Advanced microeconomics problem set 5

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1 Exercise 4.9

1.1 Original question

In a Stackelberg duopoly, one firm is a "leader" and one is a "follower". Both firms know each other's costs and market demand. The follower takes the leader's output as given and picks his own output accordingly (i.e, the follower acts like a Cournot competitor). The leader takes the follower's reactions as given and picks his own output accordingly. Suppose that firms 1 and 2 face market demand, $p = 100 - (q^1 + q^2)$. Firms costs are $c^1 = 10q^1$ and $c^2 = (q^2)^2$.

(a) Calculate market price and each firm's profit assuming that firm 1 is the leader and firm 2 the follower.

(b) Do the same assuming that firm 2 is the leader and firm 1 is the follower.

(c) How do you answers in part (a) and (b) compare with the Cournot-Nash equilibrium in this market?

1.2 Solution

Let l denote the actions of the leader, and f that of the follower. Q denotes the total output level at equilibrium

a) Firm 2 is the follower and solves

$$\Pi_2^f(q^l) = \max_{q \in \mathbb{R}_+} \left(100 - q^l - q\right) q - q^2 \tag{1}$$

An interior solution satisfies the first-order condition

$$100 - q^l - 4q = 0 \tag{2}$$

So for each q^l, the follower's response is given by

$$q_2^f(q^l) = \frac{100 - q^l}{4} \tag{3}$$

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Firm 1 being the leader, takes $q_2^f(q^l)$ as given and solves

$$\Pi_f^1(q^l) = \max_{q\in\mathbb{R}_+} \left(100 - q - q_2^f(q)\right) q - 10q \tag{4}$$

An interior solution satisfies the first-order condition

$$100 - 2q - \frac{100-2q}{4} - 10 = 0 \tag{5}$$

Note that

$$100 - 2q - \frac{100-2q}{4} - 10 = 0 \tag{6}$$
$$\iff \frac{3}{4}(100-2q) = 10 \tag{7}$$
$$\iff (100-2q) = \frac{40}{3} \tag{8}$$
$$\iff \frac{260}{3} = 2q \tag{9}$$

So firm 1 being the leader, its optimal quantity is given by

$$q_1^l = \frac{130}{3} \approx 43.33 \tag{10}$$

Then in optimum, the follower chooses

$$q_2^f(q_1^l) = \frac{170}{12} \approx 14.16 \tag{11}$$

Aggregate output and equilibrium price satisfy

$$Q = 115/2 = 57.5 \qquad p = \frac{85}{2} = 42.5 \tag{12}$$

Equilibrium profits are

$$\Pi_1^l = \frac{4225}{3} \approx 1408.33 \qquad \Pi_2^f = \frac{57800}{144} \approx 401.388 \tag{13}$$

b) Same as in part (a)
Firm 1 is the follower and solves

$$\Pi_1^f(q^l) = \max_{q\in\mathbb{R}_+} \left(100 - q^l - q\right) q - 10q \tag{14}$$

An interior solution satisfies the first-order condition

$$100 - q^l - 2q - 10 = 0 \tag{15}$$

So for each q^l, the follower's response is given by

$$q_1^f(q^l) = \frac{90-q^l}{2} \tag{16}$$

Firm 2 being the leader, takes $q_1^f(q^l)$ as given and solves

$$\Pi_f^2(q^l) = \max_{q\in\mathbb{R}_+} \left(100 - q - q_1^f(q)\right) q - q^2 \tag{17}$$

An interior solution satisfies the first-order condition

$$100 - 2q - \frac{90 - 2q}{2} - 2q = 0 \tag{18}$$

which can be solved for q. The optimal q_2^l for firm 2 is given by

$$q_2^l = \frac{55}{3} \approx 18.33 \tag{19}$$

Then in optimum, the follower chooses

$$q_1^f(q_2^l) = \frac{430}{12} \approx 35.83 \tag{20}$$

Aggregate output and equilibrium price satisfy

$$Q = 325/6 \approx 54.16 \qquad p = \frac{275}{6} \approx 45.83 \tag{21}$$

Equilibrium profits satisfy

$$\Pi_1^f = \frac{215^2}{6^2} \approx 1284.03 \qquad \Pi_2^l = \frac{4225}{3} \approx 504.17 \tag{22}$$

c) In the Nash-equilibrium, each firm's strategy must be a best-reply against the other firm's strategy. Hence, the firm's strategy conditions on the other's choice. Conditional on the other firm's choice, the first-order condition needs to hold. Hence, an equilibrium solves simultaneously the two first-order conditions

$$q_1(q_2) = \frac{90 - q_2}{2} \tag{23}$$

$$q_2(q_1) = \frac{100 - q_1}{4} \tag{24}$$

Solving this system gives

$$q_1 = \frac{260}{7} \approx 37.14 \qquad q_2 = \frac{110}{7} \approx 15.71 \tag{25}$$

The equilibrium quantity and price are given by

$$Q = \frac{370}{7} = 52.85 \qquad p = \frac{330}{7} \approx 47.14 \tag{26}$$

Equilibrium profits are given by

$$\Pi_1^c = \frac{260^2}{7^2} \approx 1379.59 \qquad \Pi_2^c = \frac{110 \times 220}{7^2} \approx 493.88 \tag{27}$$

	Firm 1 leads	Firm 2 leads	Cournot
Quantities			
q_1	43.3	35.8	37.1
q_2	14.2	18.3	15.7
Q	57.5	54.2	52.9
Prices			
p	42.5	45.8	47.1
Profits			
Π_1	1408.3	1284.0	1379.6
Π_2	401.4	504.2	493.9

Conclusion We see that compared to the Cournot oligopoly (simultaneous move-game), firm 1 generates higher profits if it leads, and lower profits if it follows. Likewise, if firm 2 is the Stackelberg leader, it earns higher profits than both under Cournot oligopoly and as a Stackelberg follower. If you take a look, at the follower's best response functions to the Stackelberg leader's action, you can see that the follower's optimal quantity decreases in the leader's quantity. Therefore, the term $\partial q^f(q)/\partial q$ in the leader's first-order condition is negative. The price decrease of an additional quantity supplied by the leader, is partially offset by the reduction in the follower's output. Therefore, the leader produces a larger quantity and consequently generates higher profits.[1]

2 Exercise 4.11

2.1 Original question

In the Cournot market of Section 4.2.1, suppose that each identical firm has cost function $c(q) = k + cq$, where $k > 0$ is fixed cost.

(a) What will be the equilibrium price, market output and firm profits with J firms in the market?

(b) With free entry and exit, what will be the long-run equilibrium number of firms in the market.

2.2 Solution

The firm solves the profit maximization problem

$$\pi(q_i, \mathbf{q}_{-i}) = \max_{q_i \in \mathbb{R}_+} \pi(q_i, \mathbf{q}_{-i})q_i - c(q_i) \tag{28}$$

An interior solution satisfies the first-order condition

$$\left(\frac{\partial p(q_i, \mathbf{q}_{-i})}{\partial q_i}\right) q_i + p(q_i, \mathbf{q}_{-i}) - \frac{\partial c({}_iq)}{\partial q_i} = 0 \tag{29}$$

If we denote $Q = \sum_{i=1} q_i$ and our functional forms for $p(\cdot)$ and $c(\cdot)$, we obtain

$$-bq_i + (a - bQ) - c = 0 = 0 \tag{30}$$

[1] A game-theory approach of thinking is that if a firm becomes the leader, then it can still choose the Cournot equilibrium output level as its choice, and the follower will also choose the Cournot equilibrium output as the best response. This is ensured because every equilibrium is a pair of "best responses". So the leader in Stackelberg game cannot earn lower profit than in Cournot game.

which implies that in optimum the firm output satisfies

$$q_i = q = \frac{a - bQ - c}{b} \tag{31}$$

Note that given that the cost structure is identical across firms, $q_i = q$ for some q for all $i \in \mathcal{I}$. All firms place the same output in optimum. Then $Q = Jq$. Multiplying both sides of the equation above gives

$$Q(J) = J\left(\frac{a - bQ - c}{b}\right) \tag{32}$$

which can be solved for Q. In optimum aggregate output is given by

$$Q(J) = \frac{J(a-c)}{b(J+1)} \tag{33}$$

Hence, a representative firm produces

$$q(J) = \frac{a-c}{b(J+1)} \tag{34}$$

which is positive if $a - c > 0$. Prices are then given by

$$p(J) = \frac{a + Jc}{J+1} \tag{35}$$

Equilibrium profits satisfy

$$\Pi(J) = \left(\frac{a-c}{\sqrt{b}(J+1)}\right)^2 - k \tag{36}$$

b) In a long-run equilibrium, J is endogenous and determined through the zero-profit condition. The long-run equilibrium number of firms $\bar{J}$ satisfies

$$\Pi(\bar{J}) = 0 \tag{37}$$

$$\iff \left(\frac{a-c}{\sqrt{b}(\bar{J}+1)}\right)^2 = k \tag{38}$$

$$\iff \bar{J} = \frac{a-c}{\sqrt{bk}} - 1 \tag{39}$$

3 Exercise 4.15

3.1 Original question

When firms $j = 1, ..., J$ are active in a monopolistically competitive market, firm j faces the following demand function:

$$q^j = (p^j)^{-2}\left(\sum_{i=1, i\neq j}^{n} (p^i)^{-\frac{1}{2}}\right)^{-2}, \quad \forall j = 1, ..., J \tag{40}$$

Active or not, each of the many firms, $j = 1, 2, ...$ has identical costs

$$c(q) = cq + k, \tag{41}$$

where $c > 0$ and $k > 0$. Each firm chooses its price to maximise profits, given the prices chosen by the other.

a) Show that each firm's demand is negatively sloped, with constant own-price elasticity, and that all goods are substitutes for each other.

b) Show that if all firms raise their prices proportionately, the demand for any given good declines.

c) Find the long-run Nash equilibrium number of firms.

3.2 Solution

a) The partial derivative of the demand function with respect to the firm's own price is given by

$$\frac{\partial q^j(\mathbf{p})}{\partial p_j} = -2(p^j)^{-3}\left(\sum_{i=1,i\neq j}^{n}(p^i)^{-\frac{1}{2}}\right)^{-2} \tag{42}$$

$$= -2(p^j)^{-1}q_j < 0 \tag{43}$$

The own-price elasticity is given by

$$\epsilon = \frac{\partial q^j}{\partial p^j}\frac{p^j}{q^j} \tag{44}$$

$$\epsilon = -2(p^j)^{-1}q_j\left(\frac{p^j}{q^j}\right) = -2 \tag{45}$$

If two goods are substitutes, then if the price of one good falls, the demand for the other good falls as well.

$$\frac{\partial q^j}{\partial p_i} = (p^j)^{-2}(p^i)^{-\frac{3}{2}}\left(\sum_{i=1,i\neq j}^{n}(p^i)^{-\frac{1}{2}}\right)^{-3} \tag{46}$$

$$= (p^i)^{-\frac{3}{2}}\left(\sum_{i=1,i\neq j}^{n}(p^i)^{-\frac{1}{2}}\right)^{-1} q_j > 0 \tag{47}$$

We can see that all goods are substitutes for each other.

b) Assume that all firms scale up their price by a factor $\lambda > 1$, that is, the new prices are given by λp_j. Let q_j^0 be the initial price. Then the new price satisfies q_j'

$$q_j' = (\lambda p^j)^{-2}\left(\sum_{i=1,i\neq j}^{n}(\lambda p^i)^{-\frac{1}{2}}\right)^{-2} \tag{48}$$

$$= (p^j)^{-2}\lambda^{-2}\left(\lambda^{-\frac{1}{2}}\sum_{i=1,i\neq j}^{n}(p^i)^{-\frac{1}{2}}\right)^{-2} \tag{49}$$

$$= (p^j)^{-2}\lambda^{-2}\lambda\left(\sum_{i=1,i\neq j}^{n}(p^i)^{-\frac{1}{2}}\right)^{-2} \tag{50}$$

$$= \lambda^{-1}q_j^0 < q_j^0 \tag{51}$$

c) Find the long-run Nash equilibrium number of firms. First, we need to determine the profit function of a firm.

$$\Pi = \max_{p_j \in \mathbb{R}_{++}} p_j q_j(p_j) - c(q_j(p_j)) \tag{52}$$

An interior solution satisfies the first-order condition

$$\frac{\partial q_j(p_j)}{\partial p_j} p_j + q_j(p_j) - \frac{\partial c(q_j(p_j))}{\partial p_j} \frac{\partial q_j(p_j)}{\partial p_j} = 0 \tag{53}$$

Rearranging gives

$$p_j = \frac{\partial c(q_j(p_j))}{\partial p_j} - \left(\frac{\partial q_j(p_j)}{\partial p_j} \right)^{-1} q_j(p_j) \tag{54}$$

We can now impose our functional form assumptions. The marginal cost is given by

$$\frac{\partial c(q_j(p_j))}{\partial q_j} = c \tag{55}$$

Using this expression together with the partial derivative in (43) gives

$$\begin{aligned} p_j &= \frac{\partial c(q_j(p_j))}{\partial q_j} - \left(\frac{\partial q_j(p_j)}{\partial p_j} \right)^{-1} q_j(p_j) && (56) \\ &= c - \left(-2(p^j)^{-1} q_j \right)^{-1} q_j(p_j) && (57) \\ &= c + \frac{p^j}{2} && (58) \end{aligned}$$

Hence, prices are given by

$$p^j = 2c \tag{59}$$

for all $j \in \{1, ..., J\}$.

$$\begin{aligned} q^j &= (2c)^{-2} \left(\sum_{i=1, i \neq j}^{n} (2c)^{-\frac{1}{2}} \right)^{-2} && (60) \\ &= (2c)^{-2} \left((J-1)(2c)^{-\frac{1}{2}} \right)^{-2} && (61) \\ &= (2c)^{-2} (J-1)^{-2} (2c) && (62) \\ &= (2c)^{-1} (J-1)^{-2} && (63) \end{aligned}$$

Equilibrium profits are then given by

$$\begin{aligned} \Pi &= (2c - c) \left((2c)^{-1} (J-1)^{-2} \right) - k && (64) \\ &= \frac{1}{2(J-1)^2} - k && (65) \end{aligned}$$

Now impose the zero profit condition. We are looking for $\bar{J}$ that satisfies $\Pi(\bar{J}) = 0$.

$$\Pi(\bar{J}) = 0 \tag{66}$$
$$\iff \frac{1}{2(\bar{J}-1)^2} - k = 0 \tag{67}$$
$$\iff \frac{1}{2k} = (\bar{J}-1)^2 \tag{68}$$
$$\iff \bar{J} = \frac{1}{\sqrt{2k}} + 1 \tag{69}$$
$$\tag{70}$$

4 Exercise 4.19

4.1 Original question

A consumer has preferences over the single good x and all other goods m represented by the utility function, $u(x,m) = \ln(x) + m$. Let the price of x be p, the price of m be unity and let income be y.

a) Derive the Marshallian demands for x and m.

b) Derive the indirect utility function, $v(p,y)$.

c) Use the Slutsky equation to decompose the effect of an own-price change on the demand for x into an income and substitution effect. Interpet your result briefly.

d) Suppose that the price of x rises from p^0 to $p^1 > p^0$. Show that the consumer surplus area between p^0 and p^1 gives an exact measure of the effect of the price change on consumer welfare.

e) Carefully illustrate your findings with a set of two diagrams: one giving the indifference curves and budget constraints on top and the other giving the Marshallian and Hicksian demands below. Be certain that your diagrams reflect all qualitative information on preferences and demands that you have uncovered. Be sure to consider the two prices p^0 and p^1, and identify the Hicksian and Marshallian demands.

4.2 Solution

a) The indirect utility function $v(p,y)$ satisfies the utility maximization problem

$$v(\mathbf{p},y) = \max_{(m,x)\in\mathbb{R}^2_+} \ln(x) + m \quad \text{s.t.} \quad xp + m \leq y \tag{71}$$

Let λ be a nonnegative multiplier associated with the budget constraint. We form the Lagrangian.

$$\mathcal{L} = \ln(x) + m + \lambda(y - xp - m) \tag{72}$$

Since the utility function is strictly increasing in both arguments, we know that the budget constraint must bind. Hence, an interior solution satisfies the first-order conditions

$$\frac{\partial \mathcal{L}}{\partial x} = x^{-1} - \lambda p = 0 \tag{73}$$
$$\frac{\partial \mathcal{L}}{\partial m} = 1 - \lambda = 0 \tag{74}$$

In addition, the budget constraint must bind.

$$xp + m = y \tag{75}$$

We see that in an interior solution $x = p^{-1}$, so the amount that the agent spends on good x is fixed at $px = 1$. Budget balance then implies that $m = y - 1$. But since we require $(x, m) \in \mathbb{R}^2_+$, in order for this bundle to be admissible, it must be that $y \geq 1$. If $y < 1$, then $x = p^{-1}y$ and $m = 0$. Therefore, the Marshallian demand functions are given by

$$x(p, y) = \begin{cases} p^{-1} & \text{if } y \geq 1 \\ p^{-1}y & \text{if } y < 1 \end{cases} \tag{76}$$

$$m(p, y) = \begin{cases} y - 1 & \text{if } y \geq 1 \\ 0 & \text{if } y < 1 \end{cases} \tag{77}$$

b) The indirect utility function is given by $v(p, y) = u(x(p, y), m(p, y))$.

$$v(p, y) = \begin{cases} y - \ln(p) - 1 & \text{if } y \geq 1 \\ \ln(y) - \ln(p) & \text{if } y < 1 \end{cases} \tag{78}$$

c) In what follows I will assume that $y > 1$, so that an interior solution exists. Recall that the Slutsky equation is defined as

$$\frac{\partial x_i(p, y)}{\partial p_j} = \frac{\partial x_i^h(p, y)}{\partial p_j} - x_j(p, y)\frac{\partial x_i(p, y)}{\partial y} \tag{79}$$

For $y > 1$, the partial derivatives of the Marshallian demand function are given by

$$\frac{\partial x_i(p, y)}{\partial p_i} = -p^{-2} \qquad \frac{\partial x_i(p, y)}{\partial y} = 0 \tag{80}$$

Demand for good x does not exhibit any income effect. The substitution effect captures the entire consumption response.

$$\frac{\partial x_i^h(p, y)}{\partial p_j} = \frac{\partial x_i(p, y)}{\partial p_j} = p^{-2} \tag{81}$$

d) Theorem 1.8 tells us that indirect utility and expenditure function satisfy

$$v(\mathbf{p}, e(\mathbf{p}, u)) = u \tag{82}$$

Exploiting this relationship, we can retrieve the expenditure function

$$e(p, u) = u + \ln(p) + 1 \tag{83}$$

Then, the compensating variation is given by

$$\begin{aligned} CV &= e(p^1, u^0) - e(p^0, u^0) &(84)\\ &= \ln[p^1] - \ln[p^0] &(85) \end{aligned}$$

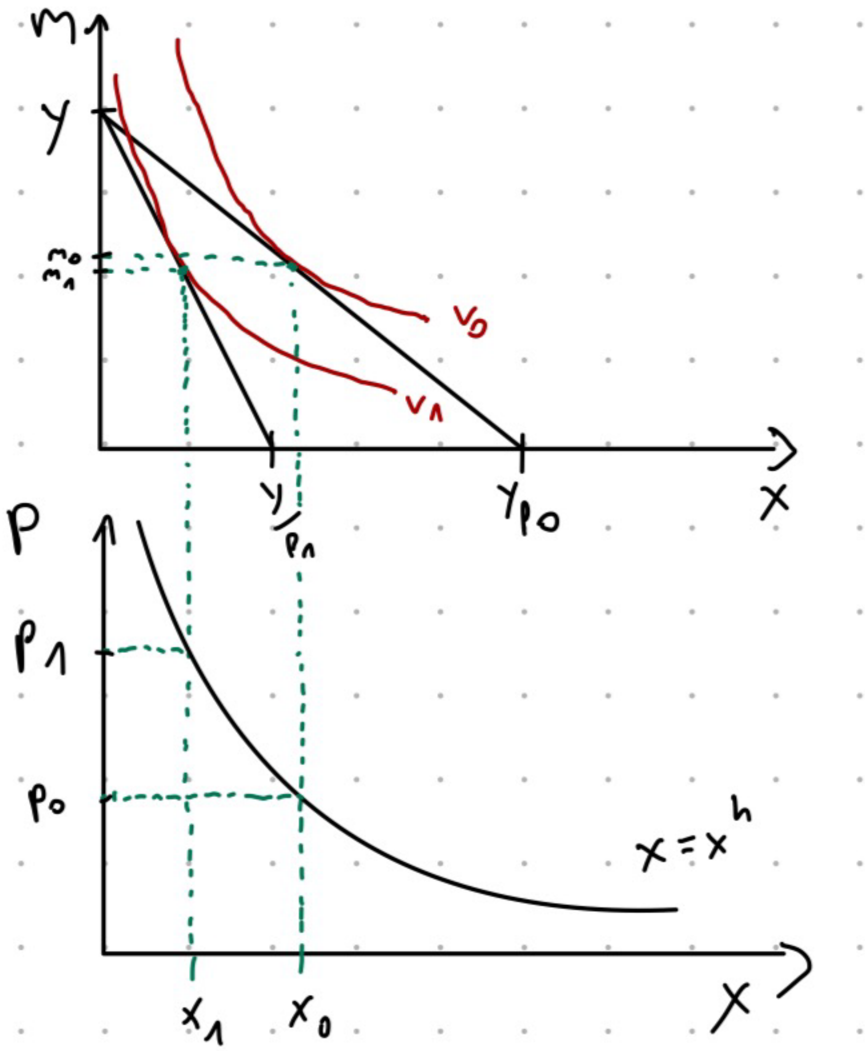

Figure 1: Figure for 4.19

The gain in consumer surplus is the change in the area under demand curve.

$$\begin{aligned}
\Delta CS &= \int_{p^1}^{p^0} x(p,y)dp && (86)\\
&= \int_{p^1}^{p^0} (p)^{-1}dp && (87)\\
&= [\ln(p)]_{p^1}^{p^0} && (88)\\
&= \ln[p^0] - \ln[p^1] && (89)\\
&= -CV && (90)
\end{aligned}$$

e) We look at figure 4.5 in textbook and do it reversely. Fig 4.5 shows what happens when price decreases. See figure 1. [2] Notice that because there is no income effect, we have no compensating variation, and thus there are only two budget lines in the graph.

[2]credit to one of the groups. If you want your name here, email me :D

5 Exercise 4.26

5.1 Original question

A competitive industry is in long-run equilibrium. Market demand is linear, $p = a - bQ$, where $a > 0, b > 0$, and Q is market output. Each firm in the industry has the same technology with cost function, $c(q) = k^2 + q^2$.

(a) What is the long-run equilibrium price? (Assume what is necessary of the parameters to ensure that this is positive and less than a.)

(b) Suppose that the government imposes a per-unit tax, $t > 0$, on every producing firm in the industry. Describe what would happen in the long run to the number of firms in the industry. What is the post-tax market equilibrium price? (Again, assume whatever is necessary to ensure that this is positive and less than a.) '

(c) Calculate the long-run effect of this tax on consumer surplus. Show that the loss in consumer surplus from this tax exceeds the amount of tax revenue collected by the government in the post-tax market equilibrium.

(d) Would a lump-sum tax, levied on producers and designed to raise the same amount of tax revenue, be preferred by consumers? Justify your answer.

5.2 Solution

(a) Notice that in the long-run, firms have marginal cost equals prices. $p = MC = c'(q) = 2q$. So we find the relation between p and q in the long-run equilibrium:

$$q = \frac{p}{2} \tag{91}$$

Then, the profit for every firm is:

$$\begin{aligned} \Pi &= pq - c(q) && (92)\\ &= p\frac{p}{2} - (k^2 + (\frac{p}{2})^2) && (93)\\ &= \frac{p^2}{4} - k^2 && (94) \end{aligned}$$

At equilibrium, firms earn zero profit, i.e. we need $\Pi = 0$. So the long-run price level is:

$$p = 2k \tag{95}$$

We can also get the corresponding output per firm, and the number of firms J at equilibrium:

$$q = \frac{p}{2} = k \tag{96}$$

$$\begin{aligned} p = a - bQ = a - bJq &= 2k && (97)\\ \to a - bJk &= 2k && (98)\\ \to J &= \frac{a-2k}{bk} && (99) \end{aligned}$$

Here, we need the assumption that $a > 2k$. We denote quantities before tax with subscripts 0, like J_0, Q_0.

(b) With the per-unit tax t, cost becomes $c(q) = k^2 + q^2 + tq$, so the marginal cost becomes:

$$MC = c'(q) = 2q + t \tag{100}$$

$p = MC = c'(q) = 2q + t$. So we find the relation between p and q in the long-run equilibrium:

$$q = \frac{p - t}{2} \tag{101}$$

Then, the profit for every firm is:

$$\Pi = pq - c(q) \tag{102}$$
$$= p\frac{p-t}{2} - (k^2 + (\frac{p-t}{2})^2 + t\frac{p-t}{2}) \tag{103}$$
$$= \frac{(p-t)^2}{4} - k^2 \tag{104}$$

At equilibrium, firms earn zero profit, i.e. we need $\Pi = 0$. So the long-run price level is:

$$p_t = 2k + t \tag{105}$$

We can also get the corresponding output per firm, and the number of firms J at equilibrium:

$$q = \frac{p-t}{2} = k \tag{106}$$

$$p = a - bQ = a - bJq = 2k + t \tag{107}$$
$$\to a - bJk = 2k + t \tag{108}$$
$$\to J = \frac{a - 2k - t}{bk} \tag{109}$$

So we see the number of firms are smaller after per-unit tax. The price increases, and quantity in total $Q_t = J_t k$ also drops. We denote quantities after per-unit tax with subscripts t, like J_t, Q_t. We also replace the assumption with $a - 2k - t > 0$.

(c) By definition of consumer surplus(for example, check figure 4.7, page 187 on book), we have:

$$CS = \int_0^Q (a - b\hat{Q})d\hat{Q} - (a - bQ)Q \tag{110}$$
$$= [a\hat{Q} - \frac{b\hat{Q}^2}{2}]|_0^Q - (a - bQ)Q \tag{111}$$
$$= aQ - \frac{bQ^2}{2} - aQ + bQ^2 \tag{112}$$
$$= \frac{bQ^2}{2} \tag{113}$$

Plugging in the corresponding total output level before tax and we have:

$$CS_0 = \frac{b}{2}(J_0 k)^2 = \frac{(a-2k)^2}{2b} > 0 \tag{114}$$

Similarly, plugging in Q_t, which is total output after per-unit tax, and we have:

$$CS_t = \frac{(a - 2k - t)^2}{2b} \tag{115}$$

And thus:

$$\Delta CS = CS_t - CS_0 = \frac{(a-2k-t)^2}{2b} - \frac{(a-2k)^2}{2b} < 0 \tag{116}$$

The absolute value of CS loss is: $|\Delta CS| = t\frac{2(a-2k)-t}{2b}$, the total tax revenue is: $T = tQ_t = tJ_t k = \frac{t(a-2k-t)}{b}$. So we see that $|\Delta CS| > T > 0$, which proves the claim in the question.

(d) Because the problem becomes significantly complicated when we look at long run equilibrium after change, Paul and I think the author's intention is to look at the short run equilibrium. So we have an additional assumption that the number of firms J is equal to that in per-unit tax equilibrium.

In the previous equilibrium we had $q = k > 0$ and $J = \frac{a-2k-t}{bk} > 0$. So we can calculate the total tax revenue as:

$$R_T = t \cdot q \cdot J = \frac{t(a-2k-t)}{b} \tag{117}$$

Then, denote T as the corresponding lump-sum tax per firm that all J firms have to pay in the new equilibrium. We have $T \cdot J = R_T$, which gives:

$$T = t \cdot k \tag{118}$$

Then the firm's cost function becomes:

$$c(q) = k^2 + q^2 + T = k^2 + q^2 + tk \tag{119}$$

The marginal cost is $MC(q) = c'(q) = 2q$, and since this is a competitive market, we have $p = MC = 2q$, and

$$q = \frac{p}{2} \tag{120}$$

The aggregate supply function is:

$$Q = Jq = \frac{a-2k-t}{bk}\frac{p}{2} \tag{121}$$

From the aggregate demand function we have:

$$Q = \frac{a-p}{b} \tag{122}$$

Combining these two equations above we have:

$$\frac{a-p}{b} = \frac{a-2k-t}{bk}\frac{p}{2} \tag{123}$$

$$\Rightarrow p = 2k\frac{a}{a-t} \tag{124}$$

So we have solved the equilibrium price when there is a lump sum tax T. Denote this price as p_{LS}, and the price with per unit tax in the previous question as $p_t = 2k + t$. We can calculate:

$$p_{LS} - p_t = \frac{2ka}{a-t} - (2k+t) \tag{125}$$

$$= t^2 + 2kt - ta \tag{126}$$

$$= t(t + 2k - a) \tag{127}$$

We saw our assumption in question (a) is $a - 2k - t > 0$, so this means $p_{LS} - p_t < 0$, and we have lower price when there is a lump-sum tax. So consumer prefers a lump-sum tax.