Discrete Geometry I Saturday, 02/05/2020 (exceptional)
Summer term 2020
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Exercise sheet 2

Due-date: Tuesday, 12/05/2020 (exceptional), as latex file via GitLab.

Exercise 1 **6 Points**
In this exercise we examine the assumptions of some theorem that we saw in the lectures.

a) Let $n \in \mathbb{N}$. Prove that there exists a finite family of convex sets $A_1, \ldots, A_m$, $m \geq n$, such that any intersection of n distinct $A_i\,'s$ is non-empty while $\cap_{i=1}^m A_i = \emptyset$ holds.

b) Give an example of a countably infinite familiy of convex sets $\{A_i\}$ in some $\mathbb{R}^d, d > 2$, such that every $d+1$ of the A_i share a common point while $\cap_{i \in \mathbb{N}} A_i = \emptyset$ holds.

Exercise 2 **2 Points**
Recall the function space $\mathcal{C}(\mathbb{R}^d)$ and show that the (ring) multiplication of the underlying $\mathbb{R}$-algebra structure is well-defined. Is $\mathcal{C}(\mathbb{R}^d)$ a finite dimensional real vector space?

Exercise 3 **4 Points**
Let $I_n : \{(x_1, \ldots, x_n) \in \mathbb{R}^n \colon 0 < x_1, \ldots, x_n < 1\}$. Determine $\chi(I_2)$ and $\chi(I_3)$.

Exercise 4 **4 Points**
Let $A_1, \ldots, A_m \subset \mathbb{R}^d$ be closed convex sets with $\cap_{i=1}^m A_i \neq \emptyset$. Prove that $\chi(\cup_{i=1}^m A_i) = 1$.