14.310x: Data Analysis for Social Scientists
Functions and Moments of a Random Variable & Intro to Regressions

Welcome to your fourth homework assignment! You will have about one week to work through the assignment. We encourage you to get an early start, particularly if you still feel you need more experience using R. We have provided this PDF copy of the assignment so that you can print and work through the assignment offline. You can also go online directly to complete the assignment. If you choose to work on the assignment using this PDF, please go back to the online platform to submit your answers based on the output produced.

Good luck!

Question 1
Assume that the random variable X has a PDF given by $f_X(x) = 1\ for\ 0 < x < 1$. What is the PDF of the random variable $Y = X^2$?

- o $f_Y(y) = \sqrt{y}\ for\ 0 < y < 1$
- o $f_Y(y) = \frac{1}{2\sqrt{y}}\ for\ 0 < y < 1$
- o $f_Y(y) = \frac{1}{2\sqrt{y}}\ for -1 < y < 1$
- o $f_Y(y) = \frac{1}{2}y^{-\frac{3}{2}}\ for -1 < y < 1$

Question 2
Suppose X has the geometric PMF $f_X(x) = \frac{1}{3}\left(\frac{2}{3}\right)^x\ for\ x = 0,1,2, \ldots$. What is the probability distribution of $Y = \frac{X}{X+1}$, its PMF? *Note that both X and Y are discrete random variables?*

- o $f_Y(y) = \frac{1}{3}\left(\frac{2}{3}\right)^{\frac{1-y}{y}}\ for\ y = 0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots, \frac{x}{x+1}, \ldots$
- o $f_Y(y) = \frac{1}{3}\left(\frac{2}{3}\right)^{\frac{(1-y)}{y}}\ for\ y = 0,1,2, \ldots$
- o $f_Y(y) = \frac{1}{3}\left(\frac{2}{3}\right)^{\frac{y}{1-y}}\ for\ y = 0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots, \frac{x}{x+1}, \ldots$
- o $f_Y(y) = \frac{1}{3}\left(\frac{2}{3}\right)^{\frac{y}{1-y}}\ for\ y = 0, 1,2, \ldots$

Question 3
Suppose the random variable X has a PDF given by $f_X(x) = \begin{cases} \frac{x-1}{2}, if\ 1 < x < 3 \\ 0, otherwise \end{cases}$. Which of the following is the monotone function $u(x)$ such that the random variable $u(X)$ has a uniform distribution between 0 and 1?

- o The monotone function does not exist.

- $\frac{(x-1)^2}{4}$
- $\frac{x(x-2)}{4}$
- $\frac{x-1}{2}$

Question 4
We have N i.i.d random variables from the uniform distribution between 0 and 1. If N=1, what is the probability that the nth order statistic is less than or equal to the value x? (In other words, what is Pr $(X_1^n \leq x)$?)

Questions 5-9 will be based on the following code:

```
#Creating a random draw of 1000 numbers
u <- runif(1000)
```

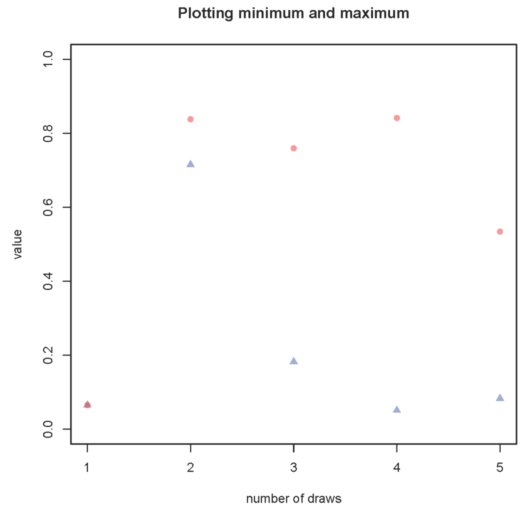
Question 5
Based on the above draw, is it possible to create from the above vector a random draw of a uniform distribution between 2 and 5?

- Yes
- No

Question 6
What is the PDF of the minimum of the draw?

- It is given by $f_{y^1}(y) = 999y^{998}$
- It is given by $f_{y^1}(y) = 1000(1-y)^{999}$
- It is given by $f_{y^1}(y) = 999(1-y)^{1000}$
- It is given by $f_{y^1}(y) = 999(1-y)^{998}$

Question 7
The following plot shows the maximum and the minimum of a uniform distribution by changing the number of draws.



A student is claiming that this plot is wrong since both the maximum and the minimum should show a monotonous relationship with the number of draws. Is this student's statement **True or False?**

- True
- False

Question 8

Use $F_X^{-1}(x)$ to transform $U \sim \mathcal{U}[0,1]$ into variable X which follows the original distribution of $\mathcal{N}(0,1)$. What is the command in R that allows you to do this?

- `pnorm`
- `rnorm`
- `dnorm`
- `qnorm`

Question 9

For each of the following, determine if the statement is true, false, or uncertain:

1. The height at a particular point of a continuous PDF can never exceed one.
2. In principle, one can always recover the joint distribution of random variables given their marginal distributions, although the calculations can sometimes be difficult.
3. Is this function a joint PDF? $f_{XY}(x,y) = \begin{cases} 1, \text{ if } 0 < x < 2, 0 < y < 1 \\ 0 \text{ otherwise} \end{cases}$

Question 10

A couple decides to continue to have children until a daughter is born. What is the expected number of children this couple will have if the probability that a daughter is born is given by p?

- The expected number of children is given by $\frac{1}{p} - 1$
- The expected number of children is given by $\frac{1-p}{p}$

- The expected number of children is given by $\frac{p}{p^3}$
- The expected number of children is given by $\frac{1}{p}$

Question 11
For each of the following expressions, find $\mathbb{E}[X]$.

$$f_X(x) = 2ax^{a-1}, 0 < x < 1, a > 0$$

$$f_X(x) = \frac{1}{n^2}, x = 1,2, \dots, n; n > 0$$

$$f_X(x) = x^3 - 4x, 0 < x < 1$$

Question 12
Suppose that the random variable Y has a binomial distribution with n trials and success probability X, where n is a given constant and X is a uniform (0,1) random variable. What is $\mathbb{E}[Y]$?

- This is given by $\frac{X}{n}$
- This is given by $\frac{n}{2}$
- This is given by $\frac{n}{3}$
- This is given by n

Question 13
As in Question 12, suppose that the random variable Y has a binomial distribution with n trials and success probability X, where n is a given constant and X is a uniform(0,1) random variables. What is $Var(Y)$?

- This is given by $\frac{n^2}{18} + \frac{n}{12}$
- This is given by $\frac{n^2}{18} + \frac{n}{6}$
- This is given by $\frac{n^2}{12} + \frac{n}{6}$
- This is given by $\frac{n^2}{6} + \frac{n}{12}$

Question 14
Assume that $y = \alpha + \beta X + U$, where $\beta = \frac{\rho_{XY}\sigma_Y}{\sigma_X}$ and $\alpha = \mu_Y - \beta\mu_X$ where μ_Y is the mean of Y and μ_X is the mean of X. What is the expected value of U?

- The expected value of U is α

- The expected value of U is μ_Y
- The expected value of U is 0
- The expected value of U is $\alpha + \beta\mu_X$

Question 15

Assume that $Y = \alpha + \beta X + U$, where $\beta = \frac{\rho_{XY}\sigma_Y}{\sigma_X}$ and $\alpha = \mu_Y - \beta\mu_X$. What is $cov(X, U)$? (Select all that apply).

- ☐ We have that $cov(X, U) = \rho_{XU}\sigma_X\sigma_U$
- ☐ We have that $cov(X, U) = var(X)$
- ☐ We have that $cov(X, U) = \sigma_X\sigma_U$
- ☐ We have that $cov(X, U) = 0$
- ☐ We have that $cov(X, U) = \sigma_X\sigma_Y$