Bounded Rationality: Iterated Reasoning

Beauty contest in the lab

Beauty contest in the field

The cognitive hierarchy model of games

Beauty Contest in the Lab (Nagel AER 1995)

Setting: A large number of players (15-18 in the experiment) simultaneously choose a number between 0 and 100. The person with the number closest to the mean of all chosen numbers multiplied by p wins a price (p is determined before the game and is common knowledge). If there is a tie the prize (5 DM≈ $3) is divided equally among the winners.

Four trials for each subject with the same group of players. Feedback about all chosen numbers, the average number and the “target number” was given after each trial; the number closest to the optimal number and the resulting payoffs were also publicly announced.

Treatments:

1. p=1/2
2. p=2/3
3. p=4/3

Theoretical Predictions in Nagel (1995)

Nash equilibrium:

P<1; all players choose 0 (infinitely repeated elimination of weakly dominated strategies).

P>1; all players choose 0 or all players choose 100.

Model of bounded rationality:

First period behavior:

Strategic of degree 0: chooses 50 (a random choice 0-100)

Strategic of degree 1: chooses 50p

Strategic of degree n: chooses $50p^n$

Behavior in periods 2-4:

Strategic of degree 0: chooses the mean of last period (r).

Strategic of degree 1: chooses rp

Strategic of degree n: chooses rp^n

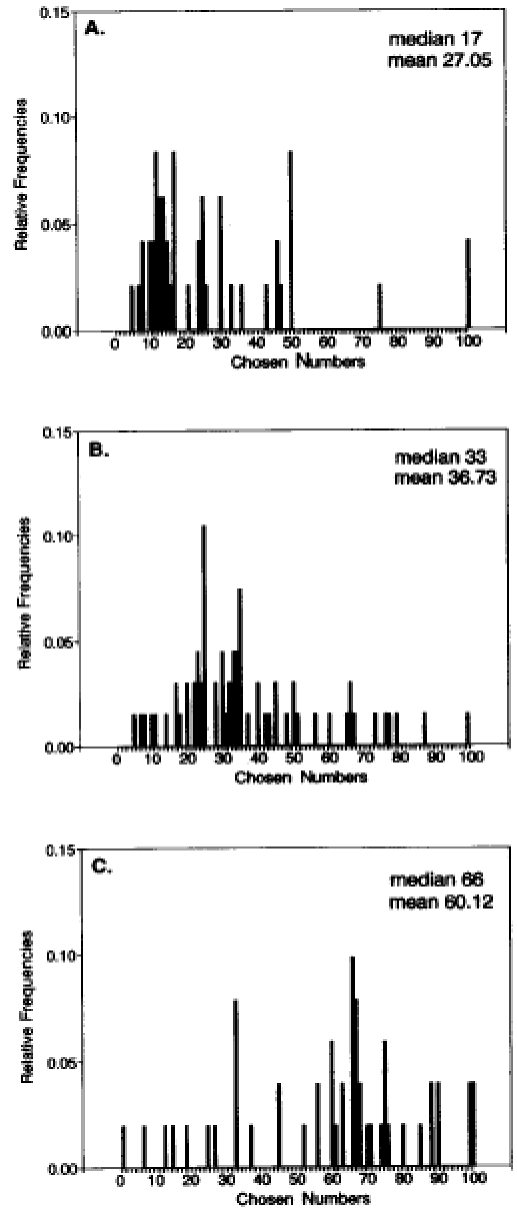

FIGURE 1. CHOICES IN THE FIRST PERIOD: A) SESSIONS 1–3 ($p = ^1/_2$); B) SESSIONS 4–7 ($p = ^2/_3$); C) SESSIONS 8–10 ($p = ^4/_3$)

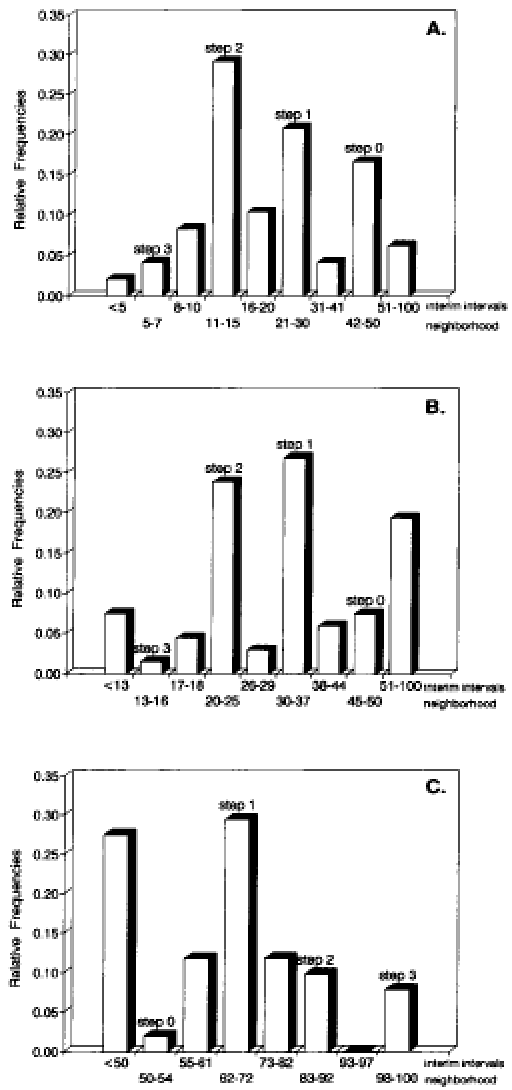

FIGURE 2. RELATIVE FREQUENCIES OF CHOICES IN THE FIRST PERIOD ACCORDING TO THE INTERVAL CLASSIFICATION WITH REFERENCE POINT 50: A) SESSIONS 1–3 ($p = 1/2$); B) SESSIONS 4–7 ($p = 2/3$); C) SESSIONS 8–10 ($p = 4/3$)

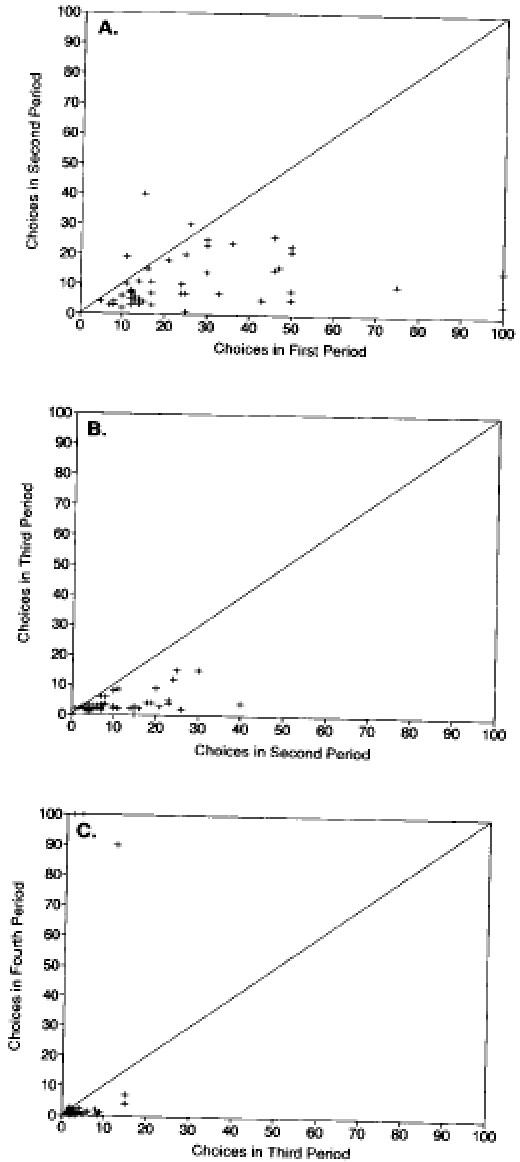

FIGURE 3. OBSERVATIONS OVER TIME FOR SESSIONS 1–3 ($p = {}^{1}/_{2}$): A) TRANSITION FROM FIRST TO SECOND PERIOD; B) TRANSITION FROM SECOND TO THIRD PERIOD; C) TRANSITION FROM THIRD TO FOURTH PERIOD

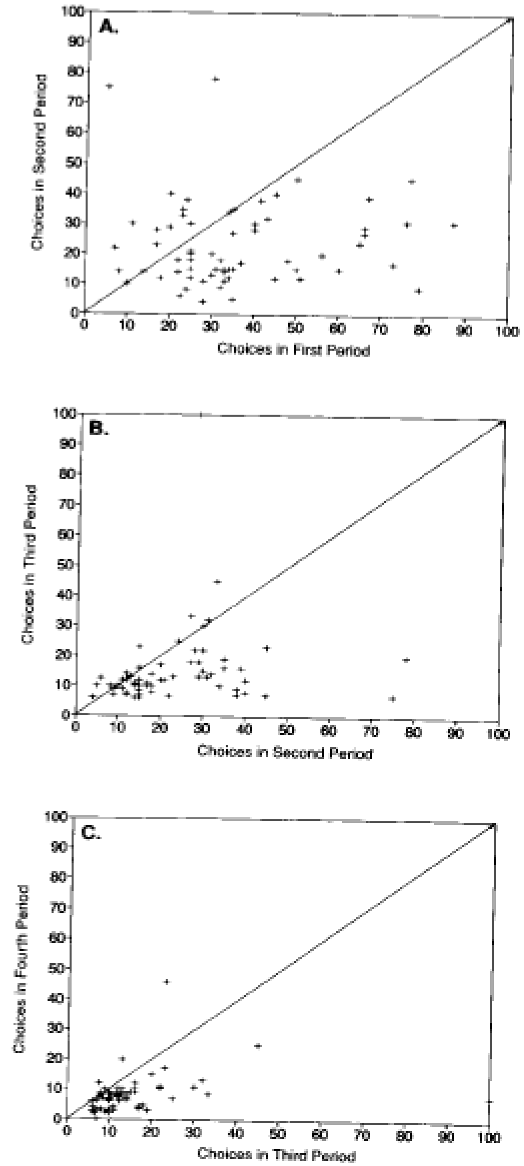

FIGURE 4. OBSERVATIONS OVER TIME FOR SESSIONS 4–7 ($p = {}^{2}/_{3}$): A) TRANSITION FROM FIRST TO SECOND PERIOD; B) TRANSITION FROM SECOND TO THIRD PERIOD; C) TRANSITION FROM THIRD TO FOURTH PERIOD

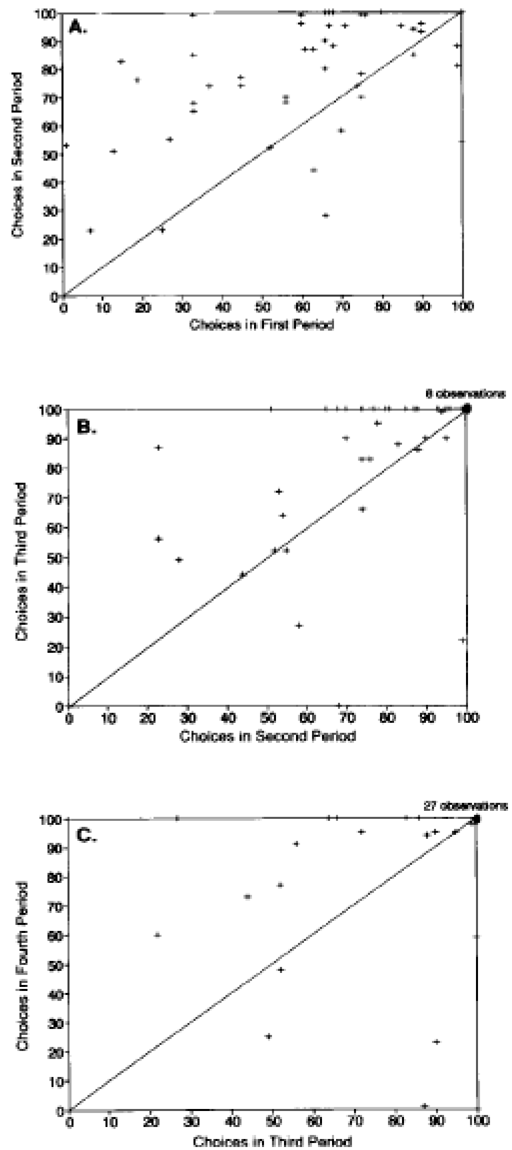

FIGURE 5. OBSERVATIONS OVER TIME FOR SESSIONS 8–10 ($p = {}^4/_3$): A) TRANSITION FROM FIRST TO SECOND PERIOD; B) TRANSITION FROM SECOND TO THIRD PERIOD; C) TRANSITION FROM THIRD TO FOURTH PERIOD

Table 2—Relative Frequencies and Areas of Periods 2–4 According to the Step-Model for Aggregated Data

	Period 2		Period 3		Period 4	
Classification	Relative frequency	Area	Relative frequency	Area	Relative frequency	Area
A. *Sessions 1–3* ($p = {}^1/_2$):						
Higher steps	4.2	2.4	4.2	1.0	20.8	0.3
Step 3	25.0	2.4	12.5	1.0	22.9	0.3
Step 2	31.3	4.9	60.4	2.0	29.2	0.7
Step 1	27.0	9.6	12.5	3.9	14.5	1.4
Step 0	2.1	7.9	4.1	3.2	4.2	1.1
Above mean_{t-1}	10.4	73.0	6.3	88.9	8.3	96.2
All	100.0	100.0	100.0	100.0	100.0	100.0
B. *Sessions 4–7* ($p = {}^2/_3$):						
Higher steps	7.5	8.9	1.5	5.8	7.5	3.8
Step 3	11.9	4.4	17.9	2.9	25.3	1.9
Step 2	31.3	6.7	46.2	4.3	47.8	2.9
Step 1	20.9	10.0	16.4	6.5	10.4	4.3
Step 0	14.9	6.7	7.5	4.4	3.0	2.9
Above mean_{t-1}	13.4	63.3	10.5	76.1	6.0	84.1
All	100.0	100.0	100.0	100.0	100.0	100.0

Beauty Contest in the Field (Bosch-Doménech et al AER 2002)

The beauty contest game is tested in three field experiments using three newspapers (Financial times in the UK, Expansión in Spain and Spektrum der Wissenschaft in Germany; p=2/3)

TABLE 1—MAIN FEATURES OF THE NEWSPAPER EXPERIMENTS

	Financial Times	*Expansión*	*Spektrum der Wissenschaft*
Number of participants	1,476 participants	3,696 participants	2,728 participants
Numbers/Interval to choose from	Integer number in [0, 100]	Number in [1, 100]	Number in [0, 100]
Explanation of "⅔ of the mean"	With an example: 5 people choose 10, 20, 30, 40, 50. The average is 30, ⅔ of which is 20. The person who chooses 20 wins.	With a definition: suppose 1000 persons participate. Sum the chosen numbers and divide them by 1000. Multiply the result by ⅔. The winning number is the closest to the last result	No explanation of mean or ⅔ of mean is given. ⅔ of mean is called "target number"
Comments asked	"Please describe in no more than 25 words the thought processes you went through in arriving at your number"	"If you want to add some comment about how you decided to choose your number, we are interested in it"	"We will be glad when you also tell us how you got to your number"
Prize	2 return Club Class tickets to New York or Chicago donated by British Airways	100.000 Pesetas (about $800), paid by *Expansión*	1000 DM (about $600) paid by *Spektrum*
Announcement of the rules	Once	Preannouncements of the game; appearance of rules on 4 consecutive days	Once in print and in their web page
Time to submit	13 days	1 week	2 weeks
Submission form	Postcards	Letters, fax, or e-mail	Letters or e-mail
Other restrictions	One entry per household, minimum age 18, resident of UK; excluded: employees of FT or close relatives, any agency or person associated with the competition	One entry per person. Personnel of Universitat Pompeu Fabra and direct family excluded	One entry per participant. Employees of *Spektrum* excluded
Cover story, context of experiment	Competition as "appetizer for the FT Mastering Finance series" ... "Contest will be discussed ... in an article on behavioral finance ... . The series will offer a mix of theory and practical wisdom on ... corporate finance, financial markets and investment management topics"	"This is an exercise, an experiment ... related to economics and human behavior. John Maynard Keynes could say that playing at the stock market is similar to participating in a Beauty-contest game ... ."	"Who is the fairest of them all? The average ... according to psychological tests. However, sometimes it helps being different from the average by the right amount." Tale about a country Hairia where the most beautiful person is the one who has ⅔ of the hair length of all contestants
Language	English	Spanish	German
Description of newspaper/magazine	Daily business paper, worldwide distribution, printed in England, with 391,000 copies per day.	Daily business paper, distributed in Spain with 40,000 copies per day	Monthly magazine, German edition of *Scientific American*, distributed in Germany, with about 120,000 copies per month.
Authors	Thaler	Bosch, Nagel	Selten, Nagel

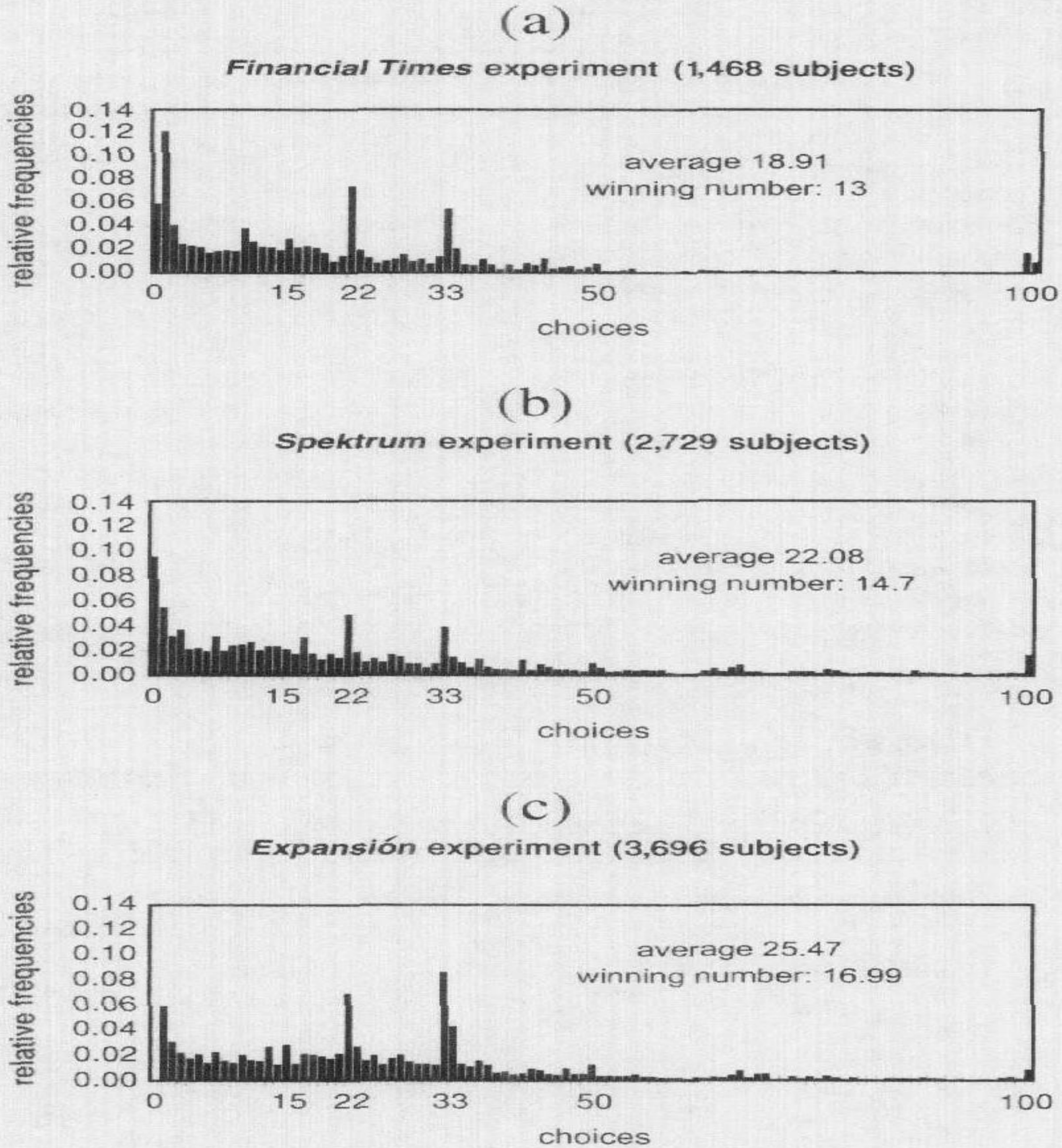

FIGURE 1. RELATIVE FREQUENCIES OF CHOICES IN THREE NEWSPAPER EXPERIMENTS

A "Cognitive Hierarchy" Model of Games (Camerer et al QJE 2004)

Based on the assumption that each player believes he/she understands the game better than the other players.

Player types (level of iterated thinking):

Level 0 players: choose randomly according to some distribution; a uniform distribution assumed (the average guess will be 50 in a [0,100] beauty contest game with p<1).

Level 1 players: assume that all other players are level 0 players (will guess p50 in a [0,100] beauty contest game).

Level 2 players: assume that all other players are level 0 or level 1 players and accurately predict the relative frequency of level 1 and level 0 players.

Level k players: assume that all other players are Level 0 to level k-1 players and accurately predict the relative frequency of level 0 to level k-1 players.

Assumed distribution of player types, f(k):

Poisson distribution; $f(k)=e^{-\tau}\tau^k/k!$

τ =parameter that characterize the Poisson distribution; equal to the mean and variance of the distribution of player types (k).

Market Entry Games and the Cognitive Hierarchy Model (Camerer et al. 2004)

Market entry game: A number of subjects decide whether to enter a market or stay out. If the fraction of entrants is equal to or below a specified fraction (d; e.g. 50%) all entrants gain \$0.5. If the fraction of entrants is over d; all entrants lose \$0.5. Equilibrium that entry equals d. Experimental results roughly consistent with equilibrium; although some overentry at low d levels and some underentry at high d levels.

Predicted entry with the cognitive hierarchy model:

Level 0 players: 50% will enter independent of d.

Level 1 players: will enter if $d>0.5$ and will not enter if $d<0.5$.

Level 2 players: the entry decision will depend on the mean of the Poisson distribution (τ). With $\tau=1.5$; the proportion of level 0 players is 0.223 and the proportion of level 1 players is 0.335; the level 2 player normalize these fractions to 1 and thinks that there are 40% level 0 players and 60% level 1 players. Will enter if $0.2<d<0.5$ or if $d>0.8$ (in these intervals the perceived entry of other players is $<d$).

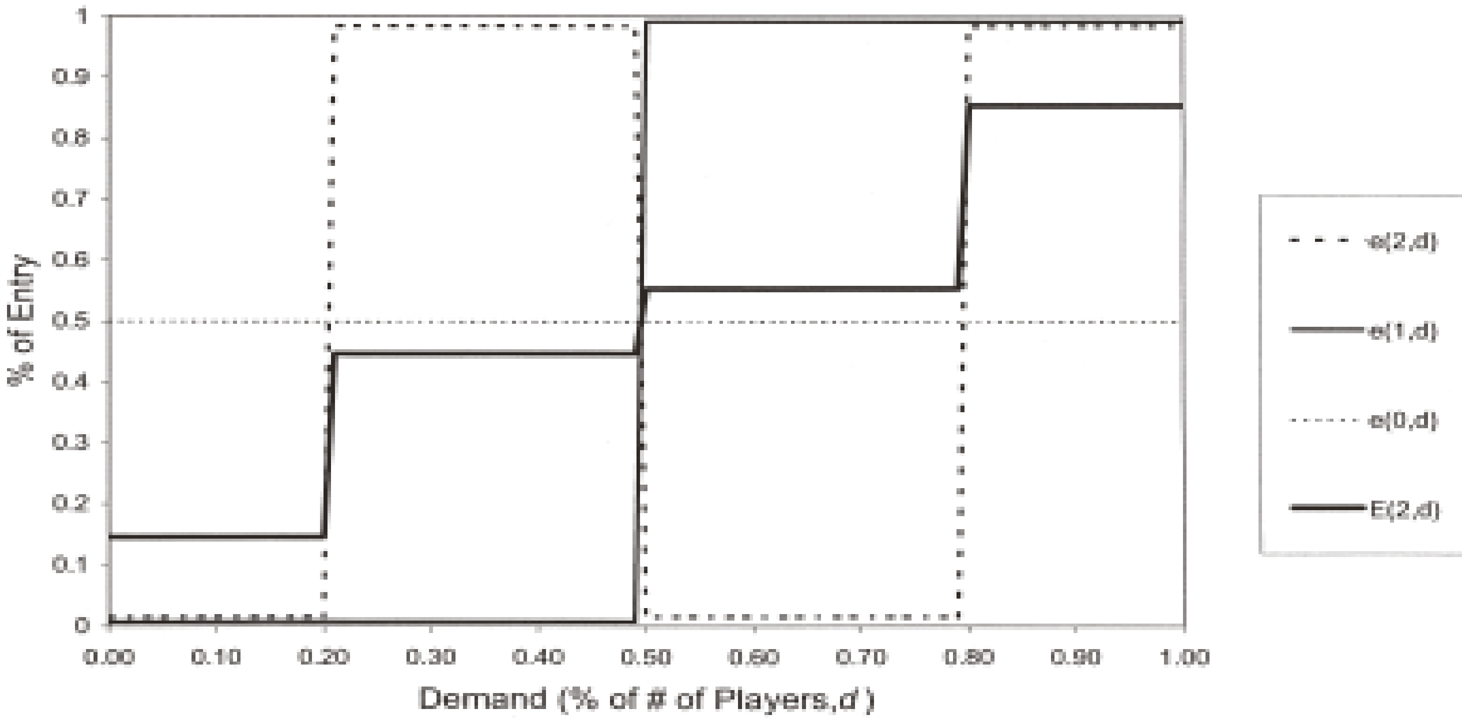

FIGURE I
Behaviors of Level 0, 1, and 2 Players ($\tau = 1.5$)

Level 0: chose randomly so that 50% enter.

Level 1: thinks that everyone else is level 0 and that the expected entry rate is 50% irrespective of d; will not enter if d is below 50%, but will enter if d is above 50%.

Level 2: thinks that 40% of other players are level 0 and 60% of other players are level 1 and that the expected entry rate is 20% when d is below 0.5 and 80% when d is 0.5 or higher; will not enter if d is 0.2 or lower; will enter if d is larger than 0.2 and lower than 0.5; will not enter when d is between 0.5 and 0.8; will enter when d is larger than 0.8.

TABLE II
DATA AND CH ESTIMATES OF τ IN VARIOUS p-BEAUTY CONTEST GAMES

Subject pool or game	Source[a]	Group size	Sample size	Nash equil'm	Pred'n error	Data			Fit of CH model					Bootstrapped 90% c.i.
						Mean	Std dev	Mode	τ	Mean	Error	Std dev	Mode	
$p = 1.1$	HCW (98)	7	69	200	47.9	152.1	23.7	150	0.10	151.6	−0.5	28.0	165	(0.0,0.5)
$p = 1.3$	HCW (98)	7	71	200	50.0	150.0	25.9	150	0.00	150.4	0.5	29.4	195	(0.0,0.1)
High $	CHW	7	14	72	11.0	61.0	8.4	55	4.90	59.4	−1.6	3.8	61	(3.4,4.9)
Male	CHW	7	17	72	14.4	57.6	9.7	54	3.70	57.6	0.1	5.5	58	(1.0,4.3)
Female	CHW	7	46	72	16.3	55.7	12.1	56	2.40	55.7	0.0	9.3	58	(1.6,4.9)
Low $	CHW	7	49	72	17.2	54.8	11.9	54	2.00	54.7	−0.1	11.1	56	(0.7,3.8)
.7(Mean+18)	Nagel (98)	7	34	42	−7.5	49.5	7.7	48	0.20	49.4	−0.1	26.4	48	(0.0,1.0)
PCC	CHC (new)	2	24	0	−54.2	54.2	29.2	50	0.00	49.5	−4.7	29.5	0	(0.0,0.1)
$p = 0.9$	HCW (98)	7	67	0	−49.4	49.4	24.3	50	0.10	49.5	0.0	27.7	45	(0.1,1.5)
PCC	CHC (new)	3	24	0	−47.5	47.5	29.0	50	0.10	47.5	0.0	28.6	26	(0.1,0.8)
Caltech board	Camerer	73	73	0	−42.6	42.6	23.4	33	0.50	43.1	0.4	24.3	34	(0.1,0.9)
$p = 0.7$	HCW (98)	7	69	0	−38.9	38.9	24.7	35	1.00	38.8	−0.2	19.8	35	(0.5,1.6)
CEOs	Camerer	20	20	0	−37.9	37.9	18.8	33	1.00	37.7	−0.1	20.2	34	(0.3,1.8)
German students	Nagel (95)	14–16	66	0	−37.2	37.2	20.0	25	1.10	36.9	−0.2	19.4	34	(0.7,1.5)
80 yr olds	Kovalchik	33	33	0	−37.0	37.0	17.5	27	1.10	36.9	−0.1	19.4	34	(0.6,1.7)
U. S. high school	Camerer	20–32	52	0	−32.5	32.5	18.6	33	1.60	32.7	0.2	16.3	34	(1.1,2.2)
Econ PhDs	Camerer	16	16	0	−27.4	27.4	18.7	N/A	2.30	27.5	0.0	13.1	21	(1.4,3.5)
1/2 mean	Nagel (98)	15–17	48	0	−26.7	26.7	19.9	25	1.50	26.5	−0.2	19.1	25	(1.1,1.9)
Portfolio mgrs	Camerer	26	26	0	−24.3	24.3	16.1	22	2.80	24.4	0.1	11.4	26	(2.0,3.7)
Caltech students	Camerer	17–25	42	0	−23.0	23.0	11.1	35	3.00	23.0	0.1	10.6	24	(2.7,3.8)
Newspaper	Nagel (98)	3696, 1460, 2728	7884	0	−23.0	23.0	20.2	1	3.00	23.0	0.0	10.6	24	(3.0,3.1)
Caltech	CHC (new)	2	24	0	−21.7	21.7	29.9	0	0.80	22.2	0.6	31.6	0	(4.0,1.4)
Caltech	CHC (new)	3	24	0	−21.5	21.5	25.7	0	1.80	21.5	0.1	18.6	26	(1.1,3.1)
Game theorists	Nagel (98)	27–54	136	0	−19.1	19.1	21.8	0	3.70	19.1	0.0	9.2	16	(2.8,4.7)
Mean									1.30					
Median									1.61					

a. HCW (98) is Ho, Camerer, Weigelt *AER* 98; CHC are new data from Camerer, Ho, and Chong; CHW is Camerer, Ho, Weigelt (unpublished); Kovalchik is data reported by Kovalchik et al. [in press].

The Poisson parameter (τ) has been estimated by finding the parameter that minimize the difference between the predicted mean (based on the CH model) and the actual mean in the data.

TABLE III
PARAMETER ESTIMATE τ FOR COGNITIVE HIERARCHY MODELS

Data set	Stahl and Wilson	Cooper and Van Huyck	Costa-Gomes et al.	Mixed	Entry
Game-specific τ					
Game 1	2.93	15.90	2.28	0.98	0.70
Game 2	0.00	1.07	2.27	1.71	0.85
Game 3	1.40	0.18	2.29	0.86	—
Game 4	2.34	1.28	1.26	3.85	0.73
Game 5	2.01	0.52	1.80	1.08	0.70
Game 6	0.00	0.82	1.67	1.13	
Game 7	5.37	0.96	0.88	3.29	
Game 8	0.00	1.54	2.18	1.84	
Game 9	1.35		1.89	1.06	
Game 10	11.33		2.26	2.26	
Game 11	6.48		1.23	0.87	
Game 12	1.71		1.03	2.06	
Game 13			2.28	1.88	
Game 14				9.07	
Game 15				3.49	
Game 16				2.07	
Game 17				1.14	
Game 18				1.14	
Game 19				1.55	
Game 20				1.95	
Game 21				1.68	
Game 22				3.06	
Median τ	1.86	1.01	1.89	1.77	0.71
Common τ	1.54	0.82	1.73	1.48	0.73

Entry: entry games (as in previous slides).
Mixed: games with mixed strategy equilibria (e.g. one strategy is played 70% of the time and another 30% of the time).
Stahl/Wilson, Cooper/Van Huyck, Costa-Gomes et al.: matrix games with 2-4 strategies and one equlibria.

TABLE IV
Model Fit (Log-likelihood LL and Mean Squared Deviation MSD)

Data set	Stahl and Wilson	Cooper and Van Huyck	Costa-Gomes et al.	Mixed	Entry
Log-likelihood					
Cognitive hierarchy (Game-specific τ)	−360	−838	−264	−824	−150
Cognitive hierarchy (Common τ)	−458	−868	−274	−872	−150
Nash equilibrium[a]	−1823	−5422	−1819	−1270	−154
Mean squared deviation					
Cognitive hierarchy (Game-specific τ)	0.0074	0.0090	0.0035	0.0097	0.0004
Cognitive hierarchy (Common τ)	0.0327	0.0145	0.0097	0.0179	0.0005
Nash equilibrium	0.0882	0.2038	0.1367	0.0387	0.0049

Shows how well the cognitive hierarchy model explains the data (either with the estimated Poisson parameter for each specific game or the estimated parameter for that class of games) compared to Nash equilibrium.

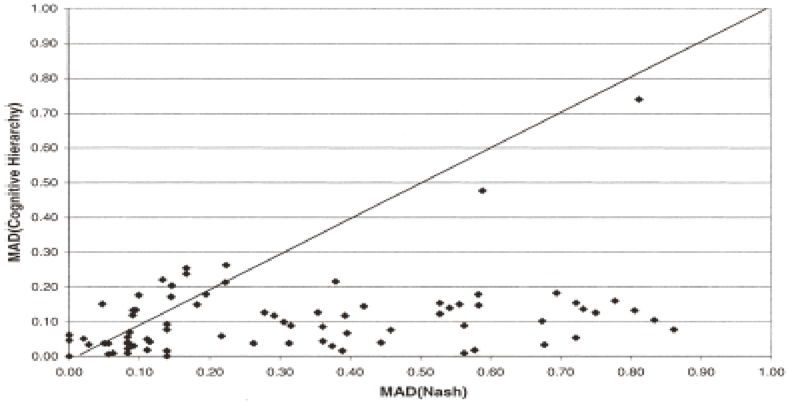

FIGURE II
Mean Absolute Deviation for Matrix Games: Nash Versus Cognitive Hierarchy (Common τ)

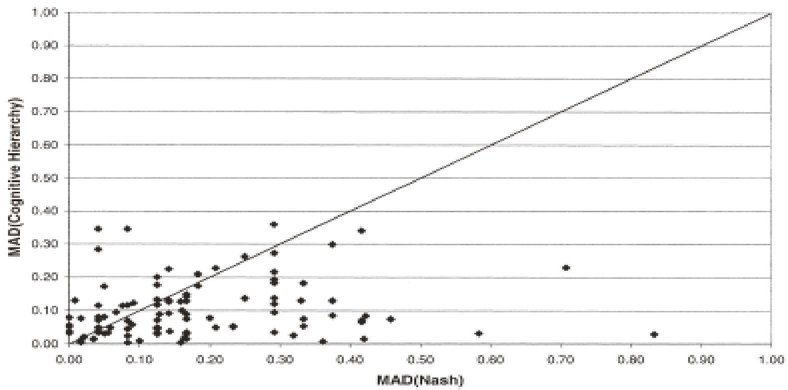

FIGURE III
Mean Absolute Deviation for Mixed Games: Nash Versus Cognitive Hierarchy (Common τ)

Shows the relationship between the prediction error for the cognitive hierarchy model and Nash equilibrium. Observations below the diagonal implies that the cognitive hierarchy model predicts better. When the error is large with Nash the error is typically small with the cognitive hierarchy model and when the error is small with Nash the cognitive hierarchy model also typically predicts well.

TABLE VI
GAME 8 FROM COSTA-GOMES ET AL. [2001]

	L	R	Data	Nash	CH
T	45, 66	82, 31	.92	1	.82
TM	22, 14	57, 55	0	0	.06
BM	30, 42	28, 37	0	0	.06
B	15, 60	61, 88	.08	0	.06
Data	.64	.36			
Nash	1	0			
CH	.55	.45			

Nash: For the row player T gives the highest payoff irrespective of the choice of the other player (i.e. predicts 100% T). Given that the row player choses T, the column player chose L (i.e. predicts 100% L).

Cognitive hierarchy model:

Row player: Level 0 chose randomly; Level 1 or higher chose the dominant strategy T. Predicts 82% T and 6% each for the other three strategies.

Column player: Level 0 chose randomly; Level 1 thinks that the row player chose randomly and therefore picks R (highest expected payoff given that the row player chose randomly); Level 2 thinks that the row player was level 1 (who chose T) or level 0 (who chose randomly); therefore chose L (that maximize the expected payoff given the beliefs about the fraction of level 0 and level 1 row players). Level 3 or higher picks L. Predicts 55% L and 45% R.

TABLE VIII
Economic Value of Various Theories

Data set	Stahl and Wilson	Cooper and Van Huyck	Costa-Gomes et al.	Mixed	Entry
Observed payoff	195	586	264	328	118
Clairvoyance payoff	243	664	306	708	176
Economic value					
Clairvoyance	48	78	42	380	58
Cognitive hierarchy (Common τ)	13	55	22	132	10
Nash equilibrium	5	30	15	−17	2
% Maximum economic value achieved					
Cognitive hierarchy (Common τ)	26%	71%	52%	35%	17%
Nash equilibrium	10%	39%	35%	−4%	3%

The economic value is the total value (in experimental payoffs) of all rounds that a "hypothetical" subject will earn using the respective model to predict other's behavior and best responds with the strategy that yields the highest expected payoff in each round.

Observed payoff: actual mean payoff.
Clairvoyance payoff: payoff of playing best reply to what the other players actually do.
Cognitive hierarchy payoff: payoff of playing best reply to the predicted play according to the cognitive hierarchy model.
Nash equilibrium payoff: payoff of playing Nash equilibrium (best reply to other players playing Nash).