Handout 2: Elasticity and Price Controls

1   Introduction

The objective of this handout is to apply the supply and demand framework to situations of price controls (floors and ceilings) and taxes and subsidies. For that, we will review the concept of *elasticity*. We've seen that the demand is downward sloping, but the elasticity is how can we *quantify* how much the quantity demanded changes when prices change. We will then show how price controls *can* affect the free-market equilibrium.

2   Elasticity

How much does supply or demand change in response to a change in price (e.g., percent change)?

2.1   Price Elasticity of Demand

Focusing just on demand: we are interested in understanding the **percentage change in quantity for a percentage change in price.** Why percentages? It doesn't really make sense to say "a little" or "a lot" or even "if price goes down by \$1, quantity demanded goes up by 1000" without understanding how that relates to the other numbers on the demand curve. If your demand curve has the point $P = 4, Q = 10,000,000$ then a change in 1000 units for \$1 is a very small change in Q for a very big change in P. This motivates the idea that we should examine **percentage changes**. We define the relevant quantity as the **price elasticity of demand**[1]

$$\epsilon_D \equiv \frac{\%\Delta Q_D}{\%\Delta P}$$

By the definition of percentage changes:

$$= \frac{\Delta Q_D/Q_D}{\Delta P/P}$$

Rearranging:

$$= \frac{\Delta Q_D}{\Delta P} \cdot \frac{P}{Q_D}$$

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[1]We use the symbol $\equiv$ to denote a definition. We are defining the elasticity of demand mathematically in this case.

Notice that this is evaluated at a point, P, Q_D, to be plugged into the rightmost part of that equation. For a **linear demand curve**, $\Delta Q_D / \Delta P$ can be found from the slope. Because we plot P on the vertical axis, this is 1 over the slope you see on the graph. You can impute the slope given any 2 points on a linear demand curve:

$$\frac{\Delta Q_D}{\Delta P} = \frac{Q_{D2} - Q_{D1}}{P_2 - P_1}$$

Notice that **demand curves slope down, so the slope will be negative**. By the same logic, if $P_2 > P_1$, and the denominator is positive, then $Q_{D2} < Q_{D1}$, and the numerator is negative. **The price elasticity of demand is always a (weakly) negative number**.

Example.

Consider the linear demand curve that passes through the points $P_0 = 10, Q_0 = 2$ and $P_1 = 5, Q_1 = 6$. Compute the price elasticity of demand at P_0, Q_0.

$$\epsilon_D = \frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q} = \frac{Q_1 - Q_0}{P_1 - P_0} \frac{P_0}{Q_0} = \frac{6-2}{5-10} \cdot \frac{10}{2} = \frac{4}{-5} \cdot \frac{10}{2} = -4$$

$\square$

Intepretation.

We just found, in that example,

$$\epsilon_D = -4$$

which means:

$$\frac{\%\Delta Q}{\%\Delta P} = -4$$

We can interpret that using the following sentence:

"A 1% increase in the price of this good will cause a 4% decrease in quantity demanded, starting from the point $P = 10, Q = 2$."

Similarly, we could say a 1% decrease in price will cause a 4% increase in quantity demanded. $\square$

2.2   Price Elasticity of Supply

Price elasticity of supply is defined identically to price elasticity of demand, except we use Q_s:

$$\epsilon_S = \frac{\%\Delta Q_s}{\%\Delta P} = \frac{\Delta Q_s}{\Delta P}\frac{P}{Q_s}$$

Because the supply curve has a positive slope, i.e. an increase in price corresponds to an increase in quantity supplied, **the price elasticity of supply is always (weakly) positive**

3   Price Controls and Subsidies

Objective.   *In this example we will apply the concepts of supply and demand to understand the effects of price controls (floors or ceilings) and subsidies. Governments establish price controls in several markets. An example of price floor is the minimum wage - which is the price of labor. An example of price ceilings is the credit market, where several countries impose a cap on the interest that can be charged on purchases. These price controls do not allow prices to adjust freely and, as a consequence, can cause excess demand (price ceiling) or excessive supply (price floor). For now, we will focus on how to find an equilibrium with price controls. In future lectures, we will discuss the consequences of these interventions for the well being of consumers and producers.*

Suppose that supply and demand in a market are respectively given by

$$Q_S = -10 + 3p, \qquad Q_D = 50 - 2p,$$

1. What are the equilibrium price and quantity in the market?

2. At the equilibrium price and quantity: is supply more elastic than demand or vice versa?

3. The government is considering setting a price ceiling on cute face masks. What would the excess demand be with a price ceiling of \$8? And with a price ceiling of \$15?

4. Alternatively, the government is considering passing a subsidy [hint: consider the subsidy is a negative tax] to producers of \$1 per unit sold. What would the new equilibrium prices and quantity be under such subsidy?

Solution.

1. Setting the quantity demanded and the quantity supplied to be equal, we recover one equation in terms of prices

$$-10 + 3P = 50 - 2P \Rightarrow 5P = 60 \Rightarrow P = 12$$

this equation pins down the equilibrium price, since this is the price that *clears* that market - that is, equates supply and demand. Solving for the price in the equation above, we find that $P^* = 12$. Replacing back in either the supply and demand gives the equilibrium quantity. In the supply, for instance:

$$Q = -10 + 3 \times 12 \Rightarrow Q^* = 26$$

Graphically, the equilibrium can be represented in the following figure
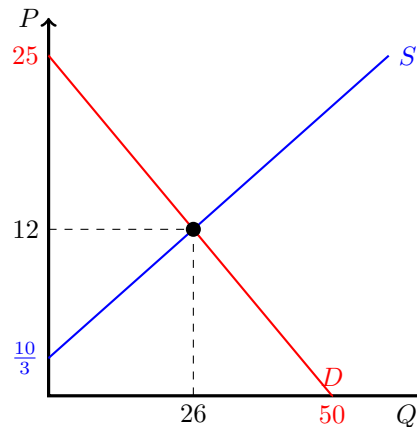
2. The price elasticity of demand represents the percent by which the quantity demanded changes with changes in prices. Mathematically :

$$\varepsilon_D \equiv \frac{\partial Q_D}{\partial P} \frac{P}{Q}$$

Note that the elasticity is defined at the point (p, Q) and for most demand curves the elasticity will change along the curve. In this example:

$$\varepsilon_D \equiv \frac{\partial Q_D}{\partial P} \frac{P^*}{Q^*} = -2 \frac{P^*}{Q^*}$$

We can compute the elasticity of supply analogously:

$$\varepsilon_S \equiv \frac{\partial Q_S}{\partial P} \frac{P}{Q}$$

In this example, the elasticity of supply in equilibrium is

$$\varepsilon_S \equiv \frac{\partial Q_S}{\partial P} \frac{P^*}{Q^*} = 3\frac{P^*}{Q^*}$$

so the **Supply is more elastic**, given that the elasticity of supply is larger in absolute value. This means that the percentage change in quantity supplied is larger (in absolute value) than the percentage change in quantity demanded for a change of prices.

3. A price ceiling of \$8 means that there will be a supply of

$$Q_S = -10 + 3 \times 8 = 14$$

and demand of

$$Q_D = 50 - 2 \times 8 = 34$$

leaving us with an excess demand of 20. The price ceiling of \$15, however, will have no effect in the equilibrium from item 1 because this price is above the equilibrium price we found in item 1. This means that the price ceiling is said *not binding*. This means that price controls *can* affect the equilibrium price and quantity, but only if it is binding, that is: a ceiling lower than the no intervention equilibrium price or a floor above it.

Graphically, the effect of the price ceilings can be seen in the following figure:
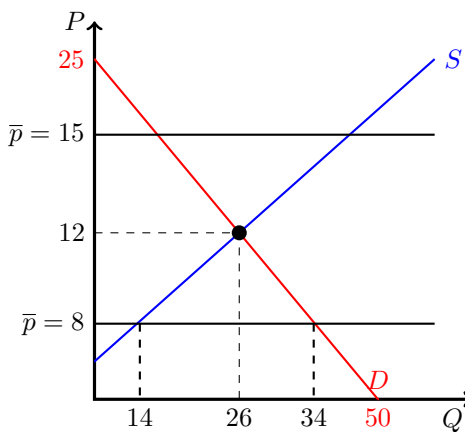
With the price ceiling of $8, the demand is much larger than the supply - and the difference is the 20 units of excess demand. The price ceiling of $15 is above the market level - and thus has no effect on the equilibrium.

4. We now use P to denote the price consumers pay. If consumers pay P, the producers will receive $P + 1$ due to the subsidy. Therefore, in equilibrium:

$$-10 + 3(P^* + 1) = 50 - 2P^* \Rightarrow P^* = \frac{57}{5} = 11.4$$

And replacing back in the supply: $Q^* = \frac{136}{5} = 27.2$. The important thing to notice here is that with subsidies, the equilibrium prices and quantities of the market will adjust. With subsidies, consumers pay 11.4 instead of 12 (the price in item 1) and producers receive 12.4 instead of 12. Both consumers and producers are better off. The economic agent that is worse off with subsidies is the Government or those who are taxed to pay for the subsidies.

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