Targeted Advertising in Elections*

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PRELIMINARY AND INCOMPLETE

LATEST VERSION

Abstract

Some elections are unwinnable for challengers because pivotal voters prefer policies on the opposite sides of the status quo. In this paper, I argue that the challenger can win any such election if he uses targeted advertising with verifiable messages. In his private ads, the challenger makes each voter believe that his policy is a sufficient improvement over the status quo and wins the election when his policy is sufficiently moderate. Targeted advertising makes the voters regret their choices and minimizes the voter welfare relative to the complete information and public advertising benchmarks. As a voter's favorite policy becomes more extreme, her dissatisfaction with the status quo grows, and she becomes persuadable by a wider range of policies. As a result, the challenger's odds of winning increase.

KEYWORDS: Persuasion, Targeted Advertising, Elections

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1. Introduction

Targeted advertising played an important role in the recent US Presidential Elections. In 2016, the Trump campaign used voter data from Cambridge Analytica to target voters via Facebook and Twitter. In 2008, the Obama campaign pioneered the use of social networks to communicate with the electorate. Even before social media, in 2000, The Bush campaign targeted voters via direct mail. Given that the winning candidate had access to better technology or voter data in all these cases, one may wonder whether targeted advertising was why these candidates won.[1] Would they have lost without targeted advertising? In other words, can targeted advertising swing electoral outcomes and help win elections that are otherwise unwinnable?

To answer these questions, I consider the following baseline model of targeted advertising in elections. There is an underlying policy space $X = [-1, 1]$, and three players: the challenger and two voters, $left$ and $right$. The challenger is privately informed about his policy $x \in X$, which is drawn from a common prior distribution with full support. The status quo policy is known and fixed at 0. The $left$ voter has bliss point $L < 0$ and the right voter has bliss point $R > 0$. Voters have quadratic preferences and dislike policies that are far from their bliss point. Specifically, if voter $v \in \{L, R\}$ approves challenger's policy $x \in X$, she gets a payoff of $-(x - v)^2$; if she rejects, she gets $-(v - 0)^2$. The challenger is office-motivated and receives a payoff of one if both voters unanimously approve his policy and zero otherwise. The challenger communicates with the voters using verifiable messages. Each message m is a statement about his policy or $m \subseteq X$. Verifiability means that each message has to include the challenger's actual policy, or $x \in m$. In words, the challenger can lie by omission, and send messages that include things other than his policy. At the same time, he cannot lie by commission and send messages that do not include his policy. The challenger know the voters' bliss points, and uses targeted advertising to send different messages to the $left$ and the $right$ voter. Each voter only observes her own message, and votes expressively for the candidate (the challenger or the status quo) whose policy she prefers the most.

Note that the baseline election is unwinnable for the challenger without targeted advertising. Specifically, his odds of winning are zero in every equilibrium, under every communication protocol that does not allow different messages to different

[1] For comparison of advertising strategies between the candidates, see Kim et al. (2018) and Wylie (2019) for the 2016 election, Harfoush (2009) and Katz, Barris, and Jain (2013) for the 2008 election and Hillygus and Shields (2014) for the 2004 election.

voters. The left voter prefers policies to the left of the status quo, while the right voter prefers policies on the right. Since the challenger's policy cannot be left and right at the same time, at most one of the voters is willing to approve it under complete information. Similarly, the challenger's policy cannot be both left and right on average, meaning that at most one of the voters is willing to approve it under common belief. As a result, the challenger definitely loses if he does not advertise, if he fully discloses his policy, or if he advertises his policy publicly.

When the challenger has access to targeted advertising, he can tell different things to different voters. In his most preferred equilibrium, the challenger makes the left voter believe that his policy is, on average, to the left of the status quo. He induces that belief by pooling this voter's favorite policies on the left with as many right policies, as possible. Similarly, in his private communication with the right voter, the challenger insists that his policy is, on average, on the right. The challenger wins the election whenever both voters approve, which happens with positive probability. That said, the challenger only benefits from private communication if his policy is sufficiently close to the status quo: the further to the right (left) his policy, the harder it becomes to convince the left (right) voter.

When a voter becomes more extreme, her dissatisfaction with the status quo grows, making her more persuadable. Consequently, as the electorate becomes more polarized, the challenger's odds of swinging an unwinnable election increase. As the right voter becomes more extreme, she becomes persuadable by a wider range of policies, including policies further to the left. As a result, the equilibrium set of unanimously approved policies shifts to the left.

Motivating Example

Suppose that $L = -0.4$ and $R = 0.2$. Also, suppose that the prior is uniform: every policy in $[-1, 1]$ is ex-ante equally likely. Consider the following strategy of the challenger. When communicating to voter L, he sends message $[-0.8, 0.4]$ whenever his policy is $x \in [-0.8, 0.4]$; otherwise, he sends message $[-1, 1]$. In words, the challenger tells the left voter that his policy is not ultra left and not moderate- to ultra right, whenever that is true, and says nothing otherwise. Similarly, when talking to voter R, he sends message $[-0.2, 0.4]$ whenever his policy is $x \in [-0.2, 0.4]$, and $[-1, 1]$ otherwise. Figure 1 illustrates the challenger's strategy.

Now, suppose that voter L receives message $[-0.8, 0.4]$. Her expected utility from approving the challenger's policy is $\int_{-0.8}^{0.4} -(x + 0.4)^2 dx = -0.192$, which happens to

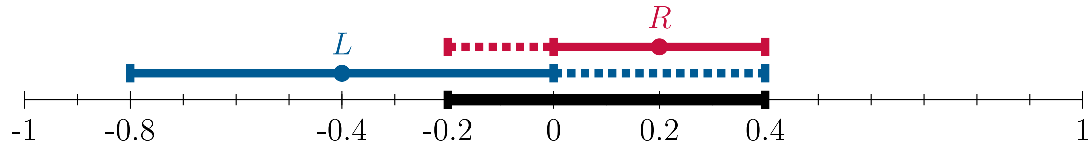

Figure 1. *Targeted messages that convince voter L (in blue) and R (in red). Each voter's convincing message includes policies that she prefers to the status quo (solid) and those that are dominated by the status quo (dashed). The challenger wins the election by unanimous approval whenever his policy is in the intersection of the convincing messages (in black).*

coincide with her expected utility from rejecting, which is $\int_{-0.8}^{0.4} -(0.4)^2 dx = -0.192$. In other words, if voter L *(i)* hears the message $[-0.8, 0.4]$ and *(ii)* knows that every challenger with the policy $x \in [-0.8, 0.4]$ sends that message with probability one, then she is indifferent between approval and rejection. Suppose that she breaks the tie in favor of approval. Note that the message $[-0.8, 0.4]$ that convinces L comprises of the policies $[-0.8, 0]$ that she prefers to the status quo as the well as policies $(0, 0, 0.4]$ that are strictly dominated by the status quo. Although the voter is indifferent when approving, the average policy that voter L approves is $-0.2 < 0$, which reflects her risk aversion. Similarly, voter R approves after the message $[-0.2, 0.4]$. Whenever each voter hears the message $[-1, 1]$, she knows that it is sent by the challenger whose policy is further away than the status quo, so she rejects.

Now, these strategies of the challenger and voters lead to the following outcome. Voter L approves the challenger's policy if and only if it belongs to the interval $[-0.8, 0, 4]$. At the same time, voter R approves the challenger's policy if it is in $[-0.2, 0.4]$, and rejects it otherwise. We conclude that the challenger wins the election by unanimous approval whenever his policy is between -0.2 and 0.4. Given the uniform prior on $[-1, 1]$, the odds of the challenger winning are 30% in this outcome. It remains to show that the players' strategies constitute a perfect Bayesian equilibrium.

To complete equilibrium characterization, we need to specify each voter's beliefs off the path. Let us impose that the voters are *skeptical*: they assume that each off path message is sent by the challenger *(i)* who can send that message and *(ii)* whose policy is the farthest away from the voter's bliss point. Now, the challenger does not have profitable deviations: for any policy, he either wins the elections and secures the highest possible payoff, or loses the election, but cannot verifiably send a message that convinces both voters. The voters do not have profitable deviations, either. Thus, the described strategies constitute an equilibrium. In fact, this is the challenger preferred equilibrium, in which the challenger's odds of winning are the highest across all equilibria.

Related Literature (In Progress)

This paper contributes to the growing literature on voter persuasion. Most of the previous work has focused on information design (Kamenica and Gentzkow, 2011, Alonso and Câmara, 2016), cheap talk (Crawford and Sobel, 1982, Schnakenberg, 2015, Jeong, 2019), and, like me, verifiable disclosure (Milgrom, 1981, Grossman, 1981, Caillaud and Tirole, 2007, Jackson and Tan, 2013).

I am not the first person to compare private and public communication. In the verifiable information literature, the closest paper to mine is Schipper and Woo (2019), who study advertising competition. They show that even with targeted advertising, the candidates tend to voluntarily disclose all their private information. This unraveling result is fairly common in the verifiable information literature on voter persuasion (Board, 2009; Janssen and Teteryatnikova, 2017), and arises because the candidates play a zero-sum game. In contrast to these papers, I consider a nonsymmetric model in which one candidate has a significant advantage over his opponent in that he is the only one who can communicate with the voters. Unraveling does not necessarily occur, and the challenger can improve his chances of winning over full disclosure.

Much progress has been made in comparing public and private disclosure in the cheap talk literature. A robust finding is that the sender often prefers to communicate in public, rather than in private (Farrell and Gibbons, 1989, Koessler, 2008, Goltsman and Pavlov, 2011, Bar-Isaac and Deb, 2014), because public communication reduces the number of possible deviations available to the sender in each state of the world. When his messages are verifiable, the sender's message space is already restricted, and there is no such effect. Consequently, my main result is the opposite: the sender strictly benefits from private advertising when his messages are verifiable to the point that he can win elections that are unwinnable otherwise.

In information design, the sender prefers private communication to public advertising (Arieli and Babichenko, 2019), even if the receivers are strategic and condition on the event of being pivotal (Bardhi and Guo, 2018, Chan et al., 2019, Heese and Lauermann, 2019). I confirm this finding: while my sender does not possess any commitment power, the sender-preferred equilibrium outcome is also a commitment outcome (Titova, 2021). Beyond that, my contribution is twofold: on the one hand, I conclude that the sender does not need commitment power to benefit from targeted advertising. On the other hand, he does not only improve his ex ante utility by communicating in private; he improves it from 0 in every equilibrium to a positive number in his most preferred equilibrium.

The model sheds more light on how political advertising, especially targeted advertising, affects electoral outcomes and why it has become widespread. DellaVigna and Gentzkow (2010) and Prat and Strömberg (2013) provide excellent surveys of the evidence of voter persuasion. First, candidates target their ads based on voters' positions on the political spectrum (George and Waldfogel, 2006; DellaVigna and Kaplan, 2007). Second, one can make a case that an increase in the availability of information catered toward certain electoral groups also counts as targeted advertising because these are the messages intended for and heard by these groups (Oberholzer-Gee and Waldfogel, 2009; Enikolopov, Petrova, and Zhuravskaya, 2011). I show that targeted political advertising may be so widespread because it allows politicians to win elections that are unwinnable otherwise.

I also contribute to the growing literature on polarization and targeted political advertising through the media. As the number of media outlets increases, they become more specialized and target voters with more extreme preferences, which leads to social disagreement (Perego and Yuksel, 2022). If the electorate is polarized to begin with, so are the candidates' chosen policy platforms (Hu, Li, and Segal, 2019; Prummer, 2020). Abstracting away from candidates choosing their policies, I find that as the electorate becomes more polarized, more challengers can swing elections that are unwinnable otherwise.

2. Baseline Election: Model

I study an interaction between a politician who challenges the status quo (the challenger, he/him) and the voters (she/her). There is an underlying policy space $X := [-1, 1]$ with policy positions ranging from far left (-1) to far right (1). The status quo policy is fixed, known, and normalized to 0. The game begins with the challenger privately observing his policy $x \in X$, which is drawn from a common prior distribution $\mu_0 \in \Delta X$ with full support.

The challenger is *office-motivated* and his goal is to win the election. In the baseline election, there are two voters, $left$ and $right$, and the challenger needs both voters to approve his proposal to win the election. I normalize his payoff from winning to 1 and losing to 0.

The challenger advertises his policy to voters through private verifiable messages. Specifically, each message m that the challenger may send *(i)* is a statement about his policy, $m \subseteq X$, and *(ii)* contains a grain of truth, $x \in m$. That is, the challenger *can lie by omission* and send messages that contain policies other than x. At the same time, he *cannot lie by commission* and send messages that do not include x. The verifiability of the messages allows the voters to draw inferences about the challenger's policy. For example, suppose that a voter hears the message $[-1/2, 0]$, or "my policy is moderately left". She concludes that the challenger's policy is not far-left or anywhere on the right. At the same time, she does not know the exact location of the challenger's policy between $-1/2$ and 0.

The voters have spatial preferences and vote expressively for the candidate they prefer the most. The voter with bliss point $v \in X$, to whom I will sometimes refer as "voter v," prefers to approve the challenger's policy $x \in X$ if she considers it a sufficient improvement over the status quo. Otherwise, she prefers to reject. Mathematically, when the challenger's policy is $x \in X$, the payoff of the voter with bliss point $v \in X$ is

$$u_v(\text{approve}, x) = -(v-x)^2, \quad u_v(\text{reject}, x) = -v^2,$$

where $d_v > 0$ for each $v \in \{L, R\}$.

To simplify the analysis, let $\alpha_v(x) := u_v(\text{approve}, x) - u_v(\text{reject}, x) = -x^2 + 2vx$ be v's <u>net payoff from approval</u>. Note that v's net payoff from approval is a downward-sloping parabola that peaks at v, which reflects that the voters are risk-averse. Now, this voter's best response is to approve the challenger's policy $x \in X$ whenever her net payoff from approval $\alpha_v(x)$ is non-negative. Also, let $A_v := \{x \in X \mid \alpha_v(x) \geq 0\}$ be her (complete information) <u>approval set</u> that includes all the policies of the challenger that she prefers to approve under complete information. I let $\lceil A_v \rceil := \max A_v$ be the largest and $\lfloor A_v \rfloor := \min A_v$ be the smallest elements of v's approval set.

In the baseline election, the *left* voter has bliss point $L \in [-1, 0)$ and the *right* voter has bliss point $R \in (0, 1]$. Figure 2 illustrates the preferences of the *left* voter.

I focus on the challenger-preferred Perfect Bayesian equilibrium of this game. Knowing his policy x, the challenger chooses verifiable messages $m_L \subseteq X$ and $m_R \subseteq X$ for voters L and R, respectively. Verifiability requires that $x \in m_v$ for all $v \in \{L, R\}$. Having observed message m_v, voter $v \in \{L, R\}$ forms a posterior belief over X. She then approves or rejects. Both voters are expressive and do not condition on the

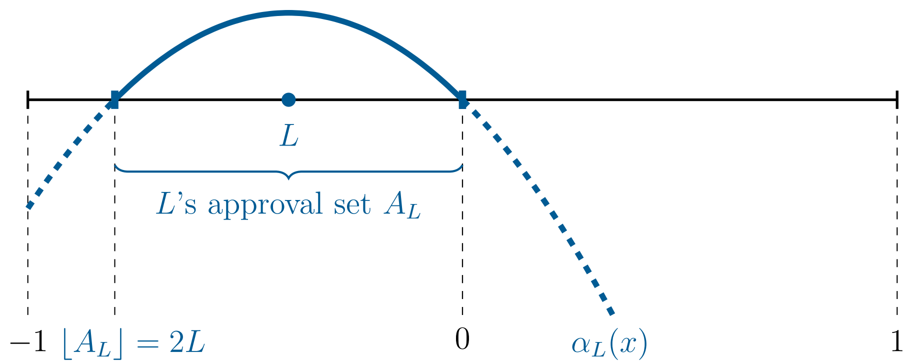

Figure 2. *The policy space $X = [-1, 1]$, the left voter with bliss point $L < 0$, her net payoff from approval $\alpha_L(x)$, and her approval set A_L. Voter L considers policies in A_L to be a sufficient improvement over the status quo.*

event of being pivotal.[2]

In equilibrium, *(i)* the challenger sends messages that maximize his payoff, *(ii)* each voter approves the challenger's policy whenever her expected net payoff from approval is non-negative under her posterior belief, *(iii)* voters' posteriors on the equilibrium path are Bayes-rational. The challenger-preferred equilibrium is one in which his odds of unanimous approval are the highest across all equilibria.

3. Baseline Election: Analysis

Incompatible Voters and Unwinnable Elections

Let us first observe that the challenger faces an electorate of voters who prefer diametrically opposing policies. As a result, the baseline election is unwinnable for him without targeted advertising.

Lemma 1. *For any common belief $\mu \in \Delta X$ such that $\mu(\{0\}) < 1$, at most one of the voters prefers to approve the challenger's proposal.*

Proof. Voter v approves under belief $\mu \in \Delta X$ only if $\mathbb{E}_\mu[\alpha_v(x)] \geq 0$. Since $\alpha_v(x)$ is

[2] That is, the voters derive utility from expressing support for their favorite candidate, and not from the policy that is implemented by the elected candidate. The theory was pioneered by Brennan and Lomasky (1993), Brennan and Hamlin (1998) and reviewed by Hamlin and Jennings (2011). There is a large body of evidence that the behavior of voters is consistent with expressive voting, e.g., in US national elections (Kan and Yang, 2001; Degan and Merlo, 2007), Spanish general elections (Artabe and Gardeazabal, 2014), Israeli general elections (Felsenthal and Brichta, 1985).

concave, we apply Jensen's inequality to get

$$\alpha_v\left(\overline{x}_\mu\right) \geq \mathbb{E}_\mu[\alpha_v(x)] \geq 0,$$

where $\overline{x}_\mu := \mathbb{E}_\mu[x]$ is the average value of x under belief μ. The former inequality is strict for any non-degenerate μ, since α_v is strictly concave.

Now, the *right* voter's net payoff from approval is $\alpha_R(x) = -x^2 + 2Rx$. Consequently, $\alpha_R(\overline{x}_\mu) \geq 0$ if and only if $\overline{x}_\mu \in [0, 2R]$. Similarly, for the $left$ voter, $\alpha_L(x) \geq 0$ if and only if $x \in [2L, 0]$. Now, for both $\alpha_L\left(\overline{x}_\mu\right) \geq 0$ and $\alpha_R\left(\overline{x}_\mu\right) \geq 0$ to be true, μ must be a degenerate distribution with $\mu(\{0\}) = 1$.

$\square$

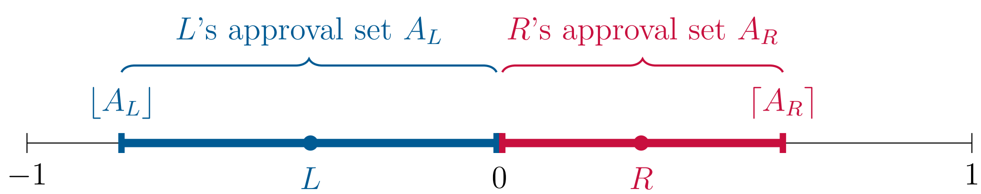

Figure 3. *Voters L and R are incompatible: L prefers to approve left policies and R prefers to approve right policies.*

Figure 3 illustrates the approval sets of the voters. Simply put, the $left$ voter prefers the *left (blue)* policies, while the *right* voter prefers the *right (red)* policies. Since the challenger's policy cannot be both left and right at the same time, at least one of the voters prefers to reject it. The same argument applies when the voters have a common belief.

Lemma 1 implies that the baseline election is *unwinnable* for the challenger *without targeted advertising*. If he does not advertise at all, the voters hold a common prior and at most one of them votes to approve. If he advertises publicly, the voters' common prior is updated to a common posterior, but again, at most one voter is convinced to approve. The only event in which the challenger wins is when his policy coincides with the status quo. However, this event has a zero prior measure.

Corollary 1. *The baseline election is almost surely unwinnable for the challenger under public disclosure.*[3] *Specifically, if he is restricted to sending the same (public) message to both voters, he loses the election with probability one in every equilibrium.*

[3]Almost surely with respect to the prior measure μ_0.

Equilibrium Outcomes under Targeted Advertising

Let us now characterize the (challenger preferred) equilibrium payoff of the baseline election game with targeted advertising. According to Titova (2021), every equilibrium is payoff equivalent to a direct equilibrium with *sets of approved policies* $W_L \subseteq X$ and $W_R \subseteq X$ that satisfy certain constraints. In the direct equilibrium, the challenger sends the message W_v to voter $v \in \{L, R\}$ if $x \in W_v$, and its complement $W_v^c := X \setminus W_v$ otherwise. When voter v hears W_v, she approves; otherwise, she rejects the challenger's policy. Therefore, we can interpret the message W_v as the challenger's *recommendation to approve* and the message W_v^c as the *recommendation to reject*.

To be implementable in equilibrium, voter v's set of approved policies W_v must satisfy two conditions. On the one hand, there is the sender's incentive-compatibility constraint, $A_v \subseteq W_v$, which guarantees that the challenger does not want to deviate towards a fully informative strategy. This constraint is automatically satisfied in the challenger preferred equilibrium because the challenger attempts to convince the voters with as many policies as possible. On the other hand, there is the receiver's obedience constraint that ensures that voter v only approves when her average net payoff from approval is non-negative:

$$\int_{W_v} \alpha_v(x) d\mu_0(x) \geq 0. \tag{obedience}$$

The challenger wins the election whenever both voters approve, or when $x \in W_L \cap W_R$, and his odds of winning are $\mu_0(W_L \cap W_R)$. Thus, the (challenger preferred) equilibrium sets of approved policies $(\overline{W}_L, \overline{W}_R)$ solve

$$\max_{W_L, W_R \subseteq X} \mu_0(W_L \cap W_R) \quad \text{subject to} \quad \int_{W_v} \alpha_v(x) d\mu_0(x) \geq 0 \text{ for each } v \in \{L, R\} \tag{1}$$

I refer to the pair $(\overline{W}_L, \overline{W}_R)$ that solves Problem (1) as the (challenger preferred) equilibrium outcome (under targeted advertising). The main result of this paper establishes that the challenger can always win an unwinnable election by advertising privately.

Theorem 1. *In equilibrium of the baseline election game, the challenger's ex-ante odds of winning are always positive.*

The proof of this result is straightforward. First, observe that each voter's approval set is guaranteed to convince this voter, i.e., $\int_{A_v} \alpha_v(x) d\mu_0(x) > 0$ for each

$v \in \{L, R\}$. Next, for each voter $v \in \{L, R\}$, select a subset $B_v \subseteq A_{-v}$ of the other voter's approval set that satisfies $\int_{A_v} \alpha_v(x)d\mu_0(x) + \int_{B_v} \alpha_v(x)d\mu_0(x) \geq 0$ and $\mu_0(B_v) > 0$. Let $W_v := A_v \cup B_v$ be voter v's set of approved policies. Although W_L and W_R may not be equilibrium sets of approved policies, they do, by construction, satisfy the constraints of Problem (1). At the same time, $\mu_0(W_L \cap W_R) = \mu_0(B_R \cup B_L) > 0$, implying that the challenger's ex ante odds of wining in equilibrium must be positive.

Before characterizing the equilibrium sets of approved policies, let us focus on the problem of maximizing the odds of convincing just one voter. Of particular interest are the cases where the voters approve *intervals* of policies, because the message "my policy is in $W_v \subseteq X$" (or "my policy is NOT in W_v^c") sounds more natural if W_v is a connected set.

One Voter's Intervals of Approved Policies

Consider a voter with bliss point $v \in X$. Let us focus on the following auxiliary problem of finding a voter's largest (in terms of prior measure) set of approved policies constrained by $l \in \big[-1, \lfloor A_v \rfloor\big]$ from the left and $r \in \big[\lceil A_v \rceil, 1\big]$ from the right.

$$\max_{W \subseteq [l,r]} \mu_0\big(W\big) \quad \text{subject to} \quad \int_W \alpha_v(x)d\mu_0(x) \geq 0. \tag{AUX}$$

The solution to the auxiliary is characterized by a cutoff value for the voter's net payoff from approval (see, for example, Alonso and Câmara, 2016 and Titova, 2021). Specifically, every policy with a not too negative payoff from approval (those $x \in X$ for which $\alpha_v(x) \geq -c_v^*$) is included in the solution I_v. Then, c_v^* is obtained from the binding obedience constraint, $\int_{I_v} \alpha_v(x)d\mu_0(x) = 0$. The set $\{x \in [l, r] \mid \alpha_v(x) \geq -c_v^*\}$ is an interval: it is the upper contour set of the concave function $\alpha_v(x)$ and is hence convex. Corollary 2 characterizes the solution of the auxiliary problem.

Corollary 2. *Consider a voter with bliss point $v \in X$. Then, the solution to Problem (AUX) with $l \in \big[-1, \lfloor A_v \rfloor\big]$ and $r \in \big[\lceil A_v \rceil, 1\big]$ is an interval $I_v(l, r)$ such that*

- *if $\int_l^r \alpha_v(x)d\mu_0(x) \geq 0$, then $I_v(l, r) = [l, r]$;*
- *otherwise, $I_v(l, r) = \big\{x \in [l, r] \mid \alpha_v(x) \geq -c_v^*(l, r)\big\}$, where $c_v^*(l, r) > 0$ is obtained from the binding constraint $\int_{I_v(l,r)} \alpha_v(x)d\mu_0(x) = 0$.*

Two special cases of the auxiliary problem will be useful in further analysis.

Firstly, there is the unconstrained version with $l = -1$ and $r = 1$. Figure 4 illustrates the largest unconstrained interval of approved policies of the *left* voter.

Definition 1. *Consider a voter with bliss point $v \in X$. Then, this voter's* largest unconstrained interval of approved policies *is* $I_v^{UC} = [a_v^{UC}, b_v^{UC}] := I_v(-1, 1)$.

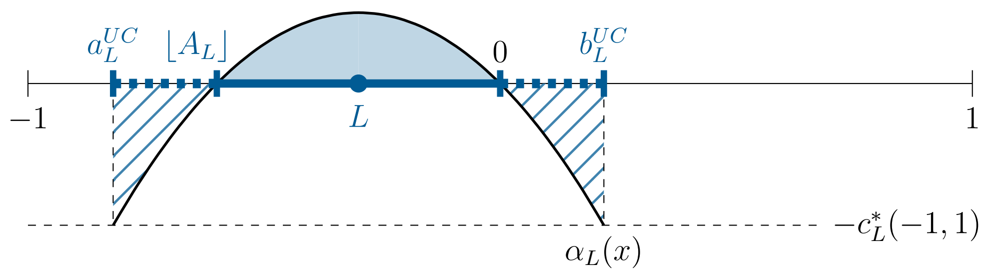

Figure 4. *$[a_L^{UC}, b_L^{UC}]$ is the left voter's largest unconstrained interval of approved policies. Under uniform prior, c_L^* is obtained from equating the solid area (expected value of $\alpha_L(x)$ over A_L) to the dashed area (expected value of $\alpha_L(x)$ outside of A_L).*

The second relevant case is the largest asymmetric interval of approved policies that includes the most policies on the opposite side of the status quo from the voter's approval set. Figure 5 illustrates the *left* voter's largest asymmetric interval of approved policies.

Definition 2.

- *The left* voter's largest asymmetric interval of approved policies *is*

$$I_L^{AS} = \left[\lfloor A_L \rfloor, b_L^{AS}\right] := I_L(\lfloor A_L \rfloor, 1).$$

- *The right* voter's largest asymmetric interval of approved policies *is*

$$I_R^{AS} = \left[a_R^{AS}, \lceil A_R \rceil\right] := I_R(-1, \lceil A_R \rceil).$$

Note that since $L < 0$ and $R > 0$, we get $b_L^{AS} > 0$ and a_R^{AS}. In words, since each voter's approval set has a positive measure, she can be persuaded by a positive measure of policies outside of her approval set.

Convincing Two Voters at the Same Time

Let us now solve Problem (1), or, put simply, attempt to convince both voters at the same time as often as possible. One thing the challenger can do is convince the *left* (*right*) voter with as many policies to the right (left) of her approval set as possible.

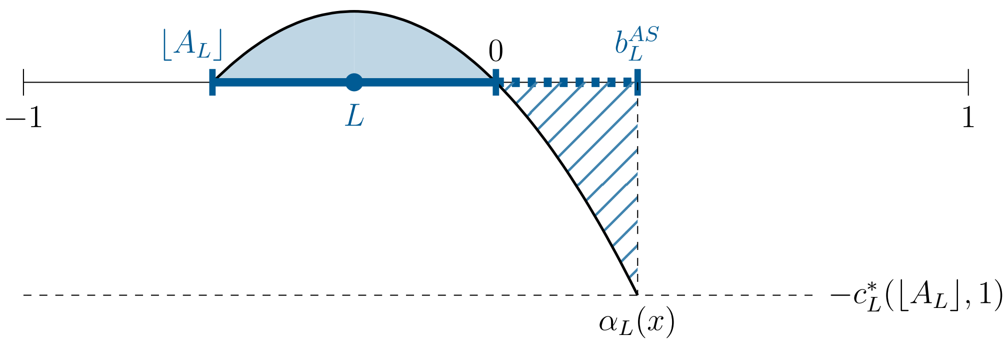

Figure 5. *$\left[\lfloor A_L \rfloor, b_L^{AS}\right]$ is the left voter's largest asymmetric interval of approved policies. Under uniform prior, c_L^* is obtained from equating the solid area (expected value of $\alpha_L(x)$ over A_L) to the dashed area (expected value of $\alpha_L(x)$ outside of A_L).*

That is, he can let each voter's set of approved policies be her largest asymmetric interval of approved policies. I illustrate the outcome (I_L^{AS}, I_R^{AS}) in Figure 6. As it turns out, (I_L^{AS}, I_R^{AS}) is often an equilibrium outcome.

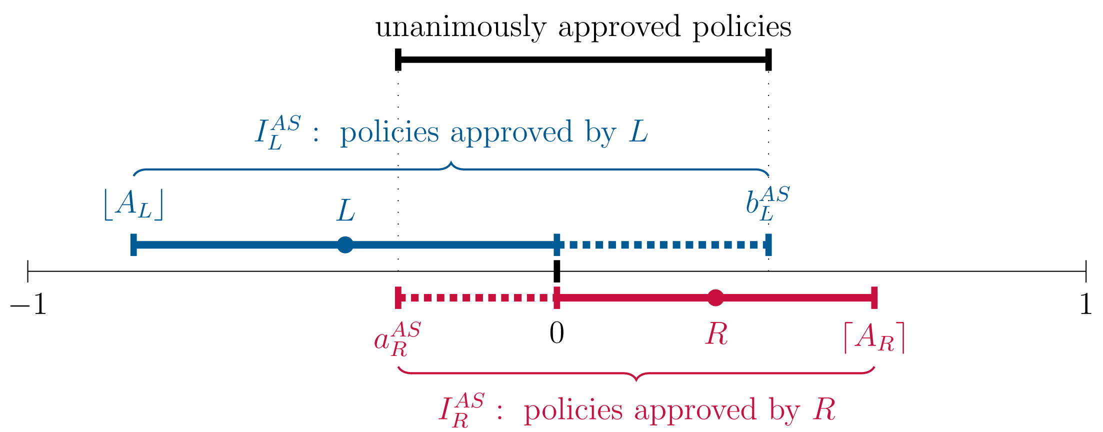

Figure 6. *The largest asymmetric intervals of approved policies of voter L (in blue) and R (in red). To convince voter L (R), the challenger pools policies that she prefers (solid) together with policies preferred by the other voter (dashed). The winning policies of the challenger (in black) are those approved by both voters.*

One case when $[a_R^{AS}, b_L^{AS}]$ may not be an equilibrium set of approved policies is if $a_R^{AS} < \lfloor A_L \rfloor$, which happens if and only if $\int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_R(x) d\mu_0(x) > 0$.[4] Intuitively, in this case, the *right* voter is so persuadable that her largest asymmetric interval of

[4] Observe that $\phi(t) := \int_t^{\lceil A_R \rceil} \alpha_R(x) d\mu_0(x)$ is strictly increasing in $t < 0$ since $\frac{\partial \phi(t)}{\partial t} = -\alpha_R(t)\mu_0(t) > 0$. Consequently, $\phi(\lfloor A_L \rfloor) > 0 = \phi(a_L^{AS})$ is possible if and only if $\lfloor A_L \rfloor > a_L^{AS}$.

approved policies includes the *left* voter's entire approval set, and then some. Note that $\int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_v(x) d\mu_0(x) > 0$ cannot hold for both $v = L$ and $v = R$ at the same time.[5]

THEOREM 2. *Suppose that the left voter is not significantly more persuadable than the right voter, i.e.* $\int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_L(x) d\mu_0(x) \leq 0$. *Almost surely, the right voter's equilibrium set of approved policies is her largest asymmetric interval of approved policies, or* $\overline{W}_R = I_R^{AS}$. *Furthermore, a.s.*

(1) if $\int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_R(x) d\mu_0(x) \leq 0$, *then*

- *The equilibrium set of approved policies of the left voter is her largest asymmetric interval of approved policies, or* $\overline{W}_L = I_L^{AS}$*;*
- *the equilibrium set of unanimously approved policies is* $\overline{W} = [a_R^{AS}, b_L^{AS}]$.

(2) if $\int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_R(x) d\mu_0(x) > 0$, *then*

- *The equilibrium set of approved policies of the left voter is the largest interval of approved policies constrained from the left by* a_R^{AS}, *or* $\overline{W}_L = I_L(a_R^{AS}, 1)$*;*
- *the equilibrium set of unanimously approved policies is* $\overline{W} = \overline{W}_L$.

The formal proof of Theorem 2 is in the Appendix, but I outline it below. Since the *right* voter is the more persuadable one, let us add as many left policies to her message, as possible. That is, have the *right* voter approve her largest asymmetric interval of policies, or $\overline{W}_R = I_R^{AS} = \left[a_R^{AS}, \lceil A_R \rceil\right]$. Now, Theorem 2 states that the equilibrium depends on how a_R^{AS} is related to $\lfloor A_L \rfloor$. Specifically, if we are in *Case (2)* of Theorem 2, then $a_R^{AS} < \lfloor A_L \rfloor$. Otherwise, we are in *Case (1)*. Let us first consider the lower values of a_R^{AS}.

Suppose first that the *right* voter is so persuadable that she is willing to approve *all* left policies, i.e. $a_R^{AS} = -1$. In this case, (I_L^{UC}, I_R^{AS}) solves Problem (1), and the set of unanimously approved policies is I_L^{UC}.[6] By construction, there is no way to increase the objective beyond $\mu_0(I_L^{UC})$ while still satisfying the *left* voter's constraint. The same argument applies whenever $I_R^{AS} \supseteq I_L^{UC}$, or for every $a_R^{AS} \in [-1, a_L^{UC}]$. This

[5] We have $\int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \big(\alpha_R(x) + \alpha_L(x)\big) d\mu_0(x) < 0$ since $\alpha_R(x) + \alpha_L(x) < 0$ for all $x \in X \setminus \{0\}$.

[6] Without the *right* voter's constraint, apply Corollary 2 to conclude that $\overline{W}_L = I_L^{UC}$.

case is illustrated in Figure 7 on the left.

Next, suppose that $a_L^{UC} < a_R^{AS} < \lfloor A_L \rfloor$. Now, (I_L^{UC}, I_R^{AS}) is no longer optimal: the challenger does not persuade the *right* voter with policies in $[a_L^{UC}, a_R^{AS})$, yet "wastes" the constraint of the *left* voter on them. Instead, select the *left* voter's message out of $[a_R^{AS}, 1]$, since the *right* voter rejects the policies outside of that interval, anyway. Now, the proposed solution is $\big(I_L(a_R^{AS}, 1), I_R^{AS}\big)$, with the set of unanimously approved policies $I_L(a_R^{AS}, 1)$.[7] The challenger cannot increase his objective beyond $\mu_0\big(I_L(a_R^{AS}, 1)\big)$: it would require unanimous approval of policies to the left of a_R^{AS}, which are strictly more expensive in terms of the *right* voter's constraint than those that she already approves. Hence, the proposed solution is optimal. This case is illustrated in Figure 7 on the right.

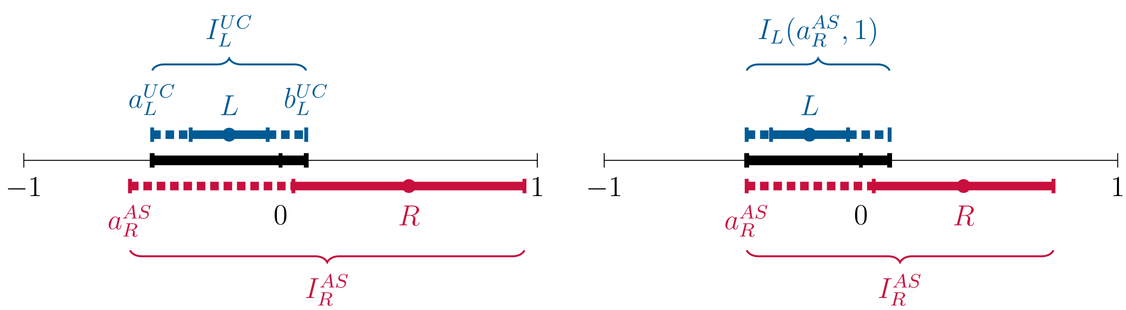

Figure 7. *Equilibrium sets of approved policies when the right voter is significantly more persuadable than the left voter.*

The last case we need to consider is when $\lfloor A_L \rfloor \leq a_R^{AS} < 0$. It remains to show that the proposed solution $(\overline{W}_L, \overline{W}_R) = (I_L^{AS}, I_R^{AS})$ with the set of unanimously approved policies $\overline{W} = [a_R^{AS}, b_L^{AS}]$ maximizes the objective of Problem (1). By contradiction, suppose that the set of unanimously approved policies is a different set $\widetilde{W} \subseteq X$. Then, it is straightforward to show that $\mu_0\big(\widetilde{W} \setminus \overline{W}\big) < \mu_0\big(\overline{W} \setminus \widetilde{W}\big)$. Indeed, if $\big(\widetilde{W} \setminus \overline{W}\big) \cap [-1, a_R^{AS})$ has a positive measure, then the *right* voter approves policies further than a_R^{AS}, which is more expensive in terms of her constraint. Similarly, if $\big(\widetilde{W} \setminus \overline{W}\big) \cap (b_L^{AS}, 1]$ has positive measure, then the *left* voter's constraint is being used on the more expensive policies.

Note that the *left* voter's constraint always binds, as she is the relatively less persuadable voter. The *right* voter's constraint binds unless $a_R^{AS} < a_L^{UC}$. It is also worth mentioning that identifying the equilibrium sets of approved policies $(\overline{W}_L, \overline{W}_R)$ requires solving at most two auxiliary optimization problems. Algorithm 1 describes

[7] Apply Corollary 2 with $l = a_R^{AS}$ and $r = 1$ to conclude that $\overline{W}_L = I_L(a_R^{AS}, 1)$.

the steps.

Algorithm 1 Calculating the equilibrium sets of approved policies $(\overline{W}_L, \overline{W}_R)$

calculate $\rho_v(\lfloor A_L \rfloor, \lceil A_R \rceil) = \int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_v(x) d\mu_0(x)$ for each $v \in \{L, R\}$

if $\rho_v(\lfloor A_L \rfloor, \lceil A_R \rceil) \leq 0$ for each $v \in \{L, R\}$ **then**

  calculate a_R^{AS} and b_L^{AS} ▷ solve two (AUX) problems

  $\overline{W}_L = I_L^{AS}$, $\overline{W}_R = I_R^{AS}$, $\overline{W} = [a_R^{AS}, b_L^{AS}]$

else if $\rho_R(\lfloor A_L \rfloor, \lceil A_R \rceil) > 0$ **then**

  $\overline{W}_R = I_R^{AS}$, $\overline{W}_L = I_L(a_R^{AS}, 1)$, $\overline{W} = \overline{W}_L$ ▷ solve two (AUX) problems

else

  $\overline{W}_L = I_L^{AS}$, $\overline{W}_R = I_R(-1, b_L^{AS})$, $\overline{W} = \overline{W}_R$ ▷ solve two (AUX) problems

The following example calculates the equilibrium illustrated in Figure 6.

Example 1 (Uniform prior, $L = -0.4$, $R = 0.3$.). In this example, $A_L = [-0.8, 0]$ and $A_R = [0, 0.6]$. First, we check the relative persuadability of each voter by calculating $\int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_v(x) dx$ for each $v \in \{L, R\}$. Both of these values are negative, so it remains to calculate b_L^{AS} and a_R^{AS}.

To find b_L^{AS}, solve $\int_{\lfloor A_L \rfloor}^{b_L^{AS}} \alpha_L(x) dx = 0$. Plugging in $\alpha_L(x) = -x^2 + 2Lx$, we arrive at the following equation: $(b_L^{AS})^3 - 3L \cdot (b_L^{AS})^2 = -4L^3$. It is not hard to check that the unique solution is $b_L^{AS} = -L = 0.4$, so that $\overline{W}_L = [-0.8, 0.4]$.[8] Similarly, we find that $a_R^{AS} = -R$ and $\overline{W}_R = [-0.3, 0.6]$.

Recall that one way to implement the equilibrium outcome is by pooling all policies in $\overline{W}_v$ into one message $\overline{W}_v$ that convinces voter $v \in \{L, R\}$. In this example, $\overline{W}_L = [-0.8, 0.4]$, meaning that the challenger says that his policy is not ultra left and not moderate to ultra right, but does not clarify any further. Furthermore, that message averages to -0.2, which is to the left of the status quo, making the $left$ voter think that the challenger's policy is aligned with her preferences.

[8] Note that this solution is applicable for any $L \in [0.5, 0)$ and any quadratic net payoff from approval $\alpha_v(x) = -d(v) \cdot (x^2 - 2vx)$ with $d(v) > 0$.

Both voters approve and the challenger wins if $x \in \overline{W}_L \cap \overline{W}_R = [-0.3, 0.4]$, that is, if his policy is sufficiently moderate. His odds of winning, calculated as the length of the interval of winning policies (0.7) relative to the length of the policy space (2), equal 0.35. We conclude that targeted advertising allows the challenger to improve his odds of winning from 0% to 35%!

Comparative Statics

Next, let us analyze what happens when the electorate becomes more polarized. Defining polarization in the baseline model with one dimension and two voters is straightforward:

Definition 3.

- *The voter with bliss point $v \in X$* becomes more extreme *if $|v|$ increases.*
- *The baseline electorate* becomes more polarized *if the left and/or the right voter becomes more extreme.*

Since the bliss points of voters must belong to the policy space, the *most extreme voter* has $|v| = 1$, and the *most polarized electorate* is $L = -1$ and $R = 1$. Note that the larger distance between the voters does not necessarily imply higher polarization. To increase polarization, one voter has to become more extreme, while the other voter has to stay fixed or also become more extreme (in the opposite direction).

Observe that when a voter becomes more extreme, she also becomes more persuadable. Using the *right* voter as an example, as R increases to R', the voter's approval set expands, extending the range of positive values of the net payoff from approval. As a result, the *right* voter's obedience constraint loosens. In particular, the voter becomes persuadable by a wider range of the left policies, as well.

Lemma 2. *As a voter becomes more extreme, her largest asymmetric interval of approved policies expands, i.e. $I_{v'}^{AS} \supseteq I_v^{AS}$. Specifically,*

- *if $L' < L$, then $\left[\lfloor A_{L'} \rfloor, b_{L'}^{AS}\right] \supseteq \left[\lfloor A_L \rfloor, b_L^{AS}\right]$, with $\lfloor A_{L'} \rfloor \leq \lfloor A_L \rfloor$ and $b_{L'}^{AS} \geq b_L^{AS}$; the latter inequality is strict unless $b_L^{AS} = 1$;*
- *if $R' > R$, then $\left[a_{R'}^{AS}, \lceil A_{R'} \rceil\right] \supseteq \left[a_R^{AS}, \lceil A_R \rceil\right]$, with $a_{R'}^{AS} \leq a_R^{AS}$ and $\lceil A_{R'} \rceil \geq \lceil A_R \rceil$; the former inequality is strict unless $a_R^{AS} = -1$.*

The technical proof is in the Appendix, but I illustrate the argument for the *right* voter in Figure 8. When her bliss point increases from R to R', two effects occur. On the one hand, her approval set expands, and her net payoff from approval over her approval set strictly increases. Consequently, the expected value of her net payoff from approval over her approval set, $\int_0^{\lceil A_R \rceil} \alpha_R(x) d\mu_0(x)$, strictly increases.[9] On the other hand, the voter becomes more risk averse as she becomes more extreme, and her net payoff from approval to the left of her approval set strictly decreases. The two changes work in opposite directions, meaning that a_R^{AS} decreases if the former effect dominates, and increases if the latter effect is stronger. As it turns out, the quadratic utility is not concave enough for the latter effect to ever be stronger.[10]

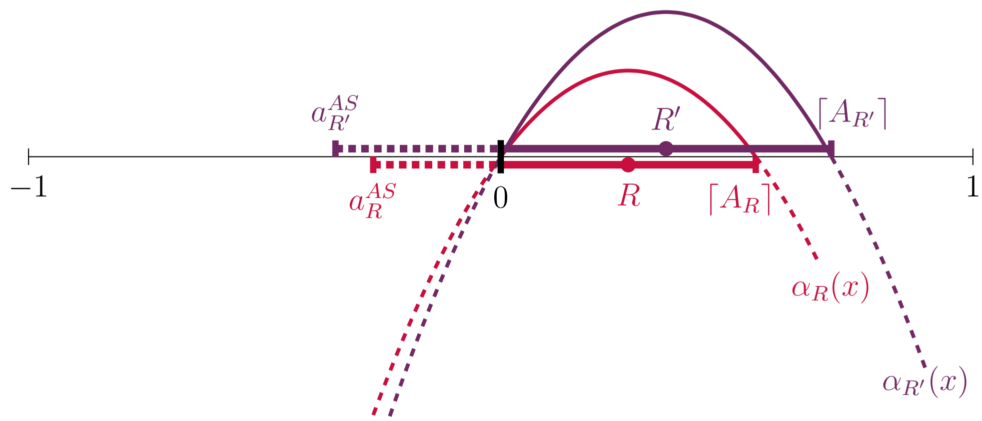

Figure 8. *The right voter becomes more persuadable by a wider range of policies as she becomes more extreme (her bliss point increases from R to R'): her approval set $[0, \lceil A_R \rceil]$ and her largest asymmetric interval of approved policies $[a_R^{AS}, \lceil A_R \rceil]$ expand.*

Now, let us consider the baseline electorate that satisfies the assumption of Theorem 2. That is, the *left* voter is not significantly more persuadable than the *right* voter. Theorem 3 describes what happens to the equilibrium sets of approved policies as the *right* voter becomes more extreme. Figure 9 illustrates.[11]

THEOREM 3. *Suppose that the left voter is not significantly more persuadable than*

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[9]If $2R > 1$, then the approval set itself remains the same, unlike in Figure 8. However, $\int_0^{\lceil A_R \rceil} \alpha_R(x) d\mu_0(x)$ strictly increases because $\alpha_{R'}(x) > \alpha_R(x)$ for $x \in [R, \lceil A_R \rceil]$.

[10]The proof of Lemma 2 applies to the net payoff v' from approval $\alpha_v(x) = -d(v) \cdot (x^2 - 2vx)$, for any $d(v) > 0$.

[11]Figure 9 presents the numerical solution for the uniform prior, $L = -0.15$, and $R_1 = 0.15$, $R_2 = 0.25$, $R_3 = 0.35$, $R_4 = 0.50$ (top to bottom). The sets of unanimously approved policies (in black) are $[-0.15, 0.15]$, $[-0.25, 0.15]$, $[-0.35, 0.1436]$, $[-0.4098, 0.1098]$, respectively.

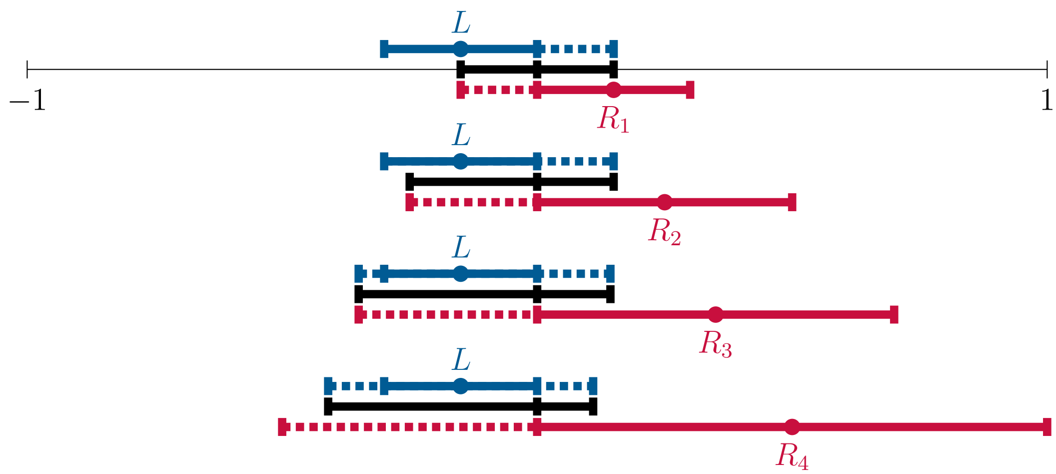

Figure 9. *Equilibrium set of approved policies as the right voter becomes more extreme (top to bottom). She becomes persuadable by a wider range of policies (in red), and the set of unanimously approved policies (in black) shifts to the left.*

the right voter, i.e. $\int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_L(x) d\mu_0(x) \leq 0$. *Then, as the right voter becomes more extreme,*

- *the challenger's odds of winning increase;*
- *the equilibrium set of unanimously approved policies shifts to the left.*

Theorem 3 compares the equilibrium outcomes of two baseline elections, fixing the $left$ voter's bliss point at L and increasing the $right$ voter's bliss point from R to R'. Assume that $\lfloor A_L \rfloor > -1$ and $a_R^{AS} > a_L^{UC}$, or else no changes will take place.[12] Let $(\overline{W}_L, \overline{W}_R)$ and $(\overline{W}'_L, \overline{W}'_R)$ be the equilibrium outcome when the $right$ voter's bliss point is R and R', respectively. Also, let $\overline{W} = \overline{W}_L \cap \overline{W}_L$ and $\overline{W}' = \overline{W}'_L \cap \overline{W}'_R$ be the equilibrium sets of unanimously approved policies before and after the change. Note that by Lemma 2, the $right$ voter's constraint is looser after the change, immediately implying that the value of the objective (the challenger's odds of winning) can only go up. Furthermore, increasing R decreases the left boundary a_R^{AS} of the $right$ voter's largest interval of approved policies (strictly so, unless $a_R^{AS} = -1$). From Theorem 2, a_R^{AS} is also the left boundary of the set of unanimously approved policies. It remains to prove that the right boundary of $\overline{W}$ decreases, as well. The general idea is that this

[12]If $\lfloor A_L \rfloor = -1$ or $a_R^{AS} \leq a_L^{UC}$, then $\overline{W} = \overline{W}_L = I_L^{UC}$ as long as the conditions of Theorem 3 are satisfied. Loosening the $right$ voter's constraint does not change the equilibrium set of unanimously approved policies because the objective cannot be improved upon $\mu_0(I_L^{UC})$ while still satisfying the $left$ voter's constraint, which does not change.

boundary cannot shift to the right, as it is determined by the $left$ voter's constraint, which does not change. The remainder of this section describes the conditions under which the decrease is strict.

Recall that Theorem 2 had two cases: one in which both voters are moderately persuadable and one in which the *right* voter is significantly more persuadable. Also, recall that increasing R increases $\lceil A_R \rceil$ and decreases $\int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_L(x) d\mu_0(x)$. That is, the $left$ voter remains moderately persuadable after the change. At the same time, $\int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_R(x) d\mu_0(x)$ increases, making the $right$ voter more persuadable. We have three cases to consider.

Case (i): the $right$ voter is moderately persuadable before and after the change, or $\int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_R(x) d\mu_0(x) < \int_{\lfloor A_L \rfloor}^{\lceil A'_R \rceil} \alpha_R(x) d\mu_0(x) \leq 0$. Applying Part *(1)* of Theorem 2, we get that $\overline{W}_v = I_v^{AS}$ for each $v \in \{L, R\}$ and $\overline{W}'_v = I_v^{AS}$ for each $v \in \{L, R'\}$. In particular, the right boundary of the set of unanimously approved policies is fixed at b_L^{AS} before and after the change. In Figure 9, *Case (i)* can be seen in the transition from the first to the second exhibit (when R_1 increases to R_2).

Case (ii): the $right$ voter is moderately persuadable before and significantly more persuadable than the $left$ voter after the change, or $\int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_R(x) d\mu_0(x) \leq 0 < \int_{\lfloor A_L \rfloor}^{\lceil A'_R \rceil} \alpha_R(x) d\mu_0(x)$. Apply Part *(1)* of Theorem 2 before the change to get $\overline{W}_v = I_v^{AS}$ for each $v \in \{L, R\}$, with $\overline{W} = [a_R^{AS}, b_L^{AS}]$. After the change, apply Part *(2)* of Theorem 2 to get $\overline{W}'_R = I_{R'}^{AS}$, $\overline{W}'_L = I_L(a_{R'}^{AS}, 1) = \left[\max\{a_{R'}^{AS}, a_L^{UC}\}, b'_L\right]$, with $\overline{W}' = \overline{W}'_L$. Now, from the $left$ voter's obedience constraint,

$$\int_{I_L^{AS}} \alpha_L(x) d\mu_0(x) = \int_{\lfloor A_L \rfloor}^{b_L^{AS}} \alpha_L(x) d\mu_0(x) = 0 \leq$$

$$\int_{I_L(a_{R'}^{AS})} \alpha_L(x) d\mu_0(x) = \int_{\max\{a_{R'}^{AS}, a_L^{UC}\}}^{b'_L} \alpha_L(x) d\mu_0(x).$$

Since $\lfloor A_L \rfloor > \max\{a_{R'}^{AS}, a_L^{UC}\}$, we must have $b_L^{AS} > b'_L$, as desired. In Figure 9, *Case (ii)* can be seen in the transition from the second to the third exhibit (when R_2

increases to R_3).

Case (iii): the *right* voter is significantly more persuadable than the *left* voter before and after the change, or $0 < \int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_R(x) d\mu_0(x) < \int_{\lfloor A_L \rfloor}^{\lceil A'_R \rceil} \alpha_R(x) d\mu_0(x)$. Applying Part 2 of Theorem 2 in both cases, we conclude that $\overline{W} = \overline{W}_L = I_L(a_R^{AS}, 1) = \left[a_R^{AS}, b_L\right]$ and $\overline{W}' = \overline{W}'_L = I_L(a_{R'}^{AS}, 1) = \left[\max\{a_{R'}^{AS}, a_L^{UC}\}, b'_L\right]$. Once again, from the *left* voter's obedience constraint, $a_L^{AS} > \max\{a_{R'}^{AS}, a_L^{UC}\} \implies b_L > b'_L$. In Figure 9, *Case (iii)* can be seen in the transition from the third to the fourth exhibit (when R_3 increases to R_4).

Welfare

Consider an outcome in which a voter with bliss point $v \in X$ approves some set of policies $W_v \subseteq X$. When v approves, her payoff is $-(v-x)^2$, and when she rejects, it is v^2. Hence, her ex-ante utility is $\mathbb{E}\big[-\mathbb{1}(x \in W_v) \cdot (v-x)^2 + \mathbb{1}(x \in W_v^c) \cdot v^2\big]$. Next, subtract $-v^2$ from that expression, to get $\int_{W_v} \alpha_v(x) d\mu_0(x)$. I use the latter object as a measure of v's welfare. I also define voter v's amount of regret as the difference between her welfare in the outcome under consideration (when she approves W_v) and under complete information (when she approves A_v).

Definition 4. *Consider a voter with bliss point $v \in X$ and her set of approved policies W_v. Then, v's*

- welfare *is* $\int_{W_v} \alpha_v(x) d\mu_0(x)$*;*
- amount of regret *is* $\int_{A_v} \alpha_v(x) d\mu_0(x) - \int_{W_v} \alpha_v(x) d\mu_0(x)$*.*

The table below compares voter welfare and challenger's odds of winning across three communication protocols. Firstly, there is the first-best *full disclosure* outcome (A_L, A_R) that delivers the complete information payoff for all players.[13] Secondly, there is the *public disclosure* outcome (W_L^{PD}, W_R^{PD}) of the baseline model with an additional restriction that the challenger must always send the same (ver-

[13]Under full disclosure, the set of approved policies of voter $v \in \{L, R\}$ is A_v. Each voter learns whether the challenger's policy is in her approval set, and thus acts as if under complete information. Note that full disclosure is the sender-worst equilibrium outcome of the baseline model.

ifiable) message to both voters.[14] Thirdly, there is the *targeted advertising* outcome $(\overline{W}_L, \overline{W}_R)$. Recall from the discussion after Theorem 2 that the obedience constraints $\int_{\overline{W}_v} \alpha_v(x)d\mu_0(x) \geq 0$ of both voters bind unless one of them is very extreme/persuadable, in which case her constraint may be loose.

	v's welfare	v's regret	challenger's odds of winning
full disclosure	$\int_{A_v} \alpha_v(x)d\mu_0(x) > 0$	0	0
public disclosure	$\int_{W^{PD}} \alpha_v(x)d\mu_0(x) \geq 0$	≥ 0	0
targeted advertising	$\int_{\overline{W}_v} \alpha_v(x)d\mu_0(x) = 0$	> 0	$\mu_0(\overline{W}_L \cap \overline{W}_R) > 0$

Notice that targeted advertising maximizes the challenger's odds of winning at the expense of minimizing voter welfare and maximizing voter regret. Interestingly, the voter's regret does not increase as she becomes more extreme: it remains the same or decreases. To see why, suppose that the *right* voter becomes more extreme and her bliss point increases from R to R'. Also, suppose that before the change, her constraint was binding.

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[14] Mathematically, voter v's set of approves policies under public disclosure W_v^{PD} solves $\max_{W \subseteq X} \mu_0(W)$ subject to $\int_W \alpha_v(x)d\mu_0(x) \geq 0$ for each $x \in \{L, R\}$. The solution is any $W^{PD} \subseteq X$ that satisfies the obedience constraints of both voters. The challenger's odds of winning are always 0 as per Corollary 1.

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Appendix: Omitted Proofs

Proof of Theorem 2

The case where $a_R^{AS} \leq a_L^{UC}$ is proved in the main text.

Suppose that $a_L^{UC} < a_R^{AS} < \lfloor A_L \rfloor$. Let $\overline{W}_L = I_L(a_R^{AS}, 1)$ and $\overline{W}_R = I_R^{AS}$. It remains to show that the challenger's odds of winning cannot be higher than $\mu_0(\overline{W}_L)$ for any other pair (W_L, W_R) that satisfies both voters' constraints. Indeed, any W_L such that $\mu_0(W_L) > \mu_0(\overline{W}_L)$ satisfies the constraint of the $left$ voter has to include a positive-measure set $Y \subseteq [-1, a_R^{AS}]$. However, every policy $y \in Y$ is more expensive in terms of R's constraint than any policy $x \in [a_R^{AS}, 0]$ (because $\alpha_R(y) < \alpha_R(x)$). Consequently, including Y in the set of unanimously approved policies increases the objective by $\mu_0(Y)$ but decreases it by more than $\mu_0(Y)$. Hence, $(I_L(a_R^{AS}, 1), I_R^{AS})$ solves Problem (1) if $a_L^{UC} < a_R^{AS} < \lfloor A_L \rfloor$, or more generally, whenever $a_R^{AS} < \lfloor A_L \rfloor$, since if $a_R^{AS} \leq a_L^{UC}$ then $I_L(a_R^{AS}, 1) = I_L^{UC}$.

The last case is where $\lfloor A_L \rfloor \leq a_R^{AS} \leq 0$. I show that the proposed solution $(\overline{W}_L, \overline{W}_R) = (I_L^{AS}, I_R^{AS})$ with the set of unanimously approved policies $\overline{W} = [a_R^{AS}, b_L^{AS}]$ maximizes the objective of Problem (1). Consider another solution $(\widetilde{W}_L, \widetilde{W}_R)$ with the set of unanimously approved policies $\widetilde{W} = \widetilde{W}_L \cap \widetilde{W}_R$. Firstly, observe that $\widetilde{W}$ cannot be left of $\overline{W}$, i.e. the set $\widetilde{W} \cap [b_L^{AS}, 1]$ must have a positive prior measure. If not, then the *right* voter's constraint has to be spent on policies further than a_R^{AS}, which decreases the objective. Specifically, from R's constraint, $\int_{\overline{W}_R \cap \widetilde{W}_R^c} \alpha_R(x) d\mu_0(x) \leq \int_{\widetilde{W}_R \cap [-1, a_R^{AS}]} \alpha_R(x) d\mu_0(x)$. Also, $\alpha_R(\overline{x}) > \alpha_R(\widetilde{x})$ for all $\overline{x} \in \overline{W}_R$ and $\widetilde{x} \in [-1, a_R^{AS}]$, which implies $\mu_0(\overline{W}_R \cap \widetilde{W}_R^c) > \mu_0(\widetilde{W}_R \cap [-1, a_R^{AS}])$. Finally, since $\widetilde{W} \subseteq [-1, b_L^{AS}]$ a.s., we have $\overline{W}_R \cap \widetilde{W}_R^c = \overline{W} \setminus \widetilde{W}$ and $\widetilde{W}_R \cap [-1, a_R^{AS}] \supseteq \widetilde{W} \setminus \overline{W}$. It follows that $\mu_0(\overline{W}) > \mu_0(\widetilde{W})$. By a symmetric argument for the *left* voter, $\widetilde{W}$ cannot be to the right of $\overline{W}$, either, and the set $\widetilde{W} \cap [-1, a_L^{AS}]$ has to have a positive prior measure.

Next, observe that $\widetilde{W} \cap \big[-1, \lceil A_L \rceil\big]$ and $\widetilde{W} \cap \big[\lfloor A_R \rfloor, 1\big]$ must be intervals that end at 0 and start at 0, respectively. Otherwise, $\widetilde{W}$ can be improved upon. For example, if $\widetilde{W} \cap \big[-1, 0\big] \neq [a, 0]$ for some $a \geq -1$, then there exist two sets $Y = [y_1, y_2] \subseteq \widetilde{W}$ and $Z = [z_1, z_2] \subseteq \widetilde{W}^c$ such that $-1 \leq y_1 < y_2 \leq z_1 < z_2 \leq 0$ and $\mu_0(Y) = \mu_0(Z)$. Then, for every $y \in Y$ and $z \in Z$, $\alpha_R(y) < \alpha_R(z) < 0$ and either $\alpha_L(y) < \alpha_L(z)$ or $\alpha_L(y) > \alpha_L(z) \geq 0$. Let $\widehat{W}_L = \big(\widetilde{W}_L \smallsetminus (Y \cap A_L^c)\big) \cup Z$ and $\widehat{W}_R = \big(\widetilde{W}_R \smallsetminus Y\big) \cup Z$. By construction, $(\widehat{W}_L, \widehat{W}_R)$ satisfies both constraints and maintains the objective at $\mu_0(\widetilde{W})$. However, since R's constraint is now loose, we can further increase the objective, a contradiction.

Proof of Lemma 2

I prove this lemma for the *right* voter whose bliss point increases from R to R'. The case of the *left* voter is symmetric. To simplify the notation, I let $a := a_R^{AS}$ and $a' := a_{R'}^{AS}$.

First, notice that $R' > R$ implies $\lceil A_{R'} \rceil \geq \lceil A_R \rceil$ since $\lceil A_{R'} \rceil = \min\{1, 2R'\} \geq \min\{1, 2R\} = \lceil A_R \rceil$. Thus, $A_{R'} \supseteq A_R$.

Next, observe that unless $a = -1$, R's constraint binds and $\int_a^{\lceil A_R \rceil} \alpha_R(x) d\mu_0(x) = 0$. If the constraint does not bind for R', then $a' = -1 < a$. If it does bind, then

$$\int_{a'}^{\lceil A_{R'} \rceil} \alpha_{R'}(x) d\mu_0(x) = 0 \iff$$

$$\int_{a'}^{a} \underbrace{\alpha_{R'}(x)}_{<0 \text{ since } x<0} d\mu_0(x) + \int_{a}^{\lceil A_R \rceil} \big(\alpha_{R'}(x) - \alpha_R(x)\big) d\mu_0(x) + \int_{\lceil A_R \rceil}^{\lceil A_{R'} \rceil} \underbrace{\alpha_{R'}(x)}_{\geq 0 \text{ since } x \in A_{R'}} d\mu_0(x) = 0.$$

Now, if $\int_a^{\lceil A_R \rceil} \big(\alpha_{R'}(x) - \alpha_R(x)\big) d\mu_0(x) \geq 0$, then $a' < a$. In what follows I show that the former inequality holds for the quadratic $\alpha_R(x)$.

Indeed, $\alpha_{R'}(x) - \alpha_R(x) = 2(R' - R)x$. Let $\overline{x}_R := \int_a^{\lceil A_R \rceil} x d\mu_0(x)$. By Jensen's inequality for the concave $\alpha_R(x)$,

$$\alpha_R(\overline{x}_R) \geq \int\limits_a^{\lceil A_R \rceil} \alpha_R(x) d\mu_0(x) = 0,$$

and $\alpha_R(\overline{x}_R) \geq 0$ if and only if $\overline{x}_R \in \big[0, \lceil A_R \rceil\big]$. Therefore, $\int\limits_a^{\lceil A_R \rceil} 2(R' - R) x d\mu_0(x) \geq 0$ and $a' < a$.