Torsion of open sections

NB.2 Internal forces due to torsion

Effects of St.Venant torsion can be neglected in sense of 6.2.7.(7) of EN1993-1-1 in case of open sections. When transverse loading on a beam of an open section is applied with the eccentricity from the shear centre the components of internal forces associated with torsion can be determined as:

$B_{Ed} = M_{Ed}\, e\, (1 - \kappa)$ $T_{t,Ed} = V_{Ed}\, e\, \kappa$ $T_{w,Ed} = V_{Ed}\, e\, (1 - \kappa)$ $K_t = L\, (GI_T / EI_w)^{0,5}$ $\kappa = 1/[\, \beta + (\, \alpha / K_t\,)^2\,]$	where B_{Ed} bimoment $T_{t,Ed}$ moment due to St'Venant torsion $T_{w,Ed}$ moment due to warping torsion M_{Ed} bending moment V_{Ed} shear force

κ coefficient taking in to account the effect of St'Venant stiffness of the cross-section. It is a function of a non-dimensional parameter in torsion K_t.

M_{Ed} and V_{Ed} is moment and shear force due to the transverse loading determined for a given member. These are determined for transverse loading with respect to the boundary condition at beam ends.

Coefficients α and β take in to account the type of loading and boundary conditions:

Boundary conditions in torsion		Torsional loading		α	β
Beam supported at both ends	Free warping	Uniformly distributed load		3,1	1,00
		Arbitrary		3,7	1,08
	Warping restrained	Uniformly distributed load	For internal force at support	8,0	1,25
			Maximum within span	5,6	1,00
		Arbitrary		6,9	1,14
Cantilever	Warping restrained	Arbitrary - internal force at support		2,7	1,11

In the case of a beam simply supported on both sides, the effect of St'Venant torsion can be neglected when $K_t \leq 1$ ($T_{t,Ed}$), while for $K_t \geq 15$ the warping torsion components (B_{Ed}, $T_{w,Ed}$) can be neglected.

NB.3 Elastic critical moment

The procedure is suitable for calculation of the critical moment of beams of constant cross-section of bi-axially symmetric cross-section, cross-sections symmetric about major axis (z) only, and a cross-section symmetric about minor axis (y) when loading passes trough the shear centre.

$$M_{cr} = \mu_{cr} \frac{\pi \sqrt{EI_z\, GI_t}}{L}$$

where non-dimensional critical moment is

$$\mu_{cr} = \frac{C_1}{k_z}\left[\sqrt{1+\kappa_{wt}^2+\left(C_2\zeta_g - C_3\zeta_j\right)^2} - \left(C_2\zeta_g - C_3\zeta_j\right)\right]$$

Non-dimensional parameter in torsion is

$$\kappa_{wt} = \frac{\pi}{k_w L}\sqrt{\frac{EI_w}{GI_t}}$$

non-dimensional parameter involving point of application of loading with respect to the shear centre:

$$\zeta_g = \frac{\pi\, z_g}{k_z\, L}\sqrt{\frac{EI_z}{GI_t}}$$

non-dimensional parameter of section assymmetry:

$$\zeta_j = \frac{\pi\, z_j}{k_z\, L}\sqrt{\frac{EI_z}{GI_t}}$$

C1, C2, C3 are coefficients accounting for loading types and end conditions, see following tables,
L is length of a beam between points where out-of plane movement is restrained,
kz and kw are buckling length coefficients,
zg = za – zs

$$z_j = z_s - \frac{0{,}5}{I_y}\int_A (y^2 + z^2)\, z\, dA$$

For sections symmetrical about y-y axis zj = 0.

za point of load application with respect to the centre of gravity
zs coordinate of a shear centre with respect to the centre of gravity
zg coordinate of load application with respect to the shear centre

For I-section with asymmetric flanges:

$$I_w = (1 - \psi_f^2) I_z (h_s / 2)^2$$

hs is a distance between the shear centres of flanges
parameter of cross-section asymmetry:

$$\psi_f = \frac{I_{fc} - I_{ft}}{I_{fc} + I_{ft}}$$

where
Ifc second moment of inertia of the compressed flange about the minor axis of the cross-section,
Ift second moment of inertia of the tension flange about the minor axis of the cross-section

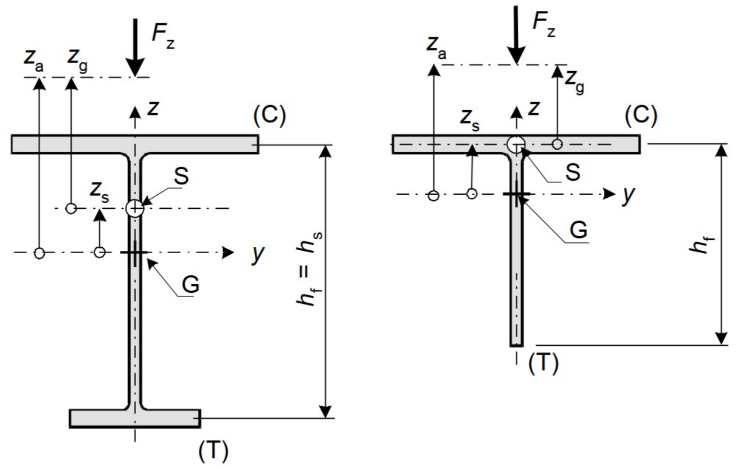
The buckling length coefficients ky, kz (describing the boundary conditions in bending) and kw (describing the torsional boundary conditions) vary from 0.5 for both ends fixed to 1.0 for the hinge for both ends, with a value of 0.7 for one end fixed and the other hinged.

The coefficient ky refers to the end rotation in the plane perpendicular to the y-y axis, the coefficient kz refers to to the end rotation in the plane perpendicular to the z-axis. These coefficients are analogous to the ratio Lcr/L of a compressed member. The factor kw relates to the end warping. Unless a special measure to prevent warping is applied, k = 1.0 may be taken.

The values of C1, C2 and C3 are given in Tables NB.3.1 and NB.3.2 for different load cases which are defined by the bending moment along the length L between the points secured against transverse buckling. The values are given as a function of the coefficient kz and, in Table NB.3.2, of the coefficient kw.

In cases where kz = 1.0, the coefficient C1 can be used for any ratio of end moments according to the table NB.3.1 can be determined approximately by the relation:

$$C_1 = (0{,}310 + 0{,}428\,\psi + 0{,}262\,\psi^2)^{-0{,}5}$$



(T) for tension part, (C) for compressed part, S for shear centre, G for centre of gravity

Figure NB.3.1 - Sign convention and definitions when applying the load Fz

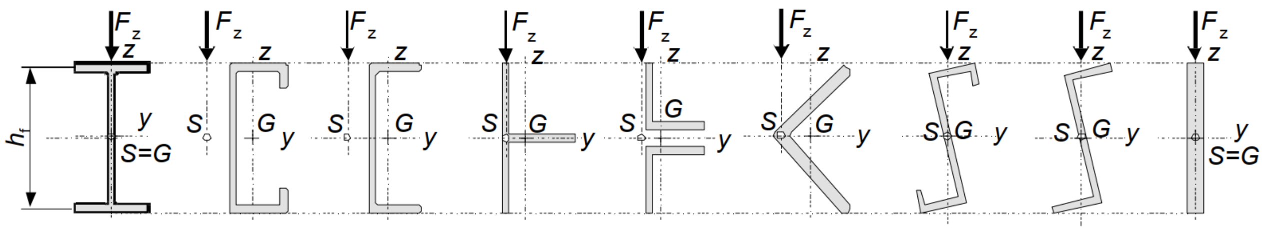


Figure NB.3.2 - Cross-sections symmetrical to the major axis or bi-axially symmetrical

The sign convention for determining z and zj, see Figure NB.3.1, is:
[1] the z coordinate is positive for the compressed flange. When zj is determined using the relation in NB.3.2(1), it points the z-axis upwards in the case of a gravity load. The z-axis points downwards in the case of a suction load;
[2] the sign of zj is the same as the sign of the cross-sectional asymmetry coefficient ψ

The sign convention for zg is:
[1] for load effects zg is positive when the load is applied above the centre of shear;
[2] in the general case zg is positive when the load is directed from the point of action to the centre of shear

Table NB.3.1 - Values of the coefficients C1 and C3 when loading the member with end moments depending on the value of the coefficient kz and the coefficients ψf and κwt

Coefficient of the end conditions of member in plane of bending ky = 1.0, in warping kW = 1.0

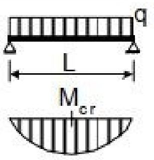
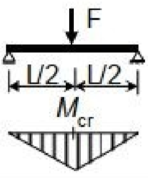
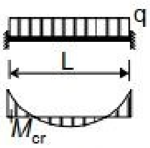
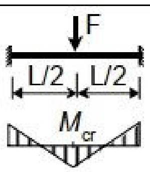
<table>
<tr><th rowspan="3">Bending moment diagram
End moment ratio (psi)</th><th rowspan="3">k_z [2]</th><th colspan="6">Coefficients c1 and C3</th></tr>
<tr><th colspan="2">C_1 [1]</th><th colspan="4">C_3</th></tr>
<tr><th>$C_{1,0}$</th><th>$C_{1,1}$</th><th>$\psi_f = -1$</th><th>$-0{,}9 \le \psi_f \le 0$</th><th>$0 \le \psi_f \le 0{,}9$</th><th>$\psi_f = 1$</th></tr>
<tr><td rowspan="4">M_{cr} $\psi = +1$</td><td>1,0</td><td>1,00</td><td>1,00</td><td colspan="4">1,00</td></tr>
<tr><td>0,7L</td><td>1,02</td><td>1,10</td><td colspan="2">1,02</td><td colspan="2">1,00</td></tr>
<tr><td>0,7R</td><td>1,02</td><td>1,10</td><td colspan="2">1,02</td><td colspan="2">1,00</td></tr>
<tr><td>0,5</td><td>1,00</td><td>1,13</td><td colspan="4">1,02</td></tr>
<tr><td rowspan="4">M_{cr} $\psi = +3/4$</td><td>1,0</td><td>1,14</td><td>1,14</td><td colspan="4">1,00</td></tr>
<tr><td>0,7L</td><td>1,21</td><td>1,31</td><td colspan="2">1,05</td><td colspan="2">1,00</td></tr>
<tr><td>0,7R</td><td>1,11</td><td>1,20</td><td colspan="4">1,00</td></tr>
<tr><td>0,5</td><td>1,14</td><td>1,29</td><td colspan="4">1,02</td></tr>
<tr><td rowspan="4">M_{cr} $\psi = +1/2$</td><td>1,0</td><td>1,31</td><td>1,32</td><td>1,15</td><td colspan="3">1,00</td></tr>
<tr><td>0,7L</td><td>1,48</td><td>1,62</td><td colspan="2">1,16</td><td colspan="2">1,00</td></tr>
<tr><td>0,7R</td><td>1,21</td><td>1,32</td><td colspan="4">1,00</td></tr>
<tr><td>0,5</td><td>1,31</td><td>1,48</td><td>1,15</td><td colspan="3">1,00</td></tr>
<tr><td rowspan="4">M_{cr} $\psi = +1/4$</td><td>1,0</td><td>1,52</td><td>1,55</td><td>1,29</td><td colspan="3">1,00</td></tr>
<tr><td>0,7L</td><td>1,85</td><td>2,06</td><td>1,60</td><td>1,26</td><td colspan="2">1,00</td></tr>
<tr><td>0,7R</td><td>1,33</td><td>1,47</td><td colspan="4">1,00</td></tr>
<tr><td>0,5</td><td>1,52</td><td>1,73</td><td>1,35</td><td colspan="3">1,00</td></tr>
<tr><td rowspan="4">M_{cr} $\psi = 0$</td><td>1,0</td><td>1,77</td><td>1,85</td><td>1,47</td><td colspan="3">1,00</td></tr>
<tr><td>0,7L</td><td>2,33</td><td>2,68</td><td>2,00</td><td>1,42</td><td colspan="2">1,00</td></tr>
<tr><td>0,7R</td><td>1,45</td><td>1,59</td><td colspan="4">1,00</td></tr>
<tr><td>0,5</td><td>1,75</td><td>2,03</td><td>1,50</td><td colspan="3">1,00</td></tr>
<tr><td rowspan="4">M_{cr} $\psi = -1/4$</td><td>1,0</td><td>2,05</td><td>2,21</td><td>1,65</td><td>1,00</td><td colspan="2">0,85</td></tr>
<tr><td>0,7L</td><td>2,83</td><td>3,32</td><td>2,40</td><td>1,55</td><td>0,85</td><td>-0,30</td></tr>
<tr><td>0,7R</td><td>1,58</td><td>1,75</td><td>1,38</td><td>0,85</td><td>0,70</td><td>0,20</td></tr>
<tr><td>0,5</td><td>2,00</td><td>2,34</td><td>1,75</td><td>1,00</td><td>0,65</td><td>-0,25</td></tr>
<tr><td rowspan="4">M_{cr} $\psi = -1/2$</td><td>1,0</td><td>2,33</td><td>2,59</td><td>1,85</td><td>1,00</td><td>$1{,}3 - 1{,}2\ \psi_f$</td><td>-0,70</td></tr>
<tr><td>0,7L</td><td>3,08</td><td>3,40</td><td>2,70</td><td>1,45</td><td>$1{,}0 - 1{,}2\ \psi_f$</td><td>-1,15</td></tr>
<tr><td>0,7R</td><td>1,71</td><td>1,90</td><td>1,45</td><td>0,78</td><td>$0{,}9 - 0{,}75\ \psi_f$</td><td>-0,53</td></tr>
<tr><td>0,5</td><td>2,23</td><td>2,58</td><td>2,00</td><td>0,95</td><td>$0{,}75 - \psi_f$</td><td>-0,85</td></tr>
<tr><td rowspan="4">M_{cr} $\psi = -3/4$</td><td>1,0</td><td>2,55</td><td>2,85</td><td>2,00</td><td>1,00</td><td>$0{,}55 - \psi_f$</td><td>-1,45</td></tr>
<tr><td>0,7L</td><td>2,59</td><td>2,77</td><td>2,00</td><td>0,85</td><td>$0{,}23 - 0{,}9\ \psi_f$</td><td>-1,55</td></tr>
<tr><td>0,7R</td><td>1,83</td><td>2,03</td><td>1,55</td><td>0,70</td><td>$0{,}68 - \psi_f$</td><td>-1,07</td></tr>
<tr><td>0,5</td><td>2,35</td><td>2,61</td><td>2,00</td><td>0,85</td><td>$0{,}35 - \psi_f$</td><td>-1,45</td></tr>
<tr><td rowspan="4">M_{cr} $\psi = -1$</td><td>1,0</td><td>2,56</td><td>2,73</td><td>2,00</td><td colspan="2">ψ_f</td><td>-2,00</td></tr>
<tr><td>0,7L</td><td>1,92</td><td>2,10</td><td>1,55</td><td>0,38</td><td>-0,58</td><td>-1,55</td></tr>
<tr><td>0,7R</td><td>1,92</td><td>2,10</td><td>1,55</td><td>0,58</td><td>-0,38</td><td>-1,55</td></tr>
<tr><td>0,5</td><td>2,22</td><td>2,39</td><td>1,88</td><td>$0{,}125 - 0{,}7\ \psi_f$</td><td>$-0{,}125 - 0{,}7\ \psi_f$</td><td>-1,88</td></tr>
</table>

Notes:

1) $C_1 = C_{1,0} + (C_{1,1} - C_{1,0})\kappa_{wt} \le C_{1,1}$, ($C_1 = C_{1,0}$ pro $\kappa_{wt} = 0$, $C_1 = C_{1,1}$ pro $\kappa_{wt} \ge 1$)

2) 0,7 L = LHS end is fixed, 0,7 R = RHS end is fixed

Table NB.3.2 - C1, C2 and C3 coefficient values for different transverse load cases depending on on the value of the coefficients k y, kz, kw and the coefficients ψf and κwt

Loading and support conditions	Buckling length coefficient			Values of C1, C2 and C3 parameters							
	k_y	k_z	k_w	C_1 [1)]		C_2			C_3		
				$C_{1,0}$	$C_{1,1}$	$\psi_f = -1$	$-0{,}9 \le \psi_f \le 0{,}9$	$\psi_f = 1$	$\psi_f = -1$	$-0{,}9 \le \psi_f \le 0{,}9$	$\psi_f = 1$
	1	1	1	1,13	1,13	0,33	0,46	0,50	0,93	0,53	0,38
	1	1	0,5	1,13	1,23	0,33	0,39	0,50	0,93	0,81	0,38
	1	0,5	1	0,95	1,00	0,25	0,41	0,40	0,84	0,48	0,44
	1	0,5	0,5	0,95	0,97	0,25	0,31	0,40	0,84	0,67	0,44
	1	1	1	1,35	1,36	0,52	0,55	0,42	1,00	0,41	0,31
	1	1	0,5	1,35	1,45	0,52	0,58	0,42	1,00	0,67	0,31
	1	0,5	1	1,03	1,09	0,40	0,45	0,42	0,80	0,34	0,31
	1	0,5	0,5	1,03	1,07	0,40	0,44	0,42	0,80	0,52	0,31
						$\psi_f = -1$	$-0{,}5 \le \psi_f \le 0{,}5$	$\psi_f = 1$	$\psi_f = -1$	$-0{,}5 \le \psi_f \le 0{,}5$	$\psi_f = 1$
	0,5	1	1	2,58	2,61	1,00	1,56	0,15	1,00	-0,86	-1,99
	0,5	0,5	1	1,49	1,52	0,56	0,90	0,08	0,61	-0,52	-1,20
	0,5	0,5	0,5	1,49	1,75	0,56	0,83	0,08	0,61	0,00	-1,20
	0,5	1	1	1,68	1,73	1,20	1,39	0,07	1,15	-0,72	-1,35
	0,5	0,5	1	0,94	0,96	0,69	0,76	0,03	0,64	-0,41	-0,76
	0,5	0,5	0,5	0,94	1,06	0,69	0,84	0,03	0,64	-0,07	-0,76

Notes:

1) $C_1 = C_{1,0} + (C_{1,1} - C_{1,0})\kappa_{wt} \le C_{1,1}$, ($C_1 = C_{1,0}$ pro $\kappa_{wt} = 0$, $C_1 = C_{1,1}$ pro $\kappa_{wt} \ge 1$)

2) Parametr ψ_f is related to the centre of span

3) Values of critical moment M_{cr} relate to the cross-section where M_{max} is applied