Capturing serial correlation in the data using AR(p) models

1. Estimate the tentative model suggested by the ACF/PACF or an Information Criteria (e.g., AIC, BIC). Assume that the tentative model is of order p^* ($0 \leq p^* \leq p$).

2. Create the residual series:

$$\hat{\varepsilon}_t = y_t - \hat{a}_0 - \hat{a}_1 y_{t-1} - \cdots - \hat{a}_{p^*} y_{t-p^*}.$$

   If the tentative model has captured all serial correlation in the data (i.e. the order of the model is set high enough) then the $\{\hat{\varepsilon}_t\}$ series should pass a White Noise (WN) test, i.e. the null hypothesis of no remaining kth-order serial correlation

$$H_0 : \rho_1 = \rho_2 = \cdots = \rho_k = 0,$$

   should *not* be rejected using the WN test (WN$\overset{approx.}{\sim} \chi^2(k)$; you may try different k-values, e.g. $k = 1, 4, 8, 12$), and where $\rho_s = Corr(\hat{\varepsilon}_t, \hat{\varepsilon}_{t-s})$.

3. If H_0 is not rejected, proceed by the AR(p^*) as the final model. If H_0 is rejected (meaning that we have not captured all serial correlation in the data), then proceed as above but base the WN test on the residuals from an AR($p^* + 1$) model:

$$\hat{\varepsilon}_t = y_t - \hat{a}_0 - \hat{a}_1 y_{t-1} - \cdots - \hat{a}_{p^*} y_{t-p^*} - \hat{a}_{p^*+1} y_{t-(p^*+1)}.$$

4. If H_0 is not rejected, proceed by the AR($p^* + 1$) as the final model. If H_0 is rejected, then proceed as above but base the WN test on the residuals from an AR($p^* + 2$) model.

$\vdots$

5. Continue until you have found a model where the null hypothesis of no remaining serial correlation for the corresponding residual series is not rejected.

Remark: It is important to capture all serial correlation in the data by your model, and going through above steps is one way ensure this. If not all serial correlation is captured by the model, the estimates of this model may be biased and also inefficient (large std.).