STEP I 2016 MARK SCHEME

Question 1 (i)

B1 for at least 3 of $q_1(x) = \dfrac{x^3+1}{x+1}$, $q_2(x) = \dfrac{x^5+1}{x+1}$, $q_3(x) = \dfrac{x^7+1}{x+1}$, $q_4(x) = \dfrac{x^9+1}{x+1}$ correct

M1 A1 for $p_1(x) = (x^2+2x+1)-3x(1)$**;** $= x^2-x+1 \equiv q_1(x)$

M1 A1 for $p_2(x) = (x^4+4x^3+6x^2+4x+1)-5x(x^2+x+1)$**;** $= x^4-x^3+x^2-x+1 \equiv q_2(x)$

M1 for attempt at binomial expansion of $(x+1)^6$ and squaring (x^2+x+1)

A1 for $(x+1)^6 = x^6+6x^5+15x^4+20x^3+15x^2+6x+1$

A1 for $(x^2+x+1)^2 = x^4+2x^3+3x^2+2x+1$

A1 for $p_3(x) = x^6-x^5+x^4-x^3+x^2-x+1 \equiv q_3(x)$ shown legitimately ⑨

M1 for valid method to show $p_4(x) \not\equiv q_4(x)$

Method I: $p_4(x) = x^8-x^7+x^6+2x^5+7x^4+2x^3+x^2-x+1$
while $q_4(x) = x^8-x^7+x^6-x^5+x^4-x^3+x^2-x+1$

Method II: partial expansion showing one pair of coefficients not equal

Method III: e.g. $p_4(1) = 2^8 - 9.1.3^3 = 13 \neq q_4(1) = \dfrac{1^9+1}{1+1} = 1$

A1 A1 for correct/valid partial working**;** completely and correctly concluded ③

Question 1 (ii) (a)

M1 M1 A1 for use of $p_1(300) = q_1(300)$**;** use of difference-of-two-squares factorisation**;** 271×331 ③

Question 1 (ii) (b)

M1 for use of $p_3\left(7^7\right) = q_3\left(7^7\right)$

M1 for identifying squares: $\left[\left(7^7+1\right)^3\right]^2 - 7^8\left(7^{14}+7^7+1\right)^2$

M1 for use of difference-of-two-squares factorisation

A1 A1 $\left[\left(7^7+1\right)^3-\left(7^{18}+7^{11}+7^4\right)\right] \times \left[\left(7^7+1\right)^3+\left(7^{18}+7^{11}+7^4\right)\right]$

or $\left(7^{21}+3.7^{14}+3.7^7+1-7^{18}-7^{11}-7^4\right) \times \left(7^{21}+3.7^{14}+3.7^7+1+7^{18}+7^{11}+7^4\right)$ ⑤

Question 2

For $y = (ax^2 + bx + c)\ln\left(x + \sqrt{1+x^2}\right) + (dx+e)\sqrt{1+x^2}$

M1 use of *Product Rule* twice

M1 A1 use of *Chain Rule* in 1st product for the log. term (allow correct unsimplified here)

$$\frac{dy}{dx} = (ax^2+bx+c)\frac{1}{x+\sqrt{1+x^2}} \times \left(1 + \tfrac{1}{2}\left[1+x^2\right]^{-\frac{1}{2}}.2x\right) + (2ax+b)\ln\left(x+\sqrt{1+x^2}\right)$$

M1 A1 use of *Chain Rule* in 2nd product (allow correct unsimplified here)

$$+\ (dx+e)\left(\tfrac{1}{2}\left[1+x^2\right]^{-\frac{1}{2}}.2x\right) + d\sqrt{1+x^2}$$

$$\frac{dy}{dx} = \frac{ax^2+bx+c}{\left[x+\sqrt{1+x^2}\right]} \times \frac{\left[\sqrt{1+x^2}+x\right]}{\sqrt{1+x^2}} + (2ax+b)\ln\left(x+\sqrt{1+x^2}\right) + \frac{x(dx+e)}{\sqrt{1+x^2}} + d\sqrt{1+x^2}$$

M1 cancelling the [-] terms

A1 A1 one mark for each term, correct and simplified

$$\frac{dy}{dx} = \frac{(a+2d)x^2 + (b+e)x + (c+d)}{\sqrt{1+x^2}} + (2ax+b)\ln\left(x+\sqrt{1+x^2}\right)$$ ⑧

Question 2 (i)

M1 A1 A1 for choosing $a = d = 0$, $b = 1$, $e = -1$ and $c = 0$ so that

$$\frac{dy}{dx} = \frac{(0)x^2 + (0)x + (0)}{\sqrt{1+x^2}} + (0+1)\ln\left(x+\sqrt{1+x^2}\right)$$

A1 and $\int \ln\left(x+\sqrt{1+x^2}\right)dx = x\ln\left(x+\sqrt{1+x^2}\right) - \sqrt{1+x^2}$ $(+\ C)$ clearly stated ④

Question 2 (ii)

M1 A1 A1 for choosing $a = b = e = 0$ and $c = d = \frac{1}{2}$ so that

$$\frac{dy}{dx} = \frac{(0+1)x^2 + (0)x + (1)}{\sqrt{1+x^2}} + (0+0)\ln\left(x+\sqrt{1+x^2}\right)$$

A1 and $\int \sqrt{1+x^2}\,dx = \frac{1}{2}\ln\left(x+\sqrt{1+x^2}\right) + \frac{1}{2}x\sqrt{1+x^2}$ $(+\ C)$ clearly stated ④

Question 2 (iii)

M1 A1 A1 for choosing $a = \frac{1}{2}$, $b = e = 0$ and $c = \frac{1}{4}$ and $d = -\frac{1}{4}$ so that

$$\frac{dy}{dx} = \frac{(\frac{1}{2}-\frac{1}{2})x^2 + (0)x + (\frac{1}{4}-\frac{1}{4})}{\sqrt{1+x^2}} + (x+0)\ln\left(x+\sqrt{1+x^2}\right)$$

A1 and $\int x\ln\left(x+\sqrt{1+x^2}\right)dx = \left(\frac{1}{2}x^2 + \frac{1}{4}\right)\ln\left(x+\sqrt{1+x^2}\right) - \frac{1}{4}x\sqrt{1+x^2}$ $(+\ C)$ clearly stated ④

Alternative: results for (i) and (ii) enable (iii) to be done using *Integration by Parts*:

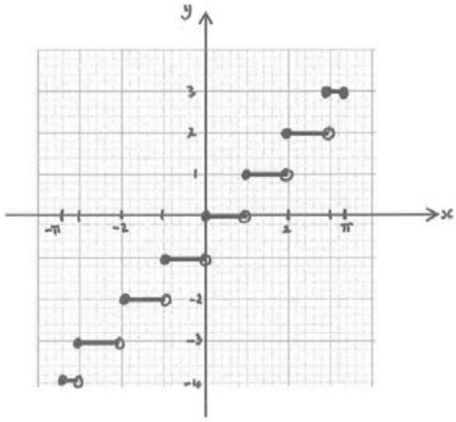
$$I_3 = \int x\,.\ln\left(x+\sqrt{1+x^2}\right)dx$$

$$= x\left\{x\ln\left(x+\sqrt{1+x^2}\right) - \sqrt{1+x^2}\right\} - \int 1.\left\{\ln\left(x+\sqrt{1+x^2}\right) - \sqrt{1+x^2}\right\}$$ **M1 A1**

$$= x^2\ln\left(x+\sqrt{1+x^2}\right) - x\sqrt{1+x^2} - I_3 + \text{(ii)}$$

M1 for turning it round, collecting I_3's etc. **A1** for final answer (**FT** (ii))

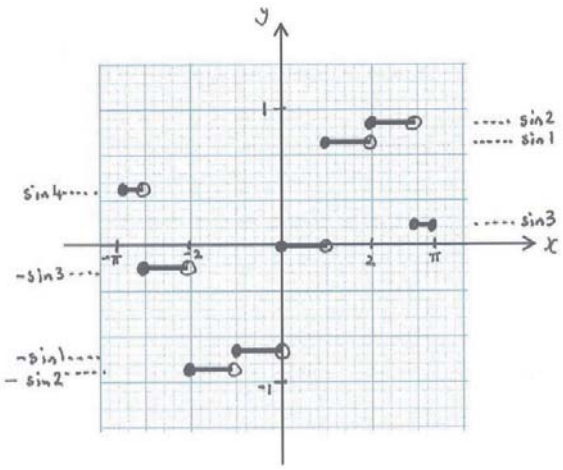
Question 3 (i)



M1 for steps
A1 y-values change at integer x-values
A1 y-values at unit heights
A1 LH ●s and RH ○s correct
(ignoring 2 at ends)
A1 for very LH & RH bits correct

⑤

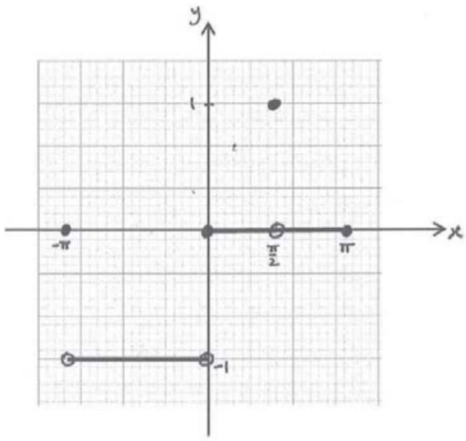
Question 3 (ii)



M1 for steps
A1 y-values change at integer x-values
A1 y-values at sin(k's), $k \in \mathbb{Z}$
A1 LH ●s and RH ○s correct
(ignoring 2 at ends)
A1 for very LH & RH bits correct

⑤

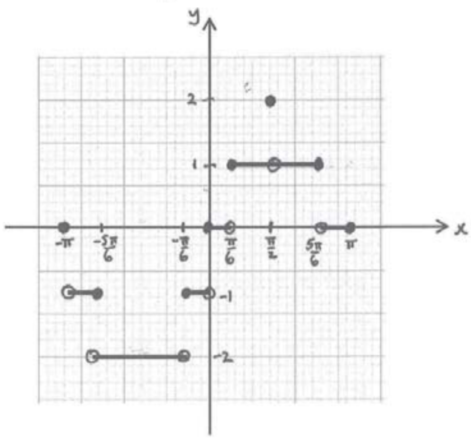
Question 3 (iii)



M1 A1 for two main steps; endpoints in right places
A1 for all endpoints correct in these two lines
B1 for ● at $\left(\frac{1}{2}\pi, 1\right)$ with clear ○ in line below
B1 for ● at $(-\pi, 0)$

⑤

Question 3 (iv)



M1 for steps at integer y-values
A1 essentially correct domains (ignoring ●s and ○s)
A1 for all lines' endpoints correct
B1 for ● at $\left(\frac{1}{2}\pi, 2\right)$ with clear ○ in line below
B1 for ● at $(-\pi, 0)$

⑤

Question 4 (i)

M1 use of *Quotient Rule* (or equivalent) on $y = \dfrac{z}{\sqrt{1+z^2}}$

A1 for correct use of *Chain Rule* for the diffl. of the denominator

$$\frac{\mathrm{d}y}{\mathrm{d}z} = \frac{\sqrt{1+z^2}.1 - z.\tfrac{1}{2}\left(1+z^2\right)^{-\frac{1}{2}}.2z}{\left(\sqrt{1+z^2}\right)^2}$$

A1 all correct and simplified: $\dfrac{1}{\left(1+z^2\right)^{\frac{3}{2}}}$ ③

Question 4 (ii)

M1 for using $z = \dfrac{\mathrm{d}y}{\mathrm{d}x}$ in $\dfrac{\left(\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right)}{\left\{1+\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right\}^{\frac{3}{2}}} = \kappa$ to get $\dfrac{\dfrac{\mathrm{d}z}{\mathrm{d}x}}{\left(1+z^2\right)^{\frac{3}{2}}} = \kappa$

M1 A1 for separating variables; correctly: $\displaystyle\int \frac{\mathrm{d}z}{\left(1+z^2\right)^{\frac{3}{2}}} = \int \kappa \, \mathrm{d}x$

A1 for correct integration using (i)'s result: $\dfrac{z}{\sqrt{1+z^2}} = \kappa(x+c)$ (+ c in any form)

M1 for re-arranging for z or z^2 : $z^2 = \kappa^2(x+c)^2(z^2+1) \Rightarrow \ldots$

A1 correct: $z = \pm\dfrac{u}{\sqrt{1-u^2}}$, $u = \kappa(x + c)$, any correct form (ignore lack of $\pm$ throughout) ⑥

M1 for attempt at $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}u}.\dfrac{\mathrm{d}u}{\mathrm{d}x}$

M1 A1 for use of the *Chain Rule* (e.g.) with $\dfrac{\mathrm{d}u}{\mathrm{d}x} = \kappa$; correct diffl. eqn. $\kappa\dfrac{\mathrm{d}y}{\mathrm{d}u} = \pm\dfrac{u}{\sqrt{1-u^2}}$ ③

M1 for separating variables: $\displaystyle\int \kappa \, \mathrm{d}y = \pm\int \frac{u}{\sqrt{1-u^2}} \mathrm{d}u$

M1 M1 A1 for method to integrate $\displaystyle\int \frac{u}{\sqrt{1-u^2}} \mathrm{d}u = -\sqrt{1-u^2}$

(by "recognition", "reverse chain rule" or substitution) ④

M1 for integrating and substituting for u : $\kappa\ y + d = \mp\sqrt{1-\kappa^2(x+c)^2}$

M1 A1 for working towards a circle eqn. : $(\kappa y + d)^2 = 1 - \kappa^2(x+c)^2$ or $\left(y + \dfrac{d}{\kappa}\right)^2 + (x+c)^2 = \left(\dfrac{1}{\kappa}\right)^2$

B1 for noting that radius of circle is the reciprocal of the curvature ④

Question 5 (i)

M1 for attempt at any of *PR*, *PQ*, *QR* using *Pythagoras' Theorem*

$PR = PQ + QR \Rightarrow \sqrt{(a+c)^2-(a-c)^2} = \sqrt{(b+a)^2-(b-a)^2} + \sqrt{(c+b)^2-(c-b)^2}$

A1 A1 A1 for correct, simplified lengths: $\sqrt{4ac} = \sqrt{4ab} + \sqrt{4bc}$

A1 given answer legitimately obtained by dividing by $\sqrt{4abc}$: $\dfrac{1}{\sqrt{b}} = \dfrac{1}{\sqrt{c}} + \dfrac{1}{\sqrt{a}}$ ⑤

M1 M1 for working suitably on RHS of (*); substituting for *b*, e.g.

$$\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)^2 = \left(\frac{1}{a}+\left\{\frac{1}{a}+\frac{2}{\sqrt{ac}}+\frac{1}{c}\right\}+\frac{1}{c}\right)^2$$

A1 $= 4\left(\dfrac{1}{a^2}+\dfrac{3}{ac}+\dfrac{1}{c^2}+\dfrac{2}{a\sqrt{ac}}+\dfrac{2}{c\sqrt{ac}}\right)$ any form suitable for comparison

M1 for working suitably on LHS of (*) *and* substituting for b^2, e.g.

A1 for correct b^2 in $2\left(\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{c^2}\right) = \dfrac{2}{a^2}+\dfrac{2}{c^2}+2\left(\dfrac{1}{a^2}+\dfrac{4}{a\sqrt{ac}}+\dfrac{6}{ac}+\dfrac{4}{c\sqrt{ac}}+\dfrac{1}{c^2}\right)$

A1 shown equal to RHS: $= \dfrac{4}{a^2}+\dfrac{12}{ac}+\dfrac{4}{c^2}+\dfrac{8}{a\sqrt{ac}}+\dfrac{8}{c\sqrt{ac}}$

⑥

Alternative: $\dfrac{1}{\sqrt{b}} = \dfrac{1}{\sqrt{c}} + \dfrac{1}{\sqrt{a}} \Rightarrow \dfrac{1}{b} = \dfrac{1}{a} + \dfrac{2}{\sqrt{ac}} + \dfrac{1}{c}$ **M1** squaring

$\Rightarrow \left(\dfrac{1}{b}-\dfrac{1}{a}-\dfrac{1}{c}\right)^2 = \dfrac{4}{ac}$ **M1 M1** rearranging and squaring again

$\Rightarrow \dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{c^2}-\dfrac{2}{ab}-\dfrac{2}{bc}+\dfrac{2}{ac} = \dfrac{4}{ac}$ **A1** correct LHS

$\Rightarrow 2\left(\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{c^2}\right) = \dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{c^2}+\dfrac{2}{ab}+\dfrac{2}{bc}+\dfrac{2}{ca} = \left(\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}\right)^2$ **M1 A1**

Question 5 (ii)

M1 If $2\left(\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{c^2}\right) = \left(\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}\right)^2$ then $\dfrac{1}{a^2}+\dfrac{1}{b^2}+\dfrac{1}{c^2} = \dfrac{2}{ab}+\dfrac{2}{bc}+\dfrac{2}{ca}$

M1 Let $x = \dfrac{1}{\sqrt{a}}$, $y = \dfrac{1}{b}$, $z = \dfrac{1}{\sqrt{c}}$ with or without actual substitution

so that $x^4+y^4+z^4 = 2x^2y^2+2y^2z^2+2z^2x^2$

M1 for recognition of conditions $b < c < a \Rightarrow y > z > x$

M1 A1 for completing the square: $\left(x^2+z^2-y^2\right)^2 = 4x^2z^2$

A1 $\Leftrightarrow x^2+z^2-y^2 = \pm 2xz$

$\Leftrightarrow \left(z \mp x\right)^2 = y^2$

A1 for the four cases $y = x - z$, $y = z - x$, $y = x + z$ or $y = -x - z$

E1 for use of conditions to *show* that only $y = x + z$ is suitable

A1 for legitimately obtaining given answer: $\dfrac{1}{\sqrt{b}} = \dfrac{1}{\sqrt{c}} + \dfrac{1}{\sqrt{a}}$ ⑨

Question 6

E1 for explanation that $\mathbf{x} = m\mathbf{a}$ since $OX \parallel OA$

B1 for $0 < m < 1$ (since X between O and A): don't penalise any equality interval endpoints

E1 for explanation that $BC \parallel OA \Rightarrow \mathbf{c} - \mathbf{b} = k\mathbf{a}$ and so $\mathbf{c} = k\mathbf{a} + \mathbf{b}$

B1 for $k < 0$ (since BC in opposite direction to OA) ④

B1 for correct set-up for $D = OB \cap AC$: $\mathbf{a} + \alpha(\mathbf{c} - \mathbf{a}) = \beta\mathbf{b}$

M1 for equating coefficients: $1 - \alpha + \alpha k = 0$ and $\alpha = \beta \left(= \dfrac{1}{1-k} \right)$

A1 for $\mathbf{d} = \dfrac{1}{1-k}\mathbf{b}$ ③

B1 for correct set-up for $Y = XD \cap BC$: $m\mathbf{a} + \alpha\left(\dfrac{1}{1-k}\mathbf{b} - m\mathbf{a} \right) = \mathbf{b} + \beta k\,\mathbf{a}$

M1 for equating coefficients: $m - \alpha m - \beta k = 0$ and $\dfrac{\alpha}{1-k} = 1$

A1 for $\mathbf{y} = km\mathbf{a} + \mathbf{b}$ from $\alpha = 1 - k,\ \beta = m$ ③

B1 for correct set-up for $Z = OY \cap AB$: $(1 - \alpha)\mathbf{a} + \alpha\mathbf{b} = \beta(km\mathbf{a} + \mathbf{b})$

M1 for equating coefficients: $1 - \alpha - km\beta = 0$ and $\alpha = \beta \left(= \dfrac{1}{1+km} \right)$

A1 for $\mathbf{z} = \left(\dfrac{km}{1+km} \right)\mathbf{a} + \left(\dfrac{1}{1+km} \right)\mathbf{b}$ (**Given Answer**) ③

B1 for correct set-up for $T = DZ \cap OA$: $\alpha\mathbf{a} = \dfrac{1}{1-k}\mathbf{b} + \beta\left(\dfrac{km}{1+km}\mathbf{a} + \dfrac{1}{1+km}\mathbf{b} - \dfrac{1}{1-k}\mathbf{b} \right)$

M1 for equating coefficients: $\alpha = \dfrac{\beta km}{1+km}$ and $0 = \dfrac{1-\beta}{1-k} + \dfrac{\beta}{1+km}$

A1 for $\mathbf{t} = \left(\dfrac{m}{1+m} \right)\mathbf{a}$ from $\alpha = \dfrac{m}{1+m}$, $\beta = \dfrac{1+km}{k(1+m)}$ ③

M1 A1 for setting up all lengths: $OA = a$, $OX = ma$, $OT = \left(\dfrac{m}{1+m} \right)a$,

$$TX = \left(\frac{m^2}{1+m} \right)a,\ TA = \left(\frac{1}{1+m} \right)a,\ XA = (1 - m)a$$

where $|\mathbf{a}| = a$, which may (w.l.o.g.) be taken to be 1

A1 for 1st correctly derived result: $\dfrac{1}{OT} = \dfrac{1}{a}\left(1 + \dfrac{1}{m} \right) = \dfrac{1}{OA} + \dfrac{1}{OX}$

A1 for 2nd correctly derived result: $OT \,.\, OA = \left(\dfrac{m}{1+m} \right)a^2 = (ma) \,.\, \left(\dfrac{1}{1+m} \right)a = OX \,.\, TA$ ④

Question 7 (i)

B1 B1 for $S \cap T = \phi$; $S \cup T$ = the set of positive odd numbers ②

Question 7 (ii)

M1 A1 for $(4a + 1)(4b + 1) = 4(4ab + a + b) + 1$ (which is in S)

M1 A1 for $(4a + 3)(4b + 3) = 4(4ab + 3a + 3b + 2) + 1$ (not necessarily as shown here)

A1 for clearly demonstrating this is **not** in T ⑤

Question 7 (iii)

M1 M1 for attempting a proof by contradiction; method for establishing contradiction

Suppose all of t's prime factors are in S

B1 for no even factors

$$t = (4a + 1)\,(4b + 1)\,(4c + 1)\ \ldots(4n + 1)$$

A1 Then $t = 4\{\ \ldots\ \ldots\ \ldots\} + 1$

E1 for convincing explanation that this is always in S

(may appeal inductively to (ii)'s result) ⑤

Question 7 (iv) (a)

B1 for writing an element of T as products of T-primes

M1 for noting that every pair of factors in T multiply to give an element of S (by (ii))

A1 so there must be an odd number of them ③

Question 7 (iv) (b)

M1 for recognisable method to find composites in S whose prime-factors are in T

M1 for recognition of the regrouping process

M1 A1 for correct example demonstrated:

e.g. $9 \times 77 = 21 \times 33$ (= 693) where 9, 21, 33, 77 are in S

and $9 = 3 \times 3$, $21 = 3 \times 7$, $33 = 3 \times 11$, $77 = 7 \times 11$ with 3, 7, 11 in T

B1 for correctly-chosen S-primes ⑤

Question 8 (i)

B1 for $f(x) = 0 + x + 2x^2 + 3x^3 + \ldots + nx^n + \ldots$

M1 for use of $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots + nx^{n-1} + \ldots$ (forwards or backwards)

A1 for **given** result correctly shown: $f(x) = x(1-x)^{-2}$ ③

M1 A1 for $x(1-x)^{-3} = x\left(1 + 3x + 6x^2 + 10x^3 + \ldots + \frac{1}{2}n(n+1)x^{n-1} + \ldots\right)$

$= 0 + x + 3x^2 + 6x^3 + \ldots + \frac{1}{2}n(n+1)x^n + \ldots$

A1 for $u_n = \frac{1}{2}n^2 + \frac{1}{2}n$ ③

M1 A1 for use of first two results: $2 \times (2^{\text{nd}}) - (1^{\text{st}})$ gives $\frac{2x}{(1-x)^3} - \frac{x}{(1-x)^2}$ with $u_n = n^2$ ②

Question 8 (ii) (a)

Method I: **B1** for $f(x) = a + (ka)x + (k^2a)x^2 + (k^3a)x^3 + \ldots + (k^na)x^n + \ldots$

M1 A1 for $a \times$ sum-to-infinity of a GP with common ratio kx : $f(x) = a\left(\frac{1}{1-kx}\right)$

B1 for showing (retrospectively) that $f(x) = a + kx\, f(x)$

Method II: **B1** for $f(x) = a + akx + ak^2x^2 + ak^3x^3 + \ldots + ak^nx^n + \ldots$

M1 $= a + kx\left(a + akx + ak^2x^2 + ak^3x^3 + \ldots + ak^nx^n + \ldots\right)$

A1 $= a + kx\, f(x)$

A1 for $f(x) = a\left(\frac{1}{1-kx}\right)$ ④

Question 8 (ii) (b)

M1 A1 for summing, and splitting off initial terms: $f(x) = 0 + x + \sum_{n=2}^{\infty} u_n x^n$

M1 for use of given recurrence relation: $= 0 + x + \sum_{n=2}^{\infty}\left(u_{n-1} + u_{n-2}\right)x^n$

M1 for dealing with limits: $= x + x\sum_{n=2}^{\infty} u_{n-1}x^{n-1} + x^2\sum_{n=2}^{\infty} u_{n-2}x^{n-2}$

A1 for re-creating f(x)'s: $= x + x\sum_{n=1}^{\infty} u_n x^n + x^2\sum_{n=0}^{\infty} u_n x^n$

A1 for correctly expressing all terms in f(x): $= x + x\{f(x) - 0\} + x^2 f(x)$

M1 A1 for re-arranging to get $f(x) = \frac{x}{1-x-x^2}$ ⑧

Question 9

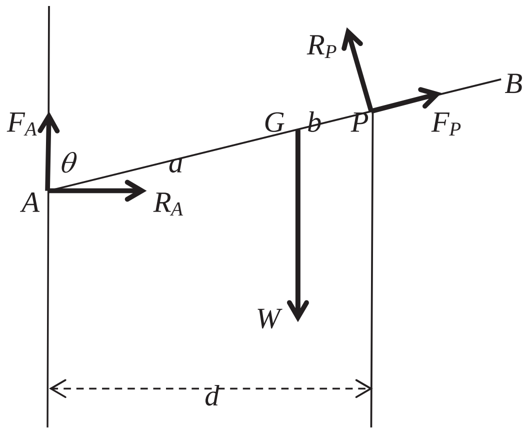


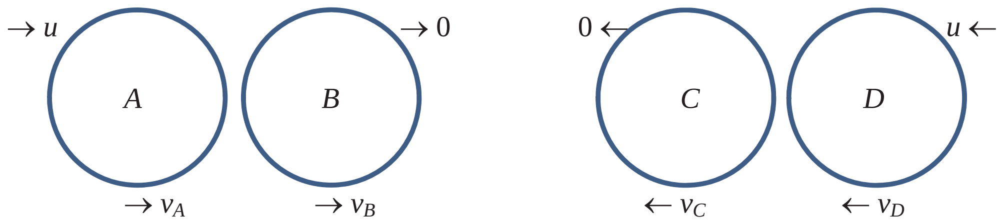
Diagram for Case 1:
G between walls;
rod about to slip down LH wall

B1	for both $F_A = \lambda R_A$ and $F_P = \mu R_P$ noted or used somewhere
M1	for resolving in one direction (with correct number of forces)
A1	e.g. **Res.**$\uparrow$ $W = F_A + R_P \sin\theta + F_P \cos\theta$
M1	for eliminating the F's (e.g.): $W = \lambda R_A + R_P \sin\theta + \mu R_P \cos\theta$
M1	for resolving in second direction (with correct number of forces)
A1	e.g. **Res.**$\rightarrow$ $R_A = R_P \cos\theta - F_P \sin\theta$
M1	for eliminating the F's (e.g.): $R_A = R_P \cos\theta - \mu R_P \sin\theta$
M1	for taking moments (with correct number of forces)
A1	e.g. $\circlearrowleft A$ $W a \sin\theta = R_P (a + b)$
M1	for correct introduction of d: $W a \sin^2\theta = R_P d$ or other suitable distance
M1 A1	for getting W in terms of one other force: e.g. $W = R_P\left(\lambda\cos\theta - \lambda\mu\sin\theta + \sin\theta + \mu\cos\theta\right)$
M1	for eliminating W and that force from two relevant equations: e.g. these last two
A1	for legitimately obtaining **given** result: $d\,\text{cosec}^2\theta = a\left([\lambda+\mu]\cos\theta + [1-\lambda\mu]\sin\theta\right)$ ⑭

For Case 2: G the other side of P; rod about to slide up LH wall …

M1 M1 M1	$F_A \to -F_A$; $F_P \to -F_P$; $a + b \to a - b$ (or switching signs of λ and μ)
A1	$W = R_P\left(-\lambda\cos\theta - \lambda\mu\sin\theta + \sin\theta - \mu\cos\theta\right)$ and $W a \sin^2\theta = R_P d$ (e.g.)
M1 A1	for obtaining $d\,\text{cosec}^2\theta = a\left(-[\lambda+\mu]\cos\theta + [1-\lambda\mu]\sin\theta\right)$ ⑥

Question 10 (i)

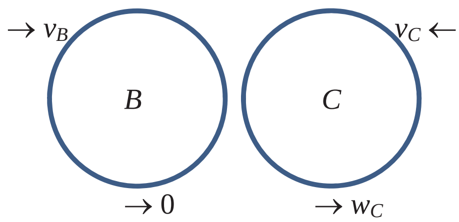


		For collision A/B	For collision C/D	
B1 B1	for CLM statements:	$m(\lambda u = \lambda v_A + v_B)$	$m(u = v_C + v_D)$	
B1 B1	for NEL/NLR statements:	$eu = v_B - v_A$	$eu = v_C - v_D$	

Watch out for different signs from alternative choices of directions

M1 solving for at least v_B and v_C

A1 A1 for $v_B = \dfrac{\lambda(1+e)}{\lambda+1}u$, $v_C = \frac{1}{2}(1+e)u$ NB $v_A = \dfrac{\lambda - e}{\lambda+1}u$ and $v_D = \frac{1}{2}(1-e)u$ not needed 7



M1 A1 A1 for CLM and NEL/NLR statements: $m(v_B - v_C) = m\,w_C$ and $e(v_B + v_C) = w_C$

M1 for substituting previous answers in terms of e and u

M1 A1 for identifying e : $e = \dfrac{\lambda - 1}{3\lambda + 1}$ **Given Answer** legitimately obtained

E1 for justifying that $e < \frac{1}{3}$ (can't just show that $e \rightarrow \frac{1}{3}$) 7

Question 10 (ii)

NB $w_C = \dfrac{(1+e)(\lambda-1)}{2(\lambda+1)}u$ correct from previous bit of work

M1 for setting $w_C = v_D$ in whatever forms they have (not just saying they are equal)

A1 correct to here: $\dfrac{(1+e)(\lambda-1)}{2(\lambda+1)}u = \frac{1}{2}(1-e)u$ **FT** previous answers

M1 for substituting for e (e.g.)

M1 A1 A1 for solving for λ and e : $\lambda = \sqrt{5}+2$, $e = \sqrt{5}-2$ 6

__Question 11__

M1 A1 for stating, or obtaining, the *Trajectory Equation*: $y = x\tan\alpha - \dfrac{gx^2}{2u^2\cos^2\alpha}$

M1 for setting $y = -h$ and re-arranging

$$\frac{gx^2}{u^2} = 2h\cos^2\alpha + 2x\sin\alpha\cos\alpha$$

A1 for legitimately obtaining **given answer** from use of double-angle formulae:

$$\frac{gx^2}{u^2} = h(1+\cos 2\alpha) + x\sin 2\alpha$$ ④

M1 A1 for differentiating w.r.t. α : $\dfrac{\mathrm{d}}{\mathrm{d}\alpha}\left(\dfrac{gx^2}{u^2}\right) = h\left(-2\sin 2\alpha\right) + \left(x.2\cos 2\alpha + \sin 2\alpha.\dfrac{\mathrm{d}x}{\mathrm{d}\alpha}\right)$

M1 for using both derivatives = 0

A1 for legitimately obtaining **given answer** $x = h\tan 2\alpha$ ④

M1 for substituting back: $\dfrac{gh^2\tan^2 2\alpha}{u^2} = h(1+\cos 2\alpha) + h\tan 2\alpha\sin 2\alpha$

M1 cancelling one h and (e.g.) writing all trig. terms in $c = \cos 2\alpha$

A1 $\dfrac{gh\left(1-c^2\right)}{u^2c^2} = 1 + c + \dfrac{1-c^2}{c} \Rightarrow gh - ghc^2 = u^2\left(c^2 + c^3 + c - c^3\right)$

M1 A1 for a quadratic in c : $0 = \left(u^2 + gh\right)c^2 + u^2c - gh$

M1 for solving attempt: $0 = \left[\left(u^2 + gh\right)c - gh\right](c+1)$

A1 for $\cos 2\alpha = \dfrac{gh}{u^2 + gh}$ ⑦

M1 for substituting $x = h\tan 2\alpha$ and $y = -h$ in $\Delta^2 = x^2 + y^2$

M1 A1 for use of relevant trig. result(s) $= h^2\sec^2 2\alpha$ i.e. $\Delta = h\sec 2\alpha$

M1 for use of previous result: $\Delta = h\,.\dfrac{u^2 + gh}{gh}$

A1 $= \dfrac{u^2}{g} + h$ correct **given answer** legitimately obtained ⑤

Question 12 (i)

M1 for some systematic approach to counting cases

A1 A1 A1 for correct cases: e.g. p(A=0).p(B=1,2,3) + p(A=1).p(B =2,3) + p(A=2).p(B=3)

M1 for some correct probabilities: $\frac{1}{4}\times\frac{7}{8}+2\times\frac{1}{4}\times\frac{4}{8}+\frac{1}{4}\times\frac{1}{8}$

A1 for correctly obtained answer, $\frac{1}{2}$

If no other marks scored, B1 for 32 outcomes ⑥

Question 12 (ii)

M1 for some systematic approach to counting cases

A1 A1 A1 for identifying the correct cases and/or probabilities

e.g. $\frac{1}{8}\times\left(\frac{4+6+4+1}{16}\right)+\frac{3}{8}\times\left(\frac{6+4+1}{16}\right)+\frac{3}{8}\times\left(\frac{4+1}{16}\right)+\frac{1}{8}\times\left(\frac{1}{16}\right)$

M1 for all cases/probabilities correct: $\frac{1}{4}\times\frac{7}{8}+2\times\frac{1}{4}\times\frac{4}{8}+\frac{1}{4}\times\frac{1}{8}$

A1 for correctly obtained answer, $\frac{1}{2}$

If no other marks scored, B1 for 128 outcomes ⑥

Question 12 (iii)

B1 for stating that, when each tosses n coins, p(B has more Hs) = p(A has more Hs) = p_2

B1 for stating that p($A_H = B_H$) = p_1

B1 for statement (explained or not) that $p_1 + 2p_2 = 1$

M1 for considering what happens when B tosses the extra coin

A1 p(B has more Hs) = p(B already has more Hs) × p(B gets T)

A1 + p(B already has more, or equal Hs) × p(B gets H)

A1 correct probs. used $= p_2\times\frac{1}{2}+(p_1+p_2)\times\frac{1}{2}$

A1 for correct answer, fully justified: $\frac{1}{2}(p_1+2p_2)=\frac{1}{2}$ ⑧

Question 13 (i)

	For the i-th e-mail,
M1	for integrating $f_i(t) = \lambda e^{-\lambda t}$
A1	for $F_i(t) = -e^{-\lambda t} + C$
M1 A1	for justifying or noting that $C = 1$ (from $F(0) = 0$)
	For n e-mails sent simultaneously,
M1 A1	for $F(t) = P(T \le t) = 1 - P(\text{all } n \text{ take longer than } t)$
B1	for $= 1 - \left(e^{-\lambda t}\right)^n$ i.e. the product of n independent probabilities
A1	for $= 1 - \lambda e^{-\lambda nt}$
M1 A1	for differentiating this: $f(t) = n\lambda e^{-\lambda nt}$ ⑩
M1	for attempt at $E(T) = \int_0^\infty t \times n\lambda e^{-\lambda nt}\, dt$
M1 A1 A1	for use of integration by parts: $E(T) = \left[-te^{-\lambda nt}\right]_0^\infty + \int_0^\infty n\lambda e^{-\lambda nt}\, dt$
A1	$= 0 + \left[\dfrac{-e^{-\lambda nt}}{\lambda n}\right]_0^\infty$
A1	for $E(T) = \dfrac{1}{n\lambda}$
	NB – anyone able to identify this as the *Exponential Distribution* can quote the Expectation (or from the Formula Book) and get 6 marks for little effort ⑥

Question 13 (ii)

M1	for observing that 2nd email is simply the 1st from the remaining $(n - 1)$ …
A1	… with expected arrival time $\dfrac{1}{(n-1)\lambda}$
E1	for careful explanation of the result
A1	for a legitimately obtained **given answer** $\dfrac{1}{n\lambda} + \dfrac{1}{(n-1)\lambda} = \dfrac{1}{\lambda}\left(\dfrac{1}{n} + \dfrac{1}{(n-1)}\right)$ ④