1 Para $f(x)=x^2$:

Sabemos que $f'(x)=2x$ y $f''(x)=2$:

$$f'(x)=\frac{-f(x+2h)+4f(x+h)-3f(x)}{2h}=\frac{-(x+2h)^2+4(x+h)^2-3x^2}{2h}$$

$$* \ f'(x)=\lim_{h\to 0}\frac{\cancel{-x^2}-4xh-4h^2+\cancel{4x^2}+8xh+8h^2-\cancel{3x^2}}{2h}$$

$$f'(x)=\lim_{h\to 0}\frac{4x\cancel{h}+4h^{\cancel{2}}}{2\cancel{h}}=\lim_{h\to 0}\left(2x+2h\right)=2x+0$$

$$\underline{f'(x)=2x}$$

$$* \ f''(x)=\frac{f(x+h)-2f(x)+f(x-h)}{h^2}=\lim_{h\to 0}\frac{(x+h)^2-2x^2+(x-h)^2}{h^2}$$

$$h \to 0 \qquad h^2$$

$$f''(x) = \frac{\cancel{x^2} + \cancel{2x}h + h^2 - \cancel{2x^2} + \cancel{x^2} - \cancel{2h}x + h^2}{h^2}$$

$$f''(x) = \lim_{h \to 0} \frac{2h^2}{h^2} = \lim_{h \to 0} 2 = 2 \Rightarrow \underline{f''(x) = 2}$$

Se cumple.

Para $f(x) = \operatorname{Sen}(x)$ sabemos que $f'(x) = \operatorname{Cos}(x)$ y $f''(x) = -\operatorname{Sen}(x)$.

$$* f'(x) = \lim_{h \to 0} \frac{-\operatorname{Sen}(x+2h) + 4\operatorname{Sen}(x+h) - 3\operatorname{Sen}(x)}{2h}$$

$$f'(x) = \lim_{h \to 0} \frac{-\operatorname{Sen}(x)\operatorname{Cos}(2h) - \operatorname{Cos}(x)\operatorname{Sen}(2h) + 4\operatorname{Sen}(x)\operatorname{Cos}(h) + 4\operatorname{Cos}(x)\operatorname{Sen}(h) - 3\operatorname{Sen}(x)}{2h}$$

$f'(x) = \frac{0}{0}$ Usamos L'hôpital:

$$f'(x) = \lim_{h \to 0} \frac{-\operatorname{Sen}(x+2h) + 4\operatorname{Sen}(x+h) - 3\operatorname{Sen}(x)}{2h} = \lim_{h \to 0} \frac{-2\operatorname{Cos}(x+2h) + 4\operatorname{Cos}(x+h) - 3 \cdot 0}{2}$$

$$f'(x) = \lim_{h \to 0} \frac{4\operatorname{Cos}(x+h) - 2\operatorname{Cos}(x+2h)}{2} = \lim_{h \to 0} 2\operatorname{Cos}(x+h) - \operatorname{Cos}(x+2h) = 2\operatorname{Cos}(x) - \operatorname{Cos}(x)$$

$$\underline{f'(x) = \operatorname{Cos}(x)}$$

$$f''(x) = \lim_{h\to 0} \frac{\operatorname{Sen}(x+h) - 2\operatorname{Sen}(x) + \operatorname{Sen}(x-h)}{h^2} \xrightarrow{\text{L'hôpital}} \lim_{h \to 0} \frac{\operatorname{Cos}(x+h) - \operatorname{Cos}(x-h)}{2h} \xrightarrow{\text{L'hôpital}} \lim_{h \to 0} \frac{-\operatorname{Sen}(x+h) - \operatorname{Sen}(x-h)}{2}$$

$$f''(x) = \lim_{h \to 0} \frac{-\operatorname{Sen}(x+h) - \operatorname{Sen}(x-h)}{2} = \frac{-\cancel{2}\operatorname{Sen}(x)}{\cancel{2}} \therefore \underline{f''(x) = -\operatorname{Sen} x.}$$