Algorithms

Lecture 19: Approximation Algorithms (Part 1)

Anxiao (Andrew) Jiang

CH 35. Approximation Algorithms

Why do we need Approximation Algorithms?

Approximation algorithms are “easier” than exact algorithms, since it requires only approximate solutions, not optimal solutions.

How to analyze approximation algorithms?

CH 35. Approximation Algorithms

Consider an optimization problem.

Let C^* be the cost of an optimal solution.

Let C be the cost of the solution found by our algorithm.

(For simplicity, assume cost > 0.)

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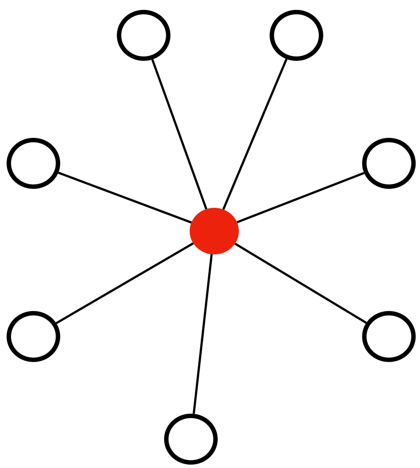
Vertex Cover Problem

Input: An undirected graph G=(V,E).

Output: A vertex cover of G of minimum size.



Vertex Cover Problem

We will show a 2-approximation algorithm.

Input: An undirected graph G=(V,E).

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Algorithm:

1. $S = \emptyset$ S is the vertex cover.
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This step seems strange.
Either u or v is enough for covering the edge (u,v).
But we choose both u and v.

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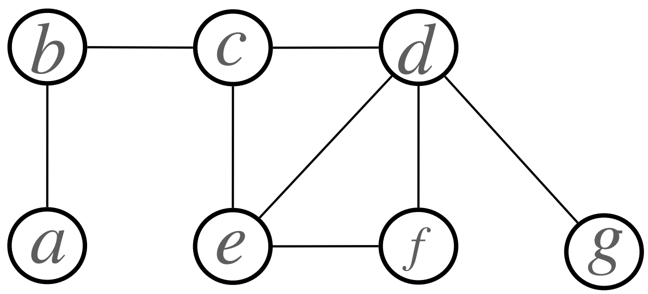

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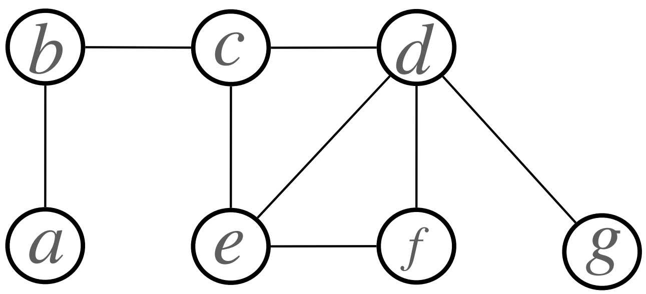

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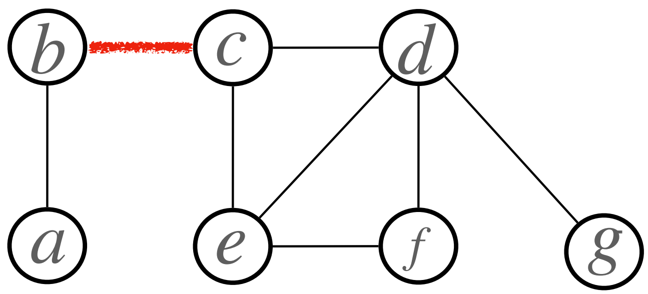

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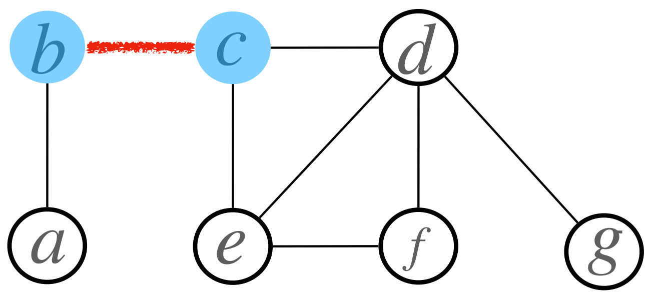

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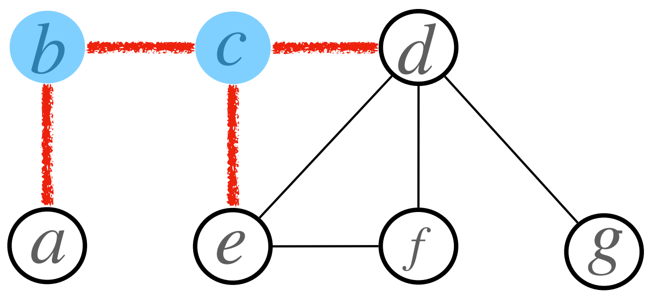

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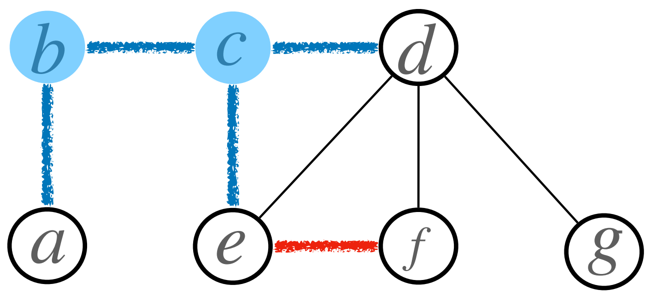

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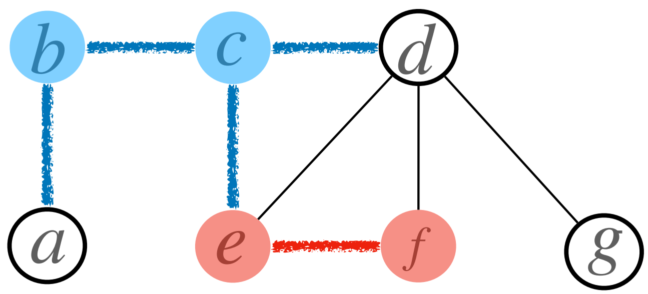

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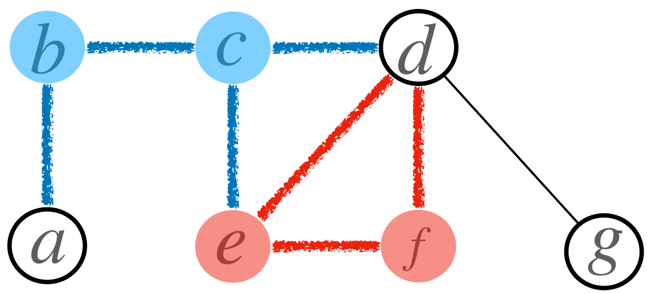

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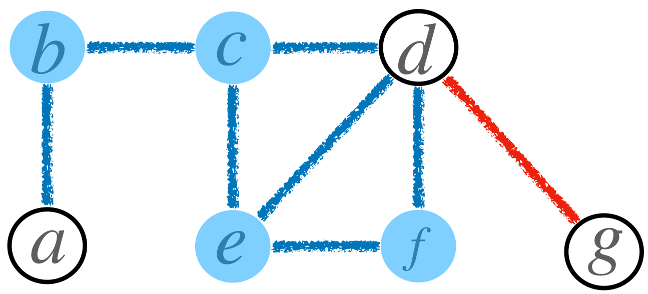

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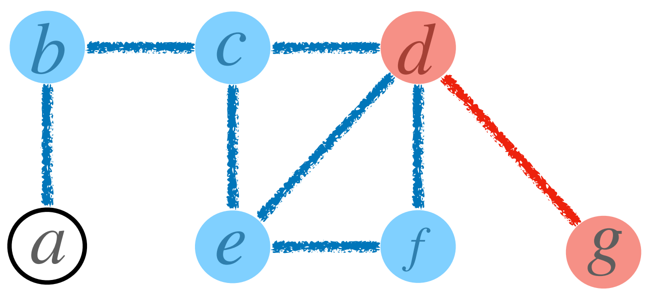

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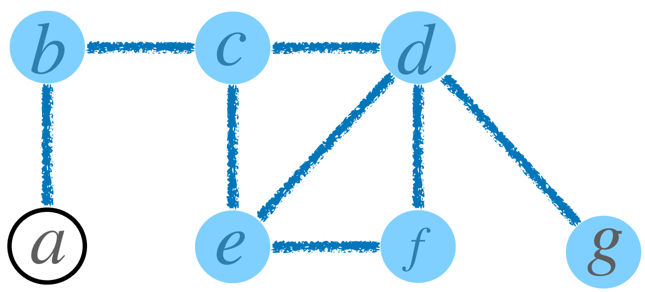

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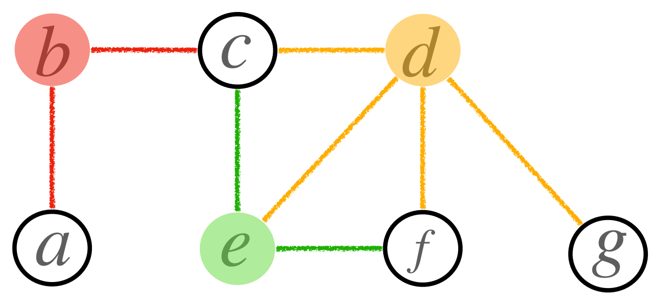

$$\frac{C}{C^*} = 2$$

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We need to show the approximation ratio is at most 2 for all instances.

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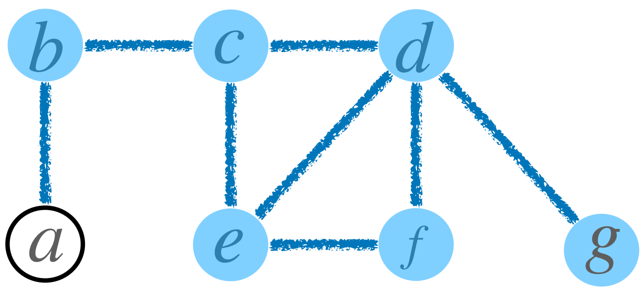

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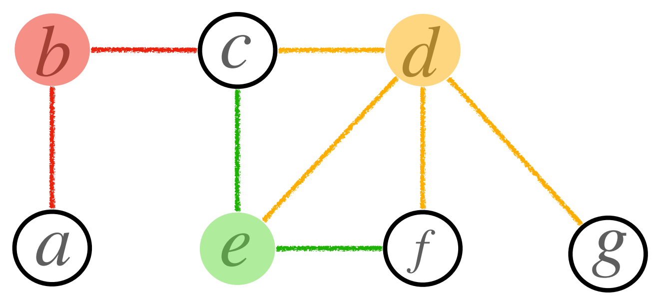

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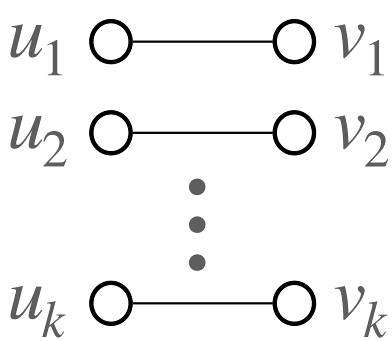

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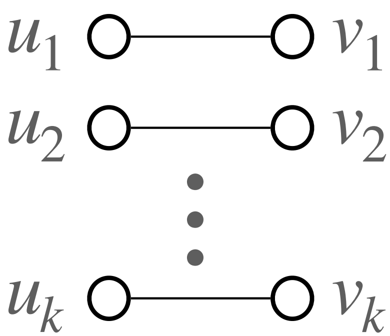
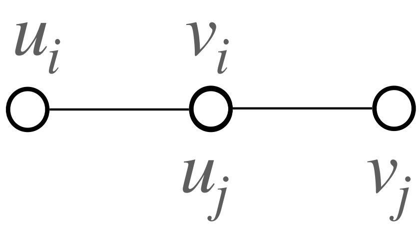


be the k edges chosen by the algorithm.

No two edges above share any common node.

Impossible:



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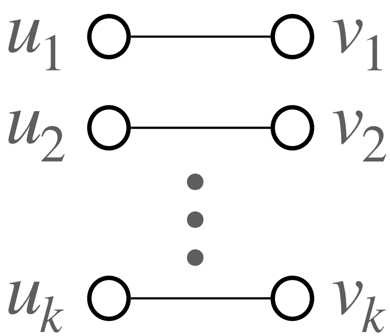

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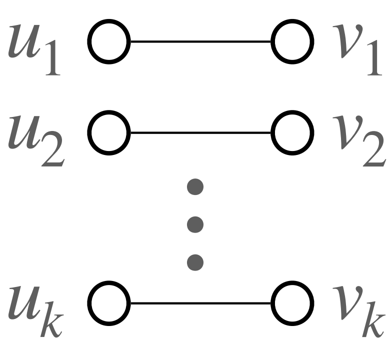

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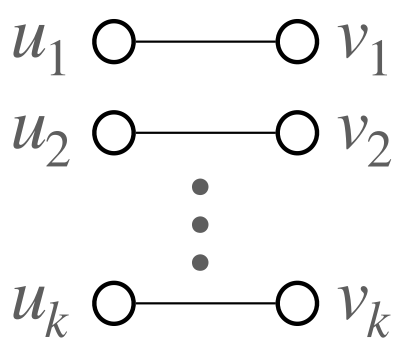

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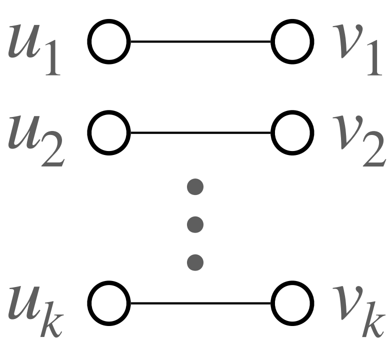

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$$\boxed{\text{So} \quad C^* \geq k}$$

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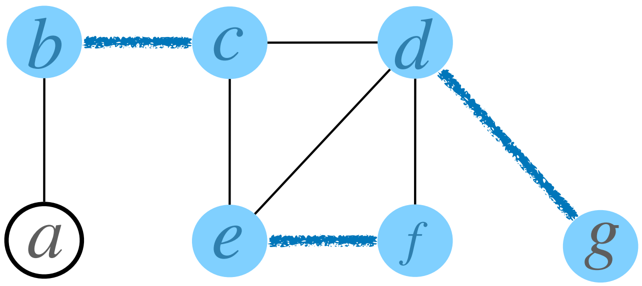
$$\boxed{\frac{C}{C^*} \leq \frac{2k}{k} = 2}$$

Let's take a look at our earlier example:



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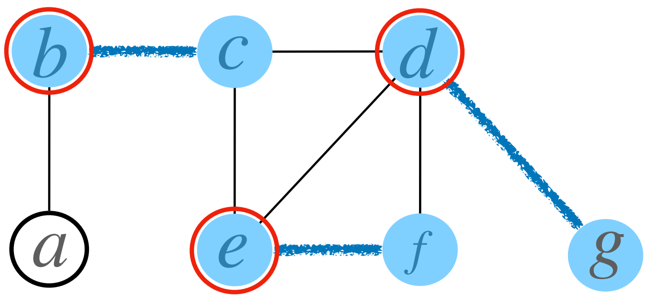

$$S = \{b,\ c,\ e,\ f,\ d,\ g\}$$
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3 disjoint chosen edges

Every vertex cover will contain at least half of those endpoints.

So the approximation ratio is 2.

How to prove $\frac{C}{C^*} \leq \rho$ without knowing C^* ?

For minimization problem, we just need a lower bound of C^* .

Remember our earlier proof:

$\boxed{\text{So } C^* \geq k}$ Lower bound

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maximization **upper bound**

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$$\boxed{\frac{C^*}{C} \leq \frac{\text{upper bound of } C^*}{C}}$$

Traveling Salesman Problem (TSP)

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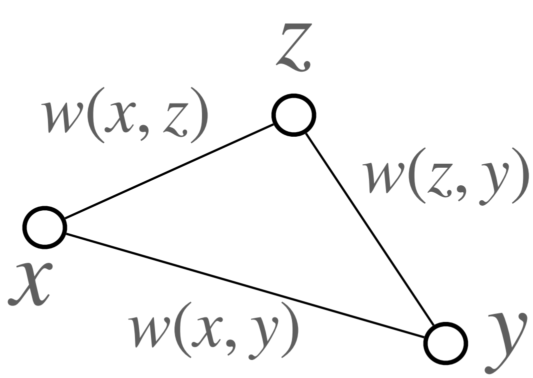

$$w(x, y) \leq w(x, z) + w(z, y)$$

Traveling Salesman Problem (TSP) with the Triangle Inequality

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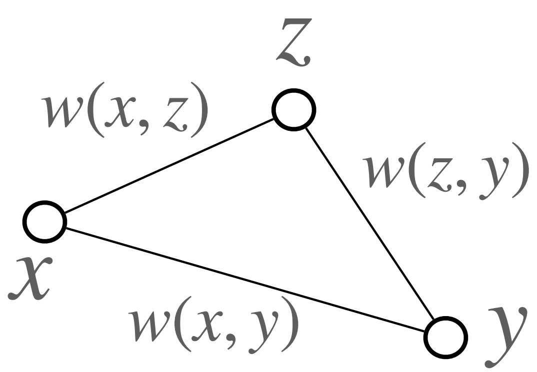

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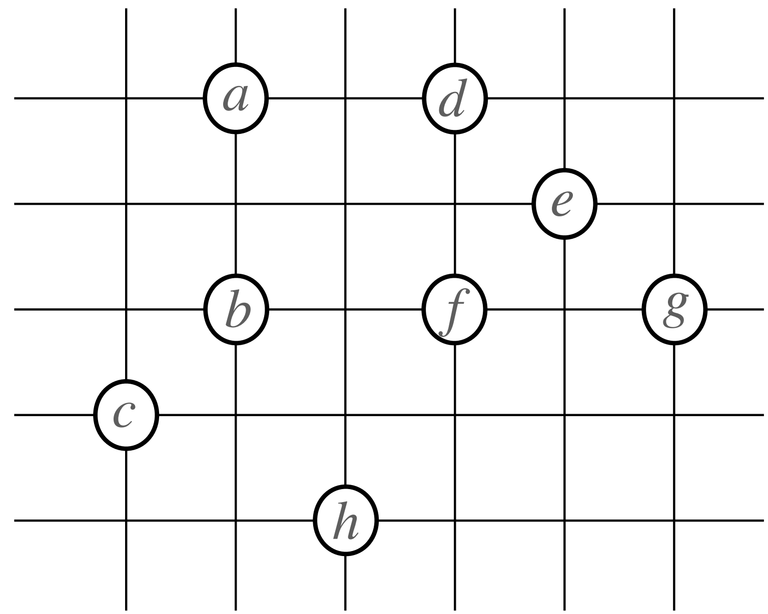
Edge weight between every two nodes: Their Euclidean distance.



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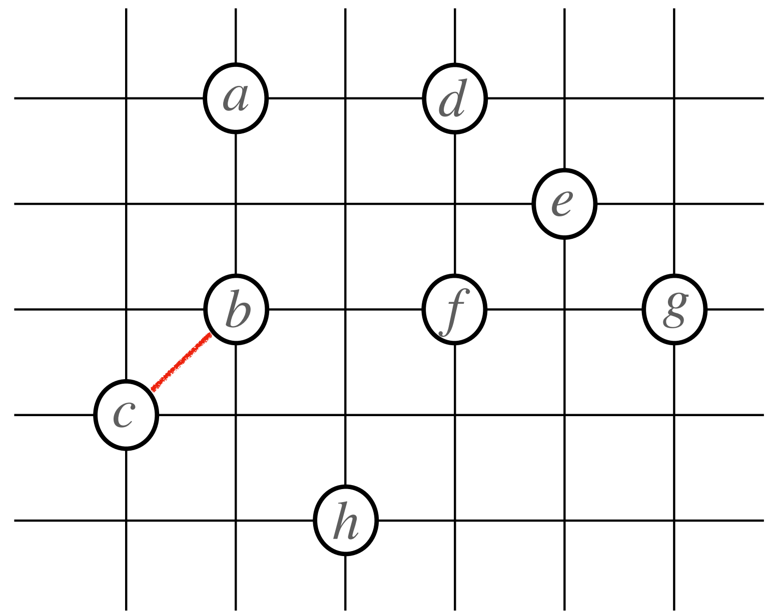

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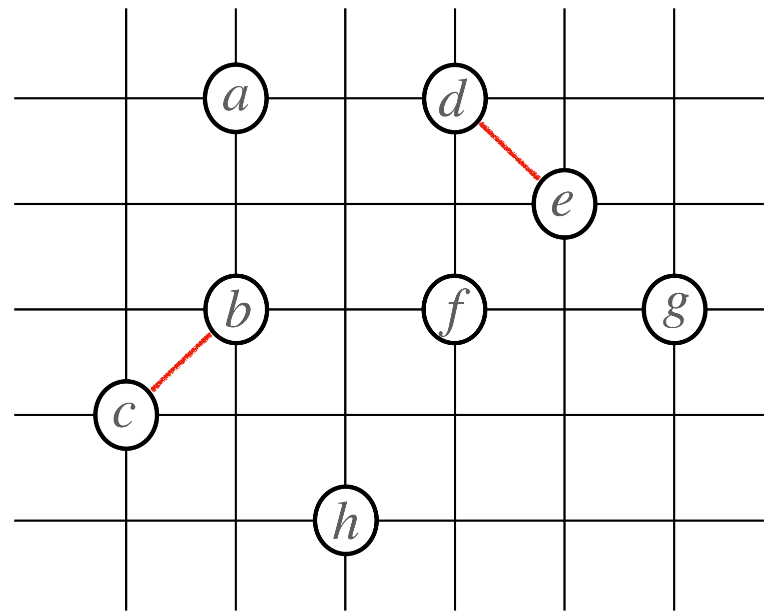

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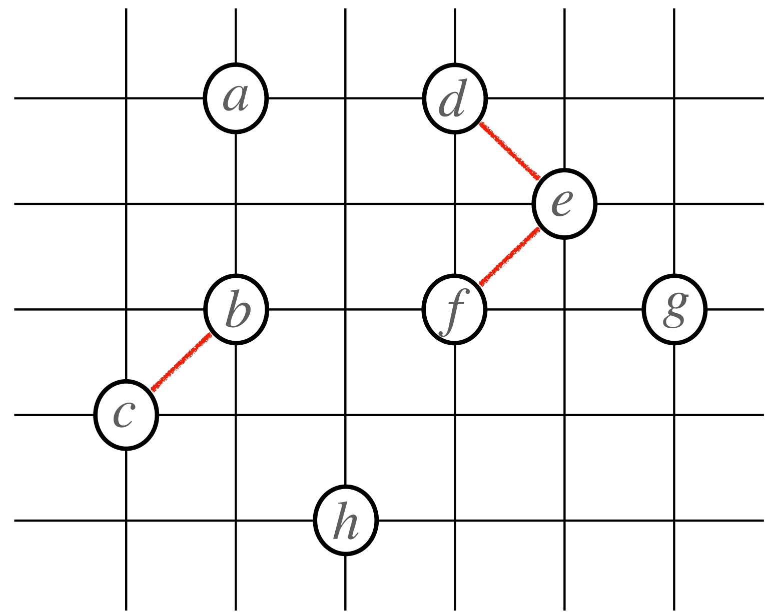

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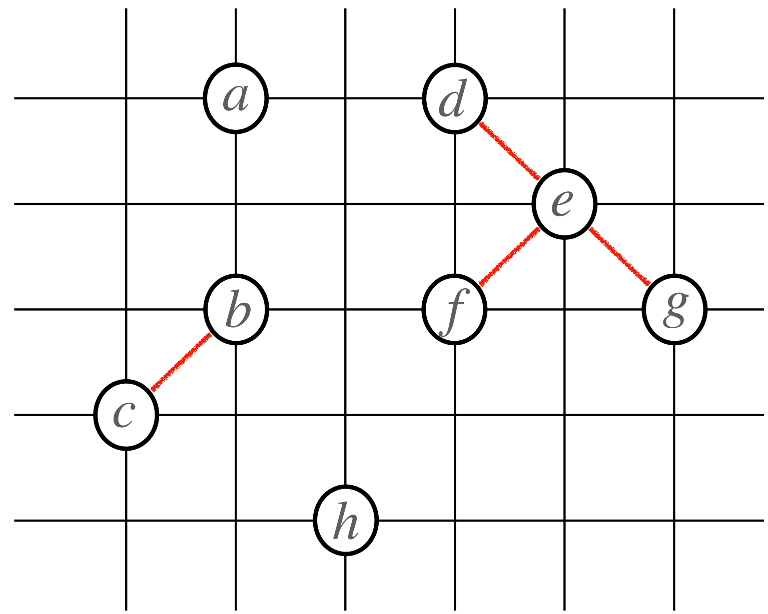

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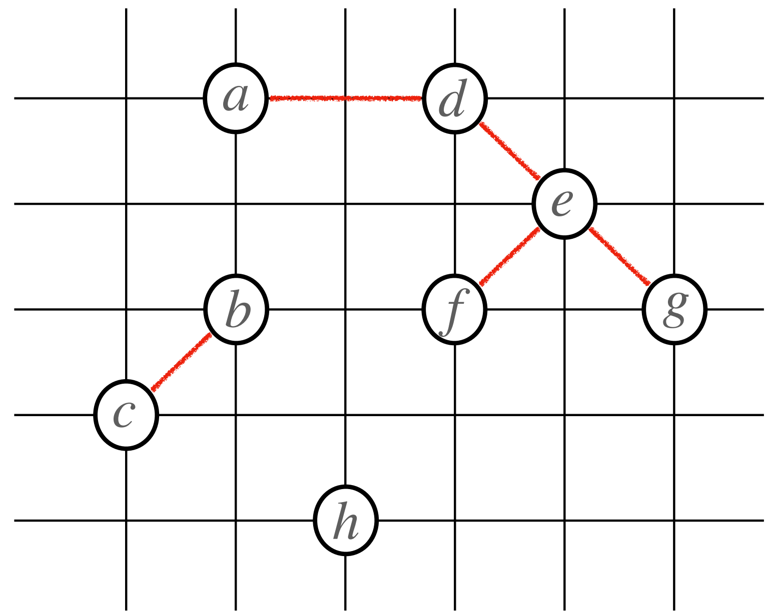

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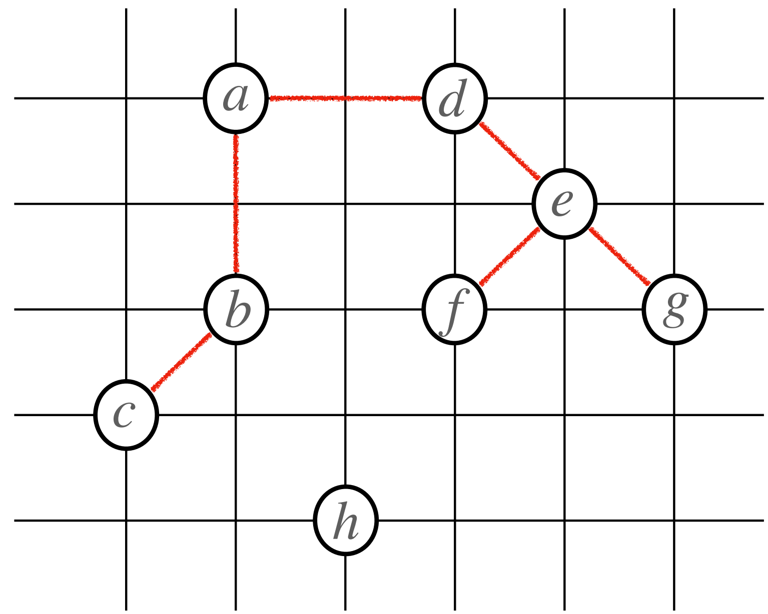

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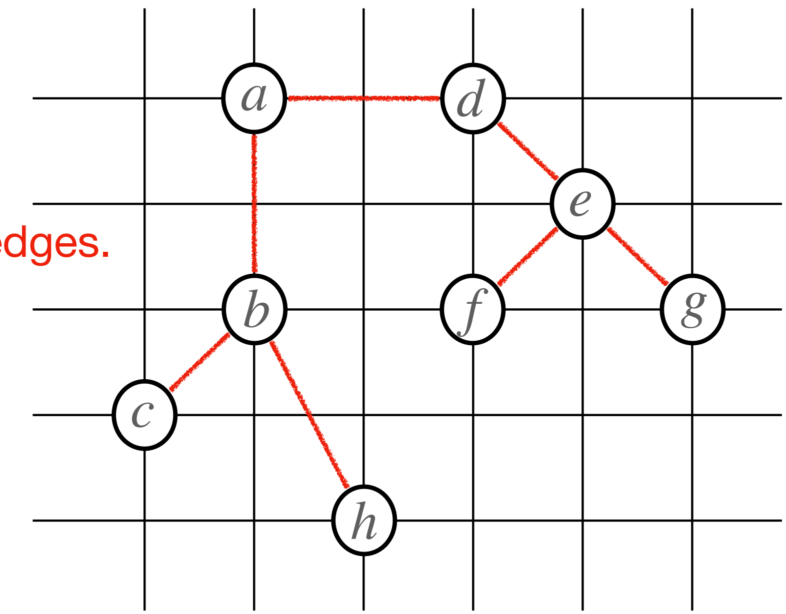
Output: A Hamiltonian cycle of minimum weight.

Edge weight between every two nodes:
Their Euclidean distance.

Idea of Algorithm:

1. Build an MST of G.
2. Get a cycle from the MST by taking a “tour" along its edges.



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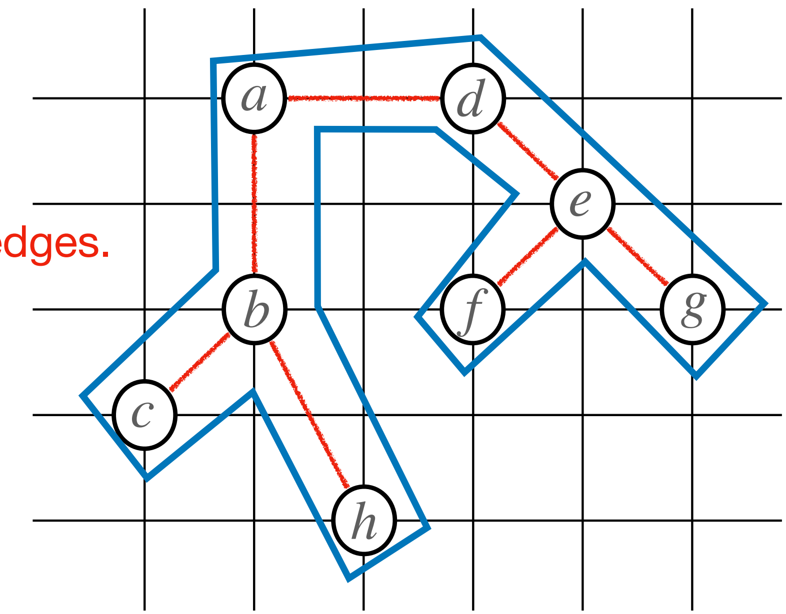

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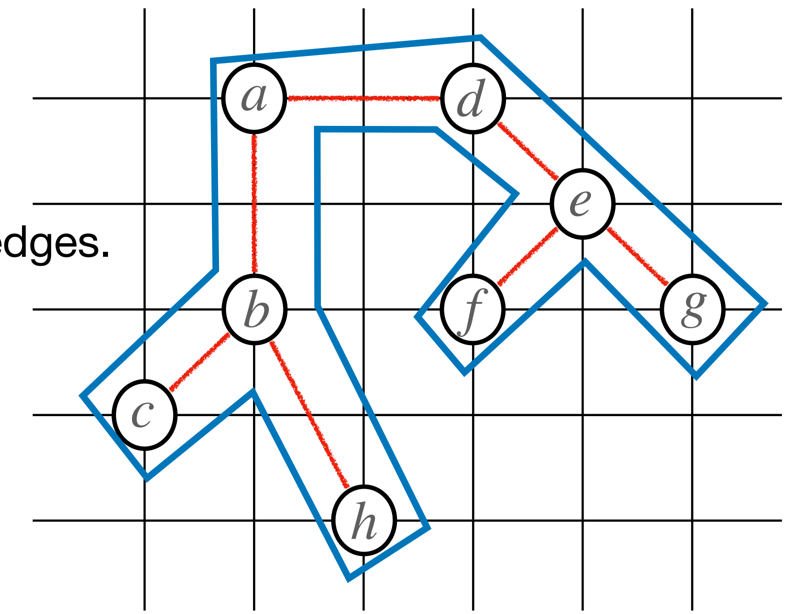

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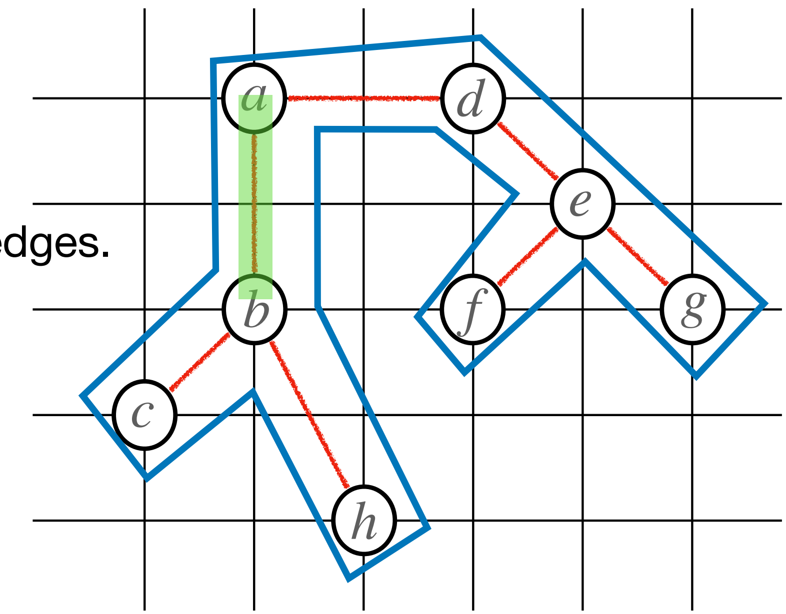

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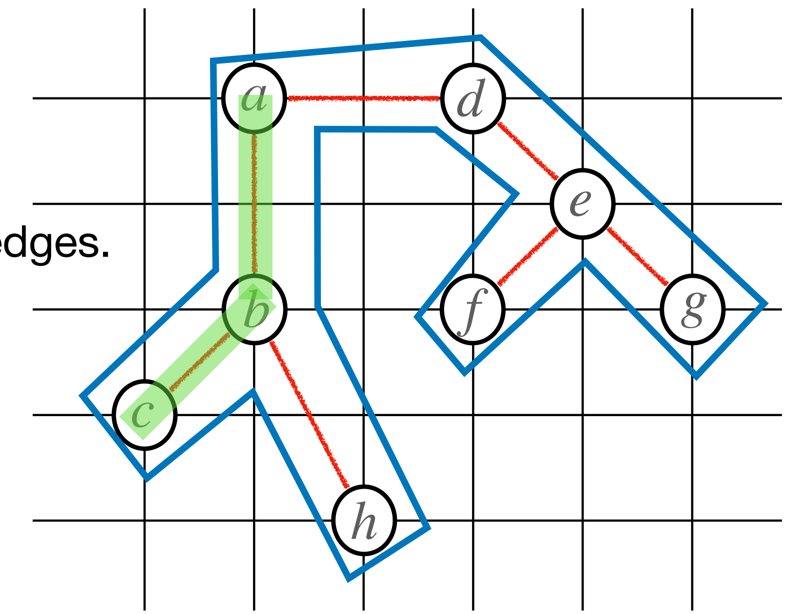

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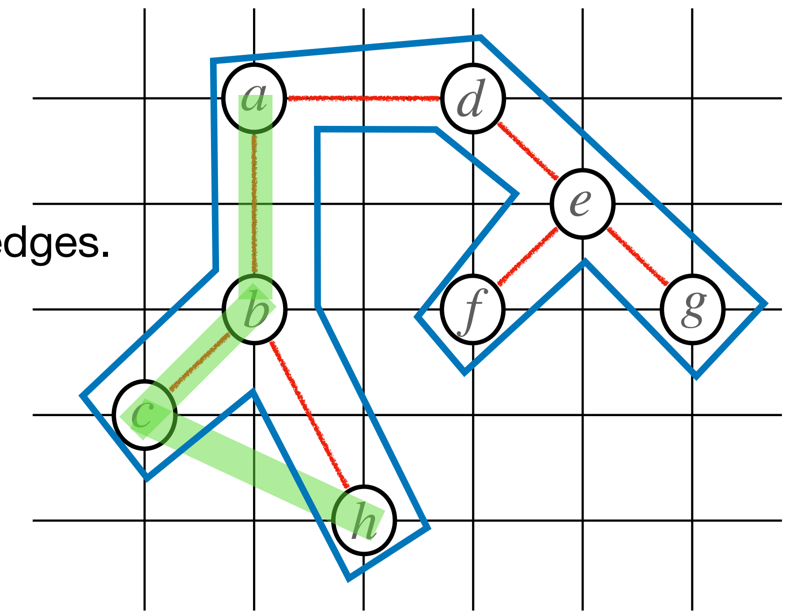

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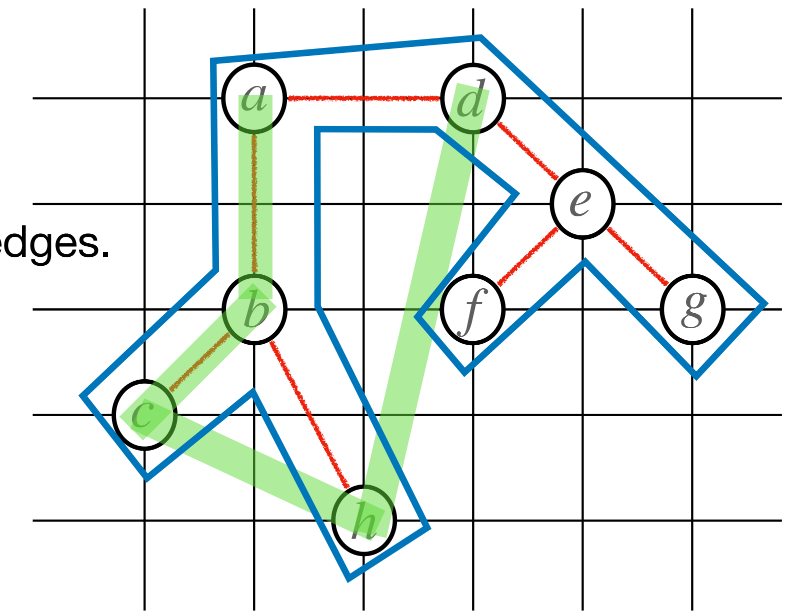

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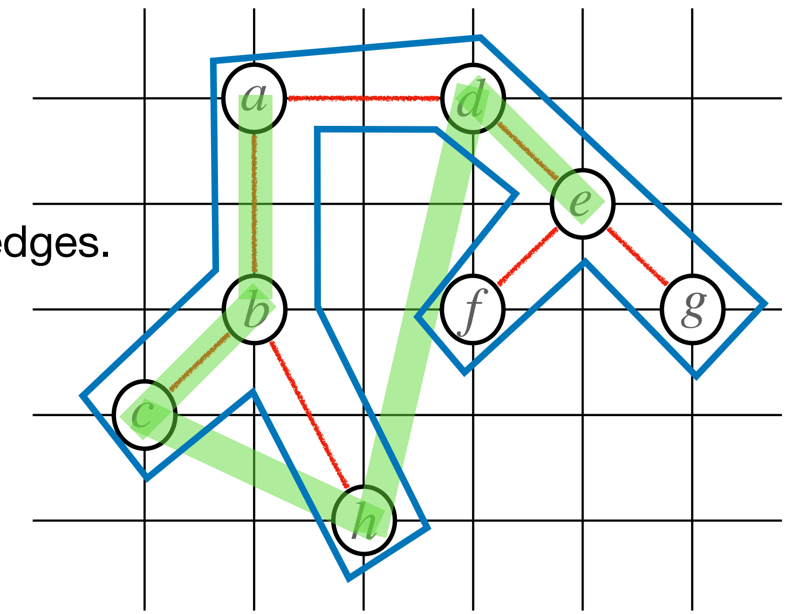

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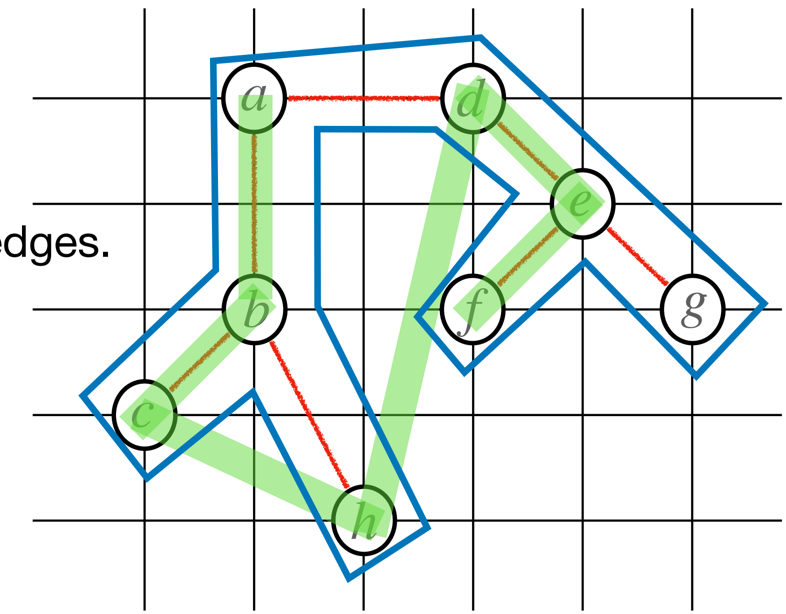

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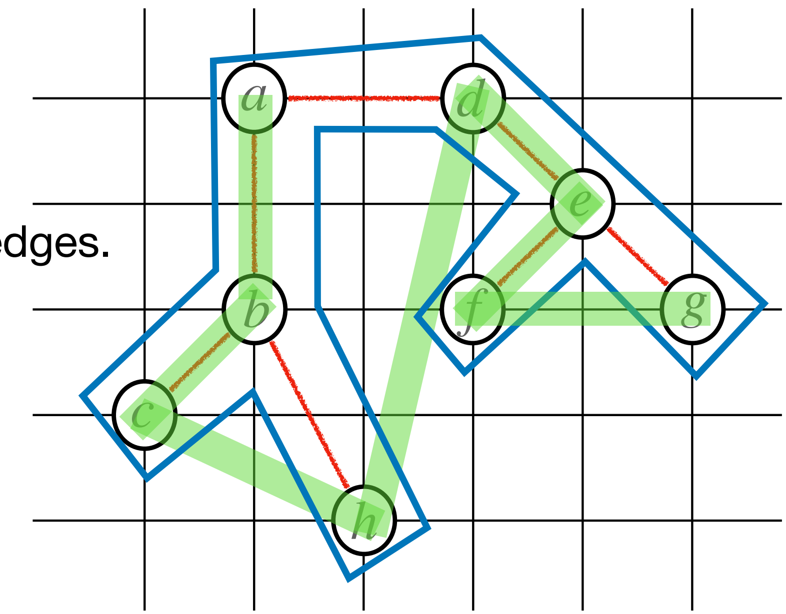

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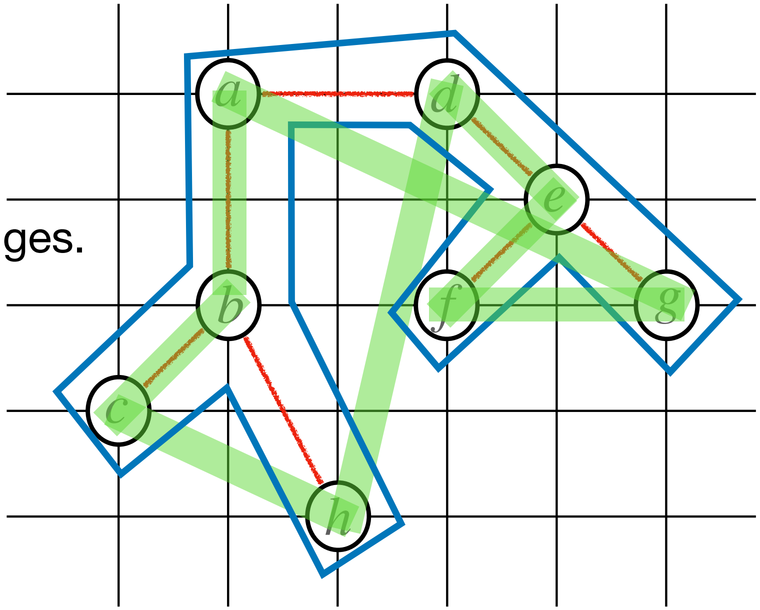

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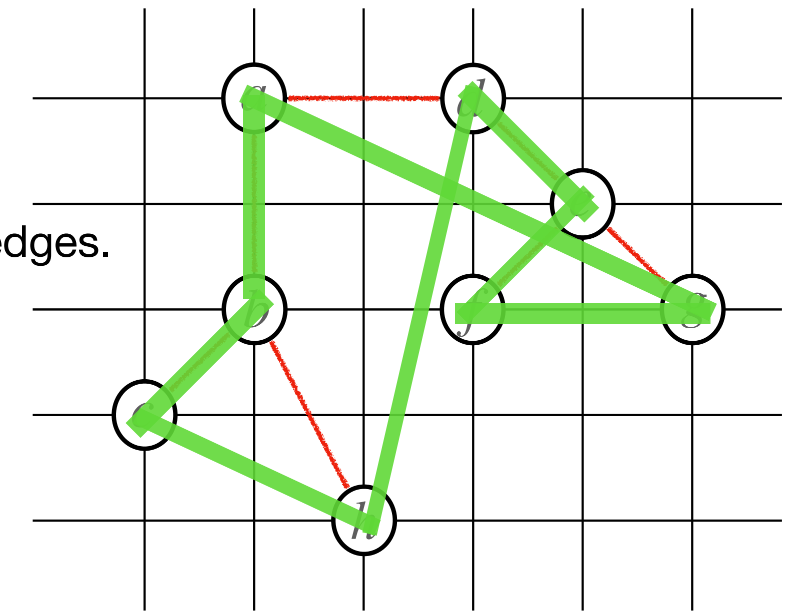
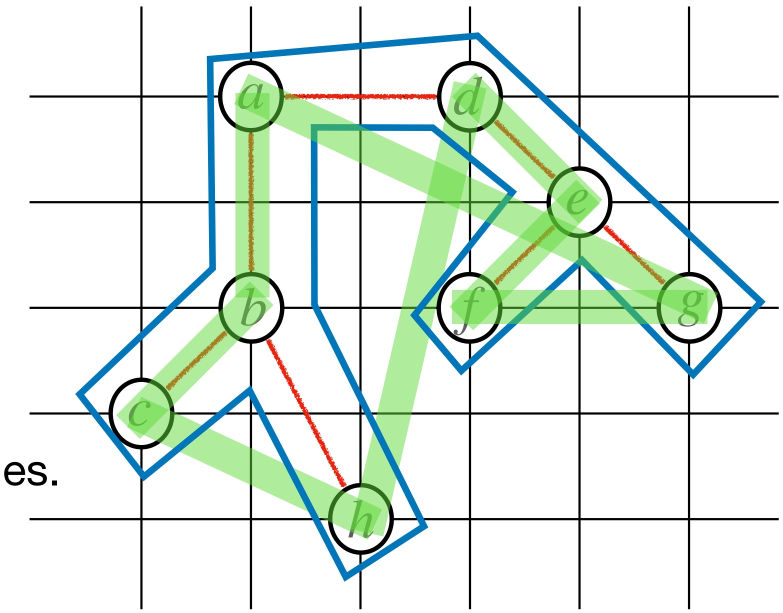
See the Hamiltonian cycle more clearly.



Theorem: The above algorithm for TSP with triangle inequality is a polynomial-time 2-approximation algorithm.

Proof:



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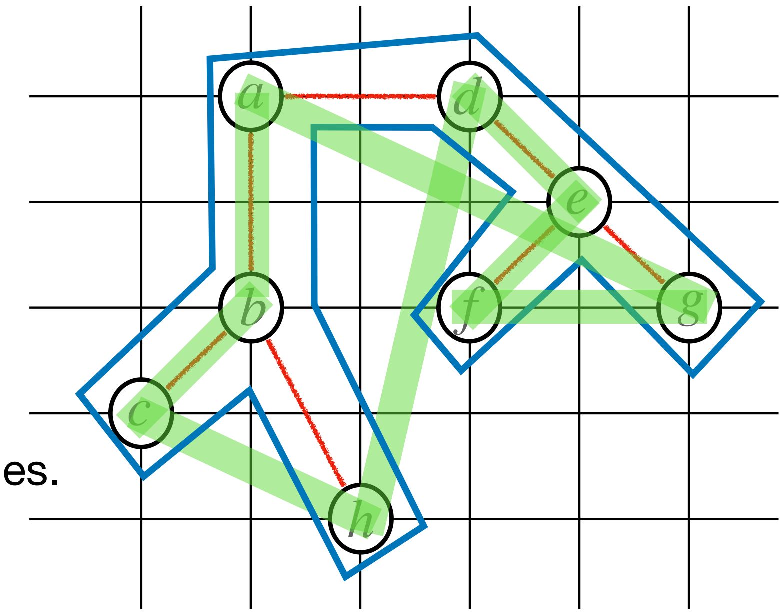

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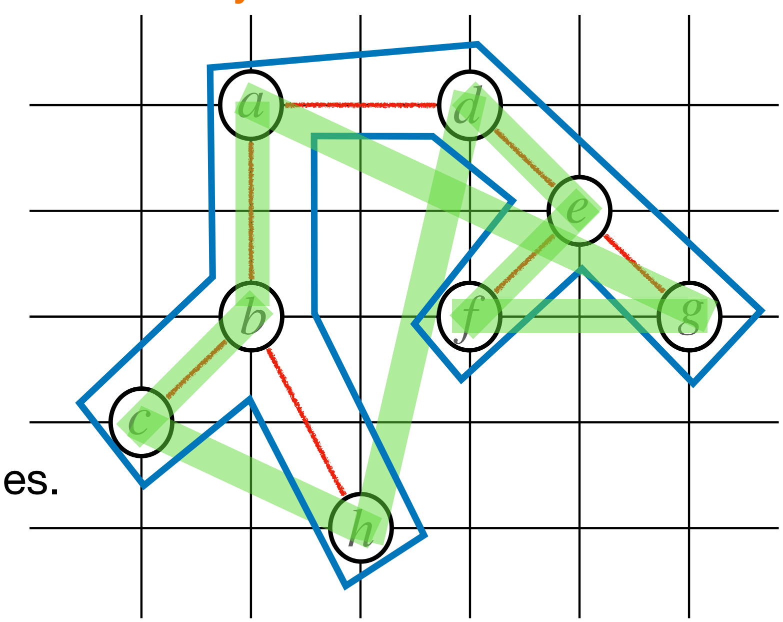
$\boxed{C_{MST} \leq C^*}$ Why?

Lower bound to C^*



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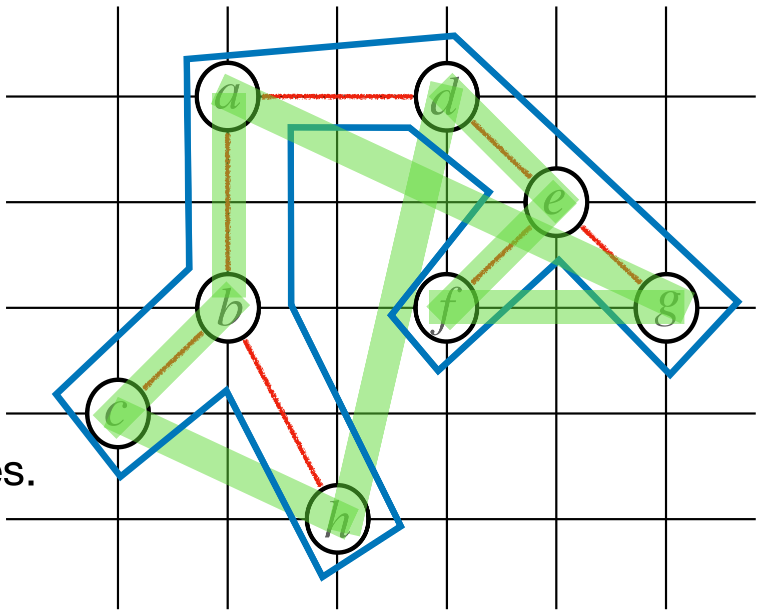

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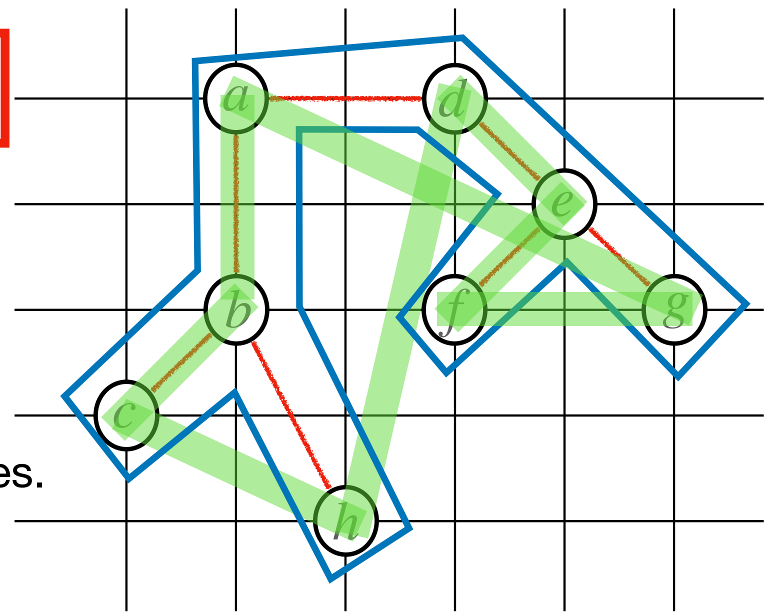

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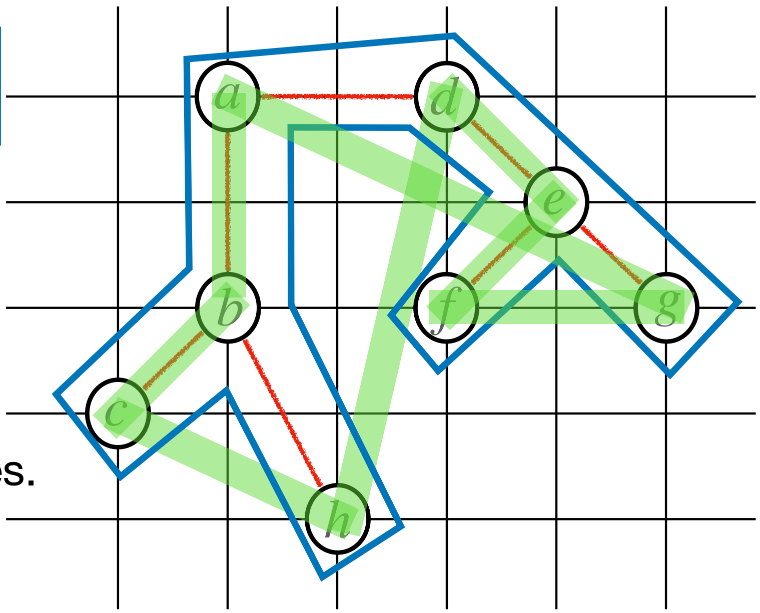

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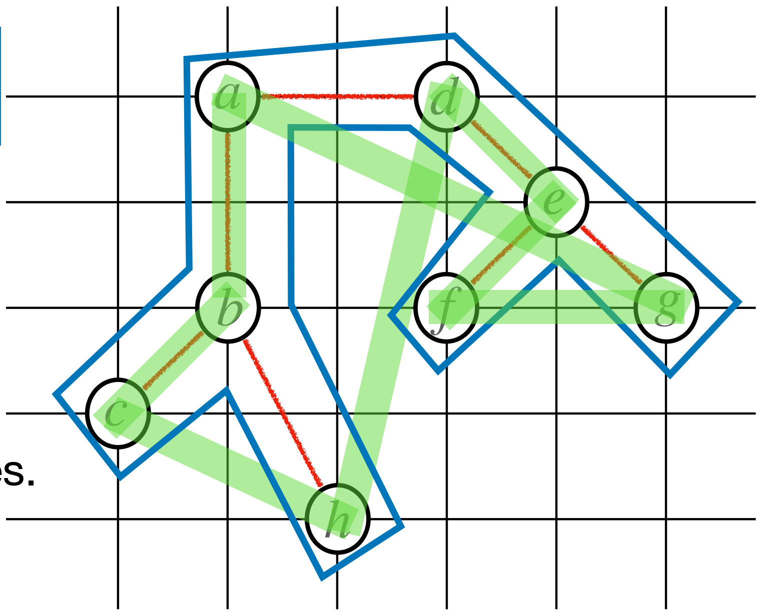
$$\boxed{C \leq C_{DFS} = 2C_{MST} \leq 2C^*} \Rightarrow \frac{C}{C^*} \leq 2$$



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