Part B: Selection on Observables

B2: Matching and Propensity Score Methods

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Outline

1. Matching
2. Propensity score methods
3. Applications

Exact matching

- When X is discrete, we can estimate CATE directly by matching treated and untreated units with the same X_i:

$$\widehat{CATE}(x) = \frac{\sum_{i:X_i=x} D_i Y_i}{\sum_{i:X_i=x} D_i} - \frac{\sum_{i:X_i=x} (1 - D_i) Y_i}{\sum_{i:X_i=x} (1 - D_i)}.$$

- Then average them into ATE (*exercise*: rewrite for ATT)

$$\widehat{ATE} = \sum_x \hat{P}(X = x) \cdot \widehat{CATE}(x) \equiv \frac{1}{N} \sum_i \widehat{CATE}(X_i)$$

- Example: Angrist (1998) study of how voluntary military service affects employment and earnings
  - X = all combinations of race, application year, years of schooling, Armed Forces Qualification Test score group, and year of birth

Matching with continuous variables

- With continuous variables, exact matching is not possible
- For each treated unit, choose R untreated ones closest to X_i (typically $R = 1$)
  - Mahalanobis distance: $d(X_i, X_j)^2 = (X_i - X_j)' \operatorname{Var}[X]^{-1} (X_i - X_j)$
  - Or diagonal version: $d(X_i, X_j)^2 = (X_i - X_j)' \operatorname{diag}\left(\operatorname{Var}[X]^{-1}\right) (X_i - X_j)$
- Matching with replacement: multiple treated units can match to the same control
- Without replacement: only match to controls previously unmatched (if $N_0 > N_1$)
  - Warning: the ordering of treated units matter
- This yields ATT
  - Rarely used for ATE but can find treated matches to each control unit

Matching caveats

- Curse of dimensionality: bias from imperfect matches is of order $N^{-1/\dim X}$ (Abadie-Imbens 2002)
  - $\dim X = 2 \Longrightarrow$ same order as SE; $\dim X > 2 \Longrightarrow$ bias dominates
  - Bias is small if $N_0 \gg N_1$; otherwise see Abadie-Imbens bias correction
- Matching is inefficient
- Asymptotic variance derived by Abadie-Imbens (2006)
  - While bootstrap fails (Abadie-Imbens 2008)
- Matching can be computationally difficult
- To reduce dimensionality, can use p-score methods...

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Propensity score results (Rosenbaum-Rubin 1983)

- Consider binary D. Recall $p(X) \equiv P(D = 1 \mid X)$
- **Proposition 1**: $D \perp X \mid p(X)$
  - i.e. propensity score balances X between treated and control groups
  - Proof: $P(D = 1 \mid X, p(X)) = P(D = 1 \mid X) = p(X)$
- **Proposition 2:** $D \perp (Y_0, Y_1) \mid X \implies D \perp (Y_0, Y_1) \mid p(X)$
  - i.e., under CIA, controlling for scalar $p(X)$ is enough
    - ⋆ A stronger version of the OVB idea
  - Proof: $P(D = 1 \mid p(X), Y_0, Y_1) = \mathbb{E}[\mathbb{E}[D \mid p(X), X, Y_0, Y_1] \mid p(X), Y_0, Y_1] = p(X)$, doesn't depend on (Y_0, Y_1)

P-score methods: Steps

1. Obtain $p(X)$
   - Known in stratified RCTs
   - Parametric estimation, e.g. logit on X
   - Non-parametric estimation
2. Assess overlap
   - Compare p-score distributions in treated & control groups
3. Verify balance
   - Within bins of $\hat{p}(X)$ compare X among treated and controls
   - If balance fails (with sufficiently many bins), make the p-score model richer
4. Adjust for p-score differences between treated and control
   - Regression, matching, blocking, reweighting

P-score adjustment methods: Regression

- With constant effects, enough to control linearly

$$Y_i = \beta D_i + \gamma p(X_i) + \text{error}$$

  - Or instrument D_i with $D_i - p(X_i)$ without any controls (E-estimator of Robins, Mark, Newey 1992)
  - *Exercise*: if $p(X)$ is estimated from a linear probability model, both ways are numerically the same as linearly controlling for X

- With heterogeneous effects, interact D with $p(X_i)$ or accept variance weighting

P-score adjustment methods: Blocking/Matching

- **Matching:** For each treated obs., find the untreated one with the closest $p(X_i)$
  - Discard untreated observations with p-score outside the range for the treated
- **Blocking** (stratifying)**:**
  - Split data into bins of $p(X_i)$
  - Estimate difference-in-means within bins
  - Average across bins weighting by # obs. (ATE) or # treated obs. (ATT)

P-score adjustment methods: Reweighting (IPW)

- In the bin with $p(X_i) = \pi$ we have fraction π of observed $Y_i(1)$ (for treated) and fraction $1 - \pi$ of comparable but missing $Y_i(1)$ (for controls)
- Horvitz-Thompson (1952) "inverse probability weighting" (IPW): reweighting by $1/\pi$ makes the sample of $Y_i(1)$ representative

$$\mathbb{E}\left[\frac{YD}{p(X)}\right] = \mathbb{E}\left[\frac{Y_1 D}{p(X)}\right] = \mathbb{E}\left[\mathbb{E}\left[\frac{Y_1 D}{p(X)} \mid X\right]\right] = \mathbb{E}\left[Y_1\right]$$

- Similarly for Y_0: $\mathbb{E}\left[\frac{Y(1-D)}{1-p(X)}\right] = \mathbb{E}\left[Y_0\right]$. Thus, under CIA+overlap:

$$ATE = \mathbb{E}\left[\left(\frac{D}{p(X)} - \frac{1-D}{1-p(X)}\right) Y\right] = \mathbb{E}\left[\frac{D - p(X)}{p(X)(1-p(X))} \cdot Y\right]$$

- *Exercise:* derive the reweighting expression for ATT

P-score adjustment methods: Reweighting (2)

- Plug-in estimator: $\widehat{ATE}_{\text{plug-in}} = \frac{1}{N}\sum_i \left(\frac{D_i}{\hat{p}(X_i)} - \frac{1-D_i}{1-\hat{p}(X_i)}\right) Y_i$
- Issue: weights on treated $(\frac{1}{N}\frac{D_i}{\hat{p}(X_i)})$ and controls $(\frac{1}{N}\frac{1-D_i}{1-\hat{p}(X_i)})$ do not exactly sum to 1
- Normalizing the weights improves performance:

$$\widehat{ATE} = \frac{\sum_i \frac{D_i Y_i}{\hat{p}(X_i)}}{\sum_i \frac{D_i}{\hat{p}(X_i)}} - \frac{\sum_i \frac{(1-D_i)Y_i}{1-\hat{p}(X_i)}}{\sum_i \frac{1-D_i}{1-\hat{p}(X_i)}}$$

- Convenient implementation: regression of Y_i on D_i with weights $\frac{D_i}{\hat{p}(X_i)} + \frac{1-D_i}{1-\hat{p}(X_i)}$ (and no controls)

P-score methods: Warnings

- P-score methods remove bias but are generally inefficient
  - E.g. in an RCT controlling for X improves efficiency
  - Although reweighting by a non-parametrically estimated p-score is efficient (Hirano, Imbens, Ridder 2003)
    - ⋆ Estimation is necessary even if true $p(X)$ is known (as in RCT)
- Finite-sample properties may be poor because of dividing by $\hat{p}(X_i)$ and $1 - \hat{p}(X_i)$
  - "Balancing" approaches estimate the inverse p-score directly (cf. Ben-Michael, Feller, Hirschberg, Zubizarreta 2021)

Inference

These estimators first estimate the p-score and then use it $\Longrightarrow$ inference is difficult

- For p-score reweighting, see Hirano-Imbens-Ridder (2003)
- For p-score matching, see Abadie-Imbens (2016)
- Bootstrap may work (except matching) although this have not been proved
- Or just ignore the error from the p-score estimation

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NSW application

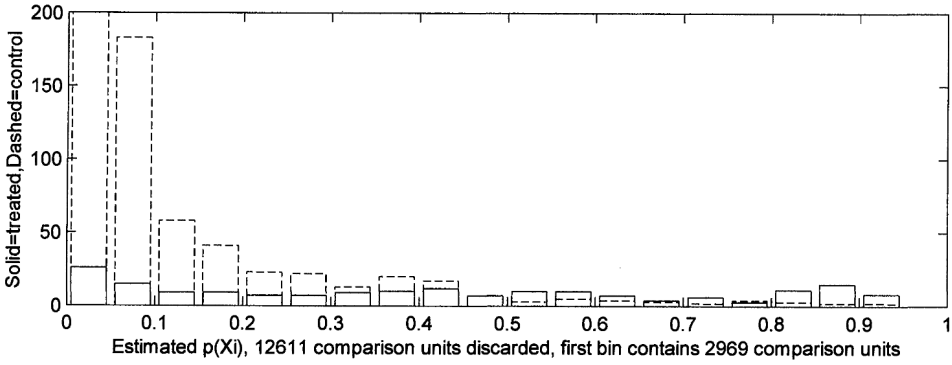
- Dehejia and Wahba (1999, 2002) show p-score methods perform much better than Lalonde's regression controls
- They search among logit specifications until they achieve balance

Table 4. Sample Means of Characteristics for Matched Control Samples

Matched samples	*No. of observations*	*Age*	*Education*	*Black*	*Hispanic*	*No degree*	*Married*	*RE74 (U.S. $)*	*RE75 (U.S. $)*
NSW	185	25.81	10.35	.84	.06	.71	.19	2,096	1,532
MPSID-1	56	26.39	10.62	.86	.02	.55	.15	1,794	1,126
		[2.56]	[.63]	[.13]	[.06]	[.13]	[.12]	[1,406]	[1,146]
MPSID-2	49	25.32	11.10	.89	.02	.57	.19	1,599	2,225
		[2.63]	[.83]	[.14]	[.08]	[.16]	[.16]	[1,905]	[1,228]
MPSID-3	30	26.86	10.96	.91	.01	.52	.25	1,386	1,863
		[2.97]	[.84]	[.13]	[.08]	[.16]	[.16]	[1,680]	[1,494]
MCPS-1	119	26.91	10.52	.86	.04	.64	.19	2,110	1,396
		[1.25]	[.32]	[.06]	[.04]	[.07]	[.06]	[841]	[563]
MCPS-2	87	26.21	10.21	.85	.04	.68	.20	1,758	1,204
		[1.43]	[.37]	[.08]	[.05]	[.09]	.08	[896]	[661]
MCPS-3	63	25.94	10.69	.87	.06	.53	.13	2,709	1,587
		[1.68]	[.48]	[.09]	[.06]	[.10]	[.09]	[1,285]	[760]

- Note small samples: most controls are discarded

NSW application: Overlap



(Dehejia and Wahba 1999, Figure 2)

- Limited overlap with the CPS control group (not shown: even worse for PSID)

NSW application: Results

Table 3. Estimated Training Effects for the NSW Male Participants Using Comparison Groups From PSID and CPS

	NSW earnings less comparison group earnings		NSW treatment earnings less comparison group earnings, conditional on the estimated propensity score					
			Quadratic in score[b]	Stratifying on the score			Matching on the score	
	(1) Unadjusted	(2) Adjusted[a]	(3)	(4) Unadjusted	(5) Adjusted	(6) Observations[c]	(7) Unadjusted	(8) Adjusted[d]
NSW	1,794	1,672						
	(633)	(638)						
PSID-1[e]	−15,205	731	294	1,608	1,494	1,255	1,691	1,473
	(1,154)	(886)	(1,389)	(1,571)	(1,581)		(2,209)	(809)
PSID-2[f]	−3,647	683	496	2,220	2,235	389	1,455	1,480
	(959)	(1,028)	(1,193)	(1,768)	(1,793)		(2,303)	(808)
PSID-3[f]	1,069	825	647	2,321	1,870	247	2,120	1,549
	(899)	(1,104)	(1,383)	(1,994)	(2,002)		(2,335)	(826)
CPS-1[g]	−8,498	972	1,117	1,713	1,774	4,117	1,582	1,616
	(712)	(550)	(747)	(1,115)	(1,152)		(1,069)	(751)
CPS-2[g]	−3,822	790	505	1,543	1,622	1,493	1,788	1,563
	(670)	(658)	(847)	(1,461)	(1,346)		(1,205)	(753)
CPS-3[g]	−635	1,326	556	1,252	2,219	514	587	662
	(657)	(798)	(951)	(1,617)	(2,082)		(1,496)	(776)

- Regression adjustment in col.3 doesn't perform well — could Oaxaca-Blinder do better?

Caveats

- IW (Lecture 2) report that p-score methods do not eliminate a significant "effect" of NSW on 1975 (pre-NSW) earnings
  - Failure of the CIA or of p-score estimation?
- Arceneaux, Gerber, Green (2006): a similar analysis of a "Get Out the Vote" program
  - Randomized phone calls but not everyone answers
  - IV methods identify the ATT — could we get it using selection on observables instead?
  - Covariates are similar to Lalonde; bigger sample allows exact matching
  - Yet, the estimates do not match the truth $\Longrightarrow$ CIA must not hold

A popular tool?

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“Sell crypto” might top “use propensity score matching as a research design” on the list of things economists would never do. Try harder, impersonators.

> **Jason Furman** @jasonfurman · 6h ·
> I have at least one impersonator here engaged in crypto scams (see this DM exchange one of them had with someone).