Seminar 11 - Solution

Review questions

- Suppose that a monopolist sells to two groups that have constant elasticity demand curves, with elasticity e_1 and e_2. The marginal cost of production is constant at c. What price is charged to each group?
  A: $p_i = \frac{ce_i}{(e_i+1)}$
- Disneyland offers a discount on admissions to residents of Southern California. What kind of price discrimination is this? What does this imply about the elasticity of demand for Disney attractions by Southern Californians?
  A: Third-degree price discrimination. Apparently, the Disneyland administrators believe that residents of Southern California have more elastic demands than other visitors of Disneyland ($e^{SC} > e^{others}$).

Exercises

1. A firm has cost function $TC = 5y^2 - 6y$. The firm can divide demand for the production into two segments, where these demand function look as follows: $D(p_1) = 100 - p_1$ and $D(p_2) = 150 - 3p_2$.
   - What are optimal outputs and prices in both segments, when the firm utilizes third price discrimination? What is the profit of the firm?
     A: Corner solution: $y_2 = 0$, $y_1 = \frac{53}{6} = 8.83 \Rightarrow p_1 = 100 - 8.83 = 91.17$, $p_2 = 50$, $\Pi = 468.2$
   - What is the output, price and profit when the firm cannot use price discrimination?
     A: The inverse demand function has two parts:
     For $y \leq 50$: $p = 100 - y$
     For $y > 50$: $p = \frac{250-y}{4}$
     When solving this problem, we need to be careful about which part of the demand curve we are on. Solving under the assumption that $y > 50$ gives us solution $y = \frac{137}{21} = 6.52$. This is in contradiction with the assumption $y > 50$.
     Thus, we look at the other part of the demand curve where $y \leq 50$, which gives us the same solution as in the first part of this exercise: $y = \dfrac{106}{12} = 8.83$, $p = 91.17$ and $\pi = 468.2$.
2. A family gets electricity from a monopolist. Demand for electricity in the family is $D(p_E) = 150 - 5p_E$. Monopolistic firm produces energy with following costs: $TC(y_E) = 6y_E + 0.1y_E^2$.
   - Calculate optimal output of the firm when producer discriminates perfectly.
     A: $y = 60$
   - What is the price for the last unit sold? What is the profit of the firm?
     A: $p_{LU} = 18$, $\Pi = 720$
   - Calculate optimal output, price and profit when the firm is not able to discriminate.
     A: $y = 40$, $p = 22$, $\Pi = 480$
3. Suppose a mail-order business has a monopoly on video games in the towns of Alexandria and Babylon. These two towns are quite a distance away from each other. The demand for video games in Alexandria is $D(p_A) = 55 - p_A$, and the demand for video games in Babylon is $D(p_B) = 70 - 2p_B$. This monopolist can produce video games at the constant marginal (and average) cost of \$5 per unit.
   - If the firm can ensure that video games sold in Alexandria are not resold in Babylon and vice versa, how many video games will it sell in these two cities? At what prices will the firm sell the games? What will its total profit be?
     A: $y_A = 25, y_B = 30$, $p_A = 30$, $p_B = 20$, $\pi = 1075$
   - Now suppose that it costs \$5 to mail a video game from Alexandria to Babylon and vice versa. How will the monopolist's behavior change?
     A: Above, the difference in prices was \$10. Under that scenario, monopolist would not sell any

games in Alexandria (games would be bought only in Babylon, and a part sent to Alexandria and re-sold for prices between \$25 and \$30). So, the monopolist is to charge prices which differ by \$5 at maximum. We also know from the previous part that people from Babylon are more price sensitive (the price is lower there). The monopolist will thus choose a profit-maximizing price for Babylon p_B and charge $p_A = p_B + 5$ in Alexandria.

- How would the answer to part 2 of this problem change if the mailing cost between the two towns was zero?
  A: The problem reduces to a single market monopoly problem with an aggregate demand function that consists of two parts:
  For $y \leq 20$: $p = 55 - y$
  For $y > 20$: $p = \dfrac{125 - y}{3}$
  We again start by solving under the assumption that $y > 20$, which gives us $y = 55$ – this result satisfies our assumption, so we can compute the price: $p = \frac{70}{3} = 23.33$.
  The total output divides between Alexandria and Babylon such that $y_A = 31.67$ and $y_B = 23.33$. $\pi = \$1008.33$.