Applied Econometric Time Series – Problem Set 2

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1   Unit Roots

Problem   State a (trend) stationarity and a unit root condition for the model (assume large samples)

$$y_t = \alpha + \beta t + u_t \tag{1.1}$$

$$u_t = \sum_{j=1}^{4} a_j u_{t-j} + \varepsilon_t \tag{1.2}$$

Problem   Show that the model in (1.1) can be written as

$$\Delta y_t = \gamma_0 + \gamma_1 y_{t-1} + \gamma_2 t + \beta_1 \Delta y_{t-1} + \beta_2 \Delta y_{t-2} + \beta_3 + \Delta y_{t-3} + \varepsilon_t \tag{1.3}$$

where you should express $(\gamma, \beta_1, \beta_2)$ in terms of $(\alpha, \beta, \{a_j\}_{j=1}^{4})$.

Problem   Explain why the unit root hypothesis in (2) can be tested by $\gamma_1 = 0$. What the is the role of γ_0 in (2) under a unit root hypothesis.

Problem   If the hypothesis $\gamma_1 = 0$ is not rejected, which model will you proceed with and which parameters are estimated? If the hypothesis $\gamma_1 = 0$ is rejected (in favor of $\gamma_1 \in (-2, 0)$), which model will you proceed with and which parameters are estimated?

Problem   Find $\mathbb{E}[y_t]$. Is the model in (1) reverting around $\mathbb{E}[y_t]$ under a unit root condition?

Problem   Could you think of a variable where economic theory supports a unit root with drift behavior?

Problem   Discuss drawbacks of detrending a difference stationary model. Discuss drawbacks of differencing a trend stationary model.

2   Unpleasant Exchange Rate Calculations

Problem   Conduct the augmented Dickey-Fuller (ADF) and the Elliott, Rothenberg, and Stock (ERS) type of unit root tests for the SEK/USD exchange rate series (*(A2_SEK_US_ExchangeRates.dta*) ranging from 1971-2019 (yearly data, $T = 49$). What do you conclude? Is the sample size a problem? Is structural change(s) a problem?

Problem   Conduct the ADF and ERS tests, and the Perron unit root tests for the US and UK Industrial Production series (*A2_IP_US.dta* and *A2_IP_UK.dta*) ranging from 1960:Q1-2019:Q4 (quarterly data, $T = 240$) and 1960:Q1-2016:Q4 (quarterly data, $T = 228$), respectively. Make suitable transformations if needed. What do you conclude?

Problem   In addition to the ADF, ERS, and Perron tests, could you see the need of other unit root tests?