TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

Prof. Dr. Yuri B. Suris
Jan Techter

Mathematical Physics I - WS 2018/2019

http://www3.math.tu-berlin.de/geometrie/Lehre/WS18/MP1/

Exercise Sheet 1

Exercise 1: (5 pts)

Consider the following IVP in $\mathbb{R}$:

$$\begin{cases} \dot{x} = |x|^{p/q}, \\ x(0) = 0, \end{cases}$$

with $p, q \in \mathbb{N} \setminus \{0\}$.

(i) Prove that it has a unique solution if $p > q$.

(ii) Prove that it has an infinite number of solutions if $p < q$.

(iii) What can you say if $p = q$?

Exercise 2: (5 pts)

Consider the following IVP in $\mathbb{R}$:

$$\begin{cases} \dot{x} = \dfrac{x^2}{x^2 + \epsilon}\sqrt{|x|}, \\ x(0) = 0, \end{cases}$$

with $\epsilon > 0$. What can you say about existence and uniqueness of its solutions? Is the solution unique if $\epsilon = 0$?

Exercise 3: (5 pts)

Consider the following IVP in $\mathbb{R}$:

$$\begin{cases} \dot{x} = t + x, \\ x(0) = 1. \end{cases}$$

Construct the sequence of Picard iterations and obtain the explicit solution.

Exercise 4: (5 pts)

Consider the following IVP in $\mathbb{R}$:

$$\begin{cases} \dot{x} = x^3 - x, \\ x(0) = \dfrac{1}{2}. \end{cases}$$

(i) What can you say about existence and uniqueness of its solutions?

(ii) Without solving the ODE, calculate $\lim_{t \to +\infty} \phi(t)$, where $\phi(t)$ is the solution.

Due Monday, October 29.