Lecture on "Intellectual Property Rights and Trade"

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Introduction

This lecture is based on

- Charles Jones (2019), "Paul Romer: Ideas, Nonrivalry, and Endogenous Growth," *Scandinavian Journal of Economics*. [Paul Romer won the 2018 economics Nobel prize]
- Paul Romer (1990), "Endogenous Technological Change," *Journal of Political Economy*. [39,485 Google Scholar] $L_{S0} = 0$ $\theta = 1$ $g_L = 0$
- Charles Jones (1995), "R&D-based Models of Economic Growth," *Journal of Political Economy*. [5,204 Google Scholar citations] $L_{S0} = 0$
- Peter Gustafsson and Paul Segerstrom (2011), "North-South Trade with Multinational Firms and Increasing Product Variety," *International Economic Review*.

- Many developing countries have pursued patent reform aimed at strengthening intellectual property rights (IPR) protection.
- During 1960-2000, IPR protection increased on average by 70 percent in a sample of developing countries (Dinopoulos and Kotarridi, 2007).
- As a result, an intense debate has arisen about the welfare implications of stronger IPR protection.

- IPR advocates argue that patent reform benefits developing countries by fostering more rapid economic growth and more rapid transfer of technology to developing countries.
- IPR opponents counter that this reform leads to neither faster economic growth nor faster international technology transfer, but mainly results in the transfer of rents to multinational patent holders in the richest countries (in particular, the US).

Economists that are IPR opponents

- Douglas Irwin (2009, *Free Trade Under Fire*): "Many developing countries complain that, unlike mutually beneficial tariff reductions, the TRIPs agreement merely transfers income from developing to developed countries by strengthening the ability of multinational corporations to charge higher prices in poorer countries."
- Jagdish Bhagwati (2004, *In Defense of Globalization*): TRIPs is "like the introduction of cancer cells into a healthy body."
- Dani Rodrik, et al (2005, *Foreign Affairs*): "An international community that presides over TRIPs and similar agreements forfeits any claim to being development-friendly. This must change: the rich countries cannot just amend TRIPs; they must abolish it altogether."

- Paul Krugman (2014, *New York Times*): "Basically, old-fashioned trade deals are victims of their own success: there just isn't much more protectionism to eliminate. Average U.S. tariff rates have fallen by two-thirds since 1960 . . . these days, "trade agreements" are mainly about other things. What they're really about, in particular, is property rights – things like the ability to enforce patents on drugs and copyrights on movies . . . Is this a good thing from a global point of view? Doubtful. The kind of property rights we're talking about here can alternatively be described as legal monopolies. True, temporary monopolies are, in fact, how we reward new ideas; but arguing that we need even more monopolization is very dubious . . . and has nothing at all to do with classical arguments for free trade."

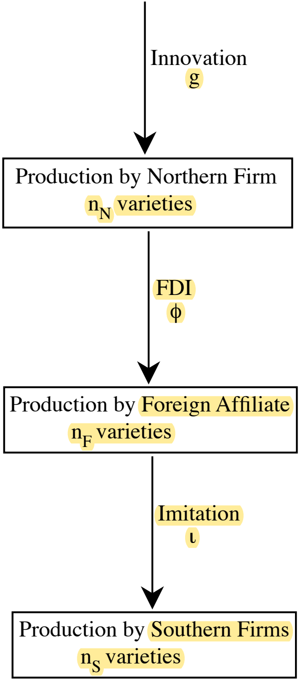
- Branstetter, Fisman and Foley (2006, QJE) examine how technology transfer within US-based multinational firms has changed in response to a series of patent reforms undertaken by 16 mainly developing countries over the 1982-1999 time period.
- They find that royalty payments for the use of intangible assets made by affiliates to parent firms increased in the wake of stronger patent regimes (by 34 percent for patent-intensive firms).
- R&D spending by affiliates also increased after patent reform (by 23 percent for patent-intensive firms).
- Branstetter, Fisman and Foley conclude that improvements in IPR protection resulted in significant increases in technology transfer from US-based multinationals to their affiliates in reforming countries.

- This new evidence represents a challenge for the literature on North-South trade with multinational firms.
- Stronger IPR protection in the South leads to a lower rate of technology transfer in Glass and Saggi (2002, JIE) and Glass and Wu (2007, JDE).
- Stronger IPR protection in the South leads to faster technology transfer but no change in R&D spending by foreign affiliates in Helpman (1993, Econometrica), Lai (1998, JDE) and Branstetter and Saggi (2011, EJ).

- Gustafsson and Segerstrom (2011, IER) presents the first North-South trade model that is consistent with the above-mentioned evidence in Branstetter, Fisman and Foley (2006, QJE) and generates large North-South wage differences for plausible parameter values.
- The model is solved numerically for the long-run welfare effects of stronger IPR protection in the South.

The Model

- global economy consists of two regions: North and South
- costless trade
- labor is the only factor of production and R&D, labor is perfectly mobile across activities but cannot move across regions, labor markets are perfectly competitive, constant returns to scale in production
- w_N is northern wage, $w_S = 1$ is southern wage
- $L_{Nt} = L_{N0}e^{g_L t} \qquad L_{St} = L_{S0}e^{g_L t}$
- $g_L > 0$ population growth rate
- Northern firms do innovative R&D to develop new product varieties
- Foreign affiliates of these northern firms then do adaptive R&D to learn how to produce in the lower-wage South
- When North-based multinationals have transferred their production to the South, they become exposed to a positive rate of imitation by southern firms


Innovation
g
Production by Northern Firm
n_N varieties
FDI
ϕ
Production by Foreign Affiliate
n_F varieties
Imitation
ι
Production by Southern Firms
n_S varieties

Households

- Households share identical preferences.
- Each household is modelled as a dynastic family that maximizes discounted lifetime utility

$$U = \int_0^{\infty} e^{-(\rho - g_L)t} \ln u_t \, dt$$

where $\rho > g_L$ is the subjective discount rate and u_t is the static utility of an individual at time t:

$$u_t = \left\{ \int_0^{n_t} x_t(\omega)^{\alpha} \, d\omega \right\}^{1/\alpha} \qquad 0 < \alpha < 1$$

where $x_t(\omega)$ is the per capita quantity demanded of the product variety ω at time t and $\sigma = \frac{1}{1-\alpha} > 1$ is the constant elasticity of substitution (CES).

- n_{Nt} equals the number of varieties produced by northern firms
- n_{Ft} equals the number of varieties produced by foreign affiliates
- n_{St} equals the number of varieties produced by southern firms
- $n_t = n_{Nt} + n_{Ft} + n_{St}$ equals the number of varieties available on the world market

- The static consumer budget constraint at time t is

$$\int_0^{n_t} p_t(\omega)x_t(\omega)d\omega = c_t.$$

- Define a new state variable as follows:

$$y(\omega) \equiv \int_0^{\omega} p_t(s)x_t(s)ds.$$

- Then

$$\dot{y}(\omega) = 1 \cdot p_t(s)x_t(s)|_{s=\omega} = p_t(\omega)x_t(\omega)$$

$$y(0) = \int_0^0 \cdots = 0$$

$$y(n_t) = \int_0^{n_t} \cdots = c_t$$

- The static consumer optimization problem can be written as

$$\max_{x_t(\cdot)} \int_0^{n_t} x_t(\omega)^\alpha d\omega \text{ s.t. } \dot{y}(\omega) = p_t(\omega)x_t(\omega),\ y(0) = 0,\ y(n_t) = c_t.$$

where $y(\omega)$ is a new state variable and $\dot{y}(\omega)$ is the derivative of y with respect to ω.

- The Hamiltonian function for this optimal control problem is

$$H = x_t(\omega)^\alpha + \gamma(\omega)p_t(\omega)x_t(\omega)$$

where $\gamma(\omega)$ is the costate variable.

- The costate equation $\frac{\partial H}{\partial y} = 0 = -\dot{\gamma}(\omega)$ implies that $\gamma(\omega)$ is constant across ω.

- $$\frac{\partial H}{\partial x} = \alpha x_t(\omega)^{\alpha-1} + \gamma \cdot p_t(\omega) = 0$$

- $\frac{\partial H}{\partial x} = \alpha x_t(\omega)^{\alpha-1} + \gamma \cdot p_t(\omega) = 0$ implies that

$$x_t(\omega) = \left(\frac{\alpha}{-\gamma \cdot p_t(\omega)}\right)^{1/(1-\alpha)}.$$

- Substituting this back into the budget constraint yields

$$c_t = \int_0^{n_t} p_t(\omega) x_t(\omega) d\omega = \int_0^{n_t} p_t(\omega) \left(\frac{\alpha}{-\gamma \cdot p_t(\omega)}\right)^{1/(1-\alpha)} d\omega$$

$$= \left(\frac{\alpha}{-\gamma}\right)^{1/(1-\alpha)} \int_0^{n_t} p_t(\omega)^{\frac{1-\alpha-1}{1-\alpha}} d\omega.$$

- Now $\sigma \equiv \frac{1}{1-\alpha}$ implies that $1 - \sigma = \frac{1-\alpha-1}{1-\alpha}$, so

$$\frac{c_t}{\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega} = \left(\frac{\alpha}{-\gamma}\right)^{1/(1-\alpha)}.$$

- $$x_t(\omega) = \frac{p_t(\omega)^{-\sigma} c_t}{P_t^{1-\sigma}} \text{ where } P_t \equiv \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$$

- A price index P_t should have the property that when all prices double, the price index doubles.
- As a check that we are not making a conceptual mistake, we show that this is the case:

$$P_t \equiv \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)}$$

$$\begin{aligned} \left[\int_0^{n_t} [2p_t(\omega)]^{1-\sigma} d\omega \right]^{1/(1-\sigma)} &= \left[2^{1-\sigma} \int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)} \\ &= 2 \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)} \end{aligned}$$

- Substituting the consumer demand function back into the consumer utility function yields

$$u_t = \left[\int_0^{n_t} x_t(\omega)^\alpha \, d\omega \right]^{\frac{1}{\alpha}} = \left[\int_0^{n_t} \frac{p_t(\omega)^{-\sigma\alpha} \, c_t^\alpha}{P_t^{(1-\sigma)\alpha}} d\omega \right]^{\frac{1}{\alpha}}$$

$$= c_t \left[\int_0^{n_t} \frac{p_t(\omega)^{-\sigma\alpha}}{P_t^{(1-\sigma)\alpha}} d\omega \right]^{\frac{1}{\alpha}}$$

or

$$\ln u_t = \ln c_t + \ln \left[\int_0^{n_t} \frac{p_t(\omega)^{-\sigma\alpha}}{P_t^{(1-\sigma)\alpha}} d\omega \right]^{\frac{1}{\alpha}} .$$

- The individual household takes the prices of all products as given, so the last bracketed expression can be ignored in solving the household's dynamic optimization problem.

- This problem simplifies to:

$$\max_{c_t} \int_0^\infty e^{-(\rho-g_L)t} \ln c_t \, dt \qquad \text{s.t.} \qquad \dot{\tilde{a}}_t = w_t + r_t \tilde{a}_t - g_L \tilde{a}_t - c_t,$$

  where $\tilde{a}_t$ represents the asset holding of the representative consumer, w_t is the wage rate and r_t is the interest rate.
- The Hamiltonian function for this optimal control problem is

$$H = e^{-(\rho-g_L)t} \ln c_t + \lambda_t \left[w_t + r_t \tilde{a}_t - g_L \tilde{a}_t - c_t \right]$$

  where λ_t is the relevant costate variable.
- The costate equation $-\dot{\lambda}_t = \frac{\partial H}{\partial \tilde{a}} = \lambda_t \left[r_t - g_L \right]$ implies that

$$\frac{\dot{\lambda}_t}{\lambda_t} = g_L - r_t.$$

- $$\partial H / \partial c_t = e^{-(\rho-g_L)t} \frac{1}{c_t} - \lambda_t = 0$$

- $\partial H / \partial c_t = e^{-(\rho - g_L)t} \frac{1}{c_t} - \lambda_t = 0$ implies that

$$e^{-(\rho - g_L)t} \frac{1}{c_t} = \lambda_t.$$

- Taking logs of both sides yields $-(\rho - g_L)\, t - \ln c_t = \ln \lambda_t$ and then differentiating with respect to time yields

$$-(\rho - g_L) - \frac{\dot{c}_t}{c_t} = \frac{\dot{\lambda}_t}{\lambda_t} = g_L - r_t$$

where the second equality follows from the costate equation.

- It immediately follows that

$$\frac{\dot{c}_t}{c_t} = r_t - \rho.$$

- Static consumer optimization $\Longrightarrow$

$$x_t(\omega) = \frac{p_t(\omega)^{-\sigma} c_t}{P_t^{1-\sigma}}$$

where c_t is individual consumer expenditure, $p_t(\omega)$ is the price charged for variety ω at time t, and

$$P_t = \left(\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega \right)^{1/(1-\sigma)}$$

is the aggregate price index.

- Dynamic consumer optimization $\Longrightarrow$

$$\frac{\dot{c}_t}{c_t} = r_t - \rho$$

where r_t is the market interest rate at time t.

- We solve for a steady-state equilibrium where w_N, w_S, c_N and c_S are all constant over time, $w_S = 1$ and $r_t = \rho$.

Product Markets

- Firms compete in prices and maximize profits
- One unit of labor produces one unit of output
- A northern firm earns the flow of global profits

$$\pi_{Nt} = (p_N - w_N)(x_{Nt}L_{Nt} + x^*_{Nt}L_{St})$$

$$= (p_N - w_N)\left(\frac{p_N^{-\sigma}c_N L_{Nt}}{P_t^{1-\sigma}} + \frac{p_N^{-\sigma}c_S L_{St}}{P_t^{1-\sigma}}\right).$$

- Maximizing π_{Nt} with respect to p_N yields the first-order condition

$$\frac{\partial \pi_{Nt}}{\partial p_N} = ((1-\sigma)p_N^{-\sigma} + w_N\sigma p_N^{-\sigma-1})\left(\frac{c_N L_{Nt} + c_S L_{St}}{P_t^{1-\sigma}}\right) = 0,$$

which implies that $(1-\sigma) + w_N\sigma p_N^{-1} = 0$ or

$$p_N = \frac{\sigma w_N}{\sigma - 1} = \frac{w_N}{\alpha}.$$

- Note that $\sigma \equiv \frac{1}{1-\alpha}$ implies that $\sigma - 1 = \frac{1-(1-\alpha)}{1-\alpha} = \frac{\alpha}{1-\alpha}$. It follows that $\frac{\sigma}{\sigma-1} = (\frac{1}{1-\alpha})/(\frac{\alpha}{1-\alpha}) = \frac{1}{\alpha}$.

- Plugging the price p_N back into the profit expression, we obtain

$$\begin{aligned}
\pi_{Nt} &= (p_N - w_N)\left(\frac{p_N^{-\sigma} c_N L_{Nt}}{P_t^{1-\sigma}} + \frac{p_N^{-\sigma} c_S L_{St}}{P_t^{1-\sigma}}\right) \\
&= \left(\frac{w_N}{\alpha} - w_N\right)\left(\frac{p_N^{-\sigma}}{P_t^{1-\sigma}}\right)(c_N L_{Nt} + c_S L_{St}) \\
&= w_N \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{p_N^{-\sigma}}{P_t^{1-\sigma}}\right)\left(\frac{c_N L_{Nt} + c_S L_{St}}{L_t}\right) L_t \\
&= w_N \left(\frac{1}{\sigma - 1}\right)\left(\frac{p_N^{-\sigma}}{P_t^{1-\sigma}}\right) \bar{c} L_t
\end{aligned}$$

where $L_t \equiv L_{Nt} + L_{St}$. It follows that

$$\pi_{Nt} = \frac{w_N \bar{x}_{Nt} L_t}{\sigma - 1} \quad \text{where} \quad \bar{x}_{Nt} = \frac{p_N^{-\sigma} \bar{c}}{P_t^{1-\sigma}}$$

is the average quantity demanded of northern varieties by world consumers and $\bar{c} \equiv (c_N L_{Nt} + c_S L_{St})/L_t$ is the average consumer expenditure in the world.

- A foreign affiliate earns the flow of global profits

$$\pi_{Ft} = (p_F - w_S)(x^*_{Ft} L_{Nt} + x_{Ft} L_{St}) = (p_F - w_S)\left(\frac{p_F^{-\sigma} c_N L_{Nt}}{P_t^{1-\sigma}} + \frac{p_F^{-\sigma} c_S L_{St}}{P_t^{1-\sigma}}\right)$$

- Maximizing π_{Ft} with respect to p_F yields the first-order condition

$$\frac{\partial \pi_{Ft}}{\partial p_F} = \left((1-\sigma)p_F^{-\sigma} + w_S \sigma p_F^{-\sigma-1}\right)\left(\frac{c_N L_{Nt} + c_S L_{St}}{P_t^{1-\sigma}}\right) = 0,$$

which implies that $(1-\sigma) + w_S \sigma p_F^{-1} = 0$ or

$$p_F = \frac{\sigma w_S}{\sigma - 1} = \frac{w_S}{\alpha}.$$

- Plugging this price back into the profit expression, we obtain

$$\begin{aligned}
\pi_{Ft} &= (p_F - w_S)\left(\frac{p_F^{-\sigma} c_N L_{Nt}}{P_t^{1-\sigma}} + \frac{p_F^{-\sigma} c_S L_{St}}{P_t^{1-\sigma}}\right) \\
&= \left(\frac{w_S}{\alpha} - w_S\right)\left(\frac{p_F^{-\sigma}}{P_t^{1-\sigma}}\right)(c_N L_{Nt} + c_S L_{St}) \\
&= w_S\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{p_F^{-\sigma}}{P_t^{1-\sigma}}\right)\left(\frac{c_N L_{Nt} + c_S L_{St}}{L_t}\right) L_t \\
&= w_S\left(\frac{1}{\sigma-1}\right)\left(\frac{p_F^{-\sigma}}{P_t^{1-\sigma}}\right)\bar{c} L_t
\end{aligned}$$

from which it follows that

$$\pi_{Ft} = \frac{w_S \bar{x}_{Ft} L_t}{\sigma - 1} \text{ where } \bar{x}_{Ft} = \frac{p_F^{-\sigma}\bar{c}}{P_t^{1-\sigma}}$$

is the average quantity demanded of foreign affiliate varieties by world consumers.

- After a foreign affiliate's product is imitated by a southern firm, price competition $\Longrightarrow$

$$p_S = w_S \qquad \pi_{St} = 0$$

- As in Vernon's (1966) description of the product life cycle, where multinational firms play a central role

$$p_N = \frac{w_N}{\alpha} > p_F = \frac{w_S}{\alpha} > p_S = w_S.$$

Innovation, FDI and Imitation

- There is free entry into innovative R&D activities in the North, with every northern firm having access to the same R&D technology.
- To innovate and develop a new product variety, a representative northern firm i must devote

$$\frac{a_N}{n_t^{\theta}}$$

units of labor to innovative R&D, where a_N is an innovative R&D productivity parameter, n_t is the disembodied stock of knowledge at time t and θ is an intertemporal knowledge spillover parameter. The disembodied stock of knowledge grows over time and is available to all firms in the world economy.
- The intertemporal knowledge spillover parameter θ can be positive or negative. For $\theta > 0$, R&D labor becomes more productive as time passes and a northern firm needs to devote less labor to develop a new variety as the stock of knowledge increases. In contrast, innovating becomes more difficult as time passes when $\theta < 0$.
- Assume $\theta < 1$ as in Jones (1995, JPE). $\theta = 1$ in Romer (1990, JPE).

- Given the innovative R&D technology, the flow of new products developed by northern firm i is

$$\dot{n}_{it} = \frac{l_{Rit}}{a_N / n_t^{\theta}} = \frac{n_t^{\theta} l_{Rit}}{a_N}$$

where $\dot{n}_{it}$ is the time derivative of n_{it} and l_{Rit} is the labor used for innovative R&D by firm i ("R" for R&D).

- Summing over individual northern firms, the aggregate flow of new products developed in the North is

$$\dot{n}_t = \frac{n_t^{\theta} L_{Rt}}{a_N}$$

where $L_{Rt} = \sum_i l_{Rit}$ is the total amount of northern labor employed in innovative activities.

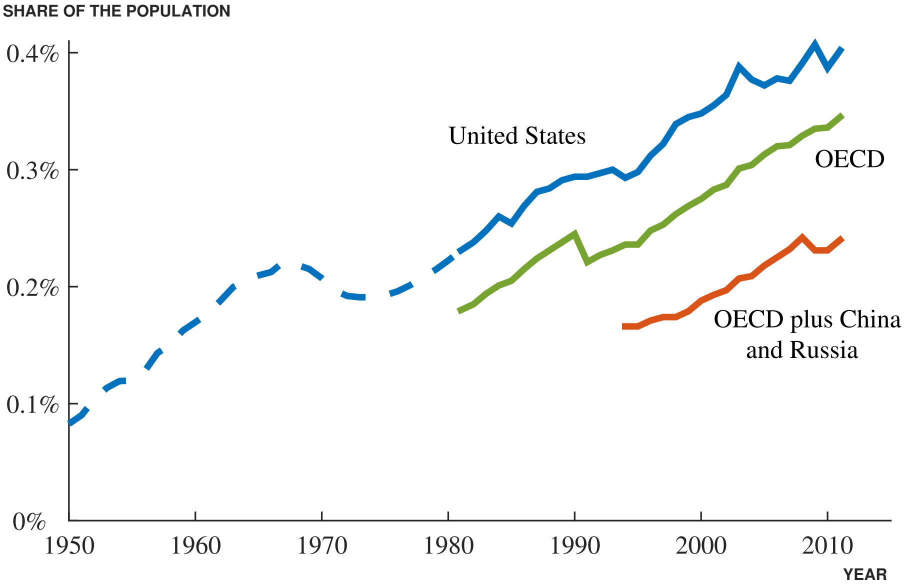
- Rewriting $\dot{n}_t = n_t^{\theta} L_{Rt}/a_N$ using $\beta \equiv 1-\theta$ yields

$$\frac{\dot{n}_t}{n_t} = \frac{n_t^{\theta-1} L_{Rt}}{a_N} = \frac{n_t^{-\beta} L_{Rt}}{a_N}$$

- Jones (2019) writes:

  There is ample evidence suggesting that Romer's original specification of the idea production function is, from an empirical standpoint, flawed. The original Romer specification [with $\theta = 1$ or $\beta = 0$] states that the growth rate of productivity is proportional to the amount of resources devoted to research. The problem with this formulation is easy to see: productivity growth rates are relatively stable over time, while the resources devoted to innovation show large exponential trends. Jones (1995) makes this point using aggregate data.

Figure 10: Research Employment Share



- Jones (2019) continues:

  More recently, Bloom, Jones, Van Reenen and Webb (2019) examine a host of evidence at different levels of aggregation confirming that the Romer specification of the idea production function is misguided. [They] find values of $\beta \approx 1/4$ for Moore's Law and semiconductor production, $\beta \approx 1$ for firm-level data, and $\beta \approx 3$ for the aggregate economy. Wherever they look, they find that research productivity – defined as the ratio of the growth rate $\dot{n}_t/n_t$ to research effort L_{Rt} – is declining rapidly. What was assumed to be constant by Romer (1990) is in fact falling sharply in the data. The typical estimate in Bloom, et al. (2019) suggests that research productivity declines at a rate of about 5% per year, meaning that the level of research productivity falls in half every twelve years.

- $$\text{research productivity} \equiv \frac{\dot{n}_t / n_t}{L_{Rt}} = \frac{n_t^{-\beta}}{a_N}$$

  is constant over time if $\beta = 0$ and is decreasing over time if $\beta > 0$.

- To learn how to produce a northern variety in the South, the foreign affiliate of a northern firm must devote a_F / n_t^θ units of labor to adaptive R&D, where a_F is an adaptive R&D productivity parameter.
- Taking into account that adaptation is followed by imitation, the number of varieties that have been successfully adapted for southern production by firm i increases over time according to

$$\dot{n}_{Fit} + \dot{n}_{Sit} = \frac{l_{Fit}}{a_F / n_t^\theta} = \frac{n_t^\theta l_{Fit}}{a_F}$$

where $\dot{n}_{Fit} + \dot{n}_{Sit}$ is the time derivative of the number of varieties that firm i is responsible for moving to the South and l_{Fit} is the labor used for adaptive R&D by firm i ("F" for FDI).

- Summing over individual foreign affiliates, the aggregate flow of varieties to the South is given by

$$\dot{n}_{Ft} + \dot{n}_{St} = \frac{n_t^\theta L_{Ft}}{a_F} \quad \text{where} \quad L_{Ft} = \sum_i l_{Fit}$$

A physics analogy

- To see that the term $\dot{n}_{St}$ should be included in the equation

$$\dot{n}_{Ft} + \dot{n}_{St} = \frac{n_t^{\theta} L_{Ft}}{a_F},$$

  it is helpful to think about a bathtub with an open drain that is being filled with water from a faucet.
- The flow of water coming out of the faucet into the bathtub equals the rate of change in the volume of water in the bathtub plus the flow of water going down the open drain.
- Likewise, the flow number of varieties that firms transfer to the South through FDI ($\dot{n}_{Ft} + \dot{n}_{St}$) equals the rate of change in the number of varieties produced by foreign affiliates ($\dot{n}_{Ft}$) plus the flow number of foreign affiliate varieties that are imitated by southern firms ($\dot{n}_{St}$).

volume of water in the bathtub $= n_{Ft}$

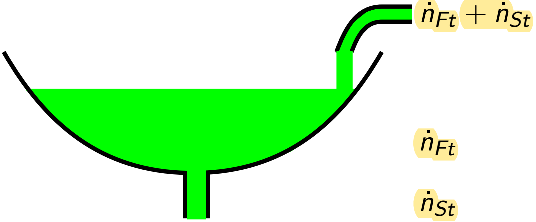


Figure: A bathtub with an open drain.

- Multinational firms that move their production to the South expose themselves to a positive rate of imitation by southern firms.
- We let
$$\iota \equiv \frac{\dot{n}_{St}}{n_{Ft}}$$
denote this imitation rate. [ι is pronounced "iota"]
- As in Helpman (1993, Econometrica) and Lai (1998, JDE), we assume that the imitation rate is exogenously given and model stronger IPR protection in the South as a reduction in the parameter ι.
[the assumption of exogenous imitation is relaxed in the second part of the Gustafsson-Segerstrom paper and the results are still the same]
- Stronger IPR protection in the South $\Longleftrightarrow \iota \downarrow$

R&D Incentives

- Let v_{Nt} denote the expected discounted profits associated with innovating in the North at time t.
- From
$$\dot{n}_t = \frac{n_t^{\theta} L_{Rt}}{a_N},$$
the R&D labor used to develop one new variety is a_N/n_t^{θ} and the cost of developing this variety is $w_N a_N/n_t^{\theta}$. [$\dot{n}_t = 1 \Rightarrow L_{Rt} = a_N/n_t^{\theta}$]
- Since there is free entry into innovative R&D activities in the North, the cost of innovating must be exactly balanced by the benefit of innovating in equilibrium:
$$v_{Nt} = \frac{w_N a_N}{n_t^{\theta}}.$$

- Let v_{Ft} denote the expected discounted profits that a foreign affiliate earns from producing a variety in the South at time t.
- From

$$\dot{n}_{Ft} + \dot{n}_{St} = \frac{n_t^{\theta} L_{Ft}}{a_F},$$

  the R&D labor that a foreign affiliate uses to transfer one variety to the South is a_F/n_t^{θ} and the cost of this transfer is $w_S a_F/n_t^{\theta}$.
  $[\dot{n}_{Ft} + \dot{n}_{St} = 1 \Rightarrow L_{Ft} = a_F/n_t^{\theta}]$
- The benefit of the transfer is not the expected discounted profits that a firm could earn from moving its production to the South v_{Ft} but the gain in expected discounted profits $v_{Ft} - v_{Nt}$, since the firm is already earning profits from producing in the North.
- Since the cost of technology transfer must be exactly balanced by the benefit in steady-state equilibrium, we obtain

$$v_{Ft} - v_{Nt} = \frac{w_S a_F}{n_t^{\theta}}.$$

- We assume that there is a stock market that channels consumer savings to firms that engage in R&D in each region and helps households to diversify the risk of holding stocks issued by these firms.
- Since there is no aggregate risk in each region, it is possible for households to earn a safe return by holding the market portfolio in each region.
- Hence, ruling out any arbitrage opportunities implies that the total return on equity claims must equal the opportunity cost of invested capital, which is given by the risk-free market interest rate ρ.

- For a northern firm i, the relevant no-arbitrage condition is

$$(\pi_{Nt} - w_S l_{Fit})dt + \dot{v}_{Nt}dt + (\dot{n}_{Fit} + \dot{n}_{Sit})dt(v_{Ft} - v_{Nt}) = \rho v_{Nt}dt.$$

- The northern firm earns the profit flow π_{Nt} during the time interval dt but also incurs the adaptive R&D expenditure flow $w_S l_{Fit}$ during this time interval.
- In addition, the firm experiences the gradual capital gain $\dot{v}_{Nt}dt$ during the time interval dt and its market value jumps up by $v_{Ft} - v_{Nt}$ for each product that it succeeds in moving to the South.
- The firm succeeds in moving $(\dot{n}_{Fit} + \dot{n}_{Sit})dt$ products to the South during the time interval dt.
- To rule out arbitrage opportunities for investors, the rate of return for the northern firm must be the same as the return on an equal sized investment in a riskless bond $\rho v_{Nt}dt$.

- Now
$$\dot{n}_{Fit} + \dot{n}_{Sit} = \frac{n_t^\theta l_{Fit}}{a_F} \text{ and } v_{Ft} - v_{Nt} = \frac{w_S a_F}{n_t^\theta}$$
implies that $(\dot{n}_{Fit} + \dot{n}_{Sit})(v_{Ft} - v_{Nt}) = w_S l_{Fit}$ and
$$v_{Nt} = \frac{w_N a_N}{n_t^\theta}.$$
implies that the gradual capital gain satisfies $\dot{v}_{Nt}/v_{Nt} = -\theta g$, where $g \equiv \dot{n}_t/n_t$ is the steady-state growth rate of varieties.
- Thus, the northern no-arbitrage condition
$$(\pi_{Nt} - w_S l_{Fit})dt + \dot{v}_{Nt}dt + (\dot{n}_{Fit} + \dot{n}_{Sit})dt(v_{Ft} - v_{Nt}) = \rho v_{Nt}dt.$$
simplifies to
$$\frac{\pi_{Nt}}{v_{Nt}} - \theta g = \rho$$
or
$$v_{Nt} = \frac{\pi_{Nt}}{\rho + \theta g} = \frac{w_N a_N}{n_t^\theta}.$$

- In the northern no-arbitrage condition

$$\frac{\pi_{Nt}}{\rho + \theta g} = \frac{w_N a_N}{n_t^{\theta}},$$

  the left-hand-side is the expected discounted profits from innovating and the right-hand-side is the cost of innovating.
- The northern firm's expected discounted profits or market value is equal to its current profit flow π_{Nt} appropriately discounted by the market interest rate ρ and the capital loss term θg (capital gain if θ is negative).

- For a foreign affiliate, the relevant no-arbitrage condition is

$$\pi_{Ft} dt + \dot{v}_{Ft} dt - (\iota dt) v_{Ft} = \rho v_{Ft} dt.$$

- The foreign affiliate earns the profit flow $\pi_{Ft} dt$ and experiences the gradual capital gain $\dot{v}_{Ft} dt$ during the time interval dt.
- However, it is exposed to a positive rate of imitation by southern firms and experiences a total capital loss if imitated, which occurs with probability ιdt.
- Equations $v_{Ft} - v_{Nt} = w_S a_F / n_t^\theta$ and $\dot{v}_{Nt} / v_{Nt} = -\theta g$ imply that the gradual capital gain satisfies $\dot{v}_{Ft} / v_{Ft} = -\theta g$.
- Thus, the no-arbitrage condition simplifies to

$$\frac{\pi_{Ft}}{v_{Ft}} - \theta g - \iota = \rho$$

or

$$v_{Ft} = \frac{\pi_{Ft}}{\rho + \theta g + \iota}.$$

- Using $v_{Ft} - v_{Nt} = w_S a_F / n_t^\theta$ and $v_{Nt} = w_N a_N / n_t^\theta$, the foreign affiliate no-arbitrage condition can be written as

$$\frac{\pi_{Ft}}{\rho + \theta g + \iota} - \frac{w_N a_N}{n_t^\theta} = \frac{w_S a_F}{n_t^\theta}$$

  where the LHS is the increase in expected discounted profits from moving production to the South and the RHS is the adaptive R&D cost.
- The expected discounted profits or market value of the foreign affiliate is equal to its current profit flow π_{Ft} appropriately discounted by the market interest rate ρ, the capital loss term θg and the imitation rate ι.

Labor Markets

- Labor markets are perfectly competitive and wages adjust instantaneously to equate labor demand and labor supply.
- In the North, labor is employed in production or in innovative R&D.
- Each innovation requires a_N/n_t^{θ} units of labor, so the total employment of labor in innovative R&D is $a_N \dot{n}_t/n_t^{\theta}$.
- Northern firms uses $\bar{x}_{Nt}L_t$ units of labor for each variety produced and there are n_{Nt} varieties produced by northern firms, so the total employment of labor in northern production is $\bar{x}_{Nt}L_t n_{Nt}$. ($\bar{x}_{Nt}$ is average quantity demanded of northern varieties by world consumers)
- As L_{Nt} denotes the supply of labor in the North, full employment requires that

$$L_{Nt} = \frac{a_N \dot{n}_t}{n_t^{\theta}} + X_{Nt}L_t$$

where $X_{Nt} \equiv \bar{x}_{Nt}n_{Nt}$ is the per capita world demand for all northern varieties.

- In the South, labor is employed in adaptive R&D, in production by foreign affiliates and in production by southern firms.
- Each variety transferred to the South requires a_F/n_t^θ units of labor, so the total employment of labor in adaptive R&D is $a_F(\dot{n}_{Ft} + \dot{n}_{St})/n_t^\theta$.
- A foreign affiliate uses $\bar{x}_{Ft}L_t$ units of labor for each variety produced and there are n_{Ft} varieties produced by foreign affiliates, so the total employment of labor in affiliate production is $\bar{x}_{Ft}L_t n_{Ft}$.
- Likewise, a southern firm uses $\bar{x}_{St}L_t$ units of labor for each variety produced and there are n_{St} varieties produced by southern firms, so the total employment of labor in southern production is $\bar{x}_{St}L_t n_{St}$.

$$\bar{x}_{st} \equiv p_S^{-\sigma}\bar{c}/P_t^{1-\sigma}$$

is average quantity demanded of southern varieties by world consumers.

- As L_{St} denotes the supply of labor in the South, full employment requires that

$$L_{St} = \frac{a_F(\dot{n}_{Ft} + \dot{n}_{St})}{n_t^{\theta}} + X_{Ft}L_t + X_{St}L_t$$

where $X_{Ft} \equiv \bar{x}_{Ft}n_{Ft}$ and $X_{St} \equiv \bar{x}_{St}n_{St}$ are the per capita world demands for all foreign affiliate and southern varieties, respectively.

- This completes the description of the model.

Solving the model

- We solve the model for a *balanced growth equilibrium* where all endogenous variables grow over time at constant (not necessarily identical) rates.
- To make progress at solving the model, we need to find endogenous variables that do not change over time and express all the equations in terms of these variables that do not change over time (that is, solve for a *steady state equilibrium* in terms of some variables).
- This is done by cleverly constructing new variables from the existing variables. For example, if variables x_t and y_t both grow at the constant rate 7% over time ($\dot{x}_t/x_t = \dot{y}_t/y_t = 0.07$), then the newly constructed variable $z_t \equiv x_t/y_t$ is constant over time ($\ln z_t = \ln x_t - \ln y_t$ implies that $\dot{z}_t/z_t = \dot{x}_t/x_t - \dot{y}_t/y_t = 0.07 - 0.07 = 0$).

- We will show that solving the North-South trade model for a balanced growth equilibrium reduces to solving a system of 6 equations in 6 unknowns, where nothing is a function of time.
- Computers are very good at solving a system of 6 equations for 6 unknowns. It takes significant time to write the Matlab code but when done, the computer solves the North-South trade model for a balanced growth equilibrium in less than 1 second.

- In any balanced growth equilibrium, the share of labor employed in R&D activities must be constant over time (in both North and South).
- Given that the supply of labor in each of the two regions grows at the population growth rate g_L, northern R&D employment L_{Rt} and foreign affiliate R&D employment L_{Ft} must grow at this rate as well.

- Since the steady-state growth rate of the number of varieties is $g \equiv \dot{n}_t/n_t$, dividing both sides of

$$\dot{n}_t = \frac{n_t^{\theta} L_{Rt}}{a_N}$$

by n_t yields

$$g = \frac{n_t^{\theta-1} L_{Rt}}{a_N}.$$

- It follows that g can only be constant over time if $(\theta - 1)g + g_L = 0$ or

$$g \equiv \frac{\dot{n}_t}{n_t} = \frac{g_L}{1-\theta}.$$

- $$g \equiv \frac{\dot{n}_t}{n_t} = \frac{g_L}{1-\theta}.$$

  has an important implication, given that the innovation rate g is proportional to the economic growth rate (as we will show later).
- This equation implies that public policy changes like stronger intellectual property rights protection (a decrease in ι) or a shift to more FDI-friendly policies (a decrease in a_F) have no effect on the steady-state rate of innovation g and hence the steady-state rate of economic growth. In this model, growth is "semi-endogenous".
- We view this as a virtue of the model because both total factor productivity and per capita GDP growth rates have been remarkably stable over time in spite of many public policy changes that one might think would be growth-promoting.
- For example, plotting data on per capita GDP (in logs) for the US from 1870 to 1995, Jones (2005, Table 1) shows that a simple linear trend fits the data extremely well.

- The FDI rate is defined by

$$\phi \equiv (\dot{n}_{Ft} + \dot{n}_{St})/n_{Nt},$$

  the rate at which northern varieties shift to the South as a result of the adaptive R&D done by foreign affiliates. [Lai (1998, JDE), Branstetter and Saggi (2011, EJ)]
- We will show that, in a balanced growth equilibrium, the FDI rate ϕ does not change over time.

- Next we derive some balanced growth equilibrium implications of the variety condition $n_{Nt} + n_{Ft} + n_{St} = n_t$.
- First, the number of varieties produced by each type of firm n_{Nt}, n_{Ft}, and n_{St} must grow at the same rate g.
- Second, the variety shares $\gamma_N \equiv \frac{n_{Nt}}{n_t}$, $\gamma_F \equiv \frac{n_{Ft}}{n_t}$ and $\gamma_S \equiv \frac{n_{St}}{n_t}$ are necessarily constant over time and satisfy $\gamma_N + \gamma_F + \gamma_S = 1$.
- Third, the FDI rate ϕ is also necessarily constant over time in any balanced growth equilibrium since

$$\phi \equiv (\dot{n}_{Ft} + \dot{n}_{St})/n_{Nt} = \frac{\dot{n}_{Ft}}{n_{Ft}} \frac{n_{Ft}}{n_{Nt}} + \frac{\dot{n}_{St}}{n_{St}} \frac{n_{St}}{n_{Nt}} = gc_1 + gc_2$$

where c_1 and c_2 are both constant over time.

- Given these properties, we can solve for γ_N.
- By differentiating the variety condition $n_{Nt} + n_{Ft} + n_{St} = n_t$, we obtain that

$$\frac{\dot{n}_t}{n_t} = \frac{\dot{n}_{Nt}}{n_{Nt}} \frac{n_{Nt}}{n_t} + \frac{\dot{n}_{Ft} + \dot{n}_{St}}{n_{Nt}} \frac{n_{Nt}}{n_t}$$

or $g = g\gamma_N + \phi\gamma_N$, and solving for γ_N yields

$$\gamma_N = \frac{g}{g + \phi}.$$

- An increase in the FDI rate ϕ naturally results in a lower share of varieties produced in the North and a higher share of varieties produced in the South.

- We can also solve for γ_F and γ_S.
- Since

$$\iota = \frac{\dot{n}_{St}}{n_{Ft}} = \frac{\dot{n}_{St}}{n_{St}} \frac{n_{St}}{n_{Ft}} = g \frac{\gamma_S}{\gamma_F},$$

we obtain $\gamma_F = g\gamma_S/\iota$ and substituting into

$$\gamma_F + \gamma_S = \frac{g}{\iota}\gamma_S + \gamma_S = \left(\frac{g+\iota}{\iota}\right)\gamma_S = \frac{\phi}{g+\phi} \quad \text{yields}$$

$$\gamma_F = \left(\frac{\phi}{g+\phi}\right)\left(\frac{g}{g+\iota}\right) \qquad \text{and} \qquad \gamma_S = \left(\frac{\phi}{g+\phi}\right)\left(\frac{\iota}{g+\iota}\right)$$

- An increase in the imitation rate ι naturally results in a lower share of varieties produced by foreign affiliates γ_F and a higher share of varieties produced by southern firms γ_S.

- From the definition of the price index $P_t \equiv \left[\int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}$, it follows that

$$\begin{aligned} P_t^{1-\sigma} &= \int_0^{n_t} p_t(\omega)^{1-\sigma} d\omega \\ &= n_{Nt} p_N^{1-\sigma} + n_{Ft} p_F^{1-\sigma} + n_{St} p_S^{1-\sigma} \\ &= \gamma_N n_t p_N^{1-\sigma} + \gamma_F n_t p_F^{1-\sigma} + \gamma_S n_t p_S^{1-\sigma} \\ &= \left[\gamma_N p_N^{1-\sigma} + \gamma_F p_F^{1-\sigma} + \gamma_S p_S^{1-\sigma}\right] n_t \end{aligned}$$

where the term in brackets in constant in balanced growth equilibrium.
$[\gamma_N \equiv n_{Nt}/n_t \quad \Longrightarrow \quad n_{Nt} = \gamma_N n_t]$

- For a northern firm,

$$\begin{aligned}
\pi_{Nt} &= w_N \left(\frac{1}{\sigma-1}\right)\left(\frac{p_N^{-\sigma}}{P_t^{1-\sigma}}\right)\bar{c}L_t \\
&= \frac{w_N}{\sigma-1}\left(\frac{p_N^{-\sigma}}{\left[\gamma_N p_N^{1-\sigma}+\gamma_F p_F^{1-\sigma}+\gamma_S p_S^{1-\sigma}\right] n_t}\right)\bar{c}L_t\frac{L_{Nt}}{L_{Nt}} \\
&= \frac{w_N}{\sigma-1}\left(\frac{p_N^{-\sigma}\bar{c}L_0/L_{N0}}{\gamma_N p_N^{1-\sigma}+\gamma_F p_F^{1-\sigma}+\gamma_S p_S^{1-\sigma}}\right)\frac{L_{Nt}}{n_t} \\
&= \left(\frac{w_N\Phi_N}{\sigma-1}\right)\frac{L_{Nt}}{n_t}
\end{aligned}$$

where

$$\Phi_N \equiv \frac{p_N^{-\sigma}\bar{c}L_0/L_{N0}}{\gamma_N p_N^{1-\sigma}+\gamma_F p_F^{1-\sigma}+\gamma_S p_S^{1-\sigma}}$$

is constant in balanced growth equilibrium.

- Likewise, the steady-state profit flow earned by a foreign affiliate is

$$
\begin{aligned}
\pi_{Ft} &= w_S \left(\frac{1}{\sigma - 1} \right) \left(\frac{p_F^{-\sigma}}{P_t^{1-\sigma}} \right) \bar{c} L_t \\
&= \frac{w_S}{\sigma - 1} \left(\frac{p_F^{-\sigma}}{\left[\gamma_N p_N^{1-\sigma} + \gamma_F p_F^{1-\sigma} + \gamma_S p_S^{1-\sigma} \right] n_t} \right) \bar{c} L_t \frac{L_{Nt}}{L_{Nt}} \\
&= \frac{w_S}{\sigma - 1} \left(\frac{p_F^{-\sigma} \bar{c} L_0 / L_{N0}}{\gamma_N p_N^{1-\sigma} + \gamma_F p_F^{1-\sigma} + \gamma_S p_S^{1-\sigma}} \right) \frac{L_{Nt}}{n_t} \\
&= \left(\frac{w_S \Phi_F}{\sigma - 1} \right) \frac{L_{Nt}}{n_t}
\end{aligned}
$$

where

$$
\Phi_F \equiv \frac{p_F^{-\sigma} \bar{c} L_0 / L_{N0}}{\gamma_N p_N^{1-\sigma} + \gamma_F p_F^{1-\sigma} + \gamma_S p_S^{1-\sigma}}
$$

is constant in balanced growth equilibrium.

$$\pi_{Nt} = \left(\frac{w_N \Phi_N}{\sigma - 1}\right) \frac{L_{Nt}}{n_t} \qquad \pi_{Ft} = \left(\frac{w_S \Phi_F}{\sigma - 1}\right) \frac{L_{Nt}}{n_t}$$

- Consequently, π_{Nt} and π_{Ft} are both proportional to L_{Nt}/n_t and only change over time based on how L_{Nt}/n_t changes over time.
- We interpret L_{Nt}/n_t as a measure of the size of the market that firms sell to.
- Population growth increases the size of the market for firms but variety growth has the opposite effect because firms have to share consumer demand with more competing firms.

- We are now in a position to define relative R&D difficulty δ_N.
- Let the term $n_t^{-\theta}$ in $v_{Nt} = w_N a_N / n_t^{\theta}$ and $v_{Ft} - v_{Nt} = w_S a_F / n_t^{\theta}$ be a measure of (absolute) R&D difficulty.
- It increases over time if $\theta < 0$ and decreases over time if $0 < \theta < 1$.
- By taking the ratio of R&D difficulty and the market size term L_{Nt}/n_t, we obtain a measure of relative R&D difficulty (or R&D difficulty relative to the size of the market):

$$\delta_N \equiv \frac{n_t^{-\theta}}{L_{Nt}/n_t} = \frac{n_t^{1-\theta}}{L_{Nt}}.$$

- Log-differentiating $\delta_N \equiv n_t^{1-\theta}/L_{Nt}$ using $g \equiv \dot{n}_t/n_t = g_L/(1-\theta)$, it follows immediately that δ_N is constant over time in any balanced growth equilibrium:

$$\ln \delta_N = \ln n_t^{1-\theta} - \ln L_{Nt} = (1-\theta)\ln n_t - \ln L_{Nt}$$

$$\frac{\dot{\delta}_N}{\delta_N} = (1-\theta)\frac{\dot{n}_t}{n_t} - \frac{\dot{L}_{Nt}}{L_{Nt}} = (1-\theta)\frac{g_L}{1-\theta} - g_L = 0.$$

- In any balanced growth equilibrium with decreasing R&D difficulty ($0 < \theta < 1$), each firm faces a shrinking market size since population growth is exceeded by variety growth (L_{Nt}/n_t decreases over time).
- If R&D difficulty increases over time ($\theta < 0$), then each firm faces an expanding market size since variety growth is exceeded by population growth (L_{Nt}/n_t increases over time).

$$\delta_N \equiv \frac{n_t^{-\theta}}{L_{Nt}/n_t} \qquad n_t \uparrow \Longrightarrow L_{Nt}/n_t \uparrow \text{ if } \theta < 0$$

- From the definition of the price index,

$$P_t^{1-\sigma} = \left[\gamma_N (p_N)^{1-\sigma} + \gamma_F (p_F)^{1-\sigma} + \gamma_S (p_S)^{1-\sigma}\right] n_t$$

  must grow over time at the rate g in any balanced growth equilibrium.
- It then follows from $\bar{x}_{Nt} \equiv p_N^{-\sigma} \bar{c} / P_t^{1-\sigma}$, $\bar{x}_{Ft} \equiv p_F^{-\sigma} \bar{c} / P_t^{1-\sigma}$ and $\bar{x}_{St} \equiv p_S^{-\sigma} \bar{c} / P_t^{1-\sigma}$ that $\bar{x}_{Nt}$, $\bar{x}_{Ft}$ and $\bar{x}_{St}$ all grow at the rate $-g$ over time.
- Consequently, $X_N \equiv \bar{x}_{Nt} n_{Nt}$, $X_F \equiv \bar{x}_{Ft} n_{Ft}$ and $X_S \equiv \bar{x}_{St} n_{St}$ must all be constant over time.

$$\begin{aligned}
\frac{\pi_{Nt}}{\rho+\theta g} &= \frac{w_N a_N}{n_t^\theta} \\
\frac{w_N \bar{x}_{Nt} L_t}{(\rho+\theta g)(\sigma-1)} &= \frac{w_N a_N}{n_t^\theta} \\
\frac{\bar{x}_{Nt} L_t}{(\rho+\theta g)(\sigma-1)} \frac{n_{Nt}}{n_{Nt}} \frac{n_t}{L_{Nt}} &= \frac{a_N}{n_t^\theta} \frac{n_t}{L_{Nt}} \\
\frac{\bar{x}_{Nt} n_{Nt}}{(\rho+\theta g)(\sigma-1)} \frac{n_t}{n_{Nt}} \frac{L_t}{L_{Nt}} &= \frac{a_N n_t^{1-\theta}}{L_{Nt}}
\end{aligned}$$

$$\frac{X_N L_0}{(\rho+\theta g)(\sigma-1)\gamma_N L_{N0}} = a_N \delta_N$$

The steady-state northern R&D condition

$$\frac{X_N L_0}{(\rho + \theta g)(\sigma - 1)\gamma_N L_{N0}} = a_N \delta_N \tag{1}$$

- The left-hand-side is the market size-adjusted benefit from innovating and the right-hand-side is the market size-adjusted cost of innovating.
- In steady-state calculations, we need to adjust for market size because market size changes over time if $\theta \neq 0$.
- The market size-adjusted benefit from innovating is higher when the average world consumer buys more of each northern variety ($X_N/\gamma_N \uparrow$), there are more consumers in the world to sell to ($L_0 \uparrow$), future profits are less heavily discounted ($\rho \downarrow$) and northern firms experiences larger capital gains over time ($\theta g \downarrow$ given $\dot{v}_N/v_N = -\theta g$).
- The market size-adjusted cost of innovating is higher when northern researchers are less productive ($a_N \uparrow$) and innovating is relatively more difficult ($\delta_N \uparrow$).

$$
\begin{aligned}
\frac{\pi_{Ft}}{\rho+\theta g+\iota} - \frac{w_N a_N}{n_t^{\theta}} &= \frac{w_S a_F}{n_t^{\theta}} \\
\frac{w_S \bar{x}_{Ft} L_t}{(\rho+\theta g+\iota)(\sigma-1)} - \frac{w_N a_N}{n_t^{\theta}} &= \frac{w_S a_F}{n_t^{\theta}} \\
\frac{\bar{x}_{Ft} L_t}{(\rho+\theta g+\iota)(\sigma-1)} \frac{n_t}{L_{Nt}} \frac{n_{Ft}}{n_{Ft}} - \frac{w_N a_N}{w_S n_t^{\theta}} \frac{n_t}{L_{Nt}} &= \frac{a_F}{n_t^{\theta}} \frac{n_t}{L_{Nt}} \\
\frac{\bar{x}_{Ft} n_{Ft}}{(\rho+\theta g+\iota)(\sigma-1)} \frac{n_t}{n_{Ft}} \frac{L_t}{L_{Nt}} - \omega a_N \frac{n_t^{1-\theta}}{L_{Nt}} &= a_F \frac{n_t^{1-\theta}}{L_{Nt}},
\end{aligned}
$$

$$
\frac{X_F L_0}{(\rho+\theta g+\iota)(\sigma-1)\gamma_F L_{N0}} - \omega a_N \delta_N = a_F \delta_N \quad \text{where} \quad \omega \equiv w_N / w_S
$$

The steady-state southern R&D condition

$$\frac{X_F L_0}{(\rho + \theta g + \iota)(\sigma - 1)\gamma_F L_{N0}} - \omega a_N \delta_N = a_F \delta_N \tag{2}$$

- The LHS is the market size-adjusted benefit from southern adaptation and the RHS is the market size-adjusted cost of southern adaptation.
- The market size-adjusted benefit is higher when the average world consumer buys more of each foreign affiliate variety ($X_F/\gamma_F \uparrow$), there are more consumers in the world to sell to ($L_0 \uparrow$), future profits are less heavily discounted ($\rho \downarrow$), foreign affiliates experiences larger capital gains over time ($\theta g \downarrow$), and foreign affiliates are exposed to a lower imitation rate ($\iota \downarrow$).
- The market size-adjusted cost is higher when foreign affiliate researchers are less productive ($a_F \uparrow$) and adaptation is relatively more difficult ($\delta_N \uparrow$).

- The definition $\delta_N \equiv n_t^{1-\theta}/L_{Nt}$ evaluated at time $t = 0$ yields $\delta_N L_{N0} = n_0^{1-\theta}$.
- Using this result, the northern labor condition becomes

$$\begin{aligned} L_{Nt} &= \frac{a_N}{n_t^\theta}\dot{n}_t + X_{Nt}L_t \\ L_{N0} &= \frac{a_N}{n_0^\theta}\frac{\dot{n}_0}{n_0}n_0 + X_{N0}L_0 \\ L_{N0} &= a_N\delta_N L_{N0}g + X_N L_0. \end{aligned}$$

- We have solved for the steady-state northern labor condition

$$L_{N0} = a_N\delta_N L_{N0}g + X_N L_0 \tag{3}$$

where $a_N\delta_N L_{N0}g$ is the northern labor devoted to R&D and $X_N L_0$ is the northern labor devoted to production.

- The southern labor condition can be rewritten as

$$
\begin{aligned}
L_{St} &= \frac{a_F(\dot{n}_{Ft} + \dot{n}_{St})}{n_t^{\theta}} + X_{Ft} L_t + X_{St} L_t \\
L_{St} &= a_F \frac{(\dot{n}_{Ft} + \dot{n}_{St})}{n_{Nt}} \frac{n_{Nt}}{n_t} \frac{n_t}{n_t^{\theta}} \frac{L_{Nt}}{L_{Nt}} + X_{Ft} L_t + X_{St} L_t \\
L_{S0} &= a_F \phi \gamma_N \delta_N L_{N0} + X_F L_0 + X_S L_0
\end{aligned}
$$

- We have solved for the steady-state southern labor condition

$$
L_{S0} = a_F \phi \gamma_N \delta_N L_{N0} + X_F L_0 + X_S L_0 \tag{4}
$$

- Now

$$\frac{X_S}{X_F} = \frac{\bar{x}_{St} n_{St}}{\bar{x}_{Ft} n_{Ft}} = \frac{p_S^{-\sigma} \bar{c} / P_t^{1-\sigma}}{p_F^{-\sigma} \bar{c} / P_t^{1-\sigma}} \frac{n_{St}/n_t}{n_{Ft}/n_t} = \left(\frac{p_F}{p_S}\right)^{\sigma} \frac{\gamma_S}{\gamma_F}$$

$$\frac{X_S}{X_F} = \left(\frac{w_S/\alpha}{w_S}\right)^{\sigma} \frac{\phi}{g+\phi} \frac{\iota}{g+\iota} \frac{g+\phi}{\phi} \frac{g+\iota}{g} = \left(\frac{1}{\alpha}\right)^{\sigma} \frac{\iota}{g}$$

- We have solved for the steady-state southern demand condition

$$\frac{X_S}{X_F} = \left(\frac{1}{\alpha}\right)^{\sigma} \frac{\iota}{g} \tag{5}$$

- Next, we define the North-South wage ratio $\omega \equiv w_N / w_S$.
- It follows that

$$\begin{aligned}
\frac{X_N}{X_F} &= \frac{\bar{x}_{Nt} n_{Nt}}{\bar{x}_{Ft} n_{Ft}} = \frac{p_N^{-\sigma} \bar{c} / P_t^{1-\sigma}}{p_F^{-\sigma} \bar{c} / P_t^{1-\sigma}} \frac{n_{Nt} / n_t}{n_{Ft} / n_t} = \left(\frac{p_F}{p_N} \right)^{\sigma} \frac{\gamma_N}{\gamma_F} \\
&= \left(\frac{w_S / \alpha}{w_N / \alpha} \right)^{\sigma} \frac{g}{g + \phi} \frac{g + \phi}{\phi} \frac{g + \iota}{g} = \omega^{-\sigma} \left(\frac{g + \iota}{\phi} \right)
\end{aligned}$$

- We have solved for the steady-state northern demand condition

$$\frac{X_N}{X_F} = \omega^{-\sigma} \left(\frac{g + \iota}{\phi} \right) \tag{6}$$

- As promised, we have shown that solving the North-South trade model for a balanced growth equilibrium reduces to solving a system of 6 equations [northern R&D, southern R&D, northern labor, southern labor, northern demand, southern demand] in 6 unknowns [ϕ, δ_N, ω, X_N, X_F, X_S], where nothing is a function of time.
- This problem is easily solved using a computer and the program *Matlab*. Computers are very good at solving systems of 6 equations in 6 unknowns.

- Once we know the balanced growth equilibrium values of the variables ϕ, δ_N, ω, X_N, X_F, X_S, we still need to solve for the remaining endogenous variables of the model.
- We proceed by solving for asset holdings, consumer expenditures, consumer utility and the economic growth rate.
- Let A_{Nt} and A_{St} denote the aggregate value of northern and southern financial assets, respectively.
- Let $A_t = n_{Nt}v_{Nt} + n_{Ft}v_{Ft}$ denote the aggregate value of all financial assets.
- Since consumer savings within the South finance the R&D investments in the South, $A_{St} = n_{Ft}(v_{Ft} - v_{Nt})$.

$$
\begin{aligned}
A_{St} &= n_{Ft}(v_{Ft} - v_{Nt}) \\
&= n_{Ft} w_S \frac{a_F}{n_t^{\theta}} \\
&= \frac{n_{Ft}}{n_t} w_S a_F \frac{n_t}{n_t^{\theta} L_{Nt}} L_{Nt} \\
&= \gamma_F w_S a_F \delta_N L_{Nt}
\end{aligned}
$$

$$
\begin{aligned}
A_{Nt} = A_t - A_{St} &= (n_{Nt} v_{Nt} + n_{Ft} v_{Ft}) - n_{Ft}(v_{Ft} - v_{Nt}) \\
&= (n_{Nt} + n_{Ft}) v_{Nt} \\
&= (n_{Nt} + n_{Ft}) \frac{w_N a_N}{n_t^{\theta}} \\
&= \left(\frac{n_{Nt}}{n_t} + \frac{n_{Ft}}{n_t} \right) w_N a_N \frac{n_t}{n_t^{\theta} L_{Nt}} L_{Nt} \\
&= (\gamma_N + \gamma_F) w_N a_N \delta_N L_{Nt}.
\end{aligned}
$$

- Let a_{it} denote the financial asset holdings of the typical consumer in region i $(i = N, S)$.
- The intertemporal budget constraint of a typical consumer in region i is

$$\dot{a}_{it} = w_i + \rho a_{it} - c_i - g_L a_{it}.$$

- In any balanced growth equilibrium where the wage rates w_i are constant over time, we must have that $\dot{a}_{it} = 0$ and it follows that

$$c_i = w_i + (\rho - g_L) a_{it}.$$

- For the typical northern consumer, $a_{Nt} = A_{Nt}/L_{Nt}$ and for the typical southern consumer, $a_{St} = A_{St}/L_{St}$.

$$
\begin{aligned}
c_N &= w_N + (\rho - g_L)\, a_{Nt} \\
&= w_N + (\rho - g_L)\, \frac{A_{Nt}}{L_{Nt}} \\
&= w_N + (\rho - g_L)\, (\gamma_N + \gamma_F) w_N a_N \delta_N \\
&= w_N \left[1 + (\rho - g_L)\, (\gamma_N + \gamma_F) a_N \delta_N\right]
\end{aligned}
$$

$$
\begin{aligned}
c_S &= w_S + (\rho - g_L)\, a_{St} \\
&= w_S + (\rho - g_L)\, \frac{A_{St}}{L_{St}} \\
&= w_S + (\rho - g_L)\, \gamma_F w_S a_F \delta_N \frac{L_{Nt}}{L_{St}} \\
&= w_S \left[1 + (\rho - g_L)\, \gamma_F a_F \delta_N \frac{L_{N0}}{L_{S0}}\right]
\end{aligned}
$$

- For the representative northern consumer, static utility at time t is given by

$$
\begin{aligned}
u_{Nt} &= \left[\int_0^{n_t} x_t(\omega)^\alpha \, d\omega\right]^{\frac{1}{\alpha}} \\
&= \left[n_{Nt}(x_{Nt})^\alpha + n_{Ft}(x_{Ft})^\alpha + n_{St}(x_{St})^\alpha\right]^{\frac{1}{\alpha}} \\
&= \left[n_{Nt}\left(\frac{p_N^{-\sigma} c_N}{P_t^{1-\sigma}}\right)^\alpha + n_{Ft}\left(\frac{p_F^{-\sigma} c_N}{P_t^{1-\sigma}}\right)^\alpha + n_{St}\left(\frac{p_S^{-\sigma} c_N}{P_t^{1-\sigma}}\right)^\alpha\right]^{\frac{1}{\alpha}} \\
&= \frac{c_N}{P_t^{1-\sigma}}\left[n_{Nt} p_N^{-\sigma\alpha} + n_{Ft} p_F^{-\sigma\alpha} + n_{St} p_S^{-\sigma\alpha}\right]^{\frac{1}{\alpha}} \\
&= \frac{c_N}{P_t^{1-\sigma}}\left[n_{Nt} p_N^{1-\sigma} + n_{Ft} p_F^{1-\sigma} + n_{St} p_S^{1-\sigma}\right]^{\frac{1}{\alpha}} \\
&= \frac{c_N}{P_t^{1-\sigma}}\left[P_t^{1-\sigma}\right]^{\frac{1}{\alpha}} = \frac{c_N}{P_t^{1-\sigma}} P_t^{-\sigma} = \frac{c_N}{P_t}
\end{aligned}
$$

since $\sigma \equiv \frac{1}{1-\alpha}$ implies that $1 = \sigma - \alpha\sigma$ and $1 - \sigma = -\alpha\sigma$.

- Likewise, for the representative southern consumer, static utility at time t is given by

$$
\begin{aligned}
u_{St} &= \left[n_{Nt} \left(\frac{p_N^{-\sigma} c_S}{P_t^{1-\sigma}} \right)^{\alpha} + n_{Ft} \left(\frac{p_F^{-\sigma} c_S}{P_t^{1-\sigma}} \right)^{\alpha} + n_{St} \left(\frac{p_S^{-\sigma} c_S}{P_t^{1-\sigma}} \right)^{\alpha} \right]^{\frac{1}{\alpha}} \\
&= \frac{c_S}{P_t^{1-\sigma}} \left[n_{Nt} p_N^{-\sigma\alpha} + n_{Ft} p_F^{-\sigma\alpha} + n_{St} p_S^{-\sigma\alpha} \right]^{\frac{1}{\alpha}} \\
&= \frac{c_S}{P_t^{1-\sigma}} \left[n_{Nt} p_N^{1-\sigma} + n_{Ft} p_F^{1-\sigma} + n_{St} p_S^{1-\sigma} \right]^{\frac{1}{\alpha}} \\
&= \frac{c_S}{P_t^{1-\sigma}} \left[P_t^{1-\sigma} \right]^{\frac{1}{\alpha}} = \frac{c_S}{P_t^{1-\sigma}} P_t^{-\sigma} = \frac{c_S}{P_t}
\end{aligned}
$$

- Now
$$P_t^{1-\sigma} = \left[\gamma_N p_N^{1-\sigma} + \gamma_F p_F^{1-\sigma} + \gamma_S p_S^{1-\sigma}\right] n_t$$
implies that
$$P_t = \left[\gamma_N p_N^{1-\sigma} + \gamma_F p_F^{1-\sigma} + \gamma_S p_S^{1-\sigma}\right]^{1/(1-\sigma)} n_t^{1/(1-\sigma)}.$$
- We conclude that in a balanced growth equilibrium where consumer expenditure is constant over time, the consumer utility growth rate is
$$g_u \equiv \frac{\dot{u}_{Nt}}{u_{Nt}} = \frac{\dot{u}_{St}}{u_{St}} = -\frac{\dot{P}_t}{P_t} = -\frac{1}{1-\sigma}\frac{\dot{n}_t}{n_t} = \frac{g}{\sigma - 1}.$$
- Since consumer utility is proportional in consumer expenditure holding prices fixed, consumer utility growth equals real wage growth and we use it as our measure of economic growth. Thus, the steady-state economic growth rate is proportional to the innovation rate g, as we earlier claimed.

- Finally, from

$$\dot{n}_{Ft} + \dot{n}_{St} = \frac{n_t^{\theta} L_{Ft}}{a_F},$$

R&D employment by foreign affiliates is

$$\begin{aligned} L_{Ft} &= \frac{a_F}{n_t^{\theta}}(\dot{n}_{Ft} + \dot{n}_{St}) \\ &= a_F \frac{\dot{n}_{Ft} + \dot{n}_{St}}{n_{Nt}} \frac{n_t^{1-\theta}}{L_{Nt}} \frac{n_{Nt}}{n_t} L_{Nt} \\ &= a_F \phi \delta_N \frac{g}{g+\phi} L_{Nt} \end{aligned}$$

Summary of how we solve the model

- It is time to summarize what we have accomplished.
- The steady-state market interest rate satisfies $r_t = \rho$ and the steady-state innovation rate $g \equiv \dot{n}_t / n_t$ is pinned down by

$$g \equiv \frac{\dot{n}_t}{n_t} = \frac{g_L}{1-\theta}.$$

- Given g, the 6 equations (1), (2), (3), (4), (5), (6) can be solved for steady-state equilibrium values of 6 variables: ϕ, δ_N, ω, X_N, X_F, X_S.
- Given the North-South wage ratio $\omega \equiv w_N / w_S$ and the numeraire choice $w_S = 1$, it follows that $w_N = \omega$.

- Then the prices charged by firms are

$$p_N = w_N/\alpha = \omega/\alpha$$

$$p_F = w_S/\alpha = 1/\alpha$$

$$p_S = w_S = 1.$$

- It also follows that

$$\gamma_N = \frac{g}{g+\phi},$$

$$\gamma_F = \left(\frac{\phi}{g+\phi}\right)\left(\frac{g}{g+\iota}\right) \qquad \text{and} \qquad \gamma_S = \left(\frac{\phi}{g+\phi}\right)\left(\frac{\iota}{g+\iota}\right)$$

  determine γ_N, γ_F and γ_S.
- Since $n_t^{1-\theta} = \delta_N L_{Nt}$, $L_{Nt} = L_{N0} e^{g_L t}$, $n_{Nt} = \gamma_N n_t$, $n_{Ft} = \gamma_F n_t$ and $n_{St} = \gamma_S n_t$, the entire equilibrium time paths of n_t, n_{Nt}, n_{Ft} and n_{St} are pinned down.

- Then the representative consumer in each region spends

$$c_N = w_N \left[1 + (\rho - g_L)(\gamma_N + \gamma_F) a_N \delta_N\right]$$

$$c_S = w_S \left[1 + (\rho - g_L)\gamma_F a_F \delta_N \frac{L_{N0}}{L_{S0}}\right],$$

the entire equilibrium time path of the price index is pinned down by

$$P_t = \left[\gamma_N p_N^{1-\sigma} + \gamma_F p_F^{1-\sigma} + \gamma_S p_S^{1-\sigma}\right]^{1/(1-\sigma)} n_t^{1/(1-\sigma)},$$

and the static utility of the representative consumer at every point in time is

$$u_{Nt} = \frac{c_N}{P_t}$$

$$u_{St} = \frac{c_S}{P_t}.$$

Choosing Parameter values

- $\rho = 0.07$ real interest rate (average real return on the US stock market over the past century as calculated by Mehra and Prescott (1985))
- $\alpha = 0.714 \implies 1/\alpha = 1.4$ (northern markup of price over marginal cost of 40 percent, $p_N = w_N/\alpha$)
- $g_L = 0.014$ (world population growth rate 1991-2000)
- $L_{N0} = 1$, $L_{S0} = 2$ ($L_{S0}/L_{N0} = 2 =$ ratio of the working age population in middle income countries to that in high income countries)
- $\theta = 0.72 \implies g_u = g_L/[(\sigma - 1)(1 - \theta)] = .02$ (2% economic growth = average US GDP per capita growth rate from 1950 to 1994)
- $\iota = 0.05$ intermediate rate of copying (one out of twenty varieties produced by foreign affiliates are copied each year)
- $a_N = 1$, $a_F = 4.21 \implies$ North-South income ratio is the same as US-Mexico GDP per worker ratio (2.17)

Numerical Results

- The main focus of this paper is on the balanced growth equilibrium effects of stronger IPR protection in the South, captured by a permanent decrease in ι.
- The Trade-Related Intellectual Property Rights (TRIPs) agreement which was signed as part of the Uruguay round of multilateral trade negotiations in 1994 calls for the establishment of minimum standards of IPR protection by all World Trade Organization (WTO) members by 2006.
- The burden of policy adjustment, however, has fallen on the shoulders of developing countries because developed countries already have higher levels of IPR protection (Maskus, 2000).
- We solve the model numerically for the effects of decreasing ι from 0.2 (very weak) to 0.05 (benchmark) to 0 (very strong). Here are the results:

- When IPR protection in the South goes from very weak ($\iota = 0.2$) to very strong ($\iota = 0$):

ι	0.2	0.05	0
w_N/w_S	2.31	1.87	1.65
δ_N	3.29	3.50	4.13
ϕ	.002	.006	.019
γ_N	0.95	0.89	0.72
γ_F	0.01	0.05	0.27
c_N/c_S	2.72	2.17	1.80
u_{N0}	5.69	6.35	8.25
u_{S0}	2.09	2.93	4.58

Preliminary Comments about the Numerical Results

- First, why do northern workers earn a higher wage rate than southern workers given that workers in both regions are equally productive (one unit of labor produces one unit of output)?
- The answer is that workers in both regions are not really equally productive since southern products sell for lower prices than northern products and this is reflected in lower wages for southern workers ($p_N = w_N/\alpha > p_F = w_S/\alpha > p_S = w_S$).
- Southern products sell for lower prices because the South has relatively few products to sell and increased production of a limited number of products drives down the prices of those products.
- And the South has relatively few products to sell because it is dependent on multinational firms transferring their production technologies.

- Second, the measure of relative R&D difficulty

$$\delta_N \equiv n_t^{1-\theta}/L_{Nt}$$

can only permanently increase if the number of products n_t temporarily grows at a faster than usual rate.
- This means that a permanent increase in δ_N is associated with a temporary increase in the northern innovation rate $\dot{n}_t/n_t$ above its steady-state value given by

$$g \equiv \frac{\dot{n}_t}{n_t} = \frac{g_L}{1-\theta}.$$

- Third, although a decrease in ι results in a permanent increase in ϕ, the question remains, what happens to R&D employment by foreign affiliates L_{Ft} in the South?
- Because

$$L_{Ft} = a_F \phi \delta_N \frac{g}{g+\phi} L_{Nt},$$

a decrease in ι permanently increases L_{Ft} because ϕ and δ_N increase and there is no permanent change in g [we are using the result that $\phi \uparrow \Longrightarrow \phi/(g+\phi) \uparrow$].

Numerical Results Again

- When IPR protection in the South goes from very weak ($\iota = 0.2$) to very strong ($\iota = 0$):

ι	0.2	0.05	0
w_N/w_S	2.31	1.87	1.65
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The numerical results show that the adoption of stronger IPR protection in the South $(\iota \downarrow)$ generates

- a permanent decrease in the North-South wage ratio $(\omega \equiv w_N / w_S \downarrow)$,
- a permanent increase in relative R&D difficulty $(\delta_N \equiv n_t^{1-\theta} / L_{Nt} \uparrow)$, which implies a temporary increase in the northern innovation rate $g \equiv \dot{n}_t / n_t$,
- a permanent increase in the rate of technology transfer from North to South $(\phi \equiv (\dot{n}_{Ft} + \dot{n}_{St}) / n_{Nt} \uparrow)$, and
- a permanent increase in R&D employment by foreign affiliates $(L_{Ft} \uparrow$ because $\delta_N \uparrow$ and $\phi \uparrow)$.

Intuition

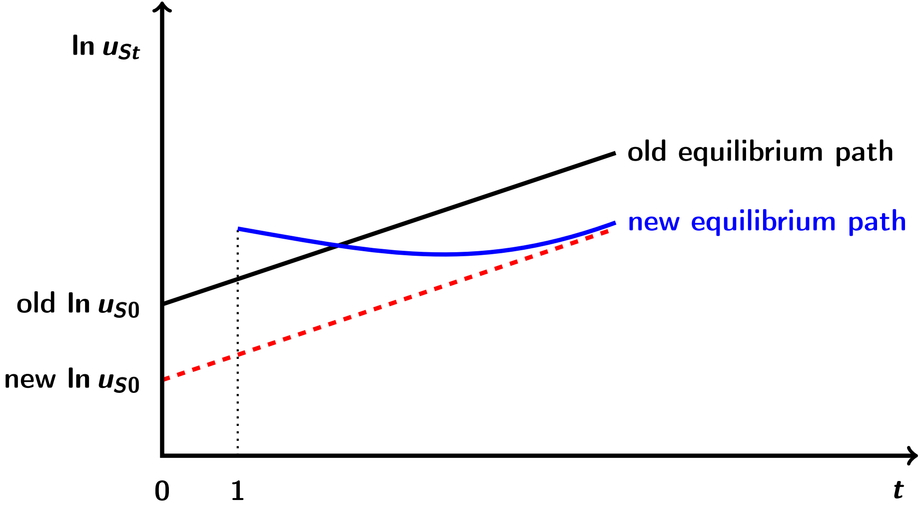
- When faced with stronger IPR protection in the South, multinational firms find it profitable to increase the adaptive R&D spending of their foreign affiliates and transfer their manufacturing production to the low-wage South more quickly ($\phi \uparrow$).
- The more rapid technology transfer from North to South in turn increases the demand for production labor in the South and decreases the demand for production labor in the North.
- These two effects cause a permanent decline in the Northern relative wage ($w_N/w_S \downarrow$) and make it more attractive for firms to engage in innovative R&D in the North.
- Firms respond by innovating more frequently, relative R&D difficulty increases and this increase causes the innovation rate to slow down.
- The permanent increase in relative R&D difficulty ($\delta_N \uparrow$) is associated with a temporary increase in the innovation rate g above its steady-state value $g = g_L/(1-\theta)$.

Numerical Results about Welfare

- Does stronger IPR protection in developing countries make people better off (as IPR advocates claim)?
- Or does patent reform in developing countries mainly benefit the owners of multinational firms in the North (as IPR opponents claim)?
- We will now look at the long-run welfare effects of stronger IPR protection in the South, that is, we will ask the question: does stronger IPR protection in the South make consumers better off in the long run?

- To answer this question, it suffices to compare steady-state utility paths before and after a policy change.
- Furthermore, since policy changes do not affect the steady-state rate of utility growth and different steady-state utility paths share a common utility growth rate, it suffices to compare steady-state consumer utility levels at time $t = 0$ to determine whether consumers benefit in the long run from a policy change.
- For example, to determine the long-run welfare effects of stronger IPR protection in the South, we solve for how the steady-state equilibrium values of u_{N0} and u_{S0} change when ι decreases.

Possible old and new equilibrium paths (consistent with the view of IPR opponents)



Numerical Results Again

- When IPR protection in the South goes from very weak ($\iota = 0.2$) to very strong ($\iota = 0$):

ι	0.2	0.05	0
w_N/w_S	2.31	1.87	1.65
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c_N/c_S	2.72	2.17	1.80
u_{N0}	5.69	6.35	8.25
u_{S0}	2.09	2.93	4.58

Five Conclusions

- First, we find that increasing IPR protection in the South has surprisingly large and positive long-run welfare effects: when IPR protection goes from very weak to very strong (ι decreases from 0.2 to 0), real consumption per capita in the South increases by 119 percent (u_{S0} increases from 2.09 to 4.58). The IPR opponents were wrong!
- Second, the model is consistent with the evidence in Branstetter, Fisman and Foley (2006, QJE) that stronger IPR protection leads to faster technology transfer within multinational firms (when ι decreases from 0.2 to 0, ϕ increases from .002 to .019). From 1995 to 2007, there was a huge 7-fold increase in R&D expenditure by non-OECD foreign affiliates (including Mexico) of US manufacturing firms.

- Third, the model is consistent with the evidence in Branstetter et al (2011, JIE) that stronger IPR protection leads to expansion of southern industrial activity and exports of new goods to the North (when ι decreases from 0.2 to 0, γ_N decreases from 0.95 to 0.72).
- Fourth, for plausible parameter values, the model can account for large wage differences between the North and the South ($w_N/w_S = 1.87$ in the benchmark parameter case). North-South trade models such as Dinopoulos and Segerstrom (2009, JDE) can only explain small wage differences between the North and the South.
- Fifth, the model is consistent with the evidence in Gould and Gruben (1996, JDE) that stronger IPR protection in developing countries is associated with faster economic growth (when ι decreases from 0.2 to 0, δ_N increases from 3.29 to 4.13 and w_N/w_S decreases from 2.31 to 1.65).