Priority Queues - Moar Heaps

CptS 223 - Fall 2017 - Aaron Crandall

Today’s Agenda

- Announcements
- Thing of the day
- Making priority queues out of heaps

Announcements

- Next MA should be out tonight.
- The leadership of Emsi is doing a presentation and student meeting session on Oct 17th, 4-5pm in Sloan 150 with pizza
  - https://www.facebook.com/events/849380565221322
- I’ve got a Heaps PA coming together, but we’ll see about timing after PA3

My brief Reddit fame:
A Linux Cake

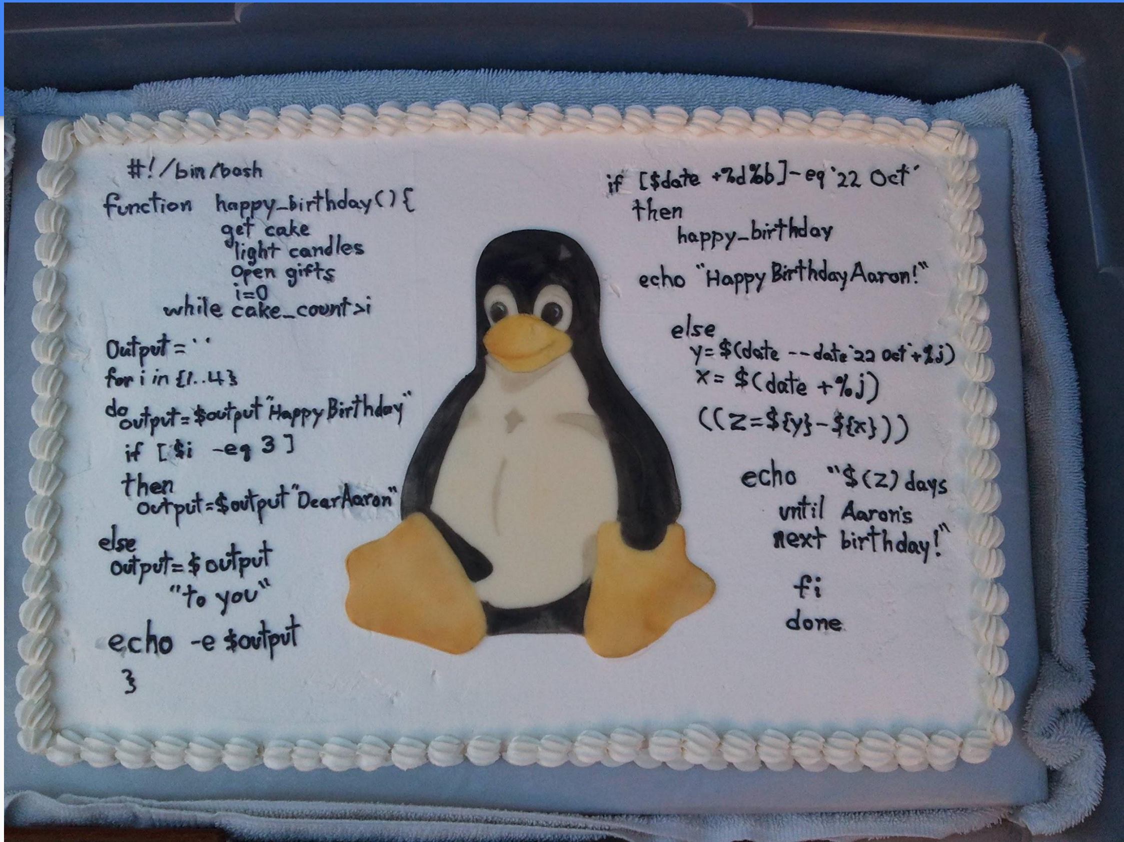
My birthday cake from a few years ago.

Posted on Reddit

4196 points, FP

The comments are geek humor galore.

Mom got it made at a baker’s in Moscow.



```
#!/bin/bash
function happy_birthday(){
                get cake
                light candles
                Open gifts
                i=0
        while cake_count>i

Output=''
for i in {1..4}
do output=$output"Happy Birthday"
  if [ $i -eq 3 ]
  then
    output=$output"DearAaron"
else
 output=$output
         "to you"
echo -e $output
 }
```

```
if [$date +%d%b]-eq'22 Oct'
  then
     happy_birthday
  echo "Happy BirthdayAaron!"

     else
       y=$(date --date'22 Oct'+%j)
       x= $(date +%j)
       ((z=${y}-${x}))

          echo "$(z) days
            until Aaron's
            next birthday!"
              fi
              done
```

Reminder: heap property

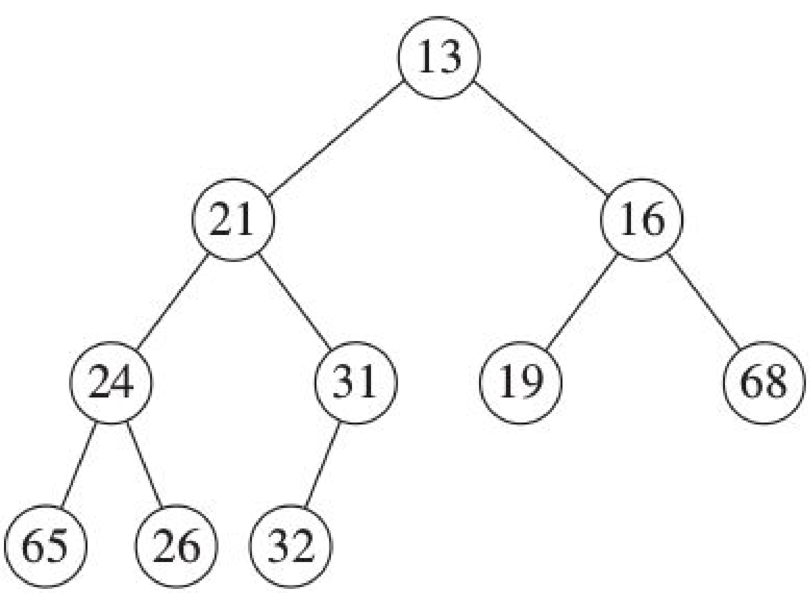
- Binary tree
- Parent key <= Children’s keys
- Height is log N
- Nodes stored in an array, not on The Heap and linked by pointers
    - Still drawn as trees to make operations clear

Remember! This is NOT a BST!

- Both children are larger than their parents in a Heap!
- Children are NOT sorted in any way
- Structure is NOT a BST

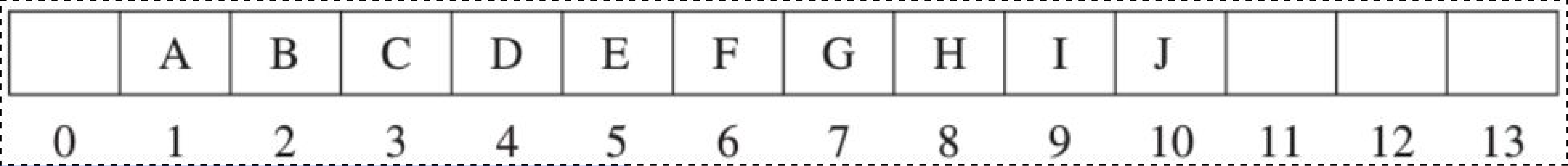
This is a valid Heap! ⇒

It's ***not*** a valid BST.



Heap-order property

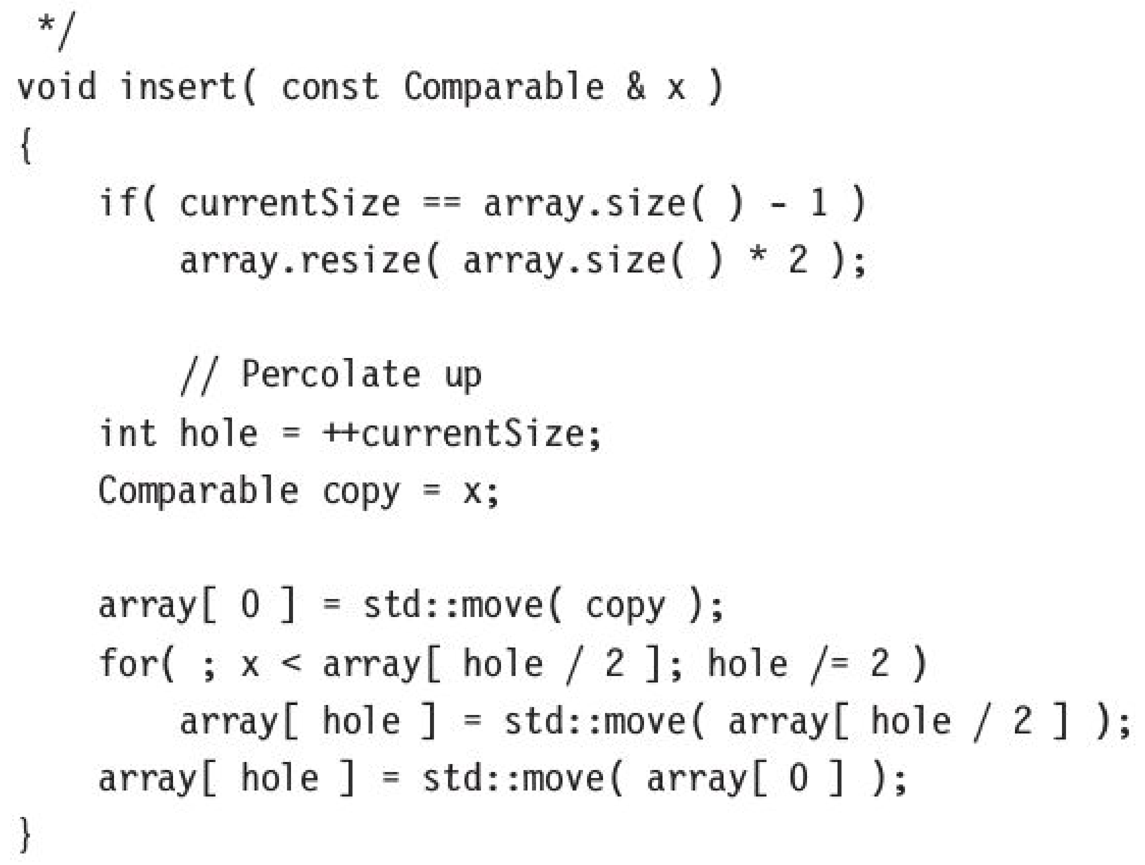
- Parents <= Children
- Root will always be smallest value
- findMin will always be in constant time



	A	B	C	D	E	F	G	H	I	J			
0	1	2	3	4	5	6	7	8	9	10	11	12	13

Short code!

- Move new item up tree until it fits heap-property

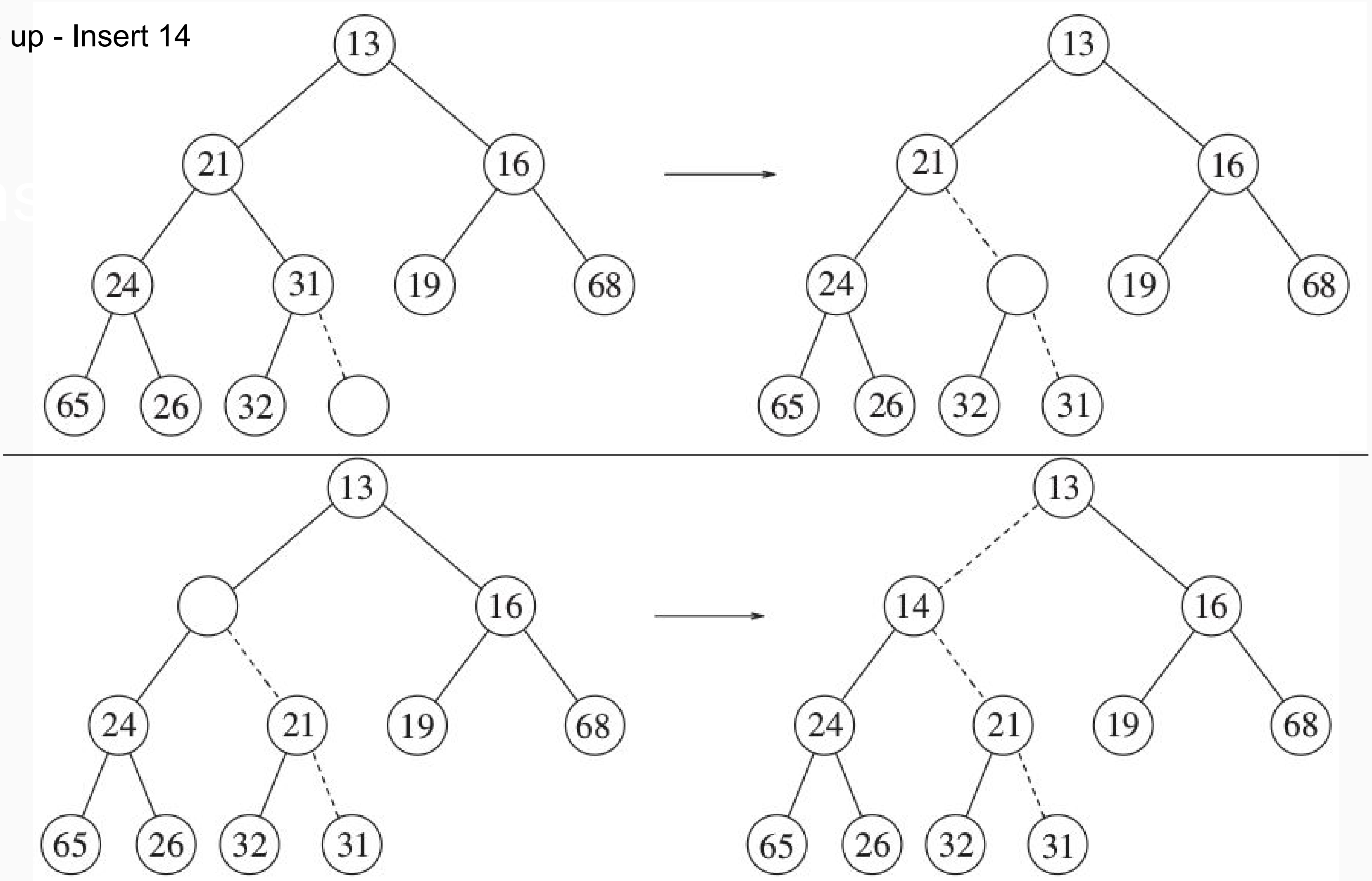


```
 */
void insert( const Comparable & x )
{
    if( currentSize == array.size( ) - 1 )
        array.resize( array.size( ) * 2 );

        // Percolate up
    int hole = ++currentSize;
    Comparable copy = x;

    array[ 0 ] = std::move( copy );
    for( ; x < array[ hole / 2 ]; hole /= 2 )
        array[ hole ] = std::move( array[ hole / 2 ] );
    array[ hole ] = std::move( array[ 0 ] );
}
```

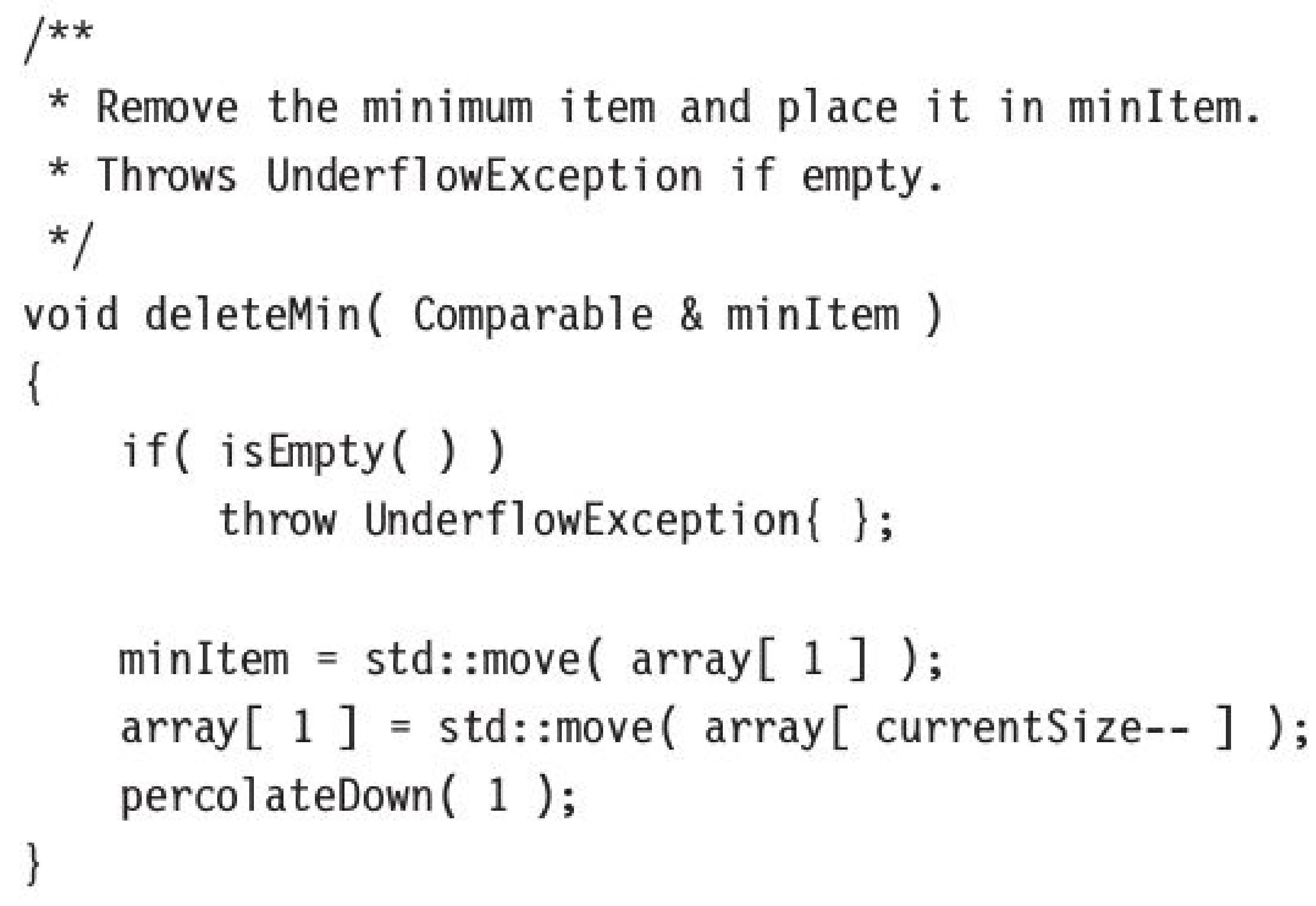
Percolate up - Insert 14



DeleteMin - Get lowest key and fix tree

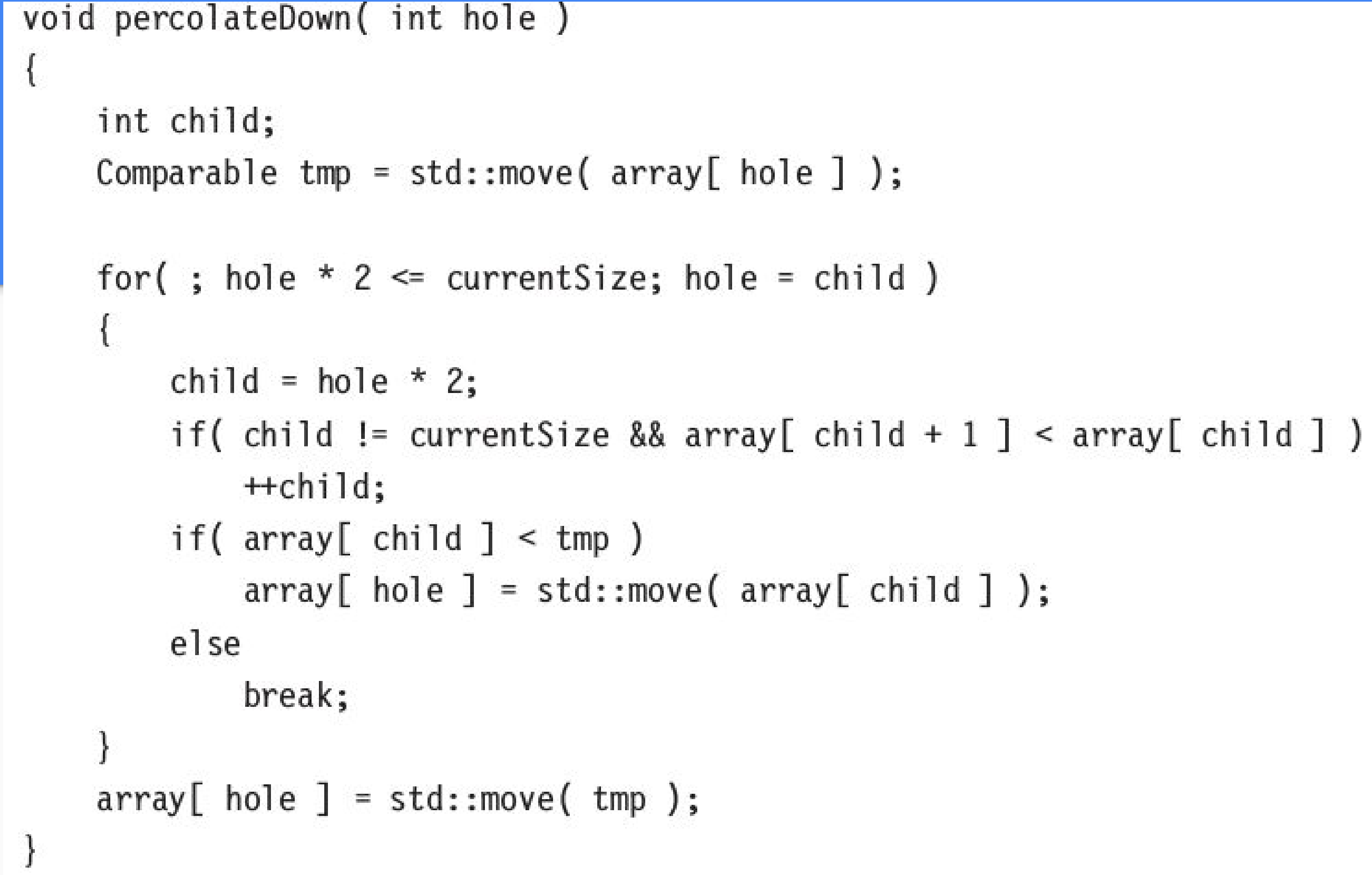
- Move off the root of the tree for returning later
- Take hole at root of tree and fill it with the last item in the tree
    - This is the last filled element in our array
- Iteratively (or recursively) try to move the root of the tree down until it satisfies the heap-order property

deleteMin



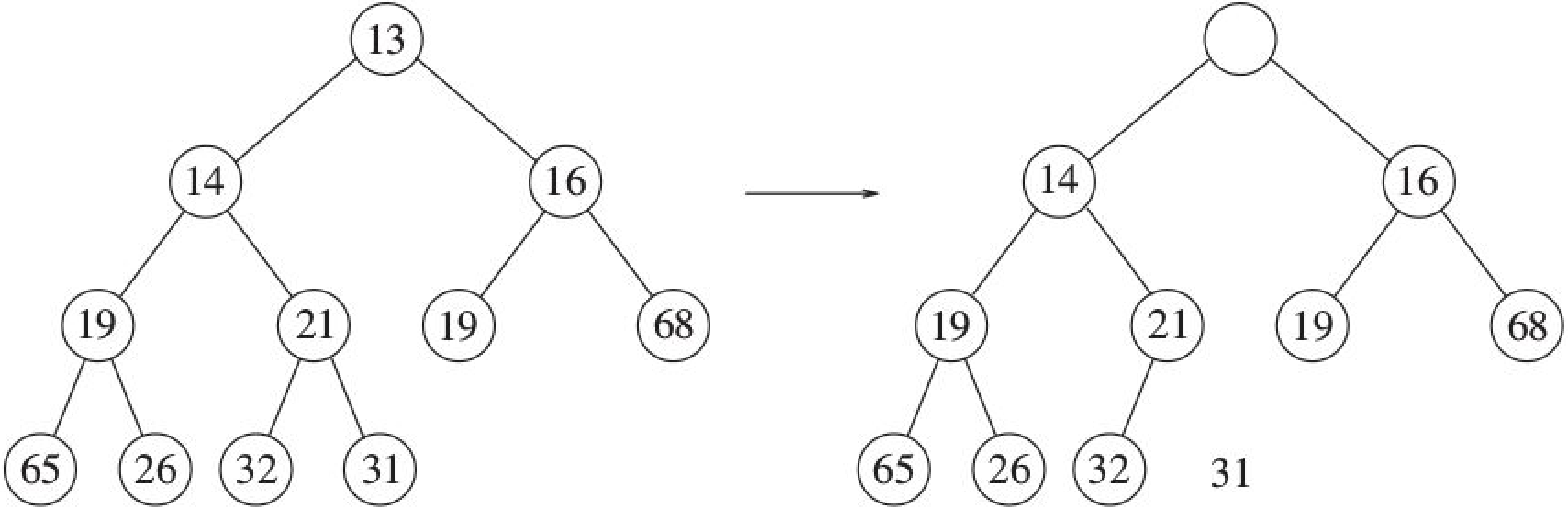
```
/**
 * Remove the minimum item and place it in minItem.
 * Throws UnderflowException if empty.
 */
void deleteMin( Comparable & minItem )
{
    if( isEmpty( ) )
        throw UnderflowException{ };

    minItem = std::move( array[ 1 ] );
    array[ 1 ] = std::move( array[ currentSize-- ] );
    percolateDown( 1 );
}
```



```
void percolateDown( int hole )
{
    int child;
    Comparable tmp = std::move( array[ hole ] );

    for( ; hole * 2 <= currentSize; hole = child )
    {
        child = hole * 2;
        if( child != currentSize && array[ child + 1 ] < array[ child ] )
            ++child;
        if( array[ child ] < tmp )
            array[ hole ] = std::move( array[ child ] );
        else
            break;
    }
    array[ hole ] = std::move( tmp );
}
```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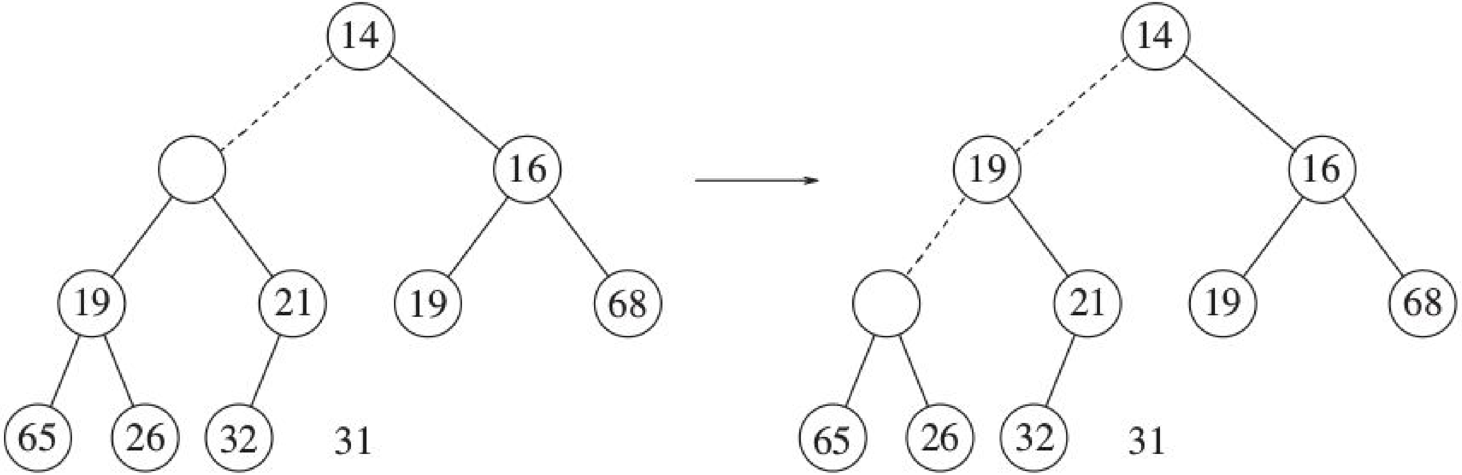
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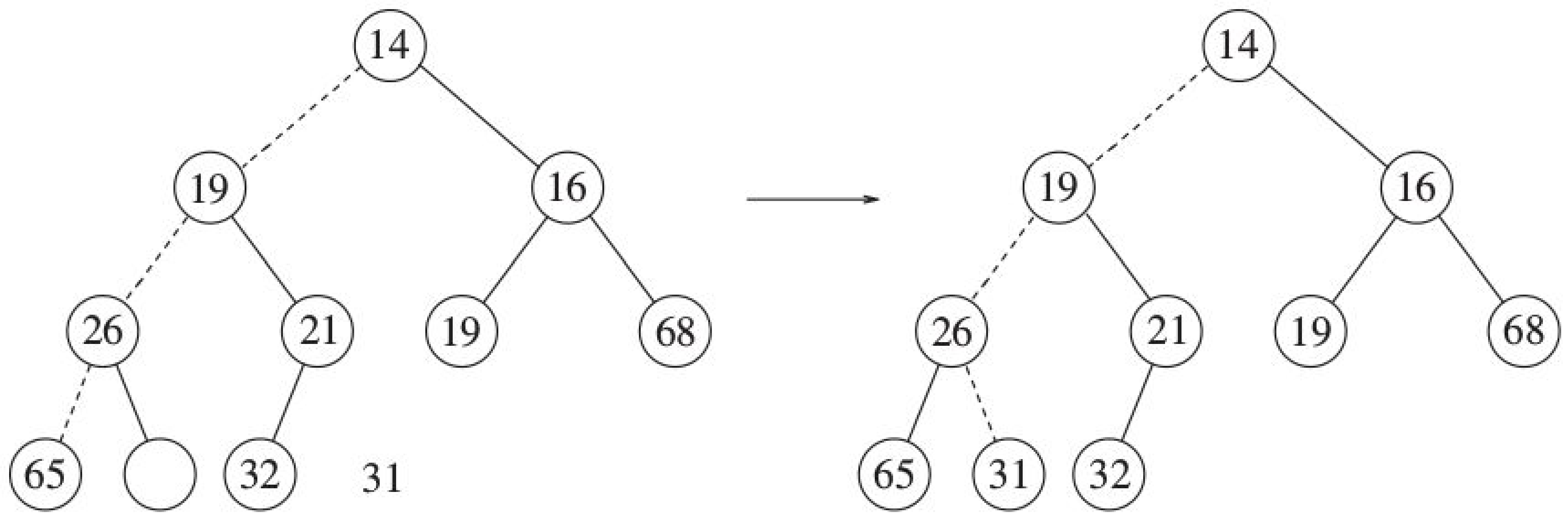
Wrap up deleteMin



Other operations

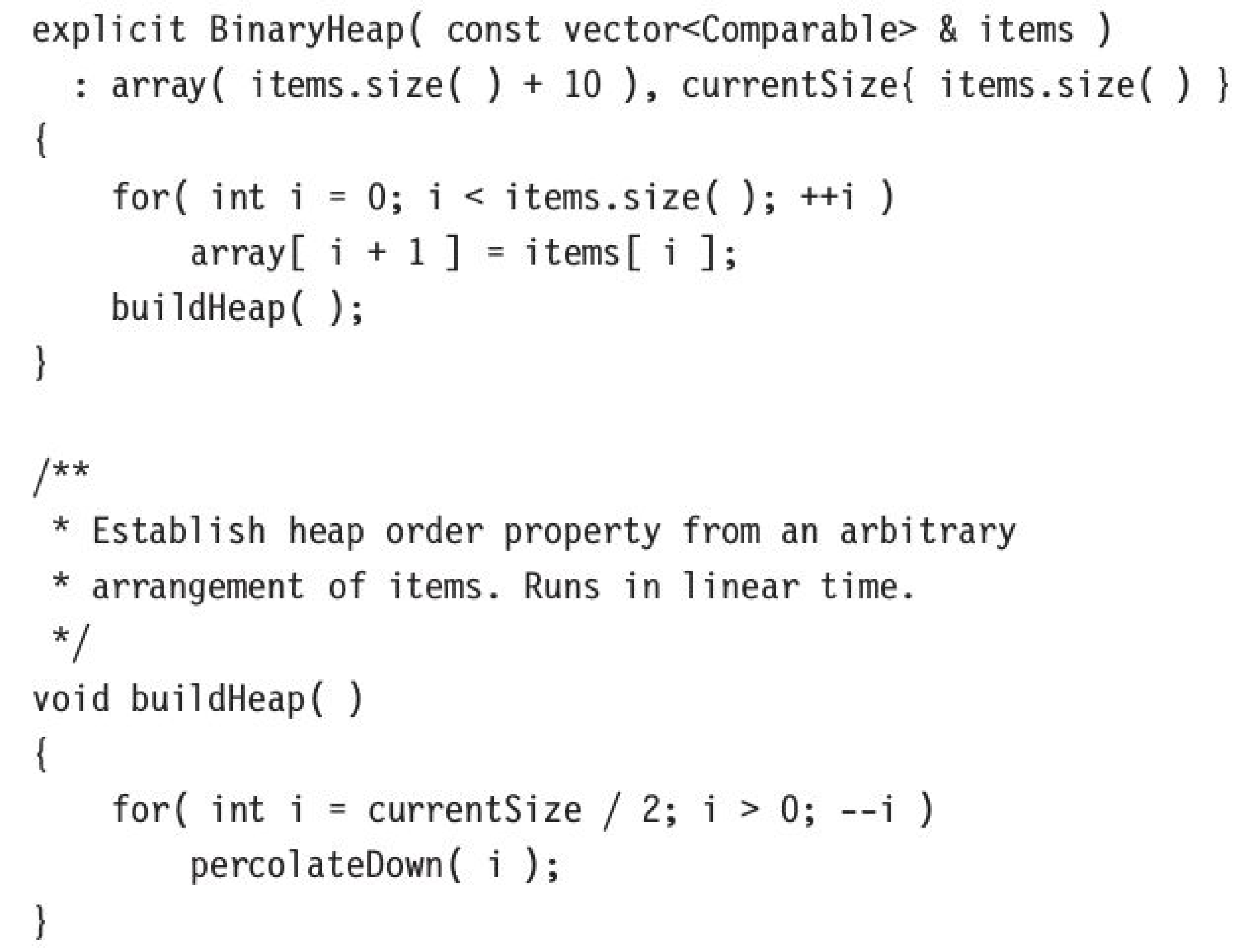
- decreaseKey(p, Δ)
  - Moves a node up the tree. Node @ p has it's key lowered by Δ. Then percolate Up(p)
  - So: heap[p].key -= Δ; percolateUp(p);
- increaseKey(p, Δ)
  - Moves a node down the tree. Node @ p has it's key raised by Δ. Then percolate Down(p)
  - So: heap[p].key += Δ; percolateDown(p);
- remove(p)
  - Removes node @ p. Set key @ p down by ∞. Percolate Up(p), then deleteMin()
  - So: decreaseKey(p, ∞); deleteMin();

Build Heap - O(N) creation of a heap!

- Actually quite simple:
  1) Start with an unordered list of nodes, based at 1 not 0 in array
  2) Start with (size of heap)/2, call percolateDown(), loop and decrement
- Inserts during build into array take O(1) average each,
  with O(log N) worst case
- Average time is: O(1) * N -> O(N)
- Worst case: O(log N) * N -> O(N log N)
- Building the heap gives us an average build of O(N) time!

Build Heap code

- Can merge two arrays?
- How long to do a merge?
    - MAGIC!

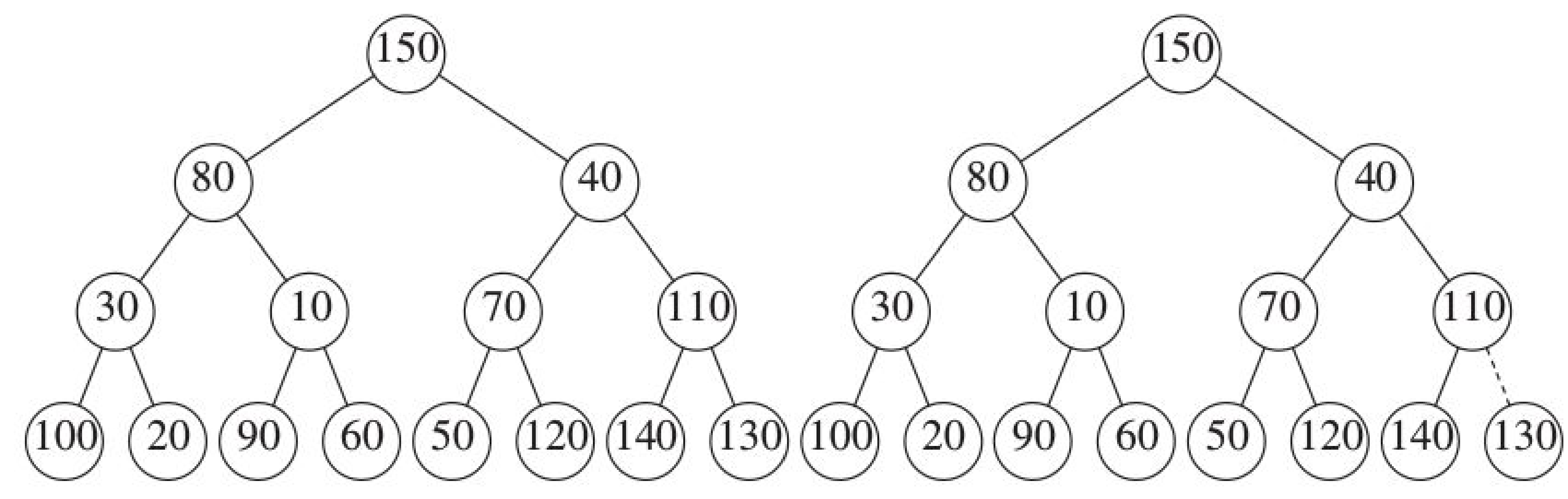


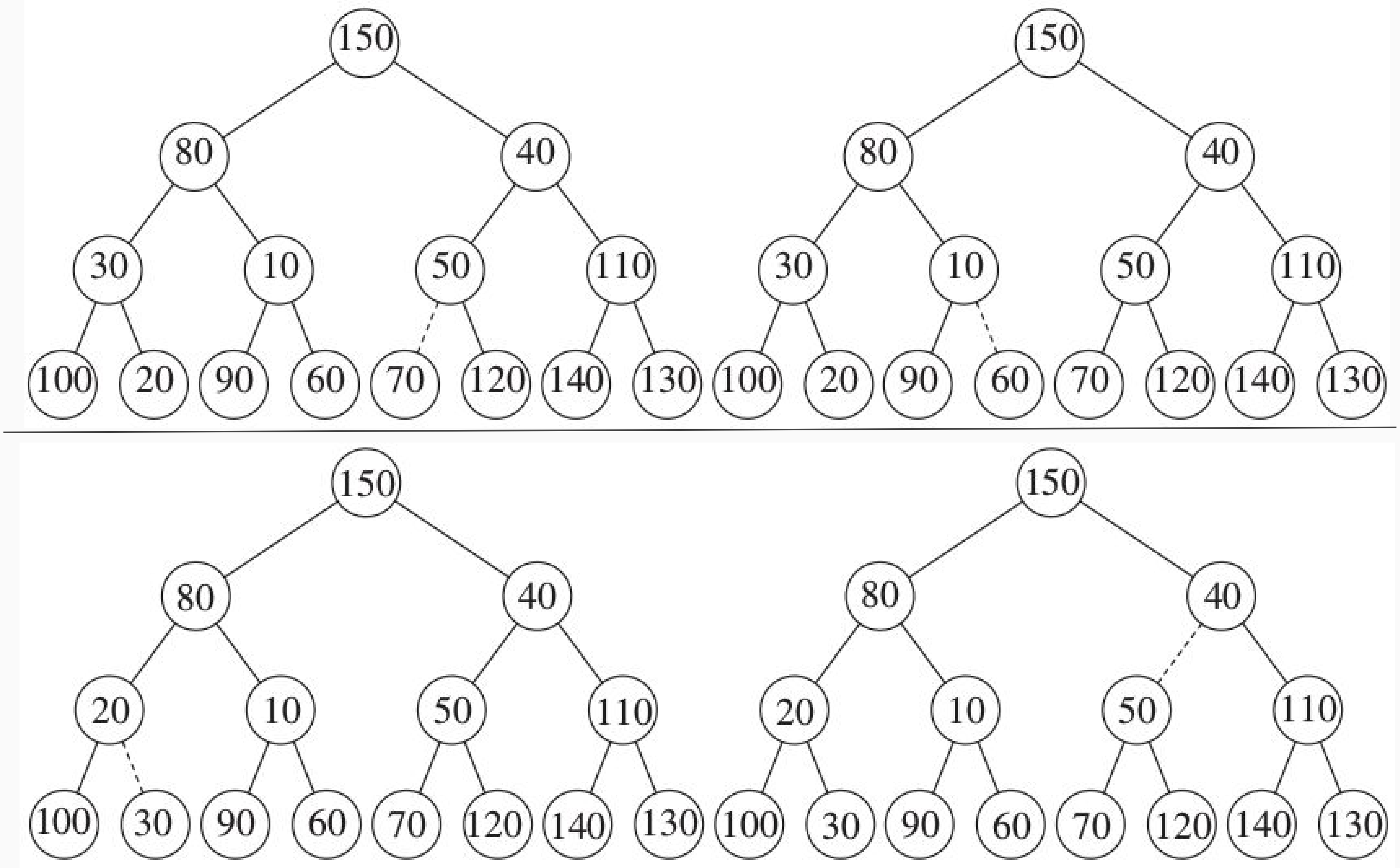
```
explicit BinaryHeap( const vector<Comparable> & items )
  : array( items.size( ) + 10 ), currentSize{ items.size( ) }
{
    for( int i = 0; i < items.size( ); ++i )
        array[ i + 1 ] = items[ i ];
    buildHeap( );
}


/**
 * Establish heap order property from an arbitrary
 * arrangement of items. Runs in linear time.
 */
void buildHeap( )
{
    for( int i = currentSize / 2; i > 0; --i )
        percolateDown( i );
}
```

Build Heap visually

Heap size = 15, so starting i = 15/2 = 7

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VisuAlgo - great heap info

- VisuAlgo has some great heap examples

  https://visualgo.net/heap

For Monday

- Last of heaps
    - D-heaps
    - Leftist heaps
- Starting sorting