Correctness of Horner's Method for Evaluating a Polynomial

```
function Horner(A[0...n], x)
    s ← A[n]
    i ← n
    while i > 0 do
        i ← i − 1
        s ← s · x + A[i]
    end while
    return s
end function
```

To prove the correctness of the algorithm, the loop invariant we will use is: $s = \sum_{j=0}^{n-i} A_{n-j} x^{n-i-j}$.

1. Just before the "while" statement starts executing, we indeed have $i = n$ and $s = \sum_{j=0}^{0} A_{n-j} x^{n-i-j} = A_n$

2. When executing the body of the loop, let us suppose at the start that $i = i_s$ and $s = A_n x^{n-i_s} + A_{n-1} x^{n-i_s-1} + \cdots + A_{i_s}$
(i_s will be used for the "start" value of i in order to avoid confusion due to the changing i.)
Then after the loop body executes, we will have $i = i_s - 1$ and:

$$s = (A_n x^{n-i_s} + A_{n-1} x^{n-i_s-1} + \cdots + A_{i_s})x + A_{i_s-1} =$$
$$A_n x^{n-i_s+1} + A_{n-1} x^{n-i_s} + \cdots + A_{i_s} x + A_{i_s-1} =$$
$$A_n x^{n-i} + A_{n-1} x^{n-i-1} + \cdots + A_{i+1} x + A_i$$

Therefore, the condition will indeed be preserved by each iteration of the loop.

3. After the loop exits, we will have $i = 0$, so $s = A_n x^n + A_{n-1} x^{n-1} + \cdots + A_1 x + A_0$ and the next "return" statement will return the correct value, according to our postcondition Q.