

Algorithms

Lecture 7: DFS

Anxiao (Andrew) Jiang

CH. 22 Elementary Graph Algorithms

22.3 Depth-First Search (DFS)

Basic idea of DFS:

- 1) go as far as possible following newly discovered nodes
- 2) when there is nowhere to go (i.e., there is no undiscovered neighbor to go to),
go back to its parent node.
- 3) repeat the above two steps until all nodes are discovered.

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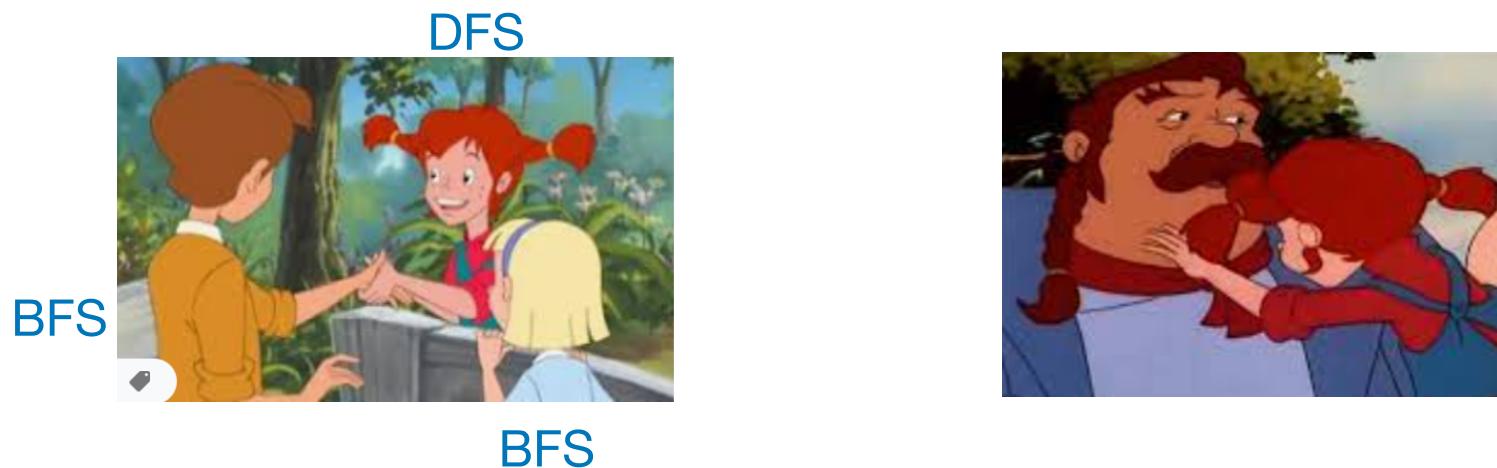


CH. 22 Elementary Graph Algorithms

22.3 Depth-First Search (DFS)

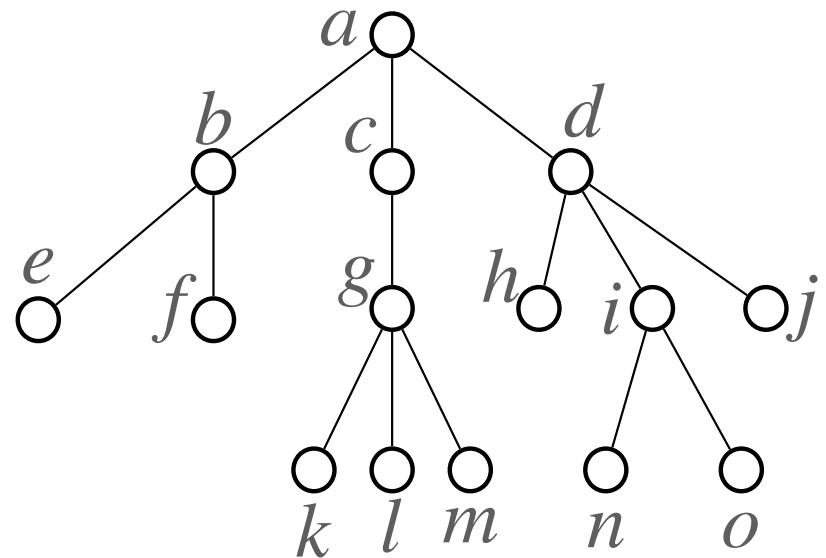
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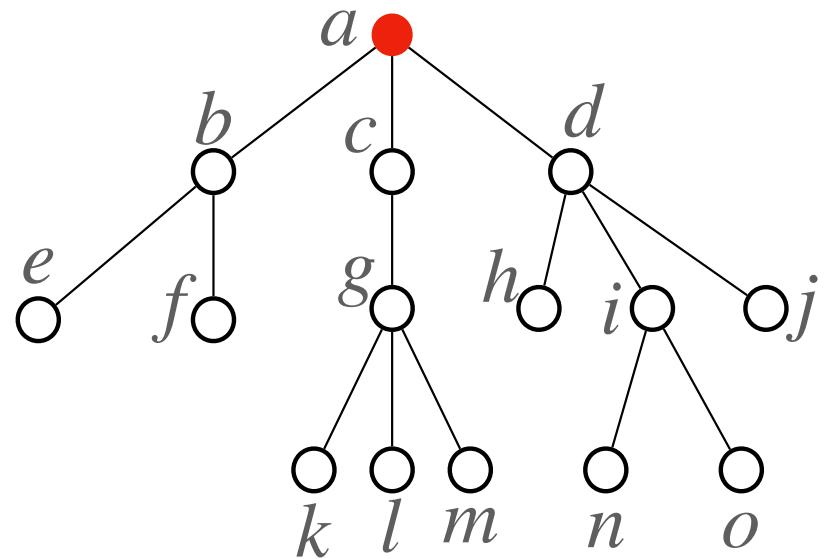
22.3 Depth-First Search (DFS)

Example of DFS on a tree:



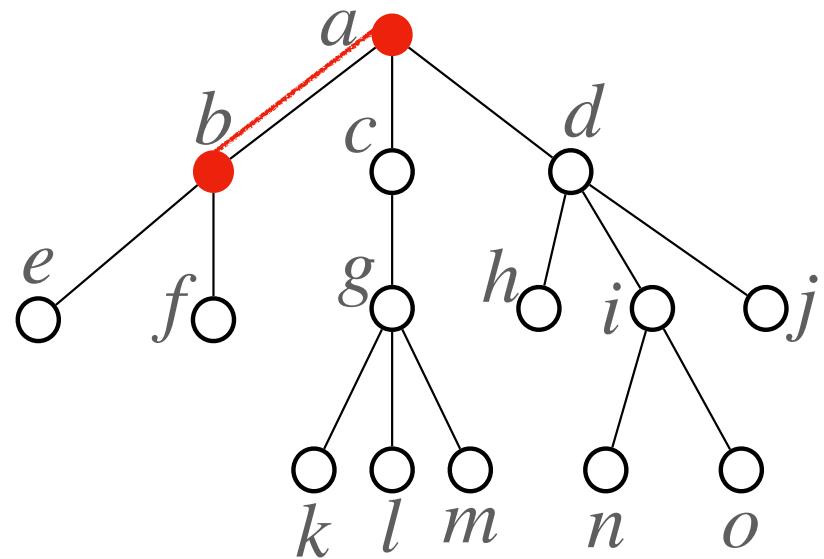
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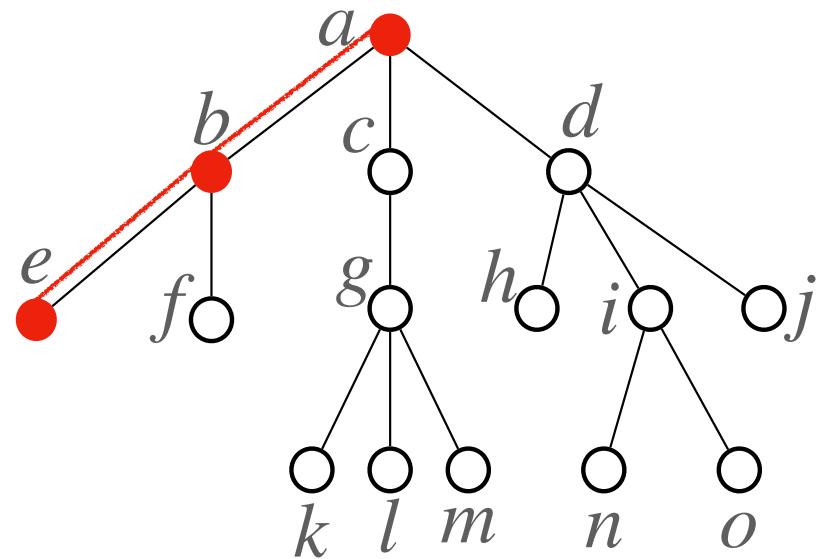
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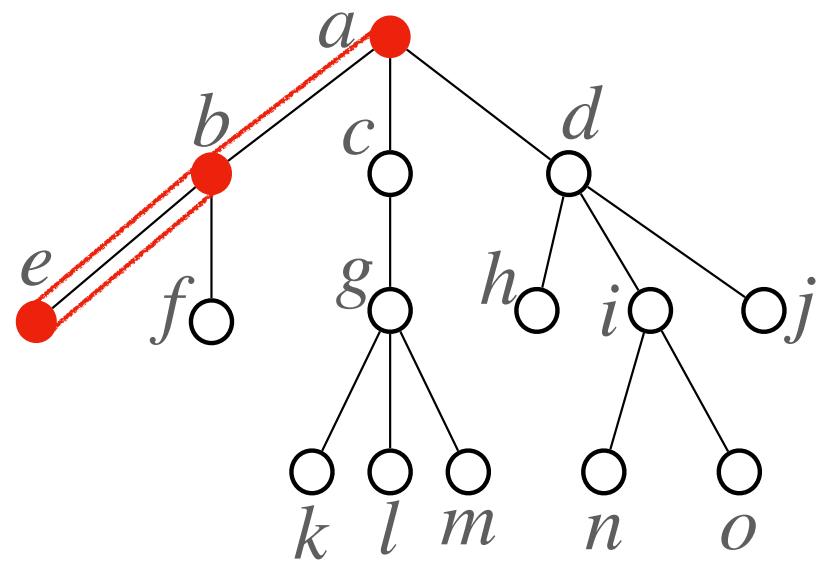
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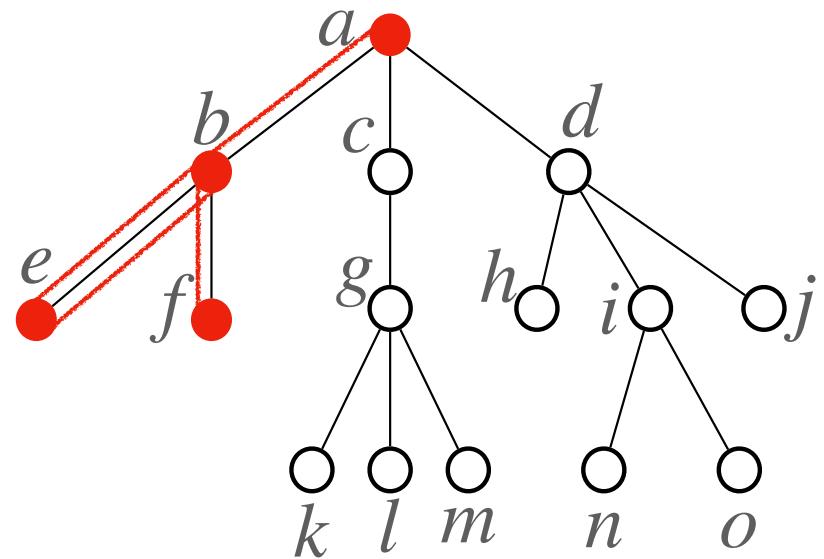
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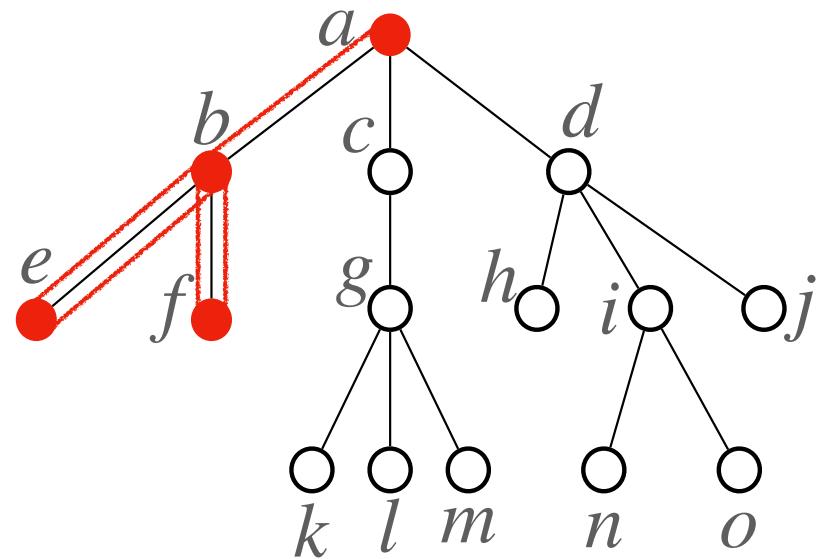
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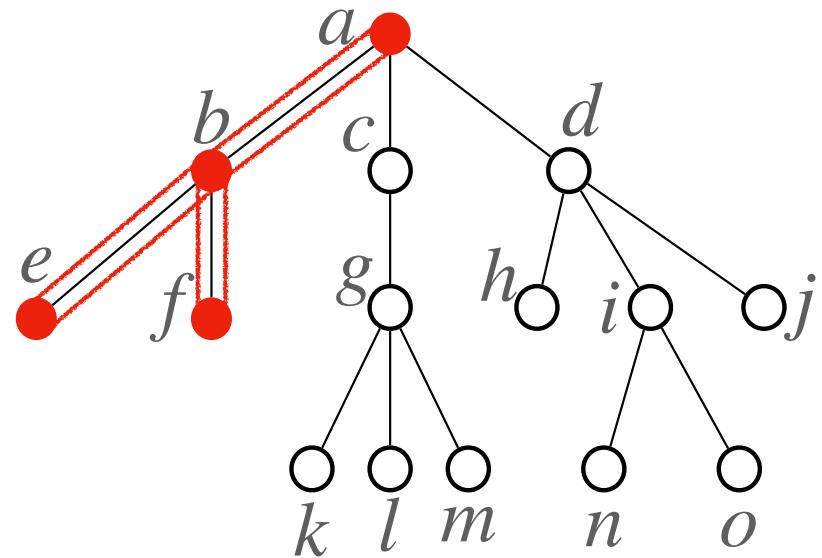
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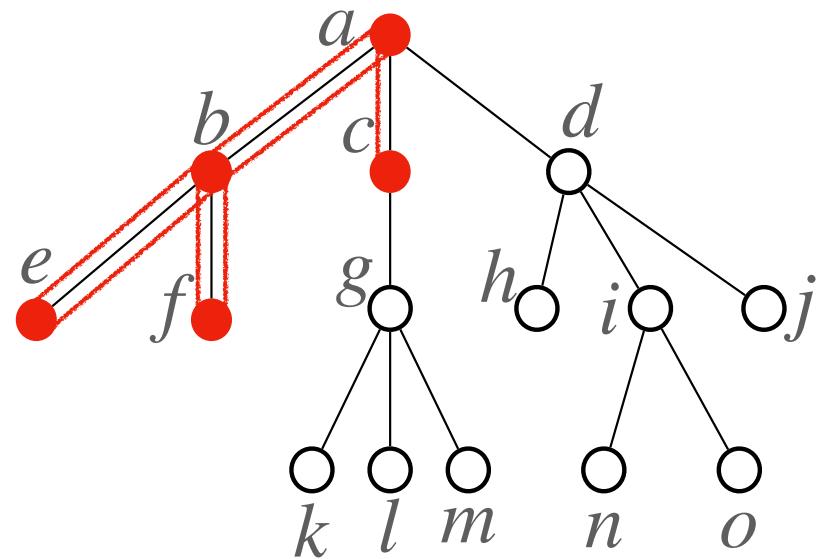
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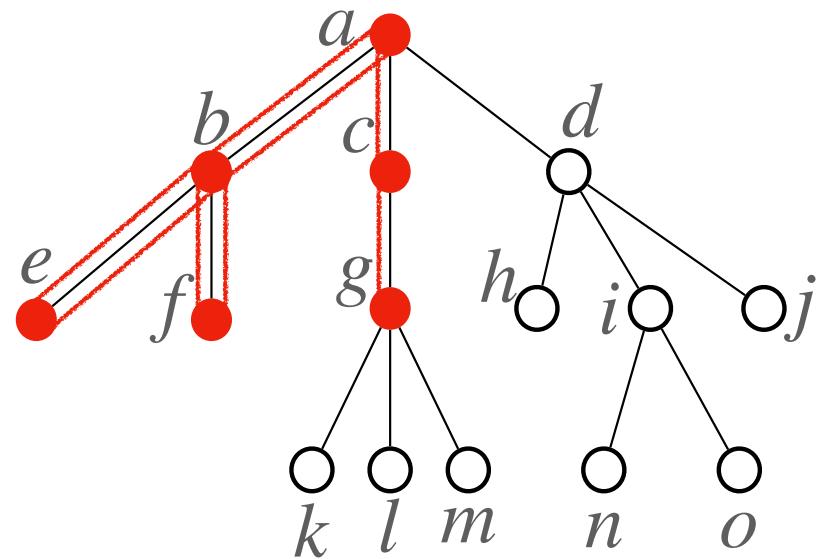
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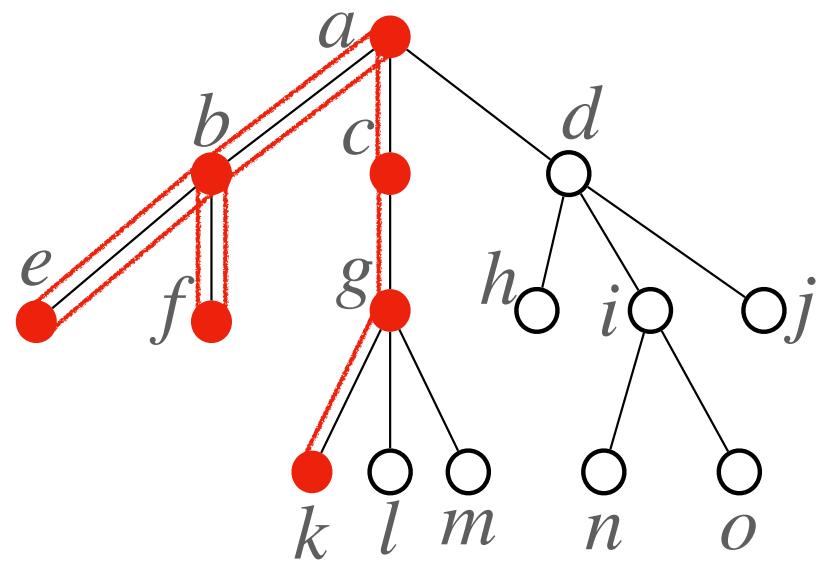
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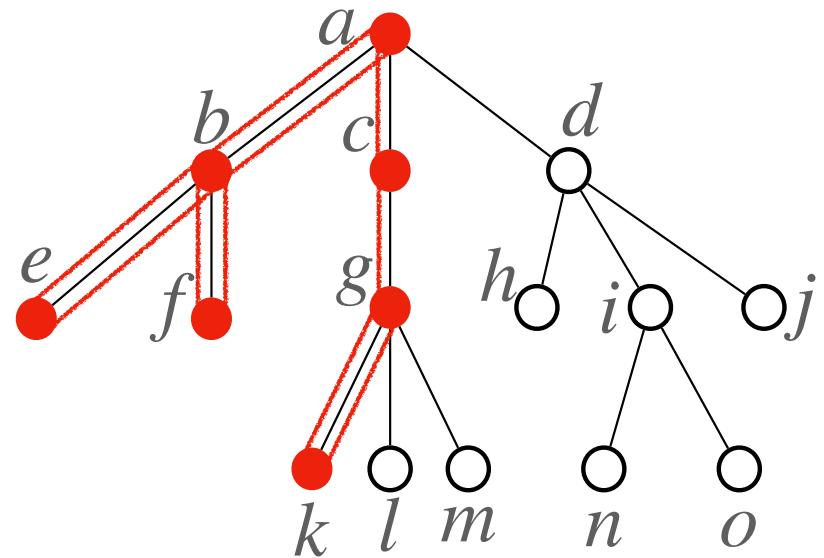
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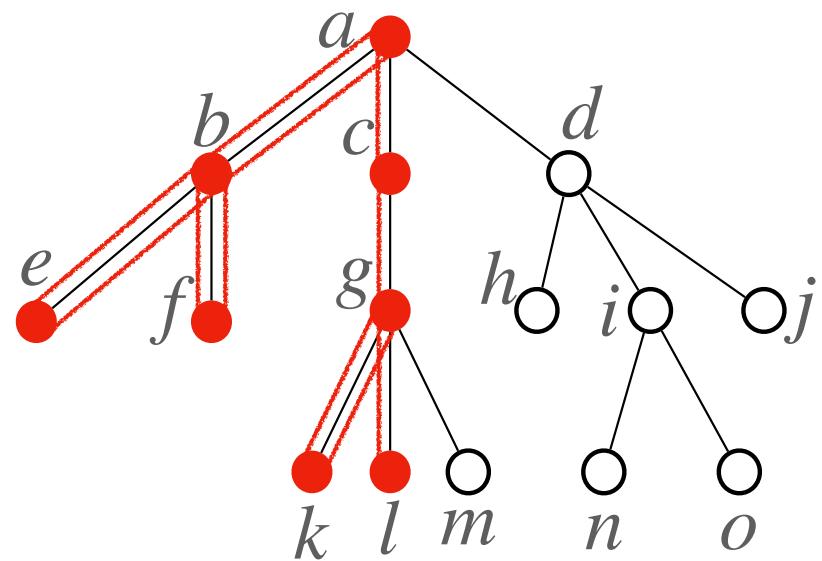
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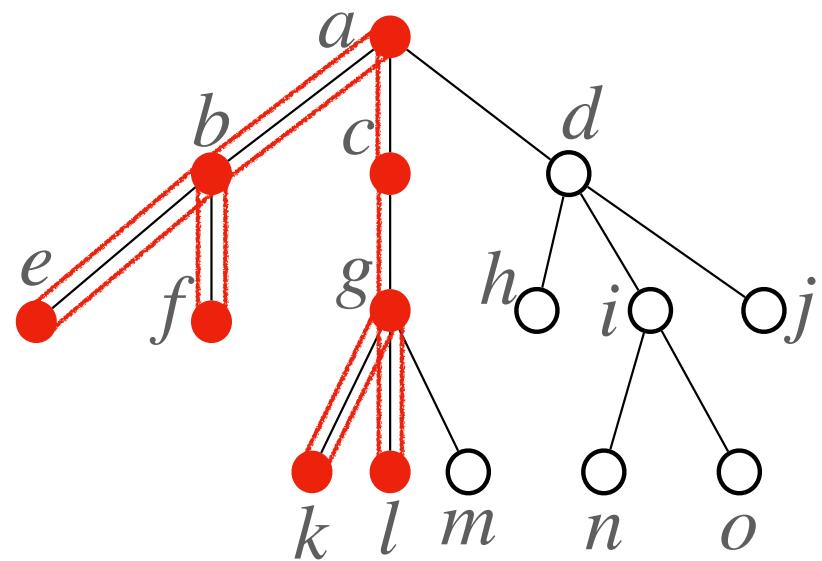
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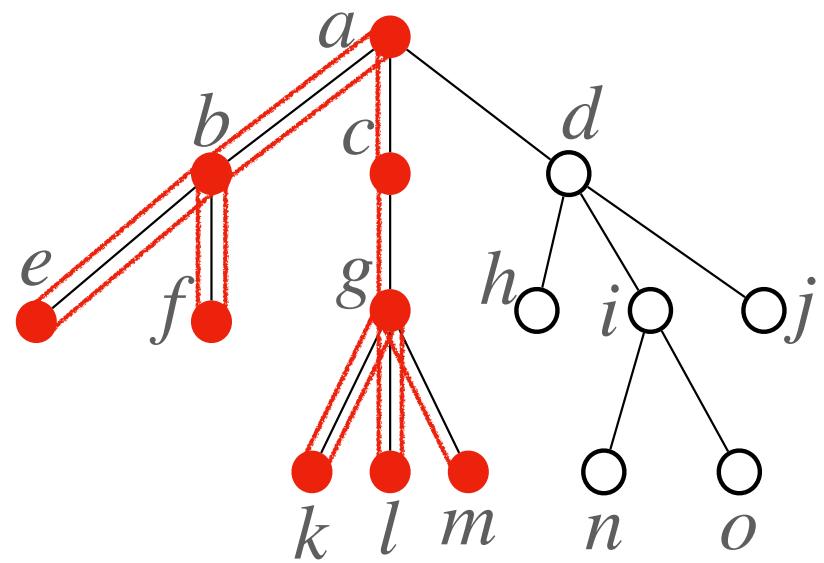
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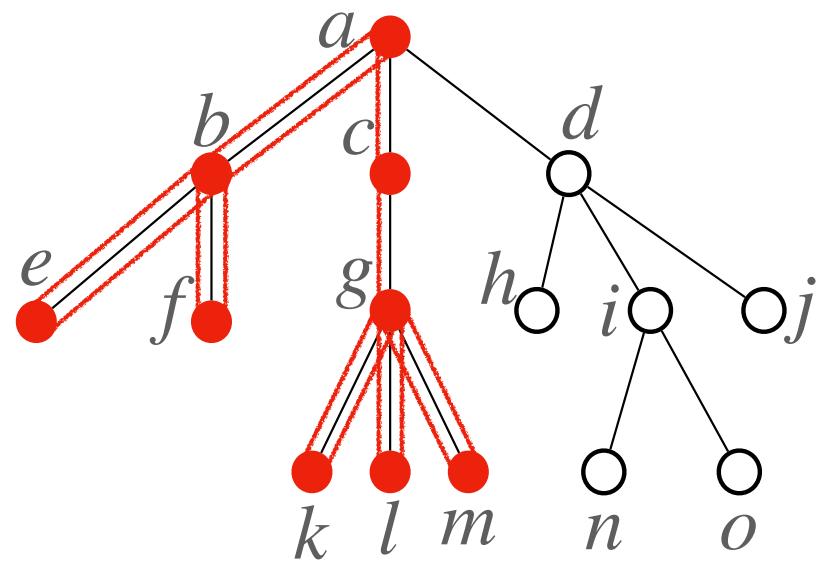
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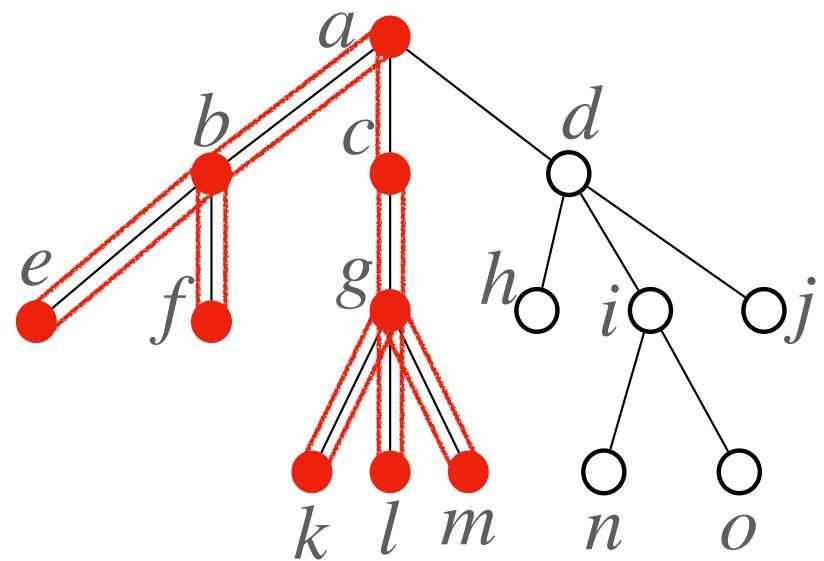
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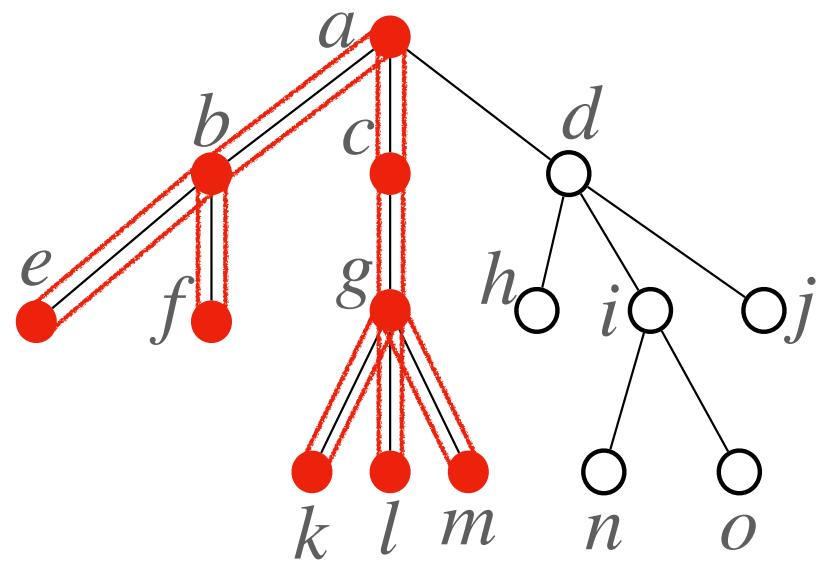
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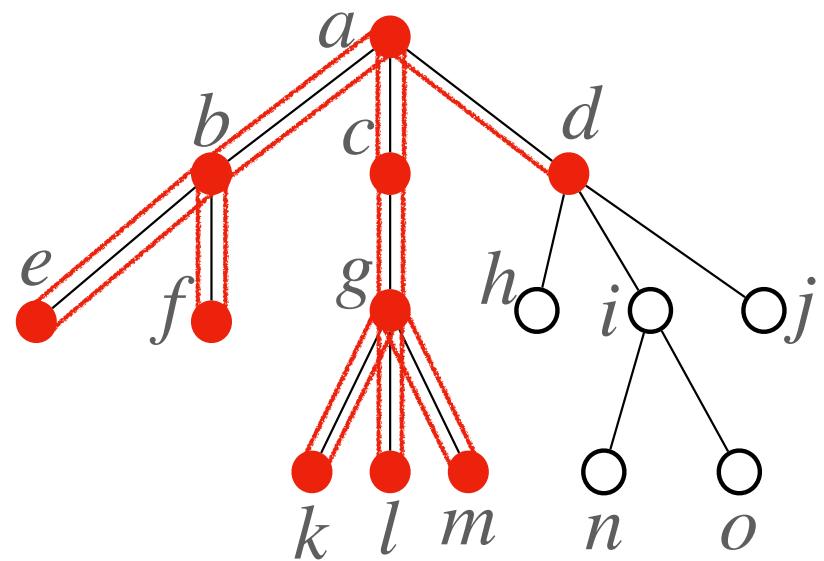
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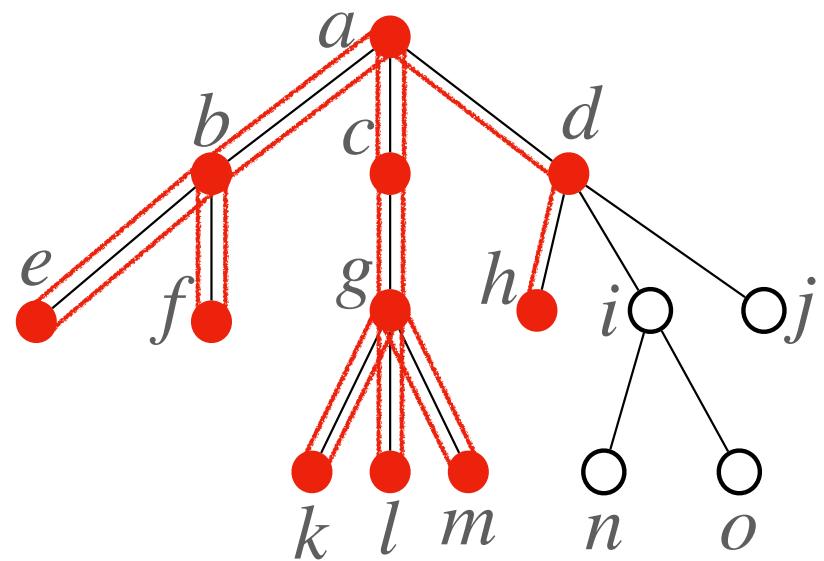
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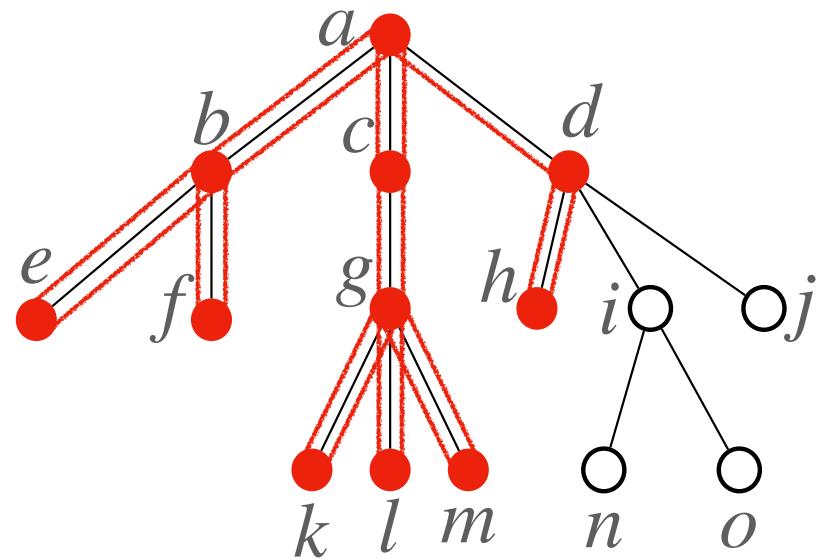
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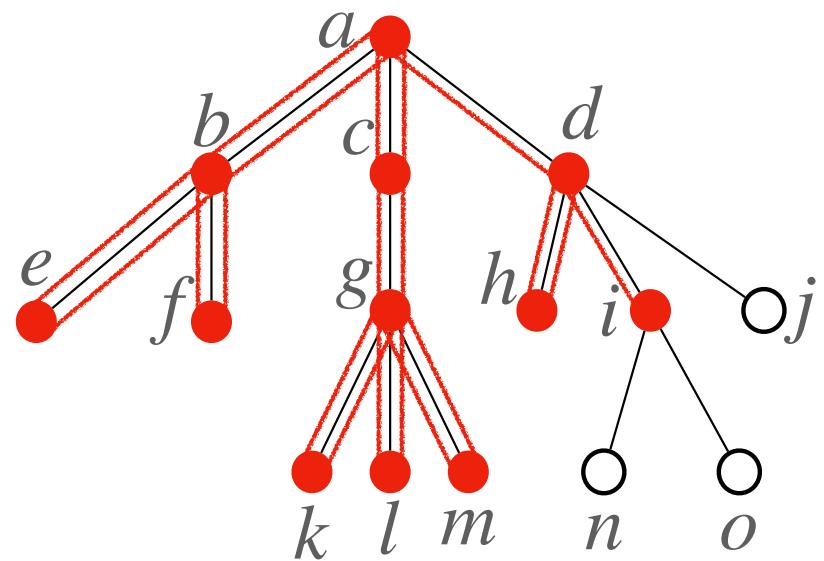
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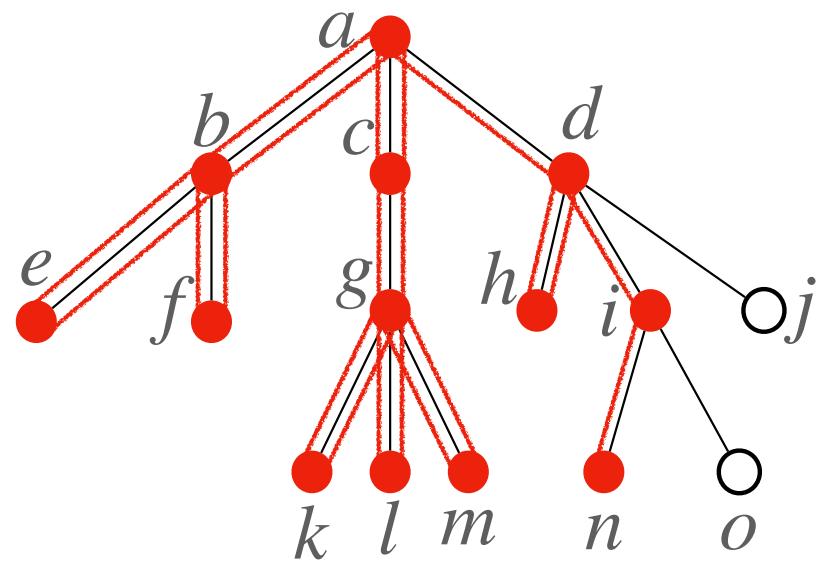
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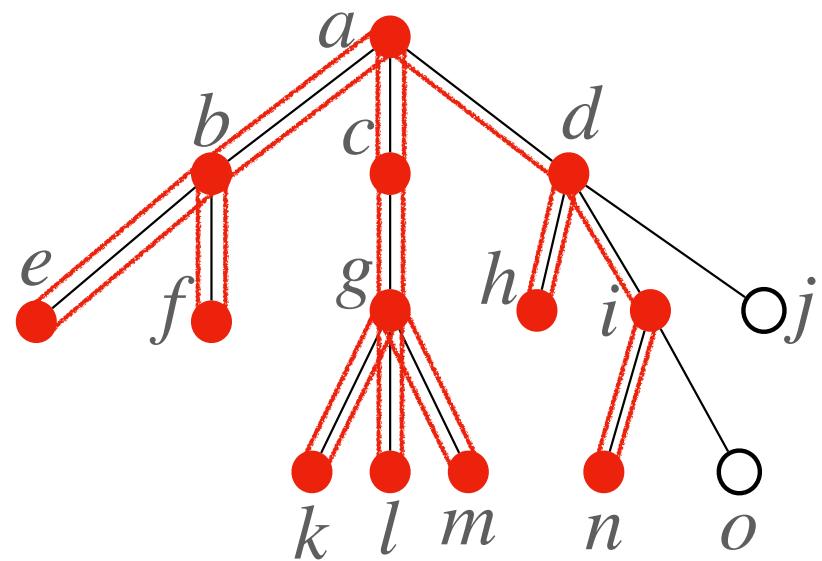
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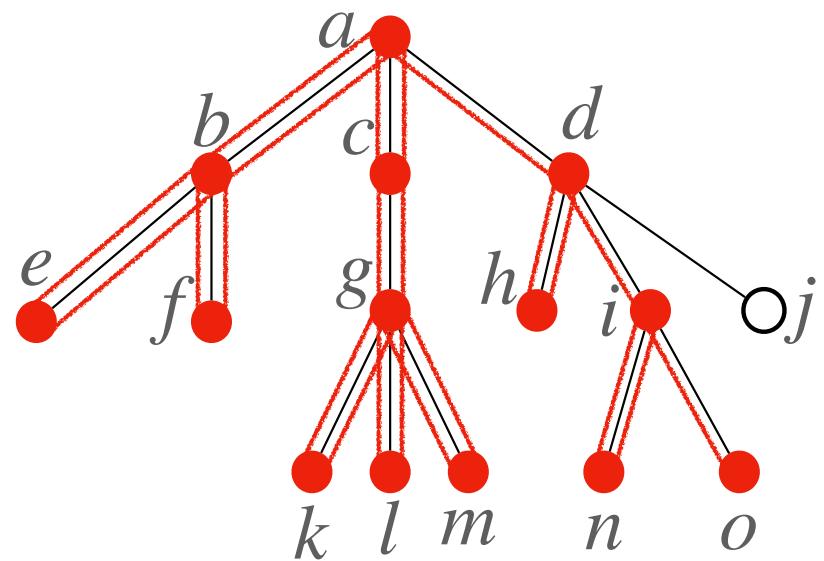
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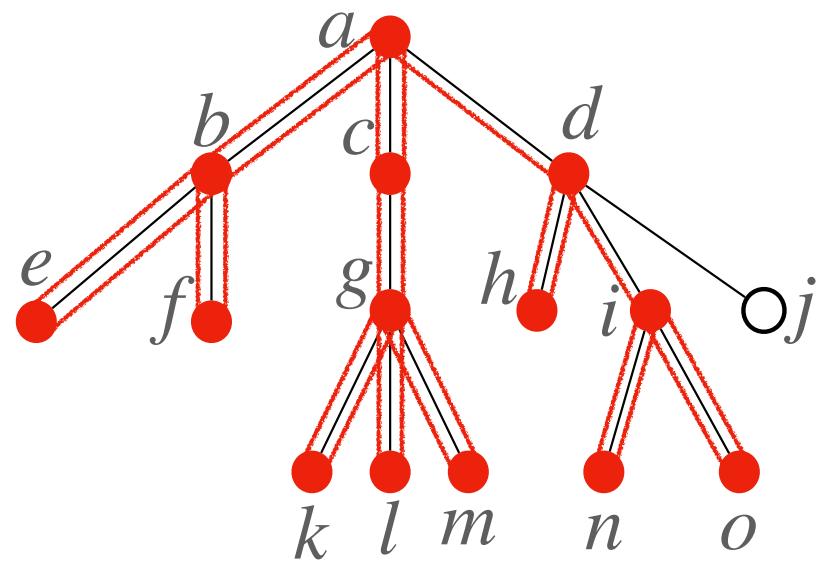
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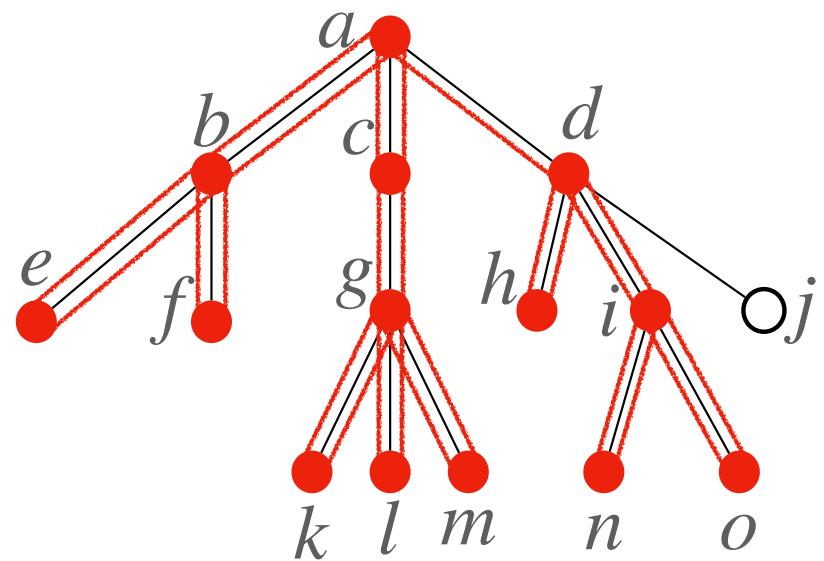
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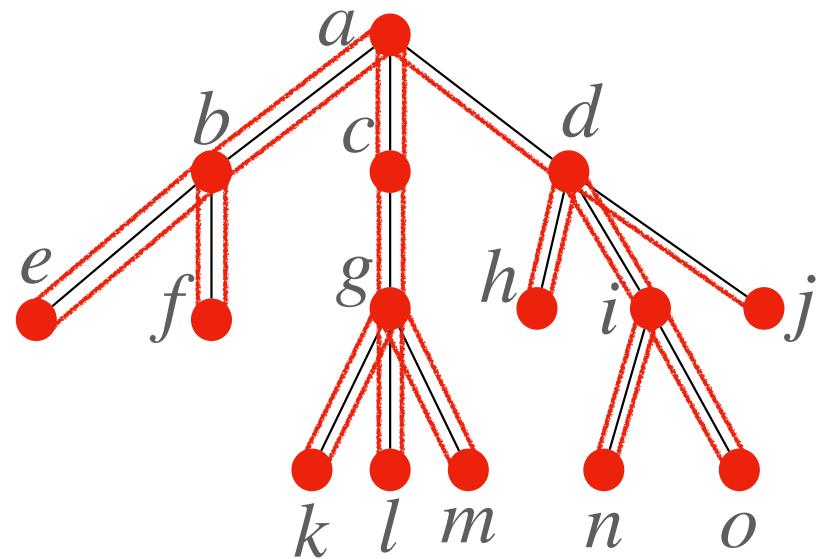
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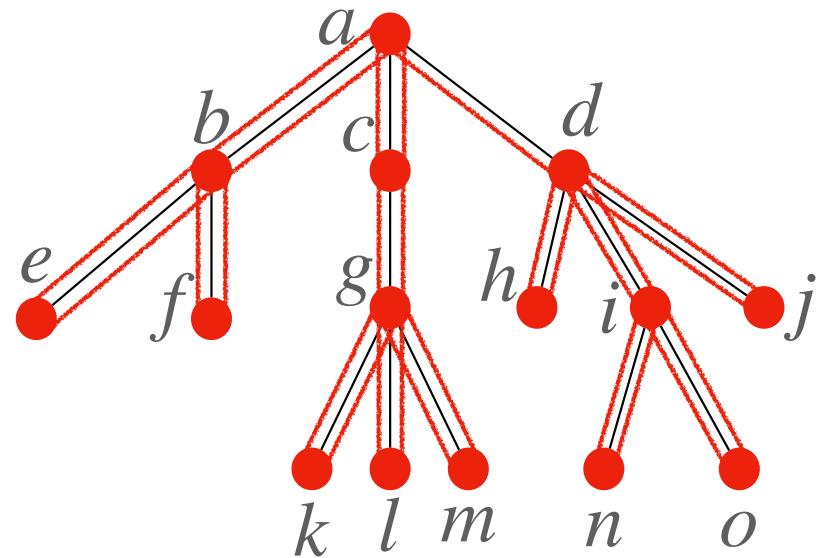
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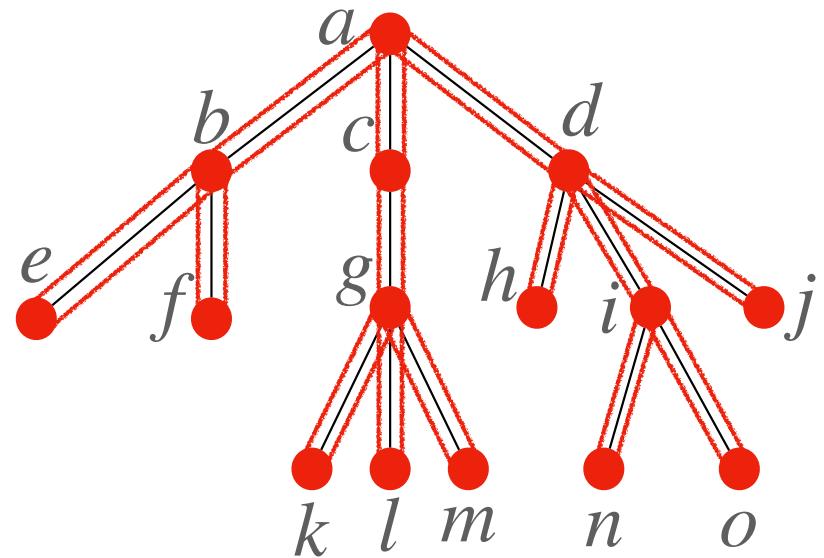
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Example of DFS on a tree:



22.3 Depth-First Search (DFS)

Example of DFS on a tree:



22.3 Depth-First Search (DFS)

Undiscovered node: white

Discovered node: gray

Finished node: black

For each node v , remember:

- 1) discover time: $d(v)$
- 2) finish time: $f(v)$

22.3 Depth-First Search (DFS)

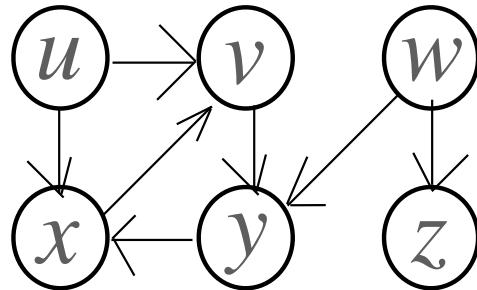
Undiscovered node: white

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Stack for discovered nodes:

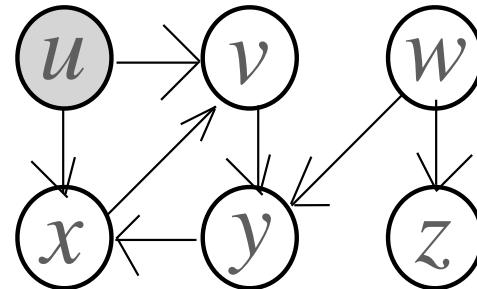
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Undiscovered node: white

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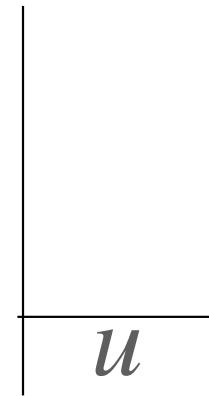
$$d(u) = 0$$



For each node v , remember:

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Stack for discovered nodes:



22.3 Depth-First Search (DFS)

Undiscovered node: white

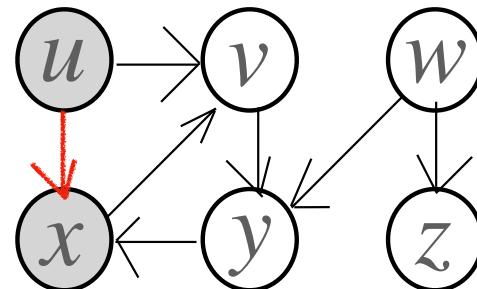
Discovered node: gray

Finished node: black

For each node v , remember:

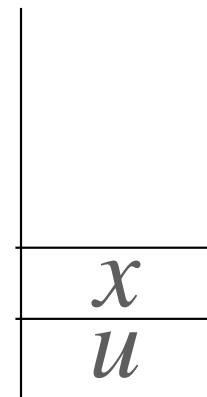
- 1) discover time: $d(v)$
- 2) finish time: $f(v)$

$$d(u) = 0$$



$$d(x) = 1$$

Stack for discovered nodes:



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Undiscovered node: white

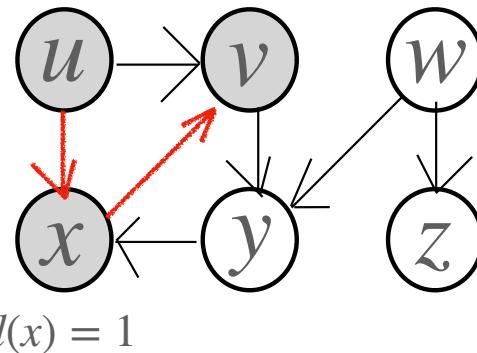
Discovered node: gray

Finished node: black

For each node v , remember:

- 1) discover time: $d(v)$
- 2) finish time: $f(v)$

$$d(u) = 0 \quad d(v) = 2$$



$$d(x) = 1$$

Stack for discovered nodes:

v
x
u

22.3 Depth-First Search (DFS)

Undiscovered node: white

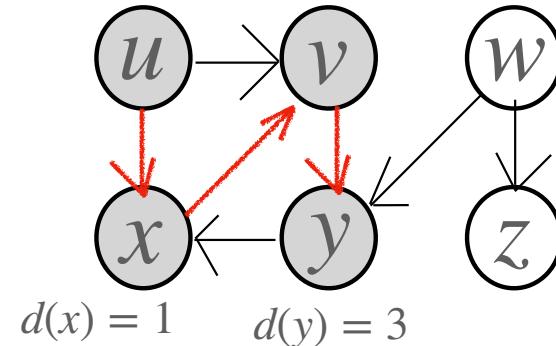
Discovered node: gray

Finished node: black

For each node v , remember:

- 1) discover time: $d(v)$
- 2) finish time: $f(v)$

$$d(u) = 0 \quad d(v) = 2$$



$$d(x) = 1 \quad d(y) = 3$$

Stack for discovered nodes:

y
v
x
u

22.3 Depth-First Search (DFS)

Undiscovered node: white

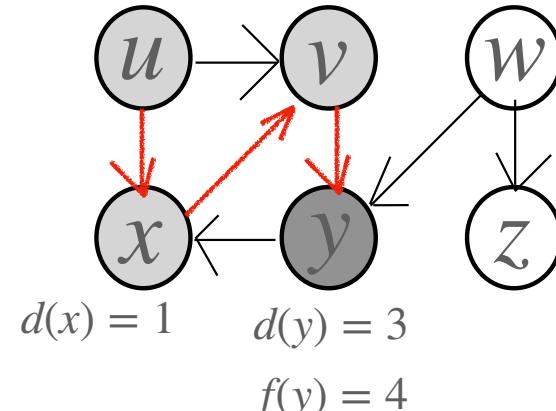
Discovered node: gray

Finished node: black

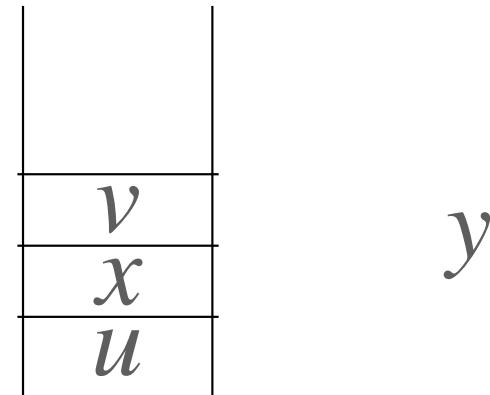
For each node v , remember:

- 1) discover time: $d(v)$
- 2) finish time: $f(v)$

$$d(u) = 0 \quad d(v) = 2$$



Stack for discovered nodes:



22.3 Depth-First Search (DFS)

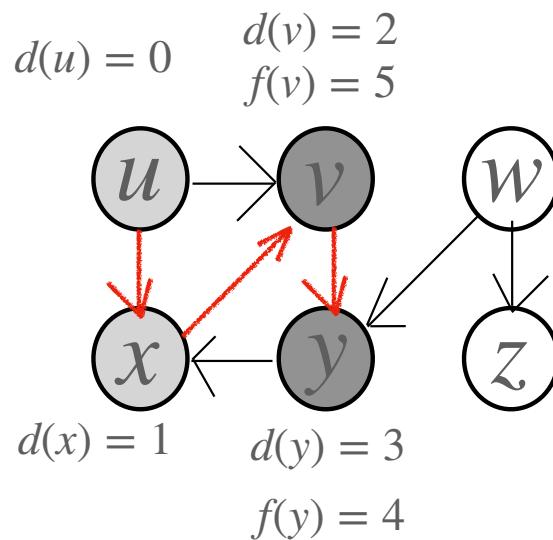
Undiscovered node: white

Discovered node: gray

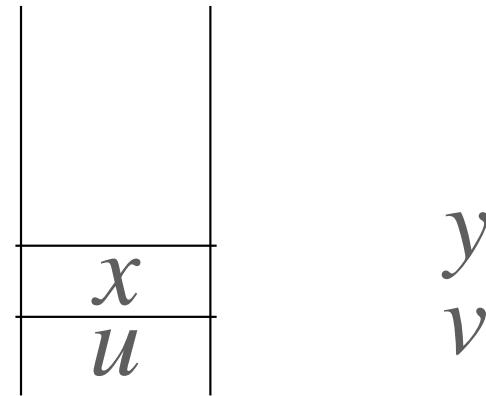
Finished node: black

For each node v , remember:

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Stack for discovered nodes:



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Undiscovered node: white

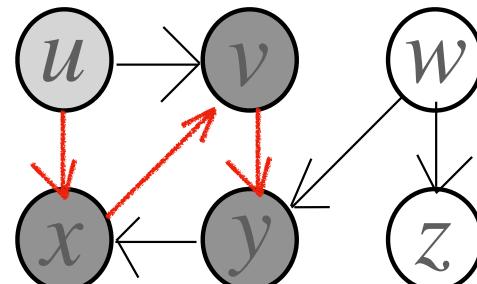
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For each node v , remember:

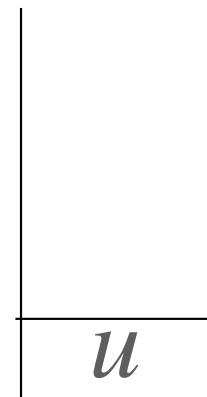
- 1) discover time: $d(v)$
- 2) finish time: $f(v)$

$$\begin{array}{ll} d(u) = 0 & d(v) = 2 \\ & f(v) = 5 \end{array}$$



$$\begin{array}{ll} d(x) = 1 & d(y) = 3 \\ f(x) = 6 & f(y) = 4 \end{array}$$

Stack for discovered nodes:



y
 v
 x

22.3 Depth-First Search (DFS)

Undiscovered node: white

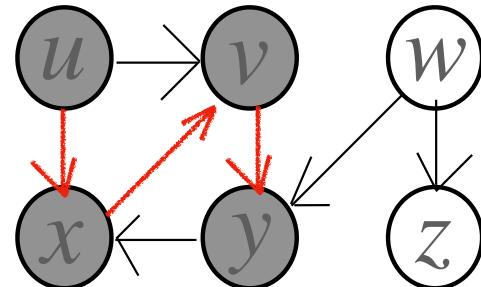
Discovered node: gray

Finished node: black

For each node v , remember:

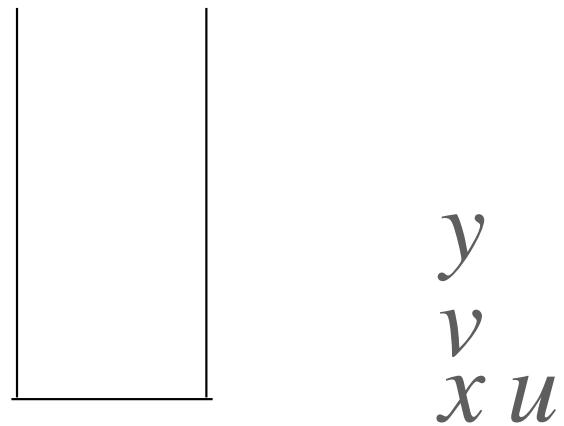
- 1) discover time: $d(v)$
- 2) finish time: $f(v)$

$$\begin{array}{ll} d(u) = 0 & d(v) = 2 \\ f(u) = 7 & f(v) = 5 \end{array}$$



$$\begin{array}{ll} d(x) = 1 & d(y) = 3 \\ f(x) = 6 & f(y) = 4 \end{array}$$

Stack for discovered nodes:



22.3 Depth-First Search (DFS)

Undiscovered node: white

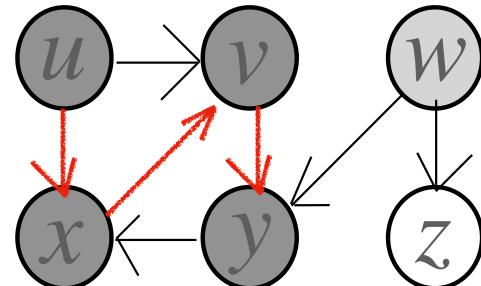
Discovered node: gray

Finished node: black

For each node v , remember:

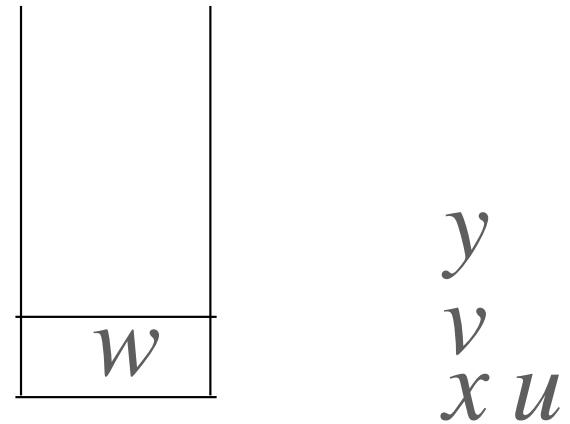
- 1) discover time: $d(v)$
- 2) finish time: $f(v)$

$$\begin{array}{lll} d(u) = 0 & d(v) = 2 & d(w) = 8 \\ f(u) = 7 & f(v) = 5 & \end{array}$$



$$\begin{array}{ll} d(x) = 1 & d(y) = 3 \\ f(x) = 6 & f(y) = 4 \end{array}$$

Stack for discovered nodes:



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Undiscovered node: white

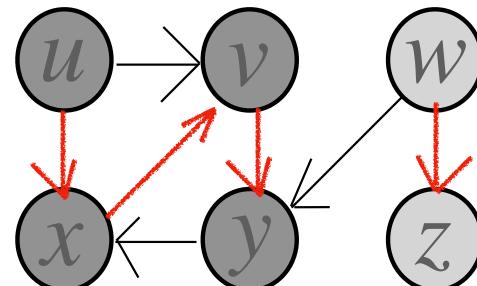
Discovered node: gray

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For each node v , remember:

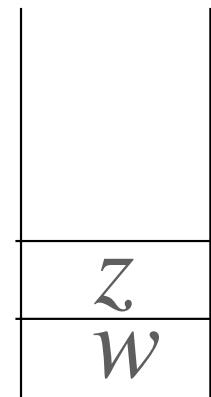
- 1) discover time: $d(v)$
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$$\begin{array}{lll} d(u) = 0 & d(v) = 2 & d(w) = 8 \\ f(u) = 7 & f(v) = 5 & \end{array}$$



$$\begin{array}{lll} d(x) = 1 & d(y) = 3 & d(z) = 9 \\ f(x) = 6 & f(y) = 4 & \end{array}$$

Stack for discovered nodes:



y
 v
 x u

22.3 Depth-First Search (DFS)

Undiscovered node: white

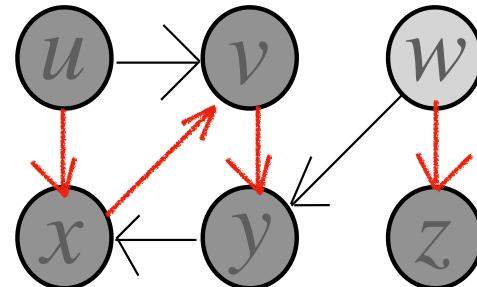
Discovered node: gray

Finished node: black

For each node v , remember:

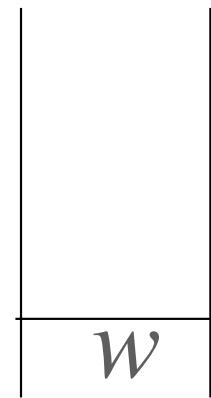
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$$\begin{array}{lll} d(u) = 0 & d(v) = 2 & d(w) = 8 \\ f(u) = 7 & f(v) = 5 & \end{array}$$



$$\begin{array}{lll} d(x) = 1 & d(y) = 3 & d(z) = 9 \\ f(x) = 6 & f(y) = 4 & f(z) = 10 \end{array}$$

Stack for discovered nodes:



y z
 v
 x u

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Undiscovered node: white

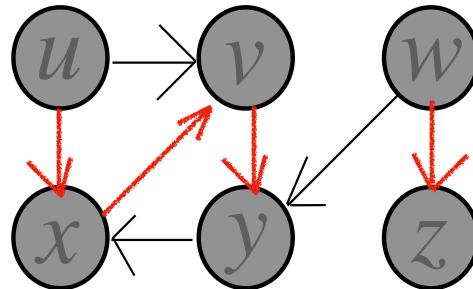
Discovered node: gray

Finished node: black

For each node v , remember:

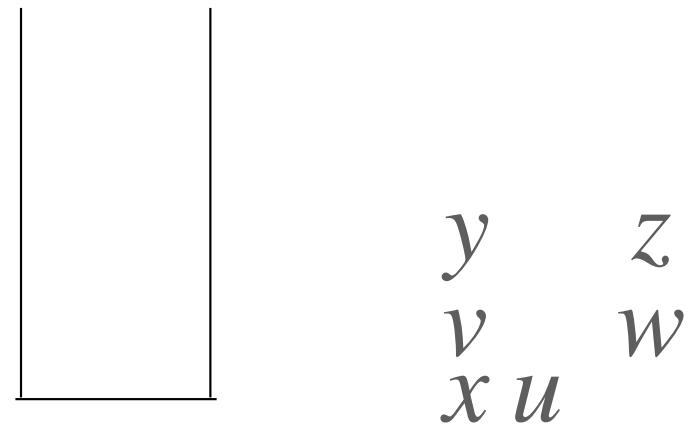
- 1) discover time: $d(v)$
- 2) finish time: $f(v)$

$$\begin{array}{lll} d(u) = 0 & d(v) = 2 & d(w) = 8 \\ f(u) = 7 & f(v) = 5 & f(w) = 11 \end{array}$$



$$\begin{array}{lll} d(x) = 1 & d(y) = 3 & d(z) = 9 \\ f(x) = 6 & f(y) = 4 & f(z) = 10 \end{array}$$

Stack for discovered nodes:



22.3 Depth-First Search (DFS)

Undiscovered node: white

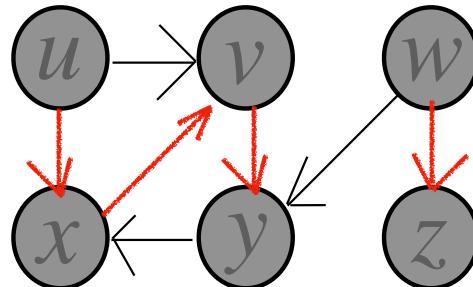
Discovered node: gray

Finished node: black

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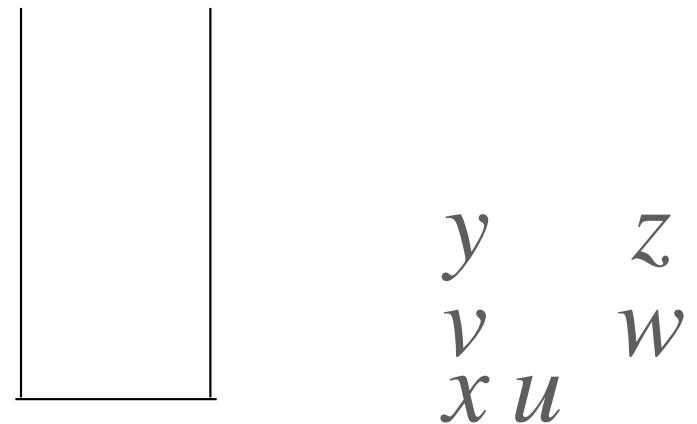
$$\begin{array}{lll} d(u) = 0 & d(v) = 2 & d(w) = 8 \\ f(u) = 7 & f(v) = 5 & f(w) = 11 \end{array}$$



DFS forest

$$\begin{array}{lll} d(x) = 1 & d(y) = 3 & d(z) = 9 \\ f(x) = 6 & f(y) = 4 & f(z) = 10 \end{array}$$

Stack for discovered nodes:



22.3 Depth-First Search (DFS)

Undiscovered node: white

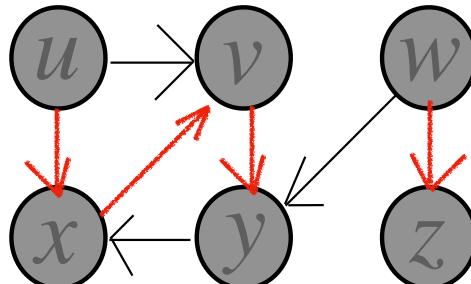
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Finished node: black

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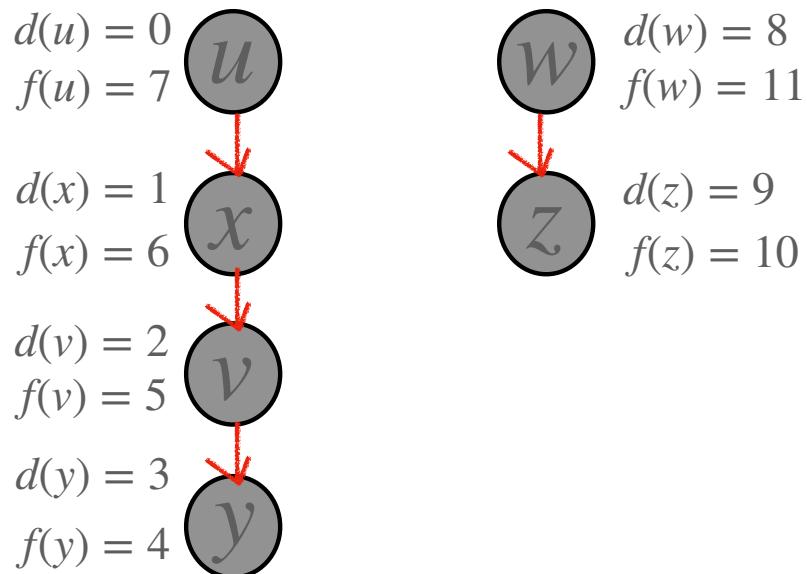
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$$\begin{array}{lll} d(u) = 0 & d(v) = 2 & d(w) = 8 \\ f(u) = 7 & f(v) = 5 & f(w) = 11 \end{array}$$



DFS forest

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22.3 Depth-First Search (DFS)

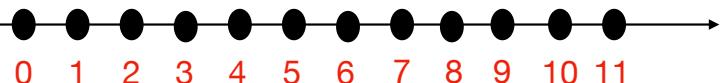
Undiscovered node: white

Discovered node: gray

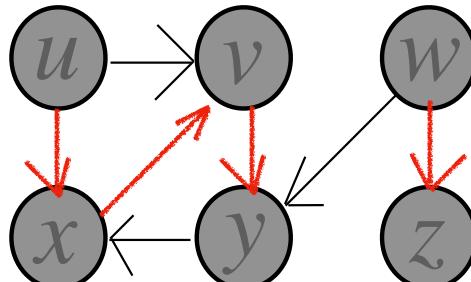
Finished node: black

For each node v , remember:

- 1) discover time: $d(v)$
- 2) finish time: $f(v)$

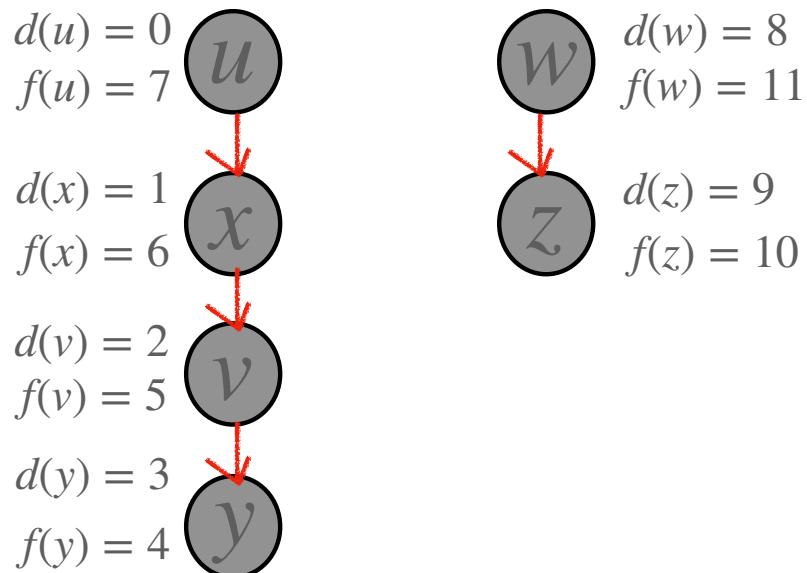


$$\begin{array}{lll} d(u) = 0 & d(v) = 2 & d(w) = 8 \\ f(u) = 7 & f(v) = 5 & f(w) = 11 \end{array}$$



DFS forest

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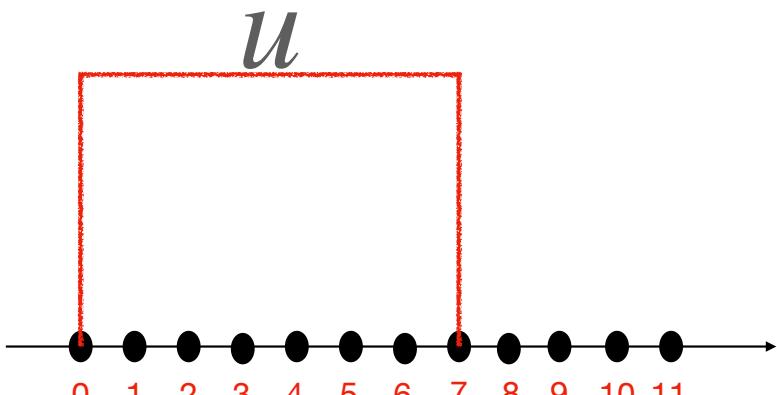
Undiscovered node: white

Discovered node: gray

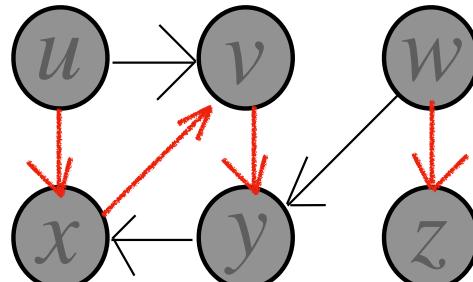
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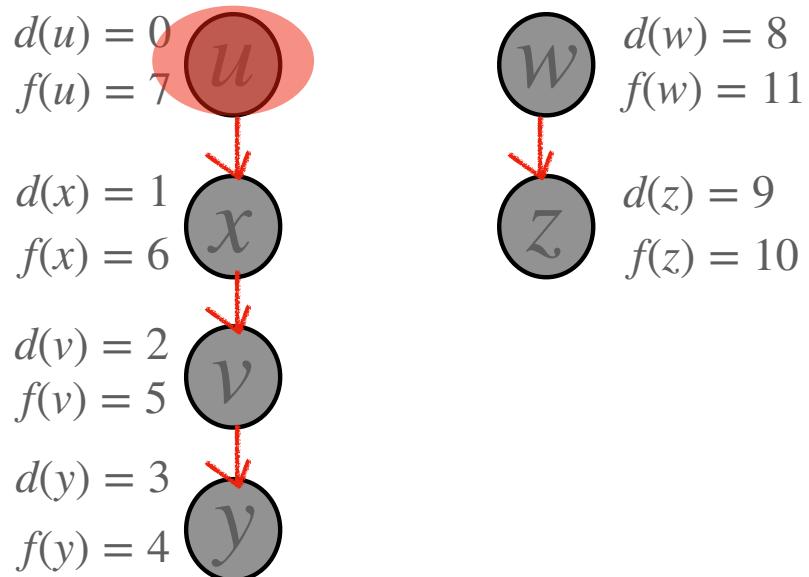


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DFS forest



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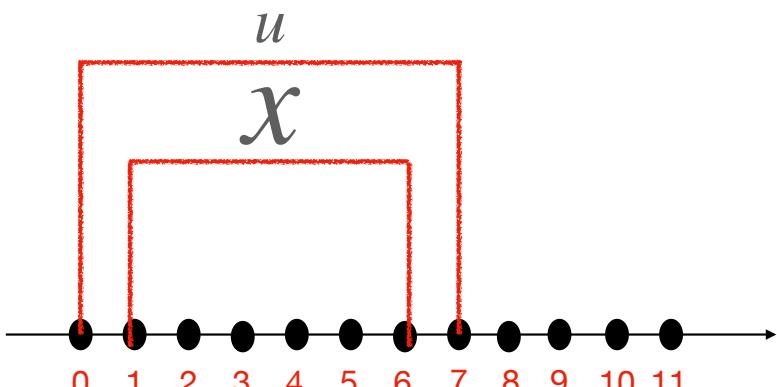
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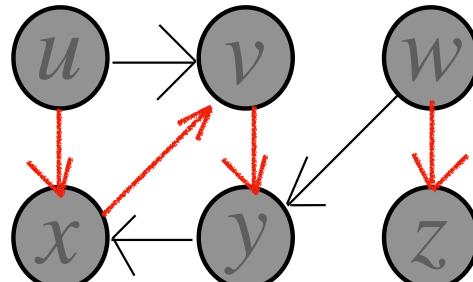
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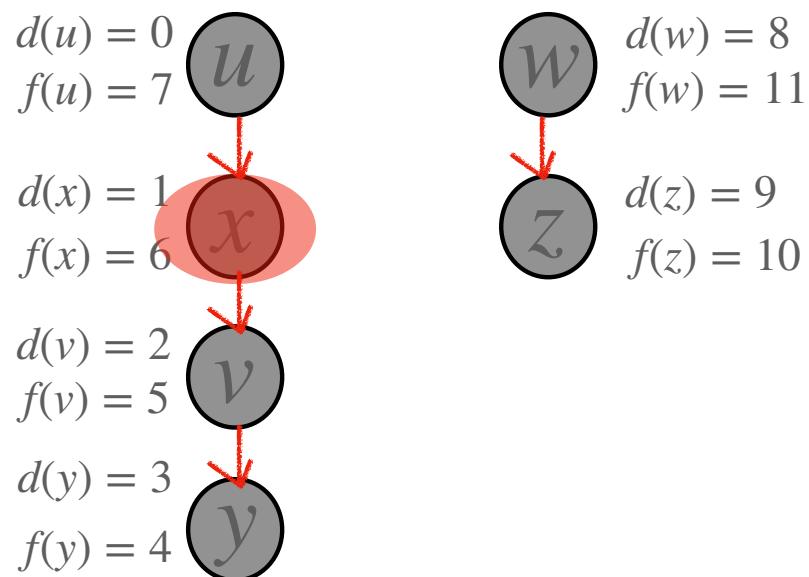


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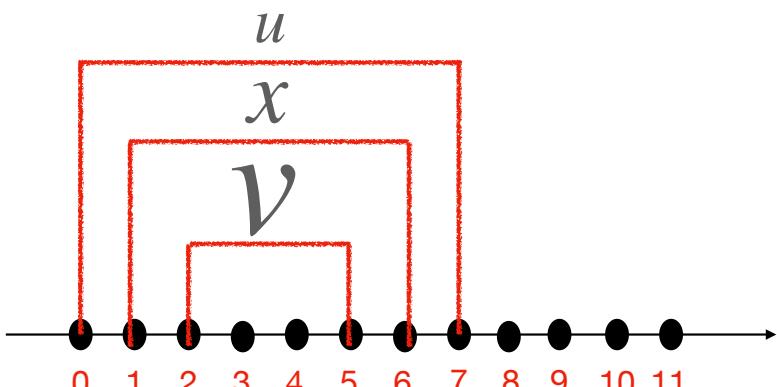
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Discovered node: gray

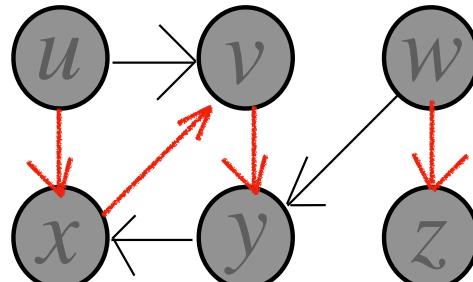
Finished node: black

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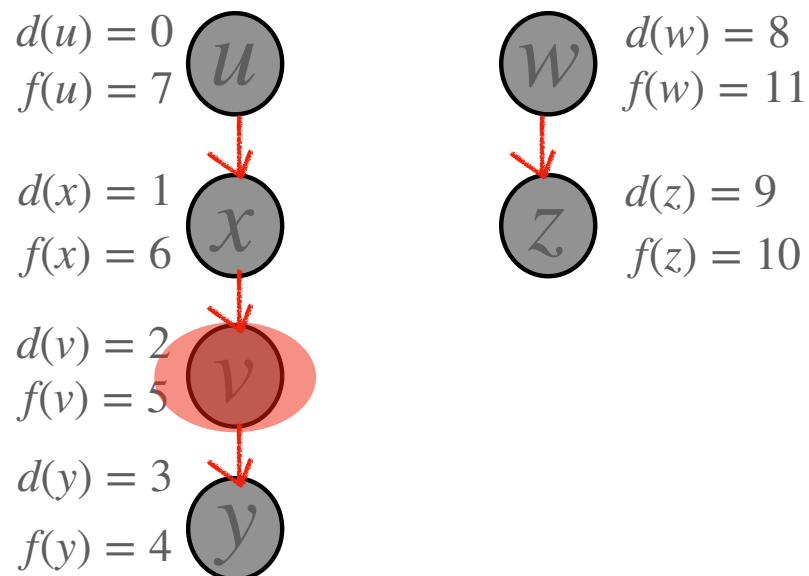


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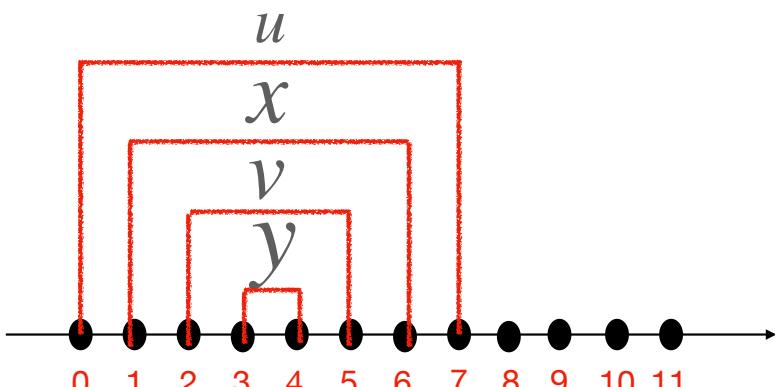
Undiscovered node: white

Discovered node: gray

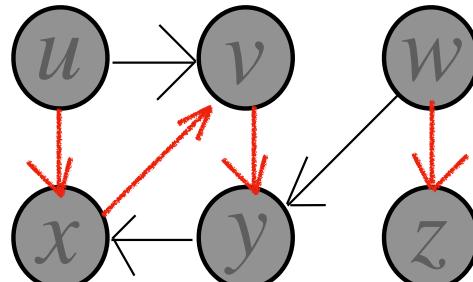
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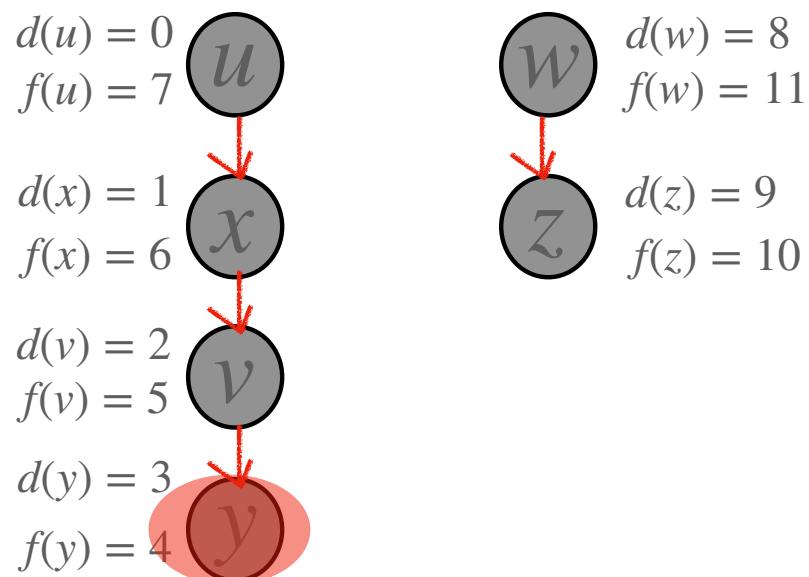


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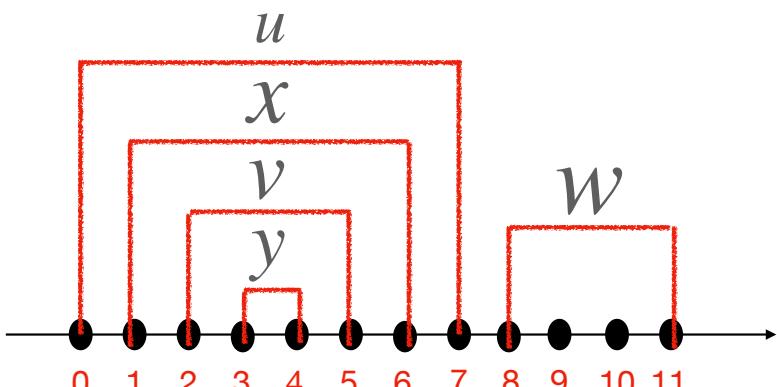
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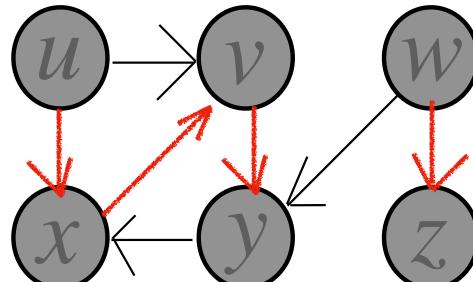
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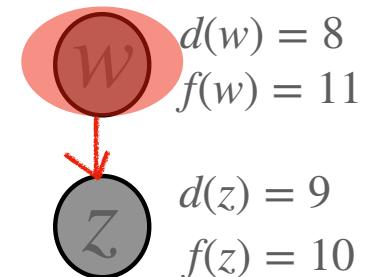
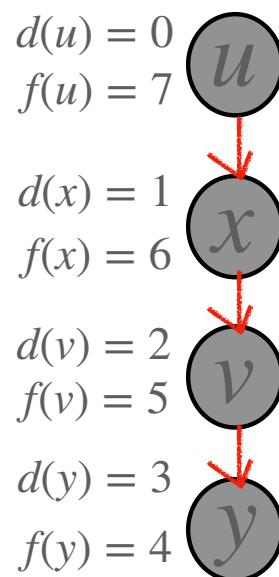


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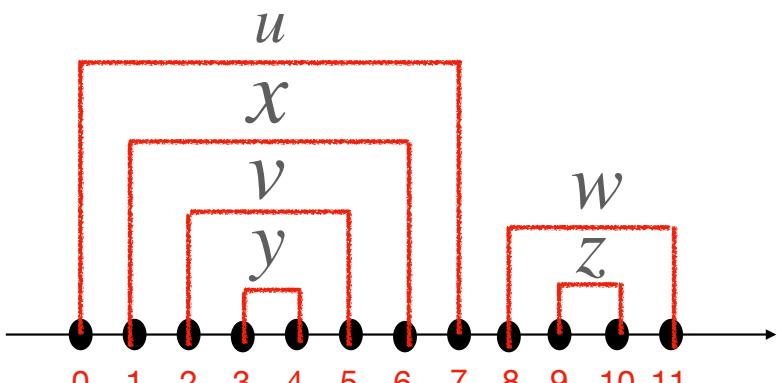
Undiscovered node: white

Discovered node: gray

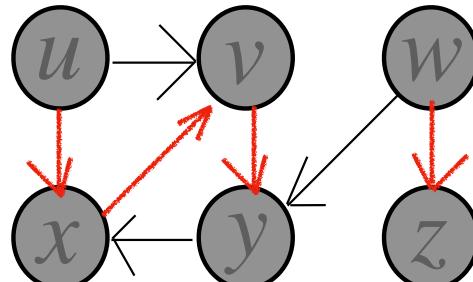
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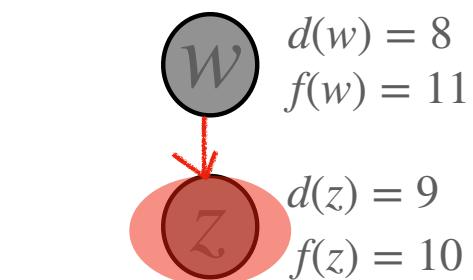
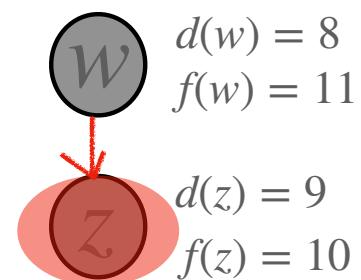
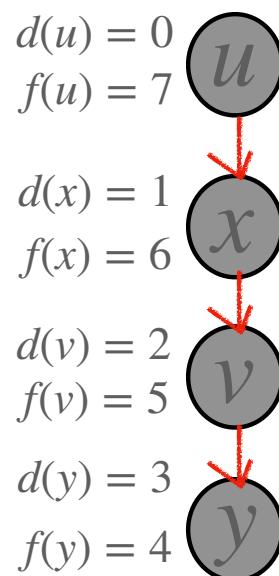


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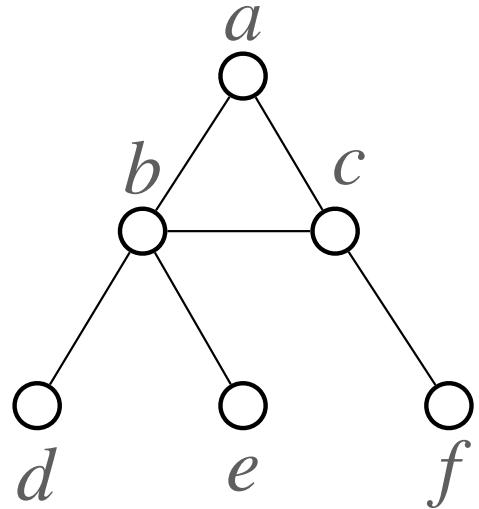


Time intervals:

- 1) Ancestor and child: ancestor's contains child's
- 2) Otherwise: disjoint time intervals

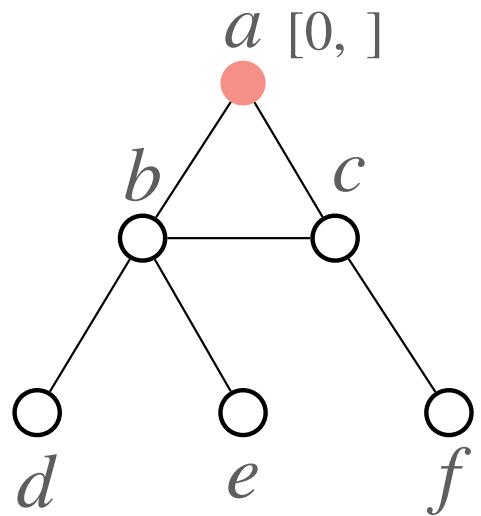
22.3 Depth-First Search (DFS)

Another example:



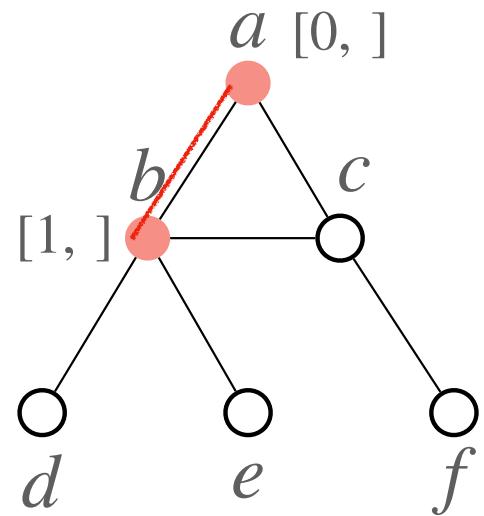
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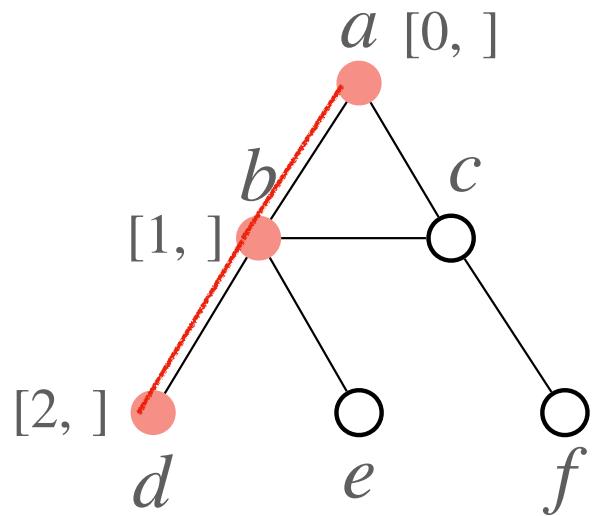
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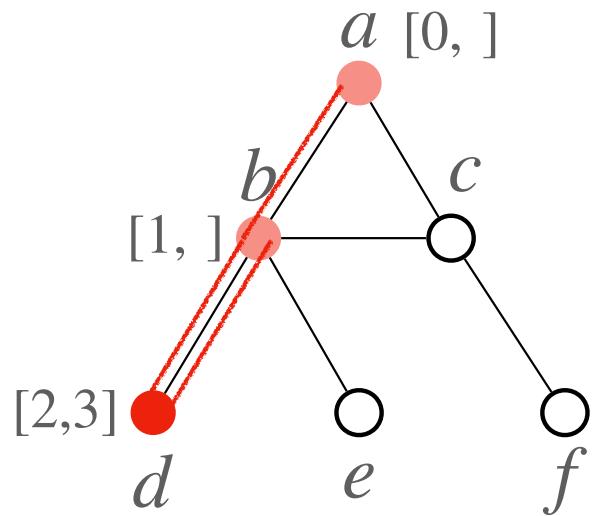
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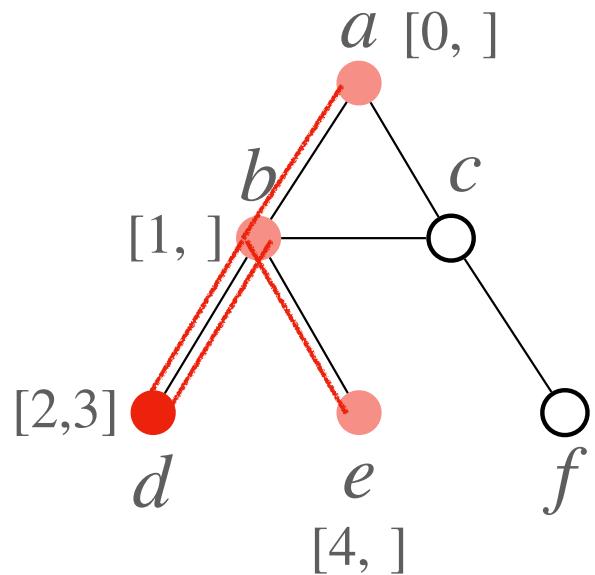
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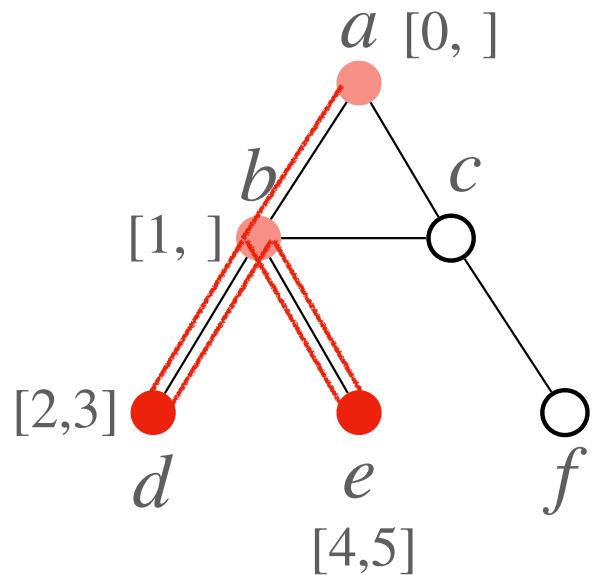
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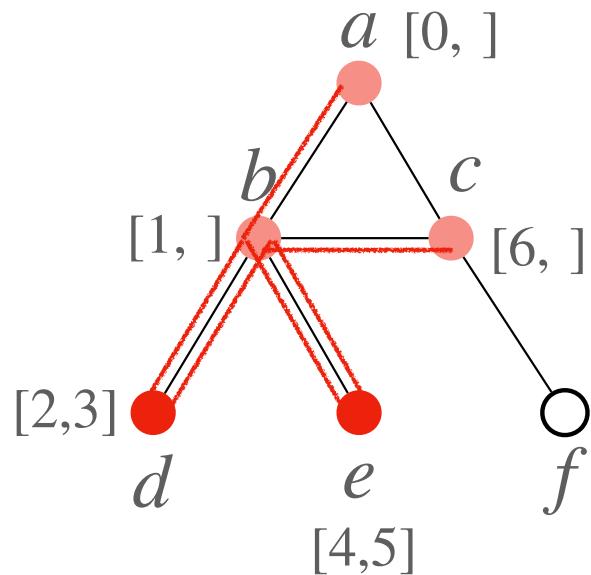
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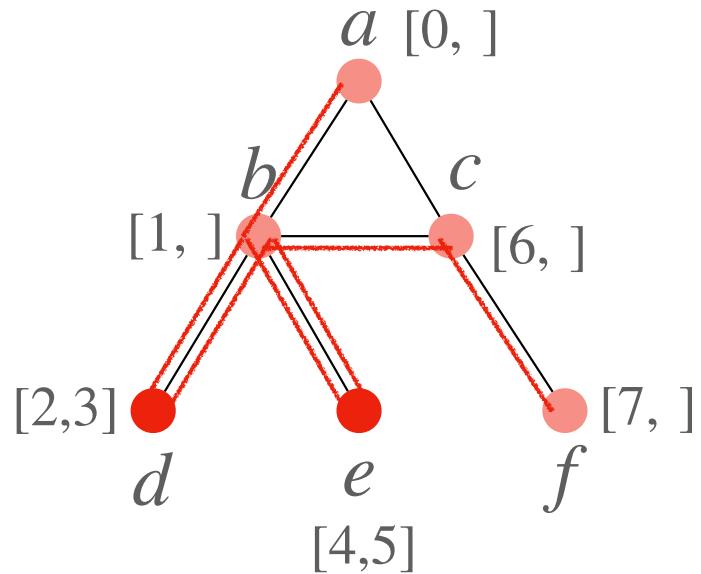
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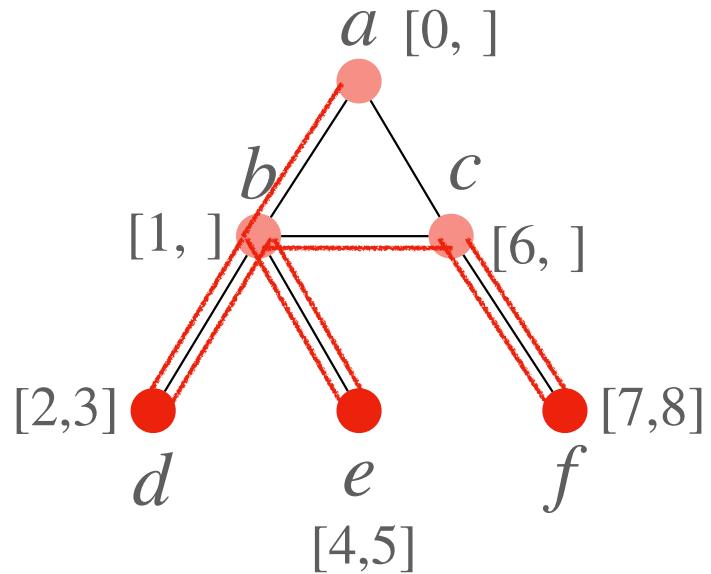
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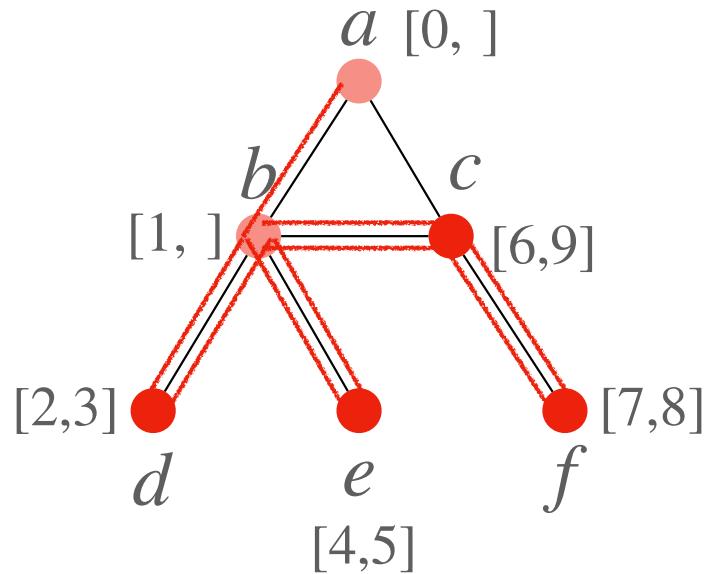
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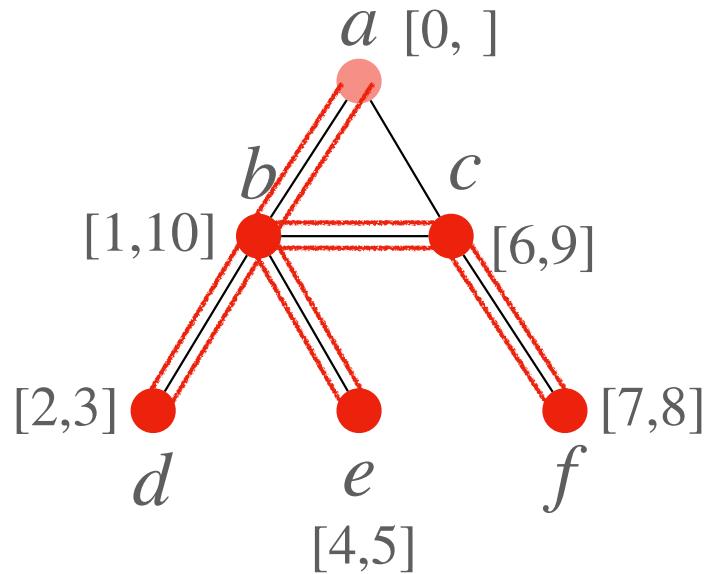
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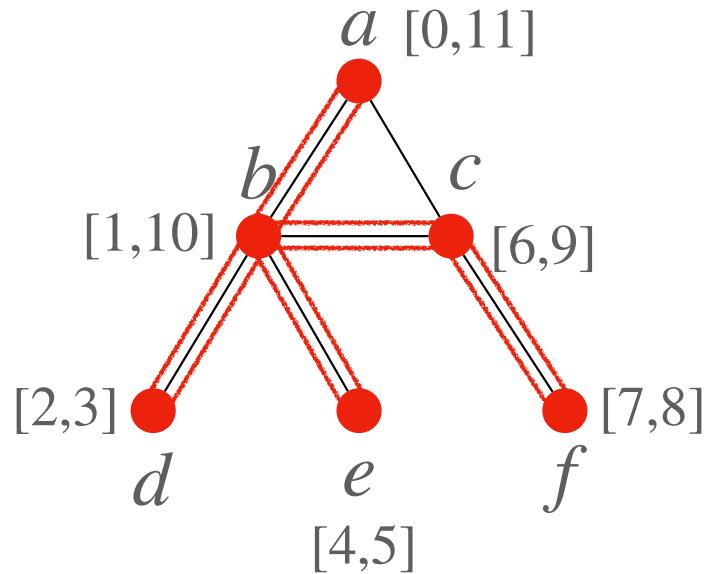
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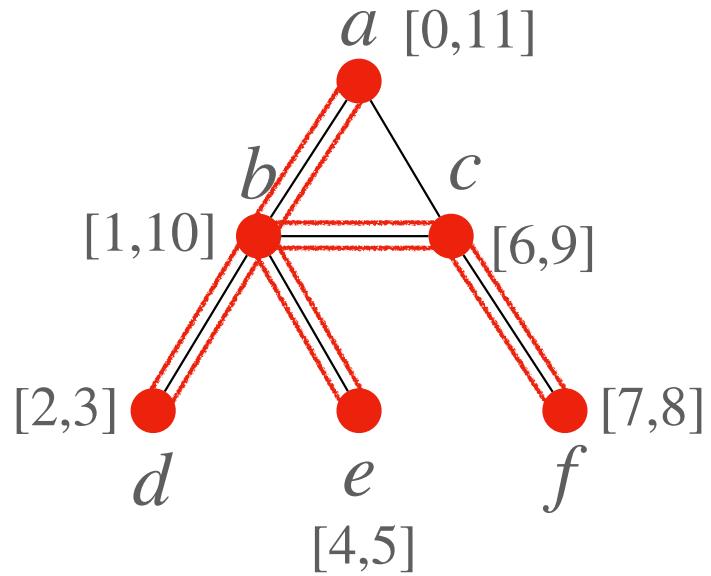
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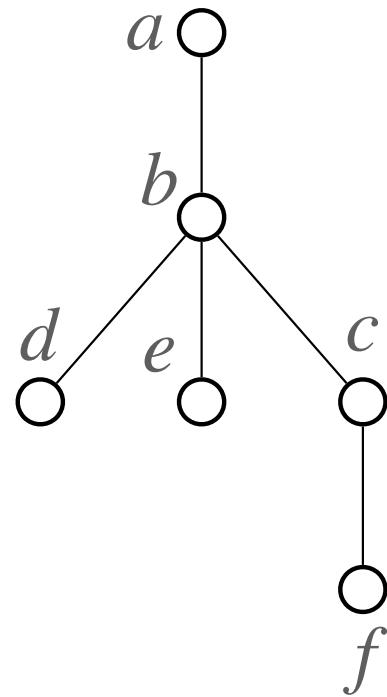


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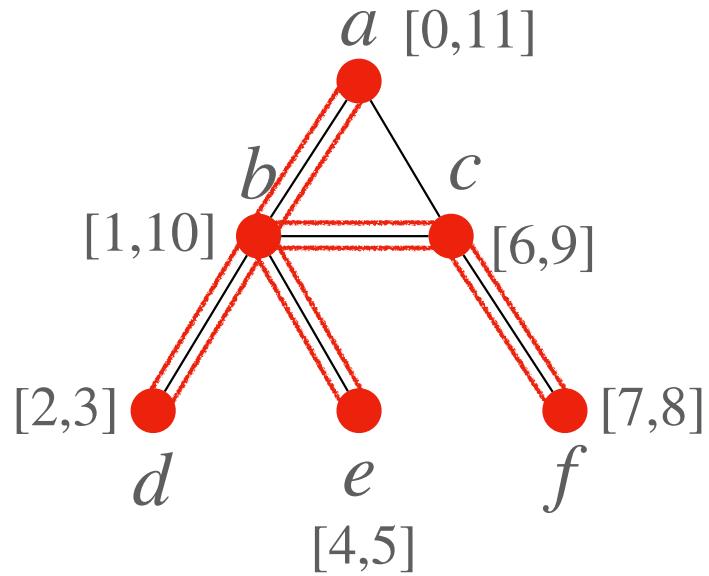


DFS tree

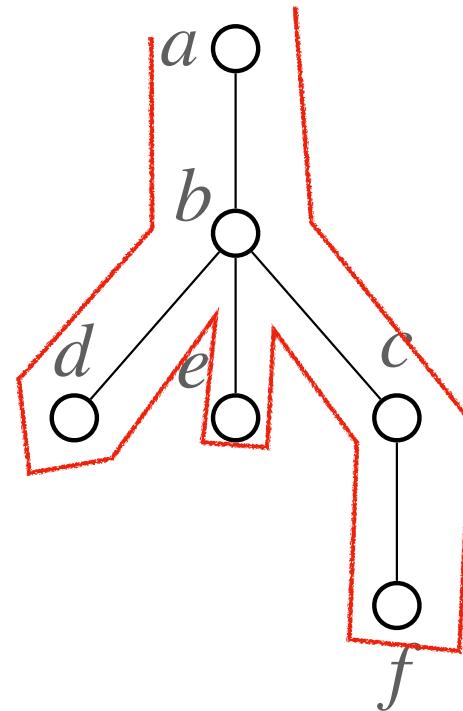


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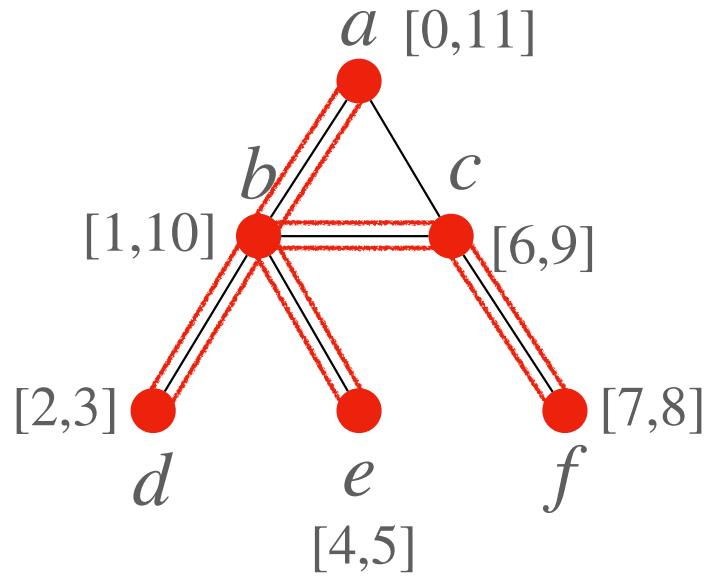


DFS tree

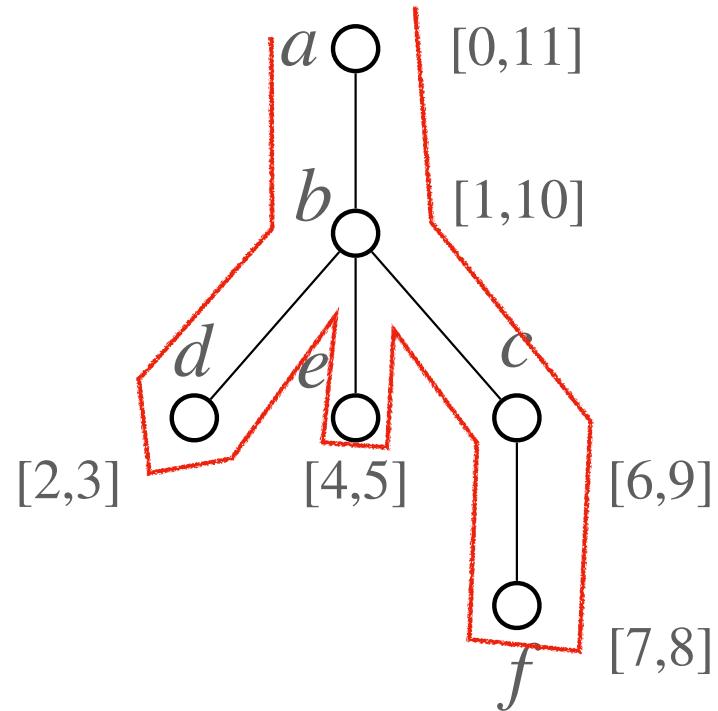


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DFS tree



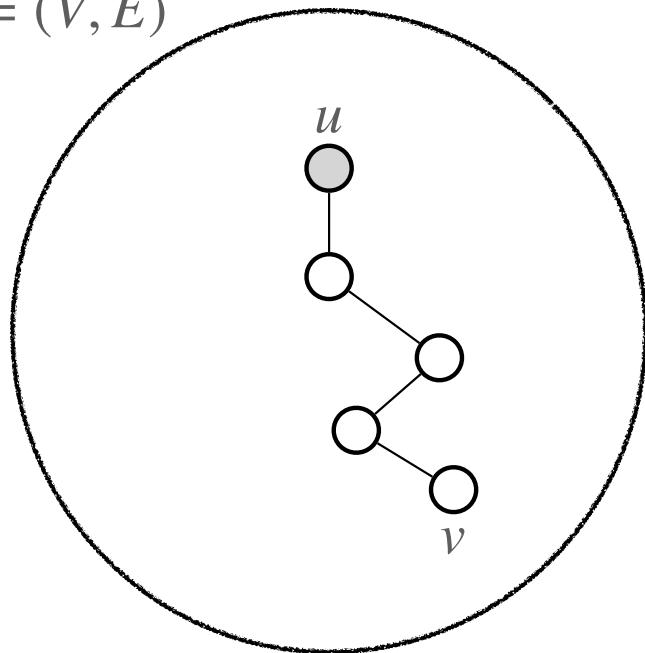
22.3 Depth-First Search (DFS)

Theorem: In a DFS forest of a graph $G=(V,E)$, vertex v is a descendant of vertex u if and only if at the time u is discovered, there is a path from u to v consisting of only undiscovered vertices.

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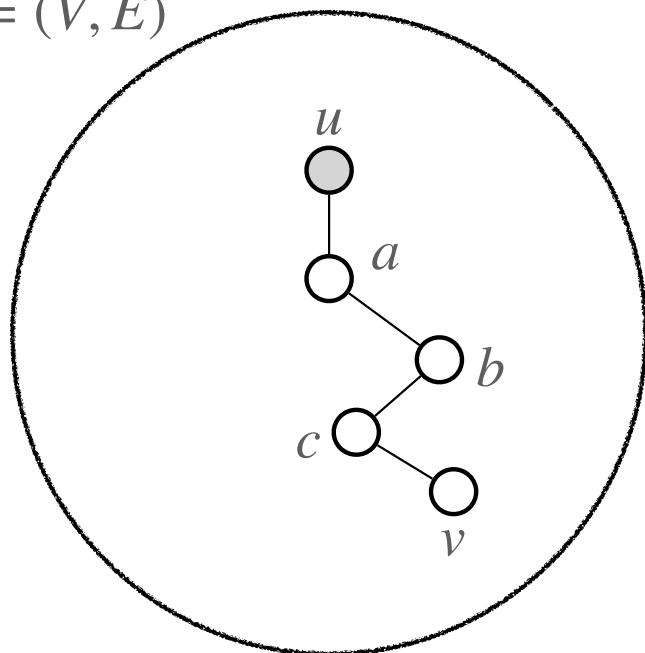
$$G = (V, E)$$



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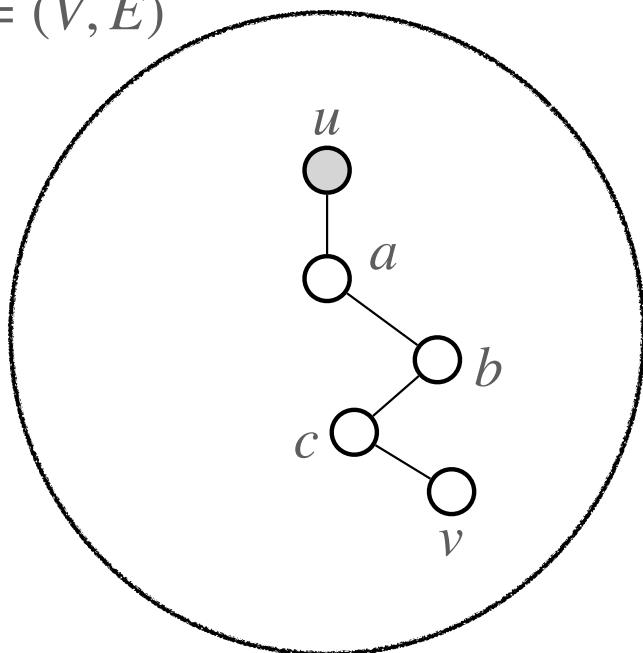
1) Direction →

That path can be the path from ancestor u to descendant v in the DFS tree.

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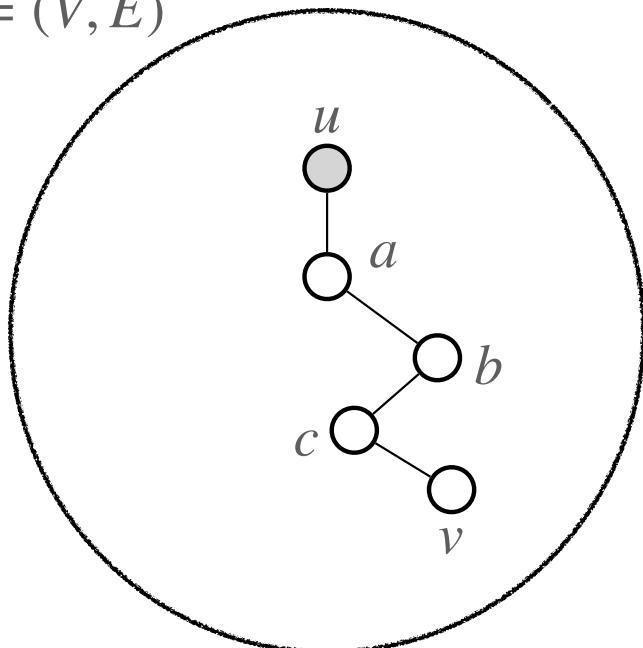
That path can be the path from ancestor u to descendant v in the DFS tree.

2) Direction \leftarrow

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$$G = (V, E)$$



1) Direction →

That path can be the path from ancestor u to descendant v in the DFS tree.

2) Direction ←

Since u can reach v through a path of undiscovered nodes, sooner or later u will discover v (and actually all those nodes in that path), and they will all become descendants of u .