Targeted Advertising in Elections

by

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Stanford GSB Political Economy Seminar, November 1, 2022

Motivation

- **Targeted Advertising** was an important part of winning campaigns in recent U.S. Presidential Elections:
  - **2016 Trump**: used voter data from Cambridge Analytica
  - **2008 Obama**: first social media campaign
  - **2000 Bush**: targeting voters by mail

can targeted advertising swing elections? → Yes

Preview of Results

- some elections are unwinnable for challengers without targeted advertising
  - (pivotal) voters prefer policies on opposite sides of status quo
  - no public message convinces them to approve challenger's policy
- any such election can be won with targeted advertising
  - challenger makes each voter believe his policy is a sufficient improvement over status quo
  - challenger wins if his policy is sufficiently close to status quo
  - voters regret their choices
- if voters become more extreme, challenger's odds of winning increase

Related Literature: Voter Persuasion

- **public voter persuasion**
  - verifiable info: Milgrom (1981), Grossman (1981), Caillaud and Tirole (2007), Board (2009), Jackson and Tan (2013), Janssen and Teteryatnikova (2017), **Titova (2022)**
  - cheap talk: Crawford and Sobel (1982), Schnakenberg (2015), Jeong (2019)
  - info design: Kamenica and Gentzkow (2011), **Alonso and Câmara (2016)**

Related Literature: Private Voter Persuasion

- **private voter persuasion**
  - verifiable info: Schipper and Woo (2019)
    - unraveling does not have to happen if only one candidate advertises
  - cheap talk: Farrell and Gibbons (1989), Koessler (2008), Goltsman and Pavlov (2011), Bar-Isaac and Deb (2014)
    - sender prefers private communication if his messages are verifiable
  - info design: **Arieli and Babichenko (2019)**, Bardhi and Guo (2018), Chan et al. (2019), Heese and Lauermann (2019)
    - sender does not need commitment to benefit from targeted advertising
    - targeting does not just improve odds of winning, it swings unwinnable elections
- **polarization & targeted advertising through media**
  - Hu, Li, and Segal (2019), Prummer (2020), Perego and Yuksel (2022)
    - more polarization → more challengers swing unwinnable elections

Baseline Model

Model Setup

- policy space is $X := [-1, 1]$
  - policies range from far-left (-1) to far-right (1)
  - status quo policy is fixed, known, normalized to 0
- **challenger** (he/him)
  - privately observes his policy $x \in X$
    - x is drawn from common atomless prior $\mu_0 \in \Delta X$ with full support
  - gets 1 if wins the election, 0 otherwise
    - winning requires <u>unanimous approval</u> of both voters

Model Setup: Communication

- challenger communicates with voter using verifiable messages
  - each message m
    - is a statement about policy: $m \subseteq X$
    - contains a grain of truth: $x \in m$
- example: $m = [-1/2, 0]$, or "my policy is moderately left"

Model Setup: Voters

- voters have spatial preferences and status quo bias
- **voter** (she/her) with bliss point $v \in X$ has

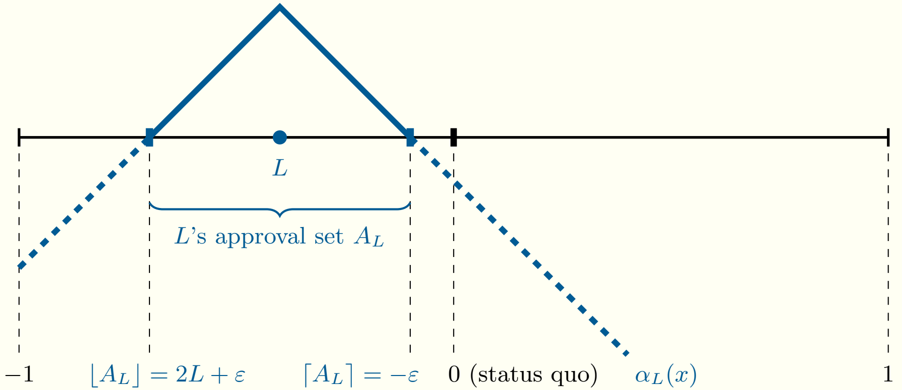
$$\text{utility of approval} \quad u_v(\text{approve}, x) = -|v - x|$$
$$\text{utility of rejection} \quad u_v(\text{reject}, x) = -|v - 0| + \varepsilon$$

net payoff from approval $\quad \alpha_v(x) := -|v - x| + |v| - \varepsilon$

approval set $\quad A_v := \{x \in X \mid \alpha_v(x) \geq 0\}$

- two voters: *left* (with $v = L < -\varepsilon$) and *right* (with $v = R > \varepsilon$)

Voter's Preferences: Illustration



Timeline and Equilibrium Concept

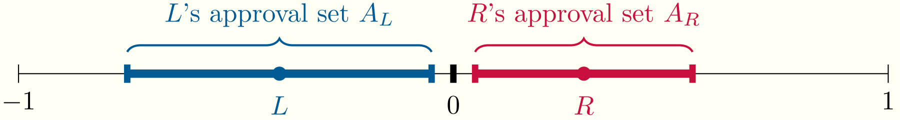
challenger learns his policy $x \in X$	$\rightarrow$	challenger selects m_L and m_R	$\rightarrow$	voter $v \in \{L, R\}$ privately observes message m_v	$\rightarrow$	voter $v \in \{L, R\}$ chooses between approval and rejection

- **Perfect Bayesian Equilibrium**
  - for every policy $x \in X$, private messages $m_L \subseteq X$ and $m_R \subseteq X$ maximize challenger's utility subject to $x \in m_L$ and $x \in m_R$
  - voter approves whenever expected net payoff from approval is non-negative under her posterior
    - does not condition on the event of being pivotal
  - voters' posteriors are Bayes-consistent

Unwinnable Election

Incompatible Voters

- *left* and *right* voters prefer policies on opposite sides of status quo



Lemma 1

If voters hold a common belief, then at most one of them prefers to approve.

Unwinnable Election

- baseline election is <u>unwinnable</u> for challenger <u>without targeted advertising</u>
  - no advertising
  - full disclosure
  - public disclosure
- true for any communication protocol (evidence, cheap talk, Bayesian persuasion) that induces *common posterior*

Equilibrium Analysis

Equilibrium Outcomes

- focus on equilibrium sets of approved policies
  - voter $v \in \{L, R\}$ approves set of policies $W_v \subseteq X$, rejects $W_v^c := X \setminus W_v$
  - direct implementation: when talking to v, challenger sends message
    - W_v if his policy is $x \in W_v \longleftarrow$ recommendation to approve
    - W_v^c if his policy is $x \notin W_v \longleftarrow$ recommendation to reject
- Titova (2022): $(W_L, W_R) \subseteq X^2$ is an equilibrium outcome iff $\forall v \in \{L, R\}$
  - $A_v \subseteq W_v$: challenger does not want to deviate to full disclosure
  - $\int\limits_{W_v} \alpha_v(x) d\mu_0(x) \geq 0$: voter's **obedience constraint**

Challenger-Preferred Equilibrium

- I focus on challenger-preferred PBE
  - one with highest odds of unanimous approval/winning
- problem:

$$(\overline{W}_L, \overline{W}_R) = \arg \max_{(W_L, W_R) \subseteq X^2} \overbrace{\int_{W_L \cap W_R} d\mu_0(x)}^{\mu_0(W_L \cap W_R)}$$

$$\text{subject to} \quad \int_{W_v} \alpha_v(x) d\mu_0(x) \geq 0 \text{ for each } v \in \{L, R\}$$

I call $(\overline{W}_L, \overline{W}_R)$ the (challenger-preferred) equilibrium outcome (under targeted advertising)

Theorem 1

Theorem 1: Swinging Unwinnable Elections

In equilibrium of the baseline election game, the challenger's ex-ante odds of winning are always positive.

idea of proof: for each voter $v \in \{L, R\}$

- observe that v always approves own approval set: $\int_{A_v} \alpha_v(x) d\mu_0(x) > 0$
- select subset $B_v \subseteq A_{-v}$ of the other voter's approval set that satisfies

$$\int_{A_v} \alpha_v(x) d\mu_0(x) + \int_{B_v} \alpha_v(x) d\mu_0(x) \geq 0 \quad \text{and} \quad \mu_0(B_v) > 0$$

- let $W_v = A_v \cup B_v$
- we have $\mu_0(W_L \cap W_R) = \mu_0(B_R \cup B_L) > 0$

$\Longrightarrow$ odds in equilibrium are positive

Convincing One Voter

Auxiliary Problem

- **question**: what is the largest subset of $[l, r] \subseteq X$ can voter v approve?

$$\max_{W \subseteq [l,r]} \mu_0(W) \quad \text{subject to} \quad \int_W \alpha_v(x) d\mu_0(x) \geq 0 \tag{AUX}$$

- **answer**: Alonso and Câmara (2016) and Titova (2022)

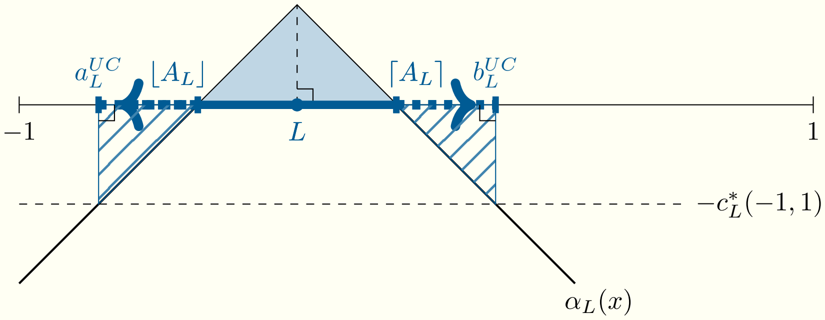
Corollary 2

Consider voter $v \in X \setminus [-\varepsilon, \varepsilon]$. Then, the solution to Problem (AUX) with $l \in \big[-1, \lfloor A_v \rfloor\big]$ and $r \in \big[\lceil A_v \rceil, 1\big]$ is an interval $I_v(l, r)$ such that

- if $\int\limits_l^r \alpha_v(x) d\mu_0(x) \geq 0$, then $I_v(l, r) = [l, r]$
- otherwise, $I_v(l, r) = \big\{x \in [l, r] \mid \alpha_v(x) \geq -c_v^*(l, r)\big\}$, where $c_v^*(l, r) > 0$ is obtained from the binding constraint $\int\limits_{I_v(l,r)} \alpha_v(x) d\mu_0(x) = 0$

Largest Unconstrained Interval of Approved Policies

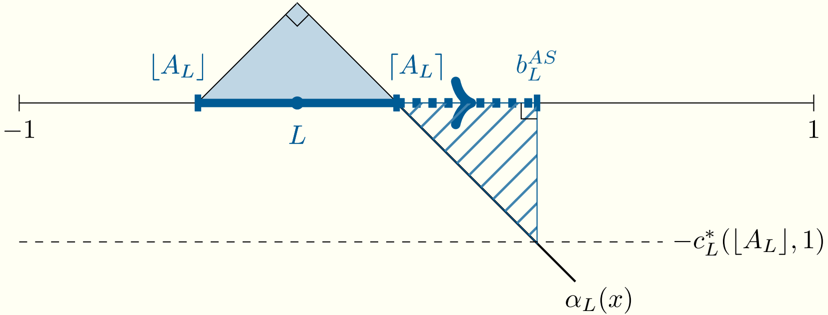
- solve (AUX) for $l = -1$ and $r = 1$ to get $I_v(-1,1) =: I_v^{UC} = [a_v^{UC}, b_v^{UC}]$
  - v's largest unconstrained interval of approved policies
- **example**: $left$ voter, $v = L$



Largest Asymmetric Interval of Approved Policies

- *left* voter: how many *right* policies can she approve?

$$I_L^{AS} = \left[\lfloor A_L \rfloor, b_L^{AS}\right] := I_L(\lfloor A_L \rfloor, 1)$$

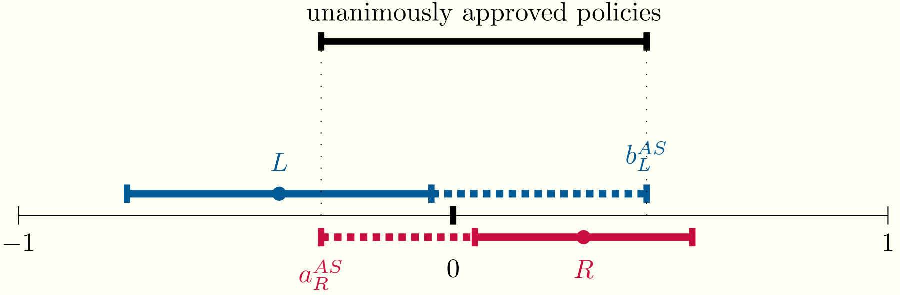


- *right* voter: how many *left* policies can she approve?

$$I_R^{AS} = \left[a_R^{AS}, \lceil A_R \rceil\right] := I_R(-1, \lceil A_R \rceil)$$

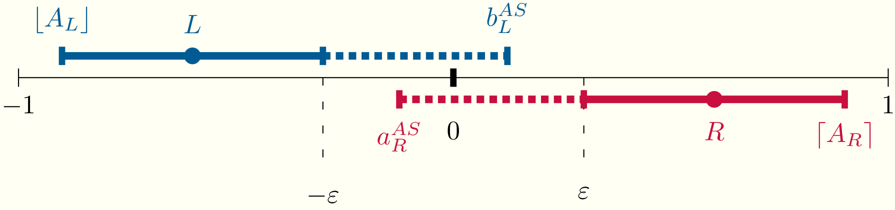
Convincing Two Voters at the Same Time

Candidate Solution



When Candidate Solution Fails, Case 1

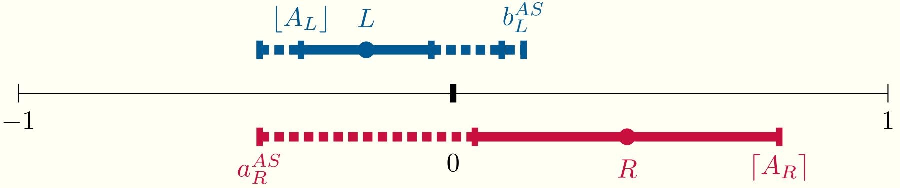
- when status quo bias ε is too large:



- **[A1]**: $\int_{\lfloor A_L \rfloor}^{\varepsilon} \alpha_L(x) d\mu_0(x) \geq 0$ and $\int_{-\varepsilon}^{\lceil A_R \rceil} \alpha_R(x) d\mu_0(x) \geq 0$ rules this out

When Candidate Solution Fails, Case 2

- *right* voter is significantly more persuadable, or $\int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_R(x) d\mu_0(x) > 0$



- **[A2]**: *left* voter is not significantly more persuadable than the *right* voter

$$\int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_L(x) d\mu_0(x) \leq 0$$

Theorem 2

Theorem 2: Equilibrium Intervals of Approved Policies

If **[A1*]** and **[A2]** hold, then $\overline{W}_L$ and $\overline{W}_R$ are intervals. Almost surely,

(1) if neither voter is significantly more persuadable than the other, then

- $\overline{W}_L = \left[\lfloor A_L \rfloor, b_L^{AS}\right]$ and $\overline{W}_R = \left[a_R^{AS}, \lceil A_R \rceil\right]$
- equilibrium set of unanimously approved policies is $\overline{W} = [a_R^{AS}, b_L^{AS}]$

(2) if *right* voter is significantly more persuadable than *left* voter, then

- $\overline{W}_R = \left[a_R^{AS}, \lceil A_R \rceil\right]$
- the *left* voter's equilibrium set of approved policies is her largest interval of approved policies constrained from the left by a_R^{AS}, or $\overline{W}_L = I_L(a_R^{AS}, 1)$
- the equilibrium set of unanimously approved policies is $\overline{W} = \overline{W}_L$

Theorem 2: Case 1

Theorem 2

If **[A1]** and **[A2]** hold, then $\overline{W}_L$ and $\overline{W}_R$ are intervals. Almost surely,

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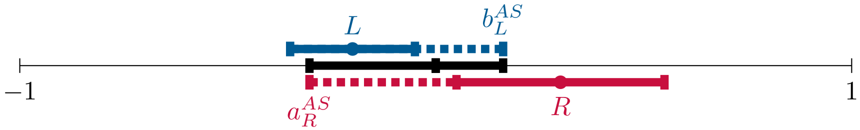
- equilibrium set of unanimously approved policies is $\overline{W} = [a_R^{AS}, b_L^{AS}]$

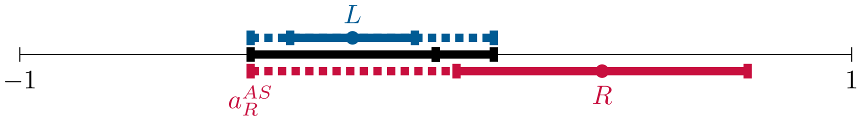


Theorem 2

If [**A1***] and [**A2**] hold, then $\overline{W}_L$ and $\overline{W}_R$ are intervals. Almost surely,

(2) if *right* voter is significantly more persuadable than *left* voter, then

- $\overline{W}_R = \left[a_R^{AS}, \lceil A_R \rceil\right]$
- the *left* voter's equilibrium set of approved policies is her largest interval of approved policies constrained from the left by a_R^{AS}, or $\overline{W}_L = I_L(a_R^{AS}, 1)$
- the equilibrium set of unanimously approved policies is $\overline{W} = \overline{W}_L$



Theorem 2

If **[A1*]** and **[A2]** hold, then $\overline{W}_L$ and $\overline{W}_R$ are intervals. Almost surely,

(2) if *right* voter is significantly more persuadable than *left* voter, then

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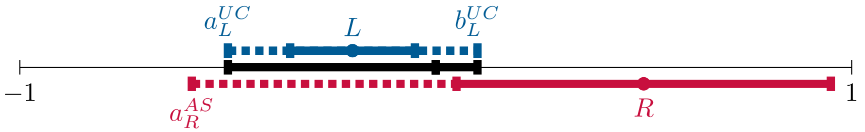


Calculating Equilibrium Intervals of Approved Policies

identifying $(\overline{W}_L, \overline{W}_R)$ requires solving 2 (AUX) problems

algorithm:

- calculate $\rho_v := \int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_v(x) d\mu_0(x)$ for each $v \in \{L, R\}$
  - if $\rho_R, \rho_L \leq 0$, neither voter is more persuadable than the other
    - solve 2x (AUX) problems to find a_R^{AS} and b_L^{AS}
  - if $\rho_R > 0 \geq \rho_L$, the *right* voter is significantly more persuadable
    - solve 1x (AUX) problem to find a_R^{AS}
    - solve 1x (AUX) problem to find $I_L(a_R^{AS}, 1)$
  - if $\rho_L > 0 \geq \rho_R$, the left voter is significantly more persuadable
    - solve 1x (AUX) problem to find b_L^{AS}
    - solve 1x (AUX) problem to find $I_R(-1, b_L^{AS})$

Comparative Statics

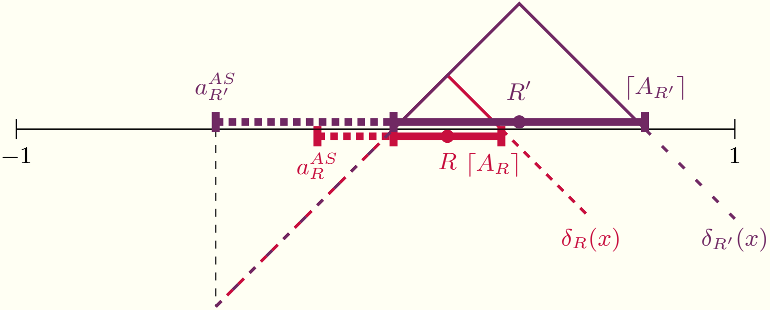
Extremism and Polarization

- voter with bliss point $v \in X \setminus [-\varepsilon, \varepsilon]$ becomes more extreme if $|v|$ increases
- baseline electorate becomes more polarized if $left$ and/or $right$ voter becomes more extreme
  - *most polarized electorate* is $L = -1$ and $R = 1$
  - larger distance between L and R doesn't always imply higher polarization

- as a voter becomes more extreme, she becomes more persuadable

Lemma 2

If $R < R' \leq 1$, then $[a_R^{AS}, \lceil A_R \rceil] \subseteq [a_{R'}^{AS}, \lceil A_{R'} \rceil]$, with $a_R^{AS} \geq a_{R'}^{AS}$ and $\lceil A_R \rceil \leq \lceil A_{R'} \rceil$; the former inequality is strict unless $a_R^{AS} = -1$.



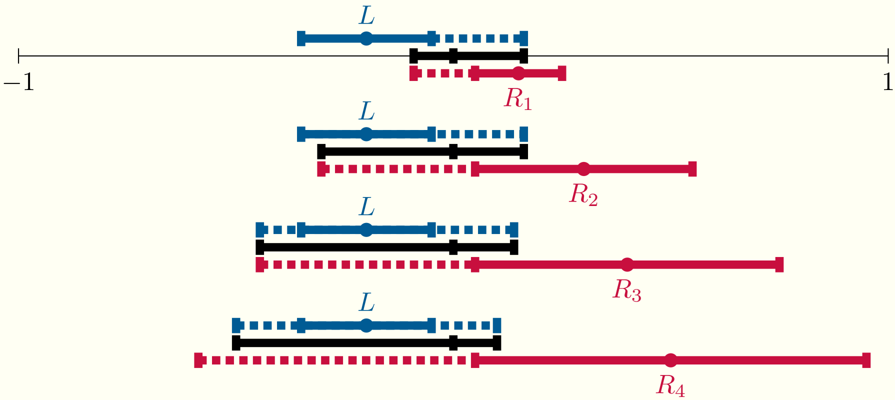
Theorem 3

Theorem 3: Comparative Statics

Suppose that L and R satisfy [**A1***] and [**A2**]. Then, as the *right* voter becomes more extreme,

- challenger's odds of winning increase;
- equilibrium set of unanimously approved policies shifts to the left.

Comparative Statics



Welfare

Welfare and Regret

- if v's set of approved states is W_v, her ex-ante utility is

$$\mathbb{E}_{\mu_0}\Big[\mathbb{1}(x \in W_v) \cdot \big(-|v-x|\big) + \mathbb{1}(x \in W_v^c) \cdot \big(-|v| + \varepsilon\big)\Big]$$

- subtract $-|v| + \varepsilon$ to get $\int\limits_{W_v} \alpha_v(x) d\mu_0(x)$

Definition

Consider $v \in X$ and her set of approved policies W_v. Then, v's

- welfare is $\int\limits_{W_v} \alpha_v(x) d\mu_0(x)$;
- amount of regret is $-\int\limits_{W_v \setminus A_v} \alpha_v(x) d\mu_0(x)$.

Communication Benchmarks

- **full disclosure** outcome (A_L, A_R)
  - also the challenger-worst equilibrium of baseline game
- **public disclosure** outcome (W_L^{PD}, W_R^{PD})
  - challenger's odds of winning are zero
- **targeted advertising** outcome $(\overline{W}_L, \overline{W}_R)$

Welfare Comparison

	v's welfare	v's regret	challenger's odds of winning
full disclosure	$\int_{A_v} \alpha_v(x) d\mu_0(x) > 0$	0	0
public disclosure	$\int_{W_v^{PD}} \alpha_v(x) d\mu_0(x) \geq 0$	≥ 0	0
targeted advertising	$\int_{\overline{W}_v} \alpha_v(x) d\mu_0(x) = 0$	> 0	$\mu_0(\overline{W}_L \cap \overline{W}_R) > 0$

Large Elections

Large Electorates and Unwinnable Elections

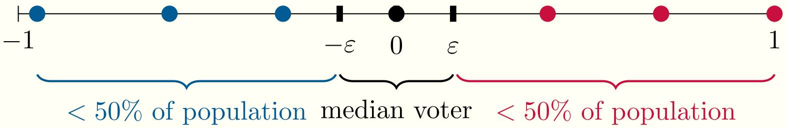
- **large electorate**: set of bliss points $V = \{v_1, \ldots, v_n\}$ with $v_1 \leq \cdots \leq v_n$
- $\mathcal{D}$ is set of <u>decisive coalitions</u>
  - challenger wins (and gets 1) iff he convinces every voter in some $D \in \mathcal{D}$

Lemma: Unwinnable Elections

The following statements are equivalent:

- election is unwinnable for the challenger without targeted advertising;
- no policy of the challenger is preferred to the status quo under complete information;
- there is no $D \in \mathcal{D}$ such that $v < -\varepsilon \ \forall v \in D$ OR $v > \varepsilon \ \forall v \in D$.

- simple majority rule – which elections are unwinnable?



(version of the) Median Voter Theorem

Under simple majority rule, election is unwinnable for the challenger without targeted advertising if and only if the median voter's bliss point is in $[-\varepsilon, \varepsilon]$.

Swinging Large Unwinnable Elections

- for any (minimal) decisive coalition D, identify
  - the left pivot: $L := \max_{v \in D \text{ s.t. } v<0} v$
    - every other voter on the left is convinced if L is convinced
  - the right pivot: $R := \min_{v \in D \text{ s.t. } v>0} v$
    - every other voter on the right is convinced if R is convinced
- solve baseline election for L and R
  - *if L or R is in $[-\varepsilon, \varepsilon]$ then election remains unwinnable under TA*
  - otherwise, use Theorem 2
- maximizing odds of winning requires doing this for every minimal winning coalition

Conclusion

- some elections are unwinnable without targeted advertising
  - (pivotal) voters prefer policies on opposite sides of status quo
- any such election can be won with targeted advertising
  - challenger makes each voter believe his policy is sufficient improvement over status quo
  - challenger wins if his policy is not too far from status quo
  - voters regret their choices
- if voters become more extreme, challenger's odds of winning increase

Thank You!