1 Some general advice on reading math texts

In the preface of my notes I write that

> this course tries to bridge the gap between the bachelor level courses that are mainly computation-driven and the abstract mathematics and proof-driven reasoning of the academic economist.

Economists formulate and study models using the language of mathematics and support the formal statements they make about these models with proofs: careful logical derivations of these conclusions from the assumptions of the model. Think about mathematics as a toolbox. Earlier in your education, you were taught how to use certain tools. But from a certain academic level onward, we not only want to use these and more advanced tools, we also need to start thinking critically about *why* they work: why do certain assumptions lead to certain conclusions in our model? This will lead to a deeper understanding about economics: which assumptions drive the conclusions? What happens if you change certain assumptions? That is why advanced texts spend much more time on including proofs of results.

But does that mean that you as a reader need to plough through each and every letter of each and every proof? No, that would — depending on your background — probably take too much time.

Math texts, like the lecture notes, are often written to cater to a range of readers with different aims. At the very least, your aim should be to get the big picture. Can you, after reading a section, summarize the key points in your own words? Take the section on vector spaces as an example. The definition of a vector space is a huge list of properties. The exam is open-book, so you can look them up if you want. What you should remember is the sentence preceding it:

> *A vector space is a set V whose elements ('vectors') you can add together and multiply with numbers ('scalars'). Addition and scalar multiplication once again produce vectors in V and satisfy standard arithmetic properties.*

Also, you can make a smaller vector space out of a larger one (subspaces) and a larger one out of smaller ones by putting copies of vector spaces next to each other (product spaces). That is a pretty good overview of that section.

The examples give particular instances of vector spaces, many of which you are probably already familiar with. And the theorems give a deeper understanding. Theorem 1.2 tells how to recognize a subspace. Theorem 1.1 tells that the long list of properties of vector spaces implies some other properties, explaining why you don't need to add those to the already long list to begin with. The proof of that theorem consists of six separate proofs. I don't count on you reading all of them, let alone being able to reproduce these from scratch (remember the magical phrase 'open-book exam'). But it is good practice to at least read one or two — preferably the earlier ones, since the later ones rely on them. If you are a budding theorist, by all means read each proof. And if you are really pressed for time, skip them altogether: I prove them so you don't have to.

So there is our range of different readers: on one side people who want to read all the proofs, on the other side people who have the assurance that I am not just peddling untruths because most proofs are there if they are unsure, but won't struggle through all of them in detail. These people, and everyone in between, will at the very least learn a lot of things to apply to specific cases.

There are very few explicit economic applications in the text. That is on purpose. After consulting with the other teachers, we recognized that there was a lot of material that needed to be covered and I could not do both the theory and a whole bunch of economic models in which to apply it. The applications come in the later courses. Moreover, admittedly, I cannot really talk with authority about economic models outside my own fields of research.

Let me try to answer some frequently-asked questions (I am cheating a bit: not all of these are FAQ's. Some are hidden recommendations from my side. . .)

2 FAQ

QUESTION: You can of course read a theorem without further reflection and use it whenever it comes in handy. But if you want a deeper understanding of a theorem or definition, what should you look out for?
ANSWER: For theorems, try to understand the reason for the different assumptions in the theorem. Can you make up examples where the theorem fails if you omit an assumption? Where in the proof are the assumptions of the theorem used?

For definitions, can you explain the concept in your own words? Often the sentences before/after a theorem or definition will help you to convey the main idea. Can you find something that satisfies the definition and something that doesn't? (For instance, for continuity: can you find your own examples of a function that is continuous and one that isn't?)

QUESTION: Will there be proofs on the exam?
ANSWER: On the front page of the exam, you will find the sentence

> To obtain a good score, write clearly and logically, starting from definitions and correctly deducing and motivating your answers.

So it's not (only) the final answer I am after, it is more about the logical steps you take to arrive at that answer. And such a sequence of logical steps that constitute a correct answer is a proof. In other words: yes, every answer you give is going to be a proof.

This insistence on including the logical steps towards your answer works in your advantage. If you without any motivation answer '5' when the correct answer is '42', there is no way to give you any credit. But if you explain your steps, starting out correctly but making a mistake along the way, there may be room for partial credit.

But I think I know what you are after: will there be proofs of general theorems, like a claim about an arbitrary vector space instead of a single particular one? There is usually one out of five exercises in the exam that is a little bit more challenging and that will involve some abstract thinking. But there is a strict time limit on the exam, so I can't expect anything long. If you need to do a general argument, it will not involve much more than two or three sentences: there isn't any time for more. In fact, my rule of thumb is that I should be able to provide a complete solution to the entire exam in two, at most two-and-a-half pages. And that includes the space-consuming solutions to the static and dynamic optimization exercises. So don't get worked up about this!

QUESTION: What level of detail should my answers have?
ANSWER: This can only be learned from experience. That is why I give you solutions to exams and problem sets from earlier years and a solutions manual to most exercises in the lecture notes. As a general rule, if you use a specific theorem or example from the notes to draw a conclusion, refer to it explicitly by its corresponding number ("By Theorem 13.3,...") or its name ("By the Extreme Value Theorem, ..."). Why? Well, if you write something like "according to a result in the notes..." I have no way of checking whether you are just bluffing. There are hundreds of results in the notes, so I need to know *which* one!