Bayes' Theorem

Joint probability

100 people in a bar…

$$\widetilde{p}(a = A, b = \bar{B}) = 40$$

	A	$\bar{A}$	
B	10	30	
$\bar{B}$	40	20	

A : Caipirinha

$\bar{A}$: Beer

B : German

$\bar{B}$: Brasilian

Joint probability

100 people in a bar…

$$\widetilde{p}(a = A, b = \bar{B}) = 40$$

Normalize so that

$$p(a,b) \propto \widetilde{p}(a,b),$$

$$\sum_{a=\{A,\bar{A}\},b=\{B,\bar{B}\}} p(a,b) = 1$$

	A	$\bar{A}$	
B	0.1	0.3	
$\bar{B}$	0.4	0.2	

A : Caipirinha

$\bar{A}$: Beer

B : German

$\bar{B}$: Brasilian

Joint probability

100 people in a bar…

$$\widetilde{p}(a = A, b = \bar{B}) = 40$$

Normalize so that

$$p(a,b) \propto \widetilde{p}(a,b),$$

$$\sum_{a=\{A,\bar{A}\},b=\{B,\bar{B}\}} p(a,b) = 1$$

$p(a,b)$: joint probability

	A	$\bar{A}$	
B	0.1	0.3	
$\bar{B}$	0.4	0.2	

A : Caipirinha

$\bar{A}$: Beer

B : German

$\bar{B}$: Brasilian

$p(a = A, b = \bar{B}) = 0.4$: probability that a randomly chosen person is Brasilian drinking caipirinha.

Joint probability

Randomly chose a single person,
then observed that she/he is drinking beer.

	A	$\bar{A}$	
B	0.1	0.3	
$\bar{B}$	0.4	0.2	

A : Caipirinha

$\bar{A}$: Beer

B : German

$\bar{B}$: Brasilian

Joint probability

Randomly chose a single person,
then observed that she/he is drinking beer.

1. Which is more likely?
   - She is German.
   - She is Brasilian.

2. Probability that she is German.

	A	$\bar{A}$	
B	0.1	0.3	
$\bar{B}$	0.4	0.2	

A : Caipirinha

$\bar{A}$: Beer

B : German

$\bar{B}$: Brasilian

Joint probability

Randomly chose a single person,
then observed that she/he is drinking beer.

1. Which is more likely?
   - She is German.
   - ~~She is Brasilian~~.
2. Probability that she is German.

	A	$\bar{A}$	
B	0.1	0.3	
$\bar{B}$	0.4	0.2	

A : Caipirinha

$\bar{A}$: Beer

B : German

$\bar{B}$: Brasilian

Joint probability

Randomly chose a single person,
then observed that she/he is drinking beer.

1. Which is more likely?
   - She is German.
   - ~~She is Brasilian~~.

2. Probability that she is German.

$$p(b = B|a = \bar{A}) = 0.6$$

	A	$\bar{A}$	
B	0.1	0.3	
$\bar{B}$	0.4	0.2	

A : Caipirinha

$\bar{A}$: Beer

B : German

$\bar{B}$: Brasilian

Joint probability

Randomly chose a single person,
then observed that she/he is $\underbrace{\text{drinking beer}}_{\text{Condition!}}$.

1. Which is more likely?
   - She is German.
   - ~~She is Brasilian~~.

2. Probability that she is German.

$$p(b = B | a = \bar{A}) = 0.6$$

	A	$\bar{A}$	
B	0.1	0.3	
$\bar{B}$	0.4	0.2	

A : Caipirinha

$\bar{A}$: Beer

B : German

$\bar{B}$: Brasilian

Joint probability

	A	$\bar{A}$	
B	0.1	0.3	
$\bar{B}$	0.4	0.2	
		0.5	

Randomly chose a single person,
then observed that she/he is $\underbrace{\text{drinking beer}}_{\text{Condition!}}$.

1. Which is more likely?
   - She is German.
   - ~~She is Brasilian~~.

2. Probability that she is German.

$$p(b = B | a = \bar{A}) = 0.6$$

$$\boxed{p(a) = \sum_{b=\{B,\bar{B}\}} p(a,b) \quad \text{: marginal probability}}$$

$p(a = \bar{A}) = 0.5$: probability that a randomly chosen person drinks beer.

A : Caipirinha

$\bar{A}$: Beer

B : German

$\bar{B}$: Brasilian

Conditional probability

$$p(b|\underbrace{a}_{\text{condition}}) = \frac{\boxed{p(a,b)}}{p(a)}$$

	A	$\bar{A}$	
B	0.1	0.3	
$\bar{B}$	0.4	0.2	
		0.5	

A : Caipirinha

$\bar{A}$: Beer

B : German

$\bar{B}$: Brasilian

Conditional probability

$$p(b \underbrace{|a}_{\text{condition}}) = \frac{\boxed{p(a,b)}}{\boxed{p(a)}}$$

	A	$\bar{A}$	
B	0.1	0.3	0.4
$\bar{B}$	0.4	0.2	0.6
	0.5	0.5	

$p(b)$

$p(a)$

A : Caipirinha

$\bar{A}$: Beer

B : German

$\bar{B}$: Brasilian

Conditional probability

$$p(b|\underbrace{a}_{\text{condition}}) = \frac{\boxed{p(a,b)}}{\boxed{p(a)}}$$

	A	$\bar{A}$	
B	0.1	0.3	0.4
$\bar{B}$	0.4	0.2	0.6
	0.5	0.5	

$$p(a|b) = \frac{p(a,b)}{p(b)}$$

A : Caipirinha

$\bar{A}$: Beer

B : German

$\bar{B}$: Brasilian

3. A randomly chosen person was Brasilian.
   What is probability that she/he is drinking Caipirinha?

Conditional probability

$$p(b|\underbrace{a}_{\text{condition}}) = \frac{\boxed{p(a,b)}}{p(a)}$$

	A	$\bar{A}$	
B	0.1	0.3	0.4
$\bar{B}$	0.4	0.2	0.6
	0.5	0.5	

A : Caipirinha

$\bar{A}$: Beer

B : German

$\bar{B}$: Brasilian

$$p(a|b) = \frac{p(a,b)}{p(b)}$$

3. A randomly chosen person was Brasilian.
   What is probability that she/he is drinking Caipirinha?

$$p(a = A|b = \bar{B}) = 0.666...$$

Conditional probability

$$p(b|a) = \frac{p(a,b)}{p(a)}$$

$$p(a|b) = \frac{p(a,b)}{p(b)}$$

Conditional probability

$$p(b|a) = \frac{p(a,b)}{p(a)}$$

$$\rightarrow \quad p(b|a)p(a) = p(a,b) = p(a|b)p(b)$$

$$p(a|b) = \frac{p(a,b)}{p(b)}$$

Conditional probability

$$p(b|a) = \frac{p(a,b)}{p(a)}$$

$$\rightarrow \quad p(b|a)p(a) = p(a,b) = p(a|b)p(b)$$

$$\downarrow$$

$$p(a|b) = \frac{p(a,b)}{p(b)}$$

$$p(b|a) = \frac{p(b)}{p(a)} p(a|b)$$

Bayes' theorem

Conditional probability

$$p(b|a) = \frac{p(a,b)}{p(a)}$$

$$p(a|b) = \frac{p(a,b)}{p(b)}$$

$$p(b|a)p(a) = p(a,b) = p(a|b)p(b)$$

$$p(b|a) = \frac{p(b)}{p(a)} p(a|b)$$

Bayes' theorem