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Standard Deviation Control Chart Based on Weighted Standard Deviation Method

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Abstract: This paper gives the method of control lines for the biased distribution based on the weighted standard deviation method which is used to construct the mean control chart for biased distribution of the unknown population, after control lines are determined, we give the estimation method for relevant parameters ,and then comparative the effectiveness of standard deviation control chart by numerical example which are based on the weighted standard deviation method and the traditional Shewhart control chart method .

Key words: biased distribution; weighted standard deviation method; standard deviation control chart

1 Introduction

[1]Short-term risk assessment is an important part of the risk of theoretical studies [1], the current study has focused on the short-term risk model described the risk of fluctuations in process, and the risk of fluctuations in short-term studies on the monitoring aspect is still rare. For short-term exposure to fluctuations in the monitoring, quality management control chart method is an important method to use for reference. Property value which reflects the short-term risk profile obey distribution that the majority of the obedience was skewed to the right characteristics (such as: the lognormal distribution) , high risk exists in the right tail of the distribution , research showing the risk of fluctuations in the monitoring of high-risk skewed to the right distribution has its important theoretical and practical significance. The quality characteristics obey normal distribution is the precondition of measuring value control chart which is included in the existing international standards ISO8258: 1991 [2] and our national standard GB/T4091-2001 [3] that is equivalent to adopt the international standard. For the monitoring problem of the mass production process, the control chart for improving product quality plays an important role . However, the variable control charts directly used for short-term risk assessment has many limitations, the reason is that the distribution of random variable which reflects the short-term risk profile mostly is a right skewed distribution, and normal distribution $N(\mu,\sigma^2)$ is symmetrical distribution, continuing to use the traditional Shewhart control chart is bound to produce a series of problems [4][5].

For partial control charts overall design studies can be roughly divided into four categories. The first method is to increase the sample size so that the sample mean is approximately normally distributed, but in many cases this method should pay a high economic cost; second approach is to assume that the overall distribution is known, established precise control chart to meet the first category of the level of risk. Ferrell [6] proposed a method that geometric

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half tolerance and geometric half poor control chart instead of Shewhart control charts under normal conditions. Nelson[7] establish median , range , scale and location of the control chart under assumed general obey Weibull distribution. Lucas [8], Vardeman and Ray [9] establish cumulative sum control chart under the premise of the overall exponential distribution, however, application of such methods has been limited under unknown of the overall distribution ; third method is to use Johnson or Pearson distribution system to transform the non-normal data , but this method is more complex and difficult to give an intuitive explanation of the control chart alarm , many quality engineers prefer using the control charts established under the normality assumption when the overall distribution is not serious biased ; fourth method is the overall distribution method is not make any assumptions , it commits the Type I error probability as close as possible to a given level with an improved way to create control charts . Cowden [10] proposed a method for separating the distribution , when the overall distribution is biased, it is divided into two parts in the mode and each part will be as a half of the different normal distribution which have the same mean and different standard deviation. Choobineh and Ballard [11] based on semi- variance approximation method proposed by Choobineh and Branting [12] coming up with weighted variance method , Bai and Choi [4] correct weighted variance method proposed by Choobineh and Ballard [11] , and provide a consistent direction and skewness of asymmetric control limits. Variance divided into two parts is the weighted variance, Bai and Choi [4] has proven to be very effective in controlling a biased process, but the establishment of the control chart is based on the variance instead of standard deviation , Chang and Bai [5] propose weighted standard deviation method to establish ideological control charts , gives the design of the mean control chart and limits and the calculation of the coefficient of the control limits , and short-term risk assessment can only control the changes of the average level using control chart alone, does not reflect the volatility of the risk. Based on this background this essay research on standard deviation control chart controlled the risk of fluctuations.

2 Weighted standard deviation method

Figure 1-6 briefly describes the main idea of the weighted standard deviation method. In figure 1, $f(x)$ is representative of the probability density function of the quality characteristics of X , and μ is the mean and σ is the standard deviation, $P_X = P\{X \le \mu\}$. In figure 2, it utilizes the right part of $f(x)$ to meet the nature of the probability density function to obtain a symmetrical probability density function $g_U(y)$. In figure 3, it utilizes the left part of $f(x)$ to obtain a symmetrical probability density function $g_L(y)$. Symmetrical distribution density function $g_U(y)$ and $g_L(y)$ have the same mean μ but different standard deviation σ_U and σ_L . For $g_U(y)$ and $g_L(y)$, it can be launched from $f(x)$

$$g_U(y) = \begin{cases} \dfrac{1}{2(1-P_X)} f(2\mu - y), y \le \mu \\ \dfrac{1}{2(1-P_X)} f(y), y > \mu \end{cases} \tag{1}$$

$$g_L(y) = \begin{cases} \dfrac{1}{2P_X} f(y), y \le \mu \\ \dfrac{1}{2P_X} f(2\mu - y), y > \mu \end{cases} \tag{2}$$

In formula (1) and (2), $1/2P_X$ and $1/2(1-P_X)$ are given weighting factor in order to satisfy the nature probability density.

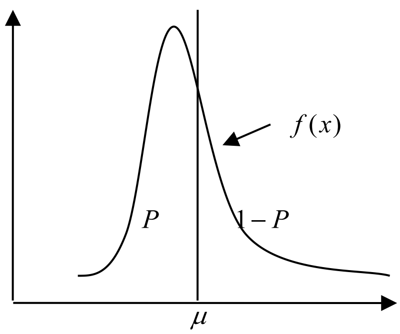
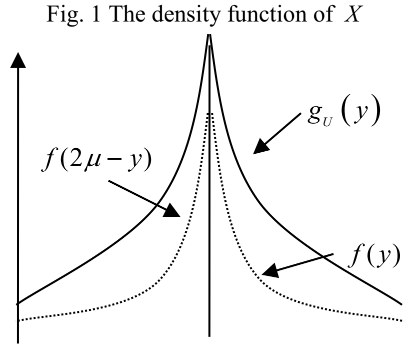


Fig. 1 The density function of X



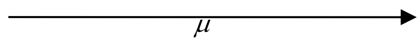


Fig. 2 The probability density function of $g_U(y)$

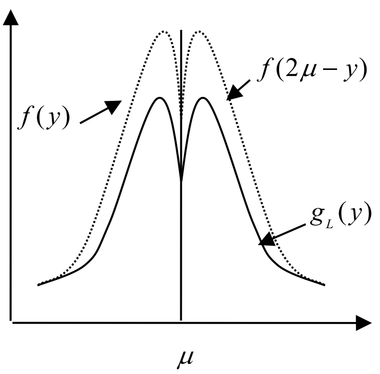


Fig. 3 The probability density function of $g_L(y)$

σ_U and σ_L can be calculated by formula (1) and (2).

$$\sigma_U = \sqrt{\frac{1}{1-P_X}\int_{\mu}^{+\infty}(y-u)^2 f(y)\,dy} \tag{3}$$

$$\sigma_L = \sqrt{\frac{1}{P_X}\int_{\mu}^{+\infty}(y-u)^2 f(y)\,dy} \tag{4}$$

Use semi-variance approximation $\int_{u}^{+\infty}(y-\mu)^2 f(y)\,dy \cong P_X\sigma^2$ launched by Choobineh and Branting(1986) [12], by the formula (3) and (4)

$$\sigma_U \cong \sqrt{\frac{P_X}{1-P_X}}\sigma \qquad \sigma_L \cong \sqrt{\frac{1-P_X}{P_X}}\sigma \tag{5}$$

In the weighted standard deviation method, the standard deviation σ can be divided into left standard deviation σ_U^W and right standard deviation σ_L^W and it meets $\sigma = \sigma_U^W + \sigma_L^W$, because the contribution of σ_U^W to σ can be as the similar of σ_U to $\sigma_U + \sigma_L$, it is reasonable to assume that

$$\sigma_U^W / \sigma = \sigma_U / (\sigma_U + \sigma_L) \quad , \quad \sigma_L^W / \sigma = \sigma_L / (\sigma_U + \sigma_L) \quad ,$$

because of $\sigma_U / (\sigma_U + \sigma_L) \cong P_X$ and $\sigma_L / (\sigma_U + \sigma_L) \cong 1 - P_X$, so it can be defined

$$\sigma_U^W = P_X\sigma \quad \sigma_L^W = (1-P_X)\sigma \tag{6}$$

Based on the above analysis, we can use the left half of normal density function

$$f_U(X) = \frac{1}{2\sigma_U^W}\Phi\left(\frac{X-u}{2\sigma_U^W}\right) \tag{7}$$

to approximate the left half of the original function $f(x)$, the shown in Figure 4 . We can use the right half of normal density function

$$f_L(X) = \frac{1}{2\sigma_L^W}\Phi\left(\frac{X-\mu}{2\sigma_L^W}\right) \tag{8}$$

to approximate right half of the original function $f(x)$, shown in Figure 5 , $\Phi(\bullet)$ represents the standard normal density function . Figure 6 shows the approximate distribution density function. If the overall distribution is symmetric, then $P_X = 1/2$ and $\sigma_U^W = \sigma_L^W = 1/2\sigma$; If the overall distribution skewed to the right, then $P_X > 1/2$, $\sigma_U^W > \sigma_L^W$; conversely left side, $P_X < 1/2$, $\sigma_U^W < \sigma_L^W$.

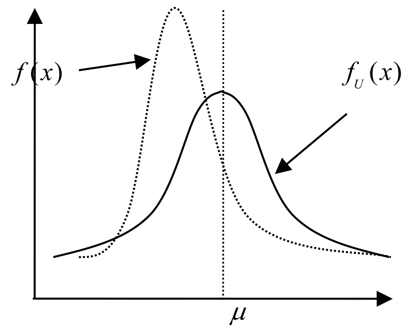


Fig. 4 The left half of the approximate $f(x)$

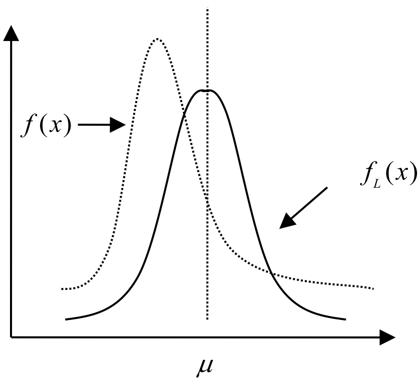


Fig. 5 The right half of the approximate $f(x)$

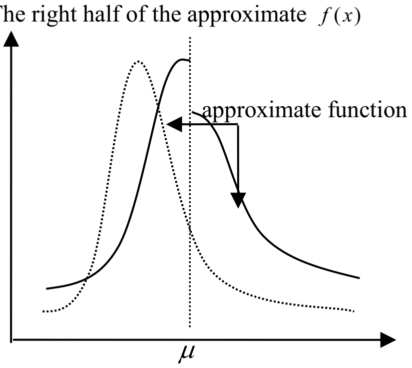


Fig. 6 Original distribution and approximate distribution

3 Based on the weighted standard deviation method of s control charts

The basic idea of the weighted standard deviation method is the presence of a skewed distribution, it can be distributed into two parts disposition in its mean, each part can produce a new symmetric distribution, two new distribution come from the original biased distribution with the same mean, but different standard deviation. The weighted standard deviation method establish control limits with these two distributions, in other words, one is used to calculate the standard deviation to establish upper control limits(UCL), another used to establish the lower control limits(LCL). If the overall control to the right side, the distance of the control upper limits to the centerline is greater than that of the lower control limits to the centerline. Similarly, if the overall control to the left side, then the distance of the lower control limits to the centerline is greater than that of the upper control limits to the centerline. For the overall symmetry, the distance of the control upper limits to the centerline is the same with that of the lower control limits to the centerline, based on the weighted standard deviation on the control chart degenerate into traditional Shewhart control charts.

Standard deviation control chart control limits is

$$\mathrm{UCL_s} = \mu_s + \sigma\sqrt{1-c_4^2}\,2P_X$$
$$\mathrm{CL_s} = \mu_s \qquad (9)$$
$$\mathrm{LCL_s} = [\mu_s - \sigma\sqrt{1-c_4^2}\,2(1-P_X)]^+$$

In the formula (9), μ_s is standard deviation for the mean, σ is standard deviation, n is the capacity of a sub-sample , and c_4 is a constant related to sample n , [a] $^+$ indicates max(a,0).

If P_X , standard deviation σ , the mean standard deviation μ_s , the capacity of a sub-sample n and constant sub-sample size c_4 is known, the formula (9) can be used to control the actual risk of fluctuations, however, in practice the parameters P_X , standard deviation σ , standard differential means μ_s is often unknown, section 4 will explore the unknown situation. Based on the formula (9), if the distribution of the overall is symmetry, $P_X = 0.5$,at this point the formula (9) degenerate to traditional Shewhart standard deviation control charts; If the overall is right side, $P_X > 0.5$, the distance of the control upper limits to the centerline is greater than that of the lower control limits to the centerline, if the overall is left side, $P_X < 0.5$, then the distance of the lower control limits to the centerline is greater than that of the upper control limits to the centerline.

4 Constant calculation of the control chart

In practice, P_X may be estimated by the number of less than equal to the total sample mean $\bar{\bar{X}}$ divided observed divided the total number of samples, as

$$\hat{P}_X = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} I(\bar{\bar{X}} - X_{ij})}{m \times n} \qquad (10)$$

In the formula (10), m and n represent the number of the sample and the sub-samples size for each sample, $I(X) = 1(X \geq 0)$, $I(X) = 0(X < 0)$.

In the theory of Shewhart control charts, generally, standard deviation of the mean $\bar{s}$ is used to estimate μ_s . Control chart use the constant c_4 to estimate σ_s ,the calculation formula is $E(s) = c_4\sigma$, $\hat{\sigma} = \bar{s}/c_4$, $\bar{s} = \sum_{i=1}^{m} s_i / m$, $\sigma_s = \sigma\sqrt{1-c_4^2}$ .However, the constant c_4 is determined under the assumption of a normal distribution, so when the distribution is biased, should adopt the c_4' reflected the impact of skewness to calculate. For skewed distribution, similar methods can be used to calculate c_4' , but the distribution of specific is given, this constant can be obtained by numerical integration, seeing the work of BaiS and Choi IS(1995) [4].

Based on c_4', in the case of unknown parameters formula (9) transform into the formula (11)

$$\mathrm{UCL_s} = \bar{s} + 3\frac{\bar{s}}{c_4'}\sqrt{1-c_4'^2}\,2\hat{P}_X = B_4\bar{s}$$

$$CL_s = \bar{s} \quad (11)$$

$$LCL_s = \bar{s} - 3\frac{\bar{s}}{c_4'}\sqrt{1-c_4'^2}\,2(1-\hat{P}_X) = B_3$$

Table 1 shows the method of weighted standard deviation calculated standard deviation control chart constants $B_3\bar{s}$ and B_4, table 1 only gives Part B_3 and B_4 in $P_X > 0.5$ situation.

Table 1: n =10, the constants of standard deviation control chart is calculated based on the weighted standard deviation method

P_X	0.6400	0.6600	0.6800	0.7000
B_3	0.0000	0.0000	0.0000	0.0000
B_4	6.2069	6.7505	7.5246	8.5521

5 Compares the control effect

This section provides a numerical example to compare methods, given the merits of comparative effectiveness of the weighted standard deviation and the traditional Shewhar control charts based on the weighted standard deviation method. Monte Carlo method generates 25 samples, each sub-sample size is 10, and the sample comes from the Weibull distribution. Considered by the random number generated $\bar{\bar{X}} = 1.0238$，$\bar{s} = 0.6917$，$\hat{P}_X$ =0.64, control limits based on the weighted standard deviation and Shewhart method are as follows:

The weighted standard deviation method: UCL$_s$= $B_4\bar{s}$ = 4.2933 CL$_s$= $\bar{s}$ = 0.6917 LCL$_s$= $B_3\bar{s}$ = 0.0000

Shewhart method: UCL$_s$= $B_4\bar{s}$ = 1.1870 CL$_s$= $\bar{s}$ = 0.6917 LCL$_s$= $B_3\bar{s}$ = 0.1964

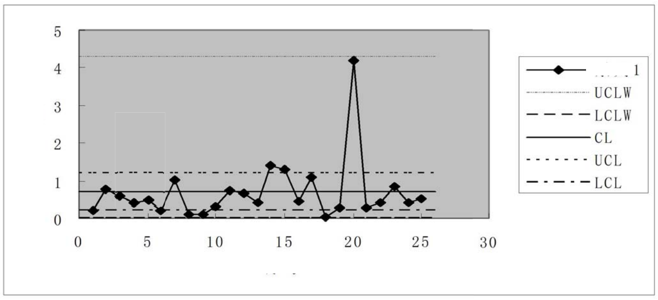


Fig. 7 Standard deviation control chart based on the weighted standard deviation and Shewhart method

Figure 7 shows standard deviation control chart based on the weighted standard deviation and Shewhart method, UCLW calculated by the weighted standard deviation method represents the upper limit, LCLW calculated by the Shewhart method represents the lower limit, CL indicates the center limits. The results showed that the distance of the control upper limits to the centerline calculated by the weighted standard deviation method establishing control charts is greater than that of Shewhart method establishing control charts. According to the criterion of control charts, all the points falling into the control limits illustrate the process in a controlled state, and there are two points fall off Shewhart mean control chart above the upper control limit, there are three points fall off Shewhart standard deviation control chart above the upper control limit, it has issued false alarms.

6 Conclusion

When the overall distribution is biased, in this paper standard deviation control chart based on a weighted standard deviation method is more effective than that of traditional Shewhart method, when the overall distribution is symmetric, improvement of the standard deviation control charts degenerate to Shewhart standard deviation control Charts. In practical application it is important to determine whether the overall distribution is biased. In addition, when the data is biased in testing process , the process must be in a controlled state. For example, if there are many sample mean drifted upward form 25 sets of samples which come from the same normal distribution , it may give a misleading biased distribution, therefore it should be checked whether the skewness is caused by a biased distribution or runaway process , the method can use the histogram to test whether there is bias in the sample, so it is inferred whether is the biased overall, and also it can use the histogram method to test whether the process is biased.

Based on the weighted standard deviation method to establish a standard deviation control chart of skewed distribution , given the constants formula of standard deviation control chart limits and the corresponding form , to a certain extent, it facilitates the application of the actual workers, aiming at the right skewed characteristics of the most of the distribution of short-term risk assessment, it is feasible that standard deviation control chart based on the weighted standard deviation method is applied to short-term risk assessment. Also in many applications it are often not fully aware of the knowledge of quality characteristics of the distribution, especially in the early product cycle, it is more suitable for the use of the such control charts , early detect unknown risks, and advance control to reduce the loss occurred.

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