Asymmetric Windows and their Application in Frequency Estimation

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ABSTRACT

Classic windows have constant time delay and linear phase because of the symmetry and the time shift causality-imposed in the time domain. And thus, all such windows have the same spectral phase response. Removal of the symmetry constraint on a classic window can result in a variable phase response and in an alterable time delay. In essence the time delay becomes shorter will bring about a lot of benefits in speech coding. Some asymmetric windows with better magnitude response also can lead to a better recognition performance if the result is relatively insensitive to phase distortion. However, it is surprising that so little attention has been paid to the asymmetric windows in past literature; and never in past history has this issue been systematically or comprehensively studied. As a result, several methods to obtain the asymmetric windows are being presented in this paper. The asymmetric windows are displayed and compared with the classic windows, concerning both in time and frequency domains. Some new asymmetric windows with good behavior are introduced. The work in this paper makes it possible for researchers to select an asymmetric window to meet their requirements, in terms of bias due to time delay or bias due to spectral behavior. Finally, the application in a new aspect, of the asymmetric windows, is displayed. The simulation demonstrates that the improved estimation method, in which the symmetric windows are replaced by the asymmetric windows, shows a stronger capability of additive noise resistance than the traditional method.

Keywords: Asymmetric window; Alterable phase; Time delay; Spectral leakage

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1. INTRODUCTION

Since the Fast Fourier Transform (FFT) algorithm has been developed, the Discrete Fourier Transform (DFT) has also been widely used across a large number of fields in signal processing. When DFT analysis is used to study the spectrum of signals, limits on resolution between different frequencies and on detectability of a small signal in the presence of a large one show up. The problem arises because the signal can only be measured for a finite time record. It will introduce discontinuities at the ends of the measured time. They will cause the signal spectrum to be spread out. That means the energy will leak into all other frequencies, instead of concentrating only on one frequency. This redistribution of energy is called spectral leakage. It will lead any given spectral component to contain not only the signal energy, but also the noise from the rest components in the spectrum, which, as contamination, will degrade the signal to noise ratio. In some cases, it is possible for the spectral leakage from a large signal component to be severe enough to mask the given spectral component.

Windows are a kind of weighting functions which are introduced to reduce the spectral leakage associated with limited observation intervals. The amount of spectral leakage depends on the amplitude of the discontinuity. Thus windowed data is smoothly brought to zero at the boundaries of each period to reduce the discontinuity amplitudes [1].Their roots can date back ,over one hundred years, to Fejer's averaging technique for a truncated Fourier series[2, 3]. In 1958, R.B. Blackman and J.W. Tukey summed up five classic pairs of windows including the rectangle (Dirichlet) window, triangle (Fejer, Bartlet) window, Hanning window, Hamming window and Blackman window[4]. Few years earlier, in 1956, N.I. Akhiezer had proposed Cauchy window and Gaussian window[1]. In 1961, E. Parzen introduced some windows, such as the Riesz window which is the simplest continuous polynomial window[5]. In 1964, N.K. Bary proposed the Riemann window and Poisson window[6]. In 1967, J.W. Turkey constructed a family of windows called Tukey window parameterized on a variable[7]. A lot of adjustable and optimal windows were proposed in this period. The Dolph-Chebyshev[8], the Kaiser-Bessel[9] and the Barcilon-Temes[10] windows are examples of classic optimal windows. In 1978, Harris[1] presented his celebrated paper in which a plethora of windows were discussed and compared. In this famous article, a family of windows formed as a sum of cosine terms was studied. Through a gradient search technique[11], the coefficients of windows achieved a minimum side-lobe level for 3- and 4-terms were evaluated. He also investigated families of windows in which the trade-off was made between the main lobe width and side-lobe levels.

In 1981, A.H. Nuttall [12] derived additional windows with very good side-lobes and optimal behavior under different constraints. In 1984, a two-parameter family of weights was derived by R.L. Streit to use in finite duration non-recursive digital filters and in finite aperture antennas[13]. The result indicated that one parameter determines the width of main-lobe and the other controls the side-lobe. In 1986, K.M.M Prabhu and H. Renganathan [14] studied a class of windows called binary window for frequency domain operation resulting in mulitplierless and memoryless implementation. In 1989, a new class of window functions was introduced for designing FIR filters by T. Saramaki [15]. These window functions are obtained from the rectangular window by using a simple frequency transformation. In 2001, A.G. Deczky[16] put forward a class of window functions that was based on the orthogonal polynomials known as the " Gegenbauer " or " Ultraspherical Polynomials ". The ultraspherical window contains three parameters. With the judicious selection of the parameters, a unique family of side-lobe patterns can be easily achieved by generating the coefficients of window through a closed-form solution[17, 18]. The applications of windows cover various aspects of traditional signal processing that included power spectral estimation, beam-forming, digital filter design and so on. In spite of their maturity, windows continue to be widely used in a variety of new aspects. Recently, windows have been employed to improve the detection of heart rate variability in electrocardiograms [19, 20], to reduce side-lobes of limited diffraction beams by Chebyshev weighting function [21], to aid in the classification of cosmic data[22, 23] and to enhance the reliability of weather prediction models [24].

However, it is astonished that so little attention was paid to the asymmetric windows in the past. Neither in the history has the issue been deeply and comprehensively studied. So far, the asymmetric windows already put forward are not as good as we expected. Therefore, the goal of this paper is to give a more systematic insight into the asymmetric windows. The main contributions include the following: (1) Four methods of constructing asymmetric windows are presented with their mathmatical formulas. A great number of asymmetric windows are obtained. Those asymmetric windows have been compared with the relevant classic windows in several aspects. (2) The figures of merit of some asymmetric windows are listed for researchers to select an asymmetric window to meet their needs. (3) The asymmetric windows were introduced in frequency estimation. The simulation illustrates that the improved method has a stronger capability of additive noise suppression, by replacing the symmetric windows with the asymmetric windows, than the old method.

2. MERITS OF WINDOWS

All the classic windows including those mentioned above are symmetric. In spite of properties of easy design and linear phase, they also imply some potential and inherent shortcomings like constant time delay and frequency response limitations[25]. Relaxing the constraint of symmetry, it can therefore create the asymmetric windows which may have some better properties for special use[25-31]. Asymmetric windows are those weighting functions whose weighting coefficient are not symmetric about the origin. The concept was firstly proposed by A. F. Florincio in 1991. Those asymmetric windows with shorter time delay presented in his article are very significant for speech coding. Some asymmetric windows can also lead to a more robust frequency response and hence have a better recognition performance[27]. Recently, the asymmetric windows have been used to facilitate the detection of closely spaced signal components as well. The asymmetric windows with properties of extended discrimination capacity, robustness against additive noise and simple generating make them very good candidates for narrow-band spectrum analysis of mechanical systems[28].

In Harris' excellent paper, he summarized some important criteria of the windows in the consideration of harmonic analysis. Windows are used here to reduce the undesirable effects coming from spectral leakage, which have a critical influence on a harmonic processor, including the impacts of attributes in detect-ability, resolution, dynamic range, confidence, and easy implementation. Those merit are still useful to evaluate the asymmetric windows. The figures of merit are listed in table 1 in the latter part.

2.1. Equivalent Noise Bandwidth

A given FFT bin includes contributions from other frequencies including accumulated broadband noise. In order to detect a narrow band signal in the presence of noise, we want to minimize the noise. This can be achieved by using a narrow bandwidth window function that is measured by the Equivalent Noise Bandwidth (ENB).

2.2. Coherent Power Gain

Coherent Power Gain measures the reduction in signal power due to the window function suppressing a coherent signal at the ends of the measurement interval. For an ideal frequency component, the noiseless signal contribution to the FFT bin is proportional to the signal amplitude. The proportionality factor is the sum of the window terms, which is just the DC gain of the window.

Table 1. Windows and figures of merit.

Window		Hightest Side-lobe Level(dB)	Side-lobe Fall-off (dB/OCT)	Coherent Gain	Equivalent Noise BW (Bins)	3.0-dB BW (Bins)	6.0-dB BW (Bins)	Scalloping Lost (dB)	Worse Case Process Loss (dB)	Normalized Average Time Delay
Rectangle		–13.2	–6	1.00	1.00	0.88	1.21	3.92	3.92	0.5
Triangle		–27.5	–12	0.50	1.33	1.27	1.77	1.82	3.07	0.5
Hanning		–32.5	–18	0.50	1.50	1.44	2.00	1.42	3.18	0.5
Minimum 3-Term		–71.5	–6	0.42	1.70	1.62	2.26	1.14	3.45	0.5
Kaiser-Bessel	$a = \sqrt{3}$	–39.8	–6	0.52	1.41	1.35	1.87	1.63	3.12	0.5
	$a = \sqrt{8}$	–65.5	–6	0.41	1.75	1.66	2.32	1.08	3.50	0.5
$[t^2]_1$- Hanning		None	–18	0.38	1.92	1.80	2.54	0.92	3.77	0.68
$[\sqrt{t}]_1$-Hanning		None	–12	0.50	1.50	1.38	1.94	1.56	3.32	0.31
$[t\sqrt{t}]_1$-Hanning		None	–18	0.44	1.69	1.60	2.24	1.15	3.43	0.61
$[\ln(t+1)/\ln 2]_1$-Hanning		None	–18	0.49	1.51	1.44	2.01	1.42	3.22	0.43
$\sin(\pi t/2)_1$-Hanning		None	–18	0.39	1.87	1.76	2.48	0.95	3.68	0.35
	$a = \sqrt{3}$	None	–6	0.52	1.41	1.28	1.81	1.82	3.31	0.32
$[\sqrt{t}]_1$-Kaiser-Bessel										
	$a = \sqrt{8}$	None	–6	0.41	1.75	1.60	2.28	1.18	3.61	0.29
	$a = \sqrt{3}$	–45.1	–6	0.46	1.55	1.45	2.05	1.42	3.33	0.61
$[t\sqrt{t}]_1$-Kaiser-Bessel										
	$a = \sqrt{8}$	–75.2	–6	0.36	1.99	1.87	2.64	0.86	3.86	0.62
$[t]_2$-Hanning		–32.3	–18	0.46	1.62	1.55	2.15	1.23	3.33	0.57
$[t]_2$-Minimum 3-Term		–70.6	–6	0.40	1.80	1.71	2.39	1.01	3.57	0.55
	$a = \sqrt{3}$	–38.2	–6	0.48	1.53	1.46	2.04	1.38	3.24	0.58
$[t]_2$-Kaiser-Bessel										
	$a = \sqrt{8}$	–62.4	–6	0.39	1.84	1.75	2.45	0.97	3.62	0.55
$[\sqrt{t}]_2$ -Hanning		–32.2	–18	0.48	1.56	1.49	2.07	1.33	3.25	0.54
$[t]_3$-Minimum 3-Term		–70.7	–6	0.41	1.76	1.67	2.33	1.07	3.51	0.53
	$a = \sqrt{3}$	–39.5	–6	0.50	1.47	1.41	1.96	1.49	3.17	0.54

Table 1 (continued)

Window		Hightest Side-lobe Level(dB)	Side-lobe Fall-off (dB/OCT)	Coherent Gain	Equivalent Noise BW (Bins)	3.0-dB BW (Bins)	6.0-dB BW (Bins)	Scalloping Lost (dB)	Worse Case Process Loss (dB)	Normalized Average Time Delay
$[\sqrt{t}]_2$ -Kaiser-Bessel										
	$a = \sqrt{8}$	–64.0	–6	0.40	1.80	1.71	2.39	1.02	3.56	0.53
$[t]_3$-Hanning		–49.2	–24	0.38	1.89	1.78	2.49	0.94	3.69	0.59
$[t]_3$-Minimum 3-Term		–89.4	–12	0.36	1.97	1.85	2.59	0.87	3.81	0.60
	$a = \sqrt{3}$	–57.1	–12	0.39	1.84	1.73	2.43	0.99	3.63	0.59
$[t]_3$-Kaiser-Bessel										
	$a = \sqrt{8}$	–85.8	–12	0.36	1.99	1.86	2.61	0.85	2.83	0.60
$[\sqrt{t}]_3$ -Hanning		–51.1	–12	0.42	1.74	1.65	2.30	1.08	3.48	0.56
$[\sqrt{t}]_3$ -Minimum 3-Term		–90.6	–12	0.41	1.80	1.71	2.38	1.01	3.56	0.56
	$a = \sqrt{3}$	–60.5	–12	0.43	1.70	1.62	2.25	1.13	3.43	0.56
$[\sqrt{t}]_3$ -Kaiser-Bessel										
	$a = \sqrt{8}$	–90.0	–12	0.41	1.81	1.72	2.39	0.99	3.57	0.56
	ε=0.6	–25.0	–18	0.50	1.50	1.43	1.99	1.44	3.20	0.54
$[\varepsilon]_4$-Hanning	ε=0.4	25.0	–18	0.50	1.50	1.43	1.99	1.44	3.20	0.46
	ε=0.3	–49.1	–18	0.50	1.50	1.41	1.96	1.59	3.25	0.42

Coherent gain equals to DC gain usually divided by the number of terms in the window. The coherent power gain is the square of this, or in other words the coherent power gain is the square of average value of the window terms.

2.3. Processing Gain

Processing gain equals the ratio of input signal to noise to output signal to noise, which is the coherent power gain divided by the noise power. For a signal made up of an ideal discrete line frequency component that is polluted by white noise, the processing gain is, amazingly, the reciprocal of the Equivalent Noise Bandwidth (ENB).

2.4. Scalloping Loss

Scalloping loss is defined as the ratio of coherent gain for a signal frequency component that is located half way between FFT bins, to the coherent gain for a signal frequency component located exactly at an FFT bin. Scalloping loss represents the maximum reduction in coherent gain due to signal frequency.

2.5. Worst Case Processing Loss

Worst case processing loss is defined as the sum of scalloping loss and processing gain. This is a measure of the reduction of output signal to noise ratio resulting from the combination of the window function and the worst case frequency location. It is of course related to the minimum tone that could be detected in broadband noise.

2.6. Spectral Leakage Revisited

Spectral leakage can pose a bias to the amplitude and position of a spectral analysis. It will have a serious interference when detecting small signals in the presence of nearby large signals. As a result, in order to reduce the effects of this bias, the window needs to have small side-lobes away from the center and meanwhile the low side-lobes should fall off rapidly. The peak side-lobe level is one of indicators that shows the ability of suppressing spectral leakage. The asymptotic rate of falloff of the side-lobes is another useful criterion as well.

2.7. Minimum Resolution Bandwidth

For harmonic analysis, it is very important to know the minimum separation distance between two equal-strength lines. The classic criterion for the resolution is the width of the window at the half power points (the 3 dB bandwidth). But this assumes incoherent addition. The DFT output is the coherent addition of frequency components weighted by the window function at each frequency. Because of the coherence, the 6-dB bandwidth defines resolution rather than the 3-dB.

2.8. Normalized Average Time Delay

All the criteria listed above are based on the consideration of harmonic analysis for symmetric windows. For asymmetric windows, some other criteria are needed for a special use. Consequently, in order to have a comparison in the ability of time delay among different asymmetric windows, normalized average time delay (NATD) has been presented. The definition of the criterion is

$$\hat{t} = \frac{1}{T}\int_0^T t f^2(t)dt \Big/ \int_0^T f^2(t)dt. \tag{1}$$

In the above definition, $f(t)$ is the asymmetric window and T is the time duration. It is the barycenter of the energy with length normalization. Similar to the continuous form, the discrete form is

$$\hat{n} = \frac{1}{N}\sum_{n=0}^{N-1} n x^2(n) / \sum_{n=0}^{N-1} x^2(n) \cdot \tag{2}$$

It is obvious that the value of all the symmetric windows is 0.5. The values of normalized average time delay with various kinds of asymmetric windows are shown in table 1.

3. METHOD OF CONSTRUCTING ASYMMETRIC WINDOW

3.1. Method of Modulation

An idea to use a composed function is presented by D.A.F. Florincio [26], of the kind $w(p(t))$, where $w(t)$ is a common continuous symmetric window, and $p(t)$ is a algebraic expression. For easy description, it is assumed that $w(t)$ is a time-shifted symmetric window with the independent variable t ranging from [0, 1]. For the continuous symmetric window $w(t)$, we can rewrite it as $w(p(t))$, where $p(t) = t$. If $p(t)$ is assigned to a different expression, a new window $\hat{w}(t)$ inherited from $w(t)$ is produced.

$$\hat{w}(t) = w(p(t)) \tag{3}$$

Therefore, it is possible to convert a symmetric window $w(t)$ to an asymmetric window $\hat{w}(t)$ in terms of modulation. Commonly, it is necessary that the $p(t)$ is a monotone function (increased or decreased) to give a guarantee of the only one extreme value in a new window. The value range of $p(t)$ should not go beyond the closed interval [0, 1]. The asymmetric windows are denoted as $[p(t)]_1$-windows.

Some examples of the method of modulation are given, where $w(t)$ is the Hanning window and $p(t)$ is assigned different elementary functions. The Hanning window and each corresponding asymmetric window are shown in Figure 1-3 for the time and the frequency domain, respectively. More examples can be obtained by replacing $p(t)$ with other modulation functions or by replacing $w(t)$ with other classic symmetric windows.

3.2. Method of Truncation

As we can see in the part 3.1, the asymmetric windows from the method of modulation are worse than the corresponding symmetric window from the aspect of spectral behavior, in which the main-lobes of these asymmetric windows become wider. Even the side-lobes fall off at a speed much less than -18dB per octave(the Hanning window) in some cases. Sometimes, we

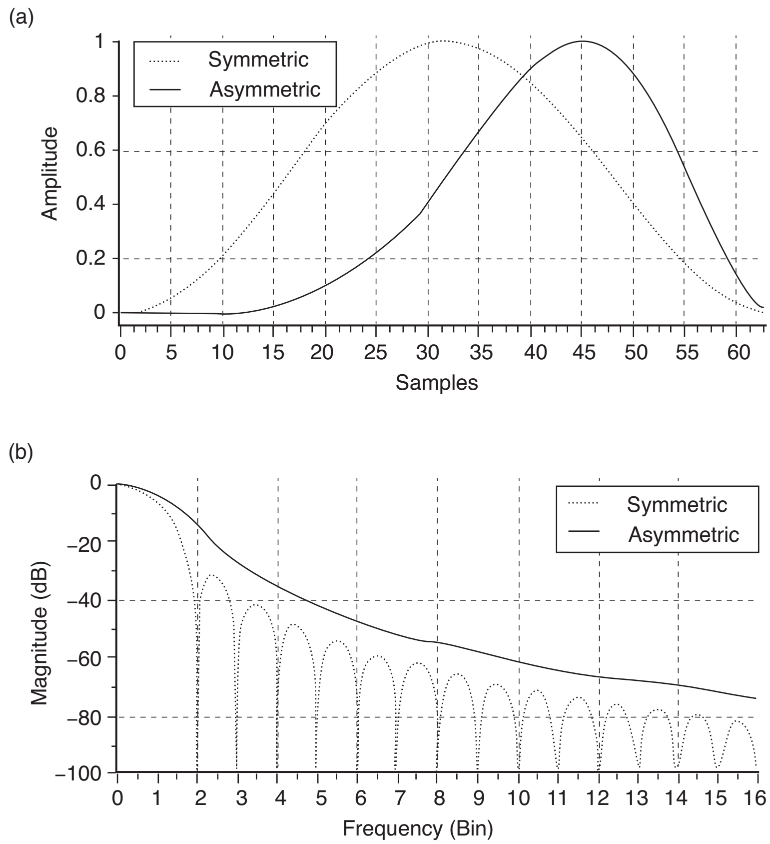


Figure 1. (a) Hanning window and $[t^2]_1$-Hanning window. (b) Normalized log-magnitude.

will need to have a good balance between the time delay and the spectral behavior. In order to obtain a finite asymmetric window with similar characteristics of the symmetric window, we can truncate some functions with the symmetric window. In other words, an asymmetric window $\hat{w}(t)$ is produced by a time-shifted symmetric window $w(t)$ multiplying by a function $s(t)$ which provides the asymmetry. These windows are denoted as $[s(t)]_T$-windows.

$$\hat{w}(t) = w(t)s(t) \tag{4}$$

One of the simplest forms of this method is to truncate a straight line by using different symmetric windows. The general form of a straight line through the origin is $s(t) = kt$, where k stands for the slope. Because of the

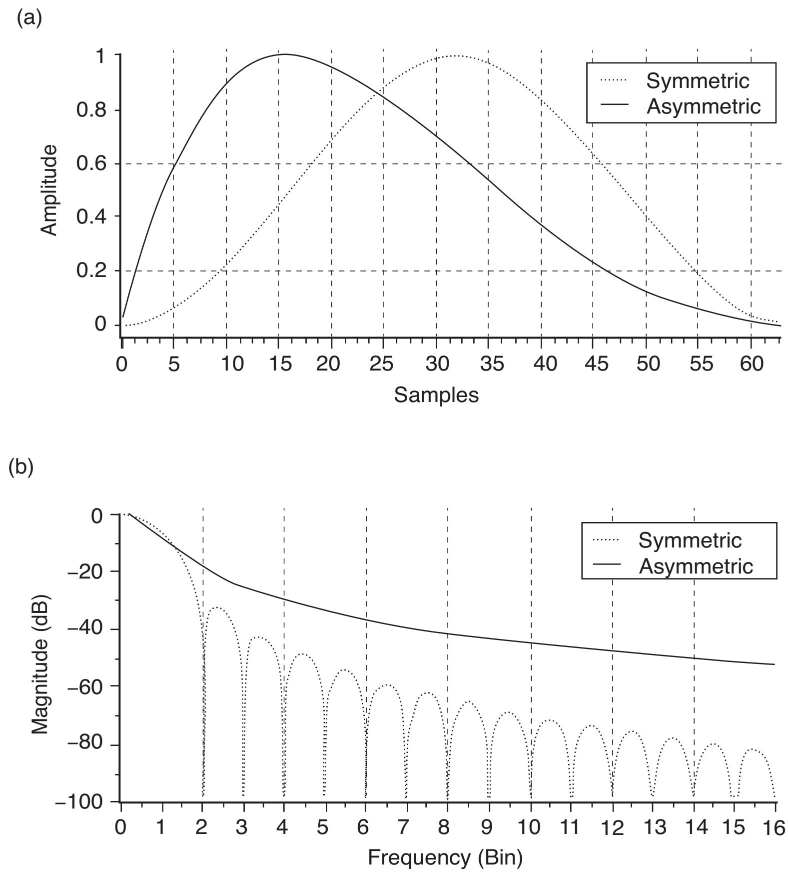
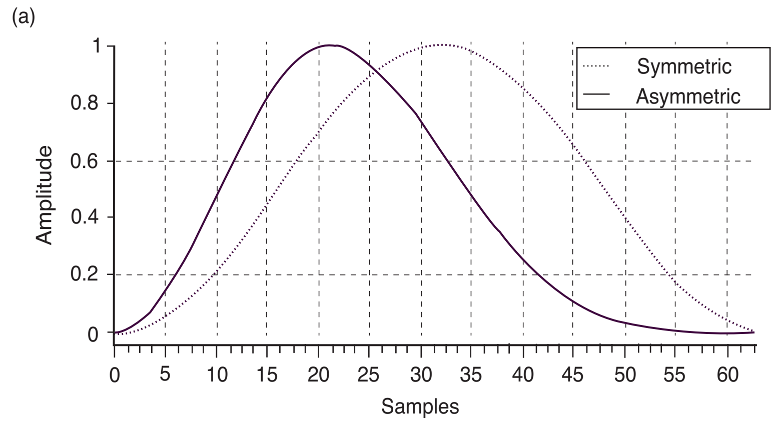


Figure 2. (a) Hanning window and $[\sqrt{t}]_1$-Hanning window. (b) Normalized log-magnitude.

linear property of the Fourier Transform, parameter k has no effect on the shape of frequency response. As a result, we simply make $s(t) = t$ and call it the $[t]_2$-window. The time domain and magnitude response of each example are shown in figures 4-6, respectively. This kind of asymmetric windows is slightly worse in some aspects than the corresponding symmetric window. They have a little wider main-lobe, and higher side-lobes. Fortunately, the asymptotic behavior of side-lobes decays as much as the corresponding symmetric windows. More examples of this kind of asymmetric windows are shown in the table 1.



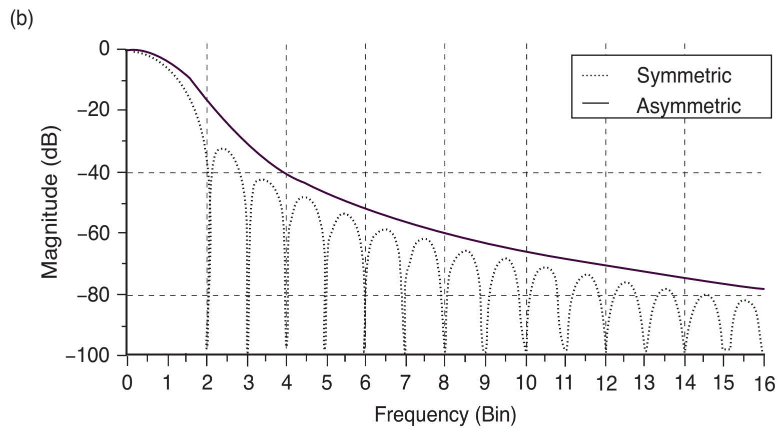


Figure 3. (a) Hanning window and $[\sin(\pi t/2)]_1$-Hanning window. (b) Normalized log-magnitude.

3.3. Method of Convolution

Convolution is the third way to get a new asymmetric window. The spectrum of the new window can be easily predicted, because the result will be the product of the spectra of the two convolved functions according to the convolution theorem. It is, therefore, reasonable to get an asymmetric window $\hat{w}(t)$ with desirable spectral behavior by convolving a function $c(t)$ with the symmetric window $w(t)$. In a simple description, the asymmetric window from convolution is denoted as $[c(t)]_3$-window.

$$\hat{w}(t) = \mathrm{w}(t)*c(t) \tag{5}$$

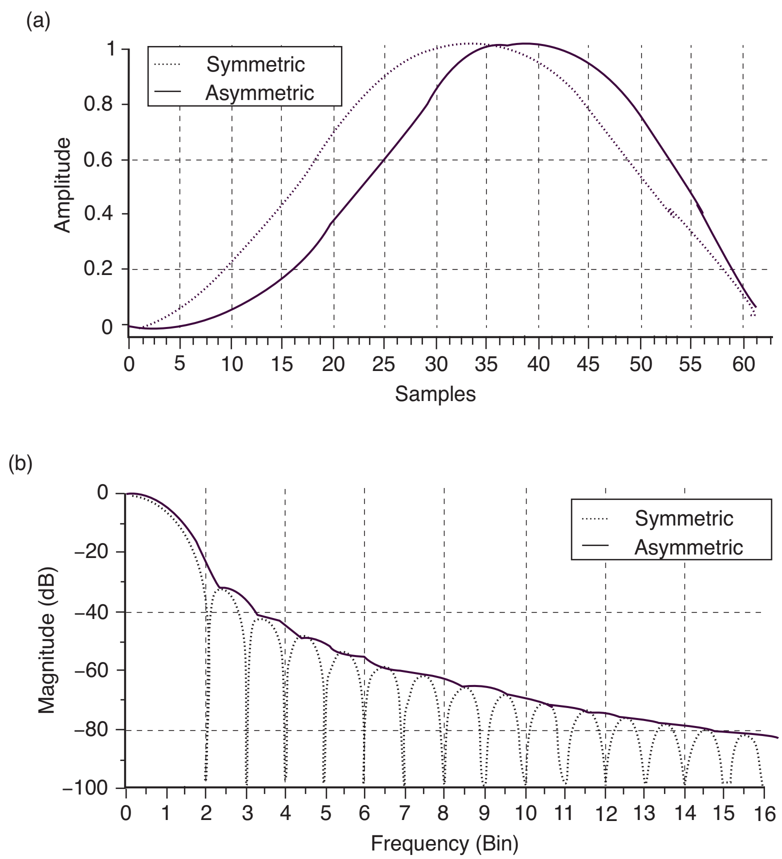


Figure 4. (a) Hanning window and $[t]_2$-Hanning window. (b) Normalized log-magnitude.

As examples, we have computed the convolution between the function $c(t)$ and the different symmetric windows. Each symmetric window has the same length as $c(t)$. The time and frequency domain of $\hat{w}(t)$ convolving separately with the function c(t) through the Hanning window, Minimum-3-term window and Kaiser window ($\alpha = \sqrt{3}$) are shown in figures 7-9, where

$$c(t) = \begin{cases} t,\ 0 \le t \le 1, \\ 0, \qquad else. \end{cases} \tag{6}$$

Compared with the corresponding symmetric window $w(t)$, the $\hat{w}(t)$ has a much wider main-lobe. But the side-lobes have a better behavior, in that the peak side-lobe level of $\hat{w}(t)$ is lower than $w(t)$,and its asymptotic decay rate of side-lobe envelope is also faster than the corresponding symmetric window.

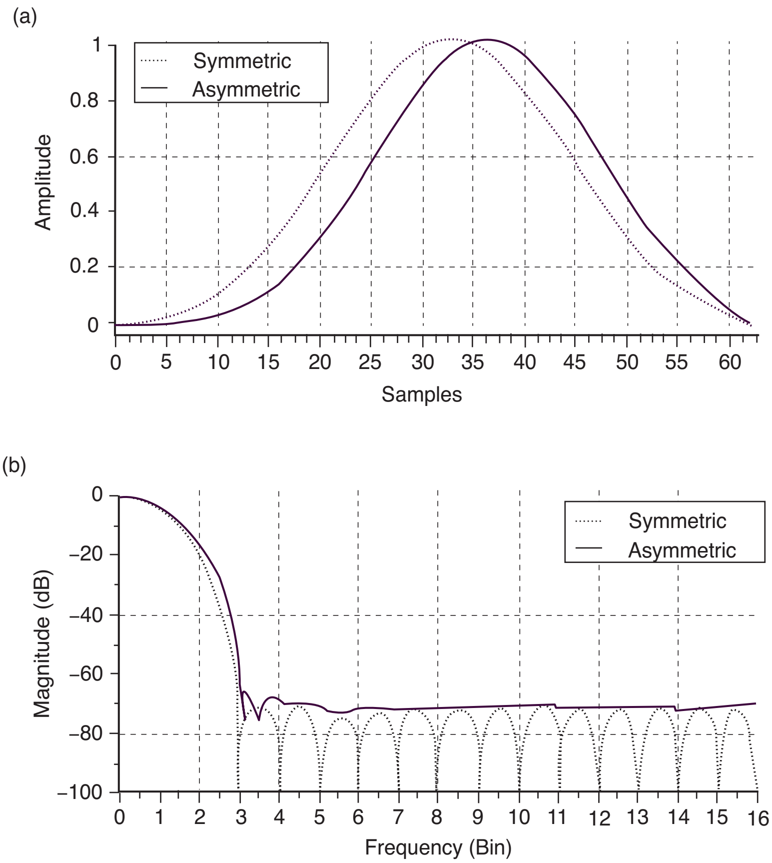


Figure 5. (a) Minimum-3-term window and its asymmetric window. (b) Normalized log-magnitude.

3.4. Method of Translation of Window Peak

The maximum value point of a symmetric window in time domain is at the middle. If the maximum value point of the window could be moved to the $\varepsilon(0 < \varepsilon < 1)$ position of the total length (ε is called the coefficient of the window maximum value) by compressing one side of the window and stretching the other, we can construct an asymmetric window. Taking the Hanning window as an example, the maximum value point of the original window was located at the half or 1/2 position of the total length. If the highest point was placed at the 3/4 position of the total length, the length of the left side needs to be stretched to its 3/2 and the right side needs to be compressed to half of itself. Finally, a new asymmetric window with the same length of the symmetric window was constructed. This type of window is denoted as $[\varepsilon]_E$-window.

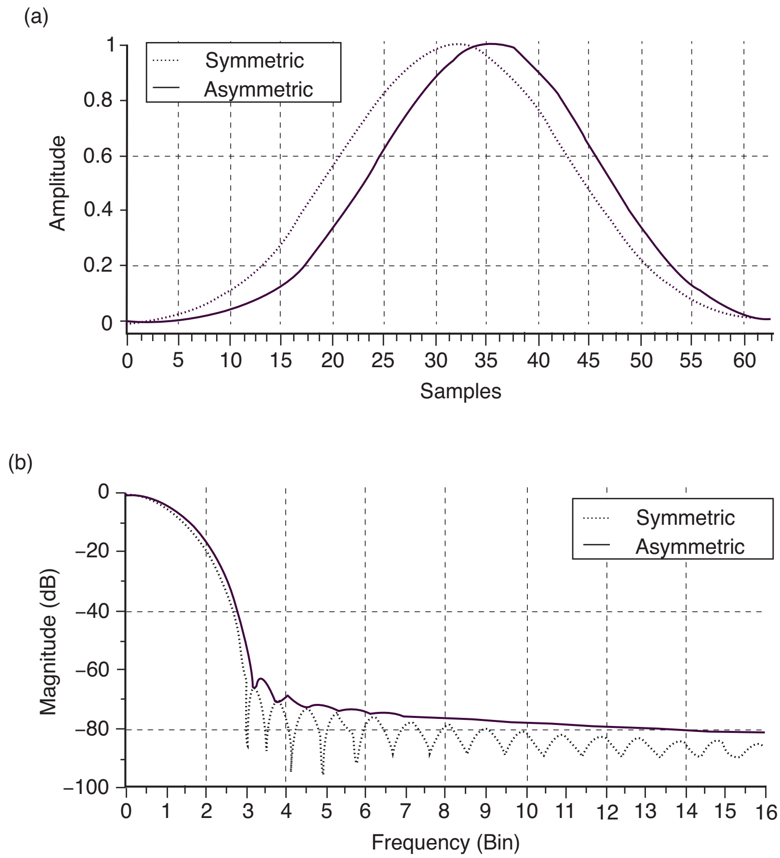


Figure 6. (a) Kaiser window and its asymmetric window with $\alpha = \sqrt{8}$. (b) Normalized log-magnitude.

$$\hat{w}(t) = \begin{cases} w(\frac{t}{2\varepsilon}), \; t \le \varepsilon, \\ w(\frac{t}{2(1-\varepsilon)}), \;\; t > \varepsilon. \end{cases} \tag{7}$$

According to (7), we can see when $\varepsilon = 0.5$, an asymmetric window regresses into a symmetric window. The most significant feature of this type of windows is that it is possible for one to adjust the asymmetry of a window function through regulating the value of ε. We can get asymmetric windows with different amplitude and phase characteristic by adjusting the coefficient of the window maximum

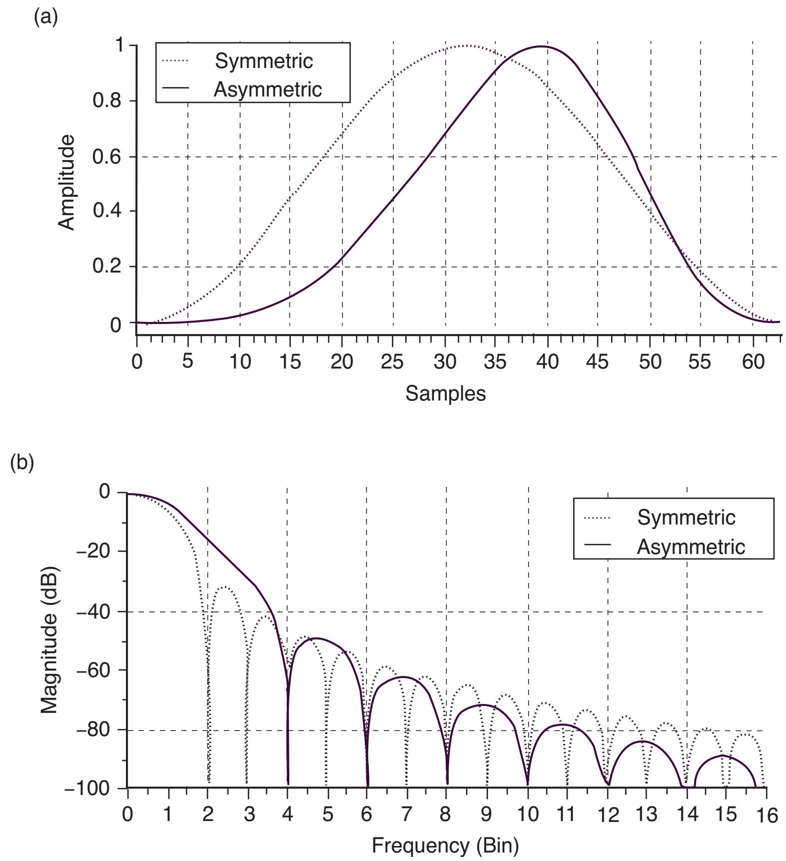


Figure 7. (a) Hanning window and $[t]_3$-Hanning window. (b) Normalized log-magnitude.

value. The closer the maximum value coefficient gets to 0.5, the weaker the asymmetry is, and vice versa. Hence, appropriate asymmetric windows can be established according to actual needs in real projects. Figures 10-12 show the time domain and magnitude response of the examples constructed from the Hanning window with $\varepsilon = 0.6$, $\varepsilon = 0.4$ and $\varepsilon = 0.3$, respectively.

When a certain asymmetric window is obtained, its time inversed form will undoubtedly be an asymmetric window as well. They have the same magnitude response but different phase response. The two asymmetric windows composing a pair will be used in next part for frequency estimation of the harmonic signal by replacing the symmetric windows. For convenient description, we nominated that the one whose normalized average time delay is less than 0.5 is P-type and the other is S-type.

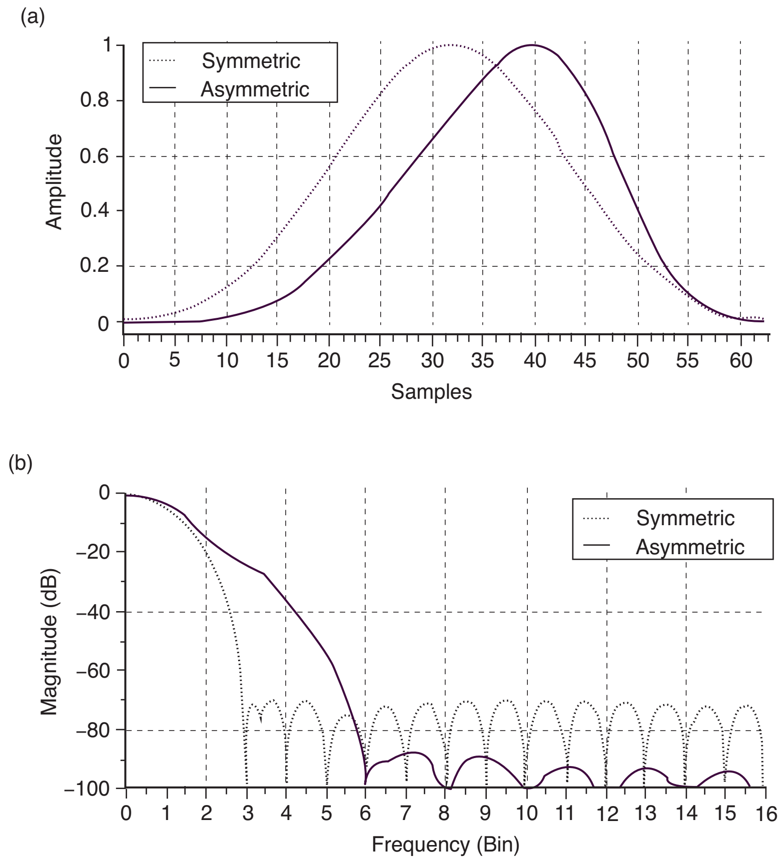


Figure 8. (a) Minimum-3-term window and its $[t]_3$-window. (b) Normalized log-magnitude.

The figures of merit of different symmetric and asymmetric windows are reported in the table 1. The asymmetric windows from modulation method can easily get a shorter (or longer) time delay at the cost of spectral behavior. We can balance them by choosing different modulation functions or by combining different modulation functions together with a weighting coefficient in order to adjust the attributes of windows. The second kind of asymmetric windows often perform well in the aspect of spectral behavior. They have not only an alterable phase response but also a good magnitude response with low peak side-lobes and the considerable rate of asymptotic decay of the side-lobes envelope. The only cost is that the main-lobe becomes slightly wider. The third method can obtain some asymmetric windows that have very good side-lobes, but the main-lobes would be much larger than the symmetric windows. The fourth method

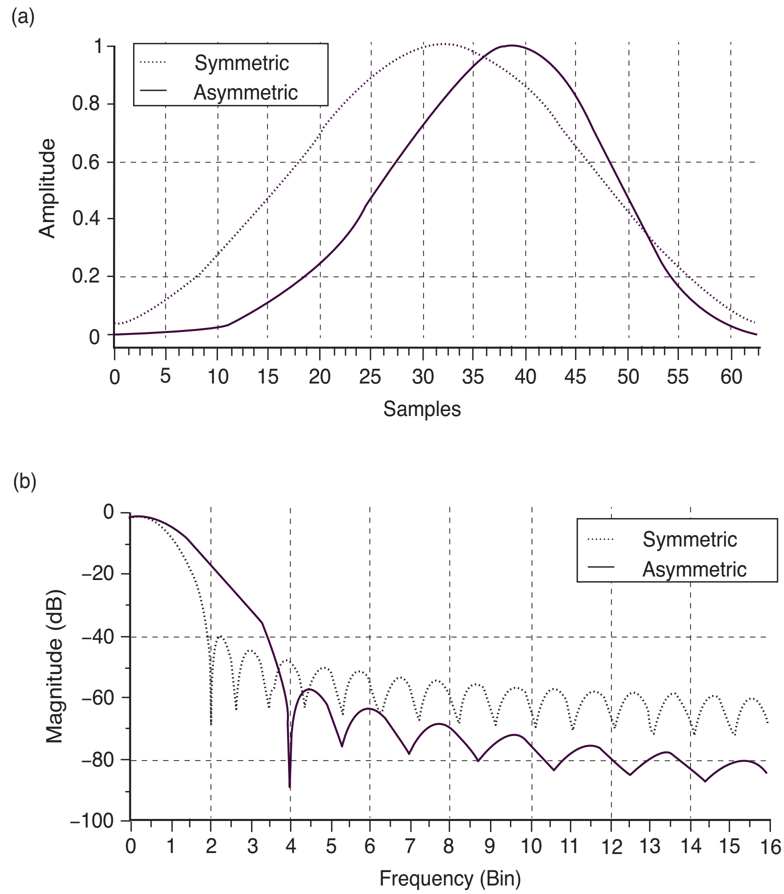


Figure 9. (a) Kaiser window and $[t]_3$-Kaiser window with $\alpha = \sqrt{3}$. (b) Normalized log-magnitude.

gives asymmetric windows by adjusting the position of the window peak in time domain. One can design asymmetric windows with the desired characteristics by choosing the coefficient of the window maximum value. As a result, it has a wider flexibility in practice use.

4. APPLICATION OF ASYMMETRIC WINDOWS IN FREQUENCY ESTIMATION

Frequency estimation of a sinusoid in additive white Gaussian noise is relevant to a wide range of areas[32], such as radar, sonar, communications, and the vibration of rotating machinery to name a few. A lot of methods have been presented to estimate the parameters in the past scientific literature. The phase

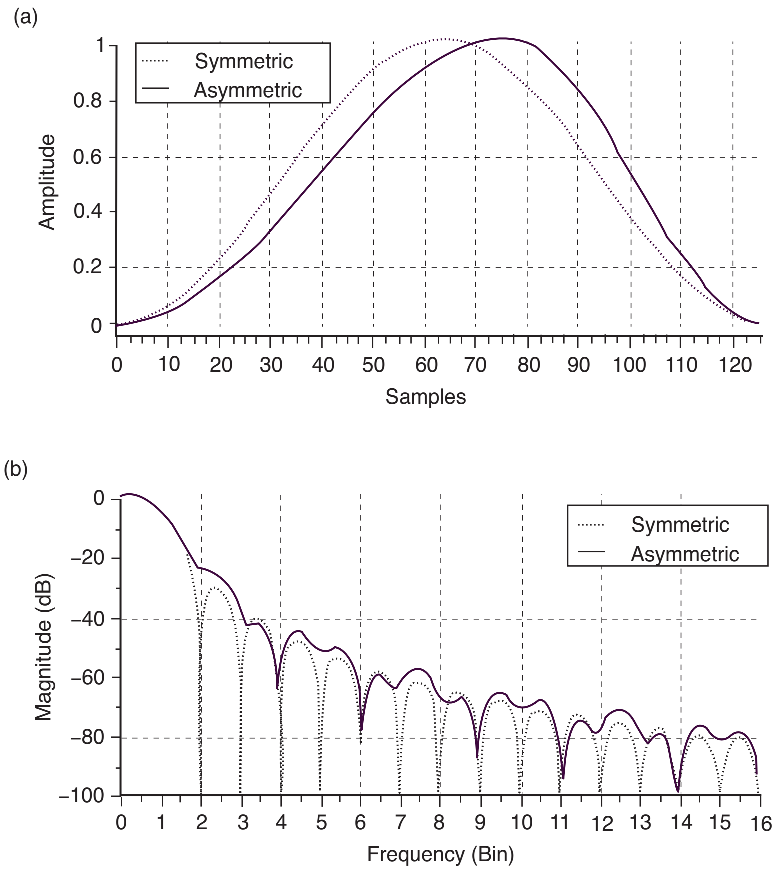


Figure 10. (a) Hanning window and $[\varepsilon]_4$-Hanning window ($\varepsilon = 0.6$). (b) Normalized log-magnitude.

difference method based on FFT is one of them. It has been reported that in the same noisy conditions the phase difference correction method will display higher resistance than other estimation methods. Here, the simulations will be presented to compare the abilities against the additive white noise of the phase difference correction methods based on symmetric and asymmetric windows.

The normalized correction value based on phase difference, obtained by translation window center, is given by [33]

$$\Delta k = -\frac{\Delta\phi(k)}{\lambda_2 - \lambda_1} \,. \tag{8}$$

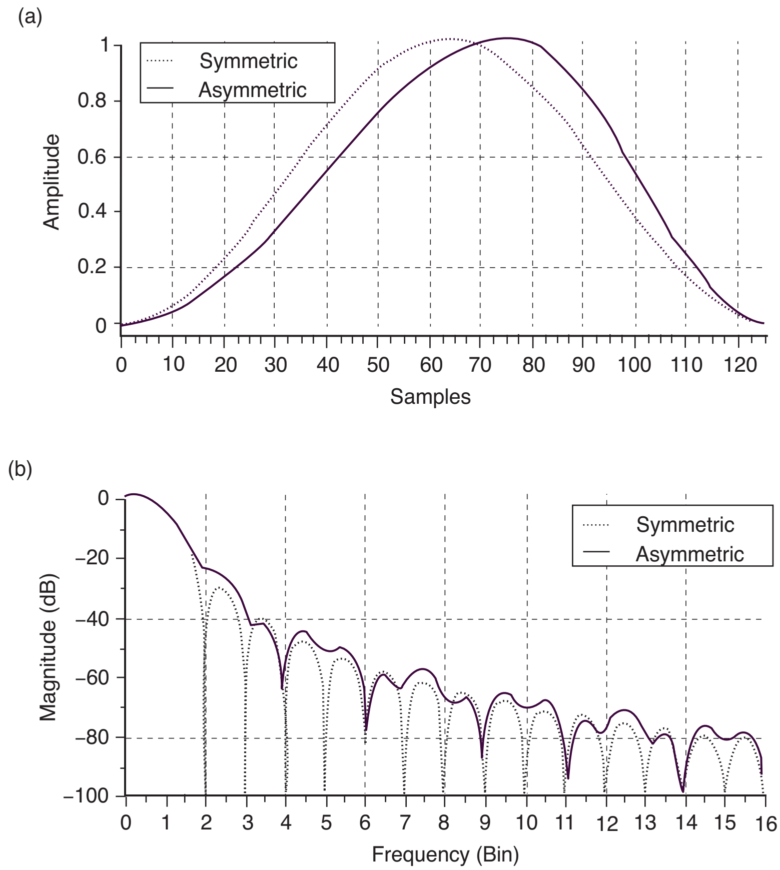


Figure 11. (a) Hanning window and $[\varepsilon]_4$-Hanning window($\varepsilon = 0.4$). (b) Normalized log-magnitude.

In above equation, $\Delta\phi(k)$ is the observed phase difference, λ_2, λ_1 are the slope of the phase line, and Δk is the normalized correction value. When the asymmetric windows are introduced, the simple linearity between the normalized correction value and the phase difference disappears because of the nonlinear phase of asymmetric windows [33]. However, we can still get the normalized frequency k_0 by solving [33]

$$\mu(k_0) = \Delta\phi(k_0) = 0. \tag{9}$$

Eq. (9) indicates that when the asymmetric windows are used the phase difference is zero when a given value is equal to the theoretical frequency.

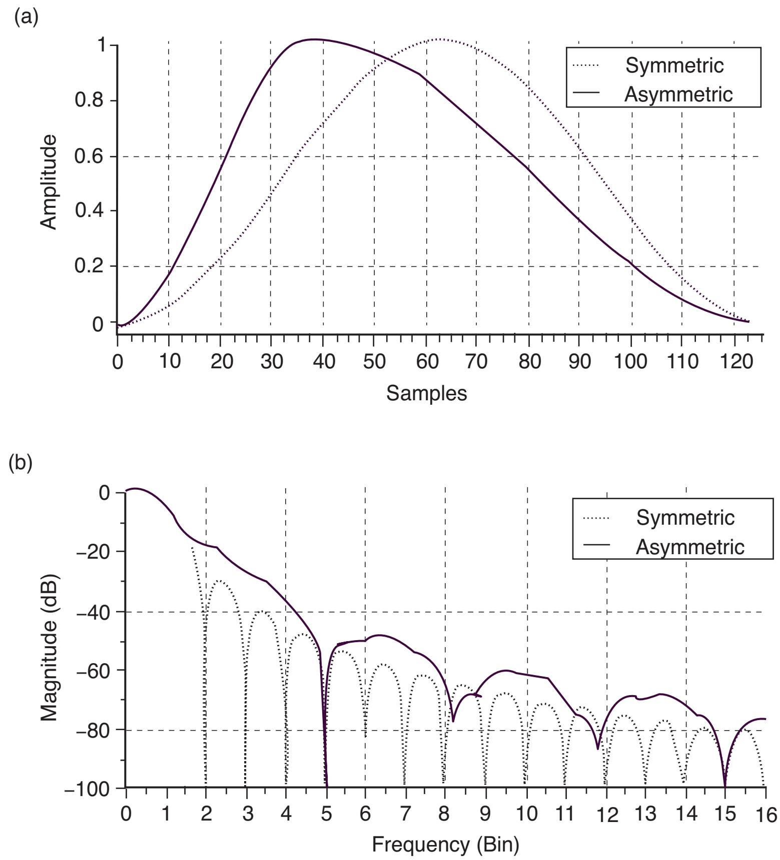


Figure 12. (a) Hanning window and $[\varepsilon]_4$-Hanning window($\varepsilon = 0.3$). (b) Normalized log-magnitude.

A single theoretical component of length $N = 1024$ samples with Gaussian noise is produced by computer according to the formula (10). The frequency f is random in the range [255.5, 256.5], the initial phase φ is random and the amplitude is one.

$$x(n) = \cos(2\pi \frac{f}{f_s} n + \varphi) + r(n) \tag{10}$$

Comparisons are made based on 100000 independent trials for each SNR, which is defined as

$$SNR = 10 \log_{10} \frac{A^2}{2\sigma^2}, \tag{11}$$

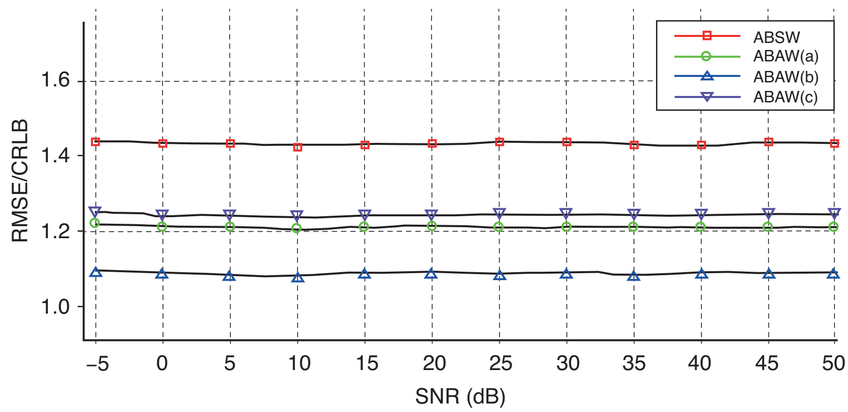


Figure 13. IRMAE for each SNR with 1000 trials (ABSW: Algorithm based on Hanning window; ABAW(a): Algorithm based on $[t^2]_1$-Hanning window; ABAW(b): Algorithm based on $[\sqrt{t}]_1$-Hanning window; ABAW(c): Algorithm based on $[t]_2$-Hanning window)

where A stands for the amplitude of the theoretical component and σ^2 is the variance of white noise. We take the root mean squared error (RMSE) of the 100000 trials into account to assess those correction methods. The root mean squared error displayed in the figure is defined as

$$\mathrm{RMSE} = \sqrt{\frac{1}{N_{tr}}\sum_{i=1}^{N_{tr}}(\hat{f}_i - f_0)^2}. \tag{12}$$

In equation (12), N_{tr} stands for the number of trials, $\hat{f}_i$ is the estimation frequency of each trial under the condition of the added white noise, and f_0 is the theoretical value of the given frequency. In each trail, the original data with the latter half part set to zero is used as the first segment, and the original data with the front half part set to zero is used as the second segment. Consequently, the added window contains 512 points. In the traditional algorithm, the Hanning window is chosen in both segments; while in the new algorithm, when a pair of asymmetric windows is chosen , its P-style is used in the first segment and the S-style is used in the second segment.

Fig.13 shows the simulation results of the traditional algorithm based on symmetric window (ABSW) and the new algorithm based on different pairs of asymmetric windows (ABAW). We can see that the RMSE/CRLB very steady in all SNR. The noticeable trend is that the indicator of new algorithms is clearly less than the old one for each SNR. For a certain SNR, RMSE/CRLBs for the ABAW(a) and the ABAW(c) are very close to each other. The value of ABAW(b) is about 1.1 much less than that of ABAW. It is clearly indicated that the algorithm based on asymmetric window has a stronger ability of noise immunity. A large number of trails and studies show that the different abilities against the noise among asymmetric windows are relative to their normalized average time delay.

5. SUMMARY

Because of their shorter time delay, alterable phase and robust frequency response, the asymmetric windows have been used in some specialized ways including speech coding, speech recognition and detecting the closely spaced signal components. In this paper, some important criteria of window are reviewed firstly. Then, four methods are proposed to create the asymmetric windows. Some examples of asymmetric windows that have good magnitude responses or considerable time delays are presented. The time and spectra domain of these asymmetric windows are separately compared to those of corresponding symmetric windows. A summary of the windows with values of merit is reported in a table for researchers to choose in their consideration. Finally, the application of the symmetric windows in frequency estimation is recommended. It is demonstrated in the simulation results that the algorithm based on asymmetric windows has a stronger capability of anti-noise interference than the old algorithm based on symmetric windows. We should stress that using asymmetric windows aims to get shortter time delay (in speech analysis) or aims to improve the ability against the noise of frequency estimation methods (in this paper). Proceeding from this angle, a slight cost of spectral behavior is acceptable and sometimes even unavoidable. The balance should be done between the time delay (or improvment in noise properties) and spectral behavior. Future work needs to be done on the clarification of the relationship between the noise resistance ability in frequency estimation and the value of normalized average time delay.

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