JEB064 2022/2023 Sample solution

1 Discrete auctions (16 pts)

Suppose $n \geq 3$ bidders bid for an item in an auction. All bidders want to sell the item on the secondary market. It is common knowledge that the price on the secondary market is $v \geq 3$, where v is an integer. Bidders can submit only integers (discrete auction). If multiple bids are winning, the winner is selected randomly (and equiprobably).

Consider only equilibria with losers (i.e., at least one bidder submits a losing bid). In other words, disregard equilibria in which all bidders submit an identical bid.

1. First-price auction: What is the equilibrium price that the winner pays? Is the winner's profit positive or not?

2. Second-price auction: What is the equilibrium price that the winner pays? Is the winner's profit positive or not?

Always find all solutions. That is, for any price that you suspect that the winner pays, construct an appropriate profile of equilibrium bids. Also remember that we are looking for Nash equilibria without additional restrictions, so we allow for weakly dominated strategies.

Sample solution

- *First-price auction*: Losers must not strictly prefer to join the winner(s) or outbid the winner(s). It means that the winning bid (also the winner's price) must be v. Any winner's profit is *zero*. Also, notice that there must be at least two winners; if not, then the winner decreases her bid below v and earns positive profits.

- *Second-price auction*: Losers must not be willing to join the winner(s) or outbid the winner(s). Therefore, the highest bid must be $\bar{b} \geq v$; when the loser matches or equals this bid, it becomes the second price (the winner's price), and this price implies a non-positive payoff. At the same time, there is no restriction on the losers' bids. They can be arbitrarily distributed over $\{0, 1, \ldots, v\}$. Therefore, also the second highest bid (second) price can be arbitrary, and therefore the auction winner can pay in the equilibrium any price from the set $\{0, 1, \ldots, v\}$. Winner's profit is either *positive* or *zero*.

2 Fiscal governance (18 pts)

Like in the class on budgetary commons, suppose the government provides collective goods of socio-economic groups $i \in \{1, \dots, n\}$ (e.g., think of roads in n cities). Spending for a collective good of group i is denoted $x_i \geq s$, where $s \in (0, 1)$ is a guaranteed minimal level that the government must provide in any case. Each group pays an identical tax $t \geq 0$. Total tax revenues cover total spending, $nt = \sum_i x_i$. Each group has a quasilinear utility function (natural log), $u_i(x_i, t_i) = \log(x_i) - t_i$.

Each group i is represented by a single minister i. Ministers are non-cooperative players (cannot form agreements). There are two budgeting regimes:

- Decentralized budgeting: Each minister i sets spending for group i. The level of spending must satisfy $x_i \geq s$.
- Delegation with a uniform spending cap X, where $X \geq s$: Using a fair lottery, a single minister out of n ministers is appointed as Minister of Finance. The Minister of Finance sets the entire government budget such that for each i, the level of its spending satisfies $x_i \in [s, X]$.

Suppose the groups are not satisfied with decentralized budgeting and discuss improving fiscal governance. They consider switching from decentralized budgeting to delegation with a uniform spending cap.

1. Conditional on delegation, which uniform spending cap do they prefer? Denote this optimal spending cap X^*.
2. Now, construct a parametrical condition under which each group prefers delegation with the uniform cap X^* to decentralized budgeting.
3. If the minimal payment for each group s increases, does each group more likely prefer delegation with the uniform cap X^* to decentralized budgeting or not? Prove analytically.
4. If the number of groups n increases, does each group more likely prefer delegation with the uniform cap X^* to decentralized budgeting or not? Prove analytically.

Sample solution

1. *Conditional on delegation, which spending cap do they prefer? Denote this optimal spending cap X^*.*

   With a binding cap $X \leq n$, the expected payoff under delegation is

$$\tfrac{1}{n} \log X + \tfrac{n-1}{n} \log s - \frac{X + (n-1)s}{n} = \tfrac{1}{n} (\log X - X) + \tfrac{n-1}{n} (\log s - s) .$$

Since $\log X - X$ is maximized at 1, the payoff-maximizing cap is $X^* = 1$.

2. *Now, construct a condition which describes when each group prefers delegation with cap X^* to decentralized budgeting.*

   Inserting $\log X^* - X^* = -1$ into the expected payoff under delegation with cap equal to 1 (see above), we obtain the expected payoff

$$-\frac{1}{n} + \frac{n-1}{n}\left(\log s - s\right).$$

   To compare, the expected payoff under decentralized budgeting is $\log n - n$. (We know that in Nash equilibrium of decentralized budgeting, each minister sets $x_i^* = n$.) After rearranging the condition such that s is on the LHS and n is on the RHS, the condition writes

$$\log s - s \geq \frac{n}{n-1}\log n - (n+1).$$

3. *If the minimal payment for each group s increases, does each group more likely prefer delegation with the optimal cap to decentralized budgeting or not? Prove analytically.*

   Yes. From the first version of the expected payoff under delegation, $\frac{1}{n}\left(\log X - X\right) + \frac{n-1}{n}\left(\log s - s\right)$, we can clearly see that the payoff is increasing in s since $\log s - s$ is increasing in s for $s < 1$. Since the payoff under decentralized budgeting is not affected by a change in s, each group *more likely* prefers delegation with the optimal cap. We can also observe that directly from the condition in Question 2 (see the positive effect on the LHS).

4. *If the number of groups n increases, does the attractiveness of delegation with the optimal cap (measured by a difference of expected payoffs) increase or not? Prove analytically.*

   Yes. We take the condition from Question 2. The derivative of the RHS with respect to n is

$$\frac{(2-n)(n-1) - \log n}{(n-1)^2}.$$

   For $n \geq 3$, both $(2-n)(n-1)$ and $-\log n$ are negative (for $n = 2$, one is zero and one is negative). Since there is no effect on LHS, the condition is more likely satisfied.

3 Wine taxation (16 pts)

Wine and beer producers have conflicting preferences over wine taxation. Wine is taxed by a tax rate $\tau \geq 0$. For simplicity, suppose that wine tax revenues T are linear in tax rate τ up to a level $\hat{\tau} > 0$ (the peak of Laffer curve), $T(\tau) = \tau$ if $\tau \in [0, \hat{\tau}]$. Suppose wine producers' bear pay full tax cost, $u_A(\tau) - u_A(0) = -T(\tau) = -\tau$ and suppose beer producers receive *all* these tax revenues, $u_B(\tau) - u_B(0) = T(\tau) = \tau$. (We implicitly assume that no additional deadweight loss arises.)

- In Stage 1, Wine producers propose tax policy $a \in [0, \hat{\tau}]$ to be implemented if they win a lobbying contest; Beer producers propose tax policy $b \in [0, \hat{\tau}]$ to be implemented if they win a lobbying contest. (Tax rates above the peak of Laffer curve are disregarded.)
- In Stage 2, the two producers engage in a lobbying contest over wine taxation. The contest is a Tullock lottery. Those producers who win the contest set the policy that they proposed in Stage 1.

Analyze the lobbying contest:

1. Find the equilibrium tax policies (a^*, b^*) that the two competing lobbies propose.
2. In equilibrium, what is the probability that wine producers win the contest?
3. Suppose now that beer producers receive only a fraction $\alpha \in (0, 1)$ of wine tax revenues. What is the probability that wine producers win the contest?

Sample solution For preferences that are linear in policy, the contest prizes are *linear* in (a, b). We suppose $0 \leq a \leq b \leq \tau$.

1. *Symmetry* ($\alpha = 1$): The prizes are $(R_A, R_B) = (b - a, b - a)$, where A is for Wine producers and B is for Beer producers. This is a symmetric Tullock contest where each group wins with probability $\frac{1}{2}$. Given the symmetric prize $R = b - a$, we know that the equilibrium profit is $\frac{R}{4} = \frac{b-a}{4}$. Therefore, $U_A(a, b) = \frac{b-a}{4} + u_A(b)$. For A, a deviation from her bliss point $\tau = a$ *decreases* her expected payoff,
$$\frac{\partial U_A(a, b)}{\partial a} = -\frac{1}{4} < 0.$$
Hence, best response of A is $a(b) = 0$. Similarly, the best response of B is $b(a) = \hat{\tau}$. Nash equilibrium is $(a^*, b^*) = (0, \hat{\tau})$.
2. *Asymmetry* ($\alpha < 1$): The prizes are now asymmetric, $R_A, R_B) = (b - a, \alpha(b - a))$. This is an asymmetric Tullock contest with prizes $(R, \alpha R)$. In this contest, we know

the ratio of equilibrium efforts is the ratio of prizes, $\frac{x_B}{x_A} = \frac{\alpha R}{R} = \alpha$. This means that Wine producers win with probability

$$p_A = \frac{x_A}{x_A + x_B} = \frac{1}{1+\alpha} > \frac{1}{2}.$$