TECHNISCHE UNIVERSITÄT BERLIN
Institut für Mathematik

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Mathematical Physics I - WS 2018/2019

http://www3.math.tu-berlin.de/geometrie/Lehre/WS18/MP1/

Exercise Sheet 4

Exercise 1: (8 pts)

Consider the following vector fields acting on $\mathbb{R}^2$:

$$\mathbf{v}_1 := x_1\, x_2 \frac{\partial}{\partial x_1} + x_2^2 \frac{\partial}{\partial x_2}, \qquad \mathbf{v}_2 := x_1 \frac{\partial}{\partial x_1}, \qquad \mathbf{v}_3 := x_2 \frac{\partial}{\partial x_1}.$$

Which of the flows Φ^t, Ψ^s, Θ^w ($t, s, w \in \mathbb{R}$) generated by $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ commute?

Exercise 2: (6 pts)

Consider the following ODE in $\mathbb{R}$:

$$\ddot{x} - 4\,\dot{x} + 2\,\alpha\, x = 0, \qquad \alpha \in \mathbb{R},$$

together with two conditions $x(0) = 0$ and $x(\pi) = 0$. Find α such that there exists a non-trivial solution.

Exercise 3: (6 pts)

Consider the following IVP in $\mathbb{R}$:

$$\begin{cases} \dot{x} = (x+\alpha)^2\,(x^2-\alpha)\,, \\ x(0) \in \mathbb{R}, \end{cases}$$

where $\alpha \in \mathbb{R}$.

(i) Find the fixed points.

(ii) Study the stability nature of the fixed points.

Due Monday, November 19.