Seminar 7 - Exercises

1. Review questions

   - What are the major assumptions characterizing perfect competition?
     A: Big number of competitors → price takers, free entry and exit, homogenous product (same cost functions)
   - In a purely competitive market a firm's marginal revenue is always equal to what? What is the profit a firm in the long run in the perfectly competitive market?
     A: Marginal revenue is equal to marginal cost; long–run profit in the perfectly competitive market is zero. $MR = p = MC$
   - Is the short-run supply curve more elastic than long-run supply curve? Why? Where do they intersect?
     A: Long-run supply curve is more elastic because of no fixed inputs.
   - Is it ever better for a perfectly competitive firm to produce output even though it is losing money? When and why?
     A: Yes, but only in the short-run and to the level of minus fixed costs.
   - A perfect competitive firm has the cost function $C(y) = (y-2)^2 + 1$. Draw a graph with AC, MC and TC and determine on what level of y do the curves intersect. Use the graph for calculation of profit when $y = 5$.
     A: $AC = y - 4 + \frac{5}{y}$, $MC = 2y - 4$, intersections: $MC = AC$ for $y = \sqrt{5}$, $TC = MC$ for $y = 3$;

2. A firm in the perfect competitive market has the cost function $LTC = 3y^3 - 8y^2 - 11y$.

   - Calculate supply curve of the firm $S^{-1}(y^*)$
     A: $9y^2 - 16y - 11 = p = S_y^{-1}(y^*)$ when $y \geq \frac{4}{3}$
   - What is the minimal price p_0 when a firm starts producing some output y_0?
     A: When $MC = AC$ then $p = -\frac{49}{3} \to p_0 = 0$
   - Derive long-run supply curve $y^* = S_y(p)$
     A: $y = \frac{8 \pm \sqrt{163+9p}}{9}$

3. A firm in the perfect competition market has the cost function $STC = y^3 - 80y^2 + 2,500y + 40,000$.

   - Calculate supply curve of the firm $S_y^{-1}(y^*)$
     A: $3y^2 - 160y + 2500 = p = S_y^{-1}(y^*)$ when $y \geq 40$
   - What is the minimal price p_0 when a firm starts producing some output y_0?
     A: $p = 900$
   - What will be the output when price is equal to £500?
     A: $p < 900 \Rightarrow y = 0$
   - Derive supply curve $y^* = S_y(p)$
     A: $S(p) = \frac{80 \pm \sqrt{3p-1100}}{3}$

4. A firm operating in the perfect competition market has the production function $f(x_1, x_2) = (min\{3x_1, 2x_2\})^2$.

   - Derive the cost function.
     A: $C = w_1 \frac{\sqrt{y}}{3} + w_2 \frac{\sqrt{y}}{2}$
   - Calculate $AC(y)$ and $MC(y)$ when $w_1 = 2$ and $w_2 = 3$. Given these values what is the supply of the firm $y = S(p)$? Can the firm produce some positive output with negative profit $\Pi^* \leq 0$?
     A: $MC = \frac{13}{12\sqrt{y}}$, $AC = \frac{13}{6\sqrt{y}}$; $S(p) = \frac{169}{144p^2}$ for $y \geq \min(AC)$.
     No, because firm has no fixed costs.
   - Calculate the profit $\Pi(1, 2, 3)$ when $p = 1$.
     A: $\Pi = -\frac{169}{144} \Rightarrow \Pi = 0$

- How will the supply of the firm $y = S(p)$ change with the price change of the first input $\frac{\partial y(p,w)}{\partial w_1}$?
  A: $\frac{\partial S(p)}{w_1} = (\frac{w_1}{3} + \frac{w_2}{2})\frac{1}{6p^2} > 0$, It means that supply will increase with the rise of w_1.

5. Consider perfect competition firm with production function $f(x_1, x_2) = (3x_1^2 + x_2^2)^{\frac{3}{2}}$ and cost function $C(y, w) = y^{\frac{1}{3}} 3^{-\frac{1}{2}}(w_1^2 + 3w_2^2)^{\frac{1}{2}}$. (This cost function is not exactly the cost function for given production function. However, it does not have any consequences for our calculations.)

   - Derive supply of the firm.
     A: $S(p) = \frac{1}{3^{\frac{9}{4}} p^{\frac{3}{2}}}(w_1^2 + 3w_2^2)^{\frac{3}{4}}$ when $y \geq \min(AC)$
   - Derive profit function $\Pi(p, w)$.
     A: $\Pi = \frac{(w_1^2+3w_2^2)^{\frac{3}{4}}}{p^{\frac{1}{2}}}(3^{-\frac{9}{4}} - 3^{-\frac{5}{4}}) < 0 \Rightarrow \Pi = 0$
   - Calculate Marshal factor demands $x_1(p, w)$ and $x_2(p, w)$.
     A: $x_1(p, w) = \dfrac{w_1(w_1^2 + 3w_2^2)^{-\frac{1}{4}}}{3^{\frac{5}{4}}\sqrt{p}}$, $x_2(p, w) = \dfrac{w_2(w_1^2 + 3w_2^2)^{-\frac{1}{4}}}{3^{\frac{1}{4}}\sqrt{p}}$

6. Assume that a certain small town contains a large number of widget-producing firms. All the firms buy oil from the same refinery. Firm 1 is situated very close to the refinery, and the other firms are located 50 miles away. Firm 1 pays £18 per barrel for the oil, while the other firms pay £18 per barrel plus a transportation charge of £0.05 a mile, or a total of £20.50 per barrel.To produce four widgets, a firm needs $\frac{1}{10}$ barrel of oil, $\frac{1}{2}$ hour of labor, and the use of one machine. The cost of labor is £10 per hour, and the necessary machine can be rented for £5 per hour. No firm has the capacity to produce more than 100 units of widgets.

   - Derive the supply curve for firm 1. Derive the supply curve for all the other firms.
     A: Each widget needs $\frac{1}{40}$ barrel of oil, $\frac{1}{8}$ hour of labor, and $\frac{1}{4}$ hour of a machine's use. Therefore, the total cost function can be written as $C = (\frac{18}{40} + \frac{10}{8} + \frac{5}{4})W = 2.95W$, where W is the number of widgets produced by firm 1. For all other firms, the cost function is $C = (\frac{20.50}{40} + \frac{10}{8} + \frac{5}{4})W = 3.01W$. Because there are a "large number" of firms, they act as price takers and equate their marginal costs to the price. Therefore, firm 1's supply curve is $P = 2.95$ and all the other supply curves are $P = 3.01$.
   - What is the equilibrium price?
     A: Because the "other" firms outnumber firm 1 by a very large number, the industry supply curve (which is derived by horizontal addition of the individual supply curves) is going to be the same as the individual supply curve of the "other" firms, i.e., $P = 3.01$, as long as the demand for widgets is greater than 100 at a price of 2.95. Because this supply curve is a horizontal line, the equilibrium price is also going to be 3.01.
   - Does any firm earn economic rent (that is, extra economic profit) in the industry?
     A: Firm 1 earns a rent of £0.06 per unit, because its average cost is £2.95 while the price is £3.01 per unit.
   - Does firm 1 affect the price of widgets in the industry? If not, why not?
     A: Firm 1 does not affect the market price as long as demand is very large because the capacity constraint allows it to produce only 100 widgets.
   - Suppose that there is no capacity limit. What will the equilibrium price be?
     A: If there is a capacity limit of 100 units for each firm, as long as the demand exceeds 100 units at a price of £3.01, the other firms have an opportunity to produce some widgets. If there were no capacity limits, firm 1 would be able to satisfy any level of demand at £2.95 and consequently, the other firms will not be able to sell any widgets at all. However, because firm 1 faces no competition when it charges less than £3.01, it will maximize its profits by charging just a little bit under £3.01, so the long-run equilibrium price will be $£3.01 - \varepsilon$.
   - Will firm 1 affect the price when there is unlimited capacity?
     A: If there is no capacity limit on output, then firm 1 indeed affects the market price, as was shown in the answer to part *e*. Firm 1 is free to set any price between £2.95 and £3.01.