A Brief Introduction into Game Theory

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JEB064 Game Theory and Applications

The language

- Non-cooperative game theory is 'the language' of modern microeconomics.
  - Key question: How do rational agents play (optimize) against each other?
- n-person rational decision-making theory, $n \geq 2$
  = constrained n-person optimization
  - decision theory: 1-person decision-making theory
- When is decision-making rational?
  - Strong incentives to correct errors in high-stakes decisions.
  - Experience accumulates in frequently repeated interactions.
  - Survival selection and imitation of successful strategies.
- Alternatives to classic non-cooperative game theory:
  - evolutionary game theory: strategies chosen by imitation and selection, not by optimization
  - behavioral game theory: systematic violations to optimization
  - algorithmic game theory: optimization reflects computational constraints

Course

- This course introduces into (relatively non-technical) essentials.
  - complete information: all agents share knowledge about fundamentals
  - both perfect and imperfect information (opponent's moves are observable or unobservable)
  - non-cooperative game theory; we will only lightly touch cooperative game theory (formation and stability of coalitions and division of profits in coalitions)
  - incomplete information (and economics of information) is covered at length in the graduate course JEM013
- Tools for applied economists:
  - rationality, normal-form games, Nash equilibria in pure and mixed strategies, weak dominance, extensive-form games, subgame perfection, infinite-horizon games

Why is incomplete information much more challenging?

- Each player must carefully assess how incentives of the opponents change when their information about fundamentals changes.

Let's Make a Deal

You are a contestant on the TV show, "Let's Make a Deal." You face three curtains, labeled A, B and C. Behind two of them are toasters, and behind the third is a car. You choose A, and the TV showmaster says, pulling curtain B aside to reveal a toaster, "*You're lucky you didn't choose B, but before I show you what is behind the other two curtains, would you like to change from curtain A to curtain C?*"
Should you switch? What is the exact probability that curtain C hides the car?

Monty Hall Problem

Let's play the game online.

The goal of the showmaster is to make a show for the spectators.

- Suppose the car is in A. The showmaster knows that the car is in A.
- We can describe the showmaster's strategy as follows:
  - Your first choice is A: The showmaker reveals randomly B or C, each with probability $\frac{1}{2}$ (we can discuss it later).
  - Your first choice is B: The showmaker reveals C.
  - Your first choice is C: The showmaker reveals B.
- The table illustrates what will be revealed under any combination of your initial choice and the true location of the car.

You/Car	A ($\frac{1}{3}$)	B ($\frac{1}{3}$)	C ($\frac{1}{3}$)
A	B ($\frac{1}{2}$), C ($\frac{1}{2}$)	C	B
B	C	A ($\frac{1}{2}$), C ($\frac{1}{2}$)	A
C	B	A	A ($\frac{1}{2}$), B ($\frac{1}{2}$)

Monty Hall Problem

- Suppose you initially chose A.
- What is the probability that the car is in A when B is revealed?
- By Bayes' rule, the (posterior/updated) belief is in our example as follows:

$$\Pr(\text{Car in A}|\text{B revealed}) = \frac{\Pr(\text{B revealed}|\text{Car in A})\Pr(\text{Car in A})}{\Pr(\text{B revealed})}$$

- Marginal likelihood:

$$\Pr(\text{B revealed}) = \Pr(\text{B revealed}|\text{Car in A}) \cdot \Pr(\text{Car in A}) +$$

$$+\Pr(\text{B revealed}|\text{Car in B}) \cdot \Pr(\text{Car in B}) + \Pr(\text{B revealed}|\text{Car in C}) \cdot \Pr(\text{Car in C})$$

$$\Pr(\text{B revealed}) = \frac{1}{2} \cdot \frac{1}{3} + 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} = \frac{1}{2}$$

- Therefore

$$\Pr(\text{Car in A}|\text{B revealed}) = \frac{1/6}{1/2} = \frac{1}{3}$$

$$\Pr(\text{Car in C}|\text{B revealed}) = \frac{1/3}{1/2} = \frac{2}{3}$$

- You *should* revise the choice.

Applications

- industrial economics (price and non-price competition, standards adoption, market entry, capacity expansion)
- managerial and organizational economics (auditing, labor tournaments)
- innovation economics (innovation contests)
- corporate finance (financial contracting)
- economic policy-making (monetary and fiscal policies, budgets, public finance)
- international economics (tax competition)

To quote just one practitioner (MIT Sloan alumnus):

> *Investment banking turned out to be more about game theory than about pure math. There are so many games happening behind the scenes!*

How about theory and empirics?

- Experiments: alternative mechanisms/games tested
- (Market and organization) design: alternative mechanisms/games proposed and tested (auctions, matching, networks, protocols)
- Reduced-form empirics: microeconomic models to explain signs of estimates (qualitative)
- Structural-form empirics: microeconomic models to derive (quantitative) parameters and then to serve for analysis of structural and policy changes (especially industrial organization)
- Business strategy: models and experience to guide managerial practices

A brief history

1930s Applied math

1940s Zero-sum games during the World War 2

1950s Deterrence to analyze the Cold War

1960s Threats and bargaining (industrial relations, politics, international relations)

1970s Evolutionary biology

1990s Auctions

2000s Market design

2010s Networks, communication

2020s Algorithms, machines

Course organization

- classes are voluntary but recommended
- slides online
- seminars/tutorials highly recommended
- homeworks individually or in couples
- homeworks take time, but these investments pay off!
- a 3-hour written closed-book exam
- the exam is based on the problem sets in class, seminars, and homeworks

Enjoy the course and good luck!