

Extending the Radial Acceleration Relation using Weak Gravitational Lensing with the Kilo-Degree Survey

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ABSTRACT

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1 INTRODUCTION

It has been known for several decades that the outer regions of galaxies rotate faster than would be expected from Newtonian dynamics, based on their luminous (baryonic) mass. This was first discovered by Rubin (1983) through measuring galactic rotation curves of optical disks, and by Bosma (1981) through measuring hydrogen profiles at radii beyond the disk. The excess gravity implied by these measurements have been generally attributed to an unknown and invisible substance named Dark Matter (DM), a term coined earlier by Zwicky (1937) when he discovered the missing mass problem through the dynamics of galaxies in clusters.

Following more recent observations using Weak gravitational Lensing (WL, Hoekstra et al. 2004; von der Linden et al. 2014; Mandelbaum 2015), Baryon Acoustic Oscillations (BAO's, Eisenstein et al. 2005; Blake et al. 2011) and the Cosmic Microwave background (CMB, Spergel et al. 2003; Planck XIII 2016), cold dark matter¹ (CDM) became a key ingredient of the current standard model of cosmology: the Λ CDM model. In this paradigm, CDM accounts for $\Omega_{\text{CDM}} = 0.266$ of the critical density in the Universe, while the cosmological constant Λ necessary to explain the accelerated expansion of the Universe (which is usually associated with dark energy; DE) accounts for $\Omega_{\Lambda} = 0.685$.

Although the Λ CDM model successfully describes the behavior of DM on a wide range of scales, no conclusive evidence for the existence of DM particles has been found so far (despite years of enormous effort; for an overview, see Bertone et al. 2005; Bertone & Tait 2018). This still leaves some room for alternative theories of gravity, such as Modified Newtonian Dynamics (MOND, Milgrom 1983) and the more recent theory of Emergent Gravity (EG, Verlinde 2016). In these theories DM does not exist, and all gravity is due to the baryonic matter (or, in the case of EG, the interaction between baryons and the entropy associated with DE). Hence, one of the main properties of these theories is that the mass discrepancy in galaxies correlates strongly with their baryonic mass distribution.

Such a correlation has indeed been widely observed. First astronomers discovered the Tully-Fisher relation (TFR, Tully & Fisher 1977) between the luminosity of a spiral galaxy and its asymptotic rotation velocity (Pierce & Tully 1988; Bernstein et al. 1994). Since this corresponds to a relation between the baryonic and the total galaxy mass, this has later been named the ‘baryonic’ TFR (BTFR, McGaugh et al. 2000; McGaugh 2012). As the radial resolution of observations increased, astronomers found a strong correlation between the observed rotation velocity $v_{\text{obs}}(r)$ as a function of galaxy radius r , and the enclosed luminous mass $M_b(< r)$ (Sanders 1986, 1996; McGaugh 2004; Sanders & Noordermeer 2007; Wu & Kroupa 2015). Since $M_b(< r)$ corresponds to the *expected* gravitational acceleration $g_b(r)$ from baryonic matter, and the observed gravitational acceleration can be calculated through $g_{\text{obs}}(r) = v_{\text{obs}}^2(r)/r$, this relation has been named the Radial Acceleration Relation (RAR)².

Particularly, the latest results from McGaugh et al. (2016) (hereafter M16) have measured the RAR relation with unprecedented accuracy, using the Spitzer Photometry and Accurate Rotation Curves (SPARC, Lelli et al. 2016) data of 153 late-type galaxies. Their results showed a tight correlation between g_{obs} and g_{bar} , which they could fit using a simple double power law (Eq. 4 in M16) depending only on g_{bar} and one free parameter: the acceleration scale g_{\dagger} where Newtonian gravity appears to break down. This sparked the interest of scientists working on alternative theories of gravity, but also of those in favor of a statistical explanation of the RAR within the Λ CDM framework (Keller & Wadsley 2017; Desmond 2017; Ludlow et al. 2017).

The latter possibility was quantified by Navarro et al. (2017) (hereafter N17) who used a range of simplifying assumptions based on galaxy observations and DM simulations, in order to create an analytical galaxy+halo model. The goal of their model was to reconstruct the RAR in galaxies, in particular the value of a_0 : the acceleration scale where the relation transitions from the baryon-dominated to the DM-dominated regime (which is equivalent to g_{\dagger}), and a_{min} : the minimum acceleration probed by galaxy disks. Based on their results, they claimed that the RAR can be explained within the Λ CDM framework, at the accelerations probed by galaxy rotation curves (i.e. $g_{\text{obs}} > a_{\text{min}}$). However, since their model relies on the fact that luminous kinematic tracers in galaxies only probe a limited radial range, N17 predicted that extending observations to radii beyond the disk (which correspond to lower gravitational accelerations) would lead to systematic deviations from the simple relation posed by M16.

The goal of this work is to extend observations of the RAR to lower accelerations, which are not measurable using galaxy rotation curves. To this end we use gravitational lensing: the perturbation of light inside a gravitational potential as described by General Relativity (GR). In particular, we use the method of Galaxy-Galaxy Lensing (GGL): the statistical measurement of the coherent image distortion (shear) of a field of background galaxies (sources) by the gravitational potential of a sample of foreground galaxies (lenses) (for examples, see e.g. Fischer et al. 2000; Hoekstra et al. 2004; Mandelbaum et al. 2006; van Uitert et al. 2016). Using GGL we can measure the average (apparent) density distribution of galaxies up to a radius of 3 Mpc, roughly a 100 times larger than the radius of the luminous disk (~ 30 kpc), corresponding to a value of g_{bar} that is roughly 3 orders of magnitude lower than those measurable with galaxy rotation curves.

First, we measure the baryonic and total density profiles of our galaxies through their luminosities and GGL profiles. These measurements will be performed using photometric data from Sloan Digital Sky Survey (Abazajian et al. 2009, SDSS,) and the VISTA Kilo-Degree Infrared Galaxy survey (Edge et al. 2013, VIKING), and weak lensing data from the Kilo-Degree Survey (KiDS, de Jong et al. 2017). We then translate these measurements into the baryonic and

¹ DM particles that moved at non-relativistic speeds at the time of recombination, as favoured by measurements of the CBM (Planck XVI 2014) and the Lyman- α forest (Viel et al. 2013).

² Another well-established name for this same relation is the

Mass-Discrepancy Acceleration Relation (MDAR), which refers to the correspondence between the observed baryonic/total mass and the inferred mass discrepancy commonly attributed to DM. Throughout this work we use the term RAR for brevity.

observed radial accelerations, g_{bar} and g_{obs} . Finally, we compare the resulting RAR to predictions from different modified gravity theories (MOND and EG) and Λ CDM.

The Λ CDM predictions will not only be provided by the N17 analytical model, but also by two mock galaxy catalogues based on two different DM simulations. One is the Marenostrum Institut de Ciències de l'Espai (MICE) Galaxy and Halo Light-cone catalog (Carretero et al. 2015; Hoffmann et al. 2015), which is based on the MICE Grand Challenge lightcone simulation (MICE-GC, Fosalba et al. 2015a,b; Crocce et al. 2015). The other mock galaxy catalog is based on a suite of large-volume cosmological hydrodynamical simulations, which is called the Baryons and HALoes of MAssive Systems (BAHAMAS) project (McCarthy et al. 2017). Our final goal is to distinguish which of the aforementioned predictions best describe the RAR at lower accelerations.

Having over ... foreground galaxies at our disposal, we are able to select specific lens samples designed to optimally test these predictions. Particularly, we note that most models (MOND, EG, and the N17 analytical DM model) focus on the description of individual, isolated galaxies. In order to test them, therefore, we select a sample of galaxies whose lensing profiles are not affected by their neighbors within the radius of our measurement. In contrast, the predictions from the MICE and BAHAMAS simulations can be tested for both isolated and non-isolated galaxy samples.

Furthermore, we note that all models give a specific prediction regarding the dependence of the RAR on baryonic galaxy mass. According to the MOND and EG theories, the relation between g_{bar} and g_{obs} should remain intact in the regime beyond the disk, independent of the disk mass. Within the Λ CDM paradigm, however, all predictions (analytical and simulated) are based on a ‘stellar-to-halo-mass relation’ which is not constant as a function of baryonic galaxy mass. By splitting our foreground galaxies into bins of increasing stellar mass, we are able to better distinguish the predictions of these different models.

Our paper is structured as follows: In Sect. 2 we introduce our main datasets: both the KiDS and GAMA galaxy surveys which are used to perform the GGL measurements, and the MICE and BAHAMAS simulations & mock galaxy catalogues to which we compare our results. Sect. 3 describes our analysis of these (mock) datasets as we select our isolated foreground galaxy sample, perform the GGL measurements, and translate the results into the RAR. Sect. 4 contains a description of the theoretical predictions to which we compare our observations: MOND, EG and the N17 analytical DM model. In Sect. 5 we present the resulting RAR measurements and model comparison. Sect. 6 contains the discussion and conclusion.

Throughout this work we adopt the WMAP 9-year (Hinshaw et al. 2013) cosmological parameters : $\Omega_{\text{m}} = 0.2793$, $\sigma_8 = 0.821$, $\Omega_{\Lambda} = 0.7207$, and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which were used as the basis of the BAHAMAS simulation. The cosmological parameters used in creating the MICE-GC simulations are: $\Omega_{\text{m}} = 0.25$, $\sigma_8 = 0.8$, $\Omega_{\Lambda} = 0.75$, and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Throughout the paper we use the reduced Hubble constant $h_{70} \equiv H_0 / (70 \text{ km s}^{-1} \text{ Mpc}^{-1})$.

2 DATA

2.1 KiDS source galaxies

Write the beginning. Need to know:

- What changes as we go to KiDS-1000 (K1000 paper?).

2.2 GAMA foreground galaxies

Although the final RAR measurements will be performed using exclusively the KiDS-1000 data, the set of foreground galaxies observed by the spectroscopic GAMA survey (Driver et al. 2011) function both as a model and validation sample for the KiDS foreground galaxies. The survey was performed by the Anglo-Australian Telescope with the AAOmega spectrograph, and targeted more than 180 000 galaxies that were selected from SDSS. For this study we use the GAMA II data release (Liske et al. 2015) consisting of three equatorial regions (G09, G12 and G15). These regions span a total area of $\sim 180 \text{ deg}^2$ on the sky, completely overlapping with KiDS.

GAMA has a redshift range of $0 < z < 0.5$, with a redshift completeness of 98.5% down to Petrosian r-band magnitude $m_r = 19.8 \text{ mag}$. We limit our GAMA foreground sample to galaxies with the recommended redshift quality: $n_Q \geq 3$. This smaller sample of spectroscopic redshifts are used to train the photometric machine-learning (ML) redshifts of our larger sample of KiDS foreground galaxies (see Sect. 2.3). Despite being smaller, GAMA’s accurate redshifts are highly advantageous when measuring the ESD profiles of galaxies (see Sect. 3.2). Also, in combination with its high redshift completeness, GAMA allows for a better application of the isolation criterion. We therefore check that the results from the KiDS-only measurements are consistent with those from KiDS-GAMA at all times.

To measure RAR with KiDS-GAMA, we need individual stellar masses M_* for each GAMA galaxy. These are measured from their *ugrizZYJHK* spectral energy distributions³ measured by SDSS and the VISTA Kilo-Degree Infrared Galaxy survey (VIKING, Edge et al. 2013), by fitting them with Bruzual & Charlot (2003) stellar population synthesis models. Following the procedure described by Taylor et al. (2011), we account for flux falling outside the automatically selected aperture using the ‘flux-scale’ correction.

2.3 KiDS foreground selection

Still need to know:

- Maciek’s GL-KiDS selection criteria for K1000.
- Angus’ stellar mass method for K1000.

2.4 MICE mock galaxies

In order to compare our observations to predictions from Λ CDM, we adopt two different DM simulations. One of these is the MICE-GC *N*-body simulation, which contains $\sim 7 \times 10^{10}$ DM particles in a $(3072 h_{70}^{-1} \text{ Mpc})^3$ comoving volume (Fosalba et al. 2015b). From this simulation the MICE

³ The spectral energy distributions are constrained to the rest frame wavelength range 3000 – 110000 Å.

collaboration constructed a $\sim 5000 \text{ deg}^2$ lightcone with a maximum redshift of $z = 1.4$. The DM halos in this lightcone are identified using a Friend-of-Friend (FOF) algorithm on the particles. Halos are considered ‘resolved’ down to 20 particles, corresponding to a halos with a mass of $6 \times 10^{11} h_{70}^{-2} M_{\odot}$. The DM halos were populated with galaxies using a hybrid Halo Occupation Distribution (HOD) and Halo Abundance Matching (HAM) prescription (Carretero et al. 2015; Crocce et al. 2015).

Every galaxy has sky coordinates, redshifts, comoving distances, apparent magnitudes and absolute magnitudes assigned to them. We use the SDSS apparent r -band magnitudes m_r , as these most closely match those from KiDS (see Brouwer et al. 2018). We can therefore limit the MICE galaxies to the same apparent magnitude as the GL-KiDS sample: $m_r < 20.2 \text{ mag}$, in order to create a GL-MICE foreground galaxy sample. We also use the same redshift limit: $z < 0.5$. The absolute magnitudes of the mock galaxies go down to $M_r < -18.9$ (*should we apply a cut here?*). In the MICECATv2.0 catalogue⁴ which we use in this work, each galaxy is also assigned a stellar mass M_* needed in order to compute the RAR (see Sect. 3.3). These stellar masses are determined from the galaxy luminosities L using Bell & de Jong (2001) M/L ratios.

In addition, each galaxy has a pair of lensing shear values associated with it (γ_1 and γ_2 , with respect to the Cartesian coordinate system). These shear values were calculated from healpix weak lensing maps that were constructed using the ‘onion shell method’ (Fosalba et al. 2008, 2015a). The lensing map of MICECATv2.0 has an improved resolution of 0.43 arcmin , which is almost twice smaller than that of MICECATv1.0. The MICE shears allow us approximate the lensing analysis we perform on our KiDS data (as described in Section 3.2) using the MICE simulation. To create a sample of MICE background galaxies for the lensing analysis, we apply limits on the MICE mock galaxies’ redshifts and apparent magnitudes which are analogous to those applied to the KiDS source sample: $0.1 < z < 0.9$, $m_r > 20$ (see Hildebrandt et al. 2017 and Sect. 2.1; note that uncertainties in the KiDS z_B are not accounted for in this selection). We also apply an absolute magnitude cut of $M_r > 19.3 \text{ mag}$, in order to resemble the KiDS source redshift distribution more closely.

The MICE-GC mock catalogue also features very accurate clustering. At lower redshifts ($z < 0.25$) the clustering of the mock galaxies as a function of luminosity is constructed to reproduce the SDSS clustering observations Zehavi et al. (2011), while at higher redshifts ($0.45 < z < 1.1$) it was validated against Cosmic Evolution Survey (COSMOS, Ilbert et al. 2009). This makes MICE especially suitable to reproduce the RAR at larger scales where neighboring galaxies start to affect the lensing signal, and to test criteria on galaxy the isolation (see Section 3.1).

Because the MICE mock catalogue consists of one large realization, it is not possible to estimate the covariance matrix on the measured lensing signal. However, we can split this realization into multiple smaller areas in order to es-

timate the ‘sample variance’: the difference between astrophysical measurements from different parts of the sky.

2.5 Bahamas mock galaxies

Written by Kyle?

3 DATA ANALYSIS

3.1 Isolated galaxy selection

Write the beginning. Still need to know:

- how to test the isolation criterion.

3.2 Lensing measurement

Write the beginning. Still need to know:

- How (if?) the GGL-pipeline changes with K1000.

We recognize that, by using the unadulterated GGL equations to measure the (apparent) density distribution around foreground galaxies, we necessarily assume that the laws of GR hold with respect to the deflection of light by a gravitational potential.

3.3 Conversion to radial acceleration

Still need to know: whether we will use the SIS assumption or linear interpolation.

- Test both methods using the Bahamas simulation.

4 THEORETICAL PREDICTIONS

4.1 Modified Newtonian Dynamics

Write everything.

4.2 Emergent Gravity

Similar to MOND, Verlinde (2016) (V16 hereafter) postulates that the ‘missing mass problem’ is not caused by an undiscovered fundamental particle, but that instead our current gravitational theory should be revised. Building on the ideas of Jacobson (1995, 2016); Padmanabhan (2010); Faulkner et al. (2014) and his previous work (Verlinde 2011), V16 abandons the notion of gravity as a fundamental force. Instead, it emerges from an underlying microscopic description of space-time, in which the notion of gravity has no a-priori meaning. A distinguishing feature of V16 is that, instead constructing EG in a static (‘anti-de Sitter’) Universe, it attempts to describe our expanding (‘de Sitter’) Universe. This results in a new volume law for gravitational entropy caused by DE, in addition to the area law normally used to retrieve Einsteinian gravity.

According to V16, energy that is concentrated in the form of a baryonic mass distribution causes an elastic response in the entropy of the surrounding DE. This results in an additional gravitational component at scales set by the ‘Hubble acceleration scale’ $a_0 = cH_0$. Here c is the speed of

⁴ The MICECATv2.0 catalogue is available through CosmoHub (<https://cosmohub.pic.es>).

light and H_0 is the current Hubble constant, which measures the Universe's expansion velocity. Because this component is predicted to explain the effects usually attributed to DM, it is often expressed as an *apparent* dark matter (ADM) distribution:

$$M_D^2(r) = \frac{cH_0 r^2}{6G} \frac{d(M_b(r)r)}{dr}. \quad (1)$$

In this form, the ADM distribution is completely defined by the baryonic mass distribution $M_b(r)$ (as a function of the spherical radius r) and a set of known physical constants.

Note that this equation is only a macroscopic approximation of the underlying microscopic phenomena described above, and is only valid for static, spherically symmetric and isolated baryonic mass distributions. For this reason, we select only the most isolated galaxies from our sample (see Sect. 3.1), such that our WL measurements are not influenced by neighboring galaxies. In addition, cosmological evolution of the H_0 parameter is not yet implemented in the theory, restricting its validity to galaxies with relatively low redshifts. However, we calculate that at our mean lens redshift ($\langle z \rangle \sim 0.2$) using an evolving $H(z)$ would result in only a 5% difference in our ESD measurement, based on the background cosmology used in this work.

Since we measure the ESD profiles of galaxies at projected radial distances $R > 30 h_{70}^{-1} \text{Mpc}$, we can assume that their baryonic component is enclosed within the minimal measurement radius (see Brouwer et al. 2017). This is equivalent to describing the galaxy as a point mass M_b , which allows us to simplify Eq. 1 to:

$$M_D(r) = \sqrt{\frac{cH_0 M_b}{6G}} r. \quad (2)$$

Now the total enclosed mass, $M_{EG}(r) = M_b + M_D(r)$, can be used to calculate the predicted gravitational acceleration $g_{EG}(r)$ as follows:

$$g_{EG}(r) = \frac{GM_{EG}(r)}{r^2} = \frac{GM_b}{r^2} + \sqrt{\frac{cH_0}{6}} \frac{\sqrt{GM_b}}{r}. \quad (3)$$

In terms of the expected baryonic acceleration $g_{bar}(r) = GM_b/r^2$, this simplifies even further to:

$$g_{EG}(g_{bar}) = g_{bar} + \sqrt{\frac{cH_0}{6}} \sqrt{g_{bar}}. \quad (4)$$

Finally, in order to test V16 using the standard WL methodology, we need to assume that the deflection of photons by a gravitational potential in this alternative theory corresponds to that in GR. This assumption is justified because, in EG's original form, Einstein's laws are meant emerge from its underlying description of space-time. The additional gravitational force described by ADM does not affect this underlying theory, which is an effective description of GR. Therefore, we assume that the gravitational potential of an ADM distribution produces the same lensing shear as an equivalent distribution of actual matter.

4.3 Analytical CDM model

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5 RESULTS

Write when the results are ready. I still need:

- The K1000 lensing catalogues with ANNz redshifts and stellar masses.
- The results from the Bahamas simulation.

5.1 Isolated galaxies

6 DISCUSSION AND CONCLUSION

Write at the end.

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