Lemma

Let $f : (a, b) \to \mathbb{R}$ be convex. Let $a < x < y < z < b$. Then, it holds

$$\frac{f(y-x)}{y-x} \leq \frac{f(z-x)}{z-x} \leq \frac{f(z-y)}{z-y}.$$

Proof. Let $x < y < z$. It holds

$$y = \frac{y-z}{x-z} + \left(1 - \frac{y-z}{x-z}\right) \quad \text{and} \quad 0 < \frac{y-z}{x-z} < 1.$$

With convexity of f it follows

$$f\left(\frac{y-z}{x-z} + (1 - \frac{y-z}{x-z})\right) = f(y) \leq$$

$\square$