Seminar 3 - Exercises

1. Review questions

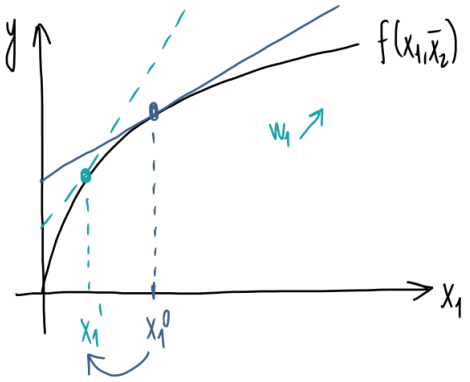
   (a) In the short run, if the price of fixed factor increases, what will happen to profit?
   A: Profit will decrease if the fixed factor is used in positive amount or remain the same if the fixed factor is equal to zero.

   (b) If $pMP_1 > w_1$, then should the firm increase or decrease the amount of factor 1 in order to increase profits?
   A: The firm should increase the amount of factor 1, as the value of additional unit of output exceeds its cost.

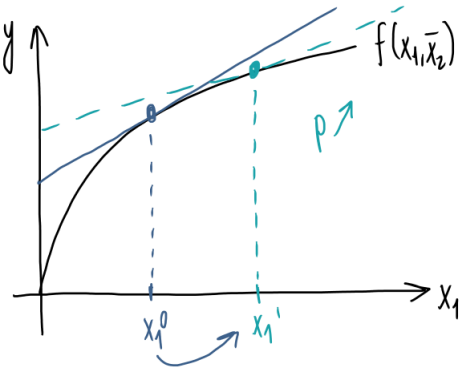
   (c) If a firm had decreasing returns to scale at all levels of output and it divided up into two equal-size smaller firms, what would happen to its overall profits?
   A: The overall profit would increase.

2. Consider production function in the form $y = (x_1 + 5)^{\frac{1}{2}}(x_2 + 5)^{\frac{1}{2}} - 2$.

   - Derive short term production function when second factor is fixed $x_2 = 4$.
     A: $y = 3(x_1 + 5)^{\frac{1}{2}} - 2$
   - For required output $y = 7$ calculate the optimal amount of variable factor x_1^*.
     A: $x_1^* = 4$
   - Determine profit of the firm when $p = 3$, $w_1 = 1$ and $w_2 = 2$.
     A: $\pi = 9$
   - What happens to the short-run profit-maximizing production plan as the variable input price w_1 increases?
     A: The amount of x_1 demanded will decrease to keep the profit maximized.



   - What happens to the short-run profit-maximizing production plan as the output price p increases?
     A: The demand for x_1 will increase.



3. Consider production function in the form $y = 2x_1^{\frac{1}{3}}x_2^{\frac{2}{3}}$.

   - Calculate factor demand functions of x_1 and x_2.
     A: For this production function we cannot calculate the factor demand functions which are dependent only on prices of inputs and outputs. We can express only relationship based on output and prices: $x_1 = \frac{py}{3w_1}$, $x_2 = \frac{2py}{3w_2}$
     This Cobb-Douglas production function ($\alpha+\beta = 1$) has constant returns to scale, therefore the supply function is not well-defined (supply derived from conditional demand is $y = 2x_1^{\frac{1}{3}}x_2^{\frac{2}{3}} = 2(\frac{py}{3w_1})^{\frac{1}{3}}(\frac{2py}{3w_2})^{\frac{2}{3}} = 2(\frac{p}{3w_1})^{\frac{1}{3}}(\frac{2p}{3w_2})^{\frac{2}{3}}y$ and output cancels out), therefore in this particular case all values of y are optimal – the profit will always be 0.
   - What is the value of x_1, x_2 when $y = 3$, $w_1 = 1$, $w_2 = 2$ and $p = 3$?
     A: $x_1 = 3$ and $x_2 = 3$. Note that this is not the unconditional profit maximizing plan. We are given the value of output that needs to be produced.

4. Consider production function in the form $y = 2(x_1 + x_2)^{\frac{1}{2}}$

   - Calculate factor demand functions of x_1 and x_2.
     A: The producers will use only the cheaper input:
     - $x_1 = (\frac{p}{w_1})^2$ if $w_1 < w_2$
     - $x_2 = (\frac{p}{w_2})^2$ if $w_2 < w_1$
     - $x_1 + x_2 = (\frac{p}{w})^2$ if $w_1 = w_2 = w$
   - Calculate profit when a firm needs to produce output $y = 5$ and prices are given as $p = 3$, $w_1 = 2$ and $w_2 = 3$
     A: $\pi = 2.5$. Note that this is not the unconditional profit maximizing plan. We are given the value of output that needs to be produced (therefore for the computation we cannot use the factor demand functions but the production function itself).
   - Sketch the situation in a graph for x_1 and x_2.