



# A Short Tutorial on Uncertainty in Deep Learning

Dr Dimity Miller

What makes a “good” predictive model in deep learning?

Let's put you to the test!













Not all predictive models are equal.

Beyond accuracy, predictions can also be associated with a “confidence” or “uncertainty”.

There are also different **types** of uncertainty.

**Epistemic**  
**(knowledge)**



**Aleatoric**  
**(data)**



# **Epistemic (knowledge)**



Can be reduced the more knowledge you have.

# **Aleatoric (data)**



Always possible and cannot be reduced.

- (1) A model that is 99.9% accurate.
- (2) A model that is 99% accurate, AND can predict when it is failing with 99% accuracy.

(1) A model that is 99.9% accurate.

0.1% failure rate

(2) A model that is 99% accurate, AND can predict  
when it is failing with 99% accuracy.

0.01% failure rate

# For robots + autonomous systems...

**Robot crushes child's finger in Russian chess match**

Science and Technology

Mon 25 Jul 2022

**Tesla self-driving car fails to detect truck in fatal crash**



By Jessica Castro   
Saturday, July 2, 2016



**Toyota Paralympics self-driving bus hits disabled athlete**

**Starship robot 'tries to run over pedestrian'**

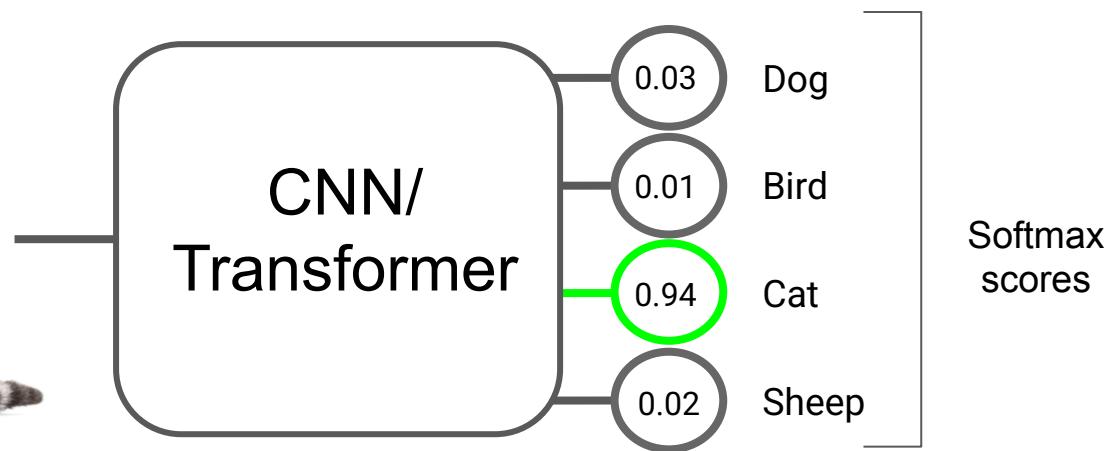
**'Worthless' Mater Dei Hospital medicine robots cause mass resignations**

What is uncertainty in deep learning?

# Predictive Uncertainty in Deep Learning

All predictions returned with an associated confidence or uncertainty.

**Classification task (e.g. image recognition)**



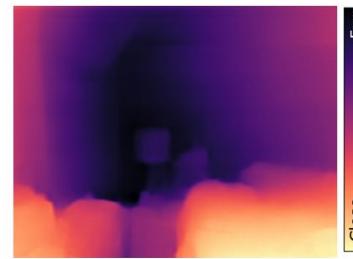
# Predictive Uncertainty in Deep Learning

All predictions returned with an associated confidence or uncertainty.

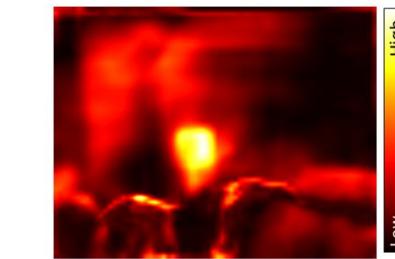
## Regression task (e.g. monocular depth estimation)



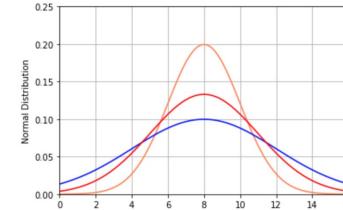
RGB Image



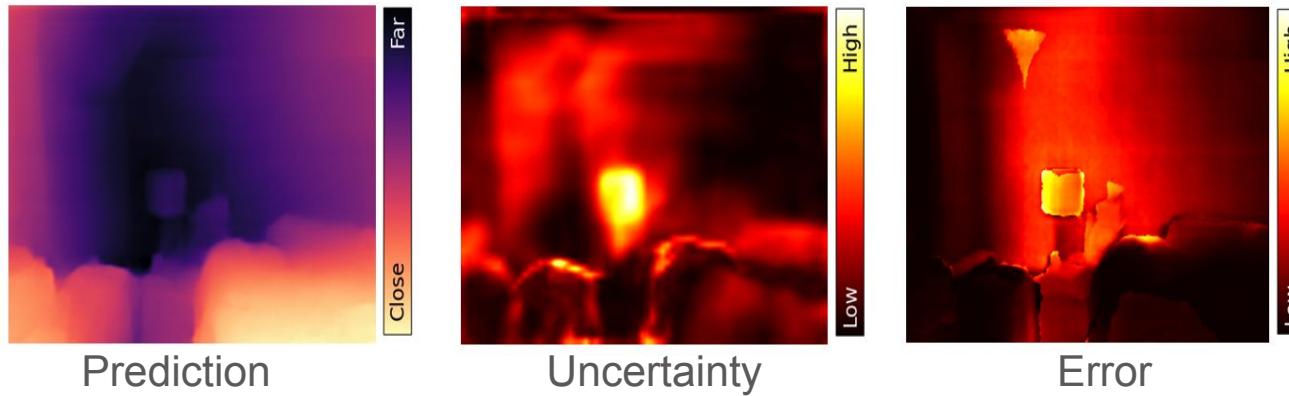
Depth Prediction



Depth Uncertainty



# Predictive uncertainty needs to be useful!



“Good” predictive uncertainty should be related to prediction error.

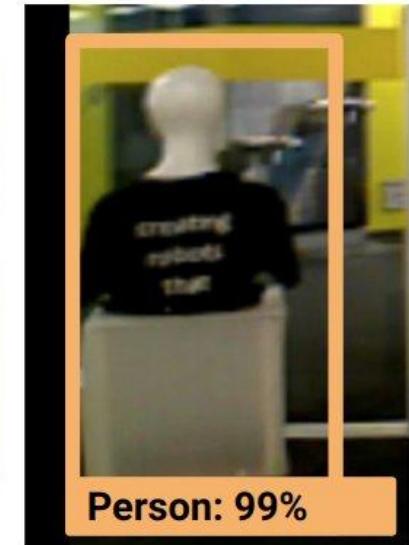
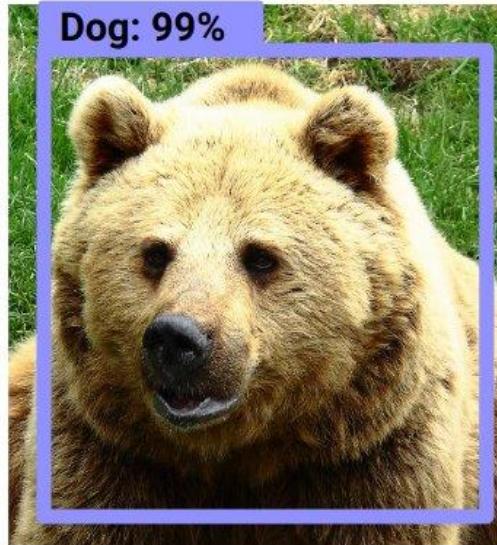
# How can we get predictive uncertainty?

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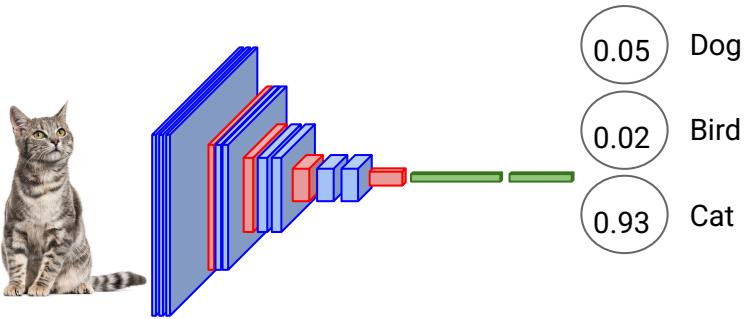
“Let’s use the softmax probability!”

# How can we get predictive uncertainty?

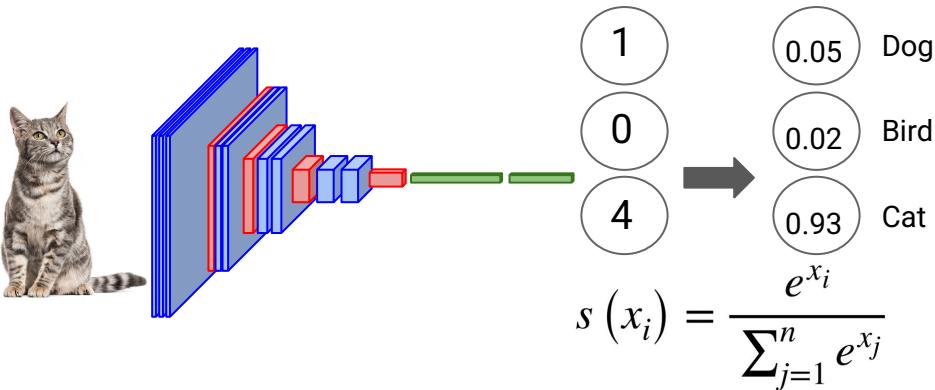
“Let’s use the softmax **probability**!”



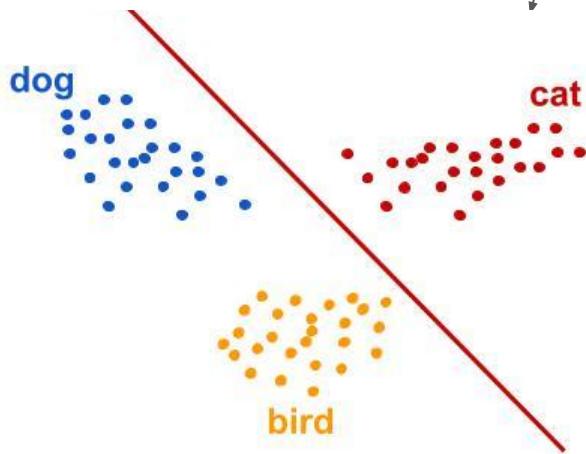
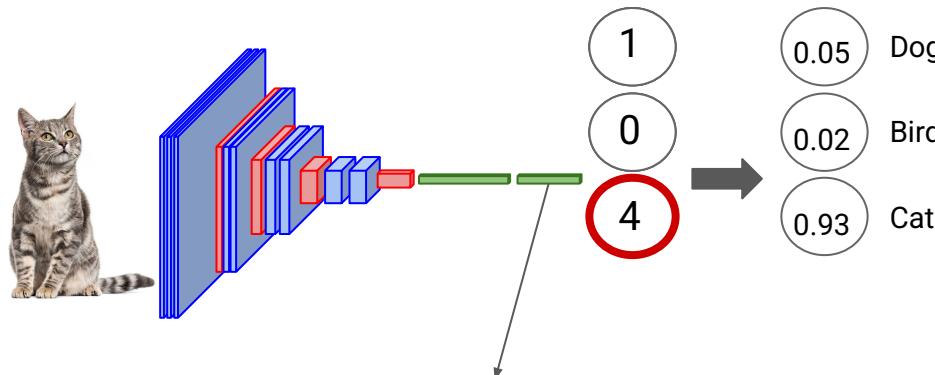
# Why Softmax often is a “bad” measure of uncertainty...



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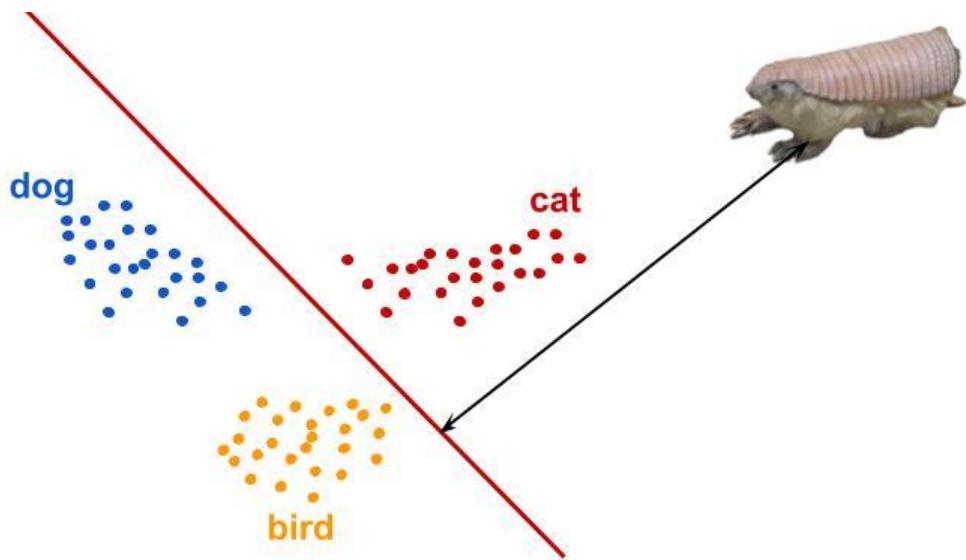


# Why Softmax often is a “bad” measure of uncertainty...



Logits are proportional to  
class-specific linear classifiers.

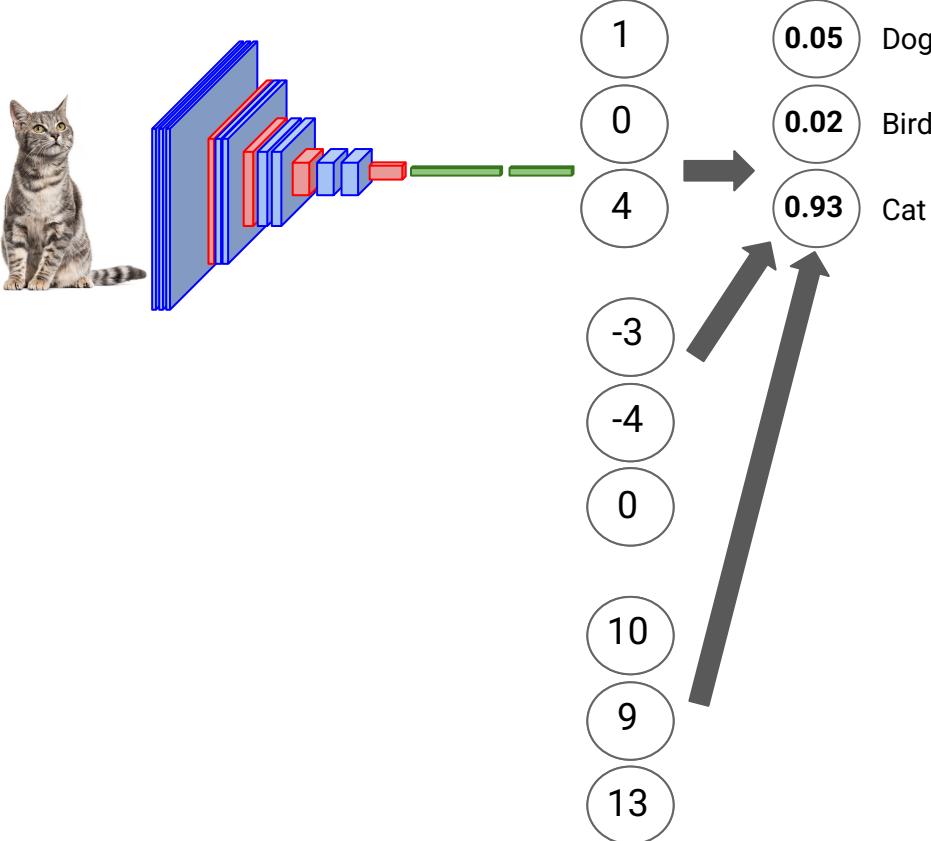
# Why Softmax often is a “bad” measure of uncertainty...



Issues:

1. Inputs can get a very high logit even if projected into “unusual” feature regions.

# Why Softmax often is a “bad” measure of uncertainty...

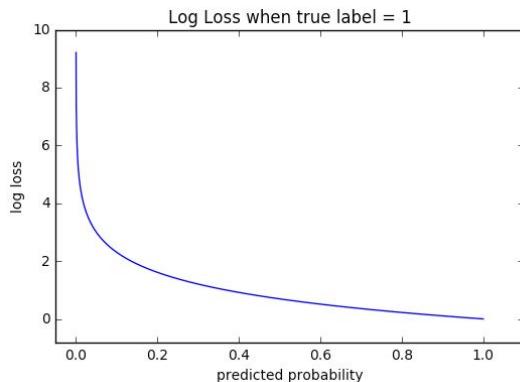


## Issues:

1. Inputs can get a very high logit even if projected into “unusual” feature regions.
2. Softmax is a relative function that encourages one-hot behaviour.

$$s(x_i) = \frac{e^{x_i}}{\sum_{j=1}^n e^{x_j}}$$

# Why Softmax often is a “bad” measure of uncertainty...



$$\text{Loss} = - \sum_{i=1}^{\text{output size}} y_i \cdot \log \hat{y}_i$$

## Issues:

1. Softmax is a relative function that encourages one-hot behaviour.
2. Inputs can get a very high logit even if projected into “unusual” feature regions.
3. Cross-entropy loss does not encourage expression of uncertainty

# How can we get **good** predictive uncertainty?

Deep Ensembles

Bayesian Neural Networks

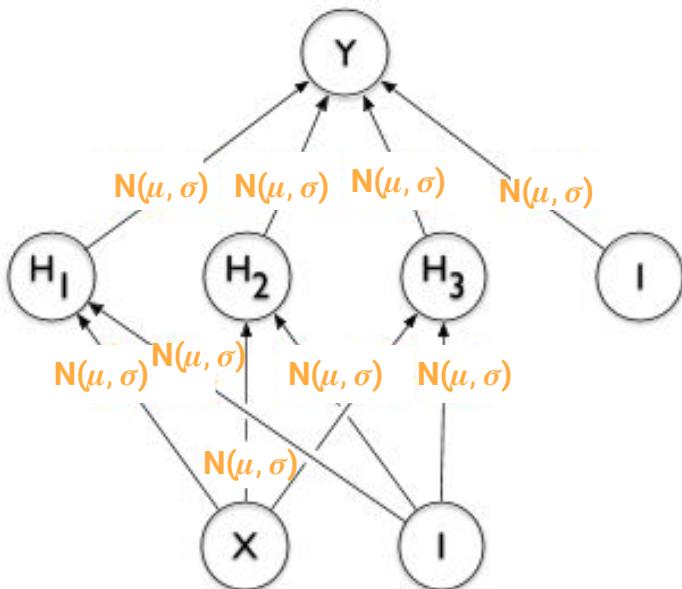
Learnt Uncertainty

Modelling “Normal”

Introspective Methods

# Bayesian Neural Networks

Output



Hidden Layer

Input

Approximate Bayesian Neural Network

(Image: Blundell et al., 2015)

Posterior

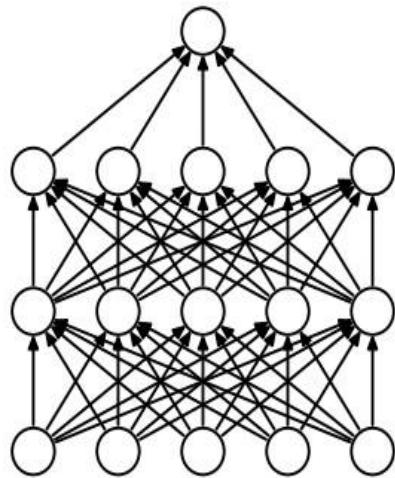
$$p(\omega | \mathbf{X}, \mathbf{Y}) = \frac{p(\mathbf{Y} | \mathbf{X}, \omega)p(\omega)}{p(\mathbf{Y} | \mathbf{X})}$$

Intractable

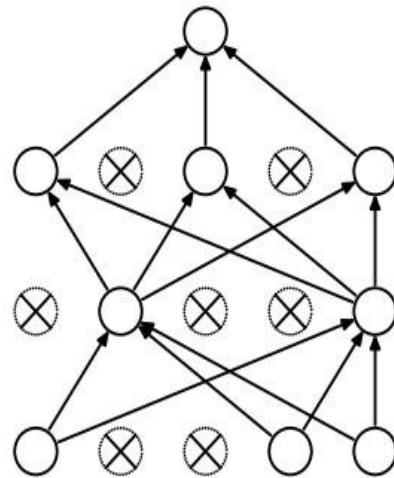
Approximate Posterior

Variational Inference

# Monte Carlo (MC) Dropout for Bayesian Neural Networks

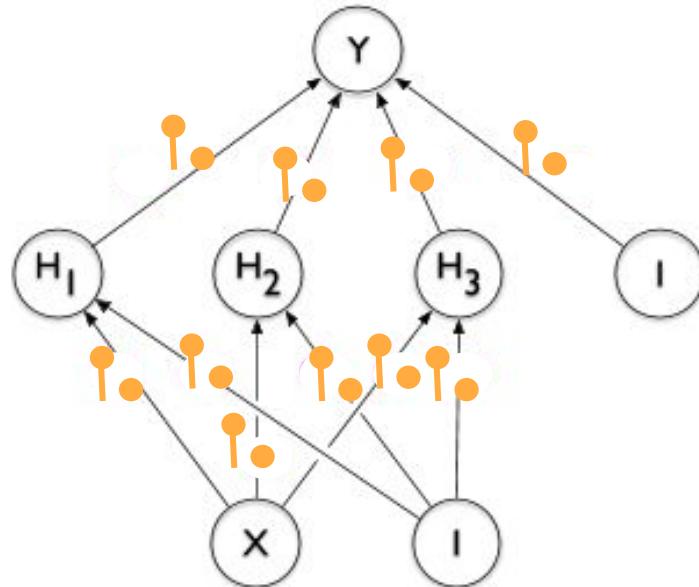


(a) Standard Neural Net

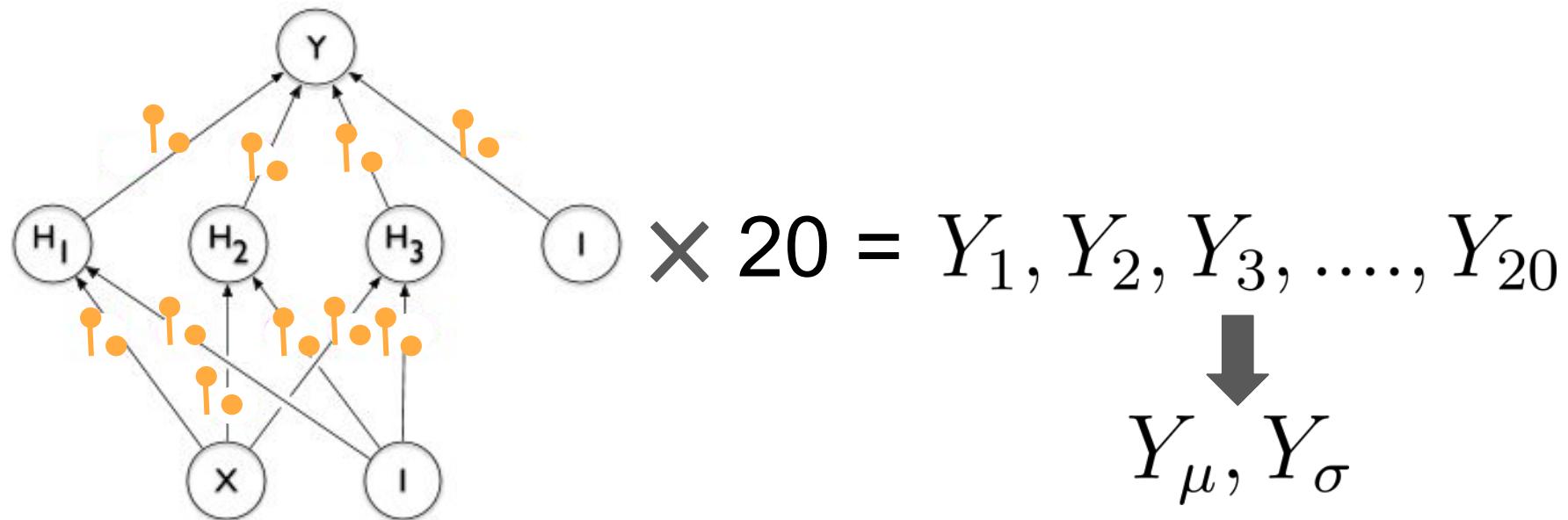


(b) After applying dropout.

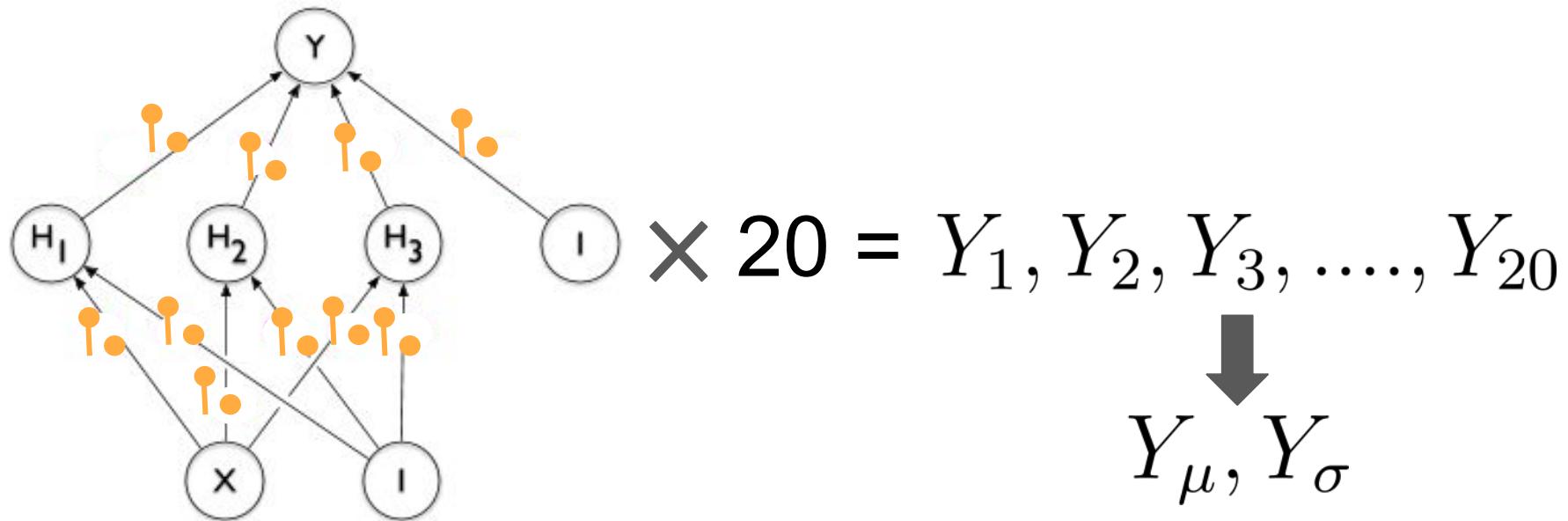
(Image: Srivastava et al., 2014)



# Monte Carlo (MC) Dropout for Bayesian Neural Networks



# Monte Carlo (MC) Dropout for Bayesian Neural Networks



See MC Layer Norm for a Transformer-friendly approximate Bayesian inference!

# Bayesian Neural Networks

## Pros:

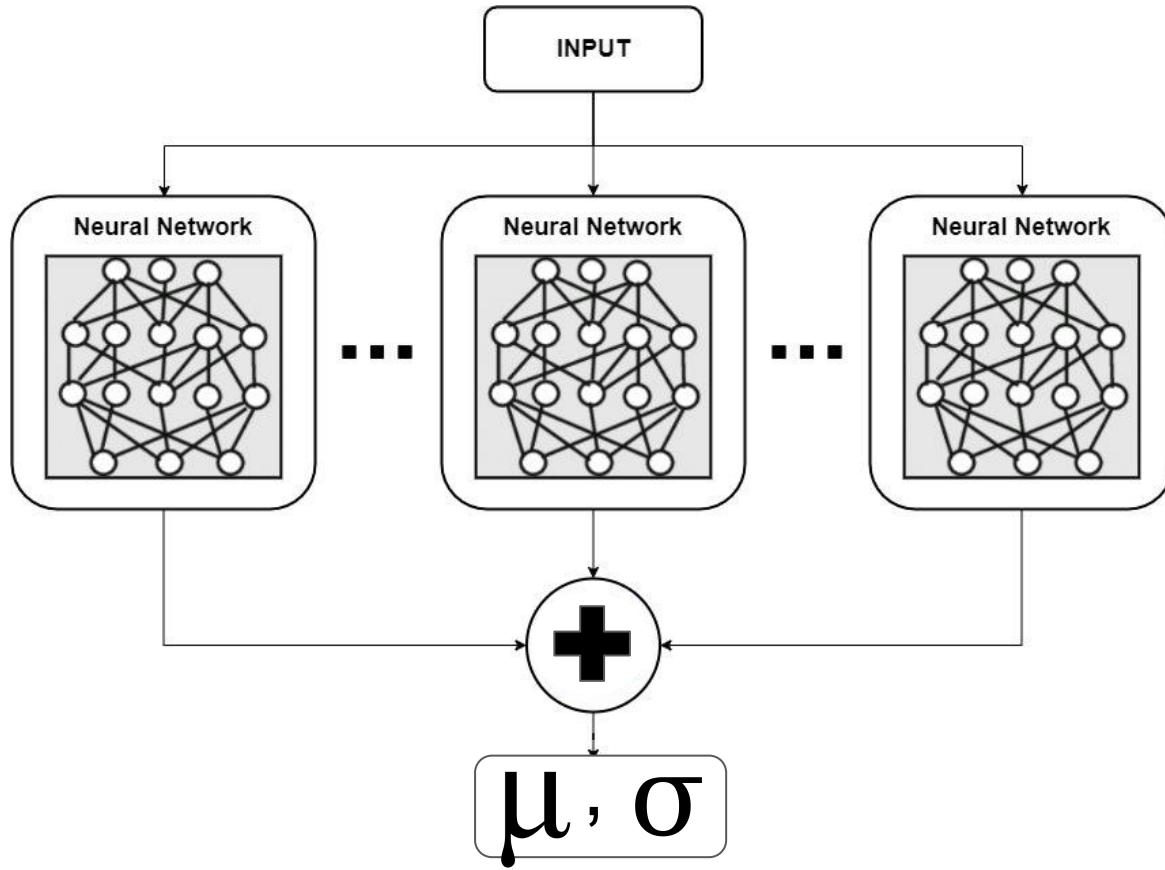
- Strong theoretical underpinnings
- Generally works well for epistemic uncertainty



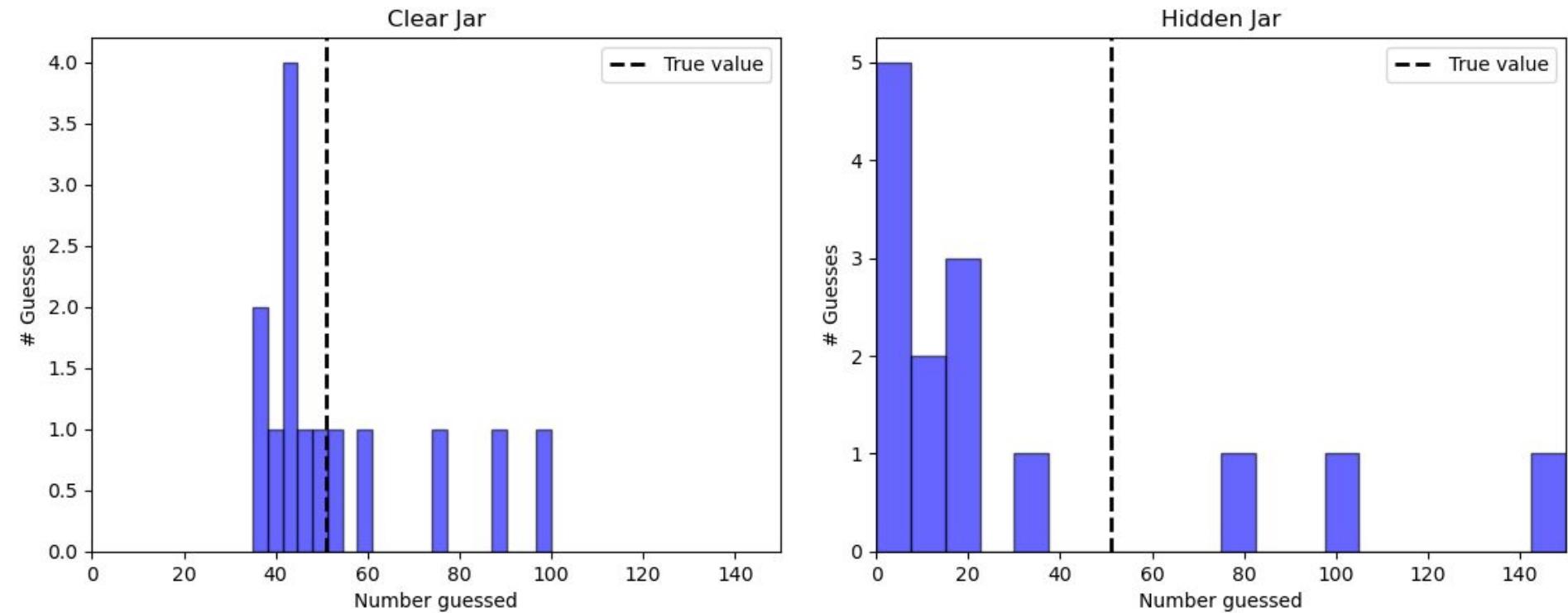
## Cons:

- Especially for large models, computationally expensive in terms of:
  - Number of parameters
  - Multiple forward passes
- Sometimes so many assumptions/approximations, doesn't actually work that well

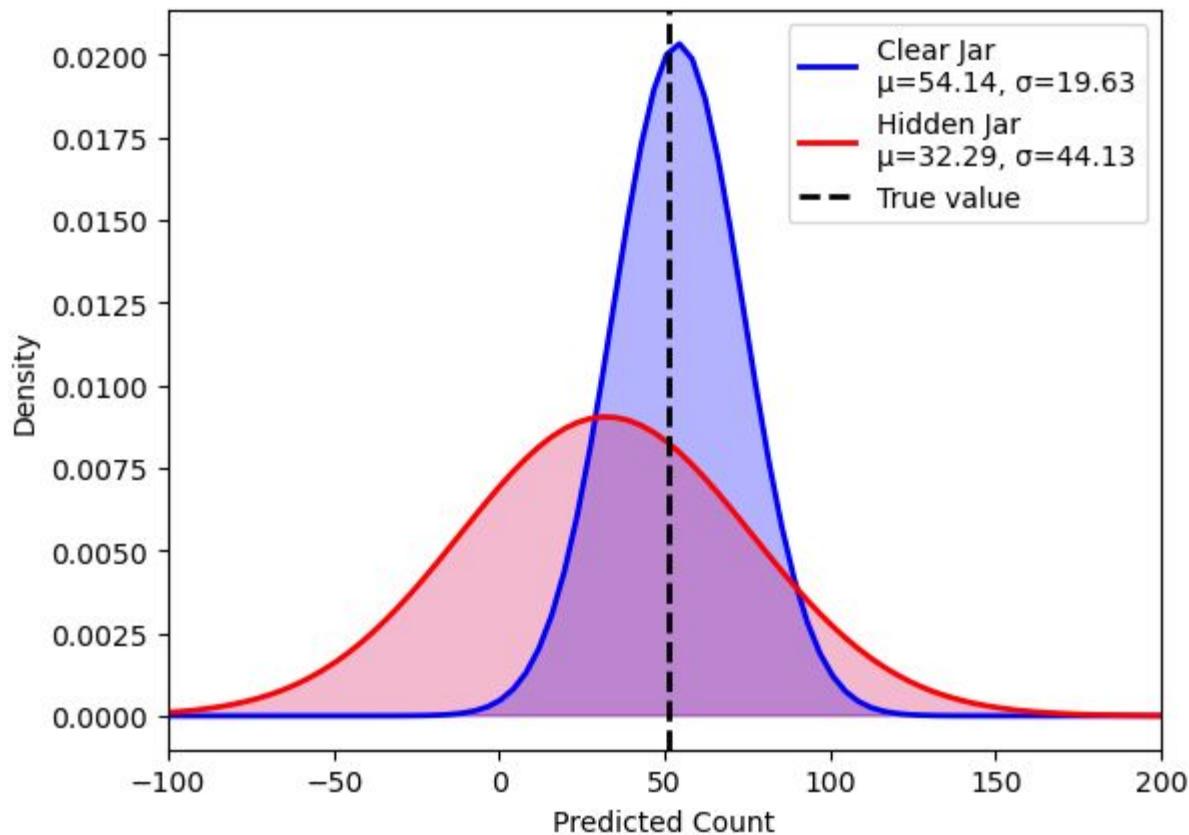
# Deep Ensembles



# Deep Ensembles: Human Edition



# Deep Ensembles: Human Edition



# Deep Ensembles

## Pros:

- Generally a strong baseline that “just works”
- Easy to implement
- Can capture epistemic and aleatoric uncertainty

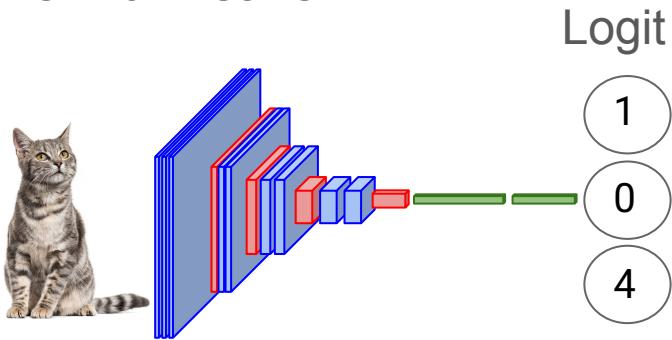


## Cons:

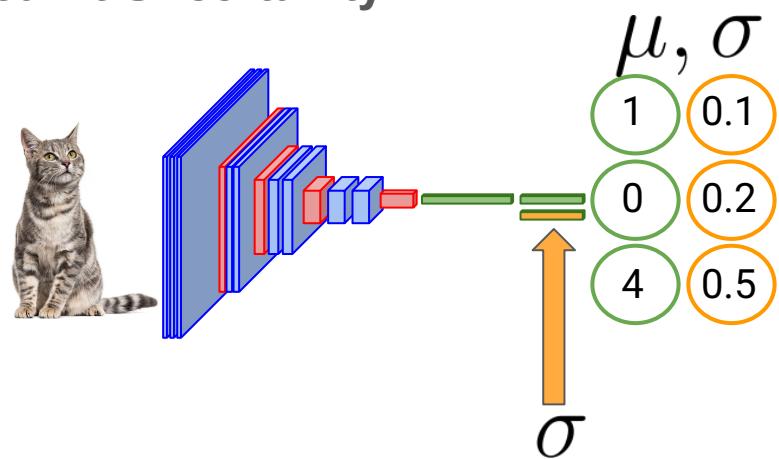
- Computationally expensive to
  - train multiple models
  - test an input through multiple models

# Learnt Uncertainty

Normal Network

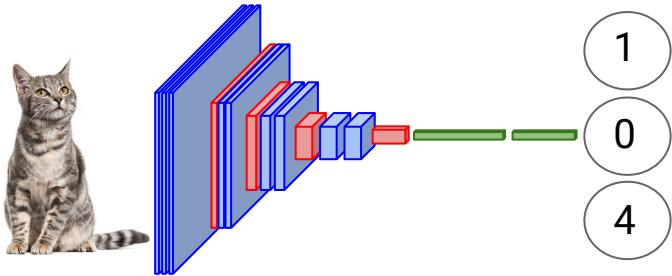


Learnt Uncertainty

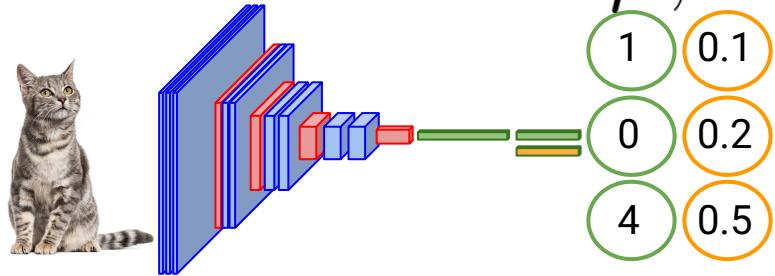


# Learnt Uncertainty

Normal Network



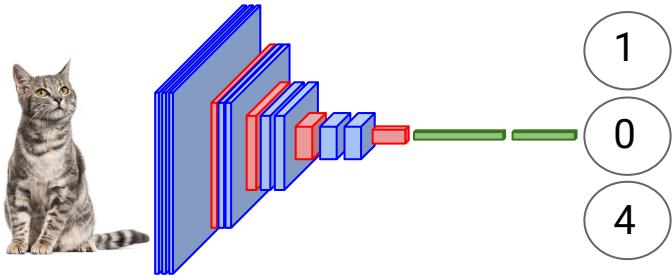
Learnt Uncertainty



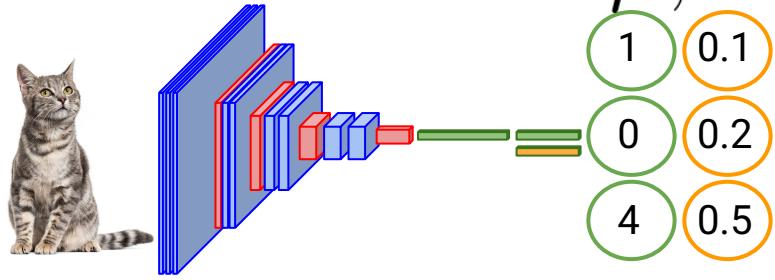
$$\mathcal{L} = \frac{||y - \mu||^2}{2\sigma^2} + \frac{1}{2}\log(\sigma^2)$$

# Learnt Uncertainty

Normal Network



Learnt Uncertainty

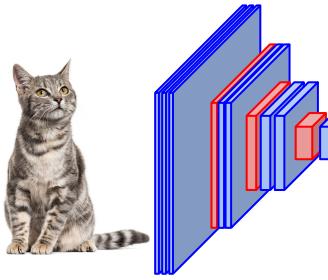


error in prediction

$$\mathcal{L} = \frac{\|y - \mu\|^2}{2\sigma^2} + \frac{1}{2}\log(\sigma^2)$$

# Learnt Uncertainty

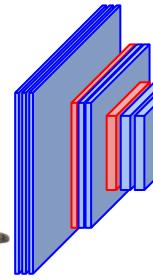
Normal Network



Logit

- 1
- 0
- 4

Learnt Uncertainty



- |               |     |
|---------------|-----|
| $\mu, \sigma$ |     |
| 1             | 0.1 |
| 0             | 0.2 |
| 4             | 0.5 |

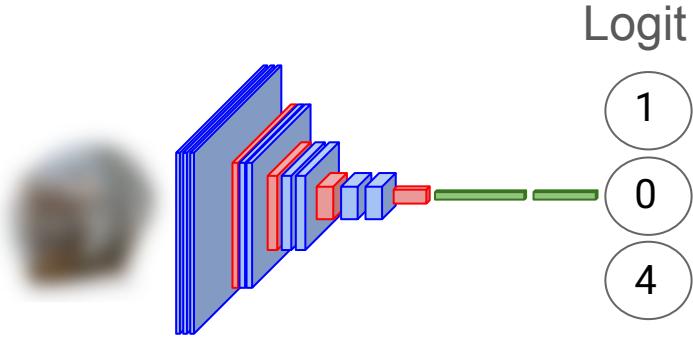
error in prediction

$$\mathcal{L} = \frac{\|y - \mu\|^2}{2\sigma^2} + \frac{1}{2}\log(\sigma^2)$$

reduce error      uncertainty penalty

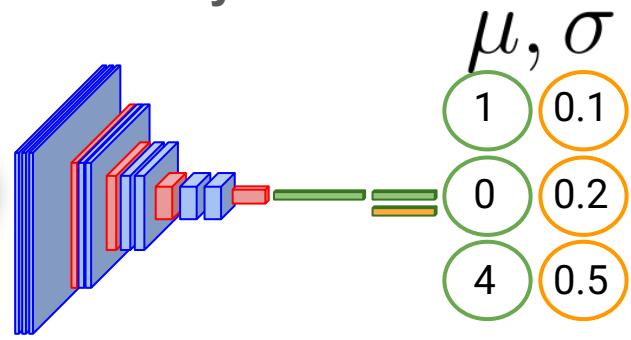
# Learnt Uncertainty

Normal Network



Logit

Learnt Uncertainty



error in prediction

$$\mathcal{L} = \frac{\|y - \mu\|^2}{2\sigma^2} + \frac{1}{2}\log(\sigma^2)$$

reduce error      uncertainty penalty

# Learnt Uncertainty

## Pros:

- Quick - single forward pass
- Can apply post-hoc to pre-trained network\*\*
- Associated with aleatoric uncertainty

## Cons:

- \*\*Performs best when network is trained alongside uncertainty head
- Sometimes doesn't reach same task performance – network can “opt out” of learning hard data by having high uncertainty

## Modelling “Normal”

Statistically model the expected distribution of the network outputs.  
At test time, flag abnormal network outputs.

## Introspective Methods

Monitor intermediate values inside the model  
(NOT just by observing outputs).

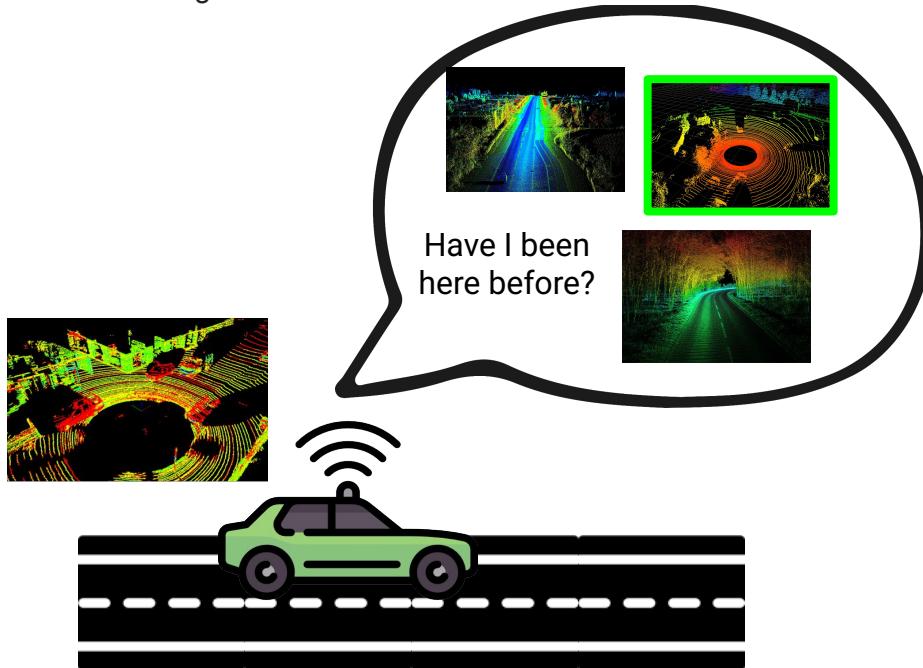
Which uncertainty technique is the best?

Some lessons learnt...

# LiDAR Place Recognition



Keita Mason, Joshua Knights, Milad Ramezani, Peyman Moghadam, and Dimity Miller. "Uncertainty-aware lidar place recognition in novel environments". In *2023 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*.



MC Dropout

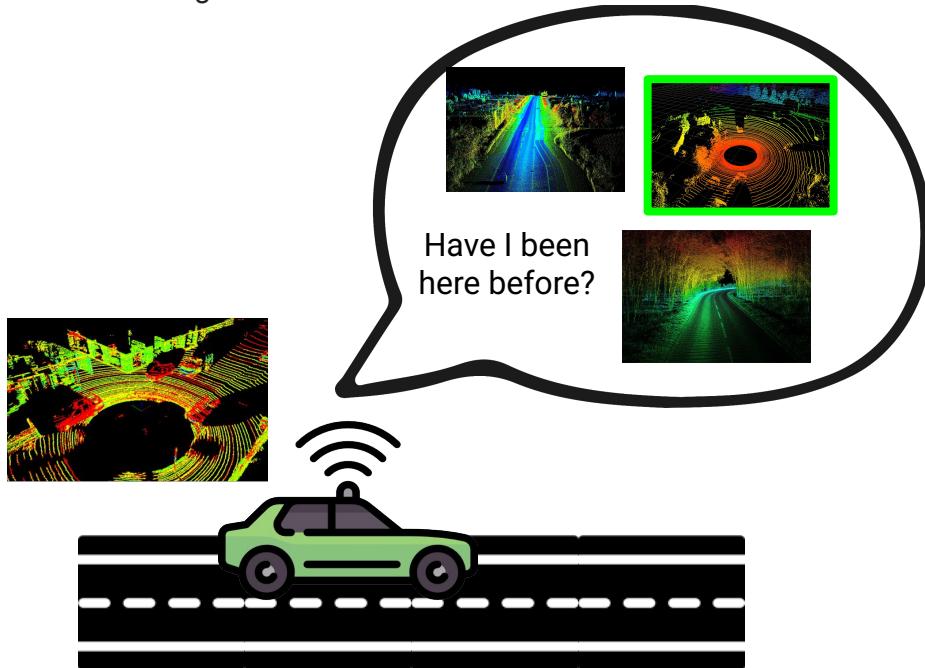
Learnt Uncertainty

Deep Ensembles

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MC Dropout



Learnt Uncertainty



Deep Ensembles



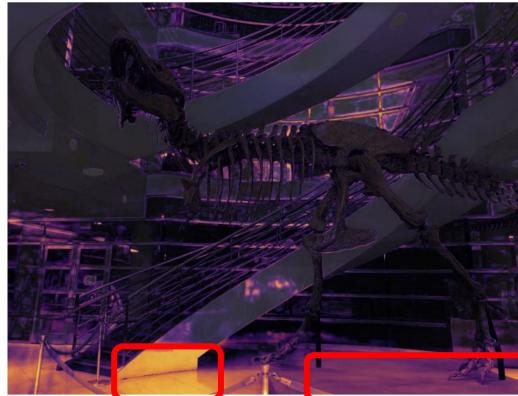
# NeRFs

Niko Suenderhauf, Jad Abou-Chakra, and Dimity Miller. "Density-aware NeRF Ensembles: Quantifying Predictive Uncertainty in Neural Radiance Fields". In *2023 IEEE International Conference on Robotics and Automation (ICRA)*.

Prediction



Error



Uncertainty



Deep Ensembles



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Prediction



Error



Uncertainty



Deep Ensembles



Introspection (Ours)



# Object detection with novel objects



Centre for  
Robotics

Dimity Miller, Niko Suenderhauf, Michael Milford and Feras Dayoub. "Uncertainty for Identifying Open-Set Errors in Visual Object Detection". In *IEEE Robotics and Automation Letters (RA-L)*, 2022.

Trained on:



Tested on:



MC Dropout

✓, 2.8fps

Deep Ensembles

✓, 4.6fps

# Object detection with novel objects



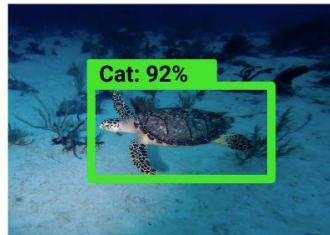
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Trained on:



Tested on:



MC Dropout



, 2.8fps

Deep Ensembles



, 4.6fps

Modelling “Normal” (Ours)



, 19.7fps



## Key take-aways:

1. Uncertainty is important, especially for robotics
2. There are different types of uncertainty
3. Softmax is NOT a probability and often not a useful estimate of uncertainty
4. The “best” uncertainty technique depends on the type of uncertainty AND characteristics of your problem