Microeconomics II - Home Assignment 2

November 2023

Home assignments are supposed to be handed out via SIS UK in .pdf format until 10.12.2023 23:59. Late submission means 0 points. File naming convention: HA2_Surname_Firstname.pdf. If you have any questions regarding the assignment, do not hesitate to email me at diana.kmetkova@fsv.cuni.cz.

Answer the following questions with proper argumentation, explaining the respective steps in your calculations. Providing only a solution will not be sufficient to earn the maximum number of points for the home assignment.

1. (2.5 p.) Consider a firm in a perfectly competitive market with the following long-term cost function:

$$LTC(y) = \frac{1}{3}y^3 - 2y^2 + 80y$$

   (a) Determine the profit-maximizing output (y^*) when the market price p is \$140.
   (b) Calculate the profit π of the firm under the given price of output.
   (c) For the general price of output p calculate the inverse supply function of the firm $S^{-1}(y^*)$.
   (d) Determine the minimal price (p_0) at which the firm starts producing some output (y_0).
   (e) Calculate the supply function of the firm, denoted as $y = S(p)$.

2. (1 p.) Assume that in a perfectly competitive market, firms have the following identical long-run cost function:

$$LTC(y) = 4y^3 - 12y^2 + 36y$$

   (a) What will be the optimal output y^* and price p^* in the long run?
   (b) Calculate the equilibrium output of the industry Q^* and the number of firms n^* in the industry for the given long-run market demand: $D_Q(p) = 234 - 2p$.

3. (1.5 p.) A monopolistic firm faces the demand curve $D(p) = 100 - \frac{p}{2}$ and has the following cost function:

$$TC(y) = 100y + 2y^2$$

   (a) Calculate MR, MC, AC and inverse demand function p(y).
   (b) Determine the profit-maximizing level of output y^* and the corresponding price p^*.
   (c) Given the optimum price, what is the profit of the monopolist?

Solution

1. (a) Profit-Maximizing Output (y^*):
   In the optimum, marginal costs are equal to the market price.

$$MC(y) = p$$

$$MC(y) = \frac{dLTC}{dy} = y^2 - 4y + 80 = p$$

Given that $p = 140$, $MC(y) = 140$ and we will solve for y.

$$y^2 - 4y + 80 = 140$$
$$y^2 - 4y - 60 = 0$$
$$y_{1/2} = \frac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot (-60)}}{2} = \frac{4 \pm \sqrt{256}}{2} = \frac{4 \pm 16}{2}$$

Then, $y_1 = 10$ and $y_2 = -6$. Because of the non-negativity of the output, the solution for the output maximizing profit for the firm is $y^* = 10$.

(b) The profit is given by the difference between total revenue (TR) and total cost (TC).

$$\pi(y) = TR(y) - TC(y)$$

$$TR(y) = p \cdot y$$
$$TR(y = 10) = p \cdot y = 140 \cdot 10 = 1400$$

$$TC(y) = LTC(y) = \frac{1}{3}y^3 - 2y^2 + 80y$$
$$TC(y = 10) = \frac{1}{3}10^3 - 2 \cdot 10^2 + 80 \cdot 10 = \frac{1000 - 600 + 2400}{3} = \frac{2800}{3} = 933,33$$

$$\pi(y = 10) = 1400 - \frac{2800}{3} = \frac{4200 - 2800}{3} = \frac{1400}{3} = 466,67 > 0$$

(c) Inverse Supply Function ($p = S^{-1}(y^*)$):
In the optimum:

$$p = MC(y) = y^2 - 4y + 80 = S^{-1}(y)$$

We need to calculate for which values of y this expression holds true, which can be done by computing the shut-down condition.

$$LMC(y) = LAC(y)$$
$$y^2 - 4y + 80 = \frac{1}{3}y^2 - 2y + 80$$
$$2y(y - 3) = 0$$

Then, $y = 0$ or $y = 3$. We also know that the inverse supply curve is on the upward-sloping part of the LMC curve, hence:

$$\frac{dLMC(y)}{dy} = 2y - 4 > 0$$
$$y > 2$$

This implies that shut down point is at $y = 3$.

$$S^{-1}(y) = \begin{cases} y^2 - 4y + 80 & y \geq 3 \\ 0 & y < 3 \end{cases}$$

(d) Minimal Price (p_0):
Set $S^{-1}(y_0) = p_0$, where y_0 is the minimal output level at which the firm starts producing.

$$S^{-1}(3) = 3^2 - 4 \cdot 3 + 80 = 9 - 12 + 80 = 77$$

Hence, the minimum price is 77.

(e) Supply Function ($y = S(p)$):
To calculate the supply function, we will express the inverse supply function $p = S^{-1}(y) = y^2 - 4y + 80$ as a function of the output price.

$$\begin{aligned} p - 80 = y^2 - 4y &= (y-2)^2 - 4 \\ p - 76 &= (y-2)^2 \\ (y-2) &= \pm\sqrt{p-76},\ p \geq 76 \\ y &= 2 \pm \sqrt{p-76} \end{aligned}$$

We know that $y \geq 3$, hence, $2 + \sqrt{p-76} \geq 3$ and $2 - \sqrt{p-76} \geq 3$.
From that, we get that the supply of the firm is:

$$\begin{aligned} S(p) &= 2 + \sqrt{p-76} && p \geq 77 \\ &= 0 && p < 77 \end{aligned}$$

2. (a) In a perfectly competitive market to find the optimal output and price in the long run, set Long-run Marginal Costs (LMC) equal to Long-run Average Costs (LAC). The optimal price will be equal to $LMC(y^*)$:

$$\begin{aligned} LMC &= 12y^2 - 24y + 36 \\ LAC &= 4y^2 - 12y + 36 \end{aligned}$$

Set LMC equal to LAC:

$$\begin{aligned} 12y^2 - 24y + 36 &= 4y^2 - 12y + 36 \\ 8y^2 - 12y &= 0 \end{aligned}$$

Then, $y^* = 0$ or $y^* = \frac{3}{2}$.
Using second-order condition, we have:

$$\begin{aligned} \frac{dLMC(y)}{dy} &= 24y - 24 > 0 \\ y &> 1 \end{aligned}$$

This implies that $y^* = \frac{3}{2}$.
The optimal price can be computed by plugging in the optimal level of output into LMC or LAC.

$$p^* = LMC(y^*) = 12(\frac{3}{2})^2 - 24\frac{3}{2} + 36 = 27 - 36 + 36 = 27$$

(b) To find the equilibrium output of the industry Q^*, we will plug in the optimal price into the long-run market demand $D_Q(p) = 234 - 2p$:

$$Q^* = D_Q(p = 27) = 234 - 2 \cdot 27 = 180$$

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To find the number of firms n^*:

$$n^* = \frac{Q^*}{y^*} = \frac{180}{\frac{3}{2}} = 120$$

3. A monopolistic firm faces the demand curve $D(p) = 100 - \frac{p}{2}$ and has the following cost function:

$$TC(y) = 100y + 2y^2$$

(a)

$$TC(y) = 100y + 2y^2$$
$$MC(y) = 100 + 4y$$
$$AC(y) = 100 + 2y$$

The marginal revenue (MR) function for a linear demand curve $p(y) = 200 - 2y$ is obtained by differentiating the total revenue (TR) function. If $TR = p \cdot y$, then $MR = \frac{dTR}{dy}$.

$$TR = (200 - 2y) \cdot y = 200y - 2y^2$$
$$MR = \frac{d(200y - 2y^2)}{dQ} = 200 - 4y$$

Market demand:

$$D(p) = y = 100 - \frac{p}{2}$$
$$\frac{p}{2} = 100 - y$$

So the inverse demand function is:

$$p(y) = 200 - 2y$$

(b) FOC:
Set $MR = MC$ and solve for y^*:

$$200 - 4y = 100 + 4y$$
$$8y = 100$$
$$y = \frac{100}{8} = \frac{25}{2} = 12.5$$

To find p^*, substitute y^* into the demand curve:

$$p^* = 200 - 2 \cdot \frac{25}{2} = 200 - 25 = 175$$

So, the profit-maximizing output (y^*) is 12.5 units, and the corresponding price (p^*) is \$175.

(c) The profit is given by the difference between total revenue (TR) and total cost (TC).

$$\pi(y) = TR(y) - TC(y)$$

$$\pi^* = p^* \cdot y^* - TC(y^*) = 175 \cdot \frac{25}{2} - (100 \cdot \frac{25}{2} + 2 \cdot (\frac{25}{2})^2)$$

$$\pi^* = \frac{4375}{2} - \frac{2500 + 625}{2} = \frac{1250}{2} = 625$$