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ABSTRACT

Strategy in Contests – an Introduction

by Kai A. Konrad *

Competition in which goods or rents are allocated as a function of the various efforts expended by players in trying to win these goods or rents is a very common phenomenon. A subset of examples comes from marketing, litigation, relative reward schemes or promotion tournaments in internal labor markets, beauty contests, influence activities, education filters, R&D contests, electoral competition in political markets, military conflict and sports. I survey here this type of competition which is sometimes called contest or tournament. I focus on the role of its various design aspects, such as prize structure, sequencing, nesting, repetition, elimination contests and many others. Some key insights about the nature and properties of this type of competition emerge from this analysis.

Keywords: Survey of contests, tournaments, conflict, strategic aspects

JEL Classification: D72, D74

ZUSAMMENFASSUNG

Strategie in Turnieren – eine Einführung

Die Arbeit behandelt strategische Entscheidungen in Turnier- und Wettkampfsituationen. Sie gibt einen theoretischen und empirischen Überblick und ergänzt die umfangreiche theoretische Literatur zu diesem Thema, besonders in Hinblick auf wiederholte Turniere und auf komplexere Wettbewerbsstrukturen, die sich aus verschiedenen gestaffelten Einzelturnieren zusammensetzen.

* I am indebted to many colleagues who discussed this project and specific topics and made most valuable suggestions, in particular, Aron Kiss, Dan Kovenock, Wolfgang Leininger, Florian Morath, Johannes Münster, Shmuel Nitzan, Stergios Skaperdas and Karl Wärneryd. The usual caveat applies.

Contents

1 An introduction to contests

There are many types of interaction in which players expend effort in trying to get ahead of their rivals. Such interactions include marketing and advertising by firms, litigation, relative reward schemes in firms, beauty contest by firms and rent seeking for rents allocated by a public regulator, political competition, patent races, pleasant activities such as sports, and also rather unpleasant events such as military combat, war and civil war are some of the examples. These have been studied in the field of contest theory both within these specific contexts and at a higher level of abstraction. The purpose of this manuscript is to survey this work, focussing on the strategic aspects of such games, their interaction with each other and within a more general decision framework.

The theory of contests, tournaments or all-pay auctions is a dynamic field. It was difficult when writing this survey to keep pace with the speed with which progress is being made. This is one of the reasons why the survey is blatantly partial, subjective and biased. Some of the biases have been introduced deliberately. I have not have done justice to the large number of empirical studies of particular contest environments and have concentrated instead on contributions that illustrate theoretical aspects of contests that seem to be generalizable and applicable to a wide set of contest environments. The work is also biased by the specific aspects and topics that I became interested in during my own past research .

There are several important lines of research which show up only marginally in this manuscript, even though whole books could be written about them. For instance, from a variety of starting points researchers study investment, savings, trade and other economic activities in a world where property rights are not exogenously defined but where players continuously struggle and expend effort to appropriate and defend resources. These 'economics of conflict' have been surveyed most recently by Garfinkel and Skaperdas (2006), and the existence of this excellent survey is perhaps a good excuse for not treating this aspect more extensively here. In addition, a whole school of researchers analysed the implications of rent-seeking in many specific contexts based on Tullock (1967) and a number of other seminal papers. Several volumes have been published over the years that collect some of the most important contributions in this field, including Buchanan, Tollison and Tullock (1980), and more recently Lockard and Tullock (2001). Many contributions of this school are surveyed in this manuscript, but rather than providing a balanced survey of this literature, the focus here is on the theoretical insights into contests developed in this literature. Further, a large theoretical and empirical literature on tournaments has been developed, particularly in the context of internal labor markets, and also in the context of patent races in industrial organization. Some theory aspects of this literature are covered in this manuscript, but the empirical studies, for instance, are not surveyed here. These are important omissions. I hope that the many many authors who have made significant contributions in these fields and whose work is not surveyed here will be lenient in their judgement.

In this chapter, I will start with a definition of the main subject of study: a contest. Then I will illustrate the wide range of examples of this type of structure. In chapter 2, three major types of contest are analysed more carefully. Chapters 3, 4, 5 and 6 will consider more complex aspects of contests and questions of contest design, keeping the focus on single contests. In chapter 7, the substructure of contests is analysed, focusing on the insight that many types of contests are part of superstructures, or consist of substructures, that have the nature of contests.

1.1 A definition

A contest can be characterized by the following elements. First, there is a prize to be allocated among the contestants who belong to a set of contestants $N = \{1, ...n\}$. Each contestant $i \in N$ can make an effort x_i, yielding a vector $\mathbf{x} = \{x_1, ..., x_n\}$ of efforts. These efforts determine which contestant will receive which prize, where, in the most simple case, only one contestant gets a positive prize of some size B and all other contestants get zero. The function that maps the vector of efforts into probabilities for the different contestants winning the prize is

$$p_i = p_i(x_1, ..., x_n). \tag{1}$$

Usually, in the contest literature this function is called *contest success function.* This suggests that, for a given vector $\mathbf{x}$ of efforts, the p_i's are between zero and one, sum up to 1, or to something smaller than one, if there is a chance that the prize is not allocated at all to one of the contestants. To end up with the contestants' payoff functions, one should note that different contestants may value the prize differently and denote $v_i(B)$ as i's value of winning. Further, contestants may differ in their cost of providing a given level of effort. The relationship between effort x_i and i's effort cost is $C_i(x_i)$. In most cases, and if not explicitly stated otherwise, we will assume that $C_i(x_i) = x_i$. Hence, depending on the effort choices, contestant i receives a payoff equal to

$$\pi_i(x_1, ..., x_n) = p_i(x_1, ..., x_n)v_i(B) - C_i(x_i). \tag{2}$$

More formally, simple contests are games that are defined by a set N of players, pure strategy spaces described by the sets of feasible pure strategies that are described as efforts $x_i \in X_i$, and by the set of payoff functions as in (2).

1.2 Examples

The number of areas in which a contest is an appropriate description of how some allocation outcomes are determined and the quantitative and qualitative significance of the phenomenon will be evident when a few areas of application are considered. Rosen (1988) emphasized the large range of applications of contest or tournament theory and mentioned applications such as examinations, college admission, quality control and medical trials, athletic competitions, elections, litigation, auctions, R&D races, work-points incentive schemes, and relative payment schemes in organizations. One may want to add advertising and other types of promotional competition, rent-seeking and appropriation conflict in which players use resources in trying to define or reallocate property rights between them, and also one of the most violent types of conflict: civil war or war between countries. We discuss a selection of these examples here and illustrate some of the research questions that emerge in this context.

1.2.1 Promotional competition

Firms try to increase their market shares by advertising campaigns and other marketing activities. Most obvious cases are newspaper advertisements or tv spots for, for example, washing powder or soft drinks, and sales agents who try and persuade customers to buy a particular product. Such activities have in common that the major share of these efforts are made up-front, prior to actual sales, and can be understood as efforts in a contest in which the effort choices determine the market shares of all firms. The contest success function does not have a probabilistic interpretation in this case, but can be seen as the share in the total market.

The expenditures on marketing and advertizing are substantial, and sometimes they are subject to regulation. In the German insurance markets, for instance, the maximum amount of promotional activities by insurance companies was regulated and constrained from above to 30 percent of premiums, and commissions paid to sales agents were limited to no more than 11 percent of premiums (Rees and Kessner, 1999). Figures for pharmaceutical companies are similar. Figures 1 and 2 depict both the advertizing-to-sales ratio

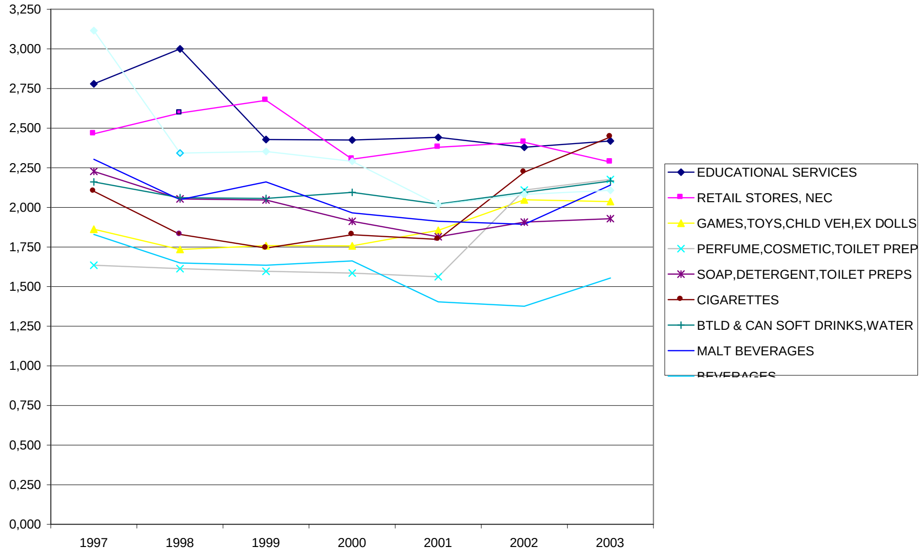

Figure 1: Advertising expenditure as a percentage of sales revenues in a number of advertising intensive sectors in the US. Data Source: www.AdAge.com, Advertising to Sales Ratios, various years.

and the marketing-to-sales ratios for the industries with the highest ratios in 2003 for the US.

The share of promotional effort in sales revenue is even higher if this is measured not only by advertising campaign costs but also by marketing effort more generally, as is shown in Figure 2.

These figures document the fact that advertising and other types of promotional or marketing activities are very commonly used tools in competition between firms. The contest nature of this type of competition was noticed quite early by Friedman (1958), Mills (1961) and Schmalensee (1976).

Firms that spend resources on advertising will generally affect their market share, but advertising may also change the total size of the market.[1] Advertising expenditures therefore have the properties of a contribution to a public good. But they also have a negative externality if such expenditures affect market shares. This specific effect makes promotional competition a contest in which the prize at stake is a function of the contestants' efforts.

[1]The two roles of marketing effort with aggregate effort affecting market size and relative effort affecting market share has also been pointed out by Bell, Keeney and Little (1975, p.136). The empirical assessment of how advertising influences market size and market shares for a particular type of consumer good is still a matter of ongoing research.

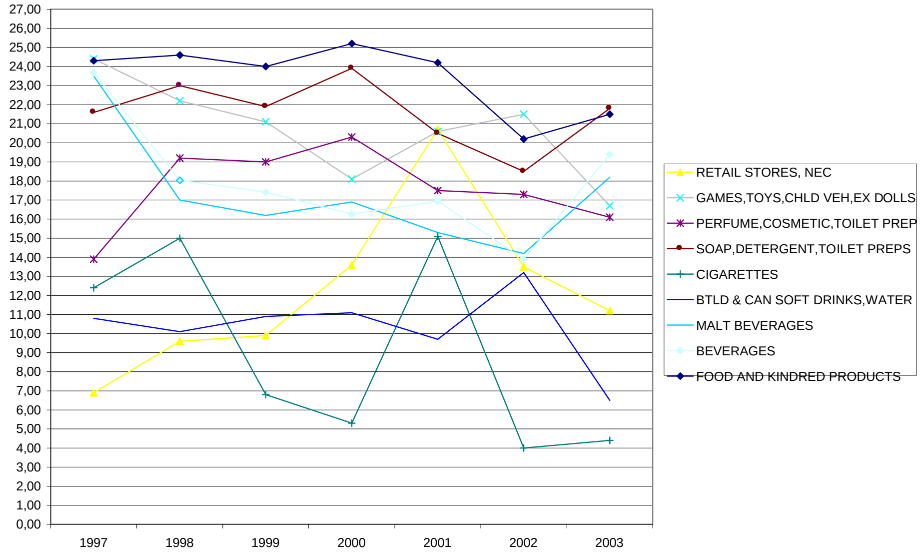

Figure 2: Marketing expenditure as a percentage of sales revenues in a number of advertising intensive sectors in the US. Data Source: www.AdAge.com, for various years.

This aspect will be considered from a theory point of view in section 4.3.[2] Of course, the advertising game is more complex than simple contest game in section 1.1 because repetition, the dynamics of advertising and the effects on the stock of consumers, possible collusive behavior between the competitors, and the size of firms within the group of largest firms may also affect the results. Advertising expenditures, and marketing activities more generally, also have the property that they may be used to hurt particular competitors, and thereby reduce one particular competitor's market share. This activity could be called sabotage and is different from effort that enhances a firm's own market size, as it will benefit not only this firm but also other firms that are not affected by the sabotage effort. The role of sabotage more generally will be considered in section 6.2.

[2] See, e.g., Piga (1998) and Gasmi, Laffont and Vuong (1992). The latter report that the soft drink producer Coca-Cola's advertizing expenditure prior to 1977 had adverse effects for the demand for Pepsi, whereas Pepsico Inc.'s advertizing expenditure had a weak stimulating effect on the demand for its rival, the Coca-Cola Company. The differential effects on market share and market size are, however, not the only possible explanations for this, as price competition, for instance, may also have played a role.

1.2.2 Litigation

Another practical example where contest effort influences the probabilities with which agents win or lose is litigation (Farmer and Pecorino, 1999, Wärneryd, 2000, Baye, Kovenock and de Vries, 2005 and Robson and Skaperdas, 2005). The international comparative evidence on the cost of litigation is not very extensive.[3] Business law, institutions, and, in particular, the institutional design of the litigation process play major roles for the quality of property rights in a country, and for the amount of resources used in litigation. Shavell (1982) has drawn attention to the importance of fee shifting rules for plaintiffs and defendants, and Spier (2005) surveys the theoretical literature. Baye, Kovenock and deVries (2005) discuss the importance of fee shifting rules for litigation effort in a contest model of litigation. They discuss the 'American rule', by which each litigant covers his own cost, the 'English rule', by which the loser pays the cost of both parties, and other cost shifting arrangements.

Litigation is a complex matter. Plaintiffs have to decide whether to make demands, and what kind these will be, defendants have to decide how to react, how to negotiate in a pre-litigation period, whether to employ laywers at that stage, and whether to enter into litigation in court, and this, again, becomes a multi-stage game with multiple options. The design of the legal system determines the rules of this game. The game has elements of contests at many stages. The theoretical analysis in the main part of the book will implicitly address many of these strategic aspects, such as the role of fee shifting rules, the entry decision, delegation, information, and information asymmetries in contests.

1.2.3 Internal labor market tournaments

Tournaments, or relative performance reward schemes, are well established in many working environments. Promotion decisions in organizations are also often based on relative performance, and sometimes firms explicitly, and repeatedly, award prizes to employees, typically rewarding individuals or a subset of their best performing employees. These schemes have the structure of a contest as the employees expend effort in trying to win a prize.

Lazear and Rosen (1981) and Rosen (1986) started the formal study of such structures in the labor market context, and the empirical importance

[3] The European Union Labour Force Survey 2003 gives the number of legal professionals for European countries. Normalizing these numbers by the number of citizens gives the number of legal professionals per 1000 citizens, and this ranges from 0.93 for France up to 3.41 for the Netherlands.

and the theoretical properties of internal labor market tournaments have been studied carefully in a large literature that had its origin in these two papers.[4] Many issues that have been analysed, and are highly relevant in these contexts, are surveyed in later sections here. Tournaments in both the labor market and organizations are typically carefully designed. The designer may consider awarding one or many prizes, may award by means of a simple or more complex structure of the tournament with multiple rounds, with or without entry barriers, and with or without elimination of some contestants at early stages of the tournament. The designer also may have various objectives in mind. In some firms, the tournament may simply serve as a reward scheme meant to induce workers to expend effort that translates into output. In other firms, the tournament may also serve as a screening device through which the firm wants to identify employees who are particularly good at pursuing superior tasks or at assuming more responsibility, and this motivation may even dominate in some firms and some internal labor market tournaments. Firms may also consider that relative rewards may induce employees to exert destructive effort, effort that does not improve their own performance, but reduces the performance of their competitors, and they may consider how to organize the tournament so as to reduce such sabotage incentives. These are some of the strategic aspects that play a role in labor market tournaments which will be considered further below.

1.2.4 Beauty contests, influence activities, and rent seeking

Economic rents are often allocated by bureaucrats or politically appointed decision makers. Accordingly, those who are the possible beneficiaries of their decisions may try to influence these decisions. Firms or consumer groups may attempt to receive favorable treatment from a regulator, firms may lobby for tariffs or other forms of import protection, and these are early examples of rent seeking that were discussed, e.g., in the classical contributions on rent seeking by Tullock (1967), Krüger (1974) and Posner (1975).

The literature on the various applied aspects of rent seeking contests is vast, and protectionist trade policy, industry regulation, privatization, development policy, and foreign aid are some of the topics that have received much attention. An early survey is that of Nitzan (1994). There is a more recent survey of the theory of rent seeking and its applications by Congleton, Hillman and Konrad (2006). Many of the aspects that are dealt with in this literature will be discussed later on, e.g., issues of who is willing to participate, and who is admitted to the contest, who defines the rents, who

[4] See, e.g., Lazear (1995) for a brief survey of the early literature.

benefits from the rent seeking expenditure, and who sets the rules of the rent seeking game.[5]

Obvious examples of beauty contests that regularly receive public attention are the games that determine the choice of locations for the Olympic games by the International Olypmic Committee (IOC), or for soccer championships by the Fédération Internationale de Football Association (FIFA). The locations that want to host these events expend considerable resources in trying to influence the decisions favorably. Steward and Wu (1997) survey some of the literature on decision making by the IOC. They report officially stated campaign costs by the cities that applied for the 2000 Olympic games: 25.2 million Australian dollars by Sydney, and 86 million D-Mark by Berlin. They also suggest that these official numbers underestimate the actual payments.

Beauty contests are also common in other contexts. For instance, the allocation of broad band telecommunication rights often occurred in beauty contests in countries in which they were or are not simply auctioned. Goree and Holt (1999) discuss the case where the U.S. Federal Communications Commission awarded 643 licenses among 320,000 applications, before switching to auctioning licenses. Hazlett and Michaels (1993) estimate that the application costs added up to about 40 percent of the value of the licenses, for which they give an estimate of 1 billion USD. The allocation of spectrum rights by way of a beauty contest has not been unusual in recent years. According to Börgers and Dustman (2003), who describe the different processes throughout Europe for awarding broad band (3G) spectrum licenses, beauty contests have been used in Finland, Spain, Sweden, Portugal, France, Ireland, and Luxemburg. Considering the rent seeking cost of these cases, influence activities could emerge both at the stage where the design of the allocation method had to be chosen, and in the actual contest between rival competitors for the spectrum rights for a given design and a given set of licenses.

1.2.5 Education filters

Education is a less obvious example of contest games. Education may serve as an input that enhances individual human capital and translates into higher labor productivity. However, education may also function as a filter which reveals the true characteristics and abilities of a person, thus allowing an improved and more productive use of the person's abilities in the assignment of tasks. The latter purpose of education was highlighted by Arrow (1973),

[5] For this aspect, see for instance, Appelbaum and Katz (1986), Hillman and Katz (1987), Ellingsen (1991), Drook-Gal, Epstein and Nitzan (2004) and Epstein and Nitzan (2004, 2006a, 2006b).

and this aspect is popular among sociologists. Hirsch (1977), for instance, considers the role of the education system in filling a number of attractive positions in a society, and acknowledges the tournament aspect of such systems. The scarcity of attractive tasks in the assignment process on the one hand, and the relative comparisons in the filtering process on the other make the assignment problem similar to a rent seeking contest. To the extent that the allocation of jobs and tasks is decided on the basis of relative ability and not absolute ability or actual productivity or skills developed, some of the effort that is expended in education could be wasteful.[6] Many of the strategic aspects discussed in later sections therefore apply to this context.

Empirically, the effort expended on education is substantial. In 2002, the average overall expenditure in the OECD was USD 5273 per student at the primary level, USD 6992 per student at the secondary level, and USD 13343 per student at the tertiary level (OECD, Eduation at a Glance, 2005, p. 161). These numbers underestimate total effort, as they mainly measure the actual resources spent on teachers and teaching institutions, and do not include the opportunity cost of time.

1.2.6 R&D contests

One of the areas of application in which the tournament character of players' interaction is also very visible, and in which the theory of contests and tournaments made major progress early on, is the area of research and development (Loury 1979, Nalebuff and Stiglitz, 1983). The firm which introduces a new product or a product improvement first will generally have some benefit from it. The firm may earn some monopoly rent as long as there are no competitors who can offer a similar product or quality, and patent protection may further increase this rent. Accordingly, firms may spend effort on research and development when chasing these rents.

Many R&D contests emerge naturally from firms' competition, the potential profitability of introducing new products, or the advantage of cost reducing innovations. However, a large number of technology prizes which several firms may contest for, are also awarded. Windham (1999) has collected a considerable list of examples. He mentions the famous contest for

[6] The tournament structure of education systems has also been recognized by contest theorists. Amegashie and Wu (2004) consider national exams as a contest that is devised to assign students to the different institutions of higher education. They find that students' selection choices about where to apply for higher education prior to preparing for the exams that produce the basis for the admission procedure can be dysfunctional. Konrad (2004d) considers the role of mobility and transition options between different, multi-stage education filters for efficiency. Fu (2006) considers handicapping of applicants from minorities in college admission competition.

a “practical and useful” means of determining longitude at sea’ (Windham 1999, p. A-3) for which the British Parliament offered a prize of GBP 20 000 in 1714, the ‘Wolfskehl Prize for proving Fermat’s Last Theorem’, the EU Information Technology prizes, and a long, and non-exhaustive, list of today’s recognition prizes.

1.2.7 Campaigning and committee bribing

Political competition is another area in which contest theory has major applications. Parties spend resources in trying to influence voters and win elections, whereas party members and politicians expend effort to advance within the party hierarchy or to be nominated for an important office. Such electoral competition and party politics resemble the promotional activities of firms who use advertising with the aim of both increasing the market - in electoral markets the aim is an increase in votes turnout, and increasing their own share in this market. The empirical significance of the phenomenon is large. One measure is, for instance, the considerable size of campaign budgets of parties or candidates who run for government or office in democratic election campaigns. Some illustrative figures are reported by Alexander (1996). According to this source, Abraham Lincoln’s campaign cost were about 100 000 USD, John F. Kennedy and Nixon spent about 9.7 million USD and 10.1 million USD respectively. Bill Clinton could draw on more than USD 130 million. But also in parliamentary systems the cost of campaign spending is significant. In a much smaller country such as Germany, the two largest parties’ expenditure were estimated to sum up to about 50 million Euro (Korte 2006).

One difference between campaigning prior to elections and firms’ ordinary promotional competition using advertising is the payoff function in elections: in markets with promotional competition, all firms typically benefit from a large market, and their payoff is typically a monotonic and smooth function of the market size and their market share. In electoral markets, the payoffs of parties or candidates are often non-continuous. Thresholds, like, e.g., the 50 percent majority of all votes, are important. This is particularly true in presidential systems or two-party competition, but thresholds also matter in electoral systems with many parties and coalition formation.[7] Moreover, parties and politicians need to collect campaign contributions in order to

[7] These thresholds are also important in another political game that could be called the committee bribing problem. In this problem, two (or more) rival players need to influence a majority of the members of committee to support their prefered policy alternative. This problem has been addressed, e.g., by Young (1978), Congleton (1984), Groseclose and Snyder (1996), Banks (2000) and others.

use them for their campaigns, which makes the political game more complex than a simple one-stage contest.[8]

This is not to say that all aspects of political competition can be mapped appropriately by a simple contest for votes with discontinuous payoffs. The political economy theory of political and electoral competition is a large field that cannot be surveyed here. However, it can, and has been, argued convincingly in the literature that the all-pay nature of campaigning and the non-trivial relationship between campaign effort and election outcomes makes contest theory a useful tool for studying this type of competition.

1.2.8 Military conflict

Arms races, or even military conflict in the form of war, or civil war, are probably among the most important and most obvious examples of contests. The rival players are typically countries, or power groups within countries. As suggested by the textbook example on economic principles, a country's resources can be used to produce 'butter', representing the set of standard consumer goods, or 'guns', representing the set of instruments that improve the country's ability to wage war, or to be the victor if attacked by another military force. Building up military force therefore has opportunity costs in terms of consumption goods sacrificed, whether or not the military goods are used, and whether or not the country is successful in an armed conflict or in negotiations taking place in the shadow of possible military conflict. From a contest theory point of view, the production of weapons constitutes effort in a contest.

Many contest problems of war or combat have been analysed in a context that is called the "Colonel Blotto" game. In this game, two (or more) rival army leaders have to make simultaneous choices about where to allocate which share of their troups among different frontiers.[9] War has also been the field of study of international policy. Researchers asked questions such as why does war take place, given the opportunity to negotiate and settle, and similarly, once war has started, why is it so difficult to terminate it. Much

[8] This is widely acknowledged in the literature. A recent example is Glazer and Gradstein (2005). Examples for contest models of electoral competition with further references are Congleton (1986), Skaperdas and Grofman (1995), who focus on negative campaigning, and Konrad (2004c) who focus on 'inverse' campaigning (see section 6.3).

[9] According to Young (1978, p. 392), '[T]he first example of a lobbying game seems to have been considered by Borel' (1938), and variants of this game have received considerable attention from theorists from time to time. Early contributions are, e.g., Blackett (1954, 1958) and Friedman (1958). Shubik and Weber (1981), and Coughlin (1992) make further contributions and provide further references. Most recent Colonel Blotto games are by Matros (2006) and Robson (2006).

of the literature, and many good answers to these questions, are surveyed in Fearon (1995) who considers asymmetric information and incomplete contracts as the main reasons for a conflict situtation turning into a war. Further contributions that explain why war may take place in a context with perfectly rational players in a full information framework are Garfinkel and Skaperdas (2000) and Slantchev (2003) who both add explanations based on incomplete contracts.

From an empirical point of view, military conflict could be studied at various levels of aggregation. Even a single battle consists of a whole set of combat events. Sets of battles are sometimes called a campaign. War is an even more complex, dynamic type of interaction in which single battles or campaigns are important components. I will consider some aspects of this complexity in the chapter on grand contests. Another important aspect of military contests is asymmetry. One type of asymmetry between the attacker and the defending player was already emphasized by Clausewitz (1832/1976): an advantage of defence. However, turning to battles and the nature of contest outcomes, this advantage is not easy to verify from the relationship between numerical superiority and battle success. Dupuy (1977) surveys evidence on ground combat in the time period 1805-1973. He surveys 42 battles. From these battles, 28 attackers and 14 defenders were successful. There were 13 numerically inferior attackers, and 12 of these were successful. In 18 of these battles the victors were numerically superior, in 24 cases the victors were numerically inferior. These data suggest that the relationship between numerical superiority and battle success is a loose one. Other qualitative aspects not well described by these numbers seemingly play a role, and these aspects may include morale, leadership, a superior strategy, technological advantage and luck. Moreover, there could be other asymmetries that also play a role. A defending player may be vulnerable at several points, and, to be successful, may need to defend all these points successfully in order to win the war, whereas an attacker may be victorious if he can surmount the defense lines of his rival successfully at one point .[10]

1.2.9 Sports

Sports are the final, and most obvious application of contest theory. Athletes spend years in training effort, months in preparing for a particular championship or event, and the actual sports event also requires that each participant expends effort that cannot be recovered even if the athlete does not win. Not surprisingly, the contest aspect of sports has received considerable

[10] Aspects of this type are considered, e.g., in Shubik and Weber (1981), and, more recently, in Clark and Konrad (2006) and in Kovenock and Roberson (2006).

attention in the literature on the economics of sports. In his careful survey, Szymanski (2003) addresses, for instance, issues such as the role of the contest success function, multiple prizes, asymmetry between contestants, the role of contest architecture, and dynamic aspects of actual contests in which the contestants repeatedly expend effort, with a feedback effect via the observed prior efforts by their rivals. He also surveys the empirical literature on individualistic sports, before he turns to team sports.

Consider, for instance, tennis. A tennis match is a complex structure of sequential contests, with points, games or sets being seen as battles in a larger contest that establishes the game. In turn, a match is only one of many battles in a tournament, and a tournament is only a battle in the grand contest for top rankings or annual awards. Sometimes it may be useful to look into this contest architecture more closely, and I will survey some work on this. However, the simple structure of the contest outlined in section 1.2 maps a central aspect of sports competition, and the analysis of single contests, which can be interpreted as a match or a whole tournament, can therefore reveal interesting insights, for instance about the role of barriers to entry or entry fees, rules that make participants in a contest more homogeneous, the number of prizes and their structure, etc. In professional golf, for instance, the 'purse' consists of a number of prizes that decline in the player's rank. Figures 3 and 4 describe the prize structure for the tournaments on the PGA tour for 42 of 47 tournaments of the European PGA tour in 2003.[11] The latter reveals that there is typically a set of prizes. Surprisingly, this set has an invariant structural pattern. The reasons for awarding multiple prizes and the structure of these prizes will be discussed in detail.

The figure shows how the size of the prize money is highest for the player who scores best and declines for players who did less well, for the

[11] For the five championships, data were not available on the internet. The tournaments included are: BMW Asian Open, the Diageo Championship at Gleneagles, Omega Hong Kong Open, Open de France, South African Airways Open, Smurfit European Open, dunhill championship, The Barclays Scottish Open, Caltex Masters presented by Carlsberg, 132nd Open Golf Championship, Heineken Classic, Nissan Irish Open, ANZ Championship, Scandic Carlsberg Scandinavian Masters, Johnnie Walker Classic, Nordic Open, Carlsberg Malaysian Open, US PGA Championship, Dubai Desert Classic, BMW Russian Open, Qatar Masters, WGC - NEC Invitational, Madeira Island Open, BMW International Open, Algarve Open de Portugal, Omega European Masters, Canarias Open de España, Trophée Lancôme, Italian Open Telecom Italia, Linde German Masters, Benson and Hedges International Open, Dunhill Links Championship, Deutsche Bank - SAP Open TPC of Europe, WGC- American Express Championship, VOLVO PGA Championship, Dutch Open, the Celtic Manor Resort Wales Open, Turespaña Mallorca Classic, the Daily Telegraph Damovo British Masters, Telefonica Open de Madrid, Aa St Omer Open, Volvo Masters Andalucia.

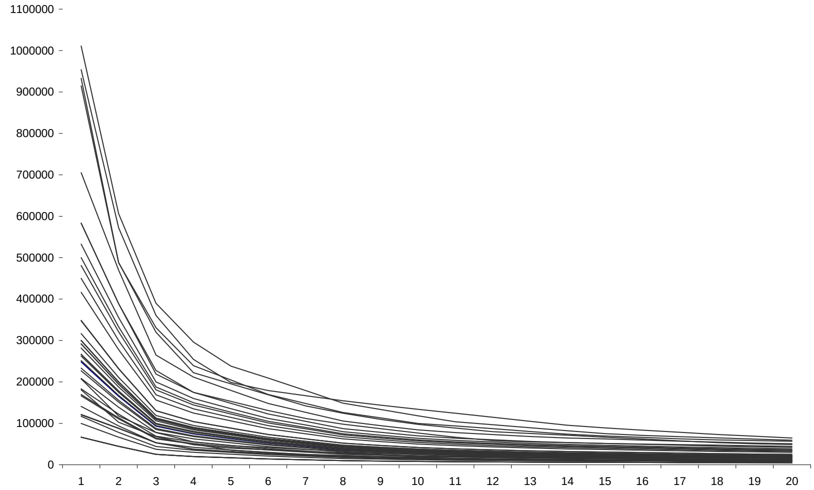

Figure 3: European Tour 2003, Prize Money in Euros. Source: Various internet homepages.

best 20 players of the respective tournaments on the European Tour 2003. Figure 4 shows prize money in percent of the total purse for the same set of tournaments.

It should be noted that this structure of monetary prizes does not fully describe the full prize structure, as there are implicit, or non-monetary benefits of performance. Winners of major tournaments have the benefits of qualifying for future tournaments, they get more media attention which can be transformed into monetary payoffs via promotion contracts with producers of brand products, they improve their score with regard to the contest for best performance in a given year, or lifetime, and they probably obtain some ego rent from winning.

Many types of sport are team sport. This implies that a team player's own effort benefits himself primarily via the improvement in his team's performance, which hints at a free-riding problem in the competition between teams. Moreover, the winning team in a championship is not fully homogenous. Not only do players have different, more or less charming appearances, and their own personalities. They also differ visibly in their contribution to their team's success. Accordingly, they do not all get the same prize from winning the championship, and it is probably true that, within teams, there is rivalry concerning who receives higher recognition and can earn higher benefits from media attention, sponsoring and ego rents. Team sports constitute an example of contests that take place both between and within the

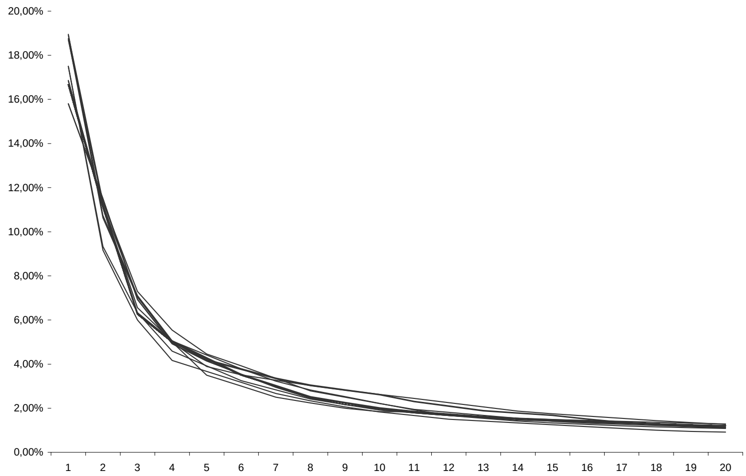

Figure 4: European Tour 2003, Prize Money in Percent. Source: data from Figure 3 and own calculations.

groups, and such structures will also be studied carefully.

2 Three types of contest

Many dimensions can be distinguished for a typology of contests. Hirshleifer and Riley (1992, chapter 10), for instance, distinguish dimensions such as whether random elements play a role in the contest success function, whether contestants are informed about their co-players' cost or their co-players' valuations of the prize, etc. With so many such dimensions classification is cumbersome. I will follow an alternative strategy and present the three types of analytical description of contests that are most prominent in the literature. A further important variant is analysed in Anderson, Goeree and Holt (1998). One type I do not consider is wars of attrition.

I will concentrate on contests in which the contestants know each other's valuation of the prize and what their efforts cost and will only touch on the theory of contests with incomplete or asymmetric information. The various information assumptions lead to a large class of all-pay auction problems. Incomplete information is one of the reasons why undesirable contests, such as fights between unions and employers or military conflict, may actually take place, and is why they are particularly important. However, as can be argued, there are also other factors, such as incomplete contracts and time consistency problems, that may, even more plausibly, be why undesirable contests take place.

2.1 The first-price all-pay auction

Consider the following imagined situation. Two cities 1 and 2 compete for becoming the location of the next Olympic games. City 1 puts a value of 10 million Euro on being chosen, whereas city 2's value is 20 million Euro. Let this be known to both cities. The choice is made by some decision maker who, for some reason, may choose the city where proposal for the organization of the games is more attractive. The cities may then spend money on architects, marketing agencies, lobbyists and other influence activities. All these types of expenditure 'improve' the city's proposal. Suppose that the cities are very similar ex ante and are capable of turning money into improvements of the proposal so that the city which spends more money wins the competition. If you are the mayor of city 1, how much money do you spend?

As stated, this is an example of a contest in which the contestant who expends the highest effort wins the prize with probability 1. This case is most relevant, for instance, if the chosen contest effort translates deterministically into an observable quality or quantity variable, and if the allocation of the prize is made on the basis of a comparison of the values of this variable for the various contestants. The relationship between a contestant's effort and the observable result of his effort is often not as clear as that, and there will often be some noise in this process, so that the reader who is impatient to consider cases with noise may jump to the next section. However, as a benchmark case, the deterministic case described in the cities' beauty contest example is particularly interesting.

Two contestants Let us concentrate on the case with two contestants that was first studied carefully by Hillman and Riley (1989). Generalizations will be discussed later. There are two contestants $i = 1, 2$ who attribute non-negative value to the prize that is allocated in the contest. Let these values be v_1 and v_2. The players know both their own valuation and the value their opponent attributes to winning the contest. By appropriate renumbering of the two, $v_1 \geq v_2 > 0$. Contestants choose their efforts $x_i \geq 0$ simultaneously, and the cost of effort is simply $C(x_i) = x_i$. Contestant 1 wins with probability

$$p_1(x_1, x_2) = \begin{cases} 1 & \text{if} \quad x_1 > x_2 \\ 1/2 & \text{if} \quad x_1 = x_2 \\ 0 & \text{if} \quad x_1 < x_2. \end{cases} \tag{3}$$

The probability that contestant 2 wins is $p_2 = 1 - p_1$.

Consider the optimal effort choices of the contestants. Each contestant i maximizes (2) subject to $x_i \geq 0$, with win probabilities defined by (3), and with $C(x_i) = x_i$. Accordingly, if contestant 1 thinks that contestant 2 will choose some x_2, the optimal effort from the perspective of contestant 1 is either some x_1 slightly above x_2 that makes 1 win the prize with certainty, or $x_1 = 0$. The latter is the case if x_2 is very high so that it does not pay to outcompete 2. The reasoning is similar for contestant 2. Hence, there is no equilibrium in pure strategies in this game. A formal proof of this is by contradiction and starts with the assumption that (x_1, x_2) is such an equilibrium, showing that x_2 can never be contestant 2's optimal reply to x_1 if x_1 is the optimal reply of contestant 1 to this x_2.

The Nash equilibrium is in mixed strategies and these mixed strategies are described by contestants' cumulative distribution functions that describe the distribution of effort choices,

$$F_1(x_1) = \begin{cases} \frac{x_1}{v_2} & \text{for} \quad x_1 \in [0, v_2] \\ 1 & \text{for} \quad x_1 > v_2 \end{cases} \tag{4}$$

and

$$F_2(x_2) = \begin{cases} \left[1 - \frac{v_2}{v_1}\right] + \frac{x_2}{v_1} & \text{for} \quad x_2 \in [0, v_2] \\ 1 & \text{for} \quad x_2 > v_2. \end{cases} \tag{5}$$

The mixed strategies that are described by (4) and (5) are mutually optimal replies, meaning that they characterize an equilibrium. This is verified by noting that contestant 1's payoff equals $v_1 - v_2$ for all choices $x_1 \in (0, v_2]$, and is smaller than this difference for all non-negative x_1 outside this interval if the contestant plays against a contestant 2 who randomizes his effort according to $F_2(x_2)$. Contestant 2's payoff on the other hand equals zero for all choices $x_2 \in [0, v_2]$, and is negative for all effort choices that are even higher than v_2 if he plays against a contestant 1 who randomizes according to the cumulative distribution function $F_1(x_1)$.

Baye, Kovenock and deVries (1996) show that this equilibrium is also unique. Their proof develops and builds on the insights that equilibrium mixed strategies cannot have mass points other than at $x_i = 0$, that the equilibrium cumulative distribution functions cannot have 'holes' along the their support, and that there can be, at most, a mass point for one of the contestants at $x_i = 0$. These insights can be verified by contradiction. Some intuition for the result is as follows. Note that 2 is never going to expend more than $x_2 = v_2$. But if contestant 2 never expends more than $x_2 = v_2$, then contestant 1 can be sure of winning with probability one if he expends $x_1 = v_2$ (or an arbitrarily small bit more than this). This defines the upper limit of reasonable efforts and reduces the problem to how the contestants randomize on the interval $[0, v_2]$. Note also that contestant 1 could guarantee an own payoff equal to $v_1 - v_2$ by simply choosing an effort that infinitesimally exceeds v_2, and, indeed, this is 1's equilibrium payoff, whereas 2's equilibrium payoff is zero. Finally, the slope of the equilibrium cumulative distribution functions can be obtained from this equilibrium payoff. The payoff of contestant 1 must be equal to this $v_1 - v_2$ for any strategy that belongs to those used in the mixed strategy. Accordingly, for all these strategies we have

$$\pi_1(x_1) = F_2(x_1)v_1 - x_1 = v_1 - v_2. \tag{6}$$

This can be transformed to (5). Similarly, if the payoff for all choices x_2 that belong to the mixed strategy for contestant 2 is zero, this means

$$\pi_2(x_2) = F_1(x_2)v_2 - x_2 = 0, \tag{7}$$

and this can be transformed to (4).

The equilibrium payoffs for contestants 1 and 2 are $v_1 - v_2$ and zero, as discussed. Their expected efforts are

$$Ex_1 = \frac{v_2}{2} \text{ and } Ex_2 = \frac{(v_2)^2}{2v_1}. \tag{8}$$

This shows that the sum of expected efforts falls short of the effort in a standard second prize auction, as the sum of efforts falls short of v_2. Also, the contest has a peculiar inefficiency property that the prize is not necessarily allocated to the contestant who has the highest valuation of the prize. This is a peculiarity that is not robust in the sense that it will disappear if one changes the order of moves between the two contestants so that they move sequentially. The prize is also efficiently allocated in some all-pay auctions with incomplete information, particularly in the symmetric independent valuation all-pay auction if the contestants know their own valuations of the prize, but not those of their competitor. Sequential moves for the all-pay auction without noise have been considered in Deneckere, Kovenock and Lee (1992) and in Jost and Kräkel (2000), and this will be re-considered in a separate section together with endogenous timing. Issues of incomplete or imperfect information will also be discussed later.

Convex cost Before turning to the case with more than two contestants, consider alternative cost functions. Let $C(x_i)$ be convex, i.e., $C(0) = 0$, $C'(x_i) > 0$ and $C''(x_i) \geq 0$. Consider two contestants with identical valuations, v, of winning the prize, and identical cost of effort. For reasons similar to the ones just discussed, an equilibrium in pure strategies does not exist. The payoff of player 1 equals

$$\pi_1 = F_2(x_1)v - C(x_1). \tag{9}$$

This equals zero if $F_2(x) = \frac{C(x_1)}{v}$. Accordingly, using the symmetry assumption, the equilibrium density becomes $F_1'(x) = F_2'(x) = \frac{C'(x)}{v}$ in the range $[0, \bar{x}]$, with $\bar{x}$ defined by $C(\bar{x}) = v$, and zero elsewhere (see, e.g., Kaplan, Luski and Wettstein 2003).

More than two contestants Baye, Kovenock and deVries (1996) give a rigorous analysis of the all-pay auction and a full characterization of the equilibria if there are more than two contestants. Their results are surveyed here. Let there be $n \geq 2$ contestants and valuations $v_1 \geq v_2 > v_3 \geq ... \geq v_n$. Then the unique equilibrium has $x_i = 0$ for all $i > 2$, and contestants 1 and 2 choose the unique equilibrium cumulative distribution functions as in the $n = 2$ case. Intuitively, note that the equilibrium play between 1 and 2 must be described by (4) and (5) if all other contestants do not make bids. But even if only contestant 1 chooses (4) and contestant 2 abstains, this makes the payoff negative for any positive effort by any contestant whose valuation of the prize is smaller than v_2.

Let there be $n \geq 2$ contestants and valuations $v_1 = v_2 = ... = v_j > v_{j+1} \geq ... \geq v_n$. Then there is a full continuum of equilibria. In any equilibrium $x_i = 0$ for all $i > j$. In the set of contestants with the highest valuation, any number between 2 and j may actively participate and make positive bids. For instance, one type of equilibrium has only two of these j contestants active. These choose their efforts according to the equilibrium cumulative densities of the two-player game and all others stay out. But there are also sets of equilibrium cumulative distribution functions in which more than two contestants make positive bids.

The cases here are not exhaustive, but make the way the reasoning for the case with many players works clear.

Other cost variants The all-pay auction with complete information and without noise has been well studied, and the equilibria are described not only for the various combinations of relative valuations of the prize. For instance, Baye, Kovenock and deVries (1998), consider the class of symmetric two-player contests along a different, interesting dimension. For this purpose, they assume that the contest success function is as in (3) and define the payoff of player 1 as

$$\begin{array}{ll} v - bx_1 - dx_2 & \text{if player 1 wins} \\ -ax_1 - tx_2 & \text{if player 1 loses,} \end{array} \tag{10}$$

half of the sum of these payoffs if both players expend the same positive effort, and zero for both players if both players expend zero effort. The payoff of player 2 is obtained by replacing all number 1 subscripts by 2 and vice versa. They give a considerable number of relevant examples. For instance, the case $b = a = 1$ and $d = t = 0$ refers to the standard contest case with $v_1 = v_2 = v$. $b = d = 0$, $a = t = 1$ refers to a system in which the contestants have to pay both their own and their opponent's effort if they lose. They refer to the British legal system as an example, where the loser pays all cost. The case $b = t = 0$, $d = a = 1$ refers to a situation in which the winner and the loser both pay the effort made by the loser. An example of this is the war of attrition. Here, both contestants choose the maximum efforts they are willing to expend over time, and expend this as a constant flow, for instance, as waiting time. Once the stock of effort that one of the players was willing to expend is used up, the other player wins and also stops expending further effort. The equilibrium cumulative distribution function is derived in Baye, Kovenock and deVries (1998) for $v_1 = v_2 = v$ as

$$G(x) = \frac{a}{a-b}\left[1 - \left[\frac{v}{v + (a-b+t-d)x}\right]^{\frac{a-b}{a-b+t-d}}\right]. \tag{11}$$

The reader may consult Baye, Kovenock and de Vries (1998) for further details and Baye, Kovenock and de Vries (2005) for an important application on litigation and fee-shifting rules.

Constraints on effort Che and Gale (1998) consider the equilibrium that emerges in an all-pay auction if there are two contestants who may have different valuations of winning the contest, but cannot choose any effort they want. The authors were motivated by the campaign contribution regulation in the US that determines the maximum campaign contribution that a single individual can make to a candidate. Other examples may emerge if the effort expended in the contest is measured by time devoted to it. Candidates may then have only a limited amount of time between the start of the contest and its end, like in many sports games or in some scoring tests in which individuals solve one or many tasks within a given time limit.

If there are n contestants and all have the same spending limit of size m as the maximum effort they can choose and if this is

$$m < (1/n)\min\{v_1, v_2, ..., v_n\} \tag{12}$$

all will simply choose the maximum effort. Intuitively, if (12) holds, contestants i have the opportunity to take part in a lottery in which their win probability is $1/n$ if they pay m and their lottery prize is equal to v_i. However, given that others pay m, in order to really hold a ticket that can win, they must pay m. This is worth it in a lottery with $n-1$ other participants even for the participant with the lowest valuation of the prize if $m < \frac{1}{n}\min\{v_1, ...v_n\}$.

This intuition structurally determines the equilibrium for a whole range of higher effort limits up to the effort limit that is determined by

$$m < (1/2)\min\{v_1, v_2\}, \tag{13}$$

where, by appropriate renumbering, v_1 and v_2 are the two highest valuations among the valuations $v_1, ...v_n$. For this whole range, only a set of players with the highest valuations will participate, with the contestant with the lowest valuation determined by the condition

$$m < (1/k)\min\{v_1, ...v_k\}. \tag{14}$$

If m becomes even larger than the largest m that fulfills condition (13), only the two contestants with the highest valuation matter, but the solution becomes more cumbersome. If the maximum feasible effort is higher than v_2 with v_2 the second highest valuation, the effort limit is non-binding in the equilibrium and can be ignored. If m is from the interval $[\frac{1}{2}\min\{v_1, v_2\}, \min\{v_1, v_2\}]$, the solution is less straightforward. As shown in more detail in Che and Gale (1998), the generic equilibrium cumulative distribution functions are as drawn in Figure 5 for $v_1 > v_2$.[12] It can be confirmed that the cumulative distribution functions of effort characterize an equilibrium by showing that contestant 1's payoff equals $v_1 - v_2$ for all effort choices from the set $(0, 2m - v_2] \cup \{m\}$ as in Figure 5 and is lower for all effort choices from outside this set, and that contestant 2's payoff equals zero for all effort choices from the set $[0, 2m - v_2] \cup \{m\}$ as in Figure 5, and is lower for other effort choices.

A related problem, treated in Che and Gale (1997), emerges if there are n contestants who face different constraints as regards their maximum effort. For instance, firms may know the profit increase from winning a particular R&D contest with certainty, but they may have a limited amount of equity and there may also be credit market imperfections due to non-contractibility problems that lead to liquidity constraints. To concentrate on the liquidity

[12] Che and Gale (1998) discuss whether additional equilibria are feasible for some non-generic parameters, particularly for $m = v_1/2$ and $m = v_2/2$. They show, for instance, that there is a continuum of equilibria for $m = v_2/2$, with $x_1 = x_2 = m$ being one of these equilibria.

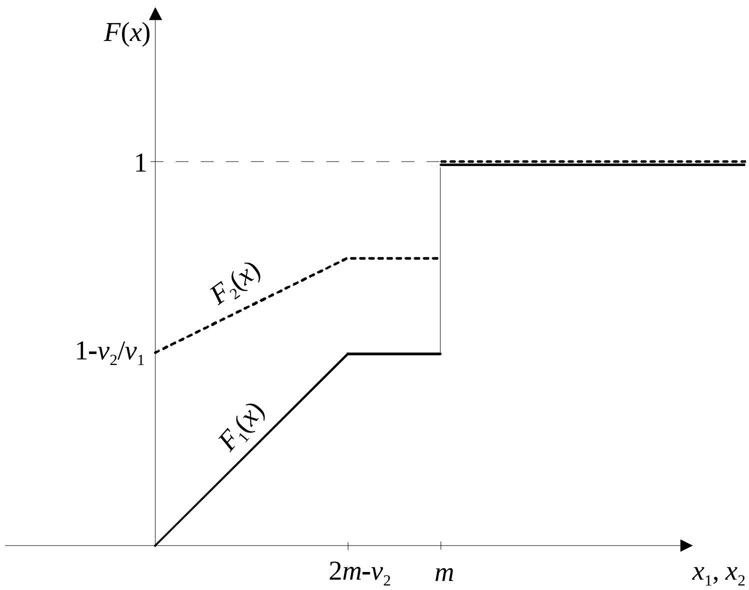

Figure 5: The equilibrium with caps

problem, let all contestants value winning the prize by v, but assume that their wealth that determines their maximum effort limit is sorted such that $w_1 > w_2 > w_3 \geq ... \geq w_n$, with $w_1 < v$. The equilibrium in this case is described by cumulative distribution functions

$$F_1(x_1) = \begin{cases} \frac{x_1}{v} & \text{for} \quad x_1 \in (0, w_2) \\ 1 & \text{for} \quad x_1 \geq w_2 \end{cases} \tag{15}$$

and

$$F_2(x_2) = \begin{cases} 1 - \frac{w_2}{v} + \frac{x_2}{v} & \text{for} \quad x_2 \in [0, w_2) \\ 1 & \text{for} \quad x_2 \geq w_2, \end{cases} \tag{16}$$

and zero effort by all other contestants (see Figure 6). The equilibrium properties of these functions can be confirmed by calculating the payoffs of the contestants. Given F_2 and zero-effort by all contestants $3, ...n$, the payoff of contestant 1 equals $(v - w_2)$ for all $x_1 \in (0, w_2]$, and is smaller for all other effort levels $x_1 \geq 0$. This makes contestant 1 indifferent with respect to all optimal effort levels $x_1 \in (0, w_2]$. He may therefore randomize according to F_1. Similarly, F_2 turns out to be an optimal reply to F_1 and to the non-participation of other contestants, and yields a payoff equal to zero to contestant 2. All other contestants $3, ..., n$ would make losses from any positive effort level given F_1 and F_2 and have zero payoff from choosing an effort level of zero.

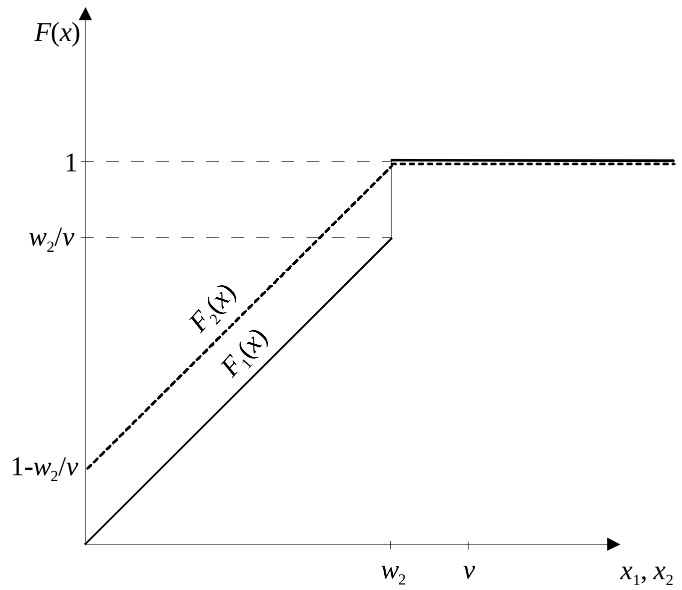

Figure 6: The equilibrium with budget constraints

Incumbency advantages In many real world contests for a prize, the contestants are not fully homogeneous. Differences with respect to contestants' valuations of the prize equivalently translate into differences in contestants' cost of providing a given level of effort. More precisely, a situation in which $C_1(x_1) = x_1$ and $C_2(x_2) = x_2$, but $v_1 > v_2$ can be mapped into an equivalent situation in which $v_1 = v_2$, but $C_1(x_1) = x_1 \frac{v2}{v1}$ and $C_2(x_2) = x_2$. This becomes clear from writing down the two contestants' objective functions for the two types of problem, noticing that one problem is obtained from the other by dividing the objective function of contestant 1 by v_1/v_2 (see Baye, Kovenock and deVries (1996) for a discussion).

A different type of asymmetry comes into play if one of the contestants has some kind of a headstart advantage; suppose, e.g., the contestants are symmetric with respect to provision cost ($C_i(x) = x$) and their valuation of the prize $v_1 = v_2 = v$, but let the contest success function be given as

$$p_1(x_1, x_2) = \begin{cases} 1 & \text{if} \quad x_1 > x_2 - \delta \\ 1/2 & \text{if} \quad x_1 = x_2 - \delta \\ 0 & \text{if} \quad x_1 < x_2 - \delta. \end{cases} \tag{17}$$

In this case, contestant 1 has a 'headstart advantage'. He needs to expend almost δ units fewer than the opponent and still wins the contest. There are many environments in which such a headstart becomes relevant. For instance, Konrad (2002) considers a situation in which an incumbent leader of a country or a firm fights with an entrant. A similar problem shows up in an R&D problem considered by Kaplan, Luski and Wettstein (2003), and in Konrad (2004c) in the context of parties' campaign contributions.

The equilibrium outcome in the problem with a headstart advantage is given by the cumulative distribution functions

$$F_1(x_1) = \begin{cases} \frac{\delta}{v} + \frac{x_1}{v} & \text{for} \quad x_1 \in [0, v-\delta) \\ 1 & \text{for} \quad x_1 \geq v - \delta \end{cases} \tag{18}$$

and

$$F_2(x_2) = \begin{cases} \frac{\delta}{v} & \text{for} \quad x_2 \in [0, \delta) \\ \frac{\delta}{v} + \frac{x_2-\delta}{v} & \text{for} \quad x_2 \in [\delta, v) \\ 1 & \text{for} \quad x_2 \geq v. \end{cases} \tag{19}$$

Again, by considering the resulting payoffs, it can be verified that these effort distributions are optimal answers to each other and establish an equilibrium.

Incomplete information Contestants often know their own valuation of winning the prize, but have only a opinion based on experience or a guess about their opponent's valuation of the prize. The all-pay auction is then one with incomplete information, and this case has attracted considerable interest in the literature for various assumptions about the distributions of types, cost functions, and bidding constraints. A selection of contributions are Glazer and Hassin (1988), Amann and Leininger (1996), Krishna and Morgan (1997), Baye, Kovenock and deVries (1998), Clark and Riis (2000), Moldovanu and Sela (2001), Feess, Muehlheusser and Walzl (2002), Gavious, Moldovanu and Sela (2002), Kura (1999), Lizzeri and Persico (2000) and Singh and Wittman (2001).

To see how the equilibrium differs from the full information case, consider the symmetric case with two contestants 1 and 2. Each values the prize of winning according to some $v_i \in [0, 1]$. The values v_i are drawn independently from a distribution $F(v)$ on the unit interval $[0, 1]$. The following is an equilibrium candidate. Let each contestant choose effort that is a function of the contestant's valuation of the prize, say, $x = \xi(v)$ such that the valuation that belongs to a given bid x is $\xi^{-1}(\xi(v))$, provided that this inverse exists. Consider now contestant 1's optimal choice if contestant 2 follows this pattern. The objective function of contestant 1 with a valuation equal to v_1 becomes

$$\pi_1(x_1) = F(\xi^{-1}(x_1))v_1 - x_1. \tag{20}$$

Maximization of this objective function yields the first-order condition

$$\pi_1'(x_1) = F'(\xi^{-1}(x_1))\frac{d\xi^{-1}}{dx_1}v_1 - 1 = 0. \tag{21}$$

Using symmetry according to which the two contestants will follow the same bid function $\xi(v)$ in the equilibrium, this can be transformed into

$$\frac{dx}{dv} = F'(v)v. \tag{22}$$

This is a differential equation. Taking into account the starting condition $\xi(0) = 0$, it can be solved for a given distribution function $F(v)$. This can, but need not, be a tricky task, depending on the distribution function. For instance, if $F(v)$ is uniform on the unit interval, then (22) becomes

$$dx = vdv \tag{23}$$

and has a simple solution:

$$\xi(v) = \frac{v^2}{2}. \tag{24}$$

The solution in this simple case is illustrated in Figure 7. It has nice properties that are robust for other distribution functions of valuations for a priori symmetric contestants. The inefficiency, according to which the prize can go to the bidder with the lower valuation that emerged in the full information case, disappears in this example. In the equilibrium, the contestant with the highest valuation of the prize wins the good. This is a nice property and also holds if the distribution of types has mass points as is illustrated in Konrad (2004a) who considers the evolutionary stability of envy and altruism as a bimorphism in a contest environment which leads to a binary distribution of valuations. The efficiency property of the allocation outcome in the all-pay auction with incomplete information is not fully robust. It will, for instance, not hold in general if the players are drawn from non-identical distributions of types.

The equilibria in the cases with complete and with incomplete information have something in common. The equilibrium effort choices of a contestant's opponent is a distribution of effort levels. In the complete information case, to overcome the non-existence problem of mutually optimal replies players need to randomize. In the incomplete information case, the randomness of types imposes randomness of one's opponent's bids, and this randomness is sufficient to make the decision problem of a contestant of a given type smooth. The duality of these two types of randomization is discussed in Amann and Leininger (1996).

One-sided asymmetric information Consider again two contestants. Suppose both contestants' valuations are drawn from the same distribution, but contestant 1's valuation v_1 is publicly observed, whereas contestant 2's

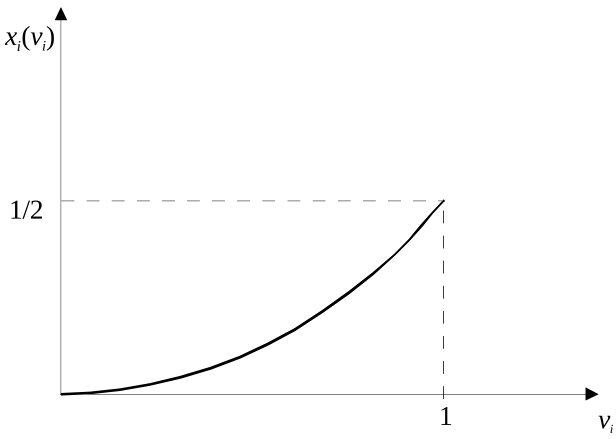

Figure 7: The equilibrium bidding function

valuation v_2 is this contestant's private information. For the class of 'smooth' contests with contest success functions that make the objective function sufficiently concave, the independent value problem has been addressed by Hurley and Shogren (1998), whereas Wärneryd (2003) has addressed a related common value problem.

For the all-pay auction, the solution of the independent value problem will be different and will typically involve mixed strategies. For illustration I consider the bivariate case here. Let v_i be drawn randomly and independently from the set $\{a, b\}$ with $a < b$, and with probabilities $q_a = 1 - q_b$ for both contestants. Let v_1 be publicly observed and let v_2 be contestant 2's private information. In this case the equilibrium has cumulative distribution functions

$$F_1^a(x_1) = \begin{cases} q_b \frac{b-a}{b} + \frac{x_1}{a} & \text{for} \quad x_1 \in [0, aq_a) \\ \frac{b-a}{b} + \frac{x_1}{b} & \text{for} \quad x_1 \in [aq_a, a] \end{cases} \tag{25}$$

for contestant 1 if $v_1 = a$ and

$$F_1^b(x_1) = \begin{cases} \frac{x_1}{a} & \text{for} \quad x_1 \in [0, aq_a) \\ q_a(1 - \frac{a}{b}) + \frac{x_1}{b} & \text{for} \quad x_1 \in [q_a a, q_a a + q_b b]. \end{cases} \tag{26}$$

if $v_1 = b$, and cumulative distribution functions for contestant 2 that depend on both contestants' types,

$$F_2^{aa}(x_2) = \frac{x_2/q_a}{a} \text{ for } x_2 \in [0, aq_a] \tag{27}$$

$$F_2^{ab}(x_2) = \frac{(x_2 - aq_a)/q_b}{a} \text{ for } x_2 \in (aq_a, a] \tag{28}$$

$$F_2^{ba}(x_2) = \frac{b-a}{b} + \frac{x_2/q_a}{b} \text{ for } x_2 \in [0, aq_a] \tag{29}$$

$$F_2^{bb}(x_2) = \frac{(x_2 - q_a a)/q_b}{b} \text{ for } x_2 \in (aq_a, q_a a + q_b b] \tag{30}$$

and zero and one respectively for x_2 to the left and to the right of these intervals with F_2^{rs} being the function for the case where contestant 1 has a publicly observed valuation of r and contestant 2 has a valuation equal to s.

The expected equilibrium payoff of the informed contestant is

$$(b-a)\, q_a q_b (2 - q_a \frac{b-a}{b}) \tag{31}$$

and the expected equilibrium payoff of the uninformed contestant is

$$q_a q_b (b-a). \tag{32}$$

The better informed contestant receives an information rent.

In the case with two-sided full information, a rent is obtained only in the cases in which the contestants' valuations differ, and the rent in this case equals $(b-a)$. Each contestant has an ex ante chance of winning this rent of $q_a q_b$. Accordingly, the total expected rent is $2q_a q_b(b-a)$ and, in expectation, it is equally distributed between the two contestants. In the case of asymmetry, the increase in the payoff is earned by the contestant who is better informed.

One may also compare this outcome with the expected equilibrium payoffs in the two-sided incomplete information case considered by Konrad (2004a). There, the contestants who have the high valuation receive an expected payoff equal to $q_a(b-a)$, and given their probability of having a high valuation, each contestant has an ex ante expected payoff equal to $q_a q_b(b-a)$, just as in the full information case. Of the three cases, the asymmetric one has the highest expected payoff in the binary distribution case. Whether this is a general pattern is unclear, but it is intuitively plausible that the better informed contestant will earn an information rent.

Conclusions The first-price all-pay auction, as the contest without noise is often called, is an interesting benchmark case. While there will be few real-world situations in which the actual effort choices of contestants translate into such a precise outcome with regard to who wins the contest, this interaction makes it very clear that the outcome in a contest strongly depends on one's own effort relative to the efforts of other contestants, and that this may cause considerable expenditure, particularly for the contestants who value the contest prize most highly, or who are most productive in generating the type of output that is the unit of comparison with respect to determining contest success. It has also been shown that asymmetry between the contestants who participate actively in the contest reduces total expenditure. Incomplete information may, but need not, reduce total contest expenditure. In particular, the binary example at the end reveals this most clearly. The first-price all-pay contest and its equilibrium under various constraints will be an important building block in the analysis of more complex situations. Before turning to these, I will analyse contest variants with two different types of noise.

2.2 Additive noise

For various reasons, noise or randomness plays a role in contests. Consider, e.g., two architects who compete for receiving the contract to build a museum. Let them both be equally talented and productive. Typically, if the prize is high, they will expend considerable manpower and come up with good project proposals. A committee then decides. The number of office hours expended on thinking about a project proposal will generally improve it, but there is no need for the architect who expends the largest number of hours to win the architecture contest for sure. The decision will generally depend on many random factors. For instance, the committee members' preferences are not perfectly known to the architects, and they may like or dislike one or the other proposal for reasons not related to the effort put into the project. The decision may also depend on the architects' outfit, mood or performance when presenting their proposals, thus adding further elements of noise.

A simple way to map this kind of noise was chosen by Lazear and Rosen (1981) and also in much of the labor market tournament literature partially surveyed in Lazear (1995). Related structures have also been analysed in the literature on contests under the 'difference-form contest success function' (see, e.g., Hirshleifer 1989, Baik 1998 and Che and Gale 2000). The underlying assumption of this contest success function is that the contest success probabilities of the set of contestants is not affected if all contestants increase their contest efforts by the same, arbitrary number, as long as the contest efforts are positive both prior to and after this shift in effort. As Skaperdas (1996) points out, this is a strong axiom. However, it does help to generate a simple framework that has been popular, particularly for studying problems in the labor market.

The two-player case Consider two contestants. They choose contest efforts x_1 and x_2. But their efforts do not translate in a deterministic way into the observable characteristics that are the basis of decision making as in the section on the first-price all-pay auction. An easy way to describe this is to allow for some additive noise: the observed characteristic for contestant i is $x_i + \epsilon_i$, where ϵ_i is some random variable, and the prize is awarded to the contestant for whom $x_i + \epsilon_i$ is largest. Written differently, and lumping the two noise parameters ϵ_1 and ϵ_2 into a single parameter $\epsilon = \epsilon_2 - \epsilon_1$, the contest success function can be written as

$$p_1(x_1, x_2) = \begin{cases} 1 & \text{if} \quad x_1 - x_2 > \epsilon \\ 1/2 & \text{if} \quad x_1 - x_2 = \epsilon \\ 0 & \text{if} \quad x_1 - x_2 < \epsilon \end{cases} . \tag{33}$$

Much now depends on the distribution G of the noise variable ϵ that is distributed on an interval $[-e, e]$, with $e > 0$.

The tournament is often designed by an organizer. The organizer may be able to determine the prize b_W obtained by the winner of the tournament, the prize b_L received by the loser of the tournament, and, possibly, the distribution $G(\epsilon)$ of noise, among other things. Let the prizes be denoted in monetary units, and let the contestants be risk neutral. This makes the value of winning the contest equal to $v_i(b_W)$ and the value of losing is $v_i(b_L)$ for $i = 1, 2$. Each single contestant then maximizes

$$p_i(x_1, x_2)(v_i(b_W) - v_i(b_L)) - C(x_i) + v_i(b_L). \tag{34}$$

Unlike in the previous section, the cost $C(x_i)$ of effort is not assumed to be linear, but instead it is a convex function of effort with $C(0) = 0$, $C'(x_i) > 0$ and $C''(x_i) > 0$. This convexity assumption brings about a situation in which the marginal effort cost for an increase in the probability of winning the prize is increasing in this probability, even if ϵ is uniformly distributed.[13] If this cost function is sufficiently convex, the contest equilibrium can simply be described by the first-order condition for contestant 1,

$$\frac{\partial G(x_1 - x_2)}{\partial x_1}[v_1(b_W) - v_1(b_L)] = C'(x_1), \tag{35}$$

and analogously for contestant 2, provided that the contestants' expected payoffs in the resulting equilibrium are larger than their payoffs from not participating in the contest, and this can always be guaranteed by a sufficiently attractive loser prize b_L.

[13] A sufficient combination of convexity of the cost function and the distribution of ϵ required for the first-order condition to describe the equilibrium. Otherwise corner solutions become important and the equilibrium will be in mixed strategies. For $C(x) = x$ and a uniform distribution of ϵ, for instance, the problem converges towards the standard all-pay auction if the support of ϵ becomes small.

The problem can be simplified if one of the following assumptions are made. First, if $v_i(b) = b$, and if both contestants have the same cost function, then $x_1 = x_2$ in the equilibrium. This reduces both contestants' first-order conditions to

$$G'(0)(b_W - b_L) = C'(x_i). \tag{36}$$

Second, if ϵ is distributed uniformly on an interval $[-e, +e]$ this causes the first-order conditions of the two contestants to be independent of each other, as this makes $\partial G(x_1 - x_2)/\partial x_1$ independent of x_2 and equal to $1/(2e)$. Accordingly, the strategic interdependence of effort choices disappears. In what follows, I will concentrate on this particularly simple and illustrative case.

The first-order condition suggests that an increase in the dispersion of ϵ, or a reduction in the value of the winner prize, will generally reduce the optimal effort of a contestant. The contestant who values the prize more highly will expend more effort than his competitor, given that they face the same function describing their cost of effort.

When the tournament is designed by an organizer who chooses the winner prize and the loser prize, the optimal design question emerges. If the distribution of noise is exogenously given, and, for simplicity, uniform the contest designer can still choose along several dimensions, for instance, the choice of the winner and loser prizes. Let us concentrate on the simple case in which $v_i(b) = b$. Two benchmark cases need to be distinguished for the contest design problem. The contest designer could be at the shorter side of the market and, much like a monopolist, may design the contest in a way that maximizes his own payoff from organizing it, or the participants in the contest could be on the short side of the market, i.e., there could be many contest designers (like firms) which compete for hiring contestants.

Consider first a firm that has monopoly power when hiring two contestants. In this case, the problem of choosing b_W and b_L is the problem of maximizing $\varphi(x_1 + x_2) - b_W - b_L$ subject to the participation constraints of the contestants, i.e., $p_i(x_1^*, x_2^*)(b_W - b_L) + b_L \geq C(x_i^*)$, where the contestant's payoff from not participating is normalized to zero, and subject to the first-order conditions determining x_1^* and x_2^*. The contest designer's benefit from the contestants' efforts x_1 and x_2 is denoted φ and is assumed to be a function of the sum of these efforts. Of course, this is only one of many alternative assumptions, some of which have been considered in the literature.

The firm can always set b_L sufficiently high to fulfill the participation constraint. If the optimization problem has an interior solution, using symmetry, the contest designer chooses $b_W - b_L$ in a way such that the $x^* = x_1^* = x_2^*$ for which

$$G'(0)(b_W - b_L) = C'(x^*) \tag{37}$$

also fulfills

$$C'(x^*) = \varphi'(2x^*). \tag{38}$$

This condition is the efficiency condition for the equilibrium effort. The contest designer can induce each of the two contestants to expend an additional unit of effort, and $\varphi'(2x^*)$ is the additional benefit this brings to the contest organizer. However, the organizer will have to motivate the contestants to do this by an increase in the contest prize. The optimal structure is reached if the additional marginal benefit generated by additional effort equals the actual marginal cost of the contestant in providing this additional effort.

Alternatively, the contestants may have all the market power. In this case, the optimal contest is characterized by the same condition (38), but the contestants will receive the maximum feasible compensation for participation that reduces the contest organizer's expected payoff from organizing the contest down to his reservation utility.

Note that the first-best optimal contest is not necessarily implemented, for a number of reasons that may come into play, like asymmetry between the contestants, risk aversion on the side of the contestants or the contest designer, and issues of incomplete information (see, e.g., O'Keefe, Viscusi and Zeckhauser 1984 for some considerations). For many of these issues, the first-best outcome will become unattainable as an equilibrium outcome.

Some considerations about the optimality of a tournament as a designed incentive mechanism when the designer cares about total output net of compensation is in Nalebuff and Stiglitz (1983). An important insight in this paper is that tournaments have useful properties as incentive contracts even if the contestants are risk averse, if the relationship between individual effort and individual output is 'disturbed' by considerable systematic noise, i.e., if the ϵ_i's that add to actual efforts x_i to determine i's observed output have a large common component. In this case, the tournament works similarly to a compensation scheme that is used to set up yardstick competition. A few other positive aspects of tournaments have been highlighted in the literature. Tournaments make it feasible for the organizer to commit to paying out a prize to at least one contestant, which may be useful if the actual output is observable, but not verifiable in court[14] or if the moral hazard problem is double sided because the principal can also affect the agent's output (see Malcomson 1984, Carmichael 1983 and a discussion in Tsoulouhas 1999). Other aspects that may improve the theoretical properties of tournaments as incentive mechanisms have been discussed. See, for instance, Quintero (2004) for

[14] The importance of this argument has also been questioned in the literature, as, once the tournament designer and the participants observe the actual output, the participants may try to bribe the designer to award the prize not according to true output.

two-sided limited liability. Gibbons and Waldman (1989) and, in particular, Lazear (1995) also survey further negative aspects of tournaments, highlighting in particular the problem of sabotage, or the disincentives to provide help to co-workers if they are rivals in a tournament, and how such problems could in theory be reduced by an appropriate design of the tournament, and consider some empirical applications.

Note that the tournament turns into the all-pay auction if the ϵ becomes degenerate. Hence, the all-pay auction without noise in the previous section can be seen as the limiting case of the tournament. This convergence, however, is not a smooth process. If the dispersion of ϵ becomes smaller and smaller, a pure strategy equilibrium disappears at some point. Mixed strategy equilibria that can emerge in this case are described for some cases in Che and Gale (2000).

2.3 The Tullock contest

Perhaps the most popular contest success function that has been suggested and used in several areas of economics assumes that a contestant i's probability of winning the contest equals the ratio between this contestant's own effort and the sum of efforts, or a variant of this. Some examples and early references in the context of promotional competition and in military applications have been discussed in Chapter 1.

The standard Tullock contest While Tullock (1980) was not the first to analyse contests with this particular structure in economic applications, the following contest success function is typically attributed to him as he was the first to use it to study the problem of rival rent-seekers who expend resources to influence the policy outcome in their favor.

$$p_i(x_1, ...x_n) = \begin{cases} \frac{x_i^r}{\sum_{j=1}^n x_j^r} & \text{if } \max\{x_1, ...x_n\} > 0 \\ 1/n & \text{otherwise.} \end{cases} \tag{39}$$

For $r = 1$, this function has also been called the 'lottery contest': the win probability equals the share of expenditure of a contestant in the total expenditure, like in a lottery in which one monetary unit buys one lottery ticket, and in which the winner is drawn from the set of all tickets with each ticket winning with the same probability. However, the parameter $r > 0$ in the function (39) allows for slightly more general types of contests. This parameter will be important for the marginal impact of an increase in a contestant's effort.

The function (39) converges towards the contest success function with no noise as $r \to \infty$. Note that p_i is a probability for all feasible combinations of effort, and that the probability for i to win the contest is increasing in i's own effort and decreasing in other contestants' effort. Together with functions

$$C_i(x_i) = x_i \tag{40}$$

describing individuals' cost of providing efforts x_i, this describes the "Tullock contest".[15]

This contest and its variants have been studied extremely carefully. The contest success function (39) has been used in the literature describing rent seeking (see Nitzan, 1994, for an early survey and Lockard and Tullock, 2001, for a more recent collection of papers). Friedman (1958) used this function for describing the relationship between the persuasive advertising of the different firms' and their respective shares in the markets. He considers the advertising game between two firms which compete in several products or product categories. Firms have given global advertising budgets and must decide how to allocate their budget to advertising expenditure for these different products. The game has been called "Colonel Blotto game" and different variants of this game have been studied in the literature (see section 1.3). Bell, Keeney and Little (1975) axiomatized this function, also in the context of advertising, Schmalensee (1976) considers more general functions describing the relationship between advertising expenditure and market shares and introduces and discusses a number of plausible properties of such functions, but considers (39) with $r = 1$ as a special case (p. 495). A more recent application in the context of promotional competition is Barros and Sørgard (2000). In sports, the outcome of sports tournaments has been described by (39) by Hoehn and Szymanski (1999) and Szymanski (2003). The function has also been used for describing R&D contests (Fullerton and McAfee 1999) and has emerged in the literature on status seeking (Congleton 1989, Konrad 1990 and Konrad 1992) in a structurally related context to describe preferences for relative standing comparisons.

The case with $r = 1$ has been particularly popular because of its analytical tractability. By adding some constants in the numerator or the denominator or by allowing for a different cost of making contributions x_j, the contestants can be made asymmetric, and handicaps for one or the other contestant can be analysed.

[15] As discussed in Michaels (1988), the effective effort x_i by contestant i can also be a function of a number of inputs which generate this effective effort. In this case, the contestant will typically choose a cost efficient mix of the different effort inputs.

Existence, Uniqueness and Comparative Statics Existence and uniqueness of the Nash equilibrium in the Tullock contest follows from the analysis in Szidarovszky and Okuguchi (1997) if x_i^r is concave in r, i.e., if $r \leq 1$. They prove this property for an arbitrary finite number of contestants and for a more general class of contest success functions.[16]

For the Tullock contest, the scope for existence and uniqueness of a Nash equilibrium can be extended. In the symmetric contest with n contestants, $r \leq \frac{n}{n-1}$ is the condition that makes the second-order condition fulfilled. If this condition is violated, a pure strategy Nash equilibrium will typically not exist. This fact has caused considerable confusion for quite some time and has raised the question of whether contestants may, on aggregate, expend more effort than the value of the prize they can win (Tullock 1980). Baye, Kovenock and deVries (1994, 1999) have shown that the equilibrium solutions that emerge if the contestants' objective functions are not concave cannot have higher expected aggregate contest efforts in an equilibrium than the value of the prize and have shown the existence of an equilibrium. They have also characterized the equilibrium which is in mixed strategies if there is a finite set of possible effort choices. Even though the equilibrium mixed strategies are different, the results regarding dissipation resemble qualitatively the all-pay auction with complete information. Given that the contestants choose mixed strategies, the total effort can, in some instances, exceed the valuation of the prize, but the expected total effort in the equilibrium cannot exceed the value of the prize. As long as contestants cannot be forced to expend positive effort or to participate in the contest, they can always abstain from expending effort, and some contestants would prefer to abstain from expending effort if participation implied an expected effort that would exceed their expected reward if they take part in the contest.

A characterization and the comparative static properties of the contest equilibrium can be obtained from the first-order conditions, where they characterize the equilibrium (see, e.g., Nti 1999).[17] For the simple case with only two contestants, the first-order conditions for the maximization problem with an objective function (2) of a contestant with contest success function (39)

[16] For a discussion of stablity proporties of the equilibrium see Xu and Szidarovszky (1999).

[17] For an early solution of a structurally equivalent problem in the context of promotional competition see Mills (1961, p.293). Mills solves a slightly more general problem which, imposing symmetry restrictions on some parameters, reduces to the Tullock problem. For comparative statics on more general contest success functions that are based on a ratio $f(x_i)/\sum f(x_j)$ see Nti (1997).

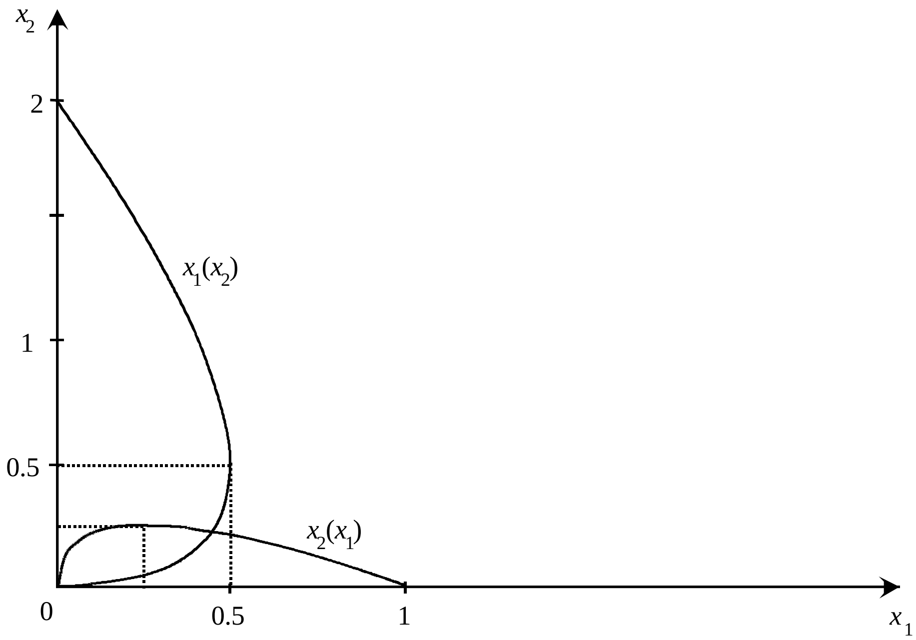

Figure 8: Reaction functions for $v_1 = 2$, $v_2 = 1$ and $r = 1$.

and a cost-of-effort function (40) are

$$\frac{r x_1^{r-1} x_2^r}{(x_1^r + x_2^r)^2} v_1 = 1 \text{ and } \frac{r x_2^{r-1} x_1^r}{(x_1^r + x_2^r)^2} v_2 = 1 \ . \tag{41}$$

The reaction functions that result from solving these first-order conditions are depicted for the case $v_1 = 2$, $v_2 = 1$ and $r = 1$ in Figure 8.[18]

The intersections determine the Nash equilibrium if the first-order conditions determine the equilibrium. The equilibrium values are

$$x_1^* = r \frac{v_2^r v_1^{1+r}}{(v_1^r + v_2^r)^2} \text{ and } x_2^* = r \frac{v_2^{1+r} v_1^r}{(v_1^r + v_2^r)^2} \tag{42}$$

$$p_1^* = \frac{v_1^r}{v_1^r + v_2^r} \text{ and } p_2^* = \frac{v_2^r}{v_1^r + v_2^r} \tag{43}$$

$$\pi_1^* = \frac{v_1^{r+1}(v_1^r + v_2^r(1-r))}{(v_1^r + v_2^r)^2} \text{ and } \pi_2^* = \frac{v_2^{r+1}(v_2^r + v_1^r(1-r))}{(v_1^r + v_2^r)^2}. \tag{44}$$

Note that the contestant with the higher valuation of the prize expends more effort and wins with a higher probability, but not with probability

[18] See Pérez-Castrillo and Verdier (1992) for more general cases.

1. The Tullock contest equilibrium does not always award the prize to the contestant who values it most, but the win probabilities are biased favorably for the contestant with the higher valuation. Also, total rent seeking effort is generally reduced if the contestants are more different in their valuation of the prize. This can be seen as follows. Consider valuations $v_1 = v + D$ and $v_2 = v - D$. The sum of efforts can be written as

$$\begin{aligned} x_1^* + x_2^* &= (v_1 + v_2)\frac{r v_1^r v_2^r}{(v_1^r + v_2^r)^2} && (45)\\ &= (v + D + v - D)\frac{r(v+D)^r(v-D)^r}{((v+D)^r + (v-D)^r)^2} && (46)\\ &= 2v\frac{r(v+D)^r(v-D)^r}{((v+D)^r + (v-D)^r)^2}. && (47) \end{aligned}$$

Differentiating $(x_1^* + x_2^*)$ with respect to D reveals after some transformations that this sum decreases in D.

Many participants Generalizing the symmetric Tullock contest with an arbitrary, but sufficiently small, r or the asymmetric Tullock contest with $r = 1$ to the case with any finite number $n > 2$ is not a difficult exercise. For comparative statics on more general contest success functions that are based on a ratio $f(x_i)/\sum f(x_j)$ but cover the case in (39) and asymmetric valuations, see Cornes and Hartley (2005) for existence, uniqueness, a characterization of the equilibrium and limit results,[19] and Stein (2002) and Meland and Straume (2005) for an elegant solution of the case (39).

The symmetric case shows that the ratio between the aggregate effort that is expended in the equilibrium and the value of the prize is increasing in the number of contestants, and converges towards r for all $r \leq 1$. In particular, the prize is fully 'dissipated' by aggregate effort, if $r = 1$.[20] The fact that the prize is not fully dissipated even with free entry and a large number of identical contestants has received some attention, as it seems to contradict the intuition that rents cannot survive with perfect competition among an infinite number of identical contestants. Some discussion of this can be found in the collected papers volume by Lockard and Tullock (2001).

[19] Cornes and Hartley (2002) consider entry fees for contests with this structure. Hillman and Katz (1984) were the first to consider risk aversion in rent seeking, and Cornes and Hartley (2001) analyse this more general structure for risk averse contestants. Earlier comparative static results on this structure are by Nti (1997) and Wärneryd (2001).

[20] These intuitive results should not simply be extrapolated to contests with other contest success functions more generally, as is shown in Wärneryd (2001).

Ellingsen (1991) carefully explains why an expected effort that exceeds the valuation of the prize or 'overdissipation' cannot occur.[21]

Gradstein and Konrad (1999) also address this issue. They show that full dissipation also results for $r < 1$ if the contest takes place not as a simultaneous Tullock-contest among all n contestants, but in a multi-stage contest in which a number of parallel contests take place at each stage, where two contestants always compete with each other in a two-player Tullock contest, and only the winner in each of the parallel stage contests is promoted to the next stage of contests, like in an pair-wise elimination tournament.

The insight used in Stein (2002), Cornes and Hartley (2005) and Meland and Straume (2005) to solve for the case with asymmetric valuations of the prize results from the observation that the payoff of player i depends only on player i's own effort and the sum of all other players' effort. For this purpose, define $X = \Sigma_{i=1}^{i=n} x_i$ and note that $p_i(x) = x_i/X$. Inserting this in the payoff function (2) and calculating first-order conditions yields

$$x_i = \max[0, X(1 - \frac{X}{v_i})]. \tag{48}$$

If all contestants expend non-zero effort in the equilibrium, this can be rearranged to characterize

$$\sum_{i=1}^{n} x_i^* = X^* = \frac{n-1}{n} \frac{n}{\Sigma_{i=1}^{i=n} \frac{1}{v_i}}. \tag{49}$$

Note that, from the first-order condition (48), corner solutions in which some contestants prefer to choose zero effort are likely to emerge if contestants are sufficiently heterogenous. In this case X^* is found by an equation similar to (49), for which n is replaced by the number of players who make non-zero bids in the equilibrium.

Why is this contest so popular? Why has this function been invented and used independently in so many different areas of applied theory? There seem to be two different types of justification.

Axiomatic reasoning Bell, Keeney and Little (1975), Skaperdas (1996), Clark and Riis (1998a) and Kooreman and Shoonbeek (1997) give systems of axioms about how conflict is decided as a function of the contestants' efforts so that these sets of axioms imply that the contest success functions are variants of the Tullock contest success function.

[21] Fabella (1995) and Keem (2001) make further additions to the framework and the result in Ellingsen (1991).

Bell, Keeney and Little (1975) were probably the first to address this problem. Their analysis is in the context of promotional competition for market shares, and they aim at an axiomatic foundation for how effort or sellers' 'attraction' translates into market shares, but can be also translated into the contest framework in which sellers' attraction is contest effort and market share corresponds with win probability.[22] They highlight the potential importance of an axiom that leads to what is called "aggregative games" in other areas of economics[23], where individual payoffs depend only on own choices and the aggregated values of all other players' efforts. Consider the two effort vectors in a contest between n players:

$$\mathbf{x}^I = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ x_k \\ x_{k+1} \\ \vdots \\ x_n \end{pmatrix} \text{ and } \mathbf{x}^{II} = \begin{pmatrix} x_k/k \\ \vdots \\ x_k/k \\ x_k/k \\ x_{k+1} \\ \vdots \\ x_n \end{pmatrix} \tag{50}$$

Both vectors $\mathbf{x}^I$ and $\mathbf{x}^{II}$ have the same total effort, denoted X. If $p_i(\mathbf{x}) = p(x_i, X)$,

$$\sum_{i=k+1}^{n} p_i(\mathbf{x}^I) = \sum_{i=k+1}^{n} p_i(\mathbf{x}^{II}), \tag{51}$$

and, accordingly,

$$\sum_{i=1}^{k} p_i(\mathbf{x}^I) = \sum_{i=1}^{k} p_i(\mathbf{x}^{II}) \tag{52}$$

Note that $p(0, X) = 0$ for $X > 0$, together with (52) and symmetry, imply $p(x_k, X) = kp(x_k/k, X)$. Hence, for given aggregate effort of all contestants including contestant i's effort, the probability of winning is linear in contestant i's own effort. This property is fulfilled by the Tullock function for $r = 1$. Moreover, Bell, Keeney and Little (1975) offer a proof by contradiction suggesting that this is the only contest success function that has this property. The proof relies on n and k being real numbers instead of integers and that there is a sufficiently large number of contestants for generating the contradiction. However, their work is pioneering in this field.

[22] Kotler (1984, p. 231) calls a function that is structurally identical to the Tullock contest success function with $r = 1$ the *fundamental theorem* of market share, and he offers a variant of this theorem for asymmetries in firms' effectiveness in marketing effort.

[23] See, e.g., Cornes and Hartley (2004).

Skaperdas (1996) derives the Tullock function for a lottery contest from several intuitive axioms. The most important axioms are on the symmetry of players, on invariance properties of the nature of the contest with respect to the number of participants, and, most importantly, a homogeneity axiom that makes contest success probabilities invariant with respect to an increase in all contestants' efforts by some given factor. The one-to-one relationship between the Tullock lottery contest and these axioms is surprising and gives strong support for the use of this contest success function in actual applications. Clark and Riis (1998a) extend Skaperdas' work. An alternative set of axioms is stated by Kooreman and Shoonbeek (1997) and leads to a modified version of the Tullock contest success function that allows for asymmetry between players. One of their axioms makes an assumption about the functional form of the derivative of the contest success function with respect to effort choices, yielding the contest success function essentially by integrating this derivative.

Microeconomic underpinnings An important reason why the Tullock contest is often used as a black box for a complicated allocation mechanism in which winning is a function of efforts comes from the literature that provides an economic underpinning for this black box approach. These probability models even make a strong case for $r = 1$. Such probability models can be found in Hirshleifer and Riley (1992), Fullerton and McAfee (1999) and Baye and Hoppe (2003).

A Tullock contest structure emerges from a simple search problem. The result can be obtained along intuitive lines of reasoning. Suppose there are two players 1 and 2. For concreteness, let us consider them as two competing interest groups who lobby for the two mutually exclusive projects 1 and 2. The choice between the projects is made by a benevolent, but uninformed, dictator who, for the sake of the argument, will decide favorably either for interest group 1 or for interest group 2, depending on who proposes the project that looks better from a welfare point of view.

Assume that the projects can be carried out with various designs, and the designs may benefit or harm a greater public by more or less, and may also (but need not) change the interest group's benefits from having their project implemented. Let x_i be the number of design proposals produced by interest group i, for all interest groups $i = 1, 2$, and suppose that the policy maker chooses the proposal of the group which produced the design that benefits a greater public most. Then, if the design proposals are independent draws from identical distributions, the probability that interest group i produces the most preferred design equals their share in the total number of proposals, i.e., (39) with $r = 1$. Moreover, let v_1 and v_2 be the interest groups' benefits from a favorable decision. These benefits may, but need not, depend on the interest groups' efforts. For instance, if the various designs do not change the interest group's benefit or cost from being granted the right to carry out their project, but do change the benefits that the project has to a greater public, v_1 and v_2 are exogenous with respect to the choices of x_1 and x_2.

More generally, the valuations of winning will be functions of x_1 and x_2 that depend on the particular framework. For instance, in the context of R&D races in which the design draws can be understood as experiments in innovating a particular product and where the firm succeeds in doing this first wins the patent, the number of experiments will be related to the overall speed of innovation, and the rent from the monopoly is likely to have a different present value if the innovation takes place earlier. Also, if the innovations are seen as improvements in product quality , with the firm that has the best product winning a contest, the number of trials may affect the expected quality of the product that finally wins the race, and, in turn, this may change the firm's valuation of winning.

Baye and Hoppe (2003) provide a formal analysis of this aspect. They assume that each contestant only tries to improve his own project or product and the contestant with the better project or product wins.[24] For concreteness, let there be two contestants who simultaneously and independently choose an amount x_i of resources. Each unit of resource is invested in research and generates a new project design with a different project value z_i for contestant i, where these valuations are independent random draws from a given distribution of project design values z with an absolutely continuous cumulative distribution function $F(z)$ with support $[0, v_{\max}]$. Each contestant then presents the project that has the highest value to him to the decision maker who chooses the project with the higher valuation.

Consider the payoff function for contestant 1 that results from this set-up. Let x_1 and x_2 be the numbers of draws chosen by the two contestants and let v_1 and v_2 be the highest project valuations. Then the probability that a given maximum valuation v_1 is higher than the highest z_2 that emerges from the other firm's draws is equal to $[F(v_1)]^{x_2}$. If contestant 1 makes only one draw that costs one unit, the expected payoff is

$$\int_0^1 F(v)^{x_2} v f(v) dv - 1. \tag{53}$$

Each additional draw costs an additional unit and yields a positive expected

[24] Related ideas have been put forward by Lagerlöf (1997, 2005) and Austen-Smith (1995, 1998). In Lagerlöf (1997) the interest groups produce competing information about their own proposals that makes the own proposal look better.

benefit if it is more successful than all other draws. Summing up all these benefits and costs yields group 1's payoff as

$$\pi_1(x_1, x_2) = x_1 \int_0^1 vF(v)^{x_2} F(v)^{(x_1-1)} f(v)dv - x_1. \tag{54}$$

Note that this makes use of the fact that a zero probability event occurs if two or more draws generate the same project quality. Making use of symmetry and defining $u(v) = F(v)^{x_2+x_1}$ and, hence, $du(v) = (x_1+x_2)F(v)^{x_2+x_1-1}f(v)dv$ yields

$$\pi_1(x_1, x_2) = x_1 \int_0^1 \frac{1}{x_1 + x_2} vdu(v) - x_1. \tag{55}$$

Integrating by parts yields

$$\pi_1(x_1, x_2) = \frac{x_1}{x_1 + x_2} v_1(x_1, x_2) - x_1 \tag{56}$$

with

$$v_1(x_1, x_2) = [1 - \int_0^1 F(v)^{x_2+x_1} dv]. \tag{57}$$

Accordingly, the groups' efforts translate into a payoff function that resembles their payoff function in a Tullock contest with the minor difference that the contest prize depends on x_1 and x_2 as well, but this dependency occurs only via the distribution function and, for appropriate distribution functions of v, it disappears.

Hirshleifer and Riley (1992) suggest a different microeconomic underpinning in one of the exercises in their chapter about contests. Assume, for instance, the prize in an R&D race is given and equal to v for both contestants. Let contestant 1 win if $q_1 x_1 > q_2 x_2$, where x_1 and x_2 are effort levels chosen by the two contestants, and q_1 and q_2 be independent draws from an exponential distribution $F(q) = 1 - e^{-aq}$. The noise that is introduced by the exponential distribution translates this all-pay auction problem into the Tullock problem as follows: for a given $\hat{q}_1$, contestant 2 wins for given (x_1, x_2) if $\hat{q}_1 x_1 < q_2 x_2$ which happens with probability

$$\begin{aligned} prob(\hat{q}_1 x_1 &< q_2 x_2) \\ &= prob(q_2 > \hat{q}_1 x_1 / x_2) \\ &= 1 - prob(q_2 \leq \hat{q}_1 x_1 / x_2) \\ &= 1 - (1 - e^{-a(\hat{q}_1 x_1 / x_2)}) \\ &= e^{-a(\hat{q}_1 x_1 / x_2)} \end{aligned} \tag{58}$$

In a next step, consider the unconditional probability for contestant 2 to win for (x_1, x_2). It can be written as

$$\begin{aligned} prob(q_1x_1 &< q_2x_2) \\ &= \int_0^\infty e^{-a(q_1x_1/x_2)} f(q_1)dq_1 \\ &= \int_0^\infty e^{-a(q_1x_1/x_2)}[ae^{-aq_1}]dq_1 \\ &= a\int_0^\infty e^{-aq_1\frac{x_1+x_2}{x_2}}dq_1 \\ &= a\left[-\frac{1}{a}\frac{x_2}{x_1+x_2}e^{-aq_1\frac{x_1+x_2}{x_2}}\right]_0^\infty \\ &= -\frac{x_2}{x_1+x_2}[0-1] \\ &= \frac{x_2}{x_1+x_2} \end{aligned} \tag{59}$$

This is a more straightforward foundation for using the ratio of efforts as a probability for winning the contest. It is based on the all-pay auction contest success function, but with some multiplicative noise that follows a particular type of distribution.

Information aspects Tullock (1980) assumed that the players who enter into the contest know everything about each other. The microeconomic foundation reveals that this structure is basically a short cut to more explicit competitive search problems in which a substantial amount of uncertainty is involved. One may consider elements of incomplete and imperfect information in the Tullock contest too. Contestants may have some uncertainty about their abilities or their valuations of the prize. For instance, each contestant may know his own valuation of the prize, but may know only the distribution from which the opponent's valuation of the prize is drawn. Malueg and Yates (2004) provide an analysis of the case in which two contestants' valuations are drawn from the same bivariate distribution but need not be the same. In their framework each player knows his own valuation of the prize.

Some players could also have superior information compared to others, and there are several problems of asymmetric information that could be distinguished. For instance, the prize value could be the same for all players, but one player may know the true valuation of the prize, where his competitor does not. The valuation of the prize could be different for two players, where only one player knows his true valuation. Similar aspects could be analysed regarding the cost of effort or the mechanism that maps effort choices into win probabilities. Wärneryd (2003) addresses a problem with asymmetric information. Two contestants' valuations are from the same distribution. Contestant 1 knows the true value of the prize. Contestant 2 knows the distribution from which this value is drawn, but does not know the true value of the prize. Hence, this is a problem with asymmetric information and common values. Wärneryd shows that, for a contest success function somewhat more general than (39), the uninformed contestant is more likely than the fully informed contestant to win the contest in the equilibrium. He also considers the special case of (39), and several specific distributions of the prize.

Hurley and Shogren (1998) address a complementary problem of asymmetric information in which the contestants' valuations of the prize are independent. The value of the prize of one of the contestants is publicly observed, like in the all-pay auction problem. They consider the impact on the contests outcome of the distribution from which the informed contestant's valuation is drawn.

2.4 Experimental evidence and evolutionary game theory

Contests also have attracted experimental game theorists. An influential paper is by Davis and Reilly (1998), who consider both Tullock's lottery contest and the all-pay auction without noise with both symmetric and asymmetric players. They confirm some of the qualitative predictions for the participation of an additional player with a higher valuation of the prize in an otherwise symmetric environment. Their most important result is that effort levels exceed the predictions resulting from the Nash equilibrium, taking effort cost and the value of the prize at face value for both the lottery contest and the all-pay auction. This 'overdissipation' is reduced if players participate more often, but it does not disappear. In Potters, deVries and Van Winden (1998), the overspending results in Davis and Reilly (1998) are critically reviewed. They find overdissipation for the lottery contest, but dissipation that is close to the predicted values for the all-pay auction without noise.

Further experiments look at whether agents who play modifications of the Tullock lottery game or the all-pay auction without noise choose the predicted equilibrium efforts. Currently the literature is expanding rapidly, as many different structures can be studied experimentally, and only a few examples are mentioned here. Millner and Pratt (1989, 1991) find mild evidence that contestants expend more effort than predicted by the equilibrium outcome if players maximize their monetary payoffs. However, in a similar treatment, in which the contestants were matched repeatedly, , this bias was not confirmed (Shogren and Baik 1991). Schmitt et al. (2004) considers a structure in which a contestant's effort in one period of contest is durable and also contributes to the contestant's probability of winning the contest in a future period of contest. Öncüler and Croson (2005) consider a prize that is awarded to the winner with only some probability. Anderson and Stafford (2003) consider participation with and without entry fees in the lottery game and also the role of players' heterogeneity. Parco, Rapoport and Amaldoss (2004) and Amaldoss and Rapoport (2005) consider two-stage lottery contests as in Amegashie (1999) and Gradstein and Konrad (1999) and find over expenditure. Rapoport and Amaldoss (2003) consider modifications of the all-pay auction, explicitly taking into consideration that the space of possible effort choices from which players choose has a finite grid, that players are resource constrained and that there is a particular tie-breaking rule if the highest effort is made by more than one player. Dechenaux, Kovenock and Lugovskyy (2003a, 2003b) draw attention to the importance of these assumptions for the possibility of multiplicity of equilibria, and draw conclusions about how the results in Rapoport and Amaldoss (2003) could be reconciled with the theory results for players who maximize their monetary payoffs. Barut, Kovenock and Noussair (1999) find that the sealed bid all-pay auction with incomplete information produces higher effort in expectation than what would be the outcome in the Bayesian equilibrium if players maximized their nominal monetary payoffs.

By and large, the perceptions in the literature are that the experimental results can often be reconciled with the theory predictions for players who maximize their monetary payoffs. If they do not, players are biased towards overdissipation. These results on overdissipation, that occur more frequently in the context of the Tullock contest than for the all-pay auction without noise, need to be explained. Potters, deVries and van Winden (1998) suggest that players may randomize uniformly across the whole range of possibly reasonable or feasible bids, and this may explain the overdissipation they found for the lottery contest case. Another explanation that can be considered, suggests that contestants gain additional psychological benefits from their choices and from the outcomes. Lugovskyy, Puzzello and Tucker (2006) mention the "desire to win", and the "desire to punish". A desire to win that adds to the monetary reward that is gained from winning in a laboratory experiment need not be ad hoc, instead it can be founded in the evolutionary game theory on contests.

Leininger (2003) and Hehenkamp, Leininger and Possajennikov (2001) use a concept for finite populations of Schaffer (1988) to consider evolutionarily stable strategies in finite contests. If the evolutionary process selects effort choices as strategies, the evolutionarily stable strategies involve effort levels that exceed the Nash equilibrium efforts.

The key argument in Leininger (2003) can be established as follows. There is a population consisting of n members in each period in a framework with many periods. In each period all members fight in a grand contest about a given amount of resources that is normalized to $v = 1$. Simplifying, suppose that they fight about shares in this prize, and that each player is awarded a share that is equal to his share in the total effort, i.e., $x_i/\Sigma_{j=1}^n x_j$. Accordingly, the payoff of each player i is simply $\pi_i = x_i/\Sigma_{j=1}^n x_j - x_i$. This 'state game' repeats in every period among the set of players that constitute this group in the respective period. The idea of evolution enters into this picture by two assumptions. First, players do not choose their efforts consciously, but there are types of players, and a player's type is essentially given by the amount of effort this player chooses. Naturally, the space of possible types in this case is $[0, \infty)$. Second, the composition of the set of n players as regards their types changes from one period to another, and this change follows a process that could be described as a random process with some drift: loosely speaking, a type that has a higher monetary payoff than another type in period t will be likely to be better represented in the period $t+1$ population than the other type. In the long run the types who have a higher monetary payoffs than their competitors will outgrow their competitors. Given the constant and finite population size, this growth will be at the expense of the number of the type of these other competitors. This already suggests that population composition will be governed by types' payoffs relative to other types' payoffs, and not by their absolute payoffs.

In line with this intuition is Shaffer's (1989) notion of evolutionary stability in such a context, which Leininger (2003) defines formally as: a strategy (or type) $\hat{x}$ can invade a population of type x if the player who expends $\hat{x}$ earns a higher monetary payoff than all the other players in the group who expend x. Moreover, a homogenous population of players consisting of type x^{ESS} cannot be invaded by players of a different type $\hat{x} \neq x^{ESS}$, if given the effort choices $\hat{x}$ by the invader and x^{ESS} by all others, the payoff of such a player expending $\hat{x}$ is not larger than the payoffs of all other group members who expend x^{ESS} , for all possible levels of effort $\hat{x} \geq 0$.

If players simultaneously choose their efforts to consciously maximize their period payoff in each given period t, the Nash equilibrium results, and given the assumed symmetry, it is characterized by $x^* = (n-1)/n^2$. The equilibrium payoff of each player is equal to $(n-1)/n^2$. This effort choice does not constitute an evolutionarily stable strategy. To confirm this suppose all players $i = 2, ...n$ are of type $x^* = (n-1)/n^2$. Consider the question whether a population of players who expend this effort can be invaded. The question can be rephrased as whether a player 1 who is of type $\hat{x}$ could attain a higher payoff than the players $1 = 2, ...n$ who all choose x^*, for some choice $\hat{x} \neq x^*$. Formally, the question is whether an $\hat{x}$ exists such that

$$\Phi(\hat{x}; x^*) \equiv \left[\frac{\hat{x}}{\hat{x} + (n-1)^2/n^2} - \hat{x}\right] - \left[\frac{(n-1)/n^2}{\hat{x} + (n-1)^2/n^2} - \frac{n-1}{n^2}\right] > 0.$$

By symmetry, $\Phi(x^*; x^*) = 0$. Moreover, note that

$$\frac{d\Phi((n-1)/n^2; (n-1)/n^2)}{d\hat{x}} = 0 + \frac{1}{n-1} > 0.$$

This shows that a type with effort slightly higher than the effort $x^* = (n-1)/n^2$ could successfully invade this group. As can be seen analogously, a population in which all players choose $x^{ESS} = 1/n$ cannot be invaded in the same way. This effort level constitutes an equilibrium in evolutionarily stable strategies. Note also that this equilibrium outcome is a sad outcome from the point of view of the group as a whole. It suggests that the evolutionarily stable outcome has efforts chosen that fully dissipate the value of the prize.

Intuitively, fitness in an evolutionary context is a relative concept, and the strategy that is relatively more successful than competing strategies makes the type of player who chooses this strategy outperform other types. Accordingly, in games with a finite number of players, a player gains not only from an increase in his own material payoff but also from a decrease in his co-players' payoffs. If all other players choose the effort that characterizes the Nash equilibrium level in the single period game, a single player's increase in effort compared to this level yields no increase in the player's payoff. By the nature of the problem, such a marginal change only has a second order effect as regards his own payoff. However, for this player this higher effort can still pay off in terms of fitness because it also reduces other players' expected reward. Loosely speaking, this deviation may reduces the player's 'absolute fitness', it may still increase his fitness relative to other members of the population.[25]

[25] A large literature in economics postulates the existence of preferences for relative income, relative performance, or status. Such preferences have far reaching consequences. They change the equilibrium wage profile in organizations (Frank 1984, 1985a), lead to excessive spending on observable goods (Frank 1985b), affect growth, may lead to overaccumulation of wealth and the separation into a class society (see, e.g., Cole et al. 1992, 1998, Konrad 1992) and have many other effects (Weiss and Fershtman 1998). Hirsch (1976) justifies the assumption of such preferences by arguing that some absolutely scarce goods may be allocated according to relative standing. Frank (1985c, p.23-26) also surveys empirical findings on sociophysiological experiments in biology and psychology that reveals connections between status and social interaction and physiological measures.

This is an interesting insight. One may go one step further, taking the complexity of the environment in which human beings interact, and their cognitive abilities into consideration. Evolution need not shape or determine particular effort choices. Evolutionary forces may shape more complex decision rules that take into consideration the particular structure and characteristics of the problem. A type's effort may not simply a scalar number, but a function of the type of contest success function, the prize structure, the number and composition of other players etc. Such types may behave as if they choose their efforts trying to maximize a given objective function, taking into consideration a set of constraints, and evolutionary forces may shape the objective functions which then make humans choose their efforts in various environments. Konrad (1990) argued that during the process of evolution a disposition for enjoying relative rewards, i.e., status preferences, could have developed that is hard wired in the biology of mankind. This is an example of what is called the 'indirect approach' in evolutionary economics. The point has been made more formally in a recent paper by Eaton and Eswaran (2003). They show that a contest environment can induce status preferences in the process of evolutionary selection of preferences.

This point can be illustrated quickly as follows. Consider the following state game. There is a set of n players, called the population, who take part in a symmetric Tullock contest with a contest success function (39) for a prize of size 1. The expected prize net of contest effort by player i is called the *absolute material payoff* of player i and is equal to

$$\pi_i = p_i - x_i \ . \tag{60}$$

The preferences of players may deviate from their material payoff. Let us allow for the class of preferences

$$u_i(\alpha) = \alpha\pi_i + (1-\alpha)[\pi_i - \frac{1}{n-1}\sum_{j\neq i}\pi_j] \tag{61}$$

The case $\alpha = 1$ describes the case in which only absolute performance matters. The case $\alpha = 0$ describes the case of pure status preferences in which only relative performance matters, defined as the difference between own absolute payoff and the average payoff of all other players. The utility function (61) is a parametric version of a more general function in which agents' utility depends on their own and their co-player's level of income, wealth or consumption that is typically assumed in analyses of economic consequences of status briefly discussed in the introduction. This particular parametric version is also frequently used. See Reiter (2000), also for further references.

Let p_i be defined by the Tullock contest success function with $r = 1$. In this case the extreme case of preferences for relative standing with $\alpha = 0$ may evolve evolutionarily in a finite population with size n of players. For defining evolutionarily stable preference strategies, denoted α^{ESS}, in a finite population in the contest, we can follow the reasoning in Schaffer (1988) regarding the equilibrium condition for ESS for $n-$player contests. The question is whether a mutant can prosper in a population in which all agents have the preference that is the candidate ESS. Strategies refer to preference types and players optimize given these preference types.

For this optimization, the perceptions of players with regard to the other players' objective functions is important, as the other players' actual preference types are not observable, that is, the equilibrium concept in the state game is Bayesian equilibrium.[26] The small-number effect will make players' evolutionary success dependent on their *relative* material payoff even if co-players' types are unobservable. The preferences for status ($\alpha < 1$) lead to choices that do not maximize absolute material payoff. Instead, players also take into consideration relative material payoff. This may be surprising, given that such preferences do not generate a strategic effect as preferences are not observable here. An intuition for the result is provided by Schaffer's (1988) observation that the material payoff in contest games with finite populations is not adequately described by the absolute payoff. Relative material payoff matters. In turn, relative material payoff is adequately described by the preferences for status or relative standing, and this may explain why such preferences turn out to be evolutionarily stable.

The result in Eaton and Eswaran (2003) needs to be compared with the outcome of the direct approach that considers the effort choices that develop evolutionarily. The evolutionarily stable actions that are determined by their 'direct' approach differ for games with different numbers of participants. Accordingly, individuals who simply choose a predetermined effort cannot optimally adjust their choices to temporary or cyclical changes in the number n of players. It needs mutations and evolutionary pressure for a population to account for a long-lasting jump in n, or a rather complete programme that makes the genetically determined effort choices a non-trivial function of all the factors that matter in a contest. Unless such a complex rule has developed, such a population does even worse with short run stochastic fluctuation or with deterministic cycles in the number of playsers.

[26]If, instead, types are observable, this leads to a strategic commitment advantage of some not narrowly selfish types of preferences in the sense that a player's preference type induces a behavior of their co-players that depends on the player's actual type. Status preference can then emerge even in infinitely large populations along arguments made by Frank (1987, 1988, 1989) or Bester and Güth (1998).

Also, such a rule is not suitable to cope with unforeseen events. Hence, agents whose efforts are predetermined by the evolutionary process behave suboptimally for environments with very reasonable temporary fluctuations, compared to agents who optimize according to a cleverly chosen objective function. Agents, who derive their actions from genetically determined preferences characterized by status preferences as in (61), behave optimally for contests with all different numbers of contestants. They spontaneously respond optimally to such fluctuations in the contest environment. Agents who choose their effort levels on the basis of preferences that are shaped by evolutionary pressure will therefore typically out perform agent types whose effort choices are shaped by evolutionary pressure directly.

2.5 Some robust results

The analysis of contests so far has revealed a few results that can be seen as fairly robust and important for considering further strategic aspects. First, contests are activities in which one player's increased effort is a negative externality for the other player. As the total win probability is given, if the probability of winning can be increased for one contestant, this necessarily decreases the win probability for someone else. With a given prize, contestants can typically increase their aggregate rents if they succeed in reducing the aggregate equilibrium effort. For the aggregate effort, not only the size of the prize but also the heterogeneity of contestants is important. If the contestants are more heterogeneous, this typically reduces the aggregate equilibrium contest efforts and increases the contestants' aggregate net-payoffs. This is true for all three major types of contests considered in this chapter. Second, the contestant with the higher valuation of the prize is typically more likely to win the prize, and also expends higher effort. This was also true for all three major types of contests. A similar relationship holds for productivity advantages in providing effort. As discussed, e.g., by Baye, Kovenock and deVries (1996), there is a close one-to-one relationship between differences in the valuation of a contest prize and differences in individual cost of producing given effort levels. Accordingly, the contestants with productivity advantages will generally make more effort and win the prize with higher equilibrium probabilities.

3 Timing and participation

3.1 Endogenous timing

The importance of commitment, and the advantage or disadvantage from irreversibly choosing one's actions prior to the actions of another player in an interactive situation, traces back at least to Stackelberg's (1934) analysis of sequential choice of prices or quantities in a duopoly. In the contest framework, the choice is about effort. The question asked by Dixit (1987) is whether there is an advantage or a disadvantage in being able to commit to an effort choice before to other contestants who can observe this choice and react to it. Sequential moves are also analysed by Pèrez-Castrillo and Verdier (1992) and Linster (1993a) and compared with simultaneous moves by Linster (1993a). Morgan (2003) also provides a discussion of the welfare effects of sequential choices, and Glazer and Hassin (2000) considers more than two players. However, unless sequential choices are imposed exogenously, why should sequential choices occur?

In line with some analysis of endogenous sequential choices in oligopoly games by Deneckere and Kovenock (1992), Hamilton and Slutsky (1990), Matsumura (1999) and Mailath (1993), who show that sequential choices of quantities in Cournot competition can be the outcome of non-cooperative play, Deneckere, Kovenock and Lee (1992) addressed this question in a pricing game that is structurally very similar to an all-pay auction. Baik and Shogren (1992) and Leininger (1993) asked whether contestants would be willing to commit on an early or late point of time for making their own choice of teffort, and whether this could lead endogenously to a sequential order of moves in a contest. They consider two dates for an irreversible effort choice: early or late. Will this lead to both contestants moving early, to both contestants moving late, or will some sequencing of choices emerge endogenously? In their framework, the contestants first make a choice of timing, and this choice is irreversible and publicly observed.

The issue can be illustrated using the functional form (39) with an exponent r equal to 1 for the case of two contestants 1 and 2 and valuations of the prize equal to v_1 and v_2 that was used by Leininger (1993). Let $v_1 > v_2$ be an appropriate numbering of the contestants. Consider the graphic approach taken by Baik and Shogren (1992) which applies for a more general contest success function.[27] The reaction functions that result from solving

[27] The case $v_1 = v_2$ is not particularly interesting as locally it does not yield strategic incentives to move early or late, and the Stackelberg and Nash equilibria coincide. This was observed by Dixit (1987).

the first-order conditions are

$$x_1(x_2) = \sqrt{x_2 v_1} - x_2 \tag{62}$$

and

$$x_2(x_1) = \sqrt{x_1 v_2} - x_1, \tag{63}$$

provided that these values are positive, and effort x_i is zero otherwise. These functions are shown in Figure 9. Reaction function $x_1(x_2)$ reaches $x_1 = 0$ for $x_2 = 0$ and for $x_2 = v_1$. It reaches its maximum at $x_2 = v_1/4$ and this maximum is equal to $x_1(v_1/4) = v_1/4$. The reaction function x_2 is described similarly. The two functions intersect at the point (x_1^*, x_2^*) for strictly positive values of effort and this intersection characterizes the Nash equilibrium that is denoted by N in Figure 9.

The set of indifference curves of each player is not drawn into the diagram, except for an indifference curve for player 1 that passes through the point S_1. All indifference curves for player 1 have the property that they are concave and have their peak along the reaction function $x_1(x_2)$. The indifference curve slope for player 1 can be made plausible by observing that the player will generally benefit from a reduction in x_2. Moreover, for a given x_2, the player will increase his payoff by moving towards $x_1(x_2)$. Accordingly, if player 1 moves away from a point on his own reaction function, for instance from S_1 towards higher x_1, this will reduce his payoff unless something pleasant happens that compensates him for the suboptimal choice in x_1. For instance, a reduction in x_2 may compensate player 1 for the reduction in utility from moving away from his own reaction curve. This explains why the indifference curves bend downwards on both sides of the reaction curve starting from a point on the reaction curve.

Consider now the choice of timing. Let there be two points in time at which contestants could irreversibly choose their contest effort. These dates are denoted as e(arly) and l(late). Let contestants first choose simultaneously whether to choose their own contest effort early or late. Once this choice is made, what follows depends on the contestants' decisions which are characterized by the pairs of timing decisions, (e, e), (e, l), (l, e), and (l, l). If both contestants have decided to choose their effort early, they enter into a contest in which their contest efforts are chosen simultaneously, and the unique Nash equilibrium at N emerges. Similarly if both decided to choose their effort late. Hence, for choices (e, e) and (l, l) the Nash equilibrium N and the associated payoffs emerge. These are denoted as π_1^N and π_2^N in this section.

If the contestants have chosen (e, l), then contestant 1 makes his effort choice first. Contestant 2 observes the given choice x_1 and then chooses the

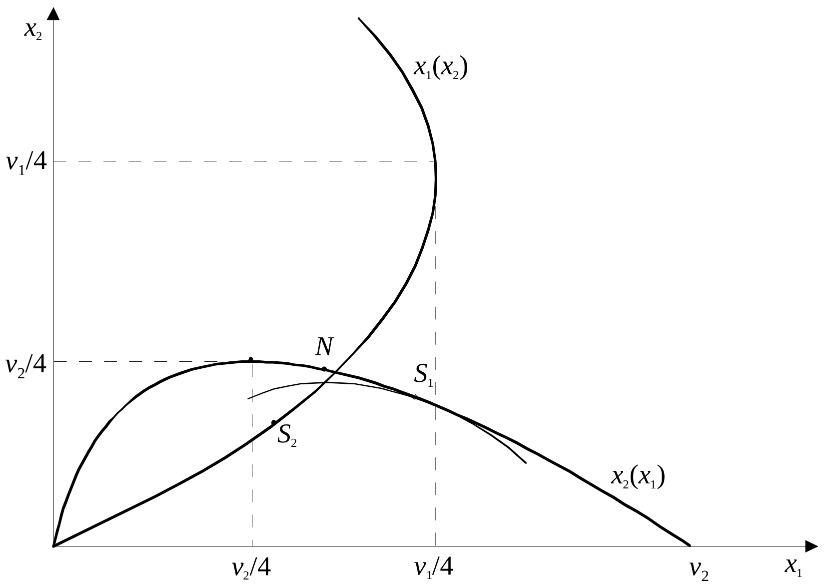

Figure 9: Reaction functions

optimal reply to this effort choice which can be found on contestant 2's reaction function $x_2(x_1)$. Given this logic, when contestant 1 makes his choice, he anticipates that contestant 2 will choose $x_2(x_1)$. Contestant 1 effectively chooses the equilibrium outcome among all points on the reaction curve of contestant 2. For contestant 1's indifference curves as in Figure 9, this equilibrium is reached at S_1 which characterizes the Stackelberg equilibrium if contestant 1 is the Stackelberg leader and contestant 2 is the Stackelberg follower. All that was needed to construct this point was that $x_1(x_2)$ intersects $x_2(x_1)$ to the right of its maximum (which is at $x_2 = v_2/4$) which follows from $v_1 > v_2$. An 'interior solution' with S_1 to the left of $(v_2, 0)$ is not required for the argument to follow. Denote the equilibrium payoffs as $\pi_1^{S_1}$ and $\pi_2^{S_1}$. As S_1 is to the right of N on contestant 2's reaction curve, $\pi_2^N > \pi_2^{S_1}$ must hold. This in turn implies that (e, l) can be ruled out as an equilibrium, as, given that contestant 1 chooses e, contestant 2 could also choose e and induce the Nash equilibrium N.

If the timing is (l, e), contestant 2 chooses the Stackelberg equilibrium S_2 along the reaction function of contestant 1. This point S_2 must be to the lower left of N. Denoting the equilibrium payoffs in S_2 as $\pi_1^{S_2}$ and $\pi_2^{S_2}$, note that $\pi_1^{S_2} > \pi_1^N$ and that $\pi_2^{S_2} > \pi_2^N$.

Turning to the stage where the contestants decide about the timing of moves, it is revealed that it is a dominant strategy for contestant 2 to move early. Given this choice, the contestant 1 has a choice between S_2 and N only, but prefers the Stackelberg equilibrium S_2.

Summarizing, the asymmetry of the two players with respect to their valuations of the prize leads to an endogenous asymmetry in their timing of moves. The player with the low valuation of the prize moves first, whereas the player with the higher valuation waits and observes this contestant's effort and then chooses the effort that maximizes his own payoff. The player with the lower valuation becomes the Stackelberg leader in this game, not by assumption, but by endogenous choice of the timing of moves.

Recall that having the lower valuation of the prize is equivalent to competing for the same given prize, but with a higher cost of making the effort. Accordingly, player 2 is the "weaker" player, or the "underdog", whereas player 1 is the "stonger" player, or the favorite. In this interpretation, the equilibrium outcome suggests that the underdog may find it in his interest to commit first to an effort that is lower than the Nash equilibrium effort, and the favorite may want to wait and observe this lower effort and then reduce his own effort in turn.

This result is interesting, because it does not conform with what is true in most commitment games. This is that it is either an advantage to commit early, in which case both players choose to commit early, or a disadvantage, in which case both players choose to commit late. Here the asymmetry between contestants translates into an asymmetry as regards their preferences for early or late commitment, which in turn reduces aggregate contest effort.

The result is not fully robust and does not hold for all contest types. However, the result does hold for the all-pay auction without noise. Consider, for instance, two contestants with $v_1 > v_2$. If contestant 1 chooses his effort first, the equilibrium has $x_1 = v_2$ and $x_2 = 0$, and the payoffs are $\pi_1 = v_1 - v_2$ and $\pi_2 = 0$, the same payoffs as with simultaneous moves. If contestant 2 chooses his effort first, the equilibrium has $x_2 = 0$ and $x_1 = 0$ and the payoffs are $\pi_1 = v_1$ and $\pi_2 = 0$. When a choice of timing precedes the choices of effort, there are several equilibria. One of these is the payoff-dominant equilibrium in which player 2 commits to choosing early, and player 1 commits to choosing late. As a result, $x_2 = 0$ and $x_1 = 0$ with $\pi_1 = v_1$ and $\pi_2 = 0$ emerges.[28]

For these results, the assumption that the contestants can commit to their timing and the observability of this commitment choice is quintessential. The weaker contestant who decided to make his effort choice early would certainly like to deviate from this choice and would typically like to add

[28] Sequential effort choices in the all-pay auction without noise have been considered, e.g., in Deneckere, Kovenock and Lee (1992) and in Jost und Kräkel (2000).

some effort at the point in time when his opponent chooses effort. As the weaker contestant expends less than his Nash effort at the early point in time, then, if a later increase in effort is feasible, this would essentially bring back the simultaneous Nash equilibrium. Romano and Yildirim (2005) consider more formally a game in which there is a stage following stage l in which both contestants make a simultaneous choice about whether they would like to add to their previously expended effort. They show that, starting from S_1 in Figure 9, or from $x_1 = x_2 = 0$ in the all-pay auction without noise, they would add to their previous efforts and revert to total efforts characterized by the Cournot-Nash equilibrium N. The same argument also applies if, once the players have chosen $x_1 = x_2 = 0$ in previous stages, they were also to revert to the Nash equilibrium in a stage that follows e and l.

3.2 Voluntary Participation

In many contests or tournaments, the choice of whether or not to participate is very limited. If one party decides to impose a conflict on another party, the decision not to fight may have severe and rather unattractive consequences, for instance, for a country leader who faces internal or external opposition.

The participation question is perhaps more interesting if participation is voluntary. If the winner in a contest is awarded a prize that is valued by a player but all losers receive nothing, intuition suggests that a player A may want to expend at least some effort trying to win the prize. However, whether this is a useful strategy will generally depend on how much other players are willing to expend. For instance, if other players value the prize more highly, and, therefore, expend considerable effort, it need not be worthwhile for a player who values the prize less to expend effort. It may be preferable to not compete at all. In the context of the all-pay auction without noise, all players who valued the prize less than the valuation of the player who values the prize second most highly were in such a position and did not expend any effort in the equilibrium. Similarly, in the Tullock contest, condition (49), together with (48), suggests that, if the group of contestants is large and heterogenous, players whose valuation of the prize is low may prefer not to make a positive effort.[29]

However, participation in a contest often has an entry fee. This fee could be explicit, or, in many cases, could consist of the opportunity cost of what a player could have otherwise done. A tennis player who decides to participate in a particular tournament may have to pay some fee, may have some

[29] An early contribution considering entry into Tullock contests is Appelbaum and Katz (1986b). Entry and participation with asymmetric players in the Tullock contest is addressed, e.g., by Stein (2002).

fixed cost of travel and accommodation, and may sacrifice the options of participating in another tournament taking place simultaneously elsewhere, or simply spending the weekend with his or her family. Such entry fees are important, as, in some contests, the expected contest effort that is expended in the equilibrium is close to, or even fully dissipates, the rents of contestants.

Entry fees may then induce some contestants to abstain, or may even lead to an entry game in mixed strategies. Hillman and Samet (1987) solve the all-pay auction without noise for the case of minimum outlays where even perfectly symmetric players have mass points on zero effort levels. Related to this, consider the following two-stage game with n players who all value winning the prize in an all-pay auction by $v_i = v$ and have cost functions of actual contest effort of $C(x_i) = x_i$. Let them decide whether to enter into this contest in stage 1, in which case they have to pay some fixed entry fee D. In stage 2, if none of them have entered, the prize is not awarded. If one of them has entered, the prize is awarded to this player, even if he expends no further effort in stage 2, and if more than one player has paid the entry fee, the standard all-pay auction takes place and in the equilibrium yields zero rents to each participating player. In this case, there are asymmetric equilibria in pure strategies in which one of the players enters and the others do not. Moreover, there is a symmetric mixed strategy equilibrium in which each player enters with a given probability. If all players enter with this probability, this makes the other player just indifferent about whether to enter or not; if a player does not enter his payoff is zero. If he does enter, his expected payoff is $(1 - q^*)^{n-1}v - D$. Accordingly, the entry probability in a symmetric mixed strategy equilibrium is

$$q^* = 1 - (\frac{D}{v})^{\frac{1}{n-1}}. \tag{64}$$

This equilibrium leads to a situation in which the number of potentially active players in stage 2 becomes a random variable.[30]

The equilibrium with randomized entry describes a possible solution to the participation problem in the symmetric all-pay auction. Note that a more extreme solution with possible non-entry applies if players are heterogeneous, as the most advantaged player can expect to receive a rent from participating in any case, and, hence, will always enter, whereas for other participants, entry becomes unattractive. However, both the case in which players randomly enter, and the solution in which only the most advantaged

[30] Unlike in this two-stage game with endogenous uncertainty, Myerson and Wärneryd (2006) analyse a situation in which there is an exogenous uncertainty about the number of active contestants that is not resolved prior to players making their choice of contest effort.

player enters, imply that prizes should frequently be observed where no active players compete for them. Alternatively, there may be cases in which there is only one contestant, particularly in the all-pay auction without noise, or in contest situations with entry fees more generally, in which the equilibrium effort may sum up to a large share of the total prize, making entry of several players unprofitable. The fact that typically we do not observe this type of mixed strategy entry behavior constitutes an entry puzzle.

One reason we typically do not observe sports tournaments in which no players, or only one player, show up is the players' incomplete dissipation and equilibria in which all active players expected to receive some positive rent. However, there is a second explanation: variation in ability over time. This has been discussed by Konrad and Kovenock (2006b). A tennis player's performance on a given day and in a given match may differ considerably from his average performance, and this random element can help in overcoming the entry puzzle. To study this, consider two players, 1, and 2, who, if they both enter, compete in an all-pay auction without noise. Let the valuation of the prize be $v_1 = v_2 = 1$ for both, but let players differ in their cost of effort: $C_1(x_1) = k_1 x_1$ and $C_2(x_2) = k_2 x_2$. For given, and known k_1 and k_2, if $0 < k_1 \leq k_2$, the unique equilibrium of the all-pay auction is described by

$$F_1(x) = \begin{cases} k_2 x & \text{for} \quad x \in (0, \frac{1}{k_2}) \\ 1 & \text{for} \quad x \geq \frac{1}{k_2} \end{cases} \tag{65}$$

$$F_2(x) = \begin{cases} 1 - \frac{k_1}{k_2} + x k_1 & \text{for} \quad x \in (0, \frac{1}{k_2}) \\ 1 & \text{for} \quad x \geq \frac{1}{k_2} \end{cases} \tag{66}$$

in line with Baye, Kovenock and de Vries (1996), and has payoffs $1 - \frac{k_1}{k_2}$ for player 1 and 0 for player 2. For $0 < k_2 \leq k_1$ the subscripts 1 and 2 in (65) and (66) need to be interchanged.

Let us now turn to the stage at which the players have not yet learned their actual productivity in expending effort in the all-pay auction. Let k_i be random variables with finite support $[0, \bar{k}]$. For the nature of the equilibrium, it is not the absolute values of k_1 and k_2, but rather their ratio that is important. Let $\alpha \equiv k_1/k_2$. Then the joint distribution of k_1 and k_2 induces a cumulative distribution function $Z(\alpha)$ on α. Let this cumulative distribution function be continuously differentiable with density function $z(\alpha)$. In this case, player 1's payoff can be stated as

$$\pi_1(Z(\alpha)) = \int_0^1 (1 - \alpha) z(\alpha) d\alpha. \tag{67}$$

The upper limit of the integral is equal to 1, as, for all $\alpha > 1$, the expected payoff of player 1 is zero. Note first, that this expected payoff (67) is non-negative and is strictly positive if the probability by which player 1 has a cost advantage is positive. Accordingly, even if player 1 is disadvantaged in expectation, i.e., if $E(\frac{k_1}{k_2}) > 1$, his expected payoff is positive, and analogously for player 2.

Intuitively, only the player who has an actual cost advantage in the all-pay auction receives a positive rent. Player 1 may be the weaker player on average, however, as abilities follow a stochastic process, player 1 may have stronger positive productivity on the day of the encounter and actually be the stronger player in that particular event. For instance, tennis players who are listed at quite different ranks in the ATP list may play against each other in a final of a tournament, and still the player who is ranked lower may be the stronger on this particular day. This need not be a result of luck or of chance, as is assumed in a contest success function with exogenous noise, but can easily be the outcome of random variations in physical or mental constitutions of the players. When play starts the players may quickly find out about their own and their adversary's constitution on a particular day and then, with these current abilities, they may play the all-pay auction with complete information.

Consider two distributions Z and $\tilde{Z}$ of α, such that $\tilde{Z}$ dominates Z by second order stochastic dominance (with or without a change in the mean). Then $\pi_1(Z) \geq \pi_1(\tilde{Z})$.

For a proof,

$$\begin{aligned} \pi_1(Z) - \pi_1(\hat{Z}) &= \int_0^1 (1-\alpha)(z(\alpha) - \tilde{z}(\alpha))d\alpha \qquad (68) \\ &= \Big[(1-\alpha)(Z(\alpha) - \tilde{Z}(\alpha))\Big]_0^1 + \int_0^1 (Z(\alpha) - \tilde{Z}(\alpha))d\alpha \\ &= \int_0^1 (Z(\alpha) - \tilde{Z}(\alpha))d\alpha \geq 0. \end{aligned}$$

The second line follows from the first by integration by parts, and the last inequality holds by the definition of second-order stochastic dominance.

The result, and its implications for tournaments or contests with multiple rounds with or without elimination of some contestants in early rounds, was derived in Konrad and Kovenock (2006a). If $\tilde{Z}$ dominates Z by second-order stochastic dominance, then the two distributions may (but need not) have the same mean, but, intuitively speaking, Z has greater probability weight in small outcomes of k_1/k_2, i.e., in states in which the advantage of player 1 is large. For instance, if $\tilde{Z}$ is obtained from Z by a mean-preserving spread as defined by Rothschild and Stiglitz (1970), player 1 will still prefer Z, i.e., the player has a preference for randomness in his own relative performance. For instance, if k_1 and k_2 are drawn from stochastically independent distributions, then, if there is a mean preserving spread in own cost k_1, this will cause a mean-preserving spread in α as well, and will generally increase player 1's payoff, and a mean preserving spread in own cost k_2 will of course increase player 2's profit for the same reason.

Where players cannot fully perceive their own strength in the actual contest, even the weaker of two players, who has a lower expected ability than the stronger player, can anticipate that he has some strictly positive expected rent from participating in the all-pay auction. When players make their entry choices, they are willing to pay an entry fee up to the amount of this expected rent.

3.3 Exclusion

An issue closely related to the decision to participate is the problem of admission of contestants. Contest designers selectively admit contestants or design rules that govern admittance. Indeed, this is a relevant issue in many designed contests. Fullerton and McAfee (1999), for instance, analyse admittance rules for an R&D tournament. On the basis of their analysis, they advocate an auction in which two contestants gain access to the actual research tournament. More obvious examples of rules for admission or elimination of contestants can be found in the literature on the economics of sports. Szymanski (2003) discusses the composition of leagues, and rules and mechanisms that have been generated to increase the homogeneity of the contestants within a league with a given, unchanged set of teams. An example is the 'rookie draft' system that is used in American Football that allocates the rights to draft new players from the pool of newly entering players in each season as a function of the teams' comparative performance. He also discusses governance structures such as hierarchies among leagues, where teams, which over- or underperform compared to the competing teams within a given league, are relegated or promoted to the next higher or lower one, respectively.

The contest designer's objective function will generally be essential for the optimal design of admittance rules in contests. Let us assume, as has frequently been done in this context, that the contest organizer cares about aggregate effort. Then the admittance rules can be used to influence key determinants of the contest, such as the players' valuations of the prize, the number of contestants, and the asymmetry of contestants. Some of the results in this context are straightforward and follow from the comparative static results in Section 2. In general, an increase in the contestants' valuations, or a decrease in their cost of effort will increase their observed effort, and there is a tendency for homogeneity among the contestants to increase their efforts, and the rules, for instance, in sports tournaments that improve the competitive balance between teams in open or closed leagues are in line with these results.

An important, counter-intuitive result in this context is found by Baye, Kovenock and deVries (1993). They show that there can be a trade off between participation by contestants with high valuations and contest homogeneity, such that it need not be optimal to admit the contestant who values the contest prize most highly. Their result can be illustrated in the context of the all-pay auction without noise. For this purpose, suppose there are three contestants with valuations $v_1 > v_2 = v_3$, and cost-of-effort functions $C_i(x_i) = x_i$.

If all three contestants are admitted to the contest, there are several equilibria. In one of the two equilibria with the highest expected aggregate effort, contestant 3 expends zero effort and contestants 1 and 2 choose the equililibrium strategies outlined in (4) and (5). It can be verified expected effort made in this equilibrium equals

$$\frac{v_2}{2}(1+\frac{v_2}{v_1}) < v_2. \tag{69}$$

Suppose now that a contest designer who cares about aggregate effort excludes contestant 1 from taking part in the contest. As a result, only contestants 2 and 3 will expend positive effort, and the equilibrium is unique and is described again by (4) and (5) with, however, v_1 replaced by v_2 and v_2 by v_3. Expected aggregate effort becomes equal to

$$\frac{v_2}{2}+\frac{v_2}{2} = v_2. \tag{70}$$

In this example, the expected aggregate effort in the equilibrium increases if the contestant who values the prize most highly is eliminated from the contest. The increase in expected effort is higher, the higher the valuation of this player with the highest valuation. Intuitively, asymmetry between players results in some discouragement effect for the weak contestant, making him choose zero effort with a considerable probability. At the same time, the weak contestant determines the effort chosen by the strong contestant. As a result, total effort is reduced by moving from a situation with $v_1 = v_2 = v$ to one with $v_2 = v$ and $v_1 = v + D$ for $D > 0$.

Of course, it is not always optimal to eliminate the strongest contestant from the set of contestants. If this were true, repeated elimination would eventually reduce the set of contestants to zero. The example is based on a trade off. Effort in an all-pay auction is high if the active bidders have a high valuation of the prize, and if the bidders are symmetric. More precisely, the bidder with the second highest valuation determines the range of possible bids, and the difference in the valuations of the prize between the two bidders with the highest valuation determines the size of the mass point of probability for which the bidder with the lower valuation bids zero. In the example, the elimination of the contestant who values the prize most does not change the range of bids as it does not reduce the second highest valuation of the prize, but it does reduce the heterogeneity between the two contestants with the highest valuation.

More generally, if the contest designer can choose among a set of contestants, who can be sorted according to their valuations of the prize as $v_1 \geq v_2 \geq ... \geq v_n$, from Baye, Kovenock and deVries (1993), the maximum expected aggregate contest effort that can be obtained from a subset $\{z_1, z_2\}$ of these contestants with $v_{z_1} \geq v_{z_2}$ is

$$\frac{v_{z_2}}{2}(1 + \frac{v_{z_2}}{v_{z_1}}). \tag{71}$$

The expected aggregate contest effort is maximized, for instance, for the set of two contestants for which this expression is largest. An optimal set of contestants can be found by computing this value for the sets $\{1, 2\}$, $\{2, 3\}$, ...$\{n - 1, n\}$, and choosing the pair that maximizes this number.

The deeper insight provided by this example is that homogeneity of the contestants can be important for generating high expected effort, and can be more important than the high valuation of the prize. The intuition holds more generally. Recall the equilibrium solution (42) in the asymmetric Tullock contest (42) for valuations $v_1 = v + D$ and $v_2 = v - D$, where it was argued that this is decreasing in D.

Heterogeneity is also an important aspect when a contest organizer has some choice with respect to the contest success function. The example above illustrates that the all-pay auction is particularly sensitive to heterogeneity among the contestants who value the contest prize most highly. The Tullock contest success function, or the all-pay auction with noise, is less sensitive to heterogeneity. This suggests that a contest designer who cannot influence the homogeneity of the contestants may want to choose the contest success function accordingly. Che and Gale (1997) illustrate this in an instructive example. This shows that the Tullock contest can induce higher aggregate effort than the all-pay auction without noise if the contestants are sufficiently heterogeneous. Suppose, for instance, that $v_1 = v + D$ and $v_2 = v - D$ with $D > 0$. The aggregate contest effort in the Tullock contest for $r = 1$ is

$$x_1^* + x_2^* = 2v\frac{(v + D)(v - D)}{4v^2}. \tag{72}$$

The expected aggregate effort in the all-pay auction without noise is

$$E(x_1^* + x_2^*) = \frac{v - D}{2}(1 + \frac{v - D}{v + D}). \tag{73}$$

Accordingly, the Tullock contest induces higher expected aggregate effort if

$$\frac{D}{v} > \sqrt{2} - 1. \tag{74}$$

Amegashie (1999, 2000) considers similar procedures called "shortlisting" in two-stage Tullock contests in which the semi-finals determine the participants in the final. These aspects are also relevant in the analysis by Gradstein and Konrad (1999) who ask how many rounds should be chosen for an elimination tournament with Tullock contests at each elimination stage if a homogeneous group of contestants is to be induced to generate maximum aggregate effort. The trade off between homogeneity and the valuation of the prize, that is the basis of the results in Baye, Kovenock and deVries (1993), does not show up in this work. Intuitively, shortlisting, or the organization of the elimination tournament in several rounds, becomes attractive there because the Tullock contest success function with $r < 1$ exhibits decreasing returns to scale in contest effort, and this makes it attractive to have a sequence of parallel contests with small numbers of participants rather than one big contest.

Note that the organizers' preferences may, but need not, be in line with the contestants' preferences. Generally, where tournaments are used to elicit effort, the organizers' and the contestants' preferences can indeed be aligned if there is sufficient competition between contest organizers. Recall the contest with additive noise in the context of labor market tournaments with perfect competition between contest organizing firms. In this framework, all rents end up in the hands of workers, and the equilibrium design is also the one that would be chosen by the workers themselves. A deviation from this design may reduce contest efforts, but this would also reduce the contest prizes that can be allocated among contestants, and reduce overall rents. Hence, a trade off between organizers and contestants mainly emerges if the prizes that are to be allocated are not endogenously determined by the output created by the contestants' efforts.

4 Cost and prize structure

4.1 Choice of cost

A natural way to influence the outcome of the contest is to influence the cost of effort of the contestants. This is easy if a contest organizer controls this aspect by setting the rules, or by using instruments such as taxes and subsidies that change the cost-of-effort functions for some, or all, contestants.[31] The change of the cost structure that is desirable will generally depend on the point of view. Contestants who want to reduce their overall equilibrium efforts might be interested in the creation of asymmetry, as will be shown in the delegation context below. The contest organizer may find high effort desirable, and, as has been discussed in the context of labor market tournaments, to some extent, this may ultimately also be in the interest of contestants if there is a positive relationship between equilibrium effort and the contest prize.

The contest designer who is interested in maximizing aggregate effort may consider handicapping one or the other contestant. This problem was addressed by Clark and Riis (2000). They consider a bureaucrat or politician who obtains the contestants' efforts for himself, and allocates some prize in an all-pay auction between two contestants who may value the prize differently. The contestants' efforts are given to the bureaucrat as bribes, and for some reason, the bureaucrat allocates the prize to the contestant who expends more effort. Clark and Riis (2000) consider this in an all-pay auction with incomplete information. They show that the bureaucrat may favor the contestant who is more likely to value the prize less, in order to make the contest more symmetric, and that handicapping may increase or decrease the efficiency of the allocation of the prize.

The contest designer's incentive to favor the contestant who attributes a lower value to winning, and the possible efficiency decrease, that may result from equilibrium handicapping, can also be illustrated for the all-pay auction with complete information. Consider the two contestants 1 and 2 with $v_1 > v_2$. The equilibrium outcome of an all-pay auction between these two is in the section characterizing the all-pay auction, and the total expected effort is $\frac{v_2}{2}(1 + \frac{v_2}{v_1})$. Suppose the bureaucrat has no other option than to let the contestants make bids in an all-pay auction, but can handicap one or the other player. For instance, the bureaucrat can allocate the prize to contestant 2 unless $x_1 > (1 + h)x_2$. As long as $v_2 < \frac{v_1}{1+h}$, using this tool will change the total expected effort to $\frac{v_2(1+h)}{2} + \frac{v_2(1+h)}{2}\frac{v_2(1+h)}{v_1}$. Contestant 2 is less likely to expend zero effort. Hence, the handicap will cause an increase in the probability that the prize is allocated to the contestant who values it less, but will increase the bureaucrat's receipts.

[31] Sometimes the change in the cost-of-effort functions in a specific contest is unintended, and is brought about by a policy change which was not motivated by the contest situation. An important example of the latter is profit taxation which changes the opportunity cost of spending resources on lobbying. See, for instance, Glazer and Konrad (1999).

4.2 Multiple prizes

Contests with more than two contestants can have more than one winner prize and one loser prize, and often do for most of the areas of application that were discussed in the first chapter.

Indeed, sports tournaments award more than these two prizes. Most obviously, there are gold, silver and bronze medals at the Olympics. In tennis tournaments, there is a scale of prize money for a number of players at the top. Often the prize structure is fairly complex. In motor sports, the best three drivers are honored and are allowed to pour champagne on each other, but even more of the drivers can earn credits for the world championship. In professional golf tournaments the 'purse' consists of a number of prizes that decline with the rank player achieves rank in the tournament. Recall the prize structure for the tournaments on the PGA tour. Note, also, that these prizes do not fully describe the prize structure, as there are implicit, or non-monetary benefits of performance. Winners of major tournaments have the benefits of qualifying for future tournaments, they get media attention that can be transformed into monetary payoffs via promotion contracts with sponsors who want to advertize their brand products, they improve their score with regard to the contest for best performance in a given year, or lifetime, and most likely they obtain some ego rents.

Similarly, as was discussed in the chapter on examples, in education tournaments, in some entry examinations for the system of higher education, or for admission to some professions, there are explicit or implicit quotas that award a winner prize to all who pass the examination. Hence, such tournaments allocate more than one winner prize.

Political competition also often has a complex prize structure. When two presidential candidates campaign and run against each other they can be seen as competing for one winner prize and one loser prize. But, in party competition with a representative system and coalitions, many outcomes other than victory and defeat are feasible. Moreover, once a party has to form the government, the party members can compete for quite a number of positions with different office rents attached to them, making this process a competition with many prizes.

Moldovanu and Sela (2001) report a number of R&D contests in which there is more than one prize. But even the numerous annual contests in which a recognition prize is awarded (see, e.g., Windham 2000 for a list) can, to some extent, be seen as contests with many prizes. Such prizes are awarded in a sequential order, particularly if it typically takes more than the period between award ceremonies to produce the effort or output which is the basis of the allocation of the respective prize.

These examples motivate questions about how the prize structure influences the contest outcome. These questions turn out to be fairly difficult to address, particularly if the contest prize structure is not caused by the nature of the game, but is deliberately chosen by some contest organizer. One needs to be careful about specifying the objective functions of a contest organizer and of the contestants. Many analyses consider the question from the point of view of an organizer who would like to maximize expected aggregate contest effort. But this is, at best, an approximation to what contest organizers care about in many applications.

How the allocation of several prizes becomes a function of the contestants efforts also has to be decided. Are multiple prizes awarded according to a given generalized contest success function on the basis of the vector of actual efforts, or are prizes allocated in sequential contests? Can a contestant win several prizes, or only one? Can contestants take part in all contests or do they have to make a selection? Can contestants choose, or are they allocated to different contests for different prizes, as is the case in league systems in sports? Are contestants constrained as regards the total effort they can expend on trying to win the various prizes? The choice of the prize structure will also generally interact with the type of contest success function, and the information status too, plays a role. The various dimensions of the multiple-prizes issue quickly generate a large research programme. Existing papers in the literature have addressed some, but not all, of these cases.[32]

Moldovanu and Sela (2001) provide a brief literature survey and give conditions when one or two prizes are optimal, focussing on the convexity properties of the cost-of-effort functions. Gavious, Moldovanu and Sela (2002) also highlight the relevance of convex cost in the context of bid caps in the all-pay auction with incomplete information. Intuitively, with convex cost-of-effort curves, it is useful to induce more than two contestants to participate

[32]For instance, Clark and Riis (1998b) consider the case with several identical prizes and heterogenous contestants who value these prizes differently, and know each others' valuations. They analyse the case in which the prizes are allocated as a function of simultaneously chosen efforts, with the contestants with the highest efforts winning the prizes, and the case in which there are several rounds of this type, where contestants who win a prize at some stage are not allowed to participate in further contest rounds. Barut and Kovenock (1998) characterize the equilibria in a related all-pay auction framework with multiple prizes of different size. Each player chooses one effort, the highest prize is awarded to the player with the highest bid, the second highest prize goes to the second highest bidder etc. Clark and Riis (1996) consider multi-prize contests with a Tullock contest success function. These approaches differ from contests in which the different contests are independent by design, but may be linked through participation decisions (see, e.g., Gradstein and Nitzan, 1989) or overall budget constraints as in Colonel Blotto games and in promotional competition games with fixed overall budgets.

in the contest, as their first units of effort are not very costly to them, but they contribute fully to the aggregate amount of effort. Hence, even if the division of prizes into several prizes reduces the cost of effort that is expended by the contestants who are most interested in winning, the increase in effort made by contestants who attribute a lower value to winning the prize may easily overcompensate this.[33]

Budget constraints will generally lead to results that parallel the results with convex cost, as they limit what the contestants who are most interested in winning can actually expend in effort. With budget constraints, a reduction in the top prize need not reduce what the contestants with the highest valuation expend, if they in any case both expend their whole budget. But the amount by which the prize is reduced can be used to establish a tournament among other less able or less eager contestants, who may expend something on contesting for this additional prize. To illustrate this, consider an all-pay auction with complete information in which four bidders compete to win. Let their cost be linear, i.e., $C_i(x_i) = x_i$, but let them have different budgets, equal to $w_1 = 10$, and $w_2 = w_3 = w_4 = 1$. The contest designer has a total prize money $b = 2$ and considers whether to make them compete for one single prize of this size, or to establish two contests with equal prizes. The situation where there is only one big prize has been considered in the subsection on constraints on effort in the section on the all-pay auction, and one of the equilibria has players 1 and 2 as the only players making positive bids. Using (15) and (16), total expected effort equals $Ex_1 + Ex_2 = 1$. Alternatively, if the contest designer chooses to establish two prizes $b_I = 1$ and $b_{II} = 1$, then one of the equilibria has players 1 and 2 compete for prize b_I and players 3 and 4 compete for prize b_{II}. The competition for each of the prizes is fully symmetric and equivalent to the standard all-pay auction without budget constraints, as the budget constraints are non-binding. The expected efforts sum up to 2. Accordingly, the choice of multiple prizes increases total effort. The example illustrates a more general principle according to which multiple smaller prizes may reduce or eliminate the effort reducing effect of budget constraints.

The general non-optimality of a single large prize, even without cost convexities or budget constraints, can be illustrated with respect to a contest many of the readers of this book are aware of - the Nobel prize. The optimality of one single big prize would imply that, disregarding aspects of risk aversion, the optimal remuneration system for economists would be to make

[33] Kräkel (2004) further explores the implications of cost convexity in a tournament set-up. He shows that splitting of the group of contestants into several subgroups and allocating a prize to each subgroup is beneficial if the total prize money available can be chosen by the tournament organizer.

the Nobel prize somewhat bigger than it is now, and to scrap all minor awards and compensation for other scientific achievements, like wage increases for successful publishing, tenure as a function of publication success, including the remuneration from publication success that stems from peer group pressure. This system would not work as the majority of economists have no positive probability of winning the Nobel prize and, disregarding intrinsic motivation for a second, they would stop publishing.[34] This suggests that a structure of major prizes and minor prizes with contestants who must choose whether to go for one or the other prize may actually increase the aggregate effort. Szymanski and Valletti (2005) discuss this point more formally. Suppose there are 1000 contestants with a large variety of (commonly known) ability, and the only benefit of participating in a contest is one single prize of given size. In this situation most of the contestants will be strongly discouraged and only a small group of top contestants will make a serious effort to win the prize. Accordingly, having several prizes of different sizes may split the set of contestants into subsets of contestants who concentrate on different sized prizes. This makes the contest in each of these sets more homogeneous. But homogeneity of contestants increases aggregate equilibrium effort. Indeed, as is shown in the paper for the case with three contestants, it could increase aggregate effort to allocate two prizes.

An example that is not identical, but is in the spirit of Szymanski and Valletti (2005), is as follows. Suppose there are four contestants with cost of making effort

$$C_1(x) = x_1, C_i(x_i) = 2x_i \text{ for } i = 2, 3, 4. \tag{75}$$

These cost functions are publicly known. A contest organizer would like to allocate total prize money that is normalized to 1 either as one big winner prize, or split into two prizes b_I and b_{II} with $b_I + b_{II} = 1$. If there are two prizes, contestants have to decide whether to make bids for one or the other prize. Finally, let the contest success function be characterized by (3).

If there is one big prize only, the effort maximizing equilibria are the ones in which only contestant 1 and one further contestant make positive bids with positive probability. Let this contestant be $i = 2$. He would never bid more than $x_2 = 1/2$, implying that contestant 1 randomizes his choice of x_1 uniformly on $(0, 1/2]$, and contestant 2 chooses zero effort with probability 1/2 and, with the remaining probability, 2 randomizes x_2 uniformly on $[0, 1/2]$. This yields expected aggregate effort

$$E(x_1 + x_2) = 3/8 \tag{76}$$

[34] This consideration was brought to my attention by Kjell Erik Lommerud many years ago.

and $x_3 = x_4 = 0$ in the equilibrium.

If there are two prizes, for instance, $b_I = b_{II} = 1/2$, then one of the effort maximizing equilibria is characterized by contestants 1 and 2 competing for prize b_I and contestants 3 and 4 competing for prize b_{II}. The equilibrium effort in the contest for b_I is equal to $E(x_1 + x_2) = \frac{3}{16}$, which is half the size of (76), and the equilibrium effort in the contest for b_{II} is equal to $E(x_3 + x_4) = \frac{1}{4}$. Summarizing these expected efforts yields

$$\frac{7}{16} > \frac{3}{8}. \tag{77}$$

The underlying idea in Szymanski and Valletti is related to the exlusion argument in Baye, Kovenock and deVries (1993). Making the homogeneous contestants 3 and 4 compete against each other for a prize that is equal to 1/2 generates even more expected effort than making contestants 1 and 2 compete for a prize of this size, as they are more homogeneous than 1 and 2, and as the valuations of the prize by the contestant who has the lower valuation among the two is the same in both contests.

Intuitively, it may be worthwhile to break up a given prize into several prizes and to move from a large contest with heterogeneous contestants to many smaller contests with more homogeneous contestants in each of these smaller contests, as this will activate some of the weaker contestants who would stay passive in the case of a big contest for just one winner prize. Note also that this argument is unrelated to, but adds to, the results on cost convexity or to budget constraints.

Sometimes contestants will take part in many contests and the outcomes in the different contests interact. A particular prize may be valuable only in combination with some other prizes. Problems of this type are important and have numerous applications. They will be treated in a separate chapter.

4.3 Endogenous prizes

A major determinant in contests is the contest prize that is awarded to the contest winner. In artificially created contests the prize is chosen by the contest organizer. The organizers of sports tournaments choose the money prize(s) and the prize structure. In research tournaments the winner prize is a matter of choice.[35] In R&D races the government can influence the size of prizes by the choice of patent laws and patent regulation more generally. But also in contests that take place in environments that have not been carefully

[35] See, for instance, Windham (1999) collected a considerable list of monetary prizes for major technological achievements.

designed for the purpose of this type of competition, contest prizes are often endogenous. Moreover, it is often the contestants who can influence the size of the prize. In this section I will consider cases in which the contestants themselves influence the size of the prize, and have two separate instruments, one of which influences the size of the prize, and another that influences their win probability. Then I will turn to cases in which one instrument influences both the win probability of a contestant and the size of the prize, and will distinguish between two situations. One situation depicts promotional competition where contest effort increases the prize. The other situation depicts the opposite case where an increase in contest effort implies a reduction in the prize at stake in the contest.

Prize moderation Contestants who can choose both the winner prize and the loser prize prior to entering into a contest, would like to move these closer together, if this does not reduce the sum of the two. Doing this will generally reduce the contestants' effort without reducing what is paid out in terms of total prizes. A more surprising result is derived by Leidy (1994) and Epstein and Nitzan (2003a, 2003b, 2004). In a more general format that allows both sides to choose their prize, Epstein and Nitzan (2004) consider two contestants who fight for different policies, like, for instance, free trade vs. a tariff with effective protection and show that both contestants may be willing to moderate their policy goals. To illustrate the point, consider two contestants who have a conflict of interest: they care about the value of a variable z that is taken from the real numbers. Their bliss points are Z_1 and Z_2, with $Z_1 > Z_2$, and their utility from an actual policy choice z equals

$$b_i(z) = -(Z_i - z)^2. \tag{78}$$

Each contestant has to choose a policy z_i which he then has to fight for. Once these values z_1, and z_2 are given and observed, the contestants expend effort x_1 and x_2 in a Tullock contest. Either z_1 or z_2 is chosen, and the probabilities for these outcomes are $\frac{x_1}{x_1+x_2}$ and $\frac{x_2}{x_1+x_2}$, respectively. Contestant 1 values winning at the difference between $b_1(z_1) - b_1(z_2)$, which is $(Z_1 - z_2)^2 - (Z_1 - z_1)^2 \equiv v_1(z_1, z_2)$, and similarly for $v_2(z_2, z_1) = (Z_2 - z_1)^2 - (Z_2 - z_2)^2$. The contest equilibrium for given prizes v_1 and v_2 is characterized in (44) with $r = 1$ and yields payoffs as a function of valuations v_1 and v_2 as

$$\pi_1(v_1, v_2) = \frac{(v_1)^3}{(v_1 + v_2)^2} \text{ and } \pi_2(v_1, v_2) = \frac{(v_2)^3}{(v_1 + v_2)^2}. \tag{79}$$

Calculating first-order conditions $\frac{d\pi_1}{dz_1} = 0$ and $\frac{d\pi_2}{dz_2} = 0$ yields

$$\frac{\frac{\partial v_1}{\partial z_1}}{\frac{\partial v_2}{\partial z_1}} = \frac{2v_1}{v_1 + 3v_2} \text{ and } \frac{\frac{\partial v_2}{\partial z_2}}{\frac{\partial v_1}{\partial z_2}} = \frac{2v_2}{v_2 + 3v_1}. \tag{80}$$

Using the parametric versions of v_1 and v_2, the left-hand side of the first-order condition for 1 becomes $-\frac{2(Z_1 - z_1)}{2(Z_2 - z_1)}$. Accordingly, $Z_1 > z_1 > Z_2$. Both contestants choose values of z_i that are somewhere between their bliss points, but closer to their own bliss point than the value chosen by their competitor.

The intuition for this moderation result is as follows. If contestant 1 departs from choosing his bliss point and chooses some z_1 slightly different from Z_1 in the direction towards Z_2, what he sacrifices if he wins compared to receiving Z_1 is of second order, as $b_1'(z_1) = 0$ at $z_1 = Z_1$. However, this departure has a positive effect that is of first-order magnitude. The move towards Z_2 reduces contestant 2's loss from losing the contest, and hence his stake in the contest by $b_2'(Z_1)$, which is strictly positive. Player 2 will then use less effort in the contest. Even if contestant 1 did not adapt his own effort to this lower effort of his competitor, contestant 1 would have a first-order gain.

The insight on moderation is important in many areas of conflict. Contestants may strengthen their position in a contest in terms of the stock of arms they accumulate, or by making choices and investment that increases their own ability to fight, but they have some tendency to moderate their own demands. In negotiations and bilateral bargaining with incomplete information a buyer may try to hide his true willingness to pay and a seller may try to overstate his reservation prize. Hence, both ask for more than what they actually need in order to be compensated. In the contest, in the absence of incomplete information, the reverse might be true. There is a tendency to narrow the gap between the two conflicting parties' demands.

A related trade-off is studied in Konrad (2002). This paper considers an incumbent who can choose how much to invest in a project if the project returns must be defended in a future contest between this incumbent and a challenger. High investment causes high future cost of defense. If the incumbent wants to avoid this, he needs to moderate his investment behavior. Unlike in a situation in which the investor receives the returns on his investment only with some exogenously given probability smaller than one, as in simple expropriation problems with exogenous expropriation probabilities, the incentive to moderate investment here is much stronger. The investor knows he can avoid expropriation of the future returns by making defense expenditures in the future. The higher the investment (and its returns) the higher is the challenger's appropriation effort, and the more defense effort is needed to fend him off. Future defense cost can be seen as being part of the investment cost, and has to be taken into consideration by the investor when he makes his investment effort.

Concordance between production and appropriation In many instances the contestants themselves influence the value of winning the prize. For instance, the analysis by Baye and Hoppe (2003) above shows that the research activity influences both the probability of winning the contest and the expected value of the prize of winning the R&D contest. A similar example emerges in the context of promotional competition and has been recognized there. Promotional effort of a firm typically changes the size of the firm's market share, but also increases the total size of the market. Promotional effort has the usual negative contest externality as a contestant steals some market share from other contestants when increasing his promotional effort, but by increasing the market size, it at the same time benefits all other competitors, and this is a positive externality. The marketing literature tries to distinguish between these two effects, and contest theory can be of help in deriving some hypotheses that are empirically testable.[36] Generally, if the prize is an increasing function in the efforts expended, the individual effort contributes to a public good (the prize), but it also has a negative externality, as it reduces the win probabilities of other contestants. Consider, for instance, a symmetric Tullock game among n players with payoffs

$$\pi_i(x_1, ..., x_n) = \frac{x_i^r}{\sum_{j=1}^n x_j^r} v_i(\sum_{j=1}^n x_j) - x_i. \tag{81}$$

that is analysed by Chung (1996). For $r = 1$ and $v' > 0$ and $v'' < 0$, imposing further regularity conditions on the problem that guarantee a unique equilibrium, he finds that the efforts chosen in the equilibrium are higher than the efforts that maximize the sum of the contestants' payoffs. He also discusses the fact that this outcome is not robust to parameter changes, as becomes clear from considering $r = 0$. For this case, the problem turns into the standard problem of private provision of a public good as in Bergstrom, Blume and Varian (1986) and is typically characterized by too small contributions x_i.[37]

[36] Huck, Konrad and Müller (2002) and Barros and Sørgard (2000), for instance, consider the profitability of merger between two firms in a market that is characterized by promotional competition.

[37] Morgan (2000) considers another case in which contest effort has a positive side effect. He studies a lottery contest in which some of the revenue from the purchases of lottery tickets are used to award a monetary prize, and some of the revenue is used to provide

In these examples, an increase in contest effort typically increases the prize, and this gives a productive element to the contest activities. The reverse may also be true in other applications in which increased effort reduces the value of the prize.

Conflict between production and appropriation An environment in which there is a trade off between production and appropriation is studied by Skaperdas (1992). He considers individual decision makers who are endowed with a given budget, e.g., an amount of time that is available and used either for productive purposes or for appropriation, which means either trying to take from others, or trying to prevent others from taking, or a mixture of the two. Let there be two individuals 1 and 2. Each of them can use a share in his endowment for producing valuable consumer goods and the remaining share for military goods. The total output of consumer goods is consumed by the two decision makers, but their consumption shares in total output are determined as a function of what they allocate to appropriation activities, for instance, to arms. Note that an increase in own appropriation expenditure will generally affect (and typically reduce) the total prize, as fewer resources can be used for productive purposes. Each contestant will face the trade off between generating a larger total output and receiving a larger share in the total output. However, the strategic situation is a bit more complicated. If a contestant decides to produce a further unit of output, this will weaken his military capacity and will also make it more attractive for the other contestant to win the output.

To consider one aspect of the problem in Skaperdas (1992) more closely, one that is a variant of what is known as the 'paradox of power' (Hirshleifer 1991), let each individual be endowed with one unit of resource. Let $y_i \in [0, 1]$ be used for producing valuable consumer goods and let $x_i = 1 - y_i$ be used for appropriation purposes. Write the total output of valuable consumer goods as $y = ay_1 + y_2$. Hence, the production amounts add to total production, and each individual has constant returns in this activity. Consider the two cases $a \in \{0, 1\}$. One case, $a = 0$, refers to the case in which contestant 1 has no specific ability to produce output, but can use his time as effectively as contestant 2 in the contest. For $a = 1$ the situation is perfectly symmetric. For ease of comparison, a linear contest success function will be used that is not very plausible, but is simple, such that $p_1(x_1, x_2) = \frac{1}{2} + \frac{x_1 - x_2}{2}$.

a public good can induce higher net contributions to the public good than in a pure private provision equilibrium as in Bergstrom, Blume and Varian (1986). Individuals who purchase lottery tickets do not directly increase the prize in the lottery as in Chung, but they contribute to a public good, and higher lottery purchases ameliorate the provision outcome.

Consider first the symmetric case. Contestant 1 maximizes

$$\pi_1(y_1, y_2) = (\frac{1}{2} + \frac{x_1 - x_2}{2})(y_1 + y_2), \tag{82}$$

and similarly for $i = 2$. Maximization of this objective function yields first-order conditions that are just fulfilled at $y_1 = y_2 = \frac{1}{2}$, i.e., for the case in which both contestants expend half of their resources on productive effort. It is interesting to compare this result with the outcome for the case $a = 0$. In this case, contestant 1 will not expend any effort on productive purposes, but will expend all effort on the contest. Contestant 2 will react to this and, hence, maximizes $\frac{x_2}{2}(1 - x_2)$, which, again, yields $y_2 = 1/2$ as the optimal choice. As a result, contestant 1 will earn a payoff equal to 3/8, and contestant 2 will earn a payoff equal to 1/8. Even though the contestant 1 has an absolute disadvantage in producing consumption goods, the comparative advantage in fighting is sufficiently strong to turn this disadvantage into an advantage. Individual 2 is more efficient in translating input into consumable output and would be much richer in the absence of conflict. In the absence of property rights, this individual expends a large share of his resources on producing consumer goods, and, hence, less on military goods. In fact, in the parametric example, $x_2 = 1/2$ and $x_1 = 1$. The individual's share in aggregate output is therefore smaller than that of individual 1: $p_2 < p_1$. The more productive individual can end up with lower consumption.

The implications of the individuals' conflict between production and defense or appropriation have been explored in a number of directions, most notably by Skaperdas and several co-workers. Some of the results are just the inverse of the standard economic results that are obtained with exogenously assumed well defined property rights. For instance, Anbarci, Skaperdas and Syropoulos (2002) consider the implications of future negotiations and the role of alternative cooperative solution concepts for investment in military capacity in earlier stages. Skaperdas and Syropoulos (2001) consider trade between agents who enter into a period of conflict later. They show that trade based on comparative advantages is not necessarily a Pareto improvement. Intuitively, relative power matters for the outcome of the contest that determines the allocation of total output. But, while trade may enhance total output, it may shift the balance of power, and, for the agent who loses power, this shift may overcompensate the benefits of the increase in output.[38] Also in contrast to the more standard intuition in economics is the result that increased competition may decrease welfare. Skaperdas (2001)

[38] Conflict is another instance in which relative position regarding some goods determines the allocation of absolutely scarce goods. Seen from this perspective, the result is reminiscent of relative standing comparisons in the literature on trade. If the country's

shows that, unlike the type of competition that brings asking prices and willingness to pay closer together, competition for the provision of protection, and, in particular, for territory or zones of influence that takes the form of a wasteful contest, mainly dissipates resources, and an increase in competition typically increases dissipation. Intuitively, this is a result of the increase in contest effort that typically goes along with an increase in competition. Further surprising results can be found in Skaperdas (2003), and Garfinkel and Skaperdas (2006) give a most recent overview over this most dynamic area of research.

5 Delegation

Delegation is known to be an important strategic option, and this is also true in the context of tournaments or contests. As discussed by Schelling (1960), delegation opens up the option to commit to future actions that are not time consistent from the perspective of the decision maker, and this may yield a strategic advantage to him. Whether such a commitment advantage also exists in the context of tournaments or auctions has been explored, e.g., by Baik and Kim (1997), Baik (2006), Wärneryd (2000) and Kräkel and Sliwka (2002) who consider delegation in the context of a Tullock contest, by Brandauer and Englmaier (2005) who consider median voter decision making on the choice of the delegated agent, by Wärneryd (2000), who adds a hidden action problem in the relationship between the principal and her delegate, and by Konrad, Peters and Wärneryd (2004), who consider delegation for the first-price all-pay auction without noise. Some of these issues are now considered, one after another.

Delegation contracts Suppose there are two individuals who care about winning a prize that is allocated in a contest. The individuals will be called principals 1 and 2. The principal i who eventually receives the prize values it by v_i with $i = 1, 2$. Principals 1 and 2 may delegate the actual bidding to the two agents, A_1 and A_2. For this purpose they write a contract with their agent. The contract can be denoted as (φ_i, b_i) and is described by the following arrangements. First, the contract transfers the right to make bids x_i from principal i to his agent A_i. This agent has a cost of making this

relative standing enters into the country's welfare function, trade that improves all countries' consumption level may still reduce the welfare of those countries who gain only little, and lose in terms of relative standing. The analysis of Skaperdas and Syropoulos (2001) can therefore also be seen as a new microeconomic underpinning for a concern for relative standing.

bid, and, for simplicity, let $C_i(x_i) = x_i$. According to the contract, only the agent is allowed to make bids. Principal i abstains from bidding. Second, if agent A_i wins the prize, the agent delivers the prize to his principal and receives a pre-specified payment b_i that cannot be re-negotiated. This price is the agent's valuation of the prize, and will therefore be called 'delegated valuation'. Third, for the contract to be valid, the agent pays an amount φ_i to the principal. This amount is transfered up front, before the actual bidding takes place and will be called 'down payment'.

Two further assumptions are made. For simplicity, the agent's reservation utility of whether to sign this contract or not is zero, implying that agent A_i is willing to sign the contract if it (φ_i, b_i) yields an equilibrium payoff that is equal to or larger than zero. Generally, φ_i and b_i can be chosen from the non-negative real numbers, but we will require that there is a maximum delegated valuation that can be agreed on between a principal and his agent, i.e., $b_i \leq \bar{b}$ for some arbitrarily large, but finite number $\bar{b}$.

Denoting $E(x_i^*)$ as the expected effort expended by agent i and p_i^* the agent's win probability in the equilibrium with delegation, the participation constraint requires

$$\varphi_i \leq b_i p_i^* - E(x_i^*). \tag{83}$$

As the choice of φ_i does not affect the actual all-pay auction once the delegation contracts are signed, the principals will offer contracts with the highest feasible up front payments. The participation constraints (83) will therefore be binding. Accordingly, principal i's payoff can be written as

$$\pi_i = v_i p_i^* - E(x_i^*), \tag{84}$$

where the equilibrium values p_i^* and x_i^* will generally depend on b_1 and b_2.[39]

Delegation in all-pay auctions Considering standard auctions with complete information, it is typically not in the bidders' interest to send someone else to the auction to make bids if this other person attributes a different valuation to the object that is auctioned. However, for the all-pay auction, this is not true, as is shown in Konrad, Peters and Wärneryd (2004). Instead, principals will choose to sign contracts with delegates and send them

[39] The delegation contract is not renegotiation proof at the stage between the publicly observed writing the delegation contract and the stage at which the delegated agent and other players choose their actual contest effort. This is a problem with delegation more generally, and is not specific to this delegation problem. Delegation contracts typically have the property that, once they are written and observed by the other players, the principal and the agent in this contract have an incentive to secretly renegotiate this contract, and the results on delegation are based on the implicit assumption that such secret re-negotiations are not feasible.

to the auction with delegated values that differ from the principals' valuation. The result relies on an asymmetry between the valuations of the delegated bidders that emerges endogenously. To study this endogenous asymmetry, it makes sense to start with a framework that is fully symmetric ex ante: two individuals who have identical valuations of winning the prize, equal to $v_1 = v_2 \equiv v$.

If these individuals make bids in a first-price all-pay auction, they will make bids according to (4) and (5) as in the standard all-pay auction. As they value the prize equally, their expected efforts and their expected benefits from winning just equalize. Their payoffs from participating in this contest if they participate and choose effort according to their own objective functions are equal to zero.

If both individuals delegate bidding to their respective agents, exactly two equilibria with pure strategy choices of contract offers exist if $\bar{b} > v/2$. The delegated valuations in these contracts are

$$(b_1^*, b_2^*) = (v/2, \bar{b}) \text{ and } (b_1^*, b_2^*) = (\bar{b}, v/2). \tag{85}$$

If, instead, $\bar{b} \leq v/2$, both buyers choose $b_1^* = b_2^* = \bar{b}$ in the equilibrium. Hence, if the maximum delegated valuation is very small, the solution is a corner solution in which both principals choose the highest feasible delegated value. Each principal would prefer not to delegate the choice of effort to an agent, he would prefer to choose the effort directly and not be subject to the agent's effort constraint. This is true, whether the other buyer delegates the effort choice or not. If the constraint on the maximum delegated valuation is weaker, two equilibria in pure strategies regarding the delegation contracts emerge in which one of the principals chooses a delegated valuation that is lower than his true valuation of the good and the other principal chooses the maximum feasible delegated valuation.

A proof is in Konrad, Peters and Wärneryd (2004) and only a heuristic argument will be given to make the outcome intuitive for the case with $\bar{b} > v/2$, showing that $v/2$ and $\bar{b}$ are optimal replies to each other. For this, suppose that principal 2 and his agent choose a delegated valuation $b_2^* = v/2$ and this is anticipated by buyer 1 and his agent. What would be the optimal contract from the perspective of principal 1 in this case? His agent 1 will bid against agent 2. Using the fact that (83) is binding, principal 1's payoff is given as $p_1^* v - E(x_1^*)$. These equilibrium values depend on the delegated valuations b_1 and b_2 that take the role of the contestants' valuations as in a standard all-pay auction. If $b_1 < b_2 = v/2$, this leads to $p_1^* b_1 = E(x_1^*) = (b_1)^2/v$. Accordingly, principal 1's payoff for this range of delegated valuations equals $p_1^*(v - b_1)$. Using $p_1^* = b_1/(2b_2) = b_1/v$ in this range, the payoff of principal 1 is strictly increasing in b_1 for $b_1 \leq b_2 = v/2$. For $b_1 > b_2 = v/2$, principal 1's payoff is simply $p_1^* v - E(x_i^*)$. As p_1^* is strictly increasing in b_1 and $E(x_i^*) = v/4$ for $b_1 > b_2 = v/2$, the optimal reply of player 1 to $b_2 = v/2$ is to choose b_1 as large as possible. Given that principal 2 chooses a delegated valuation that is lower than principal 1's true valuation, principal 1 makes his own delegate very aggressive by giving him a delegated valuation that is as high as possible. Principal 1 does not have to worry about the high b_1 that he will have to pay to his agent, because this is compensated for in the high up front fee φ_1 that A_1 will pay to his principal.

Turn now to player 2. It seems intuitively plausible that each principal makes his agent more aggressive in the bidding by giving him a high valuation, as this increases his agent's win probability. Note, however, that this strategy works only if the other principal does not choose a high delegation value as well. If both agents have high delegated valuations, e.g., if $b_1 = b_2 = b > v$, then $p_i^* b_i = E(x_i^*)$ and $\varphi_i^* = 0$. Accordingly, $\pi_i = -(b-v)/2$ in this case and both principals incur losses. This shows that both principals would be better off not delegating their bidding than in a symmetric delegation equilibrium in which both choose high delegated values. However, this still does not explain why $b_2 = v/2$ is indeed an optimal reply to $b_1 = \bar{b}$. To see why, suppose $b_1 = \bar{b}$ and consider principal 2's optimal delegated valuation. His agent will choose $E(x_2^*) = b_2 p_2^*$; hence $\varphi_2^* = 0$ and $\pi_2 = (v - b_2)\frac{b_2}{2(v/2)}$. This payoff is negative for $b_2 > v$ and positive for $b_2 < v$, and reaches its maximum for $b_2 = v/2$. Given that principal 1 has chosen a very high delegated valuation, the optimal strategy of principal 2 is to choose a fairly low delegated valuation. His agent will expend few resources on bidding in expectation, but will still win the prize with some positive probability.

The two principals earn a positive payoff, but they earn these with different margins. Principal 1 makes his agent very aggressive by giving him a high valuation of bidding. This is not very costly, as the expected bid of this agent is not determined by his own delegated valuation but instead by the opponent agent's delegated valuation. An increase in principal 1's valuation does not increase his agent's equilibrium bids, but it will deter principal 2's agent from bidding, and this benefits principal 1. In this situation, it is hopeless for principal 2 to make his agent an aggressive bidder as well, as this would lead to losses. Instead, principal 2 makes his agent a very reluctant bidder - a bidder who expends very little and still wins with some positive probability.

The outcome generalizes if the two bidders face different constraints on the maximum delegated valuation, as long as both maxima are sufficiently large. Moderate differences in the principals' true valuations of the prize also do not change the outcome qualitatively. The symmetry of the true valuations and the constraints was chosen for simplicity and because asymmetric pure strategy equilibria are more surprising as outcomes if all players are symmetric ex ante.

The mechanism that is responsible for the asymmetric delegation equilibrium also underlies a result in strategic trade policy that was shown in Konrad (2000b). He considers two firms that are located in two different countries and compete for business in a third country export market, much like in the standard framework of strategic trade policy by Brander and Spencer (1985). The main difference is that export competition is not via prices or quantities. Sales competition is organized as an all-pay auction where the two firms make simultaneous money payments (bribes) to the consumer or to the person who makes the buying decision and the decision maker awards one major contract to the firm that pays the higher bribe. The two governments in which these firms are located may change their firms' bribing incentives by trade taxes and trade subsidies. In the equilibrium, one government, say government 1, will subsidize its firm to the maximum amount feasible and endow its firm with a high valuation of winning the bribing contest. The other government 2 will optimally moderate the firm 2's incentives to bid by imposing a trade tax on this firm that has to be paid if the firm is awarded the contract. This reduces the firm's benefit from winning the export contract.

As a result, the contest will become highly asymmetric, and the total expected equilibrium bribing effort will be much lower than the value of being awarded the contract. Moreover, both countries gain from this type of strategic trade policy. The strategic trade policy alters the firms' bribing incentives in a way that is very similar to the effects of a delegation contract and puts the firms in the roles of delegated agents who bribe on behalf of their benevolent country governments.

Delegation in Tullock contests The equilibrium asymmetries that emerge in the delegation equilibria are not specific only to the all-pay auction without noise. They can also emerge in asymmetric Tullock contests as is shown in Konrad (2000b). But, typically, delegation leads to different results in Tullock contests. The strategic aspects of delegation in the Tullock contest can be studied using reaction curve diagrams. Figure 8 depicts the "true" or undelegated reaction curves $x_1(x_2)$ and $x_2(x_1)$ in a fully symmetric two-player Tullock contest that are formally derived as (41). They intersect in the symmetric Nash equilibrium N. Some representatives of the indifference curves of the two contestants 1 and 2 are also drawn in this diagram. For instance, $\pi_i(N)$ is the set of pairs of effort (x_1, x_2) that yield the payoff level $\pi_i(x_1^N, x_2^N)$ received by contestant i in the Nash equilibrium. The shape of these indifference curves has already been explained more carefully.

Suppose, now, that contestant 1 chooses to delegate the choice of x_1 to an agent who is motivated by objectives that can be completely specified in a contract and perfectly monitored as in the above framework. If the set of feasible delegation contracts is sufficiently rich, and if the agent's opportunity cost of acting as a delegate is zero, the agent will receive a net payment just equal to his equilibrium choice of effort in the contest, and the choice of the delegation contract can be understood as changing the own reaction curve to some function $x_1^D(x_2)$, without, however, changing the payoff levels associated with given pairs (x_1, x_2) of effort.

As long as contestant 2 does not delegate the effort decision to an agent, this problem resembles the problem discussed previously. Contestant 1's problem reduces to choosing his most favored point along contestant 2's reaction function $x_2(x_1)$. This point is denoted S_1 in the figure. Contestant 1 then designs a contract that will cause S_1 as the equilibrium outcome and the principal will have a payoff equal to $\pi_1(S_1)$.

If contestant 2 can also delegate his effort choice, the problem increases in complexity, particularly if the delegated reaction functions can be chosen as arbitrary functions in $\Re^+ \times \Re^+$. Wärneryd (2000) shows that the undelegated Nash equilibrium N leads to effort choices and win probabilities that also emerge as equilibrium values in a subgame perfect equilibrium in the contest with delegation. To see this, suppose that contestant 2 chooses a reaction function for his agent

$$x_2^D(x_1) \equiv x_2^N. \tag{86}$$

In this case contestant 1 can choose the equilibrium along this reaction function and prefers (x_1^N, x_2^N) among all these points. Contestant 1 can establish this outcome by a delegation contract that yields a reaction function for his agent

$$x_1^D(x_2) \equiv x_1^N. \tag{87}$$

No other delegated reaction function could yield a higher payoff to contestant 1. Of course, the same argument applies to contestant 2, given that contestant 1 chooses the delegated reaction fuction (87). Hence, delegation contracts leading to delegated reaction functions (87) and (86) constitute optimal replies to each other in the delegation stage of the game. One should also note that this is a knife edge result that disappears once the contestants are asymmetric.

Delegation and monitoring problems Wärneryd (2000) also discusses what happens if principals cannot observe the effort that is chosen by their delegated agents. The example he has in mind is litigation, where the conflicting parties usually have to delegate to lawyers the task of providing favorable evidence and of presenting it to the court. The delegation contracts between the principal and his lawyer that are used, or admitted, differ between countries, but they all have in common the fact that the actual effort of a respective lawyer can hardly be observed by the principal. This moral hazard problem leads to an equilibrium effort choice of the lawyer that is too small compared to what would maximize their joint surplus. Hence, nobody would hire a lawyer if he felt equally capable (or at least reasonably capable) of collecting evidence himself and presenting it appropriately to the court. However, in most court systems, delegation of court representation is mandatory, and, moreover, lawyers do have comparative advantages in what they do, making both sided delegation inevitable. The effort reduction that is achieved if both conflicting parties have to delegate the effort choice to their lawyers may enhance efficiency of litigation by reducing the aggregate effort that is used in litigation.

The result should be considered with some caution. If an increase in litigation effort is not simply wasteful but improves the quality of the signal to the court about what is the right or wrong decision, a trade off emerges. A reduction in effort due to delegation will then reduce the precision by which the litigation system enforces contracts or property rights. This, in turn, will generally reduce the performance of business life.

6 Externalities

Tournaments, contests and wars are games in which the effort of one player typically imposes a strong externality on all other players. These mutual negative externalities could even be seen as the constituent element of such games. However, sometimes the nature of externalities in such games is more complex and more asymmetric, and in this chapter I will turn to some examples.

6.1 Joint ownership

Unlike the designers of an all-pay auction who may be interested in the expenditure of a large amount of effort, the participants in a contest for given prizes are generally interested in a reduction in effort. They may therefore like an in increase in asymmetry. A context in which this may become feasible is that where firms with cross holdings of shares engage in market interaction that is appropriately described by a contest. Konrad (2006a) shows that one firm's share ownership in its competitor causes externalities if it wins the contest. These externalities change the owning firm's incentives to becoming the winner of the prize. Firm that owns a share in another firm is no longer indifferent about who among the other firms wins the prize. Its most prefered outcome is that the firm itself wins the prize. The second most prefered outcome is that the firm wins the prize in which it has a minority ownership share, and the least prefered outcome is that some of the other firms wins the prize. This change in incentives suggests that firms can also use shareholdings strategically in order to change the difference in their valuations of winning.

To illustrate this by way of an example, consider firms $i = 1, 2, 3$ who turn winning some contract into profits $v_1 = 210$, $v_2 = 200$ and $v_3 = 150$, and let these firms be solely owned by persons 1, 2 and 3. If the three firms compete independently in an all-pay auction, firm 1 or firm 2 will win, and only firm 1 has a positive payoff equal to $210 - 200 = 10$. Now let firm 1 purchase a 40 percent share in firm 2, and let firms behave in a way that maximizes their payoff which consists of their operating profit and their income from ownership shares in other firms. Firm 1 may quit and expend zero effort. The contest will then be between firms 2 and 3 and will earn firm 2 an expected operating profit of $200 - 150 = 50$ that is distributed between the owners according to their shareholdings. Firm 1 will therefore receive 20, which is more than the 10 received without the shareholdings, and firm 2 will receive an operating profit of 50, which exceeds the profit of zero obtained without these shareholdings. Of course, the initial owner, person 2, will receive only 30 units of this operating profit, as he sold a share of 40 percent in the firm 2 to firm 1.

In this example, the minority shareholdings reduce both the rents generated by the allocation of the prize, and the firms' aggregate equilibrium efforts. The assessment of this outcome from a social welfare point of view depends on the nature of contest effort in this example. But the example illustrates that heterogeneity can well be in the interest of the contestants if the prize that is allocated between them is independent of this heterogeneity, and that contestants may have some means of influencing their heterogeneity by creating external effects between them.

Linster (1993b) considers in the framework of the Tullock contest the problem of contests with externalities more generally. Contestants may not only care about whether they win or lose. If a contestant does not win the prize himself, he may care about who else wins the prize. Suppose players A, B and C take part in a lottery contest where A and B are brothers, and C is a stranger. Player A's willingness to purchase a further lottery ticket may be smaller if he knows that his brother holds the major share of the remaining lottery tickets than if these are owned by the stranger. The general equilibrium solution of the problem is structurally straightforward, but except for some special cases it does not lead to simple closed form solutions.

6.2 Sabotage

In a contest relative performance matters. Contestants may expend effort on improving their own performance, or they may expend effort on activities that reduce the performance of one, or some, of their competitors. Both types of activity generally increase a contestant's probability of succeeding in the contest.

Among the first researchers who highlighted the role of 'sabotage activities', i.e., activities that harm the competitor, was Lazear (1989). The possibility of sabotage is generally seen as one of the major shortcomings of tournaments as incentive mechanisms for inducing output. Lazear discusses possible counter measures that contest designers more generally, and firms in particular, may take. Pay compression, or a reduction in the spread between the payment for the winner and the loser, reduces the incentive to expend both the effort that enhances own performance and the effort that sabotages other competitors. Hence, wage compression may increase internal harmony and efficiency. However, as wage compression also reduces the stimulus to improve own performance, such pay compression runs counter to the idea of a tournament as an incentive instrument. Lazear suggests that firms could try to hire employees who are not very good in sabotaging others but who are very productive in turning own effort into own output. Such employees will not engage much in sabotage.

Lazear (1995) also reports some examples in which firms chose a structure of the tournament that reduces sabotage. He reports that, before the breakup of AT&T, the president of the corporation was usually chosen from the group of presidents of the various subsidiaries of the company. As there was some geographic distance between these subsidiaries and not too much direct exchange and communication, it was physically difficult and costly for one of them to sabotage the other, much more difficult than sabotage between various vice presidents working next to each other within the headquarters of the company. He mentions the promotion policy of Dow Chemical Corporation as a second, similar example. At this company, the competition for the top jobs at the headquarters also takes place between people in (typically different) field operations who have less opportunity to compete with each other. A similar policy that can be an effective means against sabotage is analysed by Chan (1996). He suggests that sabotage (or non-cooperation, which is a weaker form of sabotage) among n employees within a firm who compete for promotion may be reduced if the firm owner can also appoint someone from outside if the firm performance as a whole is not sufficiently good.

Kräkel (1998) discusses the role of the prize structure in sabotage effort. For instance, if there are n contestants and only one prize, the incentives to expend effort to sabotage differ from those in a tournament in which the best $n-1$ contestants receive a prize. However, this prize structure also changes the incentives to expend productive effort. Konrad (2000a) makes the point that sabotage is an activity that also has positive externalities in a context with more than two contestants. If A sabotages B, then this reduces B's relative performance and improves A's relative performance, but it also improves the relative performance of all other contestants C, D, etc. This public good aspect of sabotage makes sabotage less desirable if there are many contestants. With sufficiently many contestants, an equilibrium with very little or no sabotage becomes very likely.

Münster (2003) and Chen (2003) consider the importance of relative productivity for the question about who sabotages whom. Starting from the famous shooting contest between three "pistoleros" (Shubik 1954), in which it is most likely that the second and third best shot turn first against the best and eliminate him, Münster shows more generally that sabotage has a strongly equalizing property. The contestants who are most productive in turning effort into own output will attract the largest amount of sabotage, so that sabotage leads to an equilibrium in which the probabilities of winning are compressed compared to a world in which sabotage effort is not feasible.

6.3 Information externalities

A different type of externality was examined by Konrad (2004) in the context of electoral campaigns that was one of the important examples of contest competition. The paper shows that parties or candidates may find it in their interest to expend their campaign effort on informing voters that some of them will be better off if their rival wins the election. For this reason this type of effort is called 'inverse' campaigning. Examples of this type of behavior can be found in electoral campaigns, for instance, if a party B points out that there is a small group who would benefit from considerable tax exemptions if party A wins the election and carries through its proposed policy, or that there is are major redistribution program geared towards such elite groups as part of party A's policy programme. It is shown that negative campaigning by both parties can be an equilibrium outcome, and that the problem of optimal inverse campaigning may take the format of an all-pay auction.

The intuitive reason why inverse campaigning may take place is an information externality of such effort for the voters who are not informed if some other voters are informed about their advantages if they vote for the rival opposition. The problem can be illustrated as follows. Suppose there are only two parties A and B who compete in an election and are purely office motivated. The payoff functions of the parties are given by (2), with $v_A = v_B = 1$, and they have constant unit cost of effort. Suppose now that the parties have chosen different policy platforms a and b, and these platforms have different implications for the different voters. Let there be 100 voters, each represented by a square in the large square in Figures 10 and 11, with the number in the square being the actual payoff difference for this voter from party A being elected, compared to the outcome in which party B is elected. Let 51 among these 100 voters gain one unit of income from party A (and not B) being elected, whereas 49 voters lose 1 units of income. Let us assume further that each voter knows this distribution of gains and losses, but is ignorant about whether he belongs to the group of 51 winners or to the group of 49 losers.

1	1	1	1	1	1	-1	-1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1

Figure 10: All voters are uninformed. Each uninformed voter has a probability of 0.51 to be one of the voters who gain one unit from party A being elected, and of .49 to be one of the voters who lose one unit.

The (at least partial) ignorance of voters about their own benefit or loss from one or the other platform is a key assumption, and also plausible. Thinking, for instance, about complex tax reform proposals with many changes of which some may benefit and some may harm a particular person, and taking the complex general equilibrium repercussions of such a reform into consideration, the voter's uncertainty about whether he gains or loses from the reform is not unlikely. This is particularly true if the tax reform has mainly redistributional consequences and no major efficiency effects. Moreover, the voter's incentive to invest major reserarch effort to resolve his own ignorance is very small: rational ignorance of voters is a well known feature in political science.[40]

The decision problem for a representative voter is represented in Figure 10. If a risk neutral voter knows he is one of the voters in this grid and has the payoff of either -1 or $+1$, respectively, if party A wins rather than party B, the voter will calculate his expected payoff for both outcomes and concludes that he is better off by voting for party A, as the expected payoff if A wins is higher by $\frac{2}{100}$. Accordingly, without any campaigning, party A will win the election with 100 percent of the votes.

1	1	1	1	1	1	-1	-1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1
1	1	1	1	1	-1	-1	-1	-1	-1

Figure 11: Two voters who gain are identified (the ones in the grey boxes). This changes the distribution of gains and losses for the remaining 98 uninformed voters.

Consider now party B's incentives to use inverse campaigning. In particular, let party B have the option to identify a few voters, find out about whether they gain or lose from party A winning the election, and be able to inform these voters and a greater public about these findings. Then, if party B can identify two voters who gain one unit from party A being elected, and can credibly publicize this information to all voters, this will change the voting behavior. These two voters will now vote for party A, as they are sure

[40]The assumption is also not uncommon in the literature on electoral competition, see, e.g., Fernandez and Rodrik (1991).

to be better off if party A wins the election. However, all other voters will reconsider their decision. Their expected benefit from voting for party A is no longer $\frac{2}{100}$, but turns into a negative value, equal to $-\frac{1}{98}$. All 98 uninformed voters will then vote for party B and party B will win the election.

Of course, the choice of B to use inverse campaigning for only two voters is not the equilibrium of this game, as party A will optimally react to party B's inverse campaigning. As is verified in Konrad (2004), the structure of the problem that emerges if both parties can choose the amount of inverse campaigning is equivalent to an all-pay auction without noise.

6.4 Public goods and free riding

A different type of public good problem emerges if the prize that can be gained by winning a contest is a public good for a well defined group of recipients. Examples for this can be found and these have been shown in the federalism context. For instance, when a public facility has to be located in some municipality and generates a benefit or a cost to each member of this municipality, the municipality will then fight for or against becoming the location of this facility, depending on whether having the facility is a public good or a public bad for the inhabitants of the municipality where the facility is to be located. Other examples are groups of national producers and consumers who compete with each other about whether an import tariff should be enacted or not, or team sports where teams compete to win the tournament. Everyone in the team who wins will receive the same winner prize, but may value it differently.

Homogeneous groups Contests for a public good between groups have been analysed by Katz, Nitzan and Rosenberg (1990) more formally as follows.[41] Let there be two groups 1 and 2. Let n_1 and n_2 be the respective numbers of members of the two groups, and let all group members be identical within each group. The two groups enter into a contest, and one group wins a prize. The prize is a public good that yields each member of the winning group the same benefit that is equal to v_1 and v_2 in the two groups. Each group member can choose to contribute non-negative contest effort to the aggregate effort that is expended by his group, and x_{ij} denotes the effort expended by member j in group i. The individual contributions are summed up to aggregate group effort and the ratio between a group's aggregate effort and the sum of aggregate efforts from both groups equals the group's

[41] For related formal analyses see also Gradstein (1993) and Ursprung (1990)

probability of winning the prize, i.e.,

$$p_1(...x_{ij}...) = \begin{cases} \frac{\sum_{j=1}^{n_1} x_{1j}}{\sum_{j=1}^{n_1} x_{1j} + \sum_{j=1}^{n_2} x_{2j}} & \text{if} \quad \max\{...x_{ij}...\} > 0 \\ 1/2 & \text{otherwise.} \end{cases} \tag{88}$$

Contestants care about their expected benefit from winning and about their cost of effort. Their payoff is

$$\pi_{ij}(...x_{ij}...) = p_i(...x_{ij}...)v_i - x_{ij}. \tag{89}$$

Maximization of (89) yields first-order conditions

$$\frac{\partial \pi_{ij}}{\partial x_{ij}} = \frac{\sum_{j=1}^{n_{-i}} x_{(-i)j}}{\left(\sum_{j=1}^{n_1} x_{1j} + \sum_{j=1}^{n_2} x_{2j}\right)^2} v_i - 1 = 0 \tag{90}$$

where $-i$ denotes the group other than i.

The solution of these $n_1 + n_2$ first-order conditions typically yields a unique solution for the aggregate amounts of efforts which are expended by each group in the equilibirum. However, as can be seen from each of these first-order conditions, they do not determine how the aggregate effort of each group is allocated between the members of the group. Accordingly, the equilibirum is typically unique as regards group efforts and the winning probabilities of groups, but a multiplicity of equilibria emerges as regards individual contributions to group effort. As all first-order conditions for n_i contestants within a group are fully identical (and not just symmetric), this reduces to a system of two equations which determines $x_1 \equiv \sum_{j=1}^{n_1} x_{1j}$ and $x_2 \equiv \sum_{j=1}^{n_2} x_{2j}$ and the groups' equilibrium win probabilities as in a contest between two contestants 1 and 2 with valuations of the prize equal to v_1 and v_2. The aggregate effort of a group and its win probability in the equilibrium are hence determined by the valuations v_1 and v_2, and are fully independent of the number of members in the two groups, or the group's relative group size.

This result is in contrast to the fact that the benefits of a group from winning the contest are twice as large if the size of the group doubles. One interpretation of this result is that free riding intensifies if groups grow larger, and that the increase in free riding just compensates for the additional incentives of the group for winning the prize. However, this interpretation is misleading, as can be seen when modifying one of the assumptions made in the analysis.

Heterogeneous groups Consider the following modification in the analysis by Katz, Nitzan and Rosenberg (1990) which is discussed more formally by Baik (1993). Let the groups consist of heterogeneous members who all value the public good differently, and let $v_{ij} > 0$ be the valuation of the prize by member j in group i. Carrying through the same equilibrium analysis leads to $n_1 + n_2$ first-order conditions like (90), but with v_i replaced by the individual valuations v_{ij}. It is not possible to fulfill all these conditions for all ij, if there are at least two members of the same group whose valuation of winning the group prize differ. If all members of a group differ in their valuation of winning the group prize, the equilibrium is described by

$$x_{1k} = x_1^* = \frac{v_2^* v_1^* v_1^*}{(v_1^* + v_2^*)^2} \text{ for } k \text{ with } v_{1k} = v_1^* \text{ and } v_1^* \equiv \max\{v_{11}, ...v_{1n_i}\} \quad (91)$$

and

$$x_{1k} = 0 \text{ for all } k \text{ in group 1 with } v_{1k} < \max\{v_{11}, ...v_{1n_i}\} \quad (92)$$

and analogously for group 2 with "2" replacing "1" in (91) and (92) and vice versa. Accordingly, if the group members differ in their valuations of the public good, only the members with the highest valuation make contributions to the inter-group contest. This is a known result from the theory of public goods when the payoff functions of the voluntary contributors to the public good are linear in income, as is assumed in the context here. The result also reveals the appropriate intuition for the results in Katz, Nitzan and Rosenberg (1990). The aggregate amount of effort is independent of group size as it is determined by what the group member with the highest valuation of the prize would be willing to expend if he were the only contestant. As the group size does not change the member with the highest valuation of the prize, it does not affect the aggregate group effort in the equilibrium.

Other contest success functions The outcome that makes only the group member who likes the public good most contribute to group effort (or to a group specific public good more generally), as in the case considered by Katz, Nitzan and Rosenberg (1990) or in the case with a heterogeneous group, can be extended to the larger class of contest success functions in which the probability that group 1 wins is a function $p_1(x_1, x_2)$ of the aggregate group efforts of the various groups. Nti (1998) considers the special case $p_1(x_1, x_2) = \varphi(x_1)/(\varphi(x_1) + \varphi(x_2))$.

The result is also robust with respect to the case in which the group that makes the highest group effort wins with probability 1. This is illustrated by Baik, Kim and Na (2001). Consider, for instance, two groups 1 and 2 with n_1 and n_2 members and valuations v_{ij}, respectively, and let the size of the valuations within a group be sorted so that $v_{i1} > v_{i2} \geq ... \geq v_{in_1} \geq 0$. Assume further that $v_{11} > v_{21}$. Each individual can choose his own contest effort and group members' contest efforts add up to x_1 and x_2, respectively. Replacing the Tullock contest success function as in Katz, Nitzan and Rosenberg (1990) with the all pay auction as in (3) leads to an equilibrium in which $x_{ij} = 0$ for all $j \neq 1$ and in which group members 1 in the two groups choose mixed strategies with their efforts x_{11} and x_{21} according to the equilibrium strategies (4) and (5). Hence, if two groups bid for the allocation of a group-specific public good and each group's members make their contributions in a non-cooperative fashion, the problem reduces to a contest between the set of members who have the highest valuation in their group. All other group members choose zero effort.

The intuition for the result is as follows. Consider the equilibrium strategy for aggregate group effort that is described by $F_1(x_1)$ as in (4), with v_2 replaced by v_{21}, making use of $v_{21} = \max\{v_{21}, ...v_{2n_1}\}$. For this choice consider the member of group 2 who has the highest valuation v_{21}. If this individual anticipates that all other group members will make zero contributions, he is just indifferent about effort choices from the interval $[0, v_{21}]$ and may just randomize on this interval according to $F_2(x_2)$ as in (5), again with v_{21} replacing v_2. All other individuals would not choose any positive effort given $F_1(x_1)$ as they have a lower valuation of the prize than v_{21}. This establishes that $F_2(x_2)$ with $x_{21} = x_2$ and $x_{2j} = 0$ for all $j > 1$ is an optimal reply to $F_1(x_1)$.

The argument for why this equilibrium $F_2(x_2)$ makes player $j = 1$ in group $i = 1$ just indifferent regarding a choice of effort from the equilibrium support $[0, v_{11}]$ and makes all players $j > 1$ in this group strictly prefer $x_{1j} = 0$ follows analogous lines.

The discussion in section of the all-pay auction without noise also suggests also that this solution generalizes to more than two groups which compete for the prize, and for cases in which there is one or several groups in which there is more than one member who has the highest valuation within this group.

Making the 'group' size meaningful The result, according to which group size does not matter and effort is determined by the group members who value the public good most highly, is sensitive to, for instance, the linear cost assumption. This has been highlighted, e.g., in Konrad (1993), Esteban and Ray (2001) for the case of convex cost, and by Riaz, Shogren and Johson (1995) for the case of increasing marginal opportunity cost of individually devoting additional resources to the group contest. If, for instance, $C_i(x_i) =$

$C(x_i)$ with $C' > 0$ and $C'' > 0$, this leads to first-order conditions

$$\frac{x_2}{(x_1+x_2)^2}v_{1j} = C'(x_{1j}) \text{ and } \frac{x_1}{(x_1+x_2)^2}v_{2j} = C'(x_{2j}) \tag{93}$$

for individuals $1j$ and $2j$ in groups 1 and 2. This makes interior solutions feasible for all group members, even if they differ in their valuations of the group prize. Moreover, suppose the group effort x_1 of group 1 is given and consider an increase in the size of group 2. Let $x_2^*(x_1)$ be the aggregate effort that group 2 would expend when anticipating x_1 by group 1 in the Nash equilibrium. Then x_2^* would be an increasing function of this group's size. To see this, consider the simple case in which the group size increases from $n_2 = 1$ to $n_2 = 2$, and let the new group member have the same valuation of the public group good. Suppose both group members were to stick to the same aggregate quantity x_2^* that was optimal for $n_2 = 1$, implying that both would have to contribute only $x_2^*/2$. If the first-order condition (93) was fulfilled for x_2^* and $n_2 = 1$, then

$$\frac{x_1}{(x_1+x_2^*(x_1))^2}v_2 > C'(x_2^*/2) \tag{94}$$

would result. Both group members would like to increase their contributions until the new aggregate equilibrium effort by group 2 is reached.

In the inter-group contest the individual effort contributions to the aggregate group effort are contributions to a public good, where the public good could be defined as $p_i(x_1, x_2)$, the probability of winning the contest, and $p_i(x_1, x_2)v_{ij}$ are the individual valuations of the public good. For given (or anticipated) aggregate effort of the competing group, the voluntary contributions of effort are therefore equivalent to the problem of voluntary contributions to a public good, one which has been carefully studied.[42] The perfect substitutability of individual contributions has a prominent role in this literature. However, this literature has also emphasized that contributions to a public good could follow different patterns. For instance, Hirshleifer (1983) discusses this assumption and illustrates its crucial aspects for the equilibrium results by considering the cases in which the aggregate provision level is determined by the smallest or the highest contribution within the group. More generally, imperfect substitutability of contest efforts is a possible channel through which the group aspects of inter-group contests for public goods become meaningful, and the case in which only the group member with the highest valuation of the prize plays a role in the strategic interaction between groups must be seen as the exception rather than the rule.

[42] See, for instance, Bergstrom, Blume and Varian (1986) and Cornes and Sandler (1986).

7 Grand contests

In a contest, a set of players expends efforts up-front, trying to win some prize. The process by which the efforts translate into success probabilities is often considered a black box, and much of the analysis in this survey follows this spirit. However, if we look at contest games with some high-resolution spectacles, they often reveal a finer substructure and can be decomposed into a number of smaller battles. This is obvious with sports contests. Similarly, reseachers in the R&D context noted that research and development is not a single, one-shot event (see, e.g., Harris and Vickers, 1985, 1987). Related problems emerge, and have been analysed in the contexts of political campaigns (Klumpp and Polborn 2006), violent conflicts for territory, resources or power (Mehlum and Moene 2004, McBride and Skaperdas, 2006) and, with a multiplicity of applications in mind, by Gradstein and Konrad (1999), Amegashie (1999, 2000), McAfee (2000), Moldovanu and Sela (2006), Groh, Moldovanu, Sela and Sunde (2003), Konrad and Kovenock (2005, 2006a, 2006b) and Matros (2006).

Any particular focus on a special type of grand contest is necessarily selective. I will discuss three types of structure. One type emerges naturally from the analysis of inter-group contests if the prize a group wins is not a public good, but instead must be allocated among the members of the winning group. Such nested contests have been studied carefully, as in many contexts they emerge naturally, and have interesting efficiency properties. Second, related to this structure, the formation of groups in the inter-group contest may be endogenous, leading to the problem of formation of alliances. Finally, we consider contest structures in which players fight for themselves alone, but in which the prize is allocated according to more complex rules that possibly involve numerous contest stages. I consider three structures in particular, a race, a tug-of-war, and an elimination tournament, and then turn to discussing a fundamental principle that underlies all these structures.

7.1 Nested Contests

Conflict often takes place between groups, and examples for this have been discussed in the section on inter-group contests for public goods. Such conflict ends once the inter-group contest has determined a winner if the prize that is awarded to the winning group is a good that all group members can consume or if the allocation of the good within the group cannot be influenced by group members. In many cases, the inter-group contest is about private goods, and the conflict does not necessarily end once the contest prize is allocated to one of the groups, or once its shares are allocated to the different groups.

Examples come from war, politics, sports, federalism, corporate governance and other areas of conflict. Consider, for instance, the coalition of the US, Russia, France, and the UK who joined forces to defeat Germany, Japan, Italy and other members of this group. Once the second world war was over, the US and the Soviet Union emerged as super powers and started struggling about how to divide the world between them. War and pillage is another illustrative example. Consider a medieval army that tries to conquer a city. This is a contest between two groups: the army and the city population. Once the army has succeeded in conquering the city, the warriors will pillage the city and will devide the spoils among themselves. In politics, leading figures inside a political party often join forces prior to an election, trying to get their party into power. Once this goal is achieved, they may start struggling about who will obtain which office, and who will eventually become the party leader. The first and second triumvirate in ancient Rome is another example. The coalitions that were formed initially help those most likely to win power collectively. Later, each member of the triumvirate tried to increase his own share in this power, or to become the sole leader.

Sports contests provide further examples. Many types of sports contests take place between teams. This implies that a team player's own effort benefits himself only if it improves his team's performance. A football player can win the world championship only if his team wins the football championship. However, the winning team in a championship is not fully homogeneous. Not only do players have a different, more or less charming appearance and their own personality, they also differ visibly in their contribution to their team's success. Accordingly, they do not all get the same prize from winning the championship, and it is probably true that, within teams, there is rivalry about who receives higher recognition and can earn higher benefits from media attention, sponsoring and ego rents.

Wärneryd (1998) suggests federal structures as another example, in which players within regions may join forces in trying to obtain some share in the global budget, and once a region succeeds, there is a struggle about how to allocate it between them. Other examples comprise corporate governance issues as in Inderst, Müller and Wärneryd (2005) and Müller and Wärneryd (2001).

It seems plausible that the rules that govern the allocation of the prize will affect the group members' efforts in winning the battle. A number of different cases do need to be distinguished, but a unified framework will be used to describe them, building on the notation introduced in section 6.4, and both the Tullock contest and the all-pay auction case will be considered.

7.1.1 Exogenous sharing rules

Nitzan (1991a) considers the following nested contest. He assumes that the group members make contributions to the group's effort to win the inter-group contest, and that these very contributions also influence the allocation of the prize inside the winning group. In the example of parties which struggle for power, party members may engage actively in speeches or other promotional activities, and their effort may indeed be rewarded by a higher share in the rents accruing to the party if it gets into power. More precisely, let there be two groups 1 and 2 with n_1 and n_2 the respective numbers of members of the two groups, and let the members within each group all be identical. The two groups enter into a contest for which of them wins a prize. The prize is of size v and awarded to the group as a function of aggregate efforts of the group members; i.e., the contest success function for the inter-group contest is

$$p_i(...x_{ij}...) = \begin{cases} \frac{x_i}{x_1+x_2} & \text{if} \quad \max\{x_1, x_2\} > 0 \\ 1/2 & \text{otherwise,} \end{cases} \tag{95}$$

where $x_{ij} \geq 0$ is the effort contribution of member j in group i, and $x_i \equiv \sum_{j=1}^{n_i} x_{ij}$. Once group i wins the contest, the distribution of the prize v between the group members is also determined. Member j in group i receives a share

$$q_{ij} = \begin{cases} (1-\alpha)\frac{x_{ij}}{x_i} + \alpha\frac{1}{n_i} & \text{if} \quad \max\{...x_{ij}...\} > 0 \\ \frac{1}{n_i} & \text{otherwise.} \end{cases} \tag{96}$$

One interpretation of the allocation rule (96) is that the prize is allocated according to 'merit' with a probability $1-\alpha$ inside the group, and according to a random mechanism that gives each group member the same chance of winning with the remaining probability α. In the 'merit' regime, each group member wins the prize with a probability that equals his own share in the aggregate group effort.

Nitzan (1991a) describes the equilibrium by the first-order condition for maximizing the objective function of contestant j in group i,

$$\pi_{ij}(...x_{ij}...) = p_i q_{ij} v - x_{ij}, \tag{97}$$

and the first-order condition for $i = 1$ is

$$(1-\alpha)v\frac{x_1 + x_2 - x_{1j}}{(x_1+x_2)^2} + \alpha v\frac{x_2}{(x_1+x_2)^2}\frac{1}{n_1} - 1 = 0, \tag{98}$$

and analogously for $i = 2$.

For $\alpha = 0$, use of symmetry yields

$$x_{ij}^{*} = \frac{n_1 + n_2 - 1}{(n_1 + n_2)^2} v, \tag{99}$$

which reproduces the equilibrium effort in a symmetric Tullock contest among n_1+n_2 identical contestants. As becomes clear from the analysis in Gradstein and Konrad (1999), this structural equivalence is due to the choice of the Tullock contest success function with $r = 1$. The result also follows directly from consideration of (97) if p_i and q_{ij} are replaced by their actual values as in (95) and (96), as x_i cancels out and the payoff function as in the standard Tullock contest remains.

For $\alpha = 1$,

$$x_1^{*} = v\frac{n_2}{(n_1 + n_2)^2} \text{ and } x_2^{*} = v\frac{n_1}{(n_1 + n_2)^2}. \tag{100}$$

For $n_1 = n_2 = n$, each group' aggregate effort becomes $x_i = \frac{v}{4n}$.The aggregate effort levels are thus identical with those in a contest between two players for a prize of size $\frac{v}{n}$, which is indeed the prize that each contestant competes for in this problem.

Note also that the individual effort levels are undetermined. This result resembles the case of inter-group contests for the allocation of a prize that is a group specific public good, or the result for the private provision of a public good with homogeneous contributors with quasi-linear preferences who all have constant marginal contribution cost. Indeed, given the fixed and perfectly symmetric sharing rule, the prize is like a public good that benefits all members of the winning group equally, and here the contribution cost is assumed to be strictly linear. Moreover, if the contributors in the group had different, but constant marginal opportunity cost of effort, or had equal and constant marginal opportunity cost of effort, but value winning the prize differently, the contributor with the least cost of contributing within a group, or with the highest valuation of the prize, would became the only contributor. Like in the context of private provision of a public good, these strange features of the equilibrium outcome disappear once the marginal opportunity cost of effort is no longer constant, as has been discussed in the context of strategic aspects of public good contests between groups.

7.1.2 The choice of sharing rules

Nitzan (1991b) goes on and asks what happens if groups can choose the weights they would like to give to merit or to pure chance in the intra-group allocation of the prize. Suppose groups can choose their own α_i in (96) prior to each group member's effort choice. The first-order conditions will generally not be suitable for describing the equilibrium in the inter-group contest if the two groups have chosen sufficiently different weights α_1 and α_2.

To give an example, let $\alpha_1 = 0$ and $\alpha_2 = 1$. In this case there is an equilibrium with $x_2 = 0$ and

$$x_1 = v\frac{n_1 - 1}{n_1} \tag{101}$$

if $n_1, n_2 \geq 2$. To see this, consider the contribution incentives in group 1 if the members of group 2 do not contribute. Their contribution incentives are the same as in a simple symmetric Tullock contest with n_1 contestants. Accordingly, the aggregate contributions made by the group 1 will be $x_1 = v\frac{n_1-1}{n_1}$. For these contributions of group 1 consider now the individual incentives for members in group 2 to contribute. Suppose that all members $j = 2, ...n_2$ contribute zero. In this case the objective function for group member 1 in this group 2 is

$$\pi_{21} = \frac{x_{21}}{x_{21} + x_1}\frac{v}{n_2} - x_{21} \tag{102}$$

Now,

$$\frac{\partial \pi_{21}}{\partial x_{21}} = \frac{x_1}{(x_{21} + x_1)^2}\frac{v}{n_2} - 1. \tag{103}$$

Inserting (101) yields

$$\frac{\partial \pi_{21}}{\partial x_{21}} < 0 \text{ if } n_2 n_1 > n_1 + n_2. \tag{104}$$

Davis and Reilly (1999) characterized these corner equilibria that emerged in the problem studied by Nitzan (1991b). Their analysis highlights the importance of corner solutions in contest games with asymmetry.[43] In many instances, contestants would be wise to stay out of the contest if other contestants enter who are more motivated, either by a higher prize or by a lower cost of effort. In the case studied by Nitzan, the contestants in the group that allocates the winning prize according to 'merit' are far more motivated to expend effort than those in the group in which the prize is allocated to some egalitarian rule. Members of this group contribute to two contests simultaneously when making contributions to group effort. They make it more likely that their group will win, and they increase their own share in the prize if it does win. This latter incentive is strong enough to induce high effort choices that also count for the group's effort in the inter-group effort, even though the group does not really face any serious competition from the other group.

[43] For an analysis of possible corner solutions see also Ueda (2002).

7.1.3 Intra-group conflict

That group members' contributions to the inter-group contest influence the intra-group allocation of a prize once the prize is allocated to a group is in line with some broad views on equity and fairness. Moreover, from the perspective of the group it is efficient to use such a rule as an incentive mechanism to make the group members contribute. However, it is often more plausible that group members will struggle about the intra-group allocation of the prize, regardless how much each of them has contributed in the inter-group conflict. In this case, the intra-group allocation can be seen as the outcome of an intra-group contest in which the members' contributions to the intra-group contest are sunk and irrelevant for the intra-group allocation of the prize. The example of the conquest of a city and the pillage and plundering that occurs may illustrate this, but other examples that are more closely related to standard topics in economics have also been used to study this problem. The first paper on this topic in which the group prize is contested among the members of the winning group once the winning group has been determined is Katz and Tokatlidu (1996). They analyse this type of problem and provide the comparative statics with respect to group size.

Wärneryd (1998), who studies this problem in the context of conflict between k jurisdictions within a federation, and with m_i symmetric players in jurisdiction i, finds that this structure may have advantages compared to a one-stage contest in which no group structures exist and the prize is allocated in a single Tullock contest with $\sum_{i=1}^{k} m_i$ participants. He finds that a more hierarchical structure can be advantageous, as it tends to reduce the total effort that is expended in the various contests for allocating the prize. Müller and Wärneryd (2001) consider the role of outsiders in the ownership of a firm and show that the mere existence of such a group and its incentives to expend effort in trying to appropriate some share in the earnings, may mitigate the distributional conflict between the insiders of the firm. Inderst, Müller and Wärneryd (2005) identify similar structures in the context of allocating free cash flow for investment between divisions inside the firm.

Konrad (2004b) highlights the importance of asymmetry for the emergence of inefficient outcomes in the hierarchical structure compared to the grand single stage contest, concentrating on the all-pay auction as the relevant allocation rule. He shows that the equilibrium amount of effort and the efficiency properties of a contest structure, with an intra-group contest in the winner group following a contest between groups, depends crucially on the composition of the groups.

I will illustrate the role of a hierarchical structure for a possible reduction in overall contest effort compared to the grand single stage contest between all contestants, as in Wärneryd (1998), and then turn to the analysis of asymmetries, focussing on the all-pay auction.

Contest moderation by hierarchies Following Wärneryd (1998)[44], consider a prize that all contestants value equally as v. Suppose there are $n \geq 2$ groups that compete for the prize in an inter-group conflict. For simplicity, let each group consist of m homogenous members. Let x_{ij} be j's contribution to group i's effort in the inter-group conflict as before. Let y_j be the effort of member j in the winning group in the intra-group contest that emerges once the group is victorious in the inter-group conflict. These intra-group conflict efforts are chosen only after the inter-group conflict has been decided. Further, let the contest success function in the inter-group conflict be of the same functional form as (96) with inter-group efforts replacing the role of the intra-group efforts.

Solving the problem recursively, suppose group i was the victor and received the prize. The contest among the group members that emerges is identical with the standard Tullock contest, and individual and aggregate effort and payoffs are

$$y_j^* = \frac{m-1}{m^2}v, \; \sum_{j=1}^{m} y_j^* = \frac{m-1}{m}v, \text{ and } \pi_j^* = \frac{v}{m^2}. \tag{105}$$

Note that the members of the winning group use up considerable resources in the intra-group conflict. This reduces the value of winning the prize for the whole group from v to v/m.

Turning to the inter-group conflict, when thinking about his effort choice in the inter-group conflict each of the members of group i considers what is at stake for him. Like in a game of voluntary contributions to a public good, each member chooses his own inter-group conflict effort so as to increase their group's aggregate effort to the amount they consider individually optimal, or they abstain from making a positive effort and leave it to others to compete, if what the others contribute is above this individually optimal threshold level. Symmetry will cause all members of group i to prefer the same aggregate group effort x_i, and the objective function that guides their choices is

$$\frac{x_i}{\sum_{k=1}^{n} x_k} \frac{v}{m^2} - x_{ij}. \tag{106}$$

Taking into consideration that $\frac{\partial x_i}{\partial x_{ij}} = 1$, this yields aggregate group efforts and win probabilities in the equilibrium as in the standard Tullock contest

[44] Wärneryd (1998) considers the case with two groups only but with different sized groups. This allows him to consider the implications of asymmetry.

with $\pi_j^* = \frac{v}{m^2}$ as the winner prize. Hence,

$$x_i^* = \frac{n-1}{n^2}\frac{v}{m^2}, \sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij}^* = \frac{n-1}{nm^2}v. \quad (107)$$

Taking into consideration the equilibrium efforts in the continuation game in the winner group, the aggregate payoff that is not dissipated in the two-stage contest is

$$\left(1 - \frac{n-1}{nm^2} - \frac{m-1}{m}\right)v. \quad (108)$$

The payoffs that result for the grand single-stage contest in which all nm contestants compete with each other in a single stage, assuming that the random mechanism that determines the winner in this contest is the same lottery contest success function, is simply

$$\left(1 - \frac{nm-1}{nm}\right)v. \quad (109)$$

Comparing the two reveals that $1 - \frac{n-1}{nm^2} - \frac{m-1}{m} - 1 + \frac{nm-1}{nm} = \frac{(m-1)(n-1)}{nm^2}$. The numerator is strictly positive if there is more than one group and more than one player in each group. Hence, the hierarchical structure dissipates less effort in total than the grand simultaneous contest.

As discussed in Wärneryd (1998), there are several forces at work explaining why the hierarchical structure dissipates less rent in a more general framework. The most important effect that also emerges if the groups in the inter-group contest are of equal size, is the free rider effect. Groups do not compete according to what the group gets as a rent if the wins the contest, but only according to what each single group member expects to win if the group wins. For instance, if groups consist of two players, the group rent is $v/2$, which already takes into consideration that half of the prize is dissipated in the intra-group conflict if the group wins the prize in the inter-group contest. However, the group members behave non-cooperatively. The group's aggregate effort is the same as that which would result from coordinated group effort for a group prize of size $v/4$. Hence, the free-rider problem shelters some of the later rents from competition. For instance, if all groups in the final stage are of size $m = 2$, this will shelter one quarter of the total rent from competition and is a lower bound on dissipation for the Tullock contest with $r = 1$, regardless of how many groups compete with each other in the first stage, and how many players are involved in total.

Münster (2004) considers a similar framework, but assumes that all players have a given budget which cannot be used for anything except investing in production, in intra-group effort and intergroup effort. In his framework, a strategic effect of effort in the inter-group conflict becomes evident. As this effort predetermines intra-group effort, Münster's analysis highlights that it does matter whether the two types of conflict occur consecutively or simultaneously, as the inter-group effort can unfold commitment properties in the sequential set-up.

Asymmetric valuations The role of asymmetry among contestants with respect to their valuations of winning the prize, or differences in their marginal effectiveness in effort, are most pronounced in conflicts in which small differences in effort are decisive, i.e., if the contest success function is given by (3). Konrad (2004b) studies the role of asymmetry among players in this framework. In order to find closed form solutions, he replaces the contest success function (39) in the framework above with (3).

Consider the set of all players from all groups renumbered according to their valuation of the prize, with $h = 1$ the player with the highest valuation, and player $h = nm$ the player with the lowest valuation of the prize. Assume further that strict inequality applies, i.e., $v_h > v_{h+1}$, as this eliminates some cases with multiple equilibria and simplifies the exposition. The grand single stage contest among this set of players is the all-pay auction with nm players that has already been studied. It will give a positive payoff only to player 1 who has the highest valuation of the prize among all players, and this payoff will be equal to the difference between his valuation v_1 and the valuation v_2 of player 2. Moreover, it will elicit effort which, in expectation, sums up to $\frac{v_2}{2}(1 + \frac{v_2}{v_1})$ in the unique equilibrium.

If, instead, the players are allocated to n groups of size m, consider the incentives of players to contribute to an inter-group contest that takes place prior to the intra-group contest within the group of players that wins the prize in the inter-group contest. Players in each group know that, once their group wins the inter-group contest, only the player with the highest valuation will win some rent in the intra-group contest for this prize, and this player in group i knows that his rent will be equal to $v_{i1} - v_{i2}$, i.e., the difference between his own valuation v_{i1}, and v_{i2} which is the second highest valuation of the prize in this group i. Accordingly, all players in a group who do not have the highest valuation in their group have nothing to gain if their own group wins the prize, and they will, therefore, not contribute to their group's effort in the inter-group contest. In some sense, this reduces the inter-group contest to a contest between the players who have the highest valuation in their respective group, and defines their valuation of their group winning the inter-group contest as the respective difference $v_{i1} - v_{i2}$ for groups $i = 1, ...n$.

This sets the stage for the inter-group contest. If one assumes, for simplicity, that the groups are numbered such that the differences $v_{i1} - v_{i2}$ are descending in i and that, in addition, $v_{21} - v_{22} > v_{31} - v_{32}$ holds strictly, the inter-group contest is essentially equivalent to the all-pay auction with complete information. Only players 11 and 21 will be active bidders in the contest, and either group 1 or group 2 will win the contest.

Note that this outcome implies that, in the hierarchical contest, there is no need for the bidders who have the highest valuation of the prize to be among the players who win with a positive probability. Consider the following numerical example. Let there be 6 players who value prizes as $1001, 1000, 12, 10, 3, 1$, and let them be sorted into groups $G_1 = \{1001, 1000\}$, $G_2 = \{12, 10\}$, $G_3 = \{3, 1\}$. Note that the players in group G_1 will not actively bid in the inter-group contest, as the player who has the highest valuation in this group wins a payoff of only 1 unit in the intra-group contest if his group wins the prize, whereas the players with the highest valuations in groups G_2 and G_3 win a payoff equal to 2 if their group wins the inter-group contest. Hence, the prize will end up with group G_2 or G_3, and, by the choice of the numbers, it will end up in each of the two groups with equal probability. Once G_2 or G_3 wins the inter-group contest, the members of the respective group will compete in an all-pay auction without noise for winning the prize. It is clear that even the contestant with the smallest valuation of the prize has a considerable probability of $\frac{1}{12}$ of winning the prize in this hierarchical contest.

Conclusions on hierarchy Hierarchical structures with groups competing for the prize and later conflict within the winning group(s) have some surprising features. First, the additional layer of conflict that is added by the inter-group conflict does not necessarily increase the amount of overall contest effort. On the contrary, the public good problem that is inherent in such structures when it comes to determining the individual contributions to group effort shelters some rent. For a broad range of structures, the additional conflict layer introduced with the hierarchy will then reduce, rather than increase, the overall contest effort. A second result concerns the efficiency of conflict with respect to allocating the prize to the contestant who values it most highly. In this respect the hierarchy may introduce more distortionary elements and may cause an inferior allocation of the prize. Our discussion of this topic was in the context of a full information framework. Hierarchical contests in the context of incomplete information are a challenging field for future research. The outcome in previous conflict stages partially reveals information about the players' strength or their desire to win, which, depending on what is actually observed in previous contest stages, leads to strategic aspects of information revelation which are not discussed here.

7.2 Alliances

The strategic interaction between more than two contestants, and their incentives to coordinate their actions against other players was already noted by Shubik (1954), and some of the implications of non-cooperative play were discussed in the context of sabotage in section 6.2. A particular type of inter-group contest followed by an intra-group contest emerges if subsets of players form coalitons in a first round and fight against a common enemy and if this alliance breaks up once the common enemy has been defeated and members of the group turn against each other. Such alliances may choose their contest efforts collectively, or, as has been assumed in the types of inter-group and intra-group competition in the previous section, they may choose their efforts in the alliance non-cooperatively. In the latter case, the game structure for given alliances is the same as in the previous section and the question reduces to why do alliances form and what are their determinants.

The formation of alliances in contest games was analysed by several researchers. Skaperdas (1998) considers several 3 player contest problems in which players have been given contest resources that do not deplete during a contest. In the simplest case, players can enter an all-against-all simultaneous contest directly, or two players may decide to join forces and turn against the third player, and, if they jointly win the price, fight it out between themselves. He shows that typically two players gain only if their advantage from joining forces in the alliance is very high. The analysis in Esteban and Sakovics (2003) also suggests that alliances are unlikely to form. They also survey further literature. Garfinkel (2004) considers a richer framework where choices of conflict effort and production are endogenous and emphasizes the role of the way members behave vis-a-vis each other within an alliance.

Münster (2003) does not address an alliance problem directly. In particular, he does not allow for a sequential resolution of the game. However, he does consider individuals who may jointly, but non-cooperatively, sabotage specific rivals within a larger group. For instance, with three players, the strongest player may be sabotaged by the non-cooperative equilibrium effort choices of both weaker players. While this is not a formal alliance with an inter-group conflict followed by an intra-group conflict among the members of the winning alliance, the joint effort to sabotage the strongest player has some flavor of the formation of an alliance.

7.3 Repeated battles

I will now discuss three types of grand contests which typically have a more complex architecture, without formation of alliances or groups in early stages of the contest: a race, a tug-of-war, and an elimination contest. One could think of many other types of grand contests. Mehlum and Moene (2005), Polborn (2005), and Aidt and Hillman (2006) consider repeated games in which there is an incumbent and a rival in every period and, in some cases, these may change roles from one period to the next as an outcome of a period contest. Stephan and Ursprung (1998) and Konrad (2006b) consider asymmetric games in which only one party is initially in possession of some resource but can be attacked repeatedly up to the point where the incumbent loses a first and final time. Leininger and Yang (1994) consider rent seeking in which the contestants alternate in expending additional contest effort over many rounds, with aggregate efforts entering into the contest success function. Hillman and Katz (1987) consider a hierarchical chain of players granting favors in rent seeking contests. Gradstein (2004) and Gonzalez (2005) and Gonzalez and Neary (2005) and Eggert, Itaya and Mino (2006) consider appropriation games with an infinite time horizon and with discrete and continuous time periods, respectively.

The race A simple race between two players is depicted in Figure 12. Both players start at state (2,2). A battle at this state determines whether player 1 wins, which moves the game to state (1,2), or whether player 2 wins, which moves the game to state (2,1). At (1,2), a further battle takes place. If player 1 wins again, player 1 is awarded the prize. If player 2 wins, the game moves to state (1,1). Similarly, the game moves from (2,1), when players 1 and 2 switch roles. If state (1,1) is reached, a final battle takes place, and the winner of this battle wins the grand contest. Of course, the race could start further to the lower left, with more battle wins required for the players, and the required number of battle wins could be asymmetric such that the starting state is some state (n, m), instead of $(2, 2)$. There could also be more than two players, and various assumptions can be made about the nature of the single battles at each state,[45] about possible intermediate prizes from winning single battles, budget constraints etc.

[45]For instance, Harris and Vickers (1987), Klumpp and Polborn (2006), Ferrall and Smith (1999) and Leach (2004) consider races with noise, and Konrad and Kovenock (2006) consider the case in which each battle is an all-pay auction. Malueg and Yates (2006) consider a setup that is very similar to a race.

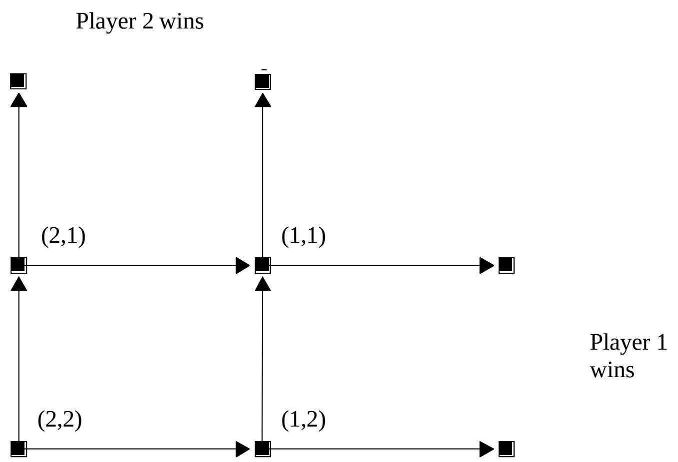

Figure 12: A race with a symmetric starting point (2,2). The game ends if player 1 wins two battles (in which case player 1 wins), or if player 2 wins two battles (in which case player 2 wins).

The tug-of-war A simple tug-of-war between two players is depicted in Figure 13. The game starts at some state $j = 0$. At this state a battle takes place, and the game moves to state $j = -1$ or $j = 1$, depending on whether player 1 or player 2 is the winner of the battle. At the new state a new battle takes place, moving the process one step further to the left or to the right. This process continues until the game reaches one of the terminal states $j = -n$ or $j = n$. Players receive prizes once a terminal state is reached. At $j = -n$ player 1 receives a winner prize and player 2 a loser prize, and vice versa at terminal state $j = n$. Players are impatient and discount effort in a future period or the delay of the distribution of prizes with a discount factor per period. Konrad and Kovenock (2005a) consider this game for positive winner prizes and loser prizes equal to zero, assuming that each battle follows the rules of an all-pay auction. McAfee (2000) considers this game for positive winner prizes but negative loser prizes. McBride and Skapedas (2006) consider a tug-of-war in which the nature of the battle contest differs between states, and where a player who leads enters into a battle which is easier for him to win.

The elimination tournament A simple example of a third structure is depicted in Figure 14. Here four players are in a grand contest. First, players 1 and 2 and players 3 and 4 are pairwise matched and compete in battles that could be called 'semi-finals'. The winners of these semi-finals are admitted to the final and compete for the prize that is awarded to the winner of this

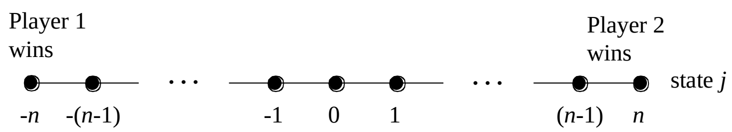

Figure 13: A tug-of-war with symmetric starting state 0 and $2n + 1$ states. State $-n$ is a terminal state in which player 1 finally wins. State n is a terminal state in which player 2 finally wins.

grand contest. Again, different assumptions can be made about the nature of the battles, players' types, their information status etc. It is clear from this picture how the framework generalizes to more than two rounds. Examples in the literature also include cases in which more than two players are matched in a battle, and in which more than one player proceeds to the next round.[46]

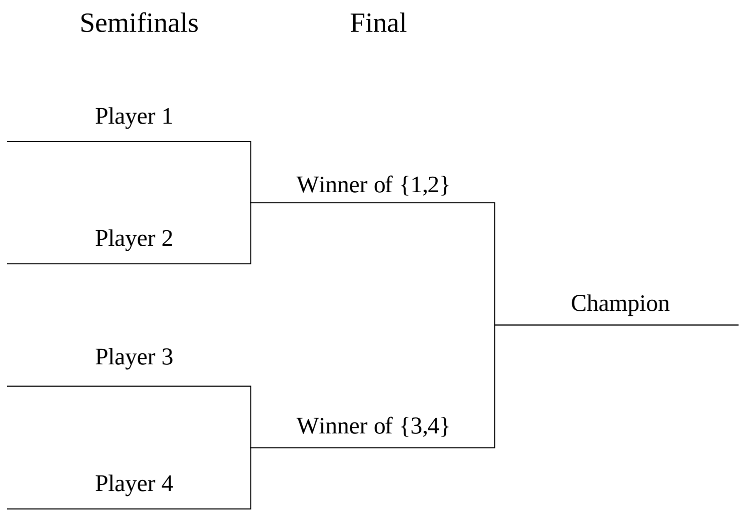

Figure 14: A simple elmination contest with four participants, two semi-finals and one final

The discouragement effect Contests with multiple rounds of the three types illustrated here have a common feature which will essentially provide

[46]The first person who studied this structure systematically and formally was Rosen (1986). More recent contributions are Gradstein and Konrad (1999), Amegashi (1999, 2000), Fu and Lu (2005), Harbaugh and Polborn (2005), Groh, Moldovanu, Sela and Sunde (2003) and Konrad and Kovenock (2006b).

an intuition for their equilibrium properties which could be termed the "discouragement effect". This effect can be illustrated for an elimination tournament as in Figure 14. Suppose, for simplicity, that the players' abilities and valuations of winning are common knowledge, and let players be perfectly symmetric for simplicity. Players 1 and 2 anticipate that a victory in the semi-final will not give them a prize, but that it allows the victorious player w_1 to enter into the final, where he expects to meet player w_2. Symmetry implies that each of the two winners of the semi-finals will receive a payoff

$$\pi_F = p(x^*, x^*) - x^* \tag{110}$$

if x^* is the symmetric equilibrium effort in the final and if the final prize is normalized to 1. This payoff is only a fraction of the prize, and, depending on the nature of the battle that constitutes the final, this fraction can be very small. Recall that, in the lottery contest with symmetric players, this fraction is $1/4$, and if the contest success function in the final is that of the all-pay auction with complete information as in (3), the fraction is even zero. Accordingly, when competing in the semi-final, players 1 and 2 compete for a possibly very small prize. The fact that most of the final prize is dissipated in contest effort in the final discourages the contestants in the semi-final from expending much effort.

Similar discouragement effects occur in other grand contests. For instance, consider the tug-of-war in Figure 13 and assume all players are symmetric, with loser prizes equal to zero and battle contests with a contest success function as in (3). It turns out that the players expend positive effort in expectation only at the symmetric state $j = 0$. Outside this state, for instance in state $j = -1, -2, ...$, player 2 essentially gives up, and player 1 wins in the following sequence of battles that leads straighforwardly to state $j = -n$. The intuition for this result is again the discouragement effect: Consider state $j = -1$: player 2 could try to win the battle and move back to state $j = 0$. However, at state $j = 0$ both players are in a symmetric contest and expect that whoever wins this battle will effortlessly win the grand contest. Accordingly, fighting intensity at state $j = 0$ is very high as –apart from discounting– the full prize of the contest is at stake, and all rents are dissipated in expectation for the all-pay auction without noise. But if nothing can be gained for player 2 by moving from state $j = -1$ to state $j = 0$, then it does not make sense for this player to expend positive contest effort at state $j = -1$ by trying to move the process to state $j = 0$, and similarly for other states to the left of $j = -1$. For states $j = 1, ...(n-1)$ the argument is similar, but players 1 and 2 switch their roles.

McAfee's (2000) results for the tug-of-war are the reverse of those just described, but they are also based on the discouragement effect. Recall that

McAfee assumes that the player who finishes in his terminal loss state receives a negative payoff. Hence, even if the player can never win, he would like to shift this negative payoff outcome into the distant or possibly infinite future. Suppose, for instance, that player 2 starts fighting very hard to avoid losing the battle at state $j = -(n-1)$, as this battle loss implies that he receives the negative loser prize at $j = -n$. Players then anticipate that fighting intensity picks up when the tug-of-war approaches one of the states next to terminal states. There is again a discouragement effect: for player 1, who is trying to win at the states $j = -(n-2)$ it is not worth much effort if the defence activity of the rival player becomes very strong in $j = -(n-1)$. A similar logic applies at the other end of the state space, with players switching roles. Alternatively, players may simply stay in the interior range and keep some distance from the wasteful states next to a terminal state, and may avoid states with high fighting intensity. In the grand contest considered by McAfee, there can be a whole interior range of states at which no player expends positive effort in the equilibrium.

Finally, consider the implications of the discouragement effect for the race. Consider, for instance, state $(1, 2)$. From here the game moves to state $(0, 2)$ at which player 1 receives the prize and player 2 receives zero and the game ends, or it moves to state $(1, 1)$. At state $(1, 1)$, the subgame remaining is a symmetric all-pay auction with complete information. Accordingly, once the game reaches state $(1, 1)$, the full value of the prize is dissipated by the players' efforts in expectation, and the continuation value of reaching this state is zero for both players. This shows that player 2 has nothing to gain at state $(1, 2)$. The intensity of conflict at state $(1, 1)$ discourages player 2 from expending any effort at $(1, 2)$. Hence, one should not expect much effort to be expended at state $(1, 2)$. A similar argument holds at state $(2, 1)$. The player who is advantaged at a state should effortlessly win the battle at this state. Turning then to state $(2, 2)$, and taking into consideration the high payoffs of 1 for reaching state $(1, 2)$, and of player 2 for reaching state $(2, 1)$, the contest at $(2, 2)$ is symmetric and the stakes are high, and this makes the contest at this state rather intense.

What prevents grand contests from becoming trivial? The discouragement effect suggests that players do not expend much effort in early rounds of an elimination tournament, or in asymmetric states in a race or in a tug-of-war, and this prediction may partially hold in the data when considering such games. But overall, we do not observe tennis players leaving the court before the game is over, we observe intense competition in early rounds of soccer world championships and in many other grand contests.

There could be at least three reasons for this.

The first reason is incomplete information. However, as was already observed by Rosen (1986), a formal treatment of incomplete information in grand contests is difficult, due to the possible signalling value of players' choices or observed outcomes in early stages.[47]

Second, grand contests with several rounds typically have a complex prize structure with prizes for the best, second best, third best, etc. There could also be intermediate prizes from winning single battles in a grand contest. Konrad and Kovenock (2006a) have analysed the race, and have shown that such intermediate prizes prevent the race from becoming trivial and boring once one of the players accumulated some advantage. Such intermediate prizes make the race 'pervasive' in the sense that, from a given state, all further feasible trajectories have a positive probability of being reached, and players who are lagging far behind may catch up, take over the lead, and finally win.

A third reason is variability of ability of players over time. As is shown in Konrad and Kovenock (2006b), if contestants' ability or their unit cost of expending effort are not time invariant but instead are random draws for each of the states of a grand contest, this will also counterbalance the discouragement effect. Moreover, they show that players with whose ability is more variable benefit more from participating in a contest and are more successful in reaching later stages of grand contests. They argue that this asymmetry in benefits has implications for the self-selection of individuals: in a population in which players differ in terms of the variability of their ability, and in which individuals have to choose whether to enter into a contest or do something else the individuals whose variability is high should be more likely to decide to enter into the contest, because their expected gain from entering is higher. Moreover, as these individuals are, in expectation, more successful in the course of a grand contest, one should observe that the ability of the set of participants in late stages of grand contests is even more variable.

8 Conclusions

The theory of contests is a field that is growing rapidly. I have been trying to survey some of the classic contributions to this theory and some more recent results. The survey reveals that contest competition is an important and widespread principle for solving allocation problems. Contests sometimes emerge naturally but they are often carefully chosen and designed. Contest

[47]See, e.g., Meyer (1991) and Münster (2006) for further considerations on repeated contests with incomplete information.

games have a large set of dimensions which it is important to consider when studying contests from a positive point of view and also when studying contest design issues. Contest competition shares properties with other types of competition which are more prevalent and have been more carefully studied in economic theory, but it also has some important differences. For instance, the fundamental principle of asymmetry figures prominently in the theory of contests. As in other types of competition, asymmetry is to the advantage of the players who compete with each other and gives them a higher rent. A second principle that is important for understanding contest competition and its role in a larger context of economic interaction, including repeated contests, is the discouragement effect that emerges from participating in a contest in the future. Despite these discouragement effects, I hope that this survey also has an encouragement effect and stimulates further work on this topic.

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