)^2}{2b} - \frac{(a-c)^2}{2b}\frac{J(J+2)}{(J+1)^2} \\
&= \frac{(a-c)^2}{2b}\left(\frac{(J+1)^2 - J(J+2)}{(J+1)^2}\right) \\
&= \frac{(a-c)^2}{2b}\left(\frac{J^2+2J+1-J^2-2J)}{(J+1)^2}\right) \\
&= \frac{(a-c)^2}{2b(J+1)^2} > 0
\end{aligned}
$$

Total surplus increases as the number of firms in the market becomes larger [since $W(\bar{q})$ increases as J increases]. As the number of firms increases, the market price converges to marginal cost. Consequently, total surplus rises toward its maximal level and the deadweight loss declines to zero as $J \to \infty$.

History of Economic Ideas

- The analysis of a single competitive market is due to Alfred Marshall (1920, *Principles of Economics*).
- Auguste Cournot presented his model of imperfect competition in his 1838 book *Researches into the Mathematical Principles of the Theory of Wealth*.
- John Nash was awarded the economics Nobel prize in 1994, largely based on his 1951 paper "Non-cooperative games" published in *Annals of Mathematics*. His life story has been made famous as a result of the Academy Award winning movie *A Beautiful Mind* where actor Russell Crowe plays John Nash.

- The concept of compensating variation CV and its relationship to consumer surplus is due to John Hicks (1956, *A Revision of Demand Theory*). Hicks was awarded the economics Nobel prize in 1972.