Lecture A2: Cameras and Image Formation

Peter Corke

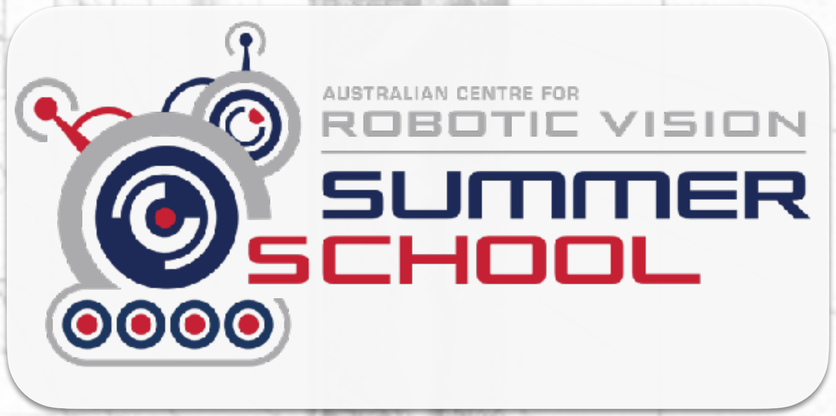


Image formation: from 3D to 2D

Cave Paintings ~40,000 years ago

Ideal City (1470)

Piero della Francesca (1415–1492)

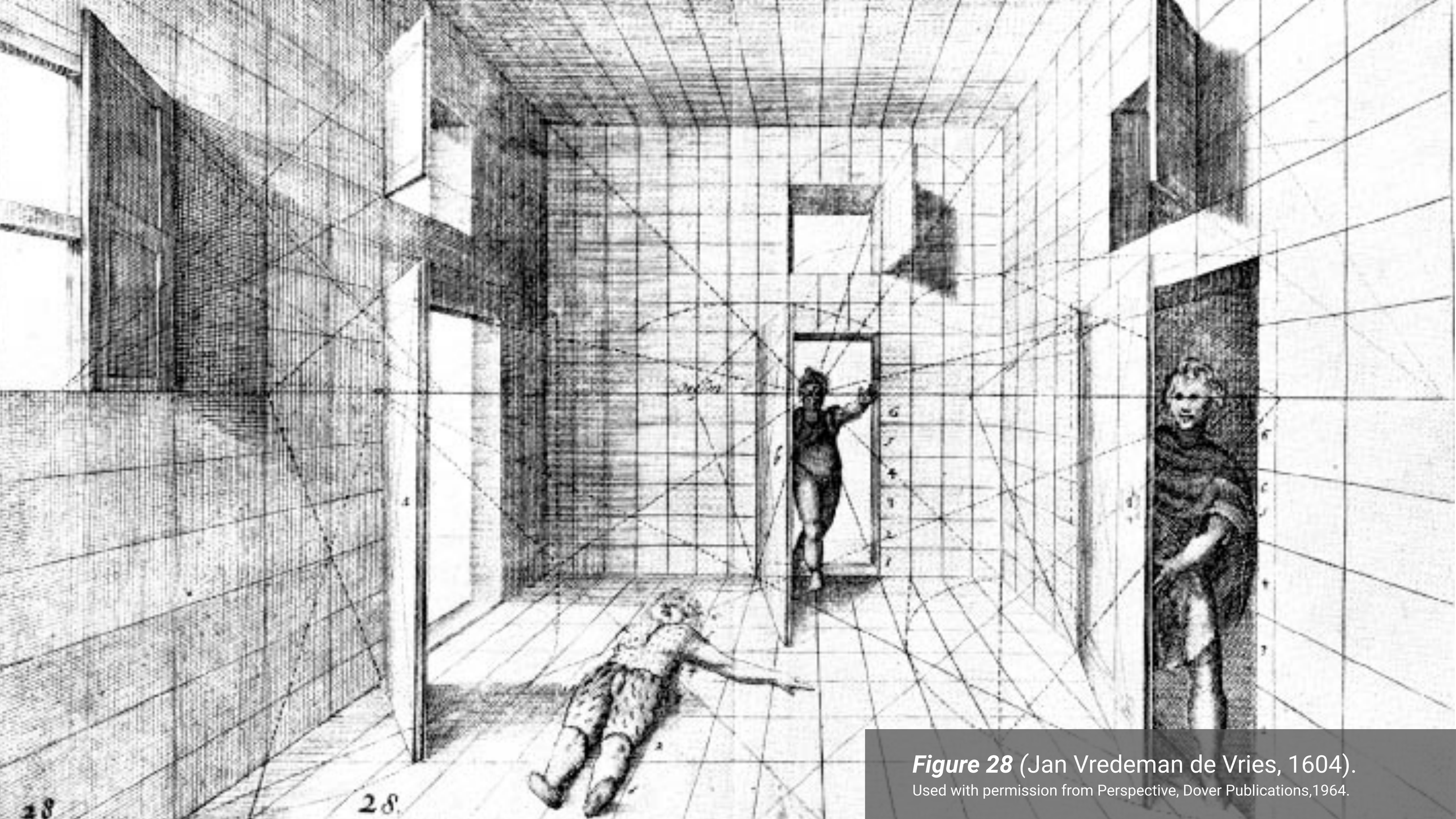


Figure 28 (Jan Vredeman de Vries, 1604).
Used with permission from Perspective, Dover Publications,1964.

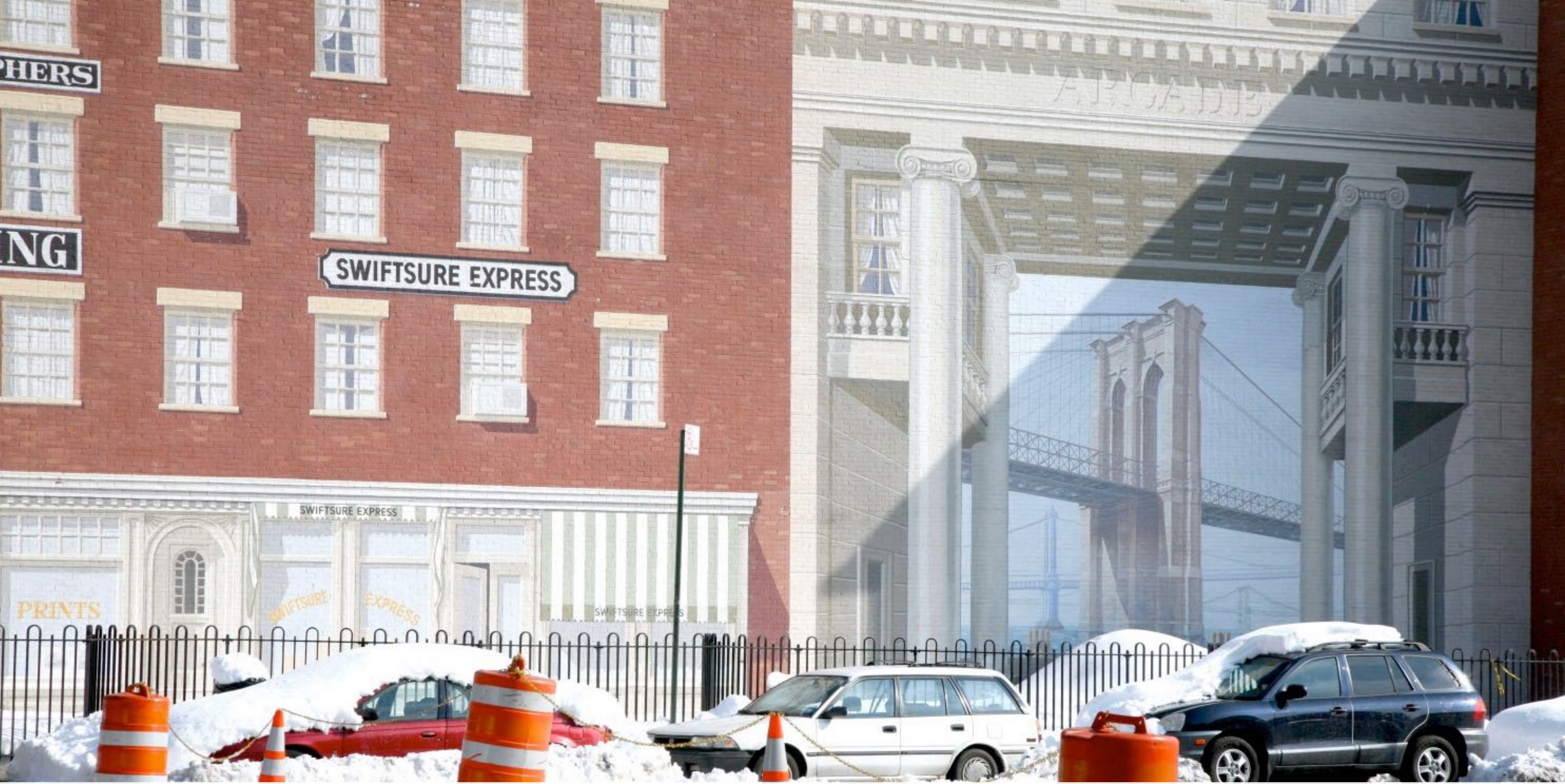
trompe l'oeil |ˌtrômp ˈloi|

noun (pl. **trompe l'oeils** pronunc. **same**)

visual illusion in art, esp. as used to trick the eye into perceiving a painted detail as a three-dimensional object.

Trompe L'oeil Tuscan Window Mural 2009

Kristin Plansky | Used with permission.



New York City, Lower Manhattan, Front St.: **Richard Haas** *Trompe l'oeil* 1975

People are actually avoiding walking in the "hole“ 2007

Joe Beever | CC A2.0

Stunning 3D chalk drawing from Zebit stops Liverpool shoppers in their tracks on Bold Street. *2012*

Bill Hunt Original art: **Zebit** | CC A2.0

Edgar Meuller http://www.metanamorph.com

Forced perspective
2011
Seongbin Im | CC A2.0

On hands *2013*
Kenzie Saunders |
CC A2.0

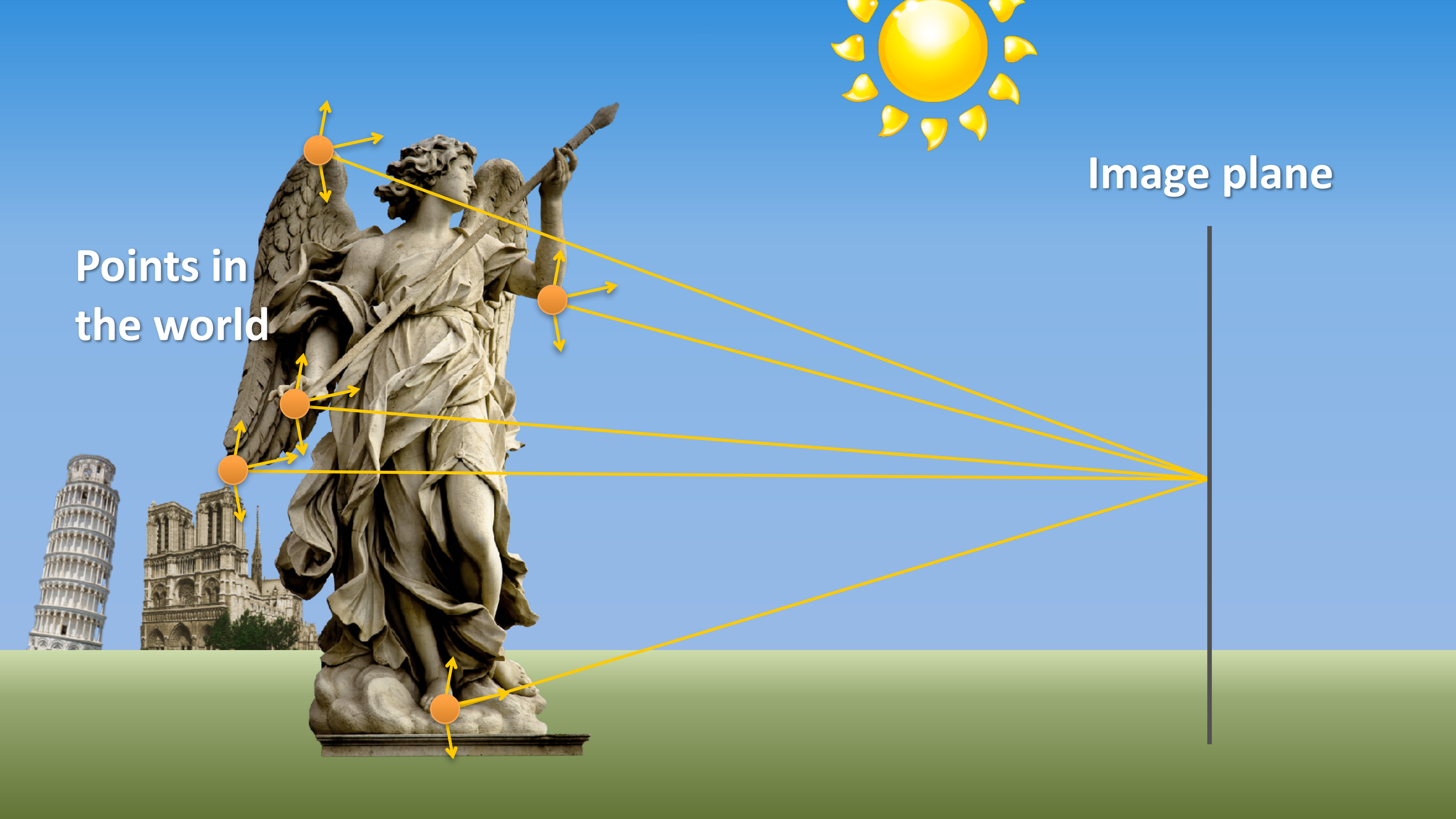


Image plane
Points in
the world

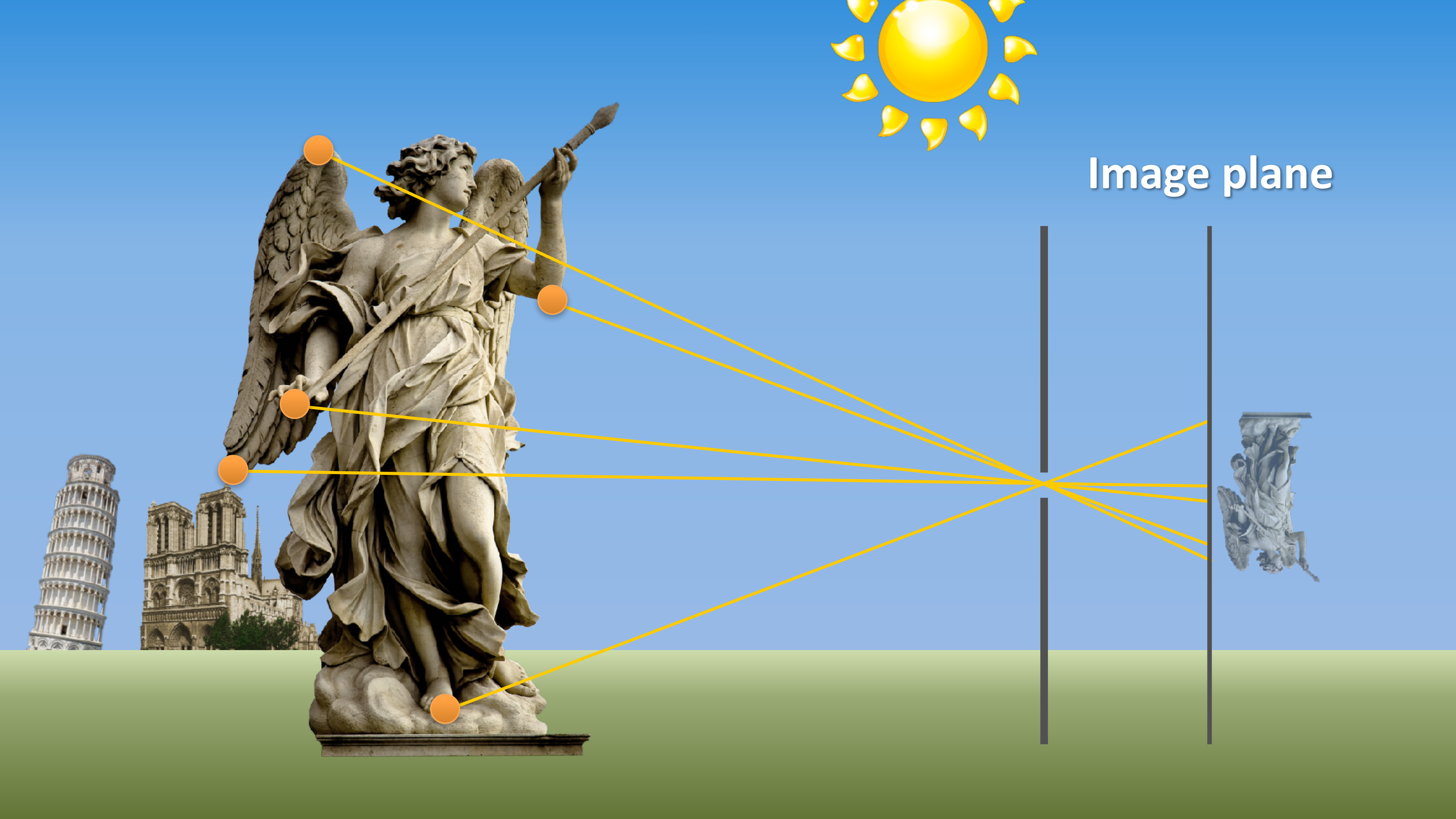
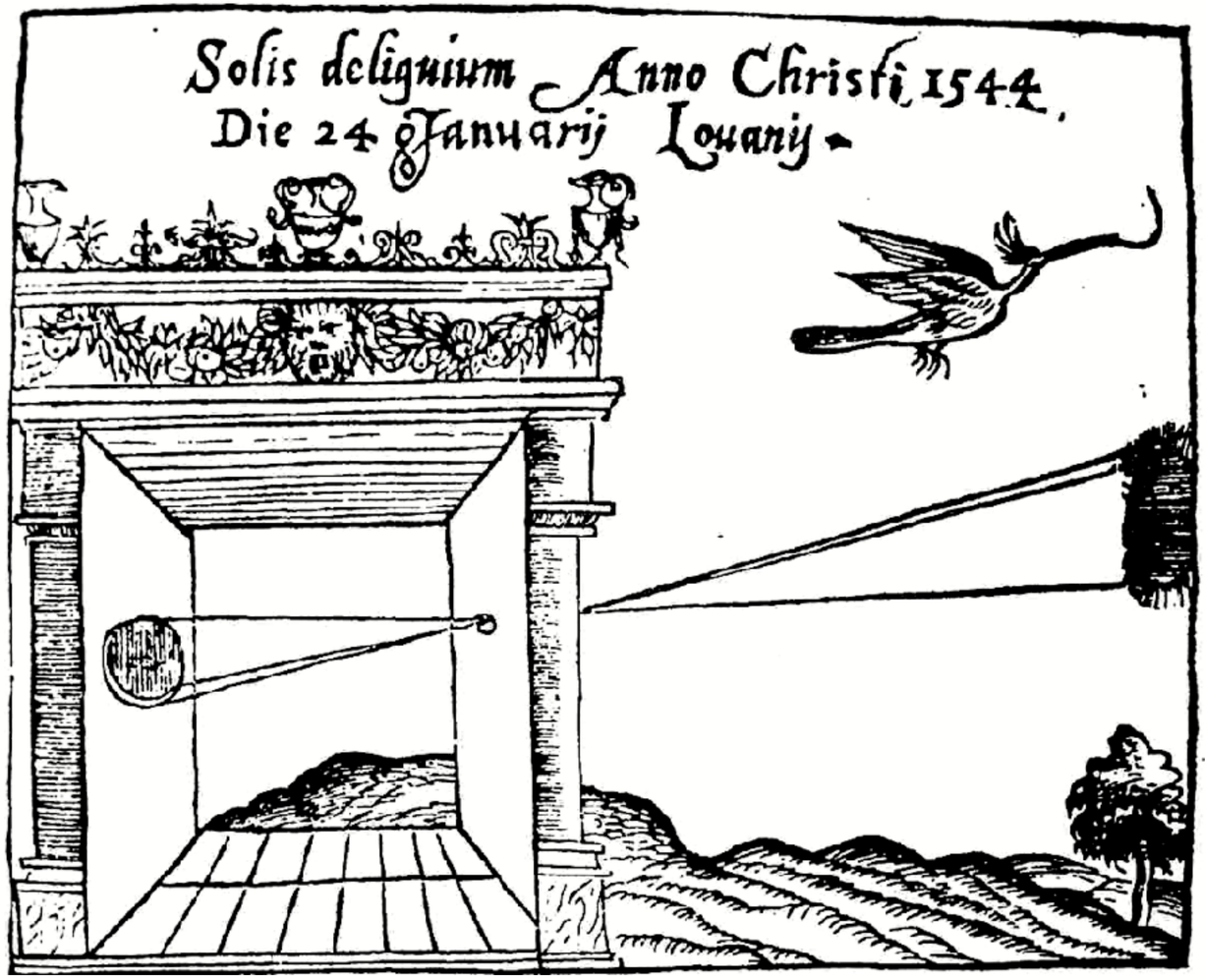
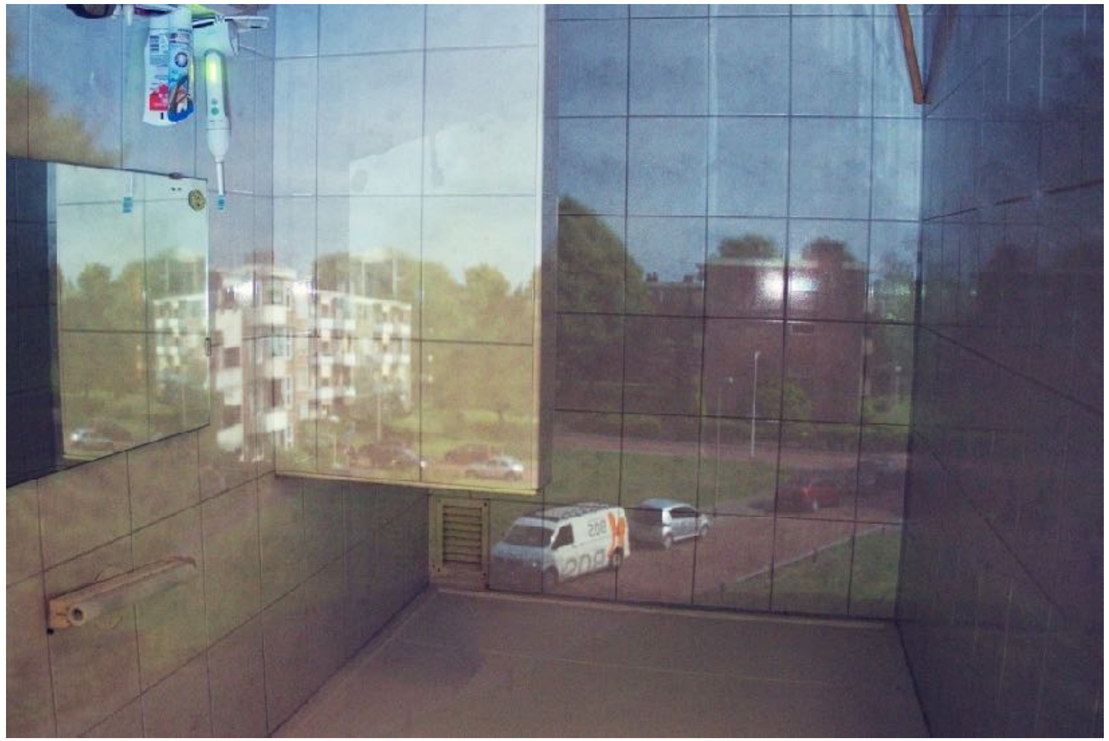


Image plane

The pinhole camera



Pinhole images



Camera obscura 2011

1banaan | CC A2.0

Camera obscura! 2011

half alive - soo zzzz | CC A2.0

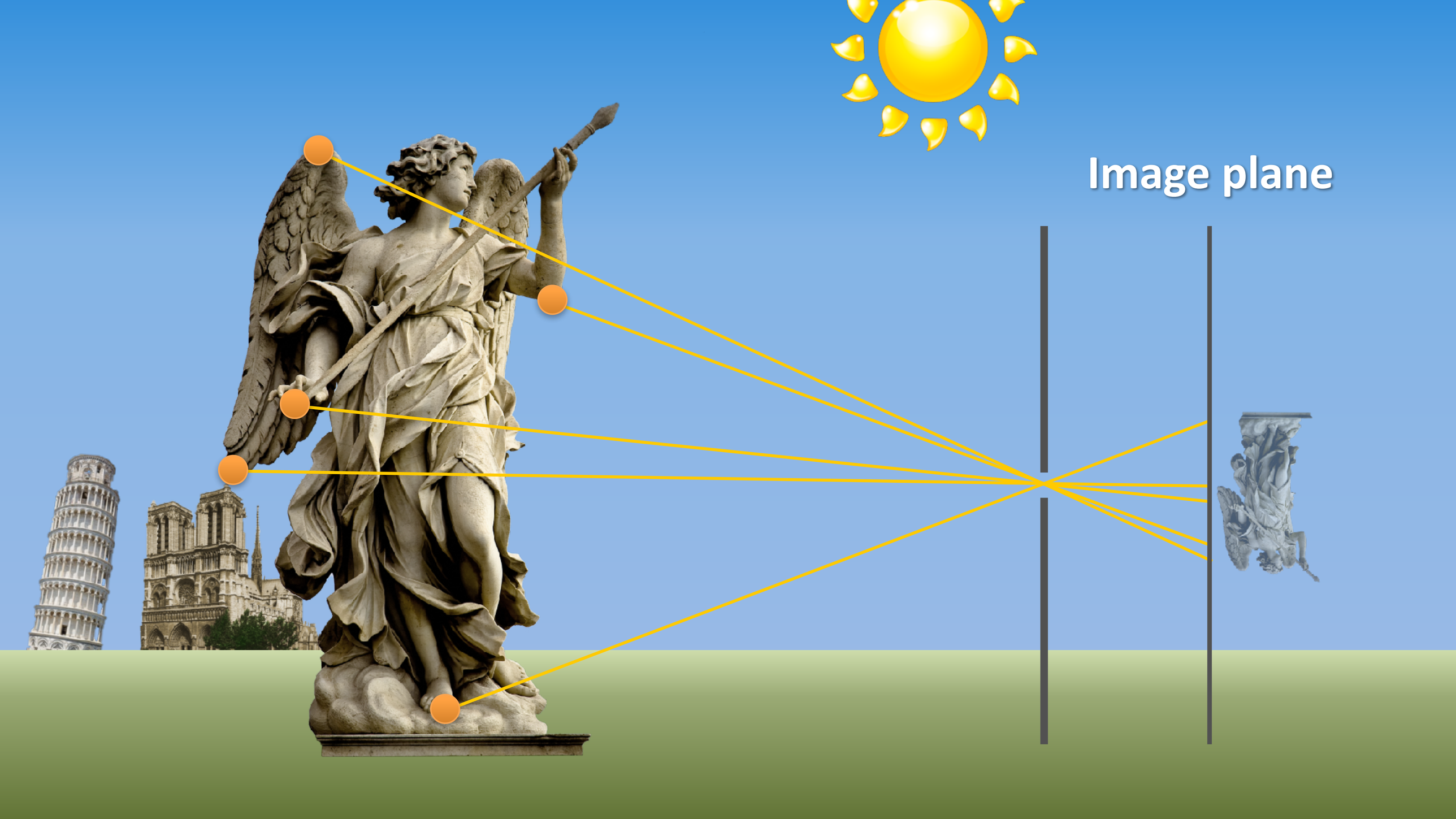
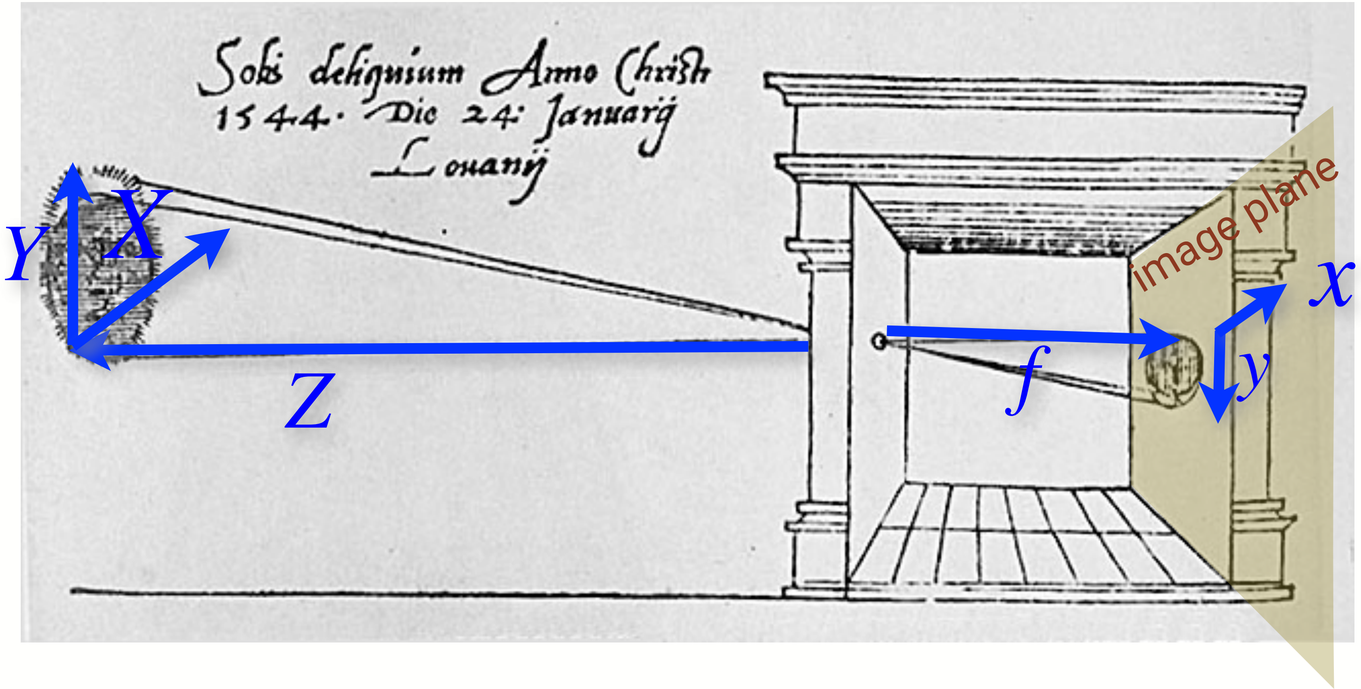


Image plane

Simple imaging

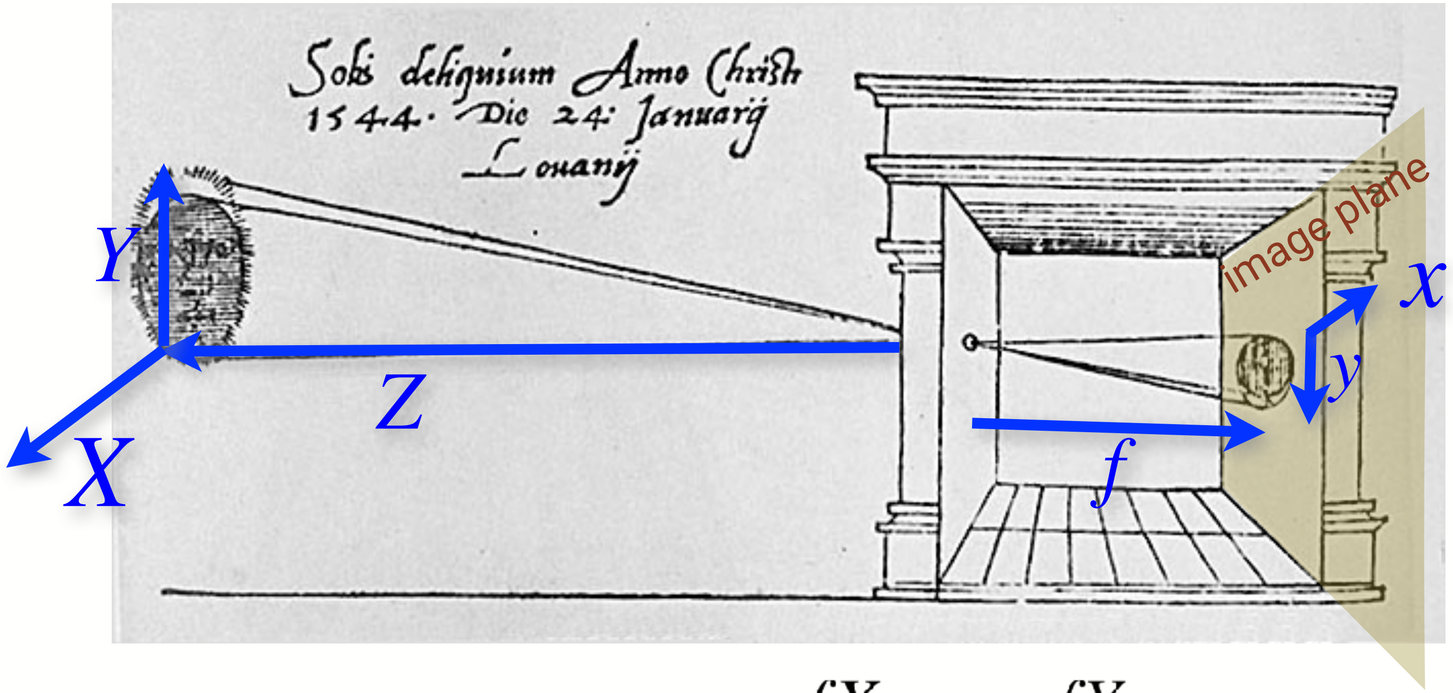


$$\frac{Y}{Z} = \frac{y}{f}$$

$$\frac{X}{Z} = \frac{x}{f}$$

- Similar triangles
- Image formation is the mapping of scene points (X,Y,Z) to the image plane (x,y)
- Image is inverted

Simple imaging



$$\frac{Y}{Z} = \frac{y}{f}$$

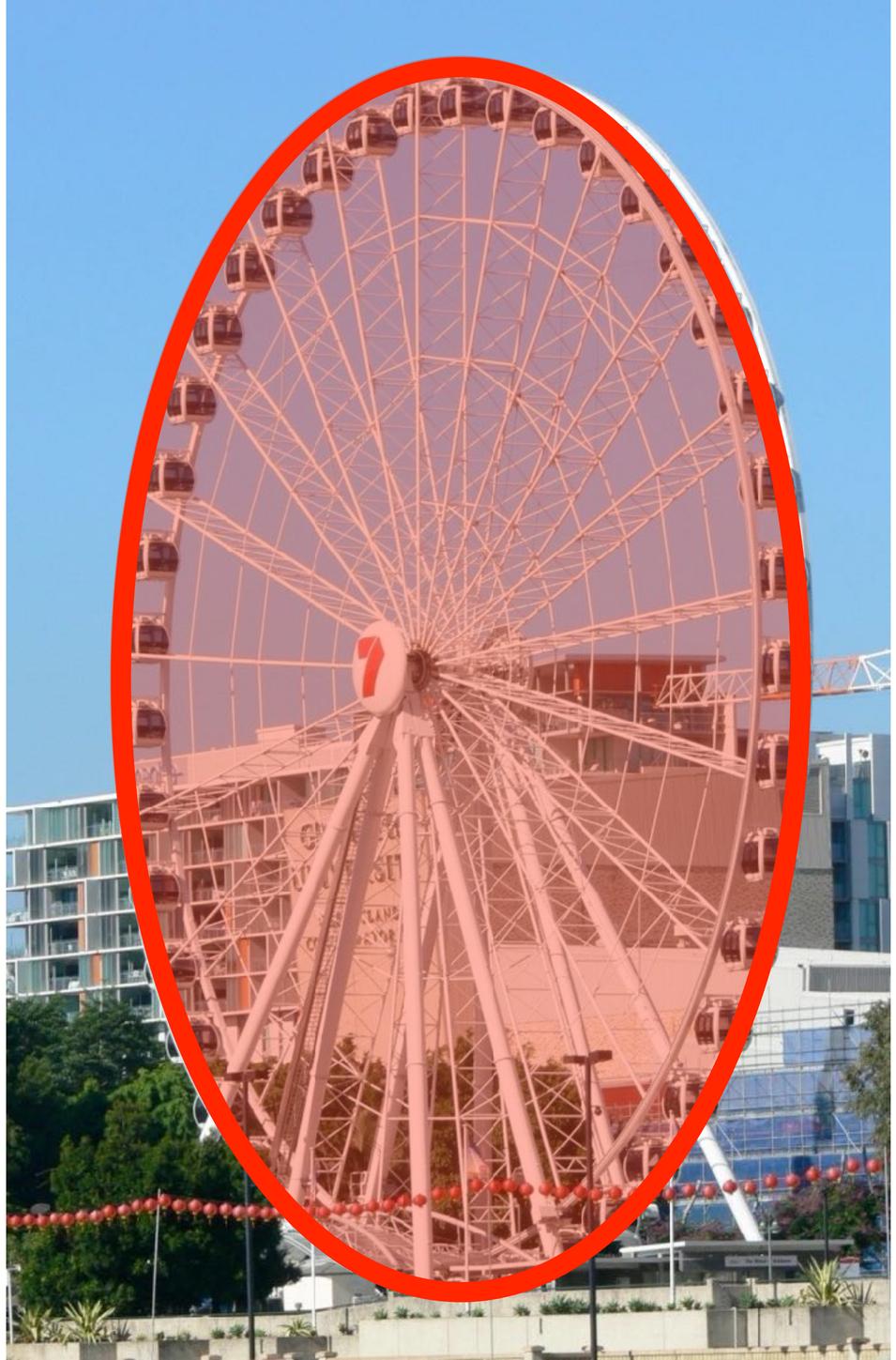
$$\frac{X}{Z} = \frac{x}{f}$$

$$x = \frac{fX}{Z}, y = \frac{fY}{Z}$$

$$(X, Y, Z) \mapsto (x, y)$$

$$\mathbb{R}^3 \mapsto \mathbb{R}^2$$

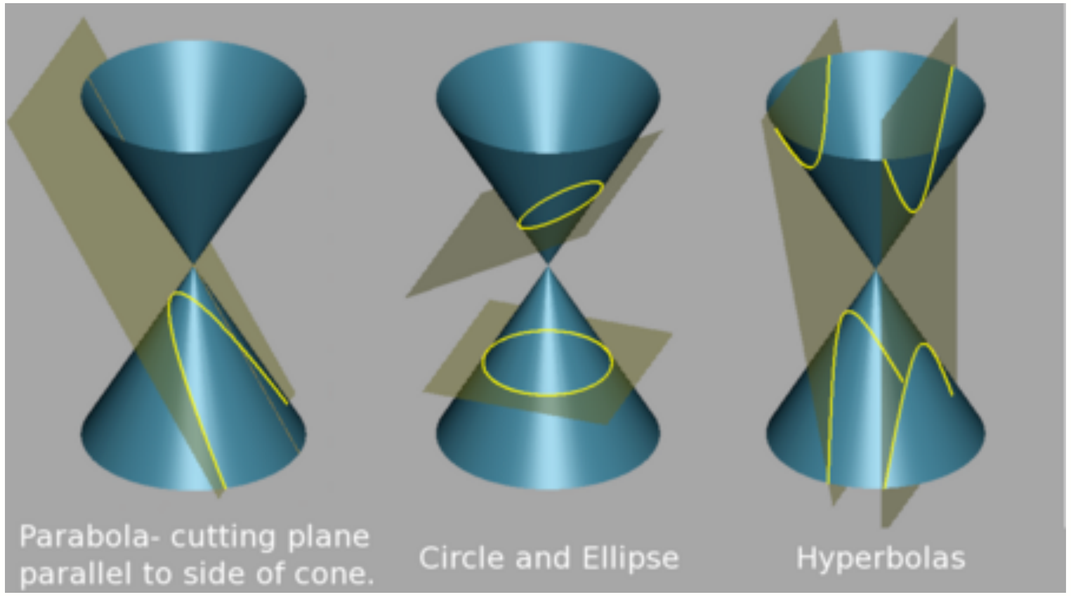
- 3D to 2D
- Perspective projection



Perspective projection

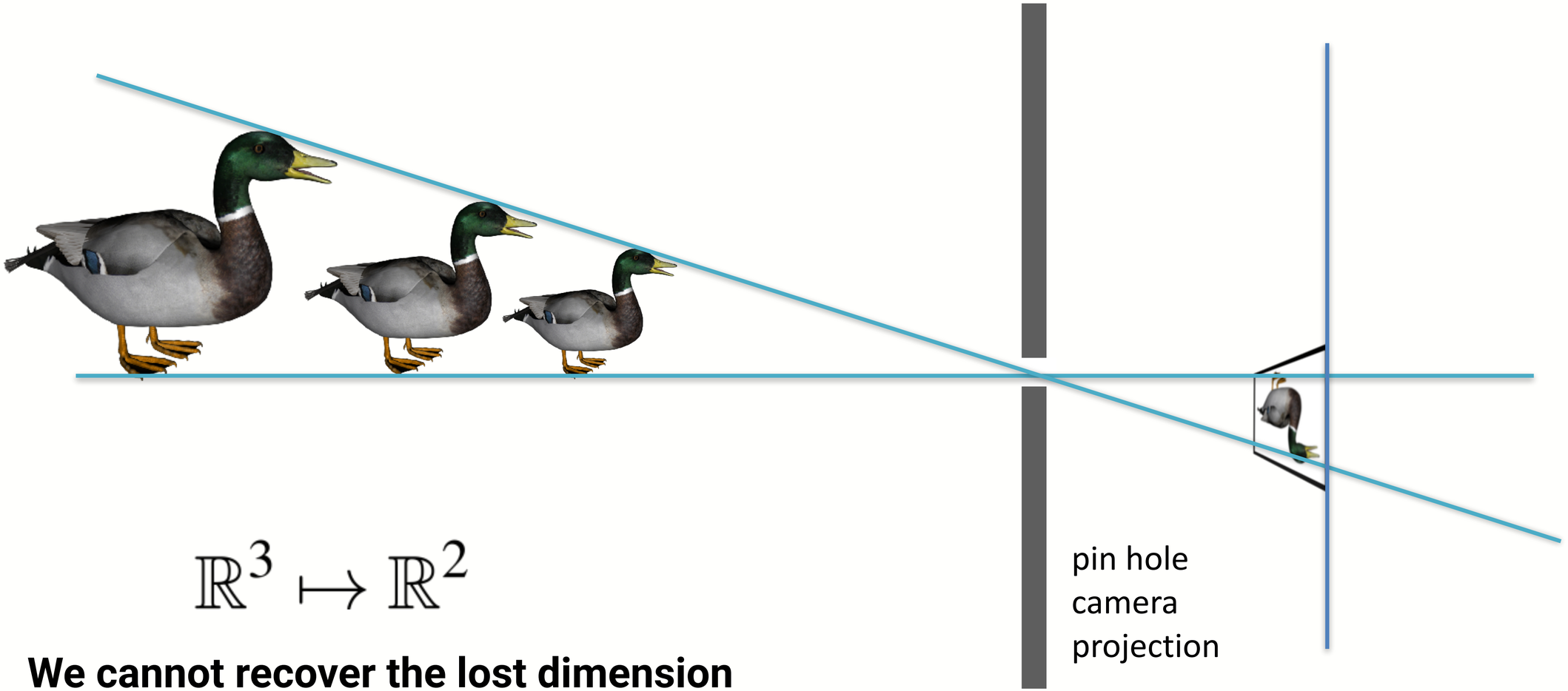
Maps

- Lines → lines
  - ↪ parallel lines not necessarily parallel
  - ↪ angles are not preserved
- Conics → conics



Duk at the English language Wikipedia

No unique inverse



$$\mathbb{R}^3 \mapsto \mathbb{R}^2$$

We cannot recover the lost dimension

- Any 2D image could be generated by one of an infinite number of possible 3D worlds

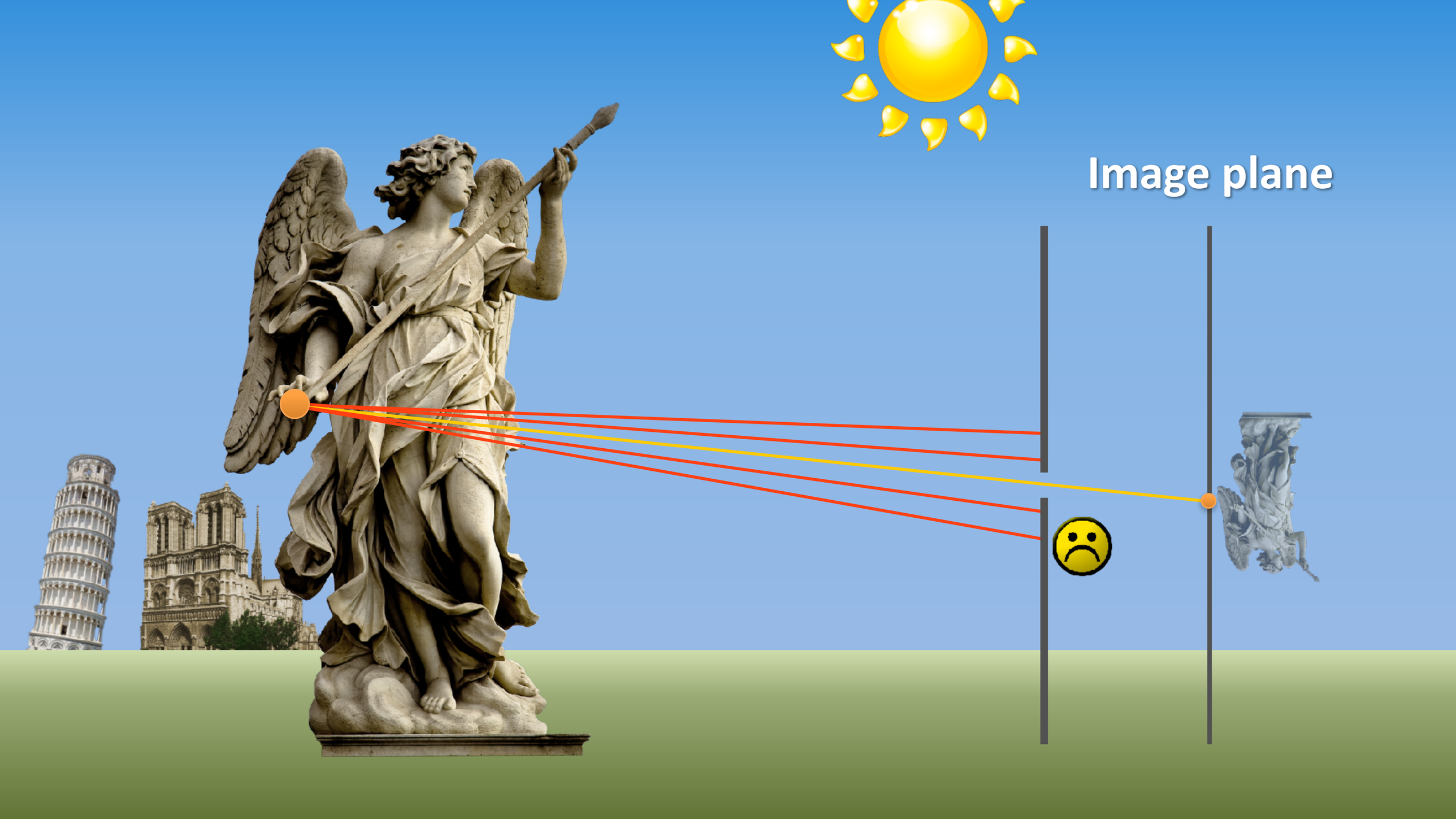
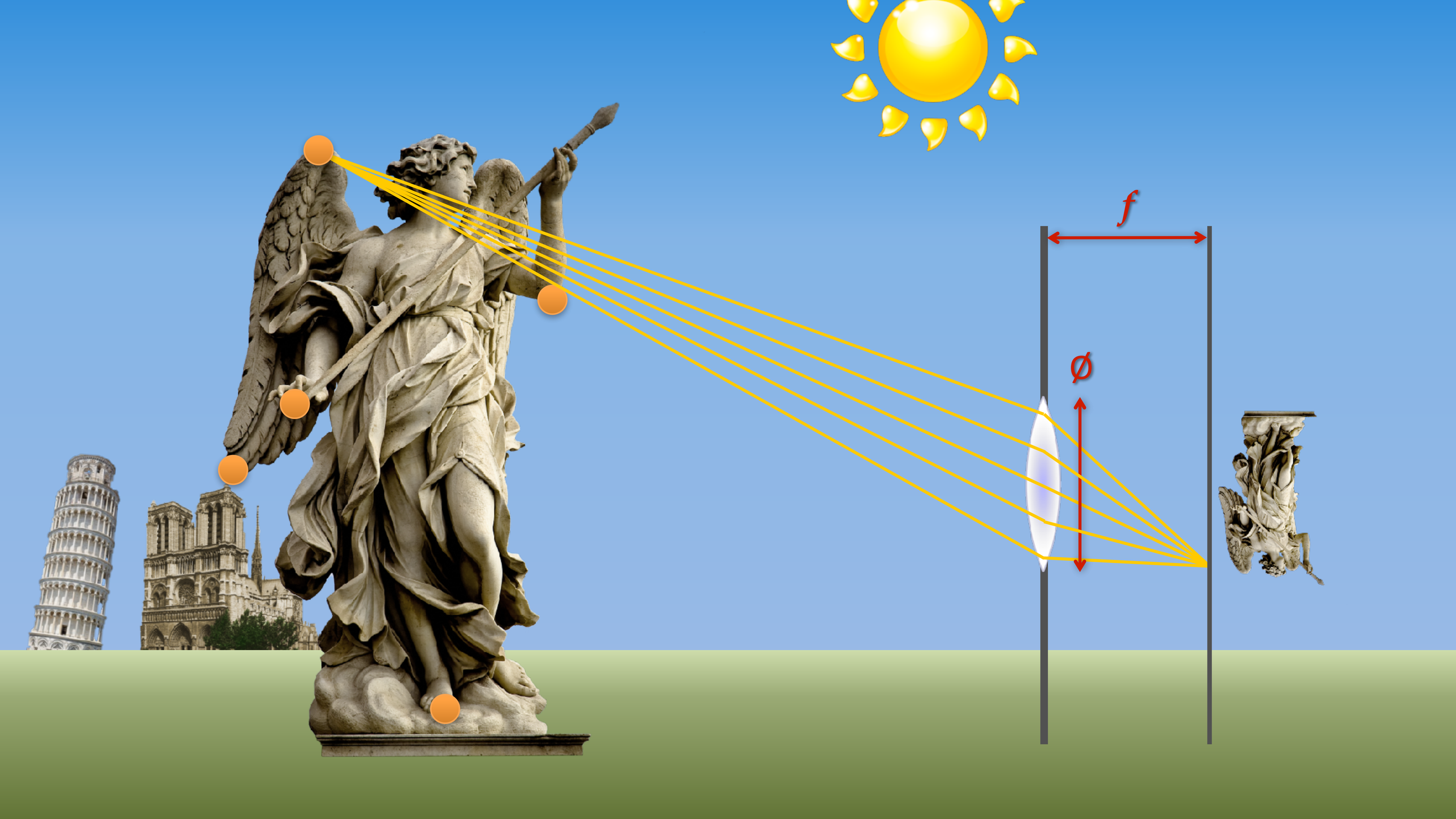


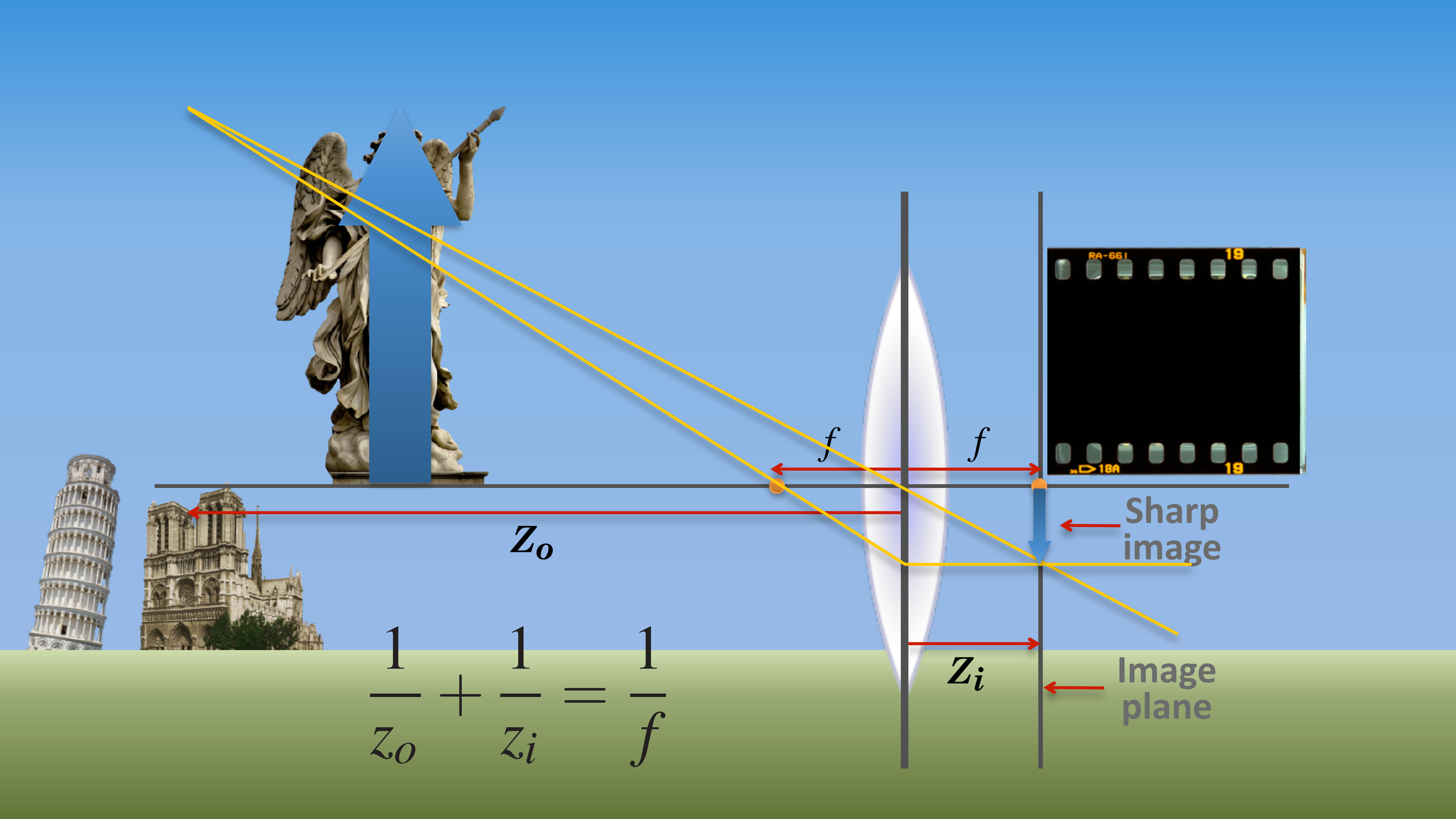
Image plane

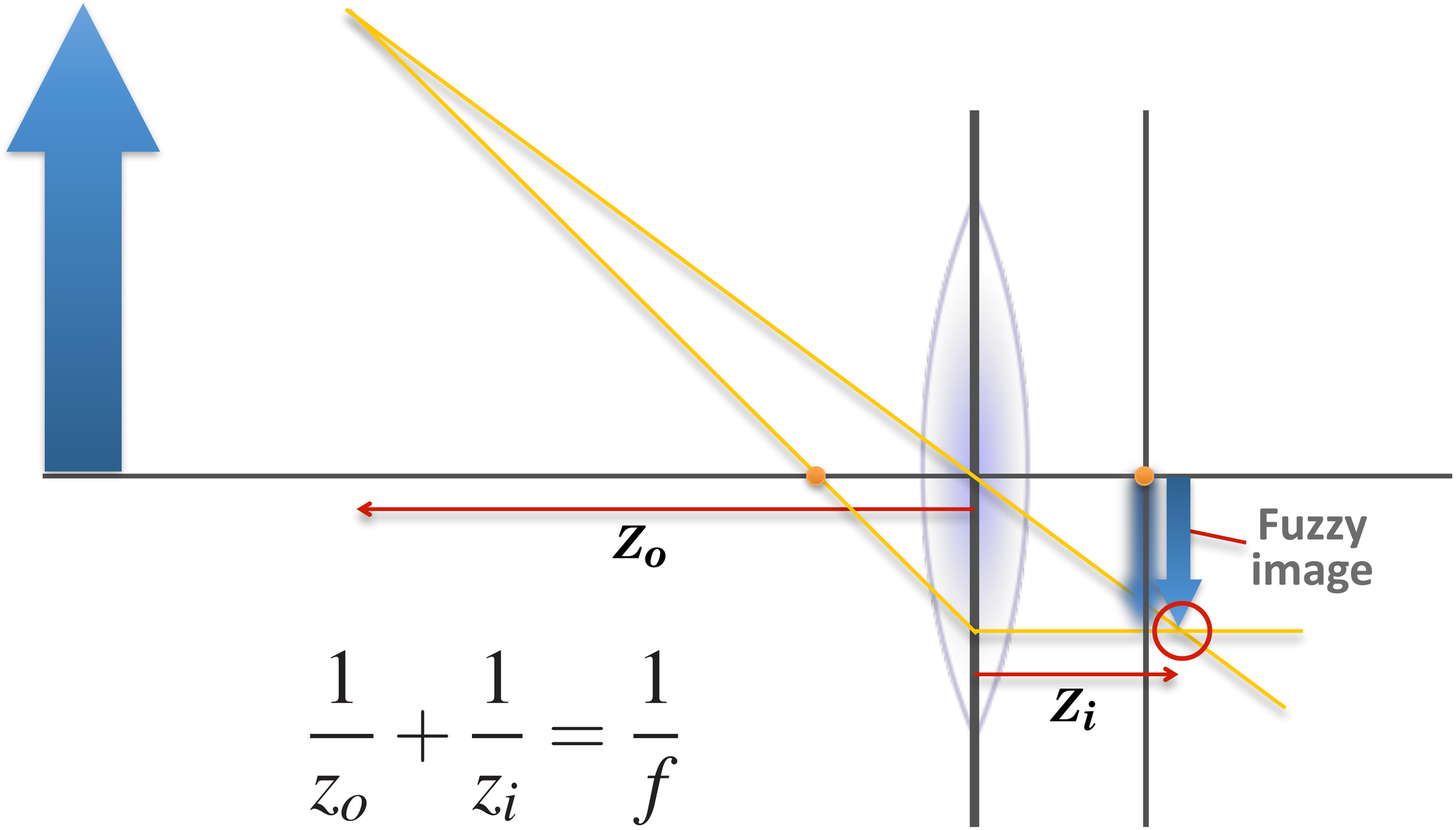
Use a lens to gather more light

George R. Lawrence 1900


f
ø

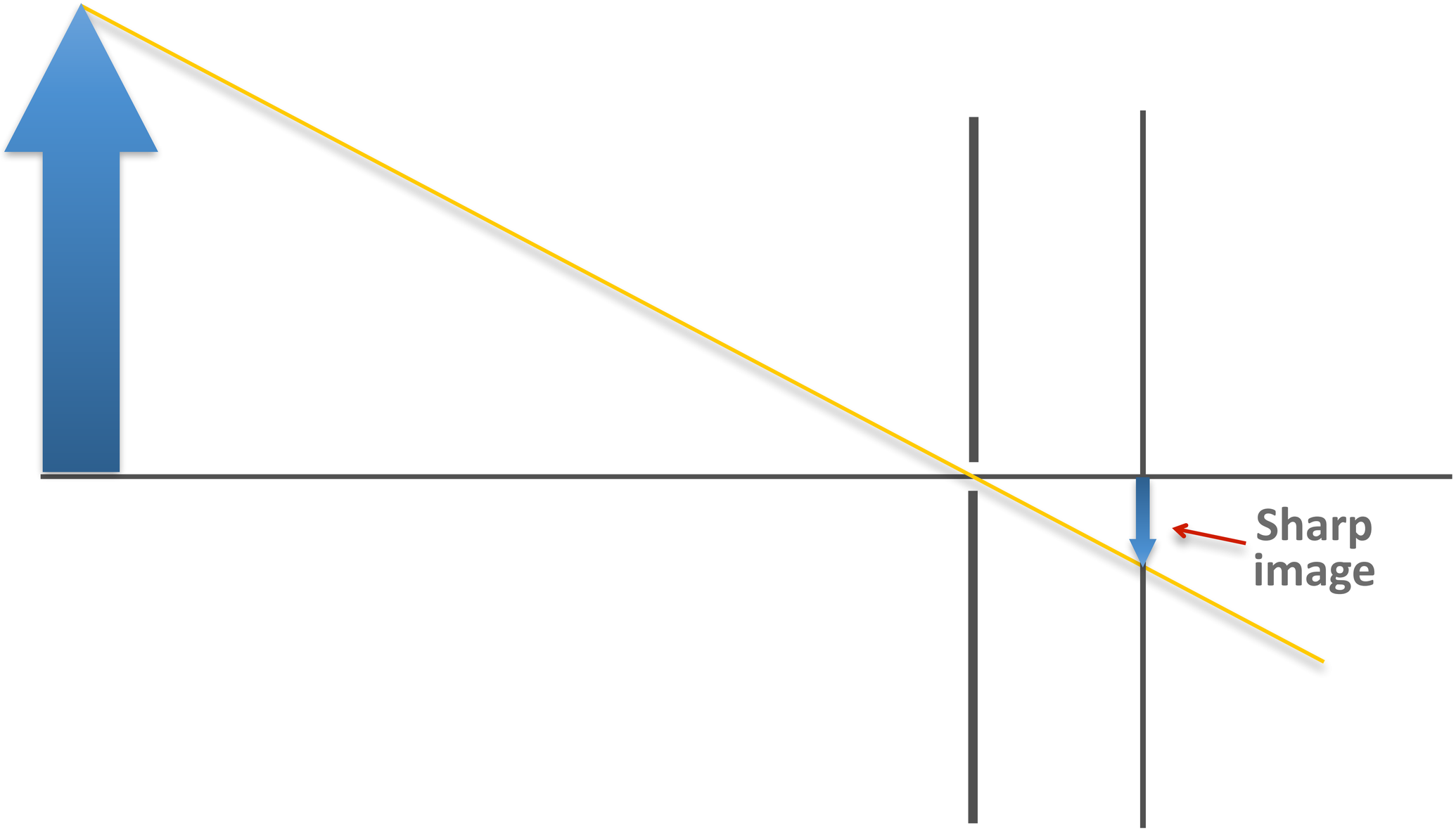

$F = \frac{f}{\phi}$
f
ø
F/1.4 lens
NIKKOR 50mm 1:1.4


f
f
z_o
z_i
Sharp image
Image plane
$$\frac{1}{z_o} + \frac{1}{z_i} = \frac{1}{f}$$

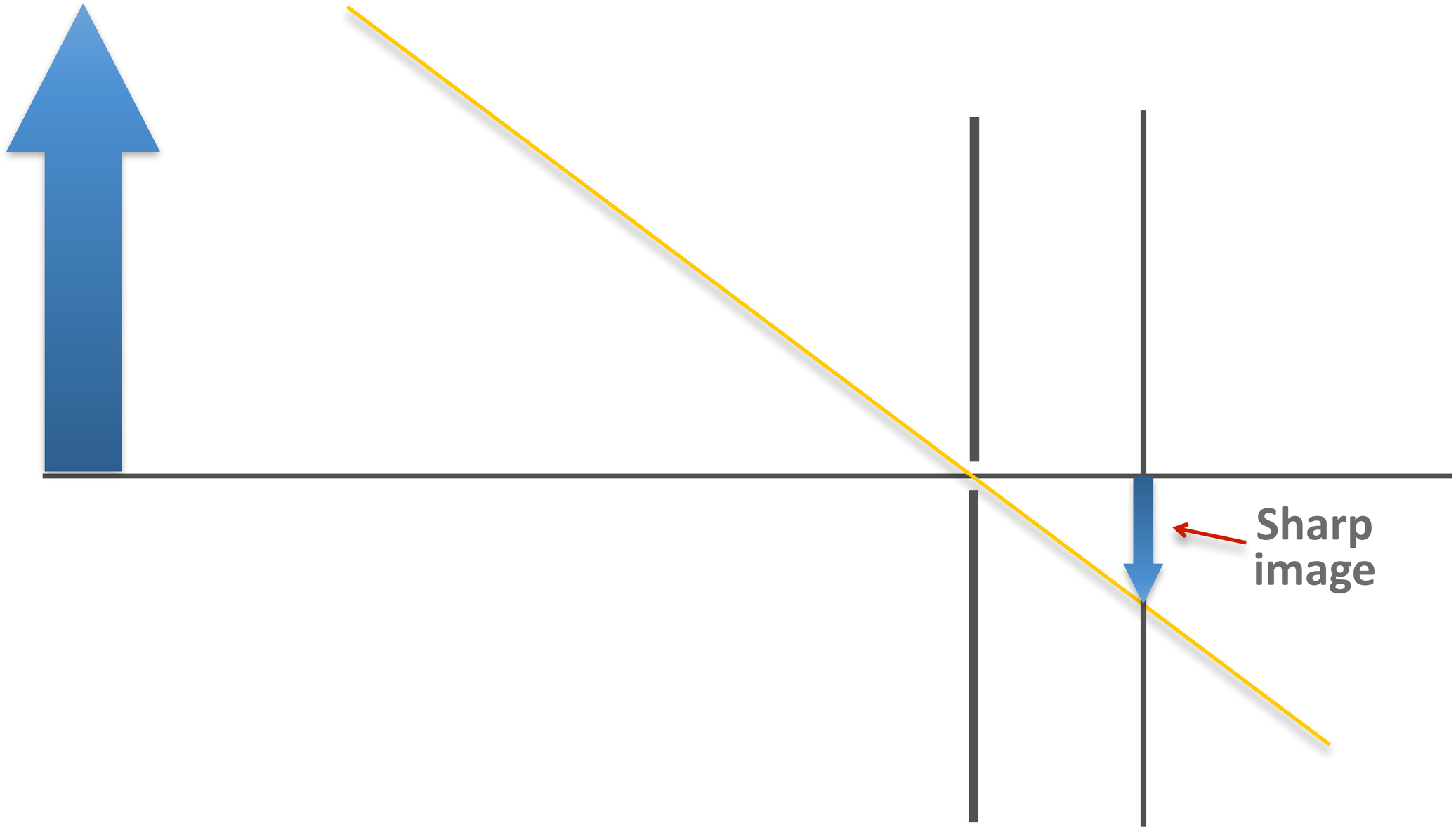


$$\frac{1}{z_o} + \frac{1}{z_i} = \frac{1}{f}$$

Pinhole camera doesn't need focus

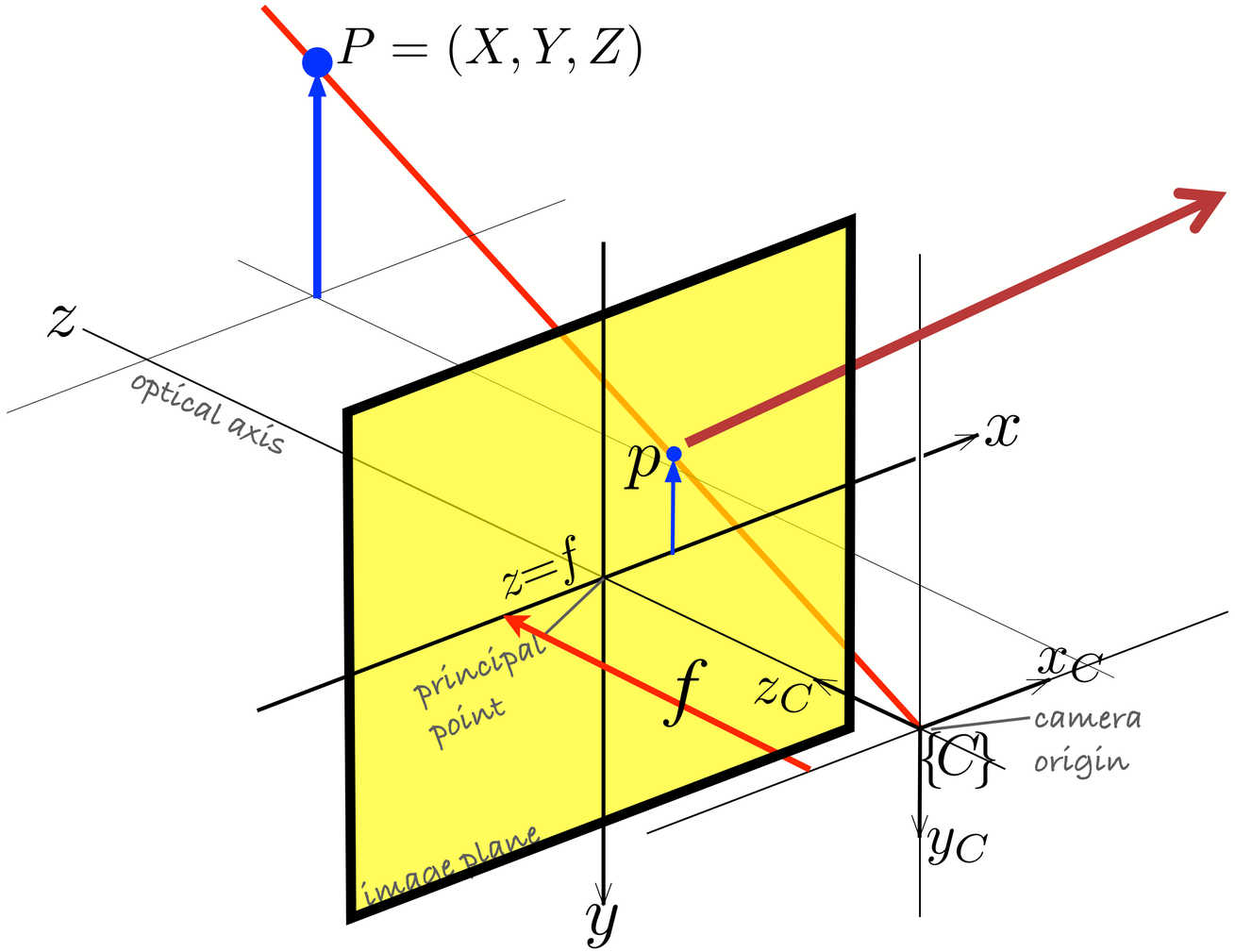


Pinhole camera doesn't need focus



Central projection camera model

Central projection model



$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

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Pin-hole model in homogeneous form

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\tilde{x} = fX, \tilde{y} = fY, \tilde{z} = Z$$

$$x = \frac{\tilde{x}}{\tilde{z}},\ y = \frac{\tilde{y}}{\tilde{z}}$$

$$\Rightarrow x = \frac{fX}{Z}, y = \frac{fY}{Z}$$

- Perspective transformation, with the pesky divide by Z, is **linear** in homogeneous coordinate form.

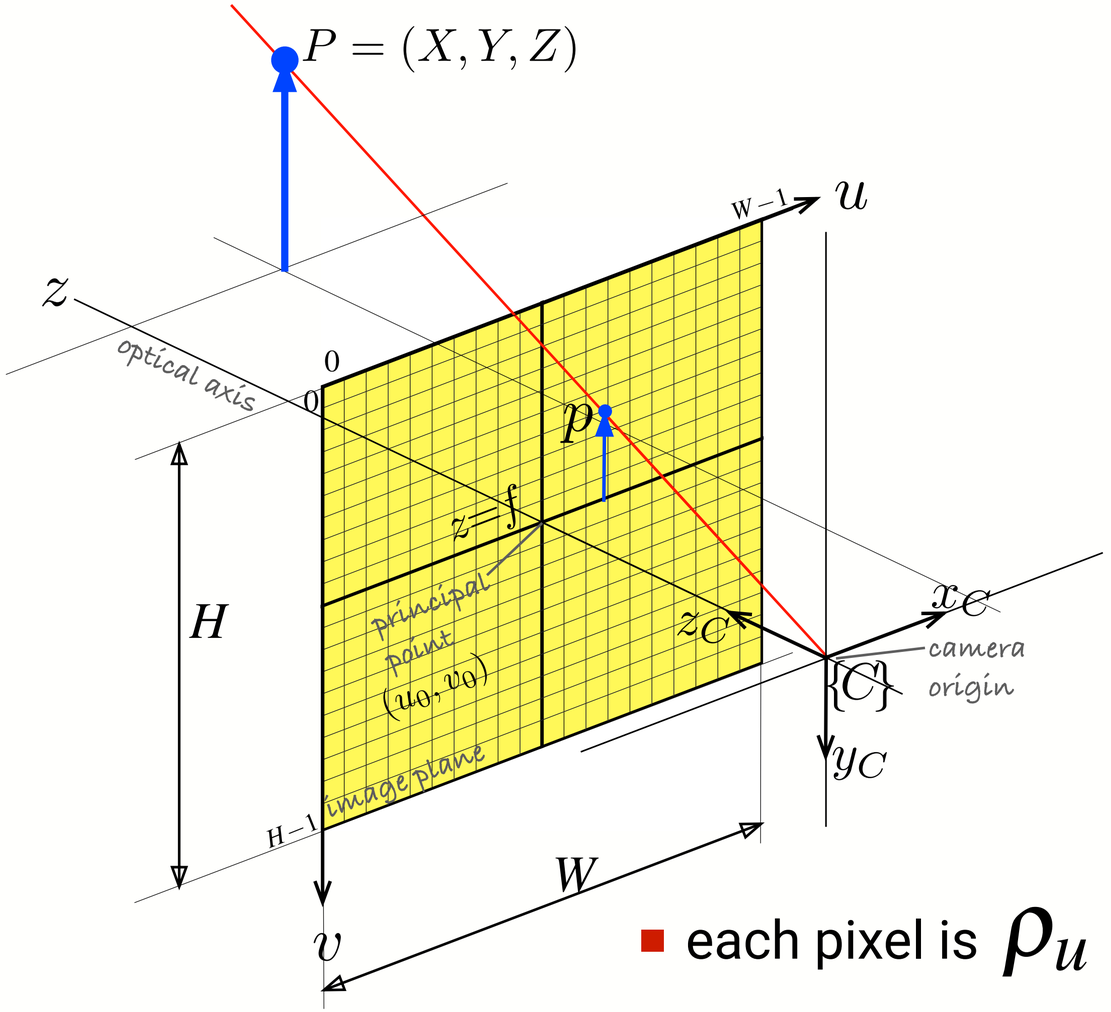
$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

3D to 2D

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

scaling/ zooming

Change of coordinates

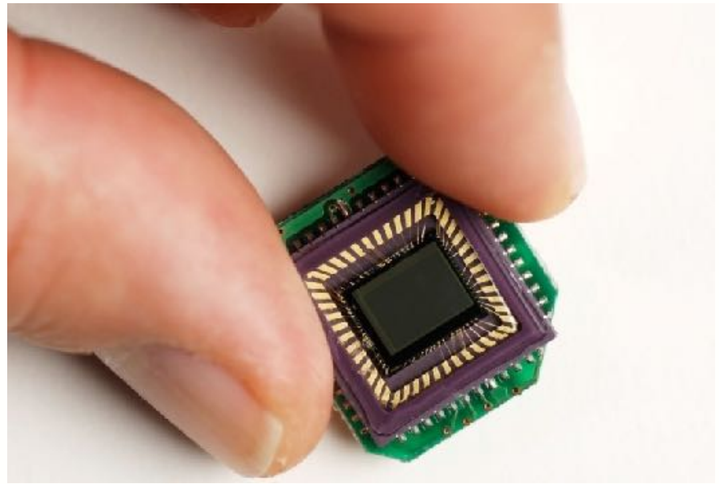


- each pixel is $\rho_u \times \rho_v$

- scale point from metres to pixels
- shift the origin to top left corner

$$u = \frac{x}{\rho_u} + u_0$$

$$v = \frac{y}{\rho_v} + v_0$$



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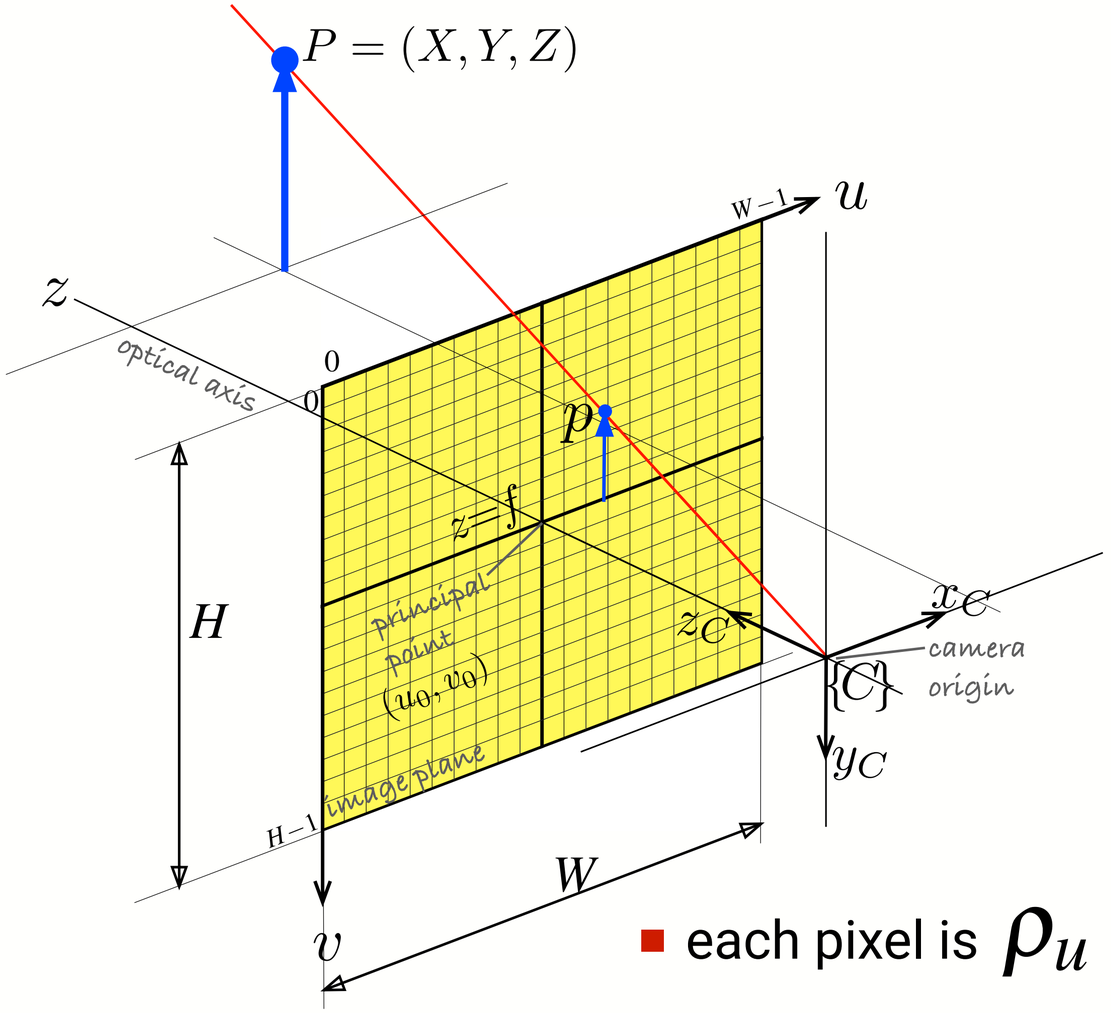
Change of coordinates



- each pixel is $\rho_u \times \rho_v$

- scale point from metres to pixels
- shift the origin to top left corner

$$u = \frac{x}{\rho_u} + u_0$$

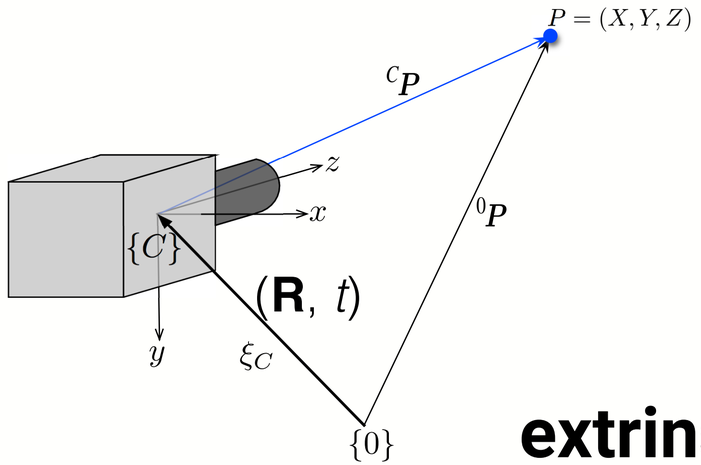
$$v = \frac{y}{\rho_v} + v_0$$

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \begin{pmatrix} \frac{1}{\rho_u} & 0 & u_0 \\ 0 & \frac{1}{\rho_v} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix}$$

$$p = \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \tilde{u}/\tilde{w} \\ \tilde{v}/\tilde{w} \end{pmatrix}$$

Complete camera model

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$



$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \underbrace{\underbrace{\begin{pmatrix} \frac{1}{\rho_u} & 0 & u_0 \\ 0 & \frac{1}{\rho_v} & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{\mathbf{K}} \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{R} & t \\ \mathbf{0}_{1\times 3} & 1 \end{pmatrix}^{-1}}_{\mathbf{C}} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \ominus \xi_C$$

extrinsic parameters

intrinsic parameters

camera matrix

Camera matrix

- Mapping points from **the world** to an **image (pixel) coordinate** is simply a **matrix multiplication** using **homogeneous coordinates**

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$u = \frac{\tilde{u}}{\tilde{w}}, v = \frac{\tilde{v}}{\tilde{w}}$$

Scale invariance

- Consider an arbitrary scalar scale factor

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \lambda \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

- $\tilde{u}, \tilde{v}, \tilde{w}$ will all be scaled by λ

- but $u = \frac{\tilde{u}}{\tilde{w}}, v = \frac{\tilde{v}}{\tilde{w}}$

- so the result is unchanged

Normalized camera matrix

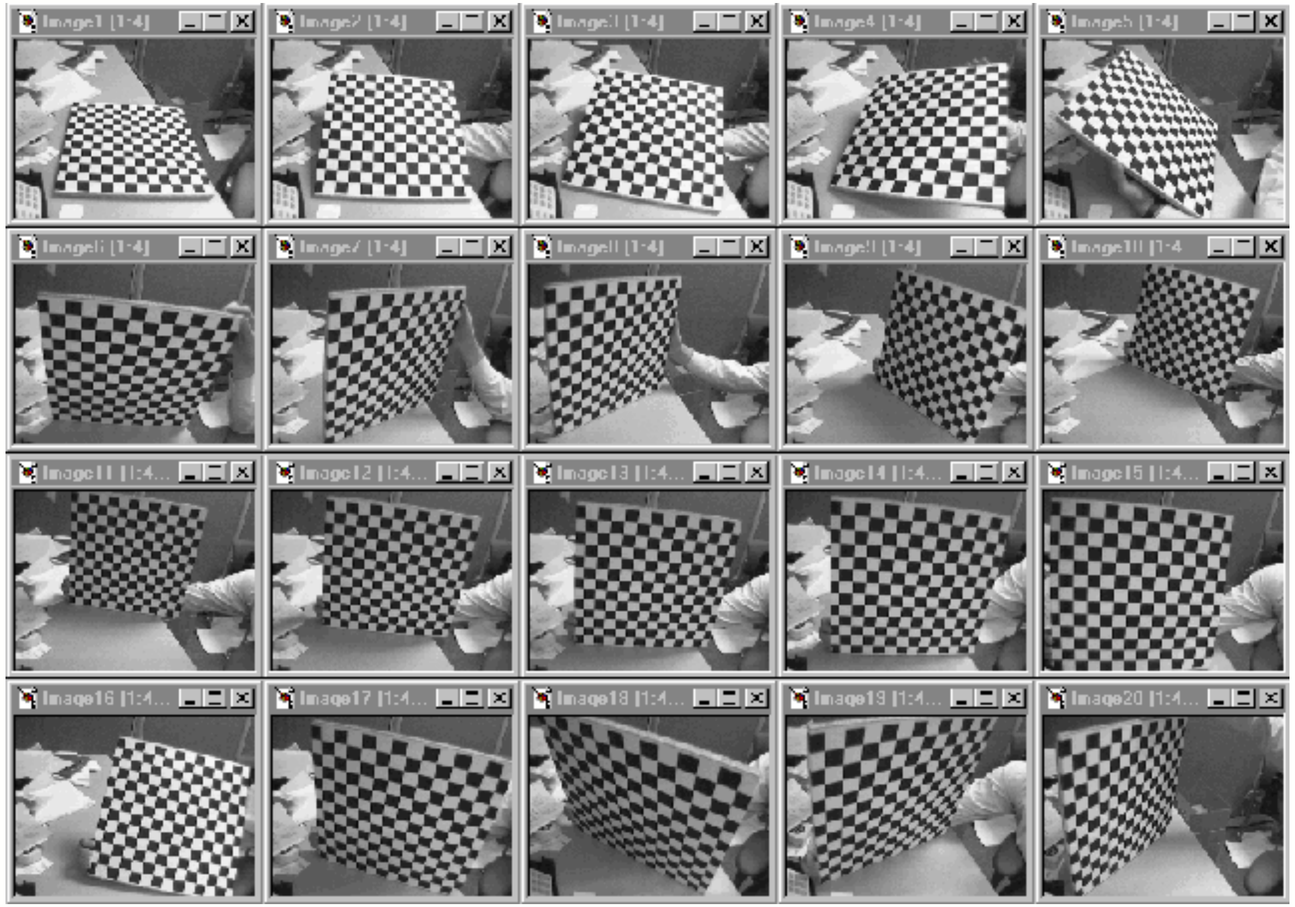
- Since scale factor is arbitrary we can fix the value of one element, typically C(3,4) to one.

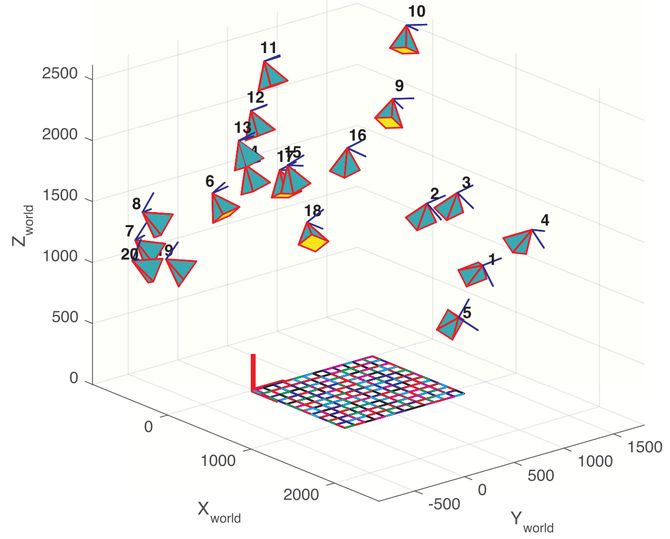
- focal length
- pixel size
- camera position
- & orientation

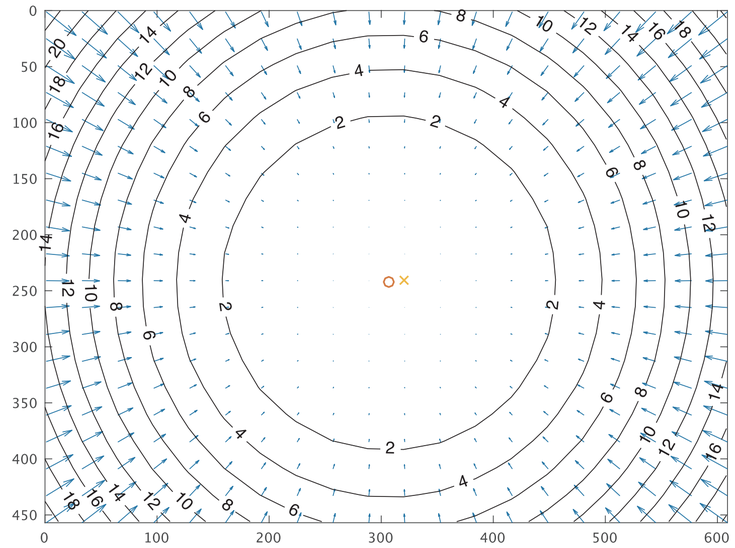
$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$u = \frac{\tilde{u}}{\tilde{w}}, v = \frac{\tilde{v}}{\tilde{w}}$$

Camera calibration





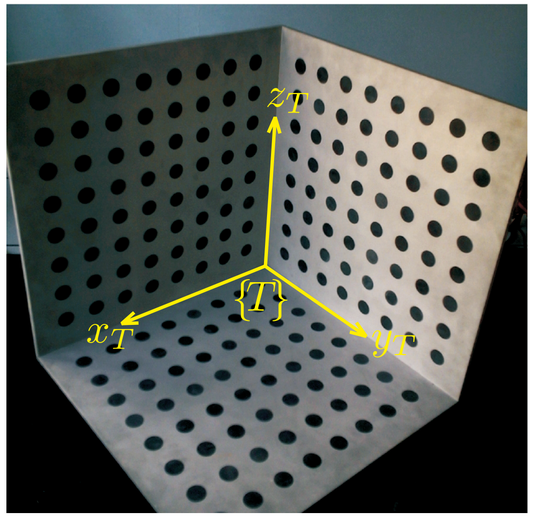


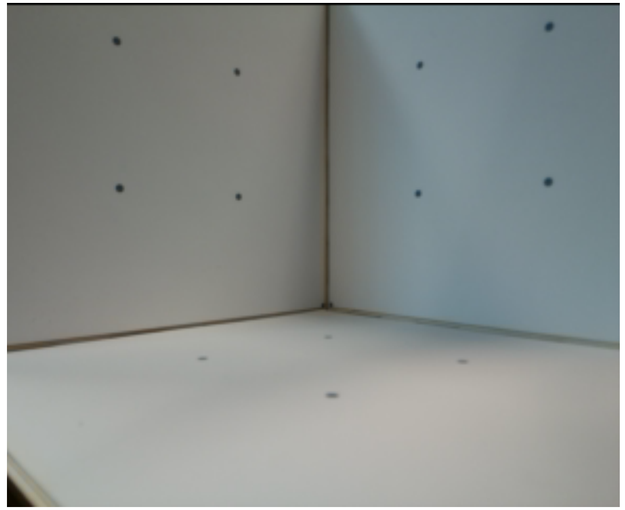
- Process to determine intrinsic and extrinsic camera parameters

For N points we stack these 2x11 matrices. To solve we need number of rows > number of columns, ie. N ≥ 6

$$\begin{pmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1Z_1 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -v_1X_1 & -v_1Y_1 & -v_1Z_1 \\ & & & & & \vdots & & & & & \\ X_N & Y_N & Z_N & 1 & 0 & 0 & 0 & 0 & -u_NX_N & -u_NY_N & -u_NZ_N \\ 0 & 0 & 0 & 0 & X_N & Y_N & Z_N & 1 & -v_NX_N & -v_NY_N & -v_NZ_N \end{pmatrix} \begin{pmatrix} C_{11} \\ C_{12} \\ \vdots \\ C_{33} \end{pmatrix} = \begin{pmatrix} u_1 \\ v_1 \\ \vdots \\ u_N \\ v_N \end{pmatrix}$$

Will fail if all the points lie on a plane



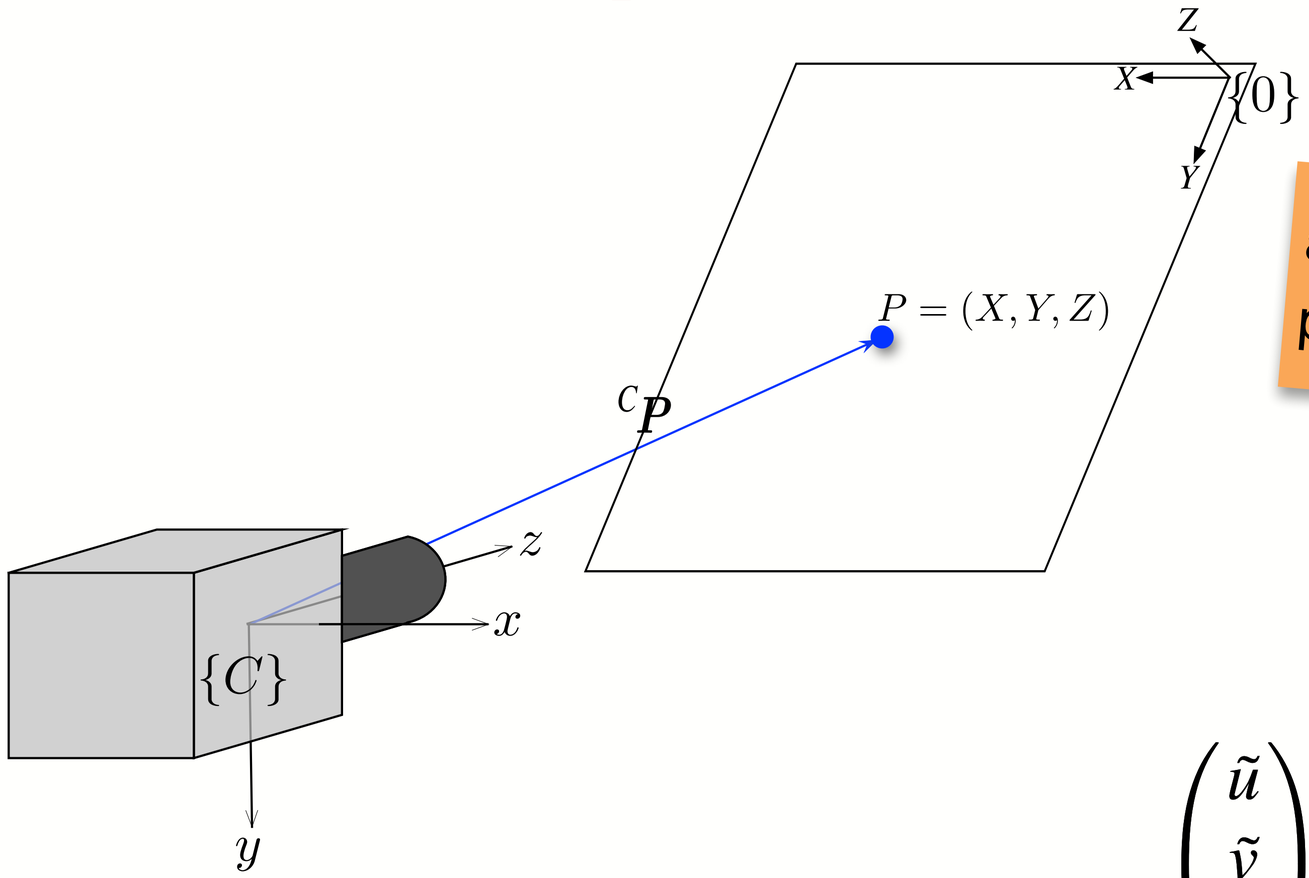


The camera matrix scrambles all the intrinsic and extrinsic parameters into 11 unique values. It is possible to extract K, R and t.

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \underbrace{\underbrace{\begin{pmatrix} \frac{1}{\rho_u} & 0 & u_0 \\ 0 & \frac{1}{\rho_v} & v_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{\mathbf{K}} \begin{pmatrix} \mathbf{R} & t \\ \mathbf{0}_{1\times 3} & 1 \end{pmatrix}^{-1}}_{\mathbf{C}} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

The factors $f/\rho_u, f/\rho_v$ cannot be untangled - focal length in units of pixels

Points on a plane

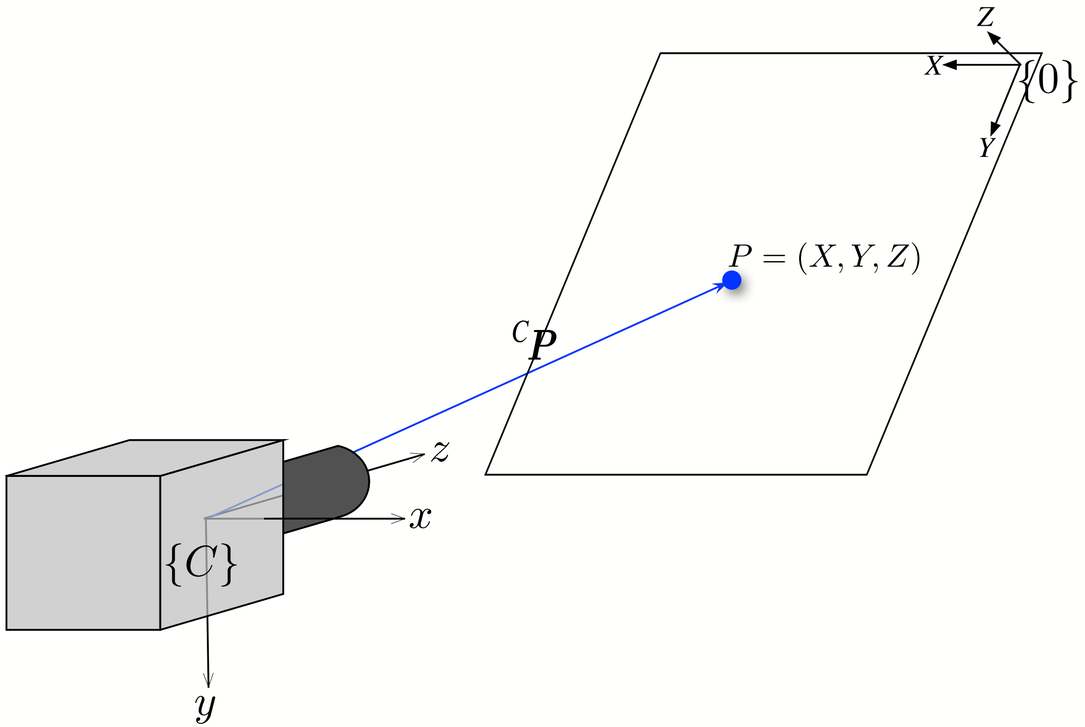


all points on the plane have z=0

3 x 3 matrix

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Planar homography



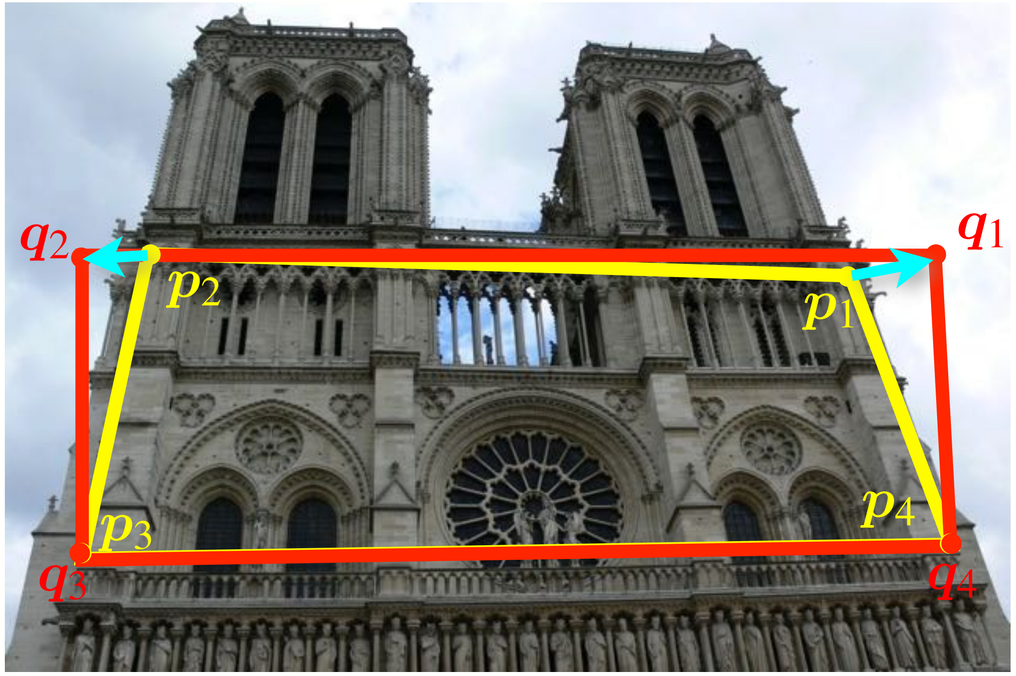
homography matrix

$$\begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

- Once again the scale factor is arbitrary
- 8 unique numbers in the homography matrix
- Can be estimated from 4 world points and their corresponding image points

$$\mathbf{H} = \mathbf{R} + \frac{\boldsymbol{t}}{d}\boldsymbol{n}^T$$

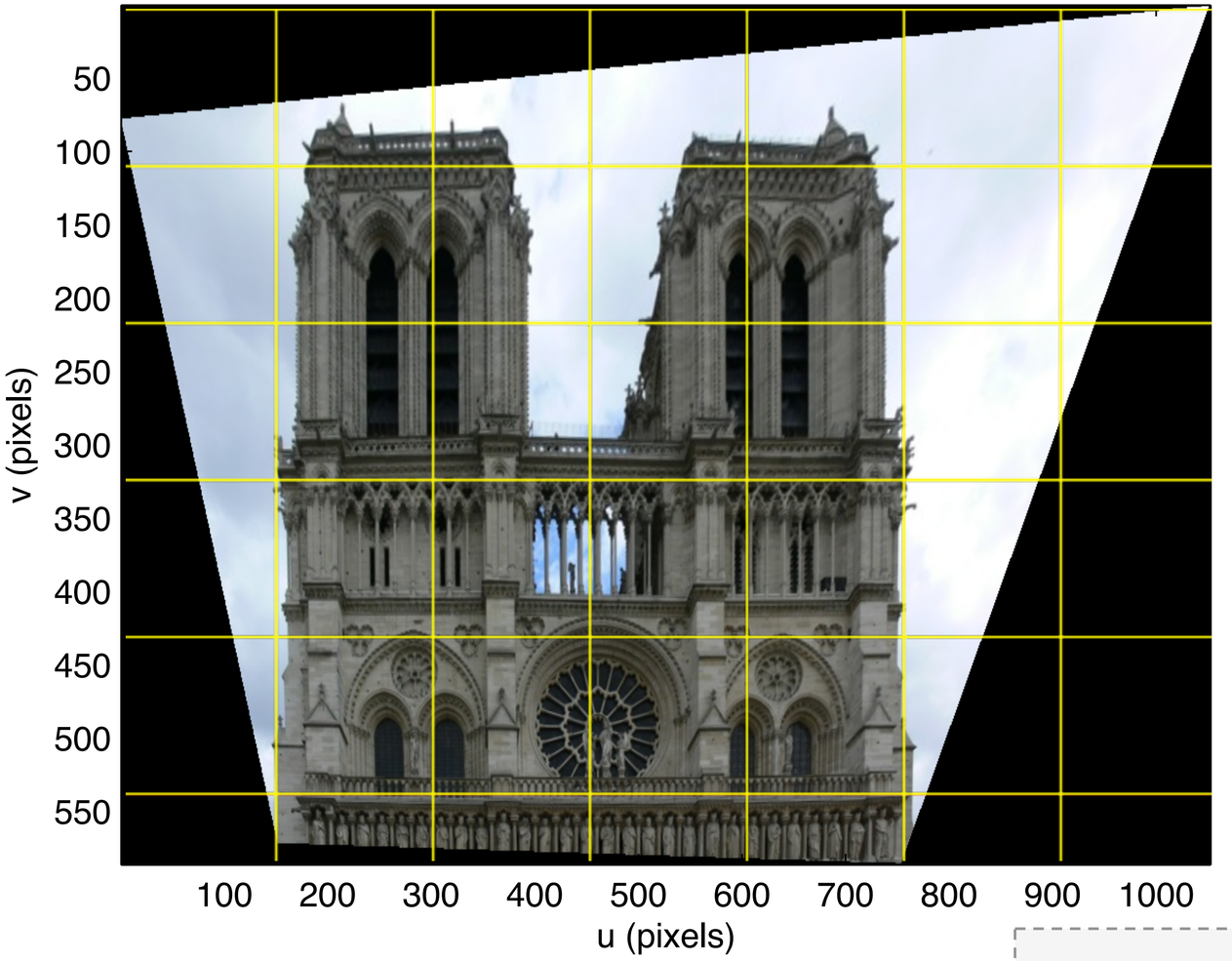
Perspective rectification



$$q = \mathbf{H}p$$

```
>>> H, _ =
CentralCamera.points2H(P, Q)
H=
 1.4003   0.3827 -136.5900
-0.0785   1.8049  -83.1054
-0.0003   0.0016   1.0000
```

Perspective rectification



Corke, P. | Reproduced with permission from Springer Science, & Business Media

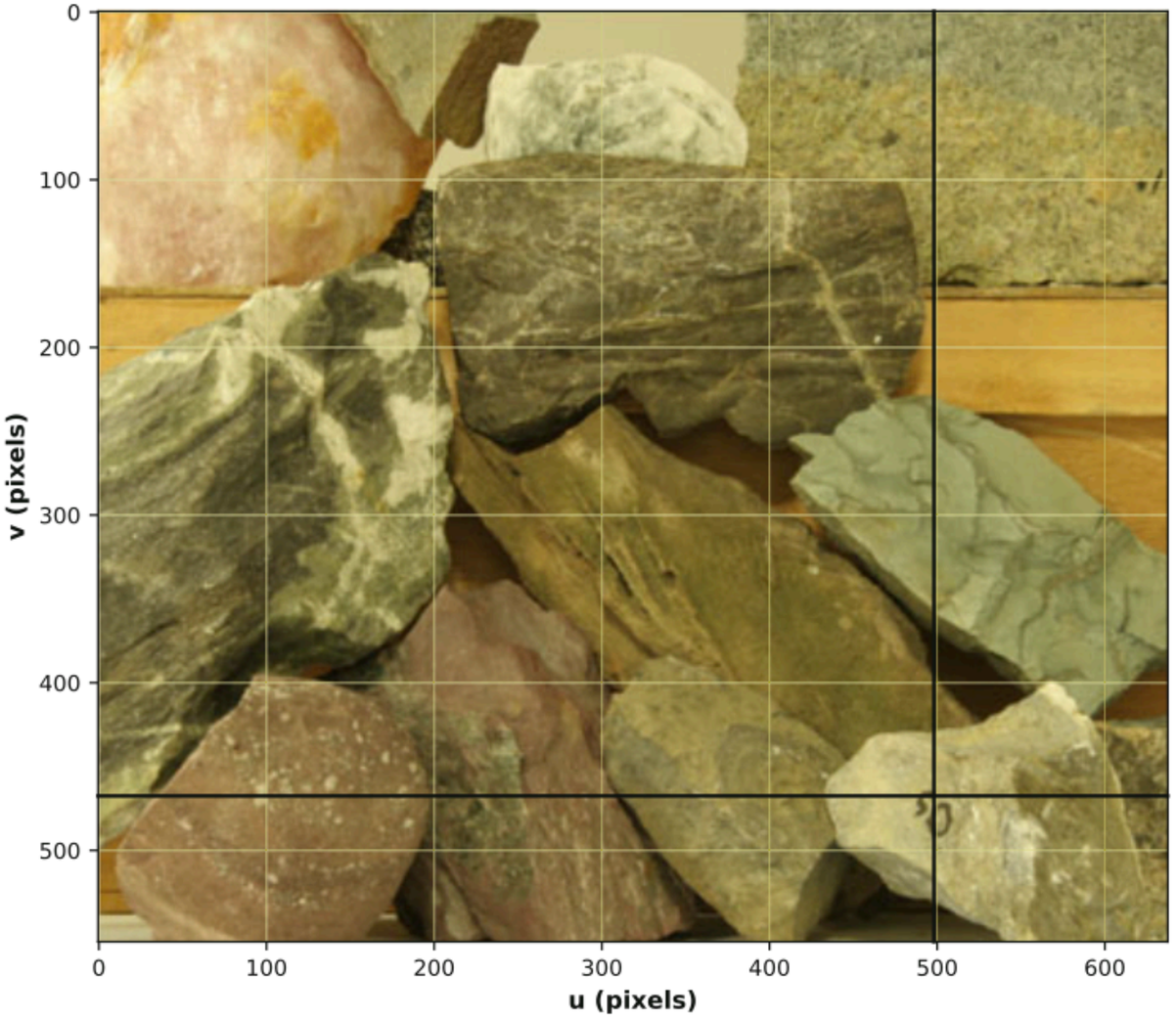
$$q = \mathbf{H}p$$

```
>>> im.warp_perspective(H).disp()
```

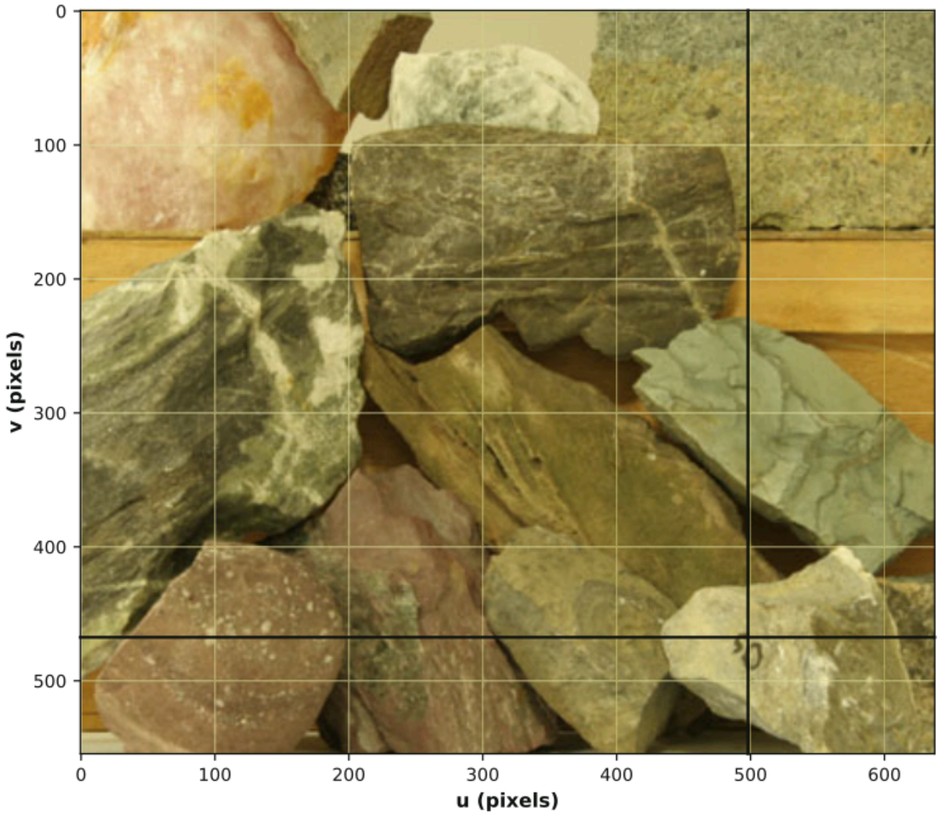

NEVO 1
TANCOK 2
MANAUDOU 3
BOVELL 4
TO 5
MOROZOV 6
DERLYUGOV 7
CHEAH 8

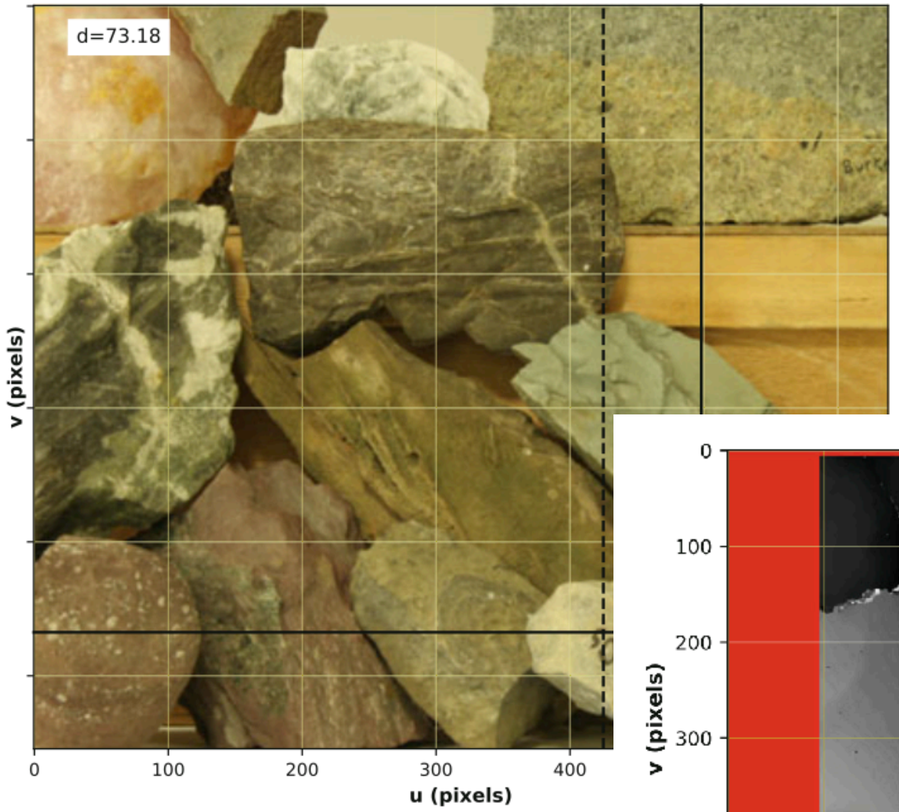
Stereo vision

How computational stereo works



How computational stereo works





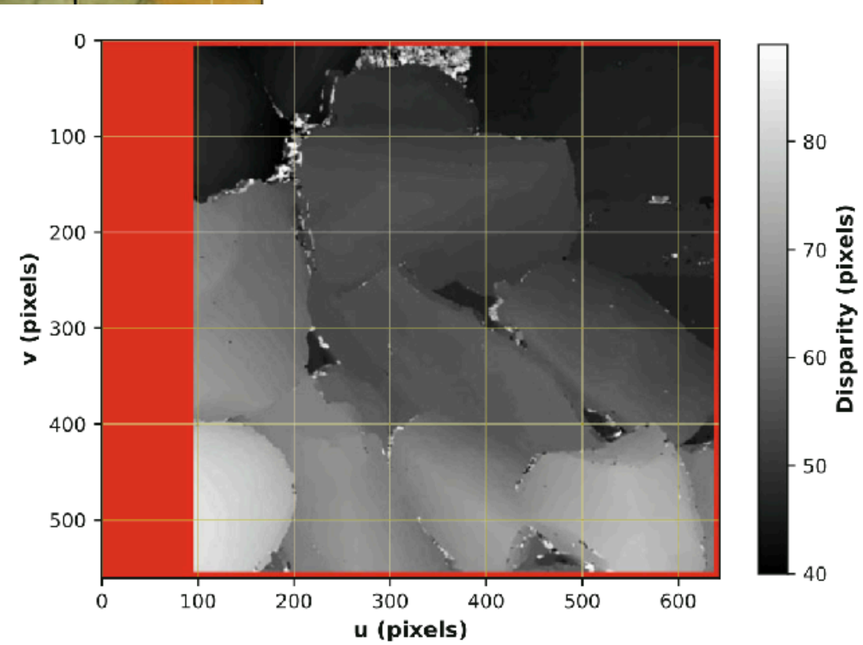


Fig. 14.25 Disparity image for the rock pile stereo pair, where brighter means higher disparity or shorter range. Pixels with a value of nan, shown in red, are where disparity could not be computed. Note the quantization in gray levels since we search for disparity in steps of one pixel

How computational stereo works

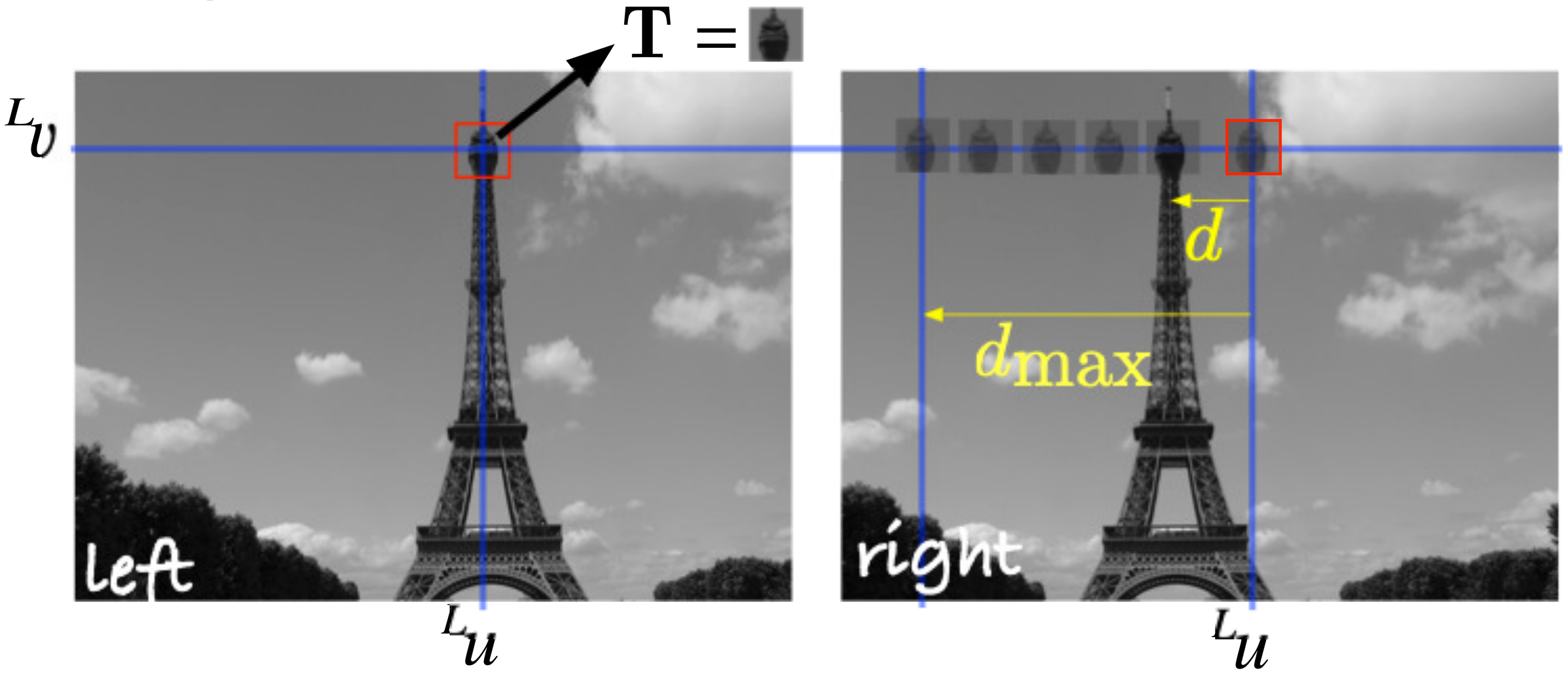


Fig. 14.24 Stereo matching. A search window in the right image, starting at $u = {}^Lu$, is moved leftward along the horizontal epipolar line $v = {}^Lv$ until it best matches the template window $\mathbf{T}$ from the left image

How computational stereo works

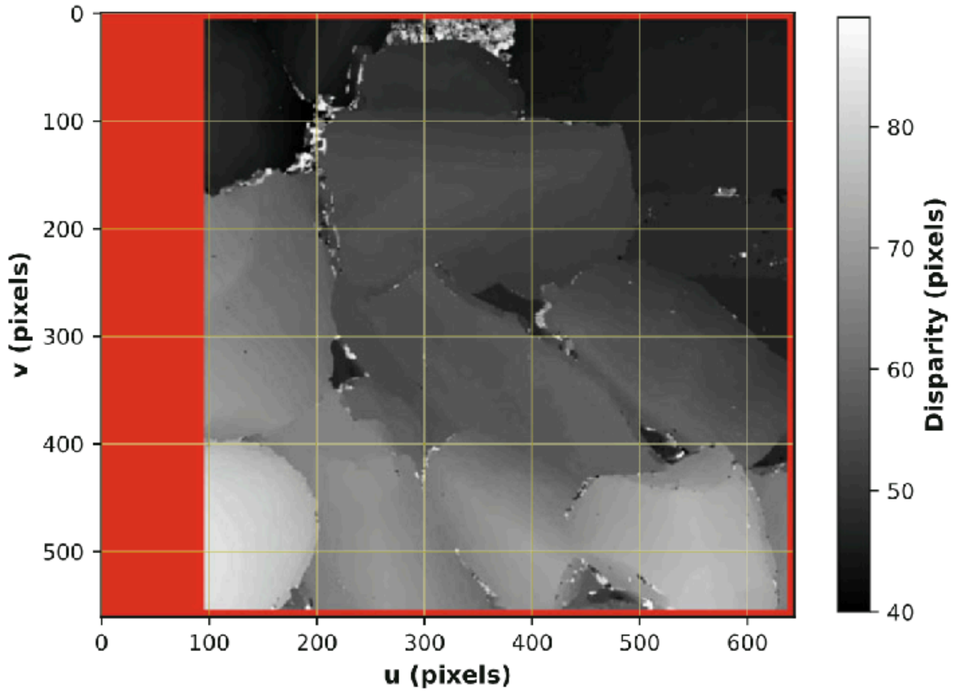
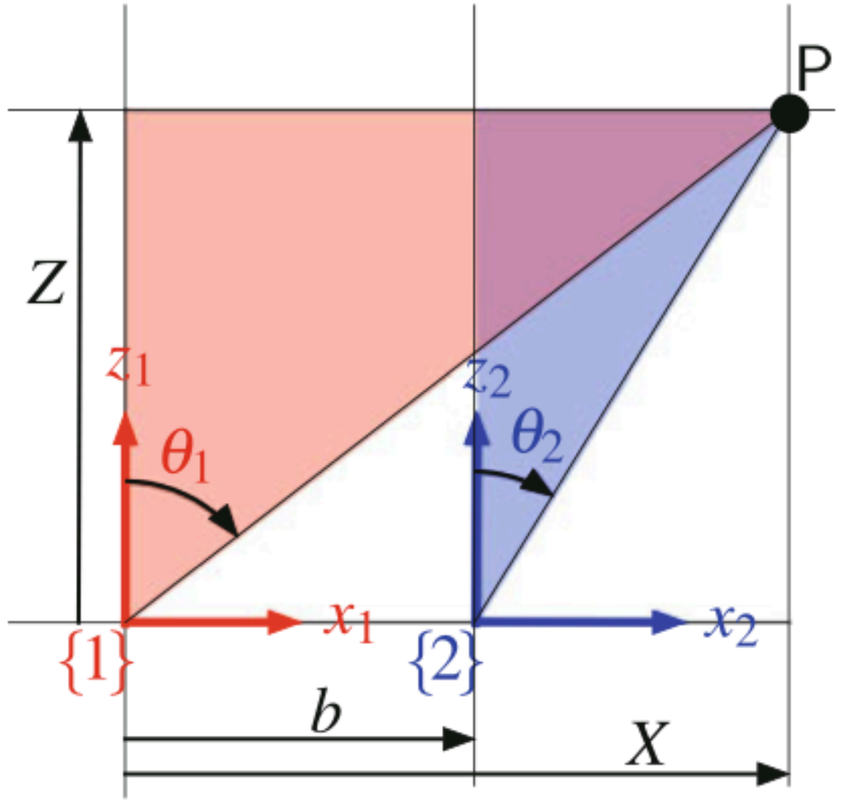


Fig. 14.25 Disparity image for the rock pile stereo pair, where brighter means higher disparity or shorter range. Pixels with a value of nan, shown in red, are where disparity could not be computed. Note the quantization in gray levels since we search for disparity in steps of one pixel



$$\boldsymbol{P} = \frac{b}{d}\left({}^{L}u - u_0, {}^{L}v - v_0, \frac{f}{\rho_u}\right)$$

RealSense camera

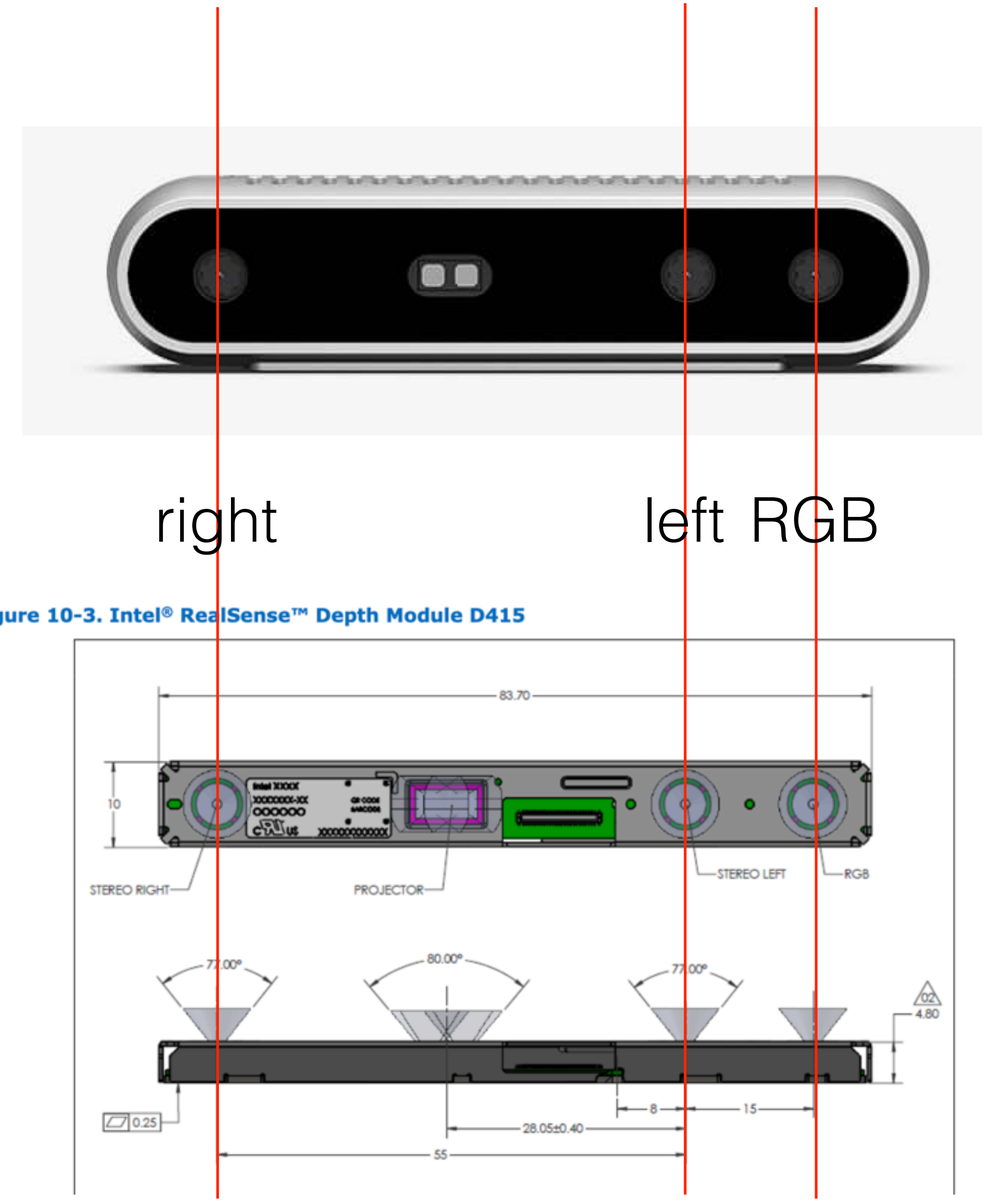
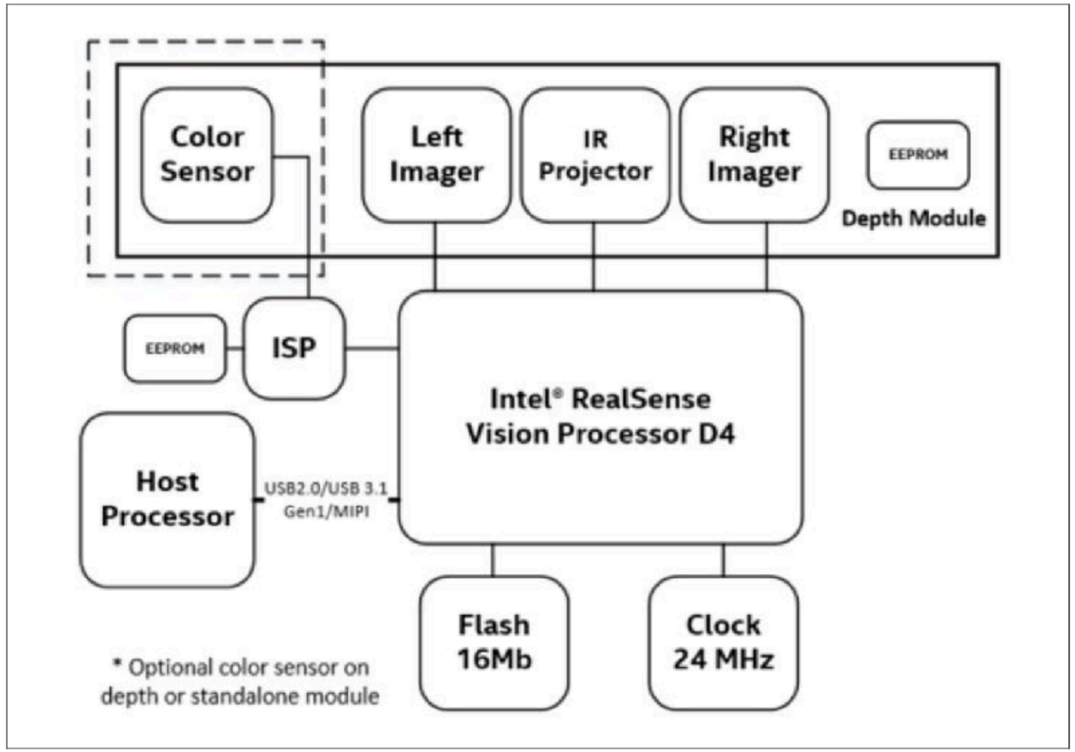
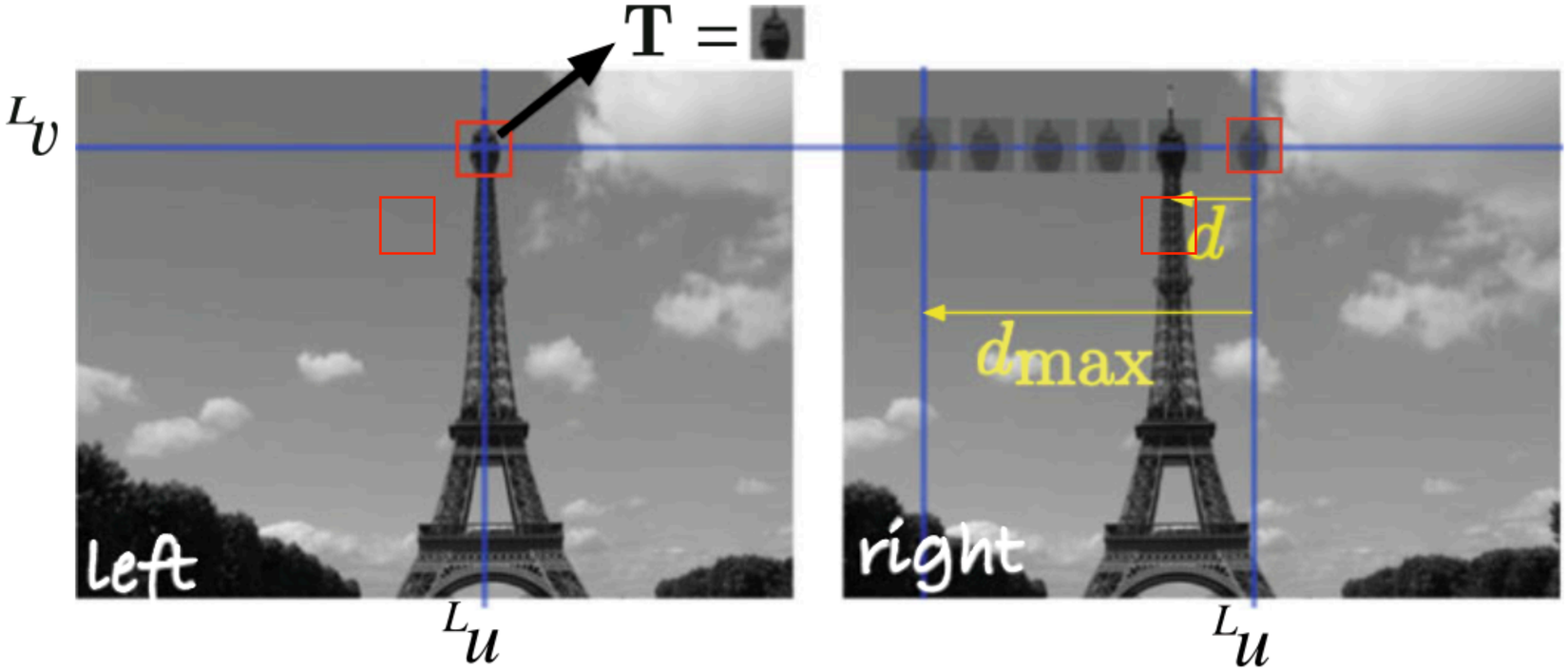


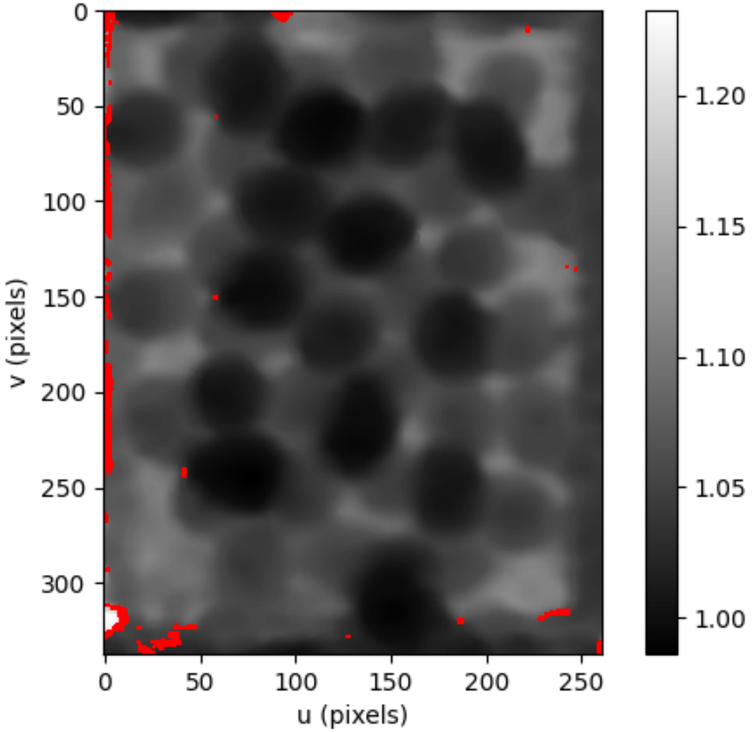
Figure 10-3. Intel® RealSense™ Depth Module D415

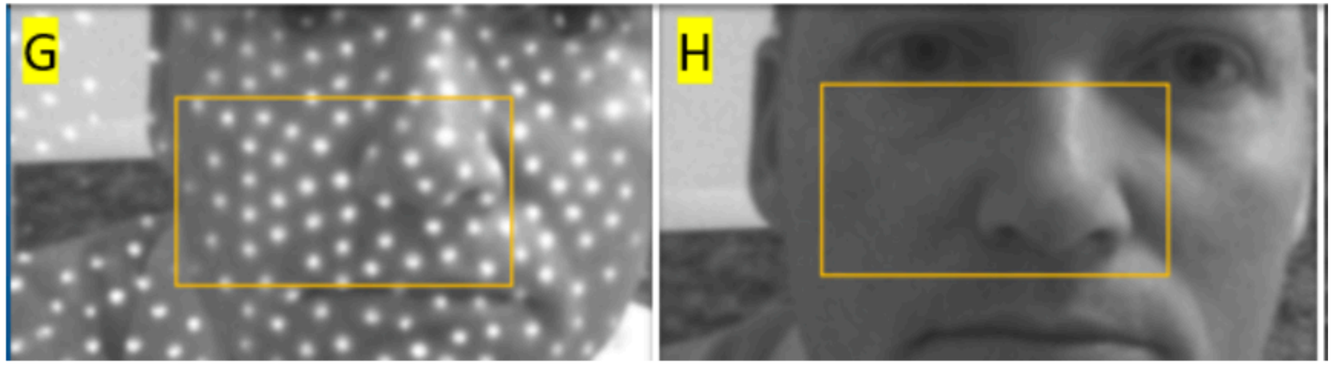
Figure 2-3. Vision Processor D4 Camera System Block Diagram



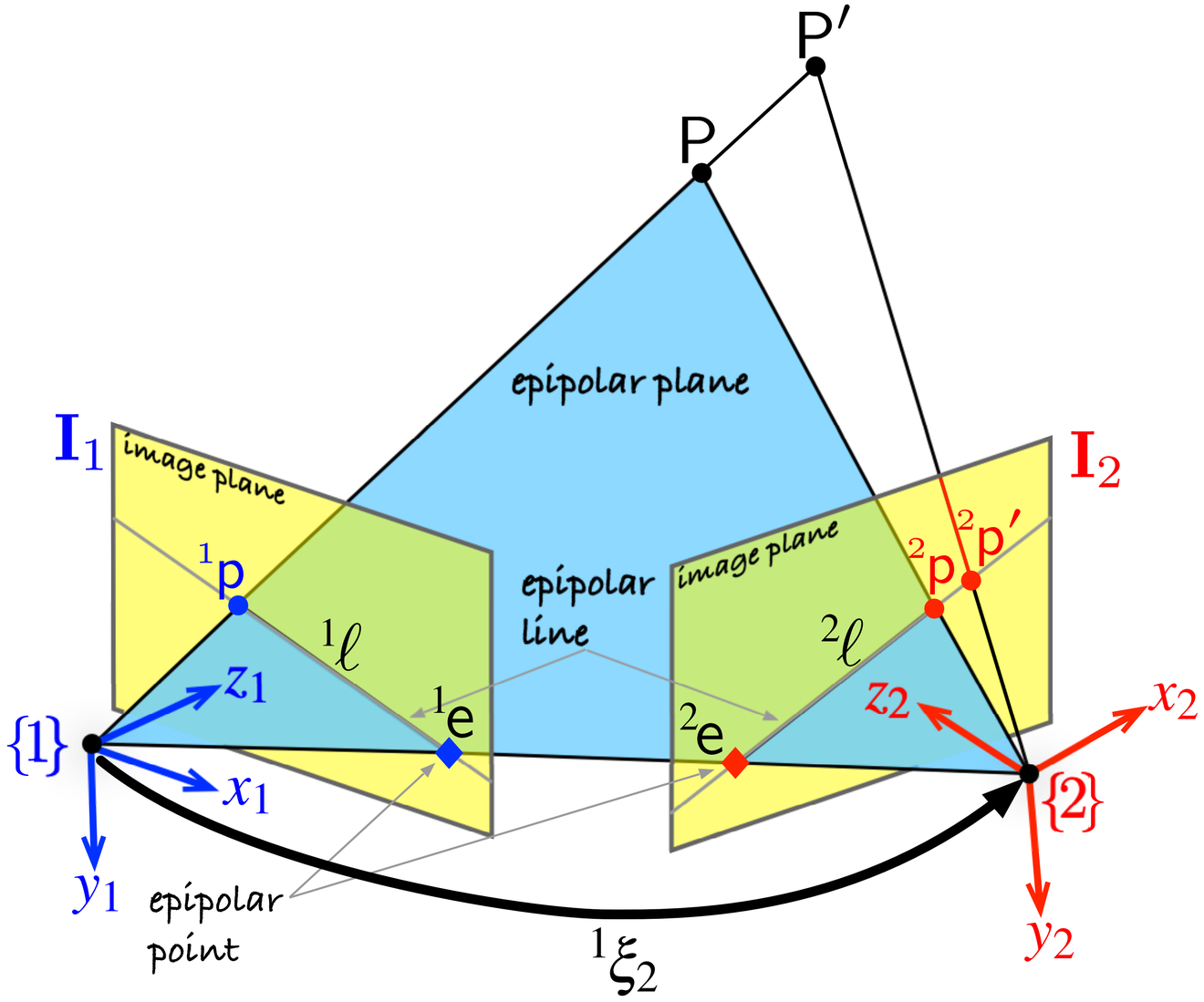
We need texture







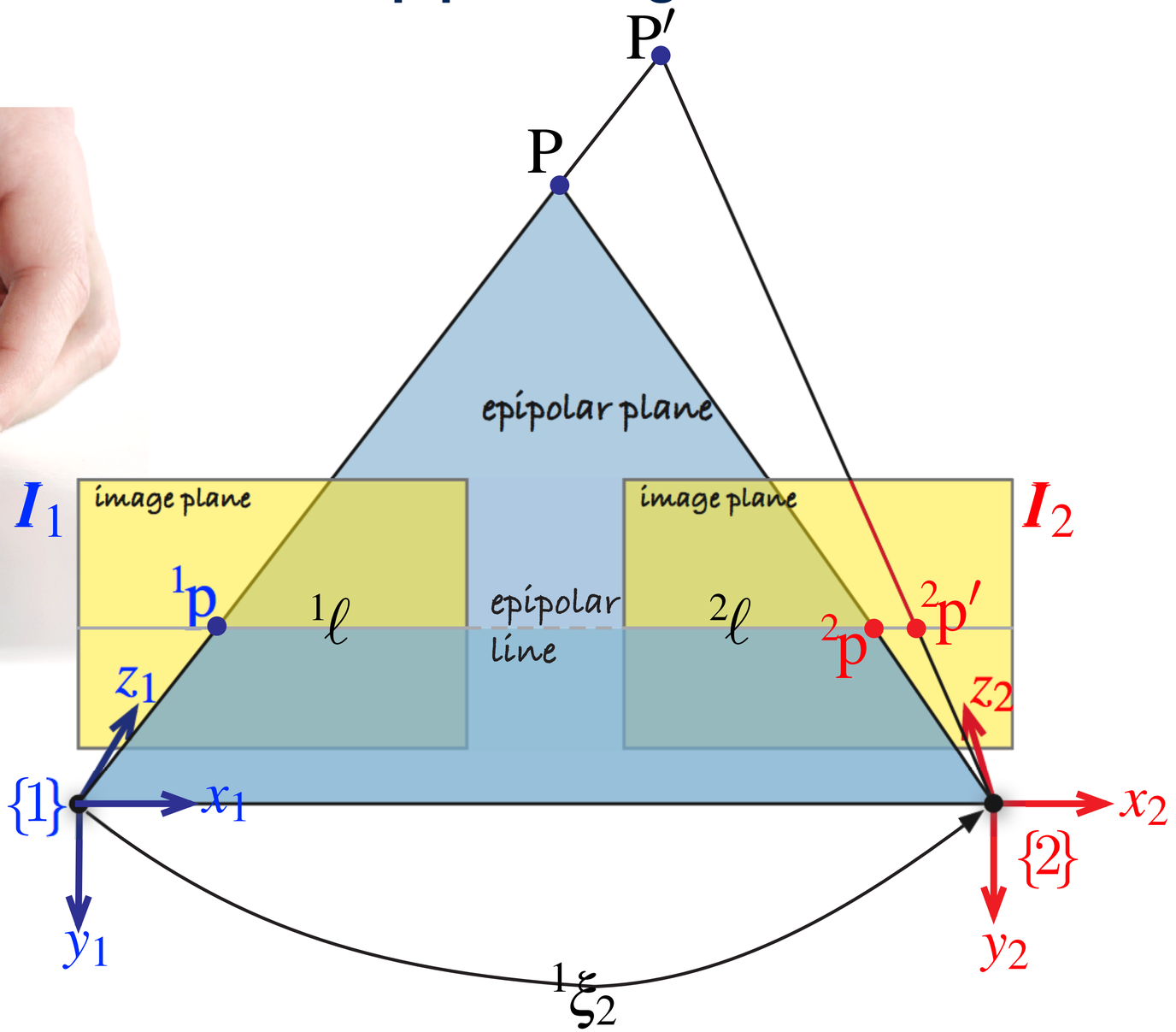
For two camera views of the same scene, a point in the left image will correspond to a point along the epipolar line in the right image



This means we only need to do a 1D search for the corresponding point.

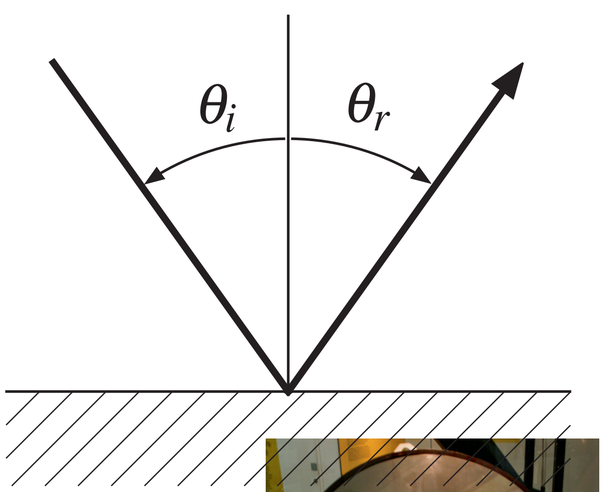
Major computational win!

If the cameras are carefully aligned the epipolar lines become horizontal. We can transform images from non-aligned cameras into epipolar aligned virtual views — this is image rectification.

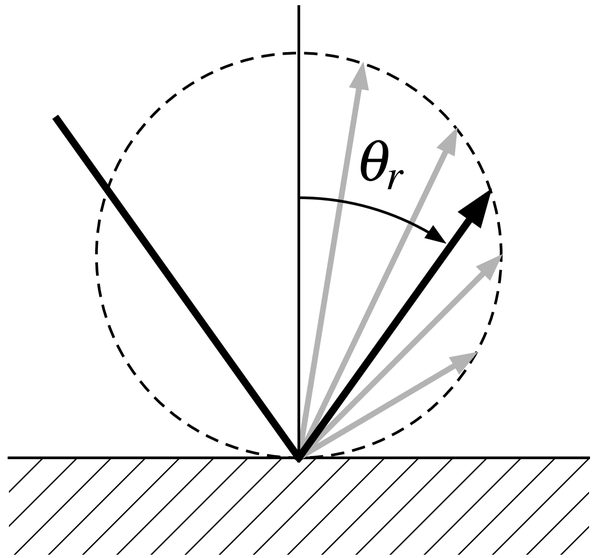


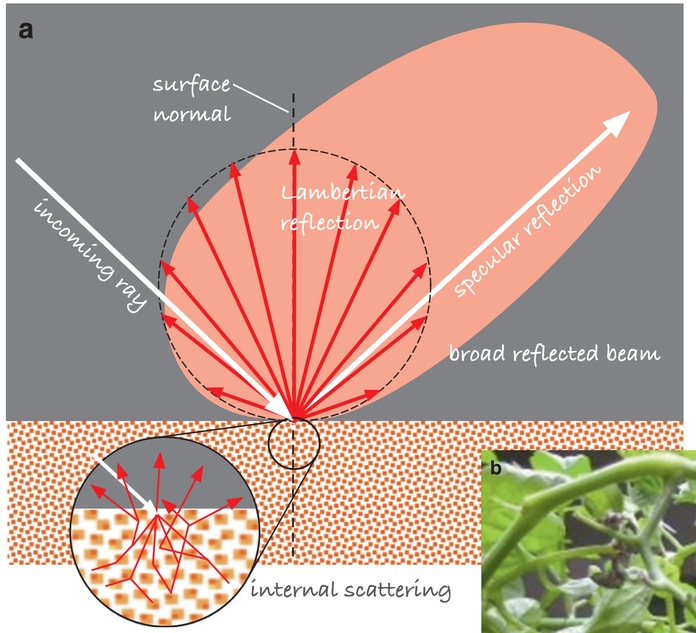
and then there is the real world…

Surface reflection is not simple. Many algorithms assume Lambertian reflection, most surfaces are not Lambertian…



speculum metal
⅔ Cu, ⅓Sn





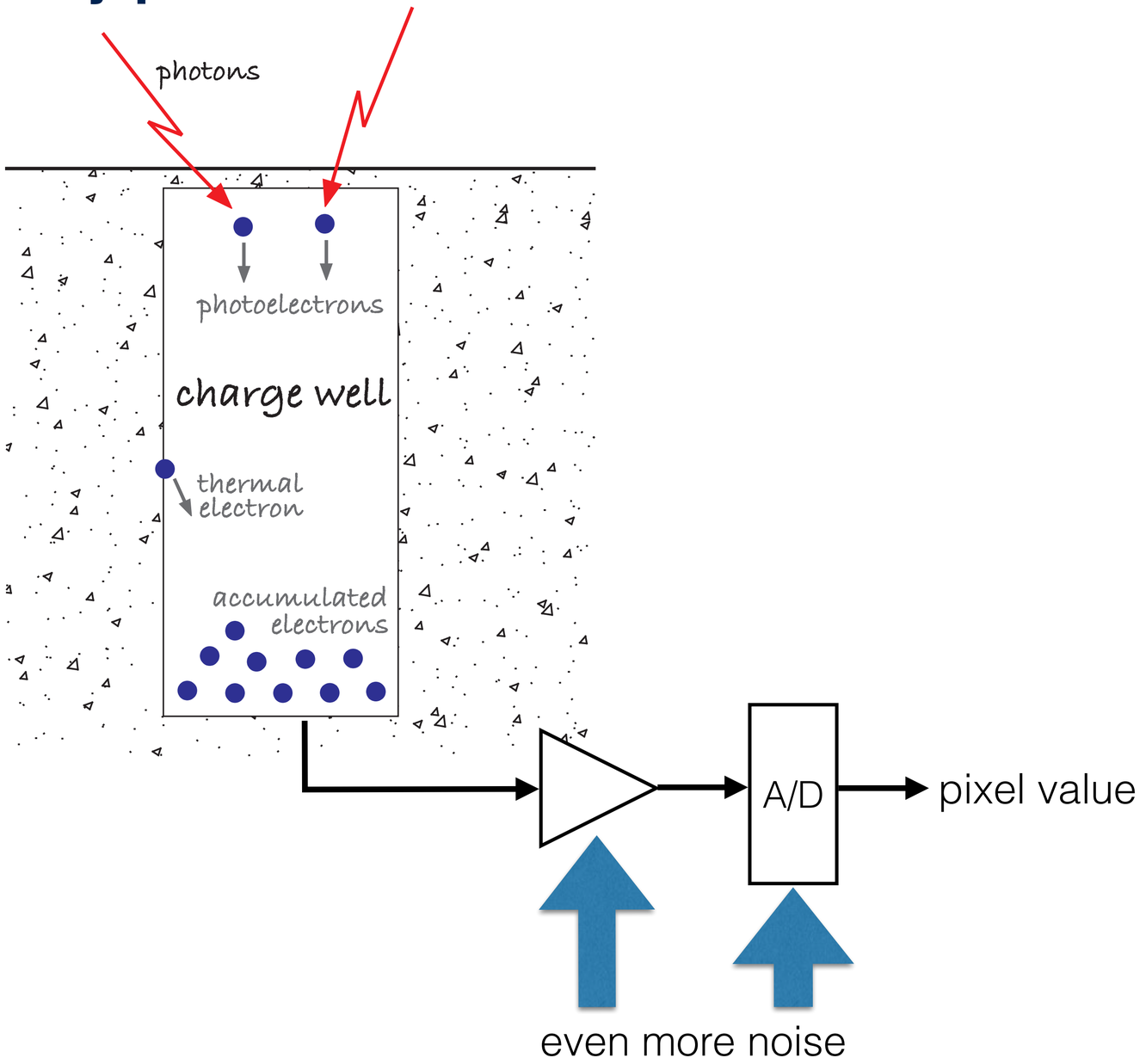
Specular Lambertian Dichromatic

A camera is fundamentally a very noisy process

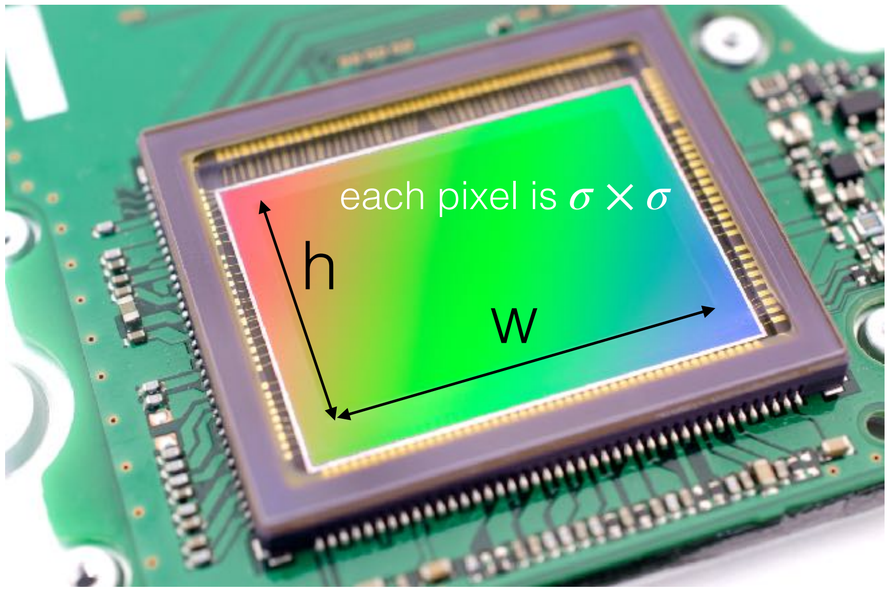
photon arrival

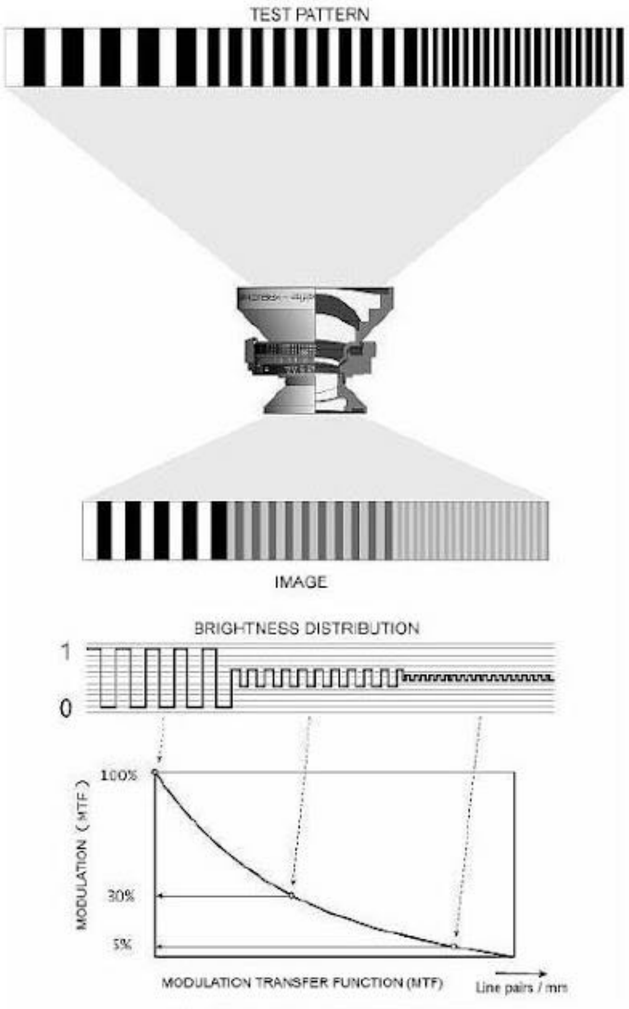
photons are discrete, the mean and variance over an interval is proportional to luminance

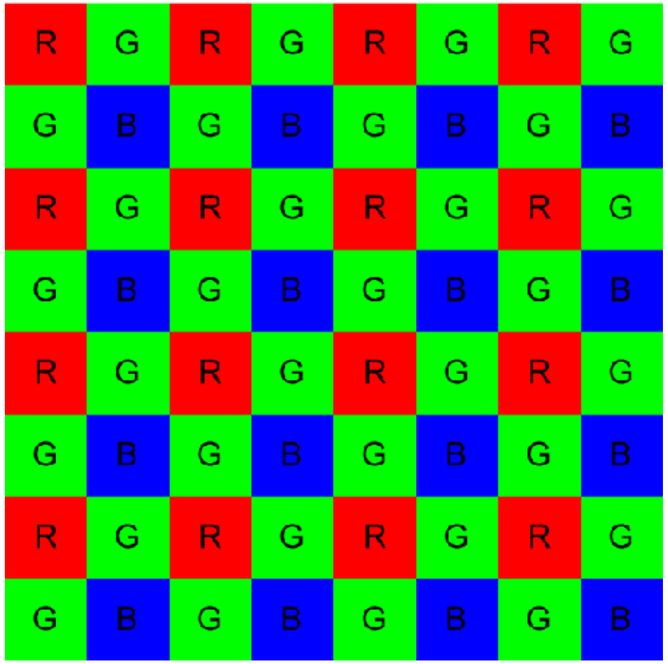
photons convert to electrons in a probabilistic way



Camera resolution is often taken as just the number of pixels, but more is not necessarily better







aspect ratio = $w : h$

linear resolution = $\frac{w}{\sigma} \times \frac{h}{\sigma}$

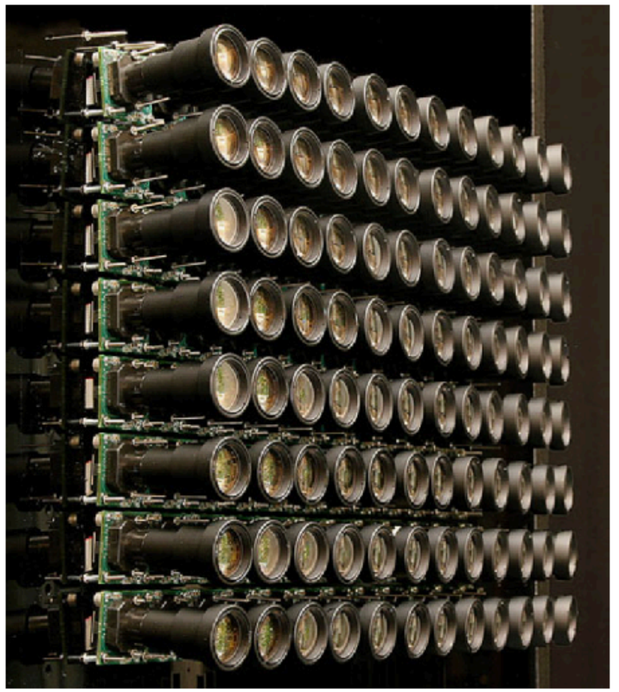
number of pixels = $\frac{wh}{\sigma^2}$

pixel area = σ^2

There are many other sorts of imaging systems

fisheye lens

catadioptric



multi-aperture