# DS 3021 EDA Assignment Q1

1. Show that $m(a+bX) = a + b \times m(X)$

- Sample mean of a variable $X \Rightarrow m(x) = \frac{1}{N}\sum_{i=1}^{N} x_i$
- Transformation $Y = a + bX \Rightarrow$
- $m(Y) = m(a+bX) = \frac{1}{N}\sum_{i=1}^{N}(a+bX_i) \Rightarrow m(Y) = \frac{1}{N}\sum_{i=1}^{N} a + \frac{1}{N}\sum_{i=1}^{N} bX_i$

  $\hookrightarrow m(Y) = a + b \times \frac{1}{N}\sum_{i=1}^{N} x_i = a + bm(X)$

  $\hookrightarrow m(a+bX) = a + bm(X)$

2. Show that $\text{cov}(X, a+bY) = b \times \text{cov}(X,Y)$

- covariance of $X$ & $Y \Rightarrow \text{cov}(X,Y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(y_i - m(y))$
- consider transformation $Z = a + bY$
- $\text{cov}(X,Z) = \text{cov}(X, a+bY) = \text{cov}(X, bY)$ → because shifting a variable by a constant a does not affect covariance

  $\hookrightarrow \text{cov}(X, a+bY) = b \times \text{cov}(X,Y)$

3. Show that $\text{cov}(a+bX, a+bX) = b^2\text{cov}(X,X)$, and in particular that $\text{cov}(X,X) = s^2$

- Sample variance of $X \Rightarrow \text{cov}(X,X) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))^2 = s^2$
- consider transformation $Z = a + bX$
- $\text{cov}(Z,Z) = \text{cov}(a+bX, a+bX) = b^2\text{cov}(X,X)$
- Since $\text{cov}(X,X) = s^2 \Rightarrow \text{cov}(a+bX, a+bX) = b^2 s^2$

4. Is a non-decreasing transformation of the median the median of the transformed variable?

- Yes, because the median is based on the ordering of values and non-decreasing transformation preserves that order. For example, if $X$ has a median $M$, then applying a non-decreasing function $g(x)$ ensures that values larger & smaller than $M$ remain larger & smaller. (applies to $2+5X$ or $\text{arcsinh}(x)$)
- applies to all quantiles since they also preserve order
- But, IQR & range may not always apply if distances between values change non-linear. (For linear transformations the median is the same)

5. Is it always true that $m(g(x)) = g(m(x))$?

- if $g(x)$ is a linear function, like $g(x) = 2+5x$, then $g(m(x)) = m(g(x))$
- for non-linear transformations this is not always true.