ELECTION GAMES WITH VERIFIABLE INFORMATION

by

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- election games with verifiable information
  - privately informed sender
    - wants his proposal approved by a **collective vote**
    - sends verifiable messages to receivers
  - group of uninformed receivers, each choosing between
    - rejecting sender's proposal
    - approving it

- wide variety of applications
  - politician challenges status quo, wants to get elected by voters
  - prosecutor convinces jury to convict defendant
  - CEO convinces board of directors to approve bonuses to executives
  - job market candidate convinces committee members to offer him a job

- **election games with verifiable information**
  - recommendation principle: can restrict attention to direct equilibria
    - sender recommends action
  - ranking of equilibrium outcomes (for sender / for receivers):
    - (worst / best): equivalent to full disclosure
    - (best / worst): equivalent to Bayesian Persuasion

APPLICATION

- **targeted advertising** in (a spatial model of) elections
  - targeting allows politician to swing elections that he would lose otherwise

MODEL SETUP

$$\Omega = [0,1]^K \text{ -- } \underline{\text{state space}}$$

$$\mathbb{V} \text{ -- finite } \underline{\text{set of receivers}}$$

- sender
  - privately observes state of the world $\omega \in \Omega$
    - ω drawn from common prior $p > 0$ over Ω
  - receives 1 if his proposal is approved, 0 otherwise (state-independent)
  - sends verifiable message $m_v \in \mathbb{M}$ to each receiver $v \in \mathbb{V}$:
    - message space $\mathbb{M} := 2^{|\Omega|}$
    - $\omega \in m_v$ – no lies of commission

consider receiver $v \in \mathbb{V}$

$\delta(v, \omega)$ is her net payoff of approval

- *under complete information* (receiver knows ω)
  - receiver v approves in state ω only if $\delta(v, \omega) \geq 0$
  - **approval set**

$$\mathcal{A}_v := \big\{\omega \in \Omega \mid \delta(v, \omega) \geq 0\big\}$$

- *under incomplete information* (receiver has belief q about ω)
  - receiver v approves under belief q only if $\mathbb{E}_q\big[\delta(v, \omega)\big] \geq 0$
  - **set of approval beliefs**

$$\mathcal{B}_v := \Big\{q \in \Delta\Omega \mid \mathbb{E}_q\big[\delta(v, \omega)\big] \geq 0\Big\}$$

- outcome of election game is decided by social choice function $f(\cdot)$:

$$f : \underbrace{2^{|\mathbb{V}|}}_{\substack{\text{receivers who} \\ \text{voted to approve} \\ \text{sender's proposal}}} \rightarrow \underbrace{\{0, 1\}}_{\substack{\text{0: proposal is rejected} \\ \text{1: proposal is approved}}}$$

satisfying unanimous agreement:

$$f(\varnothing) = 0 \qquad f(\mathbb{V}) = 1$$

- examples include
  - dictatorship: $f(V) = 1 \iff v^{dict} \in V$
  - simple majority: $f(V) = 1 \iff |V|/|\mathbb{V}| > \frac{1}{2}$
  - unanimity: $f(V) = 1 \iff V = \mathbb{V}$

- Perfect Bayesian Equilibrium (σ, α, q)
  - $\sigma\big(\{m_v\} \mid \omega\big)$ – prob. sender sends message collection $\{m_v\}$, $\forall \omega \in \Omega$
    - maximizes chances of approval under $f(\cdot)$ in every state ω
  - $\alpha := \{\alpha_v\}_{v \in \mathbb{V}}$ – receivers' voting rules
    - approve $(\alpha_v(m_v) = 1)$ if and only if $q_v(\cdot \mid m_v) \in \mathcal{B}_v$, $\forall m_v \in \mathbb{M}$
  - $q := \{q_v\}_{v \in \mathbb{V}}$ – receivers' posterior beliefs
    - $q_v(\cdot \mid m_v)$ Bayes-rational on equilibrium path

ONE RECEIVER

- direct implementation with
  - set of winning states $\mathcal{W} \subseteq \Omega$
  - set of losing states $\mathcal{L} = \Omega \setminus \mathcal{W}$

state	sender's message	receiver's belief	receiver's vote
$\omega \in \mathcal{W}$	$\mathcal{W}$	$p(\cdot \mid \mathcal{W})$	approve
$\omega \in \mathcal{L}$	$\mathcal{L}$	$p(\cdot \mid \mathcal{L})$	reject

where $p(\omega \mid W) := \frac{p(\omega)}{\int_W p(\omega')d\omega'}$, $\forall W \subseteq \Omega$ is conditional probability direct equilibrium:

- sender recommends action
  - (*approval*): winning message $\mathcal{W}$ in winning states $\omega \in \mathcal{W}$
  - (*rejection*): losing message $\mathcal{L}$ in losing states $\omega \in \mathcal{L}$
- receiver obediently follows recommendation

Theorem: Recommendation Principle

- direct implementation with set of winning states $\mathcal{W} \subseteq \Omega$ constitutes direct equilibrium if and only if
  - $\mathcal{A} \subseteq \mathcal{W}$, and
  - (obedience) constraint holds: $p(\cdot \mid \mathcal{W}) \in \mathcal{B}$
- every equilibrium is outcome-equivalent to *some* direct equilibrium

- recommendation principle allows to restrict attention to direct equilibria
  - characterized by set of winning states $\mathcal{W} \supseteq \mathcal{A}$ satisfying (obedience)
- rank equilibria by sender's ex-ante utility
  - same as his ex-ante odds of approval
  - equals $P(\mathcal{W})$

where $P(W) := \int_W p(\omega) d\omega$, $\forall W \subseteq \Omega$ is *prior measure*

- sender's odds of approval are lowest across all equilibria
  - $\mathcal{W} = \mathcal{A}$
- receiver makes fully informed choice
- outcome-equivalent to full disclosure
  - (Grossman, 1981), (Milgrom, 1981), (Milgrom and Roberts, 1986), reviewed by (Milgrom, 2008)

- maximize sender's odds of approval across all equilibria

$$\max_{\mathcal{W}} P(\mathcal{W}) \quad \text{subject to} \quad p(\cdot \mid \mathcal{W}) \in \mathcal{B}$$

  - largest (in terms of ex-ante utility) set of winning states $\mathcal{W}^*$
  - (obedience) binds: $\mathbb{E}_{p(\cdot \mid \mathcal{W}^*)}\big[\delta(v,\omega)\big] = 0$
    - receiver is indifferent when she approves

Theorem

Solutions to problems of

- finding sender-preferred equilibrium in verifiable information game
- solving sender's problem in bayesian-persuasion (BP) game

are equivalent

- verifiable information (this paper):

  sender learns $\omega \to$ sender chooses message $\to$ receiver observes message

- Bayesian Persuasion (Kamenica and Gentzkow, 2011):

  sender $\underline{\text{chooses and commits to experiment}}$ $\{\pi(\cdot \mid \omega)\}_{\omega \in \Omega}$ over S

  $\to \omega$ is realized

  $\to$ receiver observes signal $\pi(\cdot \mid \omega)$ and realization $s \in S$

same outcome $\Longrightarrow$

SENDER DOES NOT BENEFIT FROM HAVING EX-ANTE COMMITMENT POWER!

MULTIPLE RECEIVERS

- direct implementation: collection of convincing messages $\{\mathcal{W}_v\}$ such that
  - message $\mathcal{W}_v$ is recommendation to approve (sent to receiver v)

- **recommendation principle**: $\{\mathcal{W}_v\}$ constitutes direct equilibrium if and only if for every receiver
  - $\mathcal{A}_v \subseteq \mathcal{W}_v$
  - (obedience) constraint holds: $p(\cdot \mid \mathcal{W}_v) \in \mathcal{B}_v$

  every equilibrium is outcome-equivalent to *some* direct equilibrium

- ranking of equilibrium outcomes (for sender / for receivers):
  - (worst / best): equivalent to full disclosure
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APPLICATION:
TARGETED ADVERTISING IN A SPATIAL MODEL

- **Targeted Advertising** was an important part of winning campaigns in recent U.S. Presidential Elections:
  - 2016 Trump: used voter data from Cambridge Analytica
  - 2008, 2012 Obama: the first social media campaign
  - 2000, 2004 Bush: targeting voters by mail

CAN TARGETED ADVERTISING SWING ELECTIONS? → Yes

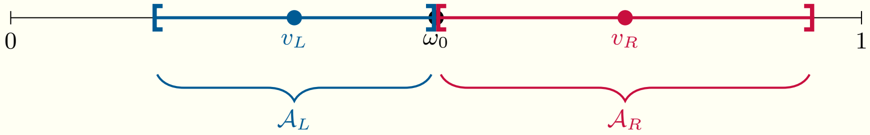
- assume **spatial model** (Downs, 1957):
  - ◇ policy space $\Omega = [0,1]$
  - ◇ $\omega^0 \in (0,1)$ is status quo policy (fixed)
  - ◇ $\mathbb{V} \subset \Omega$, $v \in \mathbb{V}$ is voter's ideal policy
    - approval set becomes $\mathcal{A}_v = \left\{\omega \in \Omega \text{ s.t. } |v-\omega| \leq |v-\omega^0|\right\}$

- compare Public Disclosure (**PD**) to Targeted Advertising (**TA**)
  - ◇ **PD**: public message (e.g. debate, tweeting)
    - *common prior + common message → common posterior*
  - ◇ **TA**: private messages (e.g. through Facebook)

- observation #1: *some voters are incompatible*

$$\text{if } v_L < \omega_0 < v_R \text{, then } \mathcal{A}_L \cap \mathcal{A}_R = \varnothing \text{ and } \mathcal{B}_L \cap \mathcal{B}_R = \varnothing$$

- observation #2: *some elections are unwinnable*
  - if incompatible voters are pivotal then sender loses almost surely
  - example: $\mathbb{V} = \{v_L, v_R\}$, unanimity rule



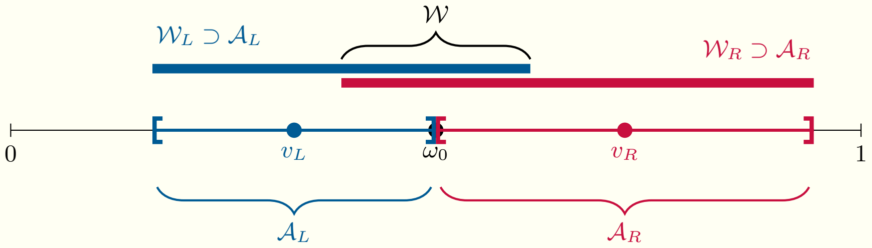
- observation #3: every unwinnable election has two incompatible voters $v_L < \omega^0$ (left pivot) and $\omega^0 < v_R$ (right pivot), convincing whom is sufficient to win under any $f(\cdot)$

Theorem: Targeting in Unwinnable Elections

In sender-preferred equilibrium of unwinnable election with left pivot $v_L < \omega^0$ and right pivot $\omega^0 < v_R$

- set of winning states is $[a, b]$ with $a < \omega^0 < b$

- $\mathbb{V} = \{v_L, v_R\}$, unanimity rule
  - must convince v_L and v_R who are incompatible $\Longrightarrow$ unwinnable under PD



- equilibrium outcome set of election games with verifiable information
    - sender-worst: full disclosure / full unraveling
    - sender-preferred: Bayesian Persuasion
        - sender does not benefit from ex-ante commitment
        - need not assume commitment, may assumer verifiable messaging

- application: targeted advertising in (a spatial model of) elections
    - targeting allows politician to swing elections that he would lose otherwise
        - only if his policy is sufficiently close to the status quo
    - *policy implications*: targeting leads to outcomes different from complete information

Thank You!