EC5301 Problem set 5

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Problem 4.9 In a Stackelberg duopoly, suppose that firm 1 and 2 face market demand $p = 100 - (q_1 + q_2)$ and costs $c_1 = 10q_1$ and $c_2 = q_2^2$.

a) Calculate market price and each firms profit assuming that firm 1 is the leader and firm 2 is the follower.

b) Do the same assuming that frim 2 is the leader and firm 1 is the follower.

c) Given your answers in (a) and (b) who would firm 1 want to be the leader in the market? Who would firm 2 want to be the leader?

d) If each firm assumes what it wants to be the case in part (c), what are the equilibrium market price and firm profits? How does this compare to the Cournot-Nash equilibrium?

Solution. Firm 1 considers firm 2 response when choosing quantity as to maximize its profit. Firm 2 response function follows from its profit maximization. The profit functions equal,

$$\pi_1 = (100 - q_1 - q_2)q_1 - 10q_1,$$
$$\pi_2 = (100 - q_1 - q_2)q_2 - q_2^2.$$

A necessary first order condition for firm 2 profit maximization is

$$\frac{d\pi_2}{dq_2} = -q_2 + 100 - q_1 - q_2 - 2q_2 = 0,$$

$$q_2 = \frac{100 - q_1}{4}.$$

The optimal quantity for firm 1 then becomes

$$\frac{d\pi_1}{dq_1} = -\frac{3}{4}q_1 + 100 - 25 + \frac{q_1}{4} - q_1 - 10 = 0,$$

$$q_1 = \frac{130}{3}.$$

Substituting firm 2 quantity into firm 1 response function pins down firm 2 response to $q_2 = 170/12$. The market price at this level of supply is

$$p = 100 - q_1 - q_2 = 100 - \frac{130}{3} - \frac{170}{12} = \frac{170}{4} = 42.5.$$

Finally, each firm earns profits

$$\pi_1 = 30 \cdot 130 - \frac{130 \cdot 130}{9} - \frac{130 \cdot 85}{3 \cdot 6} = \frac{65^2}{3} = 1408.33,$$

$$\pi_2 = \frac{170}{4} \cdot \frac{170}{12} - (\frac{170}{12})^2 = 401.38.$$

b) Change the order of the firm decision, using the same method and equations, calculate firm 1 response function

$$\frac{d\pi_1}{dq_1} = 90 - 2q_1 - q_2 = 0,$$

$$q_1 = \frac{90 - q_2}{2}.$$

Firm 2 optimal quantity is then decided by

$$\frac{d\pi_2}{dq_2} = 55 - 3q_2 = 0,$$

$$q_2 = \frac{55}{3}.$$

Implying firm 1 quantity equal to $q_1 = 215/6$. The market price is,

$$p = \frac{600}{6} - \frac{215}{6} - \frac{100}{6} = \frac{275}{6} = 45.83.$$

Firm profits become

$$\pi_1 = \frac{275}{6} \cdot \frac{215}{6} - 10\frac{215}{6} = 1284.03,$$

$$\pi_2 = \frac{275}{6} \cdot \frac{55}{3} - \left(\frac{55}{3}\right)^2 = 504.17.$$

c) In a Cournot equilibrium firm 1 selects the best response given firm 2's quantity decision and vice versa. The simplified response functions are

$$2q_1 = 90 - q_2,$$

$$4q_2 = 100 - q_1.$$

In a Nash equilibrium both equations hold. Multiply the second equation by 2 and subtract it from the first equation, this yields q_2 quantity

$$200 = 90 + 3q_2,$$

$$q_2 = \frac{110}{7} = 15.71.$$

substituting the quantity into the response function gives us q_1 equal to

$$q_1 = \frac{520}{14} = 37.14.$$

The market price is

$$p = 100 - \frac{110}{7} - \frac{520}{14} = 100 - \frac{220 + 520}{14} = \frac{660}{14} = 47.14$$

Firm profits are

$$\pi_1 = \frac{520}{14} \cdot \frac{660}{14} - 10\frac{520}{14} = 1379.59,$$

$$\pi_2 = \frac{110}{7} \cdot \frac{660}{14} - \left(\frac{110}{7}\right)^2 = 493.88.$$

The Cournot equilibrium compared to the Stackelberg equilibrium, results in a higher quantity produced and lower prices. Each firm makes lower profits in Cournot competition compared to being a leader in the Stackelberg case, but higher profits than being a Stackelberg follower.

Problem 4.11 In the Cournot market of Section 4.2.1 suppose that each identical firm has cost function $c(q) = k + cq$, where $k > 0$ is fixed cost.

a) What will be the equilibrium price, market output, and firm profits with J firms in the market?

b) With free entry and exit, what will be the long run equilibrium number of firms in the market?

Solution. a) Each firm i chooses quantity q_i as to maximize profits

$$\pi_i = (a - b\sum q_j)q_i - (k + cq_i).$$

A necessary condition for optimally is

$$\frac{d\pi}{dq_i} = -bq_i + a - b\sum q_j - c = 0.$$

Since all firms are identical, we can substitute the sum for Jq_i. Solving for quantity we find firm and therefore market output equal to

$$q_i = \frac{a-c}{(J+1)b},$$

$$q = Jq_i = J\frac{a-c}{(J+1)b}.$$

The market price p given this quantity q is

$$p = a - b\sum q_i = a - bJq_i = a - b\frac{J(a-c)}{(J+1)b} = \frac{a+Jc}{J+1}.$$

Finally, each firm makes profits

$$\pi_i = \frac{a+Jc}{J+1}\cdot\frac{a-c}{(J+1)b} - c\frac{a-c}{(J+1)b} - k = \frac{(a-c)^2}{(J+1)^2 b} - k.$$

b) In the long run firms in the market earn zero profit $\pi_i = 0$ due to entry and exit from the market. The zero profit condition implies equilibrium firm count,

$$0 = \frac{(a-c)^2}{(J+1)^2 b} - k,$$

$$J = \frac{a-c}{\sqrt{kb}} - 1.$$

Notably, for k=0 the number of firms will be infinite.

Problem 4.15. There are J firms active in a monopolistically competitive market, firm j face demand

$$q_j = \frac{1}{p_j^2}\left(\sum_{i\neq j}\frac{1}{p_i^{1/2}}\right)^{-2}.$$

Each firm have identical costs

$$c(q) = cq + k.$$

a) Show that each firms demand is negatively sloped, with constant own-price elasticity and that all goods are substitutes.

b) Show that if all firms raise thier prices proportionatley the deamnd for any given good declines.

c) Find the long-run Nash equilibrium number of firms

Solution. a) The slope of the demand curve is determined by the derivative with respect to the firms own price, it is

$$\frac{\partial q_j}{\partial p_j} = -2(p_j)^{-3}\left(\sum_{i\neq j}\frac{1}{p_i^{1/2}}\right)^{-2}.$$

The first factor is negative, the second factor is positive for all p_j such that $p_j > 0$ and the expression is only defined if $p_i > 0$ for all $i = 1, ..., J$. Therefore the derivative is negative. The own-price elasticity, expressed without absolute value, is

$$\epsilon = \frac{p_j}{q_j}\frac{\partial q_j}{\partial p_j} = \frac{p_j}{q_j}(-2)(p_j)^{-3}\left(\sum_{i\neq j}\frac{1}{p_i^{1/2}}\right)^{-2} = \frac{-2}{q_j}(p_j)^{-2}\left(\sum_{i\neq j}\frac{1}{p_i^{1/2}}\right)^{-2} = -2,$$

The own-price elasticity is constant. Lastly, the goods being substitutes is shown by fixing j and i such that $j \neq i$ and calculating the cross-price derivative

$$\frac{\partial q_j}{\partial p_i} = (-2)\frac{1}{p_j^2}\left(\sum_{i\neq j}\frac{1}{p_i^{1/2}}\right)^{-3}\cdot(-\frac{1}{2})\frac{1}{p_i^{3/2}} = \frac{1}{p_j^2}\left(\sum_{i\neq j}\frac{1}{p_i^{1/2}}\right)^{-3}\cdot\frac{1}{p_i^{3/2}},$$

all factors are positive if $p_i > 0$ for all $i = 1, .., J$, implying the goods are substitutes.

b) Fix good q_j and raise all prices to $p_j' = k \cdot p_j$, where $k > 1$ for all $j = 1, ..., J$. The demand for good q_j under the new prices is

$$q_j' = \frac{1}{(k\cdot p_j)^2}\left(\sum_{i\neq j}\frac{1}{(k\cdot p_j)^{1/2}}\right)^{-2} = \frac{1}{p_j^2}\frac{1}{k^2}\left(\frac{1}{k^{1/2}}\sum_{i\neq j}\frac{1}{(p_j)^{1/2}}\right)^{-2} = \frac{1}{k}q_j.$$

The new demand is $1/k$ times the old demand. Because $k > 1$, it follows that demand is lower under the new prices.

c) The long run equilibrium is characterized by firm profit maximization $d\pi_i/dq_i = 0$ and firms earning zero profit $\pi_i = 0$ for all $i = 1, ..., j$. The profit function is

$$\pi_j = p_j q_j(p) - cq_j(p_j) - k,$$

$$\pi_j = \frac{1}{p_j^2}\left(\sum_{i\neq j}\frac{1}{p_i^{1/2}}\right)^{-2} p_j - c\cdot\frac{1}{p_j^2}\left(\sum_{i\neq j}\frac{1}{p_i^{1/2}}\right)^{-2} - k.$$

Profit maximization implies the condition

$$\frac{d\pi_i}{dp_i} = -\frac{1}{p_j^2}\left(\sum_{i\neq j}\frac{1}{p_i^{1/2}}\right)^{-2} + 2c\cdot\frac{1}{p_j^3}\left(\sum_{i\neq j}\frac{1}{p_i^{1/2}}\right)^{-2} = 0,$$

$$p_j = 2c.$$

Substituting the price into the zero-profit condition and noting that all prices are equal gives us

$$\frac{1}{(2c)^2}\left((J-1)\frac{1}{(2c)^{1/2}}\right)^{-2} 2c - c \cdot \frac{1}{(2c)^2}\left((J-1)\frac{1}{(2c)^{1/2}}\right)^{-2} - k = 0,$$

$$\frac{1}{2c}\frac{2c}{(J-1)^2} - c \cdot \frac{1}{(2c)^2}\frac{2c}{(J-1)^2} - k = 0,$$

$$\frac{1}{(J-1)^2} - \frac{1}{2}\frac{1}{(J-1)^2} = k.$$

$$\frac{1}{2(J-1)^2} = k,$$

$$J = 1 + \frac{1}{\sqrt{2k}}.$$

The equilibrium amount of firms is $J = 1 + \frac{1}{\sqrt{2k}}$.

Problem 4.19 Solution

(a) Derive the Marshallian demands for x and m. The utility maximizing problem is given by:

max $ln(x) + m$ s.t. $xp + m = y$

Thus, the Lagrangian looks as follows:

$$L = ln(x) + m - \lambda(xp + m - y)$$

The first order conditions are:

$$\frac{\partial L}{\partial x} = \frac{1}{x} - \lambda p = 0 \tag{1}$$

$$\frac{\partial L}{\partial m} = 1 - \lambda = 0 \tag{2}$$

$$\frac{\partial L}{\partial \lambda} = -xp - m + y = 0 \tag{3}$$

From (1) and (2) follows that the Marshallian demand for x is:

$$x(p, y) = \frac{1}{p} \tag{4}$$

From (3) and (4) follows the Marshallian demand for m:

$$m(p, y) = y - 1 \tag{5}$$

Given that m can't be negative, it follows that:

$$m(p, y) = \begin{cases} y - 1 & \text{if } y \geq 1, \\ 0 & \text{else.} \end{cases}$$

(b) We get the indirect utility curve by putting the Marshallian demands into the utility function:

$$v(p, y) = u(x(p, y)) = \begin{cases} ln(\frac{1}{p}) + y - 1 & \text{if } y \geq 1, \\ ln(\frac{1}{p}) & \text{else.} \end{cases}$$

(c) Calculate the substitution effect (note that in this case Marshallian and Hicksian demand are equal as they are only dependent of p):

$$\frac{\partial x^h}{\partial p} = -\frac{1}{p^2}$$

Next, we calculate the income effect:

$$x\frac{\partial x}{\partial y} = 0$$

Thus, we can conclude that the total effect of an own-price change on the demand of x is equal to the substitution effect:

$$\frac{\partial x}{\partial p} = \frac{\partial x^h}{\partial p} = -\frac{1}{p^2}$$

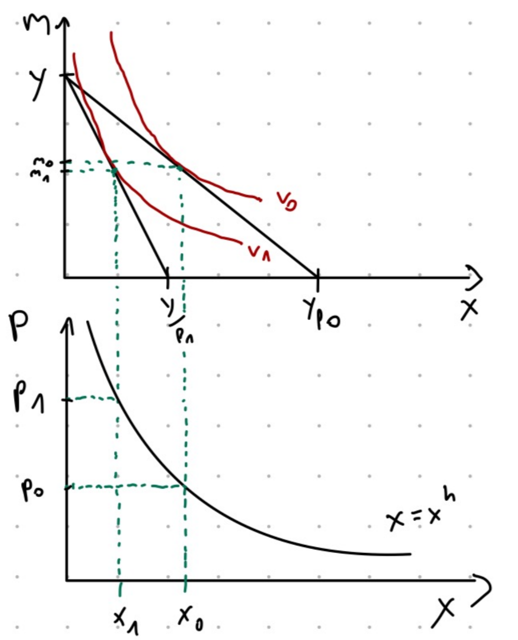
(d) The graphical interpretation of the consumer surplus is the area enclosed by the Hicksian demand curve and the interval between p_0 and p_1. It can be calculated as follows:

$$CS = \int_{p_1}^{p_0} \frac{1}{x}\,dx = ln(p_0) - ln(p_1) = ln(\frac{p_0}{p_1})$$

The change in consumer welfare is the difference between the utility functions given the parameters p_0 and p_1:

$$\Delta u = ln(\frac{1}{p_1}) + y - 1 - ln(\frac{1}{p_0}) - y + 1 = ln(\frac{p_0}{p_1})$$

Those expressions are equal.



(e)

Problem 4.26 Solution

(a) In a long-run equilibrium, firms are price takers and price equals marginal costs. Thus, we can conclude:

$$\frac{\partial c}{\partial q} = p \iff 2q = p \iff q = \frac{p}{2}$$

We also know that firms won't make any profits. Thus:

$$\Pi_j = pq - k^2 - q^2 = 0$$

Using $q = \frac{p}{2}$:

$$\frac{p^2}{2} - k^2 - (\frac{p}{2})^2 = 0 \iff k^2 = \frac{p^2}{4} \iff p = 2k$$

We will calculate the number of firms J because we will need that information in (b):

$$p = 2k \implies q = \frac{2k}{2} = k$$

Using the formula for the price ($p = a - bJq$), we get:

$$p = a - bJq \iff 2k = a - bJk \iff J = \frac{a - 2k}{bk}$$

Because J should not be negative, we assume $a > 2k$.

(b) We do analogue calculations, but include the per-unit tax t:

$$\frac{\partial c}{\partial q} + t = p \iff 2q + t = p \iff q = \frac{p - t}{2}$$

As in (a) the firms profits have to be zero:

$$\Pi_j = pq - tq - k^2 - q^2 = 0$$

Using $q = \frac{p-t}{2}$:

$$\frac{(p-t)^2}{2} - k^2 - (\frac{p-t}{2})^2 = 0 \iff k^2 = \frac{(p-t)^2}{4} \iff p = 2k + t$$

Again, we calculate the number of firms J:

$$p = 2k + t \implies q = \frac{p - t}{2} = \frac{(2k + t) - t}{2} = k$$

Using the formula for the price ($p = a - bJq$), we get:

$$p = a - bJq \iff 2k + t = a - bJk \iff J = \frac{a - 2k - t}{bk}$$

Comparing old and new J, we immediately see that the new J is smaller because $t > 0$. Accordingly, the total output decreases as well, as the quantity produced per firm stayed the same (q=k).

(c) As stated in the lecture notes on slide 70 we can calculate the difference in consumer surplus with the following formula:

$$\Delta CS = \int_{p_1}^{p_0} q(p, y)\, dp$$

In our case, this formula translates to:

$$\Delta CS = \int_{2k+t}^{2k} Q(p, y)\, dp = \int_{2k+t}^{2k} \frac{a-p}{b}\, dp$$

$$= \frac{1}{b}[\frac{4ak - (4k^2)}{2} - \frac{4ak + 2at - (4k^2 + 4kt + t^2)}{2}]$$

$$= \frac{1}{2b}[-2at + 4kt + t^2] = \frac{t}{2b}[-2a + 4k + t]$$

We now calculate the tax revenue (TR) with results from exercise (b):

$$TR = Jqt = \frac{a - 2k - t}{bk}kt = \frac{a - 2k - t}{b}t = \frac{2a - 4k - 2t}{2b}t$$

Now we can show, that the loss in CS exceeds the gain in TR:

$$\Delta CS + TR = \frac{t}{2b}[-2a + 4k + t] + \frac{2a - 4k - 2t}{2b}t$$

$$= -\frac{t^2}{2b} < 0$$

This shows that the loss in CS exceeds the revenue collected by the government.