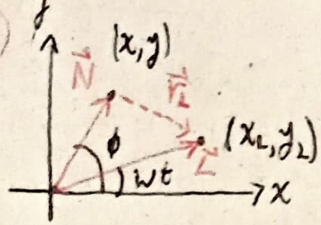
(c)



Para este esquema, por las suposiciones que hemos considerado, $x^2+y^2=d^2 : d = cte.$

Ahora la distancia entre nave (con posición $\vec N=(x,y)$) y luna ($\vec L=(x_L,y_L)$), será

$|\vec r_L|$ tq $\vec r_L = \vec L - \vec N \therefore |\vec r_L| = |\vec L-\vec N| = \left|\begin{pmatrix}x_L\\y_L\end{pmatrix}-\begin{pmatrix}x\\y\end{pmatrix}\right| = \left|\begin{pmatrix}x_L-x\\y_L-y\end{pmatrix}\right|$

$$r_L = \sqrt{(x_L-x)^2+(y_L-y)^2} = \sqrt{x_L^2-2x_Lx+x^2+y_L^2-2y_Ly+y^2} = \sqrt{\underbrace{(x_L^2+y_L^2)}_{|\vec L|^2 = d^2}+\underbrace{(x^2+y^2)}_{|\vec r(t)|^2 = |\vec N|^2}-2\underbrace{(x_Lx+y_Ly)}_{\sum x_L y = \vec L \cdot \vec N = d\cdot r(t)\cdot\cos(\phi - wt)}}$$

$$\therefore\ r_L = \sqrt{r^2(t)+d^2-2r(t)\,d\cos(\phi-wt)}.$$

(d) $\mathcal{H} = \sum(p_i\dot q_i) - L : L = T - \sum U$, $T=\frac{1}{2}m\dot q_i^2$, p_i = momento generalizado de i, $\dot q_i$ = velocidad generalizada de i

Para $\sum(p_i\dot q_i)$ vemos que sólo hay 2 coordenadas (r,ϕ) $\therefore \sum(p_i\dot q_i) = p_r\dot r + p_\phi\dot\phi \Rightarrow \mathcal{H} = p_r\dot r + p_\phi\dot\phi - L$

$= \frac{p_r^2}{\;} + \frac{p_\phi^2}{2mr^2}$

Para $L = T - \sum U$: Para $q_i = r$: $T_r = \frac{1}{2}m\dot r^2 = \frac{1}{2}\frac{m^2}{m}\dot r^2 = \frac{1}{2m}(m\dot r)^2 = \frac{p_r^2}{2m}$

Para $q_i = \phi$: $T_\phi = \frac{1}{2}m\dot\phi^2r^2 = \frac{1}{2mr^2}(mr^2\dot\phi)^2 = \frac{p_\phi}{2mr^2}$

Ahora $\sum U = U_{g_T}+U_{g_L}$, estas tienen la forma de $U_{g_j} = -\frac{Gmm_j}{r_j}$ $\therefore \sum U = -Gm\left(\frac{m_T}{r}+\frac{m_L}{r_L}\right)$

$$\Rightarrow \mathcal{H} = \frac{p_r^2}{m}+\frac{p_\phi^2}{mr^2}-\frac{p_r^2}{2m}-\frac{p_\phi^2}{2mr^2}-Gm\left(\frac{m_T}{r}+\frac{m_L}{r_L}\right)$$

$$\mathcal{H} = \frac{p_r^2}{2m}+\frac{p_\phi^2}{2mr^2}-Gm\left(\frac{M_T}{r}+\frac{M_L}{r_L}\right)$$

(e) $\frac{\partial\mathcal{H}}{\partial p_r} = \frac{2p_r}{2m}+0 = \frac{p_r}{m} = \dot r$; $\dot p_r = -\frac{\partial\mathcal{H}}{\partial r} = -\frac{-2p_\phi^2}{2mr^3}+\frac{-GmM_T}{r^2}+\frac{GmM_L}{-2r_L^3}\cdot(2r-2d\cos(\phi-wt))$

$\frac{\partial\mathcal{H}}{\partial p_\phi} = 0+\frac{2p_\phi}{2mr^2}+0 = \frac{p_\phi}{mr^2} = \dot\phi$; $\dot p_\phi = -\frac{\partial H}{\partial\phi} = -0+\frac{-GmM_L}{2r_L^3}2rd\sin(\phi-wt)$

f) Para $\tilde{r}=\frac{r}{d}$; $\tilde{P}_r=\frac{P_r}{md}$; $\tilde{P}_\phi=\frac{P_\phi}{md^2}$

$\Rightarrow$ ① $\dot{r}=\frac{P_r}{m} \longrightarrow \dot{r}=\frac{md\,\tilde{P}_r}{m}$, por $\tilde{r}=\frac{r}{d} \to \dot{\tilde{r}}=\frac{\dot{r}}{d}$ $\therefore$ $\dot{\tilde{r}}\cancel{d}=\dot{r}=\frac{\cancel{md}\tilde{P}_r}{\cancel{m}} \Rightarrow \boxed{\dot{\tilde{r}}=\tilde{P}_r}$

② $\dot{\phi}=\frac{P_\phi}{mr^2} \longrightarrow \dot{\phi}=\frac{\cancel{m}d^2\tilde{P}_\phi}{\cancel{m}r^2}$; por $\tilde{r}=\frac{r}{d}$, si elevamos al cuadrado: $\tilde{r}^2=\frac{r^2}{d^2} \to \dot{\phi}=\frac{r^2}{\tilde{r}^2}\cdot\frac{\tilde{P}_\phi}{r^2}$

$\therefore \dot{\phi}=\frac{\tilde{P}_\phi}{\tilde{r}^2}$

③ $\dot{P}_r=\frac{P_\phi^2}{mr^3}-Gm\left(\frac{M_T}{r^2}+\frac{M_L}{r_L^3}(r-d\cos(\phi-wt))\right)=\frac{m^2\tilde{P}_\phi^2 d^4}{md^3\tilde{r}^3}-Gm\left(\frac{M_T}{\tilde{r}^2d^2}+\frac{M_L}{\tilde{r}'^3 d^{\cancel{3}2}}(\tilde{r}\cancel{d}-\cancel{d}\cos(\phi-wt))\right)$

$\hookrightarrow \sqrt{r^2+d^2-2rd\cos(\phi-wt)}^{\,3}$

$\tilde{r}'^3:=d\sqrt[3]{\tilde{r}^2+1-2\tilde{r}\cos(\phi-wt)}^{\,3}$

$\therefore \dot{P}_r=\frac{md\tilde{P}_\phi^2}{\tilde{r}^3}-\frac{GmM_T}{d^2}\left(\frac{1}{\tilde{r}^2}+\frac{M_L}{M_T}\cdot\frac{1}{\tilde{r}'^3}(\tilde{r}-\cos(\phi-wt))\right)$

Si derivamos: $(\dot{P_r=\tilde{P}_r md})=\dot{P}_r=md\dot{\tilde{P}}_r$

$\therefore \dot{\tilde{P}}_r\cancel{md}=\frac{\tilde{P}_\phi^2\cancel{md}}{\tilde{r}^3}-\cancel{m}\frac{GM_T}{d^{\cancel{2}3}}\left(\frac{1}{\tilde{r}^2}+\frac{M_L}{M_T}\cdot\frac{1}{\tilde{r}'^3}(\tilde{r}-\cos(\phi-wt))\right)$

Para $\Delta=GM_Td^{3}$, $\mu=\frac{M_L}{M_T}$: $\boxed{\dot{\tilde{P}}_r=\frac{\tilde{P}_\phi^2}{\tilde{r}^3}-\Delta\left[\frac{1}{\tilde{r}^2}+\frac{\mu}{\tilde{r}'^3}(\tilde{r}-\cos(\phi-wt))\right]}$

④ $\dot{P}_\phi=-\frac{GmM_L}{r_2^3}rd\sin(\phi-wt)=-\frac{GmM_L}{\tilde{r}'^3d^3}\tilde{r}d^2\sin(\phi-wt)=-\frac{\Delta}{M_T\tilde{r}'^3}\tilde{r}\sin(\phi-wt)M_L\,md^2$

Derivando: $\frac{d}{dt}\left(\tilde{P}_\phi=\frac{P_\phi}{md^2}\right)=\dot{\tilde{P}}_\phi=\frac{\dot{P}_\phi}{md^2} \longrightarrow \cancel{md^2}\dot{\tilde{P}}_\phi=-\frac{\Delta}{\tilde{r}'^3}\cancel{md^2}\frac{M_L}{M_T}\tilde{r}\sin(\phi-wt)$

$\Rightarrow \boxed{\dot{\tilde{P}}_\phi=-\frac{\Delta\tilde{r}}{\tilde{r}'^3}\mu\sin(\phi-wt)}$