STEP 1 2013 Examiners' Report

General Comments

Around 1500 candidates sat this paper, a significant increase on last year. Overall, responses were good with candidates finding much to occupy them profitably during the three hours of the examination. In hindsight, two or three of the questions lacked sufficient 'punch' in their later parts, but at least most candidates showed sufficient skill to identify them and work on them as part of their chosen selection of questions. On the whole, nearly all candidates managed to attempt 4-6 questions – although there is always a significant minority who attempt 7, 8, 9, … bit and pieces of questions – and most scored well on at least two. Indeed, there were many scripts with 6 question-attempts, most or all of which were fantastically accomplished mathematically, and such excellence is very heart-warming.

Comments on individual questions

[Examiner's note: in order to extract the maximum amount of profit from this report, I would firmly recommend that the reader studies this report alongside the *Hints and Solutions* supplied separately.]

Q1 This question is all about using substitutions to simplify the working required to solve various increasingly complicated looking equations. It was the most popular question on the paper, essentially attempted by every candidate (as is the intention). The obvious pitfall of not realising that the square-root sign indicates the "non-negative square-root" of a quantity was clearly flagged at the outset. Thus, the only remaining hurdle to fully complete success lay in the need to check the validity of solutions once found. The mean score on this question was 14/20, and this question thus represented a successful *entré* to the paper for almost everyone.

The use of the quadratic formula and the method of completing the square appeared in almost equal measure throughout the question, although a significant minority of candidates opted to rearrange and square in both (ii) and (iii). This was not a major obstacle to success in (ii) but led to a quartic equation in (iii) with which few candidates knew how to make successful progress. The final hurdle for most candidates lay in a final justification that any roots found (up to four of them, depending upon the method chosen) were genuinely valid. It is very easy to explain, without the use of direct verification, that the two roots found via the substitution method are good, but very few candidates made any attempt to justify their results.

Q2 This was another very popular question, attempted by more than a 1000 of the candidates. The initial difficulties arose in the interpretation of the *integer-part* (or *floor*) function. Candidates' graphs revealed the difficulties and uncertainties associated with the use of such a function. In particular, the lack of "jumps" at the endpoints of each unit interval was very prevalent, and many candidates effectively assumed that the function is an *even function*. There was also considerable uncertainty in how to represent whether endpoints were "in" or "out" – the usual convention being closed dots for "included" and open dots for "not-included". Also, many candidates failed to show in their sketch that the function was zero in the interval $0 \le x < 1$, and others drew straight line segments instead of portions of a reciprocal curve in each unit segment. Pleasingly, however, (at least from the candidates' point of view), it was possible to get quite a few of these bits wrong and still go on to answer correctly many of the following parts of the question. Thus it was that the mean mark on the question, at 9/20, was still a respectable one.

In parts (ii) and (iii), it was only necessary for candidates to realise with which portion of the function they had to deal in order to be undertaking the correct algebra, and the ten marks allocated to these parts of the question were generally those from which the majority of candidates were scoring the bulk of their marks. Only the very last part of the question required much thought, and candidates were not helped by an unwillingness to set down in writing any of their underlying thoughts, merely opting for statements that seemed to come from nowhere obvious. It was unfortunate that some considered the function to be defined only on the interval $-3 \le x \le 3$, which was simply that required for the sketch.

Q3 This vector question was actually very straightforward, though its unfamiliar appearance clearly put most candidates off, with only around 350 of them making an attempt at it. There were nine marks available for the first two parts, which were technically undemanding, and it is no coincidence that the mean score on the question was around 9½/20. I suspect that, for the most part, this was considered by candidates to be one of those questions that are done towards the end of the examination in order to bump up their paper total by getting the easier marks at the beginning of the question, with no real intention of making a complete attempt. Candidates usually gave up part-way through (iii) where a stab at the "corresponding result for $X * (Y * Z)$" was required of them, which was actually just $(X * Y) * (X * Z)$. I imagine this highlights the lack of students' familiarity with such properties as distributivity when considering binary operations.

Q4 This was another very popular and high-scoring question (attracting over 1200 attempts and with a mean mark of more than 10/20). The first part to this question involved two integrals which can be integrated immediately by "recognition", although many students took a lot of time and trouble to establish the given results by substitution and surprising amounts of working. Those candidates who had found these easy introductory parts especially troubling usually did not proceed far, if at all, into part (ii). Those who did venture further usually picked up quite a lot of marks.

One of the great advantages to continued progress in the question is that the two integrals in part (ii) can be approached in so many different ways – the examiners worked out more than 25 slightly different approaches, depending upon how, and when, one used the identity $\sec^2 x = 1 + \tan^2 x$; how one split the "parts" in the process of "integrating by parts"; and even whether one approached the various secondary integrals that arose as a function of $\sec x$ or $\tan x$. This meant that, with care, most of the marks were accessible, although many candidates clearly got into a considerable tangle at some stage of proceedings. The most common "howler" was the mix-up between the definite integrals (i.e. numerical values) given in (i) and their associated *unevaluated* indefinite integrals (i.e. functions) which formed part of a subsequent integral.

Q5 This question was usually found to be amongst candidates' chosen six, attracting almost a thousand attempts, though on the whole it produced the lowest mean score of the popular pure maths questions, weighing in at under 7½/20. The initial attraction of the question was undoubtedly the obvious "circle" nature of the given quadratic form when $k = 0$, meaning that part (i) was very familiar territory. Unfortunately, there were very few marks allocated to this bit. Part (ii) drew a lot of unsuccessful work, especially as candidates seemed ill-inclined to extend the requested factorisation from that of $3x^2 + 3y^2 + 10xy$ into that of the full quadratic expression. Even amongst those who did make that extra step, there were relatively few that grasped the geometric consequences of the result that $AB = 0 \Rightarrow A = 0$ or $B = 0$ meant that the solutions amounted to a line-pair. The question's demand for a sketch of the solutions meant that most of the marks were only awarded for candidates who had made this geometric interpretation.

Part (iii) was the genuinely tough part of the question, but substantial help was offered to enable candidates to make a start on it, which most duly employed. However, working forwards and backwards through the given substitutions did not make for easy reading and it was clear that many candidates did not realise the given locus of Q is that of a standard parabola. Several marks were gained by most candidates, but few made a thorough fist of it.

Q6 Although this question attracted a few more attempts than Q3, it was the lowest-scoring of the pure maths questions. Confident use of the sigma-notation is clearly in short supply and this was, perhaps, that feature of the question that deterred most candidates from attempting it. Also, many attempts were simply from those candidates cherry-picking the opening three marks for proving the standard "Pascal's Triangle" result, mostly by proving it directly from the definition of the binomial coefficient in terms of factorials (which we had decided to allow when setting the paper). This almost invariably accounted for 3 of the 6.7 marks gained on average for the question as a whole.

Those who proceeded further than this opening result generally fell into a couple of very wide traps: a careless handling of the terms at the ends of the series (which, being 1, could be replaced by other binomial coefficients that were also 1) and a failure to consider odd and even cases separately. A final obstacle, were one needed, lay in the oversight of establishing the validity of the relationship between the B_n's and the F_n's for their starting terms – surprisingly, many candidates failed to evaluate B_0 and B_1 correctly.

Q7 Around 1300 candidates attempted this question, making it the second most popular question on the paper. It was also the second highest-scoring question on average which, if nothing else, pays tribute to the candidates' ability to spot the right questions to attempt. In hindsight, this was possibly a little too straightforward; this was undoubtedly partly due to the appearance of similar questions (on what are known as *homogeneous* differential equations) in recent years' STEPs, but also to the fact that part (ii) could be solved by the use of the given approach for part (i). It was part (iii) that required of candidates a stretch of the imagination – the use of $y = ux^2$ – but even this helped make the question more approachable, as this substitution could also be used to solve part (ii) if it turned out that candidates got imaginative a bit earlier than anticipated.

For those making essentially correct attempts at parts (ii) and (iii), the only final hurdle to complete success lay in the hoped-for statement of a domain for the functions which had been found as solutions. We allowed as obvious the taking of non-negative square-roots (since the given "initial" values of y are positive – though, in general, candidates should be encouraged to state that they recognise they are doing this) but expected candidates to indicate a suitable interval for the x values in each case: the hint lay in the given answer to part (i).

Q8 Almost 1200 candidates made an attempt at this question, making it fourth favourite, and the mean score on it was 9.3/20 which, if nothing else, suggests that it wasn't quite as easy as folks considered it to be. To begin with, there is a lot to do for the relatively few marks available, and minor slips over domains and ranges subsequently proved quite costly. Apart from the obvious errors from those candidates who thought the order of composition occurred the other way around, and the few who took "ab" to mean the product of the functions a and b, the usual slip-ups were: thinking that $\sqrt{x^2} = x$, when it is actually $| x |$, and not realising that the domain of the composite function fg is just the domain of g. In (ii), although the functions fg and gf *look* the same (both are $| x |$), their domains and ranges are different: fg has domain $\mathbb{R}$ and range $y \geq 0$, while gf has domain $| x | \geq 1$ and range $y \geq 1$.

A lack of a clear grasp of the domains and ranges of h and k in part (iii) was partly responsible for the poor sketches, although the ability to recognise the asymptote $y = 2x$ was also widespread. There were even occasions, when sketching the curve for k, that a correctly drawn asymptote was subsequently labelled as $y = -2x$ simply because of its appearance in the quadrant in which x and y are both negative.

Q9 This question was the most popular of the applied questions, drawing well over 500 responses, and the most successful of the mechanics questions, with an average score of 8.8/20. It proved to be a surprisingly good discriminator, giving a good range of marks. The use of constant-acceleration formulae for the projectile motion provided a routine and straightforward start to the question, but this was followed by the momentum equation for the collision, which proved trickier, with quite a few candidates getting to $mu\cos\alpha - Mv\cos\beta = Mw_B - mw_A$ but no further. A lot of candidates resorted to writing down the result $mu\cos\alpha = Mv\cos\beta$ without any attempt to justify it. The second result then found many candidates going round in algebraic circles, and very few indeed managed to find the answer (not given) to the very final part of the question.

Q10 This question proved to be the least popular question on the paper, eliciting a mere 150 responses. The mean score of 7.7 on it was almost entirely drawn from the first six marks allocated for obtaining the given result, and then for setting $v_n = 0$ in the following part. This does raise the thorny issue – during the setting process – of the extent to which (intermediate) answers should be given in the question, as candidates clearly find great comfort in having something to work towards, but are otherwise surprisingly weak. Here, for instance, almost any tiny slip-up in working, signs, etc., inevitably had disastrous consequences for a candidate's prospect of successful continuation with the work and very few indeed progressed much beyond the first result.

Q11 A combination of some obviously tricky trigonometry and inequalities meant that this mechanics question was both unpopular and low-scoring, despite the given answer in (i). Only 300 candidates attempted it, and they averaged a score of 5/20, with most of the marks being scored at the beginning with correct statements regarding the resolution of forces vertically and horizontally. In (ii), it was important for candidates to realise (a fact clearly indicated by the question's wording) that the condition $W > T\sin(\alpha + \beta)$ would no longer hold; those that recognised the change in the kinematics did not have too much trouble in working the problem through to its end. However, there were too few who had made it to the end of (i) intact, and these candidates had given up already without proceeding into part (ii).

Q12 This probability question drew more than 350 responses, scoring just over half-marks on average. There are many ways to go about part (i), of varying degrees of sophistication: those opting for elaborate tree diagrams tended to be the least successful. The final part of (i) was really intended as a test of whether candidates realised that this is the same situation viewed "in reverse", so the answer is the same. Very few candidates spotted the symmetry argument or got it correct by longer methods. Those who had obtained the given result of (i) by one of the more sophisticated methods had little difficulty in employing a similar argument in (ii), although some did mix up the roles of the n and the k. A few did the general method and then substituted particular values. Those who did use a general approach here then fared very well in part (iii) and they usually went on to apply Stirling's approximation correctly.

Q13 After Q10, this was the least popular question on the paper, and supplied the poorest average score on the whole paper of only 2½/20. I have little doubt that the principal reason for both these factors is the lack of any helpful structure or given answers within the question. Essentially, this problem is that of the set-up for a game of *Solitaire*, but stripped of its context. In this game, when a standard pack of playing cards, suitably shuffled, is laid out at the start, there are seven piles of cards, and each pile has its final card face up. This particular question is looking for cards of the same colour (red or black) and denomination (number or J, Q, K and Ace), giving the 26 pairs. This was, of course, entirely by-the-by as far as candidates were concerned.

Unfortunately, most attempts at this question were abandoned very early on as candidates realised they didn't really know what to do. Surprisingly, very few even took the trouble to note that the defined discrete random variable X could only take the values 0, 1, 2 or 3. Following attempts to work out the probability for any these outcomes almost invariably consisted of a jumble of fractions and factorials but without any obvious plan to them, and certainly without any explanatory indicators as to what might actually be intended. Only $P(X = 0)$, being the easiest of the four cases to evaluate, was calculated with any degree of success by any of the candidates who attempted the question.