# Stellar spectra and classification of stars

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**Abstract.** We discovered . . .

## 1. Introduction

Stellar spectra gives a lot of information about the conditions in different stars. Absorption lines in the spectra appears if the atoms in the outer layers of the stars absorbs photons emitted from the star. Therefore the absorption lines can tell something about the content of elements in the star. A classification method called the Harvard spectral sequence is based on this information from the stellar spectra. Figure 1 shows some stellar spectra illustrating the Harvard spectral sequence for wavelengths between around 3800-5000 Å. The spectra can be compared to energy level diagrams for different elements to recognize which elements that have made the different absorption lines.

The energy level diagram for hydrogen is shown in figure 2. According to this diagram, the Balmer $\beta$ line should absorb wavelengths of 4861 Å, and is the only line with a wavelength within the same range as the spectra in figure 1. At this wavelength in the spectra there is clearly, strong black line as expected for the five uppermost stars. The stars further down has no line at all for the Balmer $\beta$ line, or just a very weak one.

This article will study the Harvard spectral sequence and spectral lines. It will discuss the relationship between the population density of particles with different ionization level and the strength of the absorption lines in stellar spectra. The report will mainly focus on a fictional element called *Schadeenium*, but will also look at real elements.

### 1.1. The Boltzmann and Saha laws

The Boltzmann and Saha laws describe the population density of different ionization stages and over the discrete energy levels within each stage for a specific element. Saha's law describes how many particles of an element that occupies the different ionization stages (e.g. twice-ionized). Saha distribution is given by

$$\frac{N_{r+1}}{N_r} = \frac{1}{N_e}\frac{2U_{r+1}}{U_r}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_r/kT}, \tag{1}$$

where $T$ is the temperature, $k$ is the Boltzmann constant, $h$ is the Planck constant, $N_e$ is the electron density, $m_e$ is the electron mass, $\chi_r$ is the threshold energy needed to ionize a particle from stage $r$ to $r+1$, and $U_{r+1}$ and $U_r$ are the partition functions of the the respective ionization stages. $N_r = \sum_s n_{r,s}$ is the total particle density in all levels of ionization stage $r$. At last, the partition function $U_r$ is defined by

$$U_r \equiv \sum_s g_{r,s} e^{-\chi_{r,s}/kT} \tag{2}$$

The Boltzmann law describes the distribution of the different ionized particles over the discrete energy levels within each ionization (e.g. a neutral atom with one electron excited to the second level). The Boltzmann distribution is given by

$$\frac{n_{r,s}}{N_r} = \frac{g_{r,s}}{U_r} e^{-\chi_{r,s}/kT}. \tag{3}$$

Here $r$ and $s$ represent the level $s$ of the ionization stage $r$, $n$ is the number density in unit $m^{-3}$, $g$ is the statistical weight and $\chi$ is the excitation energy for the given level measured from the ground state $(r, 1)$.

### 1.2. Schadeenium

To analyze how the absorption lines are connected to the population density of particles with different ionization stages and excitation levels, a simple fictional element is studied. The element we call Schadeenium and it will be referred to as E. Its ionization energies are $\chi_1 = 7$ eV for neutral stage, $\chi_2 = 16$ eV for the first ionized stage, $\chi_3 = 31$ eV for the second ionized stage and $\chi_4 = 51$ eV for the this ionized stage. The excitation energies is given by $\chi_{r,s} = s - 1$ eV. This means that there is 7 excitation levels in the neutral stage, 16 in the first ionized stage etc. For Schadeenium it is also assumed that the statistical weights is $g_{r,s} = 1$ for all levels $(r, s)$.

## 2. Observations

Before analyzing the fictional element Schadeenium, an assumption needs to be made. From observations, it seems like the strength of the absorption lines scales linearly with the population density of the lower level of the corresponding transition. If the stellar outer layers have a lot of atoms with the lower level of the corresponding transition, it is natural to conclude that there would be a lot of atoms which could excite and absorb photons. This is proven to not be correct, although absorption lines do get stronger at larger lower-level population. In this report we will anyways assume that the scaling is linear. With this assumption, we can find initial rough estimates of the strength ratios of different absorption lines.

Take a neutral hydrogen atom as an example. The ratio between the strengths for the different excitation lines for hydrogen can be found using equation 3. The statistical weight for a neutral hydrogen is given by $g = 2s^2$. If we then assume we have a temperature equal to the the Sun's surface temperature ($T = 5777$ K), we get a ration between the Lyman $\alpha$ and Balmer $\alpha$ line of

$$\frac{n_{1,1}}{n_{1,2}} = \frac{2e^0}{8e^{-10.20/kT}} \approx 2 \cdot 10^8.$$

The same calculation can be done for the Balmer $\alpha$ and Paschen $\alpha$ line strength ratio,

$$\frac{n_{1,2}}{n_{1,3}} = \frac{8e^{-10.20/kT}}{18e^{-12.09/kT}} \approx 20.$$

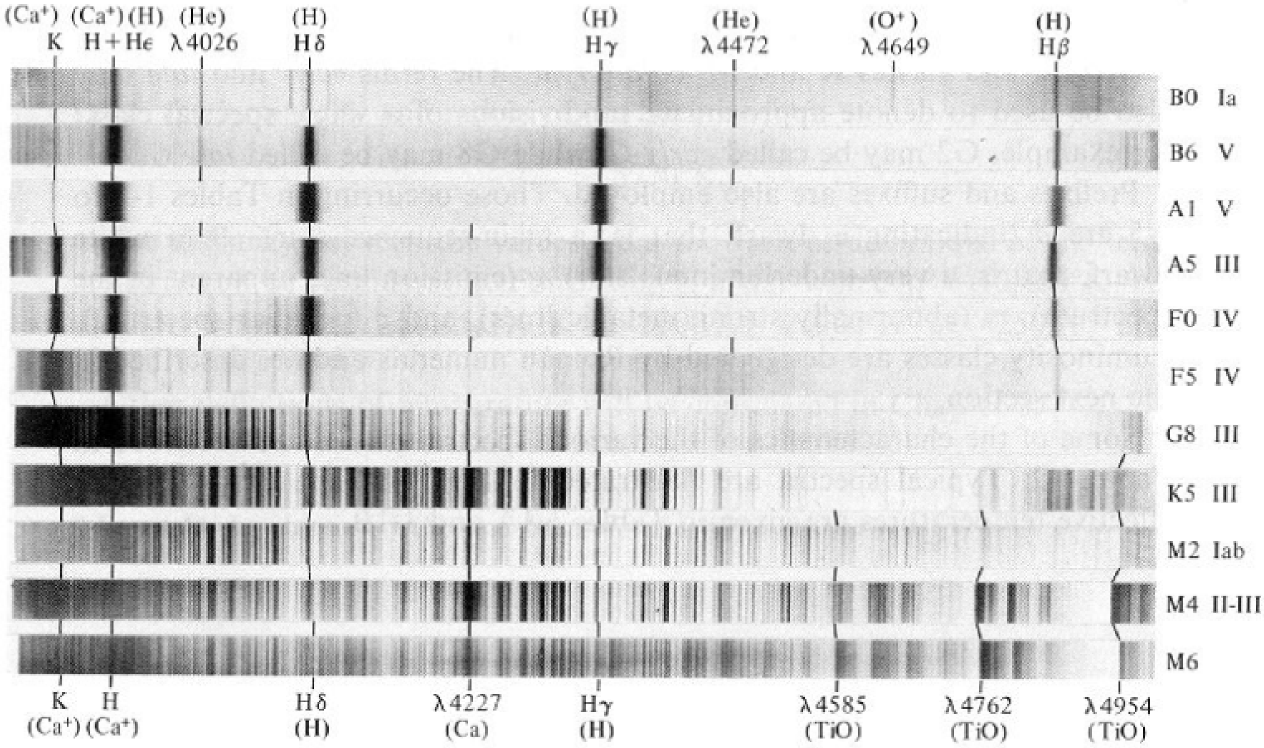


**Fig. 1.** Stellar spectra illustrating the Harvard spectral sequence. The dark lines represents the absorption lines for different wavelengths. The Harvard classification of the different stars, are written on the right. The stars are classified as either O, B, A, F, G, K or M, where the hottest stars are classified as O and the coolest as M. Novotny (1973)

| $U_r$ | 5000 K | 10 000 K | 20 000 K |
|---|---|---|---|
| $U_1$ | 1.11 | 1.46 | 2.23 |
| $U_2$ | 1.11 | 1.46 | 2.27 |
| $U_3$ | 1.11 | 1.46 | 2.27 |
| $U_4$ | 1.11 | 1.46 | 2.27 |

And for the Paschen $\alpha$ and Brackett $\alpha$ lines,

$$\frac{n_{1,2}}{n_{1,3}} = \frac{18e^{-12.09/kT}}{32e^{-12.75/kT}} \approx 2.$$

So on a sun-like star, the Lyman $\alpha$ line would be much stronger than the other lines.

## 3. Method

## 4. Results

Table 4 shows the calculations of the partition function (equation 2) for Schadeenium for different ionization stages and temperature. It shows that the partition function is almost invariant to temperature and is almost constant for different ionization stages. It is also worth noting that the partition function re of order unity.

## 5. Conclusions

The answer is 42.

*Acknowledgements.*

| $n_{r,s}/N_r$ | 5000 K | 10 000 K | 20 000 K |
|---|---|---|---|
| $s = 1$ | $9.0 \cdot 10^{-1}$ | $6.9 \cdot 10^{-1}$ | $4.5 \cdot 10^{-1}$ |
| $s = 2$ | $8.9 \cdot 10^{-2}$ | $2.2 \cdot 10^{-1}$ | $2.5 \cdot 10^{-1}$ |
| $s = 3$ | $8.7 \cdot 10^{-3}$ | $6.7 \cdot 10^{-2}$ | $1.4 \cdot 10^{-1}$ |
| $s = 4$ | $8.5 \cdot 10^{-4}$ | $2.1 \cdot 10^{-2}$ | $7.9 \cdot 10^{-2}$ |
| $s = 5$ | $8.4 \cdot 10^{-5}$ | $6.6 \cdot 10^{-3}$ | $4.4 \cdot 10^{-2}$ |
| $s = 6$ | $8.2 \cdot 10^{-6}$ | $2.1 \cdot 10^{-3}$ | $2.5 \cdot 10^{-2}$ |
| $s = 7$ | $8.1 \cdot 10^{-7}$ | $6.5 \cdot 10^{-4}$ | $1.4 \cdot 10^{-2}$ |
| $s = 8$ | $7.9 \cdot 10^{-8}$ | $2.0 \cdot 10^{-4}$ | $7.7 \cdot 10^{-3}$ |
| $s = 9$ | $7.8 \cdot 10^{-9}$ | $6.4 \cdot 10^{-5}$ | $4.3 \cdot 10^{-3}$ |
| $s = 10$ | $7.6 \cdot 10^{-10}$ | $2.0 \cdot 10^{-5}$ | $2.4 \cdot 10^{-3}$ |
| $s = 15$ | $7.0 \cdot 10^{-15}$ | $6.0 \cdot 10^{-8}$ | $1.3 \cdot 10^{-4}$ |

| $N_r/N$ | ion | 5 000 K | 10 000 K | 20 000 K |
|---|---|---|---|---|
| $r = 1$ | $E$ | $9.06 \cdot 10^{-1}$ | $4.78 \cdot 10^{-4}$ | $2.73 \cdot 10^{-10}$ |
| $r = 2$ | $E^+$ | $9.39 \cdot 10^{-2}$ | $9.45 \cdot 10^{-1}$ | $1.80 \cdot 10^{-4}$ |
| $r = 3$ | $E^{2+}$ | $8.26 \cdot 10^{-12}$ | $5.44 \cdot 10^{-2}$ | $6.32 \cdot 10^{-1}$ |
| $r = 4$ | $E^{3+}$ | $5.52 \cdot 10^{-37}$ | $8.64 \cdot 10^{-11}$ | $3.68 \cdot 10^{-1}$ |
| $r = 5$ | $E^{4+}$ | $4.61 \cdot 10^{-82}$ | $1.57 \cdot 10^{-29}$ | $1.72 \cdot 10^{-6}$ |

## References

Novotny, E. 1973, Introduction to stellar atmospheres and interiors ADS

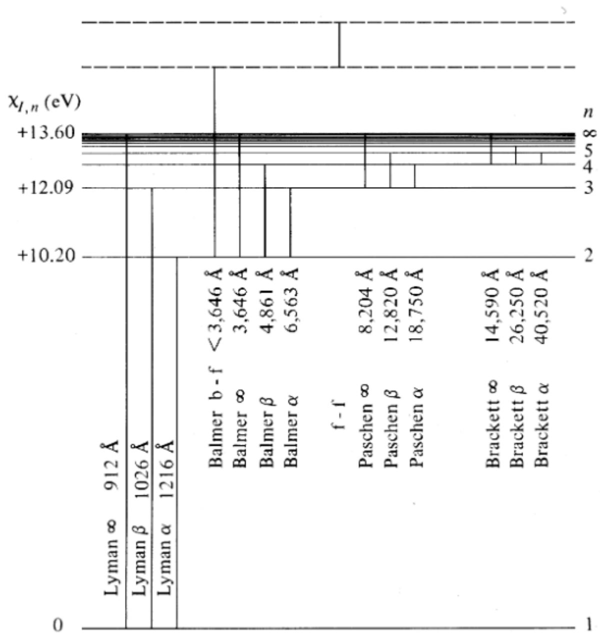


**Fig. 2.** Energy levels for hydrogen, each horizontal line represents one energy level. The excitation energies $\chi_{1,n}$ from the ground state to the respective energy levels are written in eV on the left. $n$ denotes the energy level. The figure also shows the wavelengths the energy levels represent when light is emitted or absorbed from the atom. Novotny (1973)