Discrete Geometry I Thursday, 23/04/2020
Summer term 2020
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Exercise sheet 1

Due-date: Monday, 04/05/2020 (exceptional), as latex file via gitlab.

Exercise 1 **4 Points**

As we saw in the lecture, a hyperplane is an affine subspace of codimension one.

a) Let $H_1, H_2 \subset \mathbb{R}^n$ be hyperplanes. Determine all possible values for $\dim(H_1 \cap H_2)$.

b) A hyperplane may be described by a (translated) normal vector. Consider hyperplanes $H_1, \ldots, H_7 \subset \mathbb{R}^9$ with the following normal vectors (all starting in the origin, of course):

$$\begin{aligned} n_1 &= (1,0,1,0,0,0,0,0,1) \\ n_2 &= (0,1,0,0,0,0,0,0,0) \\ n_3 &= (0,0,1,0,1,1,0,0,0) \\ n_4 &= (1,1,1,1,0,1,1,1,1) \\ n_5 &= (1,0,0,0,0,0,1,1,1) \\ n_6 &= (1,0,0,0,1,0,1,0,1) \\ n_7 &= (0,1,1,0,0,0,0,0,0)\ . \end{aligned}$$

Calculate $\dim(H_1 \cap \cdots \cap H_7)$.

Exercise 2 **2 Points**

Let $X, Y \subset \mathbb{R}^n$. Show that $\mathrm{conv}(X+Y) = \mathrm{conv}(X) + \mathrm{conv}(Y)$ holds.

Exercise 3 **6 Points**

Let $X \subset \mathbb{R}^n$. Prove or disprove the following:

a) If X is closed then $\mathrm{conv}(X)$ is closed.

b) If X is convex then $\overline{X}$ is convex.

c) If X is open then $\mathrm{conv}(X)$ is open.

Exercise 4 **4 Points**

Let $X \subset \mathbb{R}^2$ with cardinality $\#X \geq 5$ such that no three pairwise different points lie on a common line. Show that there are $x_1, \ldots, x_4 \in X$ such that $\mathrm{conv}(x_1, \ldots, x_4)$ is quadrilateral.