Reducing Interview Congestion in Two-Sided Matching Markets Using Preference Signaling

Marek Bojko*

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Abstract

The annual process of matching doctors to residency positions in the United States is preceded by costly information acquisition in the form of interviews. Interview allocation is determined in a decentralized equilibrium and often suffers from congestion. We model a tractable multi-stage game between hospitals and doctors highlighting this issue. Upon receiving applications, hospitals simultaneously extend interview offers with incomplete information about others' preferences and partial information about their own preferences, subject to a capacity constraint on their number of interviews. To reduce congestion, we consider a mechanism where each doctor can signal interest to one hospital. Whenever hospitals respond to signals in equilibrium, the signaling mechanism increases the ex-ante doctor welfare and the expected number of matches. The impact on hospital welfare, however, is ambiguous. The success of the mechanism depends on pre-interview uncertainty about the final match utilities. Hospitals are more likely to respond to signals when (i) doctors are ex-ante more similar and (ii) the pre-interview probability a hospital finds a doctor unacceptable is low.

Keywords: Two-sided Matching, Interviewing, Congestion, Preference Signaling

JEL Codes: C78, D47, D82

*Chicago Booth; marek.bojko@outlook.com, marek.bojko@chicagobooth.edu
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1 Introduction

In many matching markets, agents' preferences are formed through costly information acquisition before the final match. For example, the annual matching of doctors to residency positions in the United States, the National Residency Matching Program (NRMP), consists of multiple stages: (i) applicants apply to residency programs; (ii) hospitals screen applicants and invite them to interviews; (iii) hospitals learn their preferences based on the interviews; and (iv) a central matchmaker conducts the matching based on their reported preferences. Interviewing is costly, and hospitals choose whom to interview strategically as they face tight financial and time constraints.

Due to a recent surge in the number of applications, hospitals have problems discerning between genuinely interested candidates (Venincasa et al., 2020; Bowe et al., 2020; McMackin et al., 2020).[1] To ensure a match, candidates apply to and interview at many programs, which decreases the reported acceptance rates. Monir (2020) suggests that, as a result of low admission rates, applicants apply to even more programs to increase their perceived acceptance probability. Under the current regime, programs struggle to cope with the high volume of applications and cannot give full attention to all qualified candidates (Gardner et al., 2018).

We refer to the market's inability to reach an efficient outcome due to coordination failure and wasteful interviewing as *congestion*. This market friction imposes unambiguous costs on both sides of the market. Hospitals' positions might remain unfilled at the end of the application cycle, while well-qualified applicants leave the main round of the market without a residency position. While the aftermarket allows unmatched agents to eventually find partners, the additional interviewing and matching round imposes significant costs on the participants.[2]

This paper uses a stylized model to describe interview congestion by focusing on agents' strategic choices and suggests a signaling mechanism to remedy the market failure. Following a strand of literature on interviewing in matching markets (Lee and Schwarz, 2017; Skancke, 2021), we embed a one-to-one matching market in a multi-stage game of interviewing and matching between finite sets of hospitals and doctors. For tractability, we consider doctors to be non-strategic at the application and interviewing stage. We assume doctors know their preferences when applying and that interviewing costs are borne only by hospitals as capacity constraints on their number of interviews. Doctors find all hospitals acceptable, apply to all programs, and accept all interviews.[3] Hospitals simultaneously and strategically invite doctors to interviews with incom-

[1] Venincasa et al. (2020) report that the Centralized Application Service in the ophthalmology residency match significantly increased in the average number of applications: although the match rates have remained around 75% over the past decade, the average number of applications has increased from 48 in 2008 to 75 in 2019. It is also worth noting this trend is not unique for the hospital-residency match. For example, the Common Application system for college applications in the United States allows applicants to send many applications using the same form. Among other factors, this has driven the number of applications up significantly: 36% of students submitted seven or more applications in 2017, up from 10% in 1995.

[2] In 2015, 94.6% of residency positions unfilled in the main round were filled in the aftermarket (NRMP, 2015). This suggests the positions initially did not fill due to a lack of coordination.

[3] This is consistent with survey evidence that candidates accept almost all interviews they are offered due to the

plete information about others' preferences and only partial information about their final preferences. Upon interviewing, they privately observe their final match utility for each interviewee. At the final stage, all agents report rank-ordered lists to a centralized clearinghouse, and the final matching is obtained using the Hospital-Proposing Deferred Acceptance Algorithm (HPDA).[4] Since hospitals are on the proposing side, reporting truthfully is their dominant strategy (Dubins and Freedman, 1981). We prove that if not matching is sufficiently undesirable for doctors, they will also report truthfully in any Bayesian equilibrium.[5]

Before interviews, hospitals are endowed with private partial information about their final match values for each candidate. To learn these values, they must interview. Following Skancke (2021), we assume their pre-interview information is contained in an ordinal ranking of candidates, such that the distribution of the final match value for a candidate with more favorable pre-interview information first-order stochastically dominates that of a candidate with less favorable pre-interview record. Interview outcomes are privately observed by the interviewer. We assume hospitals only rank candidates they interview, which is largely consistent with survey evidence.[6]

We follow Coles et al. (2013), a canonical model of signaling in matching markets this paper generalizes, and assume the distribution of preferences is *block-correlated*. Hospitals can be partitioned into tiers, such that each doctor agrees on the ranking of hospitals between tiers, but preferences over hospitals within a tier are idiosyncratic and uniform. Hospitals' pre-interview rankings are idiosyncratic and drawn i.i.d. from the uniform distribution. The structure keeps the model tractable, facilitates comparison with the results of Coles et al. (2013), and to a large extent reflects real-world preferences of NRMP participants (NRMP, 2015, 2017; AAMC, 2019).

At the interviewing stage, we focus on anonymous strategies, removing any unrealistic source of coordination.[7] Hospitals can only condition their choices on their pre-interview information and beliefs about others' preferences but not doctor identities. In the game without signals, there is a unique Bayesian equilibrium in which hospitals interview their most preferred doctors according to their pre-interview ranking, up to their capacity. With a non-negligible probability, the equilibrium exhibits extreme congestion, where only a fraction of doctors receives interviews.

To alleviate congestion, we propose a signaling mechanism at the application stage. To avoid the cheap talk nature of costless signals, following Coles et al. (2013), we allow each doctor to send a single private binary signal when applying. In sequential equilibrium in anonymous strategies, doctors' signals might mix over several tiers, but whenever they signal to a tier, they signal to their most favorite hospital in the tier. Hospitals interpret such signals as being the best-in-block

high number of applications and the strong undesirability of the outside option (NRMP, 2017; AAMC, 2019).

[4]The algorithm is defined in Appendix A.

[5]Strong undesirability of the outside option for doctors is consistent with the reports in participant surveys, e.g. NRMP (2017) and AAMC (2019).

[6]AAMC (2019) reports that in 21 specialties, more than 65% of programs only rank their interviewed candidates, and 95% rank no more than one extra candidate on their submitted rank-ordered lists.

[7]Anonymous strategies are widely used in the search theoretic literature (Shimer, 2005; Kircher, 2009).

hospital for the sender. Hospitals use cutoff strategies, comparing the pre-interview information of signaling doctors to pre-determined cutoffs, which only depend on the number of received signals.

Signaling strictly increases the ex-ante expected welfare of doctors and number of matches while having an ambiguous impact on hospital welfare. When hospitals are guided by signals, their interviews are less likely to overlap. Furthermore, the number of doctors with at least one interview also increases, and the interviews are spread more evenly among doctors. Together with the fact that the total number of interviews in the market remains unchanged since hospitals fill their interview capacities in any equilibrium, coordination through signals increases the ex-ante expected number of matches. Along with receiving a position with higher probability, doctors are also more likely to interview with their favorite hospitals.

The competition effect of signals, however, stands in opposition to the information effect. A hospital interviewing a doctor who has signaled creates negative welfare spillovers on other hospitals within the same block who find the doctor ex-ante highly desirable, as their probability of matching with the doctor decreases. Depending on the magnitude of the two effects, the welfare of hospitals may increase or decrease.

The signaling mechanism has the highest value in environments with little pre-interview information to differentiate between doctors where the hospitals deem doctors generally well-qualified for their positions. We show that the doctor welfare gains increase in the probability a hospital finds a doctor fit for its position and the more ex-ante similar doctors are. Intuitively, the former regulates the competition effect of signals. As the probability increases, a hospital intending to interview a doctor who has not signaled will face tougher competition. In turn, the hospital is more likely to interview one of its signaling doctors. Moreover, when hospitals lack reliable sources of information to adjust their priors before interviews, perceived match probabilities and expected payoffs will be impacted by received signals more. Hence, hospitals are more likely to respond to signals.

Related Literature. The welfare implications of signals are in line with the results of Coles et al. (2013), which considered a decentralized market without interviews and a single round of offers and acceptances.[8] In such decentralized markets, congestion arises from miscoordination in employment offers. The iterative nature of the deferred acceptance algorithm mitigates market congestion in these instances. However, as we have highlighted, along with the preference learning

[8]Signaling mechanisms have been used in several matching markets without a centralized matchmaker. Economists are familiar with signaling on the Economics Job Market. Since 2006, the American Economic Association has used a signaling mechanism to coordinate interviews. PhD graduates are allowed to send at most two binary signals to the hiring departments to indicate an interest at the annual Allied Social Science Associations (ASSA) meetings. Coles et al. (2010) find that, in the aggregate, recruiting committees respond to signals. Signaling mechanisms have also been successfully implemented in online dating and labor markets (Lee and Niederle, 2015). Early admissions programs used by colleges can also be thought of as an example of a signaling mechanism in practice; see, for example, Avery et al. (2009) and Avery and Levin (2010) for empirical observations and game-theoretic analysis.

aspect of interviews, they also serve to coordinate matches. Hence, centralized matching markets are also prone to severe forms of congestion if the interview costs are high enough to prohibit hospitals from exploring enough candidates. To the best of my knowledge, while Lee and Schwarz (2007) were first to analyze signaling at the interviewing stage and reducing congestion through signaling has been suggested in the medical profession (e.g., Talcott and Evans (2021) and Bernstein (2017)), the current paper is the first to prove the welfare properties of a signaling mechanism in a centralized matching market with interviews.

While we leverage the parallels between our model and Coles et al. (2013) in our analysis, the results presented in this paper are not a straightforward generalization of their results. The stochastic nature of interviews and the iterative matching procedure pose additional technical difficulties. Namely, as emphasized by Lee and Schwarz (2017), hospitals evaluate not only who the interviewers of their interviewees are but also who those hospitals interview and so forth. Furthermore, as described above, how the interview assignment transforms into realized rank-ordered lists submitted to the centralized clearinghouse governs the success of the signaling mechanism.

We also borrow tools from Lee and Schwarz (2017) and especially its generalization in Skancke (2021). Skancke (2021) shows that an increased interview activity by a hospital imposes an unambiguous ex-ante welfare loss on the rest of the hospitals. In equilibrium, the welfare of both sides of the market might increase if the total number of interviews is reduced in a coordinated manner. Our results offer a complementary insight as we fix the number of interviews and show a mechanism improving coordination through informative signaling. In a continuum framework, Kadam (2021) focuses on capacities of doctors, showing that increasing capacity of some doctors benefits only the best doctors while harming others.

Several other papers model interviews in matching markets. Strategic interviewing is absent in the models of Echenique et al. (2020) and Manjunath and Morrill (2021), who consider interviewing as a coordination device. We model interviewing as a strategic game between hospitals where interviews serve both to coordinate matches and form preferences. In a large two-sided market with independent random preferences, Beyhaghi and Tardos (2021) find that with a limited number of interviews, a planner can maximize the number of matches if these interviews are spread evenly among participants. We focus on coordination through a partially decentralized process. A strand of work in the computer science literature (e.g. Rastegari et al. (2013); Drummond and Boutilier (2013); Rastegari et al. (2016)) consider adaptive algorithms designing interviewing schedules leading to stable matchings and minimizing interview costs. We consider a single round of interviewing and focus on strategic interactions.

This paper is also related to the problem of finding stable matching when preferences are a priori unknown. Gonczarowski et al. (2019) and Ashlagi et al. (2020) establish bounds on the amount of communication necessary to find stable matching in markets with private information. Ashlagi et al. (2020) also suggest a communication protocol that ensures two-sided matching markets clear efficiently. Their work is similar to ours as they consider how signaling can be used to alleviate congestion. However, their model is non-strategic.

Several recent papers have also studied congestion in matching platforms. Kanoria and Saban (2020) show that blocking one side from proposing and restricting information can make the market more efficient. Arnosti et al. (2021) and Halaburda et al. (2018) show that the designer can increase welfare in equilibrium by limiting the number of applications an agent can send or restricting consideration sets. Our results are complementary as we study how congestion can be alleviated after applications are sent without restricting capacities.

Outline. In Section 2, we provide a simple example illustrating the main intuition behind the results in this paper. We formally define the model, strategies, and equilibrium in Section 3. We characterize optimal strategies and equilibria of the games with and without signals in Sections 4.1 and 4.2. Welfare properties are analyzed in Section 5. Finally, we provide policy recommendations, discuss open questions, and conclude in Section 6.

2 An illustrative example

To illustrate the model and fix the intuition behind our main results, we next present the following simple example. Consider a one-to-one matching market between hospitals $H = \{h_1, h_2\}$ and doctors $D = \{d_1, d_2\}$. Each hospital h is endowed with private pre-interview information, yielding a ranking of candidates $\succ_h$. For each pair (h, d), post-interview hospital values are drawn from a distribution $F_{(h,d)}$, putting probability $p > 0$ on $-1/p$ and $(1-p)$ on 1 if d ranks highest on $\succ_h$ and $x \in (0, 1)$ if d ranks second. For doctors, matching yields payoffs of 1 and x. We normalize the payoff to the outside option to 0. Hospitals are constrained to make only a single interview, and doctors face no interviewing constraints. Preferences of doctors and pre-interview rankings of hospitals are random, uniform, and independent. The distribution of preferences is common knowledge.

The game without signals proceeds in two stages. First, each hospital invites a doctor for an interview, and this offer is accepted. Next, agents submit rank-ordered lists to the Hospital-Proposing Deferred Acceptance algorithm, which conducts the final match. Focusing on sequential equilibria, and since we have two agents on each side, it is a dominant strategy for each agent to report truthfully. We take this as given and examine the interview game.

Without signals, hospitals have no means to discern genuinely interested candidates. In the unique equilibrium in which hospitals cannot condition their strategies on the identity of doctors, they interview their ex-ante favorite candidate. With probability 1/2, the market is congested, as both hospitals interview the same doctor, yielding in expectation $1 - p^2$ matches and an expected payoff of $(1 - p^2)/2$ to each hospital. With probability 1/2, hospitals interview different doctors,

Equilibrium	Hospital welfare	Doctor welfare	Exp. number of matches
(respond, respond)	$\frac{5(1-p)}{8}+\frac{(1-p)(2x+p)}{8}$	$\frac{3}{4}(1-p)+\frac{1}{8}x(1-p^2)$	$\frac{3}{2}(1-p)+\frac{1}{4}(1-p^2)$
(ignore, ignore)	$\frac{3}{4}-\frac{p}{2}-\frac{p^2}{4}$	$\frac{1-p}{2}+\frac{x(1-p^2)}{4}$	$\frac{3}{2}-p-\frac{p^2}{2}$

Table 1: Equilibrium payoffs.

yielding $2(1-p)$ matches and $1-p$ in hospital welfare in expectation. In sum, the expected number of matches is $3/2-p-p^2/2$, and the expected hospital welfare is $3/4-p/2-p^2/4$. For doctors, receiving exactly one interview is equally likely to be from the first and second choice, yielding an expected payoff of $(1-p)/2+(1-p^2)/4$.

Next, we introduce a signaling mechanism in which doctors may send a single binary signal to a hospital; the signals convey no other information. We examine non-babbling sequential equilibria, in which doctors signal to their most preferred hospital and hospitals interpret signals as such.[9] We say a hospital responds to a signal if it interviews the sender.

Hospitals interview their ex-ante most preferred doctor whenever this doctor signals or they receive no signal. Next, we focus on the case when a hospital receives a signal from its ex-ante second choice. The other hospital also receives one signal. Suppose $d_1 \succ_{h_1} d_2$ and d_2 signals to h_1. Then, d_1 signaled to h_2. If h_1 interviews d_2, it receives an expected payoff of $(1-p)x$; if it interviews d_1 instead, and if h_2 responds to signals, we obtain an expected payoff of $p(1-p)$. The payoff matrix is as follows

$$\begin{array}{ll} h_1|h_2 & \begin{array}{cc}\text{respond} & \text{ignore}\end{array} \\ \begin{array}{l}\text{respond}\\ \text{ignore}\end{array} & \begin{bmatrix}(1-p)x & (1-p)x \\ p(1-p) & \frac{1-p^2}{2}\end{bmatrix}\end{array}$$

If $x > \max\{p, 1/2+p/2\} = 1/2+p/2$, there is a unique equilibrium in which hospitals respond to signals. When $p < x < 1/2+p/2$, there are two pure-strategy equilibria: both hospitals respond or ignore. If $x < p$, the unique pure strategy equilibrium is for both hospitals to ignore signals.

When signals are ignored, agents' actions are identical to the game with no signals. Equilibrium payoffs are summarized in Table 1. Observe that for any $p \in (0,1)$ and $x < 1$, doctor welfare and the expected number of matches are strictly higher in the equilibrium when both hospitals respond to signals, but that the benefit of signaling is decreasing in p for a fixed x, as illustrated in Figure 1. Whenever $p < x < 1/2+p/2$ (i.e., when there are multiple equilibria), hospitals' welfare is higher in the equilibrium in which signals are ignored.

[9] Note that there is no equilibrium in which hospitals would interpret the signals negatively. Indeed, such an equilibrium cannot be individually rational for doctors. However, there are babbling equilibria in which no information is conveyed. Since such equilibria are equivalent to having no signaling device, we will omit them from our analysis.

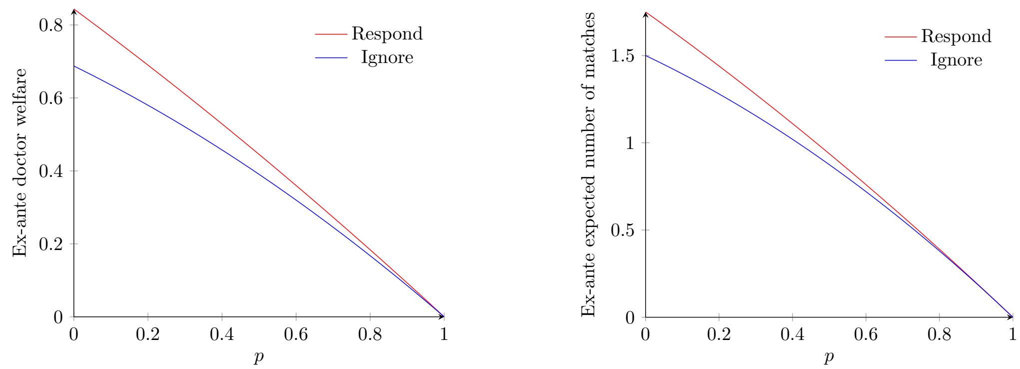

Figure 1: The relationship between the probability p and welfare measures for $x = 0.75$.

3 Model

3.1 Setup and definitions

Following Skancke (2021), we consider a matching market of finite sets of hospitals $H = \{h_1, \ldots, h_H\}$ and doctors $D = \{d_1, \ldots, d_D\}$. Each agent seeks at most one partner. The final matching is performed using the Hospital-Proposing Deferred Acceptance (HPDA) algorithm, formally defined in Appendix A. We assume that hospitals face capacity constraints on their number of interviews and that doctors bear no costs.

Hospitals. Let Θ_h be the set of all strict rank-ordered lists over D. Each hospital is endowed with private pre-interview information summarized by a vector $\theta_h \equiv (\theta_{hd_1}, \ldots, \theta_{hd_{|D|}}) \in \Theta_h$, its type, where θ_{hd} denotes h's pre-interview rank for d. We use the convention that the best doctor according to the pre-interview information has rank 1, the second best has rank 2, etc. Denote the set of all hospital type profiles by $\Theta_H \equiv \Theta_h^{|H|}$. Conditional on pre-interview information about doctor d, θ_{hd}, h's valuation of matching with d, $v_{\theta_{hd}}$, is drawn from a distribution $F_{\theta_{hd}}$. Denote by F_θ^+ the positive part of F_θ, and denote the corresponding draw from F_θ^+ by v_θ^+. The pre-interview ranking determines an ex-ante ordering of payoffs: for any $h \in H$ and $d, d' \in D$, if $\theta_{hd} \leq \theta_{hd'}$, then $F_{\theta_{hd}}$ first-order stochastically dominates $F_{\theta_{hd'}}$. Hospital valuations are drawn independently across hospitals and doctors. We assume that the payoff from staying unmatched is 0 and that the probability that h finds d unacceptable, $p \equiv \mathbb{P}(v_{\theta_{hd}} < 0) > 0$, is equal across all hospital-doctor pairs. $F_{\theta_{hd}}^+$ has a bounded expected value for any θ_{hd}: $\exists 0 < \underline{v} \leq \overline{v} < \infty$ such that $\underline{v} \leq \mathbb{E}[v_{\theta_{hd}}^+] \leq \overline{v}$. Finally, we impose symmetry on hospitals by requiring that their distributions of valuations depend only on the pre-interview ranks θ_{hd}. Formally, for any permutation ρ of doctor indices, $F_{\rho(\theta_{h\rho(d)})} = F_{\theta_{hd}}$.[10] All hospitals have equal capacities $\kappa \geq 1$ on their number of interviews.

[10]With some abuse of notation, in this paper, we apply a permutation $\rho : \{1, \ldots, |D|\} \to \{1, \ldots, |D|\}$ to the set of doctors, subsets of doctors, and preference orderings.

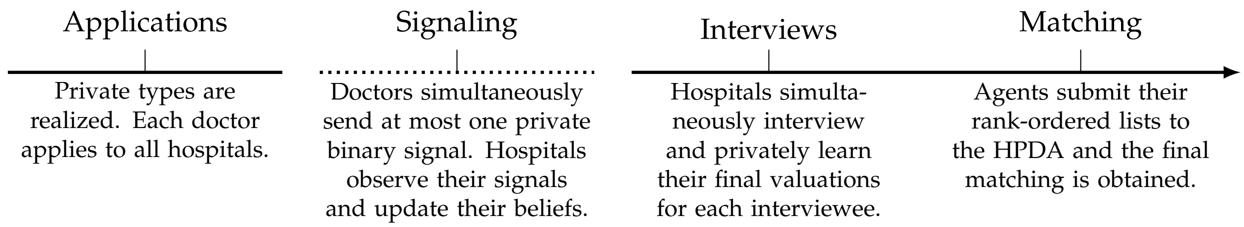

Figure 2: Timing of the game. The signaling stage is omitted in the game without signals.

Doctors. For each doctor d, let Θ_d be the set of all strict linear orders over H, where $\theta_d \in \Theta_d$ gives a vector of ranks over H. The set of all hospital type profiles is $\Theta_D \equiv (\Theta_d)^{|D|}$. We assume each doctor finds each hospital acceptable and doctor d with preference list θ_d obtains surplus $u_{\theta_{dh}} \in [\underline{u}, \overline{u}]$ for some $0 < \underline{u} \leq \overline{u} < \infty$, where again we assume the outside option has value 0. Similarly to hospital symmetry, doctors' surplus only depends on hospital rank: for any permutation ρ of hospital indices, $u_{\rho(\theta_{d\rho(h)})} = u_{\theta_{dh}}$.[11] Doctors know their valuations when applying.

Distribution of types. Similarly to Coles et al. (2013), we consider *block-correlated preferences*: there is a partition of H into $H_1, \ldots, H_B$ such that (i) if $b < b'$, for any doctor $d \in D$ and hospitals $h \in H_b$ and $h' \in H_{b'}$, $u_{\theta_{dh}} > u_{\theta_{dh'}}$; (ii) each doctor's preferences over hospitals within a block are uniform and independent; and (iii) for each hospital h, θ_h is drawn independently from the uniform distribution over Θ_h. The distribution over types T is common knowledge.

Finally, a market is a tuple $\mathcal{M} = (H, D, \Theta_H, \Theta_D, F, u, \kappa, T)$.

3.2 Strategies, equilibrium, and welfare

Figure 2 depicts the timeline. Next, we describe each stage.

The matching stage

At the matching stage, hospitals only rank doctors they interviewed and found acceptable. Without loss of generality, doctors also rank only their interviewers. While the HPDA algorithm is strategy-proof for hospitals, in general, it is not for doctors. We follow Lee and Schwarz (2017) and assume that remaining unmatched is sufficiently undesirable for doctors to guarantee truthfulness. In the rest of the paper, we assume that the conditions of the following lemma are satisfied, fixing strategies at the matching stage. The proof is contained in Appendix B.1.

Lemma 1. *If $\underline{u} > \frac{p^{1-|H|}}{1+p^{1-|H|}}\overline{u}$, there is a unique equilibrium: hospitals and doctors report truthfully.*

[11] Again, with abuse of notation, we apply a permutation $\rho : \{1, \ldots, |H|\} \to \{1, \ldots, |H|\}$ to the set of hospitals, subsets of hospitals, and preference orderings of doctors.

The interview game without signals

After obtaining pre-interview information, hospitals choose whom to interview. They may also mix over subsets of doctors that satisfy their capacity constraints, including the empty assignment $\varnothing$. We denote their menu by $\mathcal{I} \equiv \{S \in 2^D : |S| \leq \kappa\}$. Hospital h's strategy is a function $\sigma_h : \Theta_h \to \Delta(\mathcal{I})$. Denote the set of hospital h's strategies by Σ_h and a profile of hospital strategies by $\sigma_H = (\sigma_{h_1}, \ldots, \sigma_{h_H}) \in \Sigma_h^{|H|} \equiv \Sigma_H$. We assume doctors are non-strategic at the interviewing stage and that they accept all interview invitations. We define hospital h's payoff function as a function of strategies and realized types $\pi_h : \Sigma_H \times \Theta \to \mathbb{R}$. Similarly, for doctor d, $\pi_d : \Sigma_H \times \Theta \to \mathbb{R}$. With a slight abuse of notation, we will often write a pure strategy of hospitals as a set $S \subseteq D$ of interviewees.

To gain intuition behind how payoffs are computed, observe that for any type profile θ and strategy profile σ_H, $\sigma_H(\theta) \equiv (\sigma_{h_1}(\theta_{h_1}), \ldots, (\sigma_{h_{|H|}}(\theta_{h_1}))$ generates a probability distribution over *interview assignments*, many-to-many matchings of doctors and hospitals. In turn, a realized interview assignment generates a probability distribution over *rank-ordered lists* of agents supplied to the HPDA algorithm. For any type profile θ and strategy profile σ_H, denote by $\lambda_{\theta,\sigma_H}$ a matching drawn from the induced probability distribution. Note that for any pure strategy of h of type θ_h, σ_h, and any profile of strategies of the other hospitals σ_{-h},

$$\mathbb{E}_{\theta_{-h}} \left[\pi_h(\sigma_h, \sigma_{-h}, \theta) | \theta_h \right] = \sum_{\theta_{-h} \in \Theta_h^{|H|-1}} \sum_{d \in \sigma_h(\theta_h)} \mathbb{P}[\lambda_{\theta,\sigma_H}(h) = d] \, \mathbb{E}[v_{\theta_{hd}} | \lambda_{\theta,\sigma_H}(h) = d]$$

We proceed to define our equilibrium concept.

Definition 1. Strategy profile $\widehat{\sigma}_H$ is a Bayesian-Nash equilibrium of the interview game without signals if for each hospital $h \in H$ and each type $\theta_h \in \Theta_h$, $\widehat{\sigma}_h$ maximizes h's ex-ante expected payoff for type θ_h, i.e.

$$\widehat{\sigma}_h \in \arg\max_{\sigma_h \in \Sigma_h} \mathbb{E}_{\theta_{-h}} \left[\pi_h(\sigma_h, \widehat{\sigma}_{-h}, \theta) | \theta_h \right]$$

To eliminate unreasonable sources of coordination, we focus on anonymous strategies, where the interview strategies of hospitals only depend on the rank of doctors in their pre-interview ordering.

Definition 2. Strategy profile $\sigma_H \in \Sigma_H$ is *anonymous* if for any $\theta \in \Theta$ and any permutation ρ of θ, $\sigma(\rho(\theta)) = \rho(\sigma(\theta))$.[12]

The interview game with signals

At the signaling stage, doctors choose whether and to whom to signal. Denoting the no signal option by $\varnothing$, a mixed signaling strategy for a doctor d is a function $\sigma_d : \Theta_d \to \Delta(H \cup \varnothing)$. Each

[12] We only permute doctor indices and do not permute the empty set of interviews: $\rho(\emptyset) = \emptyset$.

hospital h observes its set of signaling doctors, $D_h^S \subset D \cup \varnothing$, and forms beliefs $\mu_h(\cdot|D_h^S)$ about doctors' preferences. Based on these beliefs and preferences, hospitals decide whom to interview. An interviewing strategy of hospital h is a function $\sigma_h : \Theta_h \times 2^D \to \Delta(\mathcal{I})$. For hospitals and doctors, we denote strategy spaces by Σ_h and Σ_d and strategy profiles as $\sigma_H \in \Sigma_H \equiv \Sigma_h^{|H|}$ and $\sigma_D \in \Sigma_D \equiv \Sigma_d^{|D|}$, respectively. We define hospital h's payoff function as a function of signaling strategies of doctors, interviewing strategies of hospitals, and realized types $\pi_h : \Sigma_D \times \Sigma_H \times \Theta \to \mathbb{R}$. Similarly, for any doctor d, $\pi_d : \Sigma_D \times \Sigma_H \times \Theta \to \mathbb{R}$.

Definition 3. Profile $\widehat{\sigma} = (\widehat{\sigma}_H, \widehat{\sigma}_D)$ and assessments $\{\mu_h(\cdot|D_h^S)\}_h$ for signals $\{D_h^S\}_h$ are a sequential equilibrium of the interview game with signals if for any doctor d and her realized type $\theta_d \in \Theta_d$,

$$\widehat{\sigma_d}(\theta_d) \in \arg\max_{\sigma_d \in \Sigma_d} \mathbb{E}_{\theta_{-d}} \left[\pi_d(\sigma_d, \widehat{\sigma}_{-d}, \theta)|\theta_d\right]$$

and for any hospital h of type $\theta_h \in \Theta_h$ and received signals $D_h^S \subset D \cup \varnothing$,

$$\widehat{\sigma}_h \in \arg\max_{\sigma_h \in \Sigma_h} \mathbb{E}_{\theta_{-h}} \left[\pi_h(\sigma_h, \widehat{\sigma}_{-h}, \theta)|\theta_h, D_h^S, \mu_h\right]$$

Beliefs are updated using the Bayes rule on the equilibrium path.[13]

We restrict attention to anonymous strategies for both sides of the market.

Definition 4. Hospital h's strategy σ_h is anonymous if for any θ_h and permutation of doctor indices ρ, $\sigma_h(\rho(\theta_h), \rho(D_h^S)) = \rho(\sigma_h(\theta_h, D_h^S))$. Doctor d's signaling strategy σ_d is anonymous if for any type θ_d and permutation ρ which only permutes hospital indices within blocks, $\sigma_d(\rho(\theta_d)) = \rho(\sigma_d(\theta_d))$.

Welfare measures

For welfare comparison, we consider Pareto ex-ante expected payoffs and the ex-ante expected number of matches. We define the expected number of matches for realized types as a function of strategies and realized types $m : \Sigma_H \times \Sigma_D \times \Theta \to \mathbb{R}$. For any strategy profile σ and type profile $\theta \in \Theta$,

$$m(\sigma, \theta) = \sum_{h \in H} \mathbb{P}[\lambda_{\sigma,\theta}(h) \neq \emptyset] = \sum_{d \in D} \mathbb{P}[\lambda_{\sigma,\theta}(d) \neq \emptyset]$$

With pure strategies, m yields the expected number of matches for a given interview assignment; if strategies are mixed, we average over the resulting interview assignments. The ex-ante expected number of matches is hence obtained by averaging over types.

[13]Off-equilibrium beliefs are defined as limits of completely mixed strategies.

4 Equilibrium analysis

4.1 The game without signals

First, we characterize optimal hospital strategies. The following lemma shows that whenever other hospitals use anonymous strategies, each hospital is better off interviewing a doctor with more favorable pre-interview information.

Lemma 2. *(Lemma 3.6. (Skancke, 2021)) Fix $h \in H$ of type θ_h and suppose hospitals $H \setminus \{h\}$ use anonymous strategies σ_{-h}. Let $S \subset D$ such that $|S| < \kappa$ and let $d, d' \in D \setminus S$ such that $\theta_{hd} < \theta_{hd'}$ be arbitrary. Then,*

$$\mathbb{E}_{\theta_{-h}}\left[\pi_h(S \cup \{d\}, \sigma_{-h}, \theta)|\theta_h\right] > \mathbb{E}_{\theta_{-h}}\left[\pi_h(S \cup \{d'\}, \sigma_{-h}, \theta)|\theta_h\right]$$

The marginal benefit from an additional interview is always strictly positive due to the uncertainty about doctor quality. Hence, each hospital always conducts κ interviews. Whenever the other hospitals use anonymous strategies, h's dominant strategy is to interview its κ best doctors according to θ_h,[14] leading to equilibrium uniqueness. The proof is contained in Appendix B.2.

Proposition 1. *There is a unique Baysian-Nash equilibrium in anonymous strategies in which hospitals interview their most preferred κ doctors according to their pre-interview rankings.*

4.2 The game with signals

Similarly to Coles et al. (2013), we restrict attention to *block-symmetric* equilibria, in which hospitals from the same tier use symmetric anonymous strategies and have the same beliefs, and doctors use symmetric anonymous strategies.

Doctors are isomorphic to workers in Coles et al. (2013)'s model. In particular, they know their preferences before participating in interviews, and the equivalent distribution over agent types leaves their strategic considerations unchanged despite the more complex matching procedure at the final stage. Anonymity and block-symmetry of hospital strategies imply the probabilities of receiving an interview from any two hospitals in a block are the same, both conditional on sending and not sending a signal to this block, for any doctor. We denote these probabilities by β_{db}^{S} and β_{db}^{NS}, respectively, for each doctor d. Also denote the equilibrium probability that d sends a signal to block b by α_{db}, with $\sum_b \alpha_{db} \leq 1$. The following lemma characterizes their optimal strategies.

Lemma 3. *(Proposition 2. (Coles et al., 2013)) Consider a block-symmetric sequential equilibrium that satisfies a multi-player analog of Criterion D1 of Cho and Kreps (1987). Then, either*

[14] Observe that interviewing κ most favorite doctors according to the pre-interview information is an anonymous strategy.

1. *Signals do not influence interview offers:* $\beta^{S}_{db} = \beta^{NS}_{db}, \forall d \in D, b \in \{1, \dots, B\}$

2. *Signals increase probabilities of receiving an interview invitation:* $\exists b_0 \in \{1, \dots, B\}$ *such that* $\beta^{S}_{db_0} > \beta^{NS}_{db_0}$*, and*

   - $\forall b \in \{1, \dots, B\}$ *such that* $\alpha_{db} > 0$*, we must have* $\beta^{S}_{db} > \beta^{NS}_{db}$*. Moreover, if* d *sends a signal to* b*, she sends her signal to her most favorite hospital in* b*.*

   - *For any* $b' \in \{1, \dots, B\}$ *such that* $\alpha_{db'} = 0$*,* d*'s strategy is optimal for any off-equilibrium beliefs of hospitals in block* $H_{b'}$*.*

In any non-babbling equilibrium, doctors signal only to blocks which respond to signals. Conditional on signaling to a block, they signal to their favorite hospital. Note it may also be optimal for them to mix over these hospitals.[15] We call strategies where doctors signal only to their most favorite hospital in a block (or a mixture of them) *best-in-block strategies*. Further, we call assessments of hospitals interpreting a signal as being the sender's most preferred hospital in the block as *best-in-block beliefs*.

Consider hospital h with signals $D^{S}_{h} \subset D \cup \varnothing$, and denote by $D^{NS}_{h} = D \setminus D^{S}_{h}$ the complement. The expected payoff from interviewing the ex-ante best doctor according to θ_{hd} in D^{S}_{h} and D^{NS}_{h} is strictly higher than interviewing any other doctor in the respective sets. Indeed, by block-symmetry and anonymity, for any two doctors who sent a signal, h's belief over the type distribution of these doctors and the pre-interview information of other hospitals over these doctors is identical. Hence, if h invites a signaling doctor for an interview, it should invite the signaling doctor with the most favorable pre-interview information. An analogous statement holds about non-signaling doctors. Formally stated in the following lemma, the result is proved in Appendix B.3.

Lemma 4. *Fix* $h \in H$ *with received signals* $D^{S}_{h} \subset D \cup \varnothing$*. Suppose* $H \setminus \{h\}$ *use anonymous strategies, hospitals have best-in-block beliefs, and doctors use symmetric best-in-block strategies. Let* $S \subset D$ *such that* $|S| < \kappa$ *be arbitrary. Let (i)* $d, d' \in D^{S}_{h} \setminus S$ *such that* $\theta_{hd} < \theta_{hd'}$ *or (ii)* $d, d' \in D^{NS}_{h} \setminus S$ *such that* $\theta_{hd} < \theta_{hd'}$*. Then, in both cases,*

$$\mathbb{E}_{\theta_{-h}}\left[\pi_h(S \cup \{d\}, \sigma_{-h}, \theta) | \theta_h, D^{S}_{h}\right] > \mathbb{E}_{\theta_{-h}}\left[\pi_h(S \cup \{d'\}, \sigma_{-h}, \theta) | \theta_h, D^{S}_{h}\right]$$

From h's point of view, if the best response of hospital h involves interviewing κ^{S} doctors from whom it received a signal and κ^{NS} doctors from whom it did not receive a signal, it must interview its κ^{S} top signaling doctors and κ^{NS} top non-signaling doctors according to θ_h. Similarly to Coles et al. (2013), the form of best responses motivates cutoff strategies for hospitals.

[15] For example, if all doctors were to send their signal to the same block, the benefit to one doctor from signaling to the block ranked right below it might be highly beneficial.

Definition 5. Strategy σ_h of hospital h is a *cutoff strategy* if there is a vector $\mathbf{r} \equiv (r_0, \ldots, r_{|D|}) \in [1, |D|]^{|D|+1}$ such that for any received signals D_h^S, h interviews any $d \in D_h^S$ if and only if $\theta_{hd} \leq r_{|D_h^S|}$. We call $\mathbf{r}$ h's cutoff vector.

Whenever h uses a cutoff strategy, the rank of a doctor in its pre-interview list is a sufficient statistic to determine whether she receives an interview from h, conditional on its the number of received signals. The cutoffs, in general, depend on the number of received signals, ex-ante similarity between doctors, as determined by the distributions F_θ, and the probability p. The number of received signals provides information about the number of signals received by other hospitals, affecting optimal interview offers of these hospitals, hence providing information about competition in the market. The role of the probability p and distributions F_θ is discussed in Section 5.2.

Observe that we defined cutoffs as real numbers. Whenever r_j is not an integer, we implement it as a mixture between cutoffs $\lfloor r_j \rfloor$ and $\lceil r_j \rceil$ such that $\lfloor r_j \rfloor$ is selected with probability $r_j - \lfloor r_j \rfloor$ and $\lceil r_j \rceil$ is selected with the complementary probability. By Lemma 4, any cutoff strategy with $r_j < \kappa$ for any $j \in \{0, \ldots, |D|\}$ cannot be optimal. Therefore, each element of the cutoff vector must be weakly greater than κ. Whenever each element equals κ, the hospital always interviews the κ best doctors according to its pre-interview information. Hence, the cutoff is equivalent to the dominant strategy of the interview game without signals. We denote this strategy by σ_h^0.

We also define the following partial order: cutoff strategy σ_h of hospital h is greater than cutoff strategy σ_h', denoted $\sigma_h \geq \sigma_h'$, if for any number of received signals, the corresponding cutoff under σ_h is weakly larger than the corresponding cutoff under σ_h', and the inequality is strict for at least one element. Further, we say h *responds more to signals than* h' if $\sigma_h \geq \sigma_{h'}$. Finally, we say that h *responds to signals* if h uses a strategy $\sigma_h \geq \sigma_h^0$.

Cutoff strategies are optimal. When doctors use symmetric best-in-block strategies, and hospitals use anonymous strategies and have best-in-block beliefs, for each hospital and its strategy, there is a cutoff strategy which yields a weakly higher expected surplus. To this end, note that any hospital h's belief over the type of a signaling doctor is identical across signaling doctors. Further, two sets of received signals of the same cardinality reveal no additional information about the competition for any signaling and non-signaling doctors. Combined, we obtain the following lemma, proved in Appendix B.4.

Lemma 5. *Fix $h \in H$ and suppose hospitals $H \setminus \{h\}$ use anonymous strategies σ_{-h}, hospitals have best-in-block beliefs, and doctors use symmetric best-in-block strategies. Consider two sets of received signals D_h^S and $\widetilde{D}_h^S$ such that $|D_h^S| = |\widetilde{D}_h^S|$. Then, for any $S \subset D$ such that $S \cap D_h^S = S \cap \widetilde{D}_h^S$,*

$$\mathbb{E}_{\theta_{-h}}\left[\pi_h(S, \sigma_{-h}, \theta) | \theta_h, D_h^S\right] = \mathbb{E}_{\theta_{-h}}\left[\pi_h(S, \sigma_{-h}, \theta) | \theta_h, \widetilde{D}_h^S\right]$$

Whenever hospital h finds it optimal to interview a subset I^S of signaling doctors under the set of received signals D^S, by iterative application of Lemmas 4 and 5, it must also interview a subset $I^{S\prime}$ of signaling doctors under $D^{S\prime}$ of the same cardinality if these doctors have a more favorable pre-interview record (see Lemma 8 in Appendix B.5 for a formal result). Optimality of cutoff strategies follows. The formal proof is contained in Appendix B.5.

Proposition 2. *Fix an arbitrary hospital h and suppose doctors use symmetric best-in-in-block strategies and hospitals have best-in-block beliefs. Then, for any strategy of hospital h there is a cutoff strategy that yields a weakly higher payoff for any profile of anonymous strategies of the other hospitals.*

The more other hospitals respond to signals, the riskier it is for h to interview doctors who did not signal to it. In response, it is optimal for h to also respond to signals more. The interviewing game, therefore, exhibits strategic complements. The proof of the following proposition can be found in Appendix B.6.

Proposition 3. *Suppose doctors use symmetric best-in-block strategies, hospitals have best-in-block beliefs and use cutoff strategies. Fix $h \in H$ and cutoff strategy σ_h, such that σ_h is best-responding to σ_{-h}, and suppose $\exists h' \in H \setminus \{h\}$ such that h' increases its cutoffs: $\sigma'_{h'} \geq \sigma_{h'}$. Then, h optimally plays some σ'_h with $\sigma'_h \geq \sigma_h$.*

The equilibrium existence proof is a direct adaptation of the existence proof in Coles et al. (2013). The latter part of the following result relies on the strategic complements property.

Proposition 4. *There is a block-symmetric sequential equilibrium in which doctors use best-in-block strategies, and hospitals use block-symmetric cutoff strategies. Moreover, if there is a single block of hospitals, doctors signal to their most-preferred hospital, and hospitals use symmetric pure cutoff strategies. Finally, there is a largest and smallest element of the set of equilibrium cutoff strategies.*

With a single block of hospitals, denote by σ_H^L the largest and by σ_H^S the smallest cutoff vector.

5 Welfare implications of signals

5.1 The value of a signaling mechanism

Responding to signals increases coordination, as hospitals interview doctors who are more likely to accept their offer. Since doctors are more likely to interview with hospitals they value more, their expected payoff increases. While this intuition transfers from the setting without interviews, the analysis is not a straightforward generalization due to the many-to-many structure of interviews, a multi-round matching mechanism at the final stage, and the role of the network structure of interview assignments, as suggested by Lee and Schwarz (2017) in their criterion of overlap.

While proving the following lemma for the full model presented above is work-in-progress,[16] we state the result in its general form and prove it for the following restricted case. Similarly to Kadam (2021), for each hospital h and doctor d, we assume $F^+_{\theta_{hd}}$ is a degenerate distribution placing probability 1 on $v^+_{\theta_{hd}} > 0$, such that if $\theta_{hd} < \theta_{hd'}$ for some doctor d', $v^+_{\theta_{hd}} > v^+_{\theta_{hd'}}$. The full distribution $F_{\theta_{hd}}$ is then composed by placing probability p on a strictly negative value, and the complementary probability on $v^+_{\theta_{hd}}$. In this case, the interview reveals only a binary outcome telling the hospital whether the candidate is fit for the position. While restrictive, the structure is partially consistent with the real-world characteristics of the market.[17]

Lemma 6. *Assume hospitals use cutoff strategies and have best-in-block beliefs. Also assume doctors use symmetric best-in-block strategies. For $h \in H$, consider σ_h and σ'_h, such that $\sigma'_h \geq \sigma_h$. Then,*

$$\mathbb{E}_\theta\Big[m(\sigma'_h, \sigma_{-h}, \theta)\Big] \geq \mathbb{E}_\theta\Big[m(\sigma_h, \sigma_{-h}, \theta)\Big] \quad \textit{and} \quad \mathbb{E}_\theta\Big[\pi_d(\sigma'_h, \sigma_{-h}, \theta)\Big] \geq \mathbb{E}_\theta\Big[\pi_d(\sigma_h, \sigma_{-h}, \theta)\Big], \forall d \in D$$

The proof, contained in Appendix B.7, proceeds by defining an injective mapping from the set of types for which the expected number of matches and doctor welfare strictly decrease to the set of types in which they strictly increase, and showing that the decrease in the former cannot be larger than the increase in the latter. The network structure of interview assignments plays an important role: the expected number of matches increases not only if we fill previously unfilled vacancies or doctors without an interview now obtain at least one, but also if we increase *overlap* between interview partners. To overcome these issues, we go one layer deeper and study changes in reported rank-ordered lists induced by the change in cutoff strategies that alters the resulting number of matches with a positive probability. The analysis is closely related to Blum et al. (1997)'s analysis of vacancy chains.

A strict increase is only guaranteed if there is a block with at least two hospitals that receive a signal with a strictly positive probability. The effect on hospital welfare is ambiguous due to the externalities a hospital exerts on the other hospitals in the block. When h from block b intends to interview d from whom it did not receive a signal and so does h' from a block ranked at least as high as b to which d signaled, h faces a significant amount of competition. The formal result is proved in Appendix B.8, and the ambiguous effect on hospital welfare is illustrated in the example presented in Section 2.

Theorem 1. *Consider a non-babbling block-symmetric equilibrium in which there is a block b with at least two hospitals such that doctors signal to this block with a non-zero probability and hospitals respond to signals. Compared to the unique equilibrium of the game with no signals, (i) the ex-ante expected number of matches strictly increases, (ii) the ex-ante expected welfare of doctors strictly increases, and (iii) the ex-ante expected welfare of hospitals may increase or decrease.*

[16]Please see marekbojko.github.io/research for an up-to-date version of the paper.

[17]For example, anecdotal evidence suggests hospitals often rank doctors based on their performance in exams and standardized test and use interviews to determine whether the candidate is a good cultural fit for the department or has the aptitude to perform well under the conditions present at the hospital.

With a single block of hospitals, we can compare welfare across equilibria. By Lemma 6, doctors always prefer equilibria in which hospitals respond more to signals. On the other hand, since the competition effect of signaling increases the more hospitals respond to signals, all hospitals benefit from a coordinated reduction in cutoffs.[18]

Proposition 5. *(Theorem 3, Coles et al. (2013)) Consider block-symmetric equilibria σ^1 and σ^2 and a single block s.t. $\forall h \in H, \sigma^1_h \geq \sigma^2_h$. Then: (i) each doctor obtains a weakly higher payoff in σ^1 compared to σ^2 and (ii) each hospital obtains a weakly higher payoff in σ^2 compared to σ^1.*

5.2 The role of pre-interview uncertainty

In this section, we investigate how pre-interview uncertainty about final match utilities impacts the success of the signaling mechanism. Namely, we investigate the role of the fitness probability $1-p$ and attributes of the distributions F_θ. Throughout this section, we assume there is a single block of hospitals.

The probability p limits competition for a doctor. Consider h deciding whether to interview its sender d^S or d^{NS} from whom it did not receive a signal, with $\theta_{hd^{NS}} < \theta_{hd^S}$. For low p, when other hospitals respond to signals, h faces strong competition from the receiver of d^{NS}'s signal with high probability. As p increases, the magnitude of the competition effect declines - it is less likely that any interviewer of d^{NS} will find the doctor acceptable. While the probability h finds its interviewees acceptable also decreases, the probability that h matches with d^S is a polynomial in p of lower order compared to d^{NS}. Therefore, the expected payoff from matching with d^{NS} decreases less than the expected payoff from matching with d^S. Hence, hospitals are guided by signals less for higher p.

If doctors are ex-ante more similar, hospitals respond to signals more. For example, suppose hospitals cannot conduct standardized tests before interviews, reducing the amount of pre-interview information. Changes in the perceived match probabilities caused by changes in beliefs over others' types matter more. Therefore, hospitals are guided by signals more. To formalize the intuition, we assume $||F_\theta - F_{\theta+1}||_1 = \gamma \in \mathbb{R}_+$, for any $\theta \in \{1, \ldots, |D|-1\}$, and study the market as we vary γ.

Proposition 6. *Suppose hospitals use symmetric cutoff strategies and have best-in-block beliefs, and doctors use symmetric best-in-block strategies. Fix $h \in H$ and σ_{-h}. Consider strategies σ_h and σ'_h, s.t. $\sigma'_h \geq \sigma_h$. Using superscripts to index objects by markets, for any signals $D^S_h \subset D \cup \varnothing$ and markets $\mathcal{M}$ and $\mathcal{M}'$ s.t.*

[18] For this result to hold, hospitals must suffer from an increase in the cutoffs of a single hospital. With multiple blocks, hospitals in lower-ranked blocks might benefit from an increased response to signals of a hospital from higher-ranked blocks, as it will face less competition for a doctor who signaled to it. Therefore, the statement holds only for a single block.

either (i) $p > p'$ or (ii) $\gamma > \gamma'$, we have

$$\mathbb{E}_{\theta_{-h}}\left[\pi_h^{\mathcal{M}}(\sigma_h, \sigma_{-h}, \theta) - \pi_h^{\mathcal{M}}(\sigma_h', \sigma_{-h}, \theta) | \theta_h, D_h^S\right] \geq \mathbb{E}_{\theta_{-h}}\left[\pi_h^{\mathcal{M}'}(\sigma_h, \sigma_{-h}, \theta) - \pi_h^{\mathcal{M}'}(\sigma_h', \sigma_{-h}, \theta) | \theta_h, D_h^S\right]$$

The proof is contained in Appendix B.9. It follows that whenever p or γ increase, hospitals respond to signals less. Whenever there are multiple equilibria, the smallest and largest equilibrium cutoff strategies must weakly decrease.

Corollary 1. *Suppose the conditions in Lemma 6 hold. Then, $\sigma_H^{L\mathcal{M}} \leq \sigma_H^{L\mathcal{M}'}$ and $\sigma_H^{S\mathcal{M}} \leq \sigma_H^{S\mathcal{M}'}$.*

In light of Proposition 5, whenever p or γ increase and hospitals are more guided by signals, the doctor welfare gains and the expected number of matches from the signaling mechanism must weakly increase.

6 Discussion and conclusion

Many matching markets suffer from congestion at the preference formation stage with costly information acquisition. The current paper presented a tractable framework to analyze congestion at the interviewing stage and introduced a signaling mechanism as a remedy.

When interviewing costs are borne only by hospitals, doctors seek as many interviews as possible. With bilateral private communication, hospitals would be incentivized to tell each hospital it is their first choice. Signaling, therefore, needs to be restricted. Credible signaling with a limited number of signals has been successfully used in the market for PhD economists for more than a decade (Coles et al., 2010) and has been called for in the medical community (e.g. Bernstein (2017); Talcott and Evans (2021)).

Signaling has proved promising in our model. We allow doctors to send a single costless signal. While explicitly costless, in a world where hospitals respond to signals, signaling to a particular hospital carries the opportunity cost of not signaling to another. In any non-babbling equilibrium, doctors use the signal to indicate interest in their favorite hospital in a block. Hospitals are guided by the signals, and the mechanism increases the ex-ante expected doctor welfare and number of matches while having an ambiguous effect on hospital welfare. Whether hospitals respond to signals depends on the probability a doctor is found fit for a program and the magnitude of ex-ante differences between doctors. The signaling mechanism yields the highest welfare gains for doctors when hospitals have little information to differentiate between doctors before interviews and where doctors are deemed fit for hospitals with high probability.

Since hospitals face capacity constraints on their number of interviews and the net marginal benefit of interviewing another doctor is strictly positive up to filling the capacity, hospitals interview an equal number of doctors in each equilibrium. A promising direction for future work would be to consider alternative cost structures. For example, when hospitals face positive marginal interviewing costs, they might reduce their number of interviews after receiving signals. If the probability of staying unmatched due to unsuccessful interviews is high enough, applicants' welfare might decrease, even though they are more likely to interview with their favorite hospitals. We conjecture signaling in equilibrium remains individually rational due to strategic complements: applicants find signaling more profitable the more others signal. A designer, therefore, needs to learn about the structure of the market before implementing the mechanism.

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Appendix

A The Hospital-Proposing Deferred Acceptance Algorithm

The Hospital-Proposing Deferred Acceptance (HPDA) algorithm introduced by Gale and Shapley (1962) takes the reported rank-ordered lists of hospitals and doctors and constructs the final matching as follows.

Step 1: Each hospital offers its position to its most preferred doctor. Each doctor tentatively accepts her best offer and rejects the rest.

$\vdots$

Step k: Each hospital whose offer was rejected in round $k-1$ offers its position to its most preferred doctor to whom it has not offered its position yet. Each doctor tentatively accepts her best offer and rejects the rest.

The algorithm terminates if unmatched hospitals have been rejected by all doctors or if no offers are rejected. After the final step, the tentative matching becomes the realized matching.

B Proofs

B.1 Proof of Lemma 1

Proof. Dubins and Freedman (1981) show that it is a dominant strategy for hospitals to report truthfully. We prove that under the stated assumption, it is also a dominant strategy for doctors to report truthfully, thus proving the Lemma. There are two types of deviations: truncation and reshuffling of the preference list. The former might lead to a profitable deviation by creating a rejection chain: d^1 rejects h^1's proposal; h^1 gives an offer to d^2 instead who accepts and reject some other hospital h^2, which gives an offer to d^3, . . . , until d^k rejects h^k, who finally gives an offer to d^1, such that d^1 prefers h^k to h^1.

We show such a deviation is never profitable. To this end, suppose there is a doctor d with a profitable rejection chain $d \to h \to d' \cdots \to h' \to d$. The benefit is bounded above by the measure of the domain of u: $\overline{u} - \underline{u}$. However, by rejecting h's proposal, d can end up being unmatched. In each equilibrium, doctors can received at most $|H|$ interviews. Hence, the probability that d will remain unmatched after rejecting h is at least $p^{|H|-1}$. Hence, d's opportunity cost is bounded below by $p^{|H|-1}\underline{u}$. It follows from our assumption that truncating one's preference list is strictly dominated by truthful reporting.

Next, we show that misreporting the rank-order of hospitals is also dominated for doctors. We claim that for reported rank-ordered θ, the probability of matching with the n-th ranked preferred hospital is decreasing in n. Indeed, since preferences of agents are drawn independently and private, and since each doctor observes only her own interview assignment, the probability of being given an offer by a hospital who interviewed d is equal. If we denote this probability by q, the probability of matching with the n-th choice is $(1-q)^{n-1}q$, which is decreasing in n. Since there are no indifferences in d's preferences, reporting truthfully must yield a strictly higher payoff. The result follows. $\square$

B.2 Proof of Proposition 1

Proof. Proof of equilibrium existence follows the proof of Lemma 3.3. in Skancke (2021). We prove uniqueness by showing that if all other hospitals play anonymous strategies, it is a dominant strategy for hospital h to interview its κ most favourite doctors. Indeed, fix a profile of anonymous interview strategies of the other hospitals σ_{-h} and consider an arbitrary $S \subseteq D, S \neq \{d_\theta^{(k)}\}_{k=1}^{\kappa}$, such that $|S| = \kappa$. If we list $S = \{d_S^{(1)}, \ldots, d_S^{(\kappa)}\}$ such that $\theta_{hd_S^{(j)}} < \theta_{hd_S^{(k)}}$ for $j < k$, we must have $\theta_{hd_\theta^{(j)}} \leq \theta_{hd_S^{(j)}}$ for each $j \in \{1, \ldots, \kappa\}$ with at least one strict inequality. Denote the smallest index when this happens by m, i.e.

$$m \equiv \min\left\{j \in \{1, \ldots, \kappa\} : \theta_{hd_\theta^{(j)}} < \theta_{hd_S^{(j)}}\right\}$$

Then, by Lemma 2,

$$\mathbb{E}_{\theta_{-h}}\left[\pi_h\big((S \cup \{d_\theta^{(m)}\}) \setminus \{d_S^{(m)}\}, \sigma_{-h}, \theta\big)|\theta_h\right] > \mathbb{E}_{\theta_{-h}}\left[\pi_h(S, \sigma_{-h}, \theta)|\theta_h\right]$$

Continuing inductively, we obtain

$$\begin{aligned}
\mathbb{E}_{\theta_{-h}}\left[\pi_h(\{d_\theta^{(k)}\}_{k=1}^{\kappa}, \sigma_{-h}, \theta)|\theta_h\right] &> \mathbb{E}_{\theta_{-h}}\left[\pi_h(\{d_\theta^{(1)}, \ldots, d_\theta^{(\kappa-1)}, d_S^{(\kappa)}\}, \sigma_{-h}, \theta)|\theta_h\right] \\
&\vdots \\
&> \mathbb{E}_{\theta_{-h}}\left[\pi_h(\{d_\theta^{(1)}, \ldots, d_\theta^{(m)}, d_S^{(m+1)}, \ldots, d_S^{(\kappa-1)}, d_S^{(\kappa)}\}, \sigma_{-h}, \theta)|\theta_h\right] \\
&> \mathbb{E}_{\theta_{-h}}\left[\pi_h(\{d_\theta^{(1)}, \ldots, d_\theta^{(m-1)}, d_S^{(m)}, \ldots, d_S^{(\kappa-1)}, d_S^{(\kappa)}\}, \sigma_{-h}, \theta)|\theta_h\right] \\
&= \mathbb{E}_{\theta_{-h}}\left[\pi_h(S, \sigma_{-h}, \theta)|\theta_h\right]
\end{aligned}$$

Since S was arbitrary, it follows that interviewing $\{d_\theta^{(k)}\}_{k=1}^{\kappa}$ is a dominant strategy for h. Since h was arbitrary, the result follows. $\square$

B.3 Proof of Lemma 4

Before we proceed with the proof, we prove the following lemma, which will be useful for several other results in this paper. With a slight abuse of notation, we denote by $\lambda_{\theta,\sigma}$ a matching stemming from the type profile θ and strategy profile σ.

Lemma 7. *Fix $h \in H$ with received signals $D_h^S \subset D \cup \emptyset$. Suppose $H \setminus \{h\}$ use anonymous strategies, hospitals have best-in-block beliefs, and doctors use symmetric best-in-block strategies. Let $S \subset D$ such that $|S| < \kappa$ be arbitrary. Let $d \notin S$ be arbitrary. Then,*

$$\mathbb{E}_{\theta_{-h}}\left[\pi_h(S \cup \{d\}, \sigma_{-h}, \theta)|\theta_h, D_h^S\right] - \mathbb{E}_{\theta_{-h}}\left[\pi_h(S, \sigma_{-h}, \theta)|\theta_h, D_h^S\right] =$$
$$\mathbb{E}_{\theta_{-h}}\left[\mathbb{P}\left(\lambda_{\theta,S\cup\{d\},\sigma_{-h}}(h) = d|v_{hd} > v_{h\lambda_{\theta,S,\sigma_{-h}}(h)}\right)\mathbb{E}\left[\mathbb{1}\{v_{hd} > v_{h\lambda_{\theta,S,\sigma_{-h}}(h)}\}(v_{hd} - v_{h\lambda_{\theta,S,\sigma_{-h}}(h)})\right]|\theta_h, D_h^S\right]$$

Proof. Define $\overline{\Theta}$ to be the set of agent types consistent with h's beliefs, i.e.

$$\overline{\Theta} \equiv \left\{\theta^* \in \Theta : \theta_h = \theta_h^* \text{ and } h = \operatorname*{arg\,max}_{h' \in H_b} u_{\theta^*_{dh}}, \forall d \in D_h^S\right\}$$

Observe that for any realization of types $\theta \in \overline{\Theta}$, the marginal benefit to h's payoff from matching with $\{d\}$ when interviewing $S \cup \{d\}$ versus the payoff to h from matching with $\lambda_S(h) \equiv \lambda_{\theta,S,\sigma_{-h}}(h)$ does not depend on other hospitals' and doctors' preferences, only h's own preferences, yielding

$$\mathbb{E}\left[\mathbb{1}\{v_{hd} > v_{h\lambda_S(h)}\}(v_{hd} > v_{h\lambda_S(h)})|\lambda_{S\cup\{d\}}(h) = d, \lambda_S(h), v_{h\lambda_S(h)}, d \in D_h^S\right] =$$
$$\mathbb{E}\left[\mathbb{1}\{v_{hd} > v_{h\lambda_S(h)}\}(v_{hd} > v_{h\lambda_S(h)})|v_{h\lambda_S(h)}, d \in D_h^S\right]$$

Therefore,

$$\mathbb{E}_{\theta_{-h}}\left[\pi_h(S \cup \{d\}, \sigma_{-h}, \theta)|\theta_h, D_h^S\right] - \mathbb{E}_{\theta_{-h}}\left[\pi_h(S, \sigma_{-h}, \theta)|\theta_h, D_h^S\right] =$$
$$\mathbb{E}_{\theta_{-h}}\left[\mathbb{E}\left[\mathbb{1}\{\lambda_{\theta,S\cup\{d\},\sigma_{-h}}(h) = d\}(v_{hd} - v_{h\lambda_{\theta,D,\sigma_{-h}}(h)})\right]|\theta_h, D_h^S\right] =$$
$$\mathbb{E}_{\theta_{-h}}\left[\mathbb{E}\left[\mathbb{1}\{\lambda_{\theta,S\cup\{d\},\sigma_{-h}}(h) = d\}(v_{hd} - v_{h\lambda_{\theta,D,\sigma_{-h}}(h)})|v_{h\lambda_{\theta,D,\sigma_{-h}}(h)}\right]|\theta_h, D_h^S\right] =$$
$$\mathbb{E}_{\theta_{-h}}\left[\mathbb{P}\left(\lambda_{\theta,S\cup\{d\},\sigma_{-h}}(h) = d|v_{hd} > v_{h\lambda_{\theta,S,\sigma_{-h}}(h)}\right)\mathbb{E}\left[\mathbb{1}\{v_{hd} > v_{h\lambda_{\theta,S,\sigma_{-h}}(h)}\}(v_{hd} - v_{h\lambda_{\theta,S,\sigma_{-h}}(h)})\right]|\theta_h, D_h^S\right].$$

□

We are ready to prove Lemma 4.

Proof. Fix a hospital h from tier $b \in \{1, \ldots, B\}$ with realized pre-interview information θ_h and received signals D_h^S. Let $S \subset D$ with $|S| < \kappa$ and $|D_h^S \setminus S| \geq 2$ be arbitrary. Let R^0 be a realized rank-ordered list of hospital h for the set of interviewees S. Let $d, d' \in D_h^S \setminus S$ with $\theta_{hd} < \theta_{hd'}$ be arbitrary. Construct R and R' from R^0 by keeping the relative rank of doctors in S unchanged but also ranking d and d', respectively, in the same position.

Since hospitals use best-in-block beliefs, h believes both d and d' will rank h the highest among hospitals in block b in the final matching stage, conditional on being offered an interview. Define $\overline{\Theta}$ to be the set of agent types consistent with h's beliefs, i.e.

$$\overline{\Theta} \equiv \left\{\theta^* \in \Theta : \theta_h = \theta_h^* \text{ and } h = \arg\max_{h' \in H_b} u_{\theta^*_{dh}}, \forall d \in D_h^S\right\}$$

Denote a permutation which only changes d and d's rank in θ_h by ρ. We construct a profile $\theta' \in \Theta$ as follows: (i) h's preferences are unchanged, i.e. $\theta'_h = \theta_h$, (ii) d and d' are swapped in the pre-interview ranking of the rest of the hospitals, i.e. for any $h' \in H \setminus \{h\}$, $\theta'_{h'} = \rho(\theta_{h'})$, (iii) the types of d and d' are exchanged, i.e. $\theta'_d = \theta_{d'}$ and $\theta'_{d'} = \theta_d$, and (iv) the types of the rest of the agents are unchanged. It is straightforward to check that $\theta' \in \overline{\Theta}$.

By anonymity of strategies of the rest of the hospitals, we have for any $h' \in H \setminus \{h\}$ and their signals $D_{h'}^S \subset D \cup \varnothing$

$$\sigma_{h'}(\rho(\theta_{h'}), \rho(D_{h'}^S)) = \rho(\sigma_{h'}(\theta_{h'}, D_{h'}^S))$$

Recall that d and d' are both signaling to h under both θ and θ'. Hence, they do not signal to $H\setminus\{h\}$, which immediately implies $\rho(D_{h'}^S) = D_{h'}^S$. Further, by construction, $\theta'_{h'} = \rho(\theta_{h'})$. It follows that

$$\sigma_{h'}(\theta_{h'}, D_{h'}^S) = \rho(\sigma_{h'}(\theta_{h'}, D_{h'}^S))$$

Therefore, the probability that h' interviews d under θ equals the probability that it interviews d' under θ'. Moreover, the function mapping θ to θ' presented above is a bijection. Since θ and θ' are equally likely ex-ante, the probability that h matches with d under θ is equal to the probability that h matches with d' under θ'.

$$\mathbb{P}[\lambda_{\theta,\sigma}(h) = d | d \in D_h^S, R, R'] = \mathbb{P}[\lambda_{\theta',\sigma}(h) = d' | d' \in D_h^S, R, R'']$$

This holds for any rank-ordered list R that ranks a subset of doctors in S.

By first-order stochastic dominance, since $\mathbb{1}\{x-y\}(x-y)$ is increasing in x for a fixed y, and by Lemma 7, we obtain

$$\begin{aligned}
&\mathbb{E}_{\theta_{-h}}\left[\pi_h(S \cup \{d\}, \sigma_{-h}, \theta) | \theta_h, D_h^S\right] - \mathbb{E}_{\theta_{-h}}\left[\pi_h(S, \sigma_{-h}, \theta) | \theta_h, D_h^S\right] = \\
&\mathbb{E}_{\theta_{-h}}\left[\mathbb{P}\left(\lambda_{\theta, S\cup\{d\}, \sigma_{-h}}(h) = d | v_{hd} > v_{h\lambda_{\theta,S,\sigma_{-h}}(h)}\right) \mathbb{E}\left[\mathbb{1}\{v_{hd} > v_{h\lambda_{\theta,S,\sigma_{-h}}(h)}\}(v_{hd} - v_{h\lambda_{\theta,S,\sigma_{-h}}(h)})\right] | \theta_h, D_h^S\right] > \\
&\mathbb{E}_{\theta_{-h}}\left[\mathbb{P}\left(\lambda_{\theta, S\cup\{d'\}, \sigma_{-h}}(h) = d' | v_{hd'} > v_{h\lambda_{\theta,S,\sigma_{-h}}(h)}\right) \mathbb{E}\left[\mathbb{1}\{v_{hd'} > v_{h\lambda_{\theta,S,\sigma_{-h}}(h)}\}(v_{hd'} - v_{h\lambda_{\theta,S,\sigma_{-h}}(h)})\right] | \theta_h, D_h^S\right] = \\
&\qquad \mathbb{E}_{\theta_{-h}}\left[\pi_h(S \cup \{d'\}, \sigma_{-h}, \theta) | \theta_h, D_h^S\right] - \mathbb{E}_{\theta_{-h}}\left[\pi_h(S, \sigma_{-h}, \theta) | \theta_h, D_h^S\right],
\end{aligned}$$

as required. Proof for the case (ii) is analogous and is omitted. □

B.4 Proof of Lemma 5

Proof. Fix $h \in H$ from tier b with realized type θ_h and $S \subset D$. For simplicity of exposition, we focus only on the case when D_h^S and $\widetilde{D}_h^S$ differ only by one doctor, i.e. $\exists d', \widetilde{d'} \in D$ such that $d' = D_h^S \setminus \widetilde{D}_h^S$ and $\widetilde{d'} \in \widetilde{D}_h^S \setminus D_h^S$. Define the following sets of type profiles consistent with h receiving signals from D_h^S and $\widetilde{D}_h^S$, respectively:

$$\overline{\Theta} = \left\{ \theta^* \in \Theta : \theta_h = \theta_h^* \text{ and } h = \underset{h' \in H_b}{\arg\max}\, u_{\theta^*_{dh}}, \forall d \in D_h^S \right\}$$

$$\overline{\Theta}' = \left\{ \theta^* \in \Theta : \theta_h = \theta_h^* \text{ and } h = \underset{h' \in H_b}{\arg\max}\, u_{\theta^*_{dh}}, \forall d \in \widetilde{D}_h^S \right\}$$

and consider the following function $\phi : \overline{\Theta} \to \overline{\Theta}'$ between them, constructed analogously to the mapping in the proof of Lemma 4. Let ρ be a permutation of the set of doctor indices only exchanging the respective positions of doctors d' and $\widetilde{d'}$. For any $\theta \in \overline{\Theta}$, we construct $\phi(\theta)$ as follows: (i) h's preferences are unchanged, i.e. $\phi(\theta)_h = \theta_h$, (ii) d' and $\widetilde{d'}$ are swapped in the pre-interview ranking of the rest of the hospitals, i.e. for any $h' \in H \setminus \{h\}$, $\phi(\theta)_{h'} = \rho(\theta_{h'})$, (iii) the types of d' and $\widetilde{d'}$ are exchanged, i.e. $\phi(\theta)_{d'} = \theta_{\widetilde{d'}}$ and $\phi(\theta)_{\widetilde{d'}} = \theta_{d'}$, and (iv) the types of the rest of the agents are unchanged.

The function is well-defined. Indeed, since pre-interview information of h is unchanged, doctors d' and $\widetilde{d'}$ exchange their preferences with each other, and the preferences of the rest of the doctors are unchanged by construction, implying they still signal to h. Hence, we must have that $\phi(\theta) \in \overline{\Theta}'$ for any $\theta \in \overline{\Theta}$. Moreover, observe that by construction the cardinality of the two sets is the same. Hence, ϕ is a bijection.

By anonymity of strategies of the rest of the hospitals, we have for any $h' \in H \setminus \{h\}$ and their signals $D_{h'}^S \subset D \cup \varnothing$

$$\sigma_{h'}(\rho(\theta_{h'}), \rho(D_{h'}^S)) = \rho(\sigma_{h'}(\theta_{h'}, D_{h'}^S))$$

Hence, the probability that h' interviews d' under θ equals the probability that h' interviews $\widetilde{d'}$ under $\phi(\theta)$. Further, by construction, $\forall d'' \in D \setminus \{d', \widetilde{d'}\}, \theta_{h'd''} = \phi(\theta)_{h'd''}$. Therefore, the probability that d' occupies the k-th position in h''s post-interview ranking under θ is the same as the probability $\widetilde{d'}$ occupies the k-th position of h''s post-interview ranking under $\phi(\theta)$, for any k.

Therefore, since ϕ is a bijection and $D_h^S \cap S = \widetilde{D}_h^S \cap S$, the probability distributions over the induced final matchings is the same under D_h^S and $\widetilde{D}_h^S$. Averaging over agent types, the result follows. $\square$

B.5 Proof of Proposition 2

First, we prove the following lemma. Before we do so, we introduce the following notation. Fix h and suppose $\psi : D_1 \to D_2$ for some $D_1, D_2 \subseteq D$ is an injective function such that $\forall d \in$

$D, \theta_{h\psi(d)} \leq \theta_{hd}$. Denote by $(d_{D_1}^{(1)}, \ldots, d_{D_1}^{(|D_1|)})$ the elements of D_1 ordered in a descending order of h's pre-interview information θ_h, i.e.

$$\theta_{hd_{D_1}^{(1)}} < \cdots < \theta_{hd_{D_1}^{(|D_1|)}}$$

since pre-interview ranking of each hospital is assumed to be strict. Denote by Ψ the set of such functions ψ that preserve this ordering in the image, i.e.

$$\Psi \equiv \left\{ \psi : D_1 \to D_2 | \psi \text{ is injective, } \forall d \in D, \theta_{h\psi(d)} \leq \theta_{hd} \text{ and } \theta_{h\psi(d_{D_1}^{(1)})} < \cdots < \theta_{h\psi(d_{D_1}^{(|D_1|)})} \right\}$$

We define the minimal element of Ψ, ψ_m, as follows by taking the minimum element pointwise, i.e.

$$\forall d \in D_1, \psi_m(d) \equiv \min_{\psi \in \Psi} \psi(d)$$

Lemma 8. *Suppose doctors use symmetric best-in-block strategies and hospitals have best-in-block beliefs. Fix $h \in H$ and suppose σ_{-h} is a profile of anonymous strategies. Consider two sets of received signals $D_h^S, \widetilde{D}_h^S \subset D$ of the same cardinality $|D_h^S| = |\widetilde{D}_h^S|$. Suppose that under D_h^S it is optimal for h to interview $I_h = I_h^S \cup I_h^{NS}$ such that $I_h^S \subseteq D_h^S$, $I_h^{NS} \subseteq D \setminus D_h^S$, and $|I_h| = \kappa$. Further, suppose there is an injective function $\psi : I_h^S \to \widetilde{D}_h^S$ such that $\forall d \in I_h^S, \theta_{h\psi(d)} \leq \theta_{hd}$ and consider ψ_m to be minimal such function as defined above. Then it is optimal to interview $\psi_m(I_h^S)$ if the set of received signals is $\widetilde{D}_h^S$.*

Proof. We have shown in Lemma 5 that whenever two sets of signals of the same cardinality contain the same subset of doctors, the payoff from interviewing this set does not depend on the identity of the signaling doctors. Lemma 4 says that fixing other interviewees and signals, interviewing a doctor with a strictly more favorable pre-interview information yields a strictly higher payoff. Combining these two results, we have that whenever $d' \in D_h^S$ and $d'' \in \widetilde{D}_h^S$ such that $\theta_{hd''} < \theta_{hd'}$, we must have for any $S \subseteq D$,

$$\mathbb{E}_{\theta_{-h}} \left[\pi_h(S \cup \{d'\}, \sigma_{-h}, \theta) | \theta_h, D_h^S \right] < \mathbb{E}_{\theta_{-h}} \left[\pi_h(S \cup \{d''\}, \sigma_{-h}, \theta) | \theta_h, \widetilde{D}_h^S \right]$$

Sequentially applying this result, we obtain:

$$\mathbb{E}_{\theta_{-h}} \left[\pi_h(I_h^S, \sigma_{-h}, \theta) | \theta_h, D_h^S \right] \leq \mathbb{E}_{\theta_{-h}} \left[\pi_h(\psi_m(I_h^S), \sigma_{-h}, \theta) | \theta_h, \widetilde{D}_h^S \right]$$

If $I_h^S = \psi_m(I_h^S)$, the above is an equality, and the optimal set of interviewees under $\widetilde{D}_h^S$ coincides with I_h. The remaining case is $I_h^S \neq \psi_m(I_h^S)$. Hence, the inequality must be strict. Using lemma 4, it is enough to show that the worst doctor in $\psi_m(I_h^S)$ according to θ_h must be interviewed under h's optimal strategy when the set of received signals is I_h^S. This follows because the number of interviews conducted by each hospital when best-responding is always κ, $|I_h^S| = |\psi_m(I_h^S)|$, the fact that ϕ_m is minimal mapping satisfying the listed criteria, and I_h^S was interviewed when the set of received signals is D_h^S. $\square$

We are ready to prove Proposition 2.

Proof. Fix hospital h and consider two sets of signals D_h^S and $\widetilde{D}_h^S$ such that $|D_h^S| = |\widetilde{D}_h^S|$. Suppose first that they differ only in one doctor, that is $\exists d, d' \in D$ such that $D_h^S \setminus \widetilde{D}_h^S = \{d\}$ and $\widetilde{D}_h^S \setminus D_h^S = \{d'\}$. Further, suppose that $\theta_{hd'} < \theta_{hd}$. By Lemma 8, if d is interviewed when the signals are D_h^S, d' will be interviewed when the signals are $\widetilde{D}_h^S$. It is straightforward to extend the result to the case when the two sets differ by more than one signaling doctor by repeatedly using Lemma 8.

Together with Lemma 5, these results imply that when the rest of the hospitals use anonymous strategies, h's optimal strategy is a cutoff strategy. $\square$

B.6 Proof of Proposition 3

Proof. Consider $h \in H_b \in \{1, \ldots, B\}$. First, we assume that $\sigma'_{h'}$ differs from $\sigma_{h'}$ only for profile $\widetilde{\theta}_{h'}$ and received signals $\widetilde{D}_h^S$ such that

$$\sigma_{h'}(\widetilde{\theta}_{h'}, \widetilde{D}_h^S) = \alpha I_1 + (1-\alpha) I_2 \tag{B.1}$$

$$\sigma'_{h'}(\widetilde{\theta}_{h'}, \widetilde{D}_h^S) = \alpha' I_1 + (1-\alpha') I_2 \tag{B.2}$$

$I_1 = I_1^S \cup I_1^{NS}$, $I_2 = I_2^S \cup I_2^{NS}$, $|I_1| = |I_2| = \kappa$ and $|I_1^S \setminus I_2^S| = 1, I_2^S \subset I_2^S$, that is, both strategies mix over the same set of interviewees, where the two sets of interviewees in the support differ by one doctor, where I_1 contains one more signaling doctor compared to I_2. We assume $\alpha' > \alpha$, since $\sigma'_{h'}$ has greater cutoffs.

Observe that this implies that $\sigma'_{h'}$ is not a cutoff strategy because a cutoff strategy requires that the hospital uses the same behavior for any profile for a given number of received signals. Nonetheless, we proceed proving the result for the stated assumption and note that the full result will follow by an iterated application of this claim.

Fix arbitrary pre-interview information of hospital h, $\theta_h^* \in \Theta_h$ and some signals $D_h^S \subset D \cup \varnothing$. We want to show that h's payoff from interviewing any $d \in D_h^S$ weakly increases and the payoff from any $d' \notin D_h^S$ weakly decreases whenever hospital h' uses $\sigma'_{h'}$ compared to $\sigma_{h'}$. That is, for any $S \subset D, |S| < \kappa$

$$\mathbb{E}_{\theta_{-h}}\left[\pi_h(S \cup \{d\}, \sigma_{-h}, \theta) | \theta_h = \theta_h^*, D_h^S\right] \leq \mathbb{E}_{\theta_{-h}}\left[\pi_h(S \cup \{d\}, \sigma'_{-h}, \theta) | \theta_h = \theta_h^*, D_h^S\right] \tag{B.3}$$

and

$$\mathbb{E}_{\theta_{-h}}\left[\pi_h(S \cup \{d'\}, \sigma_{-h}, \theta) | \theta_h = \theta_h^*, D_h^S\right] \geq \mathbb{E}_{\theta_{-h}}\left[\pi_h(S \cup \{d'\}, \sigma'_{-h}, \theta) | \theta_h = \theta_h^*, D_h^S\right] \tag{B.4}$$

We first prove claim B.4. To this end, we show

$$\mathbb{E}_{\theta_{-h}}\left[\pi_h(S\cup\{d'\},\sigma_{-h},\theta)|\theta_h=\theta_h^*,D_h^S\right]-\mathbb{E}_{\theta_{-h}}\left[\pi_h(S,\sigma_{-h},\theta)|\theta_h=\theta_h^*,D_h^S\right]$$
$$\geq\mathbb{E}_{\theta_{-h}}\left[\pi_h(S\cup\{d'\},\sigma'_{-h},\theta)|\theta_h=\theta_h^*,D_h^S\right]-\mathbb{E}_{\theta_{-h}}\left[\pi_h(S,\sigma'_{-h},\theta)|\theta_h=\theta_h^*,D_h^S\right]$$

Using Lemma 7, this is equivalent to

$$\begin{aligned}&\mathbb{E}_{\theta_{-h}}\Big[\mathbb{P}\left(\lambda_{\theta,S\cup\{d'\},\sigma_{-h}}(h)=d'|v_{hd'}>v_{h\lambda_{\theta,S\cup\{d'\},\sigma_{-h}}(h)},\theta_h=\theta_h^*,D_h^S\right)\\&\qquad\mathbb{E}\left[\mathbb{1}\{v_{hd'}>v_{h\lambda_{\theta,S\cup\{d'\},\sigma_{-h}}(h)}\}(v_{hd'}-v_{h\lambda_{\theta,S\cup\{d'\},\sigma_{-h}}(h)})|\theta_h=\theta_h^*\right]\Big]\geq\\&\mathbb{E}_{\theta_{-h}}\Big[\mathbb{P}\left(\lambda_{\theta,S\cup\{d'\},\sigma'_{-h}}(h)=d'|v_{hd'}>v_{h\lambda_{\theta,S\cup\{d'\},\sigma'_{-h}}(h)},\theta_h=\theta_h^*,D_h^S\right)\\&\qquad\mathbb{E}\left[\mathbb{1}\{v_{hd'}>v_{h\lambda_{\theta,S\cup\{d'\},\sigma'_{-h}}(h)}\}(v_{hd'}-v_{h\lambda_{\theta,S\cup\{d'\},\sigma'_{-h}}(h)})|\theta_h=\theta_h^*\right]\Big]\end{aligned}\tag{B.5}$$

We define the following sets

$$\Theta_+\equiv\{\theta\in\Theta|\theta_h=\theta_h^*,D_h^S,B.5\text{ has sign }<\}$$

$$\Theta_-\equiv\{\theta\in\Theta|\theta_h=\theta_h^*,D_h^S,B.5\text{ has sign }>\}$$

If $\Theta_+=\emptyset$, we are done. Suppose $\Theta_+\neq\emptyset$. Select an arbitrary $\theta\in\Theta_+$. Since the change of strategy of hospital h' changes the match payoffs, it must increase the match probability, implying that $d'=I_2^{NS}\setminus I_1^{NS}$. For h' choice to affect h, h cannot be from a block ranked higher than h', i.e. $h'\in H_{b'}$ such that $b'\leq b$. Further, observe that under θ, doctor d' has signaled to neither h nor h'.

We construct the following function $\psi:\Theta\to\Theta$ as follows. Denote by ρ a permutation only of d' and $d''\equiv I_1^S\setminus I_2^S$. For any $\theta\in\Theta$, we define $\psi(\theta)$ as follows: (i) $\psi(\theta_h)=\theta_h^*$, (ii) $\forall h''\in H\setminus\{h\},\psi(\theta)_{h''}=\rho(\theta_{h''})$, (iii) $\psi(\theta)_{d'}=\theta_{d''}$ and $\psi(\theta)_{d''}=\theta_{d'}$, (iv) $\forall d^0\in D\setminus\{d',d''\},\psi(\theta)_{d^0}=\theta_{d^0}$. That is, the type of h and any doctor other than d' and d'' remains unchanged. The types of doctors d' and d'' are exchanged, and any hospital other than h permutes these doctors in its pre-interview ranking. Observe that under θ and $\psi(\theta)$, hospital h receives the same signals and has the same pre-interview information.

By anonymity of strategies,

$$\begin{aligned}\sigma_{h'}(\psi(\theta)_{h'},\rho(D_{h'}^S))&=\sigma_{h'}(\rho(\theta_{h'}),\rho(D_{h'}^S))\\&=\alpha\rho(I_1)+(1-\alpha)\rho(I_2)\\&=\alpha I_2+(1-\alpha)I_1\end{aligned}$$

Similarly,

$$\sigma'_{h'}(\psi(\theta)_{h'},\rho(D_{h'}^S))=\alpha'I_2+(1-\alpha')I_1$$

Next, we show that $\psi(\theta) \in \Theta_-$. To this end, note that, since $\theta \in \Theta_+$, under $\psi(\theta)$, h' interviews d' more frequently when using $\sigma_{h'}$ than $\sigma'_{h'}$. Further, d' prefers h' to h under $\psi(\theta)$. Indeed, we have already noted that $b' \leq b$. If $b' < b$, the result is immediate. If $b' = b$, since d' signals to h' under $\psi(\theta)$, h' must be its most favorite hospital under $\psi(\theta)$. Therefore, the probability that h matches with d' is smaller when h' uses $\sigma'_{h'}$ than when it uses $\sigma_{h'}$. Hence, $\theta \in \Theta_-$.

Finally, observe that we get different profiles in Θ_- for each profile in Θ_+. Therefore, ψ is injective, from which it follows that $|\Theta_-| \geq |\Theta_+|$. Since all type profiles are equally likely, inequality B.4 follows.

We proceed to proving statement B.3. The proof proceeds similarly to the proof of B.4 presented above. For brevity, we provide only a sketch of the argument.

If $h \in H_b$ receives a signal from doctor d, since hospitals have best-in-block beliefs, h believes it is d's most preferred hospital in block b. In other words, d prefers h to any other hospital in block b and blocks ranked lower than b. Therefore, for the strategy of h' to affect h, it must be $b' < b$.

Hospital h' from block b' affects the match probability of the pair (h, d) only if it interviews d. However, if hospital h' responds more to signals, it interviews d with a lower probability. It follows that

$$\begin{aligned}
\mathbb{E}_{\theta_{-h}} \Big[\mathbb{P} \left(\lambda_{\theta, S\cup\{d\}, \sigma_{-h}}(h) = d | v_{hd} > v_{h\lambda_{\theta,S\cup\{d\},\sigma_{-h}}(h)}, \theta_h = \theta_h^*, D_h^S \right) \\
\mathbb{E} \left[\mathbb{1}\{ v_{hd} > v_{h\lambda_{\theta,S\cup\{d\},\sigma_{-h}}(h)} \} (v_{hd} - v_{h\lambda_{\theta,S\cup\{d\},\sigma_{-h}}(h)}) | \theta_h = \theta_h^* \right] \Big] \leq \\
\mathbb{E}_{\theta_{-h}} \Big[\mathbb{P} \left(\lambda_{\theta, S\cup\{d\}, \sigma'_{-h}}(h) = d | v_{hd} > v_{h\lambda_{\theta,S\cup\{d\},\sigma'_{-h}}(h)}, \theta_h = \theta_h^*, D_h^S \right) \\
\mathbb{E} \left[\mathbb{1}\{ v_{hd} > v_{h\lambda_{\theta,S\cup\{d'\},\sigma'_{-h}}(h)} \} (v_{hd} - v_{h\lambda_{\theta,S\cup\{d\},\sigma'_{-h}}(h)}) | \theta_h = \theta_h^* \right] \Big], \quad \text{(B.6)}
\end{aligned}$$

which is equivalent to inequality B.3.

It remains to show that statements B.4 and B.3 hold for any strategies $\sigma_{h'}$ and $\sigma'_{h'}$ where $\sigma'_{h'} \geq \sigma_{h'}$. First, we can extend the analysis from the case presented in B.1 and B.2 to the general case by noting that for any I_2 fixed, we can replace I_1 by a set of doctors I of cardinality κ such that the subset of doctors that signaled to h' among I_2 is a strict subset of the corresponding set of signaling doctors among I. The full general case of the proposition is obtained by an iterated application of the statement proved for the simplified case presented above.

Finally, it follows that when h' responds more to signals for some pre-interview information and received signals, the best response of h dictates to also increase its cutoffs. The result follows.

□

B.7 Proof of Lemma 6

Before we proceed with the proof, we show the following useful result.

Fix hospital $h \in H$ such that h ranks doctor d on the k-th position in h's rank-ordered list R_h and h does not rank doctor d' (finds her unacceptable). We say *h creates R_h' from R_h by exchanging d and d'* if under R_h': (i) h finds d unacceptable, (ii) ranks d' in the k-th position, and (iii) keeps the rest of the rank-ordered list unchanged.

Denote the set of all feasible profiles of rank-ordered lists $\mathcal{R}$. Denote the set of matchings for a market $\mathcal{M}$ by $\Lambda(\mathcal{M})$. With the rest of the primitives fixed, we will denote the set of matchings for a particular profile of rank-ordered lists R by $\Lambda(R)$. The HPDA algorithm sends R to the hospital-optimal matching in $\Lambda(R)$; we will denote this element by λ_R. We denote agent $a \in H \cup D$ being unmatched in matching λ by $\lambda(a) = \emptyset$.

We define the number of matches in λ_R, $n(\lambda_R)$ as the number of matched doctors (equivalently, the number of matched hospitals) in λ_R. More formally,

$$n(\lambda_R) = \sum_{h \in H} \mathbb{1}\{\lambda_R(h) \neq \emptyset\} = \sum_{d \in D} \mathbb{1}\{\lambda_R(d) \neq \emptyset\}$$

We obtain the following result.

Lemma 9. *Fix $h \in H$, $d, d' \in D$ and let $R = (R_h, R_{-h})$ and $R' = (R_h', R_{-h})$ be profiles of rank-ordered lists such that h ranks d in R_h and h creates R_h' from R_h by exchanging d and d'. Then:*

1. *$|n(\lambda_R) - n(\lambda_{R'})| \leq 1$*
2. *If $n(\lambda_R) - n(\lambda_{R'}) = 1$, $n(\lambda_R) = n(\lambda_{R_{-h}}) + 1$ and $n(\lambda_{R'}) = n(\lambda_{R_{-h}})$*
3. *If $n(\lambda_R) - n(\lambda_{R'}) = -1$, $n(\lambda_R) = n(\lambda_{R_{-h}})$ and $n(\lambda_{R'}) = n(\lambda_{R_{-h}}) + 1$*

Proof. Blum et al. (1997) extend the HPDA algorithm to consider an arbitrary input matching and show this algorithm converges. Moreover, a direct corollary of their Theorem 4.4. is that the following procedure produces λ_R for any $R \in \mathcal{R}$ and fixed $h \in H$:

1. Run the HPDA algorithm on a reduced profile R_{-h} after removing h.
2. Introduce h back into the market and run the HPDA algorithm with $\lambda_{R_{-h}}$ as the input matching.

Observe that Step 2. of this algorithm proceeds as follows. Hospital h offers its seat to its most preferred doctor $d^{(1)}$ according to R_h. If $d^{(1)}$ prefers $\lambda_{R_{-h}}(d^{(1)})$ to h, h's offer is rejected and it proceeds to $d^{(2)}$ according to R_h, etc., until either every doctor h finds acceptable for its position rejected its offer, or there is some acceptable doctor $d^{(k)}$ who accepts h's offer. In the latter case, $d^{(k)}$ rejects its current offer $h' \equiv \lambda_{R_{-h}}(d^{(k)})$ in favor of h. This process is hence repeated with h'. Hence, in the language of Blum et al. (1997), Step 2. generates a *vacancy chain*: h offers its position to d who rejects the offer of h' who in turn offers its position to d'', ... Such a chain ends either with a hospital with all of its offers to acceptable doctors rejected, or a doctor who previously had no offer before h's "entry" into the market but now receives a position. Observe that in the former case, $n(\lambda_R) = n(\lambda_{R_{-h}})$, whereas in the latter case $n(\lambda_R) = n(\lambda_{R_{-h}}) + 1$. The result follows. $\square$

We are ready to prove Lemma 6.

Proof. Fix hospital h and cutoff strategies σ_h and σ'_h such that σ'_h has weakly greater cutoffs. Let $D_h^S \subset D \cup \varnothing$ be an arbitrary set of received signals and denote

$$\sigma_h(\theta_h, D_h^S) = I_h^S \cup I_h^{NS}, I_h^S \subseteq D_h^S \text{ and } I_h^{NS} \cap D_h^S = \emptyset$$

and

$$\sigma'_h(\theta_h, D_h^S) = I_h^{S\prime} \cup I_h^{NS\prime}, I_h^{S\prime} \subseteq D_h^S \text{ and } I_h^{NS\prime} \cap D_h^S = \emptyset$$

By anonymity of hospital strategies and the fact that h responds more to signals under σ'_h than under σ_h, we must have $I_h^S \subseteq I_h^{S\prime}$, since the set of received signals is the same.

We prove the lemma for the case when the cutoff strategies of hospital h differ by at most 1. Hence, or any pre-interview information θ_h under which the two set of interviewees differ, $\sigma_h(\theta_h, D_h^S)$ and $\sigma'_h(\theta_h, D_h^S)$ differ only in one doctor, i.e. $\exists d^S, d^{NS} \in D$ such that $\{d^S\} = \sigma'_h(\theta_h, D_h^S) \setminus \sigma_h(\theta_h, D_h^S)$ and $\{d^{NS}\} = \sigma_h(\theta_h, D_h^S) \setminus \sigma'_h(\theta_h, D_h^S)$. Note that by construction this is equivalent to $\{d^S\} = I_h^{S\prime} \setminus I_h^S$ and $\{d^{NS}\} = I_h^{NS} \setminus I_h^{NS\prime}$. The general case will follow by a repeated application of this result.

Define the following sets of types, Θ_+ and Θ_-, where Θ_+ is the set of types consistent with the set of received signals D_h^S by h and that strictly increase the expected number of matches and Θ_- is the set of types consistent with the set of received signals D_h^S by h and that strictly decrease the expected number of matches, given the preferences and realized types of all agents. More formally,

$$\Theta_+ \equiv \left\{\theta \in \Theta : h = \arg\max_{h' \in H} u_{\theta_{dh'}}, \forall d \in D_h^S \text{ and } m(\sigma'_h, \sigma_{-h}, \theta) > m(\sigma_h, \sigma_{-h}, \theta)\right\}$$
$$\Theta_- \equiv \left\{\theta \in \Theta : h = \arg\max_{h' \in H} u_{\theta_{dh'}}, \forall d \in D_h^S \text{ and } m(\sigma'_h, \sigma_{-h}, \theta) < m(\sigma_h, \sigma_{-h}, \theta)\right\}$$

The rest of the proof proceeds as follows. We construct an bijective function $\phi : \Theta \to \Theta$ such that for any $\theta \in \Theta_-$, $\phi(\theta) \in \Theta_+$ and

$$m\big(\sigma'_h, \sigma_{-h}, \phi(\theta)\big) - m\big(\sigma_h, \sigma_{-h}, \phi(\theta)\big) \geq m\big(\sigma_h, \sigma_{-h}, \theta\big) - m\big(\sigma'_h, \sigma_{-h}, \theta\big)$$

Together with independence of types and our distributional assumptions, this will be enough to prove the result.

We construct $\phi : \Theta \to \Theta$ as follows. Fix $\theta \in \Theta$. Denote by h' the hospital to which d^{NS}

signals to under θ. Further, denote by $\mathcal{I}^{\sigma}_{d^S}$ and $\mathcal{I}^{\sigma}_{d^{NS}}$ the set of interviewees of d^S and d^{NS} under the strategy profile σ, respectively. Then, $\phi(\theta)$ is constructed as follows:

1. For any hospital $h'' \in H \setminus \{h, h'\}$, exchange the positions of doctors d^S and d^{NS} in the pre-interview ranking of h''.
2. Exchange the relative rankings of $\mathcal{I}^{\sigma}_{d^S}$ and $\mathcal{I}^{\sigma}_{d^{NS}}$ in the rankings of doctors d^S and d^{NS}, respectively. That is, $\phi(\theta)_{d^S}$ is constructed from θ_{d^S} by keeping the rank of hospitals other than $\mathcal{I}^{\sigma}_{d^{NS}}$ fixed, and setting the sub-ranking over $\mathcal{I}^{\sigma}_{d^{NS}}$ to be the same as the corresponding sub-ranking in $\theta_{d^{NS}}$. $\phi(\theta)_{d^{NS}}$ is constructed analogously.
3. For h', set $\phi(\theta)_{h'}$ to θ_h and exchange the positions of d^S and d^{NS}.
4. Set to $\phi(\theta)_h$ to θ_h and rearrange all elements other than $\{d^S, d^{NS}\}$ to match the relative ranking of these elements in $\theta_{h'}$.
5. The rest of the doctors $d \in D\{d^S, d^{NS}\}$ exchange the positions of h and h' in their preference lists if these hospitals belong to the same block. The rest of their preferences remain unchanged.

Observe that this is a bijective function, as the inverse is given by ϕ itself, i.e. $\phi(\phi(\theta)) = \theta, \forall \theta \in \Theta$.

Recall that any type profile θ^* and strategy profile σ^* induce a probability distribution over interview assignments, which in turn induces a probability distribution over rank-ordered lists reported to the clearinghouse at the matching stage. For any θ^* and σ^*, denote this probability distribution by η_{θ^*,σ^*} and its support of rank-ordered lists by $\mathcal{R}(\theta^*, \sigma^*)$. For any profile or rank-ordered lists R, denote by $n(R)$ the number of matches obtained using HPDA. For any profile of strategies σ^* and type profile θ^*, we construct a function $\psi : \mathcal{R}(\theta^*, \sigma^*) \to \mathcal{R}(\phi(\theta^*), \sigma^*)$ by an analogous contruction to ϕ, which is well-defined and a bijection by the same argument. Further, observe that by construction of ϕ and ψ, for any $R \in \mathcal{R}(\theta^*, \sigma^*)$

$$\eta_{\theta^*,\sigma^*}(R) = \eta_{\phi(\theta^*),\sigma^*}(\psi(R))$$

Next, for any θ^*, consider the following function $\varphi_{\theta^*} : \mathcal{R}(\theta^*, \sigma) \to \mathcal{R}(\theta^*, (\sigma'_h, \sigma_{-h}))$, such that for any $R \in \mathcal{R}(\theta^*, \sigma)$, we fix the rank-ordered lists of all agents other than h, and replacing d^{NS} with d^S in h's rank-ordered list. It is straightforward to show this function is well-defined and a bijection. Next, we define the following sets, for any $\theta^* \in \Theta$,

$$\mathcal{R}_+(\theta^*) \equiv \big\{R \in \mathcal{R}(\theta^*, \sigma) : n(R) < n(\phi_{\theta^*}(R))\big\}$$
$$\mathcal{R}_-(\theta^*) \equiv \big\{R \in \mathcal{R}(\theta^*, \sigma) : n(R) > n(\phi_{\theta^*}(R))\big\}$$

Fix an arbitrary $\theta \in \Theta_-$. By Lemma 9, going from R_{-h} to $R \in \mathcal{R}_-(\theta^*)$ must generate a vacancy chain which ends with a doctor; denote this chain by $\mathcal{C}(R_{-h}, R)$. Further, going from

R_{-h} to $\varphi_{\theta^*}(R)$ must generate a vacancy chain which ends with a hospital; denote this chain by $\mathcal{C}(R_{-h}, \varphi_{\theta^*}(R))$. Moreover, for the number of matches to decrease, this vacancy chain must begin with h, d^S, and d^S's partner under R_{-h}; denote this hospital by h''. In summary, we must have $n(R) - n(\varphi_{\theta^*}(R)) = 1$.

Next, we show that for any $R \in \mathcal{R}_-(\theta), \psi(R) \in \mathcal{R}_+(\phi(\theta))$. To this end, we claim that going from $\psi(R)_{-h}$ to $\psi(R)$ generates the vacancy chain $\mathcal{C}(R_{-h}, \varphi_{\theta^*}(R))$, except possibly for the first two elements. Indeed, under $\psi(R)$, by construction of the rank-ordered lists, d'' must offer its seat to d^{NS} during the execution of the HPDA. If h matches with d^{NS} under R, h matches with d^{NS} under $\psi(R)$, as well, generating a vacancy chain beggining with h, d^{NS}, h'', and continuing in the same fashion as $\mathcal{C}(R_{-h}, \varphi_{\theta^*}(R))$ since the rest of the preferences of h'' remain unchanged. In particular, the vacancy chain ends with a hospital. If h does not match with d^{NS} under R, the same chain of events nonetheless follows, by construction of the corresponding preference profiles of d^S and d^{NS} and by the fact that d^S prefers h over h'' under R (hence, d^{NS} must prefer h to any hospital that offers her employment during the execution of the HPDA algorithm). Further, going from $\psi(R)_{-h}$ to $\varphi_{\theta^*}(\psi(R))$ generates the vacancy chain $\mathcal{C}(R_{-h}, R)$ possibly except for the first two elements follows by an analogous argument. In summary, we must have $n(\varphi_{\theta^*}(R)) - n(R) = 1$.

Therefore, since $R \in \mathcal{R}_-(\theta)$ was arbitrary, $\psi(R) \in \mathcal{R}_+(\phi(\theta))$. Since ψ is a bijection, we must have

$$|\mathcal{R}_+(\phi(\theta))| \geq |\mathcal{R}_-(\theta)|$$

Moreover, since for any θ^*, σ^* and R

$$\eta_{\theta^*,\sigma^*}(R) = \eta_{\phi(\theta^*),\sigma^*}(\psi(R))$$

we must have

$$\eta_{\phi(\theta),\sigma}\big(\mathcal{R}_+(\phi(\theta))\big) \geq \eta_{\theta,\sigma}\big(\mathcal{R}_-(\theta)\big)$$

Hence, since $\theta \in \Theta_-$, and hence

$$m(\sigma_h, \sigma_{-h}, \theta) > m(\sigma'_h, \sigma_{-h}, \theta)$$

we must have

$$m(\sigma'_h, \sigma_{-h}, \phi(\theta)) > m(\sigma'_h, \sigma_{-h}, \phi(\theta))$$

That is, $\phi(\theta) \in \Theta_+$. Moreover, it also follows

$$m(\sigma'_h, \sigma_{-h}, \phi(\theta)) - m(\sigma'_h, \sigma_{-h}, \phi(\theta)) \geq m(\sigma_h, \sigma_{-h}, \theta) - m(\sigma'_h, \sigma_{-h}, \theta)$$

The statement about the expected number of matches follows.

Observe that, by the above argument, the expected number of matches increases for each doctor. Using a similar construction, for any type profile and realization of uncertainty for which a doctor loses an offer from a hospital, there is another profile and realized rank-ordered lists in which she gains a match with a hospital she prefers at least as much as the hospital she lost the offer from under the initial type profile. Such a function is again injective. Therefore, the ex-ante payoff to the doctor must increase. □

B.8 Proof of Theorem 1

Proof. Denote the strategies in the unique game without signals by σ_H^0, and let (σ_H, σ_D) be a block-correlated symmetric equilibrium of the interview game with signals. Note that the outcome of the game with strategies (σ_H^0, σ_D) is equivalent to the unique equilibrium of the interview game with no signals. Results (1) and (2) with inequalities follow by a sequential application of Lemma 6.

Consider an analogous construction of sets Θ_+ and Θ_- as above, replacing σ_H in the definitions by σ_H^0, and considering σ_H' to be σ_H, the equilibrium profile of hospital strategies in the game with signals. Recall that ϕ is bijective and that $\phi(\phi(\theta)) = \theta, \forall \theta \in \Theta$.

By assumption, there are at least two hospitals, h and h', in block H_b, that respond to signals. Consider $\theta \in \Theta_+$ such that d^S has no interview under θ and d^{NS} has interviews from h and h', the hospital she signals to under (σ_H, σ_D). By construction, we must have $\theta \in \Theta_+$. Consider $\phi(\theta)$. Then both under (σ_H^0, σ_D) and (σ_H, σ_D), the interview assignment is equivalent to the corresponding interview assignment under θ. Hence, $\theta \notin \Theta_-$, from which it immediately follows that the restriction of ϕ to Θ_- is injective but not surjective. Therefore, $|\Theta_+| > |\Theta_-|$, proving statement (1). Statement (2) follows by a similar construction. □

B.9 Proof of Proposition 6

Proof. Fix $h \in H$ of type θ_h, σ_{-h}, σ_h and σ_h' cutoff strategies such that $\sigma_h' \geq \sigma_h$, and D_h^S be arbitrary as in the statement of the Lemma. First, we prove part (i). For simplicity, we focus on the case where h uses pure cutoff strategies. The extension to mixed cutoff strategies is straightforward.

Denote by $I_h \equiv \sigma_h(\theta, D_h^S)$ and $I_h' = \sigma_h'(\theta, D_h^S)$ the sets of interviewees of hospital h under the two considered strategies. Further, we define $I_h^S \equiv D_h^S \cap I_h$, $I_h^{S\prime} \equiv D_h^S \cap I_h'$, $I_h^{NS} \equiv I_h \setminus D_h^S$, and $I_h^{NS\prime} \equiv I_h' \setminus D_h^S$. For simplicity, we consider the case where I_h and I_h' differ only in one doctor: $I_h^{S\prime} \setminus I_h^S = \{d^S\}$ and $I_h^{NS} \setminus I_h^{NS\prime} = \{d^{NS}\}$. The general case will follow by a sequential application of the result for this special case. To simplify notation, denote $S \equiv I_h \cap I_h'$.

We have

$$\begin{aligned}
&\mathbb{E}_{\theta_{-h}}\left[\pi_h\big(S\cup\{d^S\},\sigma_{-h},\theta\big)|\theta_h,D_h^S\right]-\mathbb{E}_{\theta_{-h}}\left[\pi_h\big(S\cup\{d^{NS}\},\sigma_{-h},\theta\big)|\theta_h,D_h^S\right]\\
&\qquad=\left(\mathbb{E}_{\theta_{-h}}\left[\pi_h\big(S\cup\{d^S\},\sigma_{-h},\theta\big)|\theta_h,D_h^S\right]-\mathbb{E}_{\theta_{-h}}\left[\pi_h\big(S,\sigma_{-h},\theta\big)|\theta_h,D_h^S\right]\right)-\\
&\qquad\qquad\left(\mathbb{E}_{\theta_{-h}}\left[\pi_h\big(S\cup\{d^{NS}\},\sigma_{-h},\theta\big)|\theta_h,D_h^S\right]-\mathbb{E}_{\theta_{-h}}\left[\pi_h\big(S,\sigma_{-h},\theta\big)|\theta_h,D_h^S\right]\right)
\end{aligned}$$

By Lemma 7, this expression is equal to

$$\begin{aligned}
\mathbb{E}_{\theta_{-h}}\Big[&\mathbb{P}\left(\lambda_{\theta,S\cup\{d^S\},\sigma_{-h}}(h)=d^S|v_{hd^S}>v_{h\lambda_{\theta,S\cup\{d^S\},\sigma_{-h}}(h)},\theta_h,D_h^S\right)\\
&\qquad\mathbb{E}\left[\mathbb{1}\{v_{hd^S}>v_{h\lambda_{\theta,S\cup\{d^S\},\sigma_{-h}}(h)}\}(v_{hd^S}-v_{h\lambda_{\theta,S\cup\{d^S\},\sigma_{-h}}(h)})|\theta_h\right]\\
&-\mathbb{P}\left(\lambda_{\theta,S\cup\{d^{NS}\},\sigma_{-h}}(h)=d^{NS}|v_{hd^{NS}}>v_{h\lambda_{\theta,S\cup\{d^{NS}\},\sigma_{-h}}(h)},\theta_h,D_h^S\right)\\
&\qquad\mathbb{E}\left[\mathbb{1}\{v_{hd^{NS}}>v_{h\lambda_{\theta,S\cup\{d^{NS}\},\sigma_{-h}}(h)}\}(v_{hd^{NS}}-v_{h\lambda_{\theta,S\cup\{d^{NS}\},\sigma_{-h}}(h)})|\theta_h\right]\Big],
\end{aligned}\tag{B.7}$$

where we used the linearity of the expectation operator.

Observe that F_θ^+ is unchanged for any θ when we change p. Hence, for any two interviewed doctors d and d', conditional on being find acceptable, the probability of their relative ranking in the reported rank-ordered list of any hospital is unchanged. Therefore, conditional on h finding d^S and d^{NS} acceptable, for any rank-ordered list of $R_h^{S'}$ of a subset of doctors in $S'\subseteq S$, the probability of extended rank-ordered lists $R_h^{S'\cup d^S}$ and $R_h^{S'\cup d^{NS}}$ is the same regardless of the value of p. An analogous statement holds for the other agents: the probability that a particular rank-ordered list is realized conditional on a set of realized acceptable partners is the same. Therefore, we can write the objects

$$\mathbb{P}\left(\lambda_{\theta,S\cup\{d^{NS}\},\sigma_{-h}}(h)=d^{NS}|v_{hd^{NS}}>v_{h\lambda_{\theta,S\cup\{d^{NS}\},\sigma_{-h}}(h)},\theta_h,D_h^S\right)$$

and

$$\mathbb{P}\left(\lambda_{\theta,S\cup\{d^S\},\sigma_{-h}}(h)=d^S|v_{hd^S}>v_{h\lambda_{\theta,S\cup\{d^S\},\sigma_{-h}}(h)},\theta_h,D_h^S\right)$$

as polynomials in p and consider the values as p changes. Since the probability that h finds d^{NS} and d^S is equal to p and since

$$\mathbb{E}\left[\mathbb{1}\{v_{hd^{NS}}>v_{h\lambda_{\theta,S\cup\{d^{NS}\},\sigma_{-h}}(h)}\}(v_{hd^{NS}}-v_{h\lambda_{\theta,S\cup\{d^{NS}\},\sigma_{-h}}(h)})|\theta_h\right]$$

and

$$\mathbb{E}\left[\mathbb{1}\{v_{hd^S}>v_{h\lambda_{\theta,S\cup\{d^S\},\sigma_{-h}}(h)}\}(v_{hd^S}-v_{h\lambda_{\theta,S\cup\{d^S\},\sigma_{-h}}(h)})|\theta_h\right]$$

depend on p only insofar as the considered agents are found acceptable, it is enough to show that

$$\begin{aligned}\mathbb{P}^{\mathcal{M}}&\left(\lambda_{\theta,S\cup\{d^{NS}\},\sigma_{-h}}(h)=d^{NS}|v_{hd^{NS}}>v_{h\lambda_{\theta,S\cup\{d^{NS}\},\sigma_{-h}}(h)},\theta_h,D_h^S\right)\\ &\quad-\mathbb{P}^{\mathcal{M}'}\left(\lambda_{\theta,S\cup\{d^{NS}\},\sigma_{-h}}(h)=d^{NS}|v_{hd^{NS}}>v_{h\lambda_{\theta,S\cup\{d^{NS}\},\sigma_{-h}}(h)},\theta_h,D_h^S\right)\\ &\qquad\geq\mathbb{P}^{\mathcal{M}}\left(\lambda_{\theta,S\cup\{d^{S}\},\sigma_{-h}}(h)=d^{S}|v_{hd^{S}}>v_{h\lambda_{\theta,S\cup\{d^{S}\},\sigma_{-h}}(h)},\theta_h,D_h^S\right)\\ &\qquad\quad-\mathbb{P}^{\mathcal{M}'}\left(\lambda_{\theta,S\cup\{d^{S}\},\sigma_{-h}}(h)=d^{S}|v_{hd^{S}}>v_{h\lambda_{\theta,S\cup\{d^{S}\},\sigma_{-h}}(h)},\theta_h,D_h^S\right)\end{aligned}$$

This follows by the fact that for any p,

$$\mathbb{P}\left(\lambda_{\theta,S\cup\{d^{NS}\},\sigma_{-h}}(h)=d^{NS}|v_{hd^{NS}}>v_{h\lambda_{\theta,S\cup\{d^{NS}\},\sigma_{-h}}(h)},\theta_h,D_h^S\right)$$

is a polynomial of higher degree than

$$\mathbb{P}\left(\lambda_{\theta,S\cup\{d^{S}\},\sigma_{-h}}(h)=d^{S}|v_{hd^{S}}>v_{h\lambda_{\theta,S\cup\{d^{S}\},\sigma_{-h}}(h)},\theta_h,D_h^S\right)$$

since h is d^S's first choice but d^{NS} might be interviewed by other hospitals, including the hospital she signaled to. This completes the proof of statement (i).

We proceed to proving statement (ii). Consider the same set-up as in the proof for (i) above. Note that, by construction, d^S must be the lowest-ranked doctor in I_h^S and d^{NS} must be the lowest-ranked doctor in I_h^{NS}. Further, we must have $\theta_{hd^S}>\theta_{hd^{NS}}$. By first-order stochastic dominance, and since $||F_{\theta_{hd^{NS}}}-F_{\theta_{hd^S}}||_1\geq\gamma$, for any fixed θ,

$$\begin{aligned}\mathbb{E}\Big[&\mathbb{P}\left(\lambda_{\theta,S\cup\{d^{S}\},\sigma_{-h}}(h)=d^{S}|v_{hd^{S}}>v_{h\lambda_{\theta,S\cup\{d^{S}\},\sigma_{-h}}(h)},\theta_h,D_h^S\right)\\ &\qquad\mathbb{E}\left[\mathbb{1}\{v_{hd^S}>v_{h\lambda_{\theta,S\cup\{d^{S}\},\sigma_{-h}}(h)}\}(v_{hd^S}-v_{h\lambda_{\theta,S\cup\{d^{S}\},\sigma_{-h}}(h)})|\theta_h\right]\\ &\quad-\mathbb{P}\left(\lambda_{\theta,S\cup\{d^{NS}\},\sigma_{-h}}(h)=d^{NS}|v_{hd^{NS}}>v_{h\lambda_{\theta,S\cup\{d^{NS}\},\sigma_{-h}}(h)},\theta_h,D_h^S\right)\\ &\qquad\quad\mathbb{E}\left[\mathbb{1}\{v_{hd^{NS}}>v_{h\lambda_{\theta,S\cup\{d^{NS}\},\sigma_{-h}}(h)}\}(v_{hd^{NS}}-v_{h\lambda_{\theta,S\cup\{d^{NS}\},\sigma_{-h}}(h)})|\theta_h\right]\Big]\end{aligned}$$

increases γ increases. Averaging over θ_{-h}, the result follows.

□