⑦ $\frac{du}{dt} = \alpha u, \; u(0) = u_0 \quad \longrightarrow \quad u_k = (1+\alpha\Delta t)^k u_0$

Euler: $y_{n+1} = y_n + hF(x_n, y_n)$

$$u_1 = u_0 + hF(t_0) \longrightarrow$$
$$= u_0 + \Delta t \, \alpha u(0)$$
$$= u_0(1 + \Delta t \, \alpha)$$

$$u_2 = u_1 + hF(u_1)$$
$$= u_0(1+\Delta t\alpha) + \Delta t \, \alpha(u_0(1+\alpha\Delta t))$$
$$= u_0(1 + \Delta t \alpha + \Delta t \alpha + (\alpha\Delta t)^2)$$
$$= u_0(1 + 2\Delta t\alpha + (\alpha\Delta t)^2)$$
$$= u_0(1+\alpha\Delta t)^2$$

$\downarrow$

$$u_n = u_0(1+\Delta t\alpha)^n$$

$$u_{(n+1)} = u_0(1+\Delta t\alpha)^n + u_0\Delta t\alpha(1+\Delta t\alpha)^n$$
$$= u_0(1+\Delta t\alpha)^n(1+\Delta t\alpha)$$
$$= u_0(1+\Delta t\alpha)^{n+1} \; //$$

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