CptS 223 Homework #3 - Heaps, Hashing, Sorting

Please complete the homework problems on the following page using a separate piece of paper. Note that this is an individual assignment and all work must be your own. Be sure to show your work when appropriate. Please scan the assignment and upload the PDF to Git, but also bring a printed out copy for the grading TAs. We've found that many of the scans are difficult to read and notate.

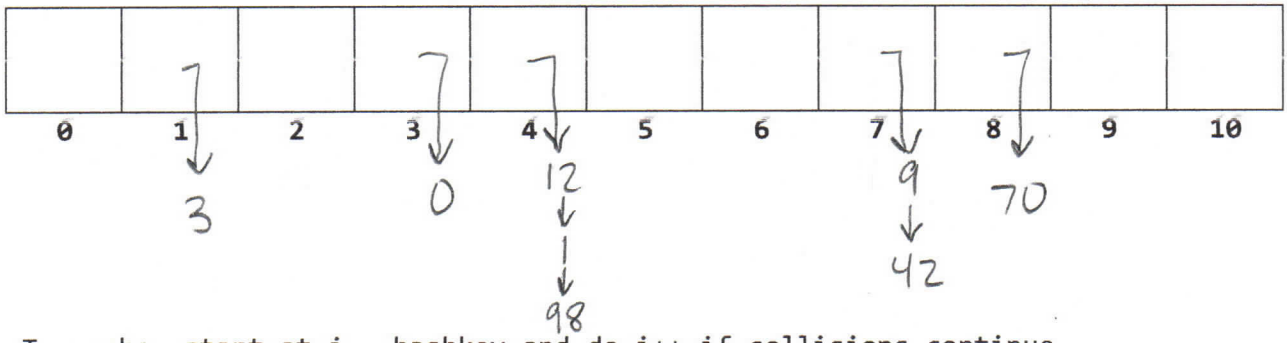
$$hashKey(key) = \lceil (key \cdot key) + 3 \rceil \ \% \ 11$$

input	output
12	4
9	7
1	4
0	3
42	7
98	4
70	8
3	1

1. [6] Starting with an empty hash table with a fixed size of 11, insert the following keys in order into four distinct hash tables (one for each collision mechanism): {12, 9, 1, 0, 42, 98, 70, 3}. You are only required to show the final result of each hash table. In the **very likely** event that a collision resolution mechanism is unable to successfully resolve, simply record the state of the last successful insert and note that collision resolution failed. For each hashtable type, compute the hash as follows:

hashkey(key) = (key * key + 3) % 11

Separate Chaining (buckets)



To probe, start at i = hashkey and do i++ if collisions continue

Linear Probing: probe(i') = (i + 1) % TableSize

0	1	2	3	4	5	6	7	8	9	10
	3		0	12	1	98	9	42	70	

Quadratic Probing: probe(i') = (i * i + 5) % TableSize

21 % 11

0	1	2	3	4	5	6	7	8	9	10
	3		0	12		70	9	98	42	1

2. [3] For implementing a hash table. Which of these would probably be the best initial table size to pick?

Table Sizes:

1 100 101 15 500

Why?

By the nature of how modulo division works, modding by a number with fewer factors results in less potential clustering if the data isn't entirely random

3. [4] For our running hash table, you'll need to decide if you need to rehash. You just inserted a new item into the table, bringing your data count up to 53491 entries. The table's vector is currently sized at 106963 buckets.

- Calculate the load factor (λ):

$$\frac{53491}{106963} \approx 0.50009 > \frac{1}{2}$$

- Given a linear probing collision function should we rehash? Why?

for linear probing threshold is $\lambda = 0.5$
so rehash happens

- Given a separate chaining collision function should we rehash? Why?

for sep. chaining threshold is $\lambda = 1.0$
so rehash does not happen

4. [4] What is the Big-O of these actions for a well designed and properly loaded hash table with N elements?

Function	Big-O complexity
Insert(x)	O(N)
Rehash()	O(N)
Remove(x)	O(N)
Contains(x)	O(N)

5. [3] If your hash table is made in C++11 with a vector for the table, has integers for the keys, uses linear probing for collision resolution and only holds strings… would we need to implement the Big Five for our class? Why or why not?

no, because strings, integers, and vectors are all native C++ data constructs, and already have their big-5 implemented, thus your table will simply use the inbuilt constructors, assignments, etc. for the above when performing operations

6. [6] Enter a reasonable hash function to calculate a hash key for these function prototypes:

```
int hashit( int key, int TS )
{
    unsigned int hashKey = 0;
    unsigned int nextDigit = 0;
    int i = 1;
    do {
        nextDigit = key % 10;
        hashKey += nextDigit * pow(37, i);
        key /= 10;
        i++;
    } while (key != 0);
    return hashKey % TS;
}
int hashit( string key, int TS )
{
    unsigned int hashKey = 0;
    for (int i=0; i < key.length(); i++) {
        hashKey += key[key.length - 1 - i] * pow(37, i);
    }
    return hashKey % TS;
}
```

7. [3] I grabbed some code from the Internet for my linear probing based hash table because the Internet's always right. The hash table works, but once I put more than a few thousand entries, the whole thing starts to slow down. Searches, inserts, and contains calls start taking *way* longer than O(1) time and my boss is pissed because it's slowing down the whole application services backend I'm in charge of. I think the bug is in my rehash code, but I'm not sure where. Any ideas why my hash table starts to suck as it grows bigger?

```
/**
* Rehashing for linear probing hash table.
*/

void rehash( )
{
      vector<HashEntry> oldArray = array;

      // Create new double-sized, empty table
      array.resize( 2 * oldArray.size( ) );
      for( auto & entry : array )
            entry.info = EMPTY;

      // Copy table over
      currentSize = 0;
      for( auto & entry : oldArray )
            if( entry.info == ACTIVE )
                  insert( std::move( entry.element ) );
}
```

not rehashing into new table, which means all the clusters are preserved in the first half of the new table, and for linear probing this devolves into O(N) behavior for inserts and searches for elements hashed into the first half of new table

8. [4] Time for some heaping fun! What's the time complexity for these functions in a binary heap of size N?

Function	Big-O complexity
insert(x)	$O(\log N)$
findMin()	$O(1)$
deleteMin()	$O(\log N)$
buildHeap(vector<int>{1...N})	$O(N \log N)$

9. [4] What would a good application be for a priority queue (a binary heap)?

If I could only purchase one cat per day but I wanted to, at any point, purchase only the cat with the best catface, I could throw in all the cats into a heap after ranking them transitively based on catface

Describe it in at least a paragraph of why it's a good choice for your example situation.

After ranking all of the cats based on catface, I could simply call getBestCat(); to determine my purchase for the day. And if new cats become available for purchase I could simply assign a score to their catface, throw them into the heap, and then still be assured that on any one day I am still purchasing the cat with the best catface.

10. [4] For an entry in our heap (root @ index 1) located at position i, where are it's parent and children?

Parent:

$i/2$ if not root

Children:

$2i$ and $2i+1$ assuming $N \geq 2i+1$

What if it's a d-heap?

Parent:

$\frac{i + d - 2}{d}$ $\quad i \neq 1$

Children:

$i \cdot d - (d-2)$, $i \cdot d - (d-3)$,

$\ldots$ $i \cdot d - (d - (d+1)) = i \cdot d + 1$

11. [6] Show the result of inserting 10, 12, 1, 14, 6, 5, 15, 3, and 11, one at a time, into an initially empty binary heap. Use a 1-based array like the book does. After insert(10):

10										

After insert (12):

10	12									

etc:

1	10	12								

1	10	12	14							

1	6	12	14	10						

1	6	5	14	10	12					

1	6	5	14	10	12	15				

1	3	5	6	10	12	15	14			

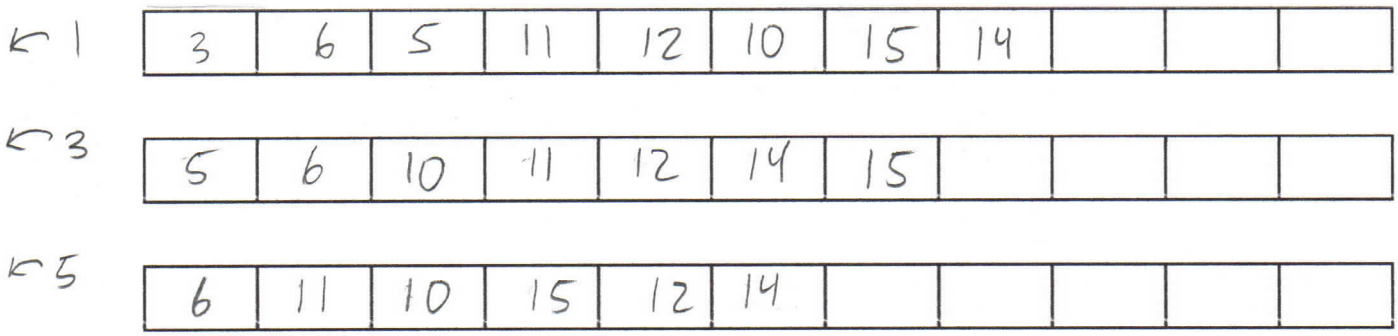
1	3	5	6	10	12	15	14	11		

12. [4] Show the same result (only the final result) of calling buildHeap() on the same vector of values: {10, 12, 1, 14, 6, 5, 15, 3, 11}

1	3	5	11	6	10	15	14	12		

13. [4] Now show the result of three successive deleteMin operations from the

prior heap:



14. [4] What are the average complexities and the stability of these sorting algorithms:

Algorithm	Average complexity	Stable (yes/no)?
Bubble Sort	$O(N^2)$	yes
Insertion Sort	$O(N^2)$	yes
Heap sort	$O(N \log N)$	no
Merge Sort	$O(N \log N)$	yes
Radix sort	$O(N \cdot w^*)$	yes
Quicksort	$O(N \log N)$	no

* # of digits (bucket recursion steps)

15. [2] What are the key differences between Mergesort and Quicksort? How does this influence why languages choose one over the other?

- If we're not implementing in-place mergesort, it takes far more memory space than quicksort.
- Mergesort does less comparisons and more copying of data (to temp array sublists). For languages that hate comparing mergesort is better. For languages that hate copying quicksort is better.
- Quicksort is not a stable sort; mergesort is.

16. [4] Draw out how Mergesort would sort this list:

24	16	9	10	8	7	20