

# Applied Econometric Time Series (Problem Set 1)

Dominik R. Wehr  
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## 1 Stochastic difference equations

**Question** Under what conditions (in terms of restrictions on  $a_2$ ) is the below AR(2) process stationary in large samples?

$$y_t = a_0 + a_2 y_{t-2} + \varepsilon_t, \quad t = 1, 2, \dots, T \quad (1.1)$$

**Question** Under the conditions derived in 1a, show that the process is not stationary in finite samples ( $y_0$  and  $y_{-1}$  are some constants).

**Question** For large samples, derive the autocovariance and autocorrelation function  $\gamma_k$  ( $k \geq 0$ ) and  $\rho_k$  ( $k \geq 0$ ), respectively.

**Question** Why is stationarity such an important concept in time-series models?

**Question** Derive the long-run equilibrium of this model (assuming stationarity and large samples)

**Question** Quantify the long-run equilibrium (assuming large samples) when  $a_0 = 2$  and  $a_2 = 0.5$ . Also, describe and quantify the adjustment mechanisms toward this equilibrium when (i)  $y_{-1} = y_0 = 2$  and  $\varepsilon_t = 0$  for all  $t$ , and (ii)  $y_{-1} = 2$  and  $y_0 = 1.5$  and  $\varepsilon_t = 0$  for all  $t$ .

**Question** If you instead let  $a_2 = 0.6$ , show that the adjustment process towards an equilibrium takes longer time than if  $a_2 = 0.5$ .

## 2 Time Series Analysis of US Interest Rates

**Question** Go through the example of interest rate spreads Section 2.10 (pp. 88-96) and try to replicate the results (you download the data from the course web; sample size  $T = 255$ ).

**Question** Conduct a Chow test for the spread series where you set the breakpoint at  $t = 1981Q4$ . What do you conclude? Will your conclusions change if you change the breakpoint?

**Question** Estimate an AR(1) process with intercept recursively for the sample sizes  $n, n+1, \dots, T-1, T$  where  $n = 10$ . Plot the estimation results for the intercept and the AR(1) coefficient with  $\pm 2$  standard deviation band (see p. 108). What do you conclude?

**Question** For the same sample sizes as in 2c, calculate the CUSUM test accompanied with  $\pm 2$  standard deviation band. What do you conclude?