Bargaining

Martin Gregor
martin.gregor AT fsv.cuni.cz

JEB064 Game Theory and Applications

There are situations where players can generate surplus/pie (we don't ask how), and it is not clear how the surplus is divided. What is an appropriate model of bargaining?

Nash demand game

- There is a 'pie' to be divided to 2 selfish kids.
- Kid 1 demands $x \geq 0$, Kid 2 simultaneously demands $y \geq 0$.
- If $x + y \leq 1$, the pie is divided and the kids receive (x, y).
- Otherwise, the pie is not divided and the kids receive $(0, 0)$.

Nash equilibria?

- Any pair $(x, 1 - x)$ where $x \in [0, 1]$.

Key features in bargaining games: (i) actions (offers, demands), (ii) horizon (static, dynamic) and (iii) equilibrium selection criteria (arguments why some equilibria are more likely than other).

What if you can design a bargaining game?

Split-and-let-choose game

- There is a pie to be divided to 2 selfish kids.
- Kid 1 splits the pie into two pieces $(x, 1-x)$.
- Kid 2 chooses a piece.

A unique subgame-perfect Nash equilibrium is $x = \frac{1}{2}$.

- For any split x, the payoffs are $(\min\{x, 1-x\}, \max\{x, 1-x\})$.

- Axiomatic bargaining: It only requires that the predicted division satisfies reasonable axioms (e.g., efficiency, anonymity). There is no game that would explain how behavior leads to the predicted surplus division. (But we can interpret the axioms as equilibrium selection criteria in Nash *demand* game.)
- Bargaining through offers: Players give *offers* to each other (1 Proposer, $n-1$ Responders) in a pre-specified order (game tree/bargaining protocol).
- Waiting games: Each player either waits or retreats. If both wait, the game is delayed. The player who *waits the longest* wins the entire pie.
- Repeated games and commitment: The players create a pie by (non-cooperatively) coordinating each other in a stage game. They play the stage game repeatedly over multiple periods. Each player can *commit to play* the same action for several subsequent stage games.

Bargaining through offers

Bargaining through offers

- Players give offers to each others.
- At each stage: 1 Proposer, $n-1$ Responders
- If all Responders agree, the game ends.
- If someone disagrees, payoffs are zero and the *bargaining protocol* determines who is next Proposer.

Bilateral bargaining: 1 offer

Ultimatum Game

Bilateral bargaining with a single take-it-or-leave-it offer.

- The offer is a surplus division $(x, 1-x)$, where $x \in [0,1]$.
- In SPNE, $x^* = 1$.
- Proposer gains full surplus and Responder gains zero.

Is there any other NE?

- Yes, if Responder threats to not accept low offers, then $x^* < 1$.
- But it is not a SPNE.
- In these NE, Responder is not sequential rational, because in the subgame where only the Responder moves, he/she is willing to accept any non-negative offer, $1 - x \geq 0$, i.e., even offers $x > x^*$.

Rubinstein bargaining

What if the offers are alternating and bargaining has infinite horizon?

- Stage 1: Offer by Player 1. If Player 2 accepts, the game ends.
- If Player 2 rejects, the game proceeds.
- Stage 2: Offer by Player 2. If Player 1 accepts, the game ends; payoffs discounted by δ.
- If Player 1 rejects, the game proceeds.
- ...

Consider *symmetric* strategy profiles in history-independent (*stationary*) strategies.

- A symmetric stationary strategy: In each stage, each Proposer offers an identical division $(x, 1-x)$ and each Responder demands $1-y$. (The pair (x, y) is history-independent.)

Which pairs (x, y) are the equilibrium pairs?

- $x < y$: The outcome is agreement on a division $(x, 1 - x)$.
  - Take any subgame.
  - Proposer deviates by increasing x (decreasing the offer $1 - x$) to obtain a better outcome $(y, 1 - y)$.
- $x = y$: The outcome is agreement on a division $(x, 1 - x)$.
  - Take any subgame.
  - Responder can reject, which delays agreement; the outcome is $(\delta(1 - x), \delta x)$.
  - Responder doesn't reject only if $1 - x \geq \delta x$, or equivalently:

$$x \leq \frac{1}{1 + \delta}$$

  - Proposer gives the maximal offer that avoids Responder's disagreement:

$$x^* = \frac{1}{1 + \delta}$$

  - At this offer, also Proposer doesn't prefer to delay, because $1 - x = \delta x$, and so $x^* > \delta(1 - x^*) = \delta^2 x^*$.

  $=$ SPNE in which the players agree

Bilateral bargaining: alternating offers

- $x > y$: The outcome is *disagreement* with payoffs $(0, 0)$.
    - Take any subgame.
    - If Responder deviates by accepting $1 - x > 0$ (lowering his/her demand from $1 - y$ to $1 - x$), the outcome is $(x, 1 - x)$.
    - If Proposer deviates by offering $1 - y > 0$, the outcome is $(y, 1 - y)$.
    - ! Unless $(x, y) = (1, 0)$, i.e., $0 = 1 - x < 1 - y = 1$, at least one strictly deviates (retreats).
- A special case of disagreement is at $(x, y) = (1, 0)$:
    - Take any subgame.
    - If Responder deviates by accepting $1 - x = 0$, the outcome is $(1, 0)$ (full surplus for Proposer).
    - If Proposer deviates by offering $1 - y = 1$, the outcome is $(0, 1)$ (full surplus for Responder).
    - $=$ A weak SPNE in which the players disagree.
- Notice Responder's zero tolerance strategy ($y_t = 0$ always) is *weakly dominated* by a strategy that sets $y_1 > 0$ (and keeps $y_t = 0$ for $t > 1$) $=$ a temporary tolerance strategy.
    - The future is identical for both strategies (disagreement).
    - With zero tolerance strategy, Responder receives 0 for $x_1 > 0$ and 1 for $x_1 = 0$.
    - With a temporary tolerance strategy $y > 0$, Responder receives 0 for $x_1 > y_1$, 1 for $x_1 = 0$, and a *positive* payoff $1 - x_1$ for any $x_1 \leq y_1$.
    - ! In fact, absolute temporary tolerance $y_1 = 1$ weakly dominates any other temporary tolerance $y_1 < 1$.

Waiting games

Waiting games

In a waiting game, the contestant who is willing to wait *the longest* wins.

- marathon dance contests
- contests to win a car by keeping their hands on it
- animal display
- labor strikes
- macroeconomic stabilization (income tax vs. spending cuts)

1 period: symmetry

Setup

- 2 players: A, B
- pure strategies: stop, wait
- mixed strategies: probability of stop is p for Player A, q for Player B
- a fixed prize 2
- if both stop: the prize is divided equally
- if both wait: the decision is postponed (delayed) and the prize is divided equally
- cost of delay $c > 1$ (depletion of resources, breakdown risk, delayed consumption)

$A \backslash B$		stop, q	wait, $1-q$
stop	p	$1, 1$	$0, 2$
wait	$1-p$	$2, 0$	$1-c, 1-c$

- This is a Chicken Game.

1 period: symmetry

Pure-strategy (asymmetric) equilibria

- $(p^*, q^*) = (0, 1)$ or $(p^*, q^*) = (1, 0)$
- no waste (ex ante Pareto efficient; maximal surplus)

Mixed-strategy (symmetric) equilibrium

- Use payoff-equalizing property of Player A: $q^* = 2q^* + (1 - q^*)(1 - c)$

$$(p^*, q^*) = \left(\frac{c-1}{c}, \frac{c-1}{c} \right)$$

- Delay cost decreases each player's probability of waiting $(1 - p^* = 1 - q^* = \frac{1}{c})$.
- Total waste $2c$ in the symmetric equilibrium with probability $\frac{1}{c} \cdot \frac{1}{c}$
- Ex ante expected waste $\frac{2}{c}$

There is an equilibrium in which 'coordination' sometimes fails.

Mixed-strategy (asymmetric) equilibrium

- Suppose Player A is more patient, $1 < c_A < c_B$.
- From the payoff-equalizing property[1], $V_A(p,q) := u_A(1,q) = u_A(0,q)$ and $V_B(p,q) := u_B(p,1) = u_B(p,0)$.

$$u_A(0,q) = 2q + (1-q)(1-c_A) = q = u_A(1,q)$$

$$u_B(p,0) = 2p + (1-p)(1-c_B) = p = u_B(p,1)$$

- The equilibrium is $(p^*, q^*) = (\frac{c_B-1}{c_B}, \frac{c_A-1}{c_A})$, hence $p^* > q^*$; Player A stops more likely even if her delay cost is lower.
- From the payoff-equalizing property, $(V_A^*, V_B^*) = (q^*, p^*)$.

! Surprisingly, the *less patient* player B waits *more* and is better off. *Patience doesn't pay off.*

- However, mixed-strategy equilibrium is no longer supported by the symmetry argument.

[1] V_i is the ex ante equilibrium value (for Player i) of the game.

Pure-strategy equilibria

- With asymmetry, asymmetric pure-strategy equilibria are now 'reasonable'.
- But which of the two pure-strategy equilibria?
- Remember: Risk dominance (Harsanyi and Selten) measures 'riskiness of miscoordination'.[2]
- For 2 NE, the *product* of an equilibrium (s_1, s_2) is

$$[u_1(s_1, s_2) - u_1(s_1', s_2)][u_2(s_1, s_2) - u_2(s_1, s_2')].$$

- Risk dominance selects the equilibrium with a *larger product*.
- Here, products are $c_B - 1 > c_A - 1$. So, $(p^*, q^*) = (0, 1)$ is selected.

! A wins the prize. *Patience pays off.*

[2] Risk dominance is supported by evolutionary game theory, which seeks the profiles to which the 'imperfectly optimizing' agents most likely converge under realistic dynamics.

Infinite horizon: symmetry

- Periods $t = 0, \ldots, +\infty$.
- In any period, if both players wait, both pay a delay cost c.
- Let V_t^i be the ex ante equilibrium value (for Player i) of the game in period t.
- By definition, this is also the payoff for a profile (wait, wait) in period $t - 1$.
- The (infinite-horizon) game in period 0:

$A \backslash B$	stop	wait
stop	$1, 1$	$0, 2$
wait	$2, 0$	V_1^A, V_1^B

- The (infinite-horizon) (sub)game in period 1:

$A \backslash B$	stop	wait
stop	$1 - c, 1 - c$	$-c, 2 - c$
wait	$2 - c, -c$	V_2^A, V_2^B

Infinite horizon: symmetry

- As usual, we only consider *stationary* (history-independent) strategies.
- The games in period 0 and 1 are equivalent if and only if $V_2^i = V_1^i - c$.
- We rewrite the game in period 0 using the equivalence $V_1^i = V_0^i - c$:

$A \backslash B$	stop	wait
stop	$1, 1$	$0, 2$
wait	$2, 0$	$V_0^A - c, V_0^B - c$

- Ex ante values of the game in period 0:

$$V_0^A = pq + 2(1-p)q + (1-p)(1-q)(V_0^A - c)$$

$$V_0^B = pq + 2p(1-q) + (1-p)(1-q)(V_0^B - c)$$

Pure-strategy equilibria

- Is there a symmetric pure-strategy equilibrium in stationary strategies?
  - Both stop ($p = q = 1$, $V_0^A = V_0^B = 1$): Each player deviates to 'wait'.
  - Both wait ($p = q = 0$): If the equivalence holds, then the value satisfies

    $$V_0^A = pq + 2(1-p)q + (1-p)(1-q)(V_0^A - c) = V_0^A - c.$$

    But this is not true since $c > 0$.[3]
- What about asymmetric pure-strategy equilibria in stationary strategies?
  - $(p, q) = (0, 1) : (V_0^A, V_0^B) = (2, 0)$
    A waits; B indeed stops since after a deviation, $V_0^B - c = -c < 0$.
  - $(p, q) = (1, 0) : (V_0^A, V_0^B) = (0, 2)$
    B waits; A indeed stops since after a deviation, $V_0^A - c = -c < 0$.

[3] Intuitively, (wait, wait) cannot be an equilibrium profile because each player would expect an infinitely small payoff associated with ever lasting delay, and would deviate to 'stop'.

Mixed-strategy equilibrium

- Is there a (symmetric) mixed-strategy equilibrium in stationary strategies?
- Consider Player A's best response in mixed strategies. At $t = 0$:

$$u_0^A(1, q) = q + (1 - q)0 = q$$

$$u_0^A(0, q) = 2q + (1 - q)(V_0^A - c)$$

- By payoff-equalizing property, $u_0^A(1, q) = u_0^A(0, q)$.
- Ex ante value of the game for Player A in period 0 is

$$V_0^A(p, q) = pu_0^A(1, q) + (1 - p)u_0^A(0, q) = u_0^A(1, q) = u_0^A(0, q) = q.$$

Mixed-strategy equilibrium

- Insert $V_0^A = q$ into $u_0^A(1,q) = u_0^A(0,q)$:

$$\begin{aligned}
2q + (1-q)(q-c) &= q \\
3q - c - q^2 + qc &= q \\
0 &= q^2 - (c+2)q + c \\
Q^*(c)_{1,2} &= \frac{c + 2 \pm \sqrt{(c+2)^2 - 4c}}{2} \\
Q^*(c) &= 1 + \frac{c}{2} - \sqrt{1 + \frac{c^2}{4}} \in (0,1)
\end{aligned}$$

- For any $t \in N$, there is a positive probability $[(1-q^*)(1-q^*)]^t$ that the game is still unresolved in t and also a positive (conditional) probability $(1-q^*)(1-q^*)$ of postponing decision to $t+1$.
- The propensity to stop increases in the *common* cost of delay:

$$\frac{dQ^*(c)}{dc} = \frac{1}{2} - \frac{c}{2\sqrt{4+c^2}} = \frac{\sqrt{4+c^2} - c}{2\sqrt{4+c^2}} > 0$$

Mixed-strategy equilibrium

- W.l.o.g., suppose A is more patient, $1 < c_A < c_B$.
- From the equalizer property, $V_0^A = q$ and $V_0^B = p$.

$$2q + (1-q)(q - c_A) = q$$

$$2p + (1-p)(p - c_B) = p$$

- We can use $Q^*(c)$ function from the symmetric case.
- Again, A is less aggressive (stops more often):

$$p^* = Q^*(c_B) > Q^*(c_A) = q^*.$$

! B is better off as $V_0^B = p^* > q^* = V_0^A$. *Patience doesn't pay off.*

Pure-strategy equilibria: risk-dominance

Product of deviations in $(0, 1)$-equilibrium is c_B.

$A \backslash B$	stop	wait
stop	$1, 1$	$0, 2$
wait	**2,0**	$2 - c_A, -c_B$

Product of deviations in $(1, 0)$-equilibrium is c_A.

$A \backslash B$	stop	wait
stop	$1, 1$	**0,2**
wait	$2, 0$	$-c_A, 2 - c_B$

- Risk dominance selects the equilibrium with the *larger product*, $c_B > c_A$.
- $(0, 1)$-equilibrium is selected.
- A wins the prize. *Patience pays off.*

Repeated games with commitment

Coordination of monetary and fiscal policy-makers

- Monetary policy-maker (a central bank, M) and fiscal policy-maker (the government, F) want to stimulate economy but also keep a stable debt level.

M \ F	passive (SQ)	active
passive (SQ)	low growth, falling real debt	stable real debt
active	stable real debt	inflation, rising real debt

- Each policy-maker prefers a stable debt level (coordination) over an unstable debt level but also prefers that her policy is **active** (expansionary) if the debt level is stable.

Monetary \ Fiscal	passive ($f = 0$)	active ($f = 1$)
passive ($m = 0$)	$1, 1$	$2, 5$
active ($m = 1$)	$5, 2$	$0, 0$

Mixed strategies:

- $m \in [0,1]$ is the probability of active monetary policy
- $f \in [0,1]$ is the probability of active fiscal policy

1 period, simultaneous moves

- 2 pure-strategy equilibria: $(m,f) = (1,0)$ where M wins, and $(m,f) = (0,1)$ where F wins
- 1 mixed-strategy equilibrium: From M's payoff-equalizing condition,

$$(1-f) + 2f = 5(1-f)$$

$$(m,f) = \left(\frac{2}{3}, \frac{2}{3}\right)$$

The mixed-strategy equilibrium suggests that we should also consider equilibria in which the surplus from coordination is not generated (waste).

1 period, policy leadership of M

- M can credibly commit to active or passive policy. F cannot commit.
- Sequential game:
    - Stage 1. M selects m.
    - Stage 2: F observes m and sets f.
    - Payoffs for (m, f) profile are realized.
- $(m, f) = (1, 0)$ is a unique subgame-perfect Nash equilibrium (SPNE).

We observe the **first-mover** advantage.

2 periods, longer horizon of M

- M disposes with a 2-period commitment.
- F disposes with a 1-period commitment.
  - Stage 1: M selects m_1 and F simultaneously selects f_1.
  - Payoffs for (m_1, f_1) profile are realized.
  - Stage 2: F observes m_1 and sets f_2.
  - Payoffs for (m_1, f_2) profile are realized.
- Insert payoffs from Stage 2 into the expected payoffs in Stage 1:

Monetary \ Fiscal	$f_1 = 0$	$f_1 = 1$
$m_1 = 0$	$1+2, 1+5$	$2+2, 5+5$
$m_1 = 1$	$5+5, 2+2$	$0+5, 0+2$

- $(m, f_1, f_2) = (1, 0, 0)$ is a unique SPNE.

We observe the **longer-commitment** advantage.

Takeaways

- In games with many equilibrium divisions of the surplus, certain divisions of the surplus disappear when we apply sequential rationality (i.e., only credible threats can be expected).
- In particular, the initial proposer has a very large bargaining power.
- Relatively longer time horizon (patience) and relatively longer commitment horizon (credibility) typically help players attain a more favorable equilibrium (= win the pie).
- Extending bargaining horizon may not have an effect if the 'history' is irrelevant.
- Some of the equilibria involve bargaining failures with a positive probability; we should look upon failure as a *necessary* condition to obtain a more balanced division of payoffs.
- In these special equilibria, patience doesn't pay off!