



# Online Estimation of Impedance Parameters for a Variable Impedance Controlled Robotic Manipulator

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**Abstract.** The aim of this work is to estimate the impedance parameters, namely the damping and stiffness, of a variable impedance dynamic system. The estimation is performed using a Constrained Extended Kalman Filter (CEKF). Comparing the various non-linear estimation techniques, Extended Kalman Filter shows a superiority with respect to speed of execution. This is a major requirement in case the estimation is used for a task involving online-tuning of the parameters of a variable impedance controlled robotic manipulator in contact with a variable impedance dynamic environment, for example, during human-robot physical interaction. In order to have a ground truth, the algorithm was experimentally tested on a system with known variable impedance, namely, a variable impedance controlled KUKA LWR. For the estimation procedure, the position of the end-effector was used as the measurement and the external force applied on it as a known input. Without giving explicit information on the dynamics of the variable impedance parameters of the controlled manipulator, the CEKF appreciably tracked the real parameters. The performance of the estimator declines in case the impedance variation is highly non-linear.

## 1 Introduction

In various dynamic systems, online estimation of the process states and parameters forms an essential part of monitoring the process conditions and model-based control. The Bayesian state estimation approach is widely used because it provides a systematic and general approach to handle the effect of various random uncertainties on the process states and measurements.

The Kalman Filter introduced by Kalman [1] is a widely used Bayesian estimation technique. This technique works best in-case the model of the system

under consideration is linear. In case the model is non-linear, which is generally the case, there are other approaches with almost the same computational complexity, but with better performance. One such approach is the Extended Kalman filter (EKF) [4]. As compared to other non-linear estimation techniques, EKF provides a better performance in terms of execution time, which is a key factor for online estimation.

The goal of this work is to estimate the impedance parameters of a variable impedance dynamic system, like a human arm [6]. The work in this paper can be useful for example in a manually guided surgery tasks where the impedance parameters of the arm of the human operator can be estimated and, based on this estimation, the impedance law of the manipulator can be varied so as to make the surgery easier for the operator.

Since it is not easy to estimate the real impedance parameters of the human arm during the execution of a task, the performance of the proposed algorithm was experimentally tested on a robotic arm. In detail, a KUKA LWR arm with known variable impedance imposed by admittance control and a known interaction force was used in the experiments, without giving explicit information on the dynamics of the variable impedance parameters of the controlled manipulator.

The remainder of the paper is organized as follows. Section 2 introduces the famous Extended Kalman Filter. Section 3 introduces an extended version of EKF, the Constrained Extended Kalman Filter (CEKF). Section 4 shows the results of the CEKF for the estimation of impedance parameters for a Variable Impedance control task. Finally, concluding remarks are drawn in Sect. 5.

## 2 Sub-optimal Solution for Online Estimation of Non-linear Systems: The Extended Kalman Filter

The general case of non-linear systems and non-linear measurement functions can be given as follows:

$$x(i+1) = f(x(i), u(i), i) + w(i) \quad (1)$$

$$y(i) = h(x(i), i) + v(i) \quad (2)$$

The vector  $f(., ., .)$  is a non-linear, time variant function of the state  $x(i)$  and the control vector  $u(i)$ ,  $w(i)$  and  $v(i)$  represent process and measurement noise respectively.

Assuming that the non-linearities of the system are smooth enough to allow linear or quadratic approximations, Kalman-like filters become within reach. These solutions are sub-optimal since there is no guarantee that the approximations are close. An obvious way to get the approximations is by application of a Taylor series expansion of the functions.

$$f(x + \varepsilon) = f(x) + F(x)\varepsilon + H.O.T. \quad (3)$$

$$h(x + \varepsilon) = h(x) + H(x)\varepsilon + H.O.T. \quad (4)$$

where, H.O.T. represent the higher order terms.

The update steps then become:

$$\hat{z}(i) = H(x(i))\bar{x}(i | i - 1) \quad (5)$$

$$S(i) = H(x(i))C(i | i - 1)H^T(x(i)) + C_v(i) \quad (6)$$

$$K(i) = C(i | i - 1)H^T(x(i))S^{-1}(i) \quad (7)$$

$$\bar{x}(i | i) = \bar{x}(i | i - 1) + K(i)(z(i) - \hat{z}(i)) \quad (8)$$

$$C(i | i) = C(i | i - 1) - K(i)S(i)K^T(i) \quad (9)$$

And the prediction step can be given as follows:

$$\bar{x}(i + 1 | i) = f(x(i), u(i), i) \quad (10)$$

$$C(i + 1 | i) = F(i)C(i | i)F^T(i) + C_w(i) \quad (11)$$

The interpretation is as follows:  $\hat{z}(i)$  is the predicted measurement. It is an unbiased estimate of actual measurement  $z(i)$  using all information from the past. The so-called innovation matrix  $S(i)$  represents the uncertainty of the predicted measurement. The uncertainty is due to two factors: the uncertainty of  $x(i)$  as expressed by  $C(i | i - 1)$ , and the uncertainty due to the measurement noise  $v(i)$  as expressed by  $C_v(i)$ . The matrix  $K(i)$  is the Kalman gain matrix. This matrix is large, when  $S(i)$  is small and  $C(i | i - 1)H^T(i)$  is large, that is, when the measurements are relatively accurate. When this is the case, the values in the error covariance matrix  $C(i | i)$  will be much smaller than  $C(i | i - 1)$ .

### 3 Constrained Extended Kalman Filter

Many a times, it is necessary to constrain the state estimation in the form of algebraic equality and/or inequality constraints. This can be necessary in case the estimated states are used in some feedback control law and unbounded states can lead to undesirable control.

The structure of the EKF does not include constraints on the states. This section presents a method to allow analytical solutions to the state constrained EKF for a class of linear constraints. The method was introduced in [2].

Consider the following linearized model of the plant:

$$x(i + 1) = F(i)x(i) + L(i)u(i) + w(i) \quad (12)$$

$$y(i) = H(i)x(i) + v(i) \quad (13)$$

Suppose at each time instant  $i$  the states are subject to following constraints:

$$D_i x(i) = d_i \quad (14)$$

The estimated state at time  $i$  by an unconstrained estimator is denoted by  $\hat{x}_i^u$ .  $\hat{x}_i^c$  be the constrained estimate taking into account Eq. 14. The associated covariances of estimation error are denoted  $P_i^u$  and  $P_i^c$ . We now apply the Principle of Projection [2] which is basically solving the constrained optimization problem:

$$\min_{\hat{x}_i^c} ((\hat{x}_i^c - \hat{x}_i^u)^T W_i^{-1} (\hat{x}_i^c - \hat{x}_i^u)) \quad s.t. \quad D_i \hat{x}_i^c = d_i \quad (15)$$

$W_i$  is a symmetric positive definite weighting matrix. The solution is obtained through the use of the Lagrange multiplier, and summarized by the following set of equations:

$$\hat{x}_i^c = \hat{x}_i^u + L_i(d_i - D_i\hat{x}_i^u) \quad (16)$$

$$P_i^c = (I_n - L_i D_i) P_i^u (I_n - L_i D_i)^T \quad (17)$$

$$L_i = W_i^{-1} D_i^T (D_i W_i^{-1} D_i^T)^{-1} \quad (18)$$

The constrained estimated state has the following properties:

- if  $W_i = P_i^u$  then it results in the minimum variance filter
- if  $W_i = I_n$  then it results in a constrained estimate that is closer to the true state than the unconstrained estimate.

Now, suppose that at each time step  $i$ , state is subject to the following linear soft inequality constraint:

$$D_i x(i) \leq d_i \quad (19)$$

A way of dealing with such a problem is using the active set method [2]. It consists in testing at each time step  $i$  the scalar inequalities of Eq. 19. For the  $k^{th}$  inequality, two scenarios can occur:

- The inequality is satisfied, and so do not have to be taken into account.
- The inequality is not satisfied. Then, an equality constraint is applied to the boundary:  $D_i x(i) = d_i$ , where for any matrix  $M_i$  at time  $i$ ,  $M_k, i$  denotes the  $k^{th}$  row of  $M_i$

Consequently, dealing with soft inequality constraints reduces to the application of the active equality constraints at each time step.

## 4 Estimation Results

Experiments to estimate the impedance parameters were performed on a Variable Impedance Controlled 7-DOF KUKA LWR manipulator which was controlled using an admittance control scheme [6].

The following 1-DOF impedance model was used:

$$\Lambda \ddot{\tilde{x}} + D_d(t) \dot{\tilde{x}} + K_d(t) \tilde{x} = F_{ext} \quad (20)$$

where,  $\tilde{x}(t) = x(t) - x_d(t)$ ,  $x \in \mathbb{R}^3$  is the Cartesian position vector of the end effector,  $x_d(t) \in \mathbb{R}^3$  is a desired configuration for the end-effector,  $K_d(t), D_d(t)$  and  $\Lambda$  are, respectively, the desired stiffness, damping and inertia matrices and  $F_{ext}$  is the external force acting on the end-effector of manipulator.  $x_{des}(t) = 0$  for the performed experiments and a sinusoidal force was applied on the end-effector for every experiment.

For the above model, the mass was considered known and was kept constant. Thus, we have four states to estimate: velocity ( $x_1$ ), position ( $x_2$ ), damping ( $x_3$ )

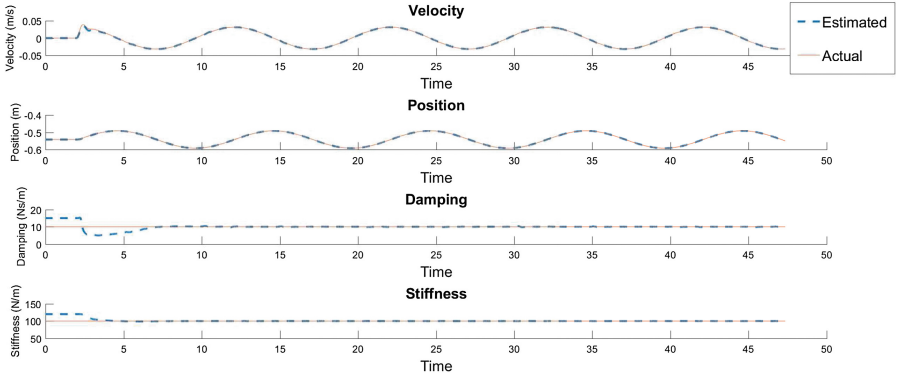
and stiffness ( $x_4$ ). The measurement is the end-effector position  $x_2$  and the input to the model is  $F_{ext}$ . In the discrete form, we have:

$$\begin{bmatrix} x_1^{t+1} \\ x_2^{t+1} \\ x_3^{t+1} \\ x_4^{t+1} \end{bmatrix} = \begin{bmatrix} \frac{dt(F_{ext} - D_d x_1 - K_d x_2)}{dt(x_1)} + x_1^t \\ x_2^t \\ x_3^t \\ x_4^t \end{bmatrix} \quad (21)$$

The dynamics of the damping and stiffness are unknown. Hence, they are considered constant.

#### 4.1 Estimation with Constant Impedance Parameters

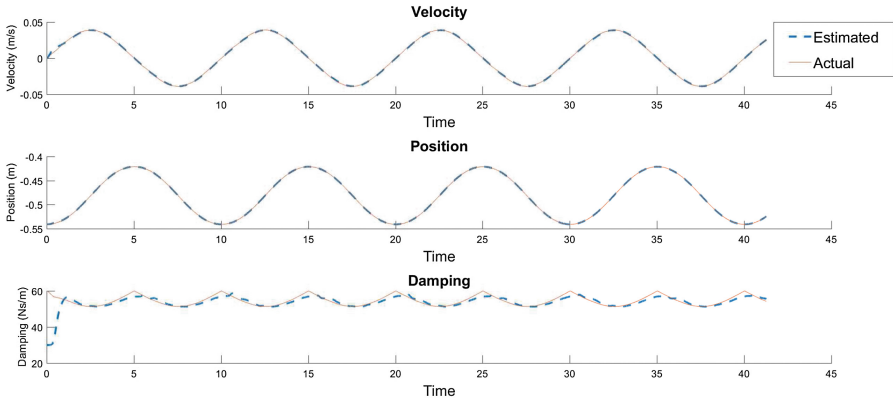
Figure 1 shows the results of estimation of constant impedance parameters ( $D_d = 10$  Ns/m and  $K_d = 100$  N/m). It can be seen that starting from initial guess different from that of the real parameters, the estimate converges to the real values.



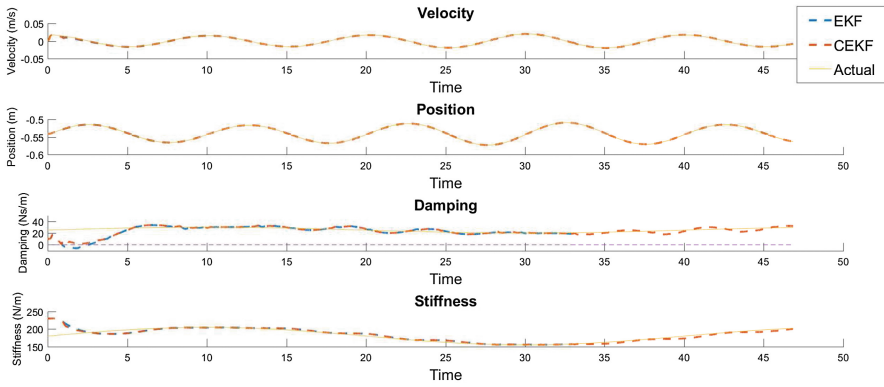
**Fig. 1.** Estimation with constant impedance parameters: mass-damping-stiffness model

#### 4.2 Estimation with Varying Impedance Parameters

Figure 2 shows the results of estimation of varying impedance parameter ( $D_d(t) = 60e^{(-4|x_1|)}$  Ns/m and no  $K_d$ ). It can be seen that starting from initial guess different from that of the real parameters, the estimate converges to the real values.



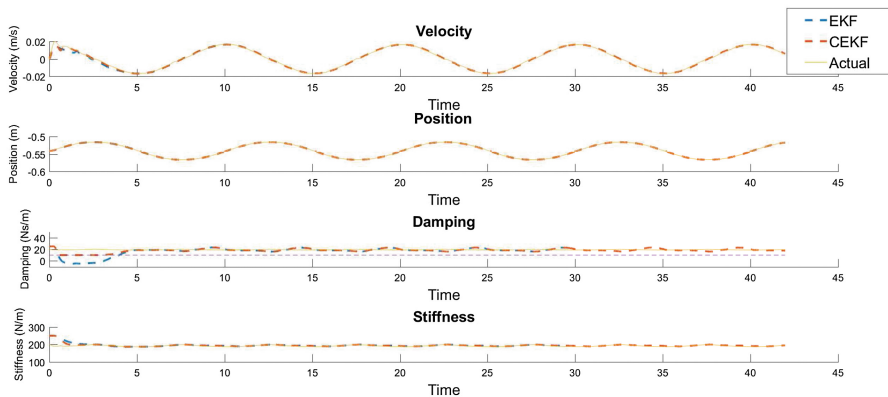
**Fig. 2.** Estimation with varying impedance parameters: mass-damping model



**Fig. 3.** Estimation with varying impedance parameters: mass-damping-stiffness model (Damping estimation is constrained to be above 0 Ns/m)

Figure 3 shows the results of estimation of varying impedance parameter ( $D_d(t) = 25 + 5\frac{\sin(2\pi t)}{40}$  Ns/m and  $K_d = 180 + 25\frac{\sin(2\pi t)}{40}$  N/m). It can be seen that starting from initial guess different from that of the real parameters, the estimate converges quite appreciably to the real values for stiffness but not damping.

Figure 4 shows the results of estimation of varying impedance parameter ( $D_d(t) = 20e^{(-4|x_1|)}$  Ns/m and  $K_d = 200e^{(-4|x_1|)}$  N/m). It can be seen that starting from initial guess different from that of the real parameters, the estimate converges quite appreciably to the real values for stiffness but the results for damping are poor.



**Fig. 4.** Estimation with varying impedance parameters: mass-damping-stiffness model (Damping estimation is constrained to be above 10 Ns/m)

## 5 Conclusion

In this work, the problem of estimating the impedance parameters of a variable impedance controlled robotic manipulator is addressed. The Extended Kalman Filter was used for this purpose because of its simplicity and speed of execution as compared to other non-linear estimation techniques.

The performance of the filter was tested for constant as well as variable impedance law. As can be expected, the estimation for the constant impedance law gives best results as they follow the state transition matrix used in the filter. In case of variable impedance law, a mass-damping and mass-damping-stiffness model was tested with inappreciable (sinusoidal) and appreciable (exponential) non-linear variation law.

In case of sinusoidal variation, the tracking is quite good but worsens in case of exponential variation of impedance law. This is because the EKF system model considers damping and stiffness as constants and goes ahead with the estimation based only on the error in the measurement data. Thus the poor performance is understandable.

It can be observed from the results that the parameter which mainly affects the dynamics, is the parameter which gets best estimated. Therefore, in case of mass-damping-stiffness model, the estimation of damping is poor as compared to that of stiffness. However, if the input to the EKF is sufficiently exciting, also the damping term becomes significant and estimation accuracy improves.

The proposed approach could be adopted for estimating the human arm impedance adopting measurements of the forces acting on the end effector and the relative kinematic variables in a manually guided robot application.

## References

1. Kalman, R.E.: A new approach to linear filtering and prediction problems. *J. Basic Eng.* **82**(1), 35–45 (1960)
2. Sircoulomb, V., et al.: State estimation under nonlinear state inequality constraints: a tracking application. In: 2008 16th Mediterranean Conference on Control and Automation. IEEE (2008)
3. Bangjun, L.: Classification, Parameter Estimation and State Estimation: An Engineering Approach Using MATLAB. Wiley, New York (2017)
4. Anderson, B.D., Moore, J.B.: Optimal Filtering, vol. 21, pp. 22–95. Englewood Cliffs, New Jersey (1979)
5. Hogan, N.: Impedance control: an approach to manipulation. In: 1984 American Control Conference. IEEE (1984)
6. Fanny, F., Villani, L., Siciliano, B.: Variable impedance control of redundant manipulators for intuitive human-robot physical interaction. *IEEE Trans. Robot.* **31**(4), 850–863 (2015)
7. Khatib, O.: A unified approach for motion and force control of robot manipulators: the operational space formulation. *IEEE J. Robot. Autom.* **3**(1), 43–53 (1987)
8. Christian, O., Mukherjee, R., Nakamura, Y.: Unified impedance and admittance control. In: 2010 IEEE International Conference on Robotics and Automation (ICRA). IEEE (2010)