Stochastic methods for finance, Report 5

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1 Introduction

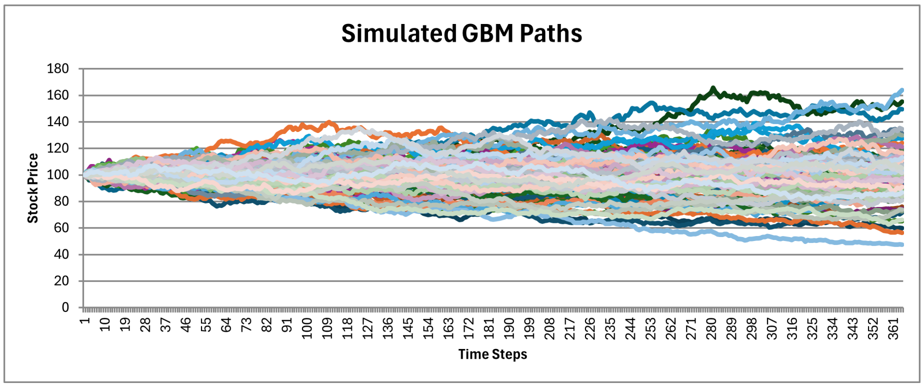
This report outlines the application of Monte Carlo simulation to price various types of financial derivatives, focusing on different option types under the Black-Scholes model. In particular the specific tasks are:

- Simulation of asset price trajectories using Geometric Brownian Motion (GBM).
- Pricing of vanilla options using a one-step Monte Carlo simulation.
- Application of multi-step Euler scheme for more accurate simulation results.
- Pricing path-dependent options like Asian options.
- Extending the model to incorporate other path-dependent options such as lookback options.
- Development of a VBA model to price a "Worst Of" option based on three different assets.

2 GBM simulation

Geometric Brownian Motion (GBM) is a stochastic process that is often used to model stock prices in the Black-Scholes framework. In this task, a VBA script was developed to simulate over N Monte Carlo iterations an asset's price over a period of one year, using the specified parameters: initial asset price $S_0 = 100$, volatility $\sigma = 20\%$, risk free rate $r = 1\%$ and time step $dt = 1$ day.

Here are the results obtained for N = 100:



3 Option pricer

A vanilla option pricer was built using a Monte Carlo simulation that calculates the option price by simulating over 500 terminal stock prices S_T at maturity and computes their average payoff, where

$$S_T = S_0 e^{(r-\frac{1}{2}\sigma^2)T+\sigma\sqrt{T}Z} \tag{1}$$

and Z is a standard normal random variable. The pricing involves both call and put options using a simple one-step approach where only the terminal stock price is simulated without accounting for the path history.

Improving upon the one-step method, a multi-step simulation was implemented to model the path more granularly with $N > 500$ steps. The option price at each step of the simulation is calculated with the Euler Scheme, which is a discretization method for Stochastic Differential Equations, using the formula:

$$S_{t+dt} = S_t + rS_t dt + \sigma S_t \sqrt{dt} Z \tag{2}$$

This approach allows the option pricing model to be more accurate by considering intermediate values of the asset's price path.

The multi-step simulation was extended to price Asian options, which are examples of path-dependent options where the payoff depends on the average price of the underlying asset. The implementation required modification of the existing Monte Carlo simulation to compute and store the average of all simulated asset prices at each step before determining the option payoff.

The existing models were further modified to handle other path-dependent options such as lookback options. The payoff for a lookback option is based on the maximum or minimum asset price reached during the option's life, necessitating the storage and comparison of asset prices at each step in the simulation.

Here I report in a table the results obtained for each option (only call options), considering that if the simulation was to be started again, the results would obviously be different, but still usually in a range of $\pm 20\%$ from those reported.

Option Type	Result
Static vanilla	8,62
Euler vanilla	8,27
Asian	5,13
Lookback	15,74

4 Worst of certificate pricer

Finally, a VBA code was written to price a "Worst Of" option written on three distinct assets. This task involved simulating the prices of three correlated assets using a Cholesky decomposition to generate correlated random paths and then determining the payoff based on the worst-performing asset. I used historical prices of three assets to calculate the daily returns for each one, in order to compute the correlation matrix as :

$$C = \begin{pmatrix} 1 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & 1 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & 1 \end{pmatrix} \tag{3}$$

Then I performed the Cholesky decomposition of the matrix C in order to use the lower triangular matrix obtained and multiply it to a random vector, composed of three standard normal random variables, so that at each step in the simulation, the three variables representing the price action of an asset each, move together accordingly to their respective correlation. For simulating the assets prices at maturity i used Euler scheme at each Monte Carlo iteration, in order to check which of the three is the worst performer for a given iteration, and use that asset to calculate the payoff, and summing that at each iteration. The result of the simulation is then obtained dividing the sum of the payoffs by the number of Monte Carlo iterations, and discounting it to the current time.

As a reference for this pricer I used an existent certificate issued by Unicredit, which can be found at Unicredit Website, so the three asset I choose are Apple, Infineon technologies and Intesa San Paolo Stocks. The Correlation matrix for these three assets turned out to be:

$$C = \begin{pmatrix} 1 & 0,079 & 0,163 \\ 0,079 & 1 & 0,081 \\ 0,163 & 0,081 & 1 \end{pmatrix} \tag{4}$$

I found the "Worst of" cerificate price to be $1,49$ euros, floating for each simulation in a range of about $\pm 30\%$ of this value.