KAZAKH BRITISH TECHNICAL UNIVERSITY

CURRENTS IN MATERIALS. DIRECT CURRENT.CIRCUITS

Bakranova Dina Igorevna

Lecture 10

- Direct current circuits.
- Battery, internal resistance.
- Resistivity
- Resistors in parallel and in series.
- Kirchhoff rules.

Battery

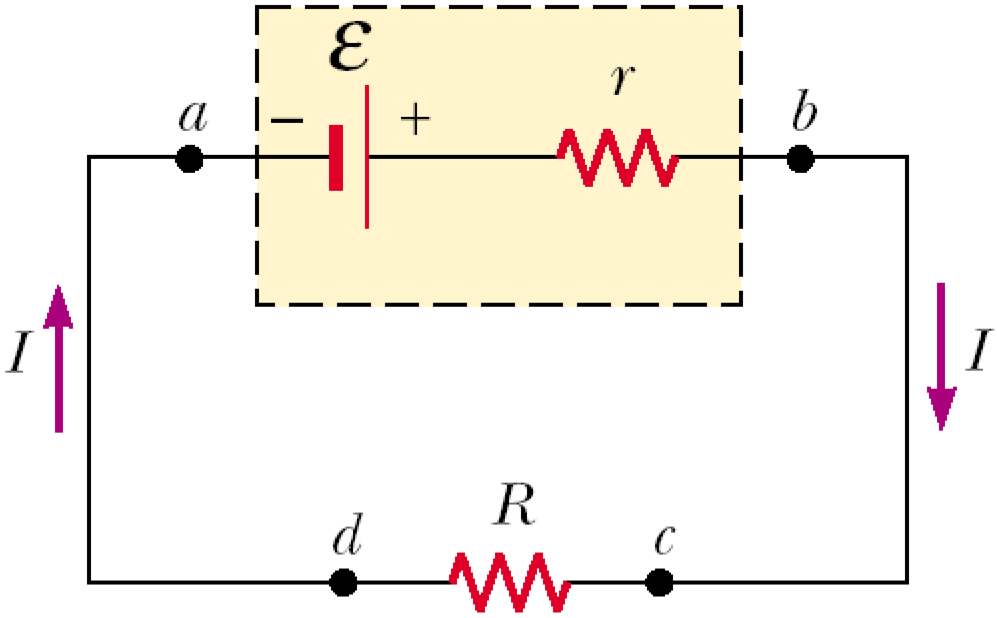
- The *emf* of a battery is the maximum possible voltage that the battery can provide between its terminals.
- Because a real battery is made of matter, there is resistance to the current within the battery.
- This resistance is called **internal resistance *r***.

Direct and Alternating current

There exist two types of current:

Direct current (**dc**) is the continuous flow of charge in only one direction. The whole lecture is devoted only to direct current circuits.

Alternating current (**ac**) is a flow of charge continually changing in both magnitude and in direction.



- ε - emf
- V – potential difference on the battery ($V= V_b-V_a$)
- r – internal resistance of emf
- R – external load

V_b-V_a: $V= \varepsilon - IR$

Circuit current: $I= \varepsilon/(R+r)$

Power output of the battery is ε*I: $\varepsilon * I = I^2R + I^2r$

Power output of a Battery

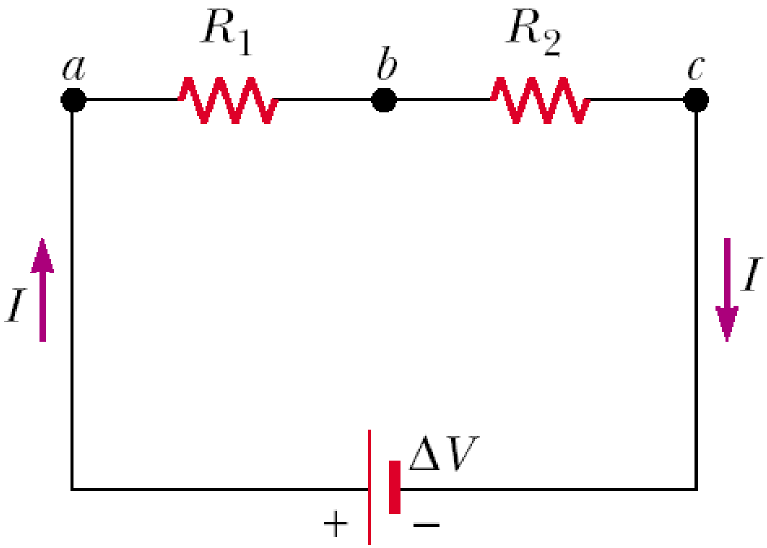
$$\varepsilon * I = I^2R + I^2r$$

- $\varepsilon * I$ - power output of the battery.
- I^2R – power transferred to the external load
- I^2r – power loss by the internal resistance
- So the power output of the battery to external resistance is accompanied by the power loss due to internal resistance.

Resistor

- Resistor is a circuit element which is used to control the current level in the various parts of the circuit. It's main property – it has constant resistivity for a wide range of potential differences.

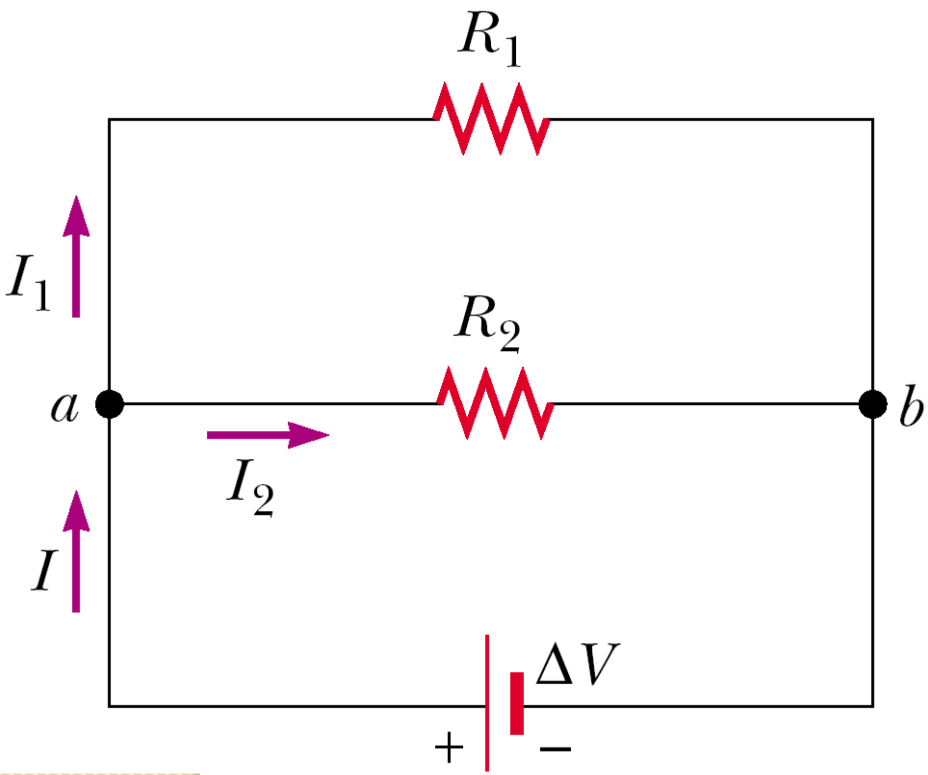
Resistors in Series



- $I_{ac} = I_1 = I_2$
- $V_{ac} = V_1 + V_2$
- $R_{ac} = R_1 + R_2$

- Currents I1 and I2 are the same in both resistors because the amount of charge that passes through must also pass through in the same time inter interval.

Resistors in Parallel



- $I = I_1 + I_2$
- $V_{ac} = V_1 = V_2$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

- When resistors are connected in parallel, the potential differences across the resistors are the same.

Any number of resistors

- **In series:**

  $I=I_1=I_2=I_3=\ldots$

  $V=V_1+V_2+V_3+\ldots$

  $R_{ac}=R_1+R_2+R_3+\ldots$

- **In parallel:**

  $I=I_1+I_2+I_3+\ldots$

  $V=V_1=V_2=V_3=\ldots$

$$\frac{1}{R_{\text{eq}}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\cdots$$

Kirchhoff's Rules for Direct Current Circuits

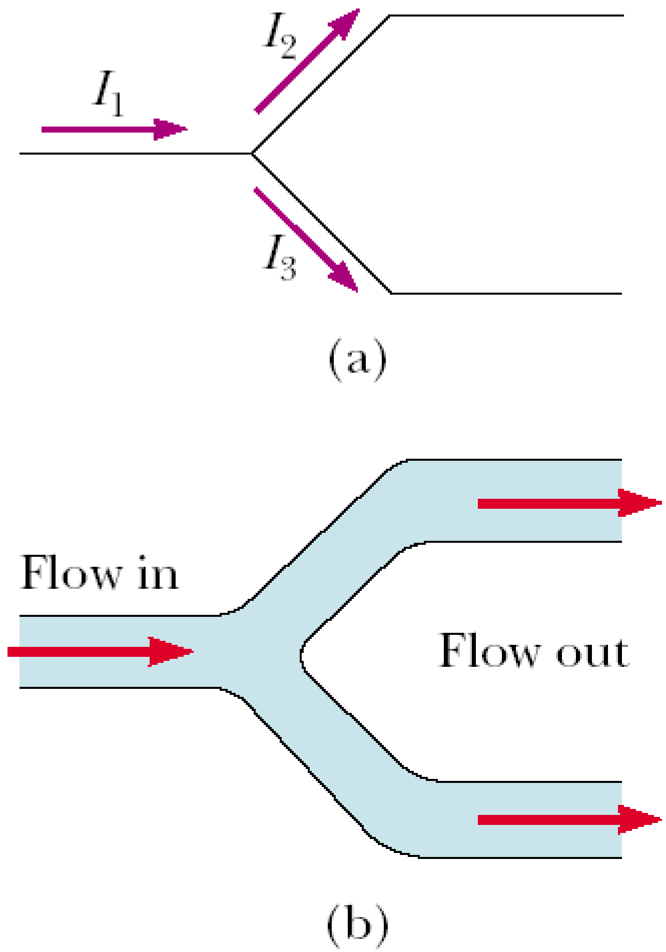
1. **Junction rule.** The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction.

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

2. **Loop rule.** The sum of the potential differences across all elements around any closed circuit loop must be zero.

$$\sum_{\substack{\text{closed}\\ \text{loop}}} \Delta V = 0$$

Junction Rule



- $I_1 = I_2 + I_3$
- The Kirchhoff's junction rule is an analogue for fluid current.
- The junction rule is a consequence of the Charge conservation law

Loop Rule Basis

- Kirchhoff's second rule follows from the law of conservation of energy. Let us imagine moving a charge around a closed loop of a circuit. When the charge returns to the starting point, the charge – circuit system must have the same total energy as it had before the charge was moved. The sum of the increases in energy as the charge passes through some circuit elements must equal the sum of the decreases in energy as it passes through other elements.
- The potential energy decreases whenever the charge moves through a potential drop $-IR$ across a resistor or whenever it moves in the reverse direction through a source of emf. The potential energy increases whenever the charge passes through a battery from the negative terminal to the positive terminal.

Kirchhoff’s rules validity

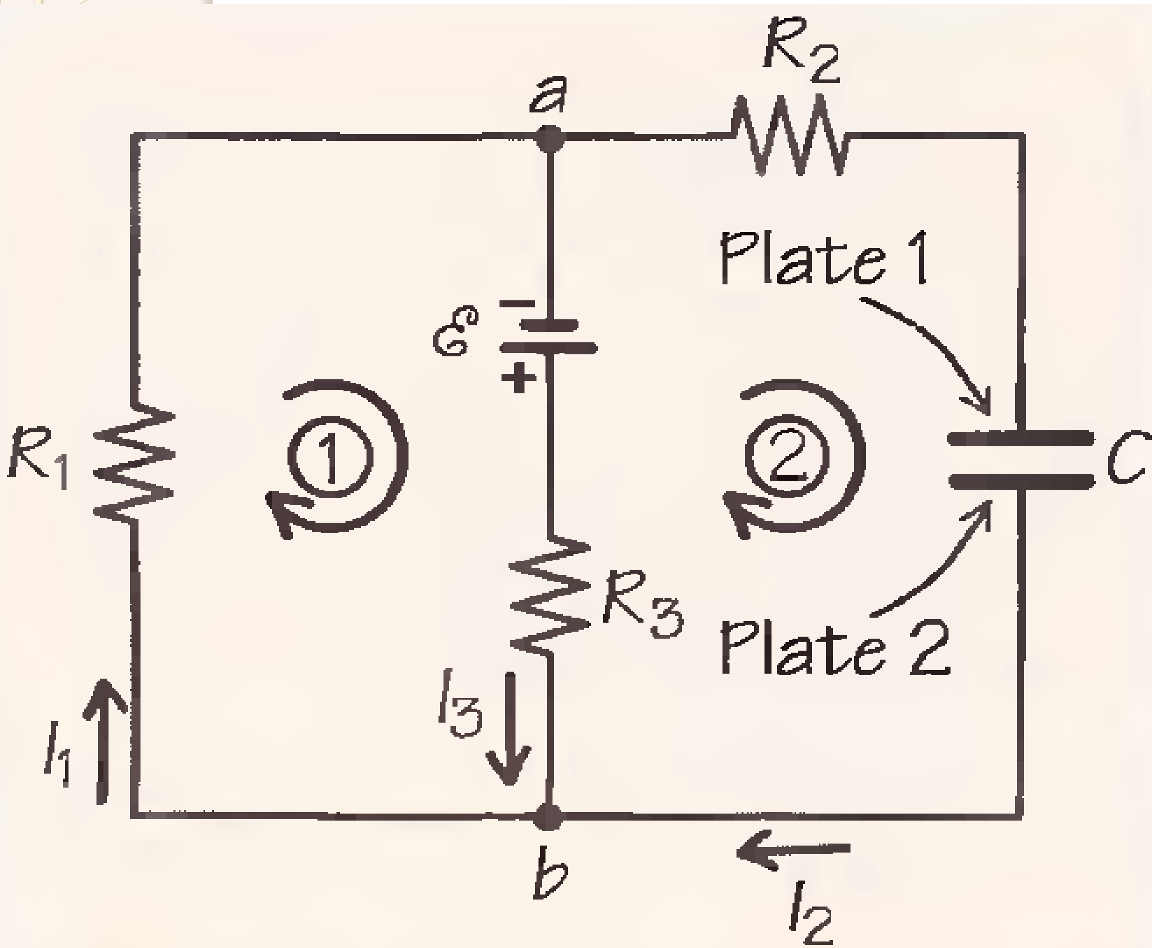
- Kirchhoff’s rules are valid only for steady-state conditions - that is, the currents in various branches are constant.
- Any capacitor acts as an open branch in a circuit; that is, the current in the branch containing the capacitor is zero under steady-state conditions.

PROBLEM-SOLVING HINTS

Kirchhoff's Rules

- Draw a circuit diagram, and label all the known and unknown quantities. You must assign a *direction* to the current in each branch of the circuit. Although the assignment of current directions is arbitrary, you must adhere rigorously to the assigned directions when applying Kirchhoff's rules.
- Apply the junction rule to any junctions in the circuit that provide new relationships among the various currents.
- Apply the loop rule to as many loops in the circuit as are needed to solve for the unknowns. To apply this rule, you must correctly identify the potential difference as you imagine crossing each element while traversing the closed loop (either clockwise or counterclockwise). Watch out for errors in sign!
- Solve the equations simultaneously for the unknown quantities. Do not be alarmed if a current turns out to be negative; *its magnitude will be correct and the direction is opposite to that which you assigned.*

Example: a multiloop circuit



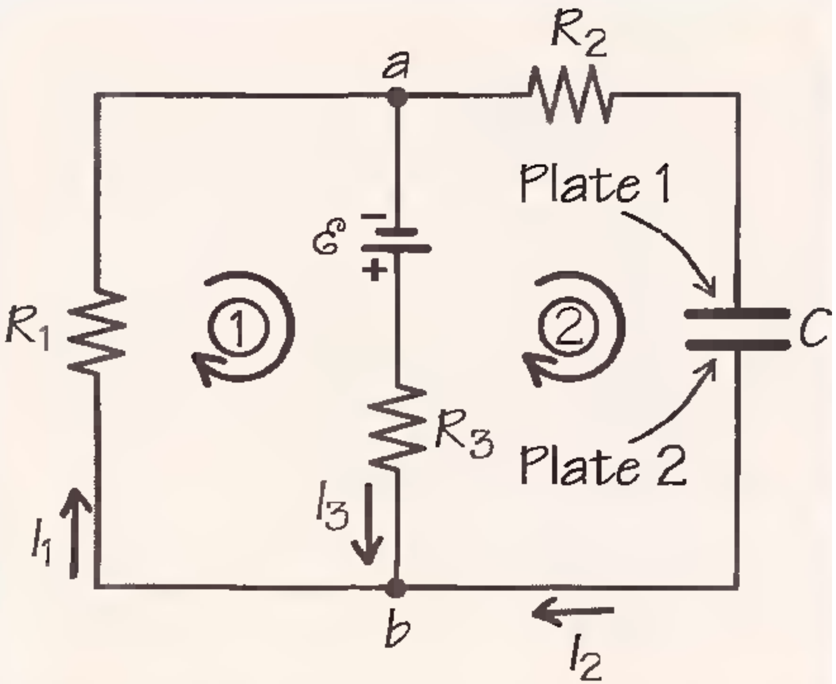
Given:

All currents are steady state, I_3=50mA,
e=6V,
R_1=100 W,
R_2= 80 W,
C=2mF.

Find:

I_1, I_2, R_3, V_C.
V_C is the capacitor's voltage

- All currents are steady state means that there is no changes in currents. In steady-state condition the capacitor acts as an open switch despite the fact that it has voltage.



I_3=50mA,
ε=6V,
R_1=100 Ω,
R_2= 80 Ω,
C=2μF.
I_1, I_2, R_3, V_C= ?

So first we choose directions in the two circuits as it shown in the picture.

$I_2=0$, as the capacitor is not charging. =>

=> For junction b: $I_3=I_1$.

For loop 1: $\varepsilon \;\; - I_3R_3 - I_1R_1$ =>

$R_3= \varepsilon/I_1 - R_1$

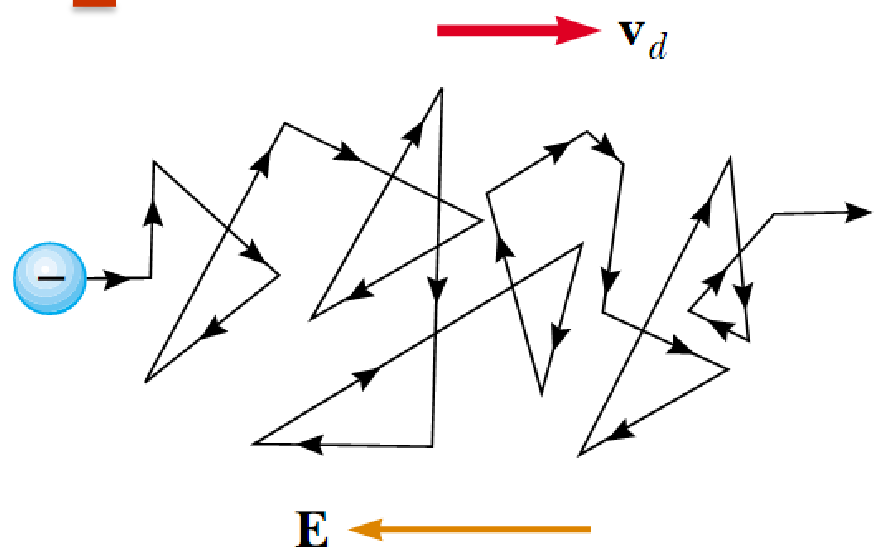
For loop 2: $\varepsilon - I_3R_3 - V_C = 0$ =>

$V_C= \varepsilon - I_3R_3 = \varepsilon - I_3R_3= I_1R_1$

Types of Conductivity

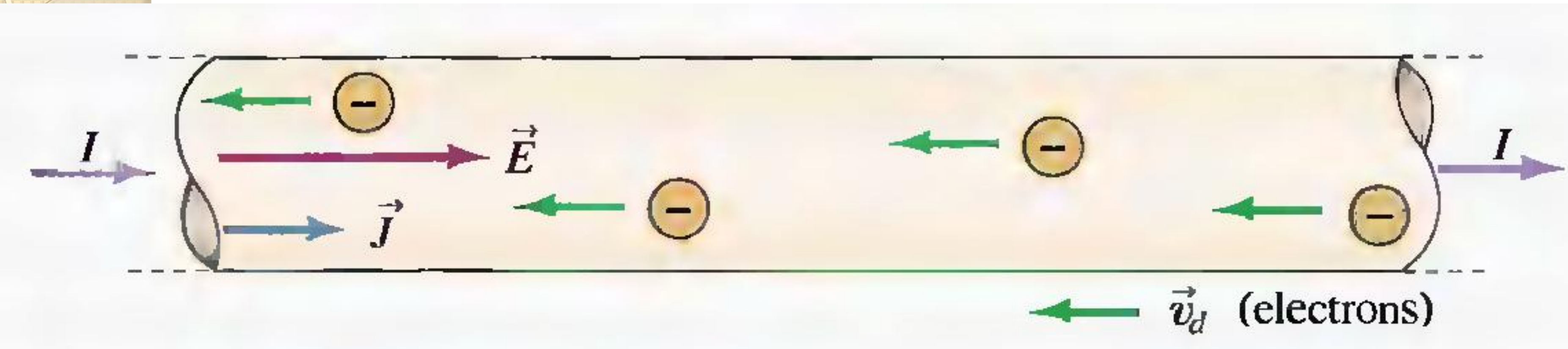
- ***Conductors*** are materials through which charge moves easily. *(metals, electrolytes, superconductors, plasmas and some nonmetallic conductors such as graphite and conductive polymers)*
- ***Semiconductors*** are materials intermediate to conductors and insulators. *(silicon, germanium, gallium arsenide, silicon carbide, orgainic cemiconductors)*
- ***Insulators*** are materials through which charge does not move easily. *(glass, paper, Teflon, rubber, rubber-like polimers and most plastics)*

Drift speed of electrons

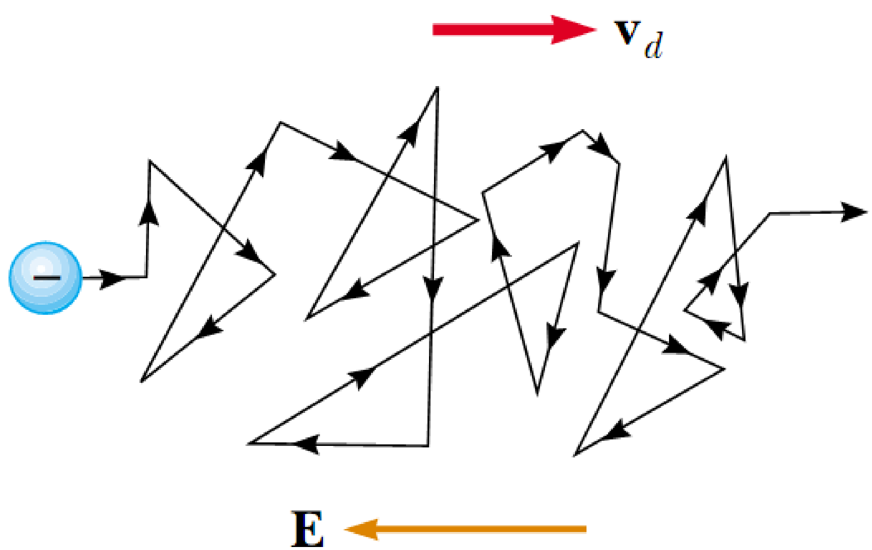


- There is a zigzag motion of an electron in a conductor. The changes in direction are the result of collisions between the electron and atoms in the conductor. The net motion – **drift speed** of the electron is opposite the direction of the electric field.

- So when we consider electric current as a flow of electrons:



in reality there happens zigzag motion of free electrons in the metal:



Current in metals

- Every atom in the metallic crystal gives up one or more of its outer electrons. These electrons are then free to move through the crystal, colliding at intervals with stationary positive ions, then the resistivity is:

$$\mathbf{r = m/(ne^2t)}$$

n - the number density of free electrons,

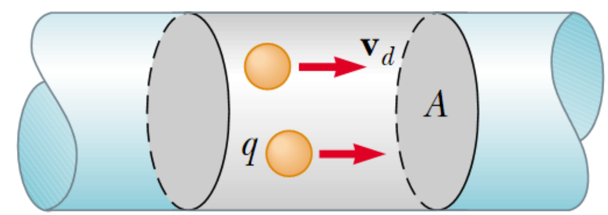
m and **e** – mass and charge of

Resistivity

- A conductor with current:

$$J \equiv \frac{I}{A} = nqv_d$$

- Current density:



$$\mathbf{J} = nq\mathbf{v}_d$$

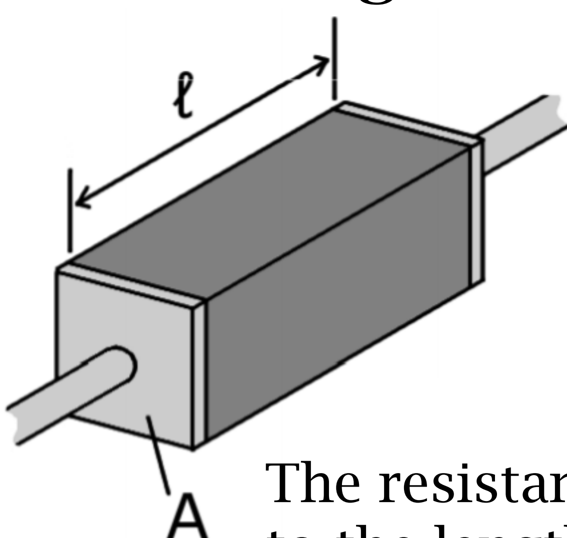
- I – electric current
- A – the cross-sectional area of the conductor
- v_d – drift speed

$$\mathbf{E} = \rho\mathbf{J}$$

ρ - resistivity

Resistivity

- All wires in a circuit also contribute to the overall resistance in a circuit. Even though the value is often small and negligible, it is often important to determine the value for the resistance of a wire if it is thick or long. This being said, the resistance is dependant y (



$$R \alpha \ell \qquad R \alpha \frac{1}{A}$$

$$\rho = \text{Resistivity Constant}$$

$$R = \frac{\rho \ell}{A}$$

The resistance of the wire is DIRECTLY proportional to the length and inversely proportional to the area. The constant of proportionality is then defined as the RESISTIVITY, which is based on material type.

Resistivity

Ohm's law → $\vec{J}$ directly proportional to $\vec{E}$.

(Intrinsic material property)

1 Ohm = 1 Ω = V/A

Resistivity: $\boxed{\rho = \dfrac{E}{J}}$

Units: Ω m = (V/m)/(A/m²) = (V/A) m

Substance		$\rho\,(\Omega\cdot\mathrm{m})$	Substance	$\rho\,(\Omega\cdot\mathrm{m})$
Conductors			**Semiconductors**	
Metals	Silver	1.47×10^{-8}	Pure carbon (graphite)	3.5×10^{-5}
	Copper	1.72×10^{-8}	Pure germanium	0.60
	Gold	2.44×10^{-8}	Pure silicon	2300
	Aluminum	2.75×10^{-8}	**Insulators**	
	Tungsten	5.25×10^{-8}	Amber	5×10^{14}
	Steel	20×10^{-8}	Glass	10^{10}–10^{14}
	Lead	22×10^{-8}	Lucite	$>10^{13}$
	Mercury	95×10^{-8}	Mica	10^{11}–10^{15}
Alloys	Manganin (Cu 84%, Mn 12%, Ni 4%)	44×10^{-8}	Quartz (fused)	75×10^{16}
	Constantan (Cu 60%, Ni 40%)	49×10^{-8}	Sulfur	10^{15}
	Nichrome	100×10^{-8}	Teflon	$>10^{13}$
			Wood	10^{8}–10^{11}

Conductivity

- A current density J and an electric field E are established in a conductor whenever a potential difference is maintained across the conducto $\mathbf{J} = \sigma \mathbf{E}$

σ is conductivity:

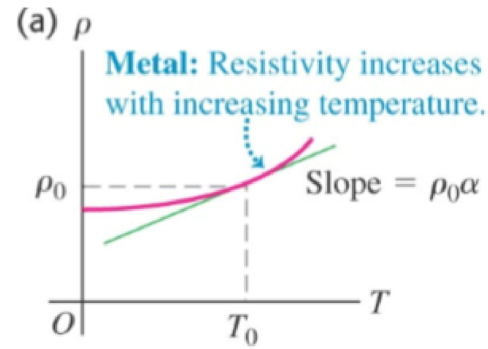
$$\sigma = 1/\rho.$$

Conductivity

Conductivity: 1/ρ

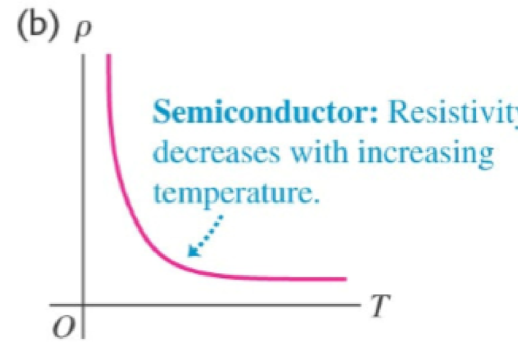
Metals: good electrical and thermal conductors. Very large difference in conductivity of metals vs. insulators → possible to confine electric currents.

Semiconductors: intermediate resistivity between metal & insulator.



Resistivity and Temperature:

$$\boxed{\rho(T) = \rho_0[1+\alpha(T-T_0)]}$$



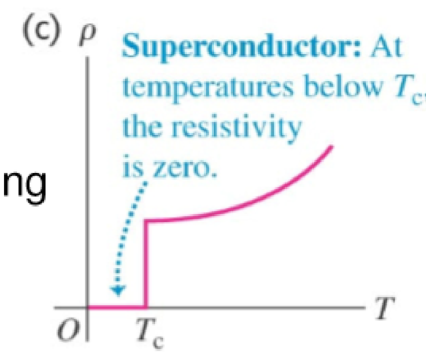
α = temperature coefficient of resistivity

Metal: ρ increases with T

Semiconductor: ρ decreases with T

Superconductor: ρ first decreases smoothly with decreasing T and becomes zero < Tc (critical T)

Highest T_c = 233 K (2009) → $Ta_5Ba_4Ca_2Cu_{10}O_x$



Temperature Coefficient of Resistivity (near Room Temperature)

Material	$\alpha\,[(°C)^{-1}]$	Material	$\alpha\,[(°C)^{-1}]$
Aluminum	0.0039	Lead	0.0043
Brass	0.0020	Manganin	0.00000
Carbon (graphite)	−0.0005	Mercury	0.00088
Constantan	0.00001	Nichrome	0.0004
Copper	0.00393	Silver	0.0038
Iron	0.0050	Tungsten	0.0045

Ohm's law again

- For many materials (including most metals), the ratio of the current density to the electric field is a constant **σ** that is independent of the electric field producing the current:

$$\mathbf{J} = \boldsymbol{\sigma}\,\mathbf{E}$$

Units in Si

- Capacitance C F=C/V
- Current I A=C/s
- Resistance R
  Ohm=V/A
- Resistivity ρ
  $\Omega \cdot m = (V/A) \cdot m$
- Electro motive
- force (emf) ε V

Control questions:

- What does EMF mean?
- What types of current are exist?
- What are total current, voltage and resistance in parallel and series connection?
- What is the first Kirchhoff's Rule for Direct Current Circuits?
- What is the second Kirchhoff's Rule for Direct Current Circuits?
- What types of Conductivity do you know?
- What are units in SI of capacitance,

Main terms

- current
- resistance
- ampere
- emf
- ohm
- rheostat
- ammeter
- voltmeter
- transient current
- source of emf
- electric power
- Ohm’s law
- resistivity
- circular mil
- DC circuit
- series connection
- parallel connection
- terminal potential difference
- internal resistance
- Kirchhoff’s first law
- Kirchhoff’s second law