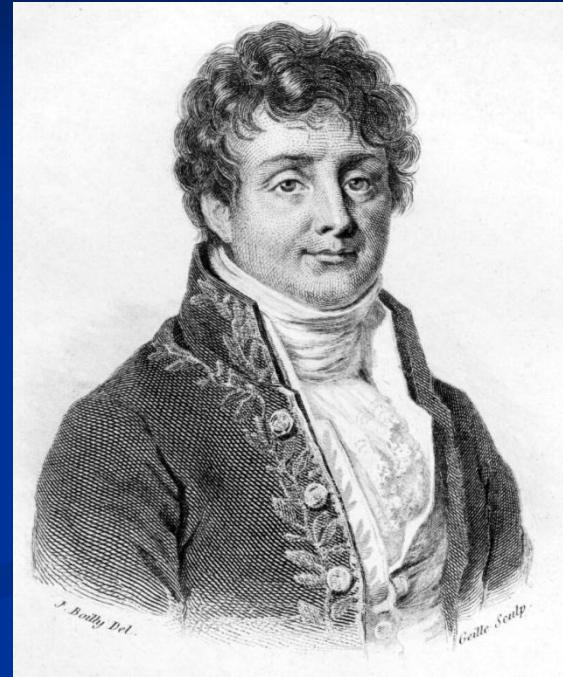


# Transformácie obrazu

Gonzales, Woods: Digital Image Processing  
kapitola: Image transforms

# Fourierova transformácia

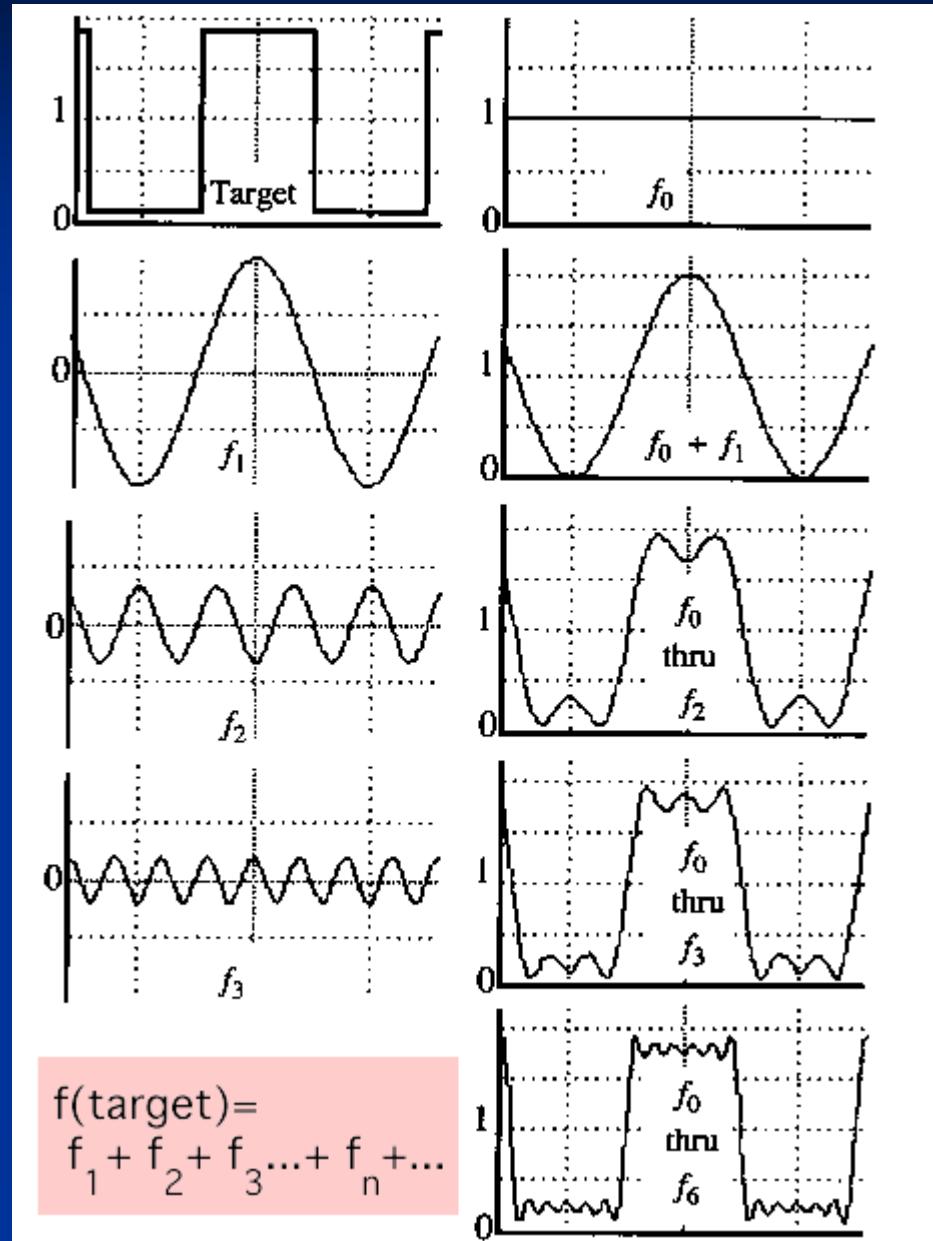
- Jean Baptiste Joseph Fourier  
(1768-1830)



- Akákoľvek funkcia  $f(x)$  môže byť vyjadrená ako vážený súčet sínusov a kosínusov

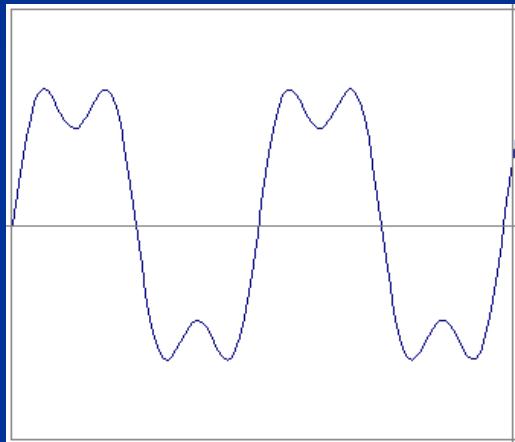
# Suma sínusov a kosínusov

■  $A \sin(\omega x + \varphi)$



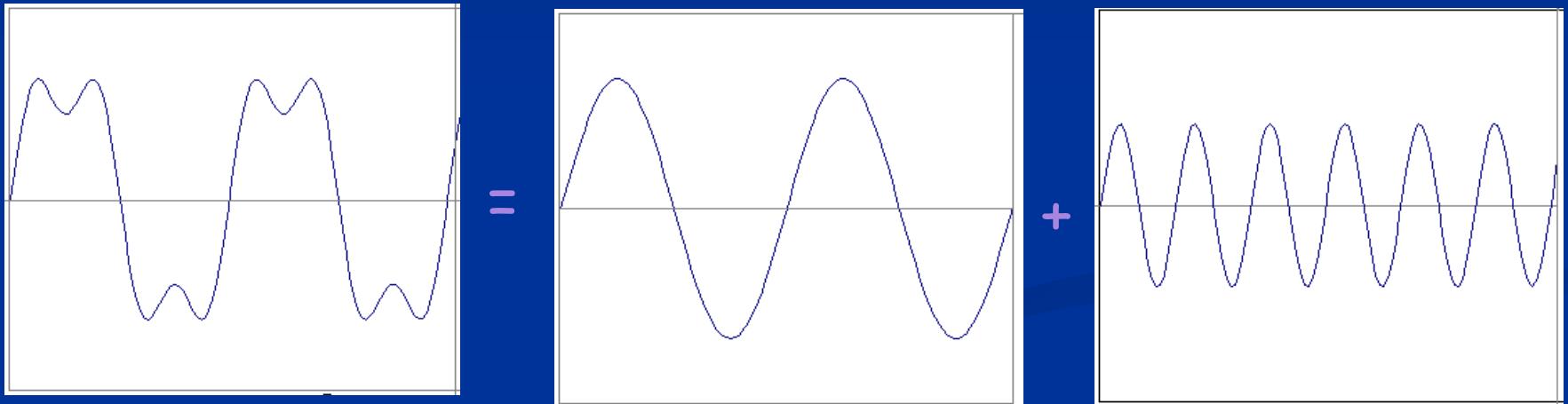
# Time and Frequency

- example :  $g(t) = \sin(2\pi f t) + (1/3)\sin(2\pi(3f)t)$



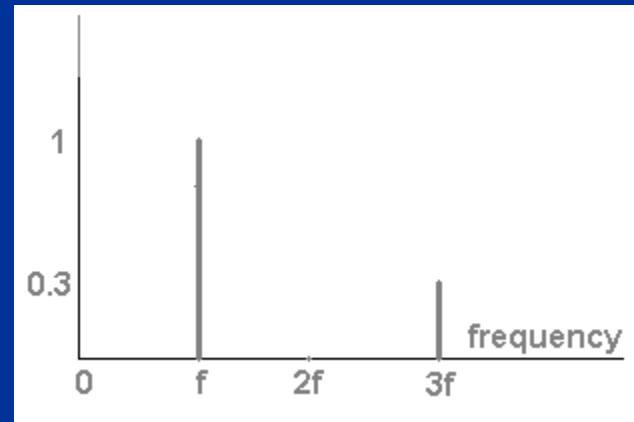
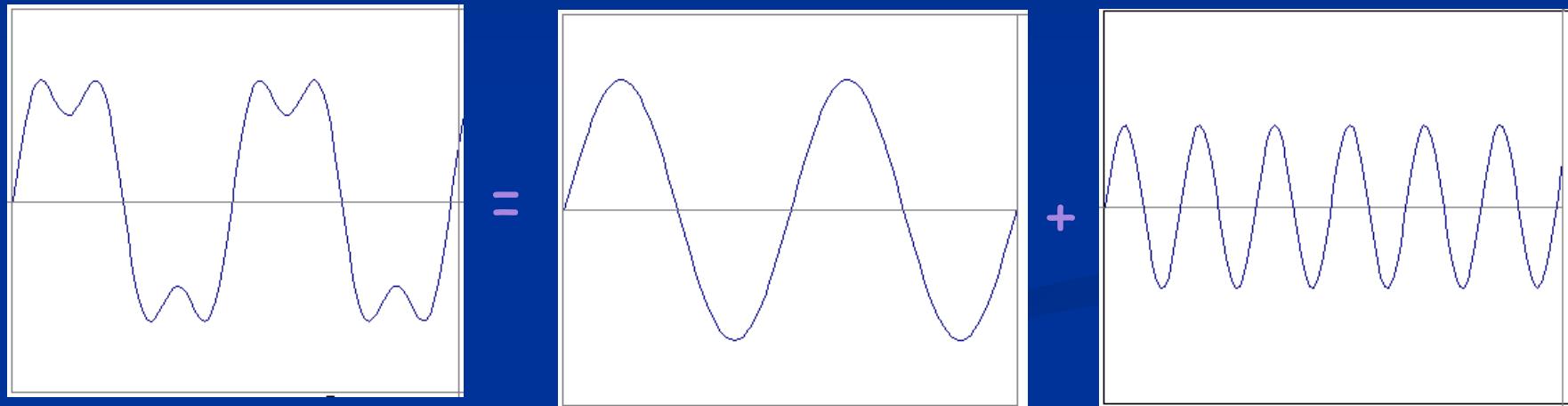
# Time and Frequency

■ example :  $g(t) = \sin(2\pi f t) + (1/3)\sin(2\pi(3f) t)$

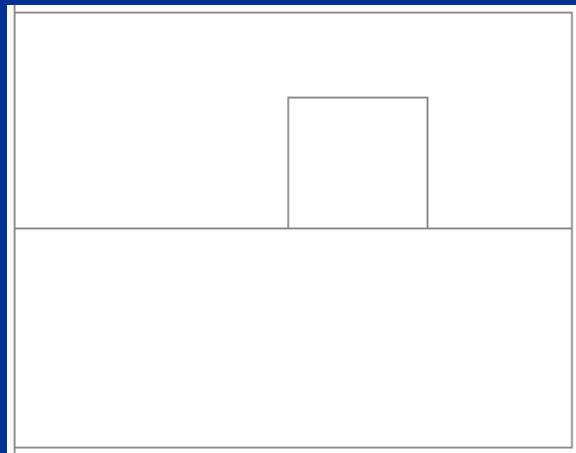


# Frequency Spectra

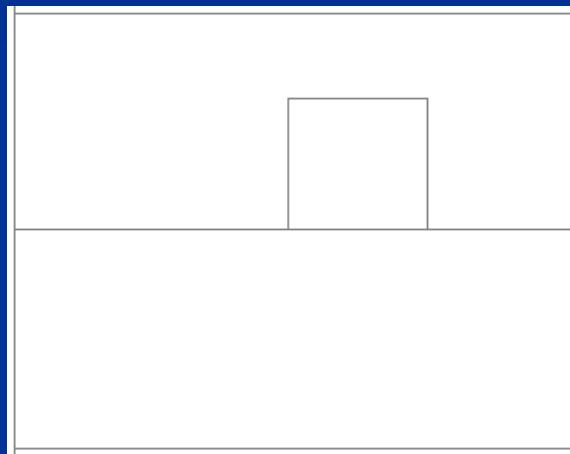
■ example :  $g(t) = \sin(2\pi f t) + (1/3)\sin(2\pi(3f)t)$



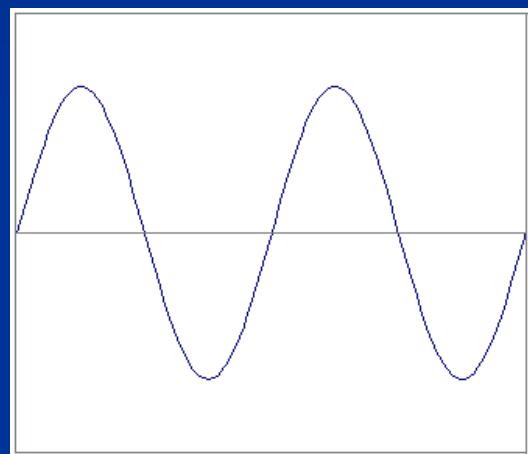
# Frequency Spectra



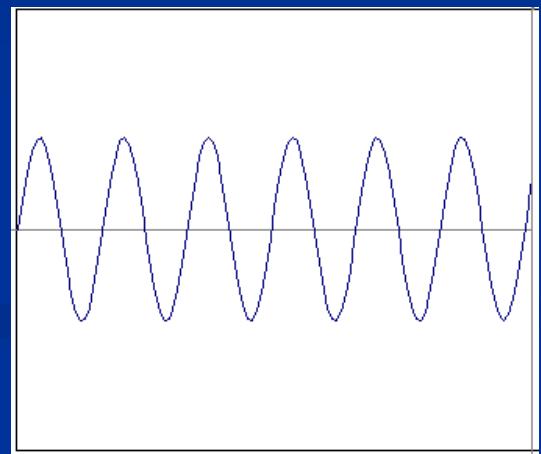
# Frequency Spectra



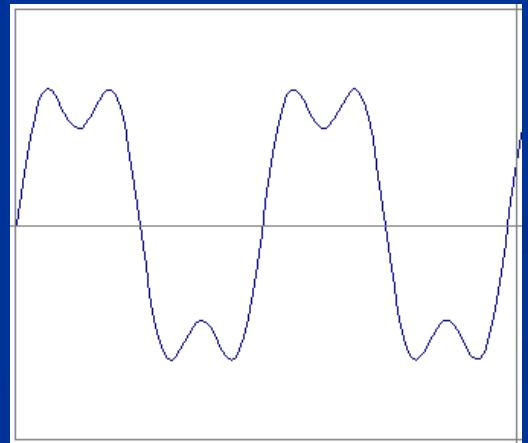
=



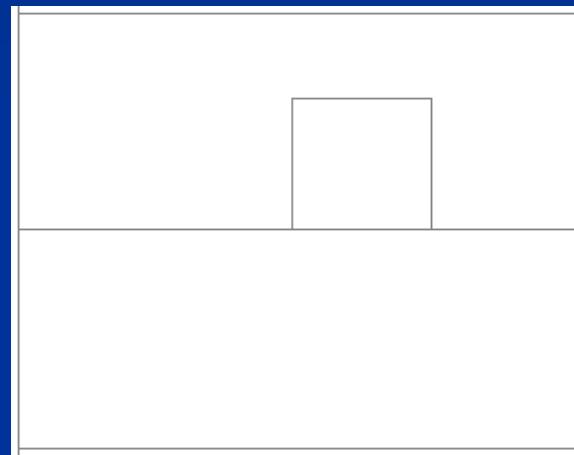
+



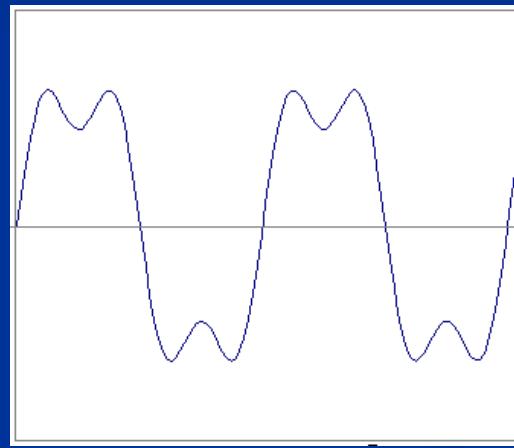
=



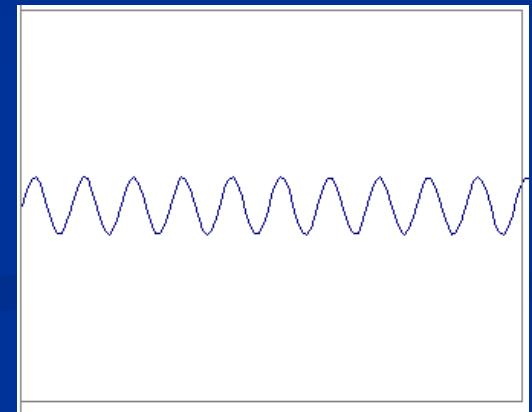
# Frequency Spectra



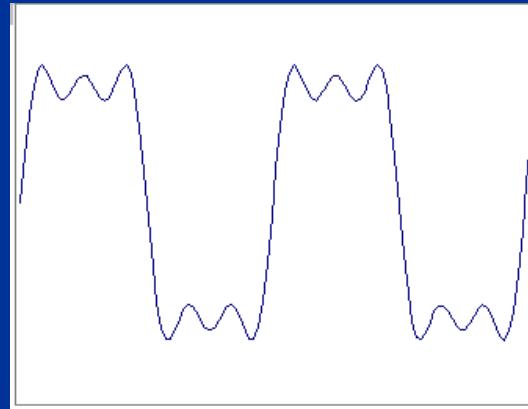
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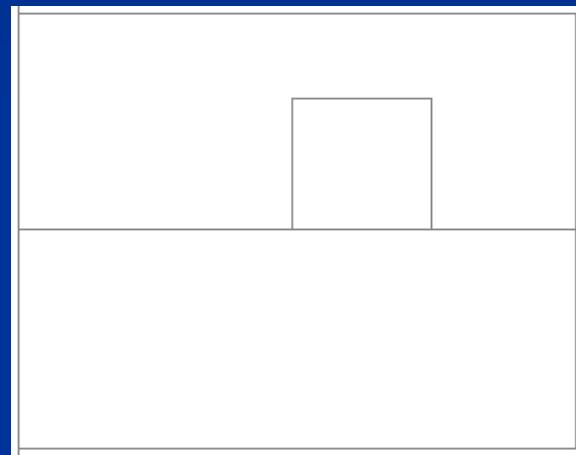
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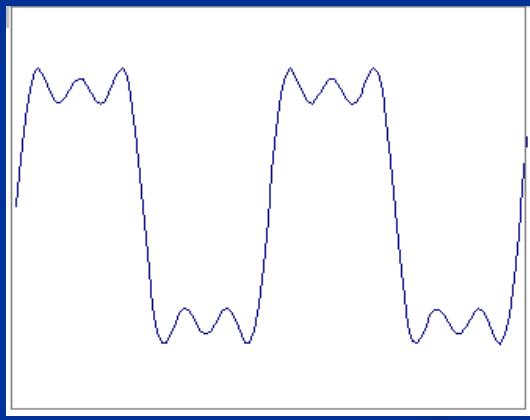
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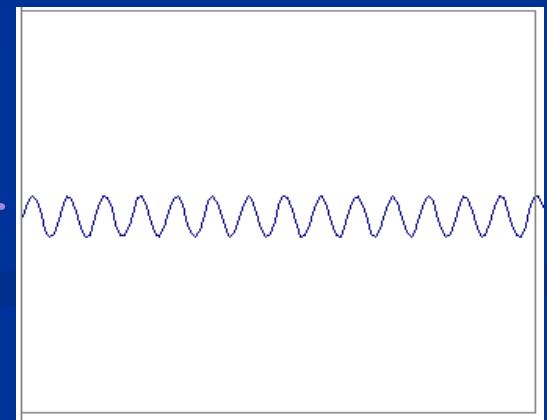
# Frequency Spectra



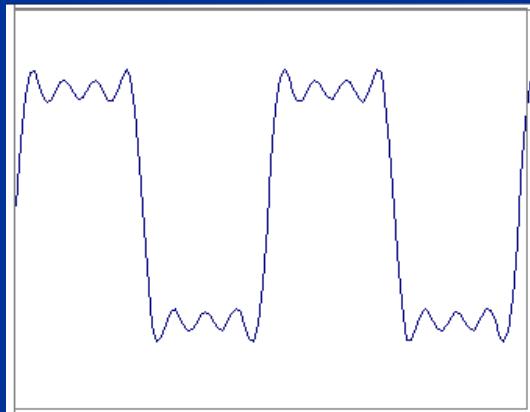
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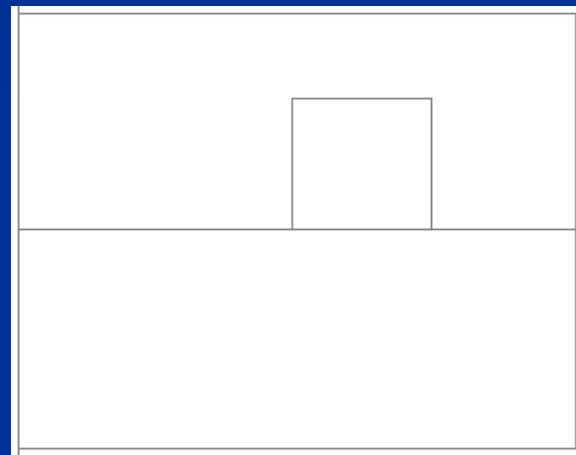
+



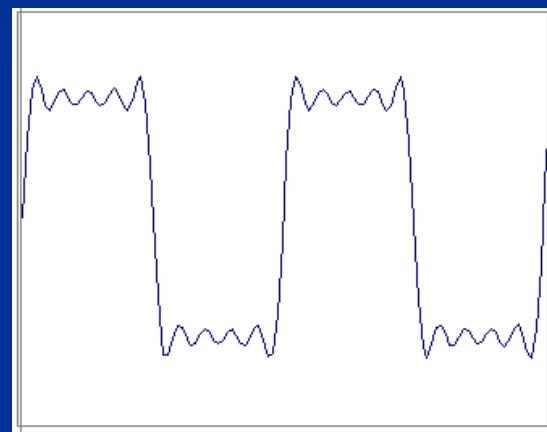
=



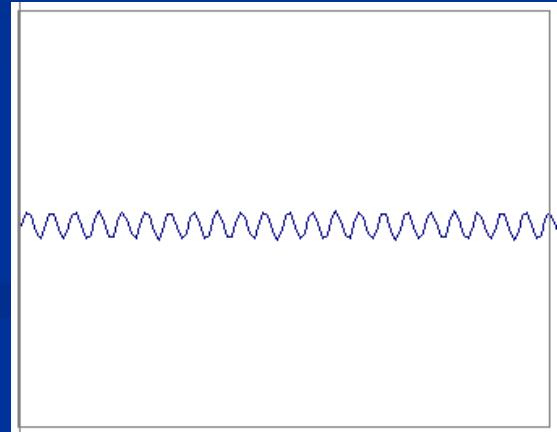
# Frequency Spectra



=



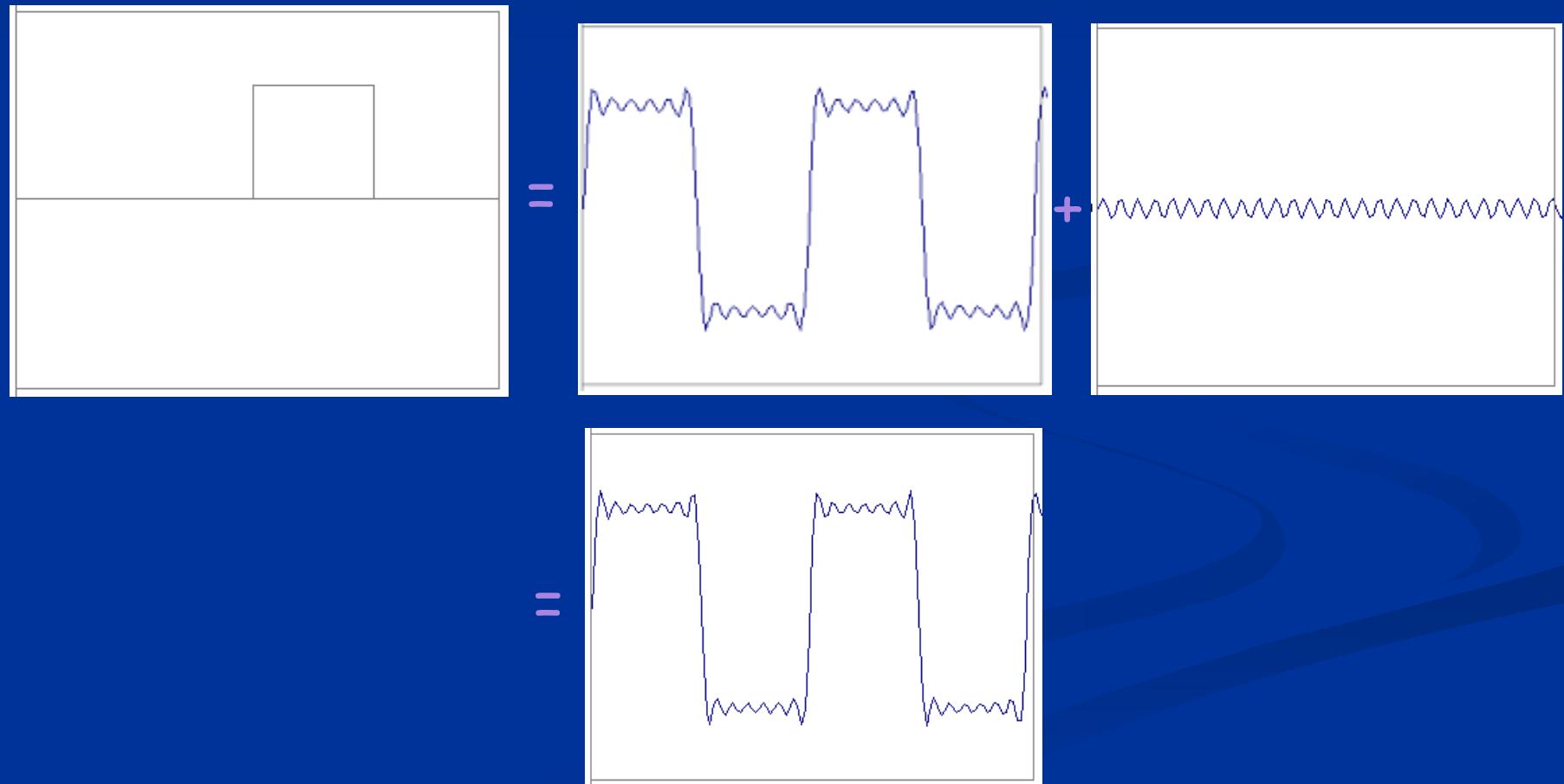
+



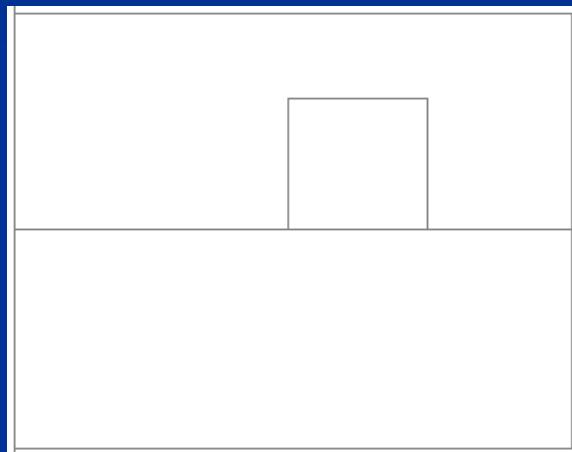
=



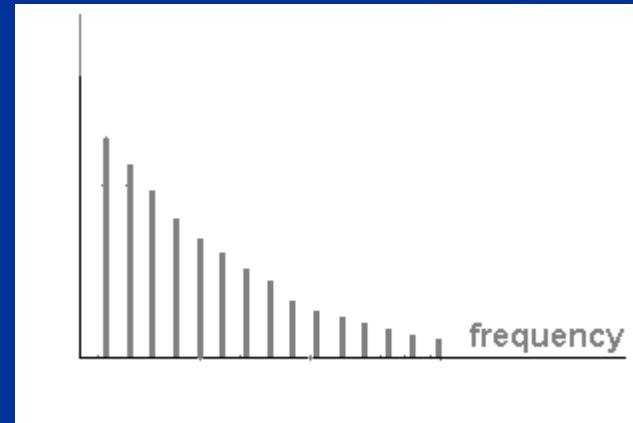
# Frequency Spectra



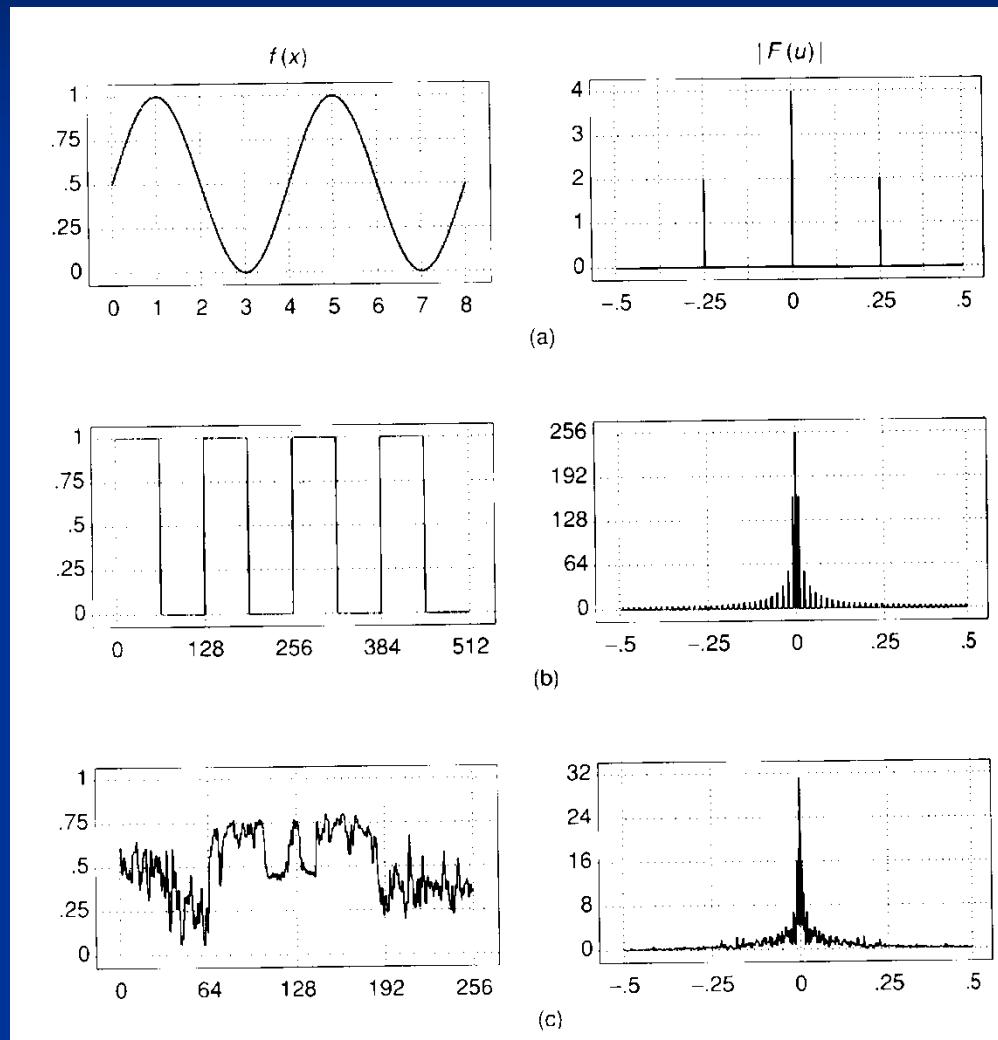
# Frequency Spectra



$$= A \sum_{k=1}^{\infty} \frac{1}{k} \sin(2\pi kt)$$

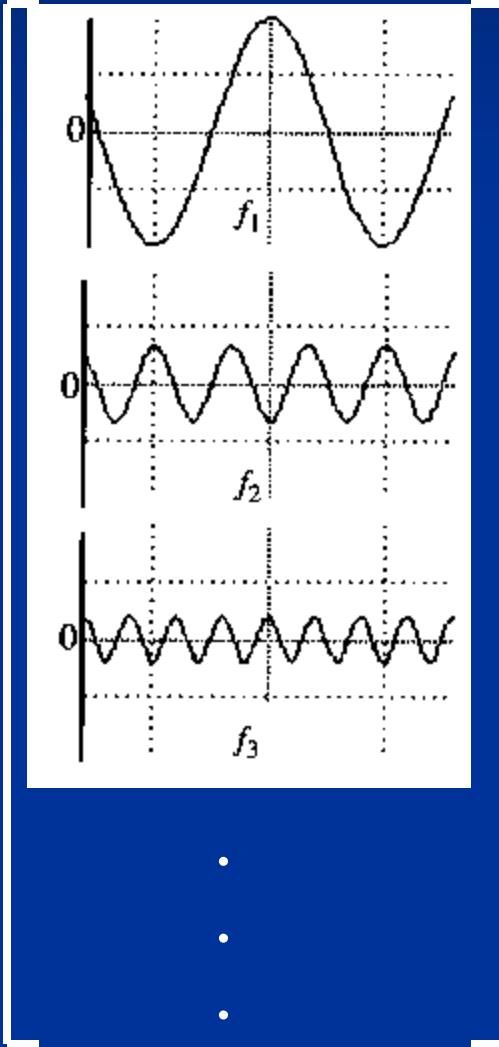


# Frequency Spectra

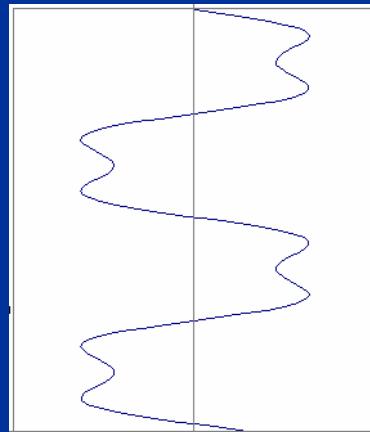


# FT: Just a change of basis

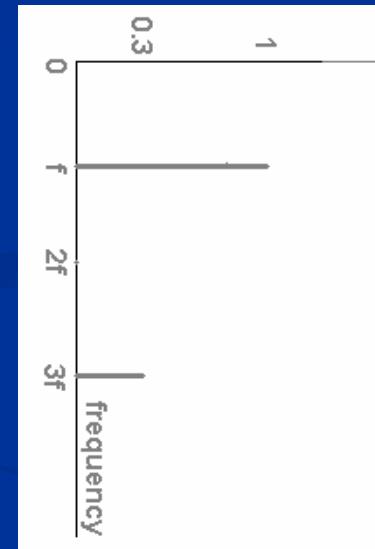
$$\mathbf{M} * f(x) = F(\omega)$$



\*

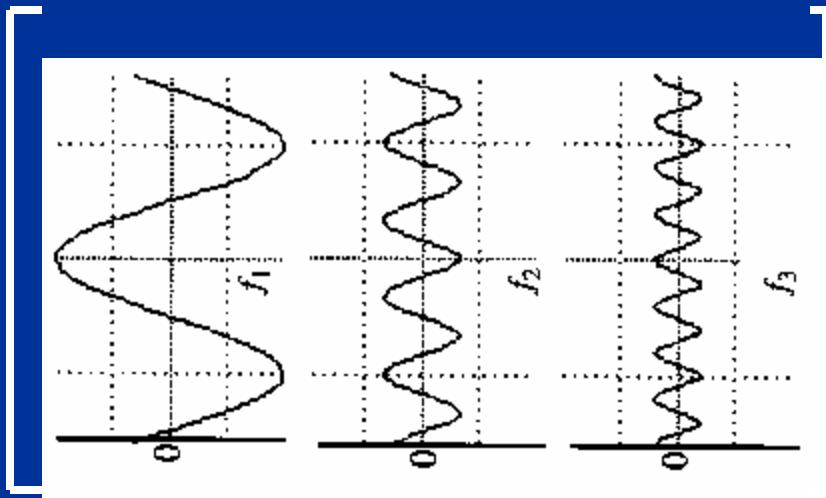


=

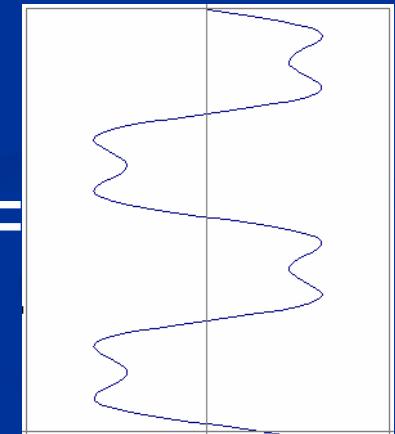
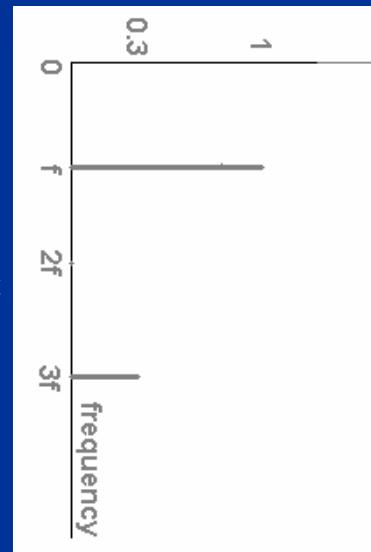


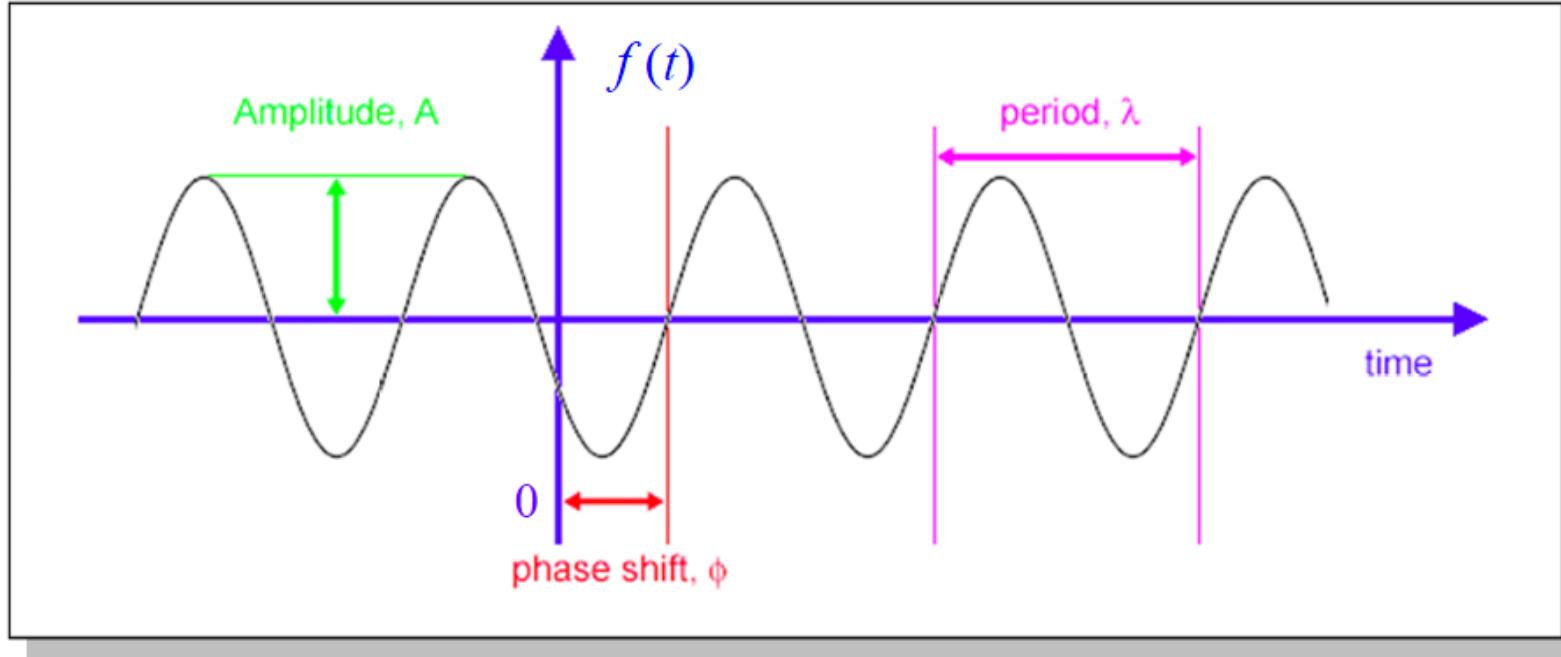
# IFT: Just a change of basis

$$\mathbf{M}^{-1} * F(\omega) = f(x)$$



\*





$$f(t) = A \sin\left(\frac{2\pi}{\lambda} t - \phi\right)$$

$1/\lambda$  is the frequency of the sinusoid (Hz).  
 $2\pi/\lambda$  is the angular frequency (radians/s).

# Fourierova transformácia

Ak  $f(x)$  je spojitá funkcia s reálnou premenou  $x$ ,  
potom Fourierovou transformáciou  $F(x)$  je

$$F(u) = \int_{-\infty}^{\infty} f(x)e^{-i2\pi ux}dx,$$

$$F(u) = \int_{-\infty}^{\infty} f(x)(\cos 2\pi ux - i \sin 2\pi ux)dx.$$

Inverznou Fourierovou transformáciou nazývame

$$f(x) = \int_{-\infty}^{\infty} F(u)e^{i2\pi ux}du.$$

pár Fourierovej  
transformácie

# Fourier Transform



Pre každé  $u$  od 0 po inf,  $F(u)$  obsahuje amplitudu  $A$  and fázu  $\phi$  odpovedajúceho sinusu

$$A \sin(ux + \phi)$$

$$F(u) = R(u) + iI(u),$$

$$A = \pm \sqrt{R(u)^2 + I(u)^2}$$

$$\phi = \tan^{-1} \frac{I(u)}{R(u)}$$



# Definitions

- $F(u)$  sú komplexne čísla:

$$F(u) = R(u) + iI(u),$$

$$F(u) = r(\sin \theta + i \cos \theta) = re^{i\theta}$$

- Magnitúda FT (spektrum):

$$|F(u)| = [R^2(u) + I^2(u)]^{1/2}$$

- Fázový uhol FT:

$$\phi(u) = \tan^{-1} \left[ \frac{I(u)}{R(u)} \right]$$

- Reprezentácia pomocou magnitúdy a fázy:

$$F(u) = |F(u)| e^{i\phi(u)},$$

- Power of  $f(x)$ :  $P(u) = |F(u)|^2 = R^2(u) + I^2(u)$

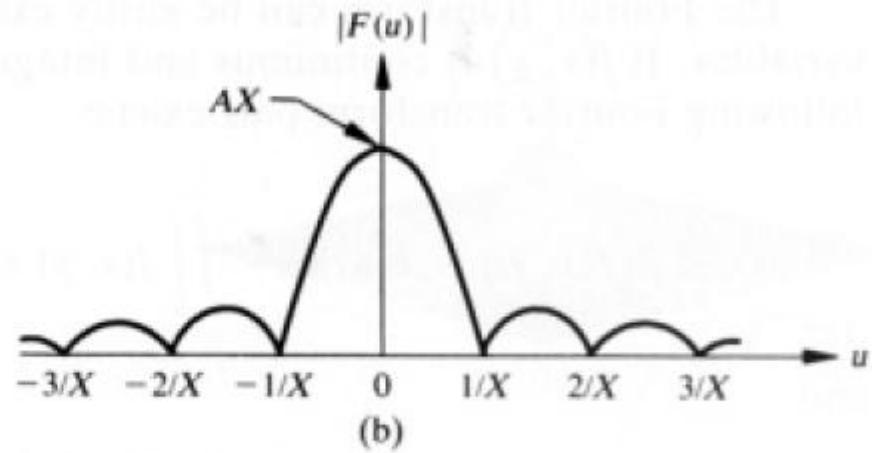
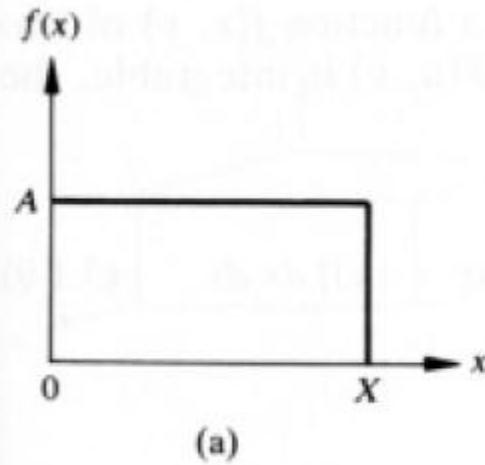


Figure 4: A simple function and its Fourier spectrum.

$$F(u) = \frac{A}{\pi u} \sin(\pi u X) e^{-i\pi u X}$$

$$|F(u)| = AX \left| \frac{\sin(\pi u X)}{(\pi u X)} \right|.$$

FT – je periodická s periódou N, to znamená, že na jej určenie stačí jedna perióda vo frekvenčnej oblasti.

# 2D Fourierova transformácia

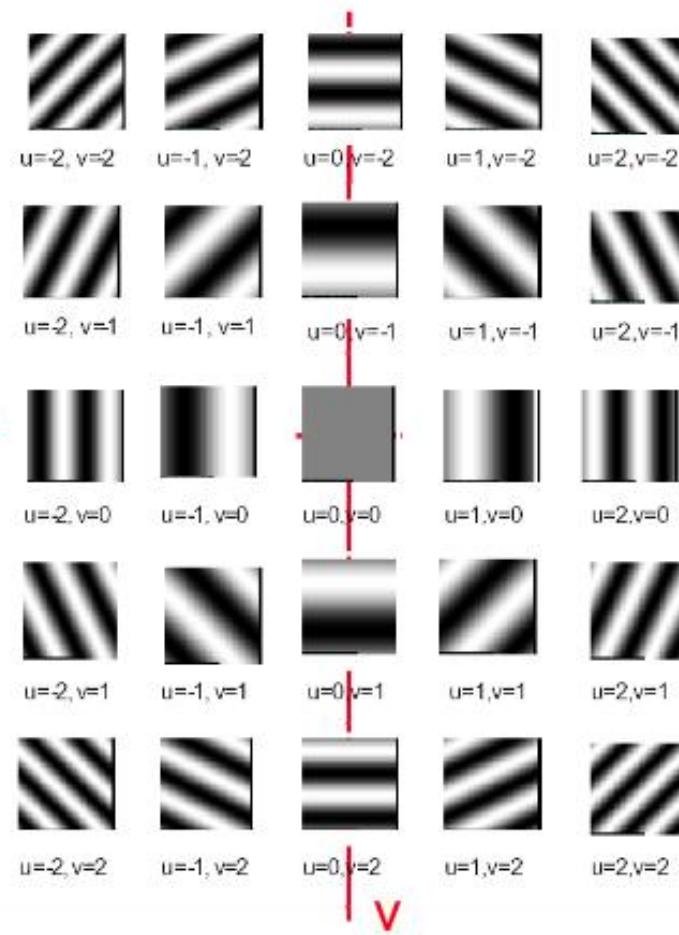
- Fourierova transformáci je ľahko rozšíritelná do 2D

$$F(u, v) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) \exp[-i 2\pi(ux + vy)] dx dy$$

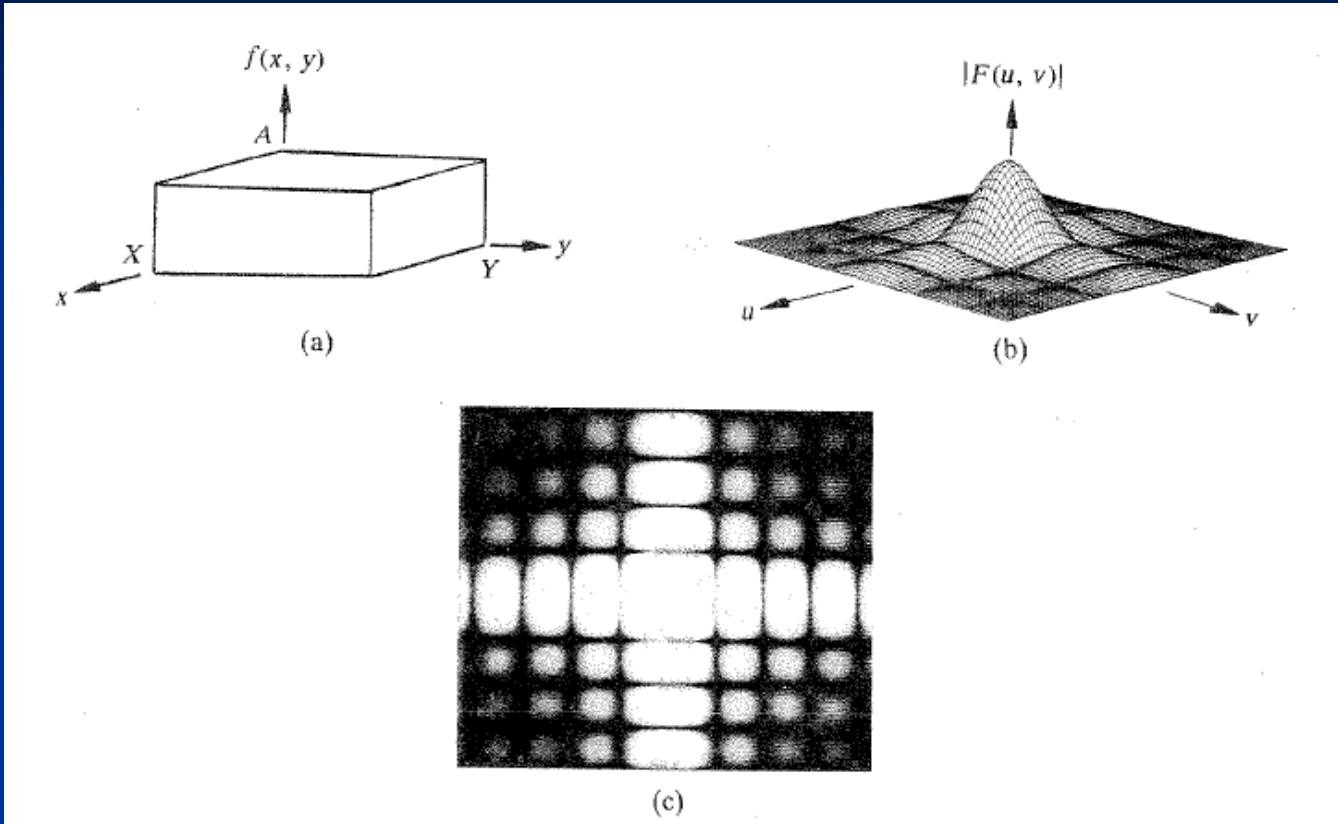
$$f(x, y) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(u, v) \exp[i 2\pi(ux + vy)] du dv .$$

# Fourier Basis

Real (cos) part			
Imaginary (sin) part			



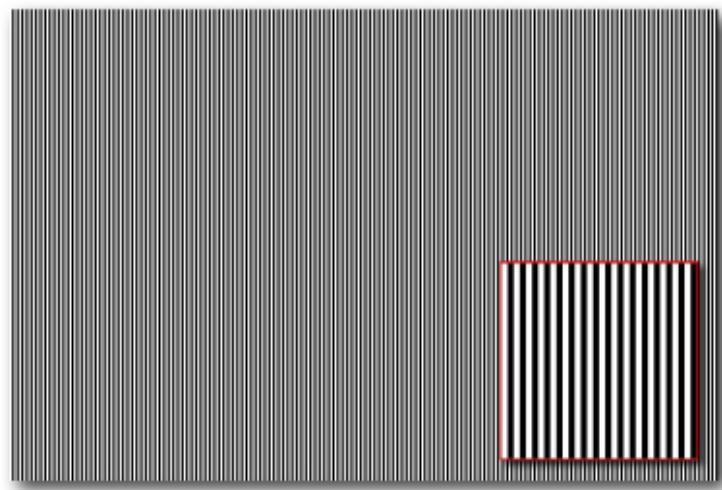
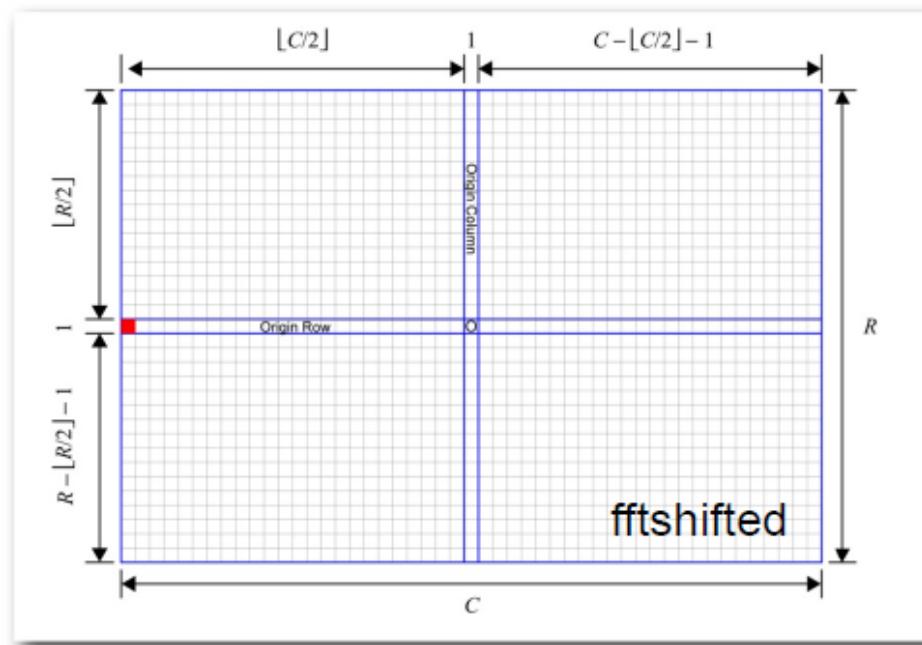
# Príklad 2D funkcie



$$F(u, v) AXY \left[ \frac{\sin(\pi u X)}{(\pi u X)} \right] \left[ \frac{\sin(\pi v Y)}{(\pi v Y)} \right]$$

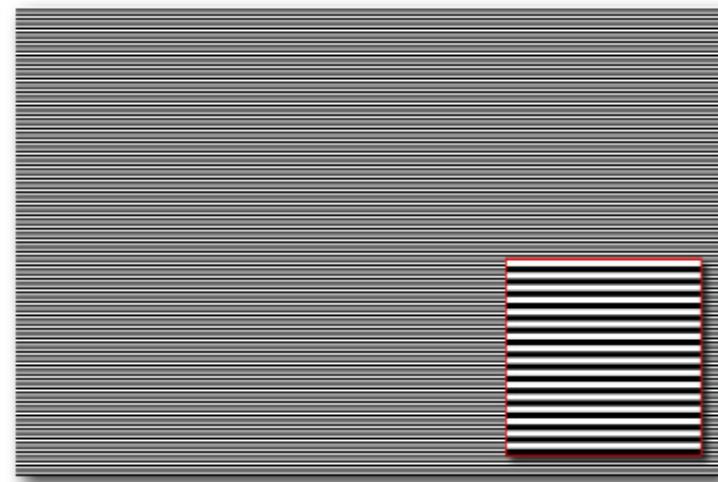
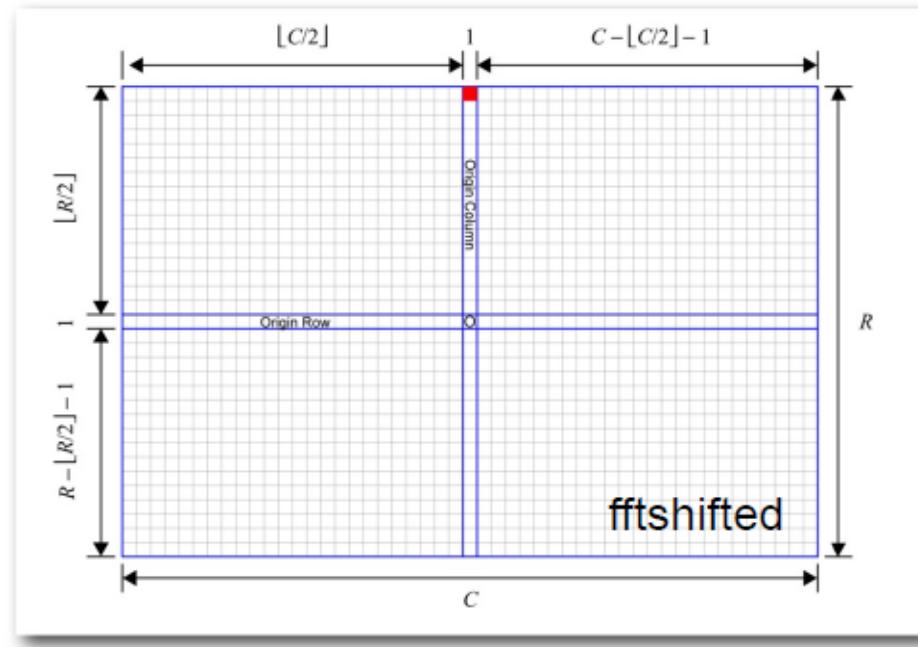
$$|F(u, v)| = AXY \left| \frac{\sin(\pi u X)}{(\pi u X)} \right| \left| \frac{\sin(\pi v Y)}{(\pi v Y)} \right|$$

"horizontal" is the wavefront direction.



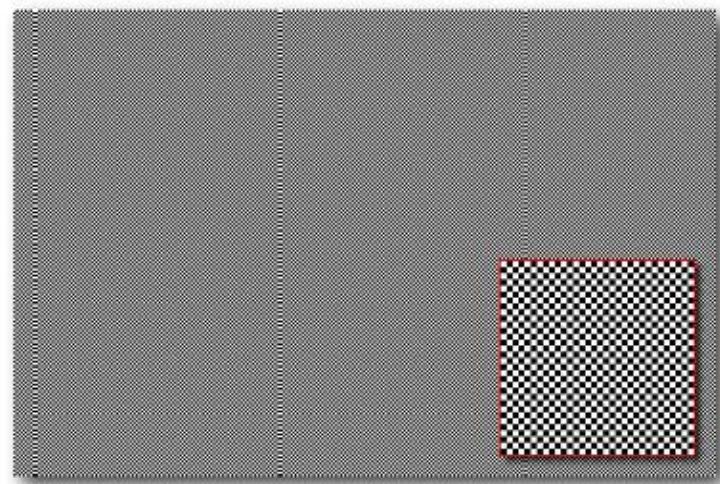
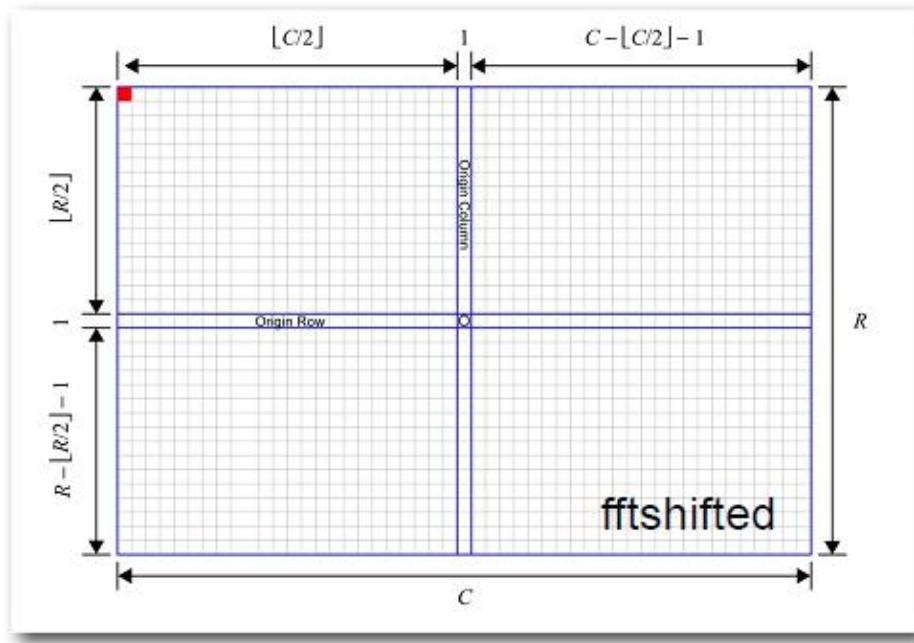
highest-possible-frequency horizontal sinusoid

"vertical" is the wavefront direction.



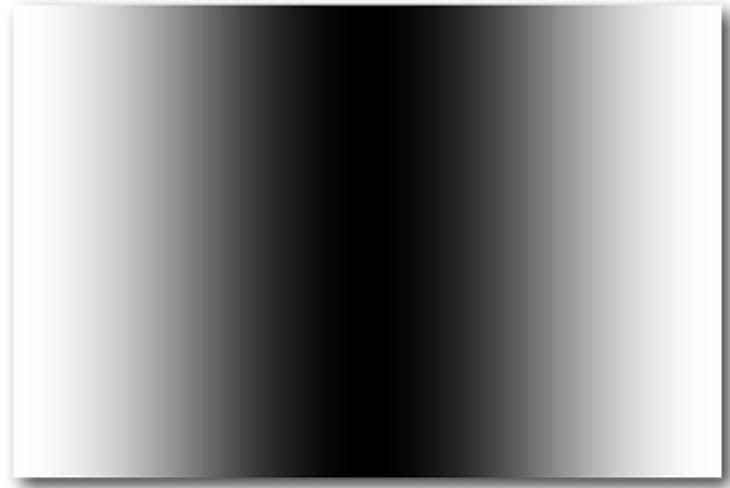
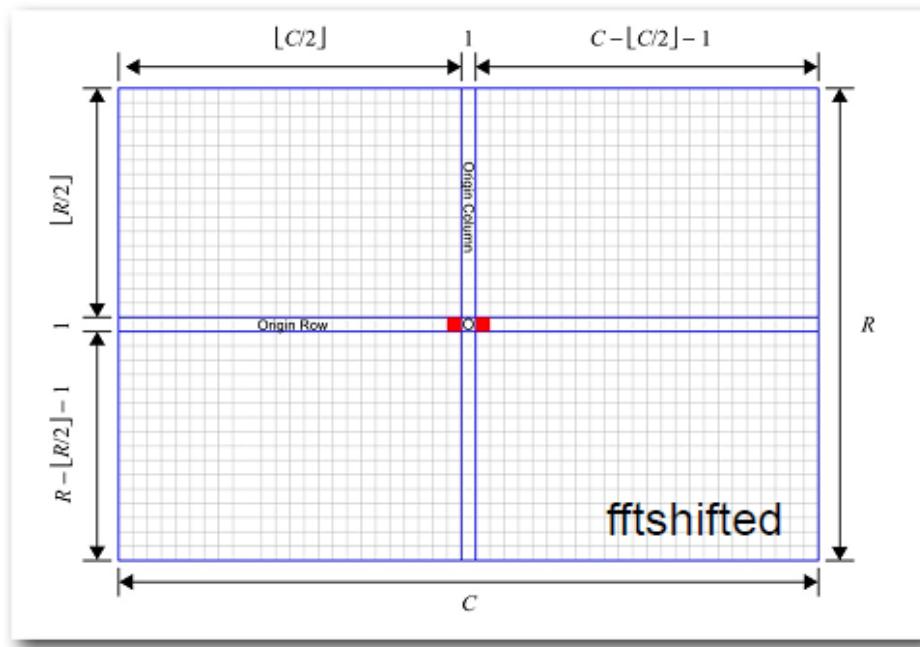
highest-possible-frequency vertical sinusoid

a checker-board pattern.



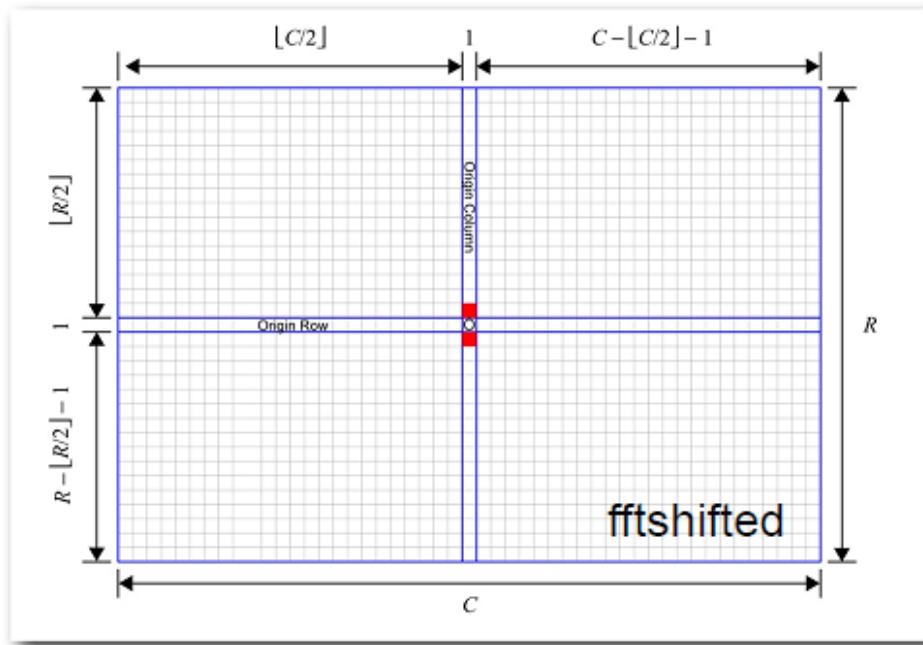
highest-possible-freq horizontal+vertical sinusoid ( $R$  &  $C$  even)

"horizontal" is the wavefront direction.



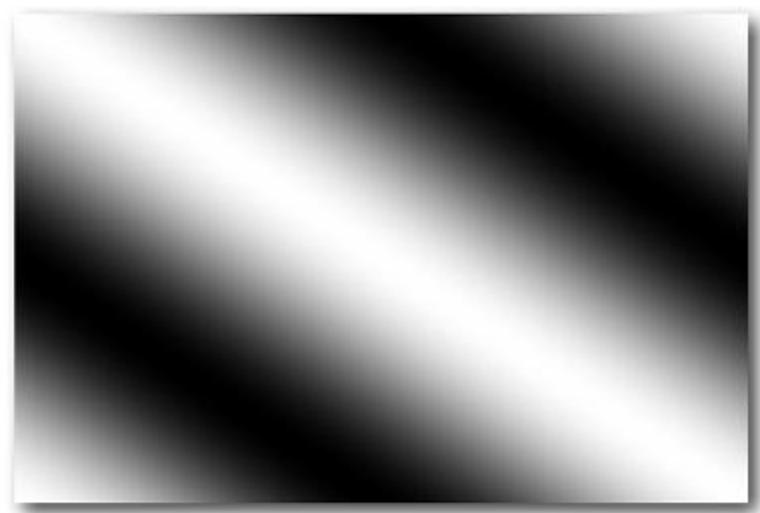
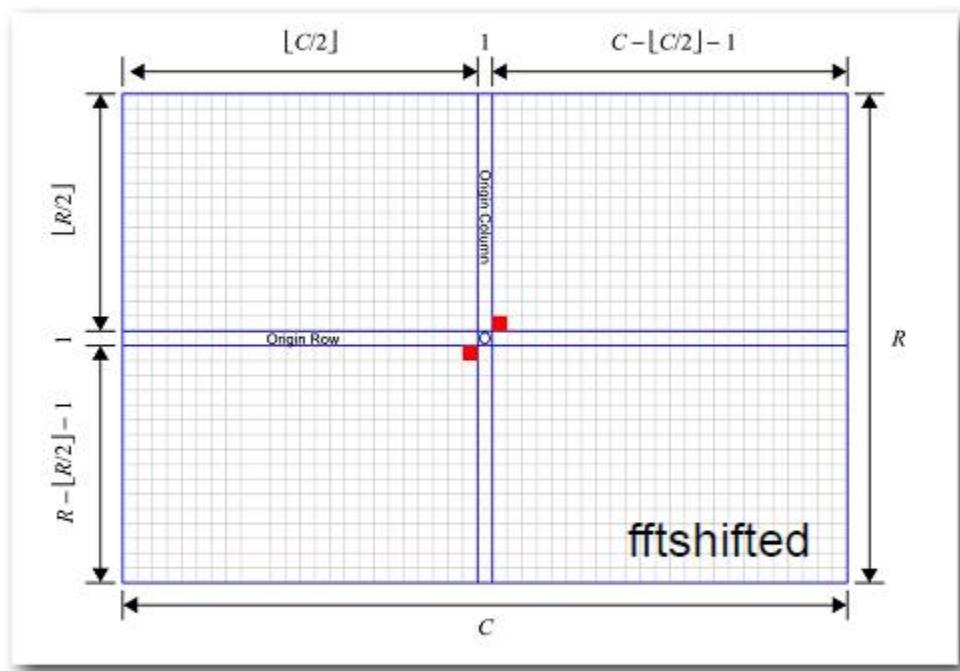
lowest-possible-frequency horizontal sinusoid

"vertical" is the wavefront direction.

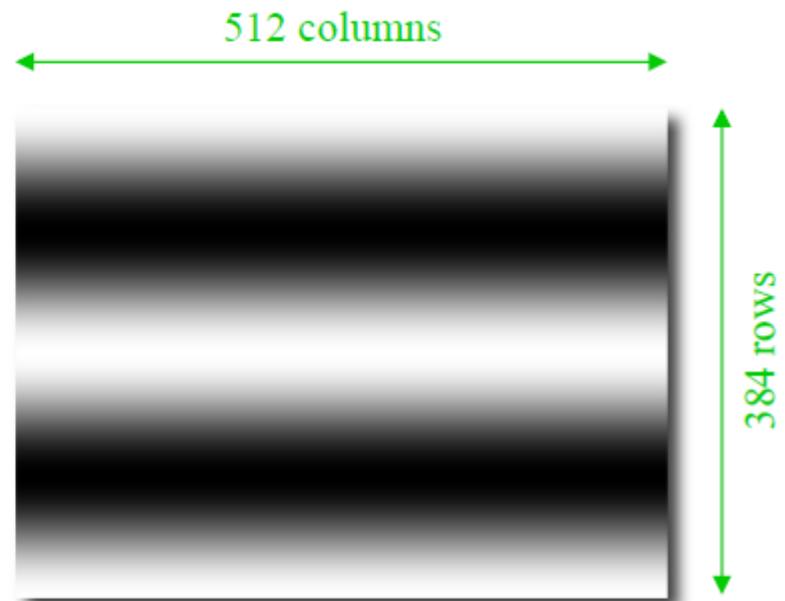
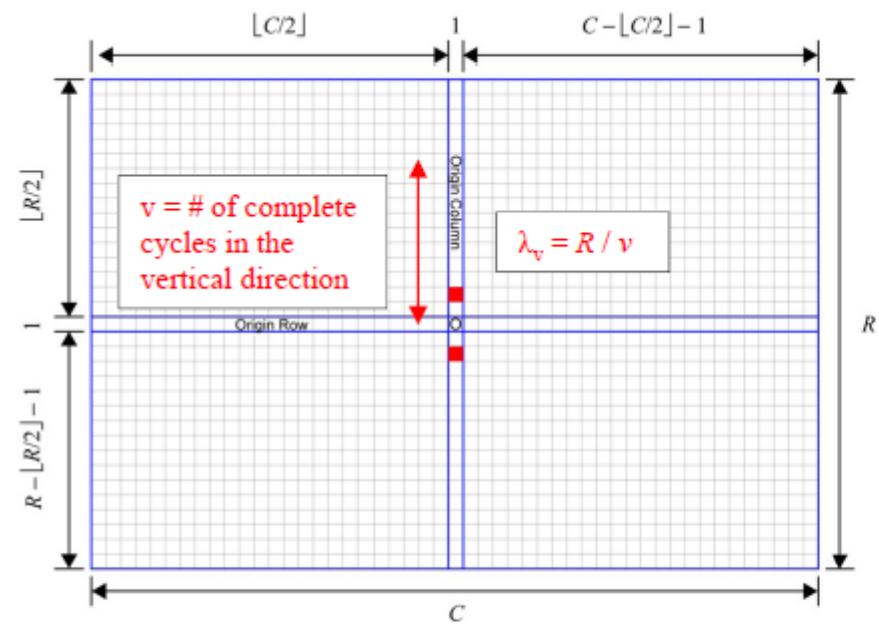


lowest-possible-frequency vertical sinusoid

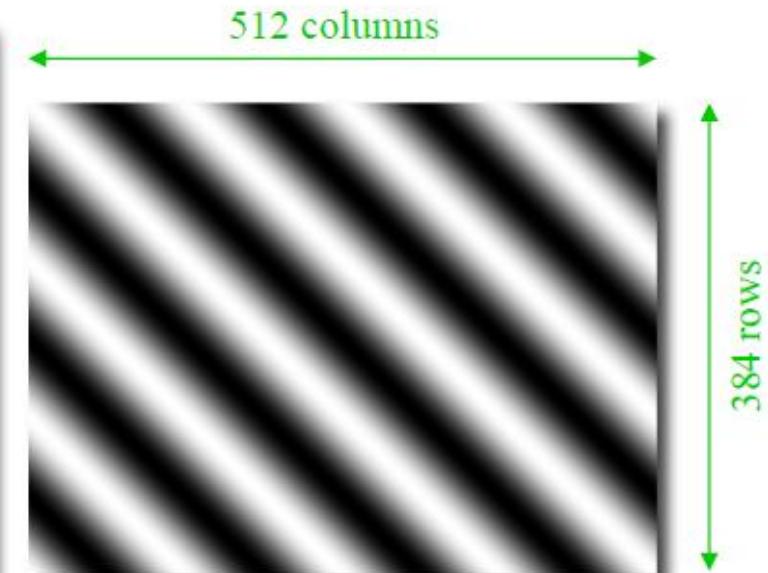
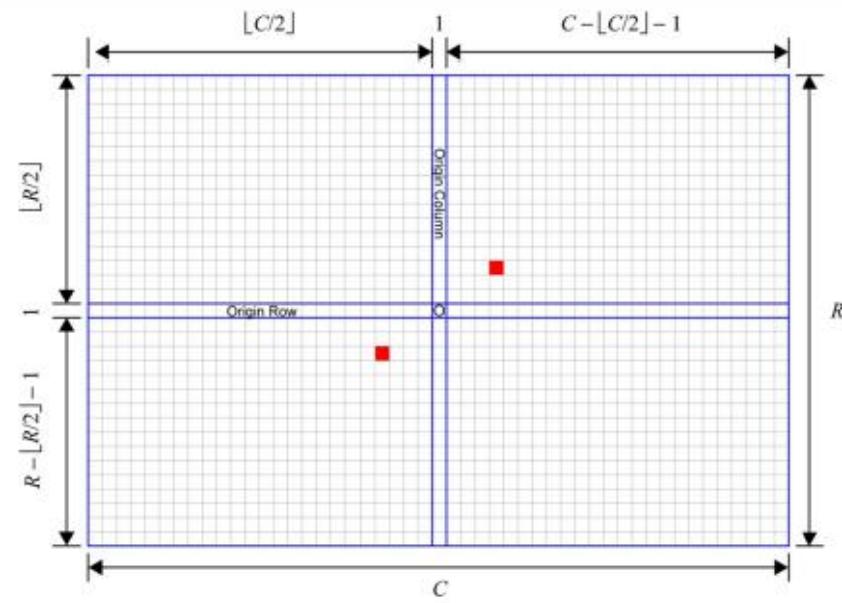
"negative diagonal" is  
the wavefront direction.



lowest-possible-frequency negative diagonal sinusoid



frequencies:  $(u, v) = (0, 2)$ ; wavelength:  $\lambda_v = 192$



frequencies:  $(u, v) = (4, 3)$ ; wavelengths:  $(\lambda_u, \lambda_v) = (128, 128)$

# Sinusoidové vzory sa zobrazia vo frekvenčnom spektre ako body.

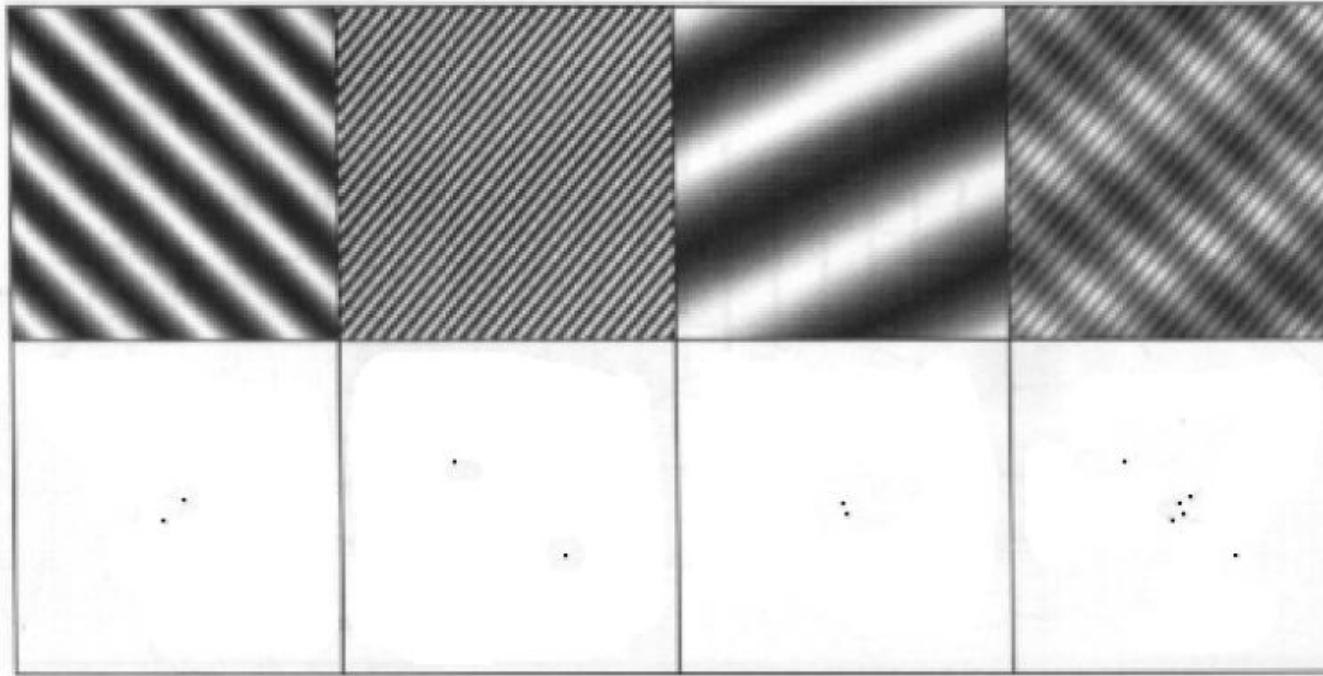


Figure 2: Images with perfectly sinusoidal variations in brightness: The first three images are represented by two dots. You can easily see that the position and orientation of those dots have something to do with what the original image looks like. The 4th image is the sum of the first three.  
(Taken from p.177 of [1].)

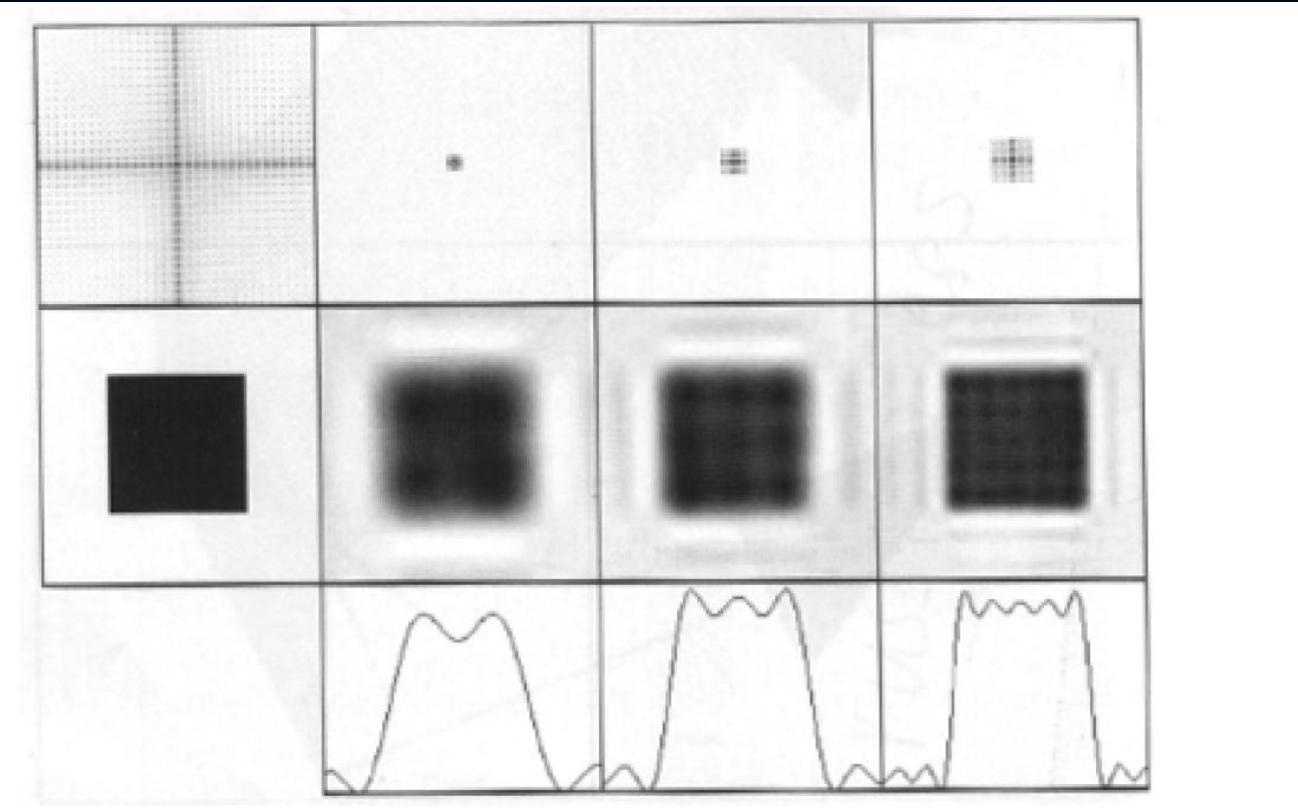


Figure 1: Images in the spatial domain are in the middle row, and their frequency space are shown on the top row. The bottom row shows the varying brightness of the horizontal line through the center of an image.

(Taken from p.178 of [1].)

- Nízke frekvencie sú pri strede a vysoké na okrajoch

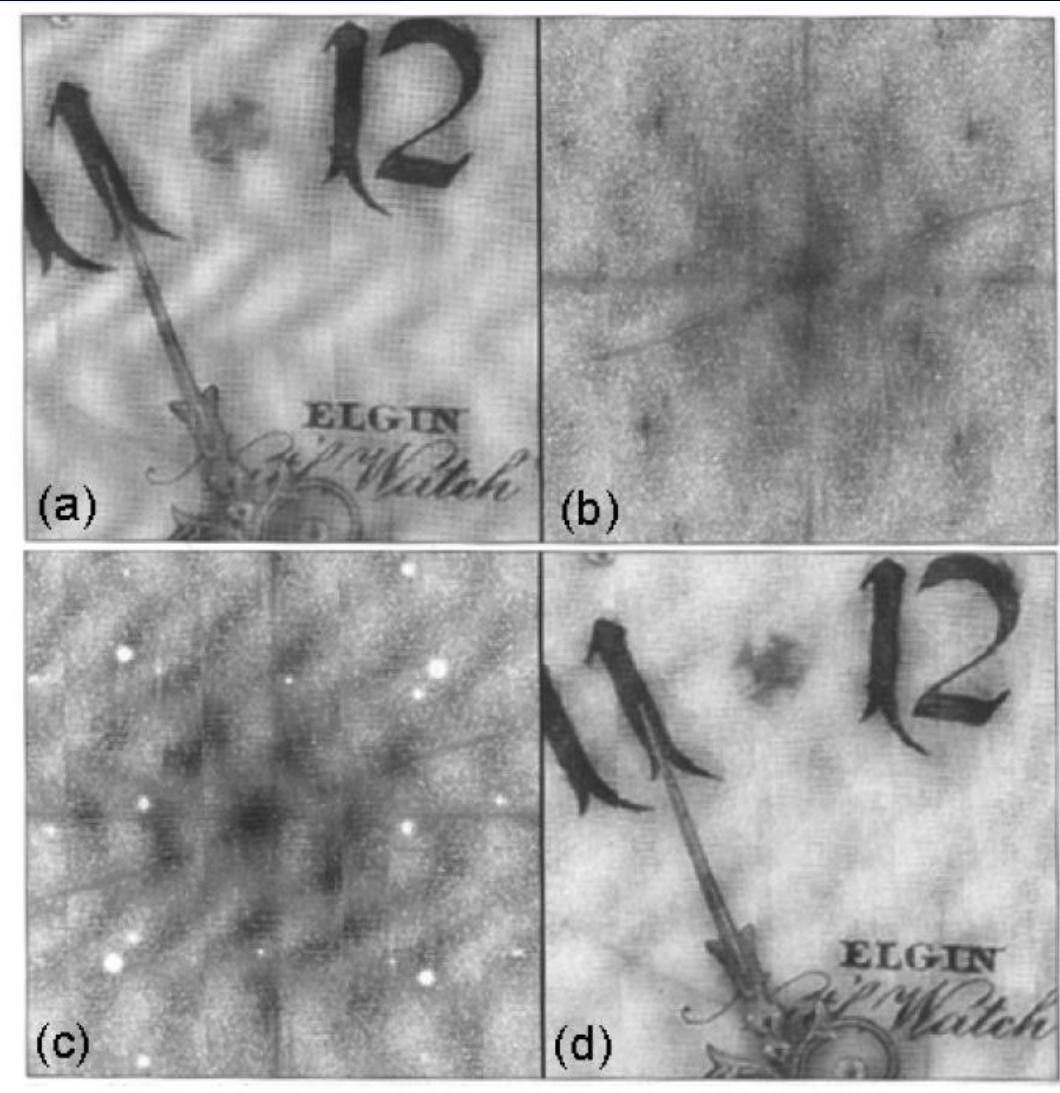
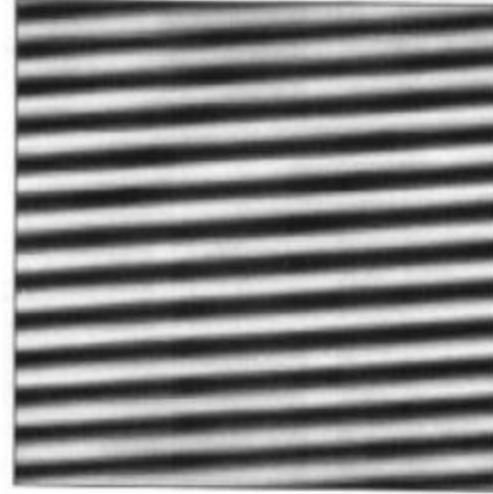
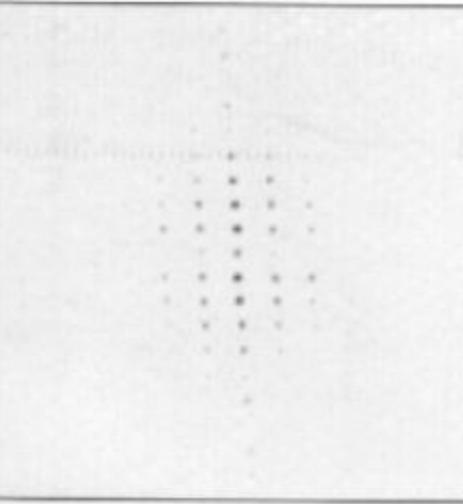
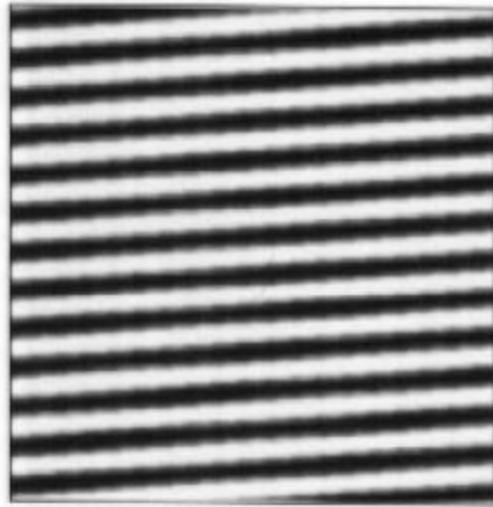
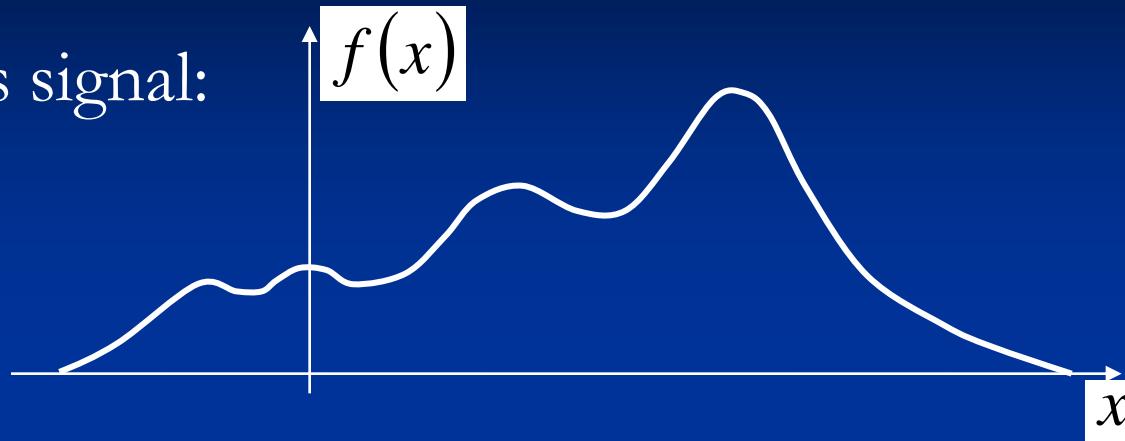


Figure 3: (a) Our dirty looking photocopied image. (b) The representation of our image in the frequency space, i.e. the star diagram. Look, you can see stars! (c) Those stars, however, do no good to the image, so we rub them out. (d) Reconstruct the image using (c) and those dirty spots on the original image are gone! (Images taken from p.204 of [1].)

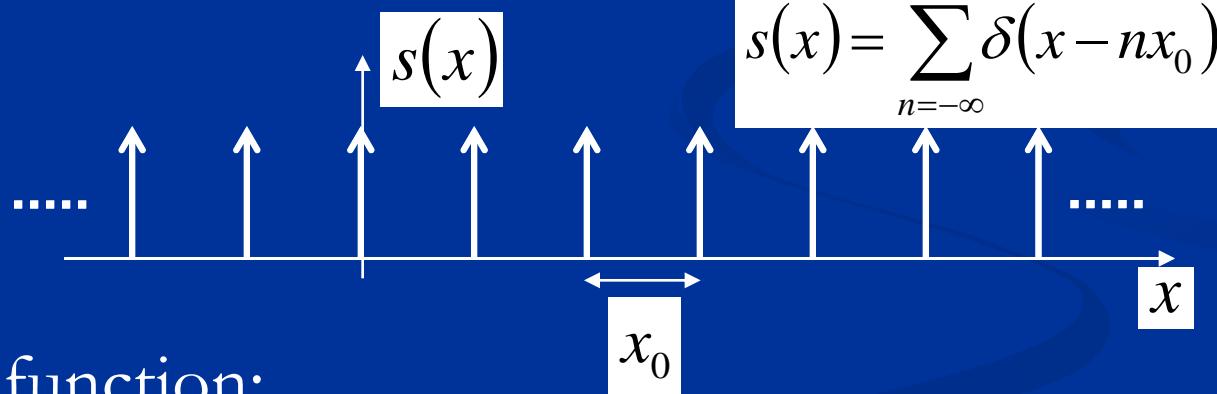


# Sampling Theorem

Continuous signal:



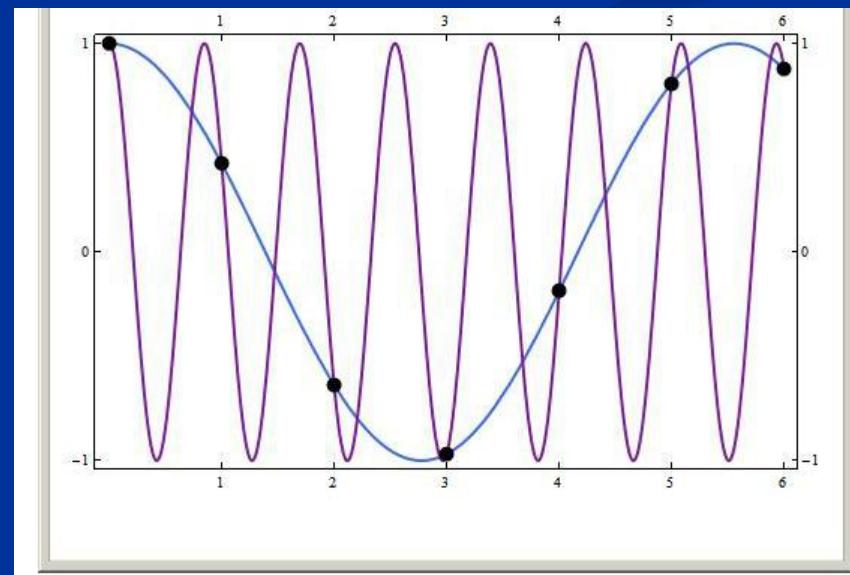
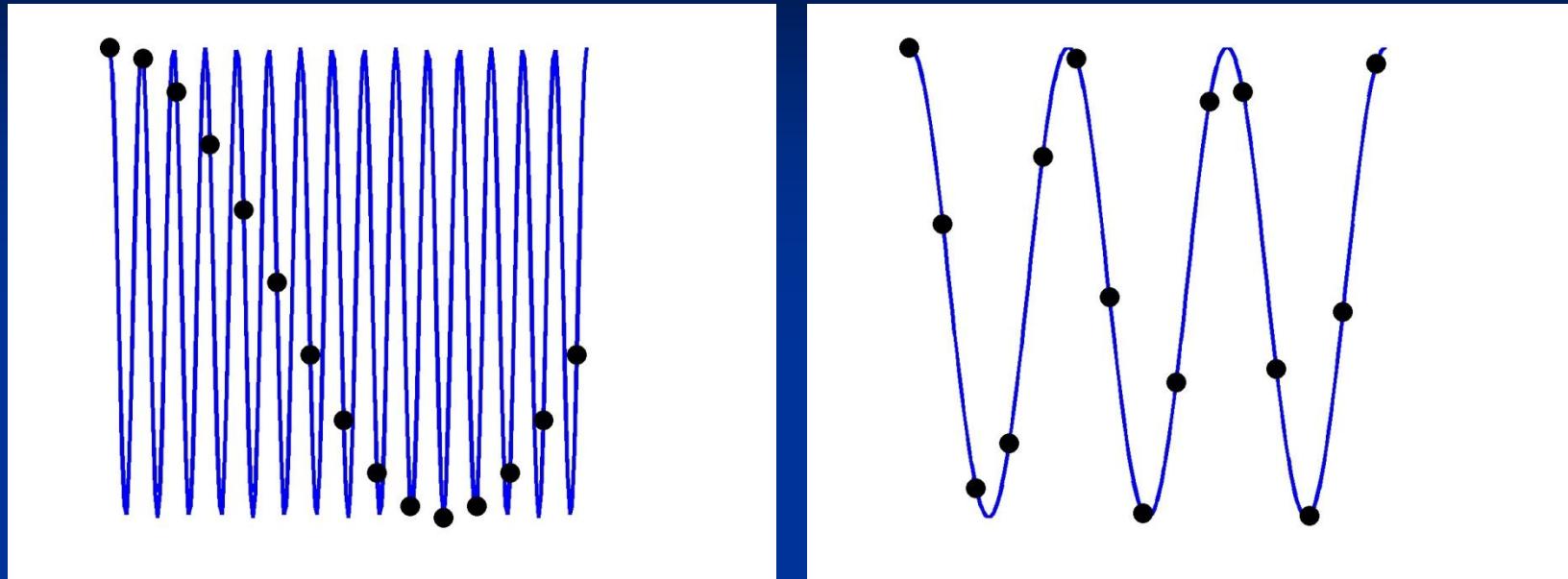
Shah function (Impulse train):



Sampled function:

$$f_s(x) = f(x)s(x) = f(x) \sum_{n=-\infty}^{\infty} \delta(x - nx_0)$$

# Sampling



# Sampling and the Nyquist rate

- Pri vzorkovaní spojitej funkcie môže vzniknúť aliasing ak vzorkovacia frekvencia nie je dostatočne vysoká
- Vzorkovacia frekvencia musí byť taká vysoká aby zachytila aj tie najvyššie frekvencie obrazu

# Sampling and the Nyquist rate

- Predíšť aliasingu:
- Vzorkovacia frekvencia  $> 2 * \text{max frekvencia v obraze}$ 
  - Treba viac ako 2 vzorky na periódu
  - Minimálna vzorkovacia frekvencia sa nazýva **Nyquist rate**

# Diskrétna Fourierova transformácia

$$F(u) = \frac{1}{N} \sum_{x=0}^{N-1} f(x) e^{-i2\pi ux/N}$$

for  $u = 0, 1, 2, \dots, N - 1$ ,

$$f(x) = \sum_{u=0}^{N-1} F(u) e^{i2\pi ux/N}$$

for  $x = 0, 1, 2, \dots, N - 1$

# Rozšírenie DFT do 2D

- Predpokladajme že  $f(x,y)$  je  $M \times N$ .

- DFT 
$$F(u, v) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(\frac{ux}{M} + \frac{vy}{N})}$$

$$(u = 0, 1, \dots, M-1, v = 0, 1, \dots, N-1)$$

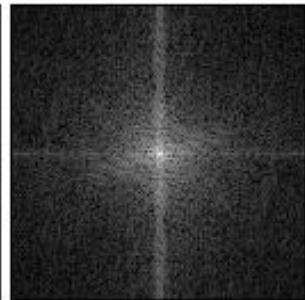
- Inverzná DFT:

$$f(x, y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) e^{j2\pi(\frac{ux}{M} + \frac{vy}{N})}$$

$$(x = 0, 1, \dots, M-1, y = 0, 1, \dots, N-1)$$

# Filtre

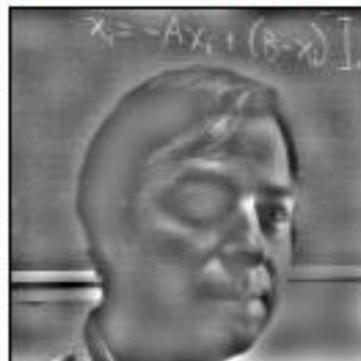
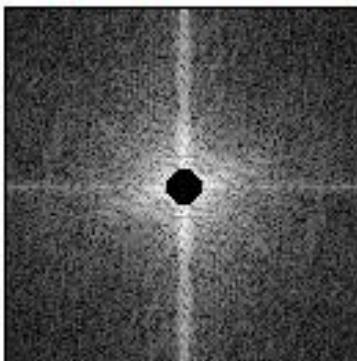
Brightness Image   Fourier Transform   Inverse Transformed



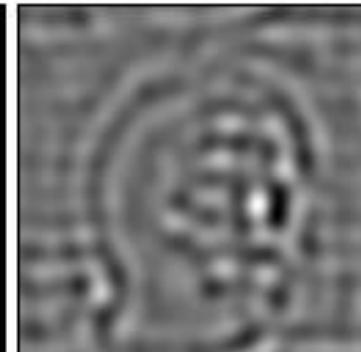
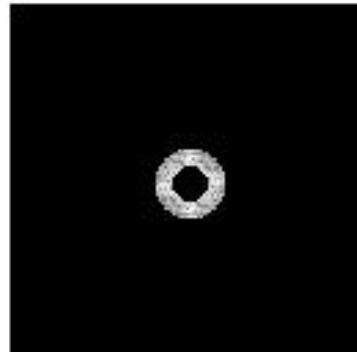
Low-Pass Filtered   Inverse Transformed



High-Pass Filtered   Inverse Transformed

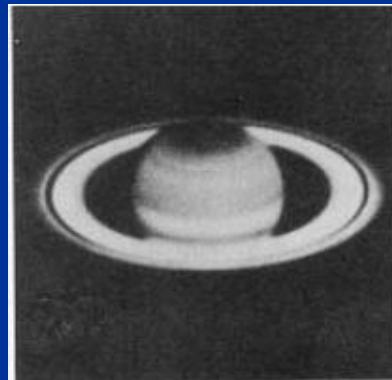


Band-Pass Filtered   Inverse Transformed



# Vizualizácia DFT

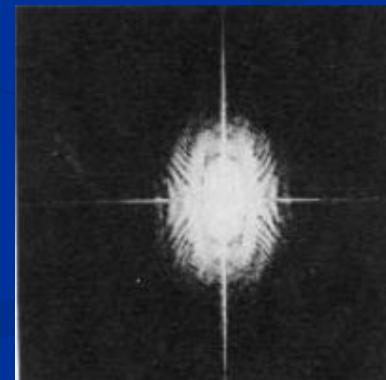
- Väčšinou zobrazujeme  $|F(u,v)|$
- Dynamický rozsah  $|F(u,v)|$  je obvykle veľmi vysoký
- Aplikujeme logaritmus: 
$$D(u, v) = c \log(1 + |F(u, v)|)$$
  
(c je konštanta)



original image



before scaling



after scaling

# DFT Properties: (1) Separability

- The 2D DFT can be computed using 1D transforms **only**:

Forward DFT:

$$F(u, v) = \frac{1}{N} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(\frac{ux+vy}{N})}$$

Inverse DFT:

$$f(x, y) = \frac{1}{N} \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} F(u, v) e^{j2\pi(\frac{ux+vy}{N})}$$

kernel is  
separable:

$$e^{-j2\pi(\frac{ux+vy}{N})} = e^{-j2\pi(\frac{ux}{N})} e^{-j2\pi(\frac{vy}{N})}$$

# DFT Properties: (1) Separability

- Rewrite  $F(u,v)$  as follows:

$$F(u, v) = \frac{1}{N} \sum_{x=0}^{N-1} e^{-j2\pi(\frac{ux}{N})} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(\frac{vy}{N})}$$

- Let's set:

$$\sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(\frac{vy}{N})} = F(x, v)$$

- Then:

$$F(u, v) = \frac{1}{N} \sum_{x=0}^{N-1} e^{-j2\pi(\frac{ux}{N})} F(x, v)$$

# DFT Properties: (1) Separability

- How can we compute  $F(x, v)$ ?

$$\sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(\frac{vy}{N})} = F(x, v) = N \left( \frac{1}{N} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(\frac{vy}{N})} \right)$$

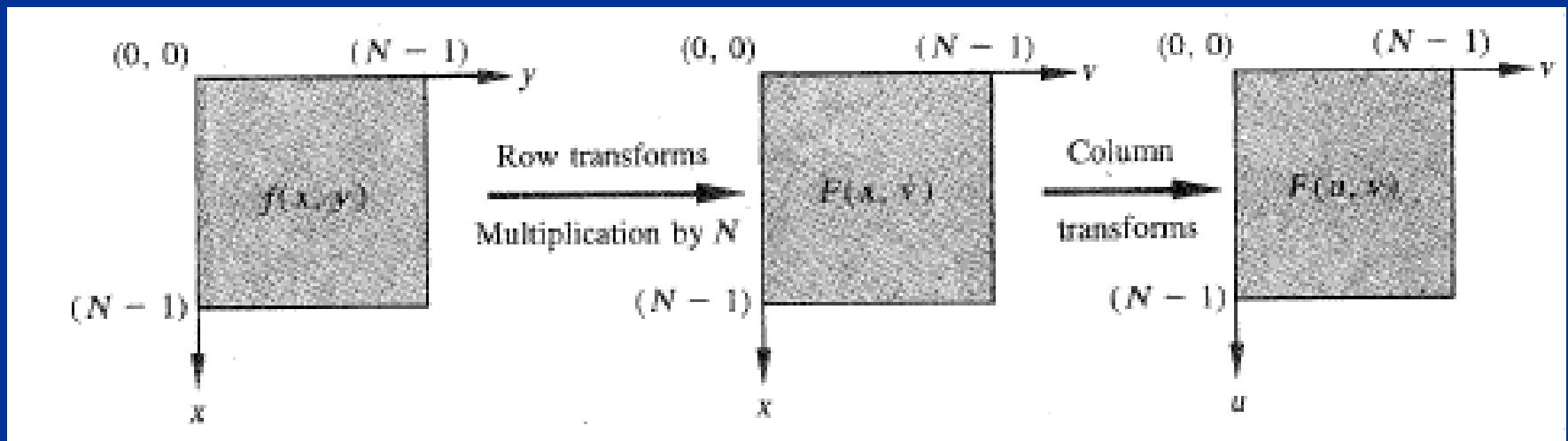
N x DFT of rows of  $f(x, y)$

- How can we compute  $F(u, v)$ ?

$$F(u, v) = \frac{1}{N} \sum_{x=0}^{N-1} e^{-j2\pi(\frac{ux}{N})} F(x, v)$$

DFT of cols of  $F(x, v)$

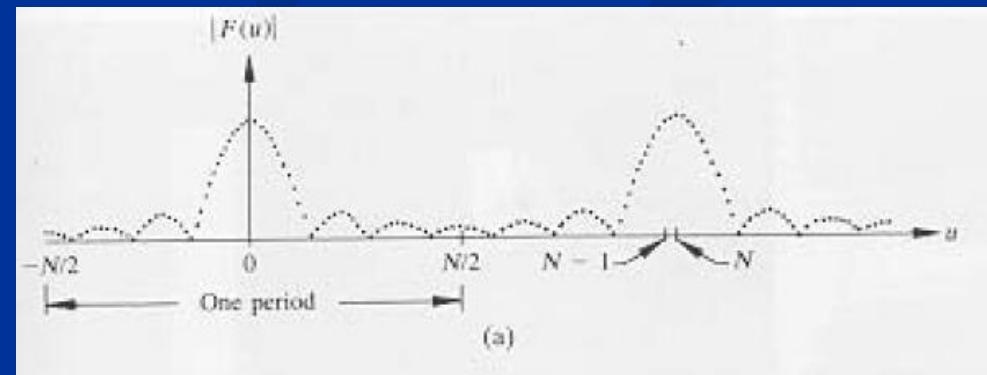
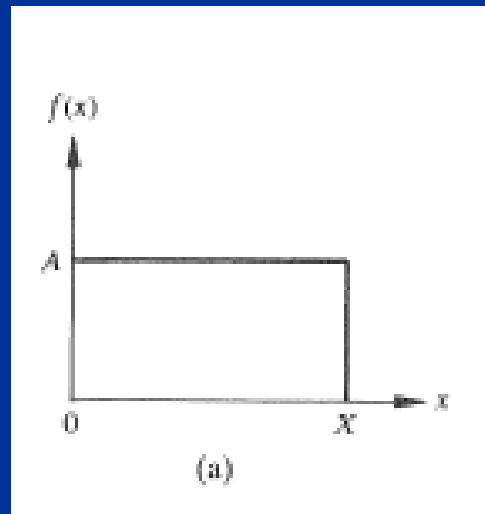
# DFT Properties: (1) Separability



# DFT Properties: (2) Periodicity

- The DFT and its inverse are periodic with period N

$$F(u, v) = F(u + N, v) = F(u, v + N) = F(u + N, v + N)$$



# DFT Properties: (3) Symmetry

- If  $f(x,y)$  is real, then

$$F(u, v) = F^*(-u, -v) \implies |F(u, v)| = |F(-u, -v)|$$

$$\begin{aligned} f(x, y) \text{ real and even} &\iff F(u, v) \text{ real and even} \\ f(x, y) \text{ real and odd} &\iff F(u, v) \text{ imaginary and odd} \end{aligned}$$

# DFT Properties: (4) Translation

$$f(x,y) \longleftrightarrow F(u,v)$$

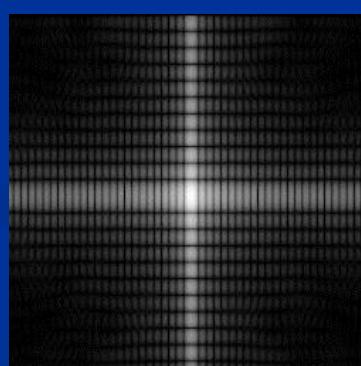
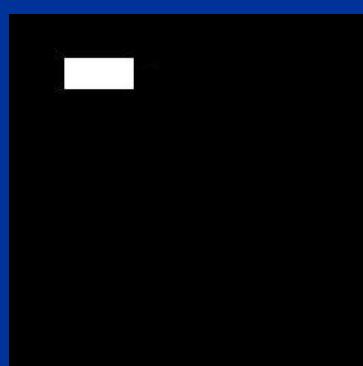
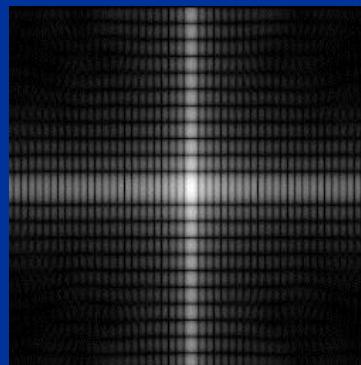
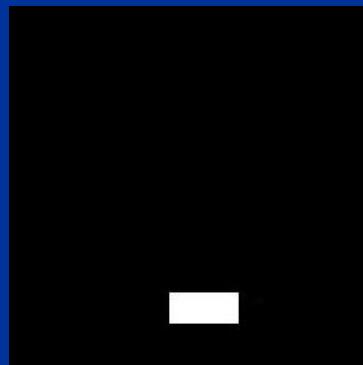
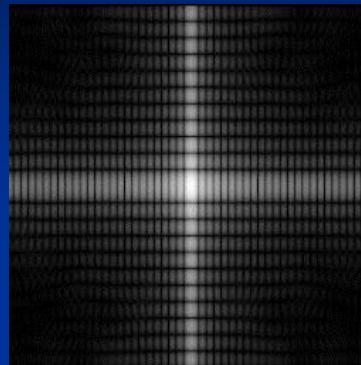
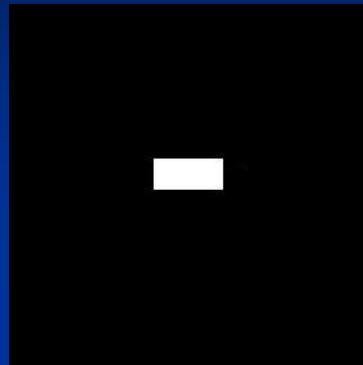
- Translation is spatial domain:

$$f(x - x_0, y - y_0) \longleftrightarrow F(u, v)e^{-j2\pi(\frac{ux_0+vy_0}{N})}$$

- Translation is frequency domain:

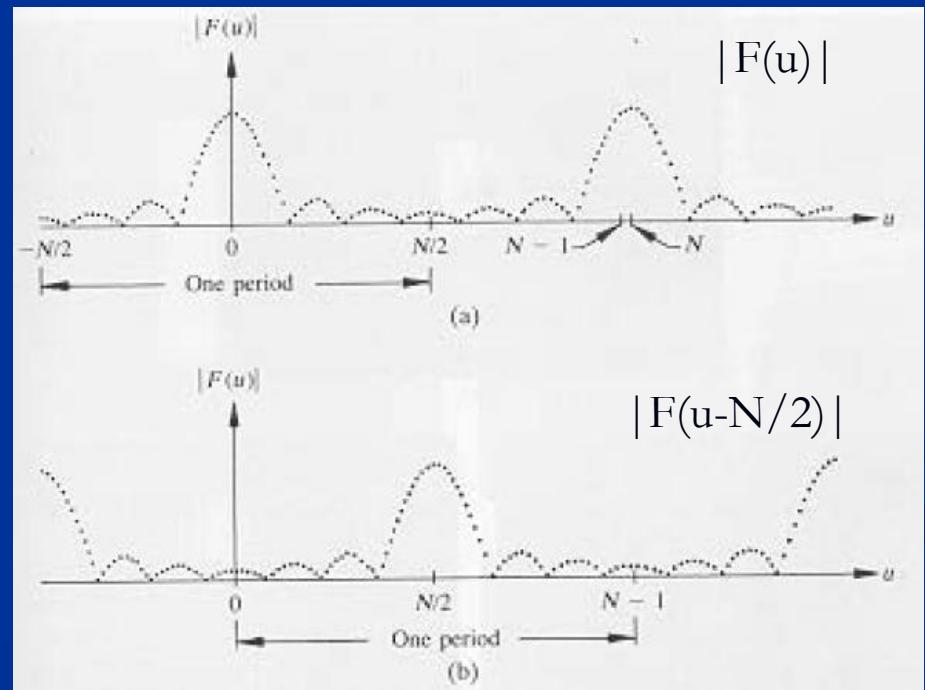
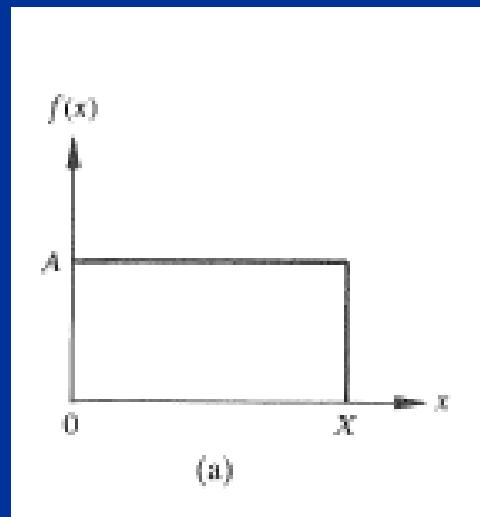
$$f(x, y)e^{j2\pi(\frac{u_0x+v_0y}{N})} \longleftrightarrow F(u - u_0, v - v_0)$$

# DFT Properties: (4) Translation



# DFT Properties: (4) Translation

Warning: to show a full period, we need to translate the origin of the transform at  $u=N/2$  (or at  $(N/2, N/2)$  in 2D)



# DFT Properties: (4) Translation

- To move  $F(u,v)$  at  $(N/2, N/2)$ , take  $u_0 = v_0 = N/2$

Using

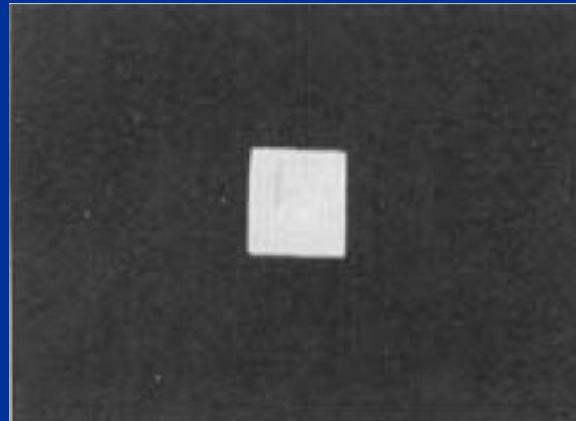
$$f(x, y)e^{j2\pi(\frac{u_0x+v_0y}{N})} \longleftrightarrow F(u - u_0, v - v_0)$$

$$e^{j2\pi(\frac{\frac{N}{2}x+\frac{N}{2}y}{N})} = e^{j\pi(x+y)} = (-1)^{x+y}$$

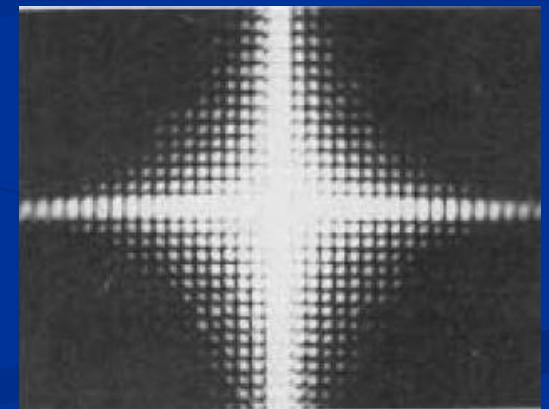
$$f(x, y)(-1)^{x+y} \longleftrightarrow F(u - N/2, v - N/2)$$

# DFT Properties: (4) Translation

$$f(x, y)(-1)^{x+y} \longleftrightarrow F(u - N/2, v - N/2)$$



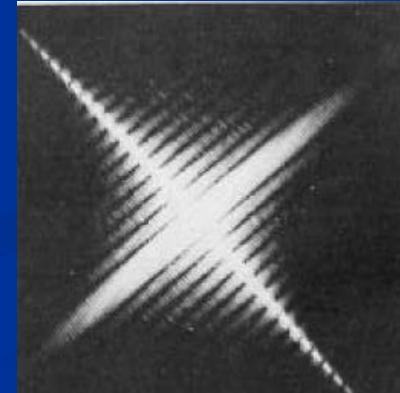
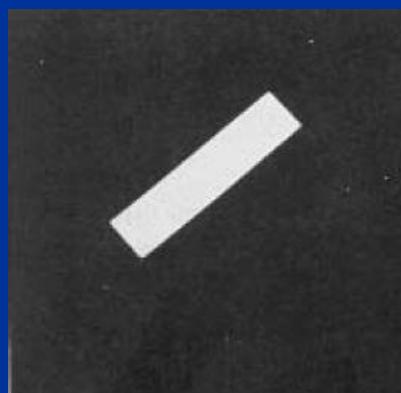
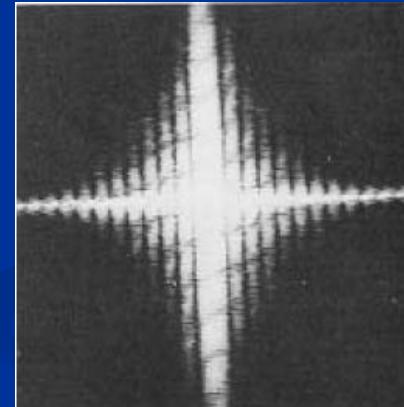
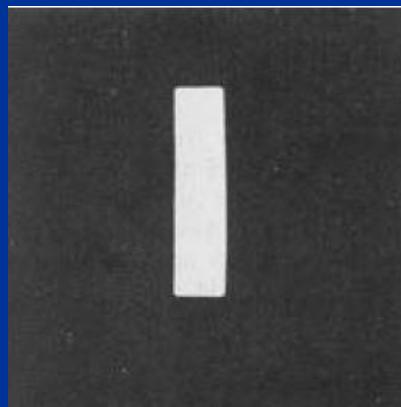
no translation



after translation

# DFT Properties: (5) Rotation

- Otočením  $f(x,y)$  o uhol  $\theta$ , sa otočí  $F(u,v)$  o ten istý uhol  $\theta$



# DFT Properties: (6) Addition/Multiplication

$$F[f(x, y) + g(x, y)] = F[f(x, y)] + F[g(x, y)]$$

but ...  $F[f(x, y)g(x, y)] \neq F[f(x, y)]F[g(x, y)]$

# DFT Properties: (7) Scale

$$af(x, y) \longleftrightarrow aF(u, v)$$

# DFT Properties: (8) Average value

Average:

$$\bar{f}(x, y) = \frac{1}{N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x, y)$$

F(u,v) at u=0, v=0:

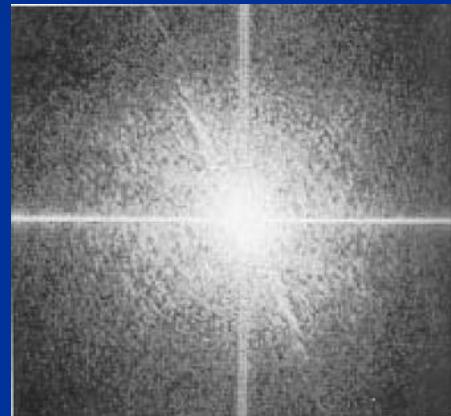
$$F(0, 0) = \frac{1}{N} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x, y)$$

So:

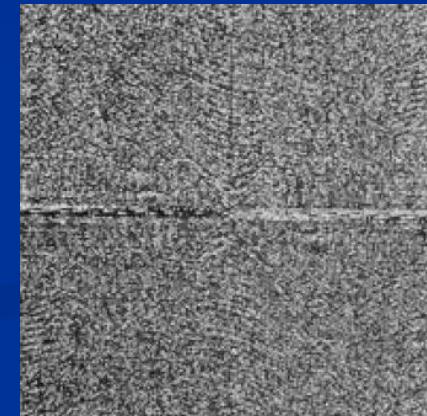
$$\bar{f}(x, y) = \frac{1}{N} F(0, 0)$$

# Magnitude and Phase of DFT

- What is more important?



magnitude



phase

- Hint: use inverse DFT to reconstruct the image using magnitude or phase only information

# Magnitude and Phase of DFT

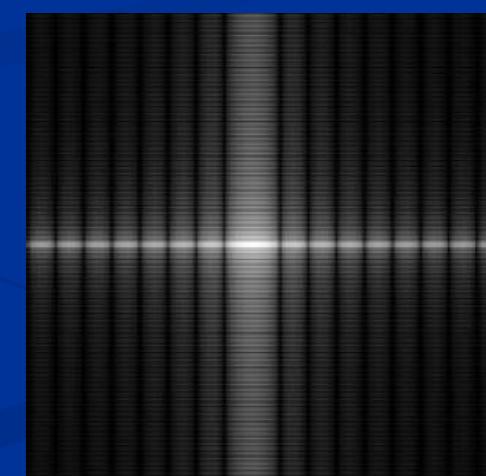
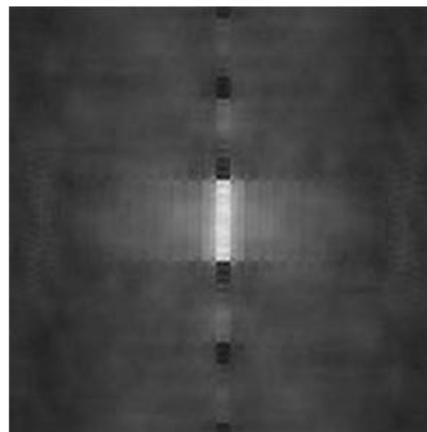
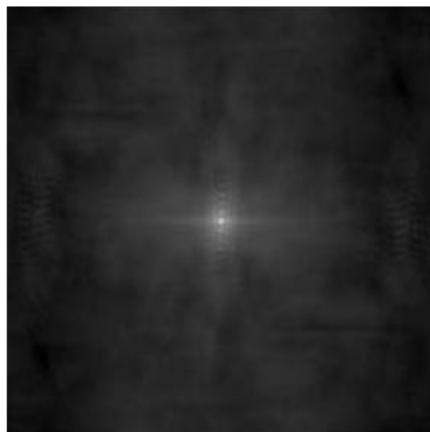
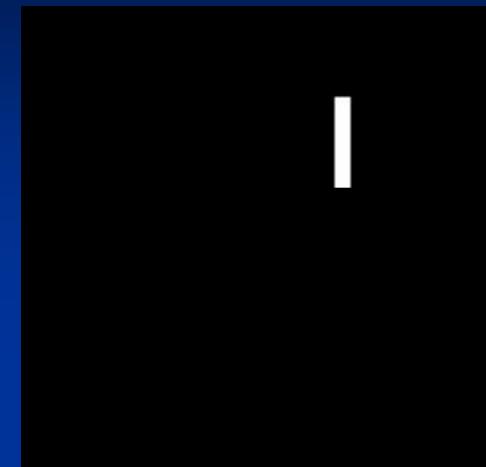
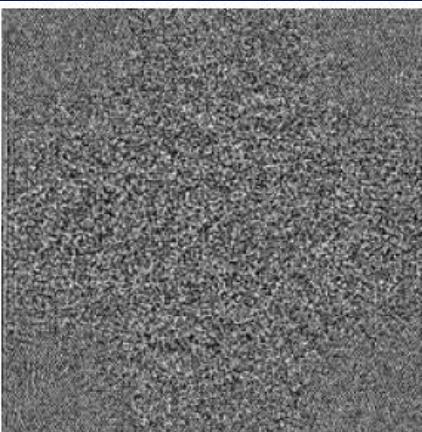


Reconstructed image using  
magnitude only  
(i.e., magnitude determines the  
contribution of each component!)



Reconstructed image using  
phase only  
(i.e., phase determines  
which components are present!)

# Magnitude and Phase of DFT



a b c  
d e f

**FIGURE 4.27** (a) Woman. (b) Phase angle. (c) Woman reconstructed using only the phase angle. (d) Woman reconstructed using only the spectrum. (e) Reconstruction using the phase angle corresponding to the woman and the spectrum corresponding to the rectangle in Fig. 4.24(a). (f) Reconstruction using the phase of the rectangle and the spectrum of the woman.

# Výpočtová náročnosť DFT

$$F_n = \sum_{k=0}^{N-1} f_k e^{-i \frac{2\pi}{N} kn}$$

*discrete Fourier transform — DFT*

$\{f_0, f_1, \dots, f_{N-1}\}$  – vstupné body

$\{F_0, F_1, \dots, F_{N-1}\}$  – výsledok Fourierovej transformácie

Výpočet pomocou 2 cyklov – > zložitosť  $O(N^2)$

Snaha zredukovať na  $O(N \log_2 N)$  – > FFT

# Fast Fourier Transform - FFT

Pre  $N=8$

$$F_n = f_0 + f_1 e^{-i \frac{2\pi}{8} n} + f_2 e^{-i \frac{2\pi}{8} 2n} + f_3 e^{-i \frac{2\pi}{8} 3n} + f_4 e^{-i \frac{2\pi}{8} 4n} + f_5 e^{-i \frac{2\pi}{8} 5n} + f_6 e^{-i \frac{2\pi}{8} 6n} + f_7 e^{-i \frac{2\pi}{8} 7n}$$

Rozdelíme na párne a nepárne členy

$$F_n = \left[ f_0 + f_2 e^{-i \frac{2\pi}{8} 2n} + f_4 e^{-i \frac{2\pi}{8} 4n} + f_6 e^{-i \frac{2\pi}{8} 6n} \right] + e^{-i \frac{2\pi}{8} n} \left[ f_1 + f_3 e^{-i \frac{2\pi}{8} 2n} + f_5 e^{-i \frac{2\pi}{8} 4n} + f_7 e^{-i \frac{2\pi}{8} 6n} \right]$$

Urobíme to isté vo zátvorkách

$$F_n = \left[ \left( f_0 + f_4 e^{-i \frac{2\pi}{8} 4n} \right) + e^{-i \frac{2\pi}{8} 2n} \left( f_2 + f_6 e^{-i \frac{2\pi}{8} 4n} \right) \right] + e^{-i \frac{2\pi}{8} n} \left[ \left( f_1 + f_5 e^{-i \frac{2\pi}{8} 4n} \right) + e^{-i \frac{2\pi}{8} 2n} \left( f_3 + f_7 e^{-i \frac{2\pi}{8} 4n} \right) \right]$$

$$F_n = \left[ \left( f_0 + f_4 e^{-i\frac{2\pi}{8}4n} \right) + e^{-i\frac{2\pi}{8}2n} \left( f_2 + f_6 e^{-i\frac{2\pi}{8}4n} \right) \right] + e^{-i\frac{2\pi}{8}n} \left[ \left( f_1 + f_5 e^{-i\frac{2\pi}{8}4n} \right) + e^{-i\frac{2\pi}{8}2n} \left( f_3 + f_7 e^{-i\frac{2\pi}{8}4n} \right) \right]$$

$$F_n = \left[ \left( f_0 + f_4 e^{-in\pi} \right) + e^{-i\frac{\pi}{2}n} \left( f_2 + f_6 e^{-in\pi} \right) \right] + e^{-i\frac{\pi}{4}n} \left[ \left( f_1 + f_5 e^{-in\pi} \right) + e^{-i\frac{\pi}{2}n} \left( f_3 + f_7 e^{-in\pi} \right) \right]$$

$$e^{i(\phi+2\pi)} = e^{i\phi}$$

Suma vo vnútorných zátvorkách je rovnaká pre  
 $n=0, 2, 4, 6$  a pre  $n=1, 3, 5, 7$

Takže máme 4 (zátvorky)  $\times 2 = 8$  súčtov na najnižšej úrovni

Ked'že  $n=1, 3, 5, 7$  zodpovedá polovici periódy  $\Pi$  a platí  $e^{i(\phi+\pi)} = -e^{i\phi}$   
 dostaneme 1 pre  $n=0, 2, 4, 6$  a  
 - 1 pre  $n=1, 3, 5, 7$

$$F_n = \left[ \left( f_0 + f_4 e^{-i\pi n} \right) + e^{-i\frac{\pi}{2}n} \left( f_2 + f_6 e^{-i\pi n} \right) \right] + e^{-i\frac{\pi}{4}n} \left[ \left( f_1 + f_5 e^{-i\pi n} \right) + e^{-i\frac{\pi}{2}n} \left( f_3 + f_7 e^{-i\pi n} \right) \right]$$

V hranatých zátvorkách je perióda exponentu n=4  
 – > n=0, 4; n=1, 5; n=2, 6 a n=3, 7.

Pre n=0, 4 faktor je 1  
 pre n=2, 6 je to -1;  
 pre n=1, 5 je -i  
 a pre n=3, 7 je i.

Takže máme 2 (zátvorky) x 4 súčtov na strednej úrovni

$$F_n = \left[ \left( f_0 + f_4 e^{-i\pi n} \right) + e^{-i\frac{\pi}{2}n} \left( f_2 + f_6 e^{-i\pi n} \right) \right] + e^{-i\frac{\pi}{4}n} \left[ \left( f_1 + f_5 e^{-i\pi n} \right) + e^{-i\frac{\pi}{2}n} \left( f_3 + f_7 e^{-i\pi n} \right) \right]$$

Na najvyššej úrovni máme 1 sumu a períodu exponentu  $n=8$   
 $- > 1 \times 8$  súčtov

Polovicu z nich môžeme vypočítat' iba zmenou znamienka

Na každej úrovni urobíme 8 operácií  $(n)$   
 Máme  $3 = \log 8$  úrovne  $(\log n)$

$\left. \right\} O(n \log n)$

# FFT - algoritmus

$$\{f_0, f_1, f_2, f_3, f_4, f_5, f_6, f_7\} \rightarrow \{f_0, f_4, f_2, f_6, f_1, f_5, f_3, f_7\}.$$

Prepare input data for summation — put them into convenient order;  
For every summation level:

For every exponent factor of the half-period:

Calculate factor;

For every sum of this factor:

Calculate product of the factor and the second term of the sum;

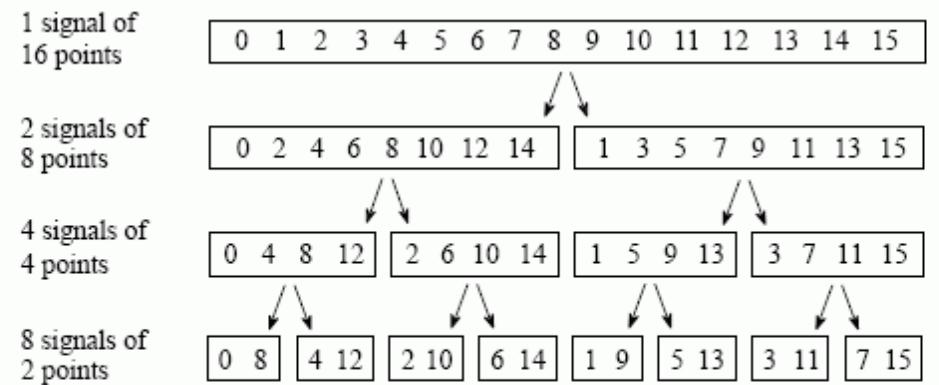
Calculate sum;

# Poradie koeficientov pre FFT

$$F_n = \left[ \left( f_0 + f_4 e^{-i\pi n} \right) + e^{-i\frac{\pi}{2}n} \left( f_2 + f_6 e^{-i\pi n} \right) \right] + e^{-i\frac{\pi}{4}n} \left[ \left( f_1 + f_5 e^{-i\pi n} \right) + e^{-i\frac{\pi}{2}n} \left( f_3 + f_7 e^{-i\pi n} \right) \right]$$

0	000	000	000	0
1	001	010	100	4
2	010	100	010	2
3	011	110	110	6
4	100	001	001	1
5	101	011	101	5
6	110	101	011	3
7	111	111	111	7

# Príklad pre N=16



Sample numbers  
in normal order

Decimal    Binary

0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

Sample numbers  
after bit reversal

Decimal    Binary

0	0000
8	1000
4	0100
12	1100
2	0010
10	1010
6	0100
14	1110
1	0001
9	1001
5	0101
13	1101
3	0011
11	1011
7	0111
15	1111

