STEP 2010 Paper I: Principal Examiner's Report

Introductory comments

There were significantly more candidates attempting this paper than last year (just over 1000), and the scores were much higher than last year (presumably due to the easier first question): fewer than 2% of candidates scored less than 20 marks overall, and the median mark was 61.

The pure questions were the most popular as usual, though there was much more variation than in some previous years: questions 1, 3, 4 and 6 were the most popular, while question 7 (on vectors) was intensely unpopular. About half of all candidates attempted at least one mechanics question, and 15% attempted at least one probability question. The marks were unsurprising: the pure questions generally gained the better marks, while the mechanics and probability questions generally had poorer marks.

A sizeable number of candidates ignored the advice on the front cover and attempted more than six questions, with a fifth of candidates trying eight or more questions. A good number of those extra attempts were little more than failed starts, but suggest that some candidates are not very effective at question-picking. This is an important skill to develop during STEP preparation. Nevertheless, the good marks and the paucity of candidates who attempted the questions in numerical order does suggest that the majority are being wise in their choices. Because of the abortive starts, I have often restricted my attention below to those attempts which counted as one of the six highest-scoring answers, and referred to these as "significant attempts".

The majority of candidates did begin with question 1 (presumably as it appeared to be the easiest), but some spent far longer on it than was wise. Some attempts ran to over eight pages in length, especially when they had made an algebraic slip early on, and used time which could have been far better spent tackling another question. It is important to balance the desire to finish the question with an appreciation of when to stop and move on.

Many candidates realised that for some questions, it was possible to attempt a later part without a complete (or any) solution to an earlier part. An awareness of this could have helped some of the weaker students to gain vital marks when they were stuck; it is generally better to do more of one question than to start another question, in particular if one has already attempted six questions. It is also fine to write "continued later" at the end of a partial attempt and then to continue the answer later in the answer booklet.

As usual, though, some candidates ignored explicit instructions to use the previous work, such as "Hence", or "Deduce". They will get no credit if they do not do what they are asked to! (Of course, "Hence, or otherwise, show ..." gives them the freedom to use any method of their choosing; often the "hence" will be the easiest, but in Question 5 this year, the "otherwise" approach was very popular.)

On some questions, some candidates tried to work forwards from the given question and backwards from the answer, hoping that they would meet somewhere in the middle. While this worked on occasion, it often required fudging, and did bring to mind a recent web comic: `http://xkcd.com/759`.

It is wise to remember that STEP questions do require a greater facility with mathematics and algebraic manipulation than the A-level examinations, as well as a depth of understanding which goes beyond that expected in a typical sixth-form classroom. STEP candidates are therefore recommended to heed the sage advice on the STEP Mathematics website, `http://www.admissionstests.cambridgeassessment.org.uk/adt/step`:

> From the point of view of admissions to a university mathematics course, STEP has three purposes. ... Thirdly, it tests motivation. It is important to prepare for STEP (by working through old papers, for example), which can require considerable dedication. Those who are not willing to make the effort are unlikely to thrive on a difficult mathematics course.

Common issues

There were a number of common errors and issues which appeared across the whole paper.

The first was a lack of fluency in algebraic manipulations. STEP questions often use more variables than A-level questions (which are more numerical), and therefore require candidates to be comfortable engaging in extended sequences of algebraic manipulations with determination and, crucially, accuracy. This is a skill which requires plenty of practice to master.

Along with this comes the need for explanations in English: a sequence of formulæ or equations with no explicit connections between them can leave the reader (and writer) confused as to the meaning: Does one statement follow from the other? Are they equivalent statements? Or are they perhaps simultaneous equations? For example, writing $x = 2$ followed by $x^2 = 4$ is not the same as writing $x = 2$ followed by $2x = 4$, and both are different from writing $x = 2$ followed by $y = 3$. In some cases, this cost marks, in particular when a candidate was required to show that "A if and only if B". Brief connectives or explanations ("thus", "so", "$\because$" or "$\Rightarrow$") would help, and sometimes longer sentences are necessary. The solutions booklet is more verbose than candidates' solutions need to be, but give an idea of how English can be used.

Another related issue is legibility. Many candidates at some point in the paper lost marks through misreading *their own writing*. Common confusions this year included muddling their symbols, the most common being: M and m; V and v; u and n; u and N; x and n (primarily among Oriental candidates); α and 2; a and 9; s, S and 5. In other years, z and 2 have also caused confusion. It is sad that, at this stage, candidates are still wasting marks because of bad writing habits.

A couple of basic (primary school) arithmetical processes caused some problems this year, namely long multiplication and fractions. While there is rarely a need for long multiplication in STEP examinations, some of the candidates attempted to use it at various points, and it was shocking to see that they were not competent with a compact hand-written method, some even still using a chunking or grid-type method. For fractions, the situation was similar, the most noticeable thing being the lack of cross-cancellation or any simplification when multiplying fractions. (For example, when calculating $\frac{3}{16} \times \frac{10}{27}$, it is far simpler to first cancel to $\frac{1}{8} \times \frac{5}{9}$ giving $\frac{5}{72}$ than to multiply directly to get $\frac{30}{432}$.)

Quadratics caused a few raised eyebrows when candidates tried to solve equations of the form (blah)$^2 = 9$ by expanding brackets and solving the resulting quadratic instead of simply saying blah $= \pm 3$.

Graph sketches were again weak. Students need much more practice with sketching graphs of interesting functions (beyond the standard A-level fare of quadratics, cubics, reciprocals of linears and x^2, and the basic trigonometrical functions). Sketching functions should involve consideration of all of their main features: their axis-crossings, their stationary points, their asymptotic behaviour and even such basics as whether they are positive or negative in various regions. Reciprocals of quadratics and cubics are good for learning this, as well as more sophisticated functions. Students preparing for STEP would be advised to make up such functions, attempt to sketch them, and then check their answers on a graphical calculator or using software such as Geogebra (which is free, and can be found at `http://www.geogebra.org/`).

Finally, a strong reminder that it is vital to draw appropriate, clear, accurate diagrams when attempting mechanics questions: it was shocking how many candidates attempted to solve the collision question without a diagram or a moments question with a tiny, rough sketch!

Question 1

This was by far the most popular question, attempted by almost all candidates. It was very pleasing to see so many perfect solutions, showing that many candidates had a good command of basic algebraic manipulation.

The first part of the question was answered very well. Some candidates failed to take enough care with their algebra, the most common errors being to either lose the $4a$ term entirely or to replace it by 4.

There were many different approaches used to tackle the second part, the majority of them being effective. The most common conceptual error was assuming that the solutions had to be integers. As the question did not say this, such attempts gained relatively few marks unless supported by a full algebraic solution.

There were a few recurring technical errors and other ineffective approaches.

The first was to implicitly assume that the second equation could be rewritten in the same way as the first without checking the consistency of all six equations (most notably, checking that $a + bc^2 = 6$).

The second was to fail to square root an equation correctly: from $(y - 2x)^2 = 4$, for example, a significant number of students gave only the single solution $y - 2x = 2$.

A significant number of students made arithmetical errors when solving the equations which left them bogged down in messy calculations, costing them only a few accuracy marks but lots of time.

Also, several took long-winded methods which involved multiplying out expressions such as $(y - 2x)^2 = 4$ to reach quadratic equations; as mentioned above, candidates should certainly be aware that they ought to simply take square roots.

Most candidates who reached a pair of simultaneous equations for $(y-2x)^2$ and $(x-y+2)^2$

correctly solved for one of the terms, but a surprising number then substituted their answer(s) back into one of the original quadratic equations in x and y to eliminate a variable, giving themselves far more algebraic slog than actually necessary.

Overall, the mean mark for this question was in excess of 14/20, making it the most successfully answered question on the paper.

Question 2

This was a fairly popular question, attempted by around two-thirds of candidates. It was pleasing to see how many of them were correctly able to differentiate the expression given at the start of the question. Again, the earlier comments on algebraic accuracy bear repeating at this point: a number of candidates became unstuck here through algebraic or sign errors, and this was a repeating theme throughout this question.

The inequalities proved challenging: setting the derivative equal to zero was an obvious step, and then the resulting quadratic is crying out for consideration of the discriminant. Even many of those who got this far failed to adequately explain their solution of the quadratic inequality, jumping straight to the given answer.

Moving on to the graph sketches, it became obvious that different candidates have different understandings of what is expected. At the very least, a graph sketch should indicate (where possible) the coordinates of stationary points and axis crossings, as well as asymptotic behaviour. Many candidates, pleased with their success at finding the x-coordinates of the stationary points, then stopped and did not calculate the y-coordinates. The lattar would have made the decision about the nature of the stationary points essentially trivial and would have helped them draw more accurate sketches, especially in part (ii).

On a positive note, most of the candidates who reached this part of the question determined the turning points and vertical asymptotes correctly, though some thought that part (ii) had no turning points (despite having shown that it does earlier on).

The mean score on this question was noticeably lower than on several of the other pure mathematics questions, suggesting that graph-sketching is an area which requires more attention from candidates during their preparation.

Question 3

This was a very popular question, and the marks were generally very encouraging. A significant majority of attempts simply rattled through the first part of the question, showing confidence and competence with their trigonometric identities. A few candidates' solutions for this part lasted several pages, but the majority were very swift and efficient.

For the main part of the question, most candidates started off very well by attempting to equate the lines' gradients, though a few thought that the gradient is given by the formula $(x_2 - x_1)/(y_2 - y_1)$. A number of candidates tried to use vector methods by showing that

$$\frac{\overrightarrow{PQ}.\overrightarrow{SR}}{|PQ|.|SR|} = 1,$$

which involves significant (and messy) algebraic manipulation. (And if they used $\overrightarrow{RS}$ instead of $\overrightarrow{SR}$, they were unlikely to try to equate to -1 instead of 1.) A few tried to show that $\overrightarrow{PQ} = \overrightarrow{SR}$, which is more restrictive than what is required. In general, the scalar (dot) product method is good for identifying perpendicular vectors, but far less useful for parallel vectors.

Most candidates who followed the gradient method then reached the intermediate conclusion that $\tan \frac{1}{2}(q+p) = \tan \frac{1}{2}(s+r)$, but did not know how to continue correctly. They either simply used the given answer or concluded that $\frac{1}{2}(q+p) + k\pi = \frac{1}{2}(s+r)$ where $k = 0$, 1 or 2. Very few said that k could be any integer or gave any justification for their restriction to the given possibilities.

Finally, almost no candidates appreciated that the question's requirement to show that the lines are parallel *if and only if* the given condition was met meant that they had to either show that their argument was reversible (using $\iff$ connectives or some other indication that it was reversible), or explain why $r + s - p - q = 2\pi$ implies that the lines are parallel.

Question 4

This was another very popular question and, in spite of the challenges it posed, was answered well by many of the candidates.

The majority of those who attempted the question knew how to perform an integration by substitution, and many were able to do so correctly in this challenging example. Almost every candidate was correctly able to determine $\mathrm{d}x/\mathrm{d}t$ and substitute for x correctly in the integrand. The next step, simplifying the resulting expression, proved more challenging, and several candidates slipped up at this point (for example by forgetting a square root sign).

Those who reached an integral of the form $\int -2/(t^2-1)\,\mathrm{d}t$ generally used the given result to perform the integration, though some ignored the note and proceeded to use partial fractions; this did not gain them any extra credit.

The majority either ignored the absolute value signs given in the note, quietly dropped them in the next step, or integrated their partial fraction expansion without the use of absolute value signs. While it is true, in this case, that they can (and should) be dropped, this did require some minimal justification.

The simplification of the logarithm expression with t substituted with $\sqrt{(x+1)/x}$ proved challenging. A good number of candidates wisely expanded the given right-hand side and showed that it gave the same as the left-hand side, while others floundered with the expression $\sqrt{1+1/x}$. This question called for a deep understanding of simplification of fractions together with an equally sophisticated understanding of rationalising denominators, and this proved too hard for many candidates.

Most candidates attempted the second part of the question, even if they had not completed the first part. A disturbing number of candidates had misremembered the formula for volume as $\int 2\pi y^2\,\mathrm{d}x$, and were therefore out by a factor of 2 throughout the rest of the question. They were only lightly penalised for this error.

There was some noticeable difficulty experienced in expanding y^2, with many also making their lives somewhat harder by writing y as a single fraction before squaring. Even among the correct expansions, fewer attempts than expected noted that the integrand could be written as $\frac{1}{x}+\frac{1}{x+1}-2\times(*)$, where $(*)$ was the integrand from the first part of the question. Therefore many candidates ended up becoming stuck at this point.

Of those candidates who reached the correct integrand, most were able to correctly substitute in the limits and get the signs correct (though this did require care, and not all managed it). The next step, simplifying the logarithms, left many struggling. While most were fairly comfortable with the rules for logs, far fewer were happy with simplifying as they went or cancelling common factors ("cross-cancelling") when multiplying fractions, leading some to work with fractions with large numerators and denominators. This approach frequently ended with arithmetical errors. Very few candidates applied the logarithm rules to logs of fractions, such as writing $\ln\frac{9}{16}=\ln 9-\ln 16=2\ln 3-4\ln 2$. This would have made the arithmetic far simpler and would have given them far more chance of reaching the correct answer.

Question 5

This was a fairly unpopular question, attempted by only about 40% of the candidates (and being one of the best six attempts of fewer than one-third of candidates). The marks were also poor, with the median mark of the significant attempts being 3/20.

About one-sixth of all attempts failed to gain a single mark, even though writing out the binomial expansion of $(1+x)^n$ using binomial coefficients would have been sufficient for this. About one-quarter of candidates failed to make any further progress beyond this point, though another quarter substituted $x=1$ and reached the result of part (i) before giving up.

Those who persevered generally did reasonably well, with many correct answers to parts (ii) and (iii), and a good number succeeding on part (iv) as well. It was surprising how many successfully answered parts (ii) and (iii) using binomial coefficient manipulations without any reference back to part (i).

In part (ii), a number of candidates attempted to differentiate 2^n to get $n.2^{n-1}$. Others, who used the binomial manipulation approach, were careless about the first and last terms of the sum or made no attempt to justify why *all* of the corresponding middle terms were equal.

On this note, some candidates expanded both sides of the identity they were trying to prove and equated the corresponding terms without any understanding or indication of why they were the same; this gained relatively few marks, especially in part (iv) where the terms do not even correspond in this way.

In part (iii), those candidates who used a calculus-based approach frequently failed to consider the constant of integration, and could not, therefore, justify the need for the -1 term.

Finally, whereas a good number of the calculus-based approaches succeeded on part (iv), no candidate who used a binomial-manipulation method managed to extend their techniques to this case.

Question 6

This was another popular and well-answered question. The median mark was 13, and around one-fifth of attempts gained full marks.

Almost all candidates were fine with the first step and showing that ($*$) holds. A significant majority were also comfortable with deducing ($**$), though there were some who had difficulties in applying the product rule twice or appreciating that u was a function of x rather than a constant.

The final part of the question, however, caused numerous difficulties for the majority of candidates. First of all, some simply did not understand what was being asked of them, and thought of v as a constant (even though they had appreciated that u itself is a function of x). Then a significant number thought that $\mathrm{d}^2u/\mathrm{d}x^2 = v^2$ rather than $\mathrm{d}v/\mathrm{d}x$, belying a lack of understanding of the meaning of a second derivative. Those who overcame these hurdles reached a correct first order ODE with no constant term, but many struggled to solve it; even those who correctly separated the variables could not figure out how to integrate $\frac{x-2}{x-1}$, even though this is a standard A-level integration question. (Some tried integration by parts, with a predicatable lack of success.)

Those who managed to integrate to determine $\ln v$ then went on to exponentiate, mostly successfully, though a number forgot about the arbitrary constant or ended up with an expression of the form $v = f(x) + c$ instead of $v = cf(x)$.

Those who reached this point generally appreciated that they now needed to write $v = \mathrm{d}u/\mathrm{d}x$ and integrate once more, and most realised that now was the time to integrate by parts. Unfortunately, many forgot to multiply their original arbitrary constant by the $-\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x$ term of the parts formula.

For the last step, a number of candidates thought that it was sufficient to simply plug $y = Ax + B\mathrm{e}^x$ into the differential equation; this, of course, gained no credit, as the question had explicitly said "Hence show that ...".

Question 7

This was a very unpopular question, attempted by only 20% of candidates and being one of the six best questions of only 10%. It was also the worst-scoring: of the significant attempts, the median mark was 1/20. However, those who managed to get beyond the start of the question generally did quite well, resulting in an upper quartile mark (among significant attempts) of 11/20. It generally indicated a very poor understanding of vectors among those students who attempted this question. Only a handful of students successfully completed the question.

Of those who attempted the question, many drew a decent diagram, which is very helpful in understanding what is being asked, though few realised that C lies strictly inside triangle OAB. Many also realised that they could simply write down the formula for $\mathbf{q}$ directly by symmetry.

Most, however, appeared to be incapable of writing down the equation of a straight line in vector form and were therefore unable to proceed any further.

Those who did often made their lives more difficult by writing the equation of OA as $\mathbf{r} = \mathbf{a} + \lambda\mathbf{a}$ rather than choosing the origin as their fixed point, giving the simpler equation $\mathbf{r} = \lambda\mathbf{a}$. While the former is clearly correct, it is messier to work with and therefore more likely to lead to errors later.

A very common confusion was to write down the equations of the lines but to call them $\overrightarrow{OA}$ and $\overrightarrow{BC}$ instead of OA and BC; this sometimes led to candidates trying to equate the *vectors* as $\overrightarrow{OA} = \overrightarrow{BC}$, which was fairly nonsensical (and other candidates tried equating the vectors without writing down the equations of the lines).

Another frequent piece of nonsense was an attempt to divide one vector by another or to add a vector to a scalar.

Few candidates used notation carefully, and many suffered for it. It is strongly recommended that students are taught to *always* distinguish their vector variables from their scalar variables by underlining, under-squiggling or using arrows above them, and that this is insisted upon. It makes it far less likely, then, that a student would write something like $\vec{a} + \lambda = \mu$ or $\underset{\sim}{a}\,\underset{\sim}{b}$ (both of which were frequently seen).

Some candidates attempted to write the formula for $\mathbf{p}$ in words and hoped that, somehow, this would be sufficient justification. This was joined by several attempts to work forwards from what was known and backwards from the desired result, which together with some glue in between was meant to provide a convincing argument for the result.

Those who successfully completed the first part were generally successful with finding the position vector of $\mathbf{r}$ as well.

The final part of the question was attempted by relatively few candidates. There was a mixed level of success. Most realised that they needed to find the position vector of $\mathbf{s}$, and this was done fairly well; the algebra was the trickiest part here, as the ideas were the same as earlier. The last step, proving the equality of the ratios, was a little trickier, and there were a few attempts to divide vectors or to ignore problems with signs.

Question 8

This was a moderately popular question and candidates obtained a broad spread of marks on it.

For the first part, almost all candidates successfully argued that a^3 is divisible by 3, but a significant number could not give a reasonably convincing explanation for why a itself is divisible by 3. We did not require a perfect argument, but there had to be a mention of 3 being prime. It was common to see this either asserted ("since $3 \mid a^3$, then $3 \mid a$") or, less commonly, something creative like: "$a = \sqrt[3]{a^3} = \sqrt[3]{3}\sqrt[3]{3c^3 - b^3}$, and as a is an integer, a must be divisible by 3".

A good number of candidates went on to correctly explain why b and c were divisible by 3, but quite a few talked about $b^3 = 3c^3 - \frac{1}{3}a^3$ and $c^3 = \frac{1}{9}a^3 + \frac{1}{3}b^3$ being divisible by 3 without any justification for these assertions; the idea of writing a multiple of 3 as $3k$ for some k was appreciated by some candidates but overlooked by others. A few candidates also made basic algebraic errors when rearranging the equation $a^3 + 3b^3 = 9c^3$; more care is needed!

It was nice to see that a fair number of candidates knew about modular arithmetic and could use it to construct effective arguments both here and in part (ii).

A number of candidates showed some serious misconceptions about divisibility, with a common error being $3 \mid (r+s) \Rightarrow 3 \mid r$ and $3 \mid s$. A few expanded $\sqrt[3]{a+b}$ as $\sqrt[3]{a}+\sqrt[3]{b}$, and there were even occurrences of $r+s=t \Rightarrow r=t/s$.

Other somewhat common failings were attempting to prove general statements by using particular examples (such as "take $a=42$"), making unsubstantiated assertions (for example "since $a^3/3$ is an integer, so is $a^3/9$"), and misuse of "similarly": just because $27 \mid a^3$, it does not follow "similarly" that $27 \mid b^3$ and $27 \mid c^3$ unless it has already been shown that $3 \mid b$ and $3 \mid c$.

The final step of the argument, the infinite descent argument, was poorly understood and served to differentiate the stongest candidates from the rest. Again, we did not expect a perfect argument, but merely some explanation that a non-zero solution would give rise to another, smaller, non-zero solution, and so on, which is impossible.

For part (ii), most candidates who attempted it realised that there were very few possibilities for the final digits of fourth powers, though a few went to the effort of calculating 8^4 and 9^4 explicitly rather than just considering final digits. Some thought that $5r^4$ could only end in 5, forgetting the possibility that it might end in 0. Nevertheless, many understood what they were meant to do and tried to argue that p and q must both be multiples of 5. Some succeeded, but others made errors in their logic and did not consider all possible cases, for example, many candidates considered several the case $5 \nmid p$ but not $5 \mid p$. Others effectively said "if $5 \mid p$, then we must have $5 \mid q$," but did not show that we must have $5 \mid p$. Finally, as in part (i), there were some who argued that $5 \mid r^4$ but did not explain how to deduce that $5 \mid r$, and many who were stuck on the final proof by infinite descent.

It was perhaps unsurprising that so many candidates appealed to Fermat's Last Theorem in their attempts to prove these results, even though it was not at all relevant to the question. (It was equally unsurprising that there was almost no use of Fermat's Little Theorem to help with the calculations required in part (ii) by stating that $a^4 \equiv 1 \pmod 5$ whenever $5 \nmid a$.)

Question 9

This was the most popular of the mechanics questions, being attempted by about one-third of candidates (though this question counted towards the final mark of only two-thirds of these). The mean mark was about 6/20.

A number of the attempts struggled to calculate the distance of the centre of mass from the wall, though most were able to do so using a quick sketch.

Unfortunately, many candidates gave up at this point, unsure of what to do next. It should be second-nature that for a large-body question, the "right" thing to do is "resolve twice, moments once". Others tried this but failed to do so correctly: first of all, many drew poor or confusing diagrams; it is vital that candidates draw diagrams which are clear enough to understand what is going on at every point. It was sadly common to see friction labelled as Fr, where this could easily be confused with $F \times r$ in some contexts: students should always be taught to use single-letter variable names. Further, many students were inconsistent with their force labelling: some labelled the reaction as R_A at A but the friction as R_B at B (where A and B are the points of contact) or similar gaffes—this, of course, led to confusion and errors later. Others used the same variable for two different reaction forces or two different friction forces. Yet others left out one of the forces.

Even with the hurdle of an accurate diagram overcome, many only resolved once rather than twice, and a few tried taking moments around two or three different points but never resolving. (The latter can be made to work, but is usually far more effort than necessary.)

Taking moments was also found to be challenging: most attempts failed by forgetting a force or by not understanding the meaning of "perpendicular distance". (For example, referring to the diagram in the sample solutions, when taking moments about B, candidates would have R_1 contributing $R_1 \times 2a$ or occasionally $R_1 \times 2a\cos\alpha$ instead of $R_1 \times 2a\sin\alpha$.) Some candidates also got their signs wrong. I personally encourage my students to always indicate the orientation in which they are taking moments (clockwise or anticlockwise, indicated with a small curved arrow as in $\mathscr{M}(\overset{\frown}{A})$) and to place all of the moments on the same side of the equation with the appropriate sign; this will also help if they ever come to learn about moments of inertia and angular acceleration, as they will then be more confident working with the formula $\sum Fd = I\ddot{\theta}$.

A surprising number switched the a and b at some point in the question.

Most of those who correctly reached this point were able to make good progress towards the required conclusions (though a few, sadly, did not attempt the final part of the question, even though it was fairly straightforward). They showed a good command of the trigonometric identities required and were confident in manipulating the equations to eliminate the forces.

Unsurprisingly, there were almost no candidates who used the Three Forces Theorem approach.

Question 10

This question was attempted by about 20% of the candidates, though many became stuck fairly early on in the question.

For the first part of the question, the majority of candidates showed a good understanding of how to differentiate a vector, though close to half got no further than finding the velocity and acceleration (often correctly, often with errors). Most of the rest then went on to evaluate scalar products to find the angle between two vectors, usually successfully. A small number took the elegant geometric-trigonometric approach very successfully.

Those who reached this point sketched the path of the particle, but with little thought for what they had just done: almost no candidates had a diagram in which the direction of motion was at 45° to the position of the particle even at the start point. Some drew a vague swirl, but most at least indicated one or two coordinates (usually correctly). Only two candidates were awarded full marks for their sketch, which required just a couple of coordinates and a clear (explicit or implicit) indication of the direction of motion somewhere along the path.

The final part of the question was very poorly done. Few even attempted it, and of those who did, many thought that a time delay of T meant that $\mathbf{r}_Q = \big(\mathrm{e}^{t+T}\cos(t+T)\mathbf{i} + \cdots\big)$ instead of using $t - T$. Those who got this far often concluded that PQ is proportional to e^t simply because they reached an equation of the form

$$|\overrightarrow{PQ}| = \mathrm{e}^t \times \sqrt{\text{some expression involving } t \text{ and } T},$$

without simplifying the square root to eliminate t.

Question 11

This was the least popular of the mechanics questions, yet the best-answered.

Nonetheless, the very standard start of the question proved to be a major stumbling-block for many candidates, with over a third unable to write down a pair of correct equations for the collision. The reason for this was very simple: about half of candidates failed to *draw a diagram*; this led to them trying to keep the directions in their heads, with the predictable consequence that most had inconsistent signs in their equations of momentum and restitution. It was almost impossible to make any significant further progress in such cases. (It also made the examiners' lives significantly harder, but they were not penalised for this!)

The majority of those who did write down two correct equations were generally able to solve them and reach correct expressions for the velocities of the two particles after the collision. (Although the question had asked for the speeds rather than the velocities, full marks were awarded for just determining the velocities.)

Many candidates who reached this point only found the conditions for one of the two particles to change direction (deducing the $2em > M - m$ required), but did not give an adequate (or any) explanation for why the lighter particle also changed direction.

A common problem in this first half was that candidates again misread their own writing, confusing M and m. In the second half, some candidates similarly confused V and v.

There were few candidates who attempted the second half of the question. Almost no candidates gave a convincing explanation for the given equation (making little or no reference to the circular track or the change of direction at every collision). Those who understood what was going on (whether or not they explained it well) and went on to try to determine v and V were frequently careless in their counting of collisions, ending up with expressions involving e^{2n} rather that e^{2n+1}.

Question 12

The two probability questions were each attempted by about 10% of candidates. This was the better-answered of them.

The majority of candidates were able to write down the definition of $\mathrm{E}(X)$ in this context, but very few figured out how to expand the definition to produce the required result.

In the second part of the question, many candidates were capable of determining $\mathrm{P}(X \geqslant 4)$ by a variety of arguments, often involving writing $\mathrm{P}(X \geqslant 4) = 1 - \mathrm{P}(X \leqslant 3)$ and then considering cases. Unfortunately, most of these arguments did not generalise easily, so that the required result of $\mathrm{P}(X \geqslant n) = p^{n-1} + q^{n-1}$ was not deduced, and this left most students stuck at this point. Also, quite a few were unable to determine $\mathrm{P}(X = 2)$ or $\mathrm{P}(X = 3)$ correctly, which did not help either.

Most candidates, even those who had become stuck earlier, attempted the final part of the question. There seemed to be a widespread understanding that pq is maximised at $p = q = \frac{1}{2}$, or that $\frac{1}{pq} \geqslant 4$, but very few actually proved this (and justification was required).

Question 13

This question was the most poorly answered on the paper, with over half of attempts scoring no marks.

Nonetheless, most candidates were capable of writing down the pdf of a Poisson distribution, but only a minority understood that they needed to consider two *different* Poisson distributions to make any progress.

Worse still, it was very common to see candidates writing things like: "Let X be the number of texts received. Then $\mathrm{P}(1 < X < 2) = \cdots$." This shows a total lack of understanding of what the Poisson distribution is doing: there is no time period given in the definition of X, and how could the number of texts lie strictly between 1 and 2? Were the candidates to have let X be the waiting time until the first text, this would have made sense, but at this level, most candidates have not yet met this concept.

Even those who progressed beyond this point and actually managed to reach the required quadratic in e^{λ} generally became stuck when trying to show that there are two positive values of λ: they showed (or tried to show) that $\mathrm{e}^{\lambda} > 0$ rather than the necessary $\mathrm{e}^{\lambda} > 1$.

Those who tried the second part generally did not appreciate that e^{λ_1} and e^{λ_2} are the two roots of the equation $p\mathrm{e}^{2\lambda} - \mathrm{e}^{\lambda} + 1 = 0$, and often used the two possible roots for *each* of e^{λ_1} and e^{λ_2}, leading to some nonsensical answers; very few reached the required expression for $\lambda_1 + \lambda_2$. Also, none of the candidates who reached this point seemed to know (or use) the result that the product of roots of $ax^2 + bx + c = 0$ is c/a; this is a very useful tool for students to have.

Finally, in the last part of the question, very few of the candidates were capable of finding an event involving the two phones equivalent to "the first text arrives between 1 and 2 hours", leaving them unable to make any meaningful progress. Drawing a Venn diagram or listing possibilities would have been of help, but there was little evidence that candidates used techniques such as these.