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Mathematical Physics I - WS 2018/2019

http://www3.math.tu-berlin.de/geometrie/Lehre/WS18/MP1/

Exercise Sheet 12

Exercise 1: (6 pts)

In the canonical phase space $\mathbb{R}^2$ consider the the parametric family of vector fields

$$f(q,p) := \left(p^\alpha q^\beta, -p^{\alpha+1} q^\delta\right),$$

with $\alpha, \beta, \delta \in \mathbb{R}$.

(i) Find the values of α, β, δ for which f is Hamiltonian.

(ii) Compute the corresponding Hamiltonians.

Exercise 2: (4 pts)

In the canonical phase space $\mathbb{R}^{2n}$ consider the transformation

$$(q,p) \mapsto (\widetilde{q}, \widetilde{p}) := (q, f(q,p)),$$

for some smooth function f. Determine the structure that f must have for the transformation to be symplectic.

Exercise 3: (4 pts)

In the canonical phase space $\mathbb{R}^2$ consider the transformation

$$(q,p) \mapsto (\widetilde{q}, \widetilde{p}) := \left(q\sqrt{1+q^2\,p^2}, \frac{p}{\sqrt{1+q^2\,p^2}}\right).$$

Show that this transformation is symplectic by proving that the canonical symplectic 2-form is preserved.

Exercise 4: (6 pts)

In the canonical phase space $\mathbb{R}^4$ consider the Hamiltonian

$$\mathscr{H}(q_1, q_2, p_1, p_2) := \frac{1}{2}\left(p_1^2 + q_1^2\, q_2\, p_2\right).$$

(i) Find an integral of motion for the flow of $\mathscr{H}$.

(ii) Find the hamiltonian vector field generated by this integral and compute its flow Ψ_s.

(iii) Check that Ψ_s is a one-parameter group of symplectic transformations

$$(q_1, q_2, p_1, p_2) \mapsto (\widetilde{q}_1, \widetilde{q}_2, \widetilde{p}_1, \widetilde{p}_2) = \Psi_s(q_1, q_2, p_1, p_2), \qquad s \in \mathbb{R},$$

which preserves the form of the function $\mathscr{H}$ for all $s \in \mathbb{R}$, i.e.,

$$\widetilde{\mathscr{H}}(\widetilde{q}_1, \widetilde{q}_2, \widetilde{p}_1, \widetilde{p}_2) := \frac{1}{2}\left(\widetilde{p}_1^2 + \widetilde{q}_1^2\, \widetilde{q}_2\, \widetilde{p}_2\right).$$

where $\widetilde{\mathscr{H}} \circ \Psi_s = \mathcal{H}$.

Due Monday, January 28.