RVSS 2026

Part I: Modelling the world

Prof Tom Drummond

Building models of the world

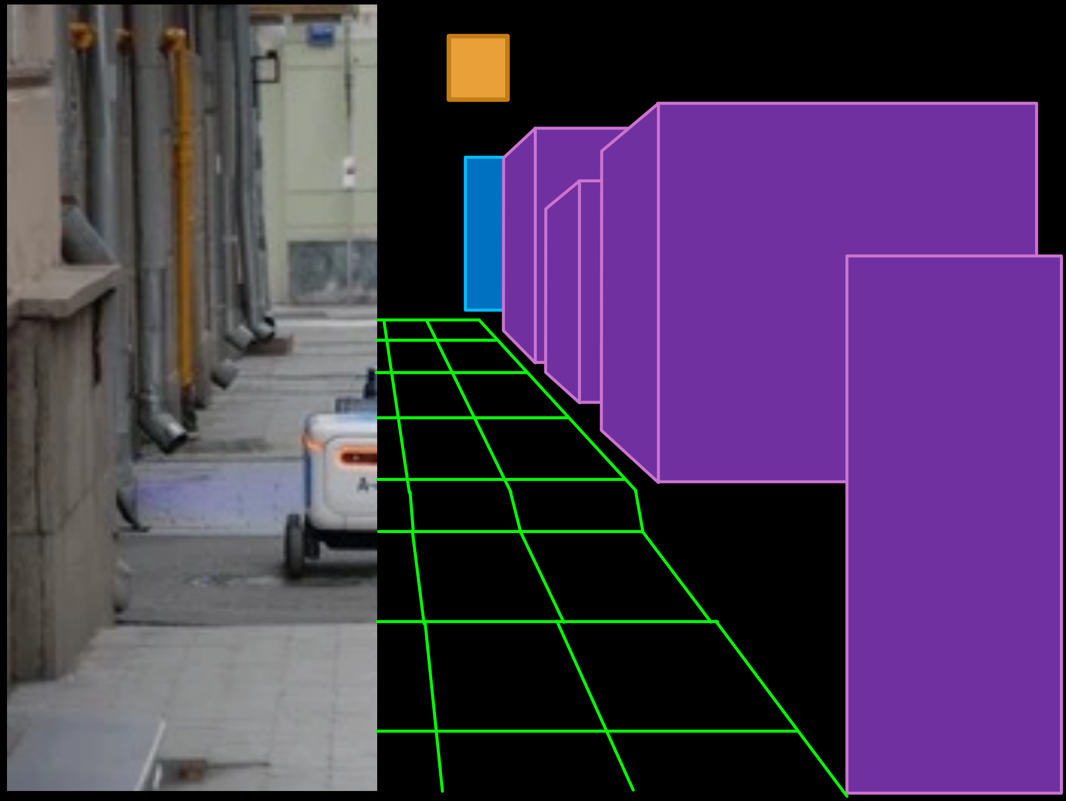
A model is some kind of digital representation of the world around the robot

BUILDING MODELS OF THE WORLD

A model is some kind of digital representation of the world around the robot

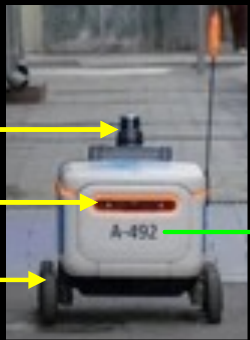
All models are wrong…

Why build models?



Building models of the world

Lidar
Vision
Odometry



Model

- Navigation
- Feedback for control
- Collision avoidance
- Prediction
- Planning

Vision = Graphics^{-1}

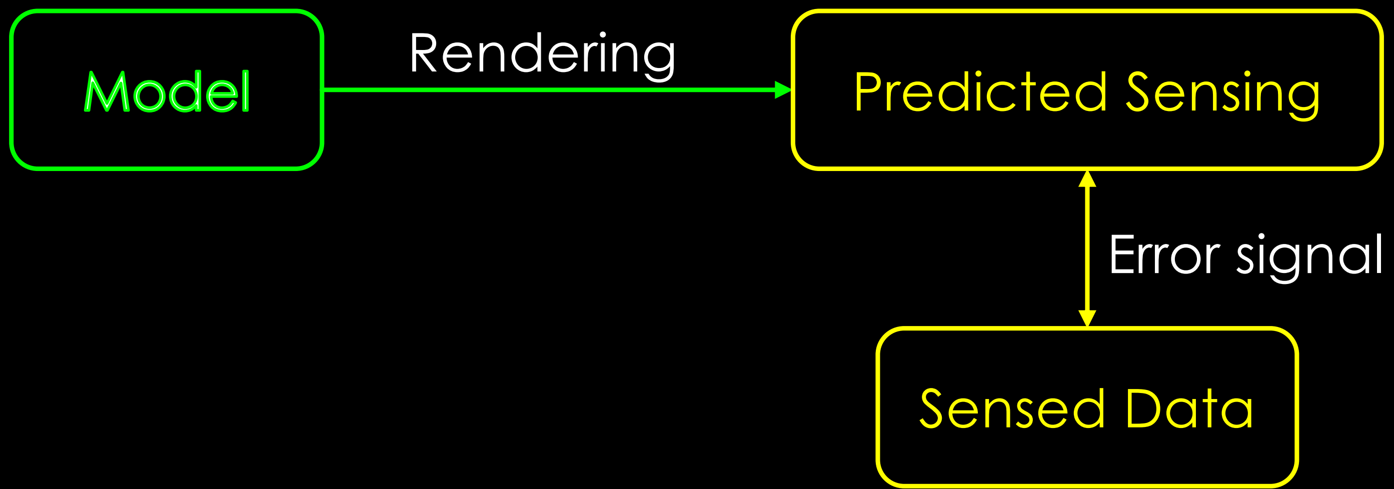
Modelling takes sensed data as input to create a model

Vision = Graphics^{-1}

Often build models by inverting the problem

Use current model to predict what we sense

Use the error to refine the model



Kinds of models

Models may contain:

- Geometry (where stuff is, usually 2D or 3D)
- Semantics (what stuff is)

Model Geometry can be:

- Sparse (just enough landmarks to compute robot motion)
- Dense (mostly complete representation of scene structure)

Model Semantics can represent:

- Stuff (continuous material: road surfaces, grass, rock, etc.)
- Things (discrete objects: cars, pedestrians, road furniture, etc.)

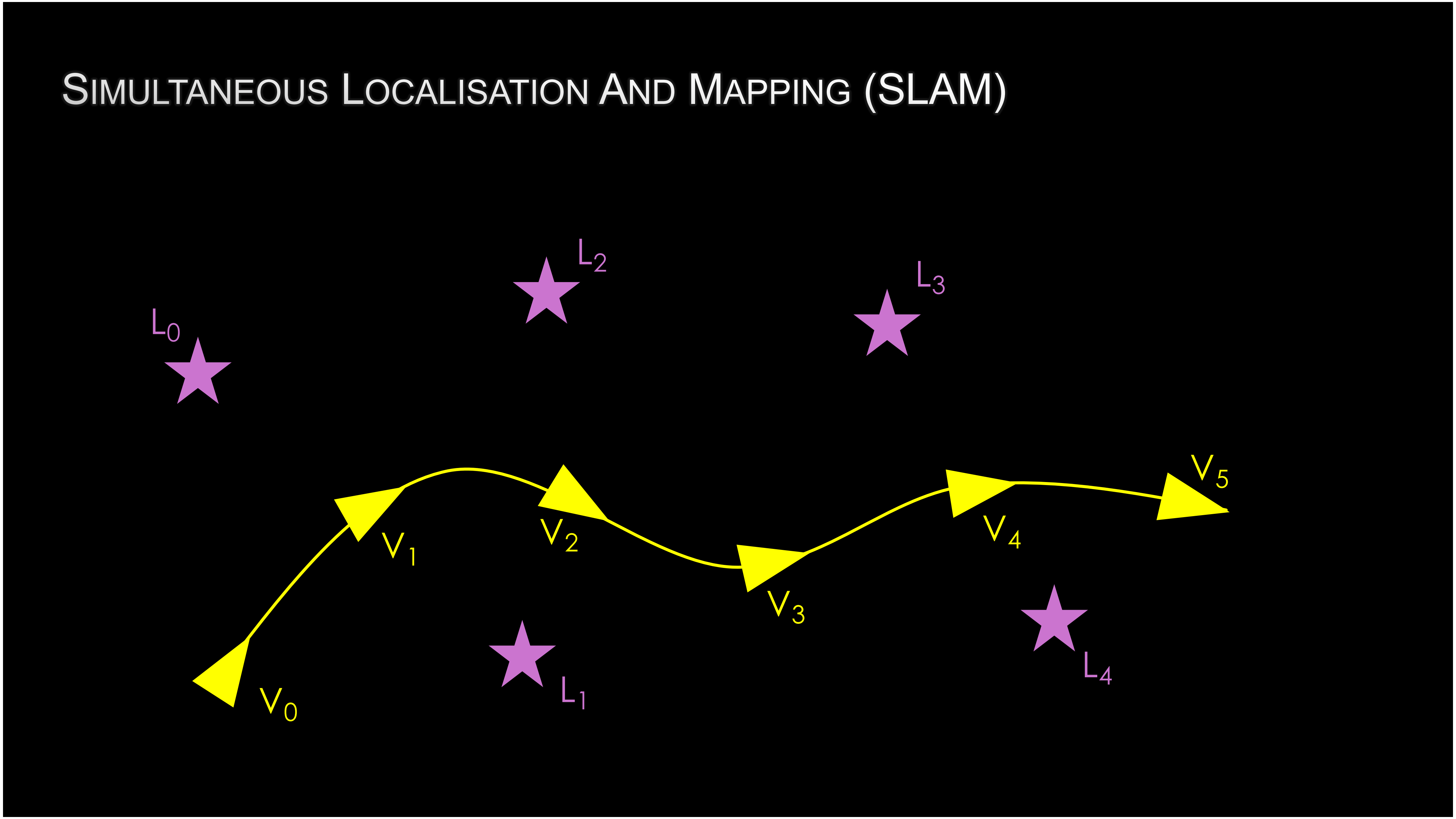
Kinds of models

Models can be used to:

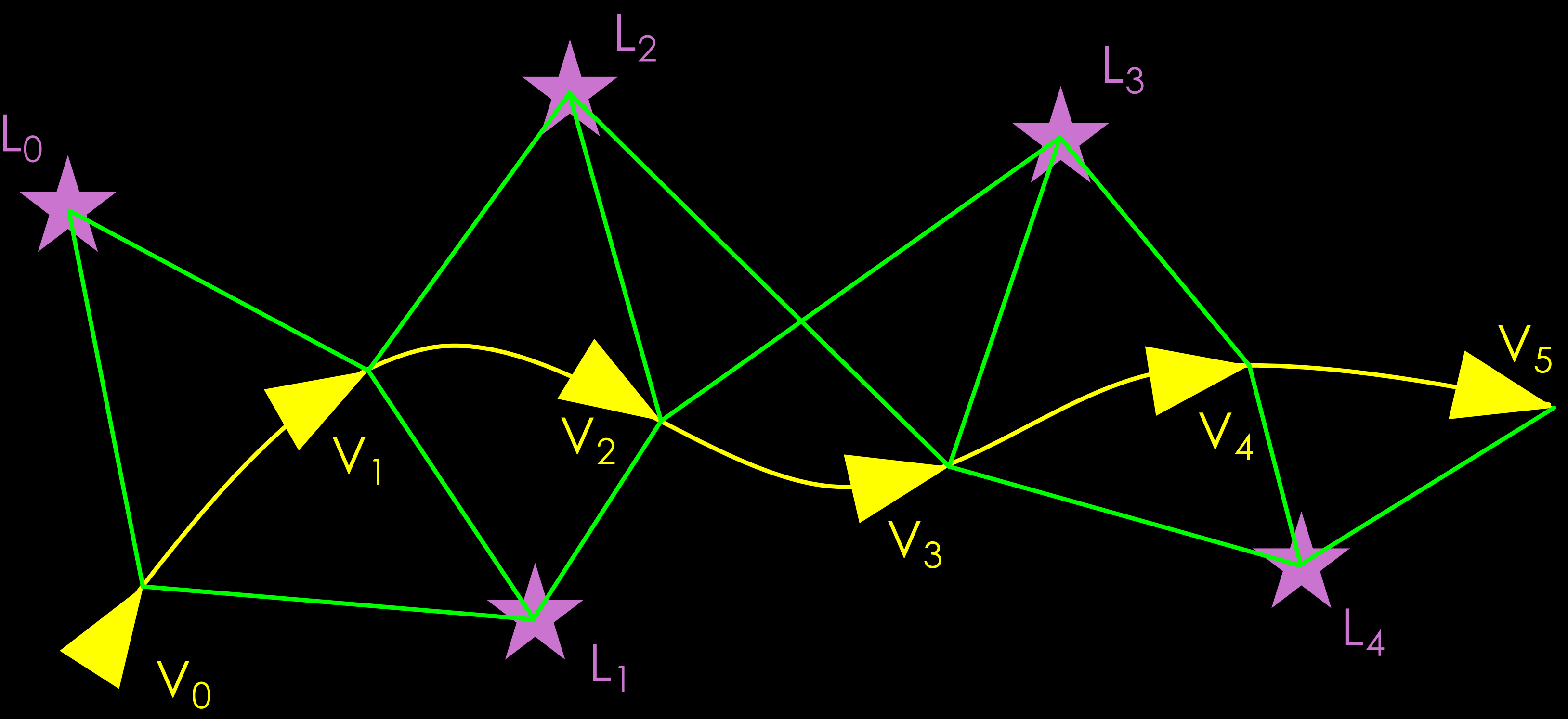
- Remember the *past* (e.g. build a map so we can navigate home)
- Represent the *present* (e.g. compute position for motion control)
- Predict the *future* (e.g. plan motion to avoid collisions with moving objects)

Models are built from:

- Point clouds (set of keypoints in space)
- + Appearance (what does each keypoint look like so we can recognize it)
- + Surface normal (what is the surface tangent around each point)
- Larger things: lines, planes, objects
- Meshes (usually triangulated, often with texture)
- Grids (2D or 3D): occupancy, signed distance function

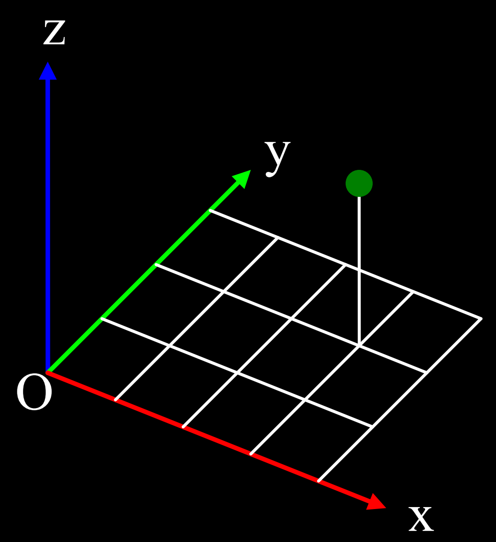

Simultaneous Localisation And Mapping (SLAM)
L0
L1
L2
L3
L4
V0
V1
V2
V3
V4
V5

SIMULTANEOUS LOCALISATION AND MAPPING (SLAM)



Point Cloud Models

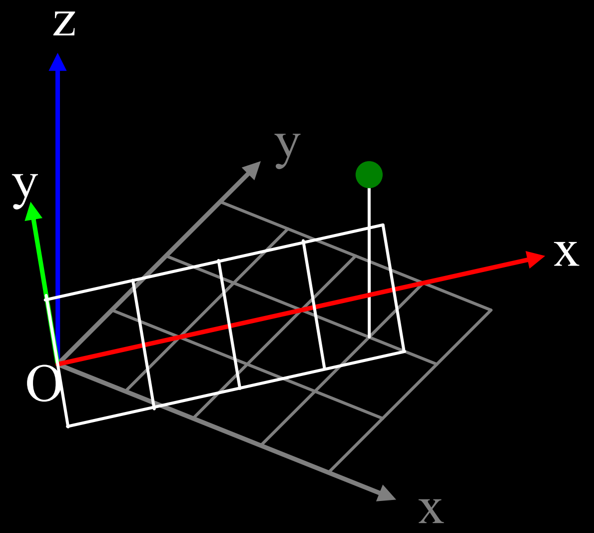
Position in space is represented using a vector for the x, y and z coordinates of a point:



For this to work, we have to agree where the origin is and the direction of each axis.

ROTATED AXES

If the axes point in different directions, then the point has different coordinates:



This rotation can be represented by a 3x3 matrix:

ROTATION MATRICES

Rotations in 3D are represented by 3x3 matrices

But not just any 3x3 matrix – what are the rules?

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

Lengths must be preserved:

Angles must be preserved:

PRESERVING LENGTHS

PRESERVING ANGLES

WHAT IS $R^T R$

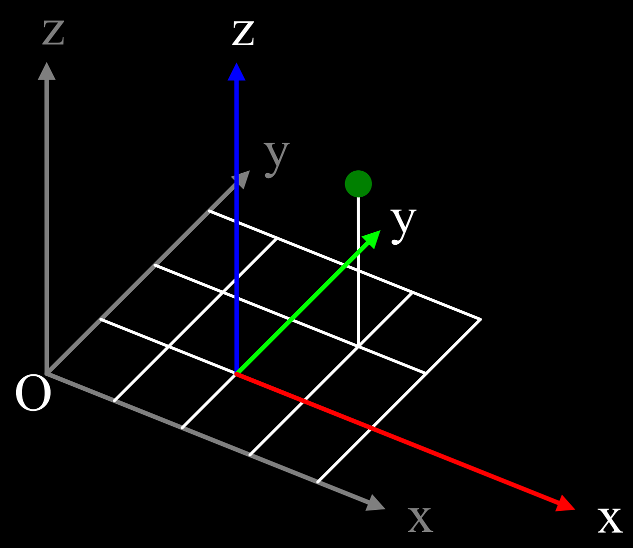
THE MATRICES FOR ROTATIONS ABOUT X, Y AND Z AXES ARE VERY SIMPLE:

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$

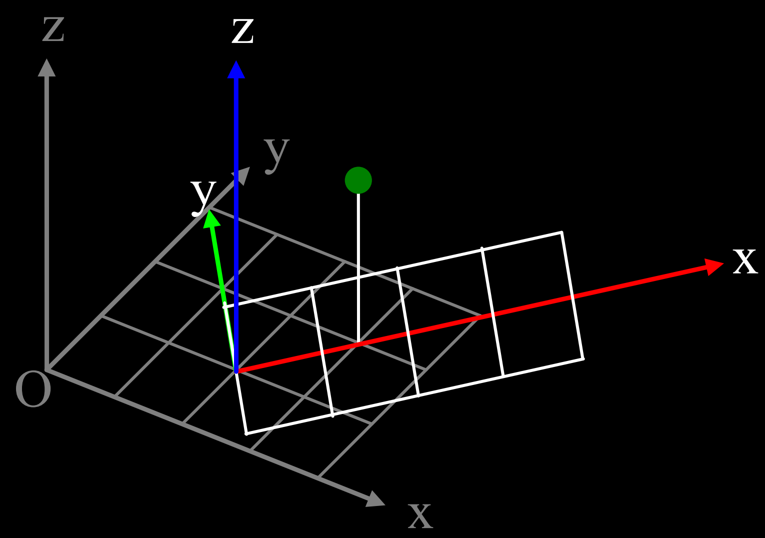
$$R_x = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

IF WE WANT TO PUT THE ORIGIN IN A DIFFERENT PLACE:



This shift (translation) can be represented by adding a vector:

WE CAN MOVE THE ORIGIN AND ROTATE THE FRAME AT THE SAME TIME



This can be represented using a rotation matrix and a vector:

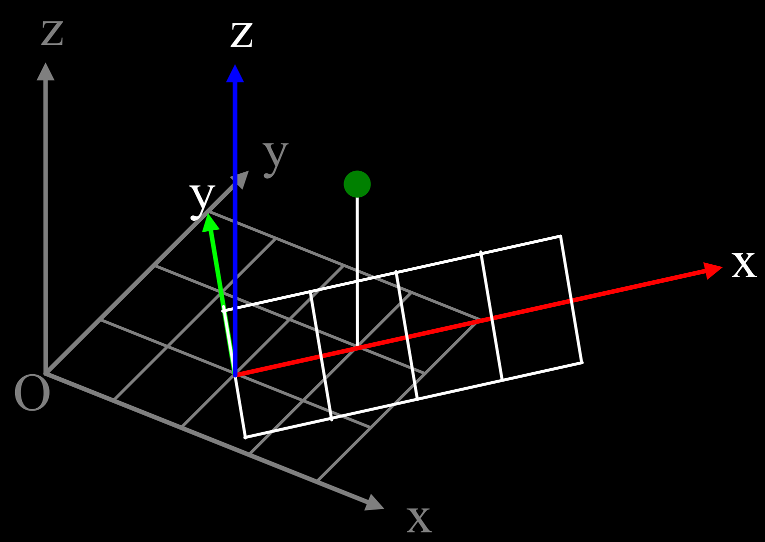
HOMOGENEOUS COORDINATES

POSITION VECTORS CAN BE EXTENDED TO 4 DIMENSIONS BY ADDING A 1 AT THE END. THIS REPRESENTATION OF POSITION IS CALLED *HOMOGENEOUS COORDINATES*.

THIS LETS US USE MATRIX MULTIPLICATION TO APPLY A TRANSLATION:

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} =$$

HOMOGENEOUS COORDINATES



$$\begin{bmatrix} 0.707 & 0.707 & 0 & -2.121 \\ -0.707 & 0.707 & 0 & 0.707 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 3 \\ 2 \\ 2 \\ 1 \end{pmatrix} =$$

$$M=\begin{bmatrix} \begin{bmatrix} & & \\ & R & \\ & & \end{bmatrix} & \begin{matrix} | \\ t \\ | \end{matrix} \\ \begin{matrix} 0 & 0 & 0 \end{matrix} & 1 \end{bmatrix}$$

$$M^{-1} =$$

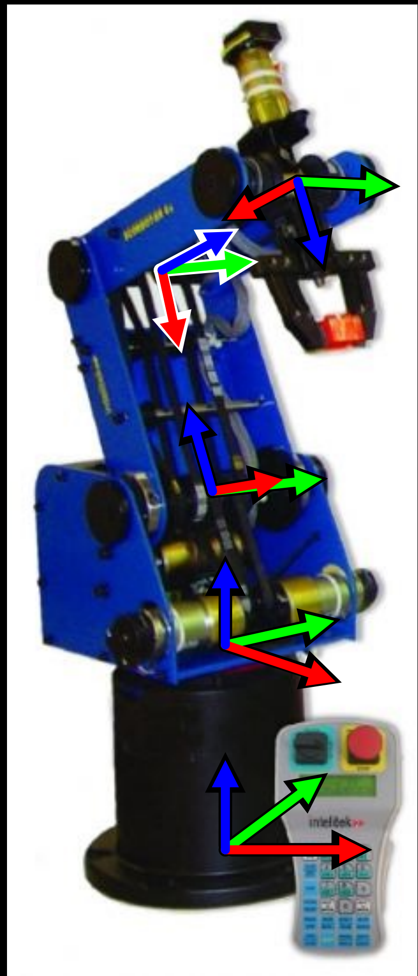
WHY DOES THIS MATTER?

THE ROTATION MATRIX AND THE TRANSLATION VECTOR DESCRIBE THE CHANGE IN COORDINATE FRAME.

THEY TELL US HOW TO CONVERT THE VECTOR THAT TURNS THE LOCATION OF A POINT IN ONE COORDINATE FRAME INTO THE VECTOR THAT GIVES THE LOCATION OF *THE SAME POINT* IN ANOTHER COORDINATE FRAME

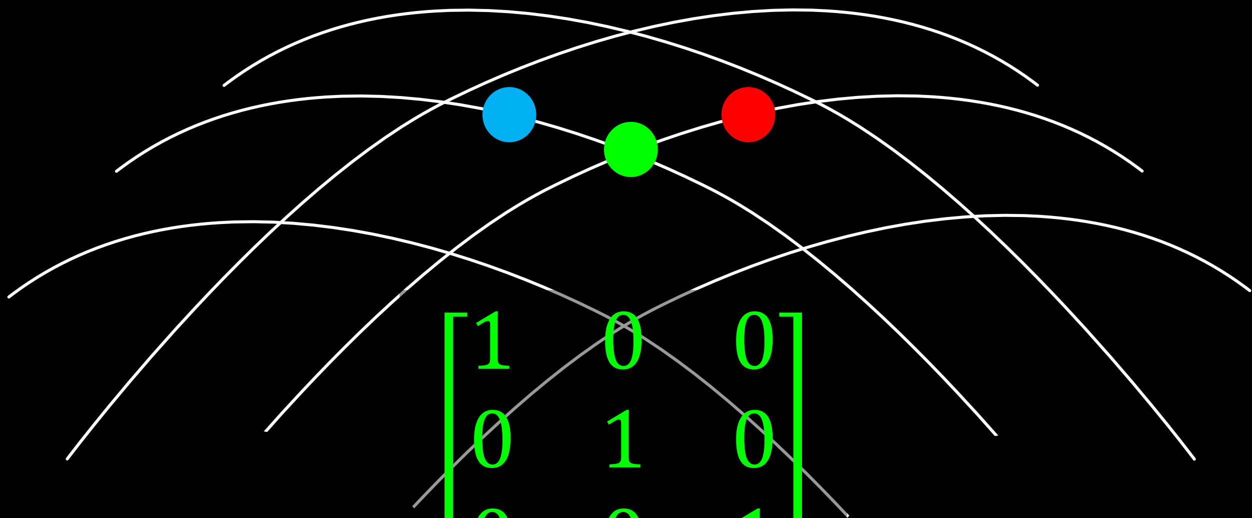
IN ROBOTICS WE OFTEN HAVE *LOTS* OF COORDINATE FRAMES

ROBOTS HAVE LOTS OF COORDINATE FRAMES



ROTATION MATRICES MAKE A 3D “SURFACE” IN 9D SPACE

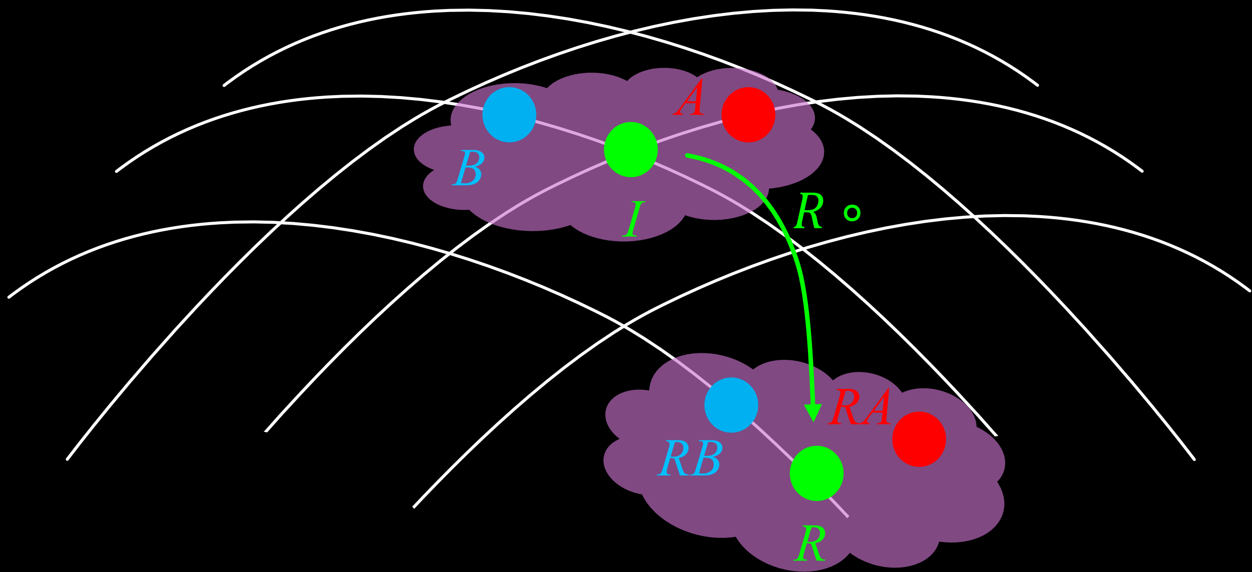
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.98 & -0.17 \\ 0 & 0.17 & 0.98 \end{bmatrix} \qquad \begin{bmatrix} 0.98 & -0.17 & 0 \\ 0.17 & 0.98 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

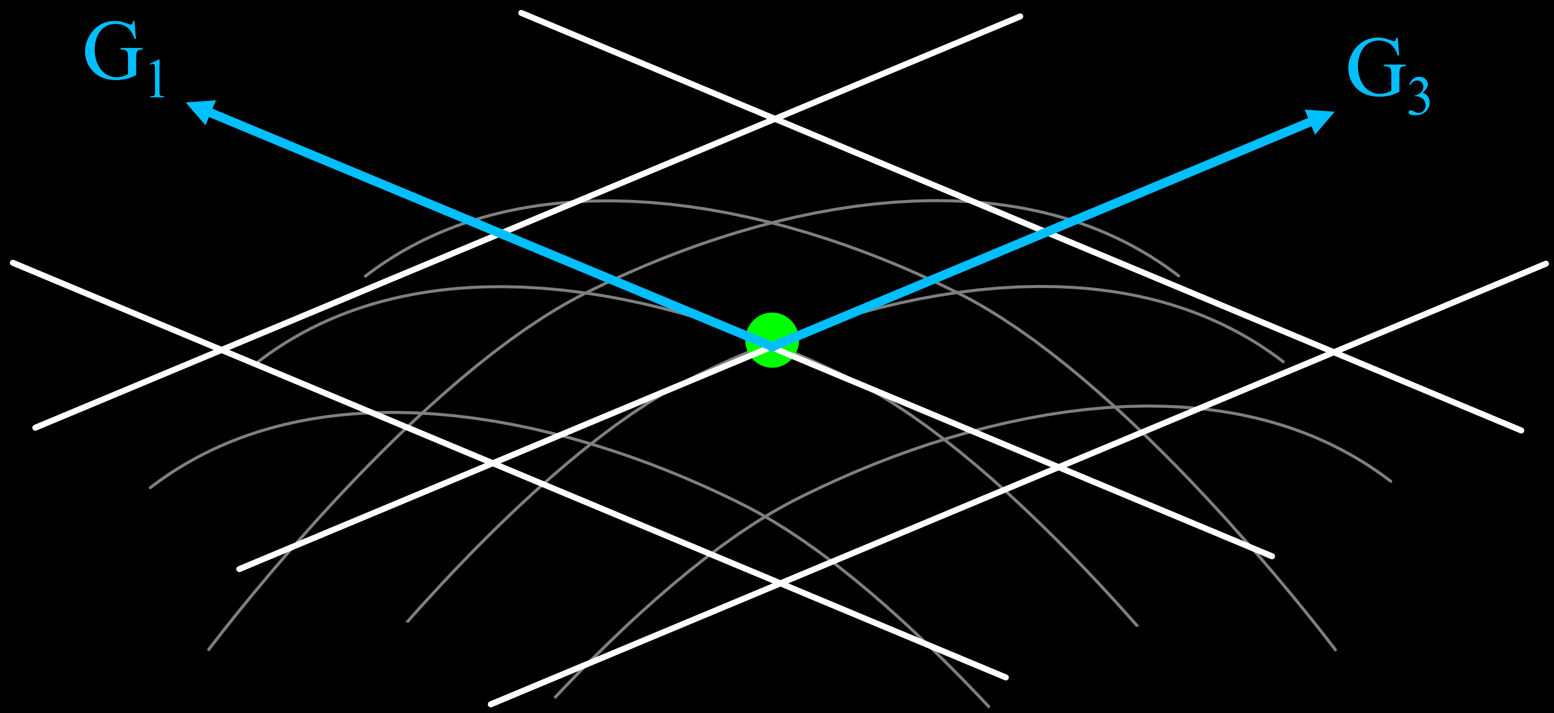
We can learn everything we need to know about the whole manifold by looking at the neighbourhood of the Identity matrix

WHY DO WE CARE SO MUCH ABOUT THE IDENTITY?



Because a neighbourhood of the Identity can be mapped to a neighbourhood of R by left multiplying by R

CAN FIT A TANGENT TO THE SURFACE AT THE IDENTITY



The tangent space of rotations is a 3D vector space

The basis axes are called generator matrices (G_1, G_2 , G_3)

Can add (infinitesimally) small amounts of these on to Identity and still have a rotation matrix

ROTATION MATRICES NEAR IDENTITY

Add small values to each element of the Identity matrix

$$R = \begin{bmatrix} 1+a & b & c \\ d & 1+e & f \\ g & h & 1+i \end{bmatrix} \qquad a, b, \ldots, i \quad \text{are small}$$

But $RR^T = I$

$$RR^T = \begin{bmatrix} 1+2a+a^2+b^2+c^2 & d+ad+b+be+cf & g+ga+bh+c+ci \\ d+ad+b+be+cf & d^2+1+2e+e^2+f^2 & dg+h+eh+f+fi \\ g+ga+bh+c+ci & dg+h+eh+f+fi & g^2+h^2+1+2i+i^2 \end{bmatrix} = I$$

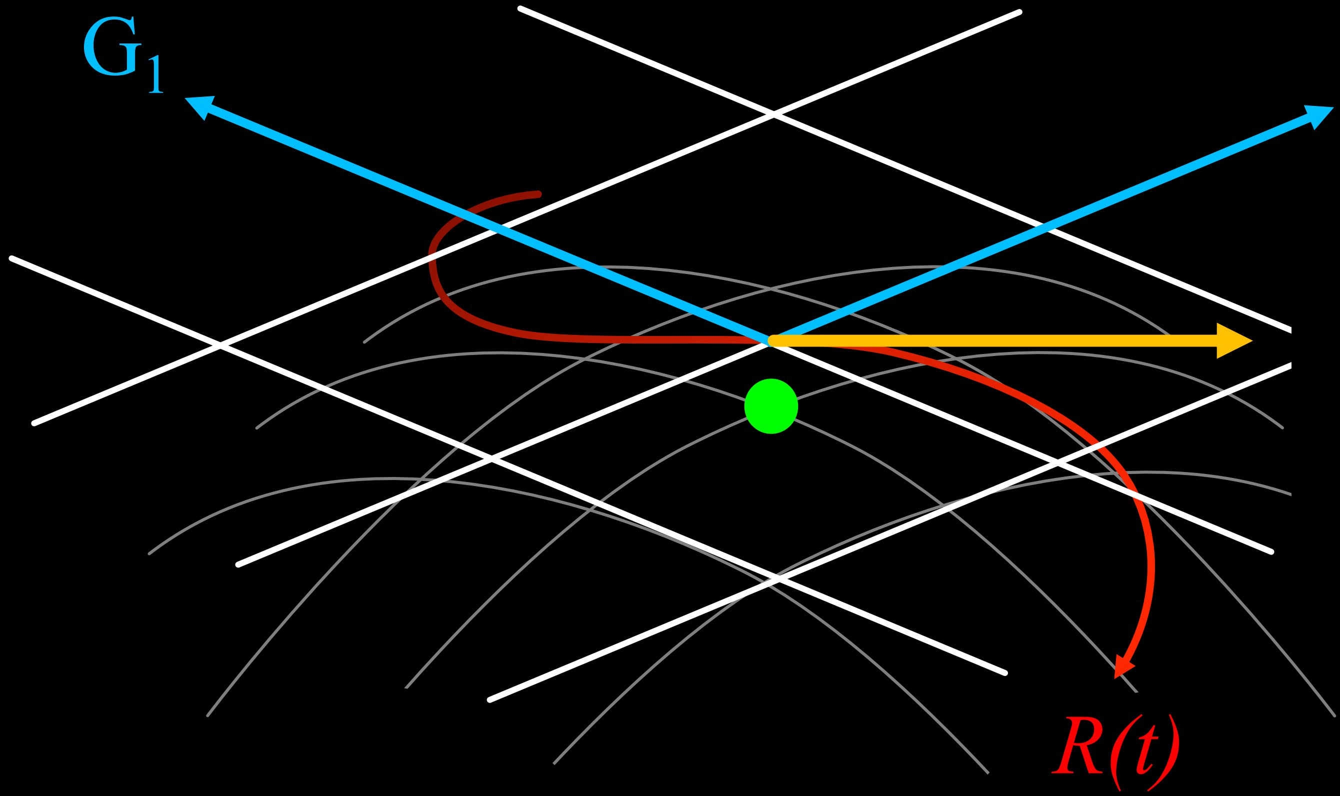
GENERATORS ARE A BASIS FOR THE TANGENT SPACE

$$G_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

TANGENT SPACE IS ALSO THE SPACE OF DERIVATIVES

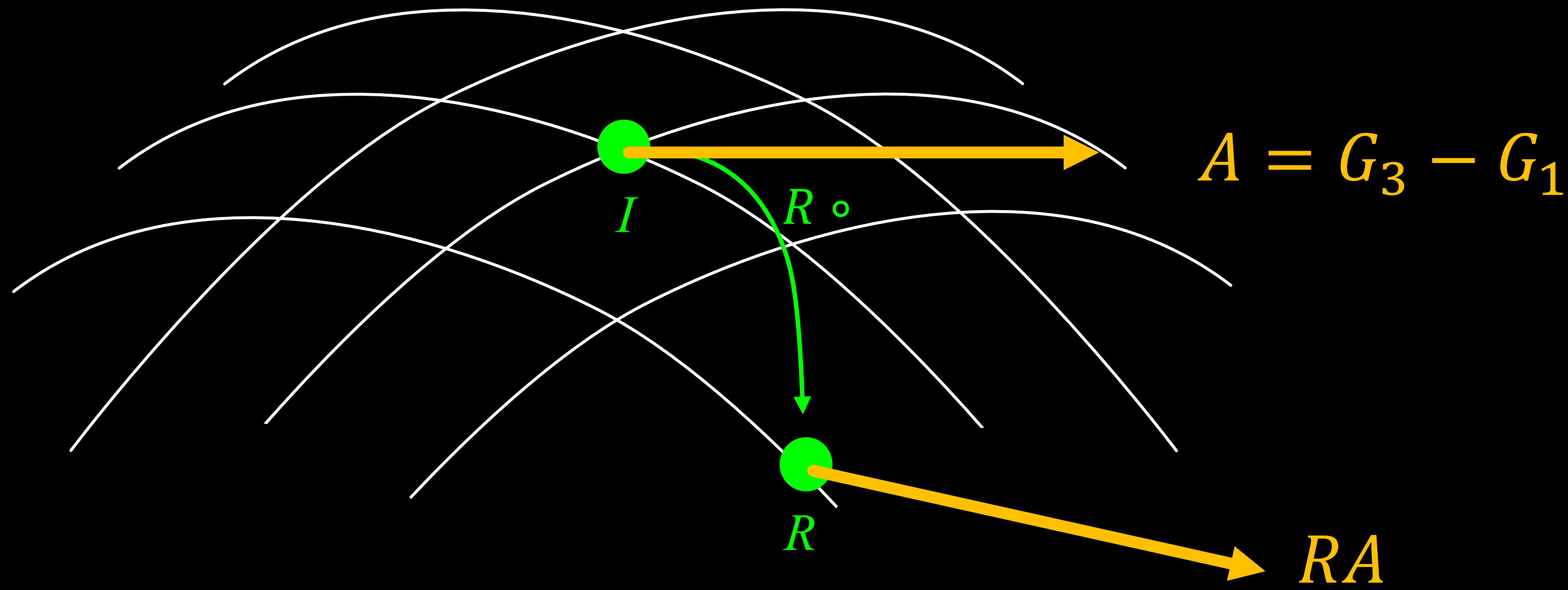


$$\frac{dR}{dt} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$= G_3 - G_1$$

Derivatives are linear combinations of Generators
(any anti-symmetric matrix)

GROUP ELEMENTS CAN MOVE DERIVATIVES TOO

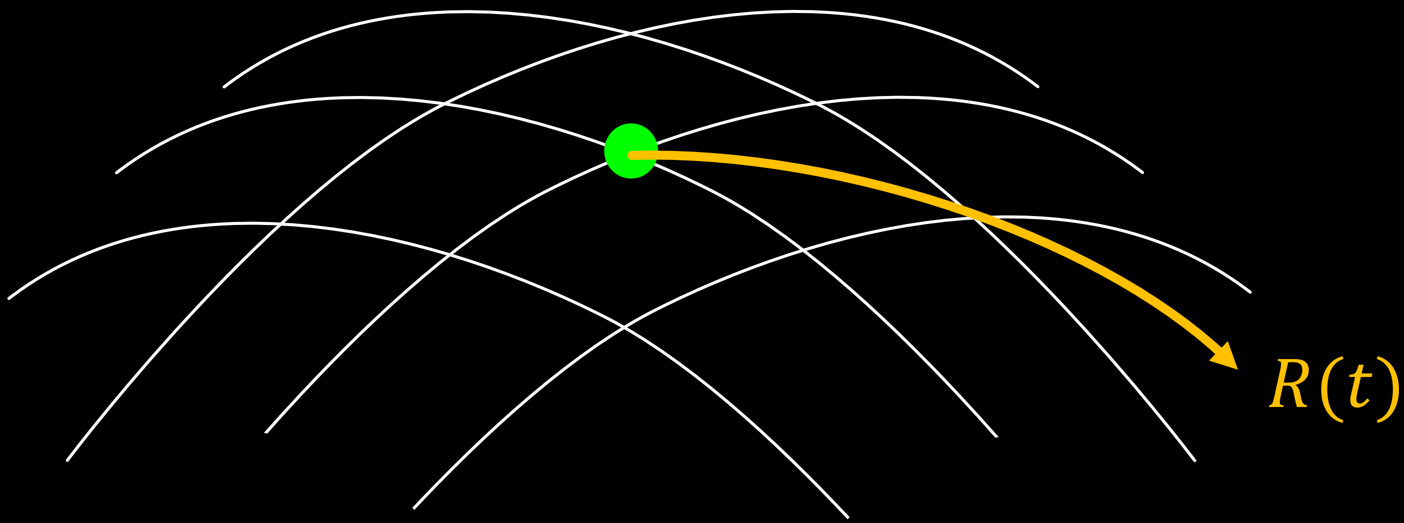


WALKING IN A STRAIGHT LINE (GEODESIC)

Start at the Identity ($R(0) = I$)

Choose a direction (derivative = A)

Move the derivative with us as we walk (derivative = RA)



$$\frac{dR}{dt} = RA$$

SIMPLE EXAMPLE:

$$e^{\begin{bmatrix} 0 & -\theta \\ \theta & 0 \end{bmatrix}} =$$

EXPONENTIALS OF MATRICES

This can be interpreted using a different expansion for exp

$$e^A = \lim_{n \to \infty} \left(I + \frac{A}{n} \right)^n$$

As n becomes large M/n becomes infinitesimal

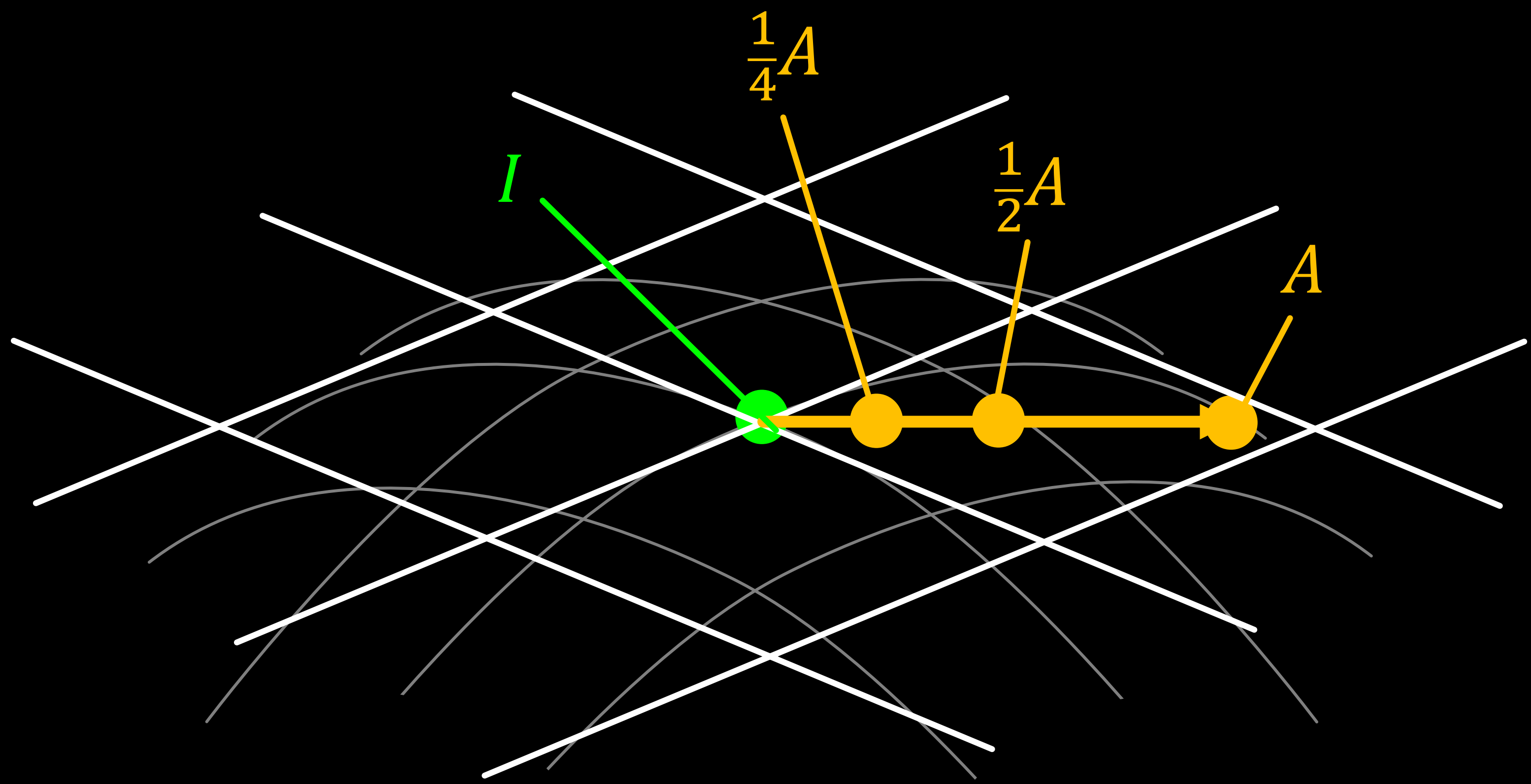
So I+M/n becomes closer to a rotation matrix

(nearer to Identity in tangent plane)

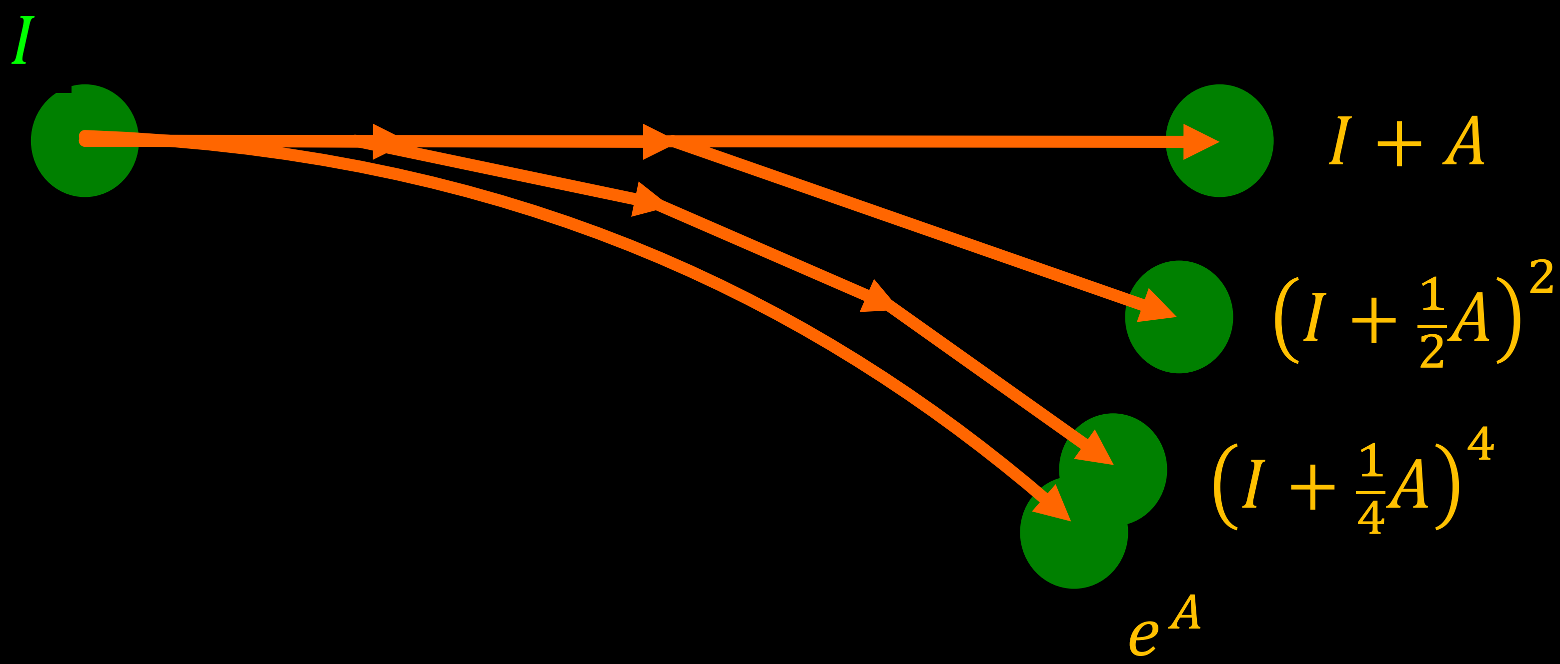
Raising to power n is just multiplying lots of rotations together

EXPONENTIALS OF MATRICES

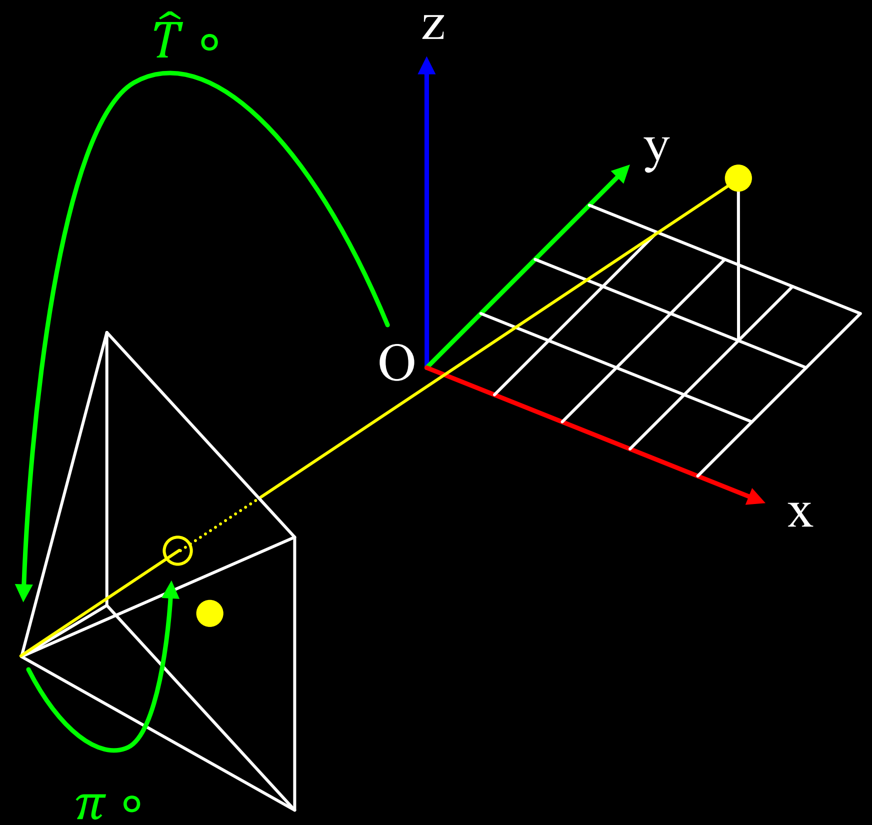
As smaller fractions of M are taken, we get closer to the manifold of rotations



EXPONENTIALS OF MATRICES

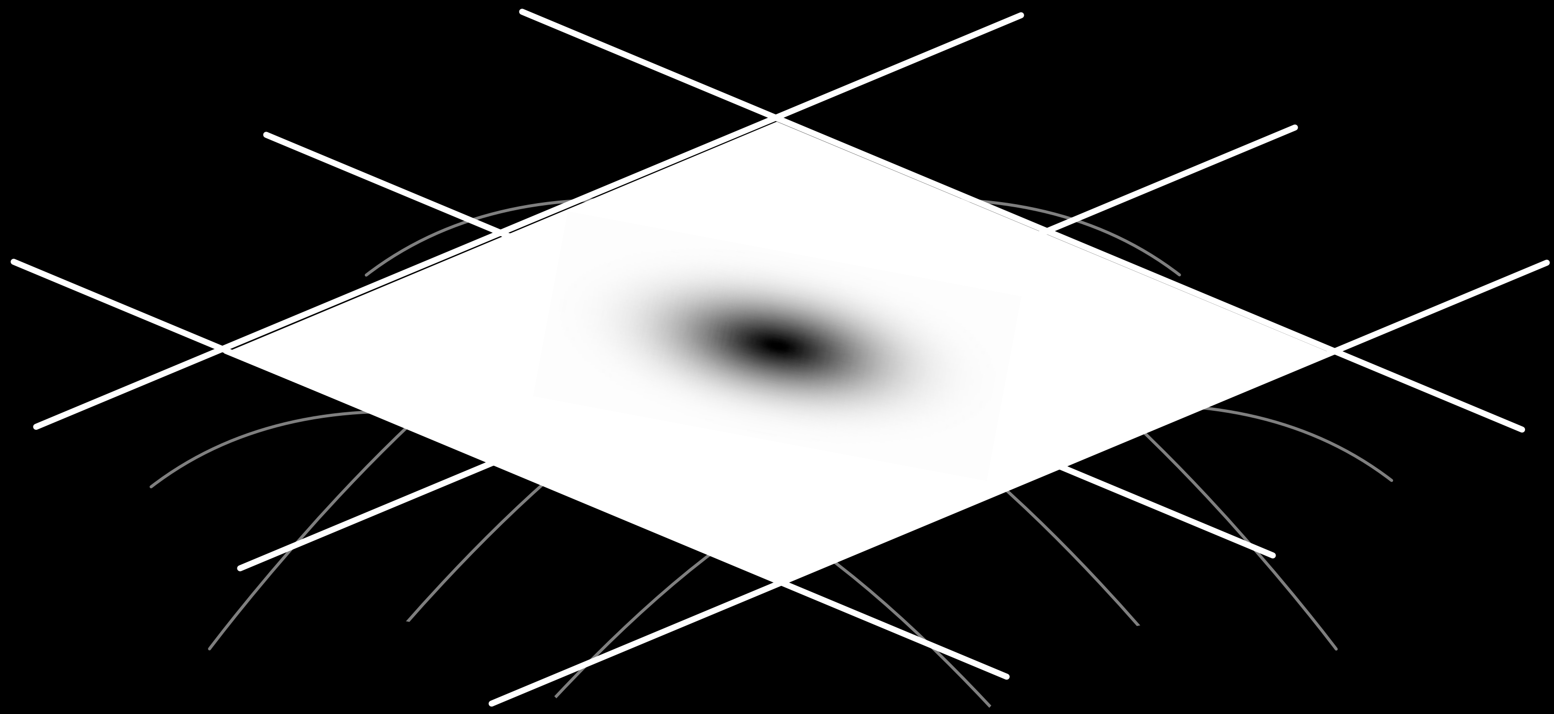


HOW DO WE USE THIS?



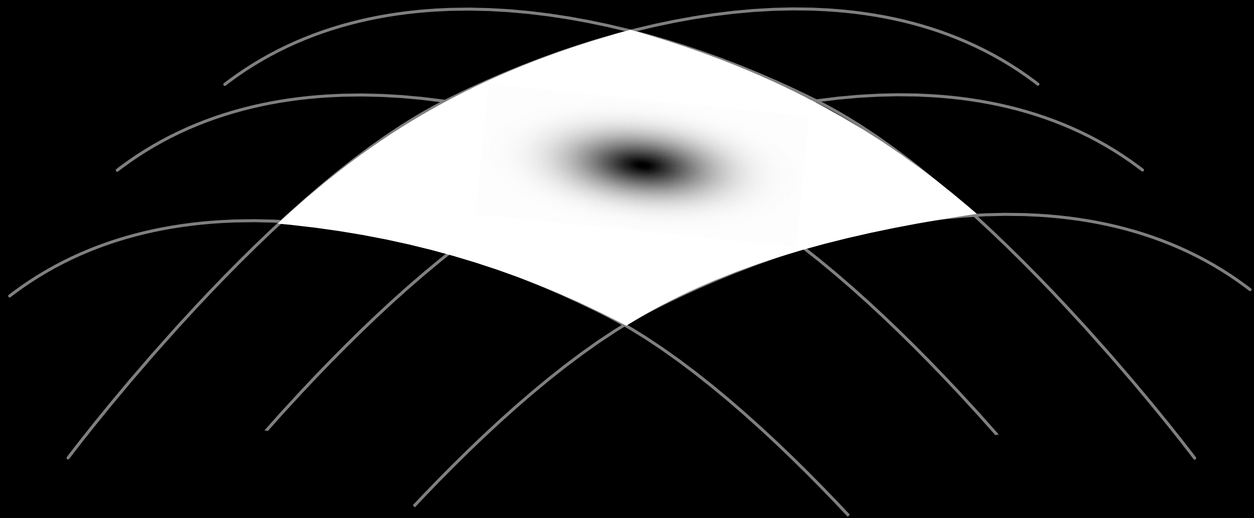
WHAT ABOUT UNCERTAINTIES?

CAN PLACE A GAUSSIAN PDF ON THE TANGENT SPACE…



EXPONENTIAL MAP PROJECTS ONTO THE GROUP

Draw samples from the tangent space, apply exponential map to each sample



AND MOVE THE CENTRE OF THE PDF BY LEFT (OR RIGHT) MULTIPLYING BY SOME GROUP ELEMENT