General Remarks

There were just under 1000 entries for paper II this year, almost exactly the same number as last year. Of this number, more than 60 scored over 90% while, at the other end of the scale, almost 200 failed to score more than 40 marks. In hindsight, many of the pure maths questions were a little too accessible and lacked a sufficiently tough ‘difficulty gradient’, so that scores were slightly higher than anticipated. This was reflected in the grade boundaries for the “1” and the “2” (around ten marks higher than is generally planned) in particular. Next year’s questions may be expected to be a little bit more demanding, but only in the sense that the final 5 or 6 marks on each question should have rather more *bite* to them: it should certainly not be the case that all questions are tougher to get into at the outset.

Most candidates attempted the requisite number of questions (six), although many of the weaker brethren made seven or eight attempts, most of which were feeble at best and they generally only picked up a maximum of 5 or 6 marks per question. It is a truth universally acknowledged that practice maketh if not perfect then at least a whole lot better prepared, and choosing to waste time on a couple of extra questions is not a good strategy on the STEPs. The major down-side of the present modular examination system is that students are not naturally prepared to approach the subject holistically; ally this to the current practice of setting highly-structured, fully-guided questions requiring no imagination, insight, depth or planning from A-level candidates in a system that fails almost nobody and rewards even the most modestly able with high grades in a manner reminiscent of a dentist giving lollipops to kids who have done little more than been brave and seen the course through, it is even more important to ensure a full and thorough preparation for these papers. The 20% of the entry who seem to be either unprepared for the rigours of a STEP, or unwittingly possessed of only a smattering of basic advanced-level skills, seems to be remarkably steady year-on-year, even in a year when their more suitably prepared compatriots found the paper appreciably easier than usual.

As in previous years, the pure maths questions provided the bulk of candidates’ work, with relatively few efforts to be found at the applied ones.

Comments on individual questions

Q1 This question was attempted by almost two-thirds of the candidature, with a mean mark of around $11\frac{1}{2}$. Whilst most attempts were very successful, a lot of marks were lost by poorly structured working, where the candidate got themselves confused in some way or another. The only two common conceptual difficulties were the oversight of the equal gradients at the point of contact and the lack of a suitable circle equation to start working with. Apart from these, most candidates’ work went smoothly and successfully, although sign errors often cost them at least one of the final three answer marks.

Q2 This was the most popular question on the paper, drawing an attempt from almst every candidate. There were several proofs of the initial trigonometric identities using *de Moivre’s Theorem* but most settled for the more standard cosine and sine of $(2x + x)$. Personally, I was against the inclusion of the given answer of $\cos^{-1}\left(-\frac{1}{6}\right)$ in (ii) as it led to what struck me as an unwelcome dichotomy of approaches. Most candidates opted to verify that the two polynomials in “c” that arose gave the same numerical answer, and this working was not entirely straightforward – in the event, lots of candidates failed to *show* the markers that they had done the working correctly for both expressions – whereas my original intention had been that they should collect terms up into a single polynomial equation and factorise it by first spotting the (repeated) factor $(c - 1)$ hinted at in (i).

There was one important mathematical oversight that many candidates made during this question, and it was due to not reading the question sufficiently carefully. The wording of the question in (ii) clearly states that Eustace's misunderstanding of the integration of powers of the sine function was for $n = 1, 2, 3, \dots$. Unfortunately, rather a lot of candidates thought that he would then have integrated $\sin x$ (i.e. the case $n = 1$) *correctly* as $-\cos x$. We concocted a mark-scheme for this eventuality which allowed candidates '*follow-through*' for 6 out of the 10 marks allocated here, but the self-imposed penalty of four marks could not be avoided as it was just no longer possible to get, for instance, the given answer.

Finally, there is a bit of an apology to make: at some final stage of the printing process, the bit of the question that identified α as lying in the range 0 to π got removed; this left candidates having to think about general solutions rather than just the two decently small ones that had been looked-for when the question was first written. Nevertheless, not only was this the most popular question for number of attempts, it was also the most successful for candidates with a man score of almost 15.

Q3 One doesn't need to be too devoted a mathematician to recognise the *Fibonacci numbers* in this question, and many candidates clearly recognised this sequence. However, they were still required to answer the question in the way specified by the wording on the paper and a lot of attempts foundered at part (ii). This was the second most frequently attempted question, yet drew the second worst marks, averaging just over 8. Most attempts got little further than (i), and many foundered even here due to a lack of appreciation of the difference of two cubes factorisation. Things clearly got much worse in (ii) when far too many folks seemed incapable of attempting a binomial expansion of $\left(1+\sqrt{5}\right)^6$; many who did manage a decent stab at this then repeated the work for $\left(1-\sqrt{5}\right)^6$. Very few sorted this out correctly and, as a result, there were relatively few stabs at part (iii).

Q4 This question received about the same number of "hits" as Q1 and came out with an average mark only fractionally lower. For the majority, the introductory work was successfully completed along with the rest of (i), although a lot of candidates' working was very unclear in the first integral, involving logarithms. One or two marks were commonly lost as the correct answer of $\frac{1}{2}$ could easily have been guessed from the initial result, and the working produced by the candidates failed to convince markers that it had been obtained legitimately otherwise. The fault was often little more than failing either to identify the relevant "f(x)" or to show it implicitly by careful presentation of the working of the log. function.

The excellent part (ii) required candidates to mimic the method used to find the opening result rather than repeat its use in a new case, and this was only accessible to those with that extra bit of insight or determination.

Q5 This was the least popular question on the paper and attracted the lowest average score of about 7. This is partly explained by the way that, like Q3 and Q6 particularly, it drew a lot of attempts from desperate weaker students who started, only to give up before too long (in order, presumably, to try yet another question in some hit-and-miss approach, scrambling for odd marks here and there). Of those who persevered, there were plenty of marks to be had. Little more was required than the use of the scalar product, a careful application of algebra, and a modest grasp of the geometrical implications of what the working represented.

Q6 Of the pure maths questions on the paper, only Q5 and this one attracted attempts from under half the candidature; this despite the fact that it is obviously (to the trained eye, at least) the easiest question on the paper. Parts (i) and (ii) require nothing more than GCSE trigonometry, and (iii) can be done in one line if one knows a little bit about geometric centres of 3-d shapes. Clearly 3-dimensional objects, and the associated trig., are sufficiently daunting to have put most folks off either completely or early on in the proceedings, and the average mark scored here was under 10.

Q7 This proved to be the second most popular question on the paper, both by choice and by success. I imagine that its helpful structure probably contributed significantly to both. Part of the problem is that there are ways to do this using methods not on single maths specifications, so it was necessary to be quite specific. Nonetheless, there were still areas where marks were commonly lost; in (i), candidates were required to show that both TPs lie *below* the x-axis and, while one of the y-coordinates was obviously negative (being the sum of three negative terms), the other one was only obviously so by completing the square. The problems found by candidates, even in the first case, just highlights the widespread difficulty found by students when dealing with inequalities.

Q8 This was the sixth most popular of the pure maths questions, with an average score of just under 11. Many of the early marks were easily gained, although the sketches were often unclear enough to warrant a loss of marks – in particular, the fact that the required function oscillates between e^{-x} and $-\mathrm{e}^{-x}$ was seldom made obvious; indeed, a clear indication that the function's zeroes occurred at (regular) intervals of π units on the x-axis was also poorly indicated. Although most candidates were happy to attempt integration by parts successfully, and then subtract areas, the limits of integration, x_n and x_{n+1}, were only occasionally correctly identified. This meant that a lot of the following work, whilst correct in method, was seldom likely to get to the correct, given answer. The final piece of work, even though it could be found using this given answer, was poorly attempted.

Q9 Despite the fact that this question required two pieces of identical working in order to obtain the given results (the second following almost immediately from the first, if reasoned appropriately) and that each could be obtained by considering either distances or times, this was a very unpopular question, eliciting only 153 attempts scoring an average of under 7 marks (the poorest average mark of all questions). The key observation was that the two particles are always at the same height, hence share a common vertical component of speed.

Q10 Eliciting exactly the same number of responses as Q9, this question was found a little easier, but only because the first part was very standard A-level "collisions" work. Applying this first result repeatedly required only clear thinking and clear presentation, and those who persevered generally scored quite highly and opened up the prospect of some straightforward log. work in the final part of (ii). The biggest hurdle to a completely successful solution usually arose in poor numerical justification of the final answer.

Q11 This was the most popular of the applied maths questions, shortly ahead of Q12 for the number of "hits" received but still well behind the popularity of any of the pure questions. Having initially expected that candidates would recognise a '*3-force problem*' and use *Lami's Theorem* or a triangle of forces, no-one did. Instead, attempts merely went for the "resolve twice and take moments" strategy, which worked very well in principle, but were often hampered by lack of care over the angles involved. Uses of the sin/cos($A \pm B$) formulae were good, although a lot of candidates got a bit confused before arriving at the given result. The final piece of work was just a bit of pure maths. Interestingly enough, just one or two candidates appealed to a result I had never heard of before: the "*Cot Rule*", which (upon investigation) turned out to be perfectly legitimate.

Q12 Only marginally less popular than Q12, and scoring marginally better on average (10.6 against 10.4), at least the first half of the question was straightforward work on a continuous probability distribution. Those who kept their nerve in (ii) when dealing with the median found many of the later marks were easily acquired also, the biggest hurdle to complete success (again) being the poor skills on display when justifying results involving inequalities.

Q13 This question worked almost exactly like Q12, in the sense that the start required some straightforward (probability) work, followed by an extension that needed only careful handling, before finishing with some poorly-handled inequalities work. The biggest problem for candidates lay in their lack of care to show that their chosen values of *p* and *q* actually satisfied any claimed conditions.

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