Handout 8: Cournot Competition

1 Introduction

In this handout we will explore various models of competition throught two worked exercises. In the first exercise, we will compare Perfect Competition, Cournot and Cartel outcomes in a model with linear demand. The objective is to show that the market structure has significant effect on prices and quantities in equilibrium. In the second exercise, we will show that some properties of the equilibrium can be computed even without solving for the equilibrium itself. We will focus in a Cournot model with N firms. We are interested in computing the equilibrium Lerner Index in the market. One way would be to solve the Cournot problem to compute how much each firm would produce, and replace it back in the demand to compute prices. Although this is feasible with equal costs for all firms, it would be very hard for models with different marginal costs for every firm (given that to do that we must solve a system of N unknowns - quantity each firm produces - and N equations - the equilibrium conditions). We will show, however, that the Lerner Index can be easily computed and interpreted.

2 Perfect Competition, Cartels and Cournot

Suppose a market has $i = 1, ..., N$ firms with marginal cost given by c and no fixed costs. The inverse demand function is given by:

$$P = a - bQ$$

Assume that $c < a$ if needed.

1. What will be the long run equilibrium price and quantity under perfect competition with free entry?
2. What will be the equilibrium price and total quantity produced if firms form a cartel?
3. What will be the equilibrium price and total quantity produced in a symmetric (where all firms produce the same quantity) Cournot equilibrium?
4. Compare the Perfect Competition, Cartel and Cournot equilibrium price and quantity.

Solution.

1. Since all firms are identical, the equilibrium price will be equal to the average total costs (ATC) in the long run under perfect competition. Since there are no fixed costs, equilibrium price will be equal to marginal cost c.

2. If firms form a cartel, they choose the total quantity of the market Q together and maximize the sum of profits of all firms - that is, they act as a monopolist. The optimization problem of the cartel is then given by

$$\max_{Q}(P(Q)-c)Q$$

which is the sum of the profits of all firms in the cartel. We can substitute in the demand function and recover:

$$\max_{Q}(a-bQ-c)Q$$

The FOC with respect to Q is given by

$$a-bQ^{Cartel}-c-bQ^{Cartel}=0\Rightarrow Q^{Cartel}=\frac{a-c}{2b}$$

The price implied by this quantity (replacing in the demand) is given by

$$P^{Cartel}=a-bQ^{Cartel}=a-b\frac{a-c}{2b}\Rightarrow P^{Cartel}=\frac{a+c}{2}$$

3. To compute the Cournot equilibrium, we find first the best response function of firm 1. The best response of firm 1 is given by maximizing the profit of firm 1 *given* what firms 2,..,N are doing:

$$\max_{q_1}\underbrace{\left(a-bq_1-b\sum_{j=2}^{N}q_j\right)}_{P}q_1-cq_1$$

The FOC is given by

$$\begin{aligned}
a-bq_1-b\sum_{j=2}^{N}q_j-bq_1 &= c\\
a-2bq_1-b\sum_{j=2}^{N}q_j &= c\\
q_1 &= \frac{a-c}{2b}-\frac{\sum_{j=2}^{N}q_j}{2}
\end{aligned}$$

Since all firms are identical, we will use here the assumption that all firms produce the same quantity (there is a way to prove this for a Cournot game, but this goes beyond the scope of this class). Using that the equilibrium will be symmetric, we know that

$$q_1 = q_2 = ... = q_N = q$$

So from the best response function we derived above for firm 1 we get

$$\begin{aligned} q &= \frac{a-c}{2b} - \frac{(N-1)\,q}{2} \\ q\left[1 + \frac{(N-1)}{2}\right] &= \frac{a-c}{2b} \\ q &= \frac{a-c}{(1+N)b} \end{aligned}$$

Then the total quantity produced is

$$\begin{aligned} Q^{PC} &= Nq \\ Q^{PC} &= \frac{a-c}{b}\frac{N}{1+N} \end{aligned}$$

and the price is

$$\begin{aligned} P &= a - bQ^{PC} \\ &= a - b\frac{a-c}{b}\frac{N}{1+N} \Rightarrow P^{PC} = a - \frac{N}{N+1}(a-c) \end{aligned}$$

4. As N increases the market quantity increases and the price decreases. As N increases, the market quantity gets divided into a larger number of smaller firms. Each firm cares only about its own profits, so when it decides how much to produce it only cares about the effect of the drop in prices on its own profits, not on the profits of all of its competitors. This effect makes the Cournot equilibrium move closer to the competitive equilibrium as N grows (you can check that as N goes to infinity, the Cournot equilibrium converges to perfect competition). For $N = 1$, the result of the Cournot is the same as in the Cartel. One firm choosing quantities as a monopolist or all firms behaving *one* yields the same results. Therefore, the Cournot model accommodates a Monopoly/Cartel outcome with $N = 1$ and approaches perfect competition with $N \to \infty$.

□

3 Number for Firms and Equilibrium Prices

Suppose a market has $i = 1, ..., N$ firms with marginal cost given by c. These firms face a demand given by:

$$Q_D(P) = P^{-\sigma}$$

1. Compute the elasticity of demand.
2. Show that the Lerner Index (a profitability measure), which is defined as

$$L \equiv \frac{P - C}{P}$$

   can be written as:

$$L = \frac{1}{N\sigma}$$

   in the Cournot equilibrium. Interpret this result.

Solution.

1. The elasticity of demand is given by:

$$\varepsilon_D = \frac{dQ_D(P)}{dP}\frac{P}{Q} = -\sigma P^{-\sigma-1}\frac{P}{P^{-\sigma}} \Longrightarrow \varepsilon_D = -\sigma$$

   This demand has a constant elasticity given by σ.

2. The maximization problem of each firm is given by:

$$\max_{q_i} \quad (P(Q_M) - c)q_i$$

   where $P(Q) = Q^{1/\sigma}$ is the inverse demand function and $Q_M = \sum_i q_i$ is the market quantity. Taking the FOC:

$$P(Q_M) + q_i\frac{dP(Q)}{dq_i} = c$$

   Therefore:

$$1 + \frac{dP(Q)}{dq_i}\frac{Q}{P}\frac{q_i}{Q} = \frac{c}{P}$$

Note that:

$$\frac{q_i}{Q} = \frac{1}{N}$$

since all firms are symmetric, they each produce $1/N$ of the market quantity. Moreover,

$$\frac{dP(Q)}{dq_i}\frac{Q}{P} = \frac{1}{\varepsilon_D}$$

since $dQ/dq_i = 1$ and the derivative of the inverse of a function is the inverse of the derivative, that is: $\frac{dP(Q)}{dq_i} = \frac{dP(Q)}{dQ} = (\frac{dQ_D(P)}{dP})^{-1}$. Therefore:

$$1 - \frac{1}{\sigma N} = \frac{c}{P} \Rightarrow \frac{P-c}{P} = \frac{1}{\sigma N}$$

This means that firms can charge more when there are less competitors and when the demand is inelastic. Think of what this implies for the equilibrium price of water during a Hurricane: less firms can supply water and people become inelastic - since they need water for survival, their quantity demanded does not react much to prices. This model would predict them a large increase in prices of water during a Hurricane - as it is often observed in reality.

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