DAGs and Bad Controls

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Core Empirical Research Methods

Selection on observables again...

Last Time

- Binary treatment D, potential outcomes (Y_0, Y_1).
- Observed outcome $Y = (1 - D)Y_0 + DY_1$.
- Selection on observables: $D \perp\!\!\!\perp (Y_0, Y_1)|\boldsymbol{X}$ for observed covariates $\boldsymbol{X}$.
- Overlap: $0 < p(\boldsymbol{X}) < 1$. (Recall that we can check this.)
- Regression Adjustment, Propensity Score weighting, Matching

Elephant in the Room

We have completely ignored the question of what to include in $\boldsymbol{X}$.

The Omitted Variables Bias (OVB) Formula[1]

- To keep things simple, assume a linear model with homogeneous effects:

$$Y = \alpha + \beta D + \gamma X + U, \quad \mathrm{Cov}(D, U) = \mathrm{Cov}(X, U) = \mathbb{E}(U) = 0.$$

- In other words:

$$Y_0 = \alpha + \gamma X + U, \quad Y_1 = Y_0 + \beta, \quad \mathrm{ATE} = \mathrm{TOT} = \beta$$

- What does a regression of Y on D identify?

$$\frac{\mathrm{Cov}(D, Y)}{\mathrm{Var}(D)} = \frac{\mathrm{Cov}(D, \alpha + \beta D + \gamma X + U)}{\mathrm{Var}(D)} = \beta + \gamma \frac{\mathrm{Cov}(D, X)}{\mathrm{Var}(X)}$$

- "Short" regression coefficient only equals β if $\gamma = 0$ or $\mathrm{Cov}(D, X) = 0$.

[1] See, e.g., Section 3.2.2 of *Mostly Harmless Econometrics*.

How *not* to interpret the OVB Formula.

- OVB Formula tells us *when* and *how* the coefficient on D differs depending on whether we include X in the regression.
- "Short" regression includes only D; "Long" regression includes both D and X.
- "Short" and "Long" coefficients for D agree if:
  1. X does not help predict Y in the "Long" regression or
  2. X is uncorrelated with D.
- Only if we **assume** that the long regression is the true causal model does this tell us whether we need to adjust for X.

Bad Advice: "Adjust for any observed variable that is correlated with D and Y."

Example 1: A prototypical bad control.

```r
set.seed(1693)
n <- 5000
d <- rbinom(n, 1, 0.4)
x <- rbinom(n, 1, 0.25 + 0.5 * d)
y <- x + rnorm(n)
mean(y[d == 1]) - mean(y[d == 0])
```

```
## [1] 0.523149
```

```r
library(broom); library(tidyverse)
xtilde <- x - mean(x)
reg <- lm(y ~ d + x + d:xtilde)
tidy(reg) |> filter(term == 'd') |>
  select(estimate, std.error)
```

```
## # A tibble: 1 x 2
##   estimate std.error
##      <dbl>     <dbl>
## 1   0.0113    0.0343
```

$$\begin{aligned}\mathbb{E}[Y_1 - Y_0] &= \mathbb{E}[Y|D=1] - \mathbb{E}[Y|D=0] \\ &= \mathbb{E}[X|D=1] - \mathbb{E}[X|D=0] = 0.5\end{aligned}$$

Why is X a bad control?

Intermediate Outcome

- Example 1: X is *itself* an outcome of D that *goes on* to cause Y.
- Adjusting for an intermediate outcome masks the true causal effect of D.
- E.g. randomized early childhood intervention causes college; college causes wage.
- In the simulation, 100% of the effect of D on Y goes through X.

Common Advice

> *Variables measured before the variable of interest [D] was determined are usually good controls. In particular, because these variables were determined before the variable of interest, they cannot themselves be outcomes in the causal nexus.*[2]

[2] From Section 3.2.3 of *Mostly Harmless Econometrics*, but similar statements are common.

Example 2: This bad control *is not* an intermediate outcome.

```r
library(mvtnorm)
R <- matrix(c(1, 0.5, 0.5, 1),
            2, 2)
errors <- rmvnorm(n, sigma = R)
u <- errors[,1]
v <- errors[,2]
x <- rbinom(n, 1, 0.5)
d <- 1 * ((-1 + 2 * x + v) > 0)
y <- -0.3 + d + u

mean(y[d == 1]) - mean(y[d == 0])
```

```
## [1] 1.511498
```

```r
xtilde <- x - mean(x)
lm(y ~ d + x + d:xtilde) |>
  tidy() |>
  filter(term == 'd') |>
  select(estimate, std.error)
```

```
## # A tibble: 1 x 2
##   estimate std.error
##      <dbl>     <dbl>
## 1     1.90    0.0355
```

Why is X a bad control?

Instrumental Variable

- Example 2: X is a valid instrument for the endogenous treatment D.
- But this is the **wrong** way to use an instrumental variable: should run IV.
- Adjusting for X soaks up the **exogenous** part of D, making the bias worse.[3]

```
library(AER)
ivreg(y ~ d | x) |> tidy() |> filter(term == 'd') |>
  select(estimate, std.error)
```

```
## # A tibble: 1 x 2
##   estimate std.error
##      <dbl>     <dbl>
## 1     1.01    0.0430
```

[3] See here for a proof.

“No causes in; no causes out.”[4]

Feeling confused?

- How can we tell which variables to adjust for and which are bad controls?
- Is is simply a matter of “I know it when I see it”?

Bad News

- Meaningful causal inference **always** requires assumptions, even in RCTs.
- Causal inference from observational data requires **even more** assumptions.

Good News

- If you make your assumptions explicit, there is a **definitive** solution.
- If it’s possible to use selection-on-observables, find the correct $\boldsymbol{X}$; if it’s not possible, show why this is so.
- Free bonus: better intuition about bad controls.

[4] Nancy Cartwright

The Birthweight Paradox[5]

The analyses in Yerushalmy's paper indicated that, among low birthweight infants of less than 2500g, maternal smoking was associated with lower infant morality. The results have been replicated in a number of studies and populations, and these seemingly paradoxical associations are now often referred to as the 'birthweight paradox'

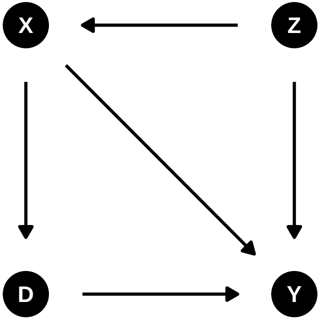
- $D = 1$ mother smokes while pregnant
- $Y = 1$ infant dies
- $X = 1$ low birthweight

Should we adjust for birthweight when studying the causal effect of maternal smoking on infant mortality?

[5] Quote from VanderWeele (2014).

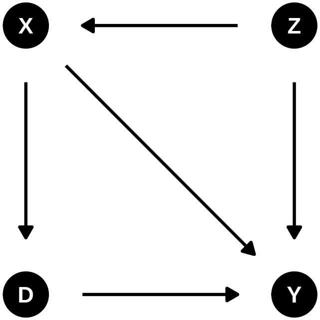
Graph: set of **nodes** connected by **edges**.

- Two nodes are **adjacent** if connected by an edge.
- Edges can be **directed** (figure) or **undirected**.
- Directed edge points from **parent** to **child**.
- **Directed graph** has only directed edges.
- **Path**: sequence of connected vertices.
- **Directed Path**: a path that "obeys one-way signs"
- Directed path points from **ancestor** to **descendant**.
- **Cycle**: directed path that returns to starting node.
- **Acyclic Graph**: a graph without any cycles.



Exercise

1. Is this graph directed?
2. Is this graph acyclic?
3. Are Z and D adjacent?
4. List all paths between D and Y.
5. List all *directed* paths from D to Y.



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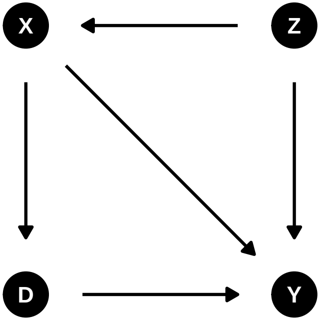


Solution

1. Yes: all edges in the graph are directed.
2. Yes: there is no directed path that takes you back to the node where you started.
3. Z and D are not adjacent: there is no edge between them.
4. There are three: $(D \rightarrow Y)$, $(D \leftarrow X \rightarrow Y)$, and $(D \leftarrow X \leftarrow Z \rightarrow Y)$.
5. There is only one: $(D \rightarrow Y)$.

Graphical Causal Models: Directed Acyclic Graphs (DAGs)

Graphical Causal Model

Directed edges encode assumptions about the "flow" of causation (edge) or lack thereof (no edge).

Potential Cause

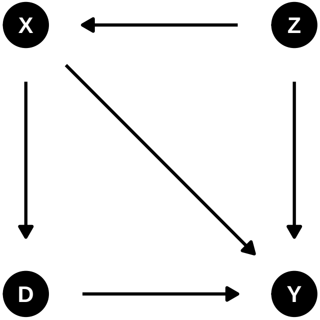
If D is an ancestor of Y, it is a **potential cause** of Y.

Direct Cause

If D is a parent of Y, it is a **direct cause** of Y.

Back Door Criterion

Can we learn $(D \to Y)$ using selection on observables? If so, what covariates should we adjust for?



"Draw Your Assumptions" – Birthweight Example

Birthweight Paradox

- Y mortality
- X birthweight
- D maternal smoking
- U unobserved: e.g. malnutrition / birth defect

Should we condition on X?

Can't adjust for U: unobserved. Should we adjust for birthweight when studying (smoking $\rightarrow$ mortality) effect?

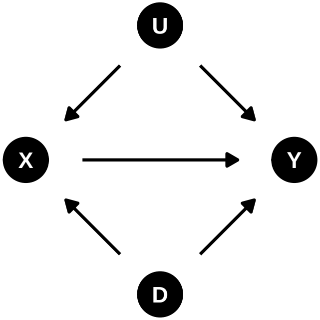


Figure 1: A possible model for the birthweight example.

Causal and Non-causal Paths

Causal Path

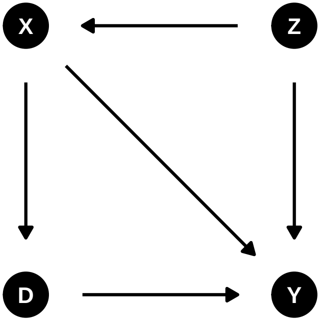
Directed path between treatment and outcome; always starts with an edge pointing *out* of treatment.

Backdoor Path

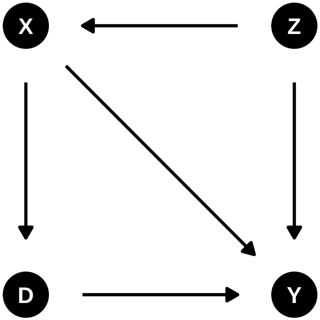
Noncausal path path between treatment and outcome; always starts with an edge pointing *into* treatment.

Exercise

1. List all causal paths from D to Y.
2. List all backdoor paths between D and Y.



Causal and Non-causal Paths



Causal Path

Directed path between treatment and outcome; always starts with an edge pointing *out* of treatment.

Backdoor Path

Noncausal path path between treatment and outcome; always starts with an edge pointing *into* treatment.

Exercise

1. List all causal paths from D to Y.
2. List all backdoor paths between D and Y.

Solution

1. $(D \rightarrow Y)$
2. $(D \leftarrow X \rightarrow Y)$, and $(D \leftarrow X \leftarrow Z \rightarrow Y)$.

Graph Surgery

Observational Distribution: $\mathbb{P}(Y|D=d)$

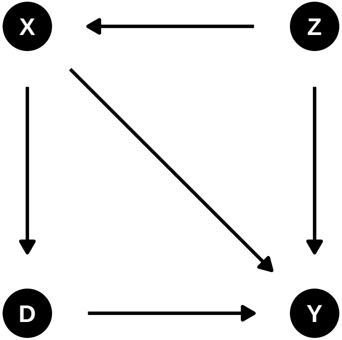
- *Actual* distribution of Y among people observed to have $D = d$.
- DAG shows the observational distribution and how it arises from our causal model.

Interventional Distribution: $\mathbb{P}(Y|\text{do}(D=d))$

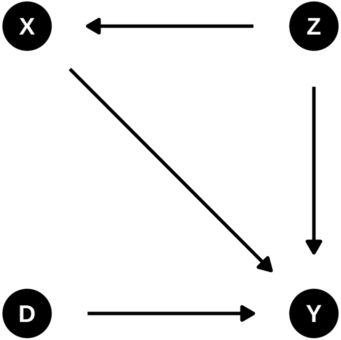
- Distribution of Y that we *would obtain* if we *intervened* and set $D = d$ for everyone.
- Obtain from DAG by removing edges pointing into D.
- Causal effect of interest is the path from D to Y in this "modified" graph.
- $\text{ATE} = \mathbb{E}(Y_1 - Y_0) = \mathbb{E}(Y|\text{do}(D=1)) - \mathbb{E}(Y|\text{do}(D=0))$
- This is what an experiment does: removes all causes of treatment!

Graph Surgery: Delete Edges Pointing Into D

Observational Distribution



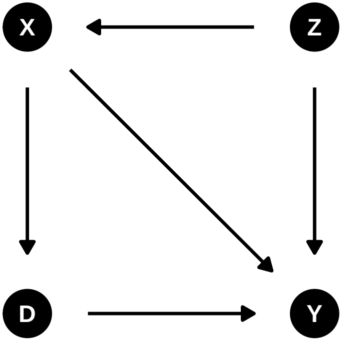
Interventional Distribution: do(D)



Interventional DAG has *no backdoor paths*. To use the observational distribution for causal inference, we will attempt to "block" the backdoor paths by conditioning.

Exercise: Draw the DAG for the do(X) Interventional Distribution

Observational Distribution



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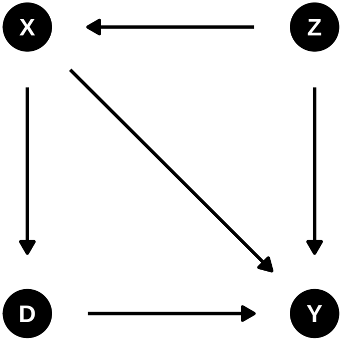
Exercise: Draw the DAG for the do(X) Interventional Distribution

Observational Distribution



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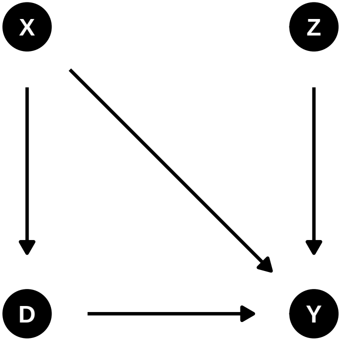


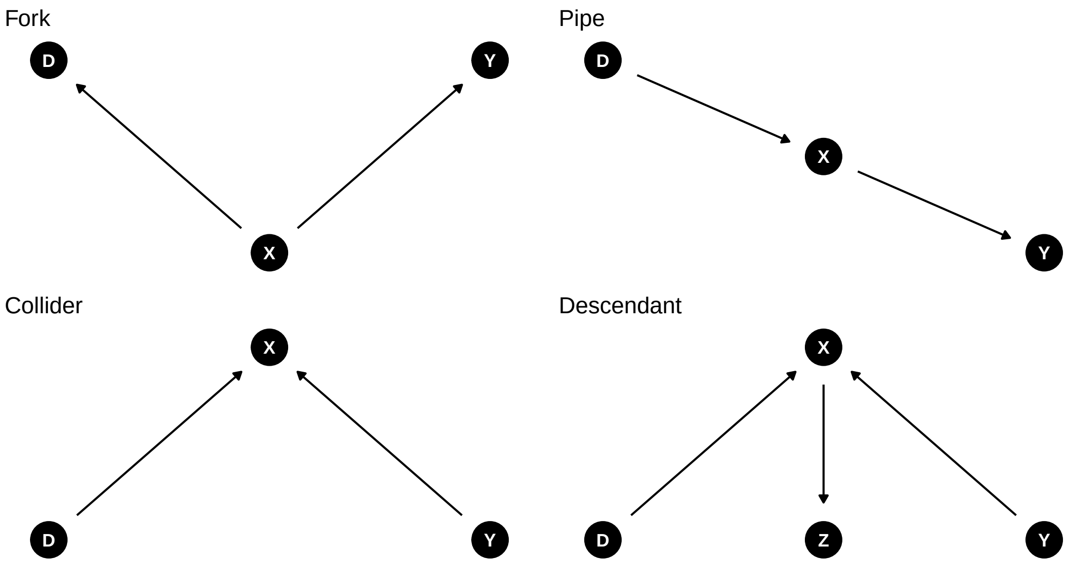


Figure 2: The Four Basic DAGs

Fork = Common Cause / Confounder

Confounder = Good Control

- D and Y are dependent: **open** path between them.
- But D doesn't cause Y: X causes D and Y.
- Conditioning on X **blocks the path** from D to Y.

Example

D is shoe size, Y is reading ability, X is age.

Fork Rule

If X is a common cause of D and Y and there is only one path between D and Y, then $D \perp\!\!\!\perp Y | X$.

"Condition on things that cause both D and Y."

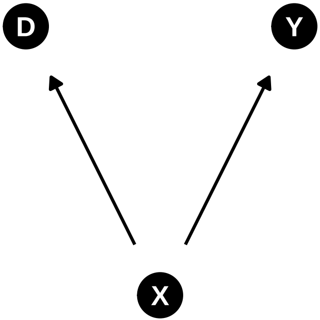


Figure 3: X is a confounder. Good control for $D \rightarrow Y$.

Pipe = Mediator

Mediator = Bad Control

- D and Y are dependent: **open** path between them.
- D causes Y through its causal effect on X.
- Conditioning on X **blocks the path** from D to Y.

Example

D is SAT coaching, X is SAT score, Y is college acceptance

Pipe Rule

If there is only one directed path from D to Y and X intercepts that path, then $D \perp\!\!\!\perp Y | X$.

"Don't condition on an intermediate outcome."

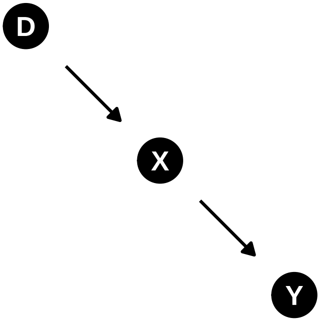
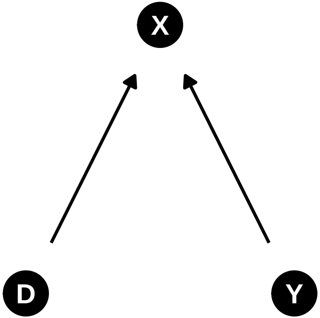


Figure 4: X is a mediator. Bad control for $D \to Y$.

Collider = Common Effect

Common Effect = Bad Control

- D and Y are independent: **blocked** path between them.
- D and Y both cause X, but neither causes the other.
- Conditioning on X **unblocks** the path between D and Y.



Example

D, Y indep. coins; X = bell rings if at least one HEADS.

Collider Rule

If there is only one path between D and Y and X is their common effect, then $D \perp\!\!\!\perp Y$ but $D \not\perp\!\!\!\perp Y | X$.

Why are brilliant researchers lousy teachers?

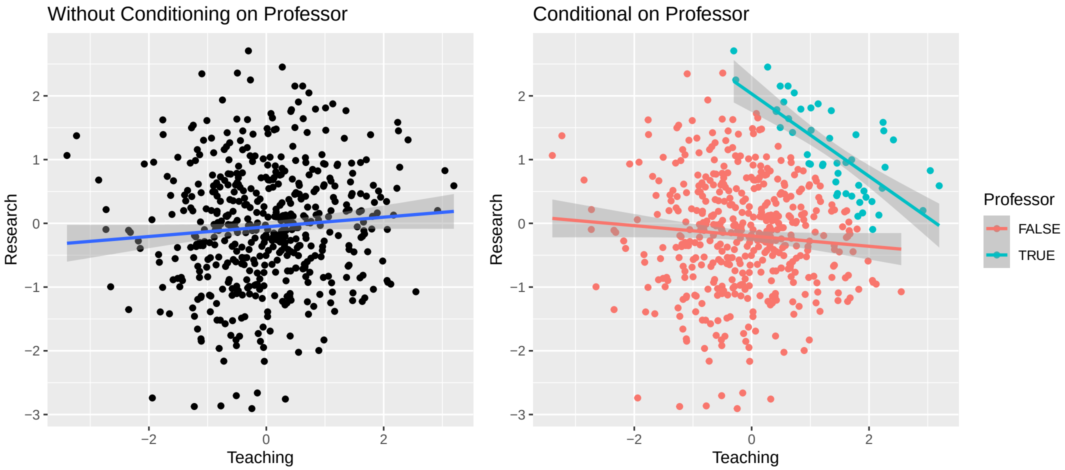


Figure 5: Teaching and Research are independent $N(0,1)$. Professor is a collider: TRUE if the sum of Research and Teaching is in the top 10th percentile of all observations.

The Descendant

Descendant Rule

Conditioning on a descendant Z of X has the effect of *partially conditioning* on X itself.

Collider Corollary

In the figure, $D \perp\!\!\!\perp Y$ but $D \not\!\perp\!\!\!\perp Y | Z$.

Discussion

- What this means depends on the situation.
- In the figure X is a collider.
- Could also have X as the middle node in pipe/fork.
- Pipe/fork: adjust for $Z \Rightarrow$ **partially block** D, Y path.

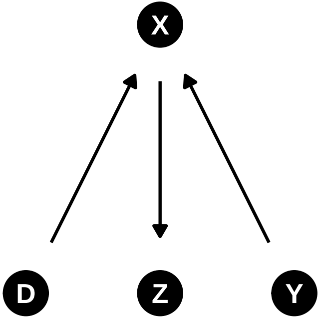


Figure 6: Z is a descendant of the collider X. Bad control for $D \to Y$

Exercise: Find all examples of the four basic DAGS.

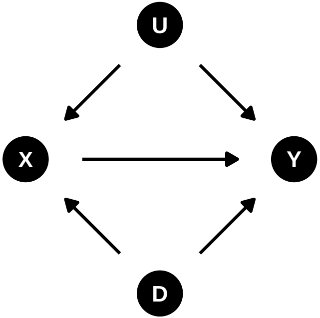


Figure 7: Birthweight DAG

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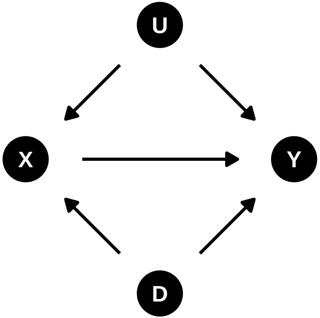


Figure 7: Birthweight DAG

Solution

1. **Forks**: $X \leftarrow U \rightarrow Y$ and $X \leftarrow D \rightarrow Y$
2. **Pipes**: $D \rightarrow X \rightarrow Y$, $U \rightarrow X \rightarrow Y$
3. **Colliders**: $D \rightarrow X \leftarrow U$ and $D \rightarrow Y \leftarrow U$.
4. **Descendant**: Y is a descendant of the collider $D \rightarrow X \leftarrow U$.

Blocking and Opening Paths in the Four Basic DAGs

Fork

$D \leftarrow X \rightarrow Y$ is an **open** path; conditioning on the **confounder** X **blocks** the path.

Pipe

$D \rightarrow X \rightarrow Y$ is an **open** path; conditioning on the **mediator** X **blocks** the path.

Collider

$D \rightarrow X \leftarrow Y$ is a **blocked** path; conditioning on the **collider** X **opens** the path.

Descendant

Conditioning on the descendant of a **confounder** / **mediator** partially blocks the open path. Conditioning on the descendant of a **collider** partially opens the blocked path.

Backdoor Criterion

Use what we know about the four basic DAGs to **block** all backdoor paths between D and Y in our "big" DAG. Obtain interventional distribution from observational data.

The Backdoor Criterion

Recall: Backdoor Path

Noncausal path between D and Y; starts with edge pointing **into** D.

Blocked Path

A set of nodes X **blocks** a path p if and only if p contains: (1) a **pipe** or **fork** whose middle node is in X or (2) a **collider** that is *not* in X and has no descendants in X.

Backdoor Criterion

A set of nodes X satisfies the back-door criterion relative to (D, Y) if no node in X is a descendant of D and X blocks every back-door path between D and Y.

A Less Formal Statement of the Back-door Criterion

1. List all the paths that connect treatment and outcome.
2. Check which of them *open*. A path is *open* unless it contains a collider.
3. Check which of them are *back-door paths*: contain an arrow pointing at D.
4. If there are no open back-door paths, you're done. If not, look for nodes you can condition on to **block** remaining open back-door paths without opening new ones.

Of course we can only condition on *observed* variables!

Important Note

In a given DAG there may be *no way* to satisfy the badk-door criterion, given what we observe. There may also be *multiple ways*!

Backdoor Theorem = Selection on observables!

Backdoor Theorem

If X satisfies the back-door criterion relative to (D, Y), then

$$\mathbb{P}(Y = y|\text{do}(D = d)) = \sum_{\text{all } x} \mathbb{P}(Y = y|D = d, X = x) \cdot \mathbb{P}(X = x)$$

What if X is empty?

Then we don't to condition on anything: $\mathbb{P}(Y = y|\text{do}(D = d)) = \mathbb{P}(Y = y|D = d)$

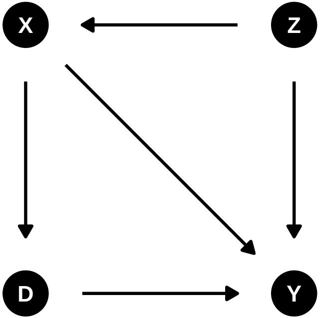
Counterfactual Interpretation

If X satisfies the back-door criterion relative to (D, Y), then $Y_d \perp\!\!\!\perp D|X$ for all d.

Translating to Potential Outcomes

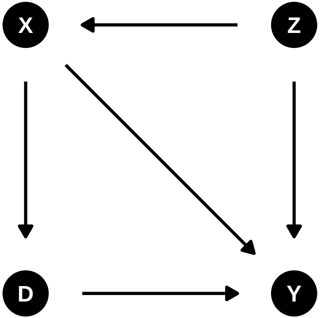
- The "counterfactuals" Y_d are our potential outcomes from earlier in this lecture.
- Back-door criterion implies selection on observables assumption for D given X.
- The formula above is nothing more than **regression adjustment**.

Exercise: What to adjust for to learn the effect of each intervention?



1. The effect of D on Y.
2. The effect of X on Y.
3. The effect of Z on Y?

Exercise: What to adjust for to learn the effect of each intervention?



1. The effect of D on Y.
2. The effect of X on Y.
3. The effect of Z on Y?

Solution

1. There are two backdoor paths. In $(D \leftarrow X \rightarrow Y)$, the middle node in a fork is X. In $(D \leftarrow X \leftarrow Z \rightarrow Y)$ the middle node in a pipe is X. Adjusting for X blocks both.
2. The only backdoor path is $(X \leftarrow Z \rightarrow Y)$, a fork with Z as its middle node. Adjusting for Z blocks this path.
3. There are no arrows pointing into Z, hence no backdoor paths. We don't have to adjust for anything.

(Possible) Solution to Birthweight Paradox

> *Among low birthweight infants... maternal smoking was associated with lower infant mortality.*

Notation

Y mortality, X birthweight, D maternal smoking, and U unobserved: e.g. malnutrition / birth defect

Birthweight is a bad control!

- Can't adjust for U because it's unobserved.
- No arrows pointing into D so no backdoor paths.
- X is a collider: conditioning on it creates spurious dependence between D and U.

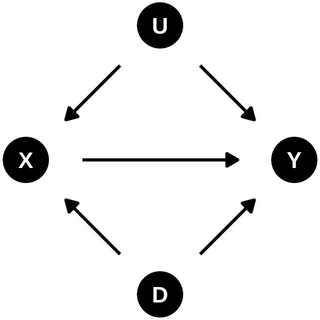


Figure 8: If we believe this model, X is a bad control.

Low birthweight infants whose mothers did *not* smoke must have an unfavorable value of U, making it appear as though smoking has health benefits.

::::

Exercise / Cancer Example Continued

- X is a **collider**: it *blocks* the back-door path between D and Y through (U, V).
- Adjusting for X *opens* this blocked path, so X is a **bad control**.
- Back door criterion is satisfied with $Z = \emptyset$: don't condition on *anything*!

```r
library(dagitty)
library(ggdag)
dagify(Y ~ D + U, D ~ V,  X ~ U + V) |>
  paths(from = 'D', to = 'Y')
```

```
## $paths
## [1] "D -> Y"                    "D <- V -> X <- U -> Y"
##
## $open
## [1]  TRUE FALSE
```