TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

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Mathematical Physics I - WS 2018/2019

http://www3.math.tu-berlin.de/geometrie/Lehre/WS18/MP1/

Exercise Sheet 5

Exercise 1: (4 pts)

Consider the following IVP in $\mathbb{R}/(2\pi\mathbb{Z})$:

$$\begin{cases} \dot{x} = \cos x - 1, \\ x(0) \in \mathbb{R}/(2\pi\mathbb{Z}). \end{cases}$$

Prove that $x = 0$ is an attracting point and find the basin of attraction.

Exercise 2: (6 pts)

Consider the following linear IVPs in $\mathbb{R}^2$:

$$\begin{cases} \dot{x}_1 = x_1, \\ \dot{x}_2 = -x_1 + x_2, \end{cases} \qquad \begin{cases} \dot{x}_1 = -x_1 + x_2, \\ \dot{x}_2 = -x_2, \end{cases} \qquad \begin{cases} \dot{x}_1 = 2x_1 + x_2, \\ \dot{x}_2 = 6x_1 + 3x_2, \end{cases}$$

with $(x_1(0), x_2(0)) \in \mathbb{R}^2$. For each of them:

(i) Find the solution.

(ii) Study the stability of the fixed points.

(iii) Sketch the phase portrait.

Exercise 3: (4 pts)

Consider the following IVP in $\mathbb{R}^2$:

$$\begin{cases} \dot{x}_1 = 2x_2\, e^{-x_1^2 - x_2^2}, \\ \dot{x}_2 = (3x_1^2 - 6)\, e^{-x_1^2 - x_2^2}, \\ (x_1(0), x_2(0)) = (0, 0). \end{cases}$$

(i) Find the fixed points.

(ii) On the basis of the “Poincaré-Lyapunov Theorem” what can you say about the stability of the fixed points?

Exercise 4: (6 pts)

Consider the following IVP in $\mathbb{R}^2$:

$$\begin{cases} \dot{x} = A\,x + \epsilon\, x\, \|x\|^2, \\ x(0) \in \mathbb{R}^2, \end{cases} \qquad A := \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

where $\epsilon \in \mathbb{R}$.

Study the stability of the fixed point $(0, 0)$ in dependence of ϵ.

Due Monday, November 26.