14.74x Land Unit
Homework Assignment

1 Sharecropping (Banerjee, Gertler, Ghatak)

This problem will walk you through a principal agent contracting problem similar to that in the Banerjee, Gertler and Ghatak paper, "Empowerment and Efficiency: Tenancy Reform in West Bengal" (2002). You will not need the paper to do this problem, but you may find it useful. Download the paper on the course website.

Consider the following setting:

- Tenant farms the land and applies effort, e
- Tenant's outside option is 0, and tenant has wealth w
- The landlord cannot observe e
- Both the tenant and the landlord are risk-neutral
- Effort is costly. The cost of effort is $\frac{1}{2}ce^2$
- Two things can happen:
  - With probability e: Output is H
  - With probability $1-e$: Output is 0 (zero)
- The tenant and the landlord write a contract which specifies a payment from the landlord to the tenant
  - Payment of h if output is H
  - Payment of l if output is 0
- Tenant and landlord maximize their expected utility, which is equal to expected income for the landlord, and expected income minus cost of effort for the tenant.

Part A: Basic and First Best

1. What are the possible differences between a sharecropping and a fixed rent contract?
   (a) Landlord gets a fixed share of the output in sharecropping contract, but a fixed amount in fixed rent contract.
   (b) Landlord gets a fixed amount in the sharecropping contract, but a fixed share in a fixed rent contract.
   (c) Tenant gets a fixed share of the output in sharecropping contract, but a fixed amount in fixed rent contract.

(d) Tenant gets a payment that depends upon the output in both the contracts

2. Now, lets find the optimal effort (first best). In the first best, we consider that there are no contractual difficulties. That is, you can specify e directly. Since the landlord can specify e, she will solve the following problem:

$$\max_{e} \Pi(e) = e \cdot H + (1-e)\,0 - \frac{1}{2}ce^2$$

What is the optimal e chosen by the landlord? Call this e_{FB}

Part B: Optimal contract without limited liability

3. **Tenant's choice of effort**: Now assume that effort is not contractible. That is, the landlord cannot specify what level of e must the tenant put. The landlord offers a contract that offers payment h when output is H and payment l when the output is 0. Given this contract, set up tentant's utility maximization problem and find his chosen level of effort e_T as a function of h, l, c

4. Now the landlord wants to ensure that $e_T = e_{FB}$. That is, she wants to set h, and l such that the tenant will always choose the first best level of effort. What will $h - l =$?

5. The landlord maximizes his income. Specifically, the landlord solves the following program:

$$\max_{h,l} e \cdot H + (1-e) \cdot 0 - (e \cdot h + (1-e) \cdot l)$$

such that

$$e \in \arg\max_{\hat{e}} \hat{e} \cdot h + (1-\hat{e}) \cdot l - \frac{1}{2}c \cdot \hat{e}^2$$

$$e \cdot h + (1-e) \cdot l - \frac{1}{2}c \cdot e^2 \geq 0$$

what is the second constraint: $e \cdot h + (1-e) \cdot l - \frac{1}{2}c \cdot e^2 \geq 0$ called?

(a) Landlord's Incentive Compatibility (IC) constraint

(b) Landlord's Participation Constraint (PC)

(c) Tenant's IC constraint

(d) Tenant's PC

6. Assume that the second constraint holds with a equality (if it did not, the landlord loses income). Note that you have already solved the first constraint

$$\arg\max_{\hat{e}} \hat{e} \cdot h + (1-\hat{e}) \cdot l - \frac{1}{2}c \cdot \hat{e}^2$$

in 3. Now, solve the landlord's optimization problem given the two constraints. What is the level of $h - l$ chosen by the landlord? Your answer should be a function of

$H, L(=0)$ and c only. [Hint: Plug in the constraints into the landlord's profit function and rewrite the maximization problem as the landlord choosing $(h-l)$ instead of h and l separately]

7. What level of effort will the tenant choose? (Again, your answer should be in terms of H and c) How does it compare with the optimal effort, e_{FB}?

   She will choose $e_T = \left(\frac{h-l}{c}\right) = \frac{H}{c} = e_{FB}$*, that is, the first best level of output.*

   Note that the landlord is able to make the tenant exert the first best level of effort even though effort is unobserved.

8. Solve for h and l in terms of H and c. To do so, plug your answers to 3 and 4 into the two constraints in 5 and solve. Call your answers h^*_{FB} and l^*_{FB}.

9. What is the tenant's utility under this contract?

Part C: Optimal Contract with Limited Liability

Now suppose that the landlord cannot take more than the tenant's wealth ($l \geq -w$). And also suppose that the l you found in 4e, call it l^*_{FB} is smaller than $-w$. That is $-w > l^*_{FB}$

10. Is the tenant's maximization problem and effort choice given h and l changed?

11. What is l chosen by the landlord? Call your answer l^*_{LL}, express it in terms of H, c, w.

12. Now, the landlords problem is

$$\begin{aligned} &\max_{h,l} e \cdot H - (e \cdot h + (1-e) \cdot l) \\ \text{such that} \\ &e = \frac{h-l}{c} \\ &l = l^*_{LL} \end{aligned}$$

    where the first constraint is the tenant's ICC and the second is the limited liability constraint (LLC). Ignore the participation constraint (PC) for now. Solve the optimization problem and find h. Call it h^*_{LL} and express it in terms of H, c, w.

13. How much effort does the tenant put in with this contract h^*_{LL}, l^*_{LL}? Call this e_{LL} and express it in terms of H, c, w. How does it compare to e_{FB}?

14. How much utility will the tenant get from this contract? Express the utility in terms of H, c, w

15. When will the tenant accept this contract?

    (a) Always

    (b) When $\frac{H^2}{8c} - w \geq 0$

(c) When $\frac{H}{2c} \geq 0$

(d) when $\frac{H^2}{8c} - w \leq 0$

16. Suppose the tenant accepts the contract. Is the tenant better-off with this contract, or the one without limited liability?

2 Land Size and Distribution of Labor

A farming family owns some land. The following information is given to you:

(i) There are six people in the family;

(ii) In any year, the equivalent of two people is needed to farm each acre of land that the family owns;

(iii) the going wage (which each person can earn if he or she so chooses) is $1000;

(iv) each acre of land produces $3000 worth of output (if it is farmed properly by two people, as earlier stated);

(v) the family is always free to lease out land (i.e., charge a rent and give up the output to a tenant), but the outside option for tenants is $1100 per person;

(vi) the family can always hire labor (instead of leasing), but hired labor is useless without supervision, and to hire one supervisor to monitor labor (irrespective of the number of laborers that you hire or acres that they work), costs $2000 per year.

1. Calculate the maximum rent per acre the family can hope to obtain by leasing out land.

2. For simplicity, suppose that the family can either lease out all the land or none of the land, and that land comes in integer units of acres (for example, a family could have 5 or 6 acres, but not 5.5). For a six-person family, what is the minimum acreage necessary for it to be optimal to lease out land? Assume that they charge the maximum rent if they lease the land. Think about how will explain your answer.

3. What is the threshold acreage after which the family wil no longer lease out land, but hire a supervisor and employ wage labor? (Please provide your answer as an integer)