STEP I 2016 REPORT

General Comments

This year, more than 2000 candidates signed up to sit this paper, though just under 2000 actually sat it. This figure is about the same as the entry figure for 2015, though the number of candidates opting to sit STEP I has risen significantly over recent years; for instance, it was around 1500 in 2013.

There is no doubt that the purpose of the STEPs is to learn which students can genuinely use their mathematical knowledge, skills and techniques in an arena that demands of them a level of performance that exceeds anything they will have encountered within the standard A-level (or equivalent) assessments. The ability to work at an extended piece of mathematical work, often with the minimum of specific guidance, allied to the need for both determination and the ability to "make connections" at speed and under considerable time pressure, are characteristics that only follow from careful preparation, and there is a great benefit to be had from an early encounter with, and subsequent prolonged exposure to, these kinds of questions.

It is not always easy to say what level of preparation has been undertaken by candidates, but the minimum expected requirement is the ability to undertake routine A-level-standard tasks and procedures with speed and accuracy. At the top end of the scale, almost 100 candidates produced a three-figure score to the paper, which is a phenomenal achievement; and around 250 others scored a mark of 70+, which is also exceptionally impressive. At the other end of the scale, over 400 candidates failed to reach a total of 40 marks out of the 120 available.

For STEP I, the most approachable questions are always set as Qs. 1 & 2 on the paper, with Q1 in particular intended to afford every candidate the opportunity to get something done successfully. So it is perfectly reasonable for a candidate, upon opening the paper, to make an immediate start at the first and/or second question(s) before looking around to decide which of the remaining 10 or 11 questions they feel they can tackle. It is very important that candidates spend a few minutes – possibly at the beginning – reading through the questions to decide which ***six*** they intend to work, since they will ultimately only be credited with their best six question marks. Many students spend time attempting seven, eight, or more questions and find themselves giving up too easily on a question the moment the going gets tough, and this is a great pity, since they are not allowing themselves thinking time, either on the paper as a whole or on individual questions.

The other side to the notion of strategy is that most candidates clearly believe that they need to attempt (at least) six questions when, in fact, four questions (almost) completely done would guarantee a Grade 1 (Distinction), especially if their score on these first four questions were then to be supplemented by a couple of early attempts at the starting parts of a couple more questions (for the first five or six marks); attempts which need not take longer than, say, ten minutes of their time. It is thus perfectly reasonable to suggest to candidates, in their preparations, that they *can* spend more than 30 minutes on a question, but only IF they think they are going to finish it off satisfactorily, although it might be best if they were advised to spend absolutely no more than 40-45 minutes on any single question; if they haven't finished by then, it really is time to move on.

Curve-sketching skills are usually a common weakness, but were only tested on this paper in Q3. The other common area of weakness – algebra – was tested relatively frequently, and proved to be as testing as usual. Calculus skills were generally "okay" although the integration of first-order differential equations by the separation of variables, as appearing repeatedly in Q4, was found challenging by many of the candidates who attempted this question. The most noticeable deficiency, however, was in the widespread inability to construct an argument, particularly in Qs. 5, 7 & 8. Vectors are often poorly handled, and this year proved no exception.

Comments on individual questions

[Examiner's note: in order to extract the maximum amount of profit from this Report, it is firmly recommend that the reader studies it alongside the *Solutions* and/or *Mark Scheme* supplied separately.]

Question 1
Marginally the most popular question on the paper, and the highest-scoring, this was a relatively carefully signposted question; thus, even though the demands were entirely algebraic, it was a good starter question and gave all candidates something to get their teeth into. It was still often the case that candidates spent a lot more time doing fairly simple things than they should have; for instance, an awful lot of attempts produced over and over again (effectively) the same work to show that $\frac{x^{2n+1}+1}{x+1} \equiv x^{2n} - x^{2n-1} + \ldots + x^2 - x + 1$ in each of the four given cases. And a similar amount of unnecessary effort was expended on what should have been some fairly simple binomial expansions. Nevertheless, most candidates made good progress for the *n* = 1, 2, 3 cases.
To show that p_4 and q_4 are not identical, it suffices to choose any one value of x for which they yield different outputs, but most approaches preferred to deal with the full polynomial expansions, which is fine but (again) not an optimal approach.
In part (ii), most candidates realised that they were to use the results of part (i), and they were generally able to do so for at least (a). Relatively few realised that the same idea (the use of *the difference of two squares factorisation*) was to be deployed in (b), presumably put off by the large numbers involved and the notion that the answer could be left in terms of powers of 7.

Question 2
This was the second most popular question of all, attempted by over 80% of candidates, though pulling in a mean score of just under half-marks. Most candidates managed the tricky differentiation; tricky, since it required the repeated use of the *Chain* and *Product* rules of differentiation. The big key to later progress in this question was the success, or otherwise, of the simplifying that was done. Those who realised that $\sqrt{1+x^2}$ needed to be re-written as $\frac{1+x^2}{\sqrt{1+x^2}}$ flew through the simplification, and this made the idea behind the three later parts much more transparent. In fact, careful differentiation and a sound simplification made choosing the appropriate values of the given constants *a* to *e* straightforward.

Question 3

It was very encouraging, on this question, to see both the number of attempts (only a few behind those for Qs.1 & 2) and the quality of them. The introduction of a new function is frequently a guarantee of poor answers, especially when it is made to act on another function (the sine function here), but many candidates seemed to cope quite well with the "int" function and the properties of its "step" graphs. Although reminded (at the end of the question) to make the endpoints of the steps completely clear, there was a lot of vagueness in candidates' answers on these points, and this was one of the principal causes of lost marks. An overall mean score of just over 12½ out of 20 is testimony to the overall success on this potentially tricky question.

Question 4

Around 60% of candidates attempted this question, but the mean score achieved on it was under 3½ out of 20. The first three marks were awarded for a correct differentiation and subsequent simplification (of a very similar kind to that in Q2 but much, much easier), but it is clear that almost no-one was able to proceed further into the question, due almost entirely to difficulties working with $\frac{dy}{dx}$ as an entity in its own right (along with *its* derivative, $\frac{d^2y}{dx^2}$). This is especially disappointing, since this is little more than a labelling matter. Even for those who did see what to do in that respect, spotting that one then had a separable-variable 1^{st}-order d.e. to deal with proved difficult.

Question 5

Attempts at this question were down to around a third of the candidature and, overall, it proved to be the lowest scoring question on the paper with a mean score of just 3 out of 20. The big problem here was the widespread inability to draw some suitable lines onto the diagram, or to realise that the lines joining the centres necessarily passed through the points of contact. Once this is done, there is an obvious right-angled triangle to be found embedded within this geometric arrangement that then requires only the use of *Pythagoras' Theorem* to make a start.

Although there were relatively few complete efforts to be found, many who did manage to make a good start found aspects of the algebra difficult: for instance, turning result (*****) into (******). Here, there are several good approaches provided one uses (*****) sensibly to eliminate (say) *b*.

Admittedly, the last part to the question *was* more demanding to deal with, but it was clear from responses that the majority of even those candidates who got this far struggled to see the logical difference between proving $A \Rightarrow B$ and proving $B \Rightarrow A$.

Question 6

This vectors question proved both unpopular and low-scoring, being attempted by only 30% of candidates and eliciting a mean score of only 5 out of 20. Once again, it was clear that candidates were, in general, doing little more than making a half-hearted attempt at the very first part of the question. The rest of the question relied only on the ability to write the vector equations of pairs of lines and then solve simultaneously by considering the coefficients of **a** and **b**. Overall this question suggests that vectors are not well understood.

Question 7

This was a rather splendid reasoning question, and many candidates responded very well to its demands. Almost half of all candidates made an attempt at it, achieving a mean score of over 7 out of 20. Whilst many attempts went little beyond the first two parts, many candidates actually did well on the question as a whole.

Part (iii) required a mixture of a proof by contradiction based on at least an informal understanding of the method of proof by induction, so it was no surprise that many gave up at this stage. The "fun" part was (v), in which candidates had to choose some numbers that demonstrated the required properties; unfortunately, it is clear this sort of request is not always well answered.

Question 8

Generating functions generally appear very late on in statistics modules whose entry numbers seldom reach three figures. Nevertheless, a mean score of almost 9 out of 20 on this question, obtained by more than a quarter of the candidates (very few, if any, of whom will have seen the idea before), suggests that it is easy to introduce the topic and that good candidates will pick it up quickly and successfully.

Apart from algebraic difficulties, the real hurdle to making a complete attempt lay in part (ii) (a); although most of the candidates who got this far went on to earn the 4 marks allocated to this part, they tended to do so by ignoring the guidance of the question. Although there are two or three perfectly good alternative approaches to the one intended, they don't necessarily point in the direction that helps the solver cope with the more imaginatively constructed part (ii) (b); and this was why most candidates struggled to know quite what to do with it.

Questions 9, 10 & 11

The three mechanics questions each received a healthy number of "hits" this year, and marks were fairly respectable also.

Question 9

In this questions – a statics question – candidates were helped enormously by the given diagram, which allowed most takers the opportunity to get most of the preliminary "force" statements down correctly: resolving twice and taking moments (choosing sensible directions and a suitable axial point), along with the straightforward use of the "$F = \mu R$" friction law, meant that 7 marks could be obtained simply for noting all the correct ingredients. Suitable choices for eliminating terms would then lead to the printed answer with little difficulty beyond the algebraic. It is fair to say, however, that the prospect of doing it all again for the second configuration proved too much for many candidates, although it actually only involved the realisation that certain forces changed directions and could thus be replaced by their negatives.

Question 10

This question was a relatively undemanding collisions question, a fact that was apparent to a lot of the candidates, almost half of whom made an attempt at the question. It turned out to be only one of three questions on the paper for which the mean achieved mark was over 10 out of 20 (after Qs. 1 & 3), and this is no doubt partly due to the fact that there are generally only three physical “laws” or “principles” to be applied on this type of mechanics problem. On this occasion, no energy considerations were involved, which meant that careful application of *Conservation of Linear Momentum* and *Newton’s (Experimental) Law of Restitution* pretty much saw the solver through the whole thing. The only minor hurdle is that candidates often get confused about which directions are being taken as positive, and this is usually down to the lack of a suitably-marked diagram. Other than that, it was only a few algebraic slips here and there that prevented full and successful attempts from being made.

Question 11

This question was also a rather pleasant question of its kind, dealing with projectile motion. It proved to be a very popular question (with 40% of all candidates attempting it, almost as popular as Q10) but with a mean score of almost 8 out of 20. There were two main obstacles to progress in this question: first, rather a lot of candidates failed to appreciate that finding the greatest something-or-other might require calculus; secondly, and more importantly, putting everything together to create a quadratic in $c = \cos 2\alpha$ proved challenging.

Question 12

This question proved to be an especially popular choice (almost 45% uptake) and a high-scoring one at that (mean score of over 9 out of 20). However, most of these medium-successful marks came from attempts at the first two (specific) cases, in which Bob threw 2 coins, then 3. There were 6 marks for successful attempts at each of these preliminary parts, and candidates were happy to work them through reasonably successfully, although some of these attempts were very poorly explained; and occasionally it was the case that there was no explanation at all. Although they may have been getting the correct answers, it is imperative to ensure that answers are sufficiently coherent for the reader to be able to follow their reasoning and/or structure and hence understand how the answers have been arrived at.

The final part of Q12 elicited little more than a few half-efforts at the general situation, but the general reasoning required proved beyond most candidates. A small number of candidates thought they were being asked for an inductive proof of some kind, using parts (i) and (ii) as base-line cases.

Question 13

This attracted the smallest amount of attention: 83 attempts, with a mean score of 5 out of 20. Candidates should recognise the (negative) exponential distribution here, and many candidates did so. It is thus a question about pdfs and cdfs and multiplying the probabilities of independent events. The only commonly scored marks were those for the expectation, gained using integration by parts. Almost no candidates made it to part (ii).