

### STEP 3 2014 Examiners' report

A 10% increase in the number of candidates and the popularity of all questions ensured that all questions had a good number of attempts, though the first two questions were very much the most popular. Every question received at least one absolutely correct solution. In most cases when candidates submitted more than six solutions, the extra ones were rarely substantial attempts. Five sixths gave in at least six attempts.

1. This was the most popular question on the paper, being attempted by approximately 14 out of every 15 candidates. It was the second most successfully attempted with a mean score of half marks. The stem of the question caused no problems, but a common mistake in part (i) was to attempt derivatives to obtain the desired result. Most candidates came unstuck in part (ii), making it much more difficult for themselves by attempting to work with expressions in  $a$ ,  $b$ , and  $c$  rather than using the log series working with  $q$  and  $r$ , and as a result making sign errors, putting part (iii) beyond reach, and although they could find counterexamples for the claim in part (iv), they did so without the clear direction that working with the expressions in  $q$  and  $r$  would have made obvious.

2. This was only marginally less popular than question 1, but was the most successfully attempted with a mean of two thirds marks. Most that attempted the question were able to do the first two parts easily, but could not find a suitable substitution to do the last part. In about a tenth of the attempts, a helpful substitution was made in part (iii) which then usually resulted in successful completion of the question. Modulus signs were often ignored, or could not be distinguished from usual parentheses, and the arbitrary constant, even though it appeared in the result for part (i), was frequently overlooked. A few did not use the correct formulae for  $\cosh 2x$ , instead resorting to the trigonometric versions. A handful of candidates attempted partial fractions in the last part having correctly factorised the quartic, but this did not use the previous parts as instructed.

3. About half of the candidates attempted this, but it was the second least successfully attempted with a mean score just below a quarter marks. Most managed the first result, with those not doing so falling foul of various basic algebraic errors. The second result of part (i) was often answered with no justification. The second part was poorly done with a variety of approaches attempted such as obtaining a distance function, and then using completing the square or differentiation, or investigating the intersections of the circle and parabola. Few considered the geometry of the parabola and its normal which would have yielded the results fairly simply.

4. Two thirds of the candidature attempted this but with only moderate success earning just a third of the marks. The very first result was frequently obtained although some fell at the first hurdle through not appreciating that they needed to use  $\sec^2 x = 1 + \tan^2 x$ , or else that there was then an exact differential. The second result in part (i) was 'only if' whereas many read it, or answered it, as 'if'. In part (ii), most spotted  $b = a$ . There were many inappropriate functions suggested for the last part of the question, many which ignored the requirement that  $y = 0, x = 1$ .

5. This was a moderately popular question attempted by half the candidates, with some success, scoring a little below half marks. There were some basic problems exposed in this question such as the differences between a vector and its length, the negative of a vector and the vector, and the meaning of 'if and only if' resulting in things being shown in one direction only throughout the question. Part (i) was generally well done, but in part (ii), it was commonly forgotten that there

were two conditions for XYZT to be a square. Approaches using real and imaginary parts (breaking into components) were not very successful.

6. The third most popular question, as well as the third most successful being only marginally behind question one in marks, having been attempted by about 70%. Many did not use the required starting point, instead resorting to monotonicity or drawing pictures (graphs) which were not proofs. In parts (i) especially and (ii) as well, candidates failed to use the result that  $f(t) > 0$ , cavalierly using  $f(t) < 0$ , or even  $f(t) < 1$  without justification. Many made complicated choices of functions for (i) and (ii), and then got lost in their differentiations, and finally there was frequent lack of care to ensure that quantities dividing inequalities were positive.

7. Roughly two fifths of the candidates attempted this with a mean score of just over three marks making it the least well attempted question on the paper. Most could do part (i), which is GCSE material, but frustratingly quite a few stated that the triangles were similar with no justification. Part (ii) was by far the most poorly attempted part with a lot of hand-waving arguments. Part (iii) was done well by virtue of only the best candidates making it past part (i) with 75% of solutions containing good proofs by contradiction for the first result and the last two parts were pretty well done.

8. Just fewer than half the candidates attempted this scoring just over a third of the marks. Many managed all but part (iii) easily but few managed that last part, and most did not try it. In part (i), having correctly used the result from the stem, there was frequently not enough care taken in extending this to the full sum. A not infrequent error of logic was that  $\sum_{r=1}^{N+1} 1/r < N + 1$  and  $\lim_{N \rightarrow \infty} N + 1 = \infty$  somehow implies that  $\sum_{r=1}^{\infty} 1/r$  does not converge.

9. A fifth attempted this, scoring at the same level as question 8. The first differentiation and the verification of the initial conditions were managed but very few bothered to check that the equation of motion was satisfied. Most obtained the first displayed result but few realised that  $\beta$  was the angle of depression rather than elevation and this generated plenty of sign errors. A few did achieve the very final result.

10. This was attempted by a quarter of the candidates with scores just below those achieved in question 5. Good candidates could obtain full marks in less than a page of working whilst weak ones spent a lot of effort trying to solve differential equations for  $x$  and  $y$  because they hadn't spotted the change of variables to  $x - y$  and  $x + y$ . The vast majority could obtain the first equation, often using the given result as a guide. However, there was frequent confusion between extension and total length of the springs. In addition, sign errors in  $\frac{d^2y}{dt^2}$  prevented the next part from working out. Lots did not realise to work with  $x - y$  and  $x + y$ , but those that saw SHM in one of these, saw it in the other. Likewise, with initial conditions, quite a few overlooked  $\frac{dx}{dt} = 0, \frac{dy}{dt} = 0$ , which prevented them solving for the constants, and also the sign was often overlooked in the condition  $y = -\frac{1}{2}a$ . In attempting the last result, some used the factor formula, which worked but was unnecessary. Quite often, they stumbled over the final step of logic ending up with apparent contradictions such as  $\sqrt{3} = 1$ , which is of course false, but did not demonstrate full understanding.

11. Just marginally more popular than question 10, it was attempted with the same level of success. Provided that a correct figure (and it didn't matter whether  $P$  was above the level of  $B$  or not) was drawn, and that resolving was correctly conducted, then candidates could obtain the two tensions in general, in which case the inequality frequently followed. However, the geometric result stumped many; the few completing it did so via the cosine rule and completing the square. At this point, the final results usually followed for candidates still on track.

12. Less than 8% tried this, scoring just over a quarter of the marks. Very few got the question totally correct, but a number got it mostly right. Nearly all managed the median of  $Y$ , but the probability density function of  $Y$  caused some to stumble. However the mode result, apart from some poor differentiation, was mostly alright. The explanation in part (iii) eluded some candidates who were otherwise strong. Applying the mode result in part (iv) to obtain  $\lambda$  surprisingly tripped up some merely through inaccurate differentiation. As one would hope, anyone that got as far as part (iv) spotted that the median of  $X$  was .

13. Just a handful of candidates more attempted this than question 12, but scoring marginally less with one quarter marks. A small number did just part (i), but otherwise candidates tended to either score zero or nearly all of the marks. There was some very shaky logic in finding the first result of part (iii) and then failing to deduce, as required, the probability result. Quite often, the working for  $\mu$  in part (iv), whilst usually correct, was extremely convoluted.