1) Definiční obor
2) Parciální derivace
3) Body, ve kterých má smysl dopočítat derivaci

$f(x, y) = |\sin(x + y)|$

1) $D(f) = \mathbb{R}^2$
2) $\frac{\partial f}{\partial x} = sgn(\sin(x + y))\cos(x + y)\,;\, y \neq -x + k\pi; k \in \mathbb{Z}$

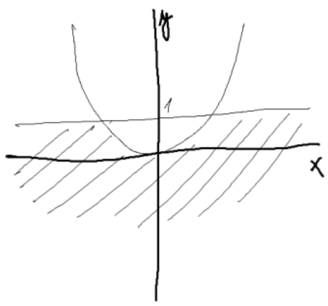
   $\frac{\partial f}{\partial y} = sgn(\sin(x + y))\cos(x + y)\,;\, y \neq -x + k\pi;\ k \in \mathbb{Z}$

3) $\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{x \to x_0^{\pm}} sgn(\sin(x - x_0 + k\pi))\cos(x - x_0 + k\pi) = \begin{matrix} x \to x_0^{+} \begin{cases} 1;\ k = 2n+1 \\ 1;\ k = 2n \end{cases} \\ x \to x_0^{-} \begin{cases} -1; k = 2n+1 \\ -1; k = 2n \end{cases} \end{matrix}$

   $y_0 = -x_0 + k\pi;\ k \in \mathbb{Z}$
   → derivace v těchto bodech neexistuje
   Analogicky pro parciální derivaci podle *y*.

$f(x, y) = \sqrt{x^2 - y}\cos\sqrt{1 - y}$

1)




2) $\frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2 - y}}\cos\left(\sqrt{1 - y}\right)$

   $\frac{\partial f}{\partial y} = \frac{-\cos\left(\sqrt{1-y}\right)}{2\sqrt{x^2-y}} + \frac{\sqrt{x^2-y}\sin\left(\sqrt{1-y}\right)}{2\sqrt{1-y}}$

3) $\frac{\partial f}{\partial x}(0,0) = \lim_{x_0 \to 0^{\pm}} \frac{x_0}{x_0^2}\cos(1) = \begin{cases} +\cos(1)\,;\ x \to 0^{+} \\ -\cos(1)\,; x \to 0^{-} \end{cases}$

   →limita v tomto bodě neexistuje

$f(x, y) = (x + y)^{|x-y|}$

1) $D(f) = \{[x, y] \in \mathbb{R}; y > -x\}$
2) $\frac{\partial f}{\partial x} = (x + y)^{|x-y|}\left[sgn(x - y)\log(x + y) + \frac{|x-y|}{x+y}\right]$

   $\frac{\partial f}{\partial y} = (x + y)^{|x-y|}\left[-sgn(x - y)\log(x + y) + \frac{|x-y|}{x+y}\right]$

3) $\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{x \to x_0^{\pm}} (x + x_0)^{|x - x_0|} \left[ sgn(x - x_0) \log(x + x_0) + \frac{|x - x_0|}{x + x_0} \right] =$

$\begin{cases} -\log(2x_0)\,; x \to x_0^+ \\ \log(2x_0)\,; x \to x_0^- \end{cases} \rightarrow \frac{\partial f}{\partial x}\left(\frac{1}{2}, \frac{1}{2}\right) = 0$, jinak limita pro $x_0 = y_0$ neexistuje

$x_0 = y_0$

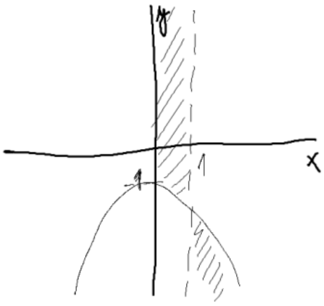
$\frac{\partial f}{\partial y}(x_0, y_0) = \lim_{y \to y_0^{\pm}} (y_0 + y)^{|y_0 - y|} \left[ -sgn(y_0 - y) \log(y_0 + y) + \frac{|y_0 - y|}{y_0 + y} \right] =$

$= \begin{cases} -\log(2y_0)\,; x \to x_0^+ \\ \log(2y_0)\,; x \to x_0^- \end{cases} \rightarrow \frac{\partial f}{\partial y}\left(\frac{1}{2}, \frac{1}{2}\right) = 0$, jinak limita pro $x_0 = y_0$ neexistuje

$x_0 = y_0$

$f(x, y) = log\,\frac{x^2 + y + 1}{1 - \sqrt{x}}$

1)



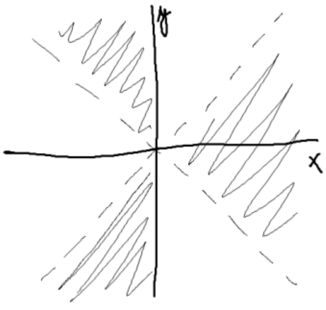
2) $\frac{\partial f}{\partial x} = \frac{2x}{x^2 + y + 1} + \frac{1}{2\sqrt{x}(1 - \sqrt{x})}$

$\frac{\partial f}{\partial y} = \frac{1}{x^2 + y + 1}$

3) Žádné body nejsou

$f(x, y) = \log\left(\frac{x}{|x| - |y|}\right)$

1)



2) $\frac{\partial f}{\partial x} = \frac{1}{x} - \frac{sgn(x)}{|x| - |y|}$

$\frac{\partial f}{\partial y} = \frac{sgn(y)}{|x| - |y|}$

3) $\frac{\partial f}{\partial y}(x_0, y_0) = \lim_{y\to 0^\pm} \frac{sgn(y)}{|x_0|-|y|} = \begin{cases} \frac{1}{|x_0|}; y \to 0^+ \\ -\frac{1}{|x_0|}; y \to 0^- \end{cases}; \frac{1}{|x_0|} \neq -\frac{1}{|x_0|}$ → limita neexistuje

$y_0 = 0$

$f(x, y) = (1 + |x|)^{|y|}$

1) $D(f) = \mathbb{R}^2$
2) $\frac{\partial f}{\partial x} = (1 + |x|)^{|y|} \frac{|y|}{1+|x|} sgn(x)$

   $\frac{\partial f}{\partial y} = (1 + |x|)^{|y|} \log(1 + |x|)\, sgn(y)$
3) $\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{x\to 0^\pm} (1 + |x|)^{|y_0|} \frac{|y_0|}{1+|x|} sgn(x) = \begin{cases} |y_0|; x \to 0^+ \\ -|y_0|; x \to 0^- \end{cases}$

   $x_0 = 0$

   → $\frac{\partial f}{\partial x}(0,0) = 0$, jinak v $[0, y]$neexistuje

   $\frac{\partial f}{\partial y}(x_0, y_0) = \lim_{y\to 0^\pm} (1 + |x_0|)^{|y|} \log(1 + |x_0|)\, sgn(y) = \begin{cases} \log(1 + |x_0|; y \to 0^+ \\ -\log(1 + |x_0|;\ y \to 0^- \end{cases}$

   $y_0 = 0$

   → $\frac{\partial f}{\partial y}(0,0) = 0$, jinak v $[x, 0]$neexistuje

$f(x, y) = \sqrt[3]{y - arctg(x)}$

1) $D(f) = \mathbb{R}^2$
2) $\frac{\partial f}{\partial x} = -\frac{1}{3} \frac{y}{1+x^2} \frac{1}{\sqrt[3]{(y-\arctan(x))^2}}$

   $\frac{\partial f}{\partial y} = \sqrt[3]{y - \arctan(x)} + \frac{y}{3} \frac{1}{\sqrt[3]{(y-\arctan(x))^2}}$
3) $\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{x\to x_0^\pm} -\frac{1}{3} \frac{\arctan(x_0)}{1+x^2} \frac{1}{\sqrt[3]{(\arctan(x_0)-\arctan(x))^2}} = \begin{cases} 0;\ x_0 = 0 \\ -\infty;\ x_0 > 0 \\ \infty;\ x_0 < 0 \end{cases}$

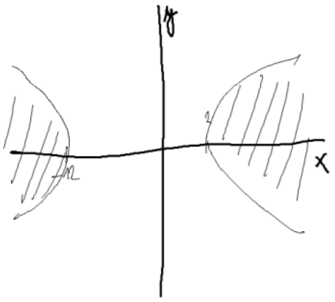
   $y_0 = \arctan(x_0)$

   $\frac{\partial f}{\partial y}(x_0, y_0) = \lim_{y\to y_0^\pm} \sqrt[3]{y - y_0} + \frac{y}{3} \frac{1}{\sqrt[3]{(y-y_0)^2}} = \begin{cases} 0;\ y_0 = 0 \\ +\infty;\ y_0 > 0 \\ -\infty;\ y_0 < 0 \end{cases}$

   → limity jsou (kromě bodu [0,0]) nevlastní

$f(x, y) = \arcsin\left(\frac{y^2+7}{x+5}\right)$

1)




2) $\frac{\partial f}{\partial x} = \sqrt{\frac{1}{1-\left(\frac{y^2+7}{x+5}\right)^2}}\frac{y^2+7}{(x+5)^2}$

$\frac{\partial f}{\partial y} = \sqrt{\frac{1}{1-\left(\frac{y^2+7}{x+5}\right)^2}}\frac{2y}{x+5}$

3) Jinde nemá smysl dopočítávat.