Research on algorithms of fractal dimension of porous media based on MATLAB

(Brief Introduction - English Ver.)

Part I

Three widely used box-counting methods for calculating fractal dimension are written in MATLAB.

(1) Process of box-counting method for fractal dimension calculation

Step 1: Obtain and preprocess the required digital image.

Step 2: Upload the preprocessed image of size $M \times M$ pixels into MATLAB.

Step 3: Turn the image into a logical (0,1) matrix.

Step 4: Choose proper sequence of box sizes $s=\{s_k\}$. There are three widely used methods for choosing sequences shown in (2), (3), and (4).

Step 5: Mesh the image by various boxes in the sequence s. Then the sequence of scaling ratio is $r=\{s_k/M\}$.

Step 6: Count boxes which contains the object of interest as $n_{ij}=1$, and the others as $n_{ij}=0$. Then the sequence of box counting number is $N_r=\{N(r_k)\}$, and

$$N(r_k) = \sum_{i=1}^{1/s_k} \left(\sum_{j=1}^{1/s_k} n_{ij} \right)$$

Step 7: Make two sequences $X=\ln(r)$, $Y=\ln(N(r))$ and perform least-squares regression of Y versus X.

Step 8: Obtain the FD by the slope of the fitted line because $N(r)$ and r have the following relation:

$$N(r) \propto (1/r)^{-D}$$

(2) Three widely used methods

1. Geometric Step Method (GSM)

The sequence of box sizes is geometric sequence s= {a^{k}, a is a positive integer.}, such as s= {1, 2, 4 …}, s= {1, 3, 9 …}. Generally, s={2^{k}} is the most popular sequence in the GSM.

Program: ***geofrac.m***

2. Arithmetic Step Method (ASM)

The sequence of box sizes is arithmetic sequence s= {ak+b, a is a positive integer, b is a natural number.}, such as s= {1, 2, 3 …}, s= {5, 10, 15 …}. Generally, s= {k} is the most popular sequence in the ASM.

Program: ***arithfrac.m***

3. Divisor Step Method (DSM)

In the DSM, all box sizes in the sequence are divisors of M (M itself is excluded), such as s={1, 2, 3, 4, 6, 8, 12}, when M=24.

Program: ***divfrac.m***

Part II

Modified algorithm is proposed based on existing box-counting methods.

(1) Coverage ratio and border effect

For box size s_k, the Coverage Ratio (CR) is the ratio of the area of covered region to the area of image area (M^2).

$$CR(s_k) = \frac{\left[\text{floor}(1/r_k)\right]^2 \times s_k{}^2}{M^2} \times 100\%$$

Where floor() means the floor function.

For example, the black part in the Fig.1 ($M \times M$) is the object of interest. The box size is set as m, but m is not a divisor of M. Hence, the border of the picture (shaded areas 1-9) cannot be covered by $m \times m$ boxes, remaining some $m \times n$ or $n \times n$ fractal boxes. Obviously, $CR(m)<100\%$. The phenomenon that fractal boxes containing object of interest are neglected, which resulting in negative effect on FD calculation, is Border Effect (BE).

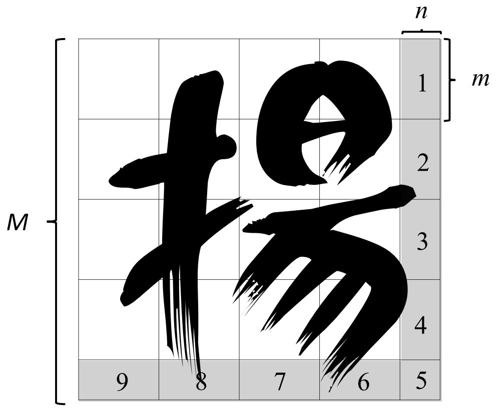

Fig.1

(2) Improvement of box size sequence

As shown in the Table 1, the GSM and the ASM result in border effect in most circumstances while the DSM, which take divisors as box sizes, usually avoid border effect. But the DSM will be ineffective when the image size is prime number or only have few divisors.

Table 1

Image size	GSM		ASM		DSM	
	s	BE	*s*	BE	*s*	BE
32×32	{1, 2, 4, 8,16}	NO	{1, 2, 3, … ,16}	YES	{1, 2, 4, 8,16}	NO
37×37	{1, 2, 4, 8,16}	YES	{1, 2, 3, … ,18}	YES	-	-
40×40	{1, 2, 4, 8,16}	YES	{1, 2, 3, … ,20}	YES	{1, 2, 4, 5, 8, 10, 20}	NO

Hence, the selection of box sizes is improved base on the DSM. As shown in the flowchart (Fig.2), more elements are added into sequence s_D by deleting the border of the image and the card(s_G) is selected as the threshold. The improvement increases the stability of the DSM but brings about the border effect.

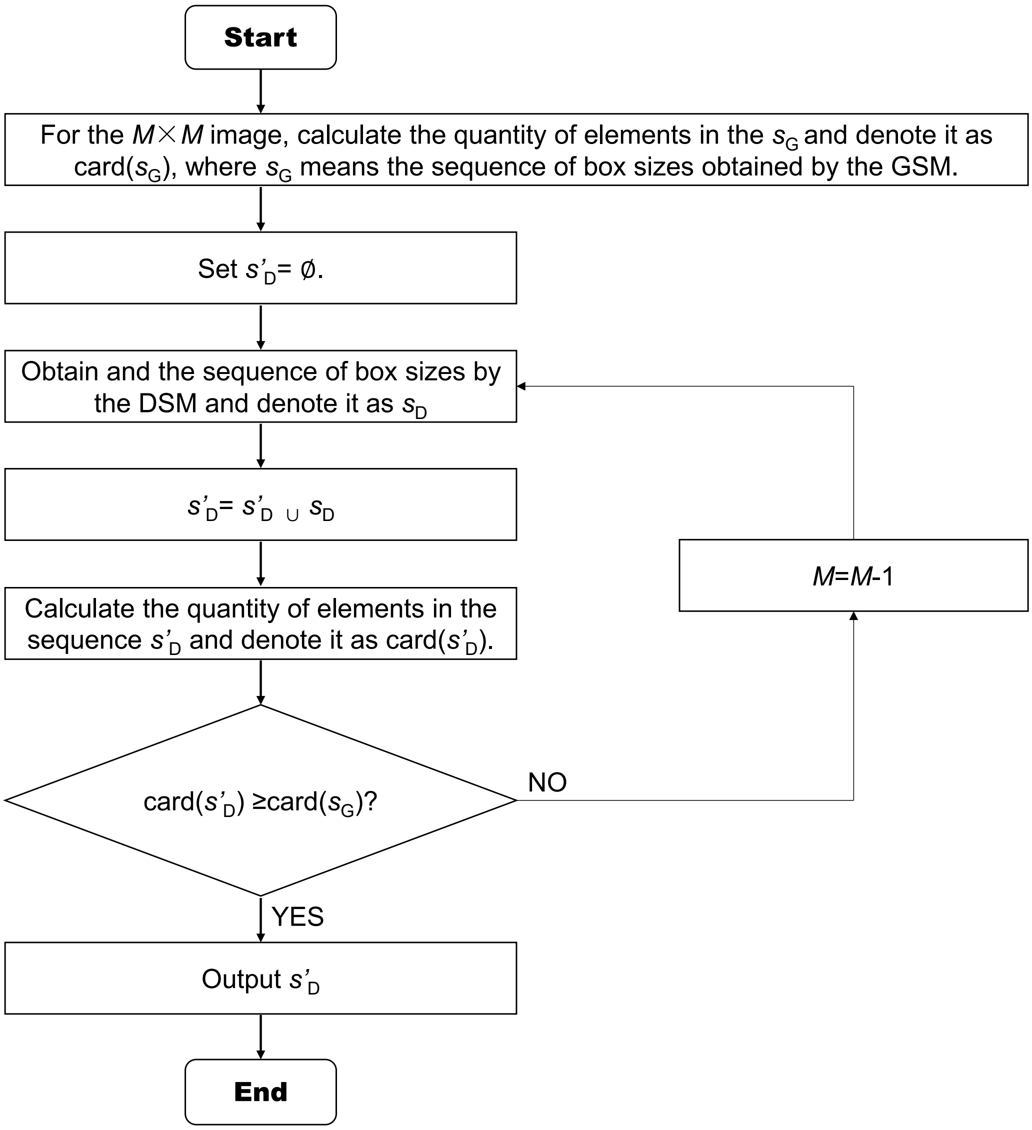

Fig.2

(3) Optimization of border box-counting

To overcome border effect, a modified border box-counting method is proposed. Because boxes on the border are fractal, n_{ij} is established as the number between 0 and 1 instead of n_{ij} =0 or 1 which mentioned before.

First of all, two factors are defined that Ratio of Area K_{area} and Ratio of Interested Infomation K_{info}.

$$K_{\text{eara}} = \frac{m \cdot n}{m \cdot m} = \frac{n}{m}$$

$$K_{\text{info}} = \min\left(\frac{I_{ij}}{I_{\text{av}}}, 1\right)$$

Where m and n was shown in Fig.1; I_{ij} means the number of interested pixels (Object of Interest) in the ijth box; I_{av} means the average number of interested pixels in all intact boxes but intact boxes without interested information should be excluded.

Thus the n_{ij} is:

$$n_{ij} = 1 \cdot K_{\text{area}} \cdot K_{\text{info}}$$

And the modified box-counting number $N'(r_k)$ is redefined as:

$$N'(r_k) = N(r_k) + N_b(r_k)$$

Where $N(r_k)$ is the box-counting number of the non-border area, and $N_b(r_k)$ is the box-counting number of the border area, which is the summary of n_{ij} of boxes on the border.

For example, in the Fig.3, I_{av}=2, and

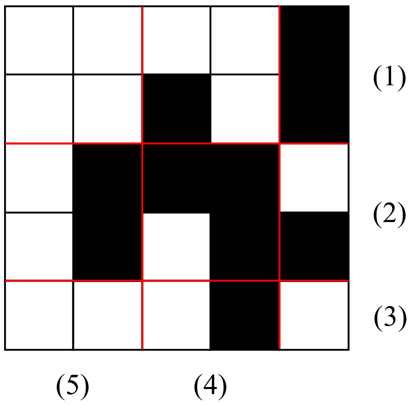

Fig.3

$K_{\text{area}(1,2,4,5)}$=0.5, $K_{\text{area}(3)}$=0.25

$$K_{\text{info}(1)} = \min\left\{\frac{2}{2},1\right\} = 1$$

$$K_{\text{info}(2)} = \min\left\{\frac{1}{2},1\right\} = 0.5$$

$$K_{\text{info}(3)} = \min\left\{\frac{0}{2},1\right\} = 0$$

$$K_{\text{info}(4)} = \min\left\{\frac{1}{2},1\right\} = 0.5$$

$$K_{\mathrm{info}(5)} = \min\left\{\frac{0}{2},1\right\} = 0$$

So $N'(r_k)$=1， $N(r_k)$=1+3=4.

*Program: **modifrac.m***

Part III

User interface for FD calculation is designed by MATLAB APP Designer.

Based on MATLAB programs above, an application which can be run independently is written by MATLAB APP Designer. In this application, users can upload images, and then FDs by four methods will be shown directly with just a click.

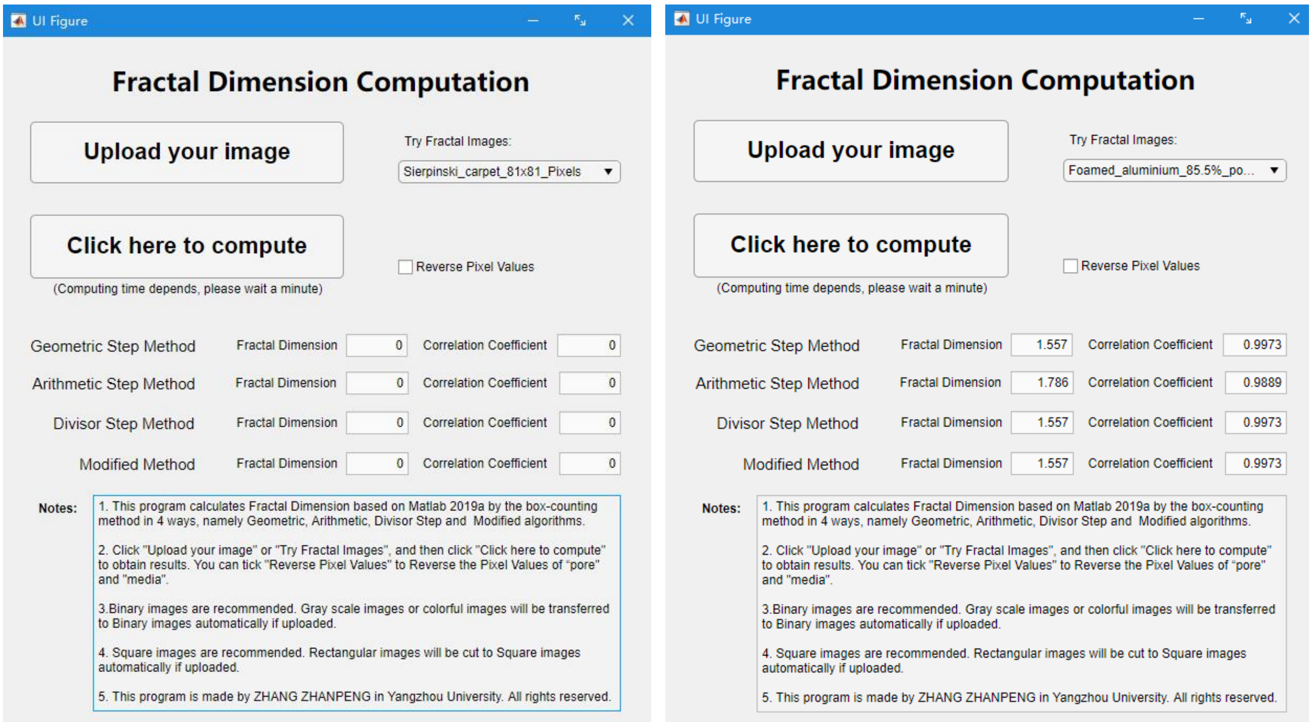

Fig.4

APP: **FDComputation.exe**

Program: **UI figure.txt**