Stochastic methods for finance, Report 6

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1 Introduction (VaR)

Value at Risk (VaR) is a widely recognized risk management tool that provides a statistical measure of the maximum expected loss that an investment might experience under normal market conditions, within a specific time frame, and at a certain confidence level. Despite its widespread usage, VaR is known also for its limitations, including its failure to fully capture tail risk, its lack of subadditivity, and its dependency on the assumed distribution of returns. Regardless, VaR remains a fundamental element in financial risk management, utilized not only to measure risk but also to implement risk control strategies, allocate capital, and conduct regulatory capital calculations.

This report presents the results from the Value at Risk analysis on an equibalanced portfolio consisting of two assets(stocks),CVX and META, supposing the portfolio Value is 1 million dollars.

The analysis covered various methods of VaR calculation:

- Parametric (or Variance-Convariance) method: Assumes that asset returns are normally distributed and that the mean and variance of returns are adequate to describe the distribution. The correlation between assets is taken into account through covariance.
- Historical: does not assume any specific return distribution and relies entirely on historical data. Assumes that historical patterns will continue into the future and that the historical data set is sufficiently comprehensive to represent potential future risks.
- Monte Carlo simulation: involves simulating a large number of potential future outcomes for asset prices based on statistical models, then calculating VaR from these simulated outcomes.

2 Parametric VaR Calculation

The objective was to calculate the average and variance of the returns for an equibalanced portfolio and determine the parametric VaR at various confidence levels and different time horizons as:

$$VaR_\alpha(T) = -(E(r)T + Z(\alpha)\sigma_p\sqrt{T}) \tag{1}$$

where $E(r)$ are the expected returns, $Z(\alpha)$ the Z-score, i.e. how many standard deviation to the left of the standard normal distribution the confidence level α is, and σ_p is the standard deviation of the portfolio.

After chosing my equibalanced portfolio, composed by 50% CVX and 50% META, i downloaded from the Refinitv workspace the historical data of the two stocks, and considering the Adjusted Close, I calculated the returns. From the returns, I computed the expected returns as the average of the returns, the standard deviation for each asset and the correlation between the two. I also calculated the standard deviation using the Exponentially Weighted Moving Avarage method, which gives more weight in the calculations to more recent returns as:

$$\sigma_{EWMA} = \sqrt{\lambda\sigma_{t-1}^2 + (1-\lambda)r_{t-1}^2} \tag{2}$$

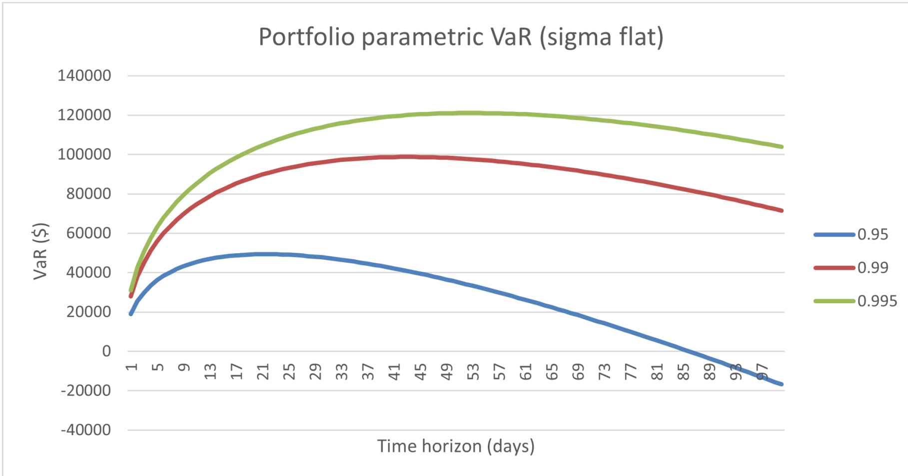
where σ_{t-1}^2 is the variance estimate for the previous period, r_{t-1} is the return for the previous period and λ is the decay factor, in our case is equal to 0.94.

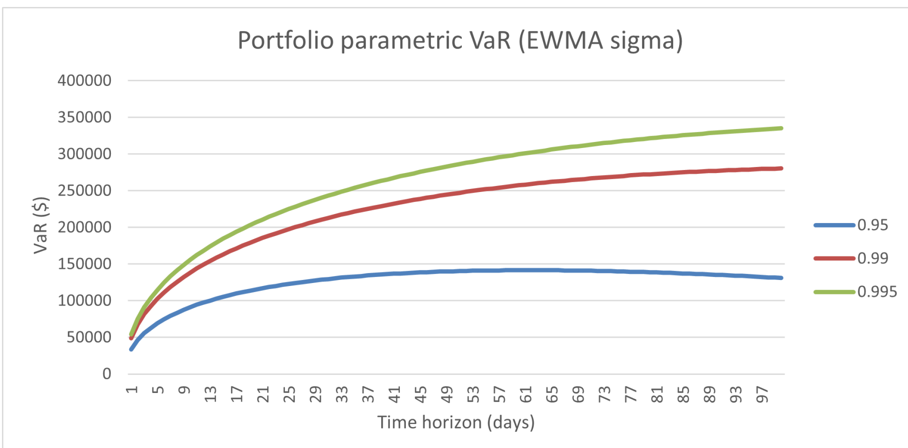
To compute the portfolio returns, since it is equibalanced i just took the mean between the returns of the two assets, and to calculate the standard deviation i used:

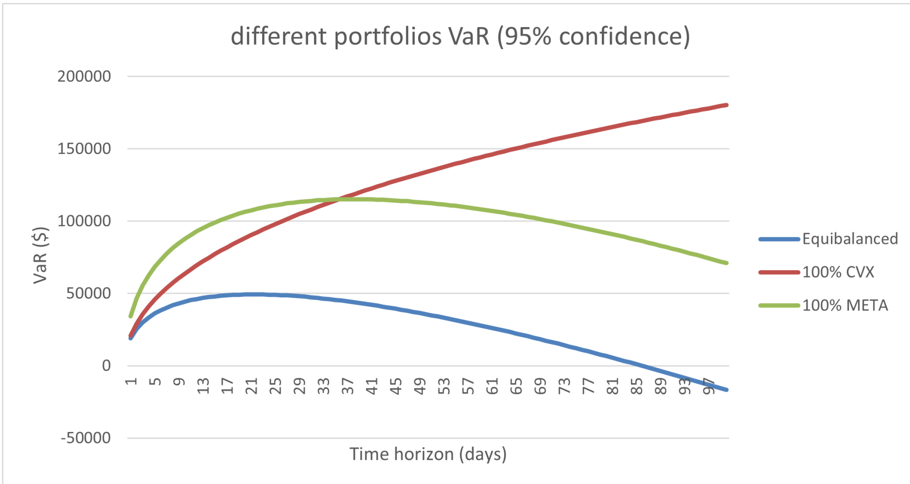
$$\sigma_p = \sqrt{(\sigma_1 w_1)^2 + (\sigma_2 w_2)^2 + 2w_1 w_2 \rho\sigma_1\sigma_2} \tag{3}$$

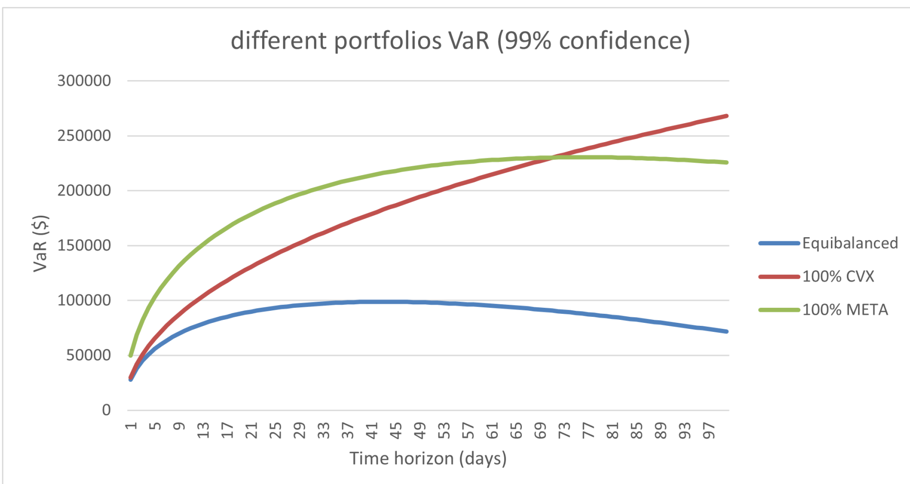
where w are the weights in the portfolio and ρ is the correlation coefficient.

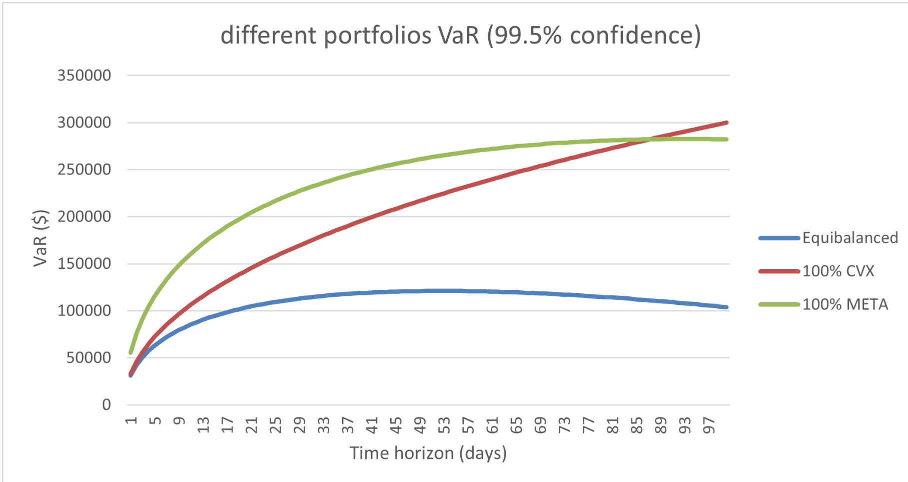
Considering investing 1 million \$, i report the result obtained calculating the VaR at different confidence levels (0.95, 0.99, 0.995) with a time horizon going from 1 to 100 days. I also considered the difference between the equibalanced portfolio and investing the same amount in the single stocks.


Portfolio parametric VaR (sigma flat)
140000
120000
100000
80000
60000
40000
20000
0
-20000
-40000
VaR ($)
1
5
9
13
17
21
25
29
33
37
41
45
49
53
57
61
65
69
73
77
81
85
89
93
97
Time horizon (days)
0.95
0.99
0.995


Portfolio parametric VaR (EWMA sigma)
400000
350000
300000
250000
200000
150000
100000
50000
0
VaR ($)
1
5
9
13
17
21
25
29
33
37
41
45
49
53
57
61
65
69
73
77
81
85
89
93
97
Time horizon (days)
0.95
0.99
0.995


different portfolios VaR (95% confidence)
200000
150000
100000
50000
0
-50000
VaR ($)
1 5 9 13 17 21 25 29 33 37 41 45 49 53 57 61 65 69 73 77 81 85 89 93 97
Time horizon (days)
Equibalanced
100% CVX
100% META


different portfolios VaR (99% confidence)
300000
250000
200000
150000
100000
50000
0
VaR ($)
1 5 9 13 17 21 25 29 33 37 41 45 49 53 57 61 65 69 73 77 81 85 89 93 97
Time horizon (days)
Equibalanced
100% CVX
100% META



As seen in these graphs, diversifying the portfolio has a very strong effect on mitigating the VaR.

3 Monte Carlo VaR Simulation

For the Monte Carlo part, I simulated the two assets price action separately, based on the average returns and volatility calculated at the previous points. I kept track of the correlation between the assets, generating two different standard random variables and multiplying them with the Cholesky lower triangular matrix obtained from the correlation matrix, wich takes the form:

$$L = \begin{bmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{bmatrix} \tag{4}$$

In order to simulate the price action with the proper correlation.
Then I simulated the returns of these two random variables at the specified time T for N monte carlo iterations (T and N free input), and obtain the portfolio returns simply by taking the average between the two.

I stored the results in an array for each asset and for the portfolio, at the end of the Monte Carlo iterations I sorted the simulated returns with the Quick Sort algorithm, the VaR is then the value present at the specified interval of confidence α percentile of the array.

Here I report in a table some of the results obtained simulating over 20000 iterations:

0.95 confidence	CVX	META	Portfolio
1 day	20795	34065	19473
10 days	63687	88579	50541
50 days	134415	113233	67571

0.99 confidence	CVX	META	Portfolio
1 day	29589	49065	27620
10 days	92353	135632	77286
50 days	196624	222370	124380

0.995 confidence	CVX	META	Portfolio
1 day	32700	55556	30909
10 days	100497	155300	87254
50 days	217691	260155	144415

Restarting the simulation this values will vary, but generally in a small interval of $\pm 1\%$ of the values reported.

4 Historical VaR

Historical VaR is a practical and straightforward approach for measuring risk that is particularly useful when the available data reflects a wide range of market conditions and when the distribution of returns is not normal. While it provides valuable insights based on actual historical performance, its effectiveness is dependent on the relevance of the historical period to future market conditions. As such, it is often used in conjunction with other methods to provide a more comprehensive risk assessment.
It is dependent only on historical returns, in this method it's sufficient to order the returns from worst to best performance, and then take the index in the array corresponding to the percentile specified by the confidence level.

Here i report the results obtained:

Confidence	CVX	META	Portfolio
0.95	22764	27303	16950
0.99	32664	42688	22255
0.995	36863	44194	23318

These are the values for a one day time horizon, to obtain other time window VaRs, it's possible to have a valid approximation(for small time windows, e.g. 10 days) just by multiplying by $\sqrt{T}$.

5 Historical Simulation VaR

For the historical simulation part, I used the statical method called "Bootstrapping",which involves repeatedly drawing samples, typically with replacement, from a dataset to create multiple "bootstrap samples." Each sample is used to calculate estimates or other statistics. I applied this method to the portfolio returns, sampling at random multiple times from the historical data, and calculating the VaR for each sample set. In particular i sampled 1000 different times a sample of 125 returns from the historical set of 252 daily returns. At the end of the simulation I took the average of the VaR found for each sample. Here are the result obtained(for the 1-day time horizon):

Confidence	CVX	META	Portfolio
0.95	21217	27572	17067
0.99	33140	46329	23902
0.995	45967	68406	31998

6 Check Additivity

The objective of checking the additivity of Value at Risk (VaR) is to examine whether the VaR for a combined portfolio equals the sum of the VaRs of its individual components. This analysis is crucial to understanding how VaR behaves in the context of portfolio diversification and whether risk measures can be linearly aggregated across different assets.
So I checked in my analysis for (sub-)additivity using:

$$VaR_{portfolio} \leq VaR_{Asset1} + VaR_{Asset2} \tag{5}$$

Subadditivity is important because it implies that diversification reduces risk, which aligns with the fundamental financial principle that diversification leads to risk reduction. For the results in this case study, VaR turned out to be always subadditive, altough, VaR is traditionally known not to be subadditive, which can lead to underestimation of risk in diversified portfolios, this check is essential for confirming the behavior in specific portfolio contexts and adapting risk management strategies accordingly.