STEP 2
General Remarks

Of the 1000+ entries for this paper, around 920 scripts actually arrived for marking, giving another slight increase in the take-up for this paper. Of this number, five candidates scored a maximum and seventy-five achieved a scoring total of 100 or more. At the other end of the scale, almost two hundred candidates failed to reach the 40-mark mark. Otherwise, marks were spread reasonably normally across the mark range, though there were two peaks at about 45 and 65 in the distribution. It is comforting to find that the 'post-match analysis' bears out the view that I gained, quite firmly, during the marking process that there were several quantum states of mark-scoring ability amongst the candidature. Many (about one-fifth of the entry) struggled to find anything very much with which they were comfortable, and marks for these candidates were scored in 3s and 4s, with such folk often making eight or nine poor efforts at different questions without ever getting to grips with the content of any one of them. The next "ability band" saw those who either scored moderately well on a handful of questions or managed one really successful question plus a few bits-'n'-pieces in order to get up to a total in the mid-forties. To go much beyond that score required a little bit of extra talent that could lead them towards the next mark-hurdle in the mid-sixties. Thereafter, totals seemed to decline almost linearly on the distribution.

Once again, it is clear that candidates need to give the questions at least a couple of minutes' worth of thought before commencing answering. Making attempts at more than the six scoring efforts permitted is a waste of valuable time, and the majority of those who do so are almost invariably the weaker brethren in the game. Many such candidates begin their efforts to individual questions promisingly, but get no more than half-a-dozen marks in before abandoning that question in favour of another – often with the replacement faring no better than its predecessor. In many such cases, the candidate's best-scoring question mark would come from their fifth, or sixth, or seventh, or …?, question attempted, and this suggests either that they do not know where their strengths lie, or that they are just not taking the time to make any sensible decision as to which questions to attempt and in what order to do them, adopting some sort of *hit-and-hope* strategy. With the pleasing number of very high totals to be found, it is clear that there are many places in which good marks were available to those with the ability to first identify them and then to persevere long enough to be able to determine what was really going on therein.

It is extremely difficult to set papers in which each question is pitched at an equivalent level of difficulty. Apart from any other factors, candidates have widely differing strengths and weaknesses; one student's algebraic nuance can be the final nail in the coffin of many others, for instance. Moreover, it has seemed enormously clear to me – more particularly so since the arrival of modular A-levels – that there is absolutely no substitute for prolonged and determined practice at questions of substance. One moment's recognition of a technique at work can turn several hours of struggle into just a few seconds of polishing off, and a lack of experience is always painfully clear when marking work from candidates who are under-practised at either the art of prolonged mathematics or the science of creative problem-solving. At the other, more successful, end of the scale there were many candidates who managed to produce extraordinary amounts of outstanding work, racking up full-, or nearly full-, marks on question after question. With the marks distributed as they were, it seems that the paper was pitched appropriately at the intended level, and that it successfully managed to distinguish between the different ability-levels to be found among the candidates.

As in previous years, the pure maths questions provided the bulk of candidates' work, with relatively few efforts to be found at the applied ones. Moreover, many of these were clearly acts of desperation.

Comments on individual questions

Q1 The first question is usually intended to be a gentle introduction to the paper, and to allow all candidates to gain some marks without making great demands on either memory or technical skills. This year, however, and for the first time that I can recall in recent years, the obviously algebraic nature of the question was enough to deter half the candidates from attempting it. Indeed, apart from Q8, it was the least popular pure maths question. This was a great pity: the helpful structure really did guide folks in the right direction, and any half-decent candidates who did try it usually scored very highly on it. There were, nonetheless, a couple of stumbling-blocks along the way for the less wary, and many candidates tripped over them. The point at which most of the less successful students started to go astray was when asked to show that *ABCD* is a rectangle. Lots of these folk elected to do so by working out distances … when the use of gradients would have been much simpler. The really disappointing thing was that many simply showed both pairs of opposite sides to be equal in length without realising that this only proved the quadrilateral a parallelogram. The next major difficulty was to be found in the algebra, in turning the area, $2\left(\alpha^2 - \beta^2\right)$, into something to do with u and v. It was quite apparent that many were unable to do so because they failed to appreciate that α and β were particular values of x and y that satisfy the two original curve equations, so that $\alpha^4 + \beta^4 = u$ and $\alpha\beta = v$. Then, squaring the area expression does the trick. Some got part of the way to grasping this idea, but approached from the direction of solving $x^4 + y^4 = u$ and $xy = v$ as simultaneous equations; the resulting surds-within-surds expressions for α and β were too indigestible for almost anyone to cope with.

Q2 Another of the less popular pure maths questions. It is clear that many A-level students are deeply suspicious of approximations and logarithms, and these plus the fact that y is a “function of a function of a function” clearly signalled to many to pass by on the other side. Of those who did take up the challenge here, almost all plumped automatically for differentiation in (i), usually by taking logs first and then differentiating implicitly. Just a few knew how to differentiate directly using the fact that $a^x = e^{x \ln a}$. However, calculus was not actually required, since the maxima and minima of y can be deduced immediately from knowledge of the sine function. It then helped candidates enormously if they were able to work generally in deciding what values of x gave these stationary points, not least because they would need some care in figuring out which to use in (iv). It was a pleasant surprise to find that (ii) was generally handled quite well, but sketches were poor – usually as a result of previous shortcomings – especially for $x < 0$; many candidates did realise, almost independently of previous working it seemed, that the right-hand ‘half’ of the curve oscillated increasingly tightly. In (iv), a lack of clarity regarding the x-values, allied to an uncertainty over dealing with the logs, proved a great hindrance to the majority. Also, it has to be said that, even amongst those *with* the right k’s to hand, a simple diagram of what they were attempting to work with would undoubtedly have saved them a lot of mark-spurning algebraic drivel.

Q3 Only marginally behind Q5 for popularity, this was a surprising hit amongst candidates. It had been anticipated that a trig. question containing lots of surds would be a bit of a turn-off, but this didn’t prove to be the case. Moreover, it turned out to be the highest scoring question on the paper too. I had expressed reservations, during the setting process, that we had been a little too helpful in flagging up what was needed at each stage of the process, and so it proved to be. Most hiccups came at the outset, where proving even a simple identity such as this one was beyond many, even those who continued very successfully. The only other trouble-spot came in (ii) when lots of candidates (who should be applauded for trying to keep in the spirit of "showing" stuff) undertook this part by rationalising the denominator (twice!) to prove the given result, while the sneakier types just multiplied across and verified it. Shame on them!

Q4 Considering the very poor marks gained on this question, it was surprisingly popular, with almost 600 "hits". Its essential difficulty lay in the fact that one can only go so far in this question before requiring the '*key of insight*' in order to progress further. And that was that, as they say. Personally, this was my favourite question, as the *key* is such a simple one once it is pointed out to you (clearly not an option in the exam., of course). Parts (i) and (ii) require candidates to find $p(1) = 1$ and to show that $p'(x)$ has $(x - 1)^4$ as a factor, and most did so perfectly satisfactorily. The (strikingly similar) information then given in (iii) **should** then suggest (surely?), to anyone with any sort of nous, that they are required to make similar further deductions. Nope – apparently not. Even amongst the few who *did* then find $p(-1) = -1$ and show that $p'(x)$ has $(x + 1)^4$ as a factor, very few knew what to do with these facts. I think that this is principally because most students work "on automatic" in examinations – a by-product of the much (and rightly) criticised modular system – simply doing as they are told at each little step of the way without ever having to stand back, even momentarily, and take stock of the situation before planning their own way forwards. This is the principal shame with modular assessment: the system prevents the very able from ever having to prove their ability whilst simultaneously persuading the only modestly able that they are fantastic mathematicians when they aren't. A moment of thoughtful reflection on the nature of this strange creature that is $p(x)$ and what we now know about it reveals all. It is a polynomial of degree 9. Its derivative **must** therefore be a polynomial of degree 8. And we know that $p'(x)$ has two completely distinct factors of degree 4. Apart from the tendency to assume that a polynomial always commences with a coefficient of 1, the rest (in principle) is just a matter of adding two 4s to get 8.

Q5 This was the most popular question on the paper, though only by a small margin, and the second highest scoring. In fact, I can be very specific and state that almost all of the really successful attempts scored 15 or 16 marks. The few marks lost were almost invariably in (ii), where so very, very few picked up the hints as to the only, minor difficulty within the question. Once again, this is almost certainly due to the mind-set of simply ploughing on regardless without stopping to think about what is actually going on. Whilst understanding that nearly all candidates will feel under considerable pressure to pick up as many marks as possible as quickly as possible, **NO-ONE** who sits this paper should be of the view that they are not going to be challenged to think. And, to be fair to the setting panel, we did put some fairly obvious signposts up for those who might take the trouble to look for such things. For future STEP candidates, this will make an excellent practice question for teachers to put their way. (If they are willing to learn from their mistakes, and you think you can catch them out … this is a marvellous question to use.) One pointer is in the change of limits, from (5, 10) to $\left(\frac{5}{4}, 10\right)$; the other is in the switch from asking for integrals-to-be-evaluated to asking for areas. The crux of the matter is that most A-level students believe that $\sqrt{x^2} = x$ rather than $|\,x\,|$. Once you realise that, the question is fiddly but otherwise rather easy.

Q6 This was another very popular question, but the one with the lowest mean mark score of all the pure questions, at about 7. I think that the initial enthusiasm of seeing something familiar in the *Fibonacci Numbers* was more than countered by the inequalities work that formed the bulk of the question. Nonetheless, I suspect that, if given the opportunity to talk it through after the event, many candidates would admit that half of the marks on the question are actually ludicrously easy to acquire and that they were really only put-off by appearances. For instance, to show that $S >$ any suitable lower-bound, one need only keep adding terms until the appropriate figure is exceeded. For those reciprocals of integers that are not easily calculated, such as $\frac{1}{13}$, it is perfectly reasonable to note something that they are greater than and us that in its place. Thus,

$$S = 1 + 1 + \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{5} + \tfrac{1}{8} + \ldots > 1 + 1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{5} + \tfrac{1}{10} + \ldots = 2 + 0.5 + 0.25 + 0.2 + 0.1 = 3.05 > 3$$

works pretty easily (though it may not have scored full marks in (i) as a particular approach was requested), and something similar could be made to work in showing that $S > 3.2$ in (ii). The approach that the question was designed to direct candidates towards was that of stopping the direct calculation at some suitable stage, and using an inequality of the form $\frac{1}{F_i} < \frac{1}{2}\left(\frac{1}{F_{i-2}}\right)$, possibly alternating with the odd- and even-numbered terms, to make the remaining sum less than a summable infinite GP. For further thoughts and possible developments of these ideas, I would refer the reader to the *Hints & Solutions* document for this paper.

Q7 In many ways, this question is little more than an academic exercise, since I can see no way in which these integrals would actually arise in any practical situation. I apologise for this. However, it *was* a good test of candidates' ability to stretch a general result in different directions, probing them for increasing amounts of insight and perseverance. For future STEP-takers, the opening result is a good one for which to find a generalisation, and it is a possibly fruitful avenue to explore the "product rule of differentiation" for three terms (etc.), say $y = pqr$ in this case. Such an approach might have helped to prevent some of the ghastly mix-ups in writing all the terms out that were to be found in the scripts. It was disappointing to see that so few candidates seemed to think that they should tidy up the answer and demonstrate that the left-over bits did indeed form a cubic polynomial, as required by the question. In the end, we gave anyone the mark who simply observed that what was left was a cubic (if indeed that was the case in their working). Thereafter, (i) is a straightforward application of the result, requiring candidates only to identify the values of n, a and b. However, even here, it was rare to see folks justifying the form of the cubic, which might have acted as a check for errors. In (ii), the polynomial term is no longer cubic, so candidates were expected to try to see if an extra factor of $(x - 1)$ could be taken out to go with the other twenty-one $(x - 1)$s, which indeed it could. Checking the cubic's terms was rather more important here. The final integral, in (iii), *was* difficult, and this was where candidates were 'found out' on this question. The obvious thing is to try and extract some $(x - 2)$ factor(s) from the quartic polynomial, but this doesn't work. Candidates may reflect that they shouldn't have found this too much of a surprise, as that would simply have been repeating the "trick" of (ii). Though only a small minority realised it, the next most obvious possibility to try, having already found in (ii) that 'the next case up' gives a quartic rather than a cubic polynomial, is to try some combination of the obvious answer and the next one up, and this turns out to be exactly what is required.

Q8 The vectors question was the least popular pure maths question by a considerable way, and only marginally more popular than most of the applied questions. In general, attempts were short-lived and candidates usually gave up when the algebra got a bit "iffy". Strangely, an awful lot of attempts began very poorly indeed; the ranges of values of λ and μ had a geometric significance relating to where P and Q lay on the lines AB and AC, and these were not well grasped. Moreover, it was surprising to see diagrams in which the lines had not even been drawn, often leaving the marker to guess whether the points were supposed to be on them or not. Many responses to the next part were so bizarre that they were almost funny: a lot of candidates thought that CQ etc. were vectors rather than lengths, and the "×" was treated variously as a scalar multiplication, the scalar product and the vector product. Oddly enough, they could all lead to the required answer, $\lambda\mu = 1$, even legitimately (with a bit of care) though we were harsh on statements that were actually nonsense. Very few made it to the later stages of the question.

Q9 Of the applied maths questions, this was (again) by far the most popular, drawing around 300 attempts. It also proved to be one of the highest-scoring of all the questions on the paper, with a mean score of 12. Finding the position of the centre of mass was sensibly used by most candidates, and the first two parts yielded high marks. The last part attracted less confident algebra, which is curious given that it involved much the same sort of work.

Q10 In general, one can be forgiven for approaching a collisions question in an automatic way; applying the *Conservation of Linear Momentum* (CLM) and *Newton's Experimental Law of Restitution* (NEL or NLR) for both initial collisions. Most candidates did the routine stuff quite well but then got bogged down in the ensuing algebra. The nice thing about this question is that it can also be done without the need for the use of CLM at all. The NEL statements give a relationship between the final velocities (v_1 to v_4, say) of the four particles, and then equating for the times to the following collisions at O uses these velocities without ever requiring to have them in terms of u. I was greatly surprised to see such a high proportion of the attempts using some such suitably concise approach, and they were almost guaranteed full marks on the question, and for very little time and trouble.

Q11 This question was the least popular question on the paper, and those trying it averaged only 6 marks on it. The most surprising aspect of it is that so few could even write a decent *N2L* statement to begin with, and they simply stood no chance thereafter. For those who made it to the first-order, variables-separable differential equation, the work was much more promising, though I suspect this is due to the fact that only the very able made it this far. The unpromising integration of $\mathrm{f}(v)\,\mathrm{d}v$, where the $\mathrm{f}(v)$ turned out to be a linear-over-linear algebraic fraction, was certainly unappealing to look at, but a simple substitution such as $s = P - (n + 1)Rv$ reduces it to a very simple piece of integration. As far as I recall it, most of the inequalities in (i) were fudged, though it was very heart-warming indeed to see those excellent few who made it right to the end. It is a pity that a last minute change to the question, prior to printing, which had been intended to help candidates by giving them the final answer, then omitted the factor $(n + 1)$ in its denominator. Fortunately, we are talking about no more than twenty of the most able (and high-scoring) candidates here; those who had explained it correctly, but then crossed-out the $(n + 1)$ since it didn't appear on the question-paper, were given the final mark. As for those who were slightly less honest and gave the proper explanation but (presumably deliberately) didn't write the missing factor in anywhere, in order to fudge it, we were mean and didn't give them the final mark.

Q12 There were only around 200 attempts to Q12, and the mean score was just 5, making it the worst question on the paper for marks. Many of the attempts failed to get very far at all, largely for finding that efforts to integrate e^{ax^2} proved difficult. Personally, I had thought it was giving too much away to ask for the sketch of the pdf at the beginning, in that it might just give the game away that what was being handled here was just half of a normal distribution. I worried in vain. Sadly, without this crucial observation, little or no progress was possible. Even with it, a little care was still needed in handling the differences between this and N(0, 1), for which the tables could be used.

Q13 This question was only marginally more popular than Q11, but those who did attempt it were usually well rewarded with marks; candidates averaging almost 11 on it. The careful use of binomial expansions, and remembering to *use* the result $q = 1 - p$ throughout, made this an eminently approachable question in principle. Those who stumbled did so over little arithmetical slips, such as with the careless handling of minus signs. Once an error has been introduced, although method marks are there to be had, the final satisfaction of getting to the answer is never to be experienced without going back to correct it.