1) Textbook page 1100, Exercise 34.5-1.

The subgraph-isomorphism problem takes two undirected graphs G1 and G2, and it asks whether G1 is isomorphic to a subgraph of G2. Show that the subgraphisomorphism problem is NP-complete.

As the certificate use the one-to-one function f from the vertices of G_1 to the vertices of G_2. Thus the length of the certificate is $O(n)$. Finally, given the certificate the verification algorithm can confirm that f is a one-to-one function and then take each edge $(u,v) \in G_1$ and verify that $(f(u), f(v)) \in G_2$. Clearly this can be done in $O(n+m)$ time. We now prove that Clique $\leq_p$ subgraph isomorphism.

Let $<G,k>$ be the input for the clique. For the subgraph-isomorphism input we let G_1 be a complete graph on k vertices and we let $G_2 = G$. Clearly this can be done in polynomial time. To see this notice that we can assume $k \leq n$ (or otherwise, clearly G does not have a clique of size k) and thus the time to create G_1 is simply $O(k^2) = O(n^2)$ which is polynomial in the number of bits to represent G.

By definition of a clique being a complete graph on k vertices, there is a clique of size k in G if and only if G_1 is a subgraph of G_2. That is if there is a clique of k vertices in G then mapping the vertices of G_1 to those vertices in G_2 gives a solution for the subgraph isomorphism problem, then the vertices in G_2 mapped to by the vertices in G_1 form a clique of k vertices.