ECON 600: Industrial Organization

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Dynamic Demand: Storable Goods

Dynamic Demand

- So far, you've studied:
  - Static models of demand (e.g. BLP).
  - Single agent Dynamic Decision Problems (e.g. Rust)
- Today we'll start putting these together.
- Demand estimation when consumers are forward looking.

Dynamic Demand 1: Durable Goods

- When are dynamics important in consumer demand?
- Two canonical cases:

1. Durable goods.
   - E.g. market for cars.
   - Value of buying a car is the PDV of expected future utility flows.

$$u_{ijt} = E\left[\sum_{t=0}^{\infty} \beta^t v_{ij(t)} + p_{j(t-1)t} - p_{j(t)} - \tau(j(t) \neq j(t-1))\right]$$

   - Where $j(t)$ is the model held at date t. τ is the transaction cost of upgrading.
   - Expectation is over future sales and purchases.
   - Value of buying model j now depends on future option value of selling it.

Dynamic Demand 1: Durable Goods

$$u_{ijt} = E\left[\sum_{t=0}^{\infty} \beta^t v_{ij(t)} + p_{j(t-1)t} - p_{j(t)} - \tau(j(t) \neq j(t-1))\right]$$

- so why can't we just run BLP and let u_{ijt} be a "reduced form" of future utility flows?
- We can (this is one way to think about what BLP is doing).
- But we can't then think about counterfactual policies that change dynamic incentives.
  - e.g. a scrappage scheme for old cars ("cash for clunkers").
  - Changes τ for old cars.
  - Need to solve dynamic model to work out how this changes u_{ijt}.

Dynamic Demand 2: Storable Goods

- When are dynamics important in consumer demand?
- Two canonical cases:

1. Durable goods.
2. Storable goods.

- What if I can buy now and consume later?
- i.e. consumers can hold **inventories** of products and wait to eat the flow utility.
  - If prices vary over time...
  - Or consumer believe prices vary over time...
  - Dynamic incentive to wait until products are on sale and then "stock up".

Dynamic Demand 2: Storable Goods

- E.g. price of 2 liter coke from Hendel and Nevo (2011)

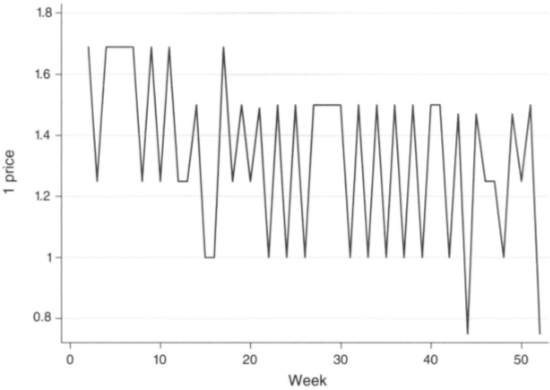

FIGURE 1. A TYPICAL PRICING PATTERN

Note: The figure presents the price of a two-liter bottle of Coke over 52 weeks in one store.

Dynamic Demand 2: Storable Goods

- E.g. price of 2 liter coke from Hendel and Nevo (2011)

TABLE 1—QUANTITY OF TWO-LITER BOTTLES OF COKE SOLD

	$S_{t-1} = 0$	$S_{t-1} = 1$	
$S_t = 0$	247.8	199.4	227.0
$S_t = 1$	763.4	531.9	622.6
	465.0	398.9	

Notes: The table presents the average across 52 weeks and 729 stores of the number of two-liter bottles of Coke sold during each week. As motivated in the text, a sale is defined as any price below one dollar.

Dynamic Demand 2: Storable Goods

- Why is this a problem from static demand estimation?
  - When supermarket cuts the price of laundry detergent for a week there is a huge increase in sales.
  - This leads us to conclude that consumers are extremely elastic to price.
  - But when price is cut permanently, there is little impact on sales!
- Standard demand estimation pools all this price variation.
- Overestimate price elasticity if lots of price variation comes from sales and products are storable.

Hendel and Nevo (2006)

- Model of dynamic demand for laundry detergent.
  - Purchase - storage - consumption.
  - How do demand estimates that account for inventories differ from static demand estimates?
- Household scanner data from 9 supermarkets in Chicago.

Hendel and Nevo (2006)

TABLE I

SUMMARY STATISTICS OF HOUSEHOLD-LEVEL DATA[a]

	Mean	Median	Std	Min	Max
Demographics					
Income (000's)	35.4	30.0	21.2	<10	>75
Size of household	2.6	2.0	1.4	1	6
Live in suburb	0.53	—	—	0	1
Purchase of laundry detergents					
Price ($)	4.38	3.89	2.17	0.91	16.59
Size (oz.)	80.8	64	37.8	32	256
Quantity	1.07	1	0.29	1.00	4
Duration (days)	43.7	28	47.3	1	300
Number of brands bought over the 2 years	4.1	3	2.7	1	15
Brand HHI	0.53	0.47	0.28	0.10	1.00
Store visits					
Number of stores visited over the 2 years	2.38	2	1.02	1	5
Store HHI	0.77	0.82	0.21	0.27	1.00

[a]For Demographics, Store visits, Number of brands, and Brand HHI, an observation is a household. For all other statistics, an observation is a purchase instance. Brand HHI is the sum of the square of the volume share of the brands bought by each household. Similarly, Store HHI is the sum of the square of the expenditure share spent in each store by each household.

Hendel and Nevo (2006)

TABLE II

BRAND VOLUME SHARES AND FRACTION SOLD ON SALE[a]

	Liquid					Powder				
	Brand	Firm	Share	Cumulative	% on Sale	Brand	Firm	Share	Cumulative	% on Sale
1	Tide	P & G	21.4	21	32.5	Tide	P & G	40	40	25.1
2	All	Unilever	15	36	47.4	Cheer	P & G	14.7	55	9.2
3	Wisk	Unilever	11.5	48	50.2	A & H	C & D	10.5	65	28
4	Solo	P & G	10.1	58	7.2	Dutch	Dial	5.3	70	37.6
5	Purex	Dial	9	67	63.1	Wisk	Unilever	3.7	74	41.2
6	Cheer	P & G	4.6	72	23.6	Oxydol	P & G	3.6	78	59.3
7	A & H	C & D	4.5	76	21.5	Surf	Unilever	3.2	81	11.6
8	Ajax	Colgate	4.4	80	59.4	All	Unilever	2.3	83	
9	Yes	Dow Chemical	4.1	85	33.1	Dreft	P & G	2.2	86	15.2
10	Surf	Unilever	4	89	42.5	Gain	P & G	1.9	87	16.7
11	Era	P & G	3.7	92	40.5	Bold	P & G	1.6	89	1.1
12	Generic	—	0.9	93	0.6	Generic	—	0.7	90	16.6
13	Other	—	0.2	93	0.9	Other	—	0.6	90	19.9

Hendel and Nevo (2006)

TABLE III

QUANTITY DISCOUNTS AND SALES[a]

	Quantity Discount (%)	Quantity Sold on Sale (%)	Weeks on Sale (%)	Average Sale Discount (%)	Quantity Share (%)
Liquid					
32 oz.	—	2.6	2.0	11.0	1.6
64 oz.	18.1	27.6	11.5	15.7	30.9
96 oz.	22.5	16.3	7.6	14.4	7.8
128 oz.	22.8	45.6	16.6	18.1	54.7
256 oz.	29.0	20.0	9.3	11.8	1.6
Powder					
32 oz.	—	16.0	7.7	14.5	10.1
64 oz.	10.0	30.5	16.6	12.9	20.3
96 oz.	14.9	17.1	11.5	11.7	14.4
128 oz.	30.0	36.1	20.8	15.1	23.2
256 oz.	48.7	12.9	10.8	10.3	17.3

Hendel and Nevo (2006): Model

- Product characteristics:
- For each brand (j), size (x) and week (t):
  1. Price p_{jxt}
  2. Quantity q_{jxt}
  3. Promotions a_{jxt} (dummies for feature and display)
- Consumer of type h has utility $u(c_{ht} + \nu_{ht}; \theta_h)$.
- Current consumption is $c_{ht} = \sum_j c_{jht}$. Not brand specific!
- i.i.d shock to marginal value of consumption ν_{ht}.
- Observed Action: $d_{hjxt} = 1$ is a purchase of brand j size x at date t by household h.

Hendel and Nevo (2006): Model

- Consumer's problem:
- State variable $s_t = \{p_t, a_t, i_t\}$.
  - p_t and a_t vectors with entry for each product.
  - i_t is the consumer's current inventory or stock of detergent.

$$
\begin{aligned}
V(s_1) & \\
&= \max_{\{c_h(s_t), d_{hjx}(s_t)\}} \sum_{t=1}^{\infty} \delta^{t-1} E\Big[u(c_{ht}, \nu_{ht}; \boldsymbol{\theta}_h) - C_h(i_{h,t+1}; \boldsymbol{\theta}_h) \\
&\qquad + \sum_j d_{hjxt}(\alpha_h p_{jxt} + \xi_{hjx} + \beta_h a_{jxt} + \varepsilon_{hjxt}) \Big| s_1 \Big]
\end{aligned}
$$

$$
\begin{aligned}
\text{s.t.} \quad & 0 \leq i_{ht}, \quad 0 \leq c_{ht}, \quad 0 \leq x_{ht}, \quad \sum_{j,x} d_{hjxt} = 1, \\
& i_{h,t+1} = i_{ht} + x_{ht} - c_{ht},
\end{aligned}
$$

  - Policy functions $c(s_t)$ and $d_{jx}(s_t)$.

Hendel and Nevo (2006): Model

- Key assumptions on state variables:
  - $i_{t+1} = t_t + x_t - c_t$.
  - ϵ_{jxt} iid type 1 extreme value. v_t distributed iid log normal.
  - p_{jxt} and a_{jxt} follow exogenous first order Markov process!
  - Price endogeneity?
    - ξ_{hjx} are fixed effects.
    - Assumption of no time varying unobservable component.
  - Still hard to explain why a 1st order Markov process is reasonable...
  - Maybe it is a good representation of consumers' beliefs?

Hendel and Nevo (2006): Likelihood

- For each consumer, we observe a sequence of price and promotion variables (p_t, a_t), and the purchase decisions d_t.
- We do not observe inventories i_t or consumption choices c_t.
- i_t is an unobserved, serially correlated state variable!
- Evolution of i_t is a function of optimal consumption choices $c_t(s_t)$ (unobserved) and purchases $d_t(s_t)$ observed.
  - "Assumption A": 1st order Markov in observed and unobserved state.
- Likelihood conditional on the initial inventory:

$$\Pr(d_1 \cdots d_T | p_1 \cdots p_T)$$
$$= \int \prod_{t=1}^{T} \Pr\big(d_t | p_t, i_t(d_{t-1}, \ldots, d_1,$$
$$\nu_{t-1}, \ldots, \nu_1, i_1), \nu_t\big)\, dF(\nu_1, \ldots, \nu_T)\, dF(i_1).$$

- This is why we can't use CCP methods for estimation here.

Hendel and Nevo (2006): Likelihood

- Model defines optimal policy functions $c(s_t)$ and $d_t(s_t)$.
- Suppose we can solve the model.
  - Optimal policy functions give us $F(i_{t+1}|i_t, d_t)$.
  - But model does not define $F(i_1)$
  - Initial conditions problem!

Hendel and Nevo (2006): Likelihood

- Hendel and Nevo's solution:
  - For each candidate parameter vector, θ, solve the model.
  - Set $i_0 = 0$. Simulate optimal choices for the first T_0 periods (using actual prices).
  - Use distribution of simulated i_{T_0} as $F(i_1)$.
  - Estimate the model on data from $(T_0 + 1) \rightarrow T$
- Assumption: by period T_0 we have reached stationary distribution.

Hendel and Nevo (2006): Model Specification

- So we can write down a likelihood if we can solve the model for $c(s_t)$ and $d_t(s_t)$.
- Back to the model...

$$V(s_t) = E_{\epsilon_t, v_t}\left[\max_{c_t d_t}\left\{u(c_t, v_t, \theta) - C(i_{t+1}; \theta) + m(d_t; \theta) + E\left[V(s_{t+1}) | s_t, d_t, c_t\right]\right\}\right]$$

- $u(c, v; \theta) = \gamma \ln(c_t + v_t)$
- $C(i_{t+1}; \theta) = \delta_1 i_{t+1} + \delta_2 i_{t+1}^2$
- $m(d_t; \theta) = \sum_{j,x} d_{jxt}\left(\alpha p_{jxt} + \xi_{jx} + \beta a_{jxt} + \epsilon_{jxt}\right)$
- In principle, can guess θ and use value function iteration to solve for optimal policy functions $c(s)$ and $d(s)$.
- Problem: large state space.
- Recall: $s_t = \{p_t, a_t, i_t\}$.
  - If there are N products, this has dimension $3N + 1$

Hendel and Nevo (2006): Estimator

- Hendel and Nevo reduce complexity by proposing a 3-step estimator:

1. Maximize likelihood of band choice conditional on size choice (static).
2. Compute inclusive value of each size and transition probabilities.
3. Solve fixed point for reduces state space (one “price index” per size)

Hendel and Nevo (2006): Step One

$$V(s_t) = E_{\epsilon_t, v_t}\left[\max_{c_t d_t}\{u(c_t, v_t, \theta) - C(i_{t+1}; \theta) + m(d_t; \theta) + E\left[V(s_{t+1})|s_t, d_t, c_t\right]\}\right]$$

- Notice that conditional on choosing a package size, x, brand choice , j, does not enter the continuation values.
- The distribution of tomorrow's state only depends on how much detergent you buy, not which band.
- So we can separate the decisions of how much to buy (dynamic) and which brand (static).
- Brand CCP:

$$P(d_{jx}|x_t, p_t) = \frac{exp(\alpha p_{jxt} + \xi_{jx} + \beta a_{jxt})}{\sum_k exp(\alpha p_{kxt} + \xi_{kx} + \beta a_{kxt})}$$

- Estimate using simple MLE.

Hendel and Nevo (2006): Step Two

$$V(s_t) = E_{\epsilon_t, v_t}\left[\max_{c_t x_t}\{u(c_t, v_t, \theta) - C(i_{t+1}; \theta) + \omega_t(x_t) + \epsilon_{xt} + E\left[V(s_{t+1})|s_t, x_t, c_t\right]\}\right]$$

- Now the problem is only about choosing x_t and c_t.
- $\omega_t(x_t)$ is the period t "inclusive value" of the static brand choice for size t:

$$\omega_t(x_t) = \log\left(\sum_k exp(\alpha p_{kxt} + \xi_{kx} + \beta a_{kxt})\right)$$

- Notice that the high dimensional state variables $\{p_t, a_t\}$ only enter the problem through ω_t.

Hendel and Nevo (2006): Step Two

$$V(s_t) = E_{\epsilon_t, v_t}\left[\max_{c_t x_t}\{u(c_t, v_t, \theta) - C(i_{t+1}; \theta) + \omega_t(x_t) + \epsilon_{xt} + E\left[V(s_{t+1})|s_t, x_t, c_t\right]\}\right]$$

- Assumption: **Inclusive Value Sufficiency**

$$F(\omega_t|s_{t-1}) = F(\omega_t|\omega_{t-1})$$

- Then we can reduce the state space to:

$$s_t = \{\omega_t, i_t\}.$$

- ω_t is a 5-vector.
- What does this mean?
  - Assume that the "quality adjusted price index" ω_t follows a 1st order Markov process.
  - Only need to keep track of an index of prices and qualities, not individual prices.
  - A substantive assumption! (testable)

Hendel and Nevo (2006): Step Two

- Compute ω_t using 1st step estimates:

$$\omega_t(x_t) = \log\left(\sum_k exp(\alpha p_{kxt} + \xi_{kx} + \beta a_{kxt})\right)$$

- Then estimate transition process:

$$F(\omega_t | s_{t-1}) = F(\omega_t | \omega_{t-1})$$

Hendel and Nevo (2006): Step Three

- Finally, can solve the model on the reduced state space using value function iteration.
- H&N approximate value function $V(s_t)$ using a smooth polynomial in s_t.
- Use policy functions to get (simulated) likelihood.
- Remaining parameters: $u(c, v; \theta) = \gamma \ln(c_t + v_t)$
- $C(i_{t+1}; \theta) = \delta_1 i_{t+1} + \delta_2 i_{t+1}^2$

Hendel and Nevo (2006): Results

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)
Price	−0.51	−1.06	−0.49	−0.26	−0.27	−0.38	−0.38	−0.57	−1.41	−0.75
	(0.022)	(0.038)	(0.043)	(0.050)	(0.052)	(0.055)	(0.056)	(0.085)	(0.092)	(0.098)
*Suburban dummy				−0.33	−0.30	−0.34	−0.33	−0.25	−0.45	−0.19
				(0.055)	(0.061)	(0.055)	(0.056)	(0.113)	(0.127)	(0.127)
*Nonwhite dummy				−0.34	−0.39	−0.38	−0.33	−0.34	−0.33	−0.26
				(0.075)	(0.083)	(0.076)	(0.076)	(0.152)	(0.166)	(0.168)
Large family				−0.23	−0.13	−0.21	−0.22	−0.46	−0.38	−0.43
				(0.080)	(0.107)	(0.080)	(0.082)	(0.181)	(0.192)	(0.195)
Feature			1.06	1.05	1.08	0.92	0.93	1.08		1.05
			(0.095)	(0.096)	(0.097)	(0.099)	(0.100)	(0.123)		(0.126)
Display			1.19	1.17	1.20	1.14	1.15	1.55		1.52
			(0.069)	(0.070)	(0.071)	(0.071)	(0.072)	(0.093)		(0.093)
Brand dummy variable		✓	✓	✓	✓					
*Demographics					✓					
*Size						✓				
Brand–size dummy variable							✓			
Brand-HH dummy variable								✓		
*Size									✓	✓

Hendel and Nevo (2006): Results

TABLE V

SECOND STEP: ESTIMATES OF THE PRICE PROCESS[a]

	Same Process for All Types				Different Process for Each Type			
	ω_{2t}	ω_{4t}	ω_{2t}	ω_{4t}	ω_{2t}	ω_{4t}	ω_{2t}	ω_{4t}
$\omega_{1,t-1}$	0.003	−0.014	0.005	0.014	−0.023	−0.005	−0.019	0.007
	(0.012)	(0.011)	(0.014)	(0.014)	(0.017)	(0.014)	(0.019)	(0.015)
$\omega_{2,t-1}$	0.413	0.033	0.295	0.025	0.575	−0.003	0.520	0.011
	(0.007)	(0.010)	(0.008)	(0.007)	(0.013)	(0.010)	(0.016)	(0.013)
$\omega_{3,t-1}$	0.003	−0.034	0.041	−0.006	0.027	−0.072	0.051	−0.018
	(0.007)	(0.007)	(0.009)	(0.009)	(0.020)	(0.016)	(0.025)	(0.020)
$\omega_{4,t-1}$	0.029	0.249	0.026	0.236	−0.018	0.336	−0.018	0.274
	(0.008)	(0.008)	(0.008)	(0.017)	(0.020)	(0.016)	(0.021)	(0.017)
$\sum_{\tau=2}^{5} \omega_{1,t-\tau}$			−0.003	−0.012			−0.008	−0.003
			(0.005)	(0.004)			(0.006)	(0.005)
$\sum_{\tau=2}^{5} \omega_{2,t-\tau}$			0.089	0.006			0.073	−0.004
			(0.003)	(0.002)			(0.005)	(0.004)
$\sum_{\tau=2}^{5} \omega_{3,t-\tau}$			−0.008	−0.009			−0.004	−0.016
			(0.003)	(0.003)			(0.008)	(0.006)
$\sum_{\tau=2}^{5} \omega_{4,t-\tau}$			−0.013	0.018			−0.008	0.056
			(0.003)	(0.003)			(0.007)	(0.005)

Hendel and Nevo (2006): Results

Household Type:	1	2	3	4	5	6
		Urban Market			Suburban Market	
Household Size:	1–2	3–4	5+	1–2	3–4	5+
Cost of inventory						
Linear	9.24	6.49	21.96	4.24	4.13	11.75
	(0.01)	(0.02)	(0.09)	(0.01)	(0.17)	(5.3)
Quadratic	−3.82	1.80	−35.86	−8.20	−6.14	−0.73
	(29.8)	(1.77)	(0.19)	(0.03)	(1.69)	(1.53)
Utility from consumption	1.31	0.75	0.51	0.08	0.92	3.80
	(0.02)	(0.09)	(0.21)	(0.03)	(0.18)	(0.38)
Log likelihood	365.6	926.8	1,530.1	1,037.1	543.6	1,086.1

Hendel and Nevo (2006): Results

- Static parameters:
  - Large effects from feature and display.
  - Demographics interact with price sensitivity (larger family, nonwhite, suburban shoppers more price sensitive).
  - Brand preference varies by size.
- Dynamic parameters:
  - Storage cost: suburban $<$ urban.

Hendel and Nevo (2006): Results

Brand	Size (oz.)	128 oz. All[b]	Wisk	Surf	Cheer	Tide	Private Label
All[b]	64	0.14	0.17	0.17	0.18	0.21	0.34
	128	1.23	0.09	0.11	0.09	0.15	0.22
Wisk	64	0.16	0.22	0.14	0.22	0.25	0.20
	128	0.08	1.42	0.08	0.13	0.18	0.11
Surf	64	0.18	0.18	0.12	0.18	0.22	0.28
	128	0.12	0.11	1.20	0.08	0.15	0.14
Cheer	64	0.14	0.24	0.16	0.14	0.22	0.24
	128	0.09	0.12	0.06	0.89	0.15	0.07
Tide	64	0.22	0.28	0.16	0.26	0.22	0.37
	128	0.11	0.16	0.08	0.13	1.44	0.31
Solo	64	0.17	0.15	0.15	0.30	0.30	0.28
	128	0.07	0.07	0.06	0.16	0.17	0.21
Era	64	0.43	0.17	0.15	0.22	0.19	0.35
	128	0.19	0.08	0.09	0.11	0.10	0.22
Private	64	0.32	0.22	0.15	0.26	0.31	0.25
label	128	0.16	0.12	0.13	0.10	0.27	1.29
No purchase		1.80	7.60	2.26	14.11	2.38	10.86

- Ratio of static elasticity to dynamic (long run) elasticity.
- Change in column price on row quantity.

Hendel and Nevo (2006): Results

- Static model ignores inventory problem

1. Larger static own price elasticity.
2. Lower static cross price elasticity.
    - In the data, response to sales comes from people going from not buying to buying the sales brand.
    - $\rightarrow$ in static model, get small cross price elasticities with respect to other products, and large cross price elasticity with respect to the outside good.

Next Time

- Gowrisankaran and Rysman (2012)
- Dynamic demand + instruments!