Discrete Geometry I Thursday, 07/05/2020

Summer term 2020

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Exercise sheet 3

Due-date: Friday, 15/05/2020, as latex file via GitLab.

Exercise 1 **2 Points**

Prove that the Minkowski sum of two polyhedra is again a polyhedron.

Exercise 2 **4 Points**

Let $P = \{x \in \mathbb{R}^n : \langle a_i, x\rangle \leq b_i, i = 1, \ldots, m\}$ be a polyhedron and let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear bijection. Prove that $Q := T(P)$ is a polyhedron given by the inequalities $\langle c_i, x\rangle \leq b_i$, $i = 1, \ldots, m$ for $c_i = (T^*)^{-1}(a_i)$ where, as usual, T^* is the linear map conjugate to T.

Exercise 3 **3 Points**

Let $A \subset \mathbb{R}^n$. Prove that A is open if and only if A is algebraically open.

Exercise 4 **5 Points**

Let $V \subset \mathbb{R}^{\mathbb{N}}$ be the vector space of all real-valued sequences that are zero almost everywhere (i.e. sequences with finite support). Let $A \subset V$ denote the sequences whose last non-zero entry is strictly positive. Prove that A is convex and that there is no affine hyperplane $0 \in H \subset V$ that isolates A. Moreover, show that A is not contained in any affine hyperplane.

Exercise 5 **2 Points**

Let $\Delta \subset \mathbb{R}^n$ be a simplex, i.e. the convex hull of $n+1$ affinely independent points. Show that $\mathrm{int}(\Delta) \neq \emptyset$.