ANA

$\int_A \operatorname{div} F \, d\lambda = \int_{\partial A} \langle F(x), \nu(x) \rangle \, d\lambda(x)$

FUNK

$v = \sum_{i=1}^{d} \lambda_i v_i$

TIONAL

$\sum_{n=0}^{\infty} \frac{1}{n!}$

$a^2 + c$

LYSIS

$\varepsilon < 0$

$[x]_\sim$

a^2+c

$a^2+Ca=8$

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LUKAS ANDRE WEISSHAUPT 391543

$\sqrt{x}=|y|$

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$\int_A f(x)\,dx \approx \pi^2$

$\varepsilon<0$ $\varepsilon<0$

$$\int_A \operatorname{div} F \, d\lambda = \int_{\partial A} \langle F(x), \nu(x) \rangle \, d\lambda(x)$$

$$\sum_{n=0}^{\infty} \frac{1}{n!}$$

$$\int_{\partial A} f(x) \, dS(x)$$

$$\mu\Big(\bigcup_{n\in\mathbb{N}} A_n\Big) \leq \sum_{n\in\mathbb{N}} \mu(A_n) \qquad f(U_\delta(x)) \subset U_\varepsilon(f(x))$$

1

Linear Spaces

Norms

Convex

— By Lukas Andre Weißhaupt

LEANING OVER ME,
HE WAS TRYING TO EXPLAIN
THE LAWS OF [*GEOMETRY*]

AND I COULDN'T HELP IT,

I JUST HAD TO KISS THE TEACHER

Exercise 1

We want to show that p satisfying the triangle inequality is equivalent to it fulfilling the condition specified in the task,

"$\Rightarrow$": Let p be a norm on X. Then let $x, y \in X$ with $p(x) \leq 1 \geq p(y)$ and $\lambda \in [0, 1]$. We then have $p(\lambda x + (1-\lambda)y) \leq \lambda p(x) + (1-\lambda)p(y) \leq \lambda + 1 - \lambda = 1$. Thus the set in question is convex.

"$\Leftarrow$": Let $A := \{x \in X : p(x) \leq 1\}$ be convex and let $x, y \in X$. Then if not both $p(x) = 0 = p(y)$ we have the following: For $\lambda = \frac{p(x)}{p(x)+p(y)} \in [0, 1]$ and $\xi = p(x) + p(y)$ we see that

$$x + y = \xi\Big(\lambda\frac{x}{p(x)} + (1-\lambda)\frac{y}{p(y)}\Big)$$

and since

$$1 = \frac{p(y)}{p(y)} = p\Big(\frac{y}{p(y)}\Big)$$

and analogously for x. Now we know that

$$\frac{p(x+y)}{p(x)+p(y)} = p\Big(\lambda\frac{x}{p(x)} + (1-\lambda)\frac{y}{p(y)}\Big) \leq 1$$

because A is convex and $\lambda \in [0, 1]$ and $x(p(x))^{-1} \in A \ni y(p(y))^{-1}$.

Thus we have shown $p(x + y) \leq p(x) + p(y)$. $\square$

Starring ABBA

2

I-p Space

I-1 Space

I-∞ Space

— By Viet Duc Nguyen

B Y   A B B A

F R O M   S U P E R   T R O U P E R

SUP-P-PER TROUP-P-PER

Exercise 2

Sketch of proof

We basically want to prove when equality holds in the Minkowski inequality. For that we reduce the Minkowski inequality to Hoelder's inequality. What we are doing next is just the proof of Minkowski inequality backwards and replacing appropriate inequalities with equalitites by using the given assumptions.

Proof

Let $p > 1$ and $p \neq \infty$. Let $(x_n)_{n\in\mathbb{N}}, (y_n)_{n\in\mathbb{N}} \in \ell_p$ with $||(x_n)|| = ||(y_n)|| = 1$ and $||(x_n) + (y_n)|| = 2$. Define $z_k := |x_k + y_k|$. It follows that

$$||(x_n)|| + ||(y_n)|| = ||(x_n) + (y_n)|| = ||(z_n)||.$$

Thus for all $m \in \mathbb{N}$ it holds

$$(\sum_{k=0}^{m} |z_k|^p)^{\frac{1}{p}} = (\sum_{k=0}^{m} |x_k|^p)^{\frac{1}{p}} + (\sum_{k=0}^{m} |y_k|^p)^{\frac{1}{p}}. \tag{1}$$

Since p is fixed, chose q such that $\frac{1}{p}+\frac{1}{q}=1$. We that $\frac{1}{p}=1-\frac{1}{q}$. So,

$$(\sum_{k=0}^{m}|z_k|^p)^{\frac{1}{p}}=(\sum_{k=0}^{m}|z_k|^p)^{1-\frac{1}{q}}=\frac{\sum_{k=0}^{m}|z_k|^p}{(\sum_{k=0}^{m}|z_k|^p)^{\frac{1}{q}}}.$$

From this equation, we can multiply (1) with $(\sum_{k=0}^{m}|z_k|^p)^{\frac{1}{q}}$ on both sides, and we obtain

$$\sum_{k=0}^{m}|z_k|^p=(\sum_{k=0}^{m}|x_k|^p)^{\frac{1}{p}}(\sum_{k=0}^{m}|z_k|^p)^{\frac{1}{q}}+(\sum_{k=0}^{m}|y_k|^p)^{\frac{1}{p}}(\sum_{k=0}^{m}|z_k|^p)^{\frac{1}{q}}$$

Now, observe that $p=q(p-1)$. Hence, we can also write former equation as

$$\begin{aligned}\sum_{k=0}^{m}|z_k|^p&=(\sum_{k=0}^{m}|x_k|^p)^{\frac{1}{p}}(\sum_{k=0}^{m}|z_k|^{q(p-1)})^{\frac{1}{q}}+(\sum_{k=0}^{m}|y_k|^p)^{\frac{1}{p}}(\sum_{k=0}^{m}|z_k|^{q(p-1)})^{\frac{1}{q}}\\&=[(\sum_{k=0}^{m}|x_k|^p)^{\frac{1}{p}}+(\sum_{k=0}^{m}|y_k|^p)^{\frac{1}{p}}](\sum_{k=0}^{m}|z_k|^{q(p-1)})^{\frac{1}{q}}\end{aligned}\tag{2}$$

We also know that

$$\sum_{k=0}^{\infty}|z_k|^p=\sum_{k=0}^{\infty}|z_k||z_k|^{p-1}=\sum_{k=0}^{\infty}|x_k+y_k||z_k|^{p-1}.\tag{3}$$

By triangular inequality we note that

$$\sum_{k=0}^{\infty}|z_k|^p\overset{(3)}{\leq}\sum_{k=0}^{\infty}(|x_k|+|y_k|)\cdot|z_k|^{p-1}.\tag{4}$$

On the other hand, we can use Hoelder's inequality on (2) to obtain

$$\begin{aligned}\sum_{k=0}^{\infty}|x_k||z_k|^{p-1}+\sum_{k=0}^{\infty}|y_k||z_k|^{p-1}&\leq[(\sum_{k=0}^{\infty}|x_k|^p)^{\frac{1}{p}}+(\sum_{k=0}^{\infty}|y_k|^p)^{\frac{1}{p}}](\sum_{k=0}^{\infty}|z_k|^{q(p-1)})^{\frac{1}{q}}\\&=\sum_{k=0}^{\infty}|z_k|^p\end{aligned}\tag{5}$$

Using (4) and (5) we find a lower and upper bound for $\sum_{k=0}^{\infty} |z_k|^p$:

$$\sum_{k=0}^{\infty} |x_k||z_k|^{p-1} + \sum_{k=0}^{\infty} |y_k||z_k|^{p-1} \leq \sum_{k=0}^{\infty} |z_k|^p \leq \sum_{k=0}^{\infty} |x_k||z_k|^{p-1} + \sum_{k=0}^{\infty} |y_k||z_k|^{p-1}.$$

Putting two and two together and we get

$$\sum_{k=0}^{\infty} (|x_k| + |y_k|)|z_k|^{p-1} = \sum_{k=0}^{\infty} |z_k|^p \overset{(3)}{=} \sum_{k=0}^{\infty} |x_k + y_k||z_k|^{p-1} \tag{6}$$

Lemma 0.1

Let $(a_n), (b_n)$ be convergent sequences in $\mathbb{C}$. If $\sum_{j=0}^{\infty} |a_j| + |b_j| = \sum_{j=0}^{\infty} |a_j + b_j|$ then $|a_j| + |b_j| = |a_j + b_j|$ for all $j \in \mathbb{N}$.

Proof. Let $(a_n), (b_n)$ be convergent sequences in $\mathbb{C}$ with

$$\sum_{j=0}^{\infty} |a_j| + |b_j| = \sum_{j=0}^{\infty} |a_j + b_j|.$$

For all $n \in \mathbb{N}$ it holds

$$|a_0| + |b_0| + \ldots + |a_n| + |b_n| \leq |a_0 + b_0| + \ldots + |a_n + b_n| \leq |a_0| + |b_0| + \ldots + |a_n| + |b_n|.$$

Assume that there exists $j \in \{1, \ldots, n\}$ such that $|a_j + b_j| \neq |a_j| + |b_j|$ Thus, it must follow by triangluar inequality $|a_j + b_j| < |a_j| + |b_j|$. So, we get

$$|a_j| + |b_j| - |a_j + b_j| + \sum_{k\in\{1,\ldots,n\}\setminus\{j\}}^{n} |a_k| + |b_k| \leq \sum_{k\in\{1,\ldots,n\}\setminus\{j\}}^{n} |a_k + b_k|,$$

which implies that $\sum_{k\in\{1,\ldots,n\}\setminus\{j\}}^{n} < \sum_{k\in\{1,\ldots,n\}\setminus\{j\}}^{n} |a_k + b_k|$. *Contradiction!* Thus, there cannot be such j and $|a_j| + |b_j| = |a_j + b_j|$ for all $j \in \{1, \ldots, n\}$. For $n \in \mathbb{N}$ was arbitrary it holds for all $j \in \mathbb{N}$ that

$$|a_j| + |b_j| = |a_j + b_j|.$$

□

Using the hint

We apply the Lemma 0.1 on (6), and we get

$$|x_k| + |y_k| = |x_k + y_k|, \qquad \forall k \in \mathbb{N}$$

Let $k \in \mathbb{N}$. Using the second hint given on the exercise sheet, we get two cases:

(1) Either it holds: $x_k = \alpha y_k$ for $\alpha > 0$, and since $||(y_k)|| = \alpha||(x_k)|| = 1$ it follows $\alpha = 1$. So $x_k = y_k$.

(2) Consider sequences $(x_k), (y_k) \in \ell_p$ with $||(x_k)|| = ||(y_k)|| = 1$ and $x_k = 0$ or $y_k = 0$ for an index $k \in \mathbb{N}$. W.l.o.g. let $x_k = 0$. For now we assume that there is exactly one member x_i, namely x_k, which is zero, i.e. we assume that $y_i \neq 0$ and $x_i \neq 0$ *(the proof can be easily generalized to the case if $x_i = 0$ for more than one $i \in \mathbb{N}$)*. Due to case (1) it holds $x_i = y_i$ for all $i \neq k$. We want to prove the contraposition of

$$\frac{||(x_k) + (y_k)||}{2} = 1 \implies (x_k) = (y_k).$$

Assume $(x_k) \neq (y_k)$. Thus, there must exist an index $l \in \mathbb{N}$ such that $x_l \neq y_l$. Because of $x_i = y_i$ for all $i \neq k$ we have $x_k = 0$ and $y_k \neq 0$. Additionally, there must exist an index $j \neq k$ such that $x_j \neq 0$ and $y_j = 0$ due to $||(x_k)|| = ||(y_k)|| = 1$ and $|x_i| = |y_i|$ for all $i \neq k$.

It holds

$$\begin{aligned}
\frac{(\sum_{i\in\mathbb{N}} |x_i + y_i|^p)}{2} &= \frac{1}{2}(\sum_{i\neq k, i\neq j}^{\infty} |x_i + y_i|^p + |x_j|^p + |y_k|^p) \\
&\Downarrow \textit{due to } |x_k| + |y_k| = |x_k + y_k| \textit{ for all } k \in \mathbb{N} \\
&= \frac{1}{2}(\sum_{i\neq k, i\neq j}^{\infty} (|x_i| + |y_i|)^p + |x_j|^p + |y_k|^p) \\
&\Downarrow \textit{due to } x_i \neq 0 \textit{ and } y_i \neq 0 \textit{ for all } i \in \mathbb{N} \setminus \{j, k\} \\
&> \frac{1}{2}(\sum_{i\neq k, i\neq j}^{\infty} |x_i|^p + |y_i|^p + |x_j|^p + |y_k|^p) \\
&= \frac{1}{2}(\sum_{i\in\mathbb{N}}^{\infty} |x_i|^p + \sum_{i\in\mathbb{N}}^{\infty} |y_i|^p) = 1.
\end{aligned}$$

Thus by contraposition it holds that $x_k = y_k = 0$.

Conclusion

Finally, it holds $(y_k)_{k\in\mathbb{N}} = (x_k)_{k\in\mathbb{N}}$. ℓ_p is strictly convex for $p \in (1, \infty)$.

Counterexamples for ℓ_1 and ℓ_∞

Consider $(x_k)_{k\in\mathbb{N}}, (y_k)_{k\in\mathbb{N}} \in \ell_1$ with

$$(x_k)_{k\in\mathbb{N}} = (\frac{1}{2}, \frac{1}{2}, 0, 0, ...) \quad \text{and} \quad (y_k)_{k\in\mathbb{N}} = (0, \frac{1}{2}, \frac{1}{2}, 0, ...).$$

We see that $||(x_k)|| = \sum_{k\in\mathbb{N}} |x_k| = 1 = \sum_{k\in\mathbb{N}} |y_k| = ||(y_k)||$ but $(x_k)_{k\in\mathbb{N}} \neq (y_k)_{k\in\mathbb{N}}$. So, ℓ_1 is not strictly convex.

Consider $(x_k)_{k\in\mathbb{N}}, (y_k)_{k\in\mathbb{N}} \in \ell_\infty$ with

$$(x_k)_{k\in\mathbb{N}} = (1, 0, 0, 0, ...) \quad \text{and} \quad (y_k)_{k\in\mathbb{N}} = (0, 1, 0, 0, ...).$$

We see that $||(x_k)|| = \sup_{k\in\mathbb{N}} |x_k| = 1 = \sup_{k\in\mathbb{N}} |y_k| = ||(y_k)||$ but $(x_k)_{k\in\mathbb{N}} \neq (y_k)_{k\in\mathbb{N}}$. So, ℓ_∞ is not strictly convex.

$\blacksquare$

Starring ABBA

3

l-p Space

l-1 Space

l-∞ Space

— Jacky Behrendt

B Y   A B B A

S O   L O N G

YOU WON'T HAVE ME TONIGHT, ALRIGHT, ALRIGHT, ALRIGHT, ALRIGHT

Exercise 3

Let $(X, ||.||)$ be a normed space and $Y \subset X$ be a finite-dimensional subspace. Prove that for any element of X there exists a projection on Y, i.e. for all $x \in X$ there exists $y_x \in Y$ such that $||x - y_x|| = \inf\{||x - y|| : y \in Y\}$. Is this projection unique ? Give a proof or a counterexample.

Since $(X, ||.||)$ is a normed space and $X \subset Y$ is a subspace Y equipped with $||.||$ is also a normed space. According to theorem 2.6 $(Y, ||.||)$ is a Banach space, since Y is finite-dimensional. Let $x \in X$. We know that $\inf\{||x - y|| : y \in Y\} < \infty$ since $0 \in Y$ and therefore

$$\inf\{||x - y|| : y \in Y\} \leq ||x - 0|| = ||x|| < \infty$$

Hence for every $\varepsilon > 0$ we can find a $y_\varepsilon \in Y$ such that

$$||x - y_\varepsilon|| < \inf\{||x - y|| : y \in Y\} + \varepsilon$$

For $(\frac{1}{n})_{n \in \mathbb{N}_{>0}}$ we find a sequence $(y_n)_{n \in \mathbb{N}_{>0}} \in Y$ such that for $n \in \mathbb{N}_{>0}$

$$||x - y_n|| < \inf\{||x - y|| : y \in Y\} + \frac{1}{n} \iff ||x - y_n|| - \inf\{||x - y|| : y \in Y\} < \frac{1}{n}$$

Hence $(||x - y_n||)_{n \in \mathbb{N}_{>0}}$ converges to $\inf\{||x - y|| : y \in Y\}$. Let $N \in \mathbb{N}_{>0}$ be fixed and $C = K_{\inf\{||x-y||:y \in Y\} + \frac{1}{N}}(x) \cap Y$. Since Y is a finite-dimensional Banach-space it is sequentially closed and this implies that it is closed. Therefore C is a bounded and closed subset of a finite-dimensional normed space, hence it is compact. It holds that for all $n \geq N$ $y_n \in C$. Since C is compact we can find a convergent subsequence $(y_{n_k})_{k \in N} \in Y$. For this subsequence follows:

$$\lim_{k \to \infty} ||x - y_{n_k}|| = \inf\{||x - y|| : y \in Y\}$$

Y is a Banach-space and therefor complete. Hence the limit y_x of $(y_{n_k})_{k \in N}$ is contained in Y.

OOOH SEE THAT GIRL, WATCH THAT SCENE, DIG IN THE DANCING QUEEN

BY ABBA

DANCIN QUEEN

Through a counterexample we will show that the projection as definied above is not unique. $(l^\infty, ||.||_{l^\infty})$ is an infinite dimensional normed space. We will now consider the space of all sequences that only differe from zero in their first two entries. These sequences are a finite-dimensional subspace of l^∞. For $x \in l^\infty$ such that $x_n = 1$ for $n \in \mathbb{N}$ and $y = (1, 1, 0, 0, ...)$ and $z = (0, 1, 0...)$ we obtain:

$$\sup_{n\in\mathbb{N}} |x_n - y_n| = 1 \qquad \sup_{n\in\mathbb{N}} |x_n - z_n| = 1$$

Therefor the projection is not unique.

4

l-p Space

l-1 Space

l-∞ Space

— Lukas André Weißhaupt

Exercise 4

Let F, G be closed subspaces of a normed space E and $d := \dim G < \infty$. Let (x_n) be a sequence in $F + G$ with $x_n \to x \in E$ for $n \to \infty$. We can find a base $(g^{(1)}, ..., g^{(d)})$ of G and we will call $\operatorname{span}\{g^{(i)}\} =: G_i$ where now $G = \bigoplus_{i=1}^{d} G_i$. Notice that for any $i \in \{1, ..., d\}$ we have either $G_i \subset F$ or $F \cap G_i = \{0\}$. In the first case $F + G_i = F$ is obviously closed.

Otherwise, $F + G_i = F \oplus G_i$ and as thus any series (y_n) in $F + G_i$ is convergent if and only if both of it's components in F and G_i respectively converge which for such a convergent sequence (y_n) provides us with another convergent sequence $(g_n^{(i)})_{n\in\mathbb{N}}$ in G with limit g' in G (because this set is closed). Then if the sequence in F converges to f' we know that $y_n \to f' + g'$ for $n \to \infty$ and thus $y_n \in F + G_i$.

This means that for $d = 1$ we have already proven the statement that $F + G$ is closed for closed linear spaces F and G. Furthermore we can see inductively that for $d > 1$ we have $F + G = F + G_1 + ... + G_d$ which is the repeated sum of a closed linear space of arbitrary dimension and closed one-dimensional spaces which each time yield another closed linear space of arbitrary dimension. Thus for any closed G with $\dim G < \infty$, $F + G$ is closed.

5

l-p Space

l-1 Space

l-∞ Space

— Jacky Behrendt

Exercise 5

Let $\mathcal{P}$ be the space of all real-valued polynomials, defined on $\mathbb{R}$. For a polynomial $p \in \mathcal{P}$, such that $p(t) = \sum_{k=0}^{n} a_k t^k$, set $||p|| := \sum_{k=0}^{n} |a_k|$. Then the set $(\mathcal{P}, ||.||)$ is a normed space. Consider the linear mappings $f_i : \mathcal{P} \to \mathbb{R}, i = 1, ..., 3$ which are given by

$$f_1(p) = \int_0^1 p(t)dt, \qquad f_2(p) = p'(0), \qquad f_3(p) = p'(1)$$

Examine for $i = 1, .., 3$ whether f_i is continuous and determine the operator norm $||f_i||$.

<u>i = 1</u>: We will show that f_1 is continuous. Let $p, \tilde{p} \in \mathcal{P}$ and $n = \max\{deg(p), deg(\tilde{p})\}$.

$$\begin{aligned}
|f_1(p) - f_1(\tilde{p})| &= |\int_0^1 p(t)dt - \int_0^1 \tilde{p}(t)dt| \\
&= |\int_0^1 p(t) - \tilde{p}(t)dt| \\
&\leq \int_0^1 |p(t) - \tilde{p}(t)|dt \\
&= \int_0^1 |\sum_{k=0}^{n}(a_k - \tilde{a}_k)t^k| \\
&\leq \int_0^1 \sum_{k=0}^{n} |a_k - \tilde{a}_k| t^k \\
&= [\sum_{k=0}^{n} \frac{|a_k - \tilde{a}_k|}{k+1} t^{k+1}]_0^1 \\
&= \sum_{k=0}^{n} \frac{|a_k - \tilde{a}_k|}{k+1} \\
&\leq \sum_{k=0}^{n} |a_k - \tilde{a}_k| = ||p - \tilde{p}||
\end{aligned}$$

Hence we obtain $|f_1(p) - f_1(\tilde{p})| \leq ||p - \tilde{p}||$. f_1 is Lipschitz-continuous and therefor contionuous. For the operator norm then follows:

$$\sup_{||p|| \leq 1} |f_1(p)| = \sup_{||p|| \leq 1} |\int_0^1 p(t)|$$

$$
\begin{aligned}
&= \sup_{||p||\leq 1} |\int_0^1 \sum_{k=0}^{n} a_k t^k| \\
&= \sup_{||p||\leq 1} |[\sum_{k=0}^{n} \frac{a_k}{k+1} t^{k+1}]_0^1| \\
&= \sup_{||p||\leq 1} |\sum_{k=0}^{n} \frac{a_k}{k+1}| \\
&\leq \sup_{||p||\leq 1} |\sum_{k=0}^{n} a_k| \\
&\leq \sup_{||p||\leq 1} \sum_{k=0}^{n} |a_k| \\
&\leq \sup_{||p||\leq 1} ||p|| \\
&\leq 1
\end{aligned}
$$

For $p = 1$ this holds equality due to $f_1(p) = \int_0^1 1dt = 1$. Therefor $||f_1|| = 1$.

i = 2: Let $p, \tilde{p} \in \mathcal{P}$.

$$
\begin{aligned}
|f_2(p) - f_2(\tilde{p})| &= |p'(0) - \tilde{p}'(0)| \\
&= |\sum_{k=1}^{n} k a_k 0^{k-1} - \sum_{k=1}^{n} k \tilde{a}_k 0^{k-1}| \\
&= |a_1 - \tilde{a}_1| \\
&\leq ||p - \tilde{p}||
\end{aligned}
$$

This shows that f_2 is continuous. Since $\sup_{||p||\leq 1} |f_2(p)| \leq ||p||$ we already know that $||f_2|| \leq 1$ and for $p = t$ it holds that:

$$|f_2(p)| = |p'(0)| = |1| = ||p||$$

Therefor $||f_2|| = 1$.

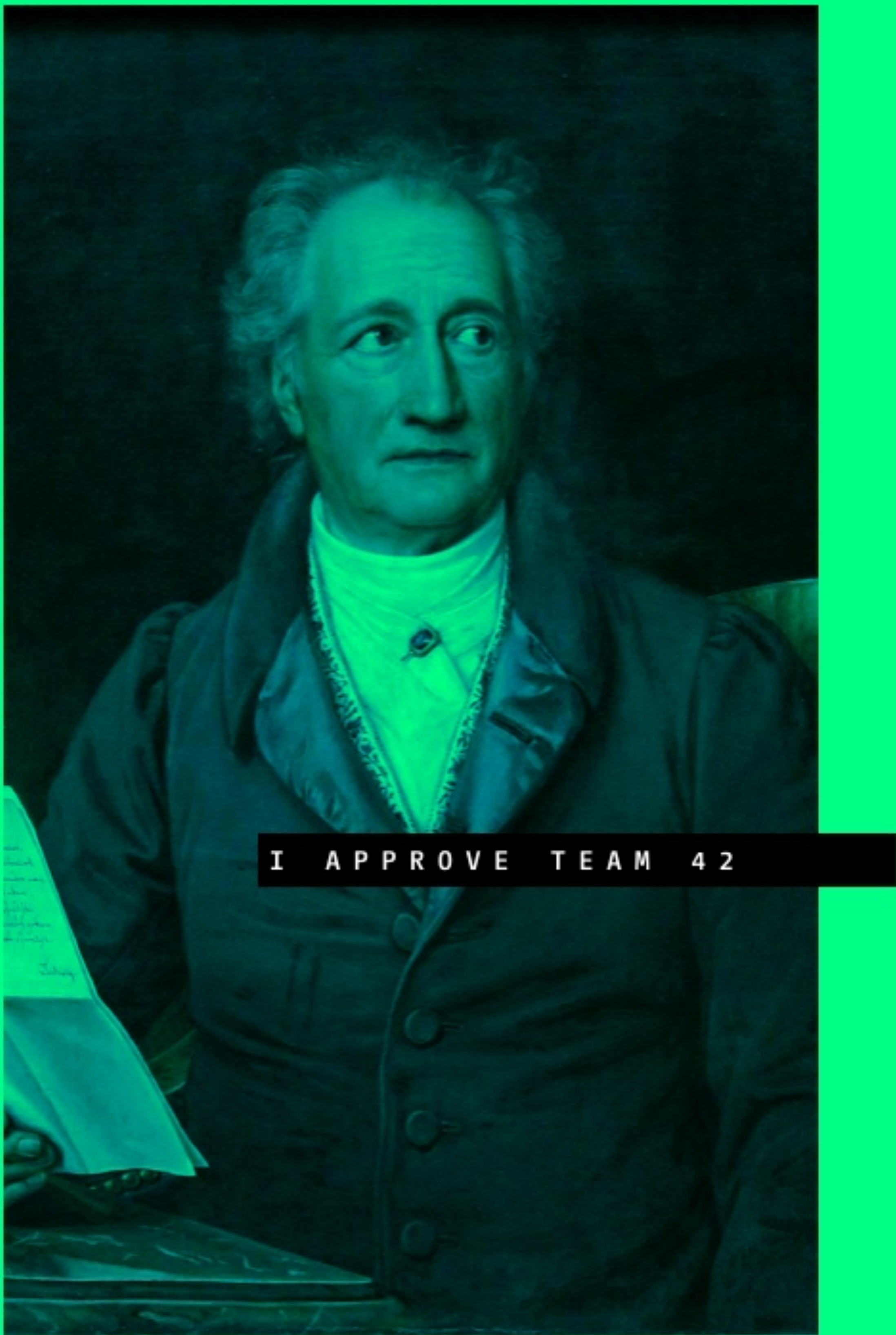
i = 3: We will prove that in this case f_3 is not contionuous. Let $p \in \mathcal{P}$ and $\delta > 0$ arbitrary. For all $\tilde{p} \in \mathcal{P}$, such that $\tilde{p} = \sum_{k=0} \tilde{a}_k t^k$ and $||p - \tilde{p}|| < \delta$ we can find $m \in \mathbb{N}$ such that $\frac{1}{m} < \delta - ||p - \tilde{p}||$ and $m > \max\{deg(p), deg(\tilde{p})\}$. Let $\dot{p} = \sum_{k=0}^{n} \tilde{a}_k t^k - \frac{1}{m} t^m$. It holds

$$||p - \dot{p}|| = |\sum_{k=0}^{n}(a_k - \tilde{a}_k) + \frac{1}{m}| \leq |\sum_{k=0}^{n}(a_k - \tilde{a}_k)| + |\frac{1}{m}| < \delta$$

And we obtain:

$$|f_3(p) - f_3(\dot{p})| = |p'(1) - \dot{p}'(1)| = |\sum_{k=1}^{n} k(a_k - \tilde{a}_k) + \frac{1}{m}m| \geq |\frac{1}{m}m| = 1$$

Therefore f_3 is not continuous and form definition 3.1 and 3.2 we obtain that the operator norm of f_3 is not bounded.



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