Applied Econometric Time Series – Problem Set 3

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1 Multi-equation Time Series Models

Consider the bivariate system

$$y_t = -b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt} \tag{1.1}$$
$$z_t = -b_{21}y_t + \gamma_{21}y_{t-1} + \gamma_{22}z_{t-1} + \varepsilon_{zt} \tag{1.2}$$

where ε_{yt} and ε_{zt} are white-noise processes with $\mathbb{E}[\varepsilon_{yt}^2] = \sigma_y^2$ and $\mathbb{E}[\varepsilon_{zt}^2] = \sigma_z^2$ for all t, and ε_{ys} and ε_{zt} are uncorrelated for all s and t.

Problem (1a) Show that $Cov(y_t, \varepsilon_{zt}) \neq 0$. In terms of estimation, what are the implications?

Problem (1b) If you let $b_{12} = 0$, does $Cov(y_t, \varepsilon_{zt}) \neq 0$ still hold?

Problem (1c) Write the system in (1) - (2) in reduced form.

Problem (1d) Let the true values for the parameters in the reduced form system be given by

$$\mathbf{A}_1 = \begin{bmatrix} .5 & .1 \\ .4 & .2 \end{bmatrix} \quad \Sigma_e = \begin{bmatrix} 1 & .5 \\ .5 & 2 \end{bmatrix} \tag{1.3}$$

Assume $b_{12} = 0$, derive and quantify the parameters in the structural VAR in (1).

Problem (1e) Let $b_{21} = 0$ and $b_{12} = -.25$. Furthermore, let the matrix of dynamics in the reduced-form system be given by

$$\mathbf{A}_1 = \begin{bmatrix} .5 & .2 \\ .4 & .3 \end{bmatrix} \tag{1.4}$$

- Show that the reduced-form system is stationary.
- Assume a unit shock in ε_{yt}, plot the impulse response functions for $s \in \{0, 1, 2, 3, 4\}$.
- Assume a unit shock in ε_{zt}, plot the response functions for $s \in \{0, 1, 2, 3, 4\}$.
- Give two examples of an $\mathbf{A}_1$ matrix such that the system in (1) contains unit roots.
- For a unit root system, plot the impulse response function for $s \in \{0, 1, 2, 3, 4\}$ assuming a unit shock in ε_{yt}. What do you conclude?

2 Applications

Do exercises 10a-10g in the textbook (p. 340). Exercise (d) is optional.