$y = x^T \cdot x \qquad x \in \mathbb{R}^n \to x = (x_1, \dots, x_n)^T$

$$x^T x = (x_1, \dots, x_n)\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = x_1 \cdot x_1 + x_2 x_2 + \dots + x_n x_n = x_1^2 + \dots + x_n^2$$

$$\frac{dy}{dx} = \operatorname{grad} y = \begin{pmatrix} \frac{\partial y}{\partial x_1} \\ \vdots \\ \frac{\partial y}{\partial x_n} \end{pmatrix} = \begin{pmatrix} 2x_1 \\ \vdots \\ 2x_n \end{pmatrix} = 2 \cdot x$$

$y = \operatorname{tr}(AB) \qquad A, B \in \mathbb{R}^{N \times N}$

$$A \cdot B = \underbrace{\begin{pmatrix} a_{11} & \dots & a_{1N} \\ \vdots & & \vdots \\ a_{N1} & \dots & a_{NN} \end{pmatrix}}_{N \times N} \underbrace{\begin{pmatrix} b_{11} & \dots & b_{1N} \\ \vdots & & \vdots \\ b_{N1} & \dots & b_{NN} \end{pmatrix}}_{N \times N} =$$

$$= \begin{pmatrix} a_{11} b_{11} + a_{12} \cdot b_{21} + \dots + a_{1N} \cdot b_{N1} & \dots & a_{11} b_{1N} + \dots + a_{1N} \cdot b_{NN} \\ a_{21} b_{11} + a_{22} \cdot b_{21} + \dots + a_{2N} \cdot b_{N1} & \dots & b_{1N} \cdot a_{21} + a_{22} \cdot b_{2N} + \dots + a_{2N} \cdot b_{NN} \\ \vdots & & \vdots \\ b_{11} \cdot a_{N1} + \dots + a_{NN} \cdot b_{N1} & \dots & b_{1N} \cdot a_{N1} + b_{2N} \cdot a_{N2} + \dots + b_{NN} \cdot a_{NN} \end{pmatrix}$$

$$y = \operatorname{tr}(AB) = a_{11} b_{11} + a_{12} \cdot b_{21} + \dots + a_{1N} \cdot b_{N1} \oplus$$
$$\oplus\, a_{21} \cdot b_{12} + a_{22} \cdot b_{22} + \dots + a_{2N} \cdot b_{N2} \oplus \dots \oplus b_{1N} \cdot a_{N1} + b_{2N} \cdot a_{N2} + \dots + b_{NN} \cdot a_{NN}$$

$$\left(\frac{dy}{dA}\right) = \left(\frac{\partial y(A)}{\partial a_{ij}}\right) = \begin{pmatrix} b_{11} & b_{21} & \dots & b_{N1} \\ b_{12} & b_{22} & \dots & b_{N2} \\ b_{1N} & \dots & & b_{NN} \end{pmatrix} = B^T$$

$$y = x^T A c \qquad A \in \mathbb{R}^{N\times N} \quad x \in \mathbb{R}^N \quad c \in \mathbb{R}^N$$

$$\underbrace{\underset{1\times N}{(x_1 \dots x_N)} \cdot \underset{N\times N}{\begin{pmatrix} a_{11} & \dots & a_{1N} \\ & \ddots & \\ a_{N1} & & a_{NN} \end{pmatrix}}}_{1\times N} \cdot \underset{N\times 1}{\begin{pmatrix} c_1 \\ \vdots \\ c_N \end{pmatrix}} = (x_1 a_{11} + \dots + x_N \cdot a_{N1} \;\dots\; a_{1N} \cdot x_1 + \dots + x_N \cdot a_{NN}) \begin{pmatrix} c_1 \\ \vdots \\ c_N \end{pmatrix} =$$

$$= c_1(x_1 a_{11} + \dots + x_N \cdot a_{N1}) + \dots + c_N (a_{1N} \cdot x_1 + \dots + x_N \cdot a_{NN})$$

$$\frac{dy}{dx} = \left(\frac{\partial y}{\partial x_i}\right) = \begin{pmatrix} c_1 a_{11} + c_2 a_{12} + \dots + c_N a_{1N} \\ \vdots \\ c_1 a_{N1} + c_2 a_{N2} + \dots + c_N a_{NN} \end{pmatrix} = A \cdot c$$

$$\frac{dy}{dA} = \left(\frac{\partial y}{\partial a_{ij}}\right) = \underset{N\times N}{\begin{pmatrix} c_1 x_1 & c_2 x_1 & \dots & c_N x_1 \\ c_1 x_2 & c_2 x_2 & \dots & c_N x_2 \\ \vdots & & & \vdots \\ c_1 x_N & c_2 x_N & & c_N x_N \end{pmatrix}} = \underset{N\times 1}{\begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix}} \cdot \underset{1\times N}{(c_1 \dots c_N)} = x \cdot c^T$$