Bayesian learning

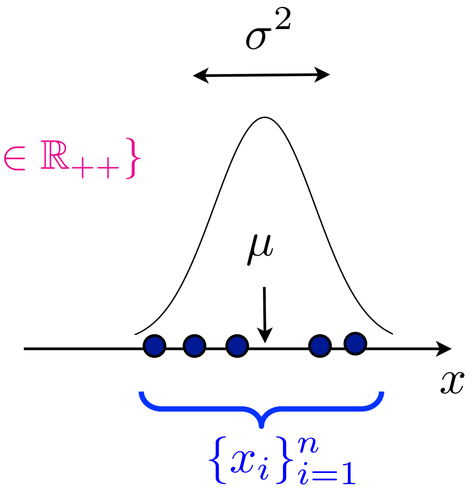
What is Bayesian learning?

Observed Data: $\{\boldsymbol{x}^{(i)}\}_{i=1}^{n}$

Parameter: $\boldsymbol{w} = \{\mu \in \mathbb{R}, \sigma^2 \in \mathbb{R}_{++}\}$

Model distribution: $p(\{\boldsymbol{x}^{(i)}\}_{i=1}^{n}|\boldsymbol{w}) = \prod_{i=1}^{n} p(\boldsymbol{x}^{(i)}|\boldsymbol{w})$

Prior distribution: $p(\boldsymbol{w}) \propto 1$



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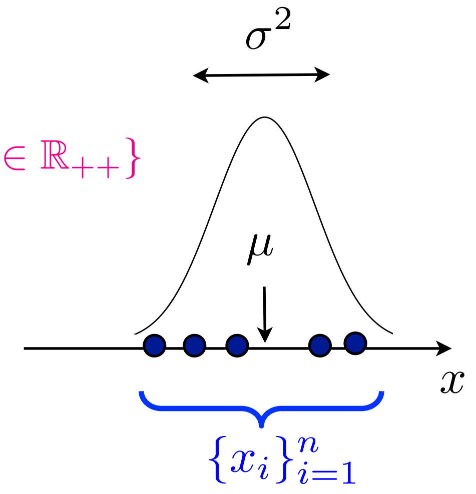

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Joint distribution: $p(\underbrace{\{\boldsymbol{x}^{(i)}\}_{i=1}^{n}}_{\text{observed}}, \underbrace{\boldsymbol{w}}_{\text{to be estimated!}}) = p(\{\boldsymbol{x}^{(i)}\}_{i=1}^{n}|\boldsymbol{w})p(\boldsymbol{w})$

We want to estimate parameter from observed data.

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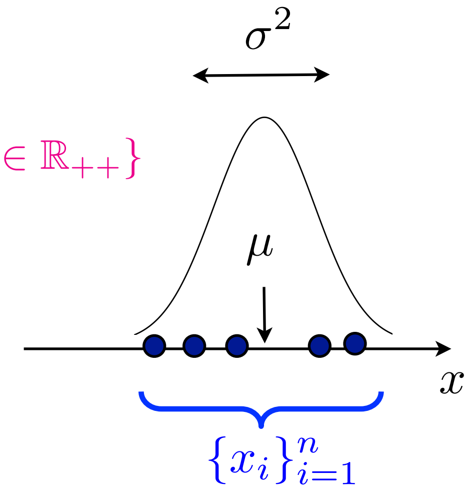

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$$p(b|a)p(a) = p(a,b) = p(a|b)p(b)$$

$a \to \{\boldsymbol{x}^{(i)}\}_{i=1}^{n},$

$b \to \boldsymbol{w}$

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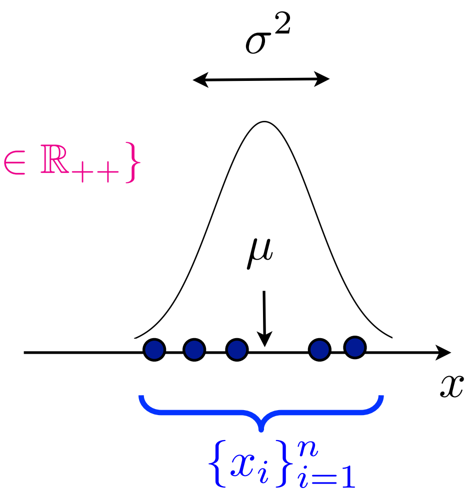

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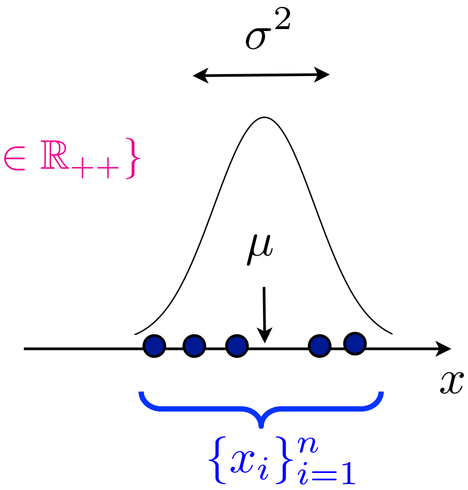

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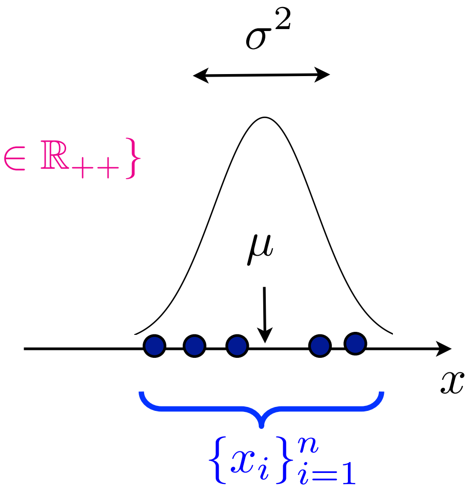

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$a \to \{\boldsymbol{x}^{(i)}\}_{i=1}^n,$

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$$\underbrace{\underbrace{p(\boldsymbol{w}|\{\boldsymbol{x}^{(i)}\}_{i=1}^n)}_{\text{posterior}}\underbrace{p(\{\boldsymbol{x}^{(i)}\}_{i=1}^n)}_{\text{marginal}}}_{\text{must compute}} = \underbrace{\underbrace{p(\{\boldsymbol{x}^{(i)}\}_{i=1}^n, \boldsymbol{w})}_{\text{joint}} = \underbrace{p(\{\boldsymbol{x}^{(i)}\}_{i=1}^n|\boldsymbol{w})}_{\text{likelihood}}\underbrace{p(\boldsymbol{w})}_{\text{prior}}}_{\text{given}}$$

Bayes posterior distribution

$$\underbrace{p(\boldsymbol{w}|\{\boldsymbol{x}^{(i)}\}_{i=1}^{n})}_{\text{posterior}}\underbrace{p(\{\boldsymbol{x}^{(i)}\}_{i=1}^{n})}_{\text{marginal}} = \underbrace{p(\{\boldsymbol{x}^{(i)}\}_{i=1}^{n}, \boldsymbol{w})}_{\text{joint}} = \underbrace{p(\{\boldsymbol{x}^{(i)}\}_{i=1}^{n}|\boldsymbol{w})}_{\text{likelihood}}\underbrace{p(\boldsymbol{w})}_{\text{prior}}$$

Posterior distribution:

marginal likelihood:

Bayes posterior distribution

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marginal likelihood:

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marginal likelihood:

Bayes posterior distribution

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marginal likelihood: $$p(\{\boldsymbol{x}^{(i)}\}_{i=1}^n) = \int p(\{\boldsymbol{x}^{(i)}\}_{i=1}^n|\boldsymbol{w})p(\boldsymbol{w})d\boldsymbol{w}$$

Bayes estimator: $$\widehat{\boldsymbol{w}} = \int \boldsymbol{w} \cdot \underbrace{p(\boldsymbol{w}|\{\boldsymbol{x}^{(i)}\}_{i=1}^n)}_{\text{posterior}}d\boldsymbol{w} = \langle \boldsymbol{w} \rangle_{p(\boldsymbol{w}|\{\boldsymbol{x}^{(i)}\}_{i=1}^n)}$$

Want to know the parameter $\boldsymbol{w}$ estimated from observation $\{\boldsymbol{x}^{(i)}\}_{i=1}^n$.

Bayes posterior distribution

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marginal likelihood: $p(\{\boldsymbol{x}^{(i)}\}_{i=1}^{n}) = \displaystyle\int p(\{\boldsymbol{x}^{(i)}\}_{i=1}^{n}|\boldsymbol{w})p(\boldsymbol{w})d\boldsymbol{w}$

Bayes estimator: $\widehat{\boldsymbol{w}} = \displaystyle\int \boldsymbol{w} \cdot p(\boldsymbol{w}|\{\boldsymbol{x}^{(i)}\}_{i=1}^{n})d\boldsymbol{w} = \langle \boldsymbol{w} \rangle_{p(\boldsymbol{w}|\{\boldsymbol{x}^{(i)}\}_{i=1}^{n})}$

Predictive distribution: $p(\boldsymbol{x}|\{\boldsymbol{x}^{(i)}\}_{i=1}^{n}) = \displaystyle\int p(\boldsymbol{x}, \boldsymbol{w}|\{\boldsymbol{x}^{(i)}\}_{i=1}^{n})d\boldsymbol{w}$

Want to predict a future $\boldsymbol{x}$ estimated from past observation $\{\boldsymbol{x}^{(i)}\}_{i=1}^{n}$.

Bayes posterior distribution

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$$\Big\downarrow \quad p(a,b) = p(a|b)p(b)$$

$$= \int p(\boldsymbol{x}|\boldsymbol{w}, \{\boldsymbol{x}^{(i)}\}_{i=1}^n)p(\boldsymbol{w}|\{\boldsymbol{x}^{(i)}\}_{i=1}^n)d\boldsymbol{w}$$

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independent!

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$$= \int \underbrace{p(\boldsymbol{x}|\boldsymbol{w})}_{\text{model distribution}}\underbrace{p(\boldsymbol{w}|\{\boldsymbol{x}^{(i)}\}_{i=1}^n)}_{\text{posterior}}d\boldsymbol{w}$$

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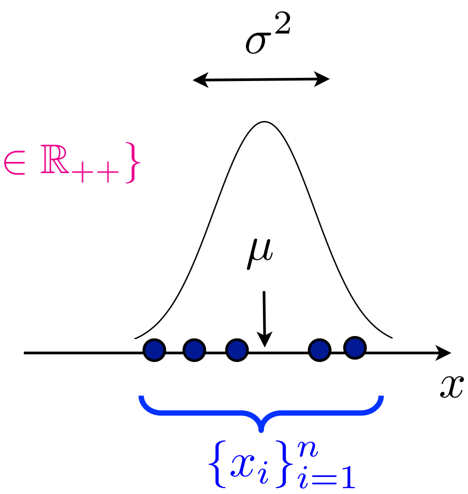

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Questions?