STEP 3 2015 Examiners' report

A very similar number of candidates to 2014 once again ensured that all questions received a decent number of attempts, with seven questions being very popular rather than five being so in 2014, but the most popular questions were attempted by percentages in the 80s rather than 90s. All but one question was answered perfectly at least once, the one exception receiving a number of very close to perfect solutions. About 70% attempted at least six questions, and in those cases where more than six were attempted, the extra attempts were usually fairly superficial

1. This was the most popular question, being attempted by 85% of candidates, it was however only moderately successful although a number achieved full marks. Quite often, candidates ignored the helpful approach suggested by the LHS of the first required result, though, of course, it was possible to start from the first defined integral and achieve the same result. Many needlessly lost marks through omitting fairly straightforward steps such as the final evaluation in the last part of the question and failing to substantiate the simplified form of the result of part (i). Some got very carried away with tan or sinh substitutions in part (i), usually unsuccessfully and leading to monstrous amounts of algebraic working. A few failed to change the limits of integration in [part (ii).

2. Nearly three quarters attempted this, though again with moderate success as the main feature of the question was proof, and this was frequently handled cavalierly. Whilst it was not a crucial aspect of the question, ignoring the fact that the question deals with sequences of positive numbers was careless. Answers to the first part suffered at times from lack of argument or backwards logic. Part (ii) was generally well answered, although there were some silly counter-examples. This part suffered from those who completely missed the point of what the question was all about, forgetting the initial definition. Whilst most appreciated that part (iv) was true, there were many different methods used to attempt to prove it, and often unsuccessfully. Whilst induction using algebra is fairly straightforward, differentiation with or without logarithms and graphical methods frequently came to grief.

3. Under 20% attempted this, making it the least popular Pure question on the paper, and it was the least successfully attempted of all questions on the paper. Candidates seemed to find it intimidating, and many gave up before part (ii). They often got confused when dealing with separate cases and did not seem to understand what was required to show $\sec\theta > 0$ in part (i). Those that did make a stab at (ii) usually omitted a factor of two and most failed to find the correct limit to use.

4. Along with questions 5 and 7, attempted by just over three quarters, this was the third most popular question, though a little less successful than the most popular question 1. The first part was frequently not well attempted, but the second part was usually mastered. Attempts at the third part suffered from arguments with poor logical structure, though many did not get a start on this part.

5. Marginally less successful than question 2, a lot of candidates earned about half of the marks. Unfortunately, many candidates approached this on the basis of their knowledge of the standard irrationality proof for root two employing rational numbers expressed in lowest terms rather than observing the specified argument. In part (i), proving step 5 was frequently beset with omissions, and simple steps like $0 < \sqrt{2} - 1 < 1$ were not acknowledged let alone justified. The first result of part (ii) caused few problems except to those that did not appreciate 'if and only if', but defining a suitable set in order to construct a similar argument to prove the irrationality of the cube roots of 2 and 2 squared was beyond most leading to mostly spurious logic.

6. About three fifths of the candidates attempted this question but without great success. The first part tripped up many through needing to prove 'if and only if'. The first part of (ii) yielded good

scoring opportunities for those that did make progress on this question, though some fell by the wayside when it came to the situation that would not generate there possible values. Some attempts at the last result failed as the counter-example was not always shown to be a counter-example.

7. This was as successful as question 2 and so was third equal most popular and second equal most successful. Usually the very first result was comfortably answered, but there were many flaws in part (i) as many could not carry out a proper formal induction. In part (ii), which saw a lot fall by the wayside, some candidates thought that x commutes with $\frac{d}{dx}$, and often, candidates invented random formulae for $D^n(1-x)^m$ from looking at the first few cases. Not surprisingly, working towards a given result, many came up with the correct result, but through spurious working such as substituting $x = 1$ in $(1-x)^m$ before using the differential operator.

8. This was a little less popular than question 1, but still was attempted by more than 80% of candidates, and was the question with highest scores. Many managed part (i) although several candidates did not realise that r was a function of θ. Part (ii) was generally fine as far as the transformed differential equation but then the correct use of partial fractions to integrate having separated variables was less frequent than it should have been. A surprising number made no attempt to sketch any solutions despite doing the rest of the question either well or perfectly. Nobody realised that the constant A was truly arbitrary in part (ii) because of the modulus signs appearing in the log terms form the integral. The sketches tested all but the best.

9. Just over 20% attempted this, making it the most popular non-Pure question, and attempts at it were slightly less successful than question 1. Quite a few found the first required equation from applying Newton's 2nd Law, when some made sign errors through not being careful with directions, and then integrating rather than from conserving energy. x_0 was found easily by the majority, and the expression for the acceleration was commonly still by Newton, though a lot of marks were lost by not substituting in $x = x_0$. The last part was poorly done in general with few getting more than an opening line, and if they made a sensible substitution, very few expanded it correctly. Not many even attempted the last part of the question.

10. Whilst this was the least popular question with just over 8% attempting it, it was only slightly less successfully attempted than question 5, though those making substantial attempts at it invariably scored half to two thirds of the marks comfortably. Most successfully wrote the position vector of one of the particles and then differentiated with respect to time to obtain the velocity correctly, though a few succeeded by adding velocities. The second displayed equation was almost always correctly derived, though many did far too much work obtaining the corresponding equations for the other particle when it could just be written down. Deducing $\ddot{x}$ and $\ddot{y}$ was fine, but $\ddot{\theta}$ frequently wasn't. At this point, finding initial values for $\dot{y}$ and $\dot{\theta}$ caused some issues, if it was realised that these were needed, and although many wrote the uniform acceleration equation for the displacement of the midpoint of the rod, the final result eluded many.

11. Marginally more popular than the Probability and Statistics questions, this was attempted by just over 10% with, on average, slightly less success than question 4, though students either got almost full marks or virtually none. Most did part (i) correctly except the final part, identifying the force from the hinge. The most common mistake was in part (ii) by those that assumed that there were no perpendicular forces acting on m_1 and m_2. Students that correctly considered the total moment for part (ii) obtained the answer. Some students got the direction of centripetal force wrong.

12. The two Probability and Statistics questions were equally popular being attempted by about 10% of the candidates with, overall, this one achieving the same sort of scores as question 6. About a fifth of the candidates attempting it got right through the question. Most however did not seem to know what a probability generating function was, and it was often confused with the probability density function. Equally there was confusion between the labels of the random variables and of the PGFs. However most were happy working with the arithmetic congruent to moduli.

13. The large majority of attempts got almost no marks, and as a consequence this was the second worst scoring question. A lot failed to draw the right sort of graph to attempt the first part of (i) or, if they did, frequently miscalculated the area to find $P(X+Y<t)$ in the case $1<t\leq 2$. The next major problem was an inability to see how to find the cumulative distribution function of $(X+Y)^{-1}$. A surprisingly large number failed to multiply by t before integrating to find the expectation. A handful of candidates got most of the question right although only one made it clear with a symmetry argument why they could write down $E\left(\frac{X}{X+Y}\right)=\frac{1}{2}$.