TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Berlin Mathematical School

Prof. Dr. Yuri B. Suris
Jan Techter

Mathematical Physics I - WS 2018/2019

http://www3.math.tu-berlin.de/geometrie/Lehre/WS18/MP1/

Exercise Sheet 6

Bonus: (5 pts)

Consider the following IVP in $\mathbb{R}^3$:

$$\begin{cases} \dot{x}_1 = 3\,x_2(x_3 - 1), \\ \dot{x}_2 = -x_1(x_3 - 1), \\ \dot{x}_3 = -x_3^3\,(x_1^2 + 1)\,, \\ (x_1(0), x_2(0), x_3(0)) \in \mathbb{R}^3. \end{cases}$$

(i) Prove that the fixed point $(0, 0, 0)$ is stable by finding a proper (quadratic) Lyapunov function.

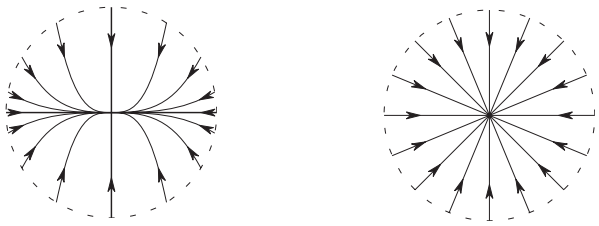
(ii) Prove that the fixed point $(0, 0, 0)$ cannot be asymptotically stable by restricting the dynamics to the invariant plane $x_3 = 0$.

Bonus: (5 pts)

Consider the following planar linear systems of ODEs:

$$\begin{cases} \dot{x}_1 = -2\,x_1, \\ \dot{x}_2 = -3\,x_2, \end{cases} \qquad \begin{cases} \dot{x}_1 = -2\,x_1, \\ \dot{x}_2 = -2\,x_2. \end{cases}$$

They have respectively the following phase portraits:



Prove that the systems are topologically conjugate by constructing explicitly the map between the orbits.

Due Monday, December 03.