Cambridge Assessment Admissions Testing

Sixth Term Examination Paper [STEP]

Mathematics 3 [9475]

2019

Examiner's Report

Hints and Solutions

Mark Scheme

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Introduction

There was a significant rise in the total entry this year with an increase of nearly 8.5% on 2018. One question was attempted by over 90%, two others were very popular, and three further questions were attempted by 60% or more. No question was generally avoided and even the least popular attracted more than 10% of the candidates. 88% restricted themselves to attempting no more than 7 questions, and only a handful, but not the very best, scored strongly attempting more than 7 questions.

Question 1

This was the third most popular question attracting 83% of candidates, though it was only the 6^{th} most successful with an average mark just under 8/20, and no candidate achieved full marks on it. Small mistakes when starting the question often resulted in very different differential equations and hence solutions which lost lots of marks. Some candidates simply did not know how to solve a differential equation and they appeared to waste a lot of time. Drawing the spiral in part (i), lots of candidates guessed that the extrema occurred on the axes losing substantial marks. Candidates should have spent a few minutes more sketching their graphs as these sketches were the main feature of the question and a lot of candidates forgot to mark salient points. Attempting the final sketch, only a handful considered the behaviour for large t.

Question 2

The most popular question, it was also the most successful with an average score of over 11/20 and many fully correct solutions. Most candidates that noticed that the equation for $f(x+y)$ implied $f(x) = 0 \ \forall x$, or $f(0) = 1$ correctly eliminated the former, but quite a few did not realise that it was a possibility to consider. Nearly every candidate successfully found

$f'(x) = f(x) \lim_{h\to 0} \left(\frac{f(h)-1}{h}\right)$ and most also proceeded correctly from there to find the required differential equation. Finding f(x) was generally successful although some did not check the boundary conditions. In part (ii), there were fewer issues demonstrating that $g(0) = 0$ than there had been with $f(0) = 1$ in part (i). The simplification in order to find the limit to obtain $g'(x)$ was usually successful. Solution of that differential equation was often well done, either using partial fractions or as a hyperbolic function, although some mistakenly identified the solution as a tan function.

Question 3

This was the 6^{th} most popular question, but the least successful of those six, indeed one of the four least successful in the whole paper scoring just under a quarter of the marks. Barely a handful of attempts scored full marks. Candidates frequently overlooked the last result in (i), or merely wrote **A=I** , presumably assuming that was hardly worth any marks. Those who got to part (iii) typically did well on it, even if they had done poorly on parts (i) and (ii). Candidates often omitted or struggled to deal with cases. Common pitfalls were unnecessary division by zero without considering if the denominator were zero, thinking that the question stated that all invariant points lie on a line leads to claims that **A** is non-linear or doesn't exist, as mentioned failure to justify that **A=I**, and use of det(**A-I**)=0 with no justification.

Question 4

Two thirds of candidates attempted this scoring, on average about half marks. The first part was often well answered by those that used the Vieta equations, though a common error was to divide by a potential zero and therefore omit one of the solutions. Those that substituted the three roots into the polynomial equation encountered equations that were more difficult to solve, and the method yielded additional solutions which were often not rejected, so generally those taking this approach did much less well. Part (ii) had many good answers from squaring the sum of roots equation, though a common error was made with inaccurate summation notation. Many argued the deduction correctly but likewise many others assumed the a s were non-zero without a reason. Part (iii) was found more difficult. Those candidates that did not realise the significance of the previous deduction were rarely successful and others gave the correct answer, but with no explanation. Those making the inductive argument to factorise powers of x rarely justified that the remaining polynomial was likewise reflexive.

Question 5

A little more popular than question 1, attempts were only a little less successful than those for question 4. The effects and consequences of the typographical error in the substitution for part (ii) are dealt with in https://www.admissionstesting.org/Images/552575-step-2019-paper-3-question-5-protocol-summary.pdf

The majority of candidates successfully drew the graph for (i), but common errors were failures to consider asymptotic behaviour and label appropriately. The vast majority used the incorrectly suggested substitution and proceeded as far as possible on the first result of (ii) using it; a few realised at this point that there was an error and then obtained the correct result. Candidates who moved on to the evaluation, scored strongly using the quoted result, even if they had not obtained it, and appreciated that the limits were needing to be changed. Most attempting the second evaluation of (ii) obtained full marks on that part as they successfully demonstrated that it was the same answer as the previous evaluation. Few attempted part (iii), and most that were successful made substitutions not involving using the previous part of the question.

Question 6

Half the candidates tried this question, but it was one of the four least successfully attempted. The majority successfully demonstrated that the locus of P was a circle with the correct centre and radius, but few made further significant progress. They usually substituted for z in terms of w but then failed to rearrange into the necessary form. If they did achieve this, then they were able to score most of the marks up until the very last part which required careful justification to earn full marks.

Question 7

Attempted by two thirds, the mean score was only about one third marks. Part (i) was not very well answered with many appearing to guess one or both of the solutions without managing to factorise, or equivalent. By contrast, part (ii) (a) was generally well-answered. Most candidates saw how to do this correctly, though a few tried to treat it as a polynomial in y and take that discriminant which got them nowhere. In part (b), most candidates did not realise what was expected of them so, for example, many wrote $x \to \infty, y \to \infty$ or found the x and y intercepts. Almost all candidates saw what was needed for part (c), however, many made small algebraic errors or didn't check all the cases; the most common error was failure to eliminate the origin. The sketch was generally badly answered, often owing to errors in previous parts or failure to use correctly the information already obtained. Candidates did appreciate what was needed for part (iii) and did this well.

Question 8

The least popular of the Pure questions it was attempted by about a quarter of the candidates. It was generally found quite challenging with many attempts receiving little or no credit and so it was one of the four least successfully attempted questions. Some struggled to handle the vectors in part (i) with some attempting to divide by vectors or confusing cross and dot products, or normals and tangents, though these sorts of errors were rare. Much more common were errors arising from incorrect signs in the projections onto the base vectors **i**, **j**, and **k** or failure to recognise whether the angle being calculated was θ or $\pi - \theta$. In general, most candidates who took the time to establish a clear vector space set-up did rather well, not just in (i) but in (ii) as well. Most who attempted the first result of (ii) did so successfully. The most challenging part of the question was found to be obtaining the expression for $\cos^2 \varphi$ which required several small insights relating to trigonometric identities and a fair amount of calculation. A pleasing proportion attempting the calculation did so successfully but again a sound coordinate based set-up rendered it manageable. A number of candidates failed to justify properly the, often elementary, steps for the final part losing credit by so failing to do. Overall the question was largely algebraic rather than geometric, but the best solutions used the interplay between these two aspects to great effect.

Question 9

Although this was the least frequently attempted question, it was tried by just over 10% scoring on average marginally under one third marks. The position and velocity in (i) were successful for most, and those considering momentum (or centre of mass) found the next result easy, although those considering forces struggled. Most struggled with (ii), for whilst considering energy, typically they forgot the kinetic energy of the hemisphere. Part (iii) evoked a number of approaches, which were usually unsuccessful, and most could not make use of the suggested method. Even candidates who were unsuccessful with the rest of the question were able to obtain the cubic equation, though hardly any could justify the final inequality.

Question 10

Comfortably the most popular applied question on the paper with two fifths of candidates trying it, it was also one of the most successfully attempted on the whole paper with an average score just shy of half marks. A significant number of candidates struggled to set up the problem correctly, but those that did generally obtained the first result of (i). Then common mistakes were using Newton's Law of Impact in this part and failure to express w in terms of the specified variables. In part (ii), most candidates correctly applied Newton's Law of Impact, but depending on their expression for w in (i) had varying levels of success obtaining the required expression. A lot of candidates did manage to gain most marks in the final part even if they had struggled earlier. Alongside trigonometric and algebraic mistakes, a common mistake was failing to express $\tan\theta$ in terms of $\tan\alpha$, as against other trigonometric ratios, and e. The commonest approach for the final result was to use differentiation, although a few candidates successfully used the AM-GM inequality. However, fewer than a handful of candidates justified the value being a global as against local maximum.

Question 11

The least successful question with a mean score of just under one quarter marks, it was attempted by a fifth of the candidates. Many candidates assumed for (i) that the number of customers taking sand would follow a Poisson distribution without giving a proof and some thought that checking the mean equalled the variance was sufficient. For (ii), some assumed $E[f(x)] = f(E[X])$ which may have been with an eye to the 'show that' so e.g. '$Y \sim Po(\lambda p)$, $e^{-k\lambda p} = E[e^{-kY}]$'. For (iii), a minority used the law of total probability and were generally successful. A popular approach was to spot that $P(assistant\ gets\ sand) = E[proportion\ of\ sand\ they\ take]$, however few were able to express this as a correct probabilistic statement. In particular, some treated the amount of sand taken/remaining as a deterministic constant equal to its mean. Many candidates struggled to differentiate $ke^{-k\lambda p}$ with respect to k, and few gave a valid justification why the stationary point was a maximum.

Question 12

A quarter of the candidates attempted this scoring on average about half marks, making it the second most successfully attempted question. Candidates using the approaches suggested by the question tended to make good progress. Many produced correct solutions using various combinatorial arguments, some of which were easier to generalise than others and not all were well-explained. In part (ii), a few candidates used arguments not permitted by the wording of the question. In part (iii), some obtained the first result by the nice method $P(A_1 \subseteq A_2) = P(\, A_1 \cap A_2{}' = \emptyset\,)$ which is of course the first result of (ii) but this was not easy to generalise. Several incorrectly assumed $P(A_1 \subseteq A_2 \subseteq A_3) = P(A_1 \subseteq A_2)\, P(A_2 \subseteq A_3)$ "by independence".