$$1)\ * \int_a^b P_1(x)dx = \frac{b-a}{2}\left(f(a)+f(b)\right)$$

Para llegar a eso, desarrollamos...

$$\int_a^b P_1(x)dx = \int_a^b \frac{x-b}{a-b}f(a) + \frac{x-a}{b-a}f(b)\,dx = \int_a^b \frac{x-b}{a-b}f(a)dx + \int_a^b \frac{x-a}{b-a}f(b)dx$$

$$= \frac{f(a)}{a-b}\int_a^b x-b\,dx + \frac{f(b)}{b-a}\int_a^b x-a\,dx = \frac{1}{b-a}\left[-f(a)\left(\frac{x^2}{2}-bx\right)\Big|_a^b + f(b)\left(\frac{x^2}{2}-ax\right)\Big|_a^b\right]$$

$$= \frac{1}{b-a}\left[-f(a)\left(\frac{b^2}{2}-b^2-\frac{a^2}{2}+ab\right)+f(b)\left(\frac{b^2}{2}-ab+\frac{a^2}{2}-a^2\right)\right]$$

$$= \frac{1}{b-a}\left[f(a)\left(\frac{b^2}{2}-ab-\frac{a^2}{2}\right)+f(b)\left(\frac{b^2}{2}-ab+\frac{a^2}{2}\right)\right] = \frac{1}{2(b-a)}\left[f(a)(b-a)^2+f(b)(b-a)^2\right]$$

$$= \frac{1}{2}\cdot\frac{(b-a)^{\cancel{2}}}{\cancel{b-a}}\left(f(a)+f(b)\right) = \boxed{\frac{b-a}{2}\left(f(a)+f(b)\right)}$$

$$3)\ * \int_a^b P_2(x)\,dx = \frac{h}{3}\left(f(a)+4f(x_m)+f(b)\right) : h = x_m - a = b - x_m,\ 2h = b-a,\ x_m = \frac{a+b}{2}$$

Para llegar a esto, desarrollamos...

$$\int_a^b P_2(x)dx = \int_a^b \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)}f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)}f(x_m) + \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)}f(b)\,dx$$

$$= \frac{f(a)}{(a-b)(a-x_m)}\int_a^b x^2 - x(x_m+b)+bx_m\,dx + \frac{f(x_m)}{(x_m-a)(x_m-b)}\int_a^b x^2-x(a+b)+ab\,dx + \frac{f(b)}{(b-a)(b-x_m)}\int_a^b x^2-x(a+x_m)+ax_m\,dx$$

$$= \frac{f(a)}{2h^2}\left(\frac{b^3}{12}-\frac{b^2a}{4}+\frac{a^2b}{4}-\frac{a^3}{12}\right) + \frac{f(x_m)}{-h^2}\left(-\frac{b^3}{6}+\frac{b^2a}{2}-\frac{a^2b}{2}+\frac{a^3}{6}\right) + \frac{f(b)}{2h^2}\left(\frac{b^3}{12}-\frac{b^2a}{4}+\frac{a^2b}{4}-\frac{a^3}{12}\right)$$

$$= \frac{1}{h^2}\left[f(a)\frac{1}{24}(b-a)^3 - f(x_m)\frac{1}{6}(a-b)^3 + f(b)\frac{1}{24}(b-a)^3\right] = \frac{1}{h^2}\left[f(a)\frac{1}{24}\cdot 8h^3 + f(x_m)\frac{1}{6}8h^3 + f(b)\frac{1}{24}\cdot 8h^3\right]$$

$$= \frac{1}{\cancel{h^2}} \cdot \cancel{h^3} \left[f(a)\,\frac{1}{3} + f(x_m)\,\frac{4}{3} + f(b)\,\frac{1}{3} \right] = \boxed{\frac{h}{3}\left(f(a) + 4f(x_m) + f(b) \right)}$$