IS-LM model

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IS-LM continuous model - simplified version (I)

- we assume **closed economy**
- **consumer expenditure** is related to disposable **income**
- **investment expenditure** is **negatively** related to the **interest rate**
- **government expenditure** is assumed to be exogenous

We therefore postulate simple linear expenditure function:

$$e(t) = a + b(1 - t_1)y(t) - hr(t) \quad (1)$$

$$a > 0, \quad 0 < b < 1, \quad 0 < t_1 < 1, \quad h > 0 \quad (2)$$

e(t) = real expenditure
a = autonomous expenditure
b = marginal propensity to consume
t_1 = tax rate
y(t) = real income
h = coefficient of investment response to r
r(t) = nominal interest rate

IS-LM continuous model - simplified version (II)

- the nominal monetary supply is assumed exogenous at $M_s = M_0$
- price level P is assumed to be constant
- we thus define real money balance (in fact money supply) $m_0 = \frac{M_0}{P}$, which is exogenous
- the **demand for real money** is assumed to be **positively related to real income** and **negatively related to interest rate**

We thus define:

$$m^d(t) = ky(t) - ur(t) \tag{3}$$

$$k > 0, \quad u > 0 \tag{4}$$

IS-LM continuous model - simplified version (III)

We assume:

- **Goods market:** income adjusts according to the excess demand
- **Money market:** interest rate adjusts according to excess demand for money

More specifically:

$$\dot{y}(t) = \alpha(e(t) - y(t)) \tag{5}$$

$$\dot{r}(t) = \beta(m^d(t) - m_0) \tag{6}$$

$$\alpha > 0, \quad \beta > 0 \tag{7}$$

Substituting from previous relations
(we drop the time variable t):

$$\dot{y}(t) = \alpha[b(1 - t_1) - 1]y - \alpha hr + \alpha a \tag{8}$$

$$\dot{r}(t) = \beta ky - \beta ur - \beta m_0 \tag{9}$$

$$\tag{10}$$

IS-LM continuous model - simplified version (IV)

We can find equilibrium lines by setting simply $\dot{y} = 0$ and $\dot{r} = 0$
For $\dot{y} = 0$ we get **IS curve**:

$$r = \frac{a - [1 - b(1 - t_1)]y}{h} \tag{11}$$

For $\dot{r} = 0$ we get **LM curve**:

$$r = \frac{-m_0 + ky}{u} \tag{12}$$

By solving these two equations for y and r we get the fixed point (y^*, r^*).

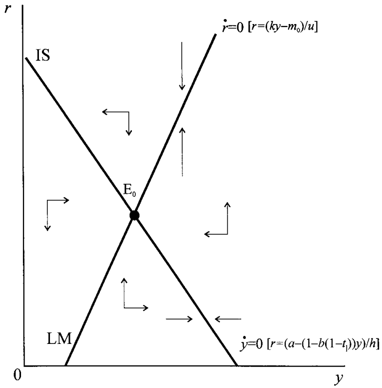
IS-LM continuous model - simplified version (V)

From $\dot{y} = 0$ equlibrium line we inspect behaviour of y close to this line. For the points to the right of the IS curve holds:

$$r > \frac{a - [1 - b(1 - t_1)]y}{h} \qquad (13)$$

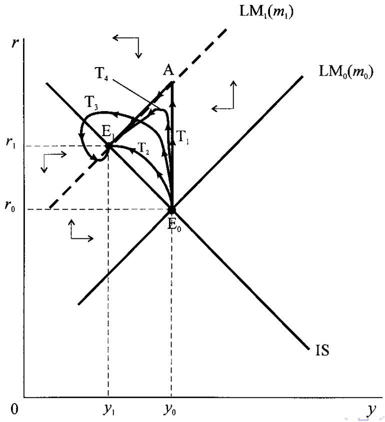
but from the equation for $\dot{y}$ it can be easily found out that such r implies $\dot{y} < 0$.

Consequently for the points on the left side of IS curve $\dot{y} > 0$. Similarly we can investigate behaviour around LM curve.



IS-LM continuous model - simplified version (V)

Consider now monetary contraction (decrease in value of m_0). We can observe (see figure) that economy converges to the new equilibrium. However trajectories (T_1, T_2, T_3) depend on specific values of parameters of model (see numerical example - next slide).



IS-LM continuous model - numerical example (I)

$$a = 50, \quad b = 0.75 \tag{14}$$

$$k = 0.25, \quad m_0 = 8 \tag{15}$$

For now we leave α and β unspecified. Thus we get system of two following differential equations:

$$\dot{y} = -0.44\alpha y - 1.53\alpha r + 50\alpha \tag{16}$$

$$\dot{r} = 0.25\beta y - 0.5\beta r - 8\beta \tag{17}$$

We assume that economy is in the equilibrium, thus initial point is equal to equilibrium of this system:

$$y_0 = y^* = 62, \quad r_0 = r^* = 15 \tag{18}$$

Now let us consider monetary contraction. We thus have new money stock $m_1 = 5$. This leads to the new equilibrium point:

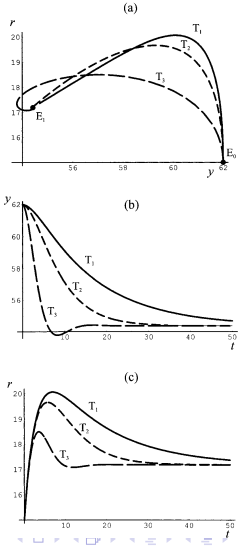
$$y_1 = 54, \quad r_1 = 17 \tag{19}$$

IS-LM continuous model - numerical example (II)

Now we are interested in different possible trajectories for different parameters α and β.

$$T_1: \quad \alpha = 0.05$$
$$\beta = 0.8$$
$$T_2: \quad \alpha = 0.1$$
$$\beta = 0.8$$
$$T_3: \quad \alpha = 0.5$$
$$\beta = 0.8$$

T_1, T_2: the goods market adjusts slowly, gradual decrease in income.
T_3: both market adjust quickly, over-shooting of income.
Overshooting in interest rate is always present.



Expendituree

- we assume **closed economy**
- **consumer expenditure** is related to disposable **income**
- **investment expenditure** is **negatively** related to the **interest rate**
- moreover **investment expenditure** is **positively** related to the **income**
- **government expenditure** is assumed to be exogenous

IS-LM continuous model (II)

We therefore postulate simple linear **expenditure function**:

$$e(t) = a + b(1 - t_1)y(t) - hr(t) + jy(t) \tag{20}$$

$$a > 0, \quad 0 < b < 1, \quad 0 < t_1 < 1, \quad h > 0, \quad j > 0 \tag{21}$$

$e(t)$ = real expenditure
a = autonomous expenditure
b = marginal propensity to consume
t_1 = tax rate
$y(t)$ = real income
h = coefficient of investment response to r
$r(t)$ = nominal interest rate
j = coefficient of investment response to y

IS-LM continuous model (III)

- the nominal monetary supply is assumed exogenous at $M_s = M_0$
- price level P is assumed to be constant
- we thus define real money balance (in fact money supply) $m_0 = \frac{M_0}{P}$, which is exogenous
- the **demand for real money** is assumed to be **positively related to real income** and **negatively related to interest rate**

We thus define:

$$m^d(t) = ky(t) - ur(t) \tag{22}$$

$$k > 0, \quad u > 0 \tag{23}$$

IS-LM continuous model (IV)

We assume:

- **Goods market:** income adjusts according to the excess demand
- **Money market:** interest rate adjusts according to excess demand for money

More specifically:

$$\dot{y}(t) = \alpha(e(t) - y(t)) \tag{24}$$

$$\dot{r}(t) = \beta(m^d(t) - m_0) \tag{25}$$

$$\alpha > 0, \quad \beta > 0 \tag{26}$$

Substituting from previous relations
(we drop the time variable t):

$$\dot{y} = \alpha[b(1 - t_1) + j - 1]y - \alpha hr + \alpha a \tag{27}$$

$$\dot{r} = \beta ky - \beta ur - \beta m_0 \tag{28}$$

$$\tag{29}$$

IS-LM continuous model (V)

We can find equilibrium lines by setting simply $\dot{y} = 0$ and $\dot{r} = 0$
For $\dot{y} = 0$ we get **IS curve**:

$$r = \frac{a - [1 - b(1 - t_1) - j]y}{h} \tag{30}$$

For $\dot{r} = 0$ we get **LM curve**:

$$r = \frac{ky - m_0}{u} \tag{31}$$

By solving these two equations for y and r we get the fixed point (y^*, r^*).

IS-LM continuous model (VI)

Recall IS curve:

$$r = \frac{a - [1 - b(1 - t_1) - j]y}{h} \tag{32}$$

The main difference compared to previous simplified version is that IS curve can have also positive slope.

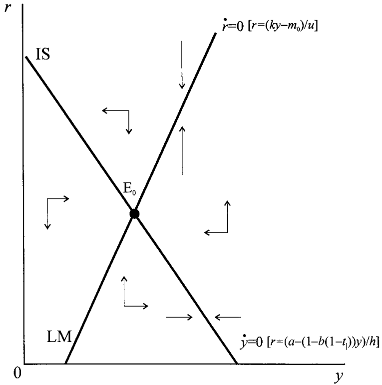
- $b(1 - t_1) + j - 1 < 0 \rightarrow$ negative slope
- $b(1 - t_1) + j - 1 > 0 \rightarrow$ positive slope

In the case that IS curve is positively slope it can be either less steep than LM curve or even steeper.

- $b(1 - t_1) + j - 1 < \frac{k}{u} \rightarrow$ IS is less steep than LM
- $b(1 - t_1) + j - 1 > \frac{k}{u} \rightarrow$ IS is steeper than LM

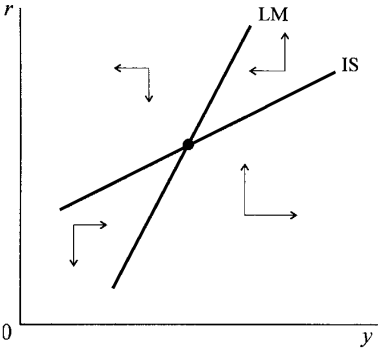
Negative slope of IS - the same as in the simplified version

From $\dot{y} = 0$ equlibrium line we inspect behaviour of y close to the IS line.
Similarly we can investigate behaviour around LM curve from $\dot{r} = 0$.
Thus we can draw phase diagram including vector forces. It is clear that the system is stable.

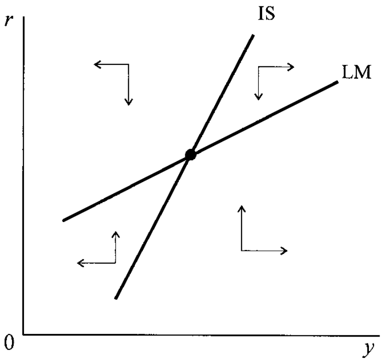


IS-LM continuous model (VII)

IS less steep than LM - spiral (stability?)



IS steeper than LM - unstable



Recall stability

Matrix and eigenvalues	Type of point	Type of stability
$\mathrm{tr}(\mathbf{A}) < 0$, $\det(\mathbf{A}) > 0$, $\mathrm{tr}(\mathbf{A})^2 > 4\det(\mathbf{A})$ $r < s < 0$	Improper node	Asymptotically stable
$\mathrm{tr}(\mathbf{A}) > 0$, $\det(\mathbf{A}) > 0$, $\mathrm{tr}(\mathbf{A})^2 > 4\det(\mathbf{A})$ $r > s > 0$	Improper node	Unstable
$\det(\mathbf{A}) < 0$ $r > 0$, $s < 0$ or $r < 0$, $s > 0$	Saddle point	Unstable saddle
$\mathrm{tr}(\mathbf{A}) < 0$, $\det(\mathbf{A}) > 0$, $\mathrm{tr}(\mathbf{A})^2 = 4\det(\mathbf{A})$ $r = s < 0$	Star node or proper node	Stable
$\mathrm{tr}(\mathbf{A}) > 0$, $\det(\mathbf{A}) > 0$, $\mathrm{tr}(\mathbf{A})^2 = 4\det(\mathbf{A})$ $r = s > 0$	Star node or proper node	Unstable
$\mathrm{tr}(\mathbf{A}) < 0$, $\det(\mathbf{A}) > 0$, $\mathrm{tr}(\mathbf{A})^2 < 4\det(\mathbf{A})$ $r = \alpha + \beta i$, $s = \alpha - \beta i$, $\alpha < 0$	Spiral node	Asymptotically stable
$\mathrm{tr}(\mathbf{A}) > 0$, $\det(\mathbf{A}) > 0$, $\mathrm{tr}(\mathbf{A})^2 < 4\det(\mathbf{A})$ $r = \alpha + \beta i$, $s = \alpha - \beta i$, $\alpha > 0$	Spiral node	Unstable
$\mathrm{tr}(\mathbf{A}) = 0$, $\det(\mathbf{A}) > 0$ $r = \beta i$, $s = -\beta i$	Centre	Stable

See MAXIMA.