Algorithms

Lecture 4: Greedy Algorithm

Anxiao (Andrew) Jiang

CH 16. Greedy Algorithms

16.3 Huffman Code

Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05

How to represent the symbols using bits, to minimize the average number of bits needed?

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Assume: we use “Fixed Length Code (FLC)”

1-bit codewords: 0, 1 (not enough)
2-bit codewords: 00, 01, 10, 11 (not enough)
3-bit codewords: 000, 001, 010, 011, 100, 101, 110, 111 (enough)

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Probability	0.45	0.13	0.12	0.16	0.09	0.05
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Compression (encoding): turn a text to bits

Decompression (decoding): turn a bit sequence back to text

Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
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Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

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000 000

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Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

000 000 001 101

Decompression (decoding): turn a bit sequence back to text

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Probability	0.45	0.13	0.12	0.16	0.09	0.05
FLC	000	001	010	011	100	101

Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

000 000 001 101 100

Decompression (decoding): turn a bit sequence back to text

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Probability	0.45	0.13	0.12	0.16	0.09	0.05
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Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

000 000 001 101 100 011

Decompression (decoding): turn a bit sequence back to text

Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
FLC	000	001	010	011	100	101

Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

000 000 001 101 100 011 010

Decompression (decoding): turn a bit sequence back to text

Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
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Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

000 000 001 101 100 011 010 000 ···

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Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

000 000 001 101 100 011 010 000 ···

Decompression (decoding): turn a bit sequence back to text

a

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Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

000 000 001 101 100 011 010 000 ···

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a a

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Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

000 000 001 101 100 011 010 000 ⋯

Decompression (decoding): turn a bit sequence back to text

$a\ a\ b$

Symbol	a	b	c	d	e	f
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Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

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Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

$000\,000\,001\,101\,100\,011\,010\,000\ \cdots$

Decompression (decoding): turn a bit sequence back to text

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Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

000 000 001 101 100 011 010 000 ⋯

Decompression (decoding): turn a bit sequence back to text

$a\ a\ b\ f\ e\ d\ c$

Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
FLC	000	001	010	011	100	101

Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

000 000 001 101 100 011 010 000 ···

Decompression (decoding): turn a bit sequence back to text

$a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
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3 bits per symbol

Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

000 000 001 101 100 011 010 000 ···

Decompression (decoding): turn a bit sequence back to text

$a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
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Variable Length Code (VLC): the codewords can have different lengths.

Prefix Code: no codeword is the prefix of another codeword.

Let's study Variable Length Prefix Code (VLPC).

Symbol	a	b	c	d	e	f
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0

Decompression (decoding): turn a bit sequence back to text

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Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

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Decompression (decoding): turn a bit sequence back to text

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3 bits per symbol

Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

0 0 1 0 1 1 1 0 0

Decompression (decoding): turn a bit sequence back to text

Symbol	a	b	c	d	e	f
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Compression (encoding): turn a text to bits

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Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

0 0 1 0 1 1 1 0 0 1 1 0 1 1 1 1 1 0 0 0 ⋯

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$a\ a\ b\ f$

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0 0 1 0 1 1 1 0 0 1 1 0 1 1 1 1 1 0 0 0 ⋯

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$a\ a\ b\ f$

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3 bits per symbol

Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

0 0 1 0 1 1 1 0 0 1 1 0 1 1 1 1 1 0 0 0 ···

Decompression (decoding): turn a bit sequence back to text

$a\ \ a\ \ b\ \ f\ \ e$

Symbol	a	b	c	d	e	f
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VLPC	0	101	100	111	1101	1100

3 bits per symbol

Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

0 0 1 0 1 1 1 0 0 1 1 0 1 1 1 1 1 0 0 0 ⋯

Decompression (decoding): turn a bit sequence back to text

$a\ \ a\ \ b\ \ f\ \ e$

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3 bits per symbol

Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

0 0 1 0 1 1 1 0 0 1 1 0 1 1 1 1 0 0 0 ⋯

Decompression (decoding): turn a bit sequence back to text

$a\ a\ b\ f\ e\ d$

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Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

0 0 1 0 1 1 1 0 0 1 1 0 1 1 1 1 0 0 0 ⋯

Decompression (decoding): turn a bit sequence back to text

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Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

0 0 1 0 1 1 1 0 0 1 1 0 1 1 1 1 0 0 0 ⋯

Decompression (decoding): turn a bit sequence back to text

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VLPC	0	101	100	111	1101	1100

3 bits per symbol

Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

0 0 1 0 1 1 1 0 0 1 1 0 1 1 1 1 0 0 0 ···

Decompression (decoding): turn a bit sequence back to text

$a\ a\ b\ f\ e\ d\ c$

Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
FLC	000	001	010	011	100	101
VLPC	0	101	100	111	1101	1100

3 bits per symbol

Text: $a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Compression (encoding): turn a text to bits

0 0 1 0 1 1 1 0 0 1 1 0 1 1 1 1 1 0 0 0 ···

Decompression (decoding): turn a bit sequence back to text

$a\ a\ b\ f\ e\ d\ c\ a\ \cdots$

Symbol	a	b	c	d	e	f	
Probability	0.45	0.13	0.12	0.16	0.09	0.05	
FLC	000	001	010	011	100	101	3 bits per symbol
VLPC	0	101	100	111	1101	1100	2.24 bits per symbol

Average codeword length for VLPC:

$$1 \times 0.45 + 3 \times 0.13 + 3 \times 0.12 + 3 \times 0.16 + 4 \times 0.09 + 4 \times 0.05 = 2.24 \text{ bits/symbol}$$

16.3 Huffman Code

Input: n symbols $s_1, s_2, \cdots, s_n$.

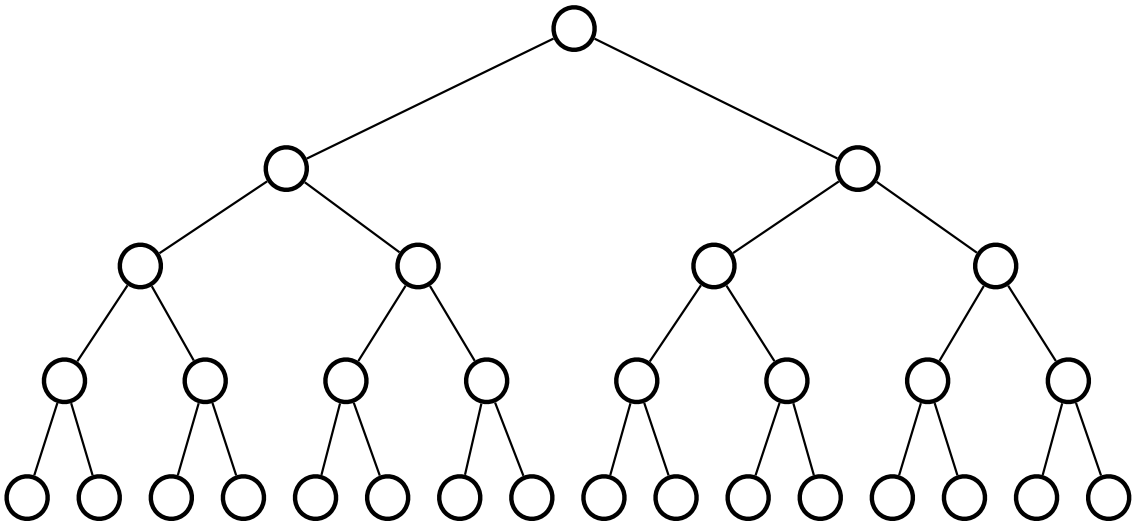
For $i = 1,2,\cdots,n,$ the symbol s_i has probability f_i.

Output: Design a prefix code for the n symbols such that the average codeword length is minimized.

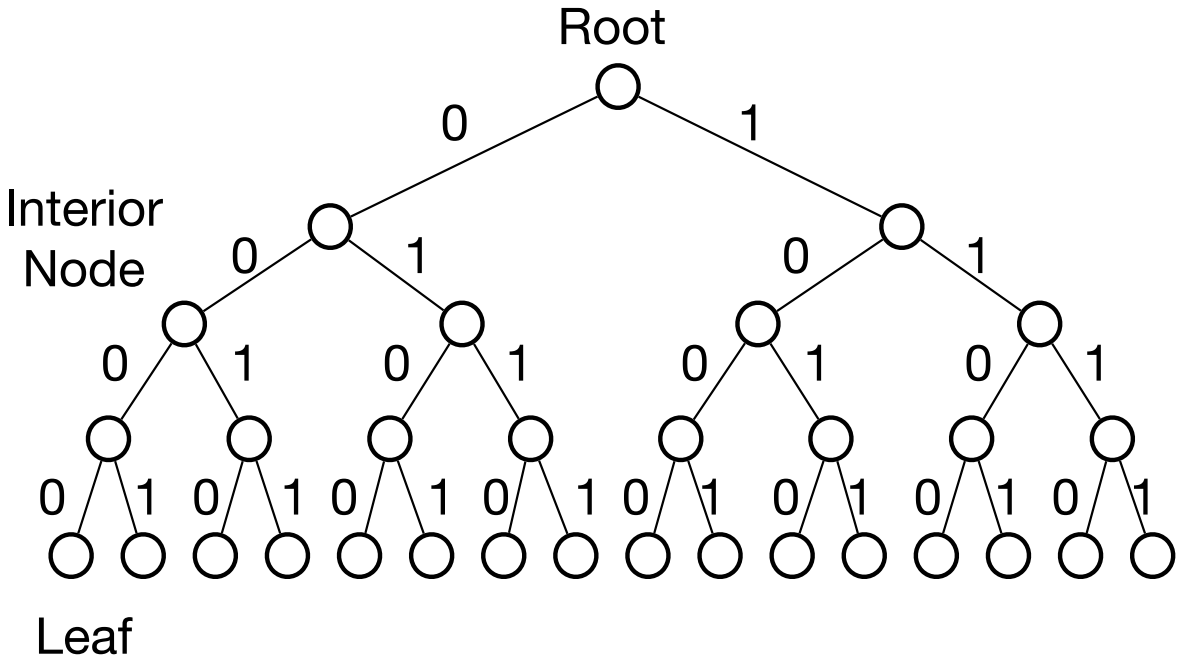
Assume the codeword for symbol s_i has L_i bits.

Average Codeword Length = $\sum_{i=1}^{n} f_i L_i$

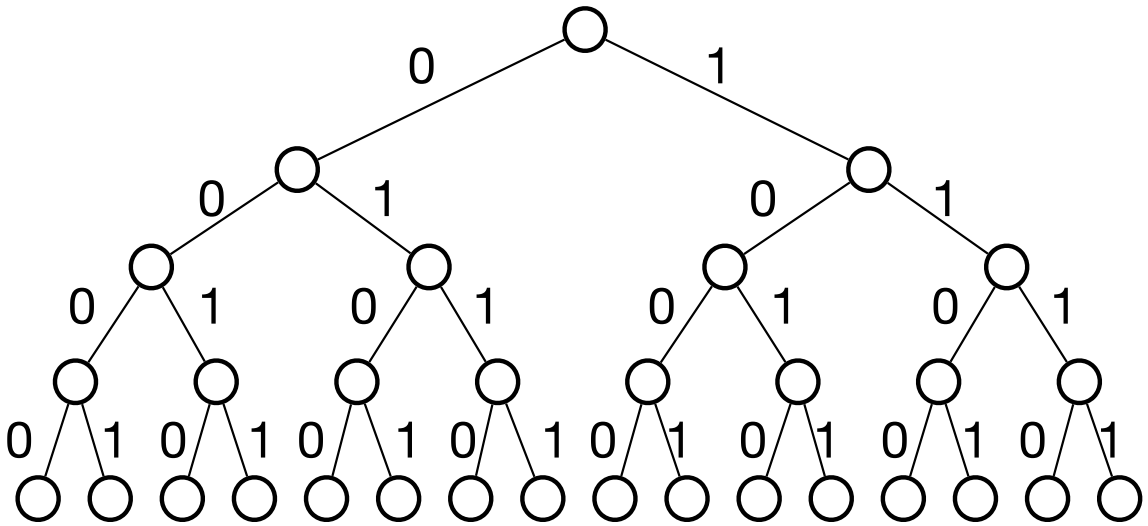
16.3 Huffman Code



16.3 Huffman Code

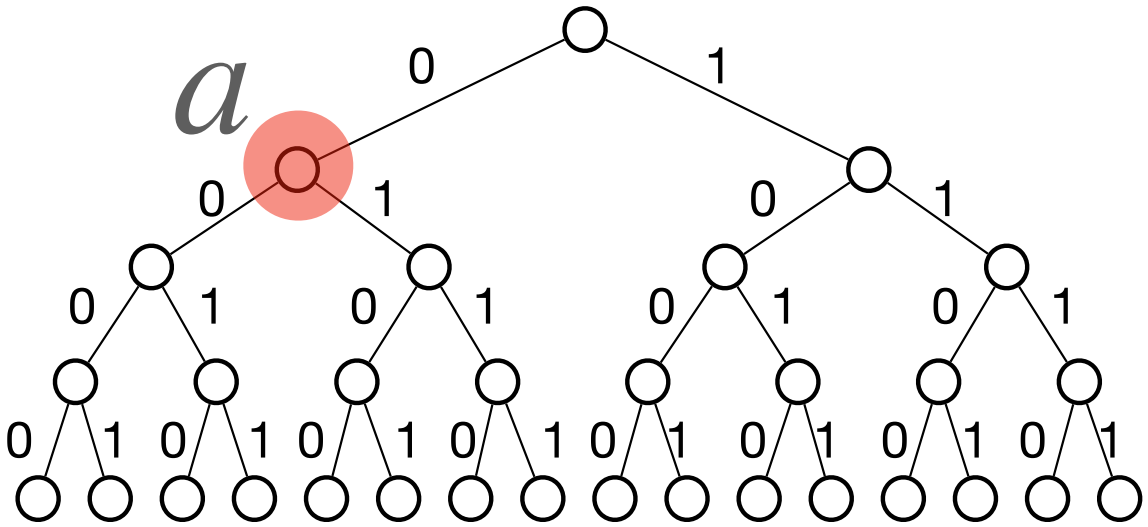


16.3 Huffman Code



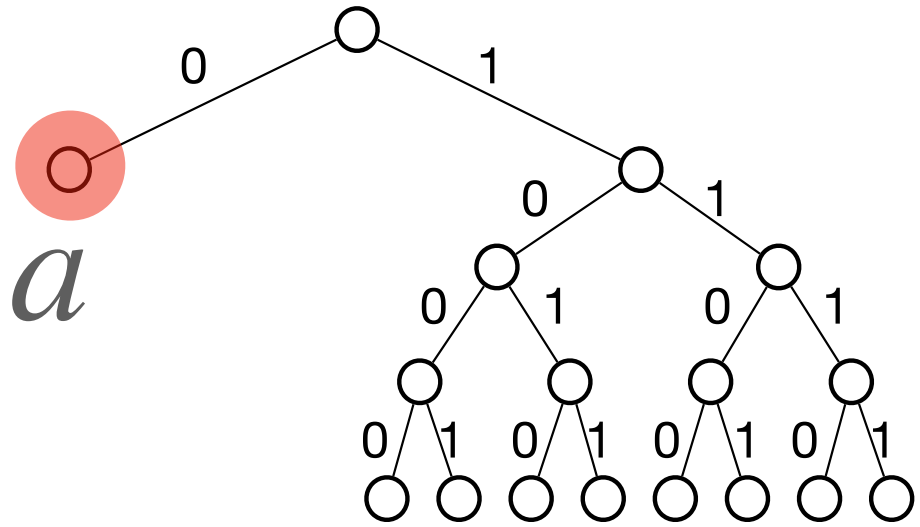
Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

16.3 Huffman Code



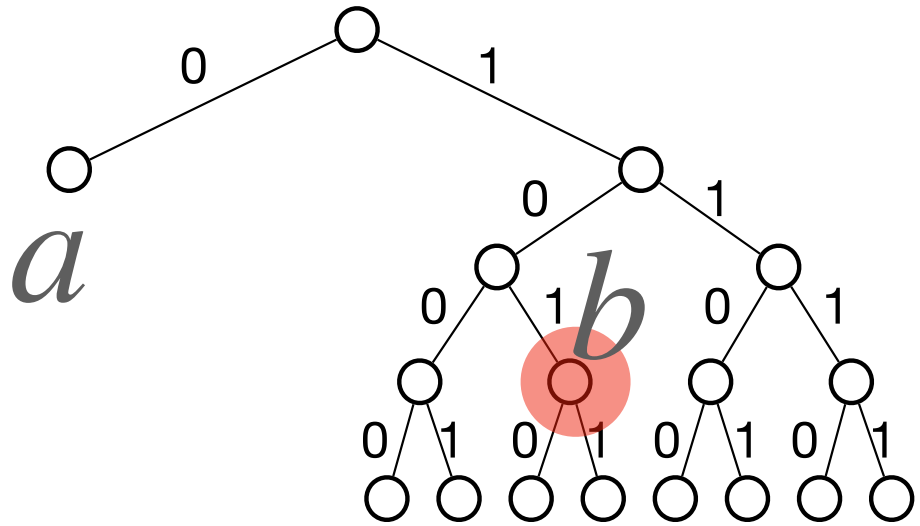
Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

16.3 Huffman Code



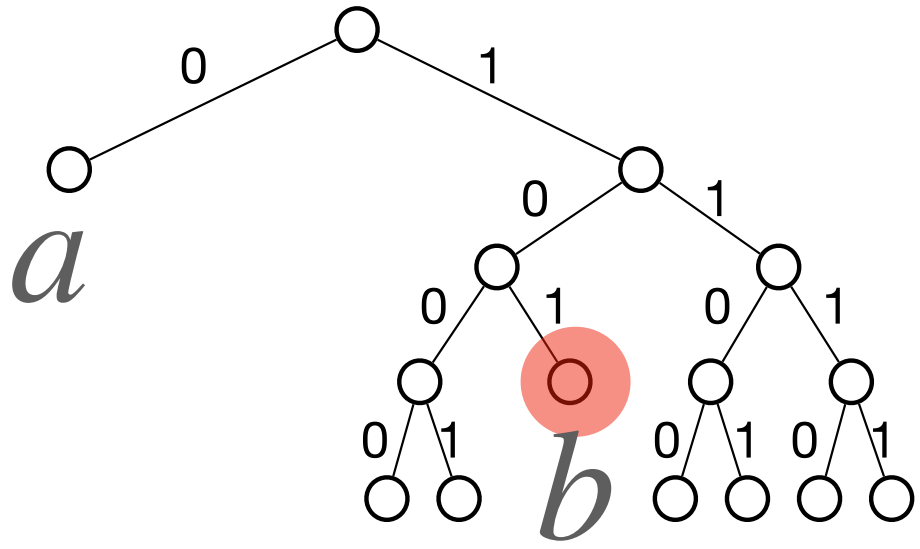
Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

16.3 Huffman Code



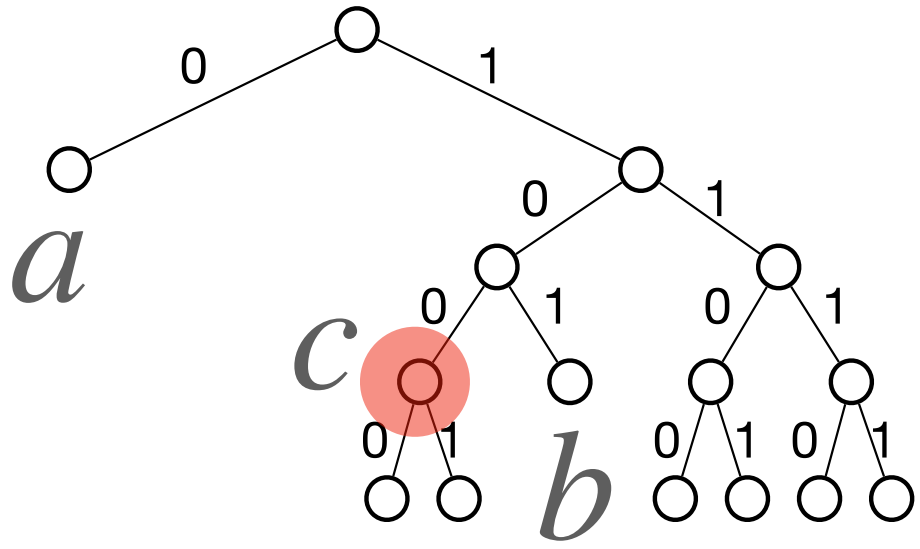
Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

16.3 Huffman Code



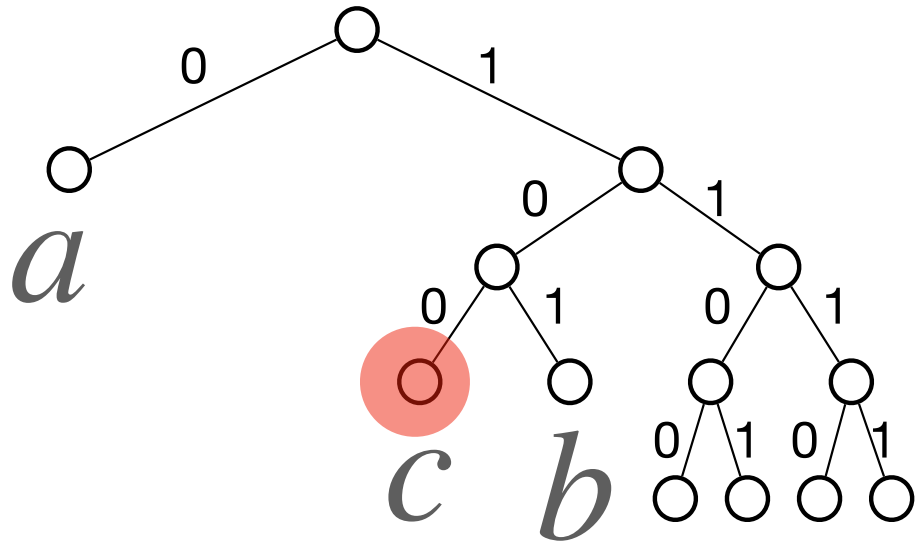
Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

16.3 Huffman Code



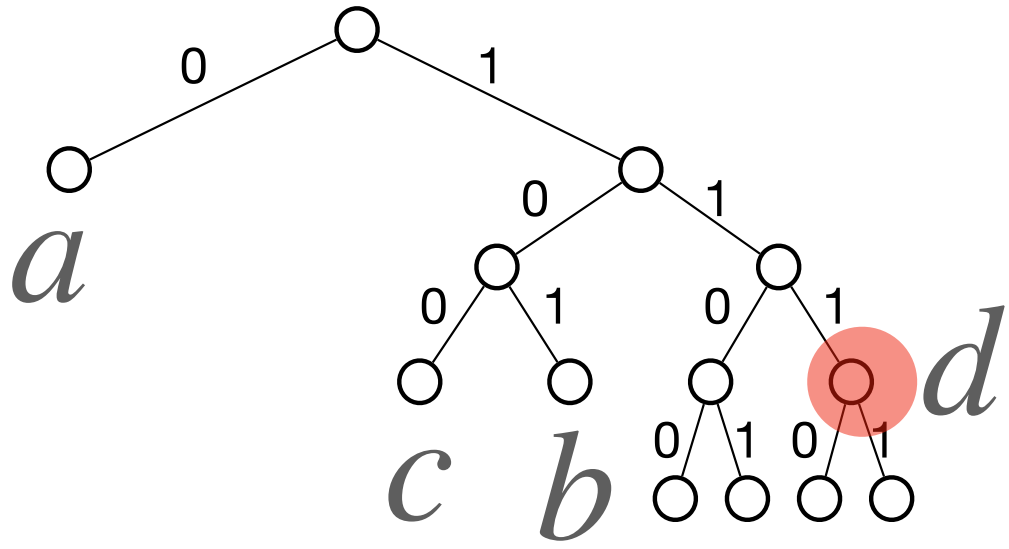
Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

16.3 Huffman Code



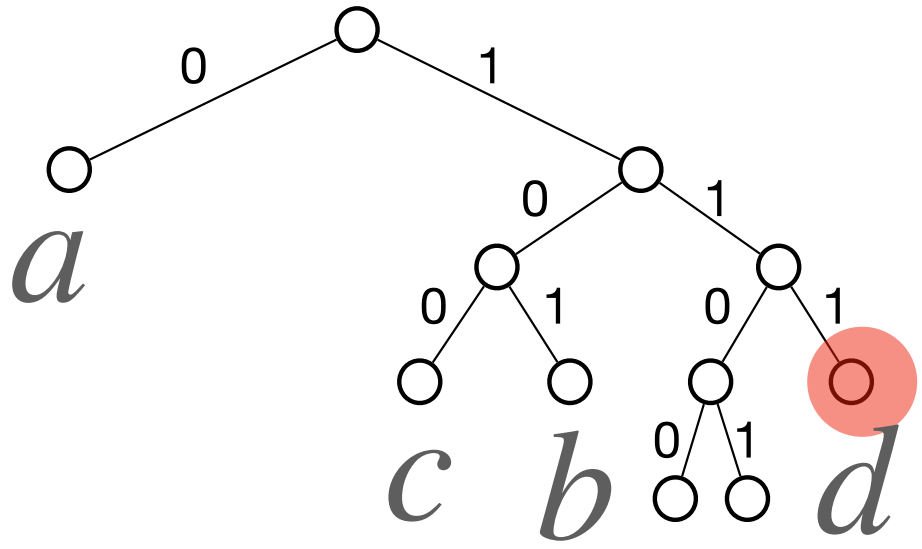
Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

16.3 Huffman Code



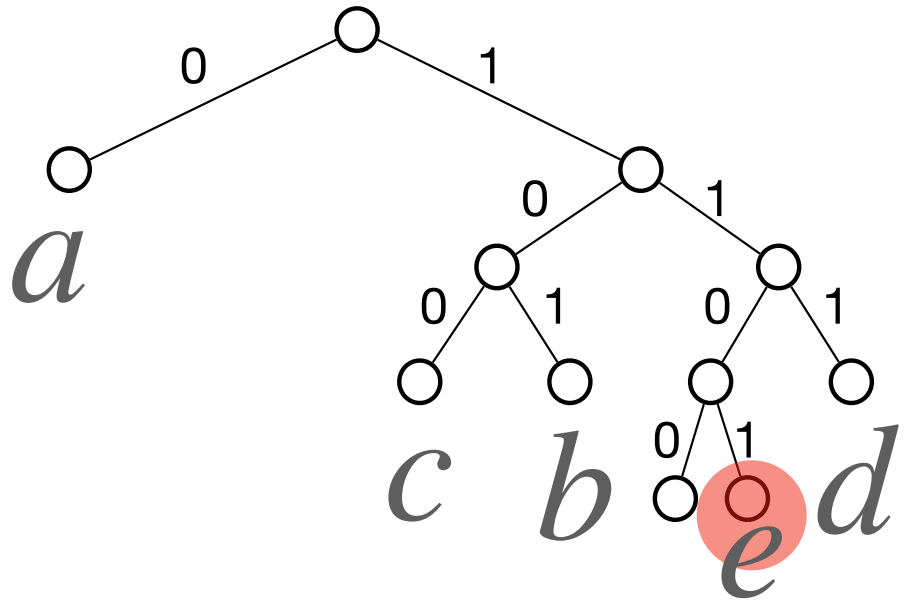
Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

16.3 Huffman Code



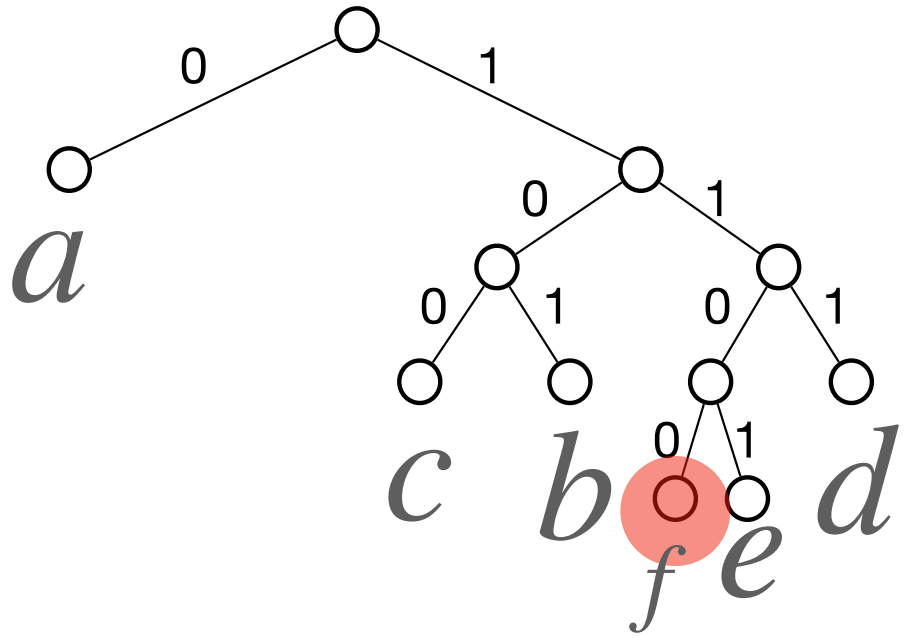
Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

16.3 Huffman Code



Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

16.3 Huffman Code

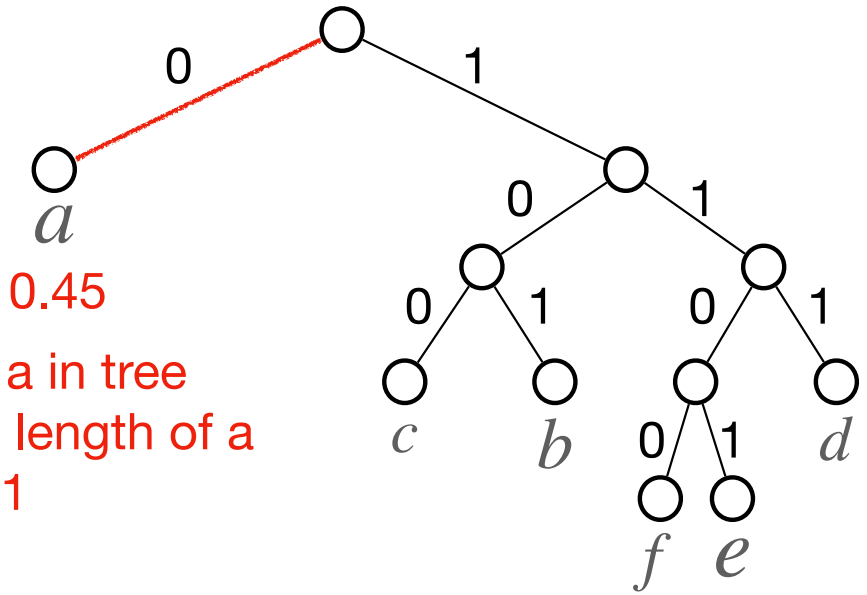


Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

Leaves of a subtree

Prefix code

16.3 Huffman Code



0.45

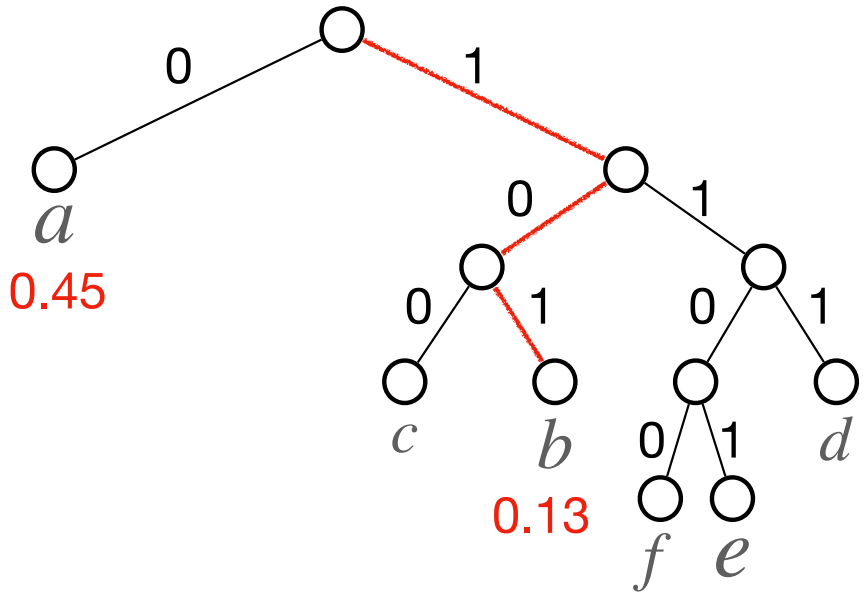
Depth of a in tree
= codeword length of a
= 1

Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

Average codeword length

Average depth of leaves

16.3 Huffman Code

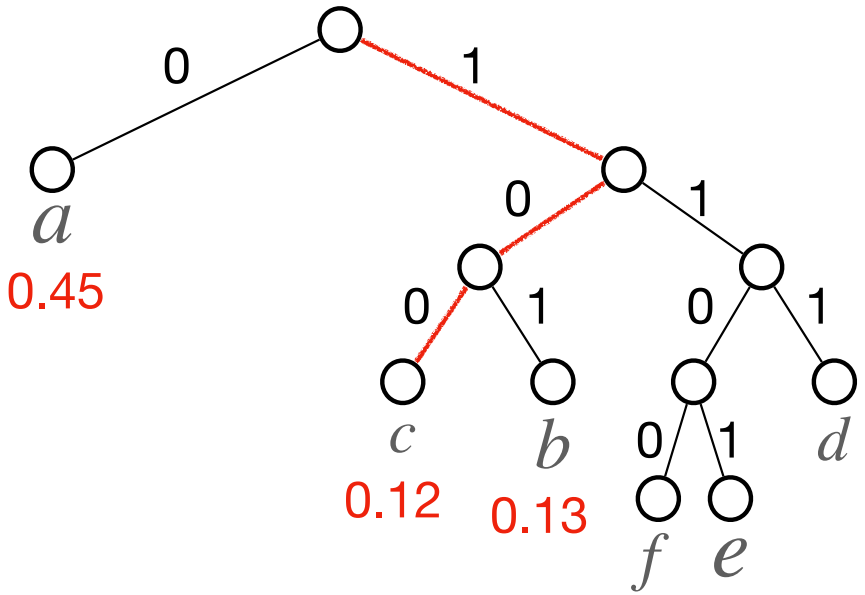


Average codeword length

Average depth of leaves

Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

16.3 Huffman Code

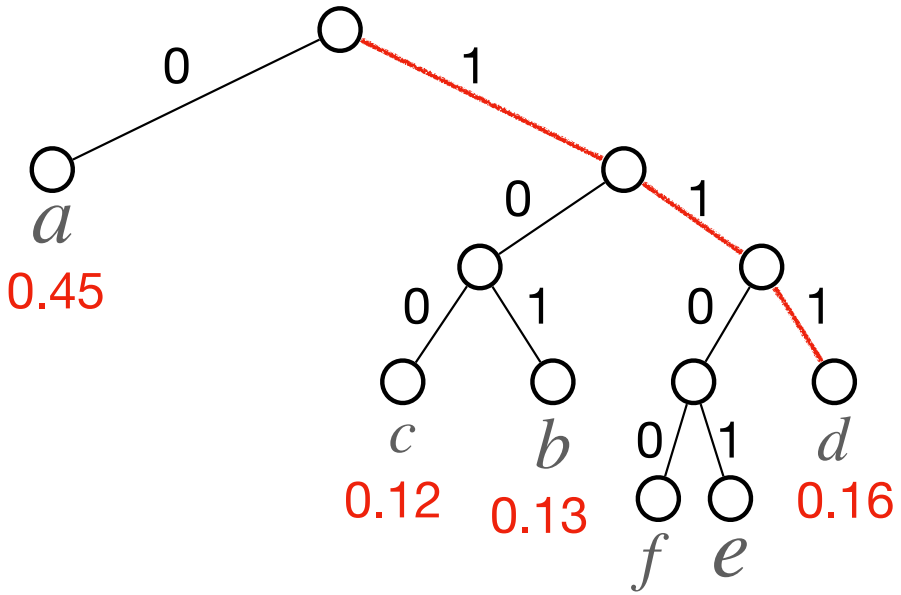


Average codeword length

Average depth of leaves

Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

16.3 Huffman Code

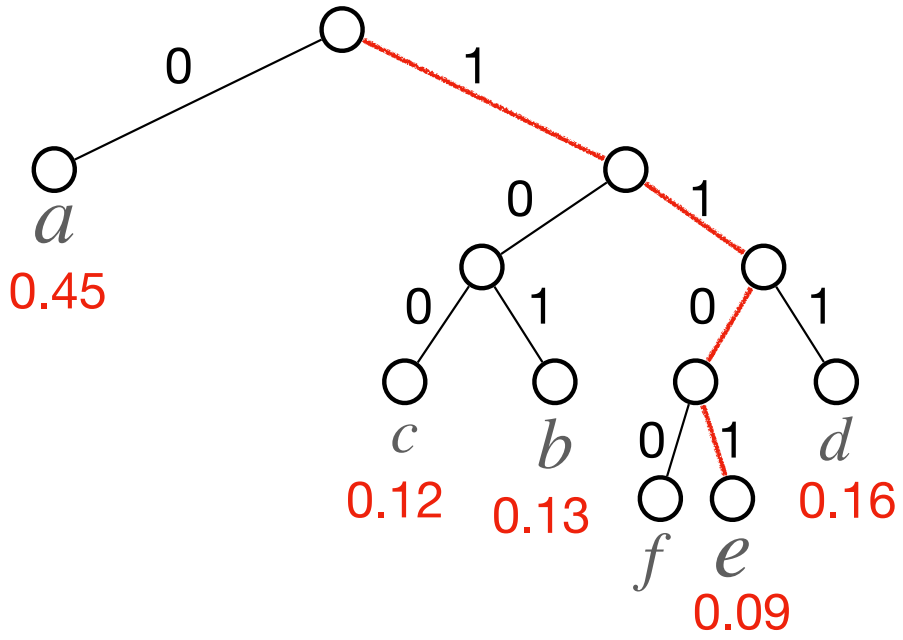


Average codeword length

Average depth of leaves

Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

16.3 Huffman Code

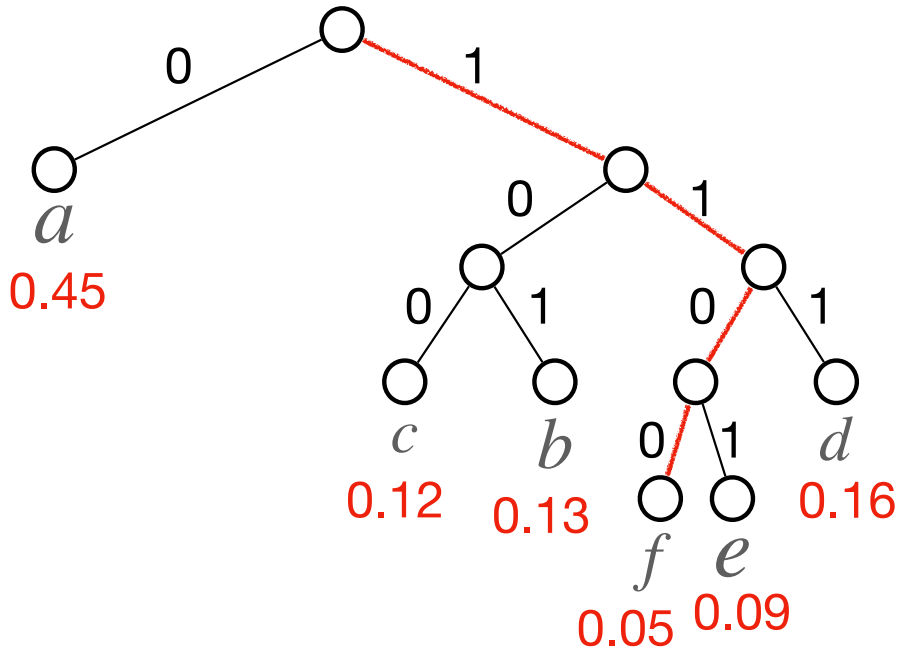


Average codeword length

Average depth of leaves

Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

16.3 Huffman Code



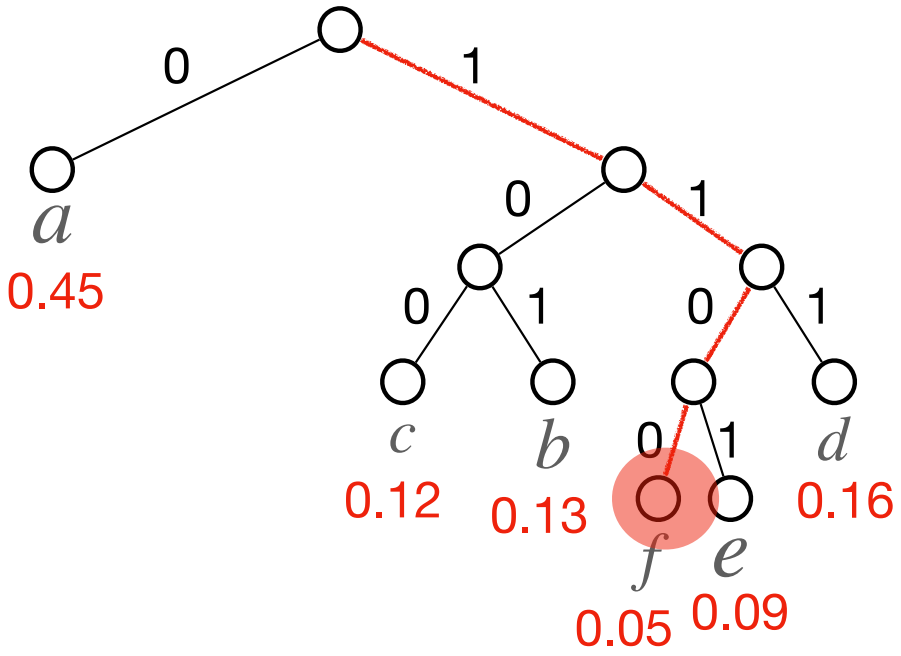
Average codeword length

Average depth of leaves

Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

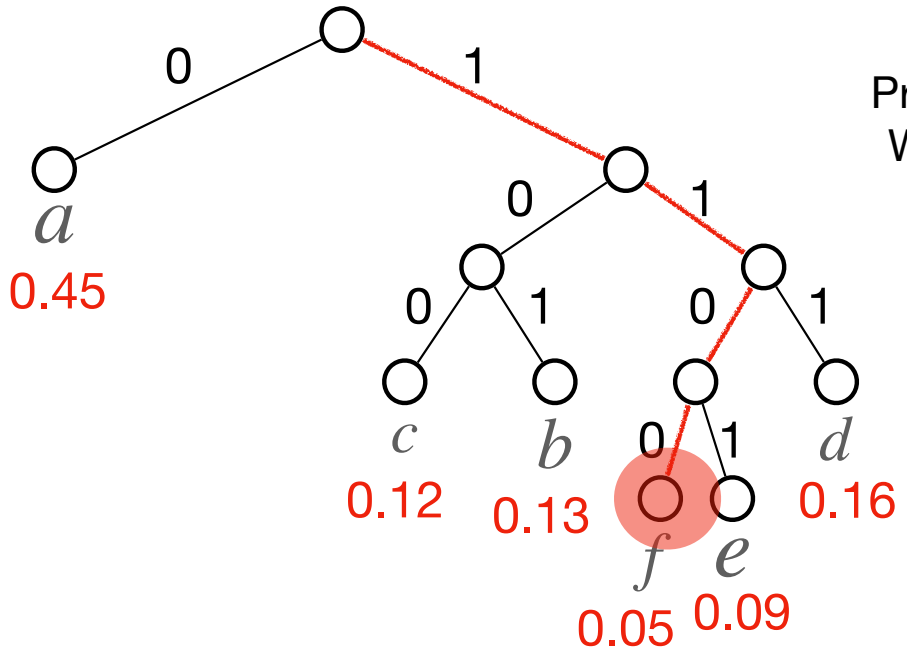
16.3 Huffman Code

Property 1 of optimal code:
The (or one) symbol of lowest probability
has the longest codeword.



Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

16.3 Huffman Code

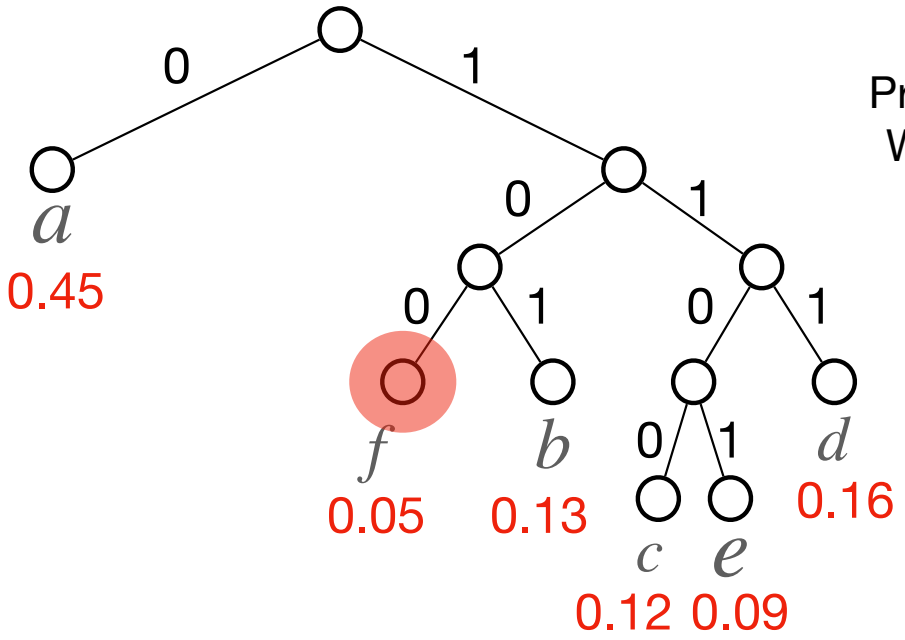


Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

Property 1 of optimal code:
The (or one) symbol of lowest probability
has the longest codeword.

Proof: If not, then we can switch its codeword
With another symbol, and get a better code.
That will be a contradiction.

16.3 Huffman Code

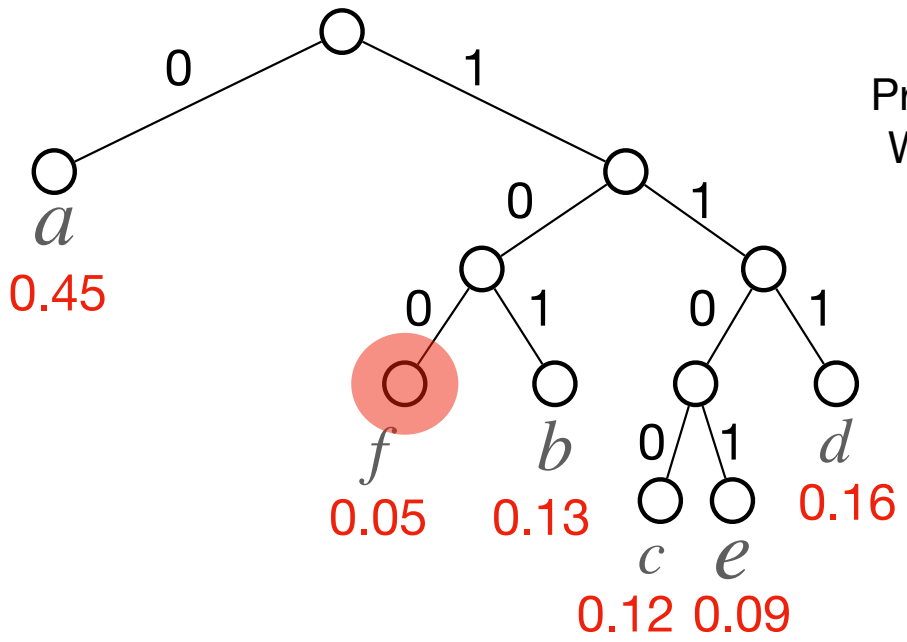


Property 1 of optimal code:
The (or one) symbol of lowest probability
has the longest codeword.

Proof: If not, then we can switch its codeword
With another symbol, and get a better code.
That will be a contradiction.

Example: Assume f does not have
the longest codeword.

16.3 Huffman Code



Property 1 of optimal code:
The (or one) symbol of lowest probability has the longest codeword.

Proof: If not, then we can switch its codeword With another symbol, and get a better code. That will be a contradiction.

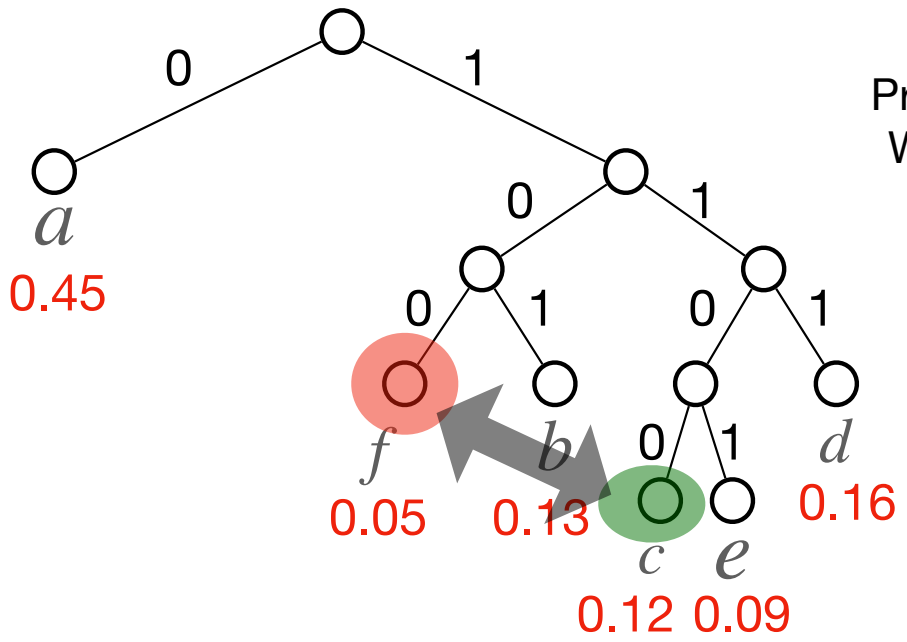
Example: Assume f does not have the longest codeword.

Average codeword length = $1 \times 0.45 + 3 \times 0.13 + 4 \times 0.12 + 3 \times 0.16 + 4 \times 0.09 + 3 \times 0.05$

16.3 Huffman Code

Property 1 of optimal code:
The (or one) symbol of lowest probability
has the longest codeword.

Proof: If not, then we can switch its codeword
With another symbol, and get a better code.
That will be a contradiction.



Example: Assume f does not have
the longest codeword.

Now switch a with c.

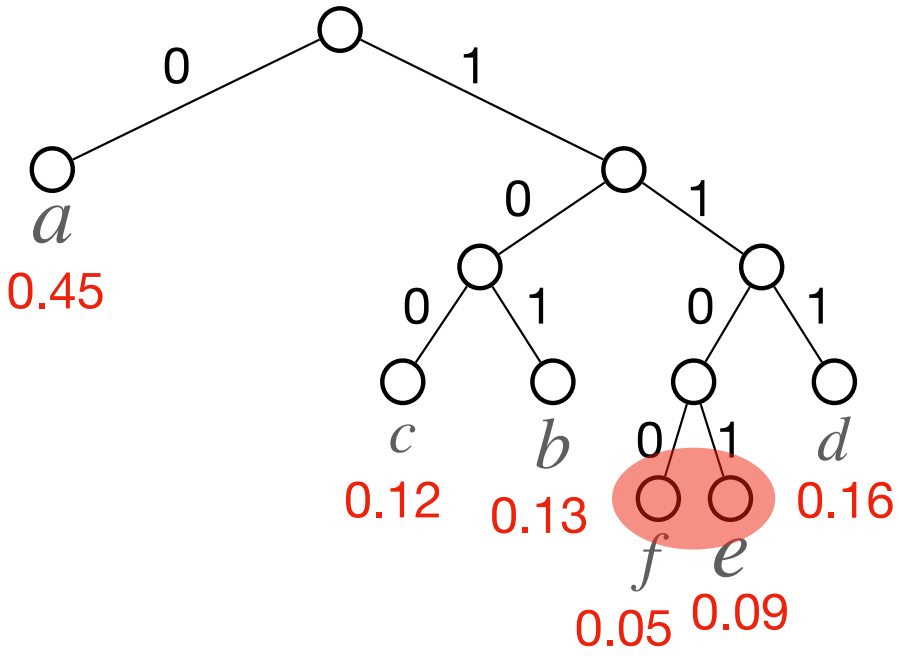
Average codeword length = $1 \times 0.45 + 3 \times 0.13 + 4 \times 0.12 + 3 \times 0.16 + 4 \times 0.09 + 3 \times 0.05$

After switch, average codeword length = $1 \times 0.45 + 3 \times 0.13 + 3 \times 0.12 + 3 \times 0.16 + 4 \times 0.09 + 4 \times 0.05$

$(4 \times 0.12 + 3 \times 0.05) - (3 \times 0.12 + 4 \times 0.05) = (4 - 3)(0.12 - 0.05) > 0$

Switch makes code better!

16.3 Huffman Code

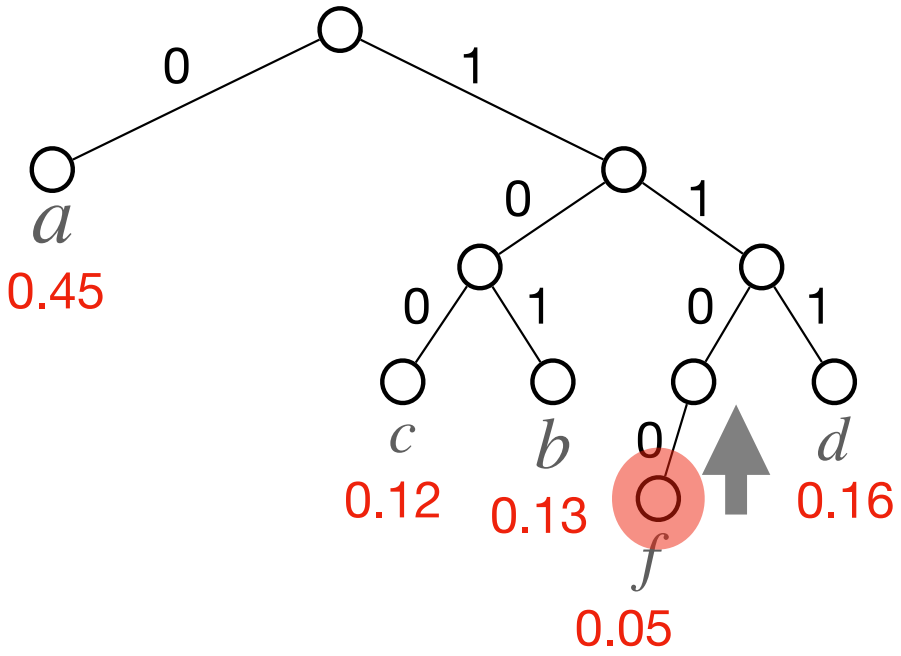


Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

Property 2 of optimal code:

The symbol of lowest probability and longest codeword has a sibling leaf node.

16.3 Huffman Code



Property 2 of optimal code:

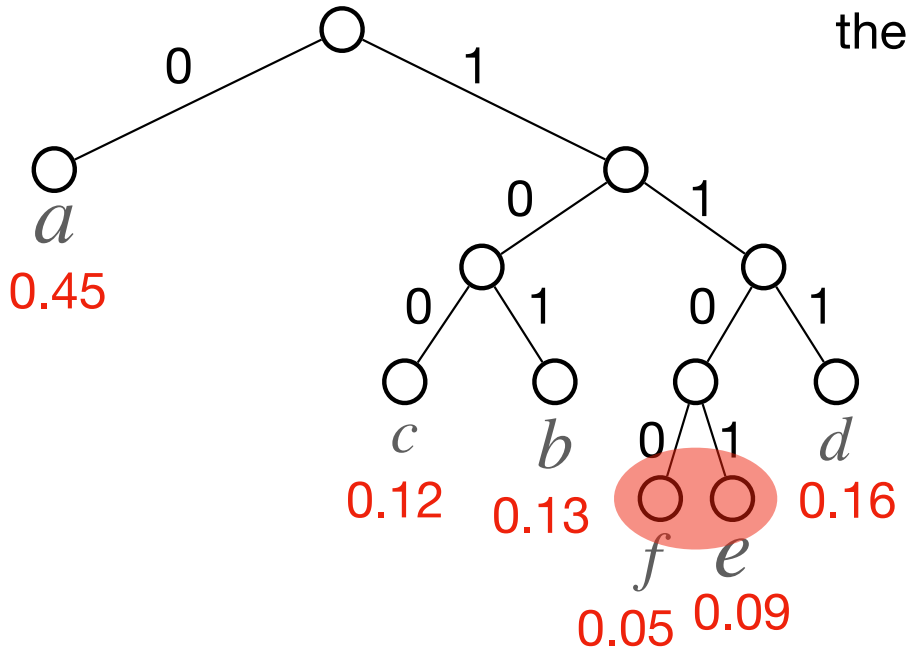
The symbol of lowest probability and longest codeword has a sibling leaf node.

Proof: If not, we can move the codeword up to make it shorter.

16.3 Huffman Code

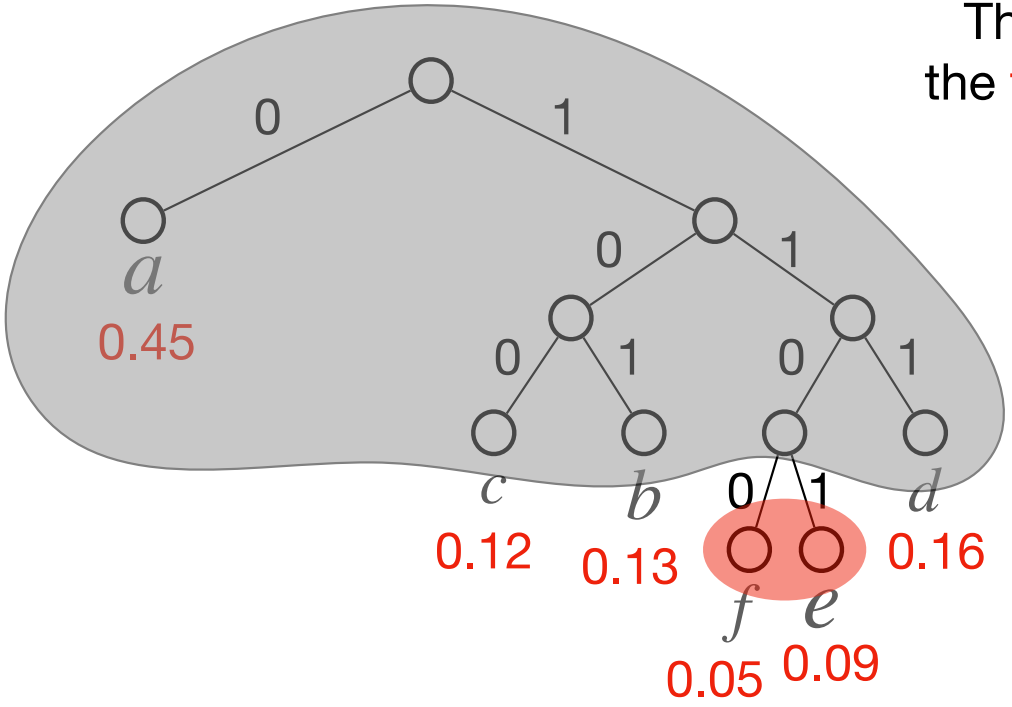
Property 3 of optimal code:

There exists an optimal code where the two symbols of lowest probabilities are siblings.



Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05
VLPC	0	101	100	111	1101	1100

16.3 Huffman Code



Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05

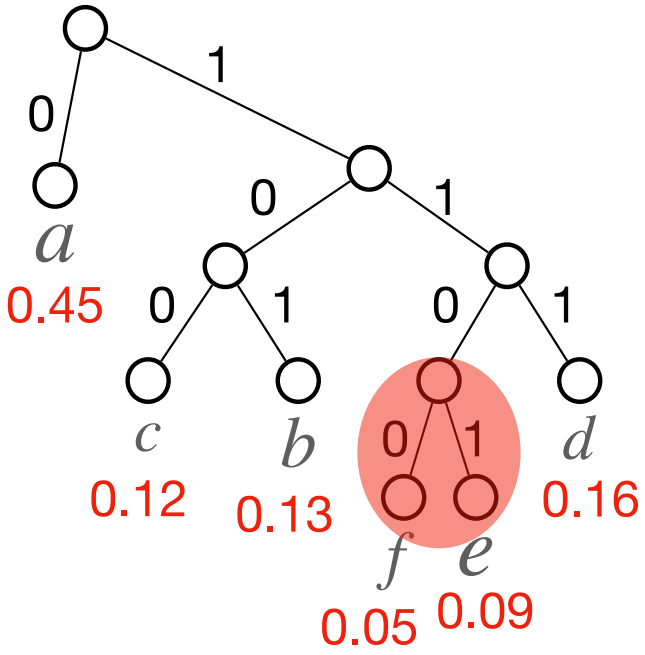
Property 3 of optimal code:

There exists an optimal code where the two symbols of lowest probabilities are siblings.

So we can build an optimal tree this way: first put the two symbols of lowest probabilities as siblings. Then figure out the rest of the tree.

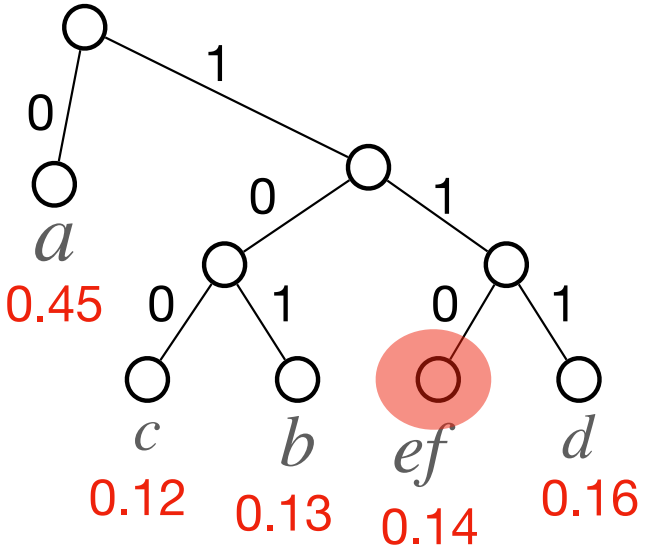
Idea: Once we put the two symbols are siblings,
see them as one symbol (node) and combine their probabilities.

Original Huffman Code



What is the relationship between the two Huffman codes?

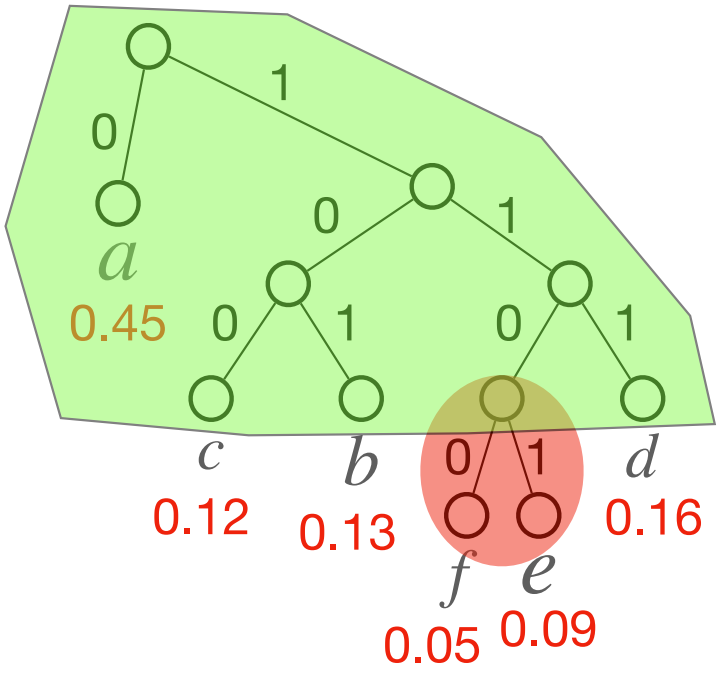
New Huffman Code



Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05

Symbol	a	b	c	d	ef
Probability	0.45	0.13	0.12	0.16	0.14

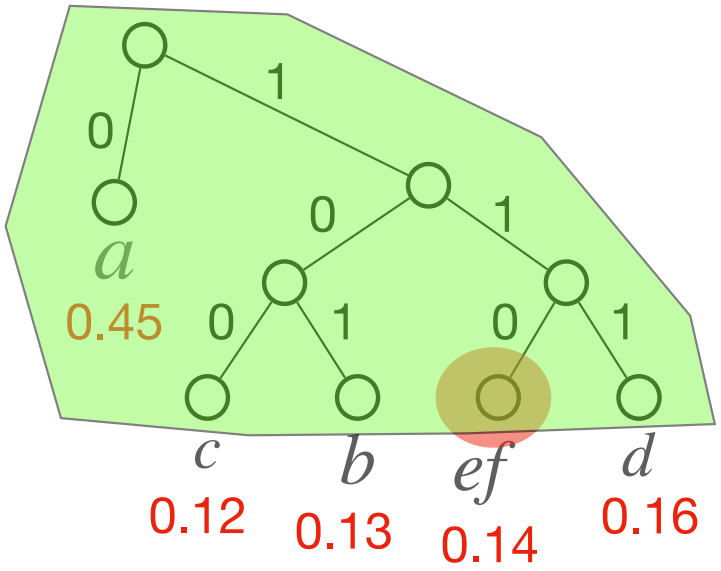
Original Code



What is
the relationship
between
the two
Huffman codes?

If we optimize the
new code,
we also optimize the
old code
(and vide versa).

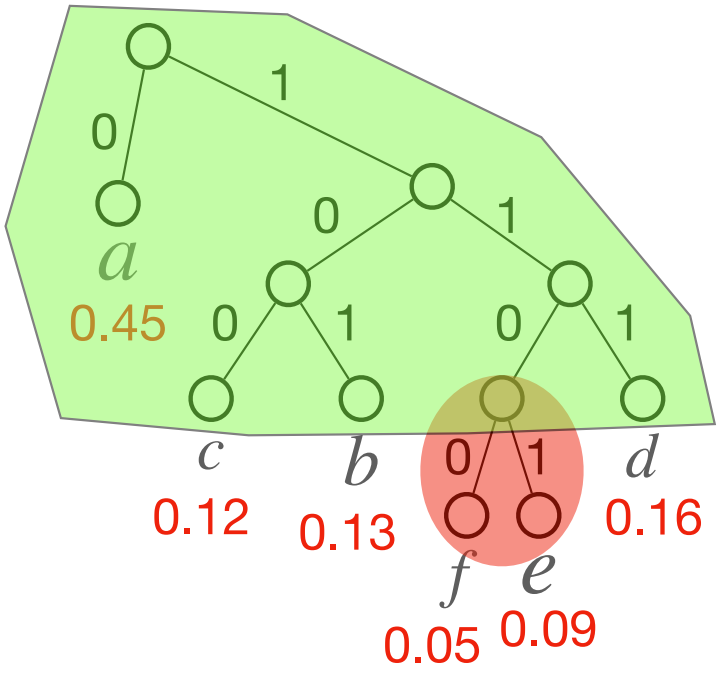
New Code



Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05

Symbol	a	b	c	d	ef
Probability	0.45	0.13	0.12	0.16	0.14

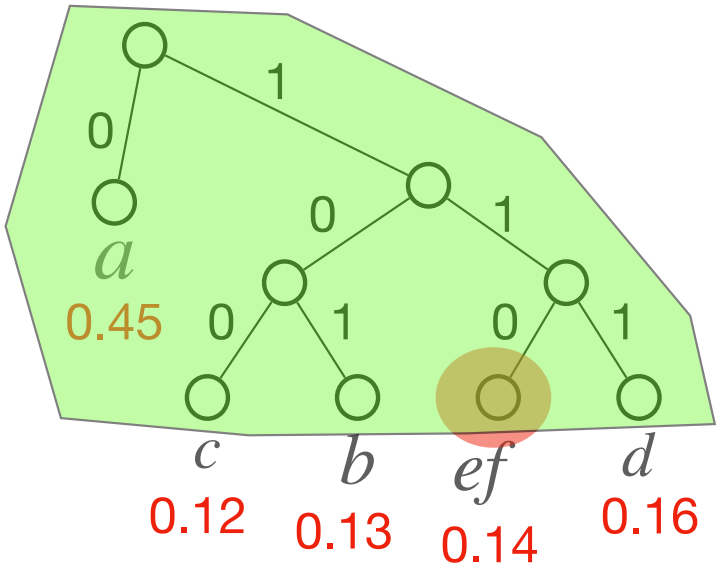
Original Code



What is the relationship between the two Huffman codes?

If we optimize the new code, we also optimize the old code (and vide versa).

New Code



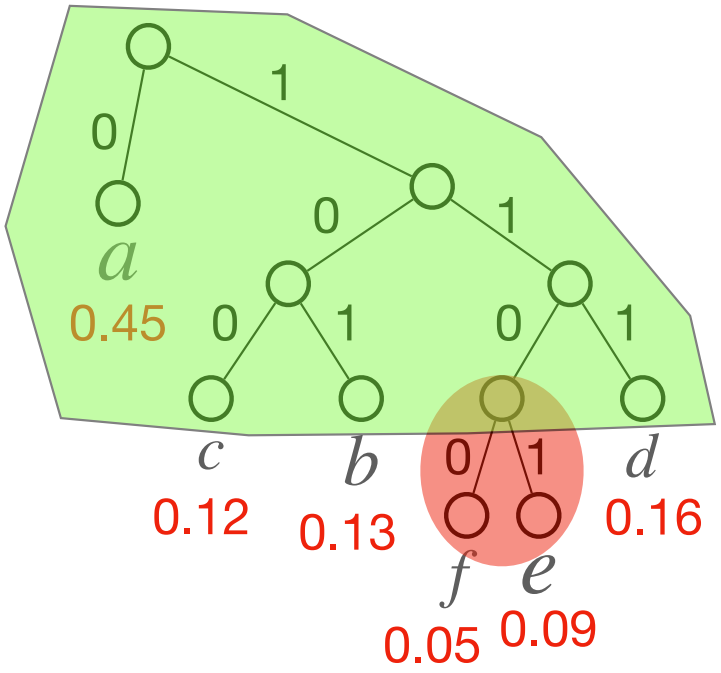
Average codeword length of original code = $1 \times 0.45 + 3 \times 0.13 + 3 \times 0.12 + 3 \times 0.16 + 4 \times 0.09 + 4 \times 0.05$

Average codeword length of new code = $1 \times 0.45 + 3 \times 0.13 + 3 \times 0.12 + 3 \times 0.16 + 3 \times (0.09 + 0.05)$

Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05

Symbol	a	b	c	d	ef
Probability	0.45	0.13	0.12	0.16	0.14

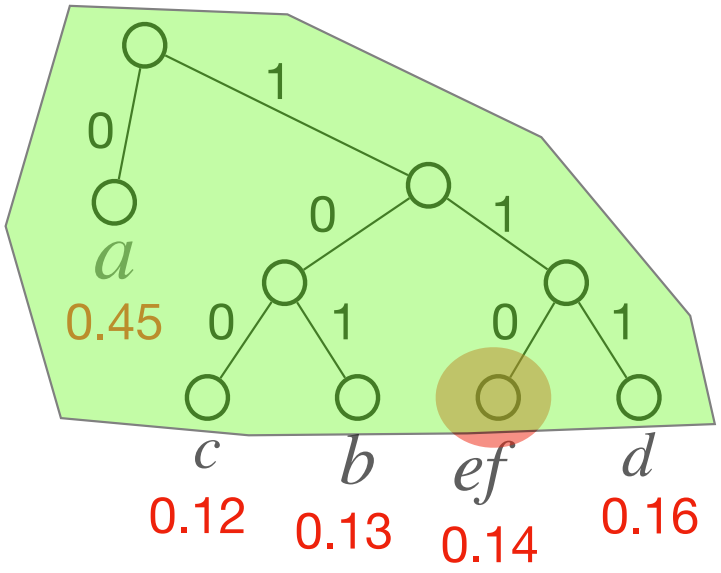
Original Code



What is the relationship between the two Huffman codes?

If we optimize the new code, we also optimize the old code (and vide versa).

New Code



They differ by $f_e + f_f$

Average codeword length of original code = $1 \times 0.45 + 3 \times 0.13 + 3 \times 0.12 + 3 \times 0.16 + 4 \times 0.09 + 4 \times 0.05$

Average codeword length of new code = $1 \times 0.45 + 3 \times 0.13 + 3 \times 0.12 + 3 \times 0.16 + 3 \times (0.09 + 0.05)$

Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05

Symbol	a	b	c	d	ef
Probability	0.45	0.13	0.12	0.16	0.14

Idea of Greedy Algorithm:

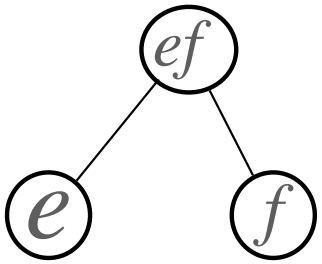
1) Make the two symbols of lowest probabilities siblings.
2) Combine them into one symbol, and repeat the above process.

Example:

Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05

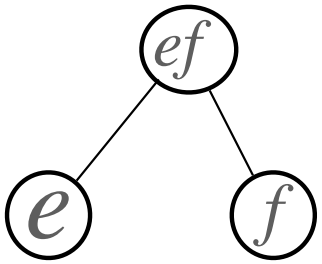
Example:

Symbol	a	b	c	d	e	f
Probability	0.45	0.13	0.12	0.16	0.09	0.05



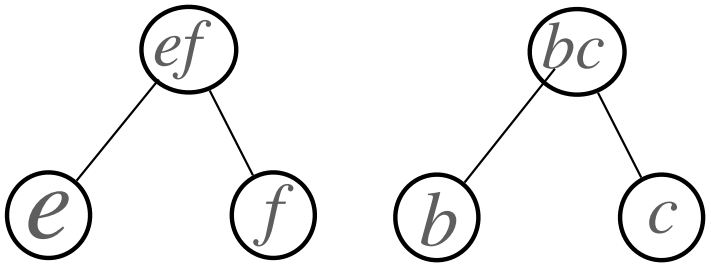
Example:

Symbol	a	b	c	d	ef
Probability	0.45	0.13	0.12	0.16	0.14



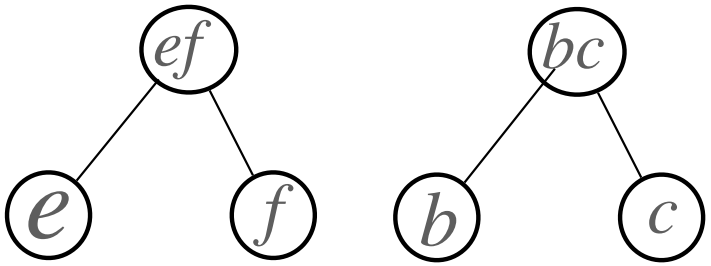
Example:

Symbol	a	b	c	d	ef
Probability	0.45	0.13	0.12	0.16	0.14



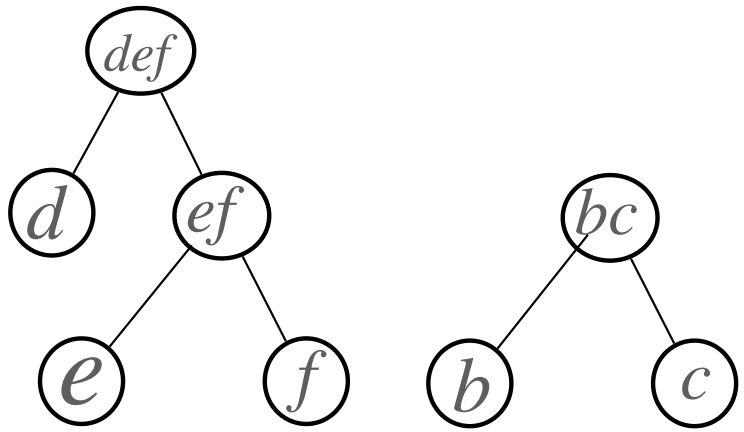
Example:

Symbol	a	bc	d	ef
Probability	0.45	0.25	0.16	0.14



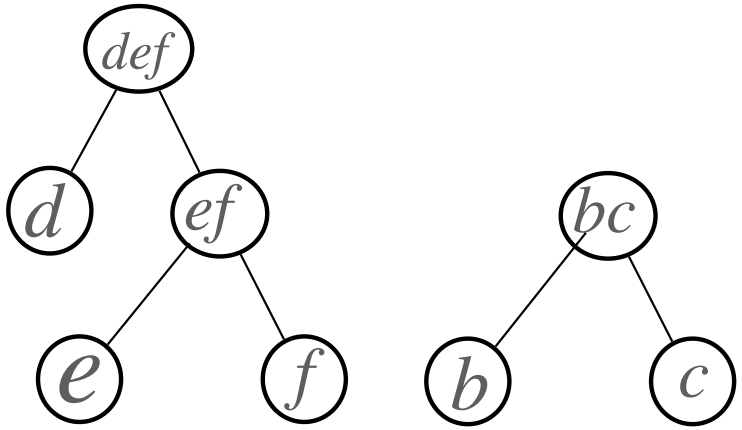
Example:

Symbol	*a*	*bc*	*d*	*ef*
Probability	0.45	0.25	0.16	0.14



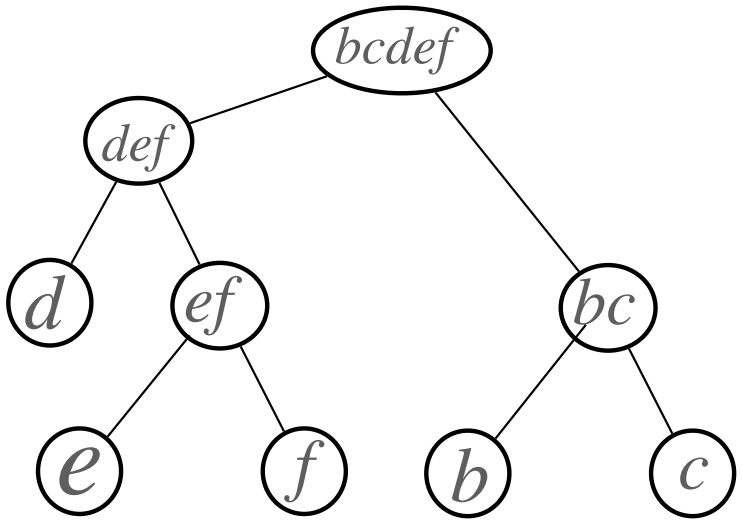
Example:

Symbol	*a*	*bc*	*def*
Probability	0.45	0.25	0.3



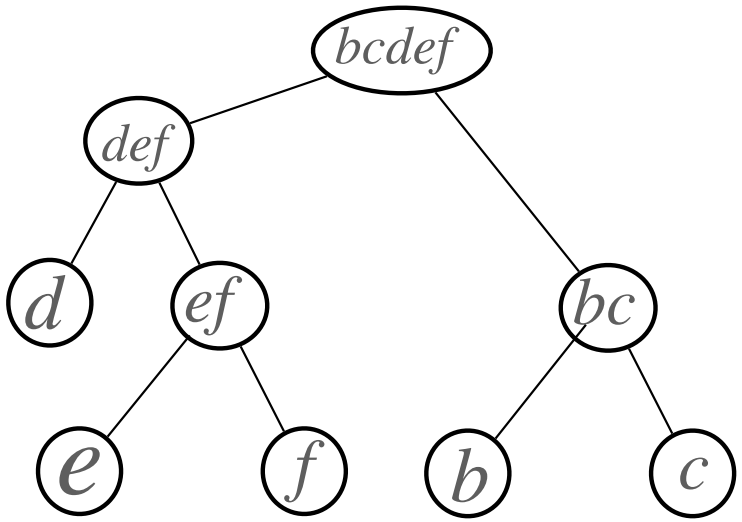
Example:

Symbol	*a*	*bc*	*def*
Probability	0.45	0.25	0.3



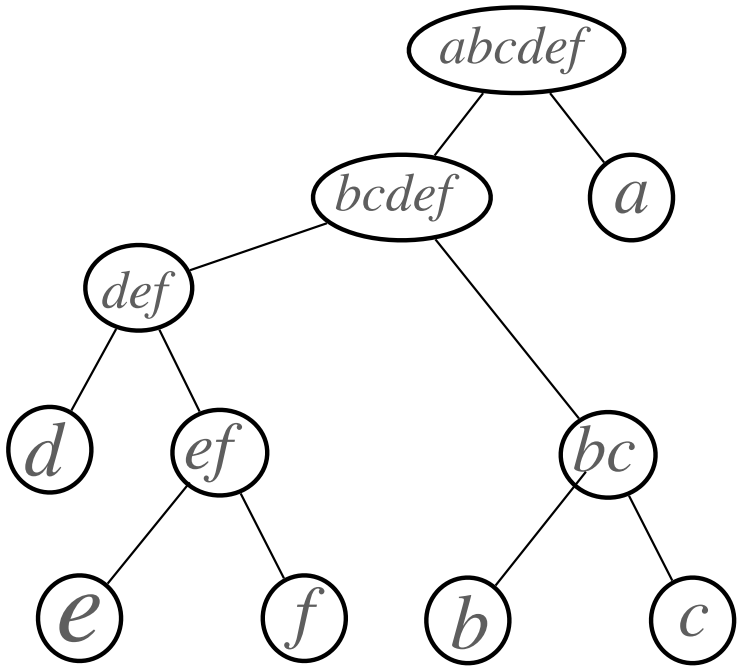
Example:

Symbol	a	$bcdef$
Probability	0.45	0.55



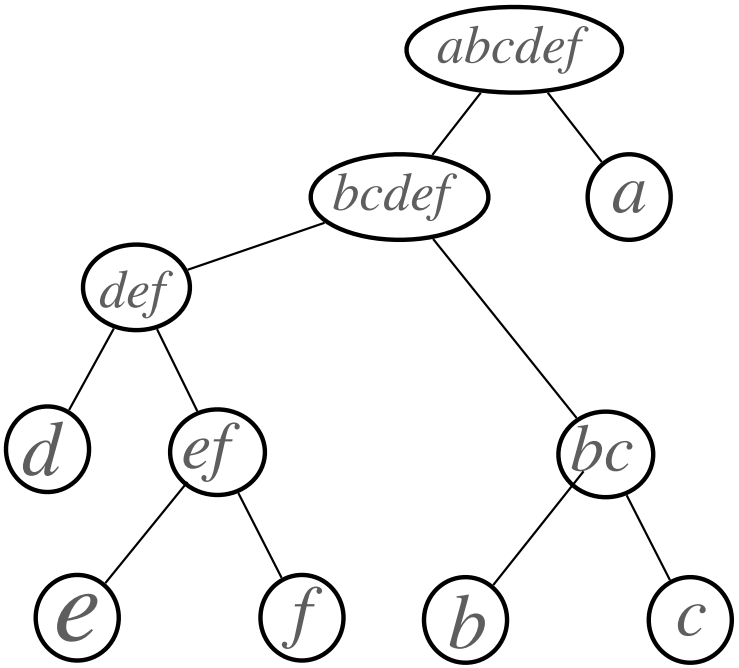
Example:

Symbol	*a*	*bcdef*
Probability	0.45	0.55



Example:

Symbol	*a*	*bcdef*
Probability	0.45	0.55



Equivalent