Research Group
Applied Functional Analysis
www.math.tu-berlin.de/afg

Functional Analysis I

Homework Assignment 6

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Exercise 1: **5 Points**

Let E and F be normed spaces, and let $\Phi \in L(E, F)$. Prove that

$$\Phi^{**}\Lambda_E = \Lambda_F \Phi.$$

Here Φ^{**} denotes the dual (or adjoint) operator of Φ^* which maps from E^{**} to F^{**}. Assume now that Φ is an isomorphism, i.e. Φ is bijective and $\Phi^{-1} \in L(F, E)$. Prove the following statements:

(i) Φ^* is also an isomorphism and $(\Phi^*)^{-1} = (\Phi^{-1})^*$.

(ii) E is reflexive if and only if F is reflexive.

Exercise 2: **5 Points**

Let E be a reflexive Banach space and F a closed linear subspace of E. Prove that $F^{\perp\perp} = \Lambda_E(F)$, where Λ_E is the canonical embedding of E in E^{**}. If E is not reflexive, is the statement then still true? Give a proof or counterexample.

Exercise 3: **4 Points**

Let E and F be normed spaces, $T \in L(E, F)$. Prove that

$$\overline{\operatorname{ran}(T)} = (\ker(T^*))_\perp, \quad \ker(T) = (\operatorname{ran}(T^*))_\perp.$$

Exercise 4: **6 Points**

Let X be a normed space and $Z \subseteq X^*$ a separable linear subspace. Prove that there is a separable linear subspace $Y \subseteq X$ suth that Z is isometrically isomorphic to a linear subspace of Y^*.

Bonus Exercise: **+5 Points**

Prove that every separable normed space (respectively Banach space) E is isometrically isomorphic to a (respectively closed) subspace of ℓ_∞.

Please submit your homework in the big exercise on Monday, May 27.