Algorithms

Lecture 18: NP Completeness (Part 4)

Anxiao (Andrew) Jiang

CH 34. NP Completeness

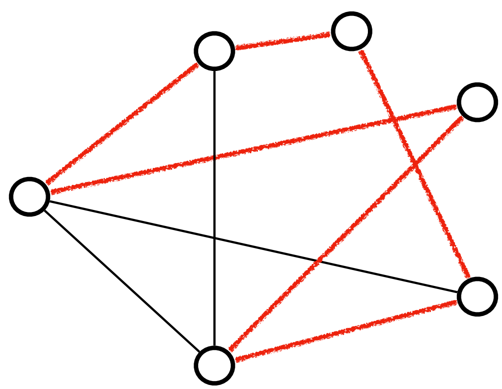
Known NPC Problems:

1) 3-CNF SAT Problem

2) Clique Problem

3) Vertex Cover Problem

Hamiltonian cycle: Give a graph G=(V,E), a Hamiltonian cycle is a cycle in G that passes every node exactly once.



Hamiltonian cycle Problem:

Input: A graph G=(V,E).

Question: Does G have a Hamiltonian cycle?

Theorem: The Hamiltonian cycle Problem is NP-Complete.

Known NPC Problems:

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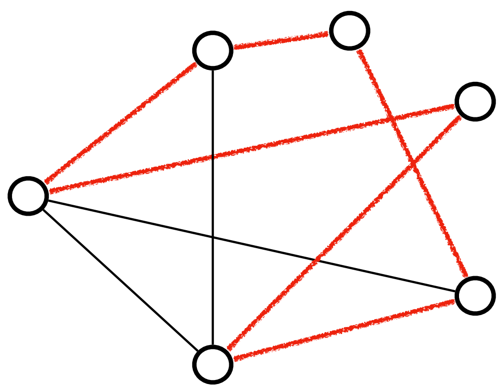
Proof: 1) Hamiltonian Cycle Problem $\in NP$.

Certificate: a Hamiltonian cycle in G.



Polynomial-time verification.

2) Which NPC problem can be reduced to the Hamiltonian Cycle Problem in polynomial time?

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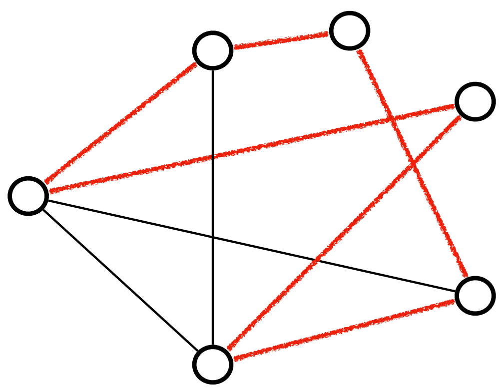

Polynomial-time verification.

2) Which NPC problem can be reduced to the Hamiltonian Cycle Problem in polynomial time?

All NPC problems can. We choose the Vertex Cover Problem.

Vertex Cover Problem

Input: An undirected graph $G = (V, E)$.
An integer k.

Question: Does G have a vertex cover of size k?

Hamiltonian cycle Problem:

Input: A graph $G' = (V', E')$.

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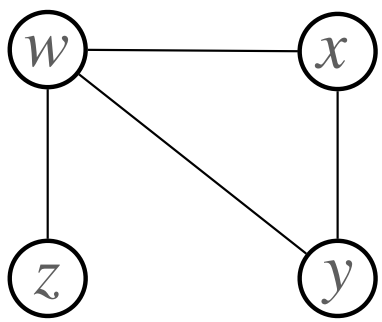

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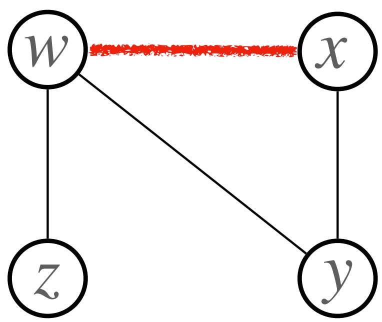

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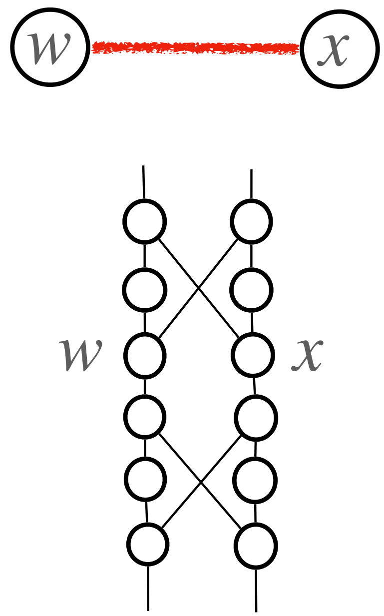

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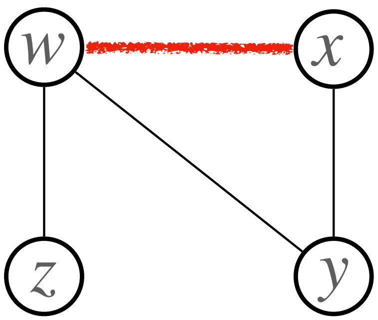

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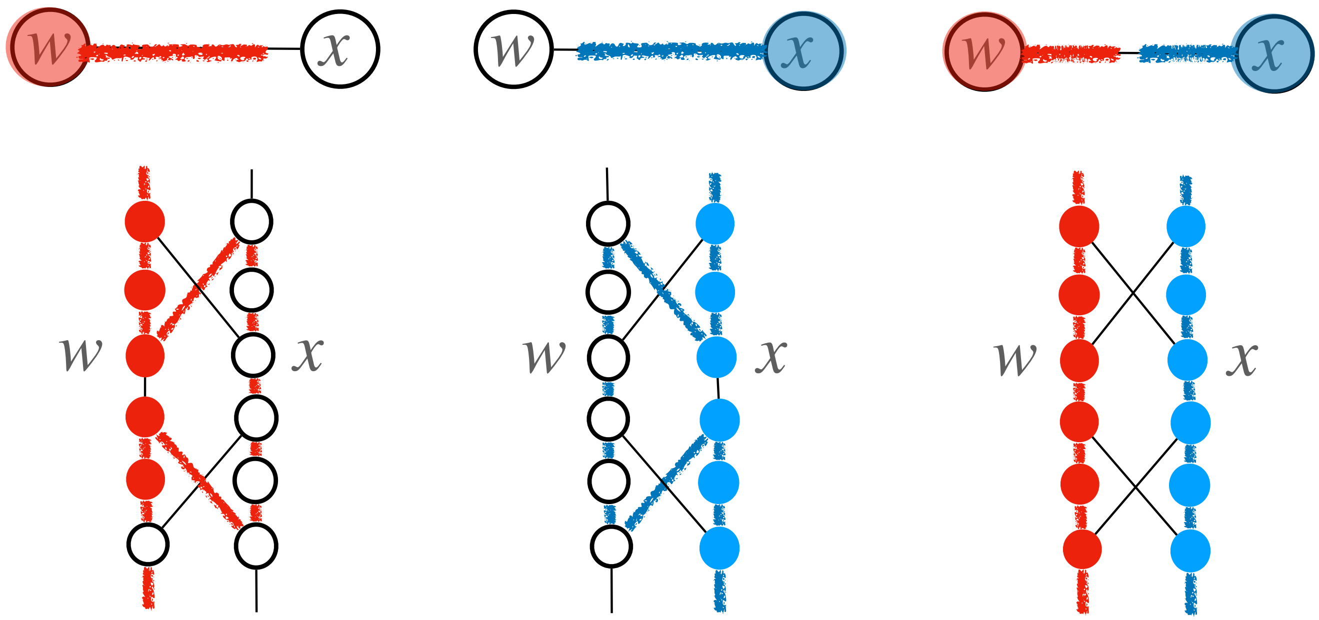
3 ways to cover an edge
3 ways to traverse the "gadget"



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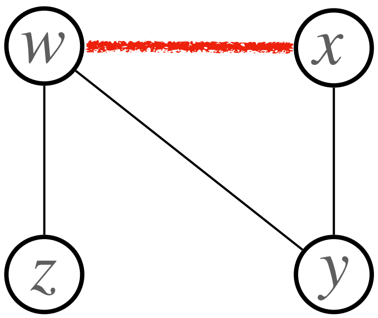

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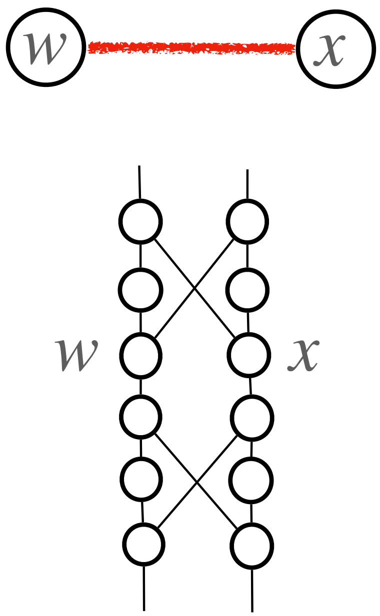

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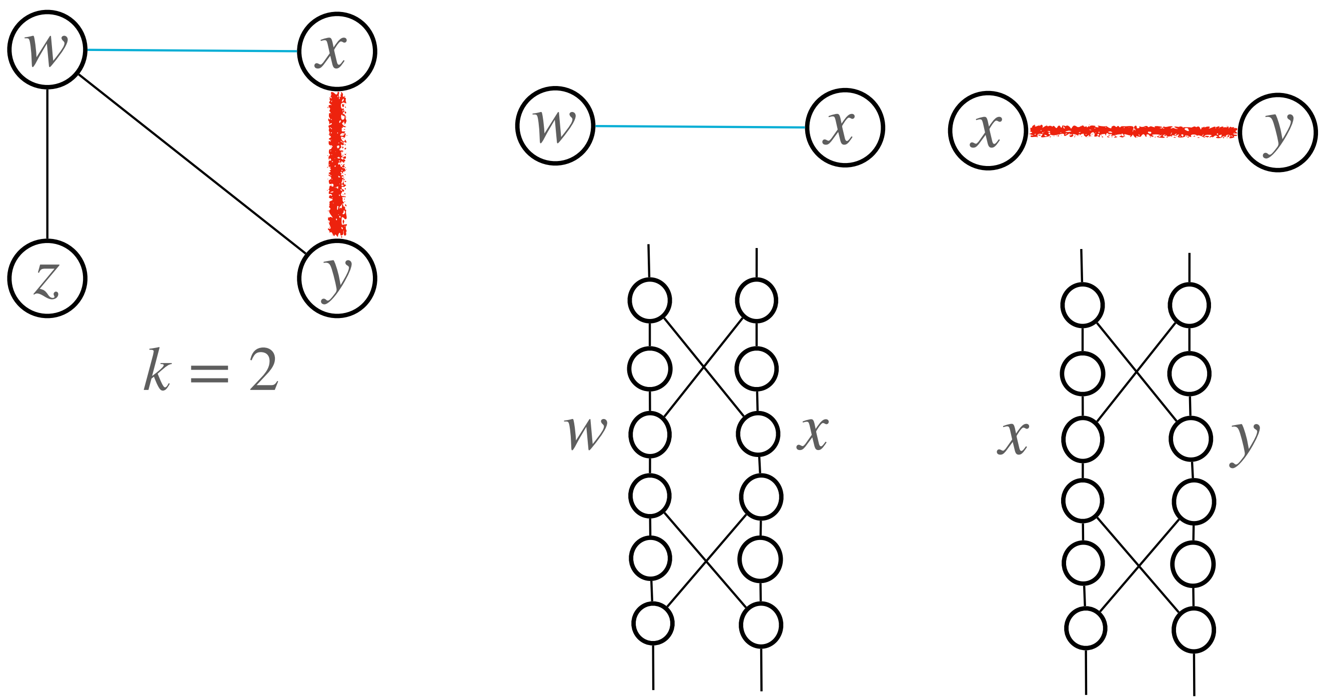

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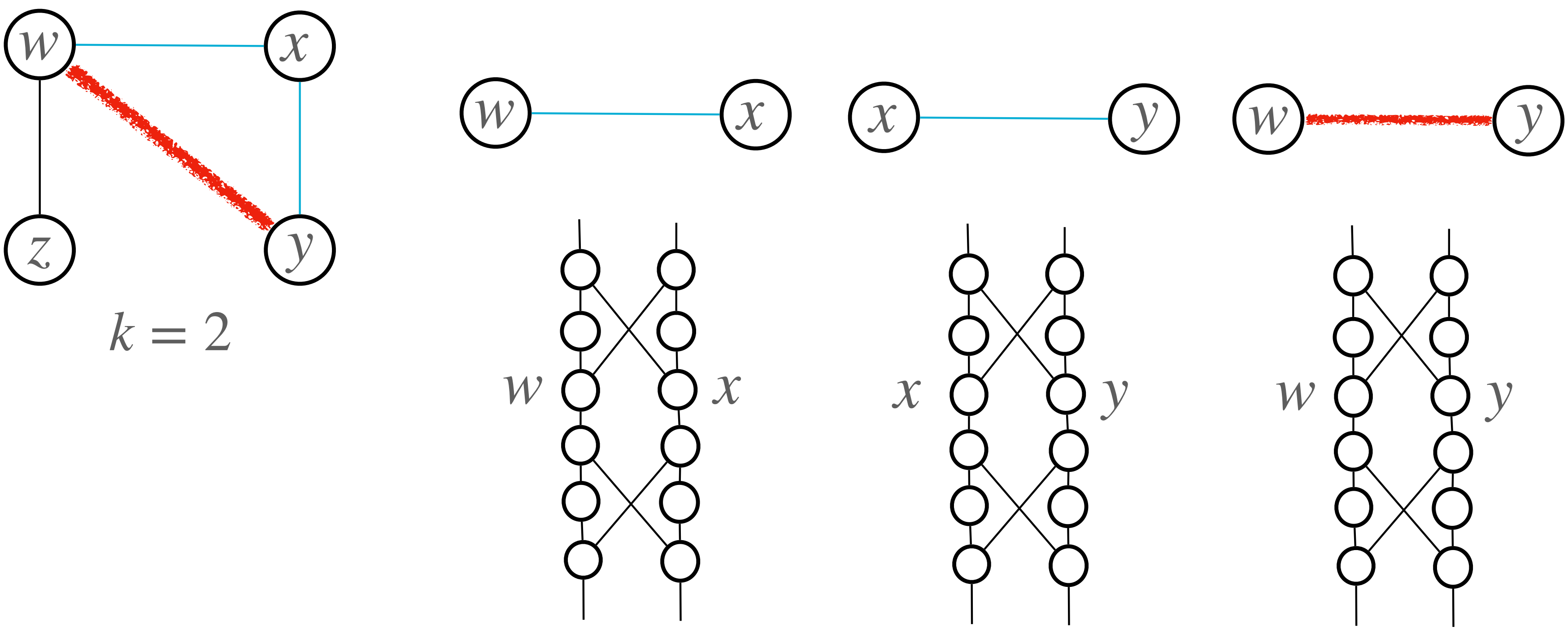

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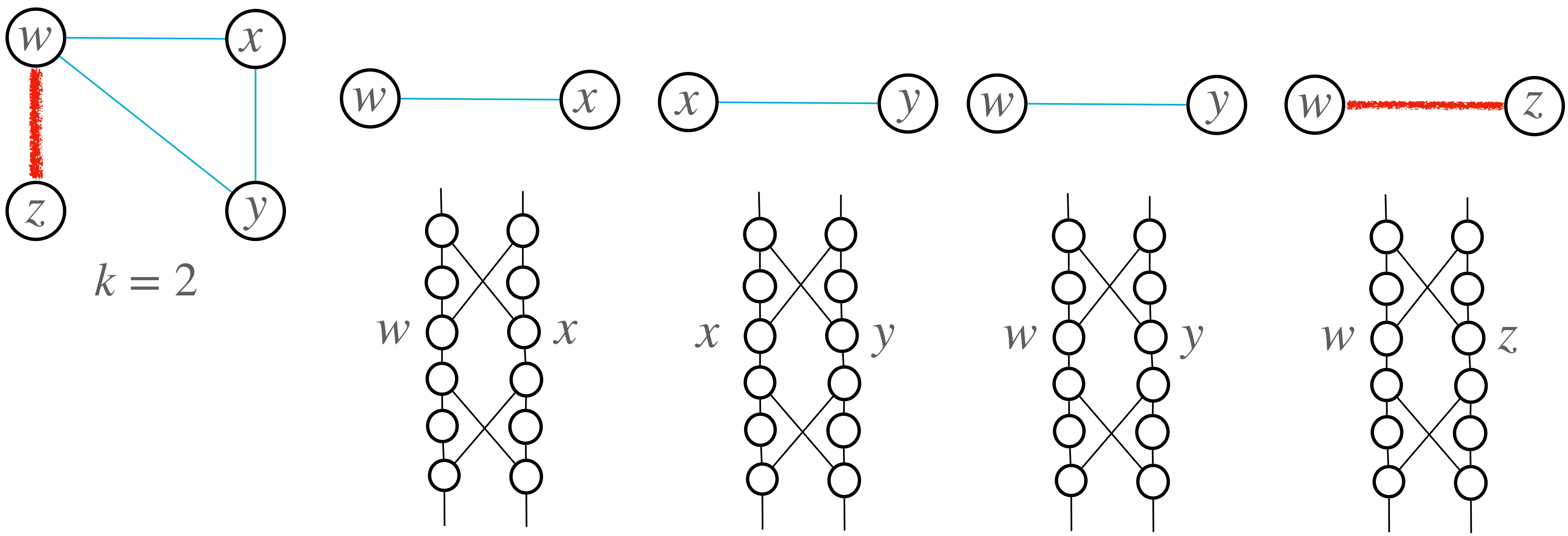

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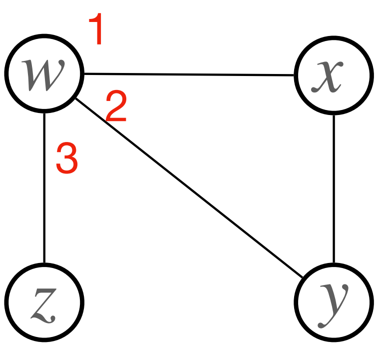

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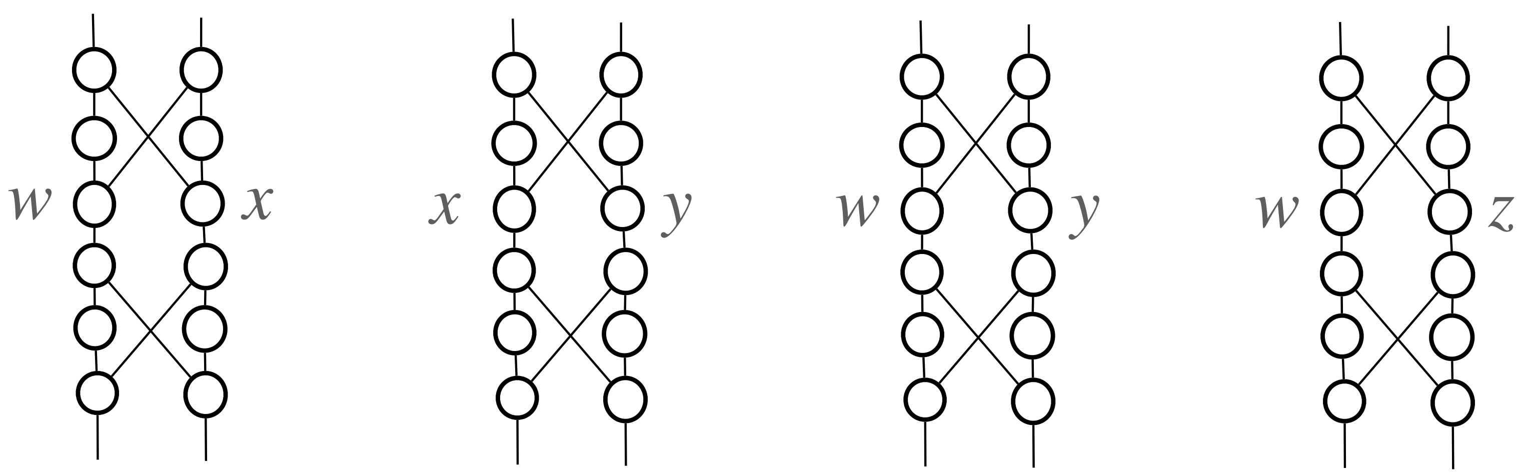
Now let's look at the neighbors of each node.
Let's order the neighbors in an arbitrary order.



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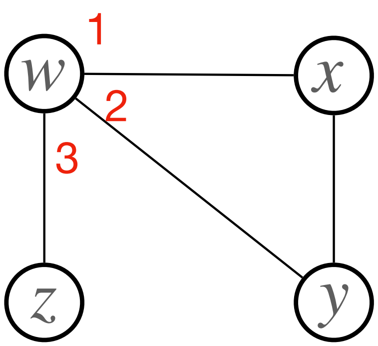

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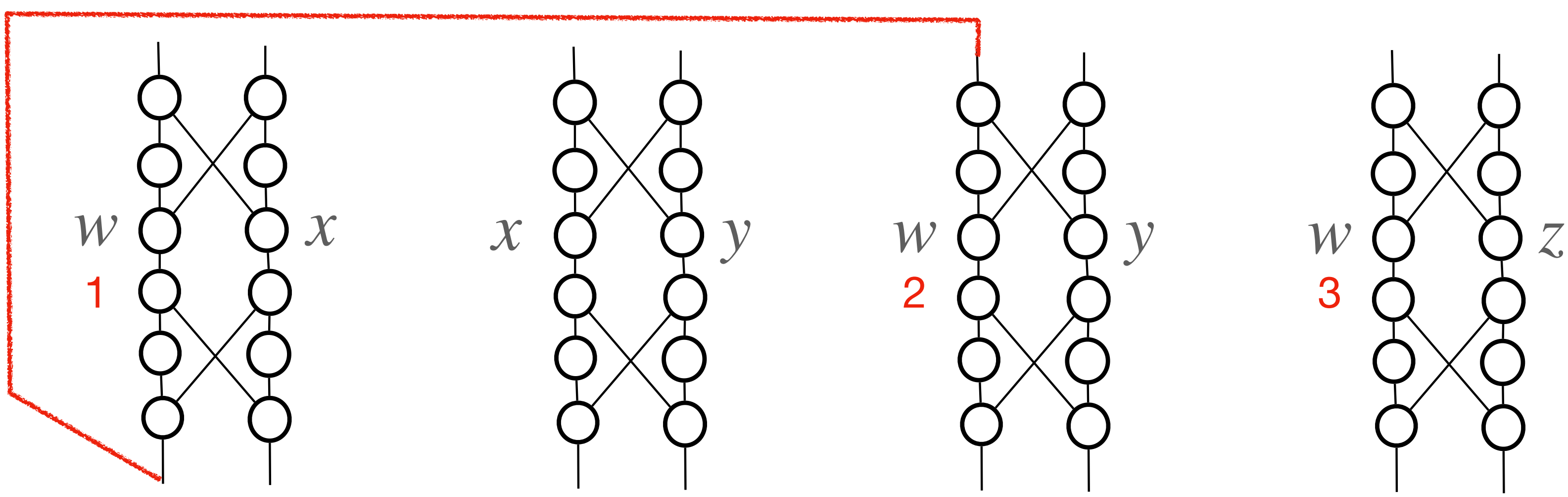

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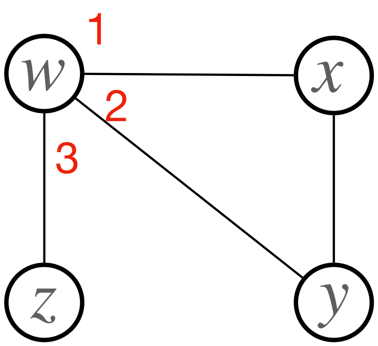

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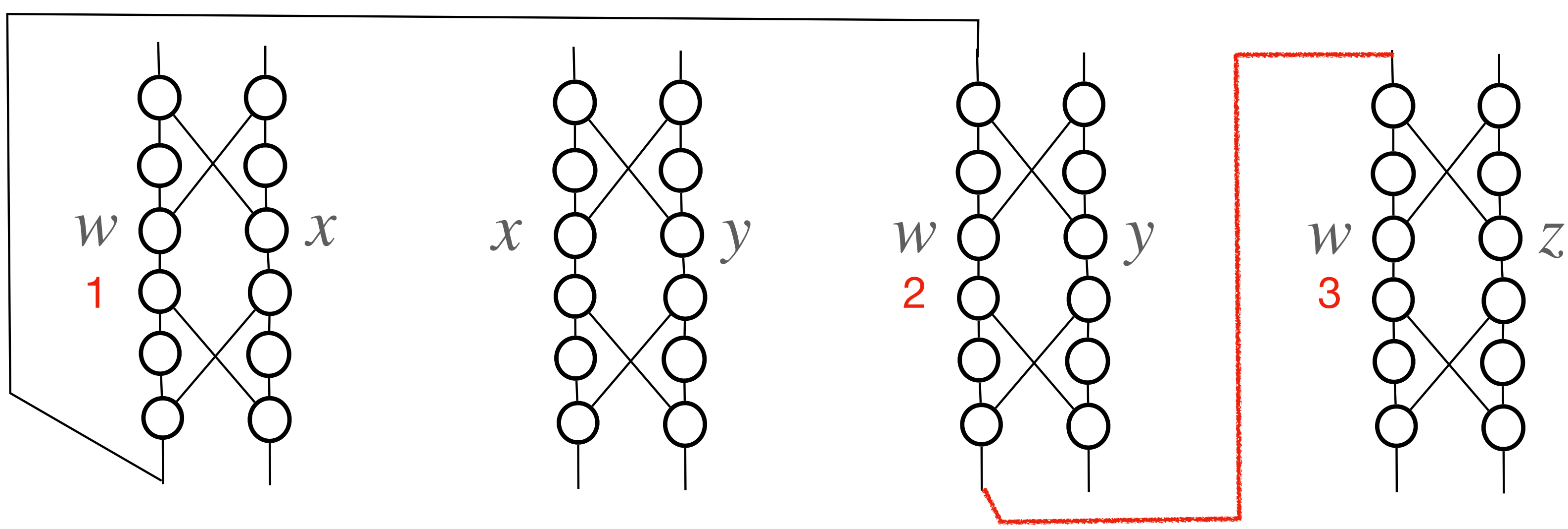

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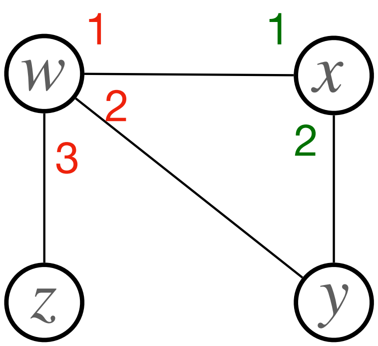

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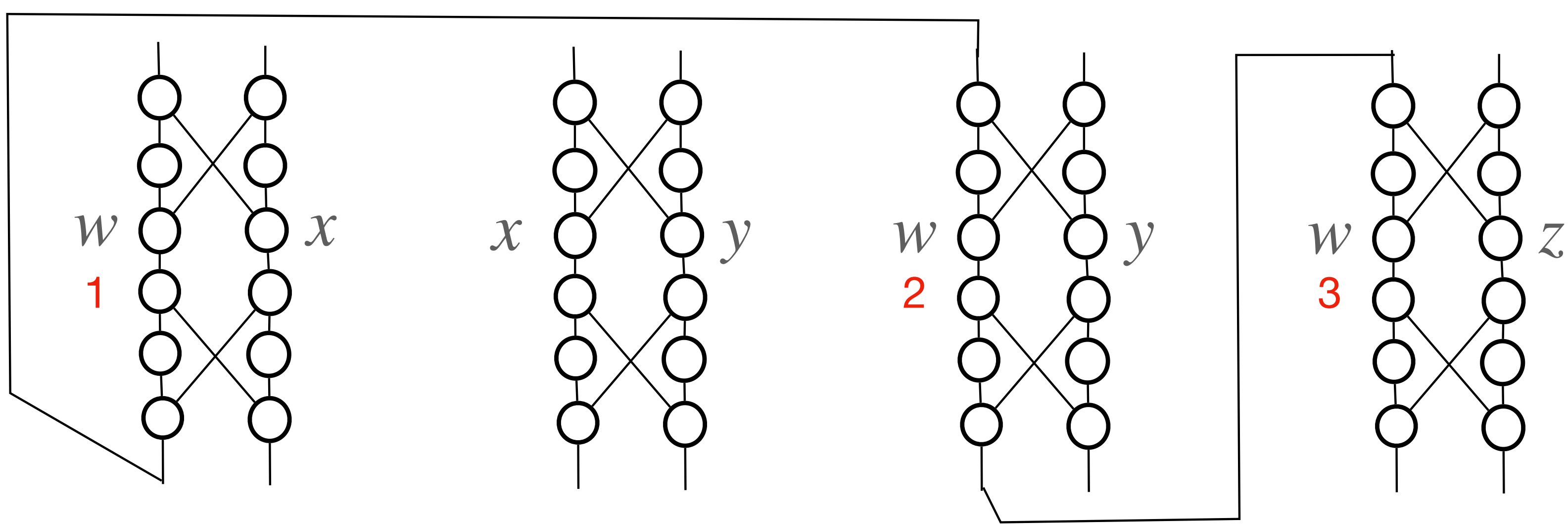

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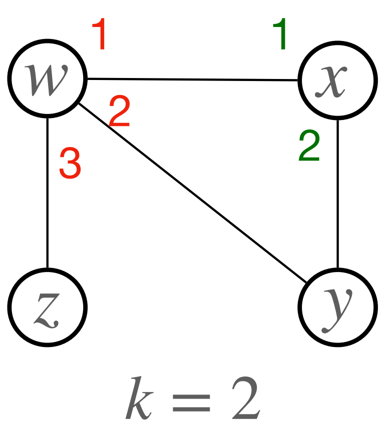

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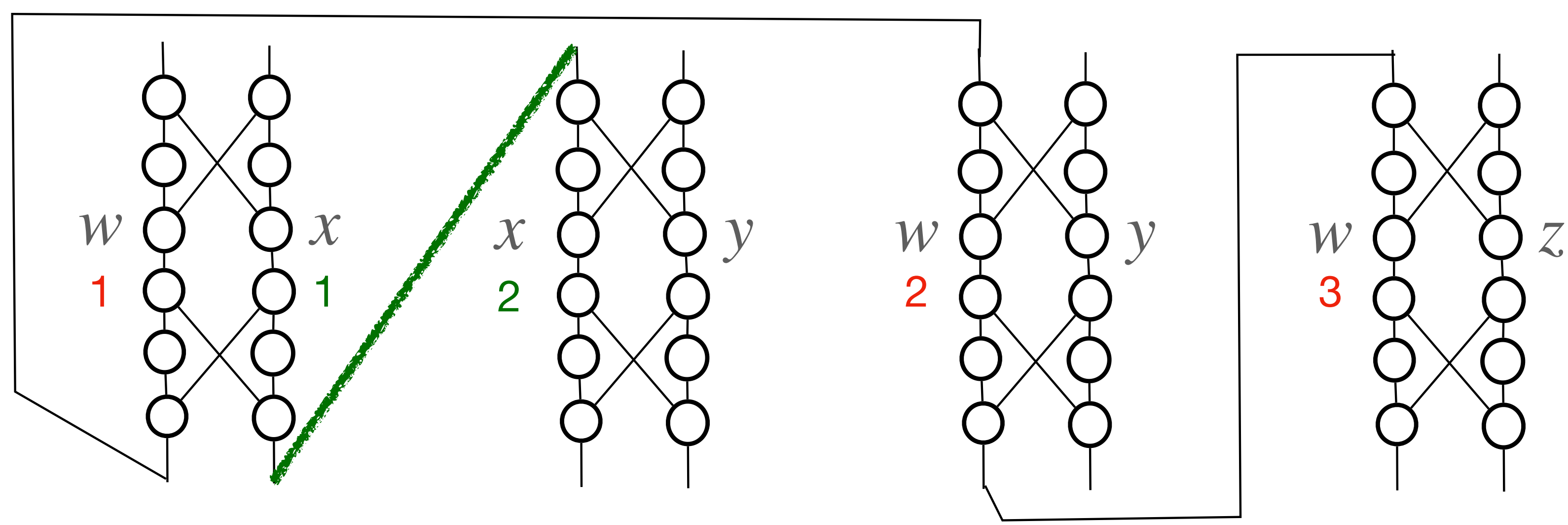

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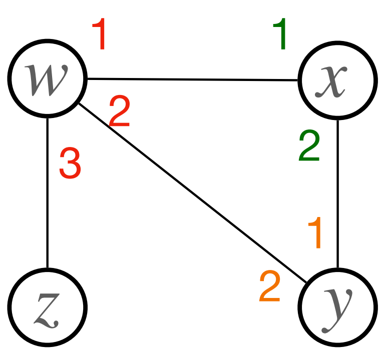

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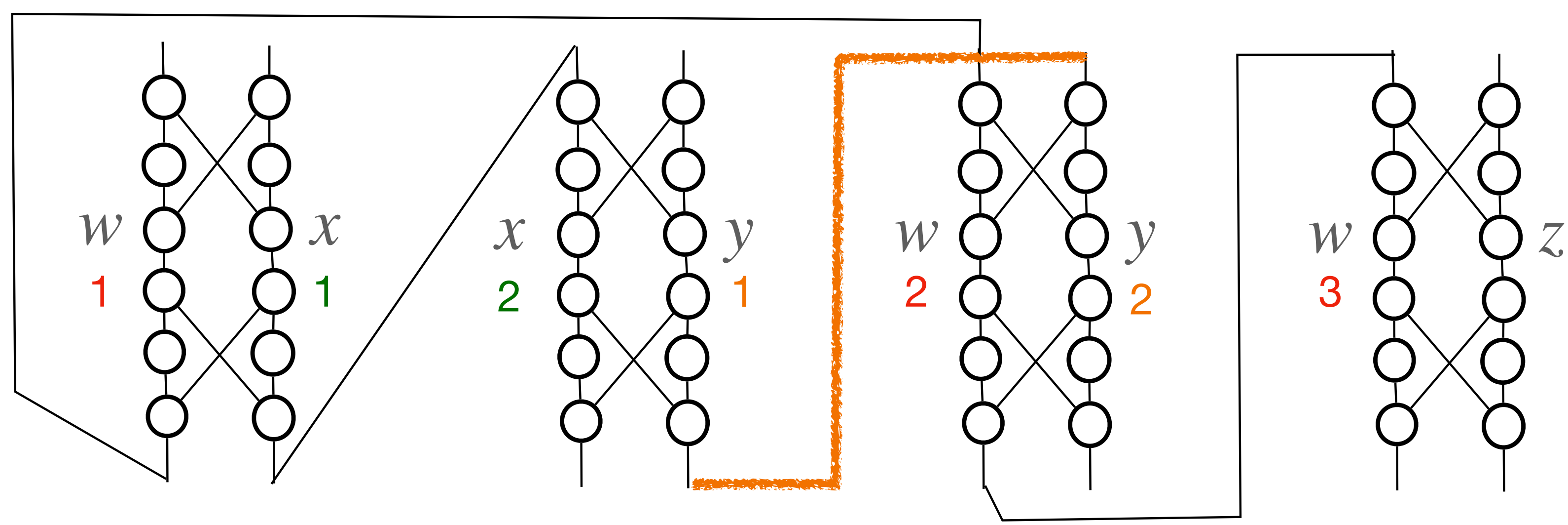

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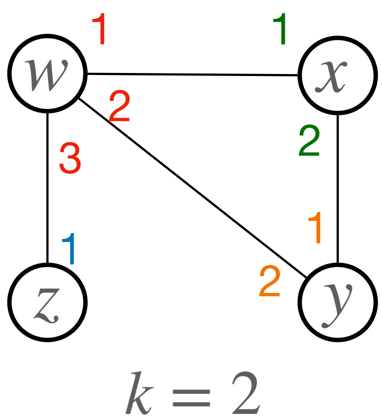

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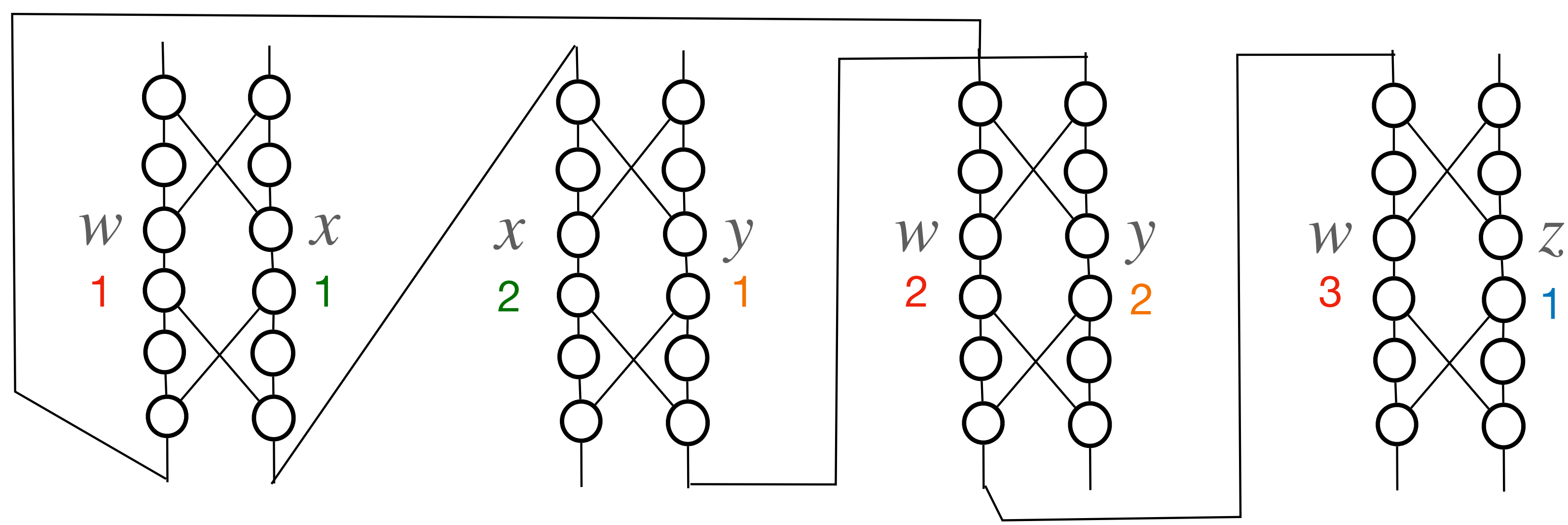

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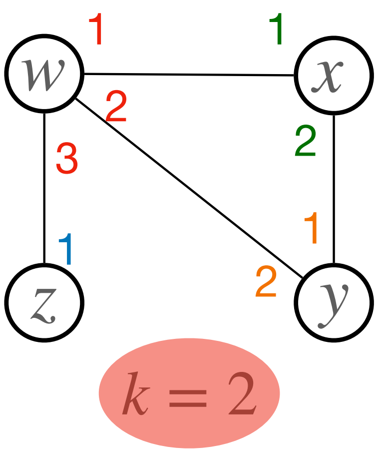
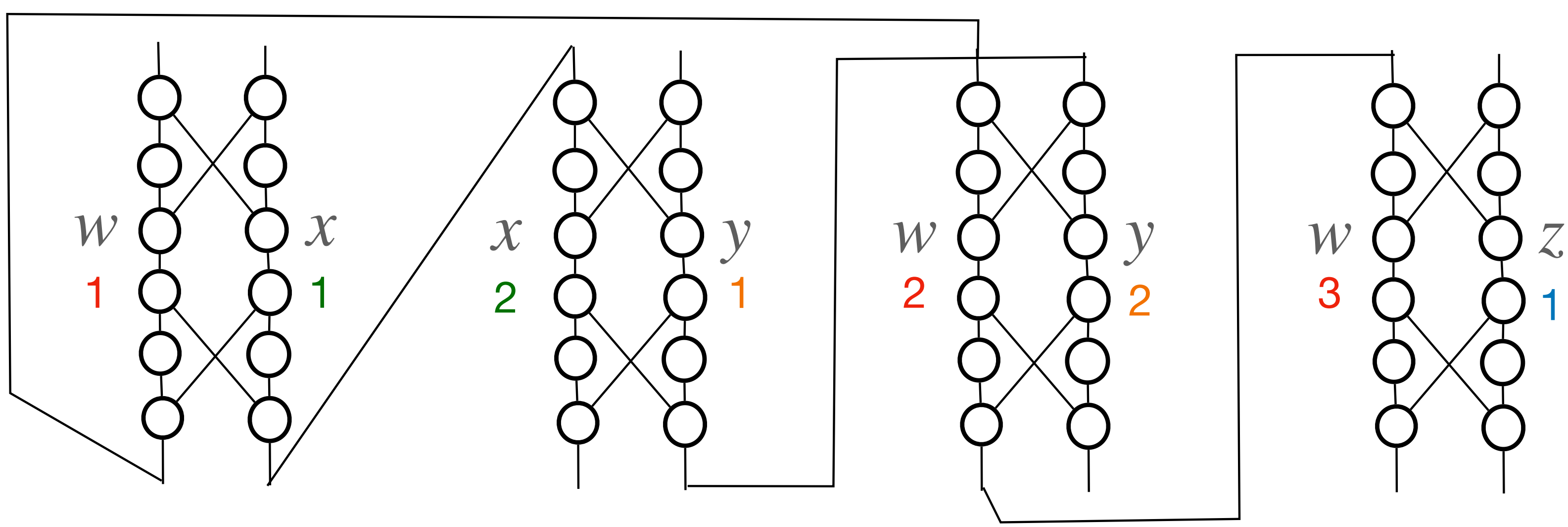


Now consider the k node selections for the vertex cover.



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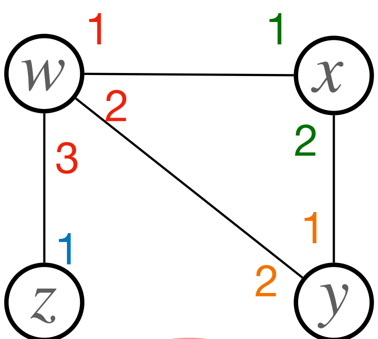
s_1 1st node in vertex cover

s_2 2nd node in vertex cover



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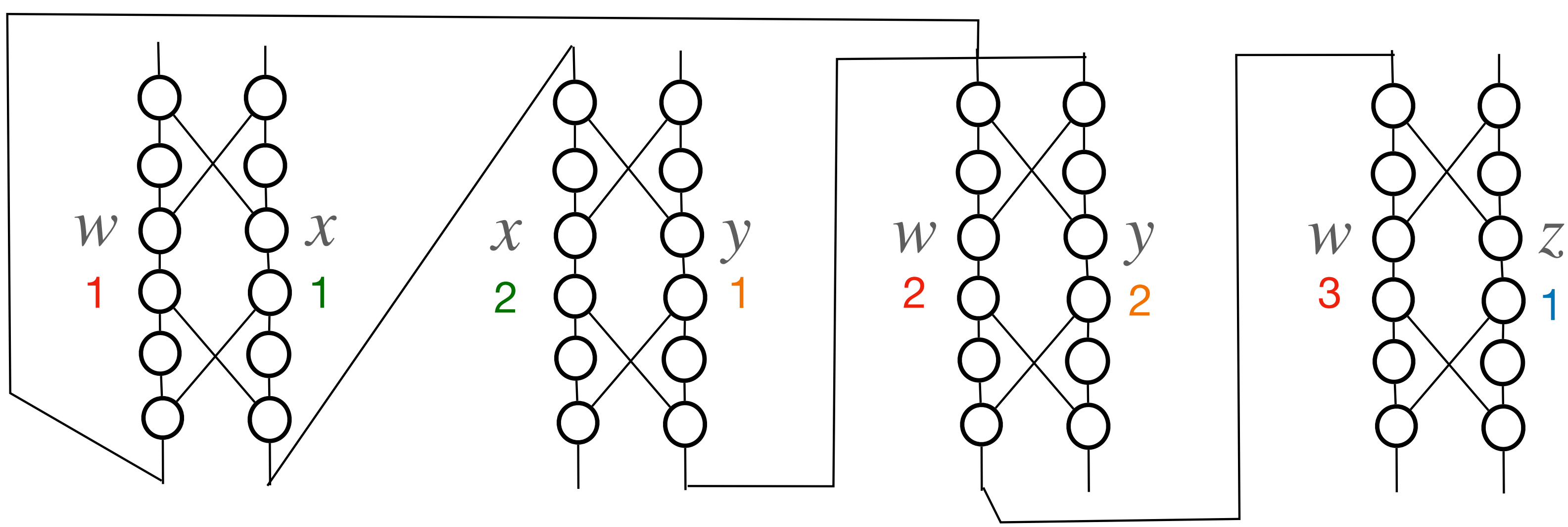

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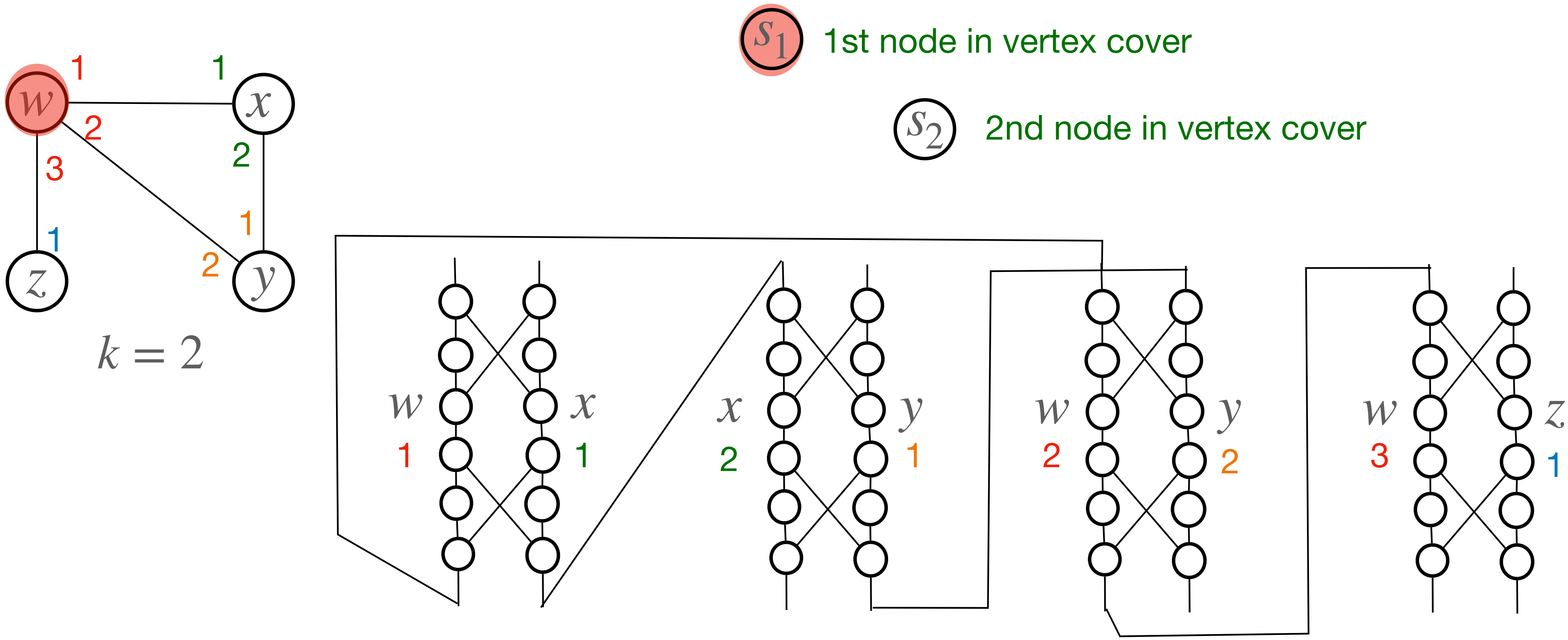

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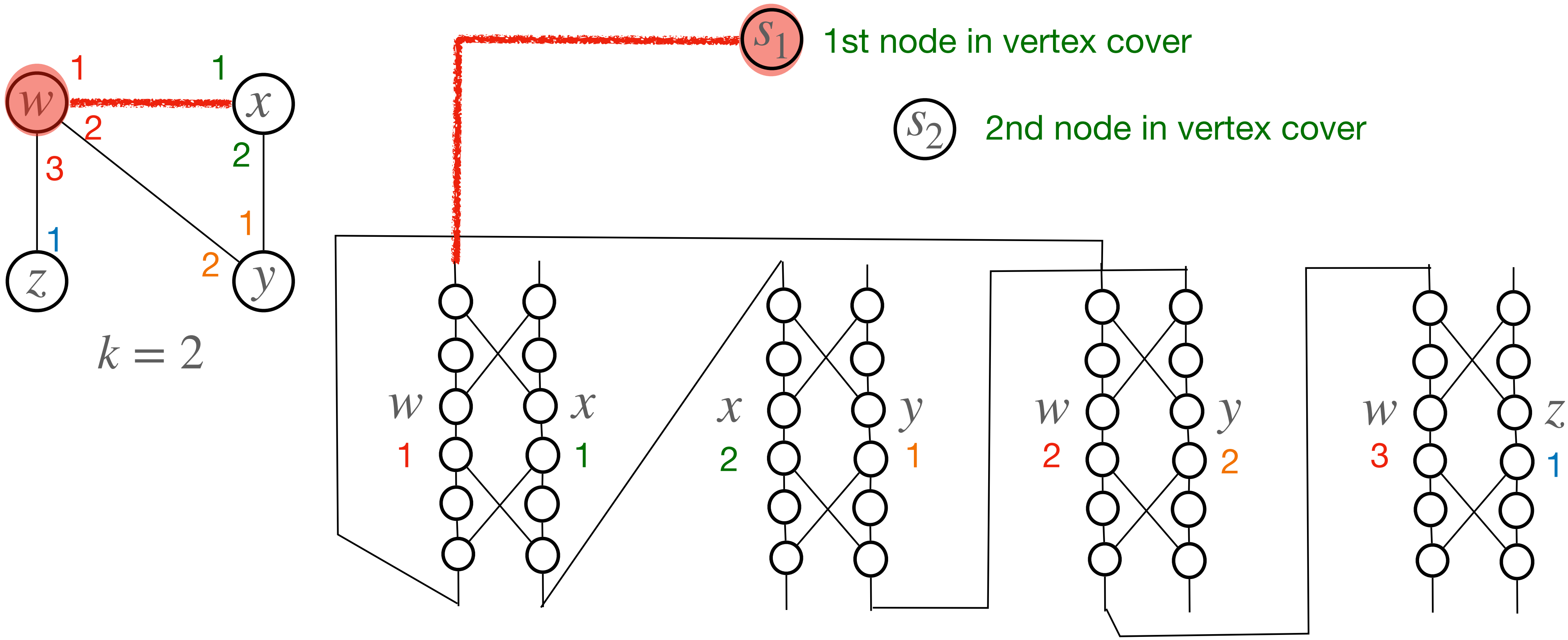

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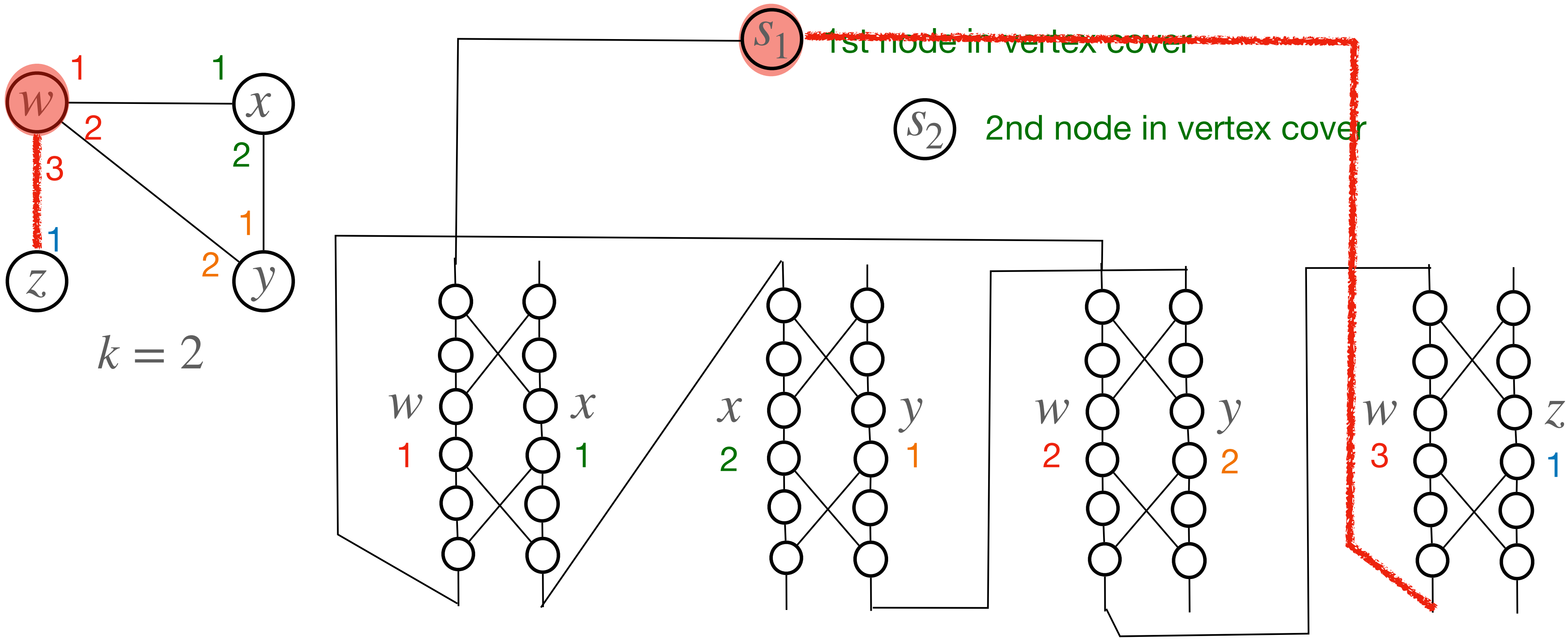

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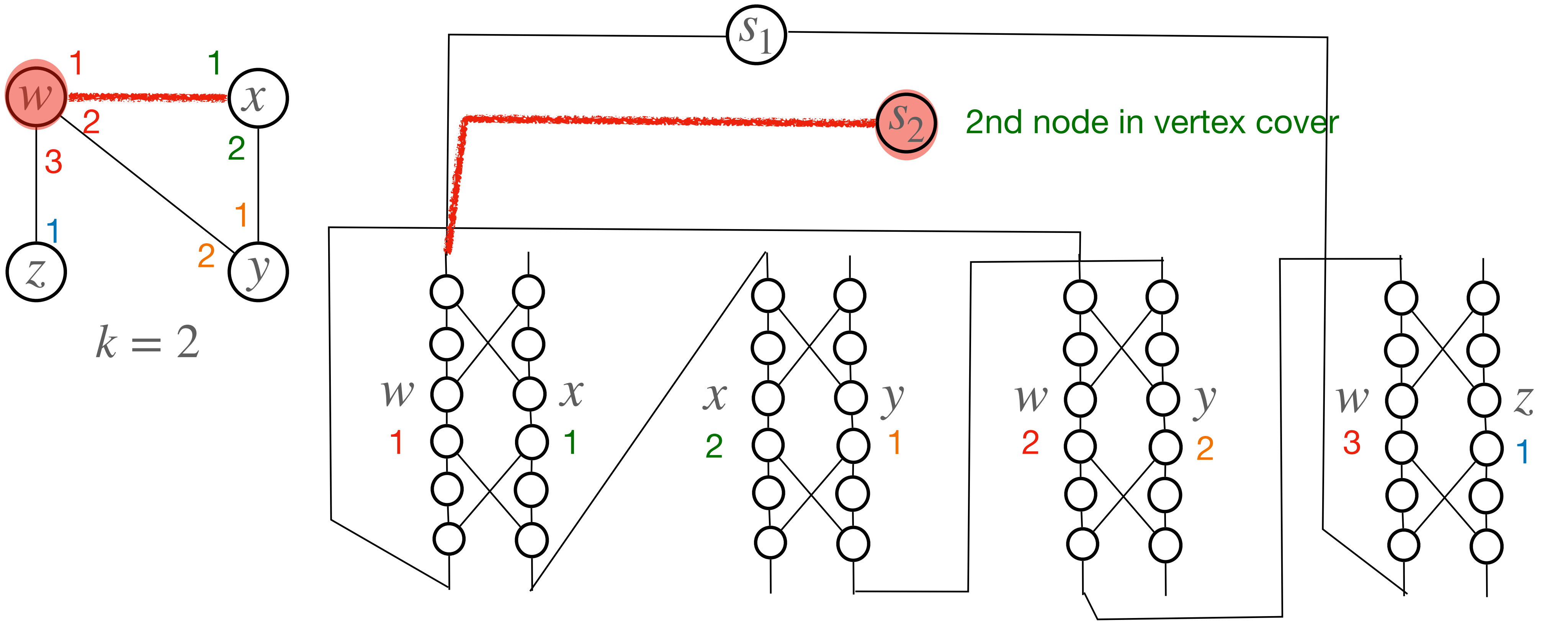

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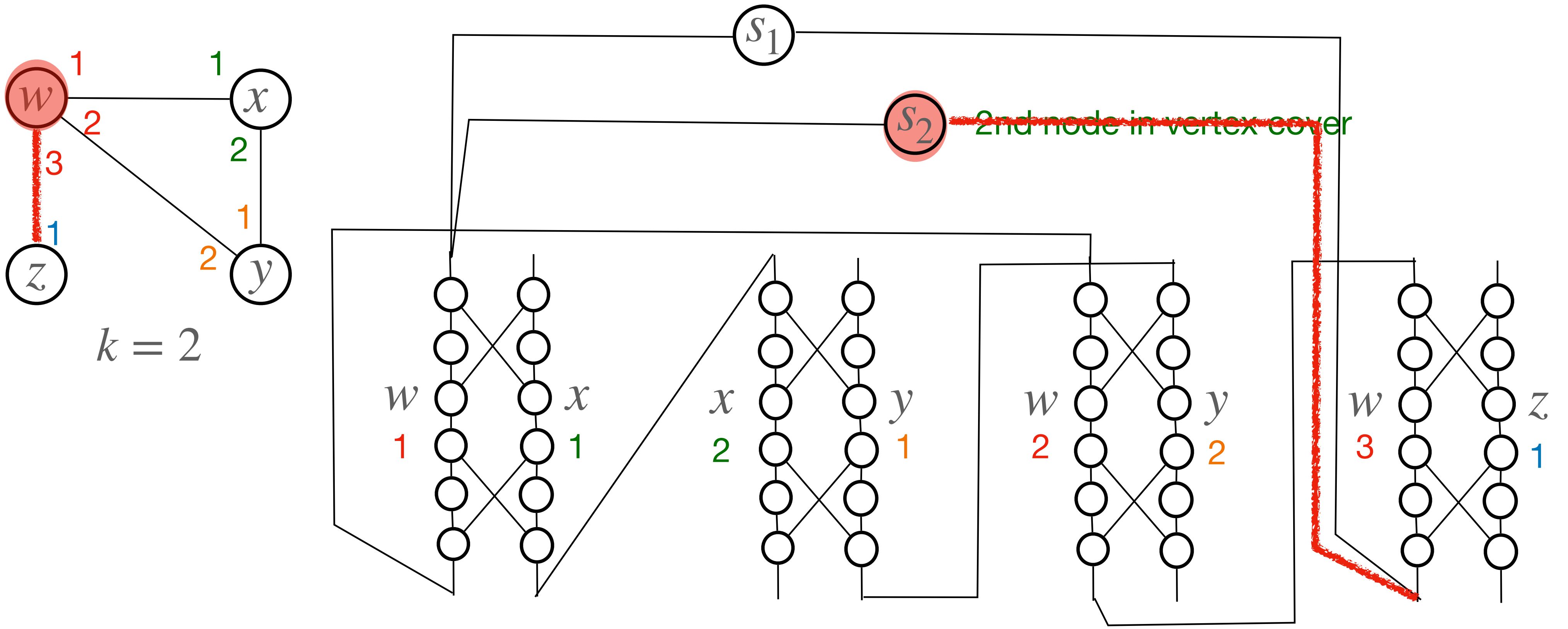

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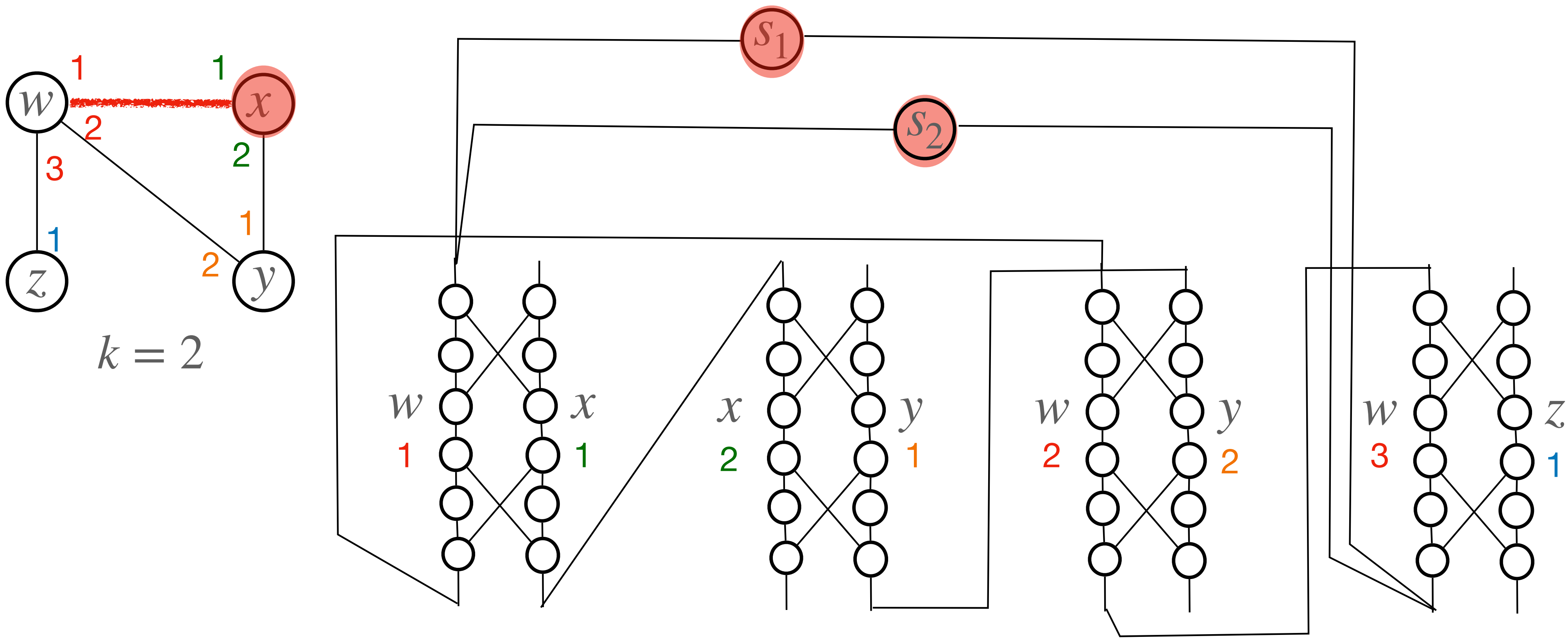

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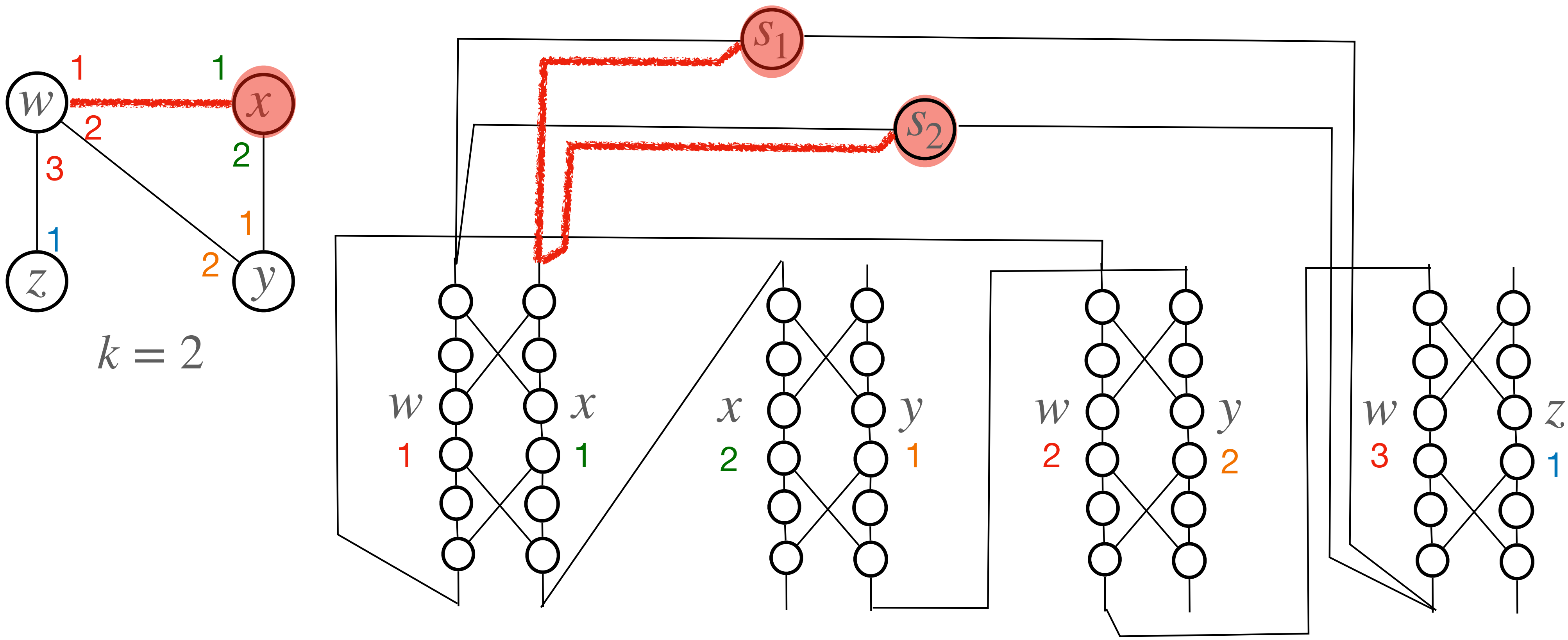

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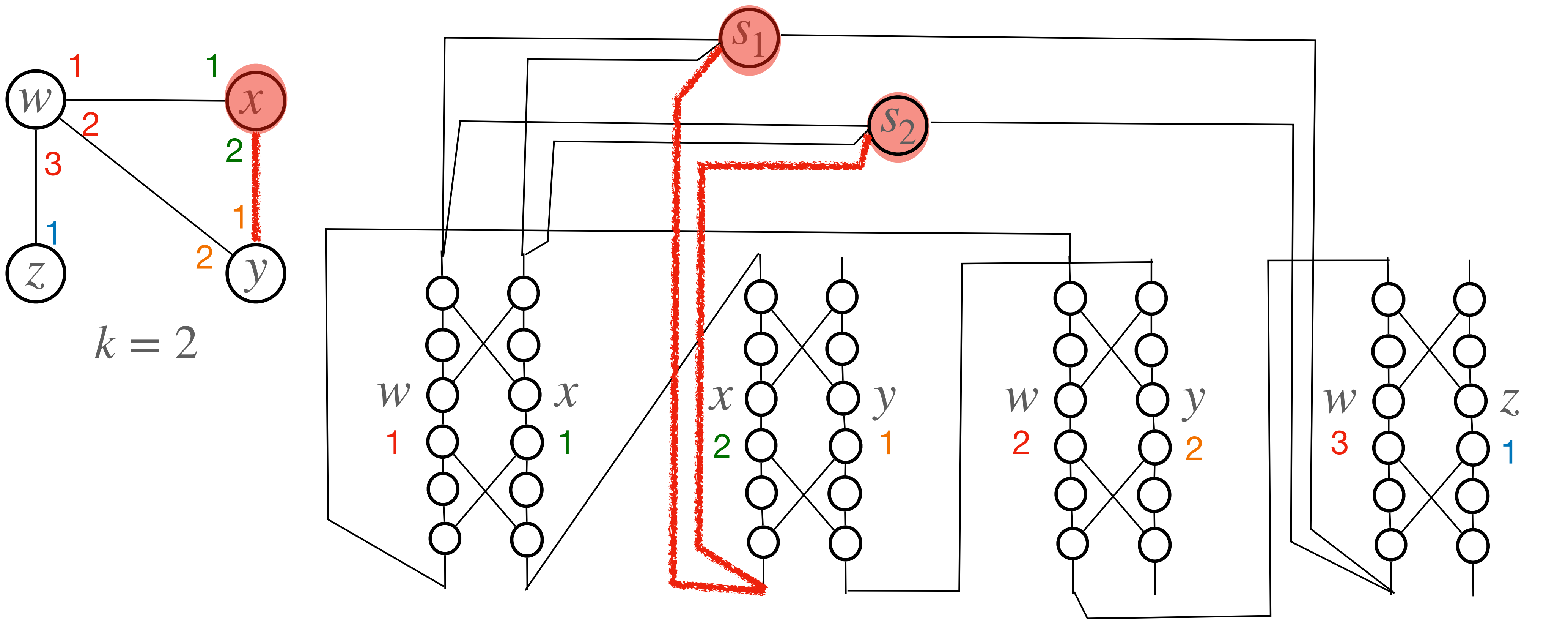

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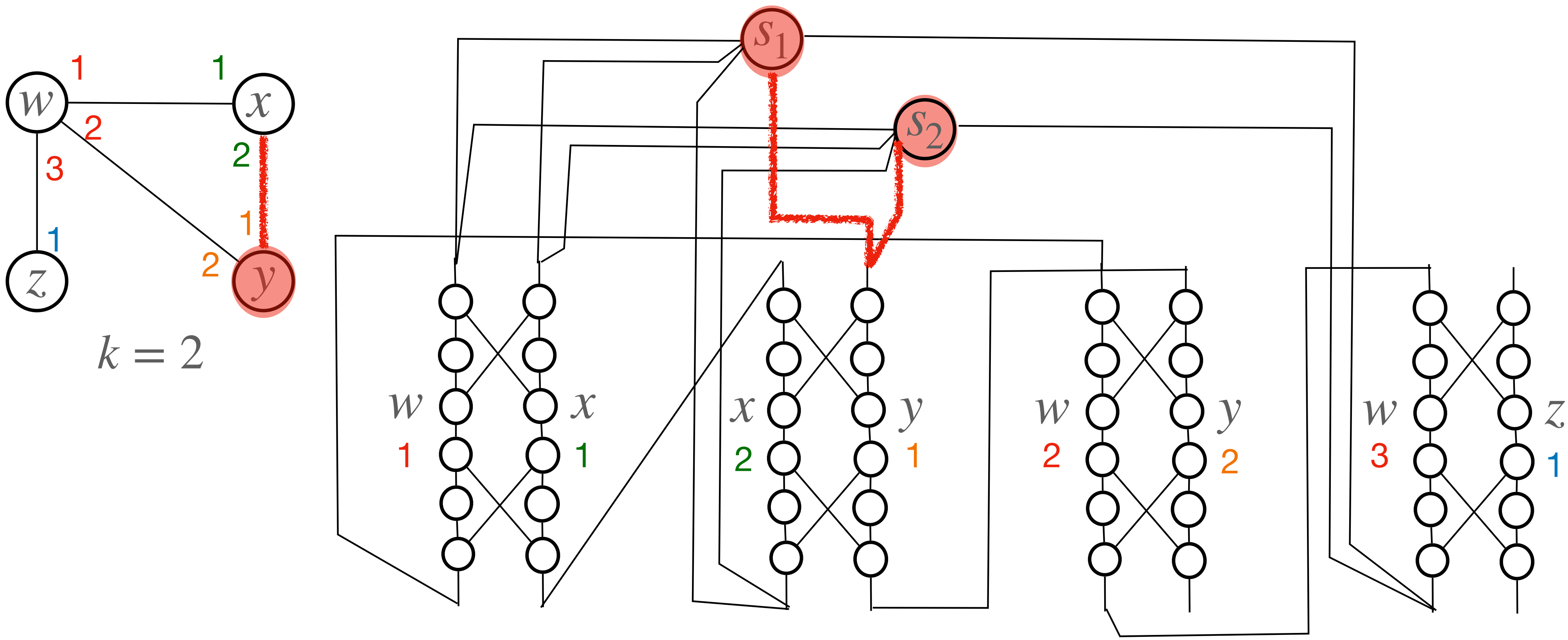

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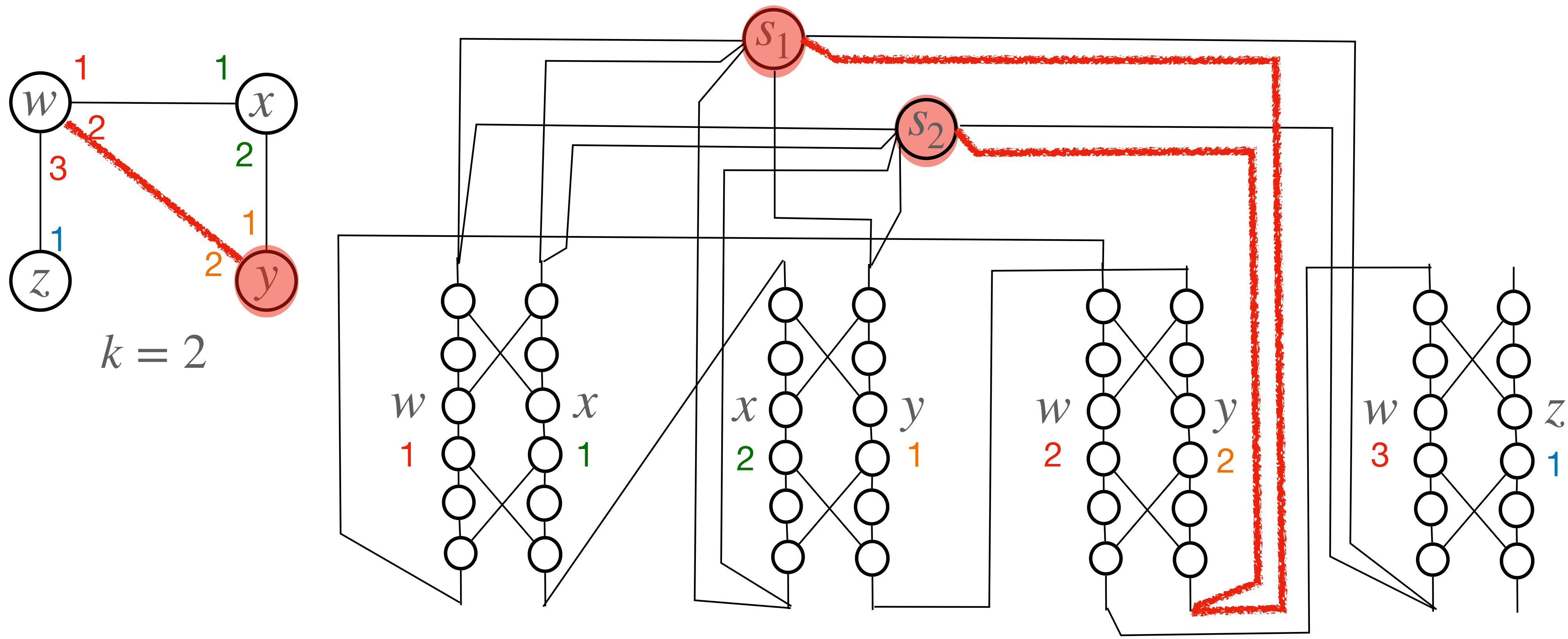

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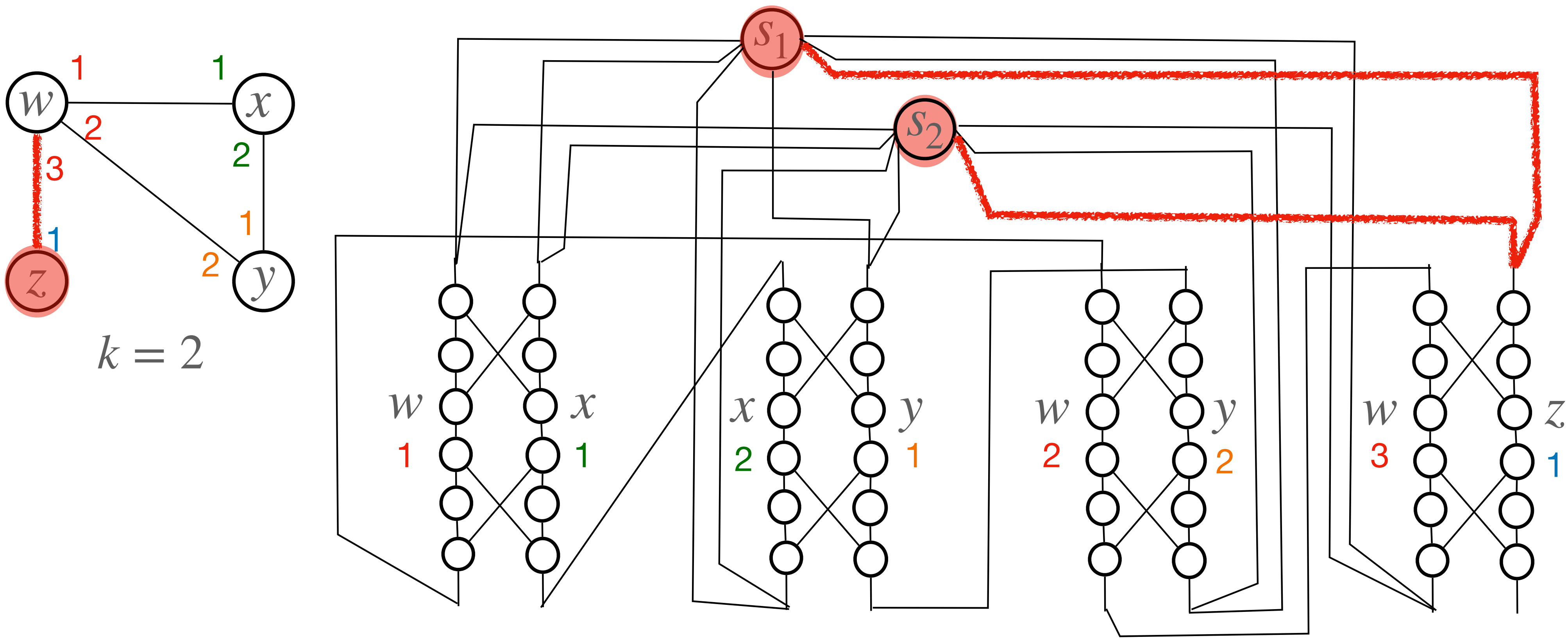

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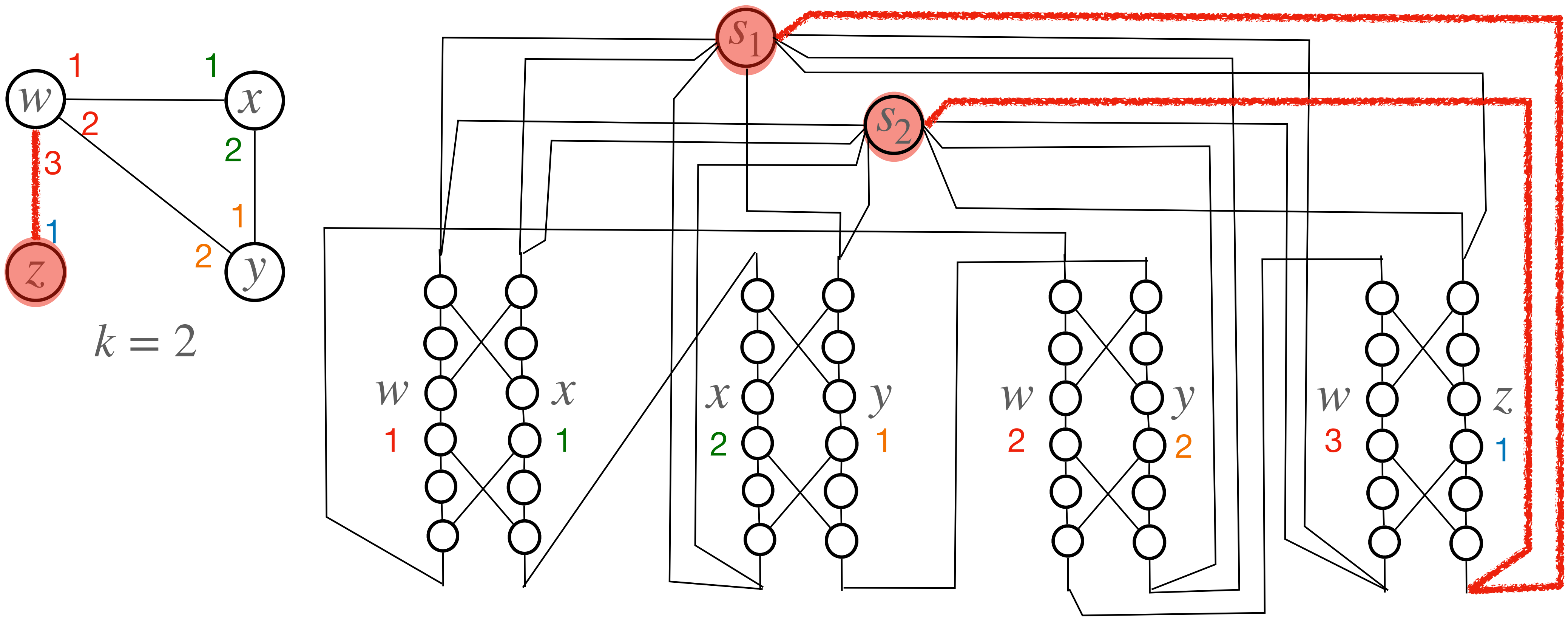
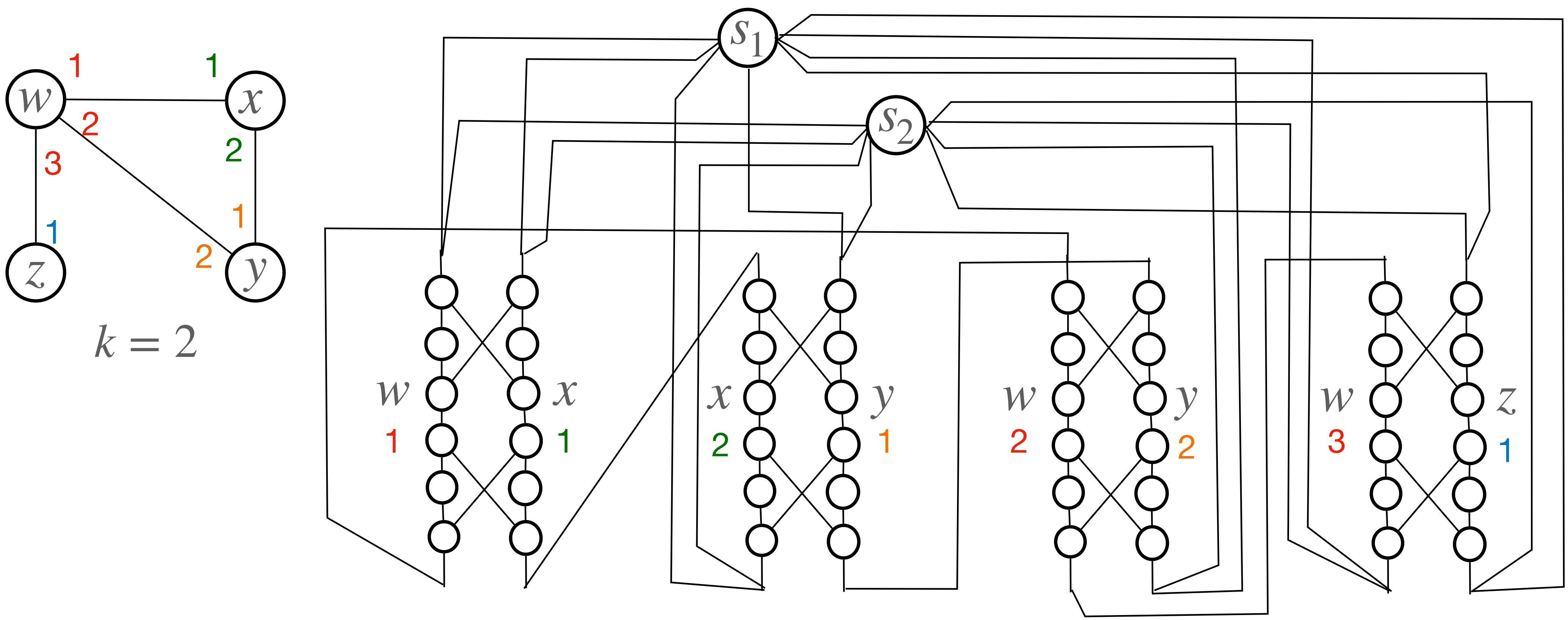
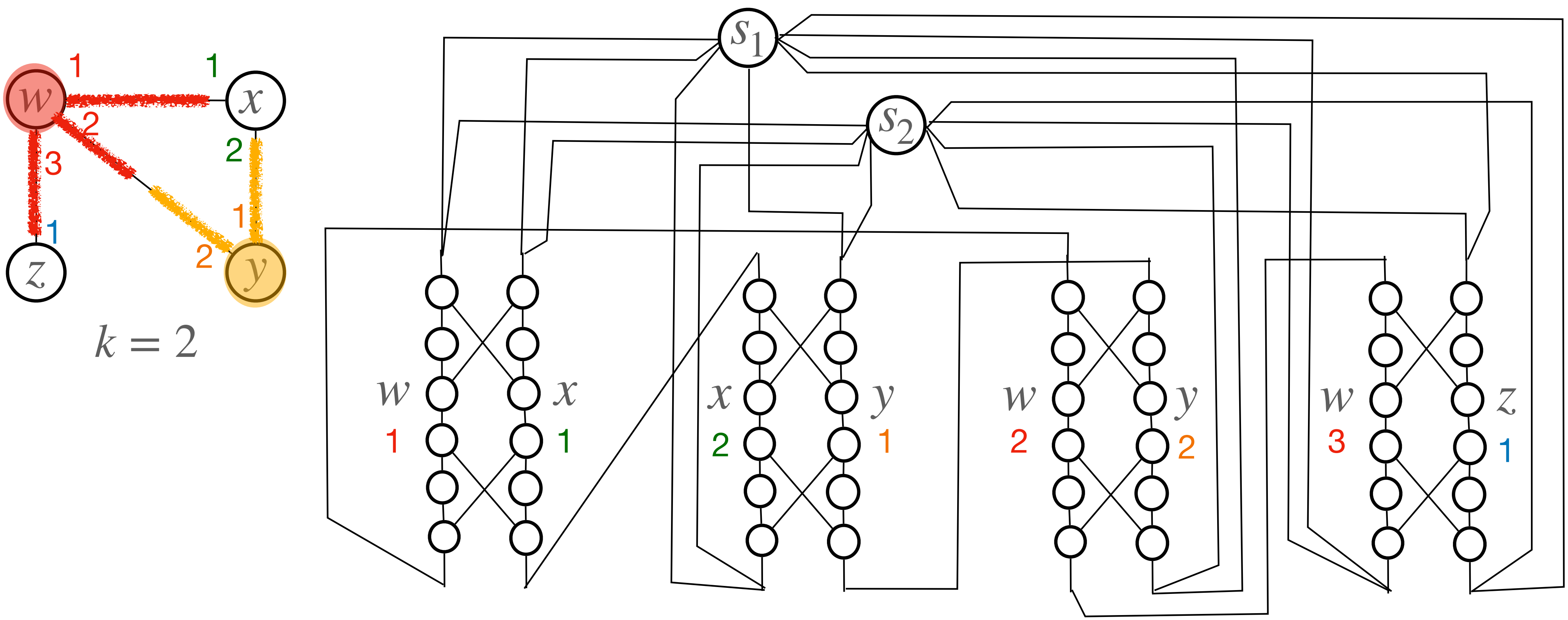


The mapping from the “Vertex Cover Problem” to the “Hamiltonian Cycle Problem” is a polynomial-time mapping.

Now let's show that it preserves the YES/NO answer.



Assume "YES" for the Vertex Cover Problem. Assume the vertex cover is {w,y}.



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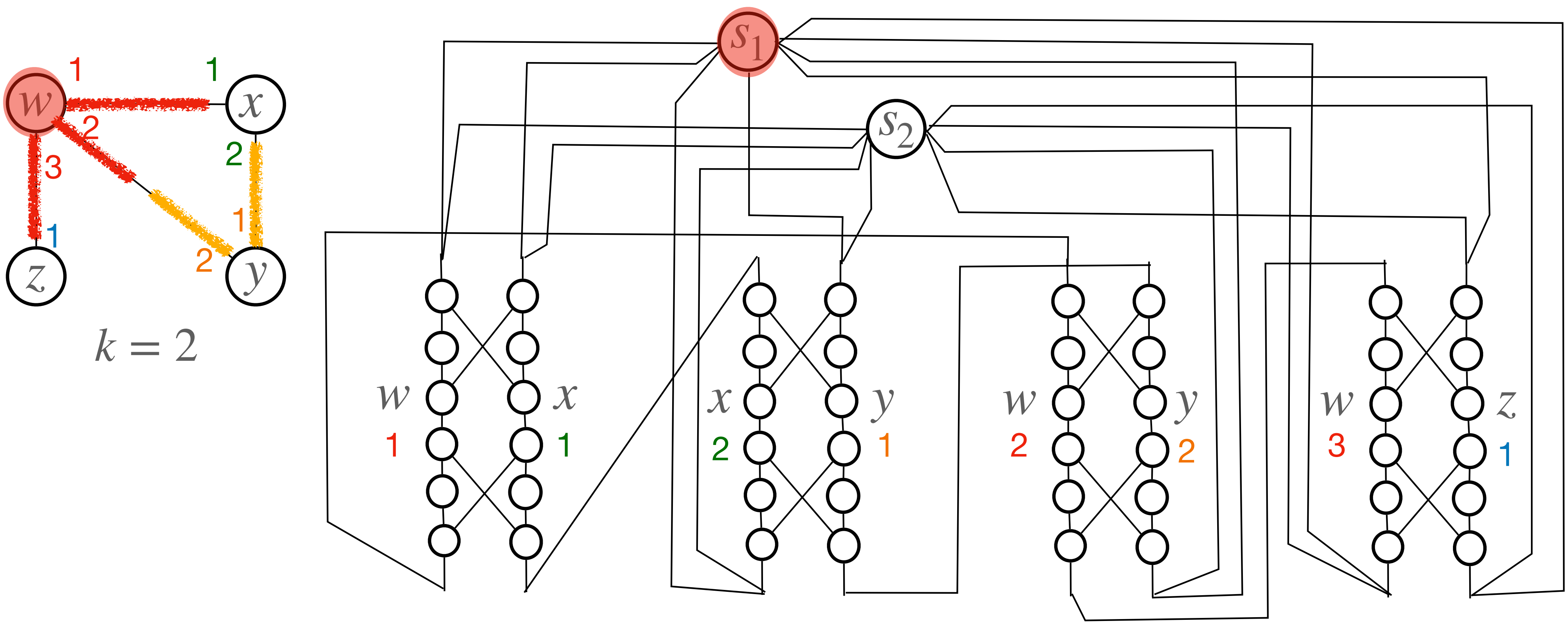
Select 1st node in the vertex cover: w

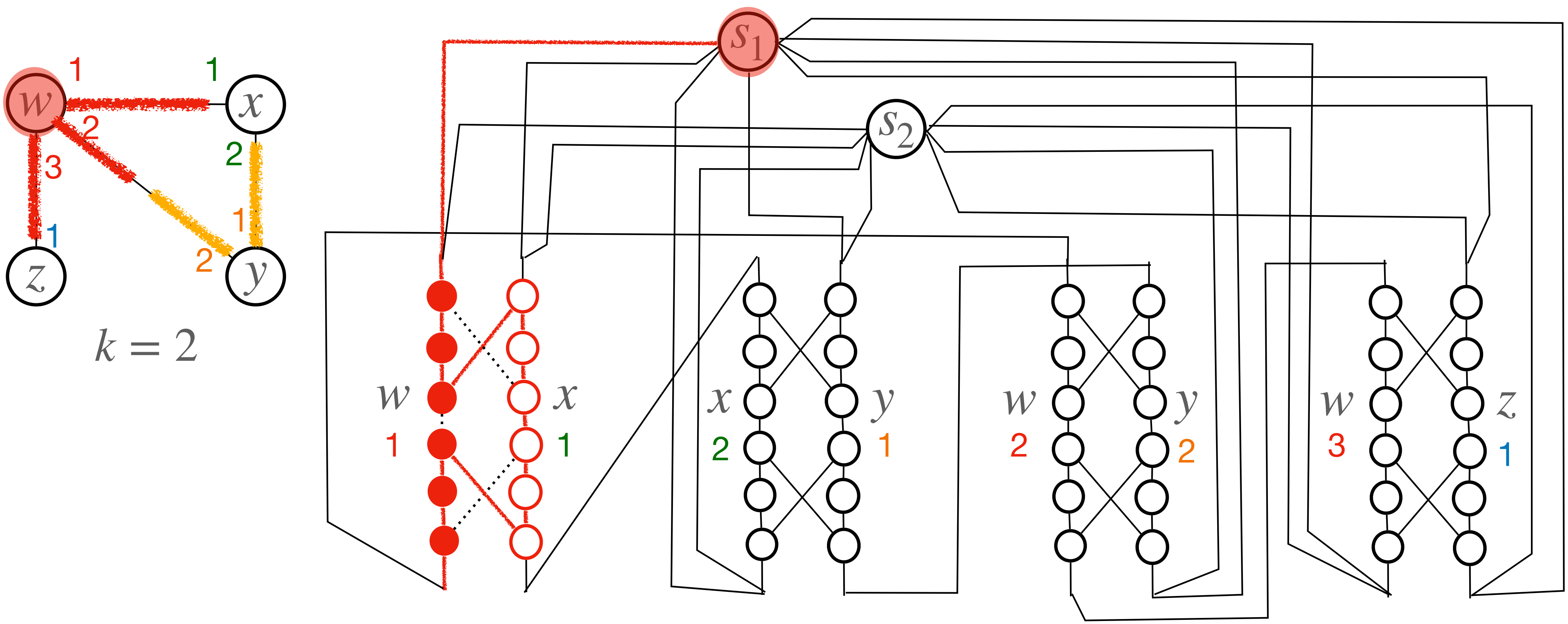


Assume “YES” for the Vertex Cover Problem.

Assume the vertex cover is {w,y}.

Select 1st node in the vertex cover: w

How does w cover its 1st edge (w,x)?

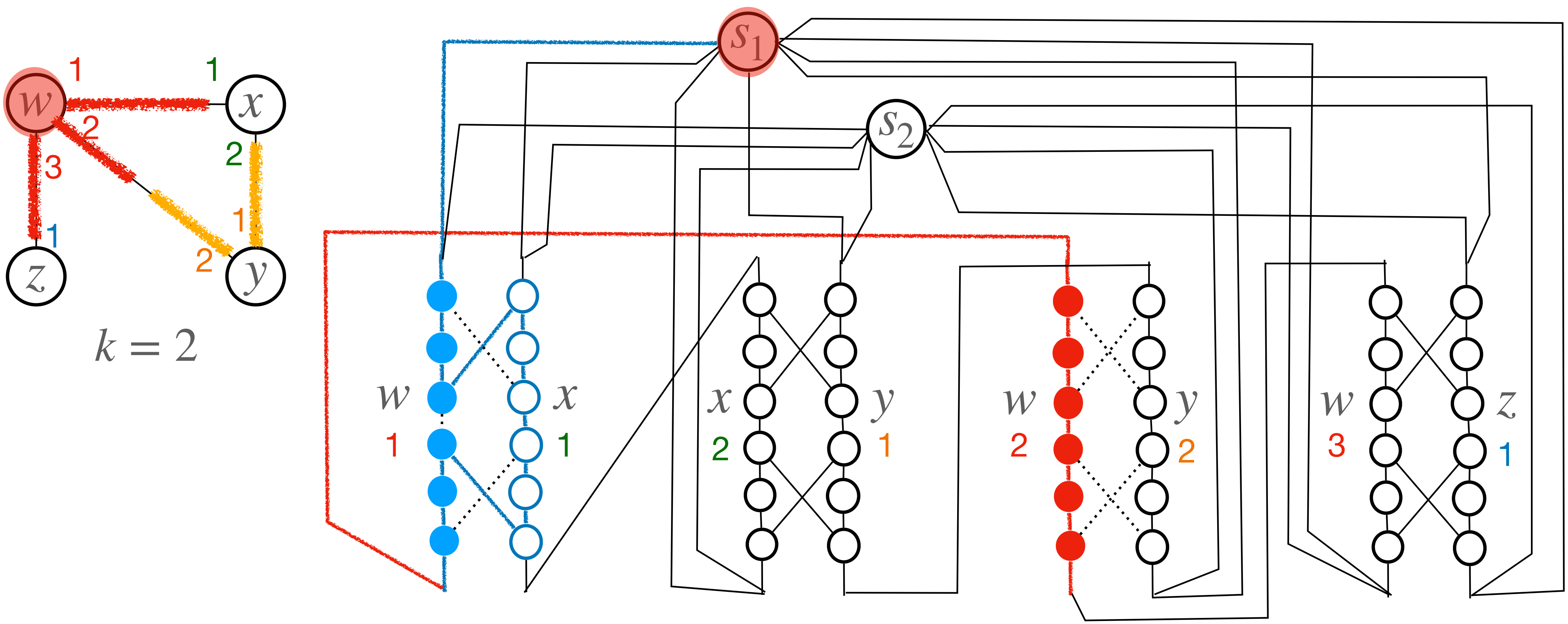


Assume “YES” for the Vertex Cover Problem.

Assume the vertex cover is {w,y}.

Select 1st node in the vertex cover: w

How does w cover its 2nd edge (w,y)?

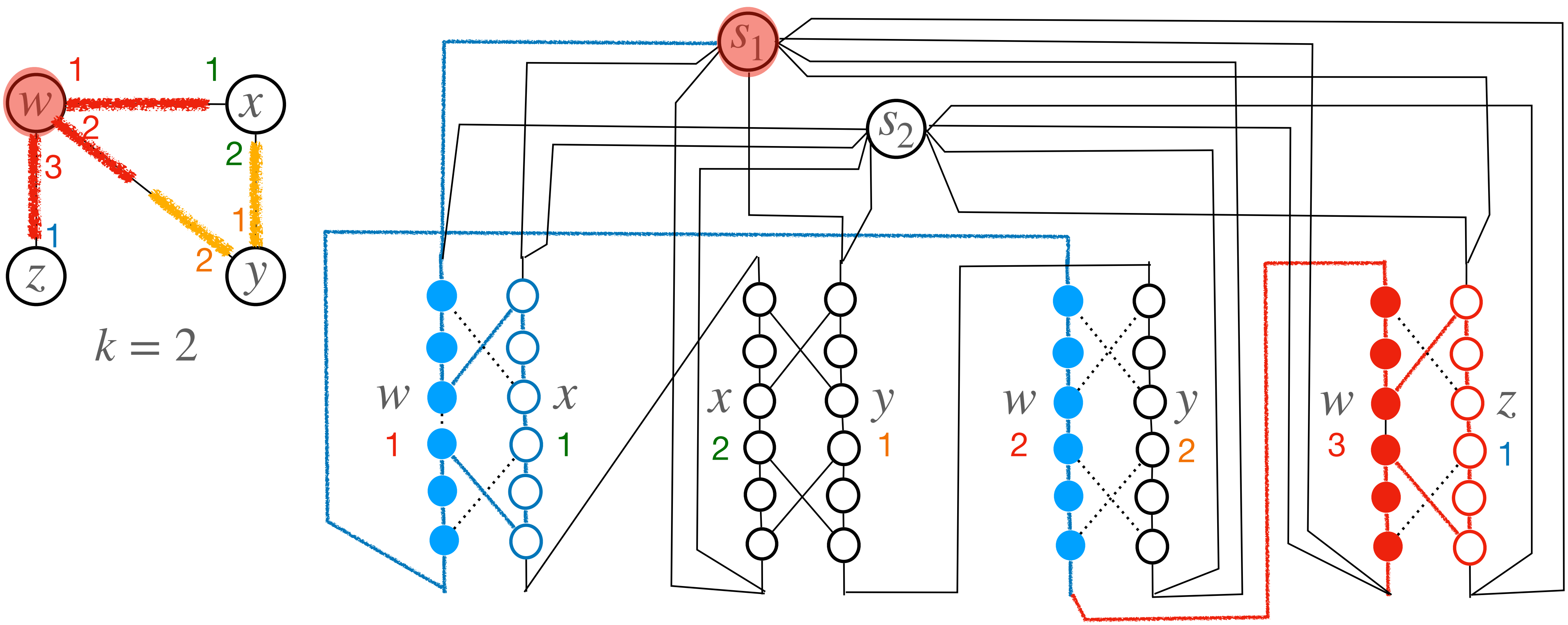


Assume “YES” for the Vertex Cover Problem.

Assume the vertex cover is {w,y}.

Select 1st node in the vertex cover: w

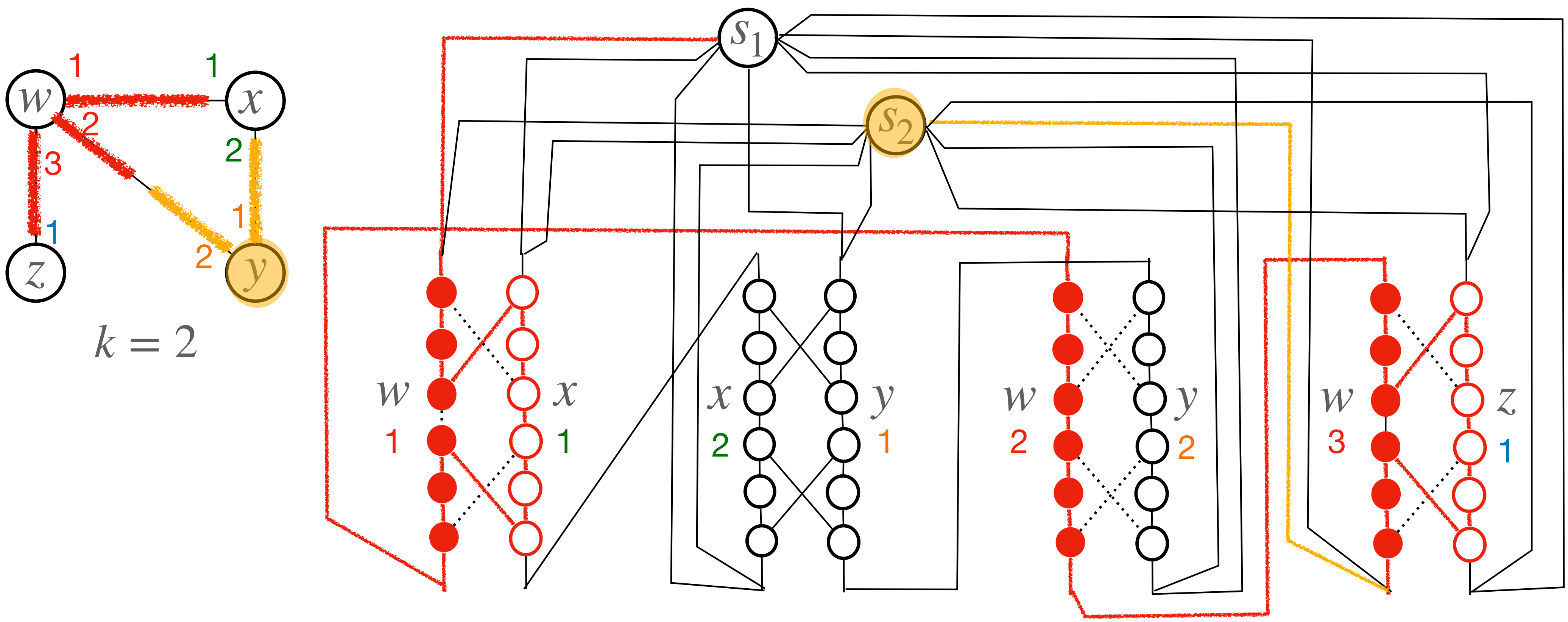
How does w cover its 3rd edge (w,z)?



Assume “YES” for the Vertex Cover Problem.

Assume the vertex cover is {w,y}.

Select 2nd node in the vertex cover: y

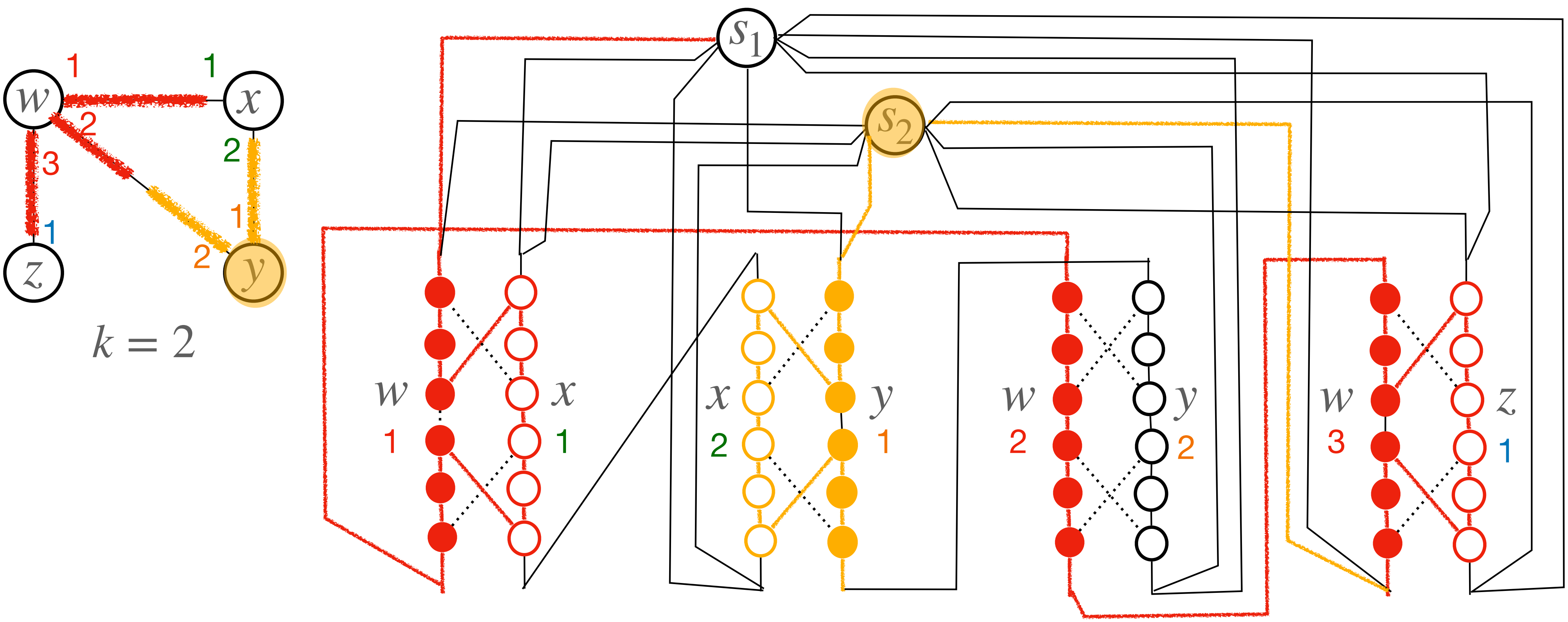


Assume “YES” for the Vertex Cover Problem.

Assume the vertex cover is {w,y}.

Select 2nd node in the vertex cover: y

How does y cover its 1st edge (y,x)?

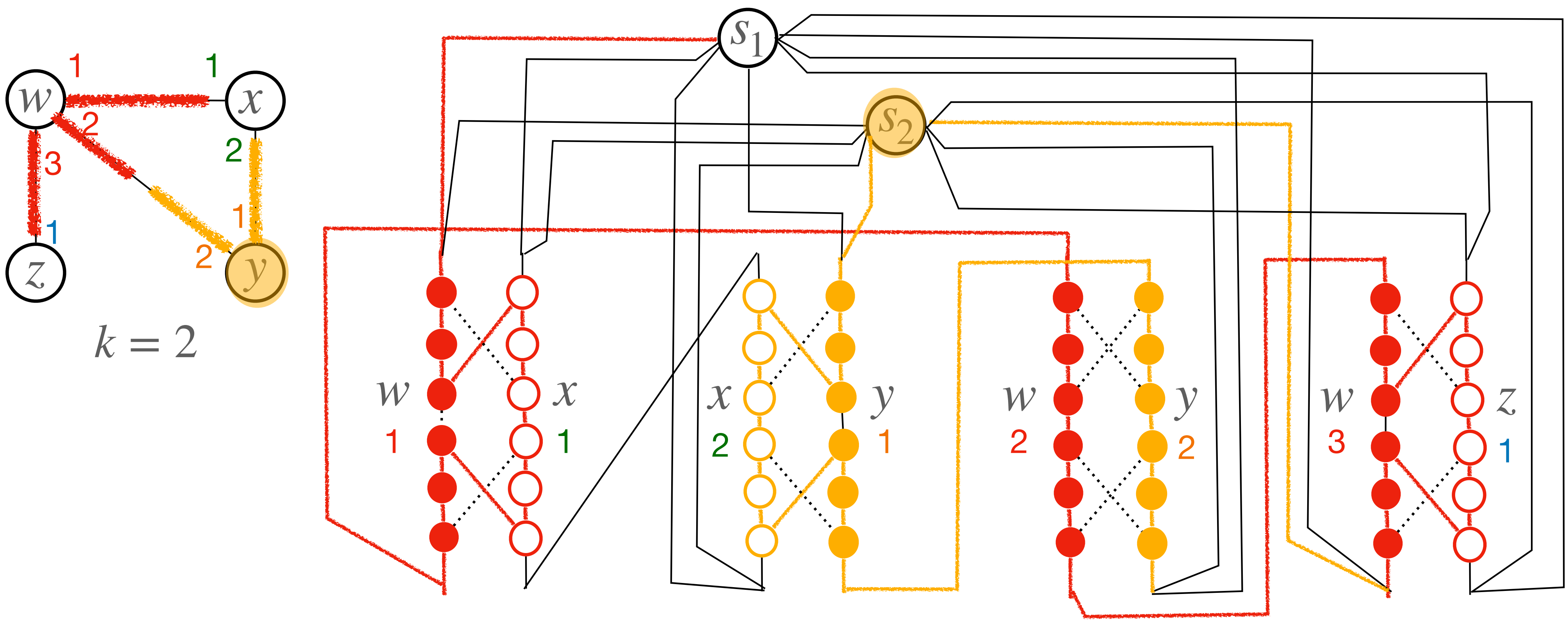


Assume “YES” for the Vertex Cover Problem.

Assume the vertex cover is {w,y}.

Select 2nd node in the vertex cover: y

How does y cover its 2nd edge (y,w)?



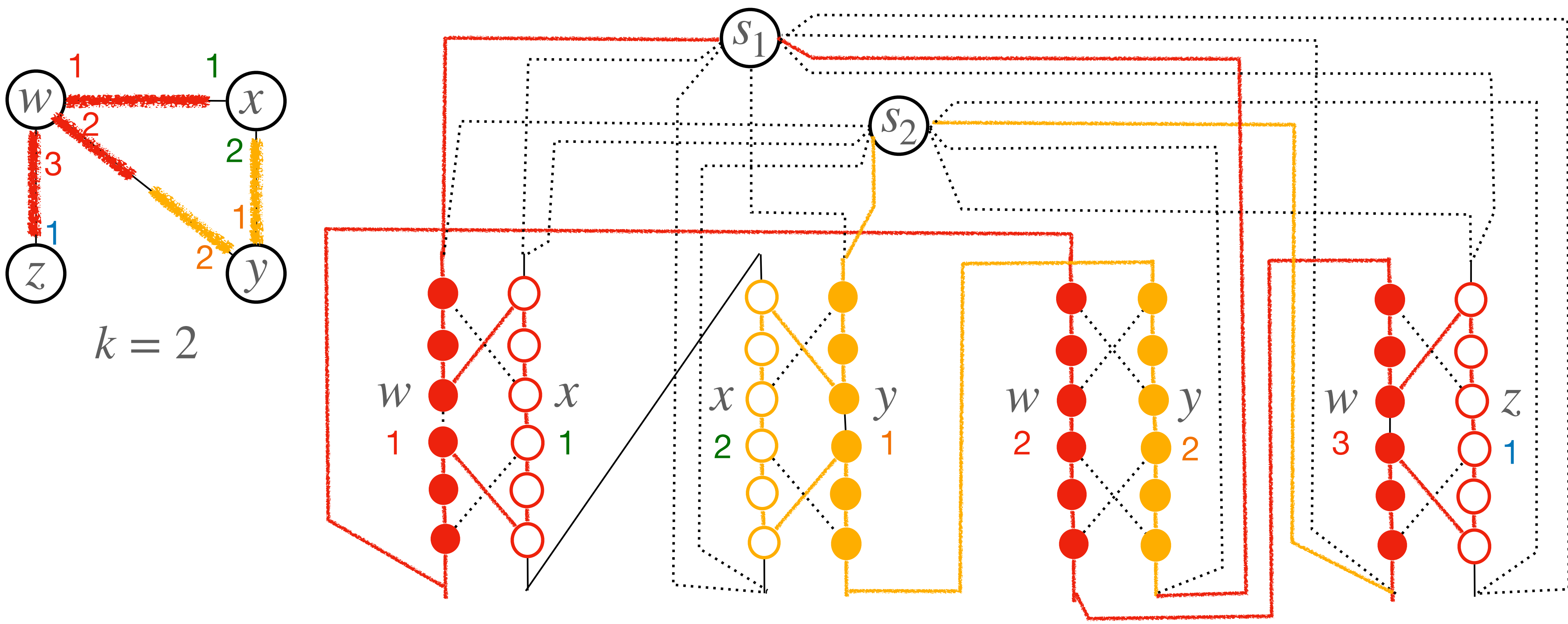
Assume “YES” for the Vertex Cover Problem.

Assume the vertex cover is {w,y}.

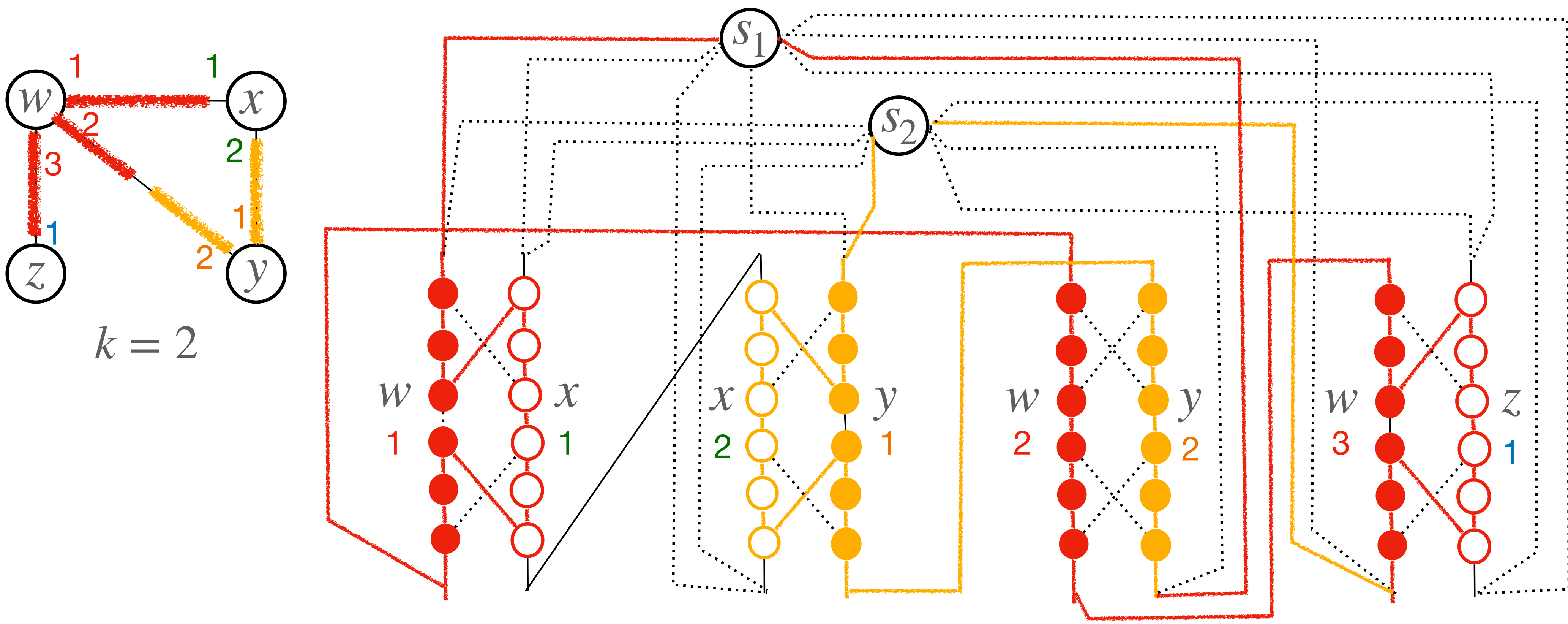
All edges are covered.

All nodes are passed through exactly once.

We got a Hamiltonian cycle!



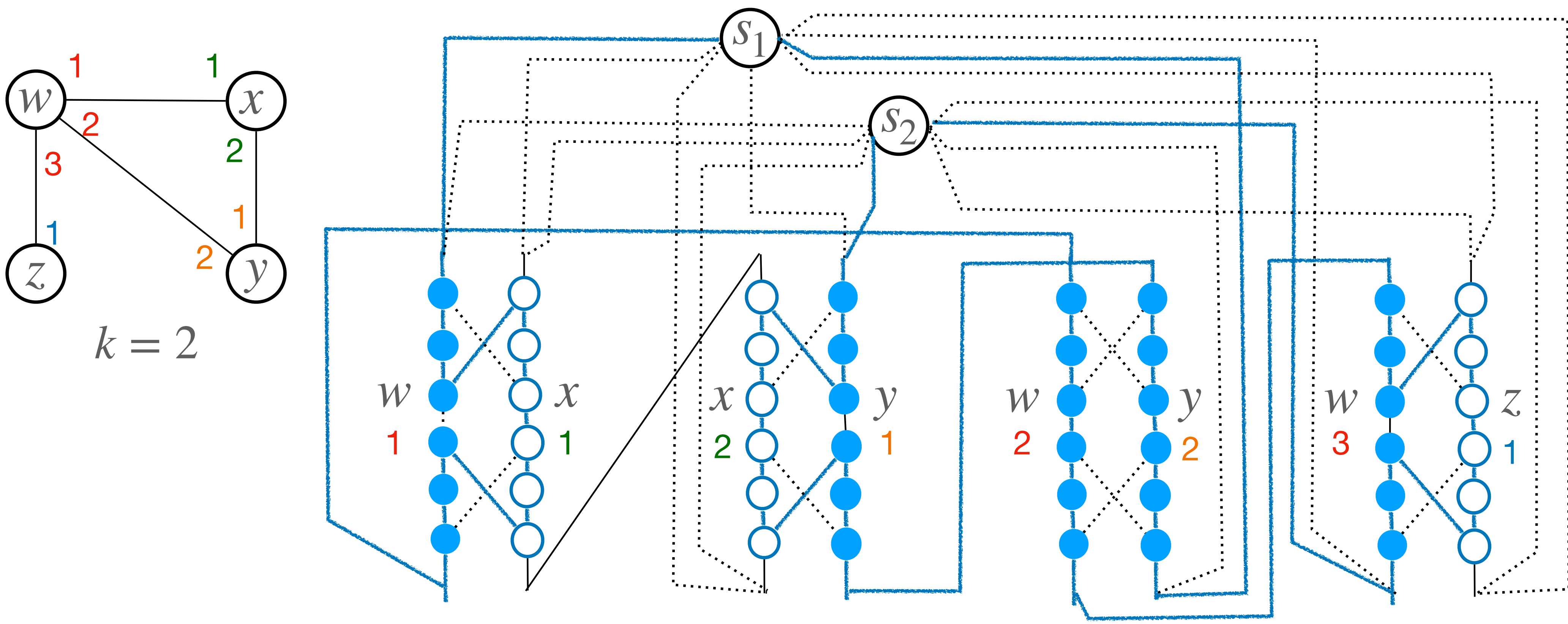
“YES” for the Vertex Cover Problem. ➡ “YES” for Hamiltonian Cycle Problem.



We now show:

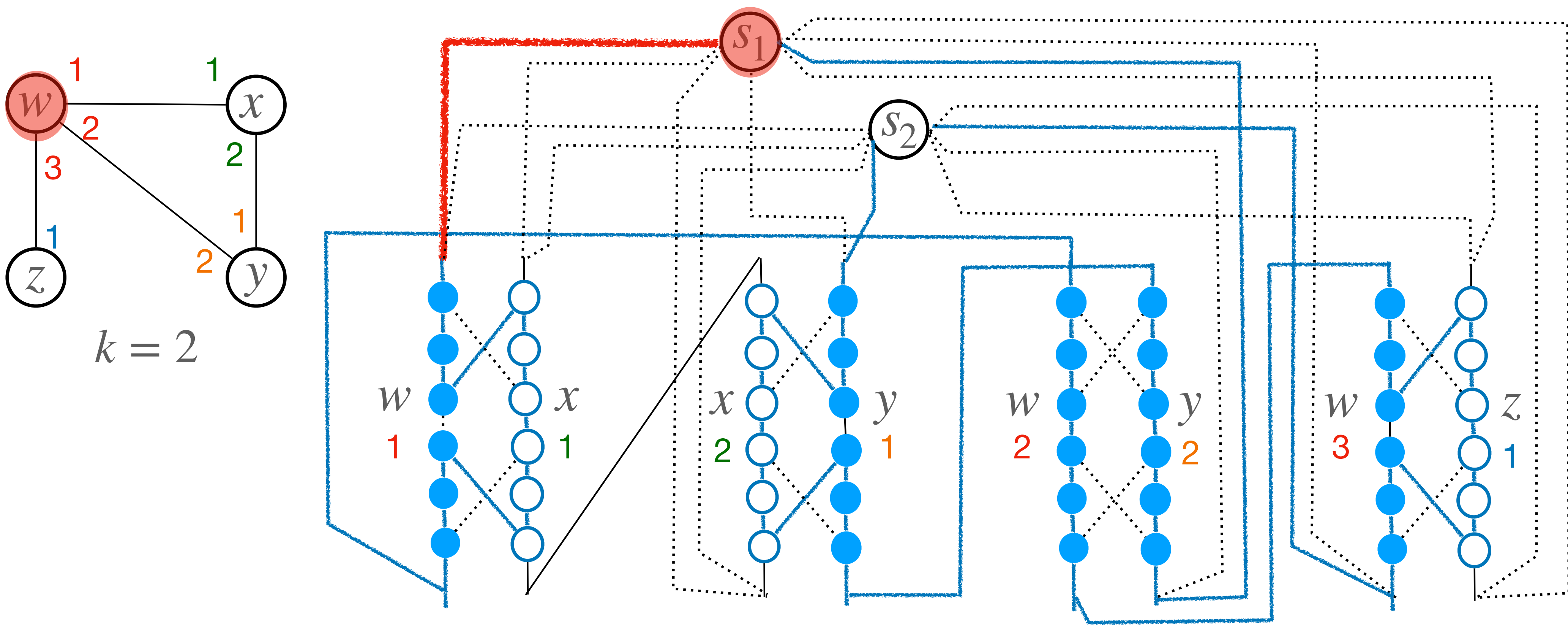
"YES" for the Vertex Cover Problem. $\Leftarrow$ "YES" for Hamiltonian Cycle Problem.

Assume “YES” for Hamiltonian Cycle Problem.



Assume “YES” for Hamiltonian Cycle Problem.

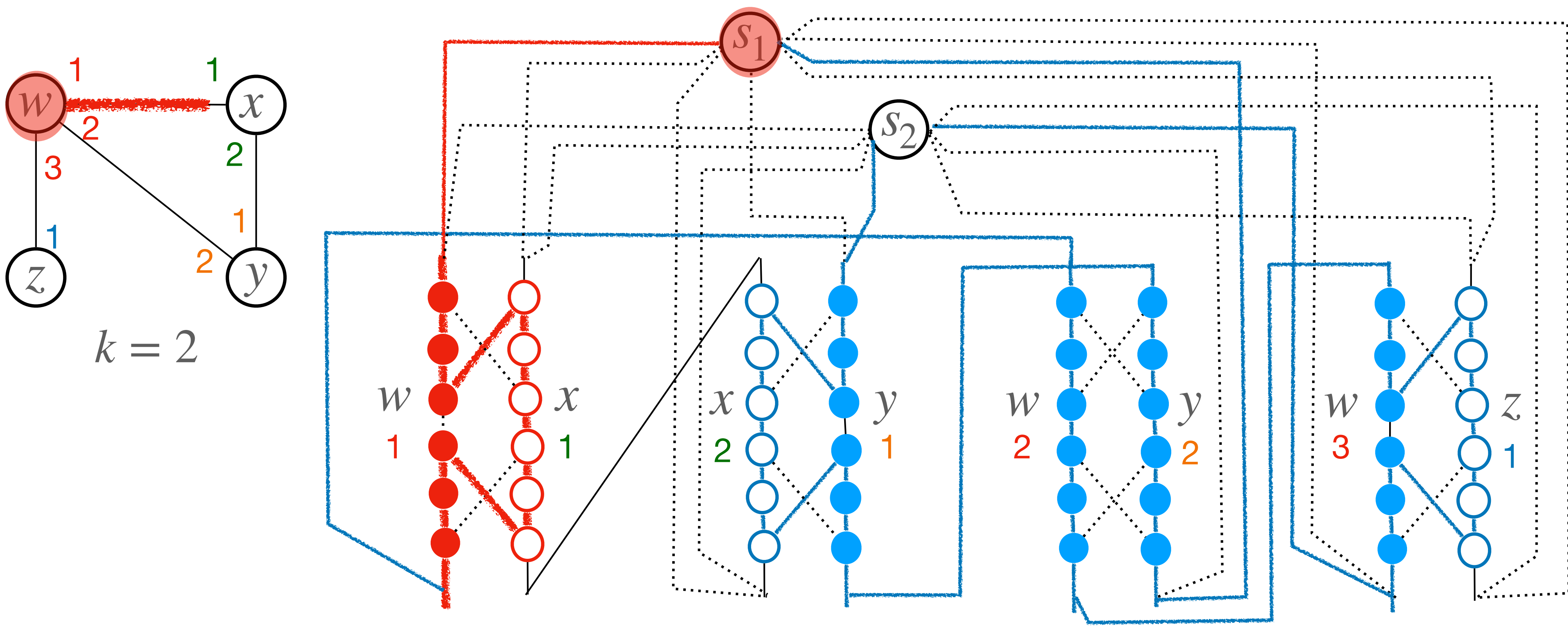
1st selected node is: w



Assume “YES” for Hamiltonian Cycle Problem.

1st selected node is: w

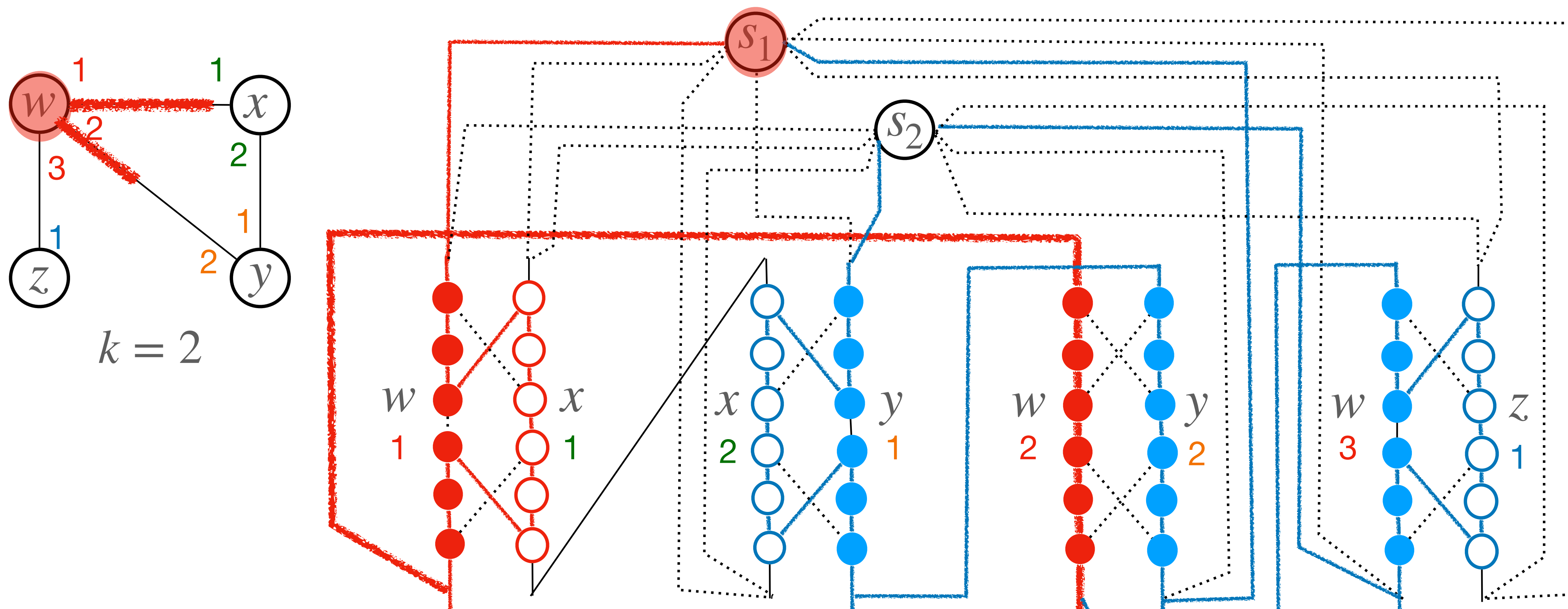
How should w cover its 1st edge (w,x)?



Assume “YES” for Hamiltonian Cycle Problem.

1st selected node is: w

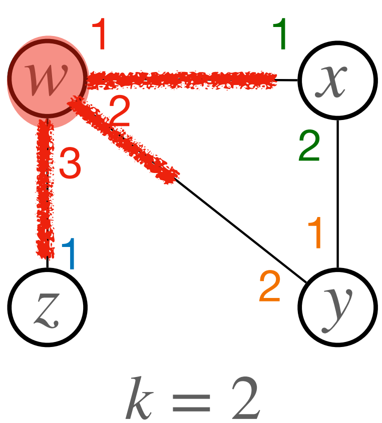
How should w cover its 2nd edge (w,y)?

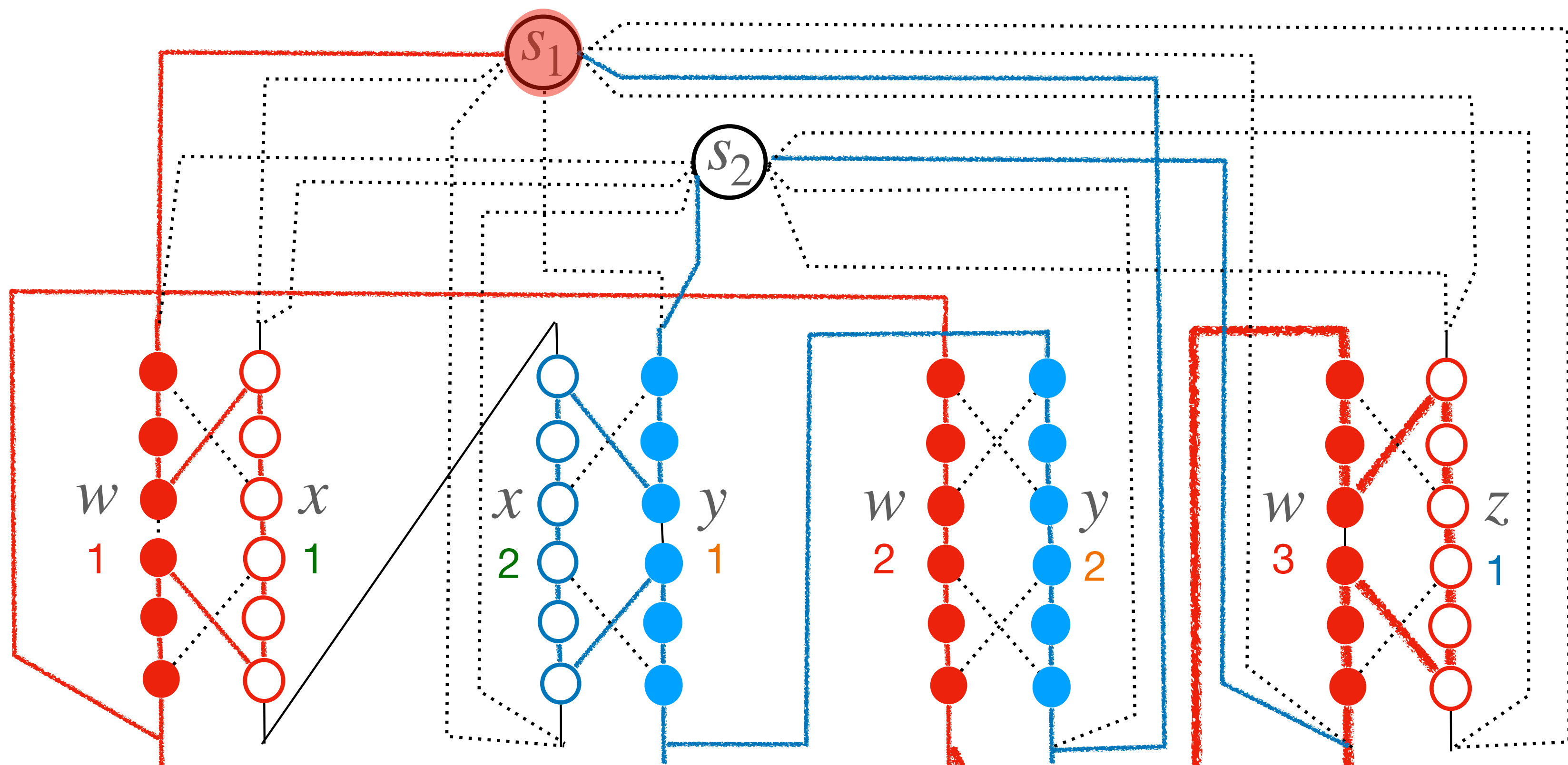


Assume “YES” for Hamiltonian Cycle Problem.

1st selected node is: w

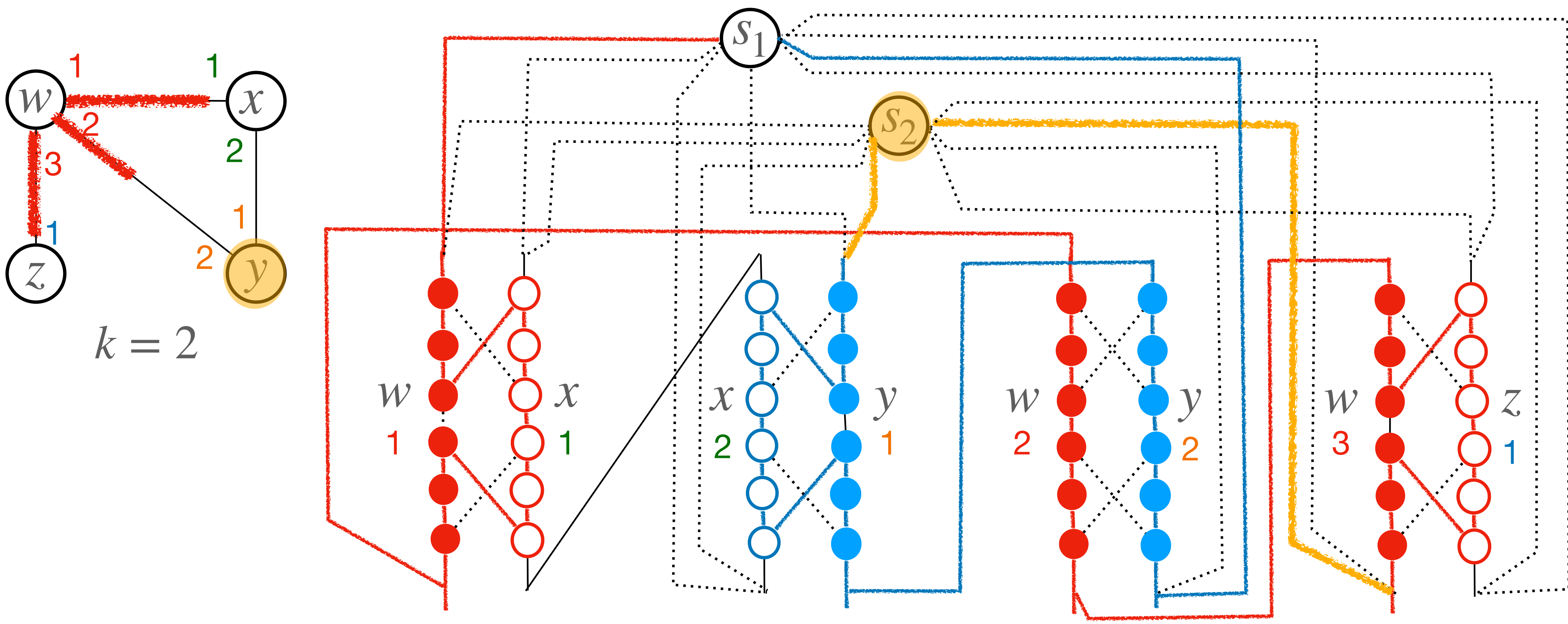
How should w cover its 3rd edge (w,z)?





Assume “YES” for Hamiltonian Cycle Problem.

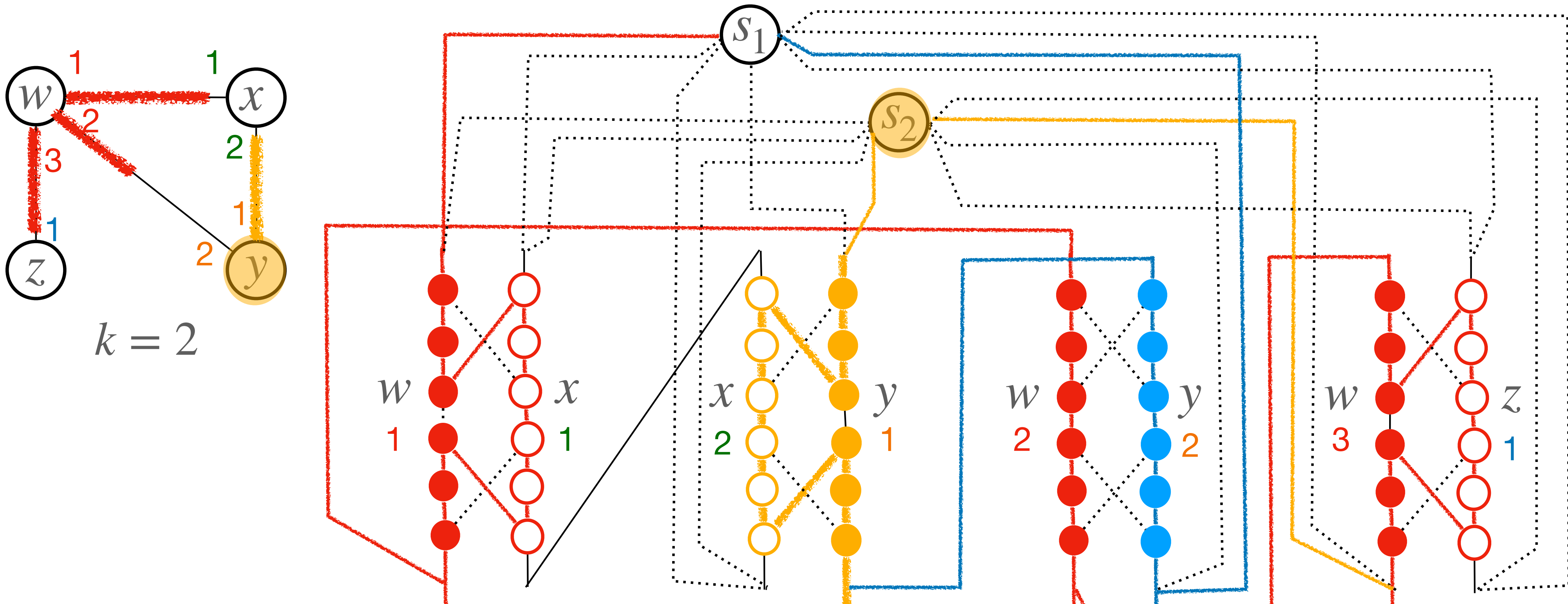
2nd selected node is: y



Assume “YES” for Hamiltonian Cycle Problem.

2nd selected node is: y

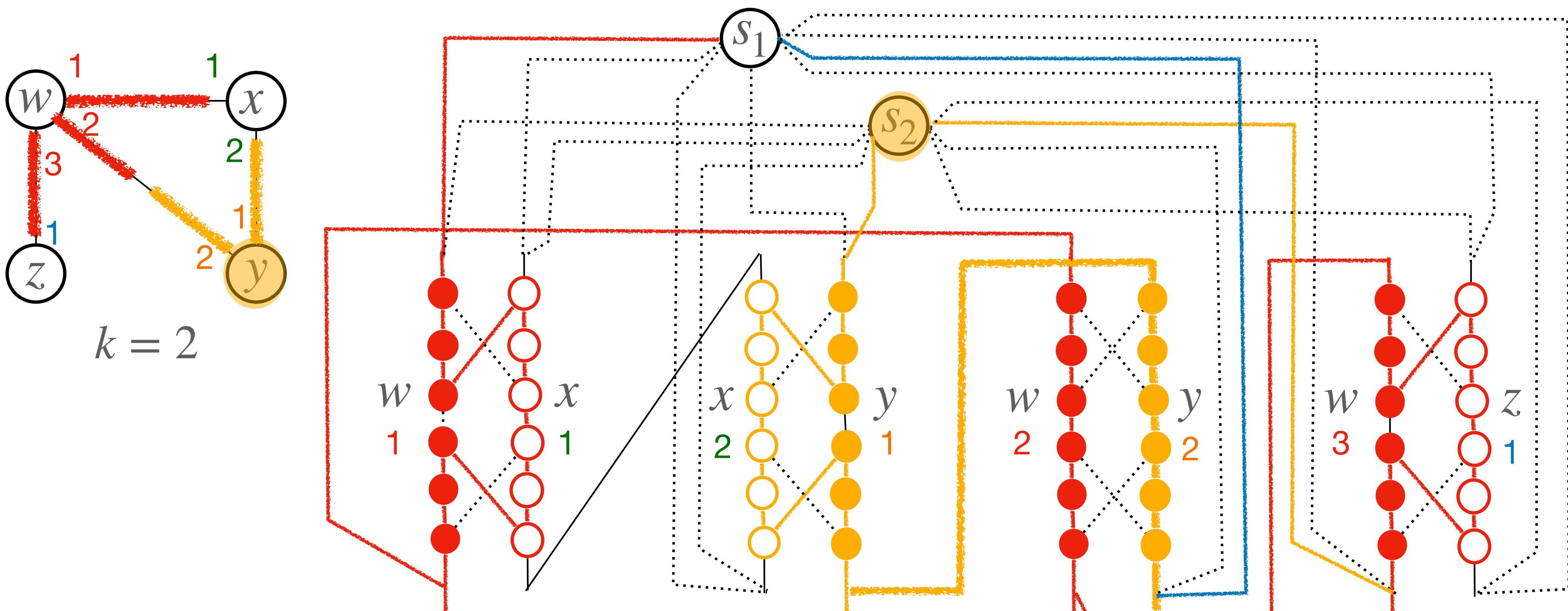
How should y cover its 1st edge (y,x)?



Assume “YES” for Hamiltonian Cycle Problem.

2nd selected node is: y

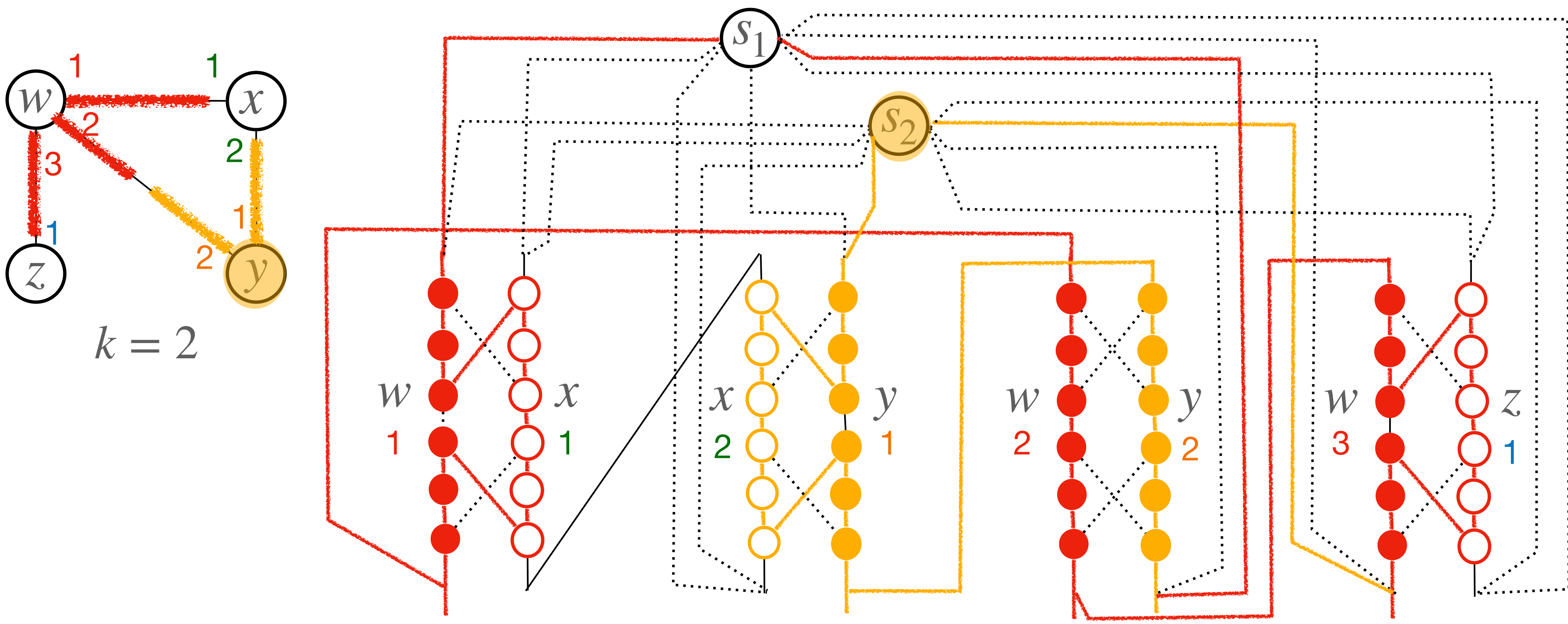
How should y cover its 2nd edge (y,w)?



“YES” for Vertex Cover Problem. ⟵ “YES” for Hamiltonian Cycle Problem.

All edges are covered.

G has a vertex cover of size 2.



“YES” for the Vertex Cover Problem. $\Longleftrightarrow$ “YES” for Hamiltonian Cycle Problem.

There is a polynomial-time reduction from “Vertex Cover Problem” to “Hamiltonian Cycle Problem”.

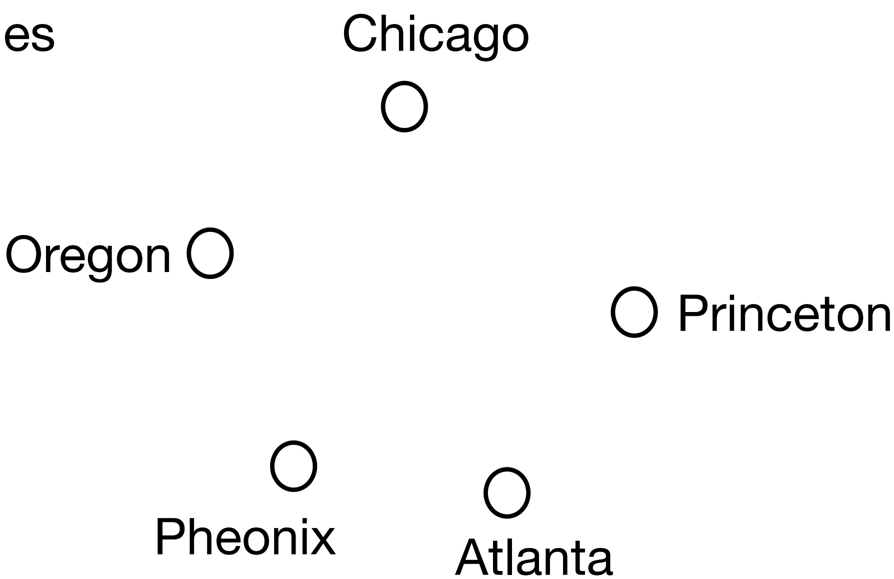
Hamiltonian Cycle Problem $\in NPC$

Traveling Salesman Problem (TSP)

Known NPC Problems:

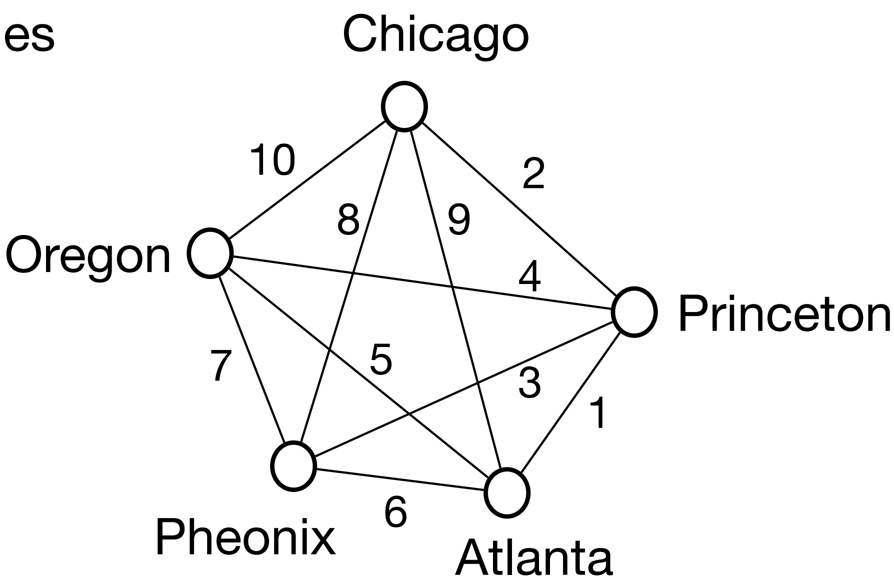
1) 3-CNF SAT Problem
2) Clique Problem
3) Vertex Cover Problem
4) Hamiltonian Cycle Problem

n cities



Traveling Salesman Problem (TSP)

n cities



Known NPC Problems:

1) 3-CNF SAT Problem

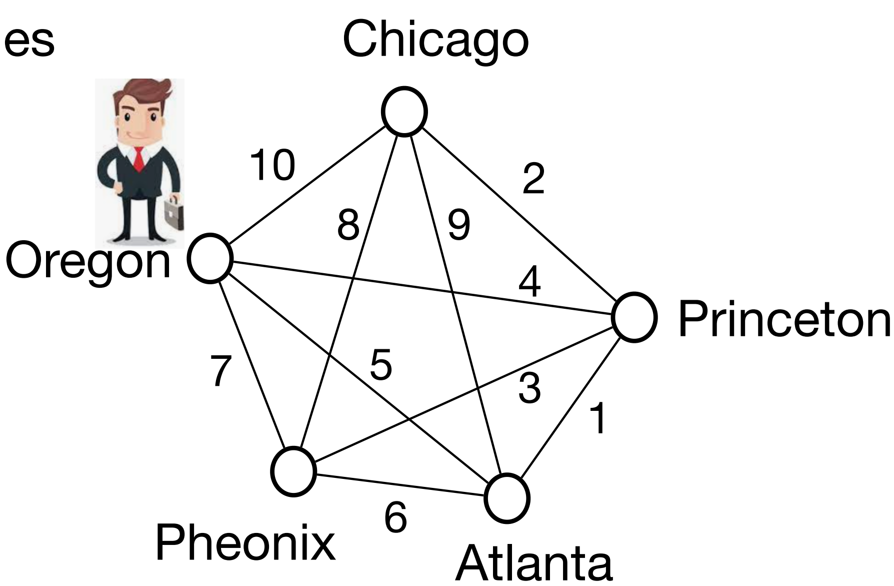
2) Clique Problem

3) Vertex Cover Problem

4) Hamiltonian Cycle Problem

Traveling Salesman Problem (TSP)

n cities



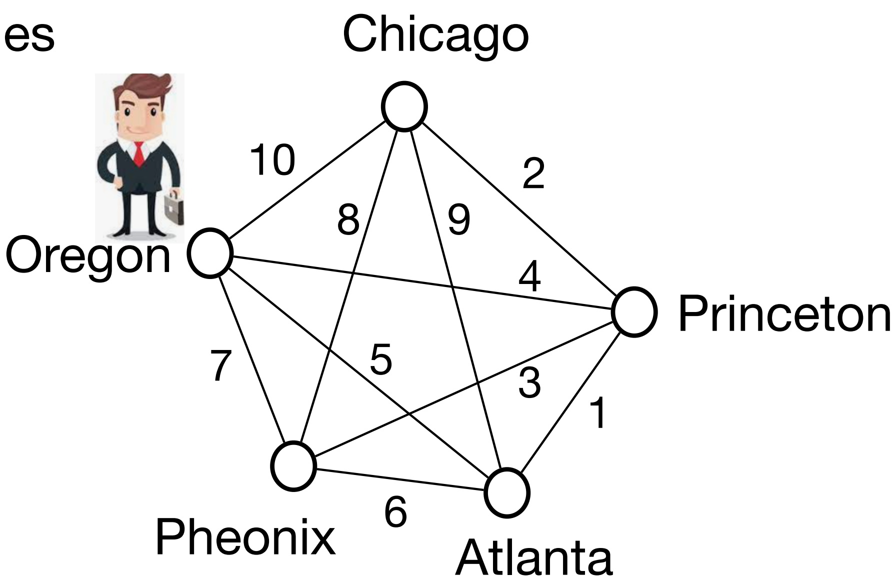
How can the salesman visit each city exactly once,
and return home via the shortest journey?

Known NPC Problems:

1) 3-CNF SAT Problem
2) Clique Problem
3) Vertex Cover Problem
4) Hamiltonian Cycle Problem

Traveling Salesman Problem (TSP)

n cities



How can the salesman visit each city exactly once, and return home via the shortest journey?

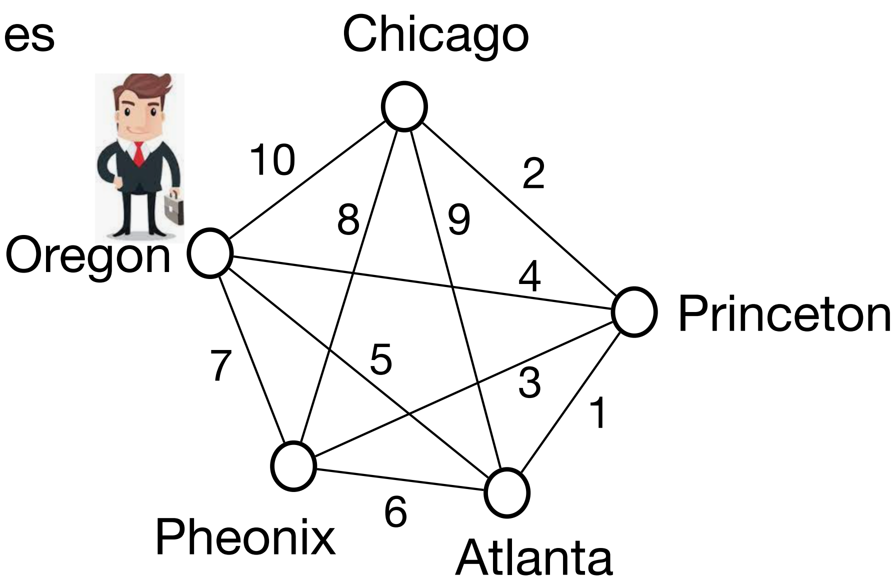
Known NPC Problems:

1) 3-CNF SAT Problem
2) Clique Problem
3) Vertex Cover Problem
4) Hamiltonian Cycle Problem

Hamiltonian cycle

Traveling Salesman Problem (TSP)

n cities



How can the salesman visit each city exactly once, and return home via the shortest journey?

Known NPC Problems:

1) 3-CNF SAT Problem
2) Clique Problem
3) Vertex Cover Problem
4) Hamiltonian Cycle Problem

Shortest

Hamiltonian cycle

Traveling Salesman Problem (TSP)

Input: An undirected complete graph G=(V,E), where every edge $(u, v) \in E$ has a non-negative integer weight $w(u, v)$. An integer $k \geq 0.$

Question: Does G have a Hamiltonian cycle of weight $\leq k$?

Known NPC Problems:

1) 3-CNF SAT Problem
2) Clique Problem
3) Vertex Cover Problem
4) Hamiltonian Cycle Problem

Theorem: $TSP \in NPC$.

Traveling Salesman Problem (TSP)

Input: An undirected complete graph G=(V,E), where every edge $(u, v) \in E$ has a non-negative integer weight $w(u, v)$. An integer $k \geq 0$.

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Known NPC Problems:
1) 3-CNF SAT Problem
2) Clique Problem
3) Vertex Cover Problem
4) Hamiltonian Cycle Problem

Theorem: $TSP \in NPC$.

Proof: 1) $TSP \in NP$.

Certificate: a Hamiltonian cycle of weight $\leq k$.

Polynomial-time verification.

2) Which known NPC problem do we want to reduce to TSP?

We want to prove: Hamiltonian Cycle Problem $\leq_p$ TSP

Hamiltonian Cycle Problem:

Input: An undirected graph $G = (V, E)$.

Question: Does G have a Hamiltonian cycle?

Traveling Salesman Problem (TSP)

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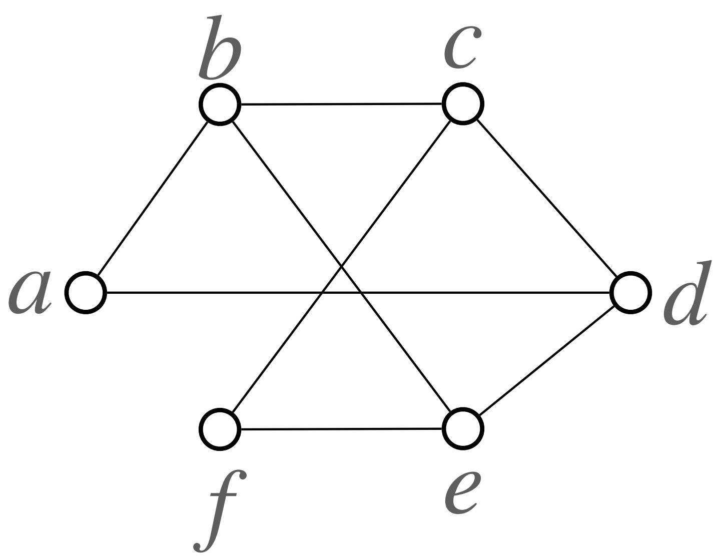
Example instance:



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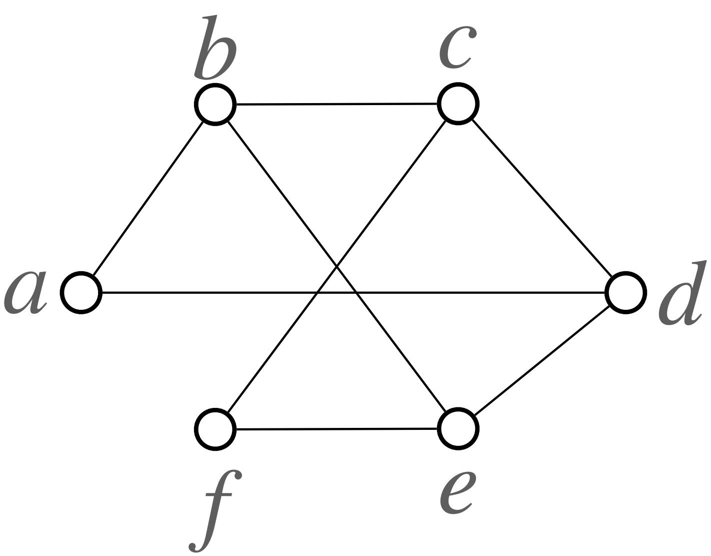

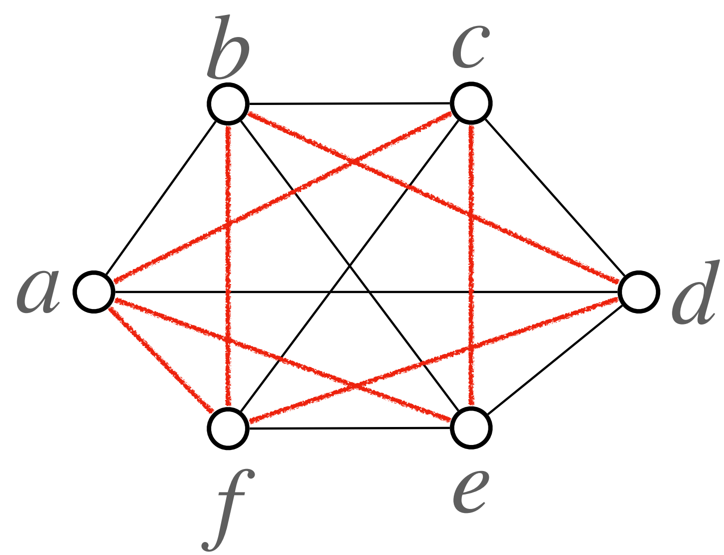

Black edges: weight 0 Red edges: weight 1

k=0

Hamiltonian Cycle Problem:

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Traveling Salesman Problem (TSP)

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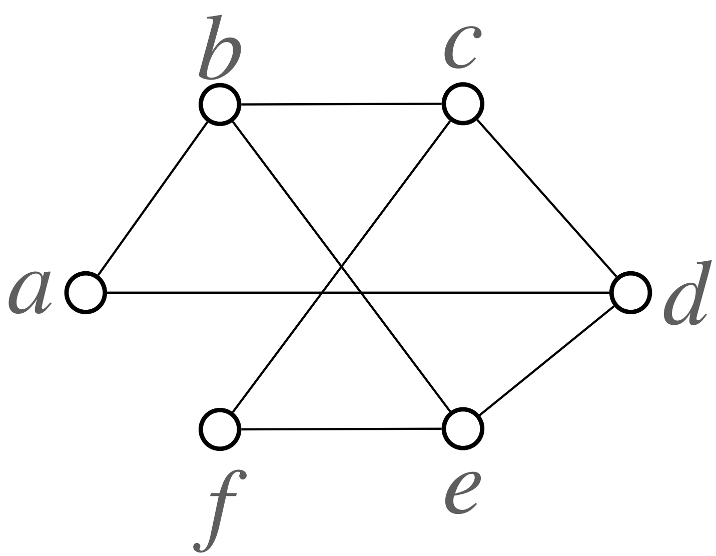
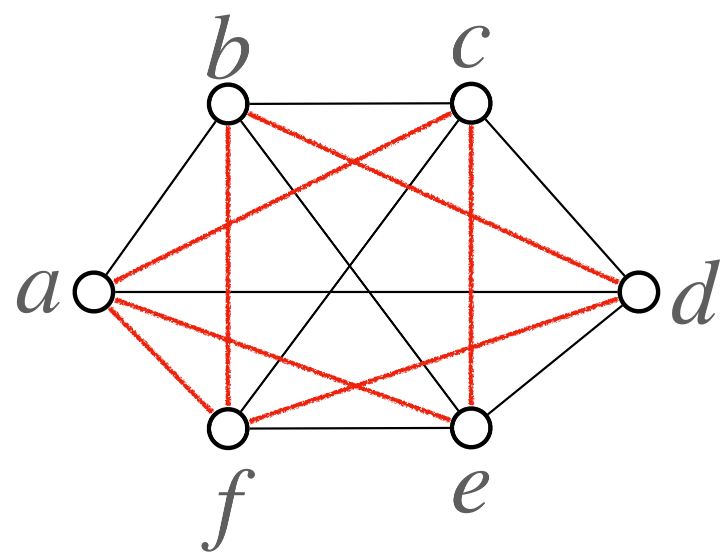
Question: Does G have a Hamiltonian cycle of weight $\leq k$?

Example instance:

Polynomial-time mapping. Does it preserve YES/NO answer?





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Hamiltonian Cycle Problem:

Input: An undirected graph $G = (V, E)$.

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Traveling Salesman Problem (TSP)

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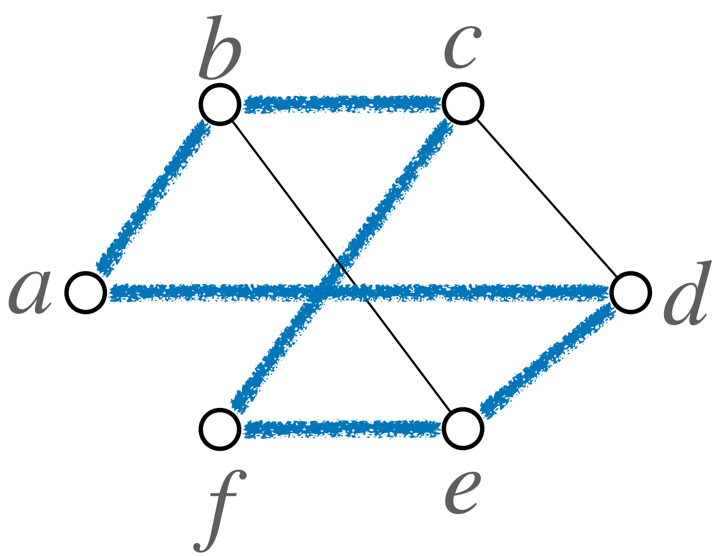
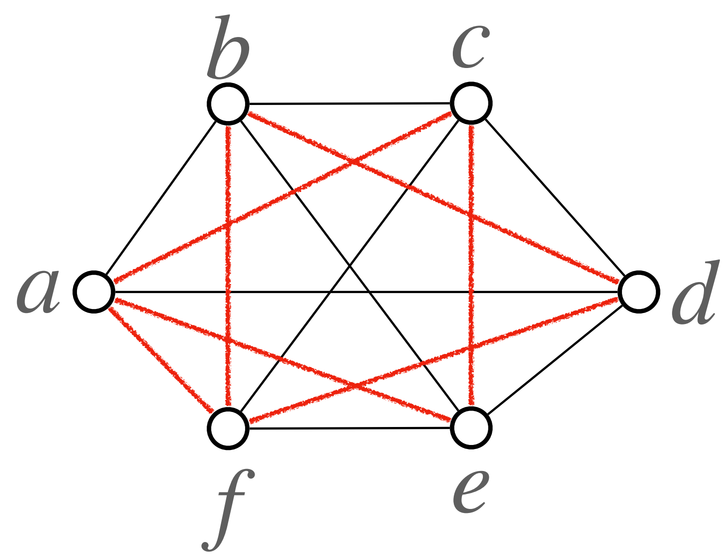
Question: Does G have a Hamiltonian cycle of weight $\leq k$?

Assume answer "YES"





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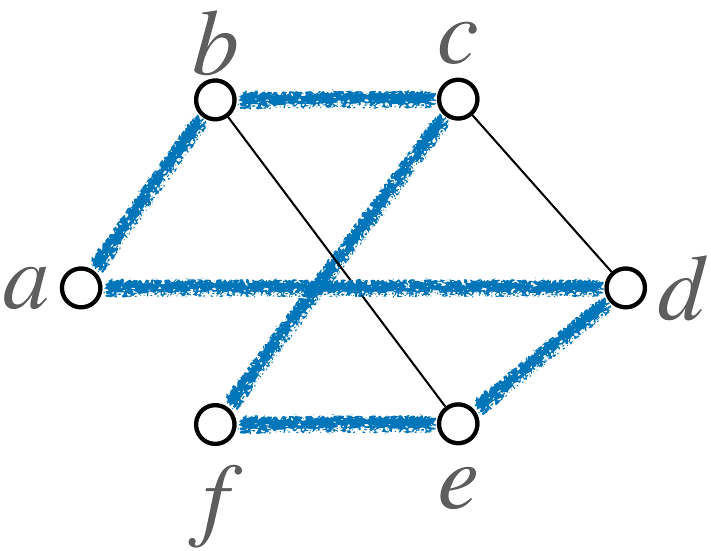
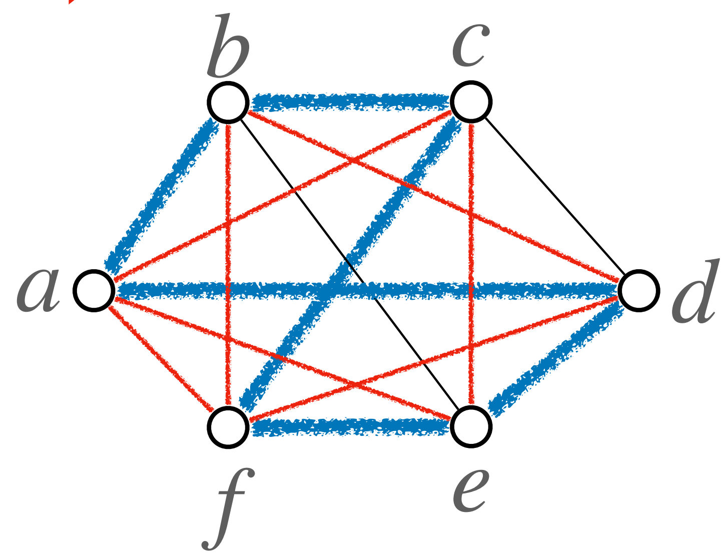


Answer "YES"



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k=0

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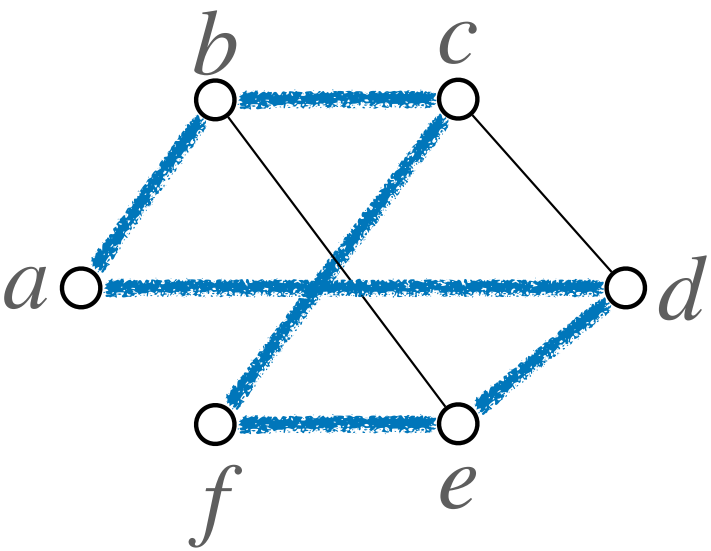
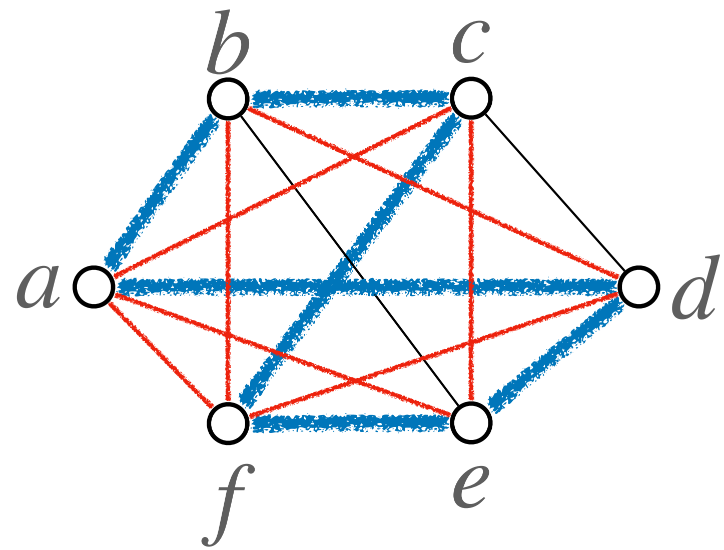
Answer "YES"



Assume answer "YES"



Black edges: weight 0

Red edges: weight 1

k=0

We have proved: Hamiltonian Cycle Problem $\leq_p$ TSP

$$TSP \in NPC$$

Subset Sum Problem

Input: A set S of positive integers.

A target integer t > 0.

Question: Does S have a subset S' whose sum equals t ?

Known NPC Problems:

1) 3-CNF SAT Problem
2) Clique Problem
3) Vertex Cover Problem
4) Hamiltonian Cycle Problem
5) Traveling Salesman Problem

Subset Sum Problem

Input: A set S of positive integers.

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5) Traveling Salesman Problem

Example: $S = \{1,2,7\}$, $t = 8$.

Subset Sum Problem

Input: A set S of positive integers.

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Question: Does S have a subset S' whose sum equals t ?

Known NPC Problems:

1) 3-CNF SAT Problem
2) Clique Problem
3) Vertex Cover Problem
4) Hamiltonian Cycle Problem
5) Traveling Salesman Problem

Example: $S = \{1,2,7\},\ t = 8.$

Answer: YES.

$S' = \{1,7\}$.

Subset Sum Problem

Input: A set S of positive integers.

A target integer t > 0.

Question: Does S have a subset S' whose sum equals t ?

Example: $S = \{1,2,7\}$, $t = 6$.

Known NPC Problems:

1) 3-CNF SAT Problem
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Subset Sum Problem

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Example: $S = \{1,2,7\}$, $t = 6$.

Answer: NO.

Subset Sum Problem

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Question: Does S have a subset S' whose sum equals t ?

Known NPC Problems:

1) 3-CNF SAT Problem
2) Clique Problem
3) Vertex Cover Problem
4) Hamiltonian Cycle Problem
5) Traveling Salesman Problem

Example: $S = \{1,2,7,14,49,98,343,686,2409,2793,16808,17206,117705,117993\}$

$t = 138457$

Subset Sum Problem

Input: A set S of positive integers.

A target integer $t > 0$.

Question: Does S have a subset S' whose sum equals t ?

Known NPC Problems:

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Example: $S = \{1,2,7,14,49,98,343,686,2409,2793,16808,17206,117705,117993\}$

$t = 138457$

Answer: Yes.

Subset Sum Problem

Input: A set S of positive integers.

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Known NPC Problems:

1) 3-CNF SAT Problem
2) Clique Problem
3) Vertex Cover Problem
4) Hamiltonian Cycle Problem
5) Traveling Salesman Problem

Theorem: Subset Sum Problem $\in NPC$.

Known NPC Problems:

1) 3-CNF SAT Problem
2) Clique Problem
3) Vertex Cover Problem
4) Hamiltonian Cycle Problem
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Subset Sum Problem

Input: A set S of positive integers.

A target integer t > 0.

Question: Does S have a subset S' whose sum equals t ?

Theorem: Subset Sum Problem $\in NPC$.

Proof: 1) Subset Sum Problem $\in NP$.

Certificate: A subset S' whose sum equals t.

Polynomial-time verification.

2) Which known NPC problem do we want to reduce to the "Subset Sum Problem"?

We want to prove 3-CNF SAT Problem $\leq_p$ Subset Sum Problem .

3-CNF SAT Problem:

Input: A CNF formula with n variables and k clauses, where each clause is the "OR" of 3 literals.

Question: Does there exist a solution to the variables that make the formula be true?

Subset Sum Problem

Input: A set S of positive integers.
A target integer t > 0.

Question: Does S have a subset S' whose sum equals t ?

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

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$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

All numbers have n+k digits

	x_1	x_2	x_3	C_1	C_2	C_3	C_4

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$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

All numbers have n+k digits

x_1	x_2	x_3	C_1	C_2	C_3	C_4
1	0	0				
1	0	0				
0	1	0				
0	1	0				
0	0	1				
0	0	1				

3-CNF SAT Problem:

Input: A CNF formula with n variables and k clauses, where each clause is the "OR" of 3 literals.

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$n = 3,\ k = 4$

All numbers have n+k digits

	x_1	x_2	x_3	C_1	C_2	C_3	C_4
x_1	1	0	0	1			
$\bar{x}_1$	1	0	0	0			
x_2	0	1	0	0			
$\bar{x}_2$	0	1	0	1			
x_3	0	0	1	0			
$\bar{x}_3$	0	0	1	1			

3-CNF SAT Problem:

Input: A CNF formula with n variables and k clauses, where each clause is the "OR" of 3 literals.

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$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

All numbers have n+k digits

	x_1	x_2	x_3	C_1	C_2	C_3	C_4
x_1	1	0	0	1	0		
$\bar{x}_1$	1	0	0	0	1		
x_2	0	1	0	0	0		
$\bar{x}_2$	0	1	0	1	1		
x_3	0	0	1	0	0		
$\bar{x}_3$	0	0	1	1	1		

3-CNF SAT Problem:

Input: A CNF formula with n variables and k clauses, where each clause is the "OR" of 3 literals.

Question: Does there exist a solution to the variables that make the formula be true?

Subset Sum Problem

Input: A set S of positive integers. A target integer t > 0.

Question: Does S have a subset S' whose sum equals t ?

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

All numbers have n+k digits

	x_1	x_2	x_3	C_1	C_2	C_3	C_4
x_1	1	0	0	1	0	0	
$\bar{x}_1$	1	0	0	0	1	1	
x_2	0	1	0	0	0	0	
$\bar{x}_2$	0	1	0	1	1	1	
x_3	0	0	1	0	0	1	
$\bar{x}_3$	0	0	1	1	1	0	

3-CNF SAT Problem:

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$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

All numbers have n+k digits

	x_1	x_2	x_3	C_1	C_2	C_3	C_4
x_1	1	0	0	1	0	0	1
$\bar{x}_1$	1	0	0	0	1	1	0
x_2	0	1	0	0	0	0	1
$\bar{x}_2$	0	1	0	1	1	1	0
x_3	0	0	1	0	0	1	1
$\bar{x}_3$	0	0	1	1	1	0	0

3-CNF SAT Problem:

Subset Sum Problem

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

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$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

All numbers have n+k digits

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	1	0	0	1	0	0	1
	$\bar{x}_1$	1	0	0	0	1	1	0
	x_2	0	1	0	0	0	0	1
	$\bar{x}_2$	0	1	0	1	1	1	0
	x_3	0	0	1	0	0	1	1
	$\bar{x}_3$	0	0	1	1	1	0	0
$2k$ rows		0	0	0	0			
		0	0	0	0			
		0	0	0	0			
		0	0	0	0			
		0	0	0	0			
		0	0	0	0			
		0	0	0	0			
		0	0	0	0			

3-CNF SAT Problem:

Subset Sum Problem

Example instance:

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$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

All numbers have n+k digits

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	1	0	0	1	0	0	1
	$\bar{x}_1$	1	0	0	0	1	1	0
	x_2	0	1	0	0	0	0	1
	$\bar{x}_2$	0	1	0	1	1	1	0
	x_3	0	0	1	0	0	1	1
	$\bar{x}_3$	0	0	1	1	1	0	0
$2k$ rows		0	0	0	1	0	0	0
		0	0	0	2	0	0	0
		0	0	0	0	1	0	0
		0	0	0	0	2	0	0
		0	0	0	0	0	1	0
		0	0	0	0	0	2	0
		0	0	0	0	0	0	1
		0	0	0	0	0	0	2

3-CNF SAT Problem:

Subset Sum Problem

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3, \ k = 4$

S has 2(n+k) numbers,
all of which have n+k digits

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2

3-CNF SAT Problem:

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Subset Sum Problem

S has 2(n+k) numbers,
all of which have n+k digits

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Subset Sum Problem

S has 2(n+k) numbers,
all of which have n+k digits

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

Polynomial-time mapping.

Does it preserve YES/NO answers?

3-CNF SAT Problem:

Assume "YES".

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

S has 2(n+k) numbers, all of which have n+k digits

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows: x_1	$v_1 =$	1	0	0	1	0	0	1
$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
x_2	$v_2 =$	0	1	0	0	0	0	1
$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
x_3	$v_3 =$	0	0	1	0	0	1	1
$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows	$s_1 =$	0	0	0	1	0	0	0
	$s_1' =$	0	0	0	2	0	0	0
	$s_2 =$	0	0	0	0	1	0	0
	$s_2' =$	0	0	0	0	2	0	0
	$s_3 =$	0	0	0	0	0	1	0
	$s_3' =$	0	0	0	0	0	2	0
	$s_4 =$	0	0	0	0	0	0	1
	$s_4' =$	0	0	0	0	0	0	2
	target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume “YES”.

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

S has 2(n+k) numbers,
all of which have n+k digits

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
$2n$ rows	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
$2k$ rows		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
	target	$t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume “YES”.

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

S has 2(n+k) numbers, all of which have n+k digits

$2n$ rows

$2k$ rows

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
x_1	$v_1 =$	1	0	0	1	0	0	1
$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
x_2	$v_2 =$	0	1	0	0	0	0	1
$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
x_3	$v_3 =$	0	0	1	0	0	1	1
$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
	$s_1 =$	0	0	0	1	0	0	0
	$s_1' =$	0	0	0	2	0	0	0
	$s_2 =$	0	0	0	0	1	0	0
	$s_2' =$	0	0	0	0	2	0	0
	$s_3 =$	0	0	0	0	0	1	0
	$s_3' =$	0	0	0	0	0	2	0
	$s_4 =$	0	0	0	0	0	0	1
	$s_4' =$	0	0	0	0	0	0	2
	target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume "YES".

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

S has 2(n+k) numbers, all of which have n+k digits

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
x_1	$v_1 =$	1	0	0	1	0	0	1
$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
x_2	$v_2 =$	0	1	0	0	0	0	1
$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
x_3	$v_3 =$	0	0	1	0	0	1	1
$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
	$s_1 =$	0	0	0	1	0	0	0
	$s_1' =$	0	0	0	2	0	0	0
	$s_2 =$	0	0	0	0	1	0	0
	$s_2' =$	0	0	0	0	2	0	0
	$s_3 =$	0	0	0	0	0	1	0
	$s_3' =$	0	0	0	0	0	2	0
	$s_4 =$	0	0	0	0	0	0	1
	$s_4' =$	0	0	0	0	0	0	2
	target $t =$	1	1	1	4	4	4	4

$2n$ rows

$2k$ rows

First n bits are already matched. Why?

3-CNF SAT Problem:

Assume "YES".

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

Sum is between 1 and 3 here. Why?

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume “YES”.

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

Sum is between 1 and 3 here. Why?

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1 $v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$ $v_1' =$	1	0	0	0	1	1	0
	x_2 $v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$ $v_2' =$	0	1	0	1	1	1	0
	x_3 $v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$ $v_3' =$	0	0	1	1	1	0	0
$2k$ rows	$s_1 =$	0	0	0	1	0	0	0
	$s_1' =$	0	0	0	2	0	0	0
	$s_2 =$	0	0	0	0	1	0	0
	$s_2' =$	0	0	0	0	2	0	0
	$s_3 =$	0	0	0	0	0	1	0
	$s_3' =$	0	0	0	0	0	2	0
	$s_4 =$	0	0	0	0	0	0	1
	$s_4' =$	0	0	0	0	0	0	2
	target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume “YES”.

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

Sum is between 1 and 3 here. Why?

$2n$ rows (rows v_1 to v_3'); $2k$ rows (rows s_1 to s_4')

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
x_1	$v_1 =$	1	0	0	1	0	0	1
$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
x_2	$v_2 =$	0	1	0	0	0	0	1
$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
x_3	$v_3 =$	0	0	1	0	0	1	1
$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
	$s_1 =$	0	0	0	1	0	0	0
	$s_1' =$	0	0	0	2	0	0	0
	$s_2 =$	0	0	0	0	1	0	0
	$s_2' =$	0	0	0	0	2	0	0
	$s_3 =$	0	0	0	0	0	1	0
	$s_3' =$	0	0	0	0	0	2	0
	$s_4 =$	0	0	0	0	0	0	1
	$s_4' =$	0	0	0	0	0	0	2
	target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume “YES”.

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3, \; k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

Sum is between 1 and 3 here. Why?

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume “YES”.

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

Sum is between 1 and 3 here. Why?

$$
\begin{array}{ll|ccccccc}
 & & x_1 & x_2 & x_3 & C_1 & C_2 & C_3 & C_4 \\
\hline
x_1 & v_1 = & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
\bar{x}_1 & v_1' = & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
x_2 & v_2 = & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
\bar{x}_2 & v_2' = & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\
x_3 & v_3 = & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\
\bar{x}_3 & v_3' = & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
 & s_1 = & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
 & s_1' = & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\
 & s_2 = & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
 & s_2' = & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\
 & s_3 = & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
 & s_3' = & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\
 & s_4 = & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
 & s_4' = & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
\hline
 & \text{target } t = & 1 & 1 & 1 & 4 & 4 & 4 & 4
\end{array}
$$

$2n$ rows (rows v_1 to v_3')

$2k$ rows (rows s_1 to s_4')

3-CNF SAT Problem:

Assume “YES”.

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

Sum is between 1 and 3 here. Why?

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume “YES”.

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

Sum is between 1 and 3 here. Why?

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume “YES”.

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

Sum is between 1 and 3 here. Why?

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Assume “YES”.

Subset Sum Problem

Answer is “YES”.

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = ?$

$x_2 = ?$

$x_3 = ?$

Subset Sum Problem

Assume "YES".

$2n$ rows

$2k$ rows

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
x_1	$v_1 =$	1	0	0	1	0	0	1
$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
x_2	$v_2 =$	0	1	0	0	0	0	1
$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
x_3	$v_3 =$	0	0	1	0	0	1	1
$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
	$s_1 =$	0	0	0	1	0	0	0
	$s_1' =$	0	0	0	2	0	0	0
	$s_2 =$	0	0	0	0	1	0	0
	$s_2' =$	0	0	0	0	2	0	0
	$s_3 =$	0	0	0	0	0	1	0
	$s_3' =$	0	0	0	0	0	2	0
	$s_4 =$	0	0	0	0	0	0	1
	$s_4' =$	0	0	0	0	0	0	2
	target $t =$	1	1	1	4	4	4	4

3-CNF SAT Problem:

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$\boxed{x_1 = 0}$

$x_2 = ?$

$x_3 = ?$

Subset Sum Problem

Assume "YES".

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
x_1	$v_1 =$	1	0	0	1	0	0	1
$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
x_2	$v_2 =$	0	1	0	0	0	0	1
$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
x_3	$v_3 =$	0	0	1	0	0	1	1
$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
	$s_1 =$	0	0	0	1	0	0	0
	$s_1' =$	0	0	0	2	0	0	0
	$s_2 =$	0	0	0	0	1	0	0
	$s_2' =$	0	0	0	0	2	0	0
	$s_3 =$	0	0	0	0	0	1	0
	$s_3' =$	0	0	0	0	0	2	0
	$s_4 =$	0	0	0	0	0	0	1
	$s_4' =$	0	0	0	0	0	0	2
	target $t =$	$\boxed{1}$	1	1	4	4	4	4

$2n$ rows

$2k$ rows

3-CNF SAT Problem:

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$\boxed{x_2 = 0}$

$x_3 = ?$

Subset Sum Problem

Assume "YES".

$2n$ rows (rows v_1 through v_3')

$2k$ rows (rows s_1 through s_4')

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
x_1	$v_1 =$	1	0	0	1	0	0	1
$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
x_2	$v_2 =$	0	1	0	0	0	0	1
$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
x_3	$v_3 =$	0	0	1	0	0	1	1
$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
	$s_1 =$	0	0	0	1	0	0	0
	$s_1' =$	0	0	0	2	0	0	0
	$s_2 =$	0	0	0	0	1	0	0
	$s_2' =$	0	0	0	0	2	0	0
	$s_3 =$	0	0	0	0	0	1	0
	$s_3' =$	0	0	0	0	0	2	0
	$s_4 =$	0	0	0	0	0	0	1
	$s_4' =$	0	0	0	0	0	0	2
	target $t =$	1	$\boxed{1}$	1	4	4	4	4

3-CNF SAT Problem:

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$\boxed{x_3 = 1}$

Subset Sum Problem

Assume “YES”.

$2n$ rows (rows v_1 to v_3')

$2k$ rows (rows s_1 to s_4')

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
x_1	$v_1 =$	1	0	0	1	0	0	1
$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
x_2	$v_2 =$	0	1	0	0	0	0	1
$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
x_3	$v_3 =$	0	0	1	0	0	1	1
$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
	$s_1 =$	0	0	0	1	0	0	0
	$s_1' =$	0	0	0	2	0	0	0
	$s_2 =$	0	0	0	0	1	0	0
	$s_2' =$	0	0	0	0	2	0	0
	$s_3 =$	0	0	0	0	0	1	0
	$s_3' =$	0	0	0	0	0	2	0
	$s_4 =$	0	0	0	0	0	0	1
	$s_4' =$	0	0	0	0	0	0	2
	target $t =$	1	1	$\boxed{1}$	4	4	4	4

3-CNF SAT Problem:

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$
$x_2 = 0$
$x_3 = 1$

Subset Sum Problem

Assume "YES".

Sum is at least 1. Why?

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	$\boxed{4}$	4	4	4

3-CNF SAT Problem:

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

Assume "YES".

Sum is at least 1. Why?

		x_1	x_2	x_3	C_1	C_2	C_3	C_4
x_1	$v_1 =$	1	0	0	1	0	0	1
$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
x_2	$v_2 =$	0	1	0	0	0	0	1
$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
x_3	$v_3 =$	0	0	1	0	0	1	1
$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
	$s_1 =$	0	0	0	1	0	0	0
	$s_1' =$	0	0	0	2	0	0	0
	$s_2 =$	0	0	0	0	1	0	0
	$s_2' =$	0	0	0	0	2	0	0
	$s_3 =$	0	0	0	0	0	1	0
	$s_3' =$	0	0	0	0	0	2	0
	$s_4 =$	0	0	0	0	0	0	1
	$s_4' =$	0	0	0	0	0	0	2
	target $t =$	1	1	1	$\boxed{4}$	4	4	4

$2n$ rows (first 6 rows); $2k$ rows (last 8 rows before target)

3-CNF SAT Problem:

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

Assume "YES".

Sum is at least 1.

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	$\boxed{4}$	4	4

3-CNF SAT Problem:

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

Assume "YES".

Sum is at least 1.

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	$\boxed{4}$	4

3-CNF SAT Problem:

Example instance:

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$

$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$

$C_4 = x_1 \vee x_2 \vee x_3$

$n = 3,\ k = 4$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

Subset Sum Problem

Assume "YES".

Sum is at least 1.

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	$\boxed{4}$

3-CNF SAT Problem:

Answer is “YES”.

Subset Sum Problem

Answer is “YES”.

Example instance:

$$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$$

$$C_1 = x_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_2 = \bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3$$

$$C_3 = \bar{x}_1 \vee \bar{x}_2 \vee x_3$$

$$C_4 = x_1 \vee x_2 \vee x_3$$

$$n = 3,\ k = 4$$

Solution:

$x_1 = 0$

$x_2 = 0$

$x_3 = 1$

			x_1	x_2	x_3	C_1	C_2	C_3	C_4
$2n$ rows	x_1	$v_1 =$	1	0	0	1	0	0	1
	$\bar{x}_1$	$v_1' =$	1	0	0	0	1	1	0
	x_2	$v_2 =$	0	1	0	0	0	0	1
	$\bar{x}_2$	$v_2' =$	0	1	0	1	1	1	0
	x_3	$v_3 =$	0	0	1	0	0	1	1
	$\bar{x}_3$	$v_3' =$	0	0	1	1	1	0	0
$2k$ rows		$s_1 =$	0	0	0	1	0	0	0
		$s_1' =$	0	0	0	2	0	0	0
		$s_2 =$	0	0	0	0	1	0	0
		$s_2' =$	0	0	0	0	2	0	0
		$s_3 =$	0	0	0	0	0	1	0
		$s_3' =$	0	0	0	0	0	2	0
		$s_4 =$	0	0	0	0	0	0	1
		$s_4' =$	0	0	0	0	0	0	2
		target $t =$	1	1	1	4	4	4	4

“YES” for 3-CNF SAT Problem $\Leftrightarrow$ “YES” for Subset Sum Problem

3-CNF SAT Problem $\leq_p$ Subset Sum Problem

Subset Sum Problem $\in NPC$