Homework 01

General

1. We show that $||A||_2 = \sqrt{\lambda_{\max}(A^H A)}$ for any matrix $A \in \mathbb{C}^{n \times m}$.

   Proof. All eigenvalues of $A^H A$ are real and nonnegative. Since $A^H A$ is HPSD we can write it as $U^H A^H A U = diag(\lambda_1, ..., \lambda_m) = D$. Then with $y = U^H x$

   $$||A||_2^2 = \max_{||x||=1} ||Ax||_2 = \max_{||x||=1} \langle A^H Ax, x\rangle = \max_{||Uy||=1} \langle A^H AUy, Uy\rangle = \max_{||y||=1} \langle Dy, y\rangle \leq \lambda_{\max}||y||_2^2$$

   We have a maximum if x equals the unit eigenvector of the largest eigenvector of $A^H A$. Then $y = e_j$ with λ_j ist the largest eigenvector. Thus, $||A||_2^2 = \lambda_{\max}$. □

Exercise 1

1. Show that $A^H A = 0 \implies A = 0$.

   Proof. Let b_{ij} denote the i, j entry of $A^H A$. It is $b_{ii} = \sum_{k=1}^{n} \bar{a}_{ki} a_{ki} = ||a_i||^2$. This means that all columns a_i must equal zero. □

Exercise 2

Exercise 3