Homework thirteen. Due: 11:59pm (Central Time) on 12/7/2021 in Canvas.

***1) Textbook page 730, Exercise 26.2-3.***

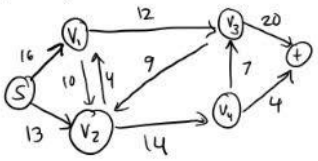
**Show the execution of the Edmonds-Karp algorithm on the flow network of Figure 26.1(a).**

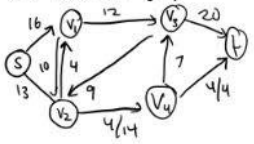
**ANSWER ON NEXT PAGE**

(1) The shortest path found in line 3 is $s \to v_2 \to v_4 \to t$. Then the minimum capacity of this path p is $c_f(p) = \min\left(c_f(s,v_2),\ c_f(v_2,v_4),\ c_f(v_4,t)\right)$

$= \min(13-0,\ 14-0,\ 4-0) = 4$

Now, augment the each edge in the augmenting path, with minimum capacity of the path p. Thus, augment the path in the residual graph. The new network is on the right side.

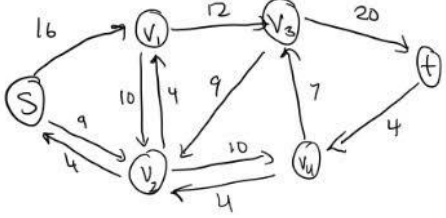


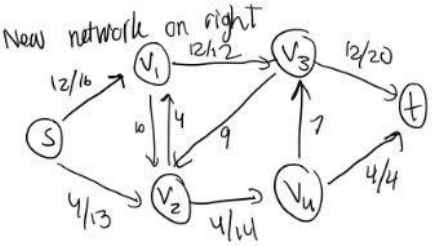




(2) The next shortest path is $s \to v_1 \to v_3 \to t$. Then the min capacity of path p is $c_f(p) = \min\left(c_f(s,v_1),\ c_f(v_1,v_3),\ c_f(v_3,t)\right)$

$= \min(16-0,\ 12-0,\ 20-0) = 12$

Therefore, the augment path is the residual graph. New network on right



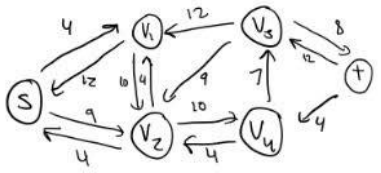



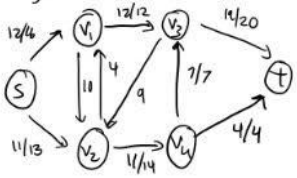

(3) The next shortest path is $s \to v_2 \to v_4 \to v_3 \to t$.

Therefore the min capacity is

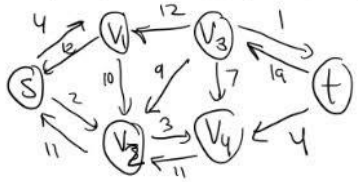
$c_f(p) = \min\left(c_f(s,v_2),\ c_f(v_2,v_4),\ c_f(v_4,v_3),\ c_f(v_3,t)\right)$

$= \min(13-4,\ 14-4,\ 7-0,\ 20-12) = 7$




Because there is no further augmenting path in the network, the algo is terminated, the final network is as follows




The max flow of the network is: $19 + 4 = \boxed{23}$