Targeted Advertising in Elections*

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PRELIMINARY AND INCOMPLETE

LATEST VERSION

Abstract

Some elections are unwinnable for challengers because pivotal voters prefer policies on the opposite sides of the status quo. In this paper, I argue that the challenger can win any such election if he uses targeted advertising with verifiable messages. In his private ads, the challenger makes each voter believe that his policy is a sufficient improvement over the status quo and wins the election when his policy is sufficiently moderate. Targeted advertising makes the voters regret their choices and minimizes the voter welfare relative to the complete information and public advertising benchmarks. As a voter's favorite policy becomes more extreme, her dissatisfaction with the status quo grows, and she becomes persuadable by a wider range of policies. As a result, the challenger's odds of winning increase.

KEYWORDS: Persuasion, Targeted Advertising, Elections

JEL CLASSIFICATION: D72, D82, D83

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1. Introduction

Targeted advertising played an important role in the recent US Presidential Elections. In 2016, the Trump campaign used voter data from Cambridge Analytica to target voters via Facebook and Twitter. In 2008, the Obama campaign pioneered the use of social media to communicate with the electorate. Even before social media, in 2000, The Bush campaign targeted voters via direct mail. Given that the winning candidate had access to better technology or better voter data in all these cases, one may wonder whether targeted advertising was why these candidates won.[1] Would they have lost without targeted advertising? In other words, can targeted advertising swing electoral outcomes and help win elections that are otherwise unwinnable?

To answer these questions, I consider the following baseline model of targeted advertising in elections. There is an underlying policy space $[-1, 1]$, and three players: the challenger and two voters. The status quo policy is fixed at 0. The two voters have bliss points $L < 0$ and $R > 0$. Each voter prefers to approve the challenger's policy whenever it is closer to her bliss point by at least $\varepsilon > 0$. The challenger is privately informed about his policy $x \in [-1, 1]$ which is drawn from a common prior distribution with full support. The challenger is office-motivated and receives a payoff of one if both voters unanimously approve his policy, and zero otherwise. The challenger communicates with the voters using messages that contain a grain of truth. Specifically, he can lie by omission, and send a message that contains more than just his policy. At the same time, he cannot lie by commission and send a message that does not include his policy.

Notice that the baseline election is unwinnable for the challenger without targeted advertising. Specifically, his odds of winning are zero in every equilibrium, under every communication protocol that does not allow different messages to different voters. The left voter prefers policies to the left of the status quo, while the right voter prefers policies on the right. Since the challenger's policy cannot be left and right at the same time, at most one of the voters is willing to approve it under complete information. Similarly, the challenger's policy cannot be both left and right on average, meaning that at most one of the voters is willing to approve it under

[1]For comparison of advertising strategies between the candidates, see Kim et al. (2018) and Wylie (2019) for the 2016 election, Harfoush (2009) and Katz, Barris, and Jain (2013) for the 2008 election, and Hillygus and Shields (2014) for the 2004 election.

common belief. As a result, the challenger definitely loses if he does not advertise, if he fully discloses his policy, or if he advertises his policy publicly.

When the challenger has access to targeted advertising, he can tell different things to different voters. In his most preferred equilibrium, the challenger makes the left voter believe that his policy is, on average, to the left of the status quo. He induces that belief by pooling this voter's favorite policies on the left with as many right policies, as possible. Similarly, in his private communication with the right voter, the challenger insists that his policy is, on average, on the right. The challenger wins the election whenever both voters approve, which happens with positive probability. That said, the challenger only benefits from private communication if his policy is sufficiently close to the status quo: the further to the right (left) his policy is, the harder it becomes to convince the left (right) voter.

When a voter becomes more extreme, her dissatisfaction with the status quo grows, which makes her more persuadable. Consequently, as the electorate becomes more polarized, the challenger's odds of swinging an unwinnable election increase. As the right voter becomes more extreme, she becomes persuadable by a wider range of policies, including policies further to the left. As a result, the equilibrium set of unanimously approved policies shifts to the left.

Related Literature (In Progress)

This paper contributes to the growing literature on voter persuasion. Most of the previous work has focused on information design (Kamenica and Gentzkow, 2011, Alonso and Câmara, 2016), cheap talk (Crawford and Sobel, 1982, Schnakenberg, 2015, Jeong, 2019), and, like me, verifiable disclosure (Milgrom, 1981, Grossman, 1981, Caillaud and Tirole, 2007, Jackson and Tan, 2013).

I am not the first person to compare private and public communication. In the verifiable information literature, the closest paper to mine is Schipper and Woo (2019), who study advertising competition. They show that even with targeted advertising, the candidates tend to voluntarily disclose all their private information. This unraveling result is fairly common in the verifiable information literature on voter persuasion (Board, 2009; Janssen and Teteryatnikova, 2017), and arises because the candidates play a zero-sum game. In contrast to these papers, I consider a non-symmetric model in which one candidate has a significant advantage over his opponent in that he is the only one who can communicate with the voters. Unraveling does not necessarily occur, and the challenger can improve his odds of winning over full disclosure.

A lot of progress has been made comparing public and private disclosure in the cheap talk literature. One robust finding is that the sender often prefers to communicate in public, rather than in private (Farrell and Gibbons, 1989, Koessler, 2008, Goltsman and Pavlov, 2011, Bar-Isaac and Deb, 2014), because public communication reduces the number of possible deviations available to the sender in each state of the world. When his messages are verifiable, the sender's message space is already restricted, and there is no such effect. Consequently, my main result is the opposite: the sender strictly benefits from private advertising when his messages are verifiable, to the point that he can win elections that are unwinnable otherwise.

In information design, the sender prefers private communication to advertising in public (Arieli and Babichenko, 2019), even if the receivers are strategic and condition on the event of being pivotal (Bardhi and Guo, 2018, Chan et al., 2019, Heese and Lauermann, 2019). I confirm this finding: while my sender does not possess any commitment power, the sender-preferred equilibrium outcome is also a commitment outcome (Titova, 2021). Beyond that, my contribution is twofold: on the one hand, I conclude that the sender does not need commitment power to benefit from targeted advertising. On the other hand, not only does he improve his ex-ante utility by communicating in private; he improves it from 0 in every equilibrium to a positive number in his most-preferred equilibrium.

The model sheds more light on how political advertising, especially targeted advertising, affects electoral outcomes and why it has become widespread. DellaVigna and Gentzkow (2010) and Prat and Strömberg (2013) provide excellent surveys of the evidence of voter persuasion. First, candidates target their ads based on voters' positions on the political spectrum (George and Waldfogel, 2006; DellaVigna and Kaplan, 2007). Second, one can make a case that an increase in the availability of information catered toward certain electoral groups also counts as targeted advertising because these are the messages intended for and heard by these groups (Oberholzer-Gee and Waldfogel, 2009; Enikolopov, Petrova, and Zhuravskaya, 2011). I show that targeted political advertising may be so widespread because it allows politicians to win elections that are unwinnable otherwise.

I also contribute to the growing literature on polarization and targeted political advertising through media. As the number of media outlets increases, they become more specialized and target voters with more extreme preferences, which leads to social disagreement (Perego and Yuksel, 2022). If the electorate is polarized to begin with, so are the candidates' chosen policy platforms (Hu, Li, and Segal, 2019; Prummer, 2020). Abstracting away from candidates choosing their policies, I find that as the electorate becomes more polarized, more challengers can swing elections that are unwinnable otherwise.

2. Baseline Election: Model

I study an interaction between a politician who challenges the status quo (the challenger, he/him) and the voters (she/her). There is an underlying policy space $X := [-1, 1]$ with policy positions ranging from far-left (-1) to far-right (1). The status quo policy is fixed, known, and normalized to 0. The game begins with the challenger privately observing his policy $x \in X$, which is drawn from a common prior distribution $\mu_0 \in \Delta X$ with full support.

The challenger is *office-motivated* and his goal is to win the election. In the baseline election, there are two voters, $left$ and *right*, and the challenger needs both voters to approve his proposal to win the election. I normalize his payoff from winning to 1 and losing to 0.

The challenger advertises his policy to the voters via private verifiable messages. Specifically, each message m that the challenger may send *(i)* is a statement about his policy, $m \subseteq X$, and *(ii)* contains a grain of truth, $x \in m$. That is, the challenger *can lie by omission* and send messages that contains policies other than x. At the same time, he *cannot lie by commission* and send messages that do not include x. Verifiability of messages allows the voters to draw inferences about the challenger's policy. For example, suppose that a voter hears message $[-1/2, 0]$, or "my policy is moderately left". She concludes that the challenger's policy is not far-left or anywhere on the right. At the same time, she does not know the exact location of the challenger's policy between $-1/2$ and 0.

The voters have spatial preferences with a status quo bias. Voter with bliss point $v \in X$, to whom I will sometimes refer as "voter v," prefers to approve the challenger's policy $x \in X$ if she considers it a sufficient improvement over the status quo. Specifically, she prefers to approve whenever x is closer than 0 to v by at least

$\varepsilon > 0$.[2] Otherwise, she prefers to reject. Mathematically, when the challenger's policy is $x \in X$, the payoff of voter with bliss point $v \in X$ is

$$u_v(\text{approve}, x) = -|v - x| - \varepsilon, \qquad u_v(\text{reject}, x) = -|v|.$$

To simplify analysis, let $\alpha_v(x) := u_v(\text{approve}, x) - u_v(\text{reject}, x) = |v| - |v - x| - \varepsilon$ be v's net payoff from approval. Now, this voter's best response is to approve the challenger's policy $x \in X$ whenever her net payoff from approval $\alpha_v(x)$ is non-negative. Also, let $A_v := \{x \in X \mid \alpha_v(x) \geq 0\}$ be her (complete-information) approval set that includes all policies of the challenger that she prefers to approve under complete information. I let $\lceil A_v \rceil := \max A_v$ be the largest and $\lfloor A_v \rfloor := \min A_v$ be the smallest elements of v's approval set.

In the baseline election, the *left* voter has bliss point $L \in [-1, -\varepsilon)$ and the *right* voter has bliss point $R \in (\varepsilon, 1]$. These conditions ensure that each voter's approval set has a positive prior measure.[3] Figure 1 illustrates the *left* voter's preferences.

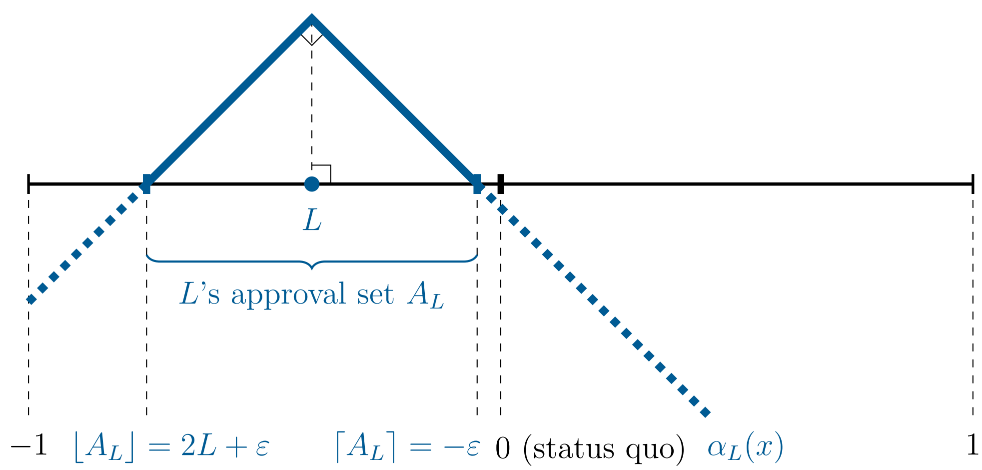

Figure 1. *The policy space $X = [-1, 1]$, the left voter with bliss point $L < -\varepsilon$, her net payoff from approval $\alpha_L(x)$, and her approval set A_L. Voter L considers policies in A_L to be a sufficient (by more than ε) improvement over the status quo.*

[2] Here, ε is the status quo bias, or the cost of voting. The cost only applies when voting to approve, as abstention is a de facto vote to reject, because unanimous approval is required for the challenger to win.

[3] Specifically, $\lfloor A_L \rfloor = \max\{-1, 2L + \varepsilon\}$, $\lceil A_L \rceil = -\varepsilon$, $\lfloor A_R \rfloor = \varepsilon$, and $\lceil A_R \rceil = \min\{1, 2R - \varepsilon\}$. If $L \in [-1, -\varepsilon)$ and $R \in (\varepsilon, 1]$ then $\int_{A_v} \alpha_v(x) d\mu_0(x) > 0$ for each $v \in \{L, R\}$.

I focus on the challenger-preferred Perfect Bayesian equilibrium of this game. Knowing his policy x, the challenger chooses verifiable messages $m_L \subseteq X$ and $m_R \subseteq X$ for voters L and R, respectively. Verifiability requires that $x \in m_v$ for all $v \in \{L, R\}$. Having observed message m_v, voter $v \in \{L, R\}$ forms a posterior belief over X. She then approves or rejects. In the baseline election, both voters are expressive and do not condition on the event of being pivotal.

In equilibrium, *(i)* the challenger sends messages that maximize his payoff, *(ii)* each voter approves the challenger's policy whenever her expected net payoff from approval is non-negative under her posterior belief, *(iii)* voters' posteriors on the equilibrium path are Bayes-rational. The challenger-preferred equilibrium is the one in which his odds of unanimous approval are the highest across all equilibria.

3. Baseline Election: Analysis

Incompatible Voters and Unwinnable Elections

Let us first observe that the challenger faces an electorate of voters who prefer diametrically opposing policies. As a result, the baseline election is unwinnable for him without targeted advertising.

Lemma 1. *If voters hold a common belief, then at most one of them prefers to approve.*

Proof. For both voters to prefer to approve under common belief $\mu \in \Delta X$, we need $\int \alpha_v(x) d\mu(x) \geq 0$ for each $v \in \{L, R\}$. That implies $\int \big(\alpha_L(x) + \alpha_R(x)\big) d\mu(x) \geq 0$, which is impossible since $\alpha_L(x) + \alpha_R(x) < 0$ for all $x \in X$.

□

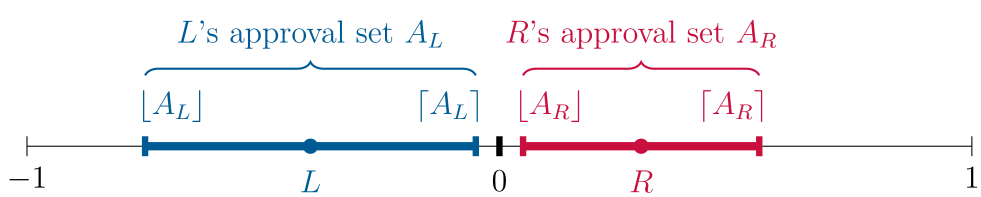

Figure 2. *Voters L and R are incompatible: L prefers to approve left policies and R prefers to approve right policies.*

Figure 2 illustrates the approval sets of the voters. Simply put, the *left* voter prefers *left (blue)* policies, while the *right* voter prefers *right (red)* policies. Since the challenger's policy cannot be both left and right at the same time, at least one of the voters prefers to reject it. The same argument applies when the voters have a common belief.

Lemma 1 implies that the baseline election is *unwinnable* for the challenger *without targeted advertising*. If he does not advertise at all, the voters hold a common prior, and at most one of them votes to approve. If he advertises publicly, the voters' common prior is updated to a common posterior, but again, at most one voter is convinced to approve.

Corollary 1. *The baseline election is unwinnable for the challenger under public disclosure. Specifically, if he is restricted to sending the same (public) message to both voters, he loses the election with probability one in every equilibrium.*

Equilibrium Outcomes under Targeted Advertising

Let us now characterize the (challenger-preferred) equilibrium payoff of the baseline election game with targeted advertising. According to Titova (2021), every equilibrium is payoff-equivalent to a direct equilibrium with *sets of approved policies* $W_L \subseteq X$ and $W_R \subseteq X$ that satisfy certain constraints. In the direct equilibrium, the challenger sends message W_v to voter $v \in \{L, R\}$ if $x \in W_v$, and its complement $W_v^c := X \setminus W_v$ otherwise. When voter v hears W_v, she approves; otherwise, she rejects the challenger's policy. We can thus interpret the message W_v as the challenger's *recommendation to approve* and the message W_v^c as the *recommendation to reject*.

To be implementable in equilibrium, voter v's set of approved policies W_v must satisfy two conditions. On the one hand, there is the sender's incentive-compatibility constraint, $A_v \subseteq W_v$, that guarantees that the challenger does not want to deviate toward a fully informative strategy. This constraint is automatically satisfied in the challenger-preferred equilibrium, because the challenger attempts to convince the voters with as many policies, as possible. On the other hand, there is the receiver's obedience constraint that ensures that voter v only approves when her average net payoff from approval is non-negative:

$$\int_{W_v} \alpha_v(x) d\mu_0(x) \geq 0. \qquad \text{(obedience)}$$

The challenger wins the election whenever both voters approve, or when $x \in W_L \cap W_R$, and his odds of winning are $\mu_0(W_L \cap W_R)$. Thus, the (challenger-preferred) equilibrium sets of approved policies $(\overline{W}_L, \overline{W}_R)$ solve

$$\max_{W_L, W_R \subseteq X} \mu_0(W_L \cap W_R) \quad \text{subject to} \quad \int\limits_{W_v} \alpha_v(x) d\mu_0(x) \geq 0 \text{ for each } v \in \{L, R\} \quad (1)$$

I refer to the pair $(\overline{W}_L, \overline{W}_R)$ that solves Problem (1) as the (challenger-preferred) equilibrium outcome (under targeted advertising). The main result of this paper establishes that the challenger can always win an unwinnable election by advertising privately.

Theorem 1. *In equilibrium of the baseline election game, the challenger's ex-ante odds of winning are always positive.*

The proof of this result is straightforward. First observe that each voter's approval set is guaranteed to convince this voter, i.e. $\int\limits_{A_v} \alpha_v(x) d\mu_0(x) > 0$ for each $v \in \{L, R\}$. Next, for each voter $v \in \{L, R\}$, select a subset $B_v \subseteq A_{-v}$ of the other voter's approval set that satisfies $\int\limits_{A_v} \alpha_v(x) d\mu_0(x) + \int\limits_{B_v} \alpha_v(x) d\mu_0(x) \geq 0$ and $\mu_0(B_v) > 0$. Let $W_v := A_v \cup B_v$ be voter v's set of approved policies. While W_L and W_R may not be equilibrium sets of approved policies, they do by construction satisfy the constraints of Problem (1). At the same time, $\mu_0(W_L \cap W_R) = \mu_0(B_R \cup B_L) > 0$, implying that the challenger's ex-ante odds of wining in equilibrium must be positive.

Before characterizing the equilibrium sets of approved policies, let us focus on the problem of maximizing the odds of convincing just one voter. Of particular interest are the cases when the voters approve *intervals* of policies, because the message "my policy is in $W_v \subseteq X$" (or "my policy is NOT in W_v^c") sounds more natural if W_v is a connected set.

One Voter's Intervals of Approved Policies

Consider a voter with bliss point $v \in X \setminus [-\varepsilon, \varepsilon]$. Let us focus on the following auxiliary problem of finding a voter's largest (in terms of prior measure) set of approved policies constrained by $l \in \big[-1, \lfloor A_v \rfloor\big]$ from the left and $r \in \big[\lceil A_v \rceil, 1\big]$ from the right.

$$\max_{W\subseteq[l,r]} \mu_0\big(W\big) \quad \text{subject to} \quad \int\limits_W \alpha_v(x)d\mu_0(x) \geq 0. \tag{AUX}$$

The solution to the auxiliary is characterized by a cutoff value for the voter's net payoff from approval (see, for example, Alonso and Câmara, 2016 and Titova, 2021). Specifically, every policy with a not too negative payoff from approval (those $x \in X$ for which $\alpha_v(x) \geq -c_v^*$) is included in the solution I_v. Then, c_v^* is obtained from the binding obedience constraint, $\int\limits_{I_v} \alpha_v(x)d\mu_0(x) = 0$. The set $\{x \in [l, r] \mid \alpha_v(x) \geq -c_v^*\}$ is an interval: it is the upper contour set of the concave function $\alpha_v(x)$, and is hence convex. Corollary 2 characterizes the solution of the auxiliary problem.

Corollary 2. *Consider a voter with bliss point $v \in X \setminus [-\varepsilon, \varepsilon]$. Then, the solution to Problem (AUX) with $l \in \big[-1, \lfloor A_v \rfloor\big]$ and $r \in \big[\lceil A_v \rceil, 1\big]$ is an interval $I_v(l, r)$ such that*

- *if $\int\limits_l^r \alpha_v(x)d\mu_0(x) \geq 0$, then $I_v(l, r) = [l, r]$;*
- *otherwise, $I_v(l, r) = \big\{x \in [l, r] \mid \alpha_v(x) \geq -c_v^*(l, r)\big\}$, and $\int\limits_{I_v(l,r)} \alpha_v(x)d\mu_0(x) = 0$.*

Two special cases of the auxiliary problem will be useful in further analysis. Firstly, there is the unconstrained version with $l = -1$ and $r = 1$. Figure 3 illustrates the largest unconstrained interval of approved policies of the $left$ voter.

Definition 1. *Consider a voter with bliss point $v \in X \setminus [-\varepsilon, \varepsilon]$. Then, this voter's* largest unconstrained interval of approved policies *is $I_v^{UC} = [a_v^{UC}, b_v^{UC}] := I_v(-1, 1)$.*

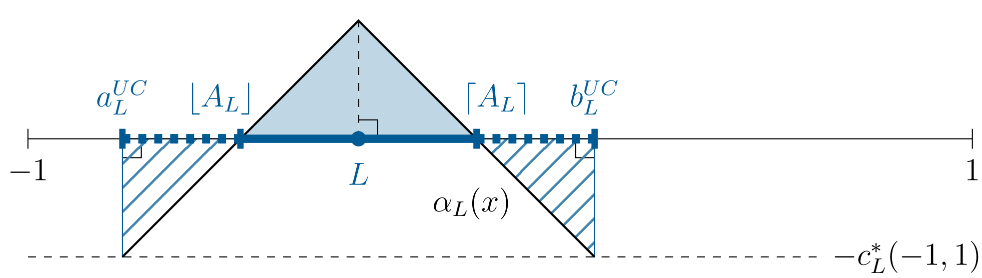

Figure 3. *$[a_L^{UC}, b_L^{UC}]$ is the $left$ voter's largest unconstrained interval of approved policies. Under uniform prior, c_L^* is obtained from equating the solid area (expected value of $\alpha_L(x)$ over A_L) to the dashed area (expected value of $\alpha_L(x)$ outside of A_L).*

The second relevant case is the largest asymmetric interval of approved policies that includes the most policies on the opposite side of the status quo from the voter's approval set. Figure 4 illustrates the *left* voter's largest asymmetric interval of approved policies.

DEFINITION 2.

- *The left* voter's largest asymmetric interval of approved policies *is*

$$I_L^{AS} = \left[\lfloor A_L \rfloor, b_L^{AS}\right] := I_L(\lfloor A_L \rfloor, 1).$$

- *The right* voter's largest asymmetric interval of approved policies *is*

$$I_R^{AS} = \left[a_R^{AS}, \lceil A_R \rceil\right] := I_R(-1, \lceil A_R \rceil).$$

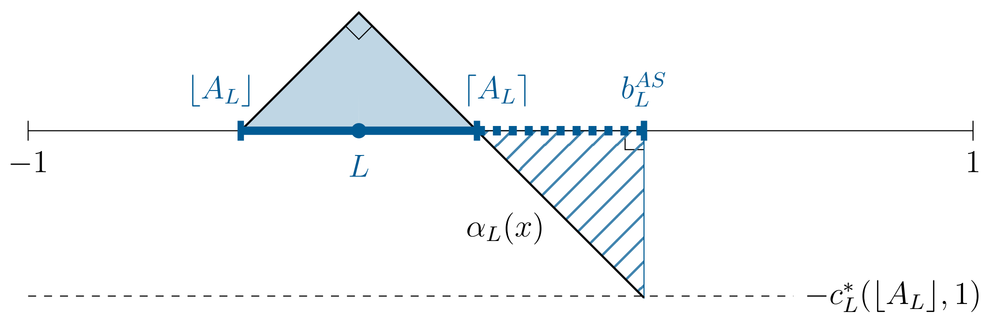

Figure 4. *$\left[\lfloor A_L \rfloor, b_L^{AS}\right]$ is the left voter's largest asymmetric interval of approved policies. Under uniform prior, c_L^* is obtained from equating the solid area (expected value of $\alpha_L(x)$ over A_L) to the dashed area (expected value of $\alpha_L(x)$ outside of A_L).*

To simplify notation, for each $v \in X \setminus [-\varepsilon, \varepsilon]$, let $\rho_v(a, b) := \int_a^b \alpha_v(x) d\mu_0(x)$ denote v's expected net payoff from approving an interval of policies $[a, b]$. Slightly abusing notation, we can conclude, for example, that $\rho_v(I_v^{UC}) \geq 0$ and $\rho_v(I_v^{AS}) \geq 0$. More generally, every interval $[a, b]$ that satisfies the voter's obedience constraint satisfies $\rho_v(a, b) \geq 0$.

Convincing Two Voters at the Same Time

Let us now solve Equation (1), or, put simply, attempt to convince both voters at the same time as often, as possible. One thing that the challenger can do is convince the *left* (*right*) voter with as many policies to the right (left) of her approval set, as possible. That is, he can let each voter's set of approved policies be her largest asymmetric interval of approved policies. I illustrate the outcome (I_L^{AS}, I_R^{AS}) in

Figure 5. As it turns out, (I_L^{AS}, I_R^{AS}) is indeed often an equilibrium outcome. The rest of this section formalizes the conditions under which the equilibrium sets of approved policies are intervals and characterizes these intervals depending on parameter values.

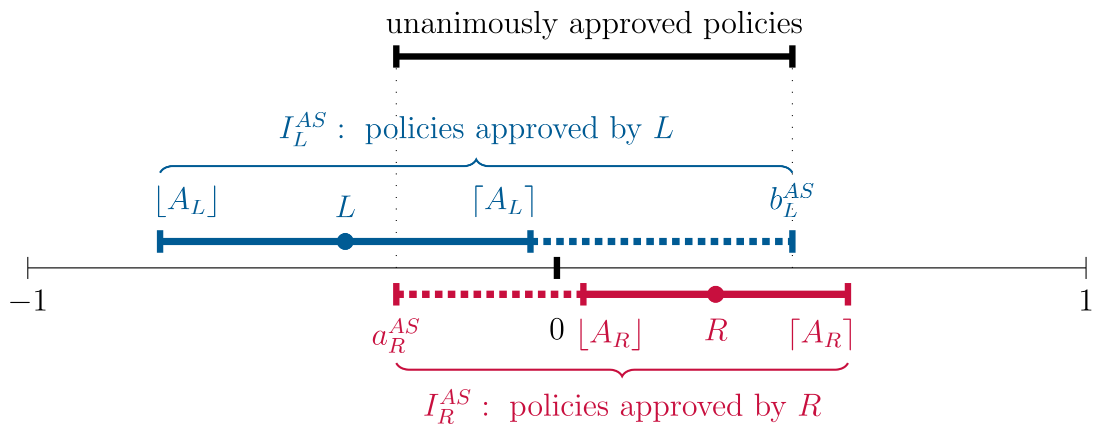

Figure 5. *The largest asymmetric intervals of approved policies of voter L (in blue) and R (in red). To convince voter L (R), the challenger pools policies that she prefers (solid) together with policies preferred by the other voter (dashed). The winning policies of the challenger (in black) are those approved by both voters.*

First of all, to guarantee that equilibrium sets of approved policies are intervals, we need to require that the status quo bias is not too large relative to $|L|$ and R. On the one hand, it is fairly straightforward to require that $\rho_L(\lfloor A_L \rfloor, \varepsilon) \geq 0$ (which implies that $b_L^{AS} \geq \varepsilon$) and $\rho_R(-\varepsilon, \lceil A_R \rceil) \geq 0$ (which implies that $a_R^{AS} \leq -\varepsilon$). Without this assumption, it is easy to show that (I_L^{AS}, I_R^{AS}) is not a solution since the challenger's odds of winning can be improved upon $\mu_0(I_L^{AS} \cap I_R^{AS})$.[4]

Assumption 1. *$\rho_L(\lfloor A_L \rfloor, \varepsilon) \geq 0$ and $\rho_R(-\varepsilon, \lceil A_R \rceil) \geq 0$.*

In some cases, to ensure an interval solution, we need to require that ε, and a voter's bliss point $v \in X \setminus [-\varepsilon, \varepsilon]$ satisfy an even stronger assumption:

Assumption 2. *$\rho_v(\lfloor A_v \rfloor - 2\varepsilon, \lceil A_v \rceil + 2\varepsilon) \geq 0$.*[5]

[4]Intuitively, at least one voter's constraint is "wasted" on policies that are not approved by the other voter. Let $Y = [\max\{a_R^{AS}, b_L^{AS}\}, \varepsilon]$, which is approved by R but rejected by L. Next, select $Z \subseteq A_L$ to satisfy $\int_Y \alpha_R(x) d\mu_0(x) = \int_Z \alpha_R(x) d\mu_0(x)$. Then, the pair $\left(I_L^{AS}, (I_R^{AS} \setminus Y) \cup Z\right)$ satisfies both constraints and improves the objective over (I_L^{AS}, I_R^{AS}) by $\mu_0(Z) > 0$.

[5]This assumption implies that $[-\varepsilon, \varepsilon] \subseteq [a_v^{UC}, b_v^{UC}]$, meaning that the voter is willing to approve at least some policies in the other voter's approval set when she is the only one who is being persuaded.

Another case when $[a_R^{AS}, b_L^{AS}]$ may not be an equilibrium set of approved policies is if $a_R^{AS} < \lfloor A_L \rfloor$, which is implied by $\rho_R(\lfloor A_L \rfloor, \lceil A_R \rceil) > 0$.[6] Intuitively, in this case, the *right* voter is so persuadable that her largest asymmetric interval of approved policies includes the *left* voter's entire approval set, and then some. Note that $\rho_v(\lfloor A_L \rfloor, \lceil A_R \rceil) > 0$ cannot hold for both $v = L$ and $v = R$ at the same time.[7] Hence, I will assume that the *left* voter is moderately persuadable, and the *right* voter could be either moderately, or significantly more persuadable than the *left* voter. The case when the *left* voter is significantly more persuadable than the *right* voter is symmetric.

THEOREM 2. *Suppose that the left voter is moderately persuadable, i.e.* $\rho_L(\lfloor A_L \rfloor, \lceil A_R \rceil) \leq 0$. *Almost surely,*[8]

(1) if Assumption 1 holds and $\rho_R(\lfloor A_L \rfloor, \lceil A_R \rceil) \leq 0$, *then*
- *each voter's equilibrium set of approved policies is her largest asymmetric interval of approved policies, or* $\overline{W}_v = I_v^{AS}$ *for each* $v \in \{L, R\}$*;*
- *the equilibrium set of unanimously approved policies* $\overline{W} = [a_R^{AS}, b_L^{AS}]$ *is sufficiently moderate, i.e.* $a_R^{AS} < -\varepsilon < \varepsilon < b_L^{AS}$*;*

(2) if L satisfies Assumption 2 and $\rho_R(\lfloor A_L \rfloor, \lceil A_R \rceil) > 0$, *then*
- *the right voter's equilibrium set of approved policies is her largest asymmetric interval of approved policies,* $\overline{W}_R = \left[a_R^{AS}, \lceil A_R \rceil\right]$*;*
- *the left voter's equilibrium set of approved policies is her largest interval of approved policies constrained from the left by* a_R^{AS}, *or* $\overline{W}_L = I_L(a_R^{AS}, 1)$*;*
- *the equilibrium set of unanimously approved policies is* $\overline{W} = \overline{W}_L$.

The formal proof of Theorem 2 is in the appendix, but I outline it below. Since the *right* voter is the more persuadable one, let us add as many left policies to her message, as possible. That is, have the *right* voter approve her largest asymmetric interval of policies, or $\overline{W}_R = I_R^{AS} = \left[a_R^{AS}, \lceil A_R \rceil\right]$. Now, Theorem 2 states that the equilibrium depends on how a_R^{AS} relates to $\lfloor A_L \rfloor$. Specifically, if we are in *Case (2)*

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[6] Observe that $\rho_R(t, \lceil A_R \rceil)$ is strictly increasing in $t < \lfloor A_R \rfloor$ since $\frac{\partial \rho_R(t, \lceil A_R \rceil)}{\partial t} = -\alpha_R(t)\mu_0(t) > 0$. Consequently, to have both $\rho_R(\lfloor A_L \rfloor, \lceil A_R \rceil) > 0$ and $\rho_R(a_R^{AS}, \lceil A_R \rceil) < 0$, we require that $a_R^{AS} < \lfloor A_L \rfloor$.

[7] We have $\int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \big(\alpha_R(x) + \alpha_L(x)\big) d\mu_0(x) < 0$ since $\alpha_R(x) + \alpha_L(x) < 0$ for all $x \in X$.

[8] "A.s." stands for almost surely with respect to the prior measure μ_0.

of Theorem 2, then $a_R^{AS} < \lfloor A_L \rfloor$. Otherwise, we are in *Case (1)*. Let us consider the lower values of a_R^{AS} first.

Suppose first that the *right* voter is so persuadable that she is willing to approve *all* the left policies, i.e. $a_R^{AS} = -1$. In this case, (I_L^{UC}, I_R^{AS}) solves Problem (1), and the set of unanimously approved policies is I_L^{UC}.[9] By construction, there is no way to increase the objective beyond $\mu_0(I_L^{UC})$ while still satisfying the $left$ voter's constraint. The same argument applies whenever $I_R^{AS} \supseteq I_L^{UC}$, or for every $a_R^{AS} \in [-1, a_L^{UC}]$. Importantly, Assumption 2 guarantees that $[-\varepsilon, \varepsilon] \subseteq I_L^{UC}$, so that $a_R^{AS} \leq a_L^{UC}$ is both necessary and sufficient for $I_L^{UC} \subseteq I_R^{AS}$.[10] This case is illustrates in Figure 6 on the left.

Next, suppose that $a_L^{UC} < a_R^{AS} < \lfloor A_L \rfloor$. Now, (I_L^{UC}, I_R^{AS}) is no longer optimal: the challenger does not persuade the *right* voter with policies in $[a_L^{UC}, a_R^{AS})$, yet "wastes" the $left$ voter's constraint on them. Instead, select the $left$ voter's message out of $[a_R^{AS}, 1]$, since the *right* voter rejects the policies outside of that interval, anyway. Now, the proposed solution is $\big(I_L(a_R^{AS}, 1), I_R^{AS}\big)$, with the set of unanimously approved policies $I_L(a_R^{AS}, 1)$.[11] The challenger cannot increase his objective beyond $\mu_0\big(I_L(a_R^{AS}, 1)\big)$: it would require unanimous approval of policies to the left of a_R^{AS}, which are strictly more expensive in terms of the *right* voter's constraint than those that she already approves. Hence, the proposed solution is optimal. This case is illustrates in Figure 6 on the right.

The last case we need to consider is when $\lfloor A_L \rfloor \leq a_R^{AS} \leq -\varepsilon$, where the last inequality is implied by Assumption 1. It remains to show that the proposed solution $(\overline{W}_L, \overline{W}_R) = (I_L^{AS}, I_R^{AS})$ with the set of unanimously approved policies $\overline{W} = [a_R^{AS}, b_L^{AS}]$ maximizes the objective of Problem (1). By contradiction, suppose that the set of unanimously approved policies is a different set $\widetilde{W} \subseteq X$. Then, some subset of $\widetilde{W}$ should lie to the right of $\overline{W}$, or else it cannot maximize the objective, because the policies to the left of $\overline{W}$ are more expensive in terms of the *right* voter's constraint. Simply put, the *right* voter is already approving as many left policies in $\overline{W}$, as

[9] Without the *right* voter's constraint, apply Corollary 2 to conclude that $\overline{W}_L = I_L^{UC}$.

[10] Without this assumption, there may not exists an interval solution. For example, if $\int_{I_L^{UC} \cup A_R} \alpha_R(x) d\mu_0(x) = 0$, then $\overline{W}_L = I_L^{UC}$, $\overline{W}_R = I_L^{UC} \cup A_R$ (which is not an interval if Assumption 2 is violated for $v = L$), $\overline{W} = \overline{W}_L$.

[11] Apply Corollary 2 with $l = a_R^{AS}$ and $r = 1$ to conclude that $\overline{W}_L = I_L(a_R^{AS}, 1)$.

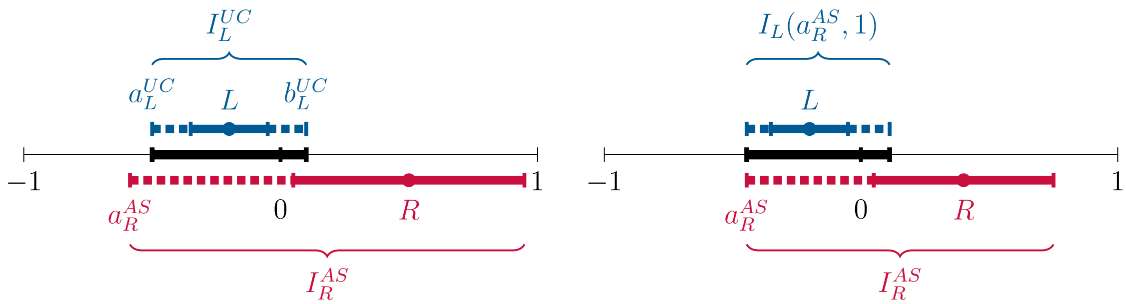

Figure 6. *Equilibrium sets of approved policies when the right voter is significantly more persuadable than the left voter.*

possible. Using a symmetric argument for the *left* voter, some subset of $\widetilde{W}$ should lie to the right of $\overline{W}$, as well. Furthermore, $\widetilde{W} \cap \big[-1, \lceil A_L \rceil\big]$ and $\widetilde{W} \cap \big[\lfloor A_R \rfloor, 1\big]$ must be intervals that end at $\lceil A_L \rceil$ and start at $\lfloor A_R \rfloor$, respectively. Otherwise, $\widetilde{W}$ can be improved upon by, for example, shifting the mass on the left toward $\lfloor A_L \rfloor$, since these policies are closer to the *right* voter's bliss point, and are therefore cheaper in terms of her constraint. At this point, $\widetilde{W} \setminus \overline{W} = [a, a_R^{AS}) \cup (b_L^{AS}, b]$ for some $a < a_R^{AS} \leq -\varepsilon$ and $b > b_L^{AS} \geq \varepsilon$, and $\overline{W} \setminus \widetilde{W} \subseteq [-\varepsilon, \varepsilon]$. Simply put, the moderate policies in $[-\varepsilon, \varepsilon]$ are outside of each voter's approval set, and it may be optimal to ignore them. Instead, we could let the *right* voter approve some policies within the *left* voter's approval set, and vice versa. As it turns out, since each $\alpha_v(x)$ is decreasing sufficiently quickly as x moves away from the approval set, that would decrease the objective.[12] This completes the outline of the proof of Theorem 2.

Note that as long as the conditions of Theorem 2 are satisfied, the *left* voter's constraint always binds, as she is the relatively less persuadable voter. The *right* voter's constraint binds unless $a_R^{AS} < a_L^{UC}$. It is also worth mentioning that identifying the equilibrium sets of approved policies $(\overline{W}_L, \overline{W}_R)$ requires solving at most two auxiliary optimization problems. Algorithm 1 describes the steps.

The following example calculates the equilibrium illustrated in Figure 5.

Example 1 (Uniform prior, $L = -0.4$, $R = 0.3$, $\varepsilon = 0.05$.). In this example, $A_L = [-0.75, -0.05]$ and $A_R = [0.05, 0.55]$. We first check the relative persuadability

[12] An extreme counterexample is when $\alpha_v(x) = \gamma > 0$ if $x \in A_v$, and $\alpha_v(x) = -\gamma$ otherwise. Suppose that $A_L = [-0.75, -0.25]$, $A_R = [0.25, 0.75]$, and the prior is uniform. If we computed the largest asymmetric intervals of approved policies, we would get $\overline{W} = [-0.25, 0.25]$ and $\mu_0(\overline{W}) = 0.25$. However, by letting $\widetilde{W}_L = \widetilde{W}_R = \widetilde{W} = [-0.75, -0.25] \cup [0.25, 0.75]$, we get $\mu_0(\widetilde{W}) = 0.5 > \mu_0(\overline{W})$.

Algorithm 1 Calculating the equilibrium sets of approved policies $(\overline{W}_L, \overline{W}_R)$

calculate $\rho_v(\lfloor A_L \rfloor, \lceil A_R \rceil) = \int\limits_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_v(x) d\mu_0(x)$ for each $v \in \{L, R\}$

if $\rho_v(\lfloor A_L \rfloor, \lceil A_R \rceil) \leq 0$ for each $v \in \{L, R\}$ **then** $\triangleright$ voters are moderately persuadable

  calculate a_R^{AS} and b_L^{AS} $\triangleright$ solve two (AUX) problems

  if $a_R^{AS} \leq -\varepsilon$ and $b_L^{AS} \geq \varepsilon$ **then** $\triangleright$ check Assumption 1

    $\overline{W}_L = I_L^{AS}$, $\overline{W}_R = I_R^{AS}$, $\overline{W} = [a_R^{AS}, b_L^{AS}]$

  else no interval solution exists

else if $\rho_R(\lfloor A_L \rfloor, \lceil A_R \rceil) > 0$ **then** $\triangleright$ *right* voter is significantly more persuadable

  if $\rho_L(\lfloor A_L \rfloor - 2\varepsilon, \varepsilon) \geq 0$ **then** $\triangleright$ check Assumption 2 for $v = L$

    $\overline{W}_R = I_R^{AS}$, $\overline{W}_L = I_L(a_R^{AS}, 1)$, $\overline{W} = \overline{W}_L$ $\triangleright$ solve two (AUX) problems

  else an interval solution may not exist

else $\triangleright$ *left* voter is significantly more persuadable

  if $\rho_R(-\varepsilon, \lceil A_R \rceil + 2\varepsilon) \geq 0$ **then** $\triangleright$ check Assumption 2 for $v = R$

    $\overline{W}_L = I_L^{AS}$, $\overline{W}_R = I_R(-1, b_L^{AS})$, $\overline{W} = \overline{W}_R$ $\triangleright$ solve two (AUX) problems

  else an interval solution may not exist

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of each voter by calculating $\int\limits_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_v(x) dx$ for each $v \in \{L, R\}$. Both of these values are negative, so it remains to calculate b_L^{AS} and a_R^{AS}.

To find b_L^{AS}, solve $\int\limits_{\lfloor A_L \rfloor}^{\lceil A_L \rceil} \alpha_L(x) dx = -\int\limits_{-\varepsilon}^{b_L^{AS}} \alpha_L(x) dx$. As illustrated in Figure 4, the former integral (the solid blue area) equals $(|L| - \varepsilon)^2$ and the latter integral (the dashed blue area) equals $\frac{(b_L^{AS} + \varepsilon)^2}{2}$. Equating them, we get $b_L^{AS} = -\varepsilon + \sqrt{2}\big(|L| - \varepsilon\big) = 0.445$ so that $I_L^{AS} = [-0.75, 0.445]$. Similarly, we find $a_R^{AS} = \varepsilon - \sqrt{2}(R - \varepsilon) = -0.304$ and $I_R^{AS} = [-0.304, 0.55]$. We confirm that $a_R^{AS} > \varepsilon$ and $b_L^{AS} < -\varepsilon$, so Assumption 1 is satisfied. We conclude that $([-0.75, 0.445], [-0.304, 0.55])$ are equilibrium sets of approved policies.

Recall that one way to implement the equilibrium outcome is by pooling all policies in $\overline{W}_v$ into one message $\overline{W}_v$ that convinces voter $v \in \{L, R\}$. In this example, $\overline{W}_L = [-0.75, 0.445]$, meaning that the challenger says that his policy is not ultra-left and not moderate- to ultra-right, but does not clarify any further. Also, that message averages out to -0.152, which is to the left of the status quo, making L think that the challenger's policy is aligned with her preferences.

Both voters approve and the challenger wins if $x \in \overline{W}_L \cap \overline{W}_R = [-0.304, 0.445]$, i.e. if his policy is sufficiently moderate. His odds of winning, calculated as the length of the interval of winning policies (0.749) relative to the length of the policy space (2), equal 0.374. We conclude that targeted advertising allows the challenger to improve his odds of winning from 0% to 37.4%!

Comparative Statics

Let us next analyze what happens when the electorate becomes more polarized. Defining polarization in the baseline model with one dimension and two voters is straightforward:

Definition 3.

- *The voter with bliss point* $v \in X \smallsetminus [-\varepsilon, \varepsilon]$ becomes more extreme *if* $|v|$ *increases.*
- *The baseline electorate* becomes more polarized *if the left and/or the right voter becomes more extreme.*

Since the voters' bliss points have to belong to the policy space, the *most extreme voter* has $|v| = 1$, and the *most polarized electorate* is $L = -1$ and $R = 1$. Note that the larger distance between the voters need not imply higher polarization. To increase polarization, one voter has to become more extreme, while the other voter has to stay fixed or also become more extreme (in the opposite direction).

Observe that when a voter becomes more extreme, she also becomes more persuadable. Using the *right* voter as an example, as R increases to R', the voter's approval set expands, enlarging the range of positive values of the net payoff from approval. As a result, the *right* voter's obedience constraint loosens. In particular, the voter becomes persuadable by a wider range of the left policies, as well.

Lemma 2. *As a voter becomes more extreme, her largest asymmetric interval of approved policies expands, i.e.* $I^{AS}_{v'} \supseteq I^{AS}_{v}$. *Specifically,*

- *if $L' < L$, then, $\left[\lfloor A_{L'} \rfloor, b_{L'}^{AS}\right] \supseteq \left[\lfloor A_L \rfloor, b_L^{AS}\right]$, with $\lfloor A_{L'} \rfloor \le \lfloor A_L \rfloor$ and $b_{L'}^{AS} \ge b_L^{AS}$; the latter inequality is strict unless $b_L^{AS} = 1$;*
- *if $R' > R$, then $\left[a_{R'}^{AS}, \lceil A_{R'} \rceil\right] \supseteq \left[a_R^{AS}, \lceil A_R \rceil\right]$, with $a_{R'}^{AS} \le a_R^{AS}$ and $\lceil A_{R'} \rceil \ge \lceil A_R \rceil$; the former inequality is strict unless $a_R^{AS} = 1$.*

The technical proof is in the appendix, but I illustrate the argument for the *right* voter in Figure 7. When her bliss point increases from R to R', her net payoff from approval $\alpha_R(x)$ remains the same for all policies to the left of R, and strictly increases otherwise. Consequently, the expected value of her net payoff from approval over her approval set, $\int_{\lfloor A_R \rfloor}^{\lceil A_R \rceil} \alpha_R(x) d\mu_0(x)$, strictly increases.[13] As a result, a larger interval of policies outside (in particular, to the left) of her approval set, now satisfies her obedience constraint.

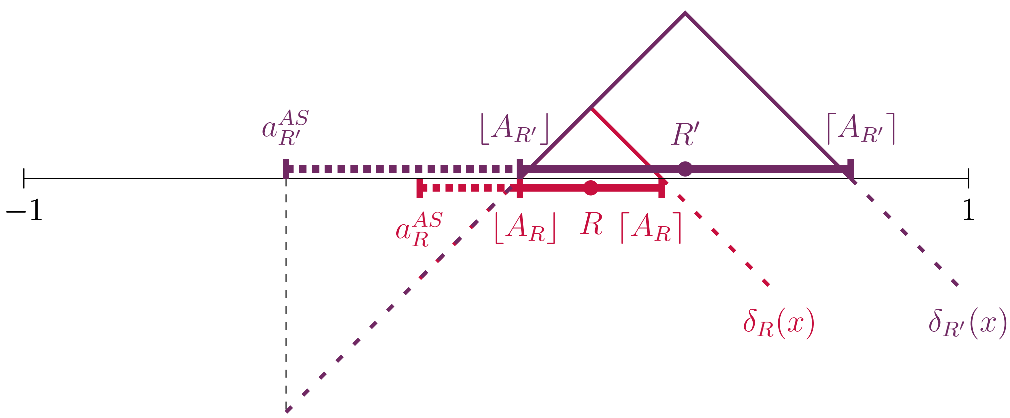

Figure 7. *The right voter becomes more persuadable by a wider range of policies as she becomes more extreme (her bliss point increases from R to R'): her approval set $\left[\lfloor A_R \rfloor, \lceil A_R \rceil\right]$ and her largest asymmetric interval of approved policies $\left[a_R^{AS}, \lceil A_R \rceil\right]$ expand.*

Now, let us consider the baseline electorate that satisfies all the assumptions of Theorem 2. That is, ε is small enough relative to $|L|$ and R, and the *left* voter is not significantly more persuadable than the *right* voter. Theorem 3 describes what happens to the equilibrium sets of approved policies as the *right* voter becomes more

[13] If $R > 0.5$, then the approval set itself remains the same, unlike in Figure 7. Yet, $\int_{\lfloor A_R \rfloor}^{\lceil A_R \rceil} \alpha_R(x) d\mu_0(x)$ strictly increases because $\alpha_{R'}(x) > \alpha_R(x)$ on $(R, R']$.

extreme. Figure 8 illustrates.[14]

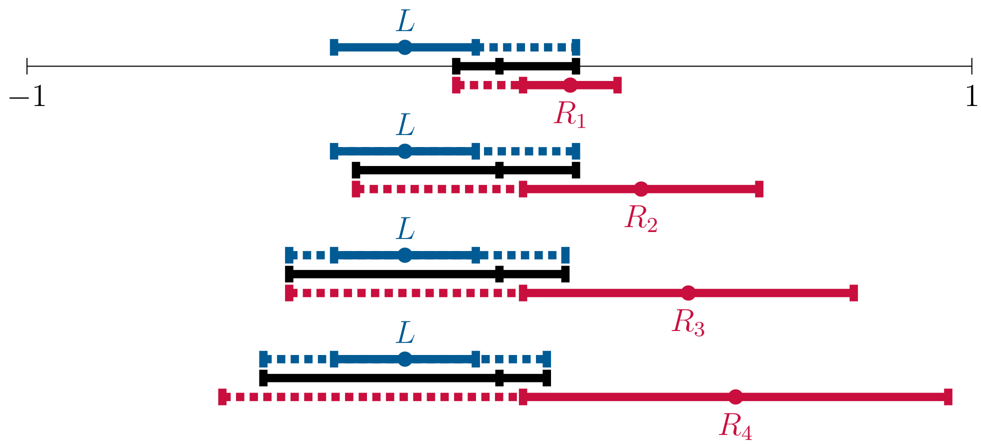

Figure 8. *Equilibrium set of approved policies as the right voter becomes more extreme (top to bottom). She becomes persuadable by a wider range of policies (in red), and the set of unanimously approved policies (in black) shifts to the left.*

Theorem 3. *Suppose that R satisfies Assumption 1, L satisfies Assumption 2, and $\rho_L(\lfloor A_L \rfloor, \lceil A_R \rceil) \leq 0$. Then, as the right voter becomes more extreme,*

- *the challenger's odds of winning increase;*
- *the equilibrium set of unanimously approved policies shifts to the left.*

Theorem 3 compares the equilibrium outcomes of two baseline elections, fixing the *left* voter's bliss point at L, and increasing the *right* voter's bliss point from R to R'. Assume that $\lfloor A_L \rfloor > -1$ and $a_R^{AS} > a_L^{UC}$, or else no changes will take place.[15] Let $(\overline{W}_L, \overline{W}_R)$ and $(\overline{W}'_L, \overline{W}'_R)$ be the equilibrium outcome when the *right* voter's bliss point is R and R', respectively. Also, let $\overline{W} = \overline{W}_L \cap \overline{W}_L$ and $\overline{W}' = \overline{W}'_L \cap \overline{W}'_R$ be the equilibrium sets of unanimously approved policies before and after the change. Note that by Lemma 2, the *right* voter's constraint is looser after the change, immediately implying that the value of the objective (the challenger's odds of winning) can only

[14] Figure 8 presents the numerical solution for the uniform prior, $L = -0.20$, and $R_1 = 0.15$, $R_2 = 0.30$, $R_3 = 0.40$, $R_4 = 0.50$ (top to bottom). The sets of unanimously approved policies (in black) are $[-0.0914, 0.1621]$, $[-0.3036, 0.1621]$, $[-0.4450, 0.1397]$, $[-0.50, 0.10]$, respectively.

[15] If $\lfloor A_L \rfloor = -1$ or $a_R^{AS} \leq a_L^{UC}$, then $\overline{W} = \overline{W}_L = I_L^{UC}$ as long as the conditions of Theorem 3 are satisfied. Loosening the *right* voter's constraint does not change the equilibrium set of unanimously approved policies because the objective cannot be improved upon $\mu_0(I_L^{UC})$ while still satisfying the *left* voter's constraint, which does not change.

go up. Furthermore, increasing R decreases the left boundary a_R^{AS} of the *right* voter's largest interval of approved policies (strictly so, unless $a_R^{AS} = -1$). From Theorem 2, a_R^{AS} is also the left boundary of the set of unanimously approved policies. It remains to prove that the right boundary of $\overline{W}$ decreases, as well. The general idea is that this boundary cannot shift to the right, as it is determined by the *left* voter's constraint, which does not change. The remainder of this section describes the conditions under which the decrease is strict.

Recall that Theorem 2 had two cases: one where both voters are moderately persuadable, and one where the *right* voter is significantly more persuadable. Also recall that increasing R increases $\lceil A_R \rceil$. Consequently, $\rho_L\big(\lfloor A_L \rfloor, \lceil A_R \rceil\big) = \int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_L(x) d\mu_0(x)$ decreases, meaning that the *left* voter remains moderately persuadable after the change. At the same time, $\rho_R\big(\lfloor A_L \rfloor, \lceil A_R \rceil\big) = \int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_R(x) d\mu_0(x)$ increases, making the *right* voter more persuadable. We have three cases to consider.

Case (i): the *right* voter is moderately persuadable before and after the change, or $\rho_R\big(\lfloor A_L \rfloor, \lceil A_R \rceil\big) < \rho_{R'}\big(\lfloor A_L \rfloor, \lceil A_{R'} \rceil\big) \leq 0$. Applying Part *(1)* of Theorem 2, we get that $\overline{W}_v = I_v^{AS}$ for each $v \in \{L, R\}$ and $\overline{W}'_v = I_v^{AS}$ for each $v \in \{L, R'\}$. In particular, the right boundary of the set of unanimously approved policies is fixed at b_L^{AS} before and after the change. In Figure 8, *Case (i)* can be seen in the transition from the first to the second exhibit (when R_1 increases to R_2).

Case (ii): the *right* voter is moderately persuadable before and significantly more persuadable than the *left* voter after the change, or $\rho_R\big(\lfloor A_L \rfloor, \lceil A_R \rceil\big) \leq 0 < \rho_{R'}\big(\lfloor A_L \rfloor, \lceil A_{R'} \rceil\big)$. Apply Part *(1)* of Theorem 2 before the change to get $\overline{W}_v = I_v^{AS}$ for each $v \in \{L, R\}$, with $\overline{W} = [a_R^{AS}, b_L^{AS}]$. After the change, apply Part *(2)* of Theorem 2 to get $\overline{W}'_R = I_{R'}^{AS}$, $\overline{W}'_L = I_L(a_{R'}^{AS}, 1) = \big[\max\{a_{R'}^{AS}, a_L^{UC}\}, b'_L\big]$, with $\overline{W}' = \overline{W}'_L$. Now, from the *left* voter's obedience constraint,

$$\int_{I_L^{AS}} \alpha_L(x) d\mu_0(x) = \int_{\lfloor A_L \rfloor}^{b_L^{AS}} \alpha_L(x) d\mu_0(x) = 0 \leq$$

$$\int_{I_L(a_{R'}^{AS})} \alpha_L(x) d\mu_0(x) = \int_{\max\{a_{R'}^{AS}, a_L^{UC}\}}^{b'_L} \alpha_L(x) d\mu_0(x).$$

Since $\lfloor A_L \rfloor > \max\{a_{R'}^{AS}, a_L^{UC}\}$, we must have $b_L^{AS} > b'_L$, as desired. In Figure 8, *Case (ii)* can be seen in the transition from the second to the third exhibit (when R_2 increases to R_3).

Case (iii): the *right* voter is significantly more persuadable than the *left* voter before and after the change, or $0 < \rho_R\big(\lfloor A_L \rfloor, \lceil A_R \rceil\big) < \rho_{R'}\big(\lfloor A_L \rfloor, \lceil A_{R'} \rceil\big)$. Applying Part 2 of Theorem 2 in both cases, we conclude that $\overline{W} = \overline{W}_L = I_L(a_R^{AS}, 1) = \big[a_R^{AS}, b_L\big]$ and $\overline{W}' = \overline{W}'_L = I_L(a_{R'}^{AS}, 1) = \big[\max\{a_{R'}^{AS}, a_L^{UC}\}, b'_L\big]$. Once again, from the *left* voter's obedience constraint, $a_L^{AS} > \max\{a_{R'}^{AS}, a_L^{UC}\} \implies b_L > b'_L$. In Figure 8, *Case (iii)* can be seen in the transition from the third to the fourth exhibit (when R_3 increases to R_4).

Welfare

Consider an outcome in which a voter with bliss point $v \in X$ approves some set of policies $W_v \subseteq X$. When v approves, her payoff is $-|v-x|-\varepsilon$, and when she rejects, it is $-|v|$. Hence, her ex-ante utility is $\mathbb{E}\big[\mathbb{1}(x \in W_v) \cdot (-|v-x| - \varepsilon) + \mathbb{1}(x \in W_v^c) \cdot (-|v|)\big]$. Next, subtract $-|v|$ from that expression, to get $\int_{W_v} \alpha_v(x) d\mu_0(x)$. I use the latter expression as a measure of v's welfare. I also define voter v's amount of regret as the difference between her welfare in the outcome under consideration (when she approves W_v) and under complete information (when she approves A_v).

Definition 4. *Consider a voter with bliss point $v \in X$ and her set of approved policies W_v. Then, v's*

- welfare *is* $\int_{W_v} \alpha_v(x) d\mu_0(x)$*;*
- amount of regret *is* $\int_{A_v} \alpha_v(x) d\mu_0(x) - \int_{W_v} \alpha_v(x) d\mu_0(x)$.

The table below compares the voter welfare and the challenger's odds of winning across three communication protocols. Firstly, there is the first-best *full disclosure* outcome (A_L, A_R) that delivers the complete information payoff for all players.[16] Secondly, there is the *public disclosure* outcome (W^{PD}, W^{PD}) of the baseline model with an additional restriction that the challenger must always send the same

[16] Under full disclosure, the set of approved policies of voter $v \in \{L, R\}$ is A_v. Each voter learns whether the challenger's policy is in her approval set, and thus acts as if under complete information. Note that full disclosure is the sender-worst equilibrium outcome of the baseline model.

(verifiable) message to both voters.[17] Thirdly, there is the *targeted advertising* outcome $(\overline{W}_L, \overline{W}_R)$. Recall from the discussion after Theorem 2 that the obedience constraints $\int_{\overline{W}_v} \alpha_v(x)d\mu_0(x) \geq 0$ of both voters bind unless one of them is very extreme/persuadable, in which case her constraint may be loose.

	v's welfare	v's regret	challenger's odds of winning
full disclosure	$\int_{A_v} \alpha_v(x)d\mu_0(x) > 0$	0	0
public disclosure	$\int_{W^{PD}} \alpha_v(x)d\mu_0(x) \geq 0$	≥ 0	0
targeted advertising	$\int_{\overline{W}_v} \alpha_v(x)d\mu_0(x) = 0$	> 0	$\mu_0(\overline{W}_L \cap \overline{W}_R) \geq 0$

Notice that targeted advertising maximizes the challenger's odds of winning at the expense of minimizing voter welfare and maximizing voter regret. Interestingly, the voter's regret does not increase as she becomes more extreme: it remains the same or decreases. To see why, suppose that the *right* voter becomes more extreme and her bliss point increases from R to R'. Also suppose that before the change, her constraint was binding.

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Appendix: Omitted Proofs

Proof of Theorem 2

The case when $a_R^{AS} \leq a_L^{UC}$ is proved in the main text.

Suppose that $a_L^{UC} < a_R^{AS} < \lfloor A_L \rfloor$. Let $\overline{W}_L = I_L(a_R^{AS}, 1)$ and $\overline{W}_R = I_R^{AS}$. It remains to show that the challenger's odds of winning cannot be higher than $\mu_0(\overline{W}_L)$ for any other pair (W_L, W_R) that satisfies both voters' constraints. Indeed, any W_L such that $\mu_0(W_L) > \mu_0(\overline{W}_L)$ that satisfies the *left* voter's constraint has to include a positive-measure set $Y \subseteq [-1, a_R^{AS}]$. However, every policy $y \in Y$ is more expensive in terms of R's constraint than any policy $x \in [a_R^{AS}, \varepsilon]$ (because $\alpha_R(y) < \alpha_R(x)$). Consequently, including Y to the set of unanimously approved policies increases the objective by $\mu_0(Y)$ but decreases it by more than $\mu_0(Y)$. Hence, $(I_L(a_R^{AS}, 1), I_R^{AS})$ solves Problem (1) if $a_L^{UC} < a_R^{AS} < \lfloor A_L \rfloor$, or more generally, whenever $a_R^{AS} < \lfloor A_L \rfloor$, since if $a_R^{AS} \leq a_L^{UC}$ then $I_L(a_R^{AS}, 1) = I_L^{UC}$.

The last case is when $\lfloor A_L \rfloor \leq a_R^{AS} \leq -\varepsilon$. I show that the proposed solution $(\overline{W}_L, \overline{W}_R) = (I_L^{AS}, I_R^{AS})$ with the set of unanimously approved policies $\overline{W} = [a_R^{AS}, b_L^{AS}]$ maximizes the objective of Problem (1). Consider another solution $(\widetilde{W}_L, \widetilde{W}_R)$ with the set of unanimously approved policies $\widetilde{W} = \widetilde{W}_L \cap \widetilde{W}_R$. Firstly, observe that $\widetilde{W}$ cannot be to the left of $\overline{W}$, i.e. the set $\widetilde{W} \cap [b_L^{AS}, 1]$ has to have a positive prior measure. If not, then the *right* voter's constraint has to be spent on policies further than a_R^{AS}, which decreases the objective. Specifically, from R's constraint, $\int\limits_{\overline{W}_R \cap \widetilde{W}_R^c} \alpha_R(x) d\mu_0(x) \leq \int\limits_{\widetilde{W}_R \cap [-1, a_R^{AS}]} \alpha_R(x) d\mu_0(x)$. Also, $\alpha_R(\overline{x}) > \alpha_R(\widetilde{x})$ for all $\overline{x} \in \overline{W}_R$ and $\widetilde{x} \in [-1, a_R^{AS}]$, which implies $\mu_0(\overline{W}_R \cap \widetilde{W}_R^c) > \mu_0(\widetilde{W}_R \cap [-1, a_R^{AS}])$. Finally, since $\widetilde{W} \subseteq [-1, b_L^{AS}]$ a.s., we have $\overline{W}_R \cap \widetilde{W}_R^c = \overline{W} \setminus \widetilde{W}$ and $\widetilde{W}_R \cap [-1, a_R^{AS}] \supseteq \widetilde{W} \setminus \overline{W}$. It follows that $\mu_0(\overline{W}) > \mu_0(\widetilde{W})$. By a symmetric argument for the *left* voter, $\widetilde{W}$ cannot be to the right of $\overline{W}$, either, and the set $\widetilde{W} \cap [-1, a_L^{AS}]$ has to have a positive prior measure.

Next, observe that $\widetilde{W} \cap \big[-1, \lceil A_L \rceil\big]$ and $\widetilde{W} \cap \big[\lfloor A_R \rfloor, 1\big]$ must be intervals that end at $\lceil A_L \rceil$ and start at $\lfloor A_R \rfloor$, respectively. Otherwise, $\widetilde{W}$ can be improved upon. For instance, if $\widetilde{W} \cap \big[-1, \lceil A_L \rceil\big] \neq [a, \lceil A_L \rceil]$ for some $a \geq -1$, then there exist two sets $Y = [y_1, y_2] \subseteq \widetilde{W}$ and $Z = [z_1, z_2] \subseteq \widetilde{W}^c$ such that $-1 \leq y_1 < y_2 \leq z_1 < z_2 \leq \lceil A_L \rceil$ and $\mu_0(Y) = \mu_0(Z)$. Then, for every $y \in Y$ and $z \in Z$, $\alpha_R(y) < \alpha_R(z) < 0$ and either $\alpha_L(y) < \alpha_L(z)$ or $\alpha_L(y) > \alpha_L(z) \geq 0$. Let $\widehat{W}_L = \big(\widetilde{W}_L \smallsetminus (Y \cap A_L^c)\big) \cup Z$ and $\widehat{W}_R = \big(\widetilde{W}_R \smallsetminus Y\big) \cup Z$. By construction, $(\widehat{W}_L, \widehat{W}_R)$ satisfies both constraints and maintains the objective at $\mu_0(\widetilde{W})$. However, since R's constraint is now loose, we can further increase the objective, a contradiction.

At this point, we can conclude that $[a, -\varepsilon] \cup [\varepsilon, b] \subseteq \widetilde{W}$ for some $a \in [-1, a_R^{AS})$ and $b \in (b_L^{AS}, 1]$. Now, let $Y_R := [a, a_R^{AS}]$, $Y_L := [b_L^{AS}, b]$, so that $\widetilde{W} \smallsetminus \overline{W} = Y_R \cup Y_L$. Also, let $M := \overline{W} \smallsetminus \widetilde{W} \subseteq [-\varepsilon, \varepsilon]$. It remains to show that $\mu_0(\overline{W}) > \mu_0(\widetilde{W})$, or $\mu_0(M) > \mu_0(Y_L \cup Y_R)$. Indeed, from the obedience constraint, $\int\limits_M \alpha_v(x) d\mu_0(x) \leq \int\limits_{Y_v} \alpha_v(x) d\mu_0(x)$ for each $v \in \{L, R\}$. Now, add this inequality for and L and R to get

$$\int\limits_M \big(\underbrace{\alpha_L(x) + \alpha_R(x)}_{=-2\varepsilon \text{ for } x \in [-\varepsilon, \varepsilon]}\big) d\mu_0(x) \leq \int\limits_{Y_R} \underbrace{\alpha_R(x)}_{=x-\varepsilon \text{ for } x \leq -\varepsilon} d\mu_0(x) + \int\limits_{Y_L} \underbrace{\alpha_L(x)}_{=-x-\varepsilon \text{ for } x \geq \varepsilon} d\mu_0(x) \implies$$

$$-2\varepsilon \cdot \mu_0(M) \leq -2\varepsilon \cdot \mu_0(Y_L \cup Y_R) + \int\limits_{Y_R} \underbrace{(x+\varepsilon)}_{<0} d\mu_0(x) + \int\limits_{Y_L} \underbrace{(-x+\varepsilon)}_{<0} d\mu_0(x) \implies$$

$$\mu_0(M) > \mu_0(Y_L \cup Y_R).$$

Proof of Lemma 2

I prove this lemma for the *right* voter whose bliss point increases from R to R'. The case of the *left* voter is symmetric. First, notice that $R' > R$ implies $\lceil A_{R'} \rceil \geq \lceil A_R \rceil$ since $\lceil A_{R'} \rceil = \min\{1, 2R' - \varepsilon\} \geq \min\{2R - \varepsilon\} = \lceil A_R \rceil$. Also, $\lfloor A_{R'} \rfloor = \varepsilon = \lfloor A_R \rfloor$. Thus, $A_{R'} \supseteq A_R$.

Next, observe that $\alpha_{R'}(x) = \alpha_R(x)$ for all $x \in [-1, R]$ and $\alpha_{R'}(x) > \alpha_R(x)$ otherwise. Indeed, if $x \leq R$, then $\alpha_R(x) = x + \varepsilon = \alpha_{R'(x)}$. Next, if $x \in (R, R']$, then $\alpha_R(x) = -x + 2R - \varepsilon < x + \varepsilon = \alpha_{R'}(x)$. Finally, if $x \in (R', 1]$, then, $\alpha_R(x) = -x + 2R - \varepsilon < -x + 2R' - \varepsilon = \alpha_{R'}(x)$. The relationship between $\alpha_R(x)$ and $\alpha_{R'}(x)$ is illustrated in Figure 7.

Now, recall that I_R^{AS} solves Problem (AUX) with $l = -1$ and $r = \lceil A_R \rceil$. If $a_R^{AS} = -1$, then $a_{R'}^{AS} = -1$. If $a_R^{AS} > -1$, then I_R^{AS} binds the constraint for R, and we have

$$\int_{a_R^{AS}}^{\lfloor A_R \rfloor} -\alpha_R(x) d\mu_0(x) = \int_{\lfloor A_R \rfloor}^{\lceil A_R \rceil} \alpha_R(x) d\mu_0(x)$$

$$< \int_{\lfloor A_{R'} \rfloor}^{\lceil A_{R'} \rceil} \alpha_{R'}(x) d\mu_0(x) = \int_{a_{R'}^{AS}}^{\lfloor A_{R'} \rfloor} -\alpha_{R'}(x) d\mu_0(x) = \int_{a_{R'}^{AS}}^{\lfloor A_R \rfloor} -\alpha_R(x) d\mu_0(x)$$

if the constraint binds for R', and $a_{R'}^{AS} = -1 < a_R^{AS}$ otherwise. The function $\phi(t) := \int_t^{\lfloor A_R \rfloor} -\alpha_R(x) d\mu_0(x)$ is strictly decreasing if $t \in \big[-1, \lfloor A_R \rfloor \big)$ because $\phi'(t) = \alpha_R(t)\mu_0(t) < 0$. We conclude that, $a_{R'}^{AS} < a_R^{AS}$.