Targeted Advertising in Elections

by

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Motivation

- **Targeted Advertising** was an important part of winning campaigns in recent U.S. Presidential Elections:
  - **2016 Trump**: used voter data from Cambridge Analytica
  - **2008 Obama**: first social media campaign
  - **2000 Bush**: targeting voters by mail

can targeted advertising swing elections? → Yes

Preview of Results

- some elections are unwinnable for challengers without targeted advertising
  - (pivotal) voters prefer policies on opposite sides of status quo
  - no public message convinces them to approve challenger's policy
- any such election can be won with targeted advertising
  - challenger makes each voter believe his policy is a sufficient improvement over status quo
  - challenger wins if his policy is sufficiently close to status quo
  - voters regret their choices
- if voters become more extreme, challenger's odds of winning increase

Related Literature

- **private vs. public voter persuasion**
  - verifiable info: Schipper and Woo (2019)
    - unraveling does not have to happen if only one candidate advertises
  - cheap talk: Farrell and Gibbons (1989), Koessler (2008), Goltsman and Pavlov (2011), Bar-Isaac and Deb (2014)
    - sender prefers private communication if his messages are verifiable
  - Bayesian persuasion: Arieli and Babichenko (2019), Bardhi and Guo (2018), Chan et al. (2019), Heese and Lauermann (2019)
    - sender does not need commitment to benefit from targeted advertising
    - targeting does not just improve odds of winning, it swings unwinnable elections
- **political ambiguity**
  - Shepsle (1972), Alesina and Cukierman (1990), Aragonès and Neeman (2000), Meirowitz (2005), Alesina and Holden (2008), Kartik, Van Weelden, and Wolton (2017), **Callander and Wilson (2008)**, **Tolvanen (2021)**
    - ambiguity allows challenger to convince multiple voters at once without lying (by commission) to any of them

Baseline Election (2 Voters)

Model Setup

- policy space is $X := [-1, 1]$
  - policies range from far-left (-1) to far-right (1)
  - status quo policy is fixed, known, normalized to 0
- **challenger** (he/him)
  - privately observes his policy $x \in X$
    - x is drawn from common atomless prior $\mu_0 \in \Delta X$ with full support
  - gets 1 if wins the election, 0 otherwise
    - winning requires unanimous approval of both voters

Model Setup: Communication

- challenger communicates with voter using verifiable messages
  - each message m
    - is a statement about policy: $m \subseteq X$
    - contains a grain of truth: $x \in m$
- example: $m = [-1/2, 0]$, or "my policy is moderately left"

Model Setup: Voters

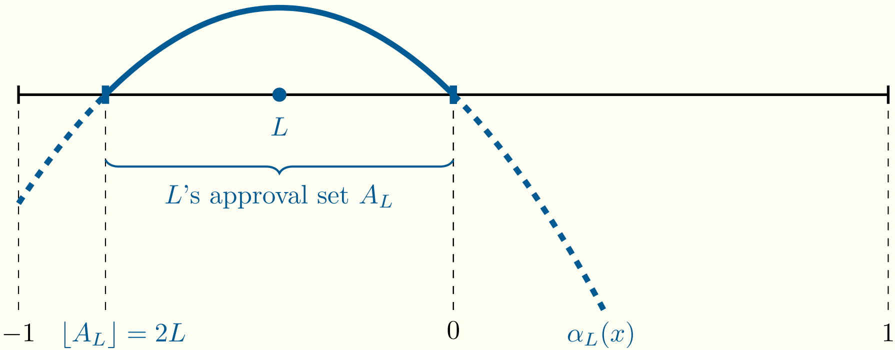
- voters have spatial preferences
- **voter** (she/her) with bliss point $v \in X$ has

$$\text{utility of approval} \quad u_v(\text{approve}, x) = -(v-x)^2$$
$$\text{utility of rejection} \quad u_v(\text{reject}, x) = -(v-0)^2$$

$$\textbf{net payoff from approval} \quad \alpha_v(x) := -(v-x)^2 + v^2$$
$$\textbf{approval set} \quad A_v := \{x \in X \mid \alpha_v(x) \geq 0\}$$

- two voters: *left* (with $v = L < 0$) and *right* (with $v = R > 0$)

Voter's Preferences: Illustration



Timeline and Equilibrium Concept

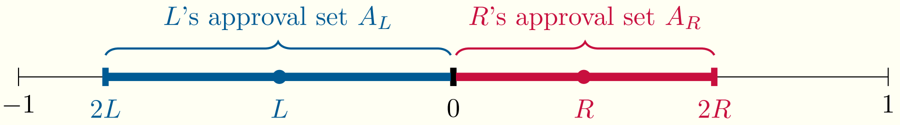
challenger learns his policy $x \in X$	$\rightarrow$	challenger selects m_L and m_R	$\rightarrow$	voter $v \in \{L, R\}$ privately observes message m_v	$\rightarrow$	voter $v \in \{L, R\}$ chooses between approval and rejection

- **Perfect Bayesian Equilibrium**
  - for every policy $x \in X$, private messages $m_L \subseteq X$ and $m_R \subseteq X$ maximize challenger's utility subject to $x \in m_L$ and $x \in m_R$
  - voter approves whenever expected net payoff from approval is non-negative under her posterior
    - expressive / does not condition on the event of being pivotal
  - voters' posteriors are Bayes-consistent

Unwinnable Election

Incompatible Voters

- *left* and *right* voters prefer policies on opposite sides of status quo



Lemma 1

If voter with bliss point $v \in X$ approves under a non-degenerate belief $\mu \in \Delta X$, then $\mathbb{E}_\mu[x]$ is strictly between 0 and $2v$.

Lemma 2

For any common non-degenerate belief $\mu \in \Delta X$, at most one of the voters prefers to approve.

Unwinnable Election

- baseline election is unwinnable for challenger without targeted advertising
  - no advertising
  - full disclosure
  - public disclosure
- true for any communication protocol (evidence, cheap talk, Bayesian persuasion) that induces *common posterior*

Equilibrium Analysis

Equilibrium Outcomes

- focus on equilibrium <u>sets of approved policies</u>
  - voter $v \in \{L, R\}$ approves set of policies $W_v \subseteq X$, rejects $W_v^c := X \setminus W_v$
  - direct implementation: when talking to v, challenger sends message
    - W_v if his policy is $x \in W_v \longleftarrow$ recommendation to approve
    - W_v^c if his policy is $x \notin W_v \longleftarrow$ recommendation to reject
- Titova (2022): $(W_L, W_R) \subseteq X^2$ is an equilibrium outcome iff $\forall v \in \{L, R\}$
  - $A_v \subseteq W_v$: challenger does not want to deviate to full disclosure
  - $\int\limits_{W_v} \alpha_v(x) d\mu_0(x) \geq 0$: voter's **obedience constraint**

Challenger-Preferred Equilibrium

- I focus on challenger-preferred PBE
  - one with highest odds of unanimous approval/winning
- problem:

$$(\overline{W}_L, \overline{W}_R) = \arg \max_{(W_L, W_R) \subseteq X^2} \overbrace{\int_{W_L \cap W_R} d\mu_0(x)}^{\mu_0(W_L \cap W_R)}$$

$$\text{subject to} \quad \int_{W_v} \alpha_v(x) d\mu_0(x) \geq 0 \text{ for each } v \in \{L, R\}$$

I call $(\overline{W}_L, \overline{W}_R)$ the (challenger-preferred) equilibrium outcome (under targeted advertising)

Proposition 1

Proposition 1: Swinging Unwinnable Elections

In equilibrium of the baseline election game, the challenger's ex-ante odds of winning are always positive.

idea of proof: for each voter $v \in \{L, R\}$

- observe that v always approves own approval set: $\int_{A_v} \alpha_v(x) d\mu_0(x) > 0$
- select subset $B_v \subseteq A_{-v}$ of the other voter's approval set that satisfies

$$\int_{A_v} \alpha_v(x) d\mu_0(x) + \int_{B_v} \alpha_v(x) d\mu_0(x) \geq 0 \quad \text{and} \quad \mu_0(B_v) > 0$$

- let $W_v = A_v \cup B_v$
- we have $\mu_0(W_L \cap W_R) = \mu_0(B_R \cup B_L) > 0$

$\Longrightarrow$ odds in equilibrium are positive

Convincing One Voter

Auxiliary Problem

- **question**: what is the largest subset of $[l, r] \subseteq X$ can voter v approve?

$$I_v(l,r) := \max_{W \subseteq [l,r]} \mu_0(W) \quad \text{subject to} \quad \int_W \alpha_v(x) d\mu_0(x) \geq 0 \qquad \text{(AUX)}$$

- **answer**: Alonso and Câmara (2016) and Titova (2022)

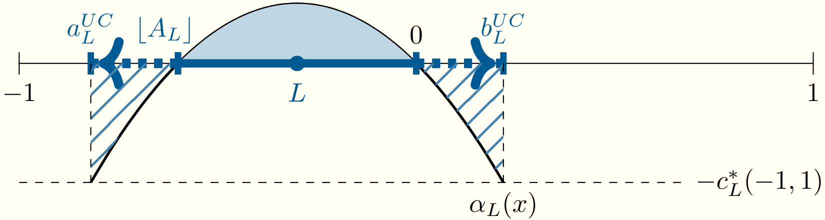
Corollary 2

Consider voter $v \in X$. The solution to Problem (AUX) with $l \in \big[-1, \lfloor A_v \rfloor\big]$ and $r \in \big[\lceil A_v \rceil, 1\big]$ is almost surely an interval such that

- if $\int_l^r \alpha_v(x) d\mu_0(x) \geq 0$, then $I_v(l,r) = [l,r]$
- otherwise, $I_v(l,r) = \big\{x \in [l,r] \mid \alpha_v(x) \geq -c_v^*(l,r)\big\}$, where $c_v^*(l,r) > 0$ is obtained from the binding constraint $\int_{I_v(l,r)} \alpha_v(x) d\mu_0(x) = 0$

Largest Unconstrained Interval of Approved Policies

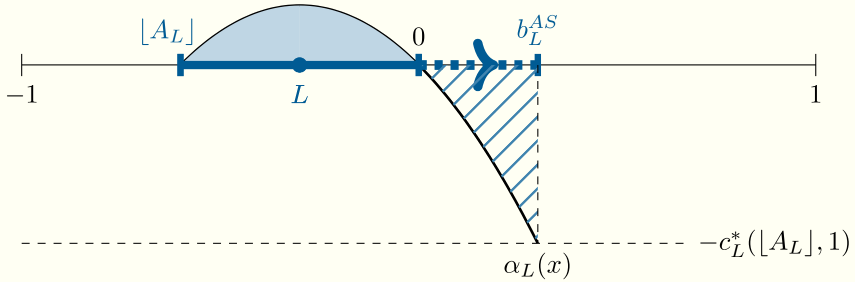
- solve (AUX) for $l = -1$ and $r = 1$ to get $I_v(-1,1) =: I_v^{UC} = [a_v^{UC}, b_v^{UC}]$
  - v's largest unconstrained interval of approved policies
- **example**: $left$ voter, $v = L$



Largest Asymmetric Interval of Approved Policies

- *left* voter: how many *right* policies can she approve?

$$I_L^{AS} = \left[\lfloor A_L \rfloor, b_L^{AS}\right] := I_L(\lfloor A_L \rfloor, 1)$$

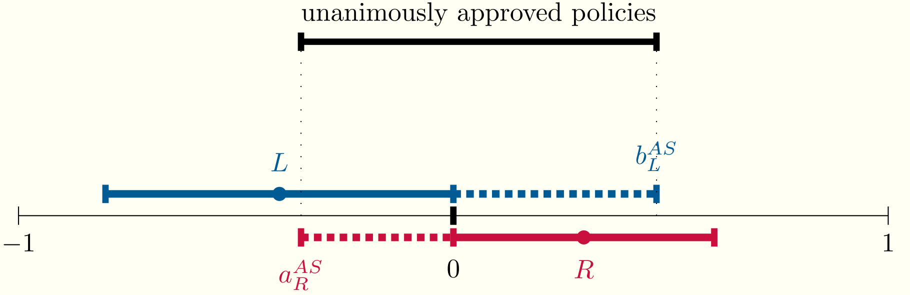


- *right* voter: how many *left* policies can she approve?

$$I_R^{AS} = \left[a_R^{AS}, \lceil A_R \rceil\right] := I_R(-1, \lceil A_R \rceil)$$

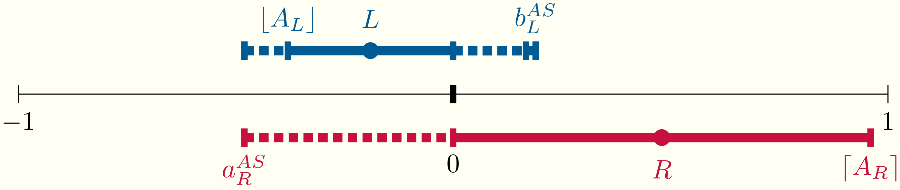
Convincing Two Voters at the Same Time

Candidate Solution



When Candidate Solution Fails

- *right* voter is significantly more persuadable, or $\int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_R(x) d\mu_0(x) > 0$



- assume *left* voter is not significantly more persuadable than *right* voter

$$\int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_L(x) d\mu_0(x) \leq 0$$

Proposition 2: Equilibrium Intervals of Approved Policies

(1) if neither voter is significantly more persuadable than the other, then

- $\overline{W}_R = \left[a_R^{AS}, \lceil A_R \rceil\right]$ and $\overline{W}_L = \left[\lfloor A_L \rfloor, b_L^{AS}\right]$
- equilibrium set of unanimously approved policies is $\overline{W} = [a_R^{AS}, b_L^{AS}]$

(2) if *right* voter is significantly more persuadable than *left* voter, then

- $\overline{W}_R = \left[a_R^{AS}, \lceil A_R \rceil\right]$
- the *left* voter's equilibrium set of approved policies is her largest interval of approved policies constrained from the left by a_R^{AS}, or $\overline{W}_L = I_L(a_R^{AS}, 1)$
- the equilibrium set of unanimously approved policies is $\overline{W} = \overline{W}_L$

Proposition 2

(1) if neither voter is significantly more persuadable than the other, then

- $\overline{W}_R = \left[a_R^{AS}, \lceil A_R \rceil\right]$ and $\overline{W}_L = \left[\lfloor A_L \rfloor, b_L^{AS}\right]$
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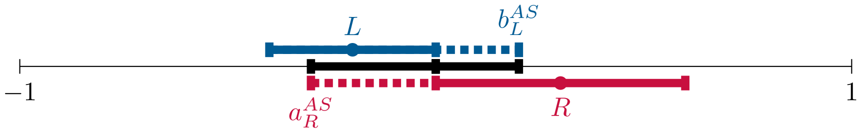
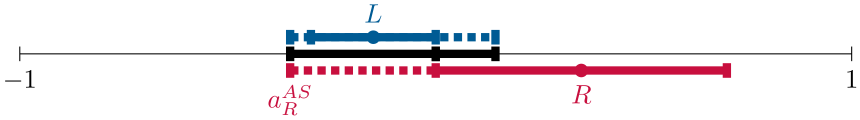


Proposition 2

(2) if *right* voter is significantly more persuadable than *left* voter, then

- $\overline{W}_R = [a_R^{AS}, \lceil A_R \rceil]$
- the *left* voter's equilibrium set of approved policies is her largest interval of approved policies constrained from the left by a_R^{AS}, or $\overline{W}_L = I_L(a_R^{AS}, 1)$
- the equilibrium set of unanimously approved policies is $\overline{W} = \overline{W}_L$



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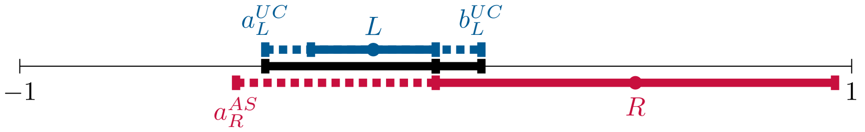


Calculating Equilibrium Intervals of Approved Policies

identifying $(\overline{W}_L, \overline{W}_R)$ requires solving 2 (AUX) problems

algorithm:

- calculate $\rho_v := \int_{\lfloor A_L \rfloor}^{\lceil A_R \rceil} \alpha_v(x) d\mu_0(x)$ for each $v \in \{L, R\}$
  - if $\rho_R, \rho_L \leq 0$, neither voter is more persuadable than the other
    - solve 2x (AUX) problems to find a_R^{AS} and b_L^{AS}
  - if $\rho_R > 0 \geq \rho_L$, the *right* voter is significantly more persuadable
    - solve 1x (AUX) problem to find a_R^{AS}
    - solve 1x (AUX) problem to find $I_L(a_R^{AS}, 1)$
  - if $\rho_L > 0 \geq \rho_R$, the left voter is significantly more persuadable
    - solve 1x (AUX) problem to find b_L^{AS}
    - solve 1x (AUX) problem to find $I_R(-1, b_L^{AS})$

Comparative Statics

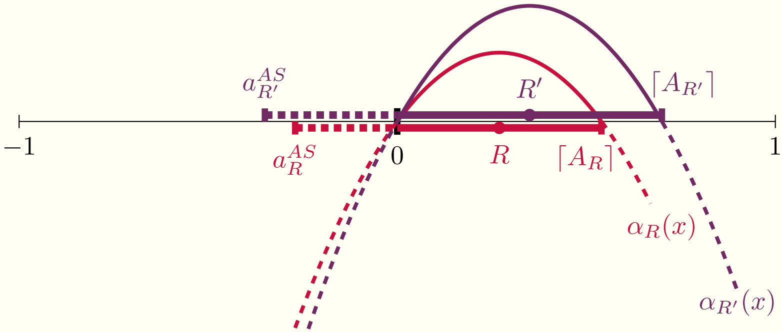
Extremism and Polarization

- *left* voter *becomes more extreme* if L decreases; *right* voter *becomes more extreme* if R increases
- baseline electorate *becomes more polarized* if the *left* and/or the *right* voter becomes more extreme
    - *most polarized electorate* is $L = -1$ and $R = 1$
    - larger distance between L and R does not always imply higher polarization

More Extreme → More Persuadable

Lemma 2

If $R' > R$, then $\left[a_{R'}^{AS}, \lceil A_{R'} \rceil\right] \supseteq \left[a_{R}^{AS}, \lceil A_{R} \rceil\right]$, with $a_{R'}^{AS} \leq a_{R}^{AS}$ and $\lceil A_{R'} \rceil \geq \lceil A_{R} \rceil$; the former inequality is strict unless $a_{R}^{AS} = -1$



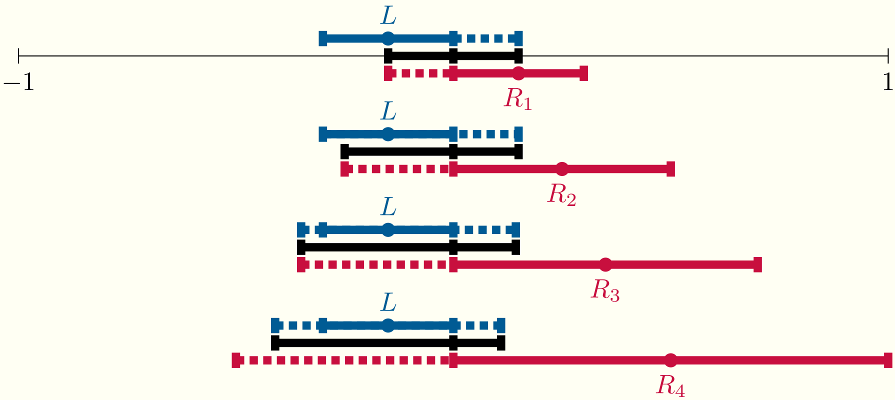
Proposition 3

Proposition 3: Comparative Statics

Suppose that the *left* voter is not significantly more persuadable than the *right* voter. Then, as the *right* voter becomes more extreme,

- the challenger's odds of winning increase;
- the equilibrium set of unanimously approved policies shifts to the left.

Comparative Statics



Welfare

Welfare and Regret

- if v's set of approved states is W_v, her ex-ante utility is

$$\mathbb{E}_{\mu_0}\Big[\mathbb{1}(x \in W_v) \cdot \big(-(v-x)^2\big) + \mathbb{1}(x \in W_v^c) \cdot \big(-v^2\big)\Big]$$

- add v^2 to get $\int_{W_v} \alpha_v(x) d\mu_0(x)$

Definition

Consider $v \in X$ and her set of approved policies W_v. Then, v's

- welfare is $\int_{W_v} \alpha_v(x) d\mu_0(x)$;
- amount of regret is $\int_{A_v} \alpha_v(x) d\mu_0(x) - \int_{W_v} \alpha_v(x) d\mu_0(x)$.

Communication Benchmarks

- **full disclosure** outcome (A_L, A_R)
  - also the challenger-worst equilibrium of baseline game
- **public disclosure** outcome $\left(W_L^{PD}, W_R^{PD}\right)$
  - challenger's odds of winning are zero
- **targeted advertising** outcome $\left(\overline{W}_L, \overline{W}_R\right)$

Welfare Comparison

	v's welfare	v's regret	challenger's odds of winning
full disclosure	$\int_{A_v} \alpha_v(x) d\mu_0(x) > 0$	0	0
public disclosure	$\int_{W_v^{PD}} \alpha_v(x) d\mu_0(x) \geq 0$	≥ 0	0
targeted advertising	$\int_{\overline{W}_v} \alpha_v(x) d\mu_0(x) = 0$	> 0	$\mu_0(\overline{W}_L \cap \overline{W}_R) > 0$

Large Elections

Large Electorates and Unwinnable Elections

- **large electorate**: set of bliss points $V = \{v_1, \ldots, v_n\}$
- $\mathcal{D}$ is set of decisive coalitions
  - challenger wins (and gets 1) iff he convinces every voter in some $D \in \mathcal{D}$

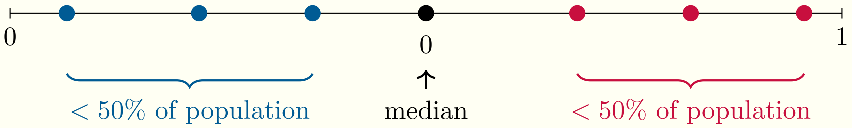
Lemma: Unwinnable Elections

The following statements are equivalent:

- election is unwinnable for the challenger without targeted advertising;
- status quo policy is a.s. socially preferred to challenger's policy under complete information;
- there is no $D \in \mathcal{D}$ such that $v < 0\ \forall v \in D$ OR $v > 0\ \forall v \in D$.

Unwinnable Elections: Example

- simple majority rule – which elections are unwinnable?



(version of the) Median Voter Proposition

Under simple majority, a large election is unwinnable for the challenger without targeted advertising if and only if the status quo is the bliss point of the median voter.

Swinging Large Unwinnable Elections

- for any (minimal) decisive coalition D, identify
  - the left pivot: $L := \max_{v \in D \text{ s.t. } v<0} v$
    - every other voter on the left is convinced if L is convinced
  - the right pivot: $R := \min_{v \in D \text{ s.t. } v>0} v$
    - every other voter on the right is convinced if R is convinced
- solve baseline election for L and R
- maximizing odds of winning requires doing this for every minimal winning coalition

Conclusion

- some elections are unwinnable without targeted advertising
  - (pivotal) voters prefer policies on opposite sides of status quo
- any such election can be won with targeted advertising
  - challenger makes each voter believe his policy is sufficient improvement over status quo
  - challenger wins if his policy is not too far from status quo
  - voters regret their choices
- if voters become more extreme, challenger's odds of winning increase

Thank You!