

23.1-6

1) Let's assume there are two MST T_1 and T_2 of the graph G . Let (u, v) be an arbitrary edge of T_1 . Consider there is a cut $(S, V-S)$ across (u, v) and (x, y) be the unique light edge cross this cut. If (x, y) not equal to (u, v) , then the spanning tree $\{T_1 \text{ minus } (u, v) \text{ plus } (x, y)\}$ has a lower weight than T_1 , which means T_1 is not MST, therefore, $(u, v) = (x, y)$. (or by Exercise 23.1-3)

Now consider there is an edge (m, n) in T_2 crossing $(S, V-S)$. using same theorem above, (m, n) must be equal to (x, y) . In another word, for the cut $(S, V-S)$, the edge (u, v) and (m, n) must be the same edge. As we choose (u, v) arbitrarily from T_1 , every edge in T_1 will be also be in T_2 . Therefore, T_1 and T_2 has to be same.

2) in this graph, the orange cut is the counterexample

