29.5 -5

The slack form of the equation will be given by : $z = x_1 + 3x_2$

$$x_3 = 8 - x_1 + x_2$$
$$x_4 = -3 + x_1 + x_2$$
$$x_5 = 2 + x_1 + 4x_2$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

The initial basic solution is *not feasible*, so form the auxiliary LP as follows maximize: $-x_0$

subject to :

$$x_1 - x_2 - x_0 \leq 8$$
$$-x_1 - x_2 - x_0 \leq -3$$
$$-x_1 + 4x_2 - x_0 \leq 2$$
$$x_1, x_2, x_0 \geq 0$$

Now, we write this LP in slack form:

$$z = -x_0$$
$$x_3 = 8 - x_1 + x_2 + x_0$$
$$x_4 = -3 + x_1 + x_2 + x_0$$
$$x_5 = 2 + x_1 - 4x_2 + x_0$$
$$x_1, x_2, x_3, x_4, x_5, x_0 \geq 0$$

PIVOT *entering* : x_0, *leaving* : x_4

$$z = -3 + x_1 + x_2 - x_4$$
$$x_0 = 3 - x_1 - x_2 + x_4$$
$$x_3 = 11 - 2x_1 + x_4$$
$$x_5 = 5 - 5x_2 + x_4$$
$$x_1, x_2, x_3, x_4, x_5, x_0 \geq 0$$

The basic solution is feasible.

PIVOT *entering* : x_1, *leaving* : x_0

$$z = -x_0$$
$$x_1 = 3 - x_0 - x_2 + x_4$$
$$x_3 = 5 + 2x_0 + 2x_2 - x_4$$
$$x_5 = 5 - 5x_2 + x_4$$
$$x_1, x_2, x_3, x_4, x_5, x_0 \geq 0$$

The basic solution is now optimal for the auxiliary LP, so now we update the objective function in the parent LP after we set x_0 to 0 :

$$z = 3 + 2x_2 + x_4$$
$$x_1 = 3 - x_2 + x_4$$
$$x_3 = 5 + 2x_2 - x_4$$
$$x_5 = 5 - 5x_2 + x_4$$
$$x_1, x_2, x_3, x_4, x_5, x_0 \geq 0$$

PIVOT *entering* : x_2, *leaving* : x_5

$$z = 5 + 7/5x_4 - 2/5x_5$$
$$x_2 = 1 + 1/5x_4 - 1/5x_5$$
$$x_1 = 2 + 4/5x_4 + 1/5x_5$$
$$x_3 = 7 - 3/5x_4 - 2/5x_5$$
$$x_1, x_2, x_3, x_4, x_5, x_0 \geq 0$$

PIVOT *entering* : x_4, *leaving* : x_3

$$z = 64/3 - 7/3x_3 - 4/3x_5$$
$$x_4 = 35/3 - 5/3x_3 - 2/3x_5$$
$$x_2 = 10/3 - 1/3x_3 - 1/3x_5$$
$$x_1 = 34/3 - 4/3x_3 - 1/3x_5$$
$$x_1, x_2, x_3, x_4, x_5, x_0 \geq 0$$

The optimal solution is $(x_1, x_2) = (34/3, 10/3)$ and the optimal value of the objective function is $z = 64/3$.