Skiplists ending

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$$E[height] \leq \sum_{i=0}^{\inf} np^i$$

Rozdelime sumu na dve casti:

$$E[height] \leq \sum_{i=0}^{L(n)-1} np^i + \sum_{i=L(n)}^{\inf} np^i$$

Cleny v prvej sume nahradime 1, lebo sme si pri nerovnostiach dovolovali prilis velky gap a vychadzaju tam prilis velke pravdepodobnosti, co nam kazi celu sumu. Nahradenim clenov v prvej sume sa zbavime toho n, co nam na cviku vychadzal.

$$\sum_{i=0}^{L(n)-1} np^i + \sum_{i=L(n)}^{\inf} np^i \leq L(n) + \sum_{i=L(n)}^{\inf} np^i$$

V druhej sume upravime index, aby siel od nuly, aby sme vedeli zratat vysledok:

$$\begin{aligned}
L(n) + \sum_{i=L(n)}^{\inf} np^i &= L(n) + \sum_{i=0}^{\inf} np^{L(n)}p^i \\
&= L(n) + np^{L(n)}\frac{1}{1-p} \\
&= L(n) + n\frac{1}{p^{log_p n}}\frac{1}{1-p} \\
&= L(n) + n\frac{1}{n}\frac{1}{1-p} \\
&= L(n) + \frac{1}{1-p} \\
&= O(lg(n))
\end{aligned}$$