Inequality, Household Behavior, & the Macroeconomy
Problem Set 1

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April 9, 2024

Part 1

1.1

Beginning with the Euler equation

$$u'(c_t) = \beta(1+r)u'(c_t+1) \quad \forall t$$

We get

$$c_t^{-\gamma} = \beta(1+r)c_{t+1}^{-\gamma}$$

And solving for c_{t+1}:

$$c_{t+1} = (\frac{c_t^{-\gamma}}{\beta(1+r)})^{-\frac{1}{\gamma}} = c_t(\beta(1+r))^{\frac{1}{\gamma}} \quad \forall t$$

1.2

Expressing c_t in terms of c_0, we see that

$$c_1 = c_0(\beta(1+r))^{\frac{1}{\gamma}}$$

And

$$c_2 = c_1(\beta(1+r))^{\frac{1}{\gamma}}$$

$$\iff$$

$$c_2 = c_0(\beta(1+r))^{\frac{1}{\gamma}} * (\beta(1+r))^{\frac{1}{\gamma}} = c_0(\beta(1+r))^{\frac{2}{\gamma}}$$

Iterating forward, we see that

$$c_t = c_0(\beta(1+r))^{\frac{t}{\gamma}} \quad \forall t$$

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1.3

The consolidated budget constraint is

$$\sum_{t=0}^{T} \frac{c_t}{(1+r)^t} = w_0 + \sum_{t=0}^{T} \frac{y_t}{(1+r)^t}$$

Plugging in $c_t = c_0(\beta(1+r))^{\frac{t}{\gamma}}$ yields

$$\sum_{t=0}^{T} \frac{c_0(\beta(1+r))^{\frac{t}{\gamma}}}{(1+r)^t} = w_0 + \sum_{t=0}^{T} \frac{y_t}{(1+r)^t}$$

$$\iff$$

$$c_0 \sum_{t=0}^{T} \frac{(\beta(1+r))^{\frac{t}{\gamma}}}{(1+r)^t} = w_0 + \sum_{t=0}^{T} \frac{y_t}{(1+r)^t}$$

1.4

Solving the above for c_0 yields

$$c_0 = (w_0 + \sum_{t=0}^{T} \frac{y_t}{(1+r)^t}) * (\sum_{t=0}^{T} \frac{(\beta(1+r))^{\frac{t}{\gamma}}}{(1+r)^t})^{-1} = (w_0 + \sum_{t=0}^{T} \frac{y_t}{(1+r)^t}) * (\sum_{t=0}^{T} \frac{(1+r)^t}{(\beta(1+r))^{\frac{t}{\gamma}}})$$

1.5

We now see that consumption is not constant over time as c_t depends on t, so this assumption does not survive. This is a function of having relaxed the no relative impatience assumption, $\beta(1+r) = 1$. Now the agents may for example be impatient and prefer to consume more in earlier periods. This fits with the fact that consumption in the real world has been shown to be not constant over the life cycle.
The assumption that consumption in time 0 is only affected by income and wealth through the present value of these resources still holds; we see that income and wealth only enter the expression for c_0 through $(w_0 + \sum_{t=0}^{T} \frac{y_t}{(1+r)^t})$. This agent still smooths her consumption somewhat in accordance with r, β, and γ.

1.6

$$\frac{c_{t+1}}{c_t} = \frac{c_0(\beta(1+r))^{\frac{t+1}{\gamma}}}{c_0(\beta(1+r))^{\frac{t}{\gamma}}} = \frac{(\beta(1+r))^{\frac{t+1}{\gamma}}}{(\beta(1+r))^{\frac{t}{\gamma}}} = (\beta(1+r))^{\frac{1}{\gamma}}$$

Then

$$\frac{\partial}{\partial(1+r)} \frac{c_{t+1}}{c_t} = \frac{\beta}{\gamma}(\beta(1+r))^{\frac{1-\gamma}{\gamma}}$$

The gross return elasticity of consumption growth is

$$\frac{\partial}{\partial(1+r)} \frac{c_{t+1}}{c_t} * \frac{(1+r)}{(\beta(1+r))^{\frac{1}{\gamma}}}$$

$$= \frac{\beta}{\gamma}(\beta(1+r))^{\frac{1-\gamma}{\gamma}} * \frac{(1+r)}{(\beta(1+r))^{\frac{1}{\gamma}}} = \frac{1}{\gamma}$$

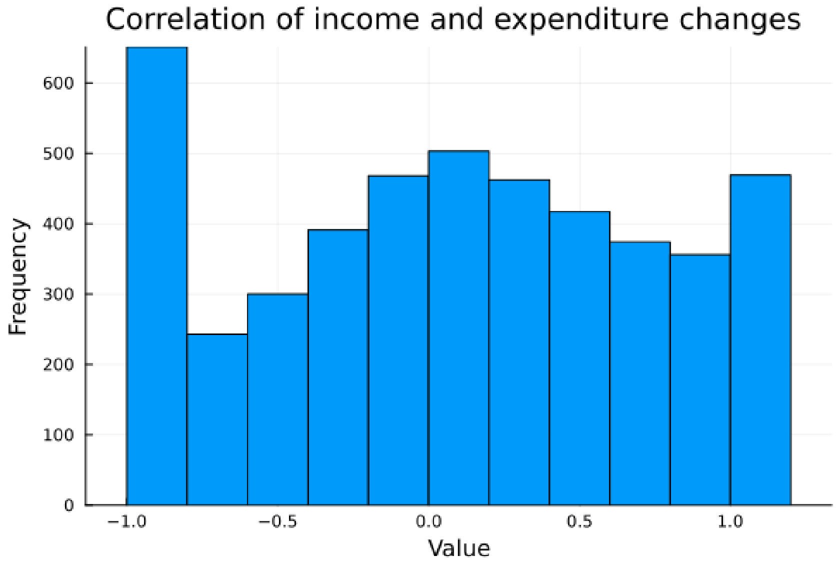
We can interpret this to mean that a 1% change in gross return is associated with a $\frac{1}{\gamma}$% change in consumption growth.
Then γ is inversely related to the percent change in consumption growth resulting from a one percent change in gross return; consumers who are more risk averse change their consumption slower in response to changes in gross return.

Part 2

1.
```
using CSV, DataFrames, Statistics, Plots
psid = DataFrame(CSV.File("psid.csv"))

psid."labor_inc_spouse" .= coalesce.(psid."labor_inc_spouse",0.0)
psid."labor_inc_family" = psid."labor_inc" .+ psid."labor_inc_spouse"

psid = psid[psid.rel_head.=="head",:]
psid = psid[psid.year.>=1999 .&& psid.family_comp_change.==0,:]
psid."expenditure_family" .= coalesce.(psid."expenditure_family",0.0)
psid = psid[:,
["id_ind","year", "expenditure_family", "labor_inc_family"]]

function lagby2!(df::DataFrame,id::String,time::String,tolag::String)
    sort!(df, [id, order(time, rev = true)])
    df[!,tolag*"_lag_2"] = missings(Float64, size(df,1))
    for i in 1:(size(df,1)-1)
        if df[i,id] == df[i+1,id] && df[i,time] == df[i+1,time] + 2
            df[i,tolag*"_lag_2"] = df[i+1,tolag]
        end
    end
end

for tolag in ["expenditure_family", "labor_inc_family"]
    lagby2!(psid,"id_ind","year",tolag)
end

for col in ["expenditure_family", "labor_inc_family"]
    psid[:,col*"_change"] = psid[:,col] .- psid[:,col*"_lag_2"]
end
```

2.
```
ids = unique(psid[:,"id_ind"])
corr =fill(0.0,size(ids,1))

for id in eachindex(ids)
    corr[id] = cor(Array(dropmissing(psid[psid."id_ind" .== ids[id],
    ["expenditure_family_change", "labor_inc_family_change"]])))[2,1]
end
```

3.
```
histogram(corr, label=false, xlabel = "Value", ylabel = "Frequency",
 title = "Correlation of income and expenditure changes")
```



In light of your findings, what can you say of Campbell and Mankiw's conclusions?

Campbell and Mankiw's assumption that half the population consumes their income every period would imply that half of the population has a correlation of income changes and expenditure changes equal to one. From the histogram, we see that significantly less than half of the population has a correlation equal to one.
This may indicate that Campbell and Mankiw overstate the proportion of hand-to-mouth consumers, but there could also be problems with the data, how the PSID measures consumption and expenditure, etc.

Part 3

1.
```
include("21_borrowinglims_solve.jl"), include("21_Zeldes.jl")

# test (ii)
beta_intercept = coef(test_i_unconstrained)[1]
beta_age = coef(test_i_unconstrained)[2]
beta_log_income = coef(test_i_unconstrained)[3]

xs = log.(constrained_sample.consumption_growth)
.− (beta_intercept .*1 .+ beta_age .*constrained_sample.age
.+ beta_log_income .* log.(constrained_sample.income))

residuals = DataFrame(xs = xs)
test_ii = lm(@formula(xs ~ 1), residuals)
println(test_ii)
                    Coef.    Std. Error       t   Pr(>|t|)
(Intercept)    0.107491    0.00365714    29.39    <1e−99
```

```
# test (iii)
residuals.income = constrained_sample.income
test_iii = lm(@formula(xs ~ 0 + log(income)), residuals)
println(test_iii)
                 Coef.     Std. Error      t    Pr(>|t|)
log(income)   -0.301341   0.0076276   -39.51    <1e-99
```

If borrowing constraints are important for consumption behavior the Lagrangian multiplier estimated by residuals $\hat{x}_{t+1}$ should be positive, which our replication of test (ii) confirms.

Further, the regression of the residuals $\hat{x}_{t+1}$ on y_{it} in test (iii) corresponds to the sign of the correlation between the (rescaled) Lagrange multiplier and income since the true error term should be uncorrelated with y_{it}. We report a negative and significant sign corresponding to the interpretation that when income is larger (all else equal), borrowing constraints should bite less. As such, our results are in line with Zeldes' expectations/findings.

2.
```
ep = EconPars(bl = median(skipmissing(data_sim.income))) # bl=1

# test (i)
test_i_constrained = lm(@formula(log(consumption_growth) ~
age + log(income)), constrained_sample)
println(test_i_constrained)
                     Coef.      Std. Error       t     Pr(>|t|)
log(income)      -0.289481    0.0215342      -13.44    <1e-37

test_i_unconstrained = lm(@formula(log(consumption_growth) ~
age + log(income)), unconstrained_sample)
println(test_i_unconstrained)
                     Coef.      Std. Error       t     Pr(>|t|)
log(income)   -0.00503646    0.00097261      -5.18    <1e-06

# test (ii)
beta_intercept = coef(test_i_unconstrained)[1]
beta_age = coef(test_i_unconstrained)[2]
beta_log_income = coef(test_i_unconstrained)[3]

xs = log.(constrained_sample.consumption_growth)
.- (beta_intercept .*1 .+ beta_age .*constrained_sample.age
.+ beta_log_income .* log.(constrained_sample.income))
residuals = DataFrame(xs = xs)
test_ii = lm(@formula(xs ~ 1 ), residuals)
println(test_ii)
                 Coef.    Std. Error      t    Pr(>|t|)
(Intercept)   0.103023   0.00535483   19.24    <1e-71

# test (iii)
residuals.income = constrained_sample.income
test_iii = lm(@formula(xs ~ 0 + log(income)), residuals)
```

```
println(test_iii)
                    Coef.    Std. Error         t   Pr(>|t|)
log(income)    -0.249647     0.0102712    -24.31      <1e-99
```

Lowering the borrowing limit (i.e. allowing the households to go into debt equal the median income of 1) and performing tests (i)-(iii) again, we note minor changes in point estimates and slight increase in standard errors (the latter likely due to reduced sample size). Yet, the overall results do not change:

(i) The parameter linked to y_{it} stays insignificant in the sample of households with a high ratio of wealth over income + significant and negative in the sample of households with a low ratio of wealth over income.

(ii) The residuals $\hat{x}_{t+1}$ stay positive, statistically significant, and quantitatively large.

(iii) The sign stays negative in the regression of the residuals $\hat{x}_{t+1}$ on y_{it}.

At first, we anticipated that the results would change. However, upon closer examination, we interpret these findings as sensible. This is because the change in parameter still left a subset of the agents in the constrained sample, relative to which the tests were conducted. Below, we demonstrate that after allowing for debt equal to 1, the constrained sample comprises approximately 2% of the total population = 1304 individuals (Compared to the default borrowing limit parameter set to zero, the constrained sample consisted of approximately 8% individuals of the total population = 4503 individuals). We note that experimenting with bl=5 led us to this argument, as it resulted in an empty set of constrained individuals.

```
println(size(constrained_sample,1)/size(data_sim,1))
0.022
println(size(constrained_sample,1))
1304
```

Finally, although our conclusions align with Zeldes (1989), we acknowledge a discrepancy between our point estimates and those reported by the author. We attribute this mainly to the difference between our simulated dataset and the one used in the paper. Additionally, the absence of instrumental variables and time-family fixed effects in our estimation of the test equations may contribute to further differences