

# Midterm #2

# Review Day

CptS 223 - Fall 2017 - Aaron Crandall



# Today's Agenda

- Announcements
- Thing of the day
- Going over the sorting quiz
- Some Linux commands
- Hashing
- Heaps
- Sorting

# Announcements



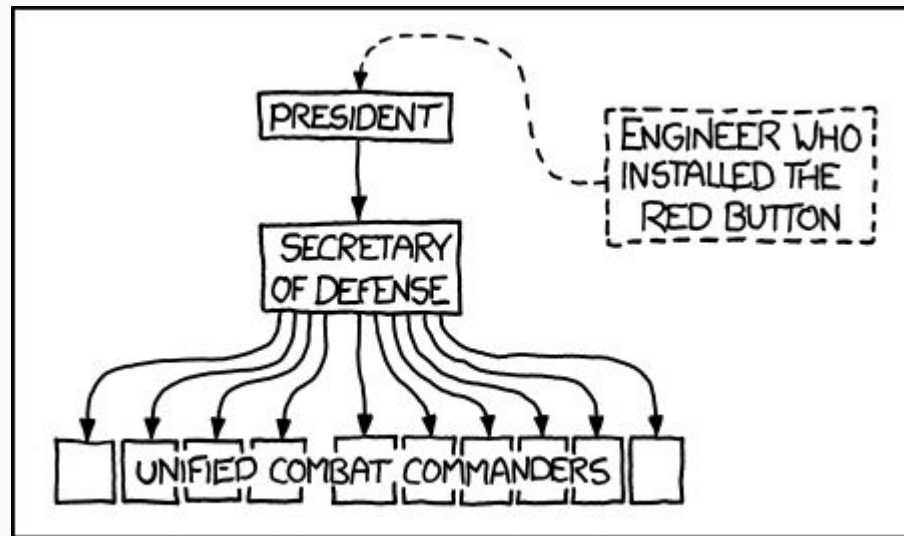
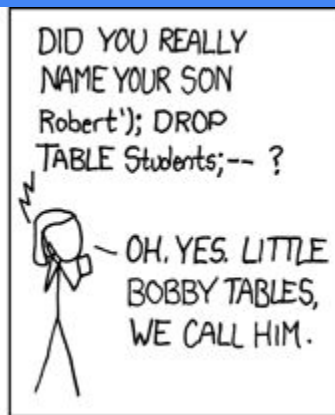
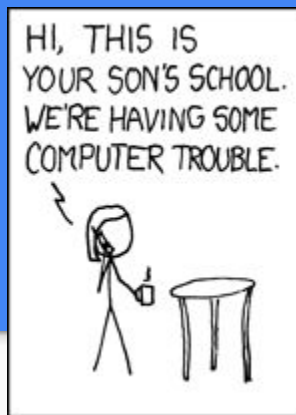
- Midterm on Friday
  - Hashing, Heaps, Sorting
- I'll put HW3 solutions up at 12:01am tomorrow



# XKCD!

## Lessons:

- Users will give you all kinds of bad inputs over the years. They are not to be trusted!
- Engineers, CS, and nerds writ large **could** run the world. We just have better things to do.



US NUCLEAR CHAIN OF COMMAND

# Going over the Sorting Quiz

# Some Linux Commands

wc

tail / head

grep

sed & awk

less

pstree

git

- This is a big topic and a wonderful tool once you learn the ins and outs of it!
- I use git often these days
- EECS has a git server for your use
  - Should be storing all programs there
  - *\*will\** be using it later to collaborate on projects

# I want to discuss GitHub.com for just a few moments

- A source control system - Primary location is github.com
- Provides you:
  - Version control
  - Branching
  - Merging
  - Issue tracking
  - Collaboration tools
  - Web-backends
  - Public and Private repositories (with your student account)
- Git is the **#1** source control tool in industry today - your public profile as a coder should be on display, and github.com is the most common way



# Midterm #2 review

- Hashing
- Heaps
- Sorting

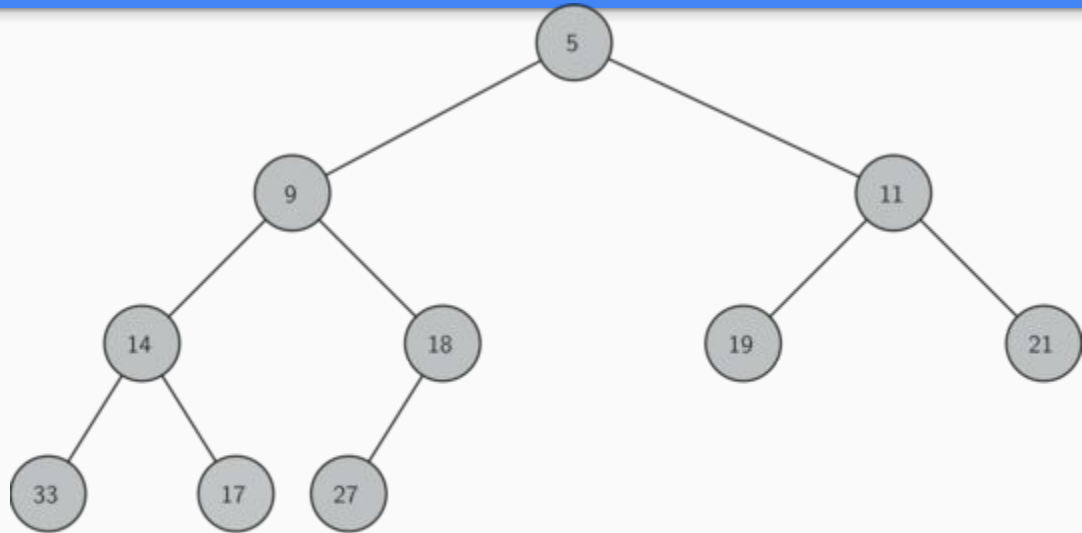


# Part #1 - Heaps (Priority Queues)

- Efficient priority queues show up in many applications
- Only rule is: parents have to be less than all of their children
- MinHeaps and MaxHeaps - just pick the one you need
- Binary heaps (though, there's n-ary heaps, with a 4-heap being more eff)
- Simple implementations: can be done in a vector
  - For 1-based indexes:
    - Children are:  $i * 2$  &  $i * 2 + 1$
    - Parent is:  $i / 2$
- Finding min/max is always  $O(1)$ , merging lists into a heap are  $O(N)$  time, other operations are  $O(\log N)$

# Why is this not a BST?

- No ordering to the children
- Only parent < children
- Tree is always balanced



	5	9	11	14	18	19	21	33	17	27	
0	1	2	3	4	5	6	7	8	9	10	11

# Functions

- Insert: Add element to end of heap, then bubbleUp() / percolateUp()
  - is  $\log(N)$  worst case, but normally only about 1.8 on average
- FindMin: Just read root of tree - is always  $O(1)$
- DeleteMin:
  - Remove (return) root of tree
  - move last element in vector to root
  - bubbleDown() / percolateDown() from root - is  $\log(N)$  worst case
- BuildHeap(List A, List B) - append two lists into a single list (call heap)
  - Then call percolateDown (adjust heap) on elements from  $(N/2..1)$
  - Works in  $O(N)$  time since it's an average of 1.5-ish swaps each call on  $N/2$  elements

# Things to practice:

- inserting
- deleteMin
- buildHeap
- Remember the heap rules!

# Heaps in the STL

- The STL has a class template for a heap
- It's called "priority\_queue" in header file queue
- Implements a max-heap rather than a min
  - Can be made min heap by changing comparison function when declaring priority\_queue

# Other heaps - d-heaps & leftist heaps

# d-Heaps

- d-ary tree instead of a 2-ary (binary) tree
- Makes height  $\log_d(N)$  instead of  $\log_2(N)$
- Makes inserts:  $O(\log_d N)$
- Makes deleteMin:  $O(d \log_d N)$
- If  $d$  is not a power of 2, it's a penalty on array implementations
  - If power of 2, can use shift register multiplication for HUGE speedups!
  - C++ Syntax: `<< >>` -- Does binary shift of bits in an integer
- Ops are still  $O(\log N)$

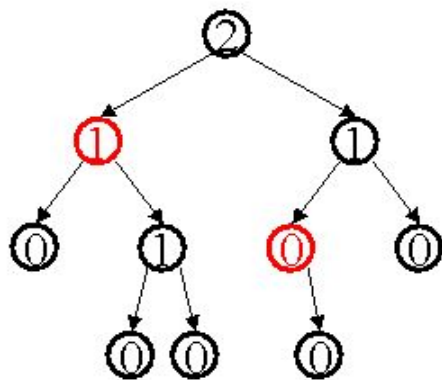
# Leftist Heaps

- Left child is height  $\geq$  right child height
- Height of node is height of shortest child + 1
- Much like AVL tree, it keeps height values in the nodes
- Guarantees left tree is taller than right
- Merge/insert are then done on right trees
- Primarily used in situations where there's lots of merging of heaps in your application - which can, and does, occur
  - See: distributed node computation where results come back in heaps to be merged

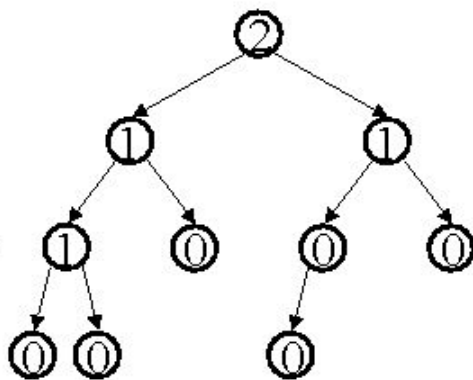


## Leftist tree examples

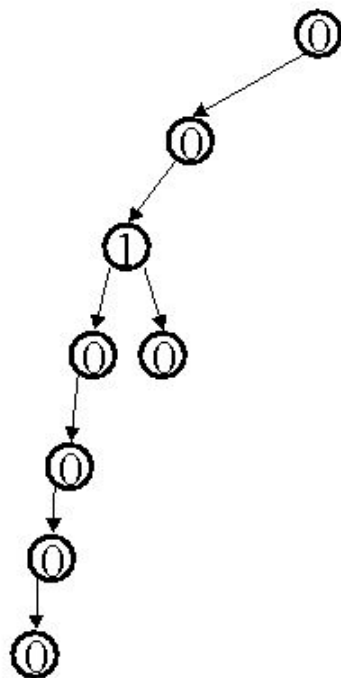
**NOT** leftist



leftist



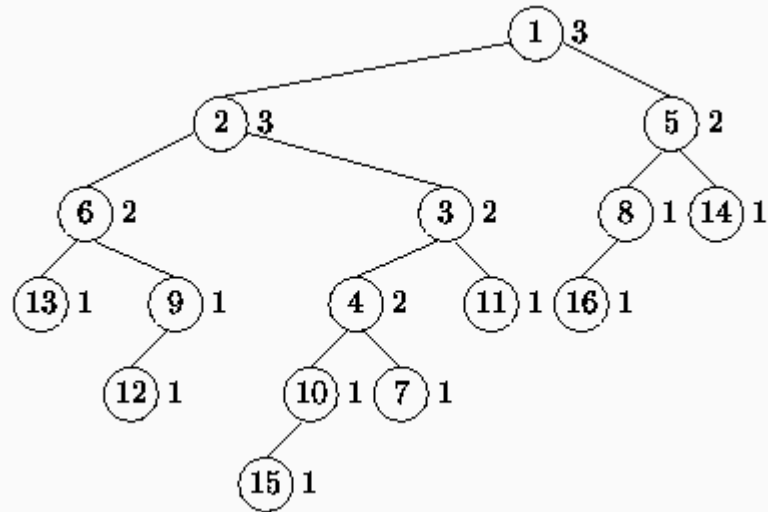
leftist



every subtree of a leftist tree is leftist, comrade!

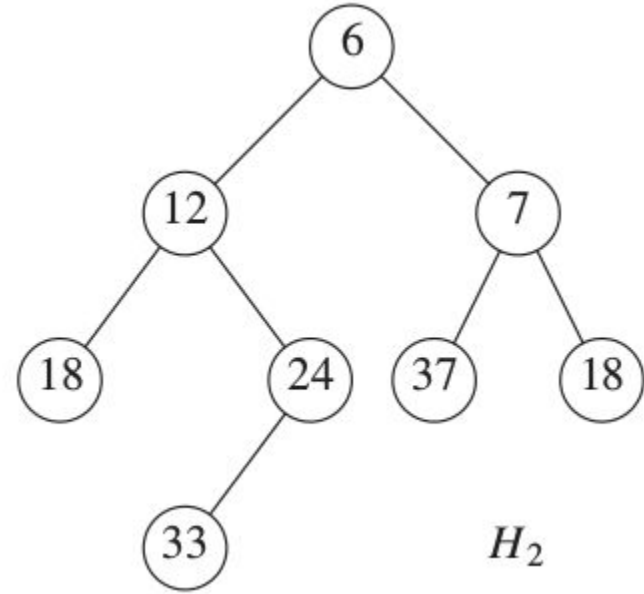
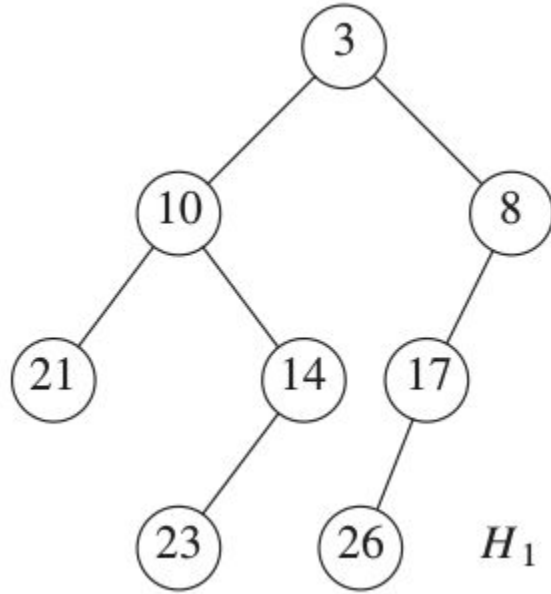
# Calculating Heights of Nodes

- Height of a node is height of shorter child + 1
- Also, left tree height  $\geq$  right tree height
- Biases left!



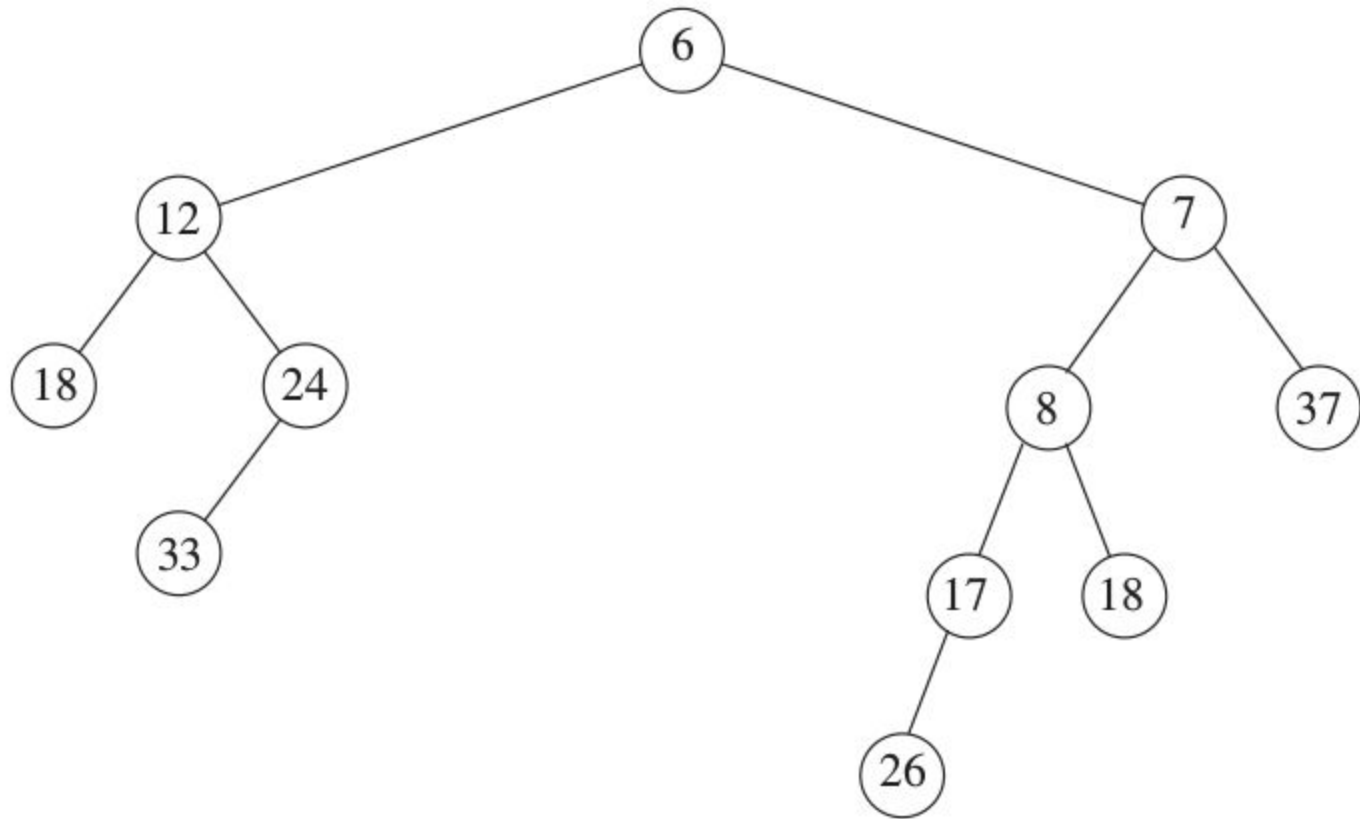
# Means right tree is shorter

- Right path will have at most  $\log(N+1)$  nodes
- This guarantees a smaller tree to work with
- Merges (Inserts are merges with a single node tree) are done on right subtree of the tree with the smaller root with the whole other tree
- After the merge, the rest of the first tree is used as the root of a new tree
- Then, heights are re-calculated
  - If a node's children violate the leftist rule, swap them

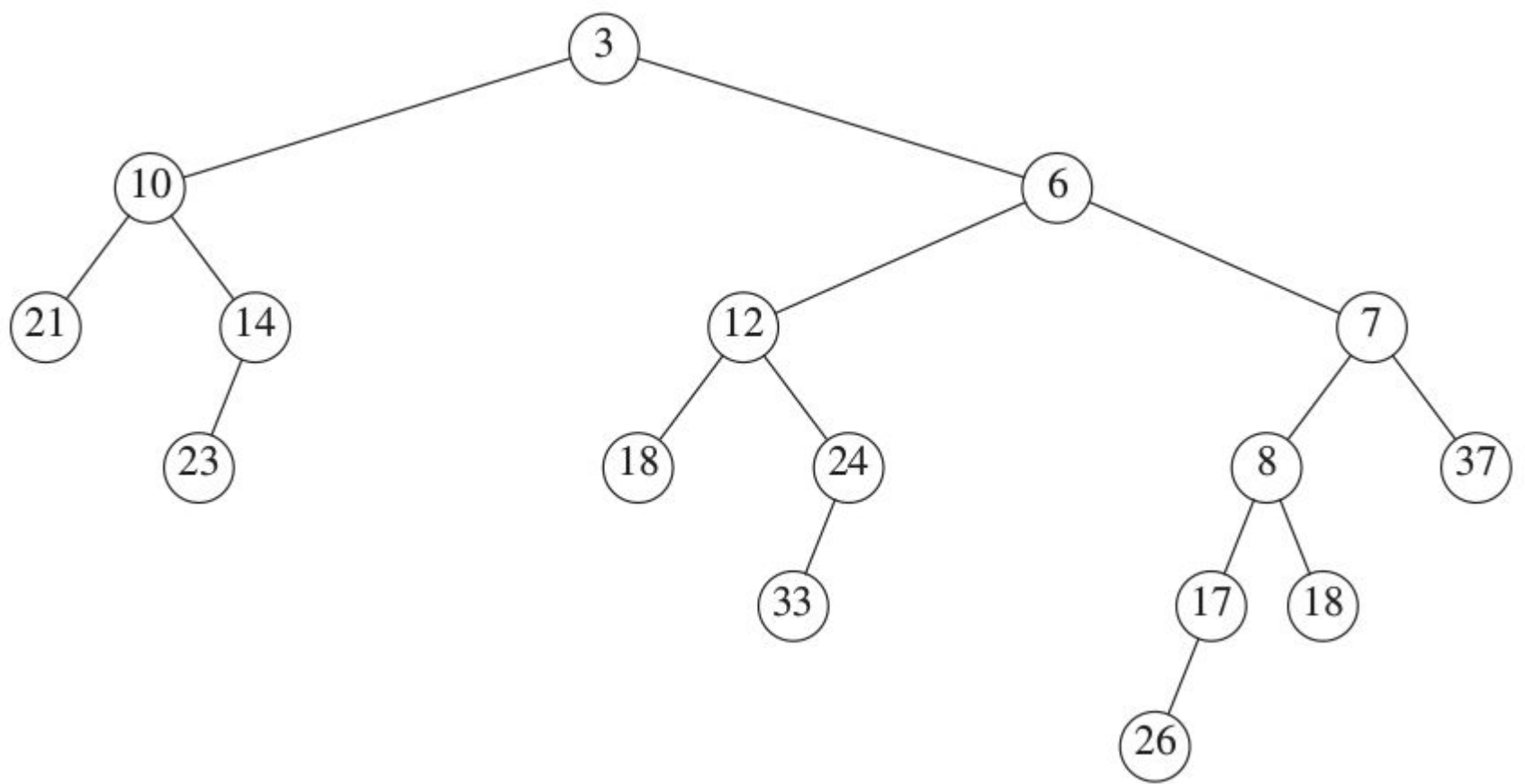


Right subtree of  $H_1$  is merged with  $H_2$

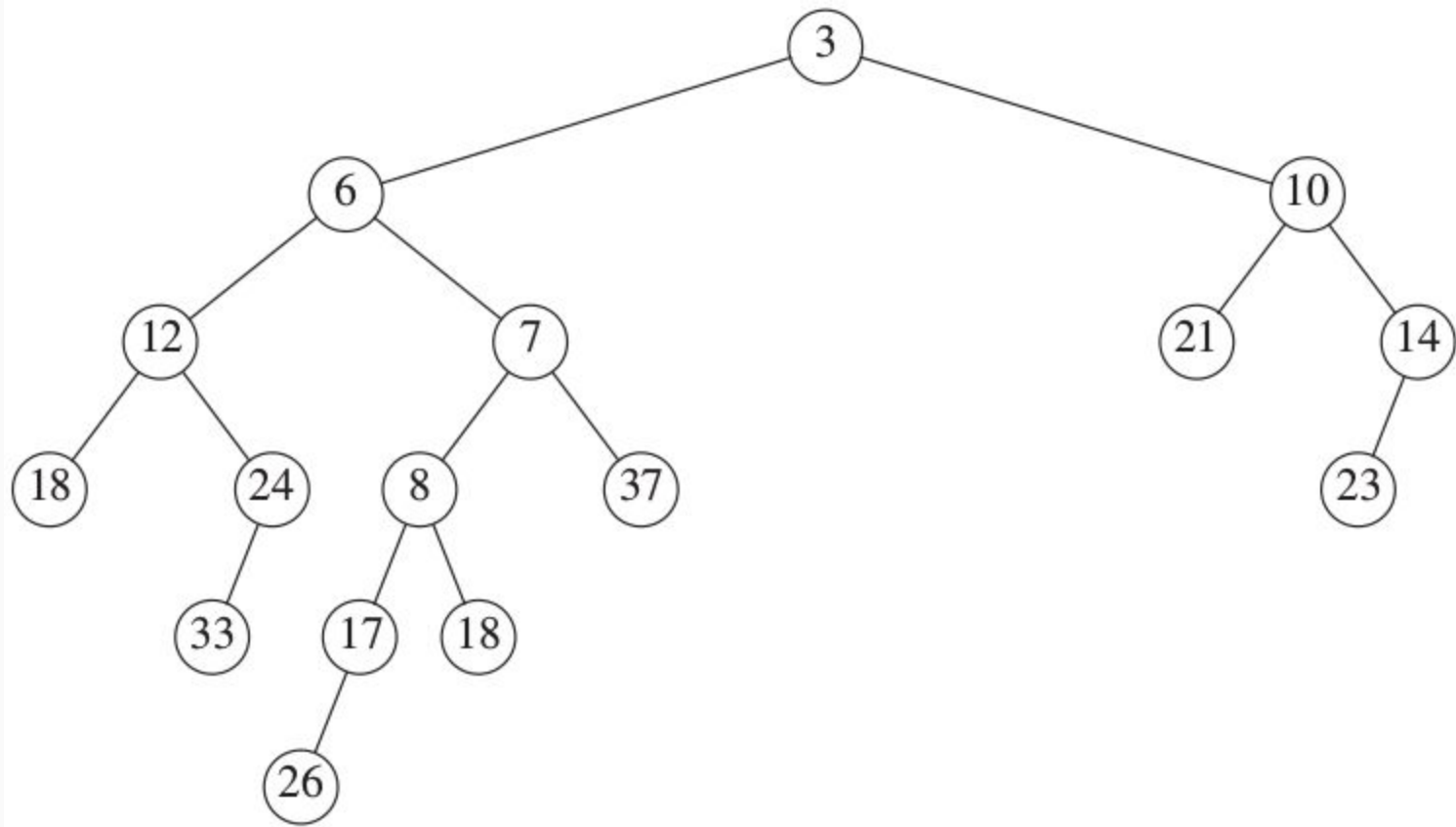
This is done recursively on each subtree as we go along



Result of top level recursive merger of subtrees



Then root and left tree from H1 are made to be the new overall root



To wrap up, the npl values of the nodes are examined and sub trees swapped as needed

# Part #2 - Hashing

- What is the goal of using hash tables?
- Major components of a hash algorithm:
  - Hash function + Hash table + Collision resolution function
- What is a hash function?
- What is a hash table?
- What is a collision resolution function?

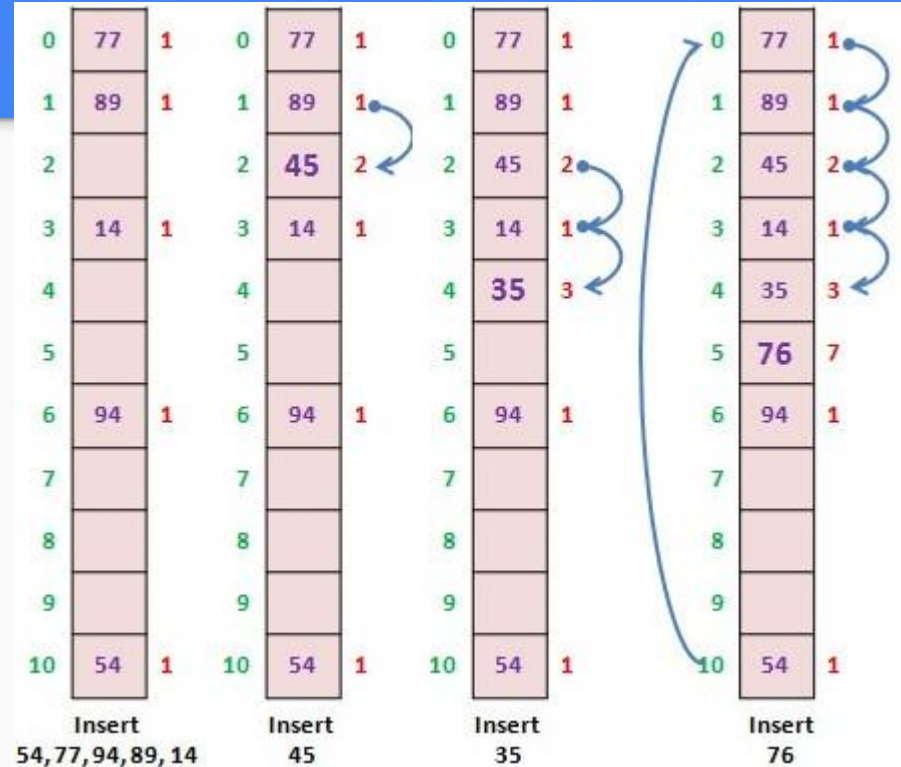


# Collision Resolution Function

- None - Just error out if there's no place for the new data
  - Could rehash to make more room and try again right away
- Linear probing
  - $\text{probe}(i) = (\text{hash}(\text{key}) + i) \% \text{table size}$
- Quadratic probing
  - $\text{probe}(i) = (\text{hash}(\text{key}) + i^2) \% \text{table size}$
- Separate chaining
  - Store buckets in linked lists - no probing
- Lazy deletion issue - probing cannot delete, only flag as “not there”
  - Things are really deleted if a rehash occurs because “not there” isn't rehashed in again

# Linear Probing

- Increment location if collision
- Works, but builds high and low density areas in the hash table
- $\text{probe}(i) = (\text{hash} + i) \% \text{TableSize}$ 
  - For  $i$  in  $1..\text{TableSize}$
- Rehash if load factor  $(\lambda) \geq 0.5$



# Quadratic Probing

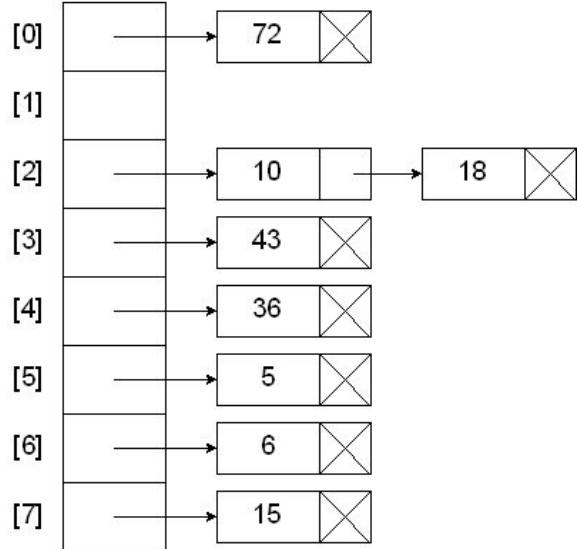
- Use a quadratic function to probe
- $\text{probe}(i) = (\text{hashkey} + i^2) \% \text{TableSize}$ 
  - For  $i$  in  $1..\text{TableSize}$
- Will eventually work: provided TS is prime and load factor isn't too high
- Rehash when load factor  $(\lambda) \geq 0.5$

# Separate Chaining

- Each table bucket is a linked list
- Inserts always work
- Searches take  $O(1)$  for to calc hash key, plus  $O(\text{list len})$  searches
- On average, this is  $O(1) + O(1.5)$
- Rehash when load factor  $(\lambda) \geq 1$

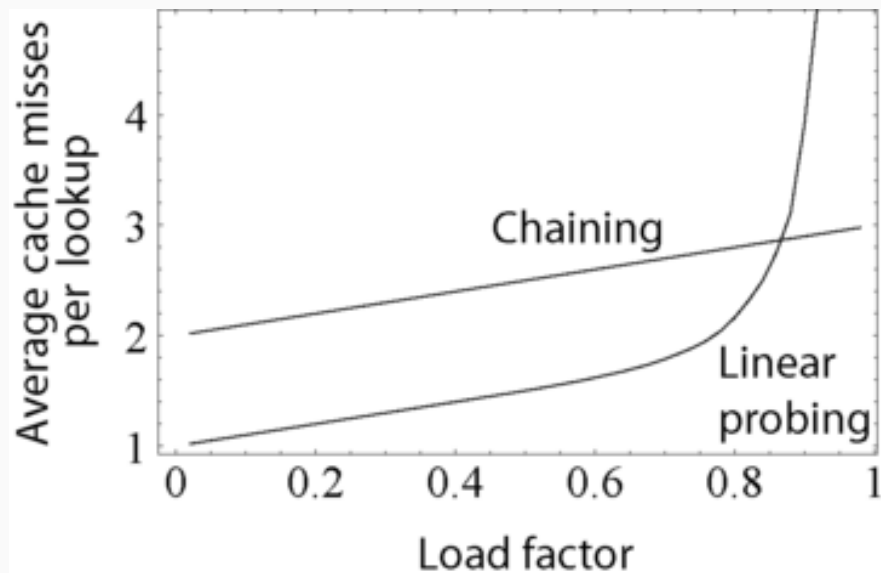
Hash key = key % table size

4	=	36	%	8
2	=	18	%	8
0	=	72	%	8
3	=	43	%	8
6	=	6	%	8
2	=	10	%	8
5	=	5	%	8
7	=	15	%	8



# Load Factor

- The load on the hash table is based on how full it is
- Normally denoted by  $\lambda$
- Calculated by  $N / \text{TableSize}$



# Why prime sized tables?

- Hashing and probing all computed with  $\text{mod}(\text{TableSize})$
- If it isn't prime, then algorithms are not guaranteed to touch all buckets for any given key value

# Rehashing

- If table is too full (or too empty?) the TableSize needs to be adjusted
- Normally, you double TableSize, then move up to next prime number
  - 11 -> 22 -> 23
  - 101 -> 202 -> 211
- Algorithm is simple, but doubles the table size memory footprint while rehashing is done

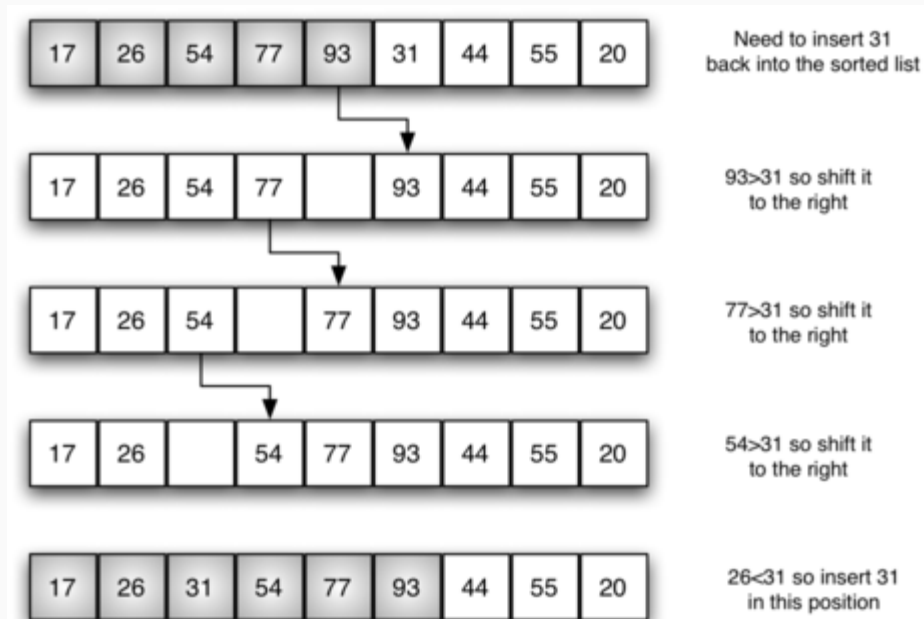
# Part #3 - Sorting

- The ordering of elements through comparison or exploiting data properties
- Many algorithms available:
  - Bubblesort, Insertionsort, Mergesort, Heapsort, Quicksort
- Simple sorts that do local comparisons operate in  $O(N^2)$  time
  - Bubblesort, Insertionsort
- Sorts that do more work per step (long distance comparisons, group comparisons) can get down to  $O(N \log N)$  time
  - Mergesort, Heapsort, Quicksort
- In special data cases you can do  $O(N)$  time sorting
  - Bucket (counting) sort, Radix sort



# Insertionsort

- Move current element until it's in correct place (less than || to right).
- Then move all elements right as needed
  - Creates lots of copying
  - Comparisons are local
- Works in  $N^2$  time
- Stable sort

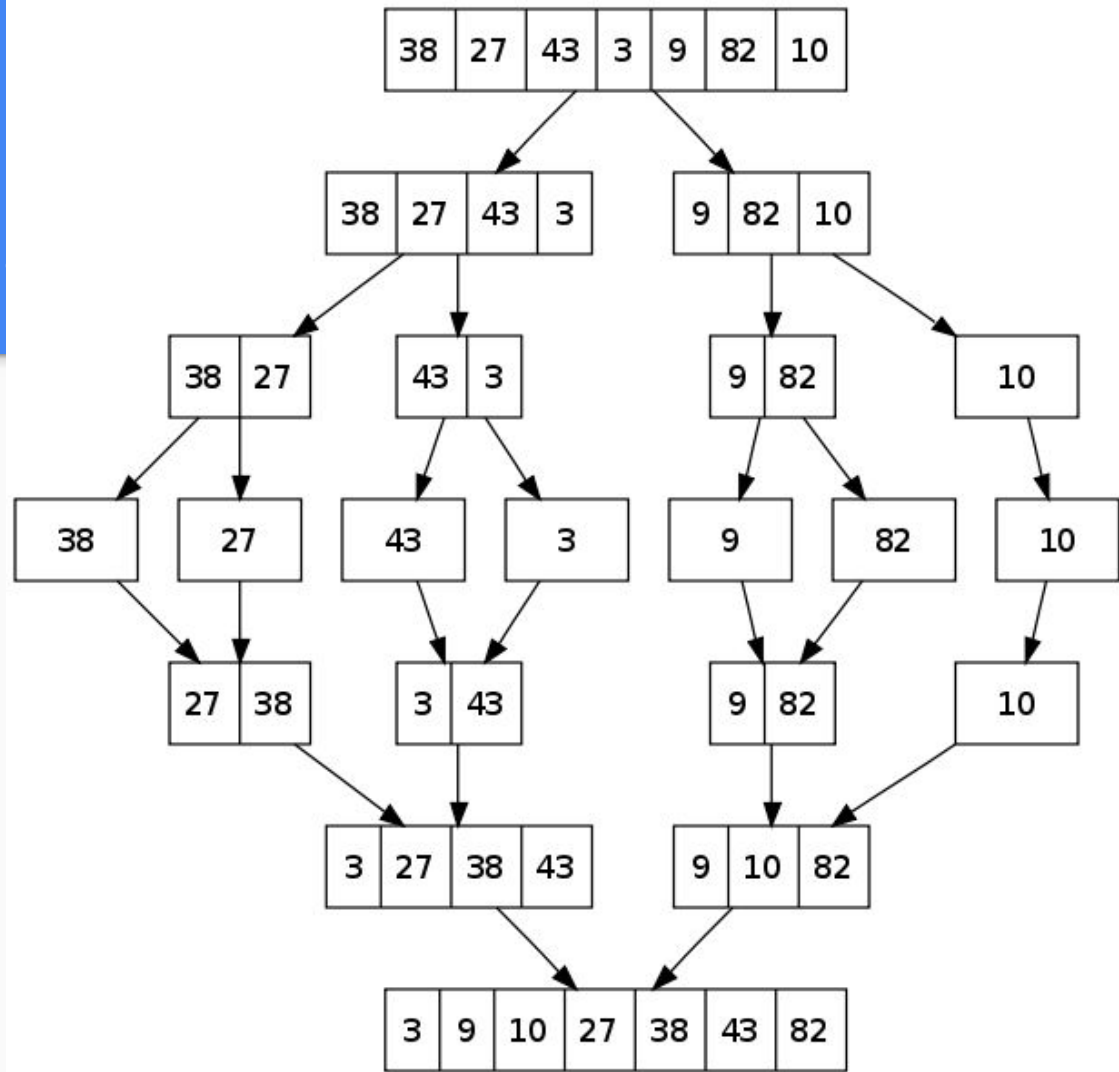


# Heapsort

- Put everything in a heap, then pull them out with deleteMin in order
  - Buildheap is  $O(N)$ , DeleteMin is  $O(\log N)$  result is  $O(N \log N)$  time
  - Takes two structures (one tree, one results vector)
- Not guaranteed to be stable

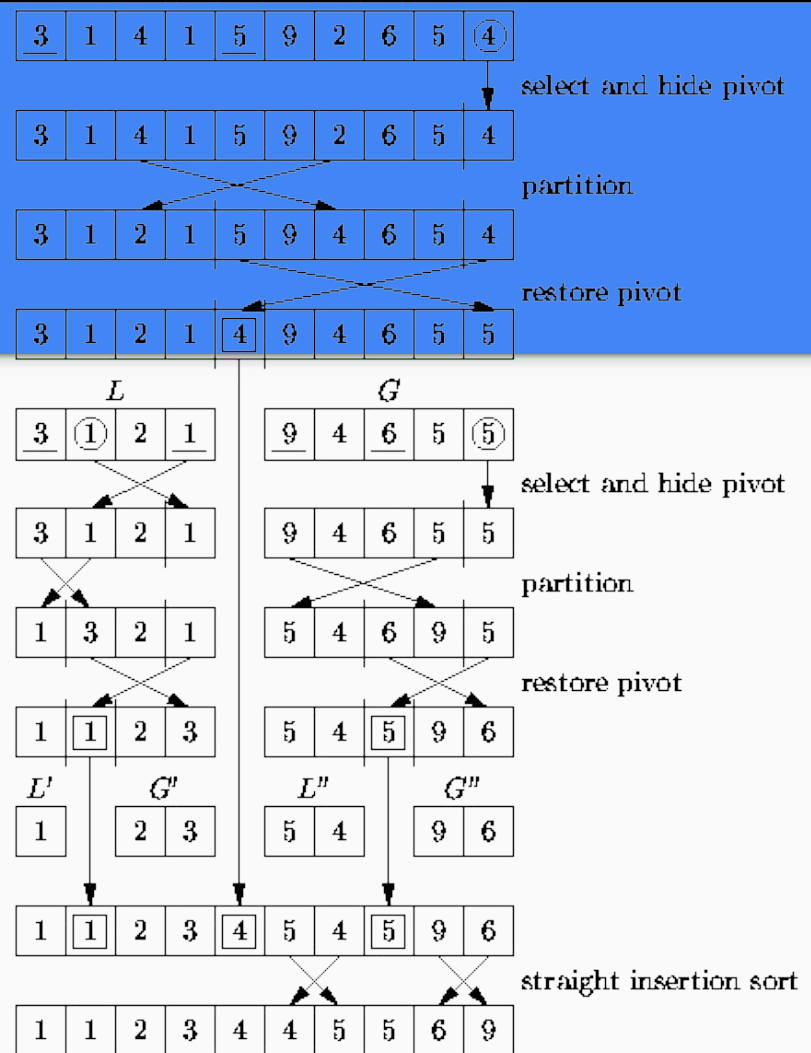
# Mergesort: Divide

- Recursively divide list into smaller sub-lists until the sub-lists are sorted
- Merge back together.  
Merging:  $O(N+M)$  time  
Division:  $O(\log N)$  time,  
Overall:  $O(N \log N)$
- low on comparisons, high on data moves
- Stable sort



# Quicksort

- Pick a pivot, move all smaller to left, and all bigger to right
- Quicksort on left
- Quicksort on right
- Return {left, pivot, right} -> WIN
- Works in  $O(N \log N)$  given good pivot values chosen
  - #1 approach: median of 3:  $[0, N/2, N]$
  - Bad if always min/max value:  $N^2$
- Stable sort



# Bucket (Counting) sort

- Linear time sort for small integers
- Given:  $Arr = [A_1, A_2, \dots, A_N]$  of positive integers smaller than  $M$ .
- Make `int counts[M]`, initialized to all 0.
- Read input: for each value in `Arr` you increment counts with: `counts[A_i]++`
- Then print out all non-zero buckets `counts[i]` times
- Results are found in  $O(M+N)$  time
  - If  $M == O(N)$ , then final result is  $O(N)$
- Power comes through  $M$ -way comparisons in  $O(1)$  time
  - Similar to hashing algorithm

# Different approach: Radix sort

- Used to sort punch cards, so also called card sort
- Can work on integers or strings
- Sorts by one indexed position at a time
  - Normally done right to left (won't work otherwise!)
- Is a stable sort
- Effectively multi-pass bucket sort

# Radix sort description

- Data must be  $N$  numbers in range  $(0..b^p-1)$ 
  - $N$  == set of numbers in array
  - $b$  == number of buckets, which is determined by the size of the alphabet in the data
  - $p$  == number of passes, which is the length of the strings/numbers
- Do passes over each digit or string position
  - Go from least significant to most significant
  - Put item into bucket as indexed by position (needs  $b$  buckets)
  - When done, read all items out in FIFO (to be a stable sort) order back to original array
  - Repeat  $p$  times, moving index from least to most significant position
- Result is done in  $O(p(N+b))$  time!

# Counting radix sort - sorting by numbers

INITIAL ITEMS:	064, 008, 216, 512, 027, 729, 000, 001, 343, 125
SORTED BY 1's digit:	000, 001, 512, 343, 064, 125, 216, 027, 008, 729
SORTED BY 10's digit:	000, 001, 008, 512, 216, 125, 027, 729, 343, 064
SORTED BY 100's digit:	000, 001, 008, 027, 064, 125, 216, 343, 512, 729

Each pass orders the items in a more significant way.

The results kind of pop into existence at the end.