9475 - MATHEMATICS III

Report for Publication to Centres

General Comments

Almost all candidates this year chose to answer questions 2, 3, 5, 6 and 7, together with question 1 or 4. Many, usually the weaker candidates, tackled more than six questions, though most of these were usually incomplete, or even barely started. Some good candidates concentrated on very full and thorough answers to fewer than six questions. Few attempts were made at the Mechanics questions and even fewer at the Probability and Statistics questions.

Some very impressive work was again seen from the best candidates, and this year there were far fewer candidates who were essentially unable to answer any of the questions. Those doing less well are typically candidates who can do effectively what they are explicitly told to but are unable to make progress where the method is not provided by the question or those who can see what do, but who are hampered by poor technical skills, especially in algebra, but also in trigonometry or calculus.

Lack of clarity about the direction of implications is often a weakness, even of good candidates: for instance (in question 1) $\cos B = \sin\left(\dfrac{\pi}{2} - B\right)$ so $\sin A = \cos B \quad \Rightarrow \quad A = \dfrac{\pi}{2} - B$, but cos is an even function so $A = \dfrac{\pi}{2} \pm B$. Many are not clear what is required to "show" a result: either sufficient working to indicate the steps taken or a written explanation of what is going on are essential. For instance, it was common to see (in question 2)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2 + y^2\right) = 2x\left(1 - \frac{c^2}{\left(x^2 + a^2\right)^2}\right) \quad \text{so} \quad \frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(x^2 + y^2\right) = 2\left(1 - \frac{c^2}{\left(x^2 + a^2\right)^2}\right) + \frac{8c^2x^2}{\left(x^2 + a^2\right)^3}$$

where candidates were asked to show the latter result, which is on the question paper: of course, it is easy to reconstruct the steps in the differentiation, but that is what the question is asking for.

Few candidates seem prepared to check their work, or to go back to look for obvious errors. It was common to see the plaintive remark "I must have made a slip somewhere" at the end of a derivation that failed to give the expected answer, where the error was a simple as the mistranscribing from one line to the next of a negative sign as a positive sign.

Comments on Specific Questions

1 This question was surprisingly unpopular, given that it was the first question on the paper – perhaps the sight of unfamiliar trigonometric graphs put candidates off. The first two parts were very poorly done: for the first result almost all only showed that $A = (4n + 1)\dfrac{\pi}{2} \pm B$ was sufficient for sin *A* = cos *B*, while in the second many looked for turning points but assumed without comment that the maximum of the modulus of a function must occur at the maximum of the function. Almost all could then use these results to show that $\sin(\sin x) = \cos(\cos x)$ had no solutions. The first two graphs were usually correctly sketched, though many had cusps on one or other curve, but the graph of $y = \sin(2\sin x)$ was very often attempted without further calculation and almost always then had a maximum at $x = \frac{\pi}{2}$ instead of a minimum there and maxima either side of this. Candidates should be aware that graph sketches in STEP papers will often require considerable working, such as determining turning points and their nature, even if this is not explicitly indicated in the question.

2 This question was attempted by almost all candidates and most managed the early parts successfully, though many used the expression $2 + 2(y')^2 + 2yy''$ for the second derivative of $x^2 + y^2$, which made this part much harder than necessary. Very few than achieved full marks for determining the closest points to the origin on the curve, noticing correctly the existence or otherwise of two turning points of $x^2 + y^2$ other than at x = 0, and showing clearly which points were minima, under the two conditions on *a* and *c*.

3 Almost all candidates attempted this question, and could complete the algebra correctly; most could also solve the quartic equation in the last part using one of the ideas from the earlier parts. Relatively few, however, understood what was meant by a necessary condition on *a*, *b* and *c* and simply gave formulae for these in terms of *p, r* and *s*, while virtually no-one, even among those who found that $a^3 + 8c = 4ab$ was necessary, could establish the sufficiency of this condition.

4 This question was one of the less popular Pure questions. There seemed to be a fairly sharp divide between the many candidates who dealt very effectively with the induction and the many who were unsure what the induction hypothesis was or what was required for the base case or who went round in circles with the recurrences. In the last part, most were able to find the required conditions successfully.

5 Almost all attempted this question and found the first two parts straightforward. Most (apart from the significant number who did not understand the phrase 'common tangent') could also identify the discriminant condition for there to be only one tangent. Disappointingly few, however, could link this accurately to the condition for the two curves to touch. The last part was almost always poorly done, with candidates either continuing to use the discriminant, which is not appropriate in this case, since the equation is linear, or being unable to state clearly what the condition is for a linear equation to have exactly one solution.

6 This question was tackled by most candidates. Almost all could do the first part; most could show that $u + \frac{b}{u}$ is a root of the equation, by a variety of methods, and relate their results to the final equation. Fewer could convincingly establish the quadratic satisfied by the other two roots of the cubic, and startlingly few could accurately solve this quadratic to get roots in terms of ω with, in particular, very many sign errors.

7 This question was popular and almost all could obtain the general result quoted, with most being able to go on to use this result on the given integrals. Many could then complete the integration for (i), but very few knew how to tackle the integral $\int \frac{\mathrm{d}u}{u\sqrt{u+1}}$ in (ii). Candidates (perhaps because this was the most recent technique they had learnt) almost all saw this as an opportunity to substitute $u = \sinh^2 t$ or $u = \tan^2 t$, which will work in principle, but are not as simple to execute correctly as $u = t^2 - 1$. Many perfectly satisfactory methods of integration not based on the general result given were also seen.

8 This was easily the least popular Pure question, though still attempted more frequently than any of the applied questions. Solutions getting beyond the first couple of parts successfully were extremely rare, with many not recognising $|a - c|$ as the radius of the circle, and hardly any being able to use the result $2aa^* = ac^* + ca^*$ to show that the conditions for B and B' to lie on the circle were equivalent, let alone the converse.

9 This was the most popular Mechanics question and most of those who attempted it seemed to know what to do, though those who had a formulaic approach to Newton's Law of restitution, writing it as $-e = \dfrac{\text{difference of final velocities}}{\text{difference of initial velocities}}$, frequently made at least one sign error in using this equation, and there were many who did not make a consistent decision about the sign convention for the final velocities, for instance by drawing a diagram. However, virtually all were defeated by the algebra required – it was very common indeed, for instance, to see the expression $(1-e)^2$ miscopied as $(1-e^2)$ at some point in the calculation, or vice versa. Unfortunately, the problem was unforgiving about this, and incorrect early results made it very difficult to complete the question successfully.

10 There were a reasonable number of attempts at this question, with some good efficient solutions, but with most candidates giving up when they did not get the required result in (i). This was usually because they had included the work done by friction on only one disc, or because they had not realised that the extension in the band is twice the distance apart of the centres, or because they assumed that there would be no elastic potential energy stored in the band if the two discs were in contact, or some combination of these.

11 This question was rarely attempted. There were a few good solutions, but most could only tackle the first part – distinguishing the two equilibria by their stability (for example by finding the value of $\sin\theta$ and $\cos\theta$ at each) was found difficult, and consequently the last part was inaccessible.

12 There were very few attempts at this question and few of these progressed beyond the first part, the result $\mathrm{E}\left[Y^2\right] = \mathrm{Var}\left[Y\right] - \mathrm{E}\left[Y\right]^2$ not being recognised as useful.

13 This was the only Probability and Statistics question attempted by more than a handful of candidates, but was still only tackled by a small minority. In the first part, most could derive a correct expression for the probability that the player wins exactly £r, but hardly any could either sum the series for the expectation or, alternatively, spot the connection with a Geometric random variable. Many did not then proceed to the second part, but those who did often found it more straightforward.

14 There were hardly any attempts at this question: most of these could successfully get to the expected value of V, but were unable to use the asterisked result to find the density function.