Research Group
Applied Functional Analysis
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Functional Analysis I

Tutorial Assignment 6

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Exercise 1: Let E be a Banach space, F a normed space, and let $T \in L(E, F)$ such that $\|Tx\| \geq c\|x\|$ for all $x \in E$, where $c > 0$ is a fixed constant. Prove that $\operatorname{ran} T$ is closed.

Exercise 2: Show that a reflexive Banach space is separable if and only if its dual is separable.

Exercise 3: Let E be a reflexive space, $\mathcal{M} \subset E^*$ a closed linear subspace. Show that $(\mathcal{M}_\perp)^\perp = \mathcal{M}$.

Exercise 4: Let E be a reflexive Banach space. Show that for each $\varphi \in E^*$ there exists $x \in E$ such that $\|x\| = 1$ and $\varphi(x) = \|x\|$. Use this to prove that $C([0, 1])$ is not reflexive, by considering the functional

$$\varphi : \mathcal{M} \to \mathbb{K}, \quad \varphi(f) = \int_0^1 f(x)dx, f \in \mathcal{M},$$

where $\mathcal{M} = \{f \in C([0, 1]) : f(0) = 0\}$.