STEP MATHEMATICS 1

2018

Examiner's Report

General Comments

In order to get the fullest picture, this document should be read in conjunction with the question paper, the marking scheme and (for comments on the underlying purpose and motivation for finding the right solution-approaches to questions) the *Hints and Solutions* document.

The purpose of the STEPs is to learn what students are able to achieve mathematically when applying the knowledge, skills and techniques that they have learned within their standard A-level (or equivalent) courses … but seldom within the usual range of familiar settings. STEP questions require candidates to work at an extended piece of mathematics, often with the minimum of specific guidance, and to make the necessary connections. This requires a very different mind-set to that which is sufficient for success at A-level, and the requisite skills tend only to develop with prolonged and determined practice at such longer questions.

One of the most crucial features of the STEPs is that the routine technical and manipulative skills are almost taken for granted; it is necessary for candidates to produce them with both speed and accuracy so that the maximum amount of time can be spent in thinking their way through the problem and the various hurdles and obstacles that have been set before them. Most STEP questions begin by asking the solver to do something relatively routine or familiar before letting them loose on the real problem. Almost always, such an opening has not been put there to allow one to pick up a few easy marks, but rather to point the solver in the right direction for what follows. Very often, the opening result or technique will need to be used, adapted or extended in the later parts of the question, with the demands increasing the further on that one goes. So a candidate should never think that they are simply required to 'go through the motions'; rather they will, sooner or later, be required to show either genuine skill or real insight in order to make a reasonably complete effort. The more successful candidates are the ones who manage to figure out how to move on from the given starting-point.

Finally, reading through a finished solution is often misleading – even unhelpful – unless you have attempted the problem for yourself. This is because the thinking has been done for you. So, when you read through the report and look at the solutions (either in the mark scheme or the *Hints and Solutions* booklet), try to figure out how you could have arrived at the solution, learn from your mistakes and pick up as many tips as you can whilst working through past paper questions.

This year far too many candidates wasted time by attempting more than six questions, with many of these candidates picking up 0-4 marks on several 'false starts' which petered out the moment some understanding was required.

There were almost 2000 candidates for this SI paper. Almost one-sixth of candidates failed to reach a total of 30 and around two-thirds fell below half-marks overall. This highlights the fact that many candidates don't find this test an easy one. At the other end of the spectrum, almost one-in-ten managed a total of 84 out of 120 – these candidates usually marked out by their ability to complete whole questions – with almost 4% of the entry achieving the highly praiseworthy feat of getting into three-figures with their overall score.

The paper is constructed so that question 1 is very approachable indeed, the intention being to get everyone started with some measure of success; unsurprisingly, Q1 was the most popular question of all, with almost all candidates attempting it, and it also turned out to be the most successful question on the paper with a mean score of more than 15 out of 20. Around 7% of candidates *didn't* make any kind of attempt at it at all.

In order of popularity, Q1 was followed by Qs. 2, 7, 4 and 3. Indeed, it was the pure maths questions in Section A that attracted the majority of attention from candidates, with the most popular applied question (Q9, mechanics) still getting fewer 'hits' than the least popular pure question (Q5). Questions 10, 11 and 13 proved to attract very little attention from candidates and many of the attempts were minimal.

Question 1

This was the most popular question and elicited the highest average score of any question on the paper. This is hardly surprising, given that it is the question that most closely resembles a long question from an A-level paper and that its demands stand out as, at least, manageable. This proved to be the case with almost all takers, the most noticeable shortcomings arising with the inequality result at the end: many candidates didn't seem to realise that it is (almost) always easier to consider the sign of some variable quantity than to prove that one variable is greater or less than another. In this case, all it takes is to turn $S > \frac{1}{3}T$ into either $S - \frac{1}{3}T > 0$ or $3S - T > 0$.

Another common fault amongst candidates is to arrive at a *given* answer (there are two here) without producing thoroughly convincing working to support it.

The final point to be made here is that a diagram (even a hastily drawn one that covers the basic features required) can be of immense value, even when not directly requested. In this question, rather a lot of solutions arose from (effectively) mistakenly having the point R on the x-axis.

Question 2

This question highlighted that most people had less than an adequate understanding of inequalities. Many students decided to turn the given statements into equalities (such as $\pi = 10$) and work through before finally, optimistically, inserting an inequality at the end. Others reversed inequalities when taking logs. Only a tiny number considered why taking logs of an inequality was acceptable.

Although there were many possible choices for the base of logarithms to work with, blindly manipulating things until the result appears was unlikely to get to the correct answer. Too many candidates produced a "stream of consciousness" including many correct statements but without any sense of creating an argument. Mathematics should be about communicating ideas. In particular, it is clear that many candidates did not know that it is flawed to start at the required result and manipulate until a known theorem is reached. Particularly when working with inequalities, these types of arguments are not always reversible.

Nonetheless, Q2 was the second most popular question and with the second highest mean score (just over half marks); this was mostly as a result of being given so much information upfront.

Question 3

Most candidates did reasonably well on part (i), getting almost full marks for that section, but there were relatively few attempts at (ii). Surprisingly, many candidates who did attempt part (ii) didn't realise that the best approach was to reverse their reasoning from part (i), and tried a completely different method, in some cases successfully.

In part (i), while the tangent solution was by far the most common successful approach, several other correct trigonometry solutions were found. However, most students who didn't use tangents made little useful progress and represented an overwhelming majority of attempts. A common slip in all these methods was cancelling a factor from both sides of an equation without mentioning that it was non-zero. There were also a very small number of elegant geometrical solutions obtained by adding another line to the diagram (either the angle bisector at S or the reflection of PS in the vertical line through P).

The paucity of attempts at part (ii) may well have been because there was no obvious way to proceed, but also possibly because some candidates confused the two directions of implication, thinking that $\beta = 2\alpha$ followed from part (i). While there was no requirement to sketch the hyperbola $y^2 = 3(x^2 - a^2)$, there were several incorrect sketches (ellipses, modified parabolae, etc.). Candidates who correctly found "$\beta = 2\alpha$ or $\beta = 2\alpha - \pi$" often did not clearly justify how they eliminated other possibilities.

Question 4

This was a very popular question, yet elicited a mean score of under 6 out of 20. The principal reason for this is that most attempts recognised that one should differentiate to start with; after that, efforts tailed off very rapidly indeed. So, in general, the first part was answered well with candidates showing a good systematic (although not always efficient) approach to differentiation. However, too many candidates thought that $(\ln x)^2 = 1$ was equivalent to $\ln x = 1$.

When integrating, the majority of candidates forgot the modulus sign in $\int \frac{1}{x}\, dx = \ln |x| + c$. It did cause many candidates to come unstuck when dealing with $\ln(-1)$ – with several candidates trying to ignore this term entirely which made the rest of the question trickier than should have been the case.

Very few candidates seemed to realise that nearly all the features of the final graph could be deduced from facts given in the question rather than the outcome of their integration. Many candidates did not link the facts given about f(x) with their sketches.

Question 5

Around a third of all candidates made an attempt at this question, yet it had the second lowest mean score of around only 4 marks out of 20.

To begin with, many candidates seemed to think that every polynomial begins with a coefficient of 1, and this led to serious difficulties with part (ii). Part (ii) also involved a *proof by contradiction* (or equivalent) and the exercise of choice … and most candidates aren't happy with finding one answer of what might be many. Those who spotted that $r = 2$ worked then frequently failed to verify it clearly.

Part (iii) was all about factors and the biggest obstacle to success lay in the widespread inability to think of integer factors as being anything other than positive; thus, the statement that 17 couldn't have four distinct factors – whilst correct – was almost invariably given because the candidate could only think of 1 and 17. In fact, 17 *does* have four integer factors, but the reason the 'proof' failed was actually because +17 and –17 couldn't both be used.

Most who persevered this far seemed to realise that factors of 16 were to be used but failed to give all the correct possibilities (or to show why there were no others) as a result of a poorly devised system for enumerating them.

Question 6

Half of all candidates attempted the question and the mean score was around 6.7 out of 20. The big issue here was that, while almost all attempts got off to a successful start, sooner or later they began to flounder as the technical demands increased. The opening results and part (i) were generally handled very well but B_n's result in (ii) required a bit more effort to finish it off, since the use of the initial result had to be supplemented with the trivial, but somewhat disguised, identity that $\cos(\pi - \theta) = -\cos\theta$ before the given answer could be obtained.

Thereafter, solutions generally fell into one of two groups: those who used trig. identities to complete the question and those who didn't. Those in the former group were much more successful than their counterparts in the latter group, most of whom made little progress and gave up.

Question 7

As with most of Qs. 3-8, this was a very popular question, but with a low mean score; in this case, of around 6 out of 20. Part (i) was generally done well although quite a lot of candidates made an algebraic error here. It was then possible to obtain full marks in the remaining sections by follow through, but few candidates followed through correctly. There were also a few candidates who found the z^3 and constant terms, but didn't actually check that the other terms cancelled. A rare alternative method to (i) was to write z in terms of x and substitute into $az^3 + b = 0$; where this method was used, it was generally well-executed.

A very large majority of candidates attempting part (ii) did so "backwards", showing instead that $d^2 > 4c^3$ follows from the relations between c, d and p, q. Those who correctly interpreted this part of the question often failed to justify strictness of the inequality.

There were more essentially correct responses to part (iii) as candidates were able to identify appropriate values of p and q, although arithmetic errors were common.

Few candidates made progress with part (iv), but those who did often got 3 marks out of 4 because they did not write the solutions in terms of c and/or d.

Question 8

Only two of the eight pure questions drew attempts from under 1000 candidates and this was one of them. Its mean score of 7.3 out of 20 made it the third most successful Section A question after Qs.1 & 2. Part of its success lay in the fact that – for those who were able to get past the sense that this had something to do with sine and cosine – it was actually a straightforward test of calculus in some 'pure' way, unadulterated by actual (known) functions.

Naturally, the differentiations in the early parts were more competently handled than the integrations of the latter parts, but there were many candidates who manipulated integrations by 'parts' and 'substitution' with considerable skill. Making all the connections, as results accumulated, wasn't too easy but those who persevered to the end scored very well indeed. However, those with "uncertainties" in their understanding of the various calculus approaches made little progress.

Question 9

This was the most popular of the applied questions, drawing more than 700 attempts. It was, after Qs.1 & 2, the highest-scoring question, one of only the three to average more than 10 out of 20.

Most candidates realised that an energy argument was useful in the first part of the question, but there were some spurious reasons given for the direction of the inequality. Candidates were then generally able to apply kinematics formulae to make progress; however, reaching the required form proved challenging. In particular, candidates were often unclear about the sign conventions they were using. When taking square roots in mechanics – either directly or in the context of the quadratic equation – the plus or minus solutions often have two physical interpretations and candidates were very bad at explaining why they were selecting one of the signs.

The algebra required to get to the given form was beyond most candidates. However, they could still have used the given form to answer the last part using some basic calculus.

Question 10

This question was neither popular nor well-handled by those starting it; part of the problem was that there appears to be too many things to consider … two engines and an unspecified number of carriages. A mean score of only just over 4 out of 20 is actually rather misleading here, since most of those attempts consisted of little more than an unused (and usually incomplete) diagram. Of those who made a serious attempt, most started this question well, considering Newton's 2nd Law for the front engine and the whole train. However, in later parts very few candidates considered a general carriage between the two engines which made any argument quite difficult.

Candidates generally had difficulty in linking the condition that $k > \frac{1}{2}n$ in any meaningful way because they were not sufficiently systematic in their consideration of carriages between the two engines and carriages after the two engines. Some candidates effectively did part (iii) before part (ii) and this was perfectly acceptable.

Question 11

A good diagram was a very important starting point for this question, but too many candidates did not use one or it was too small to be of any use. Interpreting the question using a diagram is a vital skill which we would advise requires practice. Candidates also often fooled themselves by drawing a misleading diagram – for example, assuming that BOA was a right angle.

In (ii) lots of candidates noticed that setting $\theta = 2\alpha$ gave the required result, however this selection was rarely justified in the context of the question. Equally the physical significance of the reaction force being zero was not clearly communicated by candidates.

In part (iii) candidates who resolved forces and used some trigonometric identities generally scored well, although justification for the condition being necessary was very rare.

The final part was beyond most candidates. This was the least popular question on the paper and the one with the lowest mean score.

Question 12

This was a reasonably popular question, with almost a third of the candidature making an attempt at it. Most of these candidates did (i) and (ii) well. It was quite common to state $N_1 \sim \text{Bin}(2, p)$ and write down the mean and variance of that. This was acceptable but, because the variance is given in the question, we required candidates to explicitly state the distribution rather than just giving the answers.

A lot of candidates struggled with (iii), commonly thinking it was still binomial or failing to condition on the chosen coins. While the most common successful approach was to calculate the combined probabilities of different numbers of heads, some candidates found the mean correctly by averaging the means of pairs of coins. It was very rare to obtain the variance correctly with this latter approach.

There were relatively few attempts at part (iv) which got the correct expression for the difference of the variances, and very few came up with the idea of completing the square. There was a very small number – no more than 5 – of correct solutions by the AM-GM inequality or more exotic methods. The condition for equality was often not justified.

Question 13

Part (i) was well done overall, though some candidates just listed (many) cases, often still getting the correct probabilities, and some compared expected scores (which received no credit). It was very common to assume $n = 2$ for part or all of the question and little credit was available for this (at most 5 out of 11 for part (i), 1 out of 4 for part (ii), and 4 out of 5 for part (iii)).

Many candidates did part (ii) well (marks were available here for following through wrong probabilities from (i)), but most did not consider a cancelling factor of $\frac{1}{6}$ arising from the probability of choosing a given k.

Most attempts at part (iii) were reasonably good, although many candidates got the binomial probabilities for the possible numbers of heads wrong.