PERGAMON

International Journal of Project Management 20 (2002) 475–485

INTERNATIONAL JOURNAL OF PROJECT MANAGEMENT

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A comparative morphologic analysis of benchmark sets of project networks

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Received 29 June 2000; received in revised form 28 November 2000; accepted 6 March 2001

Abstract

The performance of methods to manage projects depends heavily on the features of their project networks. This is particularly true for methods devoted to project scheduling, risk analysis and resources allocation. Therefore, a long line of research has been developed to generate benchmark sets of project networks and several sets have been proposed in the literature. Unfortunately, no comparative analyses of their features were published and hence serious doubts about the comparability of results using different benchmark sets can be raised. In this paper, a multi-dimensional taxonomy for the morphology of project networks is used and four benchmark sets are evaluated: Patterson collection of problems (Patterson JH. A comparison of exact approaches for solving the multiple constrained resource, project scheduling problem. Management Science 1984;30:854–867) and the sets produced by the generators due to Agrawal et al. Agrawal MK, Elmaghraby SE, Herroelen WS. DAGEN a generator of testsets for project activity nets. European Journal of Operational Research 1996;90:376–382, Kolisch et al. Kolisch R, Sprecher A, Drexl A. Characterization and generation of a general class of resource — constrained project scheduling problems. Management Science 1995;41:1693–1703, Tavares Tavares LV. Advanced models in project management. Kluwer, 1999 and Tavares et al. Tavares LV, Antunes Ferreira JA, Coelho JS. The risk of delay of a project in terms of the morphology of its network. European Journal of Operational Research 1999;119:510–537. Original results about the lack of representativeness of these sets are obtained showing that misleading conclusions can be deduced. The last set is, by far, that one covering most extensively the morphologic space of instances which could be foreseen because the generation of networks is carried out in terms of an wider range of parameters. This conclusion is quite useful for project managers willing to assess alternative methods to solve their problems based on project networks.

Keywords: Project management; Network generators; Network morphological aspects

1. Introduction

The performance of methods to manage projects depends heavily on the features of their project networks. This is particularly true for methods devoted to project scheduling, risk analysis and resources allocation. Therefore, a long line of research has been developed to generate benchmark sets of project networks and several sets have been proposed in the literature. A comparative analysis of these sets is essential but, unfortunately, no results testing their advantages and disadvantages have been published so far. Such comparative tests imply the use of stable and widely representative sets of problems, often called "benchmark sets". The first set widely used by many authors is due to [1] including 110 problems, selected from different sources. Recently, the improvement of generators of project networks is allowing the development of more extensive benchmark sets and two generators were proposed:

1. Kolisch, Sprecher and Drexl [2];
2. Agrawal, Elmaghraby and Herroelen [3].

These three contributions include the specification of resources constraints but in this paper just the features of the project network are considered.

More recently, Tavares [4] and Tavares et al. [5] have proposed quite a different approach based on the concept

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PII: S0263-7863(01)00022-9

of progressive level and permitting the generation of networks preserving morphologic features.

A comparative analysis of these four benchmark sets is developed and original results are presented in this paper in terms of the morphologic diversity of project networks.

The practical utility of the results presented in this paper for any project manager is quite obvious: helping him to select the benchmark set appropriate to a comparative analysis of performance of alternative methods of project management such as project scheduling, risk analysis or resources allocation.

2. The benchmark sets

2.1. A — Patterson set

The first set was proposed by Patterson [1] and includes 110 instances with a number of activities ranging from 5 to 49. There is no systematic generator and it seems that these networks were collected from different sources.

2.2. B — Kolisch et al. [2]

This generator, ProGen, adopts the Activity-on-Node (AoN) representation and attempts to preserve the number of non-redundant arcs per node (complexity indicator). Also, the consideration of another parameter has been discussed: the network restrictiveness [6]. This parameter measures the number of precedence-feasible activity sequences as a fraction of the total number of all activity sequences. This parameter is hard to be computed and Schwindt [7], has adopted an estimator which can be computed in polynomial time. However, the generator requires the specification on the maximal number of successors per activity (usually, equal to 3) rather than the introduction of the restrictiveness estimate.

The sets A and B are available for downloading from: http://www.bwl.uni.kiel.de/prod/pslib/index.html

2.3. C — Agrawal et al. [3]

This procedure, DAGEN, generates Activity-On-Arc networks. DAGEN is based on the "reduction complexity" index proposed by Bein et al. [8] measuring the non-conformity of a network to the series parallel case. DAGEN receives from the user this index, the number of nodes and arcs and tries to get a feasible solution. No dummy activities are generated and so a sub-set of possible networks is excluded from the generated networks.

The generation of large networks is quite difficult because of the strong interdependence between the lower and upper bounds of the proposed index, the number of nodes and the number of arcs.

The generated networks cover these data:

Reduction complexity	Number of networks
1	6
2	6
4	6
6	10
8	8

2.4. D — Tavares [4] and Tavares et al. [5]

This generator adopts the AoN notation and it is designed to preserve the morphology of each network in terms of three perspectives: the graphical shape, the number of non-redundant direct precedence links and the level length of the non-redundant direct precedence links.

The network's morphology is studied in terms of six indicators $I_1,\ldots, I_6$, according to Tavares [4] and a synthetic presentation is given in the Appendix.

The generated networks belong to three groups, M, S, P:

Sub-set M

I1	{25,50,75,100,150,200,250,300,400,500, 750,1000,1500,2000,3000,4000}
I2	{0.05,0.1,0.2,0.3,0.4}
I3	{0.1,0.5,0.9}
I4	{0.05,0.1,0.2,0.3,0.4}
I5	{0.05,0.1,0.2,0.3,0.4}
I6	{0.4}

Sub-set S

I1	{25,50,75,100,150,200,250,300,400,500, 750,1000,1500,2000,3000,4000}
I2	{0.5,0.6,0.7,0.8,0.9}
I3	{0.5}
I4	{0.05,0.1,0.2}
I5	{0.05,0.1,0.2}
I6	{0.05,0.1,0.2}

Sub-set P

I1	{25,50,75,100,150,200,250,300,400,500, 750,1000,1500,2000,3000,4000}
I2	{0.00,0.005,0.01,0.025}
I3	{0.5}
I4	{0.05,0.1,0.2}
I5	{0.05,0.1,0.2}
I6	{0.05,0.1,0.2}

The total number of networks is 7780.

The networks of group P are more parallel because I_2 is low and the networks of the second group, S, are more serial because they have a higher I_2.

The generation is carried out by the proposed method [4,5] allowing the production of small or large networks (up to 4000 activities) without problems. Actually, the real number of generated networks is quite close to the number of different sets of parameters specified for the generation in sets P and S.

Set	No. of different sets of data	No. of generated networks
P	1728	1503
S	2160	2008

Such difference is higher for the set M because it covers a larger sub-domain but, even so, more than 4000 networks (4269) were generated from 6000 specifications.

The maximal duration of CPU (using a Pentium II/233 MHz) to generate each network was 15 s but its average time was about one second.

The presented four benchmark sets will be studied in terms of their morphologic features (the sets C and D, are available at http://members.nbci.com/jcoelho72/psp/index.html). The format adopted for C and D is similar to that one used by Kolisch (set B). At this site, the sets A and B are also available using the same format to help any researcher to use this material. A few other sets are included in this site although not discussed in this paper because they are basically concerned with restrictions on resources.

3. Comparative analysis of results

The comparative study of benchmark sets is based on the six morphologic indicators previously defined (I_1, ..., I_6) plus an additional indicator, the Complexity Index (CI), which is often mentioned in literature. Also, the relationships between these seven indicators are thoroughly examined. It should be noted that strong correlations between different indicators may indicate that there are types of project networks not represented by the studied set.

3.1. Analysis of indicators

3.1.1. I_1

This indicator expresses the size of the network and the histograms of I_1 as well as its range for the four sets are given in Fig. 1. It is clear that just Tavares includes networks with more than 120 activities. Most of the

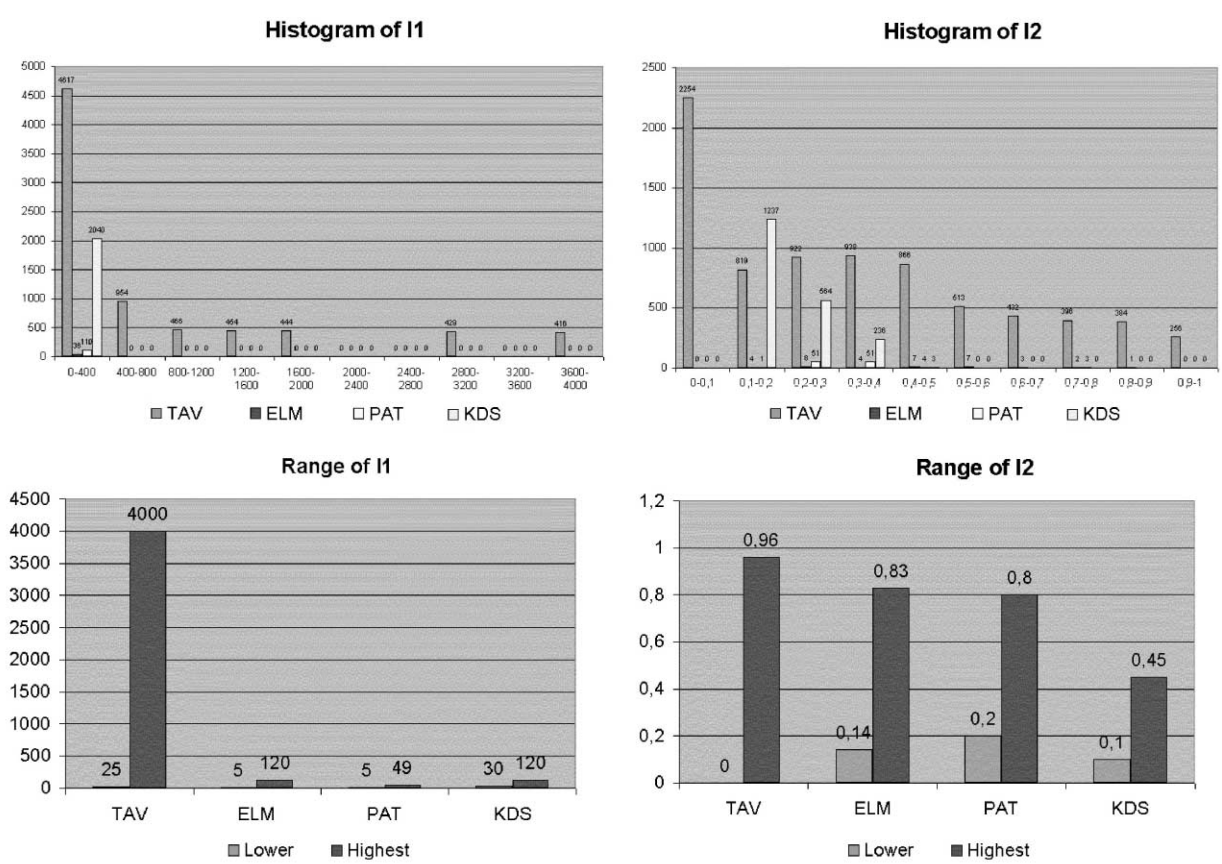

Fig. 1. Histograms and ranges for I_1 and I_2.

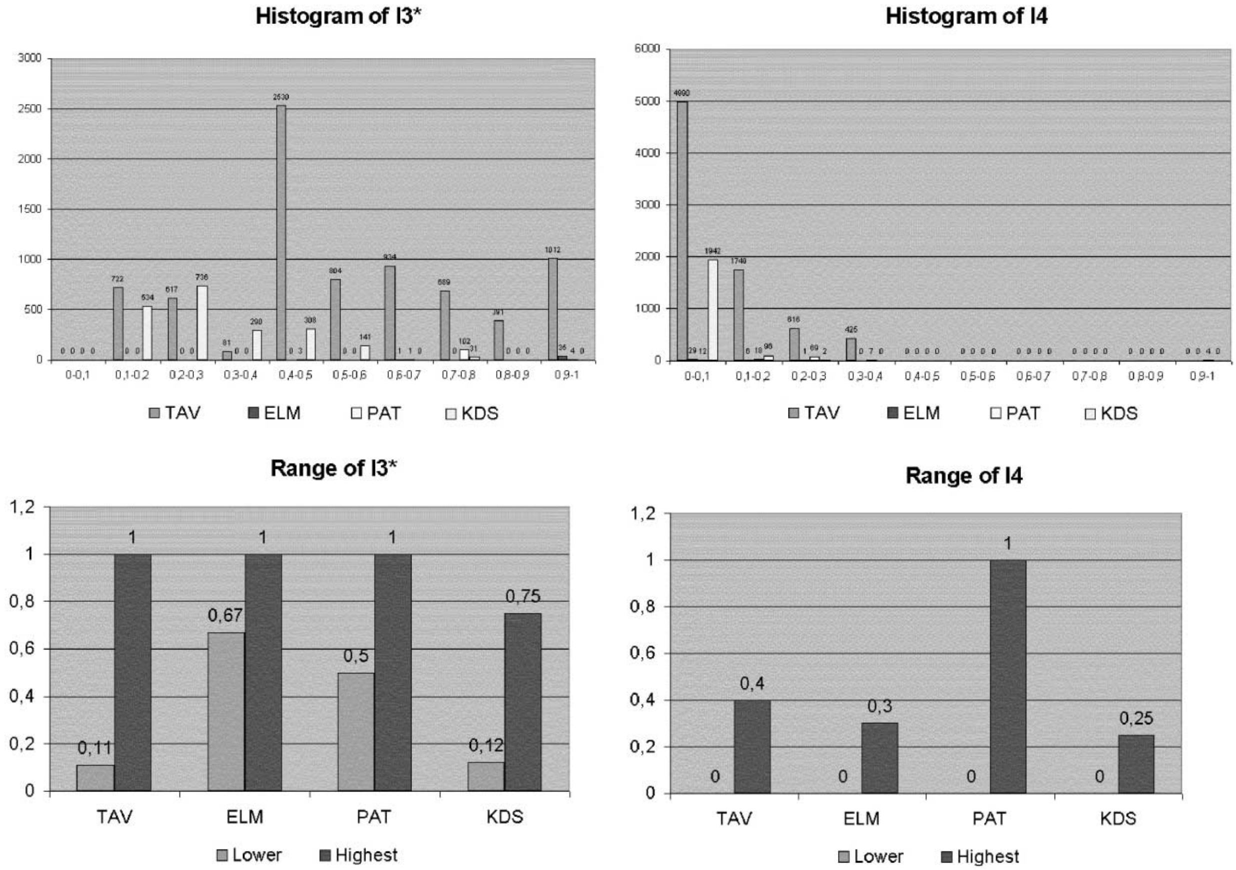

Fig. 2. Histograms and ranges for I_3 and I_4.

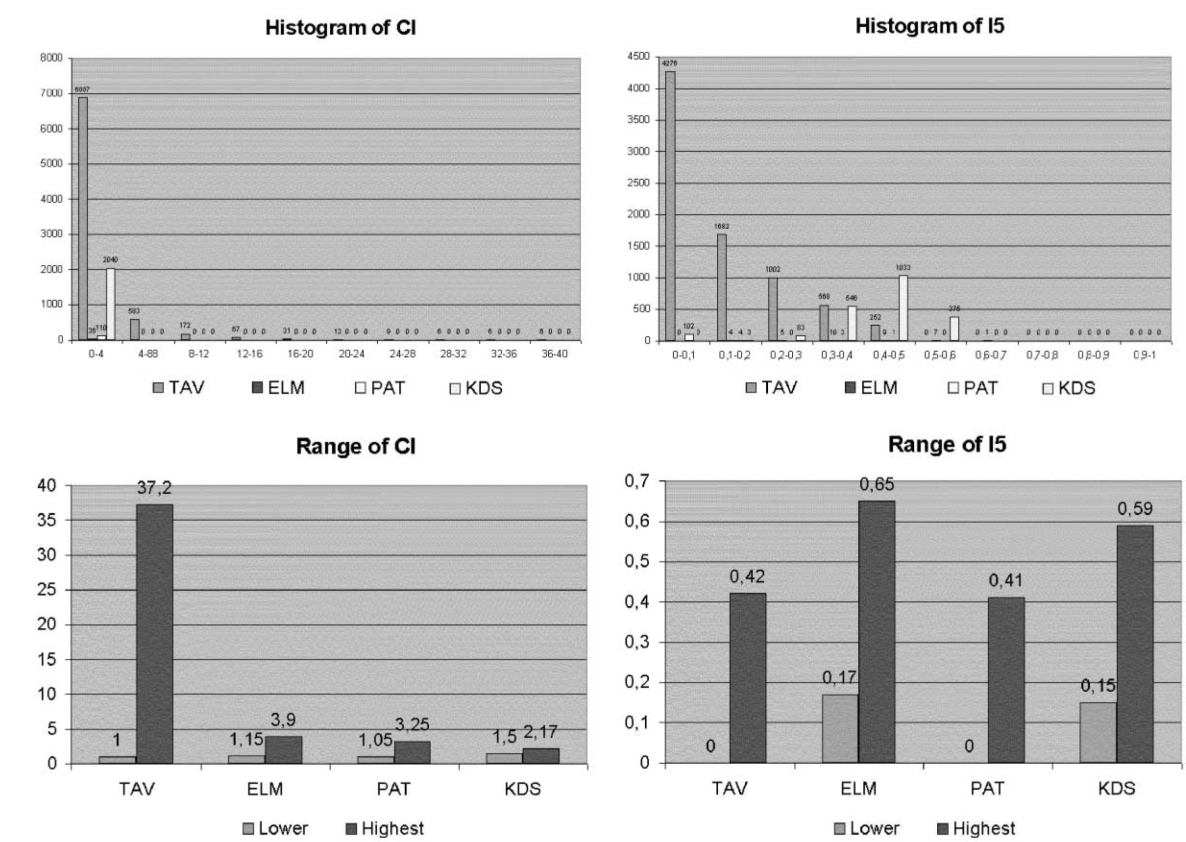

Fig. 3. Histograms and ranges for CI and I_5.

Patterson set includes small networks with no more than 50 activities.

It should be noted that project networks with more than 200 activities are most common in any engineering field.

3.1.2. I_2

This indicator describes the "depth" of the network through $(M-1)/(N-1)$ where M is the maximal progressive level, and N is the number of activities.

The results are presented in Fig. 1 showing that:

- most networks have $I_2 \leqslant 0.5$;
- the Tavares, Elmaghraby and Patterson sets cover quite a vast range between 0 and 0.80 (extended to 0.96 by the former) but I_2 for the Kolisch set does not exceed 0.46.

3.1.3. I_3

This indicator is defined by $\frac{W(1)}{MW}$ and results are presented in Fig. 2. According to the Appendix, $W(1)$ is the number of activities at progress level one and MW is the maximal number of activities within the same level. Again, Tavares sets has the widest range (0.11–1.0) and two restrictive features should be mentioned about other sets:

- Elmaghraby tends to adopt the maximal width at the first level which is quite odd and uncommon in real project networks;
- Kolisch set has $W(1)=3$ which is again a very strong limitation of the generated set.

3.1.4. I_4; CI

I_4 is defined by $[n(1)-N]/[D-N]$ and expresses how "dense" is the network in terms of links with length equal to one ($0 \leqslant I_4 \leqslant 1$). As it is presented in the Appendix, $n(1)$ is the number of non-redundant precedence links with level length one and D is its maximal possible value. The CI tells how "dense" is the network for any length but it is not bound to 0 and 1.

The results are presented in Figs. 2 and 3, showing that:

- the range of CI for the Elmaghraby, Patterson and Kolisch sets is quite narrow and just Tavares set covers an wide spectrum from 1 to 37.2.
- Kolisch set includes a high proportion of "deep" links with $l > 1$ but fewer with $l=1$. This is a consequence of the generation procedure not specifying the progressive level of each activity before constructing the precedences set. Therefore, the shape of networks tends to be similar: heavily based on activities with a small progressive level and a high number of precedences with length higher than one. This effect is mitigated by an upper bound specified for the number of activities with progressive level equal to 1 and for the maximal number of successors of each activity, both made equal to 3.
- Patterson set is in the other extreme as only links with length equal to one are used! This explains why I_4 goes up to 1.

3.1.5. I_5; I_6

I_5 expresses the average decaying rate of n (l) and I_6 is defined in terms of the maximal level length depth.

Results are presented in Figs. 3 and 4.

Again, the Tavares set has the widest range of I_6 ($0 \leftrightarrow 1$) but Patterson and Kolisch sets get close. However, the Elmaghraby set does not include networks with $I_6 < 0.5$. The upper bounds of I_5 are slightly more favourable for Elmaghraby and Kolisch than for Tavares and Patterson.

3.2. Relationship between indicators

The second type of analysis concerns the relationship between these indicators for each set. The complete collection of diagrams of Y against X where (Y, X) is a pair of indicators was produced and the following significant relationships were identified:

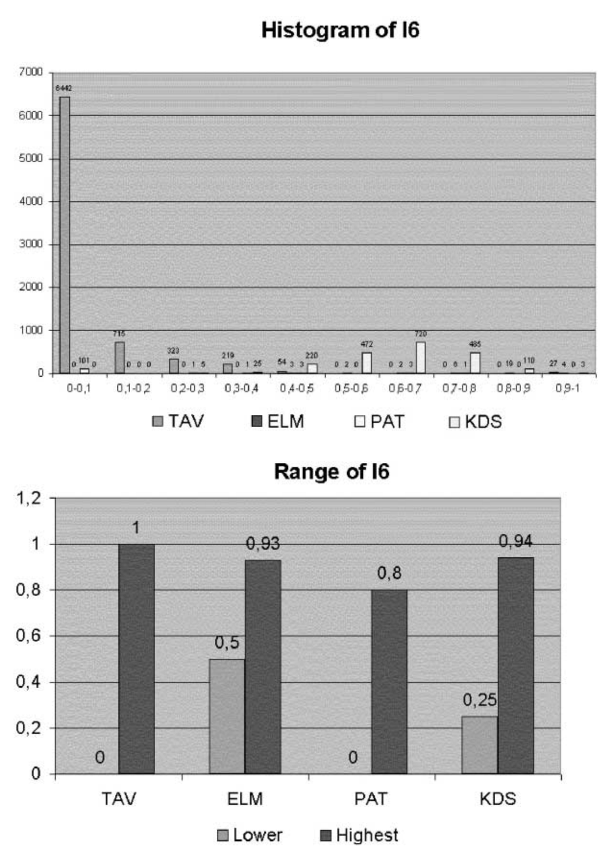

Fig. 4. Histograms and ranges for I_6.

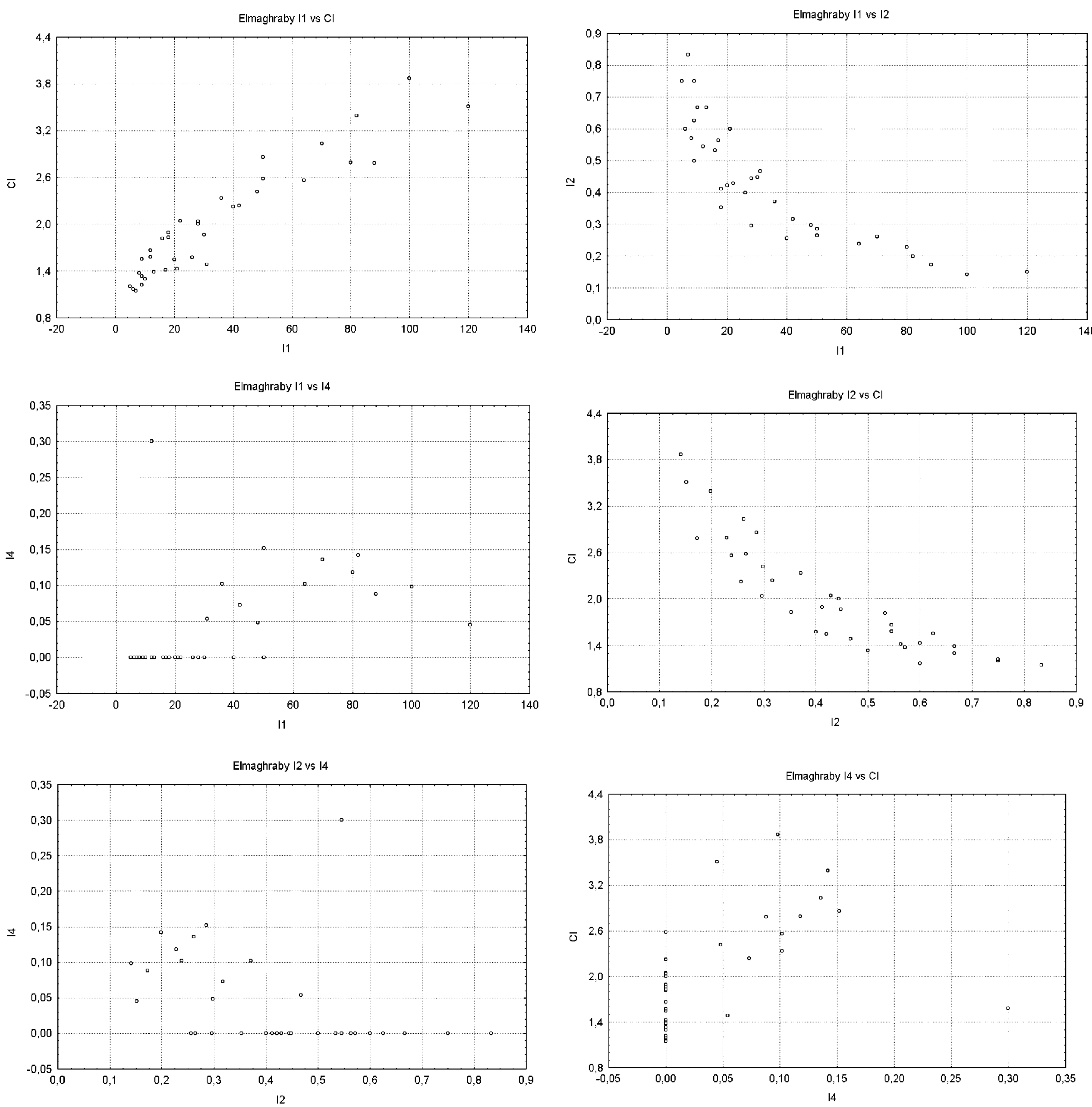

Fig. 5. Studied relationships between pairs of indicators for the Elmaghraby set.

3.2.1. Elmaghraby (Fig. 5):

I_2 with I_4 and I_2 with CI. The relationship of I_2 with CI is strongly negative and can be hardly justified because a "deeper" network can have also many links per activity.

3.2.2. Patterson (Fig. 6):

I_4 with CI which is easily explained because Patterson just uses links with $L=1$.

3.2.3. Kolisch (Fig. 7):

I_1 against I_2 and against I_4. These two negative types of relationship mean that the relative number of levels and of links ($L=1$) decrease if the size increases which means that the diversity of networks is narrowed for larger sizes.

I_2 against I_4 showing that there is a positive relationship between these indicators that should be independent.

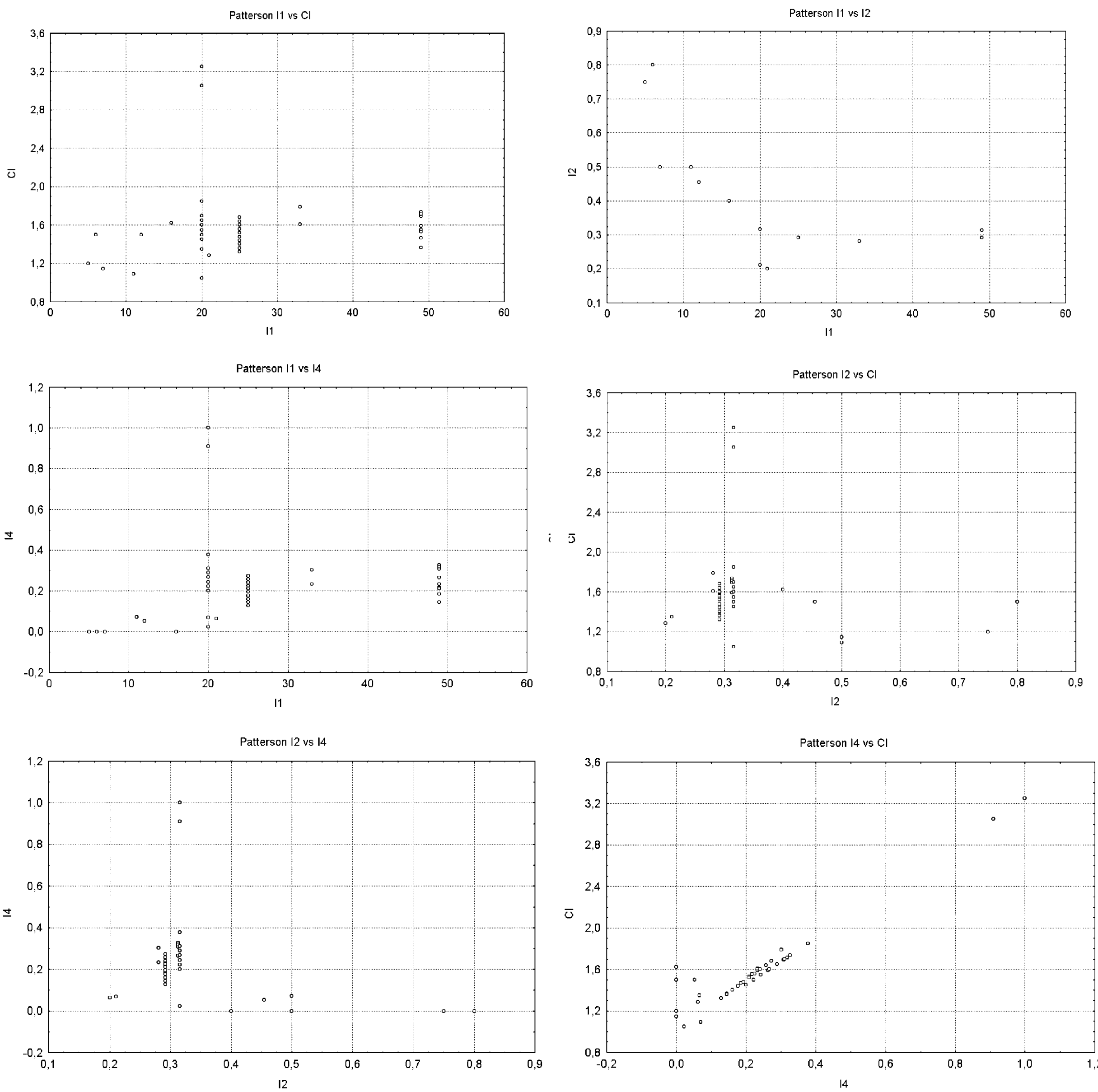

Fig. 6. Studied relationships between pairs of indicators for the Patterson set.

3.2.4. Tavares (Fig. 8):

I_2 and CI which is due to the unavoidable interdependence that exists between these two indicators. Actually, CI can be determined in terms of the six indicators obtaining

$$\mathrm{CI} = \left[1 - I_5^{1+I_1 I_2 I_6 - I_2 I_6}\right]\cdot\left[A/I_1[1 - I_5]\right] \text{ with}$$

$$A = \frac{I_1(1 - I_4) + \left[I_1(1 + I_1 I_4) - I_1^2 I_4/(I_1 I_2 - I_2 + 1)\right]/}{[I_2(I_1 - 1) + 1]}$$

if the obtained number of precedences is rounded up to an integer.

As an example, the theoretical function of CI in terms I_2 is given in Fig. 9, assuming that I_1, I_3, I_4, I_5 and I_6 are given.

4. Final comments

1. The Patterson set includes a small group of small networks with links having a length equal to l. This collection was not generated randomly, and hence, not much representativeness can be expected;
2. The Elmaghraby set covers an interesting variety of networks but it has a few restrictive features: the

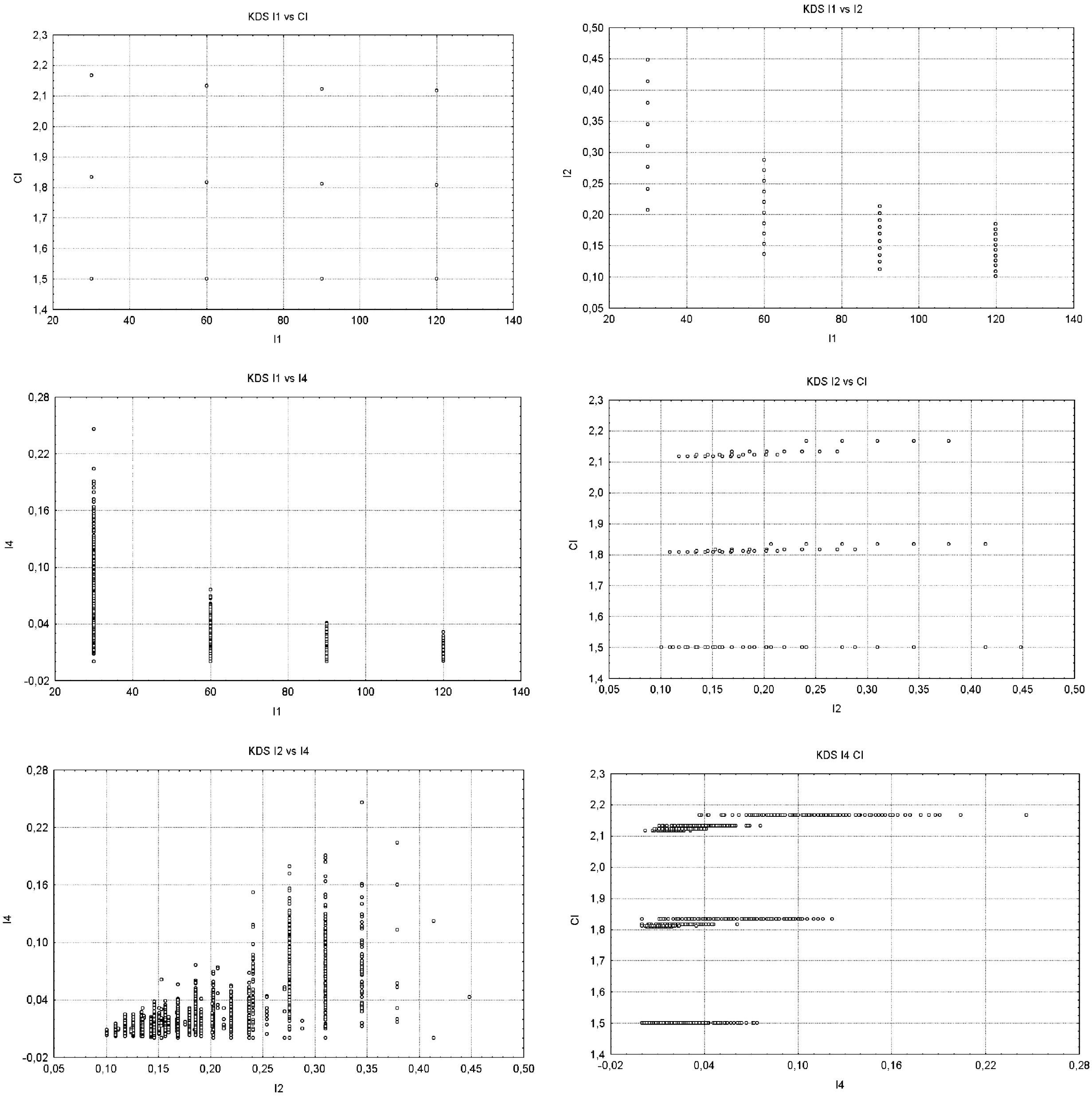

Fig. 7. Studied relationships between pairs of indicators for the Kolisch set.

widest level tends to be the first one and I_2 is correlated with I_4, CI, I_5.

3. The Kolisch set starts with $W(1) = 3$ but includes a smaller number of links with $L = 1$ and shows a positive correlation between I_1 and I_2, I_1 and I_4, I_2 and I_3, I_2 and I_4. The networks tend to have a similar shape with too many activities for low progressive levels and too few for high levels.
4. The Tavares set is, by far, the generator covering the widest range of sizes and of the other indicators. Only one case of correlation between I_2 and CI is identified which is just the result of the logic interdependence between the definition of these two indicators.
5. The comparative advantage of this last generator could be expected because its procedure is developed in terms of $I_1, \ldots, I_6$ and hence, a higher level of representativeness of different morphologies is achieved.
6. It is quite clear that the diversity of networks covered by the four sets is quite different. The benchmarking set covering a wider diversity of morphologies is, by far, that one proposed by Tavares.

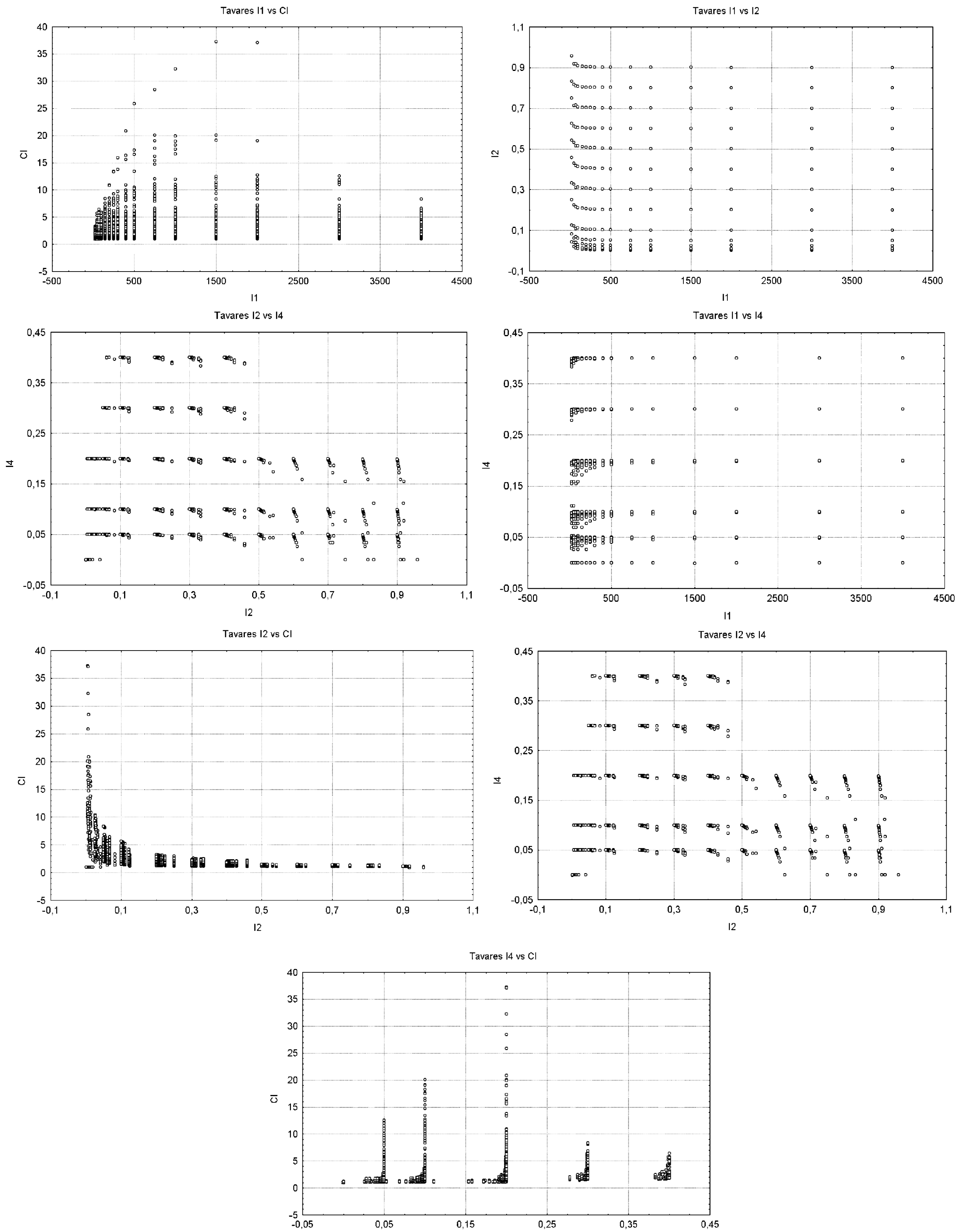

Fig. 8. Studied relationships between pairs of indicators for the Tavares set.

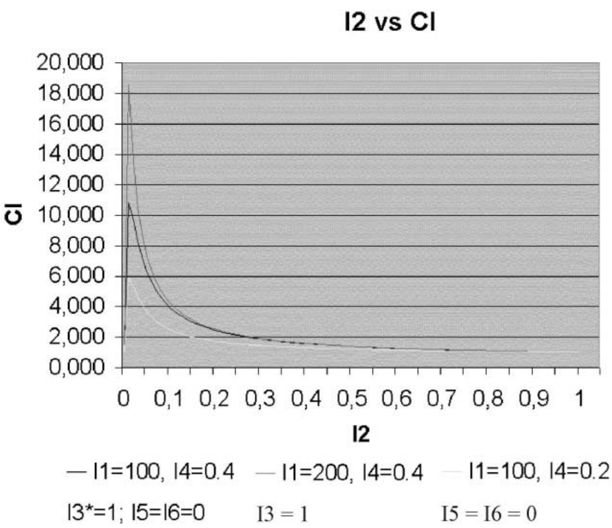

Fig. 9. CI in terms of I_2.

Appendix A. On the morphology of project networks

The concept of project network is the basic model used in Project Management. This model has been intensively studied since the seminal work of [9,10] but no quantitative attempt has been proposed to describe their different shapes until the approach developed by the authors and published in Tavares [4] and Tavares, Ferreira and Coelho [5] despite the key relevance of the shape of a project network to study most of the general objectives in Project Management such as expected duration, cost, risk, etc.

The comparative analysis of benchmarking sets presented in this paper is based on such an approach that is shortly summarized in this appendix.

According to this contribution, the morphology of a project network is described by six quantitative magnitudes that can be easily computed in terms of the number of activities, precedence links and progressive hierarchical levels. The concept of progressive hierarchical level [11] of an activity i (with $i=1, \ldots, N$ being N the number of activities of the project network) is less common than the others but can be defined by:

$$a_i = \operatorname*{Max}_{j \in J(i)} a_j + 1$$

where a_i is the progressive level of activity i and $J(i)$ is the set of activities directly preceding activity i (direct predecessors of i). The maximal progressive level is denoted by M. The precedence link connecting any pair of activities, j and i, is called a direct precedence link. If $J(i)$ is empty, then $a_i=1$ by definition (the dummy node representing the beginning of the network corresponds to $i=0$ and it has level zero).

The level length of a precedence link between j and I is given by $L=a(I) - a(j)$.

The six magnitudes are:

a. Size

The corresponding indicator I_1, is defined by the number of activities:

$$I_1 = N$$

b. Level dimension

The level dimension is defined by the dimension of the longest path measured in terms of the hierarchical levels:

$$M = \operatorname*{Max}_{i \in \Omega} a_i$$

The corresponding indicator I_2 is the relative dimension and measures how serial is the shape of the network (for $N>1$):

$$I_2 = \frac{M-1}{N-1}$$

where $0 \leqslant I_2 \leqslant 1$ being 0 the parallel case and 1 the serial one (Fig. 2).

c. Width

This magnitude can be defined by the number of activities at each progressive level, $W(a)$, with $a=1, \ldots, M$ being, $W=1, 2, \ldots$.

The maximal width is denoted by MW and several shapes can be adopted to describe the function $W(a)$

A simplified indicator can be defined by:

$$I_3 = \frac{W(1)}{MW}$$

d. Precedence density

The *maximal number*, D, *of the non-redundant precedence links with level length equal to one* is obtained by the connection of each activity, i, belonging to the level a ($a=1, \ldots, M-1$) with all the activities belonging to the level $(a+1)$. Therefore, D is given by:

$$D = W(1) + \sum_{a=1}^{M-1} W(a) \cdot W(a+1)$$

and the proposed indicator, I_4, expressing the *precedence index* for links with a length equal to 1, is defined by:

$$I_4 = \frac{n(1) - N}{D - N}$$

where $n(1)$ is the number of links with level length equal to 1, which will satisfy $0 \leqslant I_4 \leqslant 1$ as $N \leqslant n(1) \leqslant D$.

This indicator expresses how dense is the precedence structure of the network for the level length equal to one.

Another indicator often adopted to describe how rich is the network in terms of non-redundant precedence links is the Complexity Index, (CI), defined by the average number of such links per activity.

e. Decaying rate of the precedence level length

Usually, the number of links of level length L, $n\ (L)$, decreases with their level length L, and therefore the following assumption may often be adopted

$$n(L+1) = n(L) \cdot p \tag{1}$$

with $L = 1, 2, \ldots, V$ where is V the maximal level length and where p is a factor between 0 and 1. This assumption implies an exponential decrease of $n(L)$ with L being p the constant decreasing rate. It should be noted that V cannot be higher than $(M-1)$.

For a generic project network, the average rate of decay can be computed through:

$$p^* = \frac{1}{V} \sum_{k=2}^{V} \left[\frac{n(k)}{n(1)} \right]^{1/(k-1)}$$

if $V > 1$.

Hence, the proposed indicator to express this perspective is

$$0 \leqslant I_5 = p^* \leqslant 1$$

f. Maximal precedence level length

This magnitude is defined in terms of the maximal level length computed for all non-redundant precedence links, V. It measures how "deep" is the precedence structure of the project network.

It should be noted, that V cannot be higher than $(M-1)$ and so the proposed indicator is defined by

$$0 \leqslant \mathrm{I}_5 = \frac{V-1}{M-1} \leqslant 1$$

If assumption (1) holds, then the average level length, $\bar{L}$, of the direct precedence links can be determined easily by:

$$\bar{L} = \frac{V\ \mathrm{p}^V (p-1) + \left(1 - p^V\right)}{(1 - p^V)\ (1 - p)}$$

and the total number, T, of non-redundant direct precedence links is given by:

$$T = n\ (1) \cdot \ \frac{1\text{-}p^V}{1-p}.$$

In this case,

$$\mathrm{CI} = \frac{T}{N}.$$

Furthermore, the maximal value which can be given to V is related with the number of links with length equal to 1 because:

$$n(V) \ = n(1) \cdot p^{V-1} \geqslant 1$$

or

$$\ln n(1) + (V-1) \ln p \geqslant 0$$

and therefore

$$V \leqslant 1 + \frac{\ln n(1)}{(-\ln p)}$$

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