Problem Set 1

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- Groups of up to four students may submit one set of solutions. Write each group member's name and student number clearly on the first page of your solutions. Group compositions are allowed to change from one home assignment to another.
- To obtain a good score, write clearly and logically, starting from the definitions and correctly deducing and motivating your answers.
- Please submit your answer **before 13:15 on September 8th**.

Exercise 1 Suppose that $S = \{v_1, v_2, v_3\}$ is a set of three vectors from $\mathbb{R}^n$. Prove that the set $T = \{v_1 - 8v_2 + 5v_3, 2v_1 - v_2 + v_3, v_1 + 2v_2 - v_3\}$ is linearly dependent.

Exercise 2 Consider the vectors $v_1 = (1, 3, 5), v_2 = (-1, 0, 2), v_3 = (0, 5, 1), v_4 = (0, 3, -1)$ in R^3.

(a) Show that $\{v_1, v_2, v_3, v_4\}$ is linearly dependent.

(b) Show that $\{v_1, v_2, v_3, v_4\}$ spans R^3.

(c) Find a subset of $\{v_1, v_2, v_3, v_4\}$ that is a basis for R^3.

Exercise 3 Determine whether the following sets are subspaces of $\mathbb{R}^3$ under the addition and scalar multiplication defined on $\mathbb{R}^3$.

(a) $W_1 = \{(\alpha_1, \alpha_2, \alpha_3) \in \mathbb{R}^3 : \alpha_1 = 3\alpha_2,\ \alpha_3 = -\alpha_2\}$

(b) $W_2 = \{(\alpha_1, \alpha_2, \alpha_3) \in \mathbb{R}^3 : 2\alpha_1 - 7\alpha_2 + \alpha_3 = 0\}$

Exercise 4 Let $\mathbb{R}^2$ be the x-y-plane. Then $\mathbb{R}^2$ is a vector space. A line $\ell \subset \mathbb{R}^2$ with slope m and y-intercept b is defined by

$$\ell = \{(x, y) \in \mathbb{R}^2 | y = mx + b\}$$

Show that ℓ is only a subspace of $\mathbb{R}^2$ if and only if $b = 0$.

Exercise 5 Let $n \geq 2$. Are the following functions norms?

(a) $h : \mathbb{R}^n \to \mathbb{R}$, with $h(x) = (\sum_{i=1}^{n} |x_i|^{\frac{1}{3}})^3$

(b) $g : \mathbb{R}^n \to \mathbb{R}$, with $g(x) = \min\{x_1, \cdots, x_n\}$