Persuasion with Verifiable Information*

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LATEST VERSION

Abstract

This paper studies persuasion with verifiable information. An informed sender with state-independent preferences sends private verifiable messages to multiple receivers attempting to convince them to approve a proposal. I find that every equilibrium is outcome equivalent to a direct equilibrium, in which the sender tells each receiver what to do, and receivers obediently follow their recommendations. This allows me to characterize the full equilibrium set. The sender-worst equilibrium outcome is one in which information unravels, and receivers act as if under complete information. The sender-preferred equilibrium outcome is the commitment outcome of the Bayesian persuasion game. In the leading application, I study targeted advertising in elections and show that by communicating with voters privately, a challenger may win elections that are unwinnable with public messages. As the electorate becomes more polarized, the challenger can swing unwinnable elections with a higher probability.

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1. INTRODUCTION

Suppose the sender attempts to convince a group of receivers to take his favorite action. The only tool available to him is hard evidence. What he can do is choose how much of it to reveal. On average, what is the best outcome that the sender can hope for?

Persuasion with verifiable information plays an essential role in electoral campaigns, product advertising, financial disclosure, and job market signaling, among many other economic situations. In politics, a challenger convinces voters to elect him over the status quo by sending fact-checked ads about his policy position on some relevant socio-economic issues, saying nothing about other issues. In business, a firm convinces consumers to adopt its product by advertising some product characteristics, not mentioning others. In finance, a CEO convinces the board of directors to approve managerial compensation by presenting some financial indicators and statements, omitting others. In labor markets, a job candidate convinces committee members to offer him a job by attaching to his application selected evidence of his qualifications.

I consider the following formal model of persuasion with verifiable information. There is an underlying continuous space of possible states of the world, which is assumed to be a unit interval for simplicity. The sender is fully informed about the state of the world, but his preferences do not depend on it. Receivers are uninformed about the state of the world, which to them is payoff-relevant. The sender sends a verifiable message to each receiver. Verifiability means that the message contains the truth (hard evidence is presented), but it could be vague (not all the evidence is presented). Each receiver independently chooses between two options: to approve the proposal or to reject it. There are no information spillovers between the receivers: each receiver only hears her own private message.

How does the sender convince one receiver with verifiable information? Rather than looking at the sender's messages and the receiver's beliefs, I focus on what the receiver does in every state of the world. Since she chooses between two options, one can partition the state space into two subsets: the set of states in which the proposal is approved and its complement. The first result states that if some set of approval states is induced in equilibrium, then there exists a direct equilibrium, in which the sender simply tells the receiver what to do. More specifically, if the state of the world is within the set of approval states, he tells the receiver to approve the proposal; otherwise, he tells her to reject it. Every direct equilibrium is characterized by the set of approval states that satisfies two constraints. The *receiver's obedience constraint* ensures that when the sender tells the receiver to approve the proposal, the receiver must rationally expect that her net payoff of approval is non-negative. The *sender's incentive-compatibility constraint* (IC) ensures that the sender does not wish to deviate toward a fully informative strategy that induces the receiver to act as if fully informed. Equipped with the direct implementation result, I characterize the full equilibrium set by restricting attention to sets of approval states that satisfy these two constraints.

In the sender's least preferred equilibrium, his ex-ante odds of approval are minimized across all equilibria. The sender voluntarily discloses whether the state of the world is in the receiver's complete information approval set; the receiver learns everything relevant for her decision and makes a fully-informed choice.

In the sender's most preferred equilibrium, his odds of approval are maximized subject to the receiver's obedience constraint. The solution features a cutoff state: if the net payoff of approval is greater than the cutoff value, then the receiver approves the proposal, and if not, then she rejects it. The cutoff value is negative: the sender convinces the receiver to approve his proposal when she prefers not to approve it under complete information, thus improving his odds upon full disclosure. The sender obfuscates the state of the world so long as the receiver follows through on the recommendation. Consequently, the receiver is indifferent between approval and rejection when she is recommended to approve. This observation makes a connection between verifiable disclosure and Bayesian persuasion (Kamenica and Gentzkow, 2011). The sender-preferred equilibrium outcome is also a commitment outcome of the Bayesian persuasion game. As a result, the challenger need not benefit from having ex-ante commitment power and can persuade the receiver with hard evidence.

With many receivers, one can also restrict attention to direct equilibria, in which the sender privately tells each receiver what to do, and each receiver obediently follows her recommendation. The sender-worst equilibrium features full disclosure, and the sender-preferred equilibrium outcome is a commitment outcome.

The leading application of the model allows me to make predictions about the effectiveness of targeted advertising in electoral campaigns. The state space is now a one-dimensional policy space with positions ranging from ultra-left (0) to ultra-right (1). The voters choose between the challenger, whose policy is unknown, and the status quo policy, which is fixed and known. Each voter prefers to vote in favor of the policy that is closest to her bliss point. In his electoral campaign, the challenger sends verifiable messages to the voters to inform them about his policy and convince them to elect him.

Suppose that winning an election requires convincing two voters, L and R, whose bliss points are located to the left and the right of the status quo policy, respectively. Observe that unless the challenger can privately advertise to each of these voters, he always loses this election. As long as these voters hold a common belief, which they do under full disclosure, no disclosure, or public disclosure by the challenger, only one of these voters expects the challenger's policy to be closer to her bliss point than the status quo. Consequently, the challenger certainly loses this election when the voters hold a common belief. I call this election unwinnable for the challenger. Whether an election is unwinnable depends on the institution (the social choice function) and the ideology of the electorate (bliss point of the voters). For example, under the majority rule, I show that an election is unwinnable if and only if the status quo is the median voter's bliss point.

When the challenger has access to targeted advertising, he can tell different things to different voters. Recall that in his most-preferred equilibrium, the sender improves his odds of approval upon full disclosure. In particular, the challenger manages to convince voter L (R) even when his policy is slightly to the right (left) of the status quo. Consequently, he can convince both voters at the same time and win unwinnable elections with positive probability. That said, the challenger only benefits from private communication if his policy is sufficiently close to the status quo: the further to the right (left) his policy is, the harder it becomes to convince voter L (R).

When a voter's bliss point moves away from the status quo, she becomes less satisfied with the status quo, and that makes her more persuadable. Consequently, when the electorate becomes more polarized, which happens when one of the voters' positions becomes more extreme, the challenger has higher odds of swinging an unwinnable election.

As voter R's position moves further to the right, she becomes more persuadable also by policies further to the left of the status quo. Consequently, when voter R's bliss point shifts to the right, the challenger-preferred set of winning policies shifts to the left, toward the policies preferred by the less extreme voter L.

RELATED LITERATURE

I assume that the sender uses hard evidence to communicate with the receivers. This verifiable information communication protocol was introduced by Milgrom (1981) and Grossman (1981). Other communication protocols include cheap talk by Crawford and Sobel (1982); Bayesian persuasion by Kamenica and Gentzkow (2011); signaling by Spence (1973). Relative to these other models of communication, Bayesian persuasion makes the sender better off because it endows him with ex-ante commitment power. Lipnowski and Ravid (2020) find that the sender's maximal equilibrium payoff from cheap talk is generally strictly lower than his payoff under commitment. Consequently, a cheap-talk sender values commitment.[1] In contrast to their result, I show that the sender does not necessarily benefit from commitment if he possesses the hard evidence to verify his messages.

There is extensive literature on applications of Bayesian persuasion models. It includes settings in which schools persuade employers to hire their graduates (Ostrovsky and Schwarz, 2010; Boleslavsky and Cotton, 2015); pharmaceutical companies persuade the FDA to approve their drug (Kolotilin, 2015); matching platforms persuade sellers to match with buyers (Romanyuk and Smolin, 2019); politicians persuade voters (Alonso and Câmara, 2016; Bardhi and Guo, 2018); governments persuade citizens through media (Gehlbach and Sonin, 2014; Egorov and Sonin, 2019). My contribution states that in all these applications, one can replace the assumption that the sender has commitment power with the assumption that the sender has hard evidence.

The leading application contributes to the growing literature on voter persuasion. My results are in line with the recent findings in the information design literature on private

[1] Lipnowski (2020) also notes that the sender reaches the commitment outcome with cheap talk if his value function is continuous in the receiver's posterior belief. That assumption is very restrictive: when receivers choose between two options and the sender's preferences are state-independent, the sender's value function must be constant, meaning that no communication takes place under cheap talk, verifiable information, and Bayesian persuasion. I thank Elliot Lipnowski for this insight.

persuasion of strategic voters. In particular, Chan et al. (2019) confirm that the politician does better when private disclosure is allowed, and Heese and Lauermann (2019) confirm that the politician needs very little commitment power to achieve the desired outcome. In the verifiable-information literature, electoral competition usually results in the full unraveling of information (Board, 2009; Janssen and Teteryatnikova, 2017; Schipper and Woo, 2019) because the candidates play a zero-sum game and that pushes them to disclose all information voluntarily. In contrast to these papers, I consider a non-symmetric model in which one candidate has a significant advantage over his opponent in that he is the only one who can communicate with the voters. Unraveling does not necessarily occur, and the challenger can improve his odds of winning over full disclosure.

The leading application sheds more light on how political advertising, especially targeted advertising, affects electoral outcomes and why it has become widespread. DellaVigna and Gentzkow (2010) and Prat and Strömberg (2013) provide excellent surveys of the evidence of voter persuasion. First, candidates target their ads based on voters' positions on the political spectrum (George and Waldfogel, 2006; DellaVigna and Kaplan, 2007). Second, one can make a case that an increase in the availability of information catered toward certain electoral groups also counts as targeted advertising because these are the messages intended for and heard by these groups (Oberholzer-Gee and Waldfogel, 2009; Enikolopov et al., 2011). I show that targeted political advertising may be so widespread because it allows politicians to win elections that are unwinnable otherwise.

I also contribute to the growing literature on polarization and targeted political advertising through media. As the number of media outlets increases, they become more specialized and target voters with more extreme preferences, which leads to social disagreement (Perego and Yuksel, 2018). If the electorate is polarized to begin with, so are the candidates' chosen policy platforms (Hu et al., 2019; Prummer, 2020). Abstracting away from candidates choosing their policies, I find that as the electorate becomes more polarized, more challengers can swing elections that are unwinnable otherwise.

This paper is organized as follows. Section 2 introduces the model. Section 3 describes equilibrium outcomes in the game with one receiver. Section 4 generalizes the model to many receivers. Section 5 studies targeted advertising in elections. Section 6 concludes.

2. MODEL

There is a state space $\Omega := [0,1]$ and a finite set of receivers $I := \{1,\ldots,n\}$. The game begins with the sender (him) observing the realization of the random state $\omega \in \Omega$, which is drawn from an atomless common prior distribution $p > 0$ over $\Delta\Omega$.[2] Having observed the state, the sender transmits a verifiable message $m_i \in \mathbb{M} := 2^{|\Omega|}$, such that $\omega \in m_i$, to each receiver (her) $i \in I$.[3] Receiver i observes her private message m_i, but not the state ω, and decides whether to approve the sender's proposal (take action 1) or reject it (take action 0). Receiver i's preferences are described by a utility function $u_i : \{0,1\} \times \Omega \to \mathbb{R}$, and her net payoff of approval of the sender's proposal in state ω is $\delta_i(\omega) := u_i(1,\omega) - u_i(0,\omega)$.

Receiver $i \in I$ approves the proposal when the net payoff of approval is non-negative.[4] Her preferences under complete information are summarized by her approval set

$$\mathcal{A}_i := \{\omega \in \Omega \mid \delta_i(\omega) \geq 0\}.$$

The sender's payoff depends only on the subset of receivers who approve his proposal and is described by utility function $u_s : 2^n \to \mathbb{R}$. I assume that the sender weakly prefers that more receivers approve his proposal: given two sets of receivers $T, S \subseteq I$, $u_s(T) \leq u_s(S)$ if $T \subseteq S$.

The following example introduces the leading application of the model.

EXAMPLE 1 (ELECTIONS). The sender is a politician (the challenger) who challenges the status quo. The set of voters I is the electorate. The state space Ω is the policy space (e.g. on one socio-economic issue), with 0 being the ultra-left position and 1 being ultra-right. The challenger gets a payoff of 1 if he wins the election and 0 otherwise. With that, the

[2] For a compact metrizable space S, ΔS denotes the set of all Borel probability measures over S. For any distribution $q \in \Delta\Omega$ and any measurable subset of the state space $W \subseteq \Omega$, $Q(W) := \int_W q(\omega)d\omega$ is the probability measure and if $Q(W) > 0$, $q(\omega \mid W) := \frac{q(\omega)}{Q(W)}$ is the conditional probability distribution.

[3] I borrow the definition of a verifiable message as a subset of the state space that includes the true realization from Milgrom and Roberts (1986). This method satisfies normality of evidence (Bull and Watson, 2004), which means that it is consistent with both major ways of modeling hard evidence in the literature.

[4] The assumption that the receiver breaks ties in favor of the sender guarantees the existence of the sender-preferred equilibrium and plays no role in other equilibria.

sender's utility function u_s assumes the role of the social choice function that aggregates individual votes into the election outcome. Examples of social choice rules include

- dictatorship: $u_s(T) = 1 \iff i \in T$;
- simple majority: $u_s(T) = 1 \iff |T| > n/2$;
- unanimity: $u_s(T) = 1 \iff T = I$.

Voters have spatial preferences à la Downs (1957): voter $i \in I$ has ideal policy $v_i \in \Omega$ and evaluates all other policies based on how far they are from that bliss point.[5] Specifically, voter i compares the challenger's unknown policy ω to a known status quo policy $\omega_0 \in \Omega$. Her net payoff of voting for the challenger with policy ω is $\delta_i(\omega) = |v_i - \omega_0| - |v_i - \omega| - \varepsilon$, where $\varepsilon > 0$.[6] Her approval set comprises of policies that are closer to her bliss point than the status quo by at least ε: $\mathcal{A}_i = \left\{\omega \in \Omega \text{ s.t. } |v_i - \omega| \leq |v_i - \omega_0| - \varepsilon\right\}$. Figure 1 illustrates this setup.

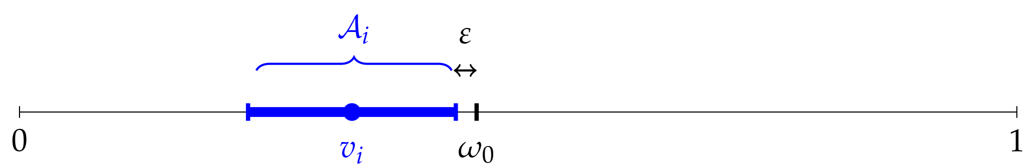

Figure 1: *policy space $\Omega = [0, 1]$, status quo policy ω_0, voter i's bliss point v_i, her approval set $\mathcal{A}_i$. Under complete information, voter i votes for the challenger if his policy is in the solid blue region.*

EQUILIBRIUM OUTCOMES

I consider Perfect Bayesian Equilibria (henceforth just *equilibria*) of this game. Under incomplete information, receiver i approves the proposal if her expected net payoff of approval is non-negative. Thus, her set of approval beliefs is

$$\mathcal{B}_i := \{q \in \Delta\Omega \mid \mathbb{E}_q[\delta_i(\omega)] \geq 0\}.$$

[5] I assume that voting is sincere (also known as expressive). That is, voters derive utility from expressing support for one of the candidates (the challenger or the status quo), and not from the policy that is implemented by the elected candidate. The theory was pioneered by Brennan and Lomasky (1993), Brennan and Hamlin (1998) and reviewed by Hamlin and Jennings (2011). There is a large body of evidence that the behavior of voters in large elections is consistent with sincere voting, e.g., in U.S. national elections (Kan and Yang, 2001; Degan and Merlo, 2007), Spanish General elections (Artabe and Gardeazabal, 2014), Israeli General elections (Felsenthal and Brichta, 1985).

[6] ε is the status quo bias; $\varepsilon > 0$ rules out situations wherein the challenger with the status quo policy always wins the election.

DEFINITION 1. *An* <u>*equilibrium*</u> $(\sigma, \boldsymbol{a}, \boldsymbol{q})$ *consists of the messaging strategy of the sender* $\sigma : \Omega \to \Delta(\mathbb{M}^I)$ *and the profiles of approval strategies* $\boldsymbol{a} := \{a_i : \mathbb{M} \to \{0,1\}\}_{i \in I}$ *and posterior beliefs* $\boldsymbol{q} := \{q_i : \mathbb{M} \to \Delta\Omega\}_{i \in I}$ *of the receivers, such that*

(i) $\forall \omega \in \Omega$, $\sigma(\cdot \mid \omega)$ *is supported on* $\arg\max_{m_1,\ldots,m_n} u_s\big(\{i \in I \mid a_i(m_i) = 1\}\big)$, *s.t.* $\omega \in m_i$, $\forall i \in I$.

The following conditions must hold for every receiver $i \in I$*:*

(ii) $\forall m \in \mathbb{M}$, $a_i(m) = \mathbb{1}\big(q_i(\cdot \mid m) \in \mathcal{B}_i\big)$;

(iii) $\forall m \in \mathbb{M}$ *such that* $\int\limits_{\Omega} \sigma_i(m \mid \omega) d\omega > 0$, $q_i(\omega \mid m) = \frac{\sigma_i(m \mid \omega) \cdot p(\omega)}{\int_\Omega \sigma_i(m \mid \omega') \cdot p(\omega') d\omega'}$, *where* σ_i *is the marginal distribution of messages heard on the equilibrium path by receiver* i;

(iv) $\forall m \in \mathbb{M}$, $\operatorname{supp} q_i(\cdot \mid m) \subseteq m$.

In words, *(i)* states that the sender puts positive probability only on collections of messages that maximize his utility; *(ii)* states that each receiver is sequentially rational and approves the proposal if and only if her expected net payoff of approving is non-negative under her posterior belief; *(iii)* states that receivers' posterior beliefs are Bayes-rational on the equilibrium path; *(iv)* states that the receivers' posterior beliefs on and off the path are concentrated on the states in which the message is available to the sender.

I analyze the set of equilibrium outcomes of this game via a *state-based approach*. In every state ω, I am interested in which action each receiver takes, and with what probability.

DEFINITION 2.

- *An* <u>*outcome*</u> $\alpha_i(\omega) \in [0,1]$, *for every* $i \in I$ *and* $\omega \in \Omega$, *specifies the probability that receiver* i *approves the sender's proposal in state* ω.
- *An outcome is an* <u>*equilibrium outcome*</u> *if it corresponds to some equilibrium.*[7]

Some outcomes are simpler than others in that every receiver takes a particular action with probability one in each state.[8] Put differently, each receiver essentially partitions the state space into two subsets, the states of approval and the states of rejection. Formally,

[7] Specifically, if there exists equilibrium $(\sigma, \boldsymbol{a}, \boldsymbol{q})$ such that for every receiver $i \in I$ and state $\omega \in \Omega$, $\alpha_i(\omega) = \int\limits_{\mathcal{M}_i(\omega)} \sigma_i(m \mid \omega) dm$, where $\mathcal{M}_i(\omega) := \{m \in \mathbb{M} \mid \sigma_i(m \mid \omega) > 0 \text{ and } a_i(m) = 1\}$ is the set of messages transmitted from state ω that convince receiver i to approve the proposal.

[8] Although each receiver breaks ties in favor of approval, the sender may be playing a mixed strategy in state ω, and then in that state the receiver may be approving the proposal with probability between 0 and 1.

Definition 3.

- *an outcome is* <u>*deterministic*</u> *if* $\alpha_i(\omega) \in \{0,1\}$ *for every* $i \in I$ *and* $\omega \in \Omega$*;*
- *if an outcome is deterministic, then for every receiver* $i \in I$, $W_i := \{\omega \in \Omega \mid \alpha_i(\omega) = 1\}$ *is the* <u>*set of approval states*</u>*.*

Some equilibria are simpler than others in that the sender plays a pure strategy of directly recommending action to each voter.

Definition 4. *Equilibrium* $(\sigma^D, \boldsymbol{a}^D, \boldsymbol{q}^D)$ *is a* <u>*direct equilibrium*</u> *with sets of approval states* $(W_1, \ldots, W_n)$*, if for every receiver* $i \in I$ *and message* $m_i \in \{W_i, \Omega \setminus W_i\}$

$$\sigma_i^D(m_i \mid \omega) = \mathbb{1}(\omega \in m_i), \qquad a_i^D(m_i) = \mathbb{1}(m_i = W_i).$$

In a direct equilibrium, the sender sends message W_i to receiver i if the realized state is $\omega \in W_i$, and message $\Omega \setminus W_i$ otherwise. In the first case, he effectively recommends that the receiver approves the proposal, while in the second case, he recommends rejection. Notice that W_i is both the *set of states* in which the proposal is approved and the *message* that convinces the receiver to approve the proposal. Given the sender's simple pure strategy, each receiver's Bayesian updating is straightforward: she conditions the prior distribution on the message she receives. Consequently, for receiver $i \in I$ to correctly interpret message W_i as a recommendation to approve the proposal, message W_i must satisfy the following constraint in any direct equilibrium:

$$p(\cdot \mid W_i) \in \mathcal{B}_i. \tag{obedience}$$

The sender can always fully reveal the state of the world $\omega \in \Omega$ by sending the fully informative message $\{\omega\}$. According to the equilibrium condition *(iv)*, when a receiver hears this message, her posterior belief places probability one on ω, meaning that she learns the true state of the world. Consequently, the sender can guarantee that receiver i approves the proposal in states in which she approves it under complete information. That gives rise to the sender's incentive-compatibility constraints, $\forall i \in I$,

$$\mathcal{A}_i \subseteq W_i. \tag{IC}$$

VALUE OF COMMITMENT

I compare the *equilibrium outcomes* of the described verifiable information game to the *commitment outcomes* of the Bayesian persuasion game. In the Bayesian persuasion game, the sender commits to a signaling policy $\sigma^{BP} : \Omega \to \Delta(\Theta_1, \dots, \Theta_n)$, where Θ_i is the private signal set of receiver i. Once state $\omega \in \Omega$ is realized, a collection of signals $\{\theta_i\}_{i \in I}$ is generated according to σ^{BP}, and receiver i observes her private signal realization θ_i.

DEFINITION 5.

- $(\sigma^{BP}, a^{BP}, q^{BP})$ *is a* <u>*commitment protocol*</u> *if it satisfies equilibrium conditions (ii) and (iii).*
- *An outcome is a* <u>*commitment outcome*</u> *if it corresponds to some commitment protocol.*

Since a commitment protocol satisfies two out of four equilibrium conditions, the sender must do weakly better in the Bayesian persuasion game relative to the game with verifiable information.

3. ONE RECEIVER

Let us first focus on the case with one receiver, i.e. $I = \{1\}$. For ease of exposition, I drop all receiver-relevant subscripts i. I assume that the sender's utility is strictly increasing in the receiver's action: the sender gets a payoff of 1 if he convinces the receiver to approve the proposal, and 0 otherwise. The first result of this paper establishes an equivalence between the set of equilibrium outcomes and the set of direct equilibrium outcomes. It also provides a simple characterization for both in terms of two constraints.

DIRECT IMPLEMENTATION

THEOREM 1. *Suppose $n = 1$. Then, every equilibrium outcome is deterministic. Moreover, the following statements about $W \subseteq \Omega$ are equivalent:*

1. *W is an equilibrium set of approval states;*
2. *W is a set of approval states in a direct equilibrium;*
3. *W satisfies the receiver's* (obedience) *and the sender's* (IC) *constraints.*

The proof of Theorem 1 can be found in the appendix, along with all other proofs. Here I describe the intuition behind this result. First, every equilibrium outcome is deterministic and induces a partition of the state space into W, where the proposal is approved, and $\Omega \setminus W$, where the proposal is rejected. Suppose, on the contrary, that in some state the sender induces both approval and rejection with positive probability. Since the sender has access to a message that induces approval, nothing prevents him from deviating to that message and inducing approval with certainty. Consequently, the sender always plays a pure strategy of recommending one of the actions with probability one.[9]

Next, observe that *(1)* implies *(3)*. If W is an equilibrium set of approval states, it has to satisfy the sender's (IC) constraint, or else the sender has a profitable deviation to full disclosure. To see why W also satisfies the receiver's (obedience) constraint, imagine that this set of approval states is implemented directly. Specifically, rather than playing the (possibly mixed) strategy prescribed in the original equilibrium, the sender uses a direct pure strategy of sending message W from states $\omega \in W$, and message $\Omega \setminus W$ from states $\omega \in \Omega \setminus W$. Since every message coming from state $\omega \in W$ induces approval in the original equilibrium, so does the pooling message W in the direct equilibrium.

Next, notice that *(3)* implies *(2)*. An outcome partition that satisfies the two constraints describes the path of a direct equilibrium. Because of (IC), the sender does not have profitable deviations toward full disclosure. Due to verifiability, the sender cannot deviate to sending the convincing message W from states outside of W. Because of (obedience), the receiver is best responding. Off the equilibrium path, let the receiver be "skeptical" and assume that any unexpected message comes from the worst possible state.

Finally, since a direct equilibrium is an equilibrium, *(2)* implies *(1)*.

Note that the equivalence between statements *(1)* and *(2)* of Theorem 1 can be viewed as a version of the communication revelation principle for games with verifiable information. The communication revelation principle for mediated sender-receiver games, introduced by Myerson (1986) and Forges (1986), states that any equilibrium outcome may be implemented truthfully and obediently. In the present context, it translates into *(i)* the sender truthfully revealing the state of the world to the mediator, *(ii)* the mediator translating this report into an action recommendation for the receiver, and *(iii)* the receiver obe-

[9] This is a purification argument – when the state space is continuous and the information structure is atomless the existence of pure strategy equilibria is often guaranteed (Radner and Rosenthal, 1982; Aumann et al., 1983; Milgrom and Weber, 1985).

diently following her recommendation. Which equilibrium outcome is implemented is decided by the mediator at step *(ii)*. Conveniently, statement *(3)* of Theorem 1 provides the necessary and sufficient conditions for a set of approval states to be implementable in equilibrium.

EQUILIBRIUM RANGE AND VALUE OF COMMITMENT

According to Theorem 1, when characterizing the full equilibrium set of the game with one receiver, one may restrict attention to direct equilibria. Each direct equilibrium is characterized by the set of approval states W that satisfies the sender's (IC) and the receiver's (obedience) constraints. The advantage of the state-based approach lies in that the sender's ex-ante utility in a direct equilibrium equals $P(W)$ and depends only on the set of approval states W and the prior measure $P(\cdot)$.

In the sender-worst equilibrium, the set of approval states is the smallest in terms of the sender's ex-ante utility, binds the sender's (IC) constraint, and is $\underline{W} = \mathcal{A}$. In this equilibrium, the receiver effectively learns all the relevant information (whether the true state is within her approval set or not), thus making her decision as if under complete information. This equilibrium is outcome-equivalent to *full disclosure* (also known as *full unraveling*), salient in the verifiable information literature.

In the sender-preferred equilibrium, the set of approval states $\bar{W}$ maximizes the sender's ex-ante utility across all equilibria. Mathematically, $\bar{W}$ solves

$$\max_{W \subseteq \Omega} P(W), \quad \text{subject to} \quad \begin{aligned} \mathcal{A} &\subseteq W, \\ p(\cdot \mid W) &\in \mathcal{B}. \end{aligned}$$

To find the sender-preferred equilibrium, one would increase the ex-ante measure of the set of approval states W for as long as message W convinces the receiver to approve the proposal. Because the state space is continuous, W is chosen to make the receiver exactly indifferent between approval and rejection, thus binding her obedience constraint. The resulting sender-preferred set of approval states $\bar{W}$ is described in the following theorem.

THEOREM 2. *When $n = 1$, the sender-preferred equilibrium outcome is a commitment outcome. The sender-preferred set of approval states $\bar{W}$ is characterized by a cutoff value $c^* > 0$ such that*

- *sender's proposal is approved if $\delta(\omega) > -c^*$ and rejected if $\delta(\omega) < -c^*$;*

- *whenever the sender's proposal is approved, the receiver's expected net payoff of approval equals zero:* $\mathbb{E}_p\left[\delta(\omega) \mid \bar{W}\right] = 0$.

To understand the intuition behind this result, recall that an outcome specifies probability $\alpha(\omega)$ that the receiver approves the proposal in state ω. According to Kamenica and Gentzkow (2011), to find a commitment outcome, one may restrict attention to *straightforward* signals with realizations that are interpreted by the receiver as recommendations to take particular actions. Consequently, $\alpha(\omega)$ becomes the probability that the recommendation to approve comes from state ω, and the sender's problem rewrites itself in terms of finding the optimal outcome subject to an obedience-like constraint of the receiver.

Because the receiver ranks states according to her net payoff of approval $\delta(\omega)$, the solution features a set of cutoff states.[10] To see why, observe that by adding state ω to the set of approval states $\bar{W}$ one increases the objective by $p(\omega)$, at the cost (in terms of the obedience constraint) of $-\delta(\omega) \cdot p(\omega)$. States that are approved under complete information have a non-negative net payoff of approval (and thus a negative cost). These are added first, automatically satisfying the sender's (IC) constraint. If some state ω such that $\delta(\omega) < 0$ is added, it must be the case that every state with a higher (but possibly also negative) net payoff of approval is also added since those states are "cheaper" in terms of the constraint. States are added until the obedience constraint of the receiver binds or until $\bar{W} = \Omega$.

While the logic above applies to both equilibrium and a commitment outcomes, a commitment outcome may not be deterministic. Suppose that in some states the sender is making a mixed recommendation and induces both actions with positive probabilities. By the argument above, the receiver's net payoff of approval must be constant in all these states. Rather than making a mixed recommendation, partition the set of these states in two and let the challenger recommend one action on each subset with certainty. Due to the continuity of the state space, such partitioning does not affect the objective function or the obedience constraint of the receiver. This deterministic commitment outcome is an equilibrium outcome because the receiver's obedience constraint is the same with and without commitment. It imposes that the receiver correctly interprets the *straightforward signal* or the *convincing message* as a recommendation to take the appropriate action.

[10] The solution under commitment is due to Alonso and Câmara (2016).

Example 2 (Dictatorial Elections). Consider the setting from Example 1 with just one voter (call him the dictator), convincing who is necessary and sufficient for the challenger to win the election. According to Theorem 1, one can restrict attention to direct equilibria characterized by the set of approval states W that satisfies two constraints. First, the sender's (IC) constraint $\mathcal{A} \subseteq W$ ensures that the challenger does not have a profitable deviation toward fully revealing his policy. Second, the receiver's (obedience) constraint $p(\cdot \mid W) \in \mathcal{B}$ ensures that the dictator interprets message W as a recommendation to vote for the challenger. In the present context, W is the set of winning policies of the challenger as well as the message that convinces the dictator to vote for the challenger.

In the sender-worst equilibrium, the set of winning policies coincides with the dictator's approval set, $\underline{W} = \mathcal{A}$. The voter learns all the information relevant for her decision, i.e. whether the challenger's policy is preferred to the status quo under complete information.

In the sender-preferred equilibrium, the challenger's odds of winning $P(W)$ are maximized subject to the receiver's (obedience) constraint. According to Theorem 2, the resulting sender-preferred set of winning policies $\bar{W}$ is the same as in the Bayesian persuasion game, implying that the challenger need not benefit from having commitment power. The sender-preferred set of winning policies, illustrated in Figure 2, is characterized by a cutoff value c^* of the dictator's net payoff of voting for the challenger, such that

- challenger with policy ω is elected if and only if $\delta(\omega) \geq -c^*$;
- $c^* > 0$: some challengers with policies outside of the voter's approval set are elected; for small enough ε, challengers with policies further than the status quo are elected;
- when voting for the challenger, the dictator is indifferent between the challenger and the status quo: $\mathbb{E}_p\left[\delta(\omega) \mid \bar{W}\right] = 0$.

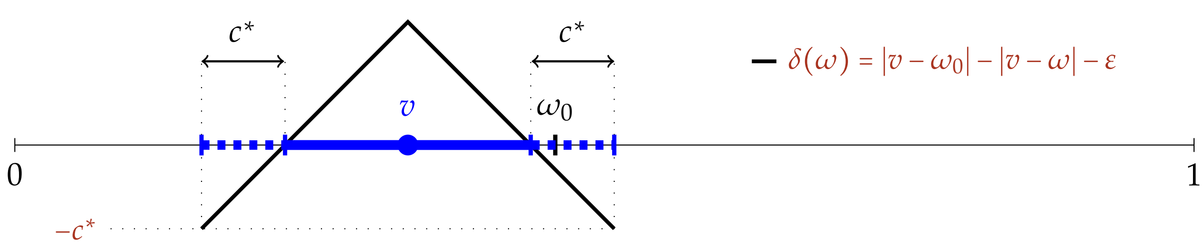

Figure 2: *the sender-preferred set of winning policies $\bar{W}$ (in blue) consists of the voter's approval set (solid) and policies outside of the approval set (dotted).*

4. Many Receivers

Recall that with one receiver every equilibrium outcome is deterministic because the sender has a strict incentive to convince the receiver in every state. As a result, Theorem 1 establishes the equivalence between the sets of direct equilibrium outcomes and all equilibrium outcomes. With many receivers, there may exist non-deterministic equilibria, which means that the outcome equivalence breaks down. However, an equilibrium may only be non-deterministic when the sender does not have a strict incentive to convince some receiver in some state. That means that the set of direct equilibria still determines the full range of equilibrium ex-ante utilities of the sender.

THEOREM 3. *The following statements about the sender's ex-ante utility $\bar{u}_s$ are equivalent:*

1. *$\bar{u}_s$ is reached in equilibrium;*
2. *$\bar{u}_s$ is reached in a direct equilibrium;*
3. *$\bar{u}_s$ is given by*

$$\bar{u}_s = \int_{\Omega} u_s(\{i \in I \mid \omega \in W_i\}) \cdot p(\omega) d\omega,$$

where for every receiver $i \in I$, $W_i \subseteq \Omega$ is her set of approval states, which satisfies

- *receiver's obedience constraint $p(\cdot \mid W_i) \in \mathcal{B}_i$;*
- *sender's incentive-compatibility constraint $\mathcal{A}_i \subseteq W_i$.*

When studying the multiple-receiver case, I rely on the assumptions of *(i)* no information spillovers between the receivers and *(ii)* receivers' utilities being independent of other receivers' actions. Receivers are essentially solving separate utility maximization problems and are thus considered individually. The proof of the theorem follows the same steps as the proof of Theorem 1. The only substantial difference arises in proving that *(1)* implies *(3)* because not all equilibria are deterministic. If an equilibrium outcome is not deterministic, then the sender induces multiple actions of receiver i in state ω, and it is unclear whether ω should be added to the approval recommendation W_i or not. Notice that if the sender's utility in state ω strictly increases in receiver i's action, then the sender would never convince her with a probability between 0 and 1 in equilibrium. Once a receiver is "convincible," the sender should be convincing her with certainty. Thus, removing all the states in which the sender makes a mixed recommendation to receiver i from this receiver's set of approval states is inconsequential to the sender's ex-ante utility.

According to Theorem 3, when characterizing the sender's equilibrium ex-ante utility, one can restrict attention to collections of sets of approval states $(W_1, \ldots, W_n)$, each of which satisfies the receiver's obedience and the sender's incentive-compatibility constraints. Moreover, the sender's ex-ante utility only depends on the sets of approval states and the prior distribution.

Once again, in the sender-worst equilibrium, in which the sender's ex-ante utility is minimized across all equilibria, the sender does as well as under full disclosure. The set of approval states of receiver $i \in I$ is $\underline{W}_i = \mathcal{A}_i$, each receiver learns all the relevant information and makes her decision as if under complete information.

The sender-preferred equilibrium outcome is characterized by the collection of sets of approval states that maximizes the sender's ex-ante utility across all equilibria, i.e. subject to every receiver's obedience constraint and every incentive-compatibility constraint of the sender. When there are many receivers, the sender need not benefit from having commitment power, either.

THEOREM 4. *The sender-preferred equilibrium outcome is a commitment outcome.*

The proof boils down to showing that there exists a deterministic commitment outcome and that every receiver's set of approval states satisfies the sender's incentive-compatibility constraint. The former is possible because of the continuity of the state space. The latter is true because adding state $\omega \in \mathcal{A}_i$ to the induced set of approval states loosens receiver i's obedience constraint and weakly increases the value of the objective.

Finding the sender-preferred equilibrium is computationally hard when the sender's preferences are not separable in receivers' actions.[11] In some special cases, particularly when the state space is binary, solutions to the problem under commitment are available. Most notably, Arieli and Babichenko (2019) solve the case of the supermodular utility of the sender, and Babichenko and Barman (2016) show that when the sender's utility is submodular, the problem is NP-hard and provide an approximation. In the following section, I

[11] If sender's utility is separable in receivers actions, then the sender can determine the optimal signal receiver by receiver and faces a set of independent problems of a single-receiver variety, as observed by Kamenica (2019).

return to the setup of the spatial model of elections. That setup is simple enough to characterize the solution, make a meaningful comparison between public and private disclosure, and analyze comparative statics.

5. Targeted Advertising in Elections

Let us come back to the setting introduced in Example 1 and first study what happens in the absence of targeted advertising. In particular, consider the cases of complete information, no disclosure (the voters vote under the prior belief), and public disclosure (the challenger sends a public message to all the voters). A common feature of these cases is that the voters hold a common belief.

Observe that the preferences of the electorate can be summarized by the preferences of at most two voters. To see this, first notice that as a voter's bliss point moves away from the status quo, her approval set expands. As a result, the left (right) voter whose bliss point is closest to the status quo represents the preferences of all the voters on the left (right). Second, voters with bliss points on opposite sides of the status quo have incompatible preferences. Specifically, if these voters hold the same belief, then only one of them would expect that the challenger's policy is closer to her bliss point than the status quo. These observations are summarized in Corollary 1 and illustrated in Figure 3.

Corollary 1. *Let* $L = \arg\max_{i \in I,\, v_i < \omega_0} v_i$ *and* $R = \arg\min_{j \in I,\, v_j > \omega_0} v_j$. *Then,*

1. L *is the representative voter on the left:* $\forall i \in I$, *if* $v_i < \omega_0$, *then* $\mathcal{B}_L \subseteq \mathcal{B}_i$; R *is the representative voter on the right:* $\forall j \in I$ *if* $v_j > \omega_0$, *then* $\mathcal{B}_R \subseteq \mathcal{B}_j$;
2. *approval sets and sets of approval beliefs of voters* L *and* R *do not intersect, i.e.* $\mathcal{A}_L \cap \mathcal{A}_R = \varnothing$ *and* $\mathcal{B}_L \cap \mathcal{B}_R = \varnothing$.

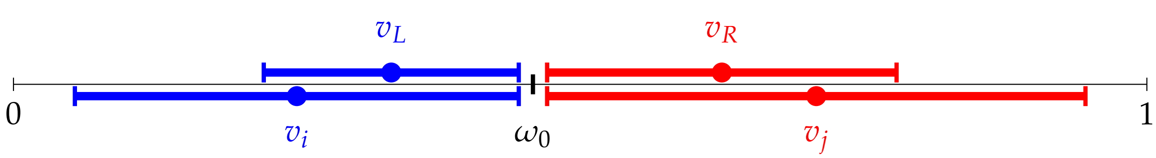

Figure 3: *voter* i *is convinced if voter* L *is convinced: her approval set includes* L*'s approval set (solid blue lines). Voters* L *and* R *have incompatible preferences: their approval sets do not intersect.*

Part 2 of Corollary 1 implies that voters L and R *never* both vote for the challenger when they hold the same belief. If voters L and R are jointly pivotal, the challenger always loses the election under common belief.

DEFINITION 6. *Election with representative voters L and R is unwinnable for the challenger under common belief, if for all $T \subseteq I$, $u_s(T) = 1$ if and only if $\{L, R\} \in T$.*

Whether an election is unwinnable is determined by the institution (the social choice function) and the ideology (bliss points of the voters). For example, under the simple majority rule, we arrive at a version of the median voter theorem.[12] Intuitively, for an election to be unwinnable, there may not be a majority of voters located on either side of the status quo.

COROLLARY 2. *Under the simple majority rule, an election is unwinnable for the challenger when the voters hold a common belief if and only if ω_0 is the median voter's bliss point.*

SWINGING UNWINNABLE ELECTIONS

With targeted advertising, the challenger can say different things to different voters. The voters will no longer hold the same belief, which opens up a possibility of winning (with positive probability) an unwinnable election. Here I show how the challenger can convince representative voters L and R, persuading who is sufficient to win *any* unwinnable election. I focus on the best-case scenario for the challenger and thus consider the sender-preferred equilibrium.

Using Theorem 3, one may restrict attention to a pair of sets of convincing policies (W_L, W_R), one for each voter, where W_i is also the message that convinces voter i, $i \in \{L, R\}$. In the sender-preferred equilibrium, his odds of winning are maximized subject to each voter's (obedience) constraint:

$$\max_{W_L, W_R} P(W_L \cap W_R)$$
$$\text{subject to } p(\cdot \mid W_i) \in \mathcal{B}_i, \text{ for } i \in \{L, R\}.$$

The solution to this problem is described in the following theorem.

[12] Black (1948) states the median voter theorem as "If Ω is a single-dimensional issue and all voters have single-peaked preferences defined over Ω, then ω_0, the median position, could not lose under majority rule."

THEOREM 5. *In the sender-preferred equilibrium of an unwinnable election with representative voters L and R,*

- *message $\bar{W}_i$ that convinces voter $i \in \{L, R\}$ is an interval $[a_i, b_i] \supset \mathcal{A}_i$;*
- *if ε is small enough, the challenger has positive ex-ante odds of winning this election; the set of winning policies is $[a_R, b_L]$ with $a_R < \omega_0 < b_L$.*

To understand the intuition behind this result, recall that when voter L (R) is a dictator, the challenger can convince her to elect him even when his policy is slightly to the right (left) of the status quo. That was illustrated in Figure 2 of Example 2. One thing that the challenger can do under private communication is treat each voter as a dictator. If his policy is close enough to the status quo and ε is small enough, the challenger convinces both voters at the same time and swings an unwinnable election. However, he can do even better. Rather than sending each voter a message centered around this voter's bliss point, skew her message toward the other voter. More precisely, start with voter L's (R's) approval set and then add policies to the right (left) of the status quo until this voter's obedience constraint binds. This solution is illustrated in Figure 4.

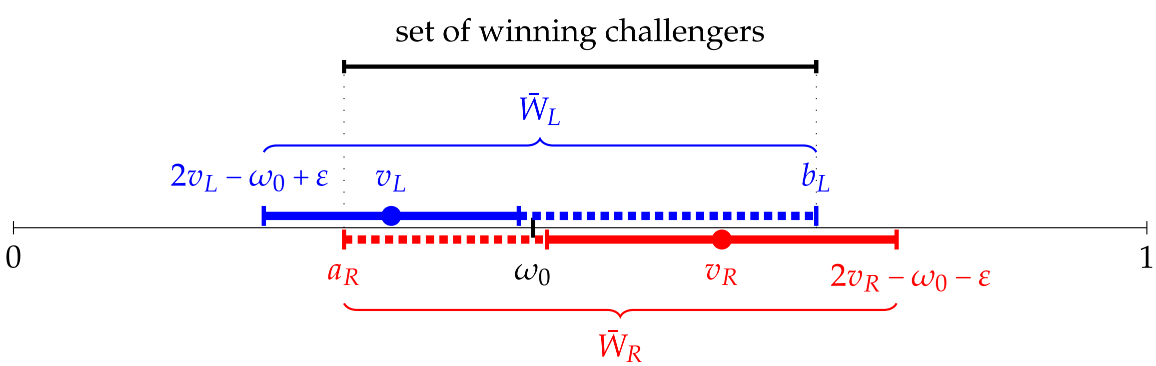

Figure 4: *sender-preferred messages $\bar{W}_L$ (in blue) and $\bar{W}_R$ (in red) that convince voters L and R. $\bar{W}_i$ consists of voter i's approval set (solid) and policies preferred by $j \neq i$ (dotted). Challenger wins the election by convincing both voters when his policy is in $\bar{W} = \bar{W}_L \cap \bar{W}_R$.*

COMPARATIVE STATICS

If at least one of the obedience constraints does not bind when $b_L = 2v_R - \omega_0 - \varepsilon$ or when $a_R = 2v_L - \omega_0 + \varepsilon$, then the solution may look different, but both convincing messages will still be intervals. Assume for the rest of this section that the prior is uniform.[13] Notice that

[13] The prior is chosen to be uniform for ease of exposition. Similar results hold for any prior distribution.

the distance from a voter's bliss point to ω_0 measures measures this voter's persuadability.

DEFINITION 7. *Assume uniform prior. Then,*

- *voter i is more persuadable than voter j if $|v_i - \omega_0| > |v_j - \omega_0|$, where $i, j \in I$;*
- *consider electorates I and I' with representative voters $\{L, R\}$ and $\{L', R'\}$. I' is more polarized than I if $v'_L \leq v_L < \omega_0 < v_R \leq v'_R$.*

In words, the further from the status quo the voter's bliss point is, the less satisfied she is with the status quo policy, and that makes her more persuadable. I say voter $i \in I$ becomes more persuadable if $|v_i - \omega_0|$ increases. The electorate becomes more polarized when either representative voter becomes more persuadable. Figure 5 illustrates the dynamics of the numerical solution to the problem of finding sender-preferred equilibrium as voter R becomes more persuadable (the electorate becomes more polarized). Theorem 6 summarizes the comparative statics.

THEOREM 6. *Assume uniform prior. In the sender-preferred equilibrium of an unwinnable election with representative voters L and R,*

- *as R becomes more persuadable, the challenger's odds of winning $P(\bar{W}_R \cap \bar{W}_L)$ increase;*
- *suppose $|v_L - \omega_0| = |v_R - \omega_0|$, meaning that neither voter is more persuadable than the other. Then, as R becomes more persuadable, the set of winning policies $\bar{W} = [a_R, b_L]$ shifts to the left, i.e. a_R and b_L decrease.*

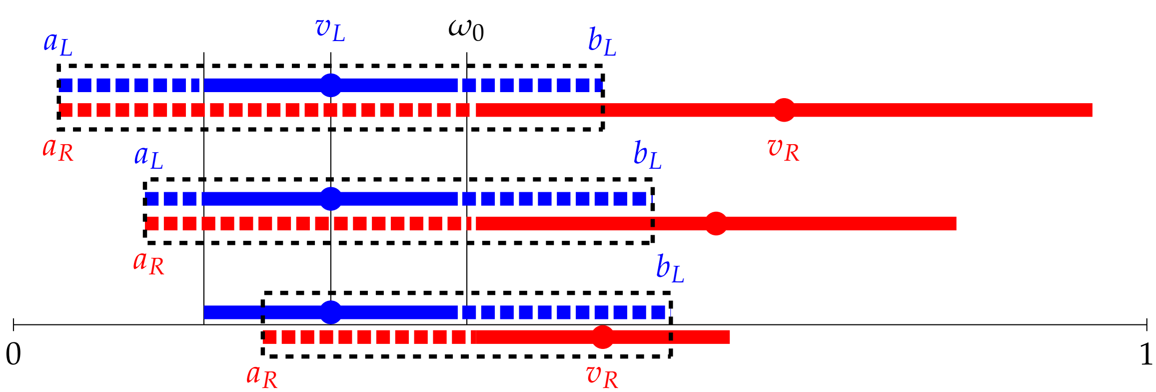

Figure 5: *comparative statics as voter R moves to the right (from bottom to top): her approval set (solid red area) expands; she becomes more persuadable by policies to the left (dashed red component of her convincing message expands); set of winning policies (black dashed area $[a_R, b_L]$) moves to the left, and the length of that interval increases.*

In words, as voter R becomes more persuadable, it becomes easier for the challenger to swing the election by targeting, in the sense that his ex-ante odds of winning increase. Furthermore, R becomes more persuadable by policies further to the left, meaning that the set of winning policies shifts to the right, also. When voter R is far enough to the right, her obedience constraint no longer binds (as in the top exhibit of Figure 5), and the sender-preferred set of winning policies is the same as if voter L was a dictator.

6. CONCLUSION

This paper argued that the sender need not benefit from having commitment power, and can persuade the receivers with verifiable information only. This result is useful in applications, especially in the context of elections, where assuming that the sender has hard evidence is more plausible than assuming that the sender has commitment power.

While illustrated in the simplified framework, the observation that targeted advertising helps challenger swing elections holds for more than two voters, more than one dimension, and any social choice rule. Because targeting leads to election outcomes that are different from the complete-information outcomes, one can argue that targeted advertising is bad for democracy. Certain policy implications, especially concerning restricting the collection and use of personal data by the candidates in their electoral campaigns, should be considered.

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Appendix: Omitted Proofs

Lemma A.1. *In equilibrium $(\sigma, \boldsymbol{a}, \boldsymbol{q})$, for every receiver $i \in I$, let*

- $\mathcal{M}_i := \{m \in \mathbb{M} \mid a_i(m) = 1\}$ *be the set of messages that convince the receiver to approve;*
- $\mathcal{W}_i := \{\omega \in \Omega \mid \exists m \in \mathcal{M}_i \text{ s.t. } \omega \in m\}$ *be the set of all states in which at least one convincing message is available. Note that $\mathcal{A}_i \subseteq \mathcal{W}_i$: if $\omega \in \mathcal{A}_i$, then $\{\omega\} \in \mathcal{M}_i$ because $q_i(\cdot \mid \{\omega\}) \in \mathcal{B}_i$;*
- $X_i := \{\omega \in \Omega \mid \alpha_i(\omega) = 1\} \subseteq \mathcal{W}_i$ *be the set of states in which this receiver approves the proposal with probability 1. To convince the receiver in state ω with certainty, the sender must be sending her convincing messages, and convincing messages only, i.e.*

$$\forall \omega \in X_i, \quad \int_{\mathcal{M}_i} \sigma_i(m \mid \omega) dm = 1.$$

Claim: set $X_i \cup \mathcal{A}_i$ satisfies receiver i's (obedience) constraint $p(\cdot \mid X_i \cup \mathcal{A}_i) \in \mathcal{B}_i$.

Proof. Every message in $\mathcal{M}_i$ convinces the receiver to approve the proposal:

$$\forall m \in \mathcal{M}_i, \;\; q_i(\cdot \mid m) \in \mathcal{B}_i \iff \int_{\operatorname{supp} q_i(\cdot \mid m) \subseteq m \subseteq \mathcal{W}_i} \delta_i(\omega) \cdot q_i(\omega \mid m) d\omega \geq 0,$$

which, on the equilibrium path, becomes

$$\int_{\mathcal{W}_i} \delta_i(\omega) \cdot \frac{\sigma_i(m \mid \omega) \cdot p(\omega)}{\int_{\mathcal{W}_i} \sigma_i(m \mid \omega') \cdot p(\omega') d\omega'} d\omega \geq 0 \iff \int_{\mathcal{W}_i} \delta_i(\omega) \cdot \sigma_i(m \mid \omega) \cdot p(\omega) d\omega \geq 0.$$

Integrate the above inequality over all $m \in \mathcal{M}_i$:

$$\int_{\mathcal{M}_i}\int_{\mathcal{W}_i} \delta_i(\omega)\cdot\sigma_i(m \mid \omega)\cdot p(\omega)d\omega dm \geq 0.$$

Next, partition $\mathcal{W}_i$ into X_i, $\mathcal{A}_i \setminus X_i$, and $\mathcal{W}_i \setminus (X_i \cup \mathcal{A}_i)$ and observe that

$$\int_{\mathcal{M}_i}\int_{X_i} \delta_i(\omega)\cdot\sigma_i(m \mid \omega)p(\omega)d\omega dm = \int_{X_i} \delta_i(\omega)p(\omega)\underbrace{\int_{\mathcal{M}_i}\sigma_i(m \mid \omega)dm}_{=1,\forall\omega\in X_i} d\omega = \int_{X_i}\delta_i(\omega)p(\omega)p\omega;$$

$$\int_{\mathcal{M}_i}\int_{\mathcal{A}_i} \delta_i(\omega)\sigma_i(m \mid \omega)p(\omega)d\omega dm = \int_{\mathcal{A}_i} \underbrace{\delta_i(\omega)}_{\geq 0\forall\omega\in\mathcal{A}_i} p(\omega)\underbrace{\int_{\mathcal{M}_i}\sigma_i(m \mid \omega)dm}_{\leq 1} d\omega \leq \int_{\mathcal{A}_i}\delta_i(\omega)p(\omega)p\omega;$$

$$\int_{\mathcal{M}_i}\int_{\mathcal{W}_i\setminus(X_i\cup\mathcal{A}_i)} \underbrace{\delta_i(\omega)}_{\leq 0\forall\omega\notin\mathcal{A}_i} \sigma_i(m \mid \omega)p(\omega)d\omega dm \leq 0.$$

As a result,

$$\int_{\mathcal{M}_i}\int_{\mathcal{W}_i} \delta_i(\omega)\cdot\sigma_i(m \mid \omega)\cdot p(\omega)d\omega dm \geq 0 \Longrightarrow \int_{X_i\cup\mathcal{A}_i} \delta_i(\omega)p(\omega)p\omega \geq 0 \iff p(\cdot \mid X_i \cup \mathcal{A}_i) \in \mathcal{B}_i.$$

Proof of Theorem 1 (See page 11).

Theorem 1. *Suppose $n = 1$. Then, every equilibrium outcome is deterministic. Moreover, the following statements about $W \subseteq \Omega$ are equivalent:*

1. *W is an equilibrium set of approval states;*
2. *W is a set of approval states in a direct equilibrium;*
3. *W satisfies the receiver's* (obedience) *and the sender's* (IC) *constraints.*

Proof. *Part I*, every equilibrium outcome is deterministic. Consider equilibrium (σ, a, q). Suppose, on the contrary, that $\alpha(\omega) \in (0,1)$ for some $\omega \in \Omega$. In words, the sender is inducing both approval and rejection with positive probabilities in state ω. Then, the sender has a profitable deviation to exclusively sending the messages that induce approval.

Part II, (1) ⇒ (3): consider equilibrium (σ, a, q) with set of approval states W. W must satisfy the sender's (IC) constraint, or else the sender can deviate to full disclosure. Furthermore, since the sender's utility strictly increases in receiver's action, $\alpha(\omega) > 0$ implies $\alpha(\omega) = 1$, or else the sender could deviate to one of the available convincing messages. In terms of Lemma A.1, the set of all states in which at least one convincing message is available is simply W, so one can conclude that set W satisfies the (obedience) constraint.

<u>*Part III*</u>, *(3) ⇒ (2)*: assume that $W \subseteq \Omega$ satisfies (IC) and (obedience). To complete the characterization of the direct equilibrium with this set of approval states, one needs to specify off-path beliefs of the receiver, i.e. following any message $m \in \mathbb{M} \setminus \{W, \Omega \setminus W\}$. There are two restrictions on these beliefs: *(i)* the density must be supported on the set of states in which the message was available to the sender: $\forall m \in \mathbb{M}$, $\operatorname{supp} q^D(\cdot \mid m) \subseteq m$, *(ii)* the sender may not have profitable deviations in states $\Omega \setminus W$. One way to ensure that does not happen is to impose "skeptical beliefs"

$$\forall m \subseteq \mathcal{A}, \;\; \operatorname{supp} q^D(\cdot \mid m) \subseteq m, \text{so that } q^D(\cdot \mid m) \in \mathcal{B},$$

$$\forall m \nsubseteq \mathcal{A}, m \neq \mathcal{W} \;\; \operatorname{supp} q^D(\cdot \mid m) \subseteq m \setminus \mathcal{A}, \text{so that } q^D(\cdot \mid m) \notin \mathcal{B}$$

that assign positive probability to states within the approval set if and only if the message comprises of these states only.

<u>*Part IV*</u>, *(2) ⇒ (1)* is trivially true.

PROOF OF THEOREM 2 (SEE PAGE 13).

THEOREM 2. *When $n = 1$, the sender-preferred equilibrium outcome is a commitment outcome. The sender-preferred set of approval states $\bar{W}$ is characterized by a cutoff value $c^* > 0$ such that*

- *sender's proposal is approved if $\delta(\omega) > -c^*$ and rejected if $\delta(\omega) < -c^*$;*
- *whenever the sender's proposal is approved, the receiver's expected net payoff of approval equals zero: $\mathbb{E}_p\big[\delta(\omega) \mid \bar{W}\big] = 0$.*

PROOF. Consider the problem of finding the optimal commitment protocol $(\sigma^{BP}, a^{BP}, q^{BP})$. According to Kamenica and Gentzkow (2011), that problem may be simplified to finding an optimal *straightforward* experiment σ^{BP} that is supported on set $\{s^+, s^-\}$, where s^+ induces posterior $q^+ \in \mathcal{B}$ and recommends that the receiver approves the sender's proposal and s^- induces posterior $q^- \notin \mathcal{B}$ and recommends rejection. The outcome takes form of $\alpha(\omega) =$

$\text{Prob}(s^+ \mid \omega)$, and the sender's problem under commitment becomes

$$\max_{\alpha} \int_{\Omega} \alpha(\omega) p(\omega) d\omega, \quad \text{subject to}$$

- $\forall \omega \in \Omega, 0 \leq \alpha(\omega) \leq 1$;
- the receiver's obedience constraint $q^+ \in \mathcal{B}$, which becomes

$$\int_{\Omega} \delta(\omega) \cdot q^+(\omega) d\omega = \int_{\Omega} \delta(\omega) \cdot \frac{\alpha(\omega) \cdot p(\omega)}{\int_{\Omega} \alpha(\omega') \cdot p(\omega') d\omega'} d\omega \geq 0$$

$$\iff \int_{\Omega} \delta(\omega) \cdot \alpha(\omega) \cdot p(\omega) d\omega \geq 0.$$

Observe that $\forall \omega \in \mathcal{A}$, $\alpha(\omega) = 1$, or else increasing $\alpha(\omega)$ relaxes the constraint (because $\delta(\omega) \geq 0$) and increases the objective function. Next, if the receiver approves the proposal under the prior belief, then the optimal experiment features no disclosure, i.e. $p \in \mathcal{B}$ implies $\alpha(\omega) = 1$ for every $\omega \in \Omega$. If $p \notin \mathcal{B}$, then the receiver's obedience constraint has to bind, or else the value of the objective can be increased. Assume for the rest of this proof that $p \notin \mathcal{B}$.

If $\alpha(\omega) \in (0, 1)$ for some $\omega \in \Omega$, then $\alpha(\omega') = 1$ for all $\omega' \in \Omega$ such that $\delta(\omega') > \delta(\omega)$, since these types are "cheaper" in terms of the obedience constraint and hence should be added the set of approval states first. Consequently, there must exist a threshold value $c^* < 0$ of the receiver's net payoff of approving the sender's proposal, and $\delta(\omega) = c^*$ for all ω such that $\alpha(\omega) \in (0, 1)$. Let $\bar{\mathcal{D}} := \{\omega \in \Omega \text{ s.t. } \delta(\omega) \geq c^*\}$; $\mathcal{D} := \{\omega \in \Omega \text{ s.t. } \delta(\omega) > c^*\}$; $\partial \mathcal{D} := \bar{\mathcal{D}} \setminus \mathcal{D} = \{\omega \in \Omega \text{ s.t. } \delta(\omega) = c^*\}$. The solution $\alpha(\cdot)$ takes form

$$\alpha(\omega) = \begin{cases} 1, & \omega \in \mathcal{D}; \\ (0, 1), & \omega \in \partial \mathcal{D}; \\ 0, & \omega \in \Omega \setminus \bar{\mathcal{D}}. \end{cases}$$

The (binding) obedience constraint is thus

$$\int_{\mathcal{D}} \delta(\omega) \cdot p(\omega) d\omega + \int_{\partial \mathcal{D}} \alpha(\omega) \cdot \underbrace{\delta(\omega)}_{= c^*, \forall \omega \in \partial \mathcal{D}} \cdot p(\omega) d\omega = 0.$$

If measure of set $\partial \mathcal{D}$ is not zero, partition it into two sets, $X \subseteq \partial \mathcal{D}$ and $Y = \partial \mathcal{D} \setminus X$. Let

$\tilde{\alpha}(\omega) = \mathbb{1}(\omega \in X)$, where X solves

$$\int_{\partial\mathcal{D}} \alpha(\omega) \cdot p(\omega) d\omega = \int_{\partial\mathcal{D}} \tilde{\alpha}(\omega) \cdot p(\omega) d\omega = P(X).$$

By changing $\alpha(\omega)$ to $\tilde{\alpha}(\omega)$ on $\partial\mathcal{D}$, the objective function of the sender does not change:

$$\int_{\partial\mathcal{D}} \alpha(\omega) p(\omega) d\omega = \int_{\partial\mathcal{D}} \tilde{\alpha}(\omega) p(\omega) d\omega = P(X);$$

and the obedience constraint does not change, either:

$$\int_{\partial\mathcal{D}} \alpha(\omega) \cdot \underbrace{\delta(\omega)}_{=c^*, \forall \omega \in \partial\mathcal{D}} \cdot p(\omega) d\omega = c^* \cdot \int_{\partial\mathcal{D}} \tilde{\alpha}(\omega) p(\omega) d\omega = c^* \cdot P(X).$$

Hence, the commitment outcome can be expressed as $\alpha(\omega) = \mathbb{1}(\omega \in \mathcal{D} \cup X)$, and the sender's problem under commitment can be stated as

$$\max_{W \subseteq \Omega} \int_{\Omega} \mathbb{1}(\omega \in W) \cdot p(\omega) d\omega \;\; \text{subject to} \int_{\Omega} \delta(\omega) \cdot \mathbb{1}(\omega \in W) \cdot p(\omega) d\omega,$$

which is the same problem as finding the sender-preferred equilibrium in the game with verifiable information. By the argument above, the solution is characterized by the cutoff value of receiver's net payoff of approval c^*.

Proof of Theorem 3 (See page 16).

Theorem 3. *The following statements about the sender's ex-ante utility $\bar{u}_s$ are equivalent:*

1. *$\bar{u}_s$ is reached in equilibrium;*
2. *$\bar{u}_s$ is reached in a direct equilibrium;*
3. *$\bar{u}_s$ is given by*

$$\bar{u}_s = \int_{\Omega} u_s(\{i \in I \mid \omega \in W_i\}) \cdot p(\omega) d\omega,$$

where for every receiver $i \in I$, $W_i \subseteq \Omega$ is her set of approval states, which satisfies

- *receiver's obedience constraint $p(\cdot \mid W_i) \in \mathcal{B}_i$;*
- *sender's incentive-compatibility constraint $\mathcal{A}_i \subseteq W_i$.*

PROOF. *Part I*, *(1) $\Rightarrow$ (3)*: consider equilibrium $(\sigma, \boldsymbol{a}, \boldsymbol{q})$ with the ex-ante utility of the sender $\bar{u}_s$. Let $X_i := \{\omega \in \Omega \mid \alpha_i(\omega) = 1\}$ be the set of states in which the sender convinces receiver $i \in I$ to approve the proposal with certainty. For very $i \in I$, set $W_i = X_i \cup \mathcal{A}_i$ satisfies the sender's (IC) constraint, and by Lemma A.1, W_i also satisfies receiver i's (obedience) constraint.

If $(W_1, \ldots, W_n)$ is the collection of the receivers' sets of approval states, then the sender's ex-ante utility equals

$$\int_{\Omega} u_s(\{i \in I \mid \omega \in W_i\}) \cdot p(\omega) d\omega,$$

because receiver i approves the proposal if and only if $\omega \in W_i$. What remains to show is that this expression equals $\bar{u}_s$, the ex-ante utility of the sender in the original equilibrium. That is true because if in state $\omega \in \Omega$ receiver $i \in I$ is convinced

- with certainty, then $\omega \in W_i$;
- with probability less than 1 and $\omega \in \mathcal{A}_i$, then her action is inconsequential to the sender's utility; adding ω to W_i does not change the sender's ex-ante utility;
- with probability less than 1 and $\omega \notin \mathcal{A}_i$, then her action is inconsequential to the sender's utility; removing ω to W_i does not change the sender's ex-ante utility.

As a result, $\bar{u}_s$ equals the expression above.

Part II, *(3) $\Rightarrow$ (2)*: consider collection $(W_1, \ldots, W_n)$ of receivers' sets of approval states, each of which satisfies the sender's (IC) and receiver's (obedience) constraints. In the direct equilibrium, the strategy of the sender is to send message W_i to receiver i if $\omega \in W_i$, and the complementary message $\Omega \setminus W_i$ otherwise. Sender's incentive-compatibility and receivers' obedience constraints ensure that the sender cannot deviate to full disclosure and the receivers interpret the two messages they receive on the path as recommendations to take appropriate actions. Off-the-path, the same "skeptical beliefs" apply as in the one-receiver case, and they ensure that the sender cannot deviate to an off-path message, either.

Part III, *(2) $\Rightarrow$ (1)* is trivial because a direct equilibrium is an equilibrium.

PROOF OF THEOREM 4 (SEE PAGE 17).

THEOREM 4. *The sender-preferred equilibrium outcome is a commitment outcome.*

PROOF. Consider the problem of finding the optimal commitment protocol $(\sigma^{BP}, a^{BP}, q^{BP})$. According to Kamenica and Gentzkow (2011), the problem may be simplified to finding an optimal *straightforward* experiment σ^{BP} that is supported on set $(S_1, \ldots, S_n)$, where $S_i = \{s_i^+, s_i^-\}$ is the private set of *straightforward* signal realizations of receiver $i \in I$. Signal realization s_i^+ induces posterior $q_i^+ \in \mathcal{B}_i$ and recommends that receiver i approves the proposal and s_i^- induces posterior $q_i^- \notin \mathcal{B}_i$ and recommends rejection. Commitment outcome is then $\alpha_i(\omega) = \text{Prob}(s_i^+ \mid \omega)$, $\forall i \in I$ and $\omega \in \Omega$, and it solves

$$\max_{\alpha_i, \forall i \in I} \sum_{T \subseteq 2^I} \int_\Omega \alpha(T, \omega) \cdot u_s(T) \cdot p(\omega) d\omega, \text{ subject to } \forall i \in I$$

- $\forall \omega \in \Omega$, $0 \leq \alpha_i(\omega) \leq 1$;
- receiver i's obedience constraint $q_i^+ \in \mathcal{B}_i$, which is $\int_\Omega \delta_i(\omega) \cdot \alpha_i(\omega) \cdot p(\omega) d\omega \geq 0$,

where $\alpha(T, \omega) := \prod_{i \in T} \alpha_i(\omega) \cdot \prod_{j \in I \setminus T} \big(1 - \alpha_i(\omega)\big)$ is the probability that receivers in $T \subseteq I$ approve the proposal and the receivers in $I \setminus T$ reject it. Notice that if $\alpha_i(\omega) = \mathbb{1}\big(\omega \in W_i^j\big)$ for all $i \in I$, then $\alpha(T, \omega) = \mathbb{1}\big(T = \{i \in I \mid \omega \in W_i\}\big)$, and the sender's problem becomes

$$\max_{W_i \subseteq \Omega, \forall i \in I} \int_\Omega u_s(\{i \in I \mid \omega \in W_i\}) \cdot p(\omega) d\omega,$$

subject to receiver i's obedience constraint $p(\cdot \mid W_i) \in \mathcal{B}_i$, for all $i \in I$. What remains to show is that *(i)* there exists a deterministic commitment outcome, and *(ii)* every set of approval states W_i induced by that outcome satisfies the sender's (IC) constraint. I construct a deterministic commitment outcome $\tilde{\alpha}_i$, $\forall i \in I$, in a sequence of steps.

Step 1: if, for some $i \in I$ and $\omega \in \mathcal{A}_i$, $\alpha_i(\omega) < 1$, then let $\tilde{\alpha}_i(\omega) = 1$. This weakly increases the objective, loosens receiver i's obedience constraint, and does not alter other receivers' obedience constraints. Note that this case only arises when the sender's utility does not strictly increase in receiver i's action in state ω;

Step 2: if, for some $i \in I$, this receiver's obedience constraint does not bind, then let $\tilde{\alpha}_i(\omega) = 0$ for every ω such that $\alpha_i(\omega) < 1$. In those states, the sender could have increased $\alpha_i(\omega)$ by tightening receiver i's obedience constraint, but did not do so because convincing this receiver in this state is inconsequential to the sender's utility;

Step 3: if, for some receiver $i \in I$ and set $\mathcal{D} \subseteq \Omega$, $\alpha_i(\omega) \in (0,1)$ for every $\omega \in \mathcal{D}$, and this receiver's obedience constraint binds, then we follow the steps on the proof of Theorem 2. The (binding) obedience constraint of receiver i becomes

$$\int_{\mathcal{D}} \delta_i(\omega) \cdot \alpha_i(\omega) \cdot p(\omega) d\omega = -\int_{\Omega \setminus \mathcal{D}} \delta_i(\omega) \cdot \alpha_i(\omega) \cdot p(\omega) d\omega := \mathcal{I}_i.$$

Since $\alpha_i(\omega) \in (0,1)$ on $\mathcal{D}_i$, then $\delta_i(\omega)$ and is constant on $\mathcal{D}_i$. Otherwise, if $\delta_i(\omega) > \delta_i(\omega')$ for some $\omega, \omega' \in \mathcal{D}_i$, then increasing $\alpha_i(\omega)$ and decreasing $\alpha_i(\omega')$ increases the value of the objective and loosens receiver i's obedience constraint. Next, partition $\mathcal{D}_i$ into sets X_i and $\mathcal{D}_i \setminus X_i$, where X_i solves

$$\int_{\mathcal{D}_i} \alpha_i(\omega) \cdot p(\omega) d\omega = \int_{X_i} p(\omega) d\omega = P(X_i).$$

Letting

$$\tilde{\alpha}_i(\omega) = \begin{cases} 1, & \omega \in X_i, \\ 0, & \omega \in \mathcal{D}_i \setminus X_i \end{cases}$$

does not alter the value of the objective or any receiver's obedience constraint, similarly to the one-receiver case;

Step 4: if for $i \in I$ and $\omega \in \Omega$, $\alpha_i(\omega) \in \{0,1\}$, then let $\tilde{\alpha}_i(\omega) = \alpha_i(\omega)$.

At this point, $\tilde{\alpha}_i$, $\forall i \in I$, is a deterministic commitment outcome that satisfies all of the sender's (IC) constraints. Consequently, this outcome also solves the problem of finding the sender-preferred equilibrium in the verifiable information game.

PROOF OF COROLLARY 1 (SEE PAGE 18).

COROLLARY 1. *Let* $L = \arg\max_{i \in I,\, v_i < \omega_0} v_i$ *and* $R = \arg\min_{j \in I,\, v_j > \omega_0} v_j$. *Then,*

1. *L is the representative voter on the left: $\forall i \in I$, if $v_i < \omega_0$, then $\mathcal{B}_L \subseteq \mathcal{B}_i$; R is the representative voter on the right: $\forall j \in I$ if $v_j > \omega_0$, then $\mathcal{B}_R \subseteq \mathcal{B}_j$;*
2. *approval sets and sets of approval beliefs of voters L and R do not intersect, i.e. $\mathcal{A}_L \cap \mathcal{A}_R = \varnothing$ and $\mathcal{B}_L \cap \mathcal{B}_R = \varnothing$.*

PROOF. 1. By contradiction, suppose $\exists q \in \mathcal{B}_L$ such that $q \notin \mathcal{B}_i$. Notice that because $v_i <$

$v_L < \omega_0$, we have $|v_i - \omega| = |v_i - v_L| + |v_L - \omega_0|$, that is, v_L is located between v_i and ω_0. Then,

$$q \notin \mathcal{B}_i \iff \mathbb{E}_q\big[|v_i - \omega|\big] > |v_i - \omega_0| - \varepsilon = |v_i - v_L| + |v_L - \omega_0| - \varepsilon \overset{q \in \mathcal{B}_L}{\geq} |v_i - v_L| + \mathbb{E}_q\big[|v_L - \omega|\big].$$

We have arrived at a violation of the triangle inequality, which for every $\omega \in \Omega$ states that $|v_i - \omega| \leq |v_i - v_L| + |v_L - \omega|$. The proof for the right pivot R is analogous.

2. By the definition of the set of approval beliefs, for every $i \in \{L, R\}$

$$q \in \mathcal{B}_i \iff \int_\Omega |v_i - \omega| \cdot q(\omega) d\omega \leq |v_i - \omega_0| - \varepsilon.$$

Adding up the right-hand sides for $i \in \{L, R\}$,

$$q \in \mathcal{B}_L \cap \mathcal{B}_R \Longrightarrow \int_\Omega [|v_L - \omega| + |\omega - v_R|] \cdot q(\omega) d\omega \leq |v_L - \omega_0| + |\omega_0 - v_R| - 2\varepsilon < |v_L - v_R|.$$

The right hand side violates the triangle inequality, which states that $|v_L - \omega| + |\omega - v_R| \geq |v_L - v_R|$ for every $\omega \in \Omega$. This proves that $\mathcal{B}_L \cap \mathcal{B}_R = \varnothing$. Since $\mathcal{B}_i$ includes beliefs that put probability 1 on $\omega \in \mathcal{A}_i$ for $i \in \{L, R\}$, $\mathcal{A}_L \cap \mathcal{A}_R = \varnothing$.

PROOF OF THEOREM 5 (SEE PAGE 20).

THEOREM 5. *In the sender-preferred equilibrium of an unwinnable election with representative voters L and R,*

- *message $\bar{W}_i$ that convinces voter $i \in \{L, R\}$ is an interval $[a_i, b_i] \supset \mathcal{A}_i$;*
- *if ε is small enough, the challenger has positive ex-ante odds of winning this election; the set of winning policies is $[a_R, b_L]$ with $a_R < \omega_0 < b_L$.*

PROOF. Recall that $\delta_i(\omega) = |v_i - \omega_0| - |v_i - \omega| - \varepsilon$ is voter i's net payoff from voting for the challenger. Her (obedience) constraint is:

$$p(\cdot \mid W_i) \in \mathcal{B}_i \iff \int_{W_i} \delta_i(\omega) p(\omega) d\omega \geq 0$$

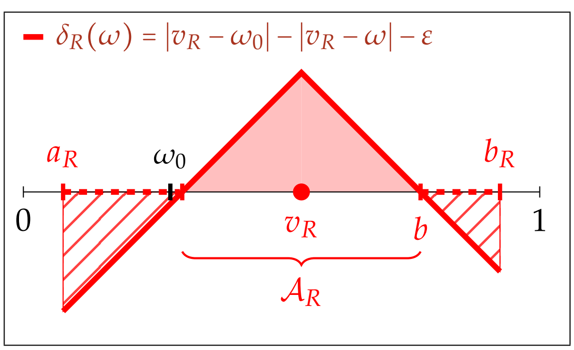
$$\iff \int_{W_i \setminus \mathcal{A}_i} -\underbrace{\delta_i(\omega)}_{<0, \forall \omega \notin \mathcal{A}_i} \cdot p(\omega) d\omega \leq \int_{\mathcal{A}_i} \underbrace{\delta_i(\omega)}_{>0, \forall \omega \in \mathcal{A}_i} p(\omega) d\omega := \mathcal{I}_i.$$

Notice that when $\omega \notin \mathcal{A}_i$, $-\delta_i(\omega)$ reflects the distance from point ω to the approval set of voter i. The voter's obedience constraint states that the expected distance from the challenger to the voter's approval set must not exceed a known quantity $\mathcal{I}_i$, which reflects how persuadable this voter is. For example, Figure 6 – part *(a)* illustrates how under uniform prior, voter R's obedience constraint states that the area under the function $\delta_R(\omega)$ over the approval set (it equals $\mathcal{I}_R$) must exceed the area over the same function outside of the approval set. Adding point x to $W_L \cap W_R$ increases the objective function by $p(x)$ and costs $-\delta_i(x)p(x) \cdot \mathbb{1}(x \notin \mathcal{A}_i)$ to each voter $i \in \{L, R\}$. Consequently, $x \notin \mathcal{A}_i$ is "cheaper" in terms of i's obedience constraint than $y \notin \mathcal{A}_i$ if $\delta_i(x) \geq \delta_i(y)$. Points in the approval set of the voter are "free" in terms of the obedience constraint of that voter.

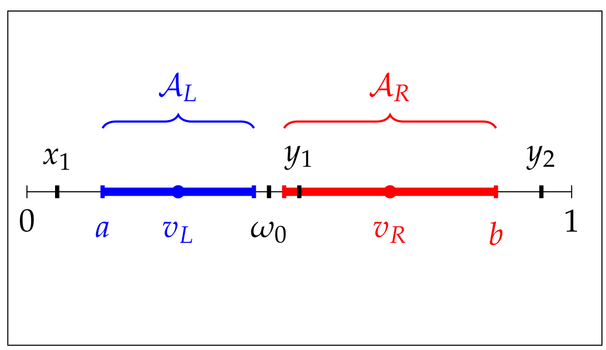
Relying on these observations, the following arguments, illustrated in Figure 6, part *(b)*, prove that $W_L = [a_L, b_L]$ with $a_L \leq a$ and $b_L > \omega_0 - \varepsilon$. Letting $a = 2v_L - \omega_0 + \varepsilon$ and $b = 2v_R - \omega_0 - \varepsilon$,

- $[a, \omega_0 - \varepsilon] \subseteq W_L$ because it is the approval set of voter L;
- if $x_1 \in [0, a)$ and $x \in W_L$, then $\forall y_1 \in [\omega_0 - \varepsilon, b]$ such that $|a - x_1| \geq |y_1 - \omega_0 + \varepsilon|$, $y_1 \in W_L$;
- if $x_1 \in [0, a)$ and $x \in W_L$, then $\forall x \in (x_1, a]$, $x \in W_L$;
- if $y_1 \in (\omega_0 - \varepsilon, b]$ and $y_1 \in W_L$, then $\forall y \in [\omega_0 - \varepsilon, y_1)$, $y \in W_L$;
- if $y_2 \in (b, 1]$ and $y_2 \in W_L$, then $\forall y \in [\omega_0 - \varepsilon, y_2)$, $y \in W_L$.

Finally, $b_L > \omega_0 - \varepsilon$ because $\mathcal{I}_L > 0$, and for small enough ε, $b_L > \omega_0$.



(a) *Voter R's net payoff of voting for the challenger. Under uniform prior, her obedience constraint states that the solid area exceeds the dashed area.*



(b) *Approval sets of the voters and points x_1, y_1, y_2.*

Figure 6: *why sender-preferred convincing messages are intervals.*

PROOF OF THEOREM 6 (SEE PAGE 21).

THEOREM 6. *Assume uniform prior. In the sender-preferred equilibrium of an unwinnable election with representative voters L and R,*

- *as R becomes more persuadable, the challenger's odds of winning $P(\bar{W}_R \cap \bar{W}_L)$ increase;*
- *suppose $|v_L - \omega_0| = |v_R - \omega_0|$, meaning that neither voter is more persuadable than the other. Then, as R becomes more persuadable, the set of winning policies $\bar{W} = [a_R, b_L]$ shifts to the left, i.e. a_R and b_L decrease.*

PROOF. Given convincing message $[a_R, b_R] \supseteq [\omega_0 + \varepsilon, 2v_R - \omega - \varepsilon] = \mathcal{A}_R$, voter R's obedience constraint becomes

$$\int_{a_R}^{\omega_0+\varepsilon} (\omega_0 - \omega)p(\omega)d\omega + \int_{2v_R-\omega_0-\varepsilon}^{b_R} (\omega_0 - \omega - 2v_R)p(\omega)d\omega$$

$$\leq \int_{\omega_0+\varepsilon}^{v_R} (\omega - \omega_0)p(\omega)d\omega + \int_{v_R}^{2v_R-\omega_0-\varepsilon} (2v_R + \omega - \omega_0)p(\omega)d\omega.$$

The derivative of the left-hand side of this inequality with respect to v_R is negative and equals $-2P\big([2v_R - \omega_0 - \varepsilon, b_R]\big)$, while the derivative of the right-hand side with respect to v_R is positive and equals $2P\big([v_R, 2v_R - \omega_0 - \varepsilon]\big)$. Consequently, as v_R increases, voter R's obedience constraint loosens, and that is true for any prior distribution. Hence, the solution, specifically, the challenger's ex-ante odds of winning, can only improve.

Now suppose $|v_L - \omega_0| = |v_R - \omega_0|$ and let $a = 2v_L - \omega_0 + \varepsilon$ be the left boundary of L's approval set, and let $b = 2v_R - \omega_0 - \varepsilon$ be the right boundary of R's approval set. Voters' (obedience) constraints are symmetric about ω_0, implying that the solution is symmetric, as well, i.e. $|b_L - \omega_0| = |\omega_0 - a_R|$. Here, a_R solves voter R's obedience constraint $-\int_{a_R}^{\omega_0+\varepsilon} \delta_R(\omega)d\omega = \int_{\omega_0+\varepsilon}^{b} \delta_R(\omega)d\omega > 0$. For small enough ε, $a_R < \omega_0 - \varepsilon$ (from obedience, $a_R < \omega_0 + \varepsilon$) and $b_R > \omega_0 + \varepsilon$, implying that $a_R < \omega_0 < b_L$ and the challenger swings the election with a positive probability.

As v_R increases, voter R's obedience constraint loosens, while voter L's obedience constraint remains the same. An increase in the value of the objective function is thus obtained by decreasing both a_R and b_L because

- b_L cannot increase because it is obtained from the binding obedience constraint of voter L that was not affected by the change;
- for high enough v_R, $\int_{\omega_0+\varepsilon}^{b} \delta_R(\omega)d\omega > -\int_{a_R}^{\omega_0+\varepsilon} \delta_R(\omega)d\omega$, meaning that the optimal message that convinces voter L has to be optimally shifted to the left and becomes $[a_L, b'_L]$, with $a_L < a$ and $b'_L < b_L$;
- voter L's obedience constraint becomes $\int_{a}^{\omega_0-\varepsilon} \delta_L(\omega)d\omega \geq -\int_{a_L}^{a} \delta_L(\omega)d\omega - \int_{\omega_0-\varepsilon}^{b_L} \delta_L(\omega)d\omega$. Because b_L is further from v_L than a is, removing $b_L - d$ from the message that convinces voter L and replacing it with $a - d$ (for some $d > 0$) loosens voter L's obedience constraint and keeps the value of the objective the same. That means that as b_L decreases, a_L decreases even more;
- the above argument stops working when $b_L - \omega_0 = a - a_L$. At that point, voter R is so persuadable that only voter L's constraint binds. The problem boils down to persuading just voter L, is characterized in Theorem 2, and no further dynamics are observed.