SI-2015/Q1

(i) $y = e^x(2x-1)(x-2)$ **B1** Correct factorisation of quadratic term (or formula, etc.)

$(\frac{1}{2}, 0)$ & $(2, 0)$ **B1** Noted or shown on sketch

$\frac{dy}{dx} = e^x(2x^2 - x - 3)$ **M1** Derivative attempted and equated to zero for TPs

$= e^x(2x-3)(x+1)$

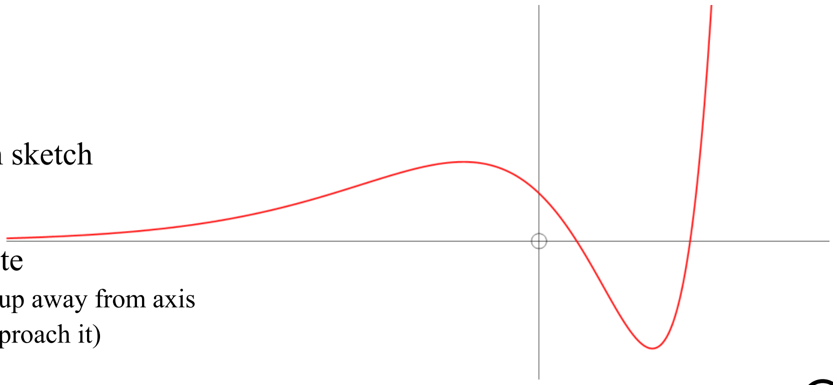
$(\frac{3}{2}, -e^{1.5})$ & $(-1, 9e^{-1})$ **A1 A1** Noted or shown on sketch

(if y-coords. missing, allow one A1 for 2 correct x-coords.)

G1 Generally correct shape

G1 for (0, 2) noted or shown on sketch

G1 for negative-x-axis asymptote
(penalise curves that clearly turn up away from axis or that do not actually seem to approach it)



⑧

Give **M1** for either 0, 1, 2 or 3 solutions *OR* clear indication they know these arise from where a horizontal line meets the curve (e.g. by a line on their diagram) – implied by any correct answer(s)

Then $y = k$ has	NO solutions for $k < -e^{1.5}$	**A1**
	ONE solution for $k = -e^{1.5}$ and $k > 9e^{-1}$	**A1 A1**
	TWO solutions for $-e^{1.5} < k \le 0$ and $k = 9e^{-1}$	**A1 A1**
	THREE solutions for $0 < k < 9e^{-1}$	**A1**

FT from their y-coords.of the Max. &Min. points.

⑦

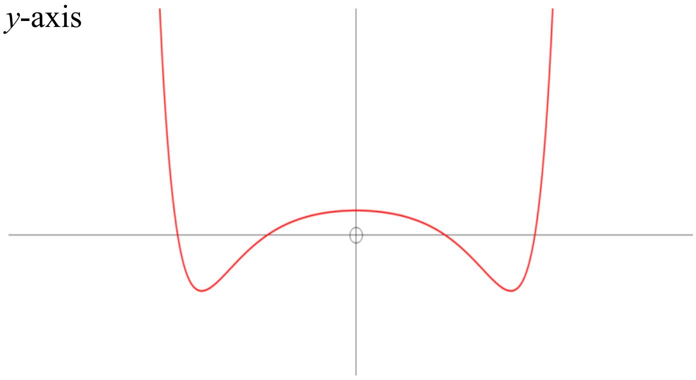
(ii) **G1** Any curve clearly symmetric in y-axis

G1 Shape correct

G1 A Max. TP at (0, 2) **FT**

G1 Min. TPs at $(\pm\sqrt{\frac{3}{2}}, -e^{1.5})$ **FT**

G1 Zeroes at $x = \pm\sqrt{\frac{1}{2}}, \pm\sqrt{2}$ **FT**



⑤

SI-2015/Q2

(i) **M1** Use of cos($A - B$) formula with $A = 60^{\circ}$, $B = 45^{\circ}$ *OR* $A = 45^{\circ}$, $B = 30^{\circ}$ **or** $2\cos^2 15^{\circ} - 1$ etc.

A1 Exact trig.values used (visibly) to gain $\cos 15^{\circ} = \frac{\sqrt{3}+1}{2\sqrt{2}}$ *legitimately* (**Given Answer**)

M1 Similar method *OR* $\sin = +\sqrt{1-\cos^2}$ (as 15° is acute, no requirement to justify +vesq.rt.)

A1 $\sin 15^{\circ} = \frac{\sqrt{3}-1}{2\sqrt{2}}$ (however *legitimately* obtained) ④

(ii) **M1** Use of cos($A + B$) formula *and* double-angle formulae *OR* de Moivre'sThm. (etc.)

A1 $\cos 3\alpha \equiv 4\cos^3\alpha - 3\cos\alpha$

A1 Justifying/noting that $x = \cos\alpha$ is thus a root of $4x^3 - 3x - \cos 3\alpha = 0$

M1 For serious attempt to factorise $4(x^3 - c^3) - 3(x - c)$ as linear × quadratic factors **or** via *Vieta's Theorem* (roots/coefficients)

A1 $(x-c)\{4(x^2 + cx + c^2) - 3\}$

M1 Solving $4x^2 + 4cx + (4c^2 - 3) = 0$ **FT** their quadratic factor

Remaining roots are $x = \frac{1}{2}\left(-c \pm \sqrt{c^2 - (4c^2 - 3)}\right)$

M1 Use of $s = \sqrt{1-c^2}$ to simplify sq.rt. term

A1 $x = \frac{1}{2}\left(-\cos\alpha \pm \sqrt{3}\sin\alpha\right)$

⑧

(iii) **M1** $\frac{1}{2}y^3 - \frac{3}{2}y - \frac{\sqrt{2}}{2} = 0$

A1 $4\left(\frac{1}{2}y\right)^3 - 3\left(\frac{1}{2}y\right) - \frac{\sqrt{2}}{2} = 0$

M1 $\cos 3\alpha = \frac{\sqrt{2}}{2} = \cos 45^{\circ}$

A1 $\Rightarrow \alpha = 15^{\circ}$

M1 $\frac{1}{2}y = \cos\alpha$, $\frac{1}{2}\left(-\cos\alpha + \sqrt{3}\sin\alpha\right)$, $\frac{1}{2}\left(-\cos\alpha - \sqrt{3}\sin\alpha\right)$ with their α

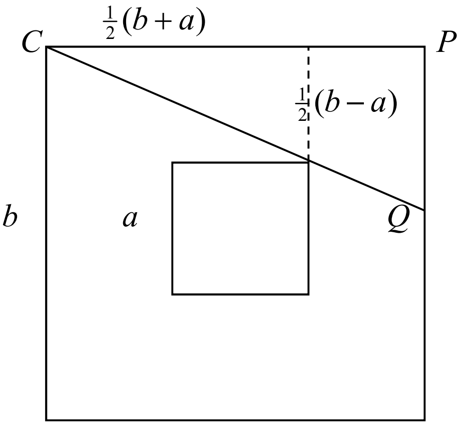
A1 $y = 2\cos 15^{\circ} = \frac{\sqrt{3}+1}{\sqrt{2}}$

A1 $\sqrt{3}\sin 15^{\circ} - \cos 15^{\circ} = -\frac{\sqrt{3}-1}{\sqrt{2}}$

A1 $-\sqrt{3}\sin 15^{\circ} - \cos 15^{\circ} = -\sqrt{2}$

⑧

SI-2015/Q3



B1 For correct lengths in smallerΔ

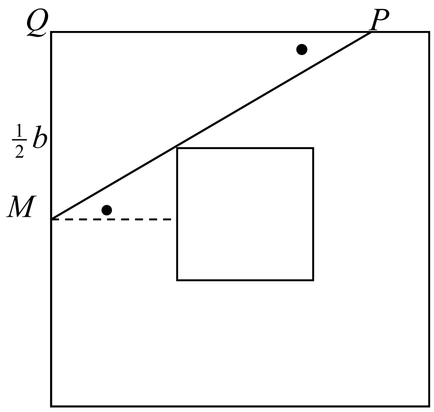
M1 By similar Δs (*OR* trig.*OR*coord.geom.)

A1 $\dfrac{PQ}{b} = \dfrac{\frac{1}{2}(b-a)}{\frac{1}{2}(b+a)} \Rightarrow PQ = \dfrac{b(b-a)}{b+a}$

M1 so a guard at a corner can see $2(b + PQ)$

A1 $= \dfrac{4b^2}{b+a}$ (might be given as all but $\dfrac{4ba}{b+a}$ or as a fraction of the perimeter)

5



Lengths $\frac{1}{2}a$ and $\frac{1}{2}(b-a)$ in smaller Δ

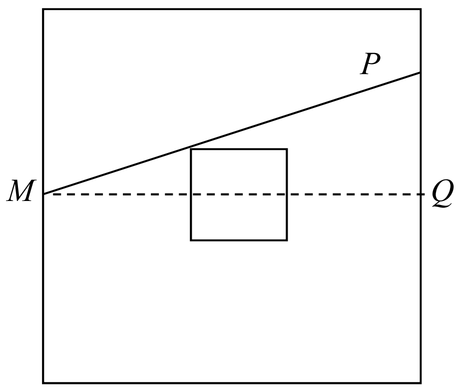
M1 By similar Δs (*OR* trig.*OR*coord.geom.)

A1 $\dfrac{\frac{1}{2}b}{PQ} = \dfrac{\frac{1}{2}a}{\frac{1}{2}(b-a)} \Rightarrow PQ = \dfrac{b(b-a)}{2a}$

M1 so a guard at a midpoint can see $b + 2PQ$

A1 $= \dfrac{b^2}{a}$ (might be given as all but $\dfrac{b(4a-b)}{a}$ or as a fraction of the perimeter)

4



Lengths $\frac{1}{2}a$ and $\frac{1}{2}(b-a)$ in smaller Δ

M1 By similar Δs (*OR* trig. *OR* coord.geom.)

A1 $\dfrac{PQ}{b} = \dfrac{\frac{1}{2}a}{\frac{1}{2}(b-a)} \Rightarrow PQ = \dfrac{ba}{b-a}$

M1 so a guard at a midpoint can see $4b - 2PQ$

A1 $= \dfrac{2b(2b-3a)}{b-a}$ (might be given as all but $\dfrac{2ba}{b-a}$ or as a fraction of the perimeter)

4

B1 Recognition that $b = 3a$ is the case when guard at M / C equally preferable
(P at corner in the two M cases)

M1A1 Relevant algebra for comparison of one case $\dfrac{4b^2}{b+a} - \dfrac{b^2}{a} \equiv \dfrac{b^2}{a(b+a)}(3a-b)$

A1 Correct conclusion: Guard stands at C for $b < 3a$ and at M for $b > 3a$

M1A1 Relevant algebra $\dfrac{4b^2}{b+a} - \dfrac{2b(2b-3a)}{b-a} \equiv \dfrac{2ba}{(b+a)(b-a)}(3a-b)$

A1 Correct conclusion: Guard stands at C for $b < 3a$ and at M for $b > 3a$

7

Overall, I am anticipating that most attempts will do the Corner scenario and ***one*** of the Middle scenarios. This will allow for a maximum of 12 = 5 (for the Corner work) + 4 (for the Middle work) + 3 (for the comparison). In this circumstance, it won't generally be suitable to give the **B1** for the $b = 3a$ observation.

SI-2015/Q4

M1 When P is at $(x,\ \frac{1}{4}x^2)$ ... and makes an angle of θ with the positive x-axis

A1 ... the lower end, Q, is at $\left(x - b\cos\theta,\ \frac{1}{4}x^2 - b\sin\theta\right)$

M1 Also, $y = \frac{1}{4}x^2 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}x = \tan\theta$

A1 $\Rightarrow x = 2\tan\theta$ i.e. $P = \left(2\tan\theta,\ \tan^2\theta\right)$

A1A1 so that $Q = \left(2\tan\theta - b\cos\theta,\ \tan^2\theta - b\sin\theta\right)$ obtained *legitimately* (**Given Answer**)

⑥

M1A1 When $x = 0$, $2\tan\alpha = b\cos\alpha \Rightarrow b = \frac{2\tan\alpha}{\cos\alpha}$

M1A1 Substg. into y-coordinate $\Rightarrow y_A = \tan^2\alpha - 2\tan\alpha\frac{\sin\alpha}{\cos\alpha} = -\tan^2\alpha$

④

M1A1 Eqn. of line AP is $y = x\tan\alpha - \tan^2\alpha$

M1A1 Area between curve and line is $\int\left(\frac{1}{4}x^2 - \left[x\tan\alpha - \tan^2\alpha\right]\right)\mathrm{d}x$

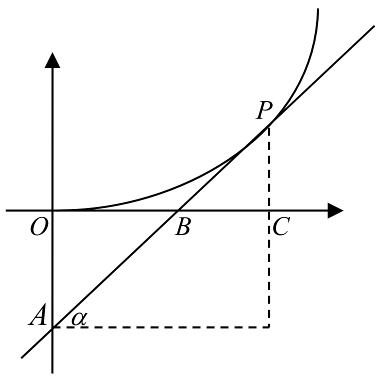
B1 Correct limits (0, 2tan α)

A1A1 $= \left[\frac{1}{12}x^3 - \frac{1}{2}x^2\tan\alpha + x\tan^2\alpha\right]$ (Any 2 correct terms; all 3)

A1A1 $= \frac{2}{3}\tan^3\alpha - 2\tan^3\alpha + 2\tan^3\alpha$ (Any 2 correct terms; all 3 **FT**)

A1 $= \frac{2}{3}\tan^3\alpha$ obtained *legitimately* (**Given Answer**) ⑩

ALTERNATIVE



M1 A1 for obtaining the "conversion factor" $b\cos\alpha = 2\tan\alpha$ **or** $\tan^2\alpha = \frac{1}{2}b\sin\alpha$

M1 A1 for distances $OB = BC$ $(=\frac{1}{2}b\cos\alpha)$ and so $PC = OA = \tan^2\alpha$

M1 A1 giving $\Delta OAB \equiv \Delta CPB$

A1 $\Rightarrow$ Area is $\int\frac{1}{4}x^2\ \mathrm{d}x$

B1 Correct limits (0, 2 tanα) used

A1 A1 Correct integration; correct **Given Answer**

ALTERNATIVE Translate whole thing up by $\tan^2\alpha$ and calculate $\int_0^{b\cos\alpha}\left(\frac{1}{4}x^2 + \tan^2\alpha\right)\mathrm{d}x - \Delta$

SI-2015/Q5

(i) **M1A1** $f(x) = \left[\dfrac{(t-1)^x}{x} \right]_1^3$

A1 $= \dfrac{2^x}{x}$ 3

M1 Differentiating by use of *Quotient RuleOR* taking logs.anddiffg. implicitly)

B1 for $\dfrac{d}{dx}\left(2^x\right) = 2^x . \ln 2$ seen at any stage

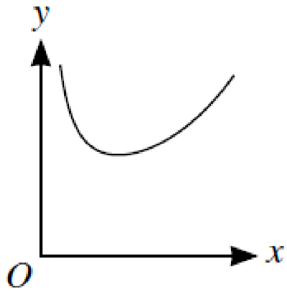
A1 $\dfrac{dy}{dx} = \dfrac{x.2^x . \ln 2 - 2^x}{x^2}$

A1 TP at $\left(\dfrac{1}{\ln 2}, (e \ln 2) \right)$ (y-coordinate not required)

B1 Jusitfying that the TP is a minimum 5

G1 Generally correct ∪-shape

G1 Asymptotic to y-axis and TP in **FT** correct position



2

(ii) **M1** Let $u^2 = 1 + x^2 - 2xt$

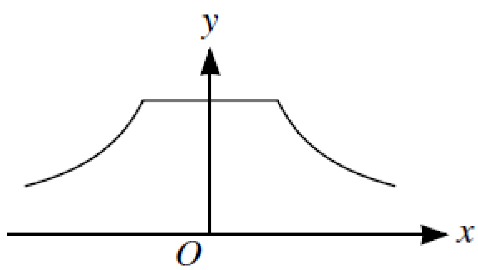
A1 $2u \, du = -2x dt$

B1 t**:** $(-1, 1) \rightarrow u$**:** $(| 1 + x |, | 1 - x |)$ Correct limits seen at any stage

M1A1 Full substn. attempt; correct $g(x) = \dfrac{-1}{x} \int 1 . du$

A1 $g(x) = \dfrac{1}{x}\left(|1 + x| - |1 - x| \right)$ ∫n. may be done directly, but be strict on the limits

or $g(x) = \begin{cases} -\dfrac{2}{x} & x < -1 \\ 2 & -1 \le x \le 1 \\ \dfrac{2}{x} & x > 1 \end{cases}$



(Must have completely correct three intervals: $x < -1$, $-1 \le x \le 1$, $x > 1$)

M1 Graph split into two or three regions

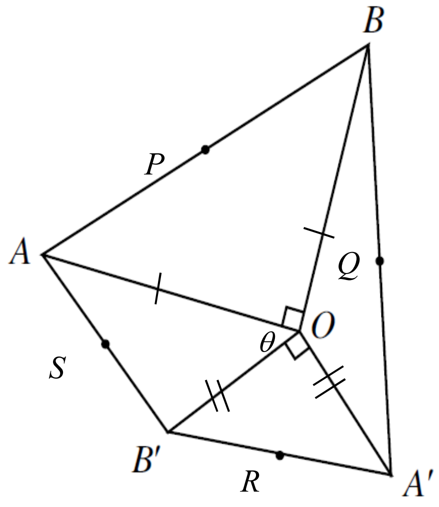
A1 A1 Reciprocal graphs on LHS & RHS (must be asymptotic to x-axis)

(Allow even if they approach y-axis also)

A1 Horizontal line for middle segment

10

SI-2015/Q6



Let P, Q, R and S be the midpoints of sides (as shown)

Then

M1A1 $\quad$ $\mathbf{p} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$, $\mathbf{q} = \frac{1}{2}\mathbf{b} + \frac{1}{2}\mathbf{a}'$,

$\mathbf{r} = \frac{1}{2}\mathbf{a}' + \frac{1}{2}\mathbf{b}'$, $\mathbf{s} = \frac{1}{2}\mathbf{b}' + \frac{1}{2}\mathbf{a}$

and

M1A1 $\quad$ $\overrightarrow{PQ} = \overrightarrow{SR} = \frac{1}{2}(\mathbf{a}' - \mathbf{a})$

A1 $\quad$ $\overrightarrow{QR} = \overrightarrow{PS} = \frac{1}{2}(\mathbf{b}' - \mathbf{b})$

A1 $\quad$ so that $PQSR$ is a //gm.

(opposite sides // and equal)

⑥

M1 $\quad$ $\overrightarrow{PQ} \bullet \overrightarrow{QR} = \overrightarrow{PQ} \bullet \overrightarrow{QS} = \frac{1}{2}(\mathbf{a}' - \mathbf{a}) \bullet \frac{1}{2}(\mathbf{b}' - \mathbf{b})$ $\quad$ for use of the scalar product

A1 $\quad$ $= \frac{1}{4}(\mathbf{a}' \bullet \mathbf{b}' - \mathbf{a} \bullet \mathbf{b}' - \mathbf{a}' \bullet \mathbf{b} + \mathbf{a} \bullet \mathbf{b})$ $\quad$ Do not accept $\mathbf{a}'\mathbf{b}'$ etc.

M1 $\quad$ Use of perpendicularity of OA, OB and OA', OB'

$= -\frac{1}{4}(\mathbf{a} \bullet \mathbf{b}' + \mathbf{a}' \bullet \mathbf{b})$

M1 $\quad$ $\angle AOB' = \theta \Rightarrow \angle A'OB = 180^\circ - \theta$; and $\cos(180^\circ - \theta) = -\cos\theta$

A1 $\quad$ $= 0$ since $\mathbf{a} \bullet \mathbf{b}' = ab'\cos\theta$ and $\mathbf{a}' \bullet \mathbf{b} = -a'b\cos\theta$

and we are given that $a = b$ and $a' = b'$

A1 $\quad$ so that $PQRS$ is a rectangle (adjacent sides perpendicular)

⑥

B1 $\quad$ $PQ^2 = SR^2 = \overrightarrow{PQ} \bullet \overrightarrow{PQ} = \frac{1}{4}\left(a^2 + (a')^2 - 2\mathbf{a} \bullet \mathbf{a}'\right)$

B1 $\quad$ $QR^2 = PS^2 = \frac{1}{4}\left(b^2 + (b')^2 - 2\mathbf{b} \bullet \mathbf{b}'\right)$

M1 $\quad$ Since $a = b$, $a' = b'$ and $\mathbf{a} \bullet \mathbf{a}' = aa'\cos(90^\circ + \theta)$, $\mathbf{b} \bullet \mathbf{ab}' = bb'\cos(90^\circ + \theta)$

A1 $\quad$ it follows that $PQRS$ is a square (adjacent sides equal)

④

M1A1 $\quad$ Area $PQRS = \frac{1}{4}\left(a^2 + (a')^2 - 2aa'\cos[90^\circ + \theta]\right)$

M1 $\quad$ ... which is maximal when $\cos[90^\circ + \theta] = -1$

A1 $\quad$ i.e. when $\theta = 90^\circ$

④

SI-2015/Q7

M1 $f'(x) = 6ax - 18x^2$

$= 6x(a - 3x)$

A1A1 $= 0$ for $x = 0$ and $x = \frac{1}{3}a$

A1A1 $f(0) = 0$ $f(\frac{1}{3}a) = \frac{1}{9}a^3$

A1 (Min. TP) (Max. TP) since $f(x)$ is a 'negative' cubic

(f(0) = 0 and the TPs may be shown on a sketch – award the marks here if necessary)

⑥

M1 Evaluating at the endpoints

A1A1 $f(-\frac{1}{3}) = \frac{1}{9}(3a + 2)$; $f(1) = 3a - 6$

③

M1 $\frac{1}{9}(3a + 2) \geq \frac{1}{9}a^3 \Leftrightarrow a^3 - 3a - 2 \leq 0$

M1 $\Leftrightarrow (a + 1)^2(a - 2) \leq 0$

A1 and since $a \geq 0$, $a \leq 2$

M1 $\frac{1}{9}a^3 \geq 3a - 6 \Leftrightarrow a^3 - 27a + 54 \geq 0$

M1 $\Leftrightarrow (a - 3)^2(a + 6) \geq 0$

A1 which holds for all $a \geq 0$

M1 $\frac{1}{9}(3a + 2) \geq 3a - 6 \Leftrightarrow 3a + 2 \geq 27a - 54$

$\Leftrightarrow 8(3a - 7) \leq 0$

A1 $\Leftrightarrow a \leq \frac{7}{3}$ (which, actually, affects nothing, but working should appear)

⑧

Thus

B1B1B1 $M(a) = \begin{cases} \frac{1}{9}(3a+2) & 0 \leq a \leq 2 \\ \frac{1}{9}a^3 & 2 \leq a \leq 3 \\ 3a - 6 & a \geq 3 \end{cases}$ (Ignore 'non-unique' allocation of endpoints) ③

(Do not award marks for correct answers unsupported or from incorrect working)

SI-2015/Q8

(i) $S = 1 + 2 + 3 + \ldots + (n-2) + (n-1) + n$

M1 $S = n + (n-1) + (n-2) + \ldots + 3 + 2 + 1$ Method

M1 $2S = n \times (n+1)$ Adding

A1 $S = \frac{1}{2}n(n+1)$ obtained *legitimately* (**Given Answer**)

(Allow alternatives using induction or the *Method of Differences*, for instance, but **NOT** by stating that it is an AP and just quoting a formula; ditto Δ-number formula)

③

(ii) $(N-m)^k + m^k$ (k odd)

M1A1 $= N^k - \binom{k}{1}mN^{k-1} + \binom{k}{2}m^2N^{k-2} - \ldots + \binom{k}{k-1}m^{k-1}N - m^k + m^k$

E1 which is clearly divisible by N (since each term has a factor of N)

(Allow alternatives using induction, for instance)

③

Let $S = 1^k + 2^k + \ldots + n^k$ an odd no. of terms

M1 $= 0^k + 1^k + 2^k + \ldots + n^k$ an even no. of terms

M1 $= \left[(n-0)^k + 0^k\right] + \left[(n-1)^k + 1^k\right] + \ldots + \left[(\frac{1}{2}n + \frac{1}{2})^k + (\frac{1}{2}n - \frac{1}{2})^k\right]$

(no need to demonstrate final pairing but must explain fully the pairing up *or* the single extra n^k term)

E1 and, by (ii), each term is divisible by n.

③

For $S = 1^k + 2^k + \ldots + n^k$ an even no. of terms

M1 $= 0^k + 1^k + 2^k + \ldots + n^k$ an odd no. of terms

M1 $= \left[(n-0)^k + 0^k\right] + \left[(n-1)^k + 1^k\right] + \ldots + \left[(\frac{1}{2}n + 1)^k + (\frac{1}{2}n - 1)^k\right] + \left(\frac{1}{2}n\right)^k$

(no need to demonstrate final pairing but must explain the pairing and note the separate, single term)

and, by (ii), each paired term is divisible by n

E1 and the final single term is divisible by $\frac{1}{2}n \Rightarrow$ required result

③

M1 By the above result … for n even, so that $(n+1)$ is odd

A1 $(n+1) \mid 1^k + 2^k + \ldots + n^k + (n+1)^k$

E1 $(n+1) \mid S + (n+1)^k \Rightarrow (n+1) \mid S$

M1 By the above result … for n odd, so that $(n+1)$ is even

A1 $\frac{1}{2}(n+1) \mid 1^k + 2^k + \ldots + n^k + (n+1)^k$

E1 $\frac{1}{2}(n+1) \mid S + (n+1)^k \Rightarrow \frac{1}{2}(n+1) \mid S$ (as $\frac{1}{2}(n+1)$ is an integer)

E1 Since $\text{hcf}(n, n+1) = 1 \Rightarrow \text{hcf}(\frac{1}{2}n, n+1) = 1$ for n even

E1 and $\text{hcf}(n, \frac{1}{2}(n+1)) = 1$ for n odd

So it follows that $\frac{1}{2}n(n+1) \mid S$ for all positive integers n ⑧

SI/15/Q9

M1	Time taken to land (at the level of the projection) (from $y = ut\sin\alpha - \frac{1}{2}gt^2$, $y = 0$, $t \neq 0$)
A1	is $t = \dfrac{2u\sin\alpha}{g}$ (may be implicit)
M1	Bullet fired at time $t\left(0 \le t \le \dfrac{\pi}{6\lambda}\right)$ lands at time
A1	$T_L = t + \dfrac{2u}{g}\sin\left(\dfrac{\pi}{3} - \lambda t\right)$
M1A1	$\dfrac{\mathrm{d}T_L}{\mathrm{d}t} = 1 - \dfrac{2\lambda u}{g}\cos\left(\dfrac{\pi}{3} - \lambda t\right) = \dfrac{1}{k}\left\{k - \cos\left(\dfrac{\pi}{3} - \lambda t\right)\right\}$
A1	$= 0$ when $k = \cos\left(\dfrac{\pi}{3} - \lambda t\right)$
M1A1	Horizontal range is $R = \dfrac{2u^2\sin\alpha\cos\alpha}{g}$ (from $y = ut\sin\alpha - \frac{1}{2}gt^2$ with above time)
A1	$\Rightarrow R_L = \dfrac{2u^2}{g}k\sqrt{1-k^2}$ obtained *legitimately* (**Given Answer**) ⑩
M1A1	$\dfrac{\mathrm{d}^2T_L}{\mathrm{d}t^2} = -\dfrac{2\lambda^2 u}{g}\sin\left(\dfrac{\pi}{3} - \lambda t\right) < 0 \Rightarrow$ maximum distance
M1A1	$0 \le t \le \dfrac{\pi}{6\lambda}$ in $k = \cos\left(\dfrac{\pi}{3} - \lambda t\right) \Rightarrow \dfrac{1}{2} \le k \le \dfrac{\sqrt{3}}{2}$ ④
M1	If $k < \frac{1}{2}$ then $\dfrac{\mathrm{d}T_L}{\mathrm{d}t} < 0$ throughout the gun's firing …
A1	… and T_L is a (strictly) decreasing function.
M1	Then T_L max. occurs at $t = 0$
A1	i.e. $\alpha = \dfrac{\pi}{3}$
M1A1	and $R_L = \dfrac{2u^2}{g} \times \dfrac{1}{2} \times \dfrac{\sqrt{3}}{2} = \dfrac{u^2\sqrt{3}}{2g}$ ⑥

SI-2015/Q10

B1	Speed of rain relative to bus is $v\cos\theta - u$ (or $u - v\cos\theta$ if negative)
M1A1	When $u = 0$, $A \propto hv\cos\theta + av\sin\theta$ (width of bus and time units may be included as factors)
E1	When $v\cos\theta - u > 0$, rain hitting top of bus is the same, and rain hits back of bus as before, but with $v\cos\theta - u$ instead of $v\cos\theta$
E1	When $v\cos\theta - u < 0$, rain hitting top of bus is the same, and rain hits front of bus as before, but with $u - v\cos\theta$ instead of $v\cos\theta$
A1	Together, $A \propto h\,\lvert v\cos\theta - u\rvert + av\sin\theta$ Fully justified (**Given Answer**)

⑥

M1	Journey time $\propto \dfrac{1}{u}$ so we need to minimise
A1	$J = \dfrac{av\sin\theta}{u} + \dfrac{h\,\lvert v\cos\theta - u\rvert}{u}$ (Ignore additional constant-of-proportionality factors)
M1	For $v\cos\theta - u > 0$, if $w \le v\cos\theta$, we minimise $J = \dfrac{av\sin\theta}{u} + \dfrac{hv\cos\theta}{u} - h$
E1	and this decreases as u increases
E1	and this is done by choosing u as large as possible; i.e. $u = w$
M1	For $u - v\cos\theta > 0$, we minimise $J = \dfrac{av\sin\theta}{u} - \dfrac{hv\cos\theta}{u} + h$
E1	and this decreases as u increases if $a\sin\theta > h\cos\theta$
E1	so we again choose u as large as possible; i.e. $u = w$

[Note: minimisation may be justified by calculus in either case or both.]

⑧

M1	If $a\sin\theta < h\cos\theta$, then J increases with u when u exceeds $v\cos\theta$
A1	so we choose $u = v\cos\theta$ in this case

②

M1A1	If $a\sin\theta = h\cos\theta$ then J is independent of u, so we may as well take $u = w$ ②

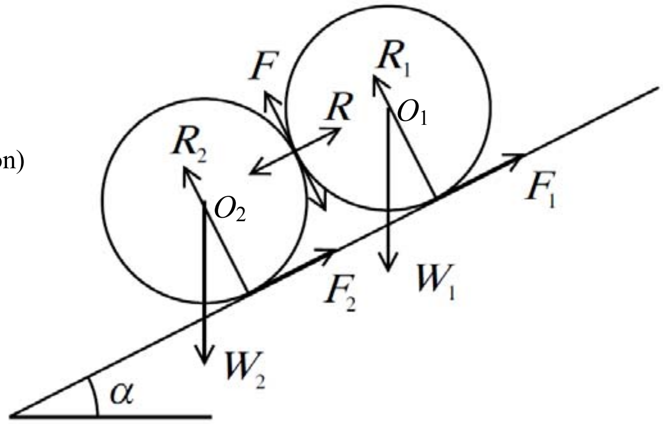
M1	Replacing θ by $180^\circ - \theta$ gives $J = \dfrac{av\sin\theta}{u} + \dfrac{hv\cos\theta}{u} + h$
A1	Which always decreases as u increases, so take $u = w$ again

②

SI-2015/Q11

(i) **B1** $\circlearrowleft O_1$**:** $F = F_1$

$\circlearrowleft O_2$**:** $F = F_2$ (Both, with reason)



①

(ii) **B1** Res^g. ||plane (for C_1)**:** $F_1 + R = W_1 \sin\alpha$ ①

B1 Res^g.$\perp^r$. plane (for C_1)**:** $R_1 + F = W_1 \cos\alpha$ ②

B1 Res^g.||plane (for C_2)**:** $F_2 - R = W_2 \sin\alpha$ ③

B1 Res^g.$\perp^r$. plane (for C_2)**:** $R_2 - F = W_2 \cos\alpha$ ④

Max 4 marks to be given for four independent statements (though only 3 are required).
One or other of

Res^g.||plane (for system) **:** $F_1 + F_2 = (W_1 + W_2)\sin\alpha$

Res^g.$\perp^r$. plane (for system) **:** $R_1 + R_2 = (W_1 + W_2)\cos\alpha$

may also appear instead of one or more of the above.

(F_1 and F_2 may or may not appear in these statements as F, but should do so below)

④

M1A1 Equating for $\sin\alpha$ **:** $\dfrac{F+R}{W_1} = \dfrac{F-R}{W_2}$ using ① and ③

M1A1 Re-arranging for F in terms of R**:** $F = \left(\dfrac{W_1 + W_2}{W_1 - W_2}\right)R$

M1 Use of the Friction Law, $F \le \mu R$

A1 $\Rightarrow \dfrac{W_1 + W_2}{W_1 - W_2} \le \mu$ obtained *legitimately* (**Given Answer**)

⑥

M1A1 (e.g.) ①÷② $\Rightarrow$ $\tan\alpha = \dfrac{F+R}{R_1+F}$

M1A1 Subst^g. for R $= \dfrac{F + F\left(\dfrac{W_1 - W_2}{W_1 + W_2}\right)}{R_1 + F}$ using $R = \left(\dfrac{W_1 - W_2}{W_1 + W_2}\right)F$

A1 $= \dfrac{F\left(\dfrac{2W_1}{W_1 + W_2}\right)}{R_1 + F_1}$

M1A1 Subst^g. for R_1 (correct inequality) using Friction Law $F_1 \le \mu_1 R_1 \Leftrightarrow R_1 \ge \dfrac{F_1}{\mu_1}$

$$\le \frac{F\left(\dfrac{2W_1}{W_1 + W_2}\right)}{\dfrac{F_1}{\mu_1} + F_1}$$

M1 Tidying-up algebra $= \dfrac{F\left(\dfrac{2W_1}{W_1 + W_2}\right)}{F\left(\dfrac{1+\mu_1}{\mu_1}\right)}$

A1 $\Rightarrow \tan\alpha \le \dfrac{2\mu_1 W_1}{(1+\mu_1)(W_1 + W_2)}$ obtained *legitimately* (**Given Answer)**

⑨

SI-2015/Q12

(i) **M1A1** P(exactly r out of n need surgery) = $\binom{n}{r}\left(\frac{1}{4}\right)^r\left(\frac{3}{4}\right)^{n-r}$ (A binomial prob. term; correct)

2

(ii) **M1** $\text{P}(S=r)=\sum_{n=r}^{\infty}\frac{e^{-8}8^n}{n!}\times\frac{n!}{r!(n-r)!}\left(\frac{1}{4}\right)^r\left(\frac{3}{4}\right)^{n-r}$ Attempt at sum of appropriate product terms

B1B1A1 Limits✓✓ All internal terms correct; allow ${}^{n}C_r$ for the A mark

M1 $=\frac{e^{-8}}{r!}\sum_{n=r}^{\infty}\frac{8^n}{(n-r)!}\times\left(\frac{1}{4}\right)^r\left(\frac{3}{4}\right)^{n-r}$ Factoring out these two terms

M1 $=\frac{e^{-8}}{r!}\sum_{n=r}^{\infty}\frac{8^n}{(n-r)!}\times\frac{3^{n-r}}{4^n}$ Attempting to deal with the powers of 3 and 4

A1 $=\frac{e^{-8}}{r!}\sum_{n=r}^{\infty}\frac{2^n\times 3^{n-r}}{(n-r)!}$ Correctly

M1 $=\frac{e^{-8}\times 2^r}{r!}\sum_{n=r}^{\infty}\frac{6^{n-r}}{(n-r)!}$ Splitting off the extra powers of 2 ready to …

M1 $=\frac{e^{-8}\times 2^r}{r!}\sum_{m=0}^{\infty}\frac{6^m}{m!}$ … adjust the lower limit (i.e. using $m=n-r$)

A1 $=\frac{e^{-8}\times 2^r}{r!}\times e^6$ i.e. $\frac{e^{-2}\times 2^r}{r!}$

A1 … which is Poisson with mean 2 (Give **B1** for noting this without the working) 11

(iii) **M1** $\text{P}(M=8\mid M+T=12)$ Identifying correct conditional probability outcome

A1A1A1 $=\dfrac{\frac{e^{-2}\times 2^8}{8!}\times\frac{e^{-2}\times 2^4}{4!}}{\frac{e^{-4}\times 4^{12}}{12!}}$ One A mark for each correct term (& no extras for 3rd A mark)

A1A1 $=\frac{2^{12}\times 12!}{4^{12}\times 8!\times 4!}$ Powers of e cancelled; factorials in correct part of the fraction – (unsimplified is okay at this stage)

A1 $=\frac{495}{4096}$ 7

SI-2015/Q13

Reminder

A : the 1^{st}6 arises on the n^{th} throw
B : at least one 5 arises before the 1^{st}6
C : at least one 4 arises before the 1^{st}6
D : exactly one 5 arises before the 1^{st}6
E : exactly one 4 arises before the 1^{st}6

(i) **M1A1** $\quad P(A) = \left(\frac{5}{6}\right)^{n-1}\left(\frac{1}{6}\right)$ ②

(ii) **M1A1** $\quad$ By symmetry (either a 5 or a 6 arises before the other), $P(B) = \frac{1}{2}$ ②

(iii) **M1** $\quad$ The first 4s, 5s, 6s can arise in the orders **<u>456</u>**, 465, **<u>546</u>**, 564, 645, 654

A1 $\quad \Rightarrow P(B\cap C) = \frac{1}{3}$ (i.e. "by symmetry" but with three pairs)

②

(iv) **M1A1A1** $\quad P(D) = \left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \binom{2}{1}\left(\frac{1}{6}\right)\left(\frac{4}{6}\right)\left(\frac{1}{6}\right) + \binom{3}{1}\left(\frac{1}{6}\right)\left(\frac{4}{6}\right)^2\left(\frac{1}{6}\right) + \ldots$

M1 for infinite series with 1^{st}term ✓; A1 for 2^{nd} term ✓; A1 for 3^{rd} term and following pattern ✓

M1 $\quad = \left(\frac{1}{36}\right)\left\{1 + 2\left(\frac{2}{3}\right) + 3\left(\frac{2}{3}\right)^2 + \ldots\right\}$ $\quad$ For factorisation and an infinite series

M1 $\quad = \left(\frac{1}{36}\right)\left(1 - \frac{2}{3}\right)^{-2}$ $\quad$ Use of the given series result

A1 $\quad = \frac{1}{4}$

⑥

(v) **M1** $\quad P(D\cup E) = P(D) + P(E) - P(D\cap E)$ $\quad$ Stated or used

B1 $\quad P(E) = P(D) =$ answer to (iv) $\quad$ Stated or used anywhere

M1A1A1 $\quad P(D\cap E) = \left(\frac{2}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \binom{3}{1}\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \binom{4}{2}\left(\frac{3}{6}\right)^2\left(\frac{2}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) + \ldots$

M1 for infinite series with 1^{st}term ✓; A1 for 2^{nd} term ✓; A1 for 3^{rd} term and following pattern ✓

M1 $\quad = \left(\frac{1}{108}\right)\left\{1 + 3\left(\frac{1}{2}\right) + 6\left(\frac{1}{2}\right)^2 + \ldots\right\}$ $\quad$ For factorisation and an infinite series

M1 $\quad = \left(\frac{1}{108}\right)\left(1 - \frac{1}{2}\right)^{-3}$ $\quad$ Use of the given series result

A1 $\quad \Rightarrow P(D\cup E) = \frac{1}{2} - \frac{2}{27} = \frac{23}{54}$

⑧