STEP 2011 Paper 2 Report

General Remarks

There were just under 1000 entries for paper II this year, almost exactly the same number as last year. After the relatively easy time candidates experienced on last year's paper, this year's questions had been toughened up significantly, with particular attention made to ensure that candidates had to be prepared to invest more thought at the start of each question – last year saw far too many attempts from the weaker brethren at little more than the first part of up to ten questions, when the idea is that they should devote 25-40 minutes on four to six *complete* questions in order to present work of a substantial nature. It was also the intention to toughen up the final "quarter" of questions, so that a complete, or nearly-complete, conclusion to any question represented a significant (and, hopefully, satisfying) mathematical achievement. Although such matters are always best assessed with the benefit of hindsight, our efforts in these areas seem to have proved entirely successful, with the vast majority of candidates concentrating their efforts on four to six questions, as planned. Moreover, marks really did have to be earned: only around 20 candidates managed to gain or exceed a score of 100, and only a third of the entry managed to hit the half-way mark of 60.

As in previous years, the pure maths questions provided the bulk of candidates' work, with relatively few efforts to be found at the applied ones. Questions 1 and 2 were attempted by almost all candidates; 3 and 4 by around three-quarters of them; 6, 7 and 9 by around half; the remaining questions were less popular, and some received almost no "hits". Overall, the highest scoring questions (averaging over half-marks) were 1, 2 and 9, along with 13 (very few attempts, but those who braved it scored very well). This at least is indicative that candidates are being careful in exercising some degree of thought when choosing (at least the first four) 'good' questions for themselves, although finding six successful questions then turned out to be a key discriminating factor of candidates' abilities from the examining team's perspective. Each of questions 4-8, 11 & 12 were rather poorly scored on, with average scores of only 5.5 to 6.6.

Comments on individual questions

Q1 The first question is invariably set with the intention that everyone should be able to attempt it, giving all candidates something to get their teeth into and thereby easing them into the paper with some measure of success. As mentioned above, this was both a very popular question and a high-scoring one. Even so, there were some general weaknesses revealed in the curve-sketching department, as many candidates failed to consider (explicitly or not) things such as the gradient of the curve at its endpoints and, in particular, the shape of the curve at its peak (often more of a vertex than a maximum). It was also strange that surprisingly many who had the correct domain for the curve *and* had decided that the single point of intersection of line and curve was at $x = 1$ still managed to draw the line $y = x + 1$ **not** through the endpoint at $x = 1$. Most other features – domain, symmetry, coordinates of key points, etc. – were well done in (i). Unfortunately, those who simply resort to plotting points are really sending quite the wrong message about their capabilities to the examiners.

Following on in (ii), the majority of candidates employed the expected methods and were also quite happy to plough into the algebra of squaring-up and rearranging; however, there were frequently many (unnecessarily) careless errors involved. The only other very common error was in the sketch of the **half**-parabola $y = 2\sqrt{1-x}$, due to a misunderstanding of the significance of the radix sign.

Q2 Personally, this was my favourite question, even though it was ultimately (marginally) deflected from its original purpose of expressing integers as sums of two rational cubes. Given that the question explicitly involves inequalities (which are, as a rule, never popular) and cubics rather than quadratics, it was slightly surprising to find that it was the most popular question on the paper. However, although the average score on the question was almost exactly 10, these two issues then turned out to be the biggest stumbling-blocks to a completely successful attempt as candidates progressed through the question, both in establishing the given inequalities and then in the use of them. In particular, it was noted that many candidates "proved" the given results by showing that they implied something else that was true, rather than by *deducing* them from something else known to be true; such logical flaws received little credit in terms of marks. The purpose of this preliminary work was to enable the candidates to whittle down the possibilities to a small, finite list and then provide them with some means of testing each possibility's validity. This help was often ignored in favour of starting again. In general, though, part (i) was done reasonably well; as was (ii) by those who used (i)'s methodology as a template.

Only a very few candidates were bold enough to attempt (ii) successfully without any reference to (i)'s methods; indeed, this arithmetic approach was how the question was originally posed (as part (i), of course) before proceeding onto the algebra. Noting that the wording of the question does not demand any particular approach in order to find the required *two* solutions to the equation $x^3 + y^3 = 19z^3$, a reasonably confident arithmetician might easily note that

$$19 \times 2^3 = 152 = 3^3 + 5^3 \text{ and } 19 \times 3^3 = 513 = 1^3 + 8^3$$

and it isn't even necessary to look very far for *two* solutions. For 10 marks, this is what our transatlantic cousins would call " a steal".

Q3 Again, despite the obvious presence of inequalities in the question, this was another very popular question, and was generally well-handled very capably in part (i), where the structure of the question provided the necessary support for successful progress to be made here. Part (ii) was less popular and less well-handled, even though the only significant difference between this and (i)(c) was (effectively) that the direction of the inequality was reversed. Although the intervals under consideration were clearly flagged, many candidates omitted to consider that, having shown the function *increasing* on this interval, they still needed to show something simple such as $f(0) = 0$ in order to show that $f(x) \geq 0$ on this interval. A few also thought that $f'(x)$ increasing implied that $f(x)$ was also increasing.

Q4 This question was the first of the really popular ones to attract relatively low scores overall. In the opening part, it had been expected that candidates would employ that most basic of trig. identities, $\sin A = \cos(90^\circ - A)$, in order to find the required values of θ, but the vast majority went straight into double-angles and quadratics in terms of $\sin\theta$ instead, which had been expected to follow the initial work; this meant that many candidates were unable to explain convincingly why the given value of $\sin 18^\circ$ was as claimed.

Despite the relatively straightforward trig. methods that were required in this question, with part (ii) broadly approachable in the same way as the second part of (i), the lack of a clear-minded strategy proved to be a big problem for most attempters, and the connection between parts (ii) and (iii) was seldom spotted – namely, to divide through by 4 and realise that $\sin 5\alpha$ must be $\pm\frac{1}{2}$. Many spotted the solution $\alpha = 6^\circ$, but few got further than this because they were stuck exclusively on $\sin 30^\circ = +\frac{1}{2}$.

Q5 This vectors question was neither popular nor successful overall. For the most part this seemed to be due to the fact that candidates, although they are happy to work with scalar parameters – as involved in the vector equation of a line, for instance – they are far less happy to interpret them geometrically. Many other students clearly dislike non-numerical vector questions. Having said that, attempts generally fell into one of the two extreme camps of 'very good' or 'very poor'. More confident candidates managed the first result and realised that a "similarity" approach killed off the second part also, although efforts to tidy up answers were frequently littered with needless errors that came back to penalise the candidates when they attempted to use them later on. Many candidates noted that D was between A and B, but failed to realise it was actually the midpoint of AB. In the very final part, it was often the case that candidates overlooked the negative sign of $\cos\theta$, even when the remainder of their working was broadly correct.

Q6 This was another very popular question attracting many poor scores. There were several very serious errors on display, including the beliefs that

$$\int \mathrm{f}(x)^n \,\mathrm{d}x = \frac{\mathrm{f}(x)^{n+1}}{(n+1)} \quad \text{or} \quad \frac{\mathrm{f}(x)^{n+1}}{(n+1)\mathrm{f}'(x)}.$$

The understanding that the original integral needed to be split as $\int \mathrm{f}'(x)\times\left(\mathrm{f}'(x)\mathrm{f}(x)^n\right)\mathrm{d}x$ before attempting to integrate by parts was largely absent, with many substituting immediately for $\mathrm{f}(x)\mathrm{f}'(x)$ in terms of $\mathrm{f}''(x)$, which really wasn't helpful at all. Those who got over this initial hurdle generally coped very favourably with the rest of the question.

In (i), it was quite common for candidates to omit verifying the result for $\tan x$.

Q7 The initial hurdle in this question involved little more than splitting the series into separate sums of powers of λ and μ, leading to easy sums of GPs. Many missed this and spent a lot of wasted time playing around algebraically without getting anywhere useful. In (ii), many candidates applied (i) once, for the inner summation, but then failed to do so again for the second time, and this was rather puzzling. Equally puzzling was the lack of recognition, amongst those who had completed most of the first two parts of the question successfully, that the sum of the odd terms in (iii) was *still* a geometric series. Almost exactly half of all candidates made an attempt at this question, but the average score was only just over 5/20.

Q8 This was the least popular of the pure maths questions, probably with good reason, as it included a lengthy introduction *and* a diagram. In the first part, despite showing candidates that the point where the string leaves the circle is in the second quadrant, the necessary coordinate geometry work provided a considerable challenge. The second part, finding the maximum of x by standard differentiation techniques, proved to be relatively straightforward and a lot of candidates managed to get full marks for this work. The third part presented the core challenge of this question, in the sense that not many candidates seemed to have understood how to set the limits of the parametric integral, and '*benefit of the doubt*' had to be fairly generously applied to those who switched signs when it suited them. The next part of the question involved applying integration by parts in order to evaluate the integrals but surprisingly few candidates managed to do so entirely successfully. Some of the common issues were the signs, that now needed to be fully consistent, and the application of parts twice after using double-angle formulae. The notion of the "total area swept out by the string" was also not so well understood, with only a very few realising that they needed to integrate from $t = 0$ to $t = \frac{1}{2}\pi$ as well. Most remembered to subtract the area of the semi-circle though.

Q9 Almost half of all candidates attempted this question, and scores averaged over 12/20. In the majority of cases, the first two parts of the question proved relatively straightforward conceptually, although there was the usual collection of errors introduced because of a lack of care with signs/directions. It was only the final part of the question that proved to be of any great difficulty: most candidates realised that they had to show that the given expression for *B*'s velocity was always positive, but a lot of their efforts foundered on the lack of appreciation that the term $1-4e^2$ could be positive or negative.

Q10 This was the second most popular of the mechanics questions. The first couple of parts to the question were fairly routine in nature, but then the algebra proved too demanding in many cases, principally when it came to dealing with a quadratic equation in t which had non-numerical coefficients. Candidates also found it a struggle to know when to use g and H instead of u and θ in the working that followed. A good number of candidates understood the nature of the problem as the two particles rose and fell together, although it transpired (unexpectedly) that there was another difficult obstacle to grasp in working with two distances. Even amongst essentially fully correct solutions, very few indeed arrived at the correct final answer for $\tan\theta$.

Q11 This was the least popular of all the questions on the paper, receiving under 40 "hits". The fact that it clearly involved both vectors and 3-dimensions was almost certainly responsible for the reluctance of candidates to give it a go. Those who managed to get past the initial stage of sorting out directions and components usually did very well, but most efforts foundered in the early stages. It was not helpful that some of these efforts confused angles to the vertical with those to the horizontal. Almost no-one verified that the given vector in (i) was indeed a unit vector.

Q12 Around a quarter of all candidates made an attempt at this question, though the average score was very low. Parts (ii) and (iii) were managed quite comfortably, on the whole, but it was (i) that proved to be difficult for most of those who attempted the question. The real difficulty lay in establishing the given result for *w*, as the event to which it corresponded was defined recurrently. As it happens, most wayward solutions left the straight-and-narrow by misreading the rules of the match to begin with.

Q13 This question was almost as unpopular as question 11, receiving under 70 attempts, very few of which ventured an opening opinion as to what *skewness* might measure. Those who could handle expectations lived up to them and scored well; the rest just found the question a little too overwhelming in its demands.

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