STEP II 2017 Examiner's Report

This year's paper was, perhaps, slightly more straightforward than usual, with more helpful guidance offered in some of the questions. Thus the mark required for a "1", a Distinction, was 80 (out of 120), around ten marks higher than that which would customarily be required to be awarded this grade. Nonetheless, a three-figure mark is still a considerable achievement and, of the 1330 candidates sitting the paper, there were 89 who achieved this. At the other end of the scale, there were over 350 who scored 40 or below, including almost 150 who failed to exceed a total score of 25.

As a general strategy for success in a STEP examination, candidates should be looking to find *four* "good" questions to work at (which may be chosen freely by the candidates from a total of 13 questions overall). It is unfortunately the case that so many low-scoring candidates flit from one question to another, barely starting each one before moving on. There needs to be a willingness to persevere with a question until a measure of understanding as to the nature of the question's purpose and direction begins to emerge. Many low-scoring candidates fail to deal with those parts of questions which cover routine mathematical processes - processes that should be standard for an A-level candidate. The significance of the "rule of four" is that four high-scoring questions (15-20 marks apiece) obtains you up to around the total of 70 that is usually required for a "1"; and with a couple of supporting starts to questions, such a total should not be beyond a good candidate who has prepared adequately.

This year, significantly more than 10% of candidates failed to score at least half marks on any one question; and, given that Q1 (and often Q2 also) is (are) specifically set to give all candidates the opportunity to secure some marks, this indicates that these candidates are giving up too easily.

Mathematics is about more than just getting to correct answers. It is about communicating clearly and precisely. Particularly with "show that" questions, candidates need to distinguish themselves from those who are just tracking back from given results. They should also be aware that convincing themselves is not sufficient, and if they are using a result from 3 pages earlier, they should make this clear in their working.

A few specifics:

In answers to mechanics questions, clarity of diagrams would have helped many students. If new variables or functions are introduced, it is important that students clearly define them.

One area which is very important in STEP but which was very poorly done is dealing with inequalities. Although a wide range of approaches such as perturbation theory were attempted, at STEP level having a good understanding of the basics – such as changing the inequality if multiplying by a negative number – is more than enough. In fact, candidates who used more advanced methods rarely succeeded.

Question 1

Almost all candidates attempted this question, making it the most popular on the paper; it was also the highest-scoring, with a mean score of 15. Indeed, the careful structure meant that its direction was clear to almost all candidates and it was only the rather tricky induction proof in (iii) that prevented the question from being completely transparent.

Question 2

This was the second most popular question of all, attempted by over 80% of candidates; but scoring relatively poorly with a mean score of under 10. It was, of course, heavily algebraic and this meant that many candidates found it challenging, getting lost in the algebra. In most cases, this was largely avoidable: the simple device of calling the first term "*X*" (say) would have prevented a lot of unnecessary subscripts from cluttering up the working. A few moments of thought from those candidates who simply embarked on the (potentially) intricate algebra could have saved a lot of trouble. The point of a sequence's periodicity is that it is the smallest cycle over which terms repeat; it should be noted that the condition for each term to be equal (a constant sequence) must clearly be embedded in any condition that gives $x_{n+2} = x_n$. Similarly, in order to satisfy $x_{n+4} = x_n$, we must automatically have the cases when all terms are the same *and* every other term equal present somewhere. This makes any ensuing factorisations much easier to deal with.

It could be noted that the requirement for $x_{n+4} = x_n$ can be thought of as a two-stage sequence using every other term; and this situation has just been sorted out.

Question 3

Attempts fell to around the 50% figure with marks scored by those who attempted the question averaging about 10 out of 20. There is not much to this question beyond the baseline realisation that $\sin y = \sin x$ does not necessarily imply that $y = x$. In essence, it is all about "quadrants" work, where candidates need to consider the two solutions, x and $\pi - x$ in one period of the sine function, and then adding or subtracting multiples of 2π as necessary. Once one has done this, the accompanying straight-line segments are straightforward marks in the last part of (i).

A lot of marks were gained in (ii), as candidates were clearly attracted by the familiar "differentiate this couple of times" demand; most of them were quite happy with the differentiation, performed either implicitly or directly using arcsines.
The drawings required in (ii) and (iii) then relied on an appreciation of the symmetries of the sine function, along with the use of the identity $\cos y \equiv \sin\left(\frac{1}{2}\pi - y\right)$.

Question 4

This is the first question where the difference between "attempts" and "serious attempts" arises to any significant extent: there were just over 800 of the former but well under 500 of the latter. This is also a good point at which to raise a key issue in respect of *strategy* for candidates sitting a STEP. Spending a few minutes of reading time, at some particular time during the examination, could be a significant asset, especially to those candidates who have particular strengths and weaknesses to play to or to avoid. A very brief analysis of this question, on first reading, should help one recognise that a result is being **given** (with no requirement to establish it in any way) and all that is required is to use it. Part (i) then clearly directs part of the way, and the required limits are rather obviously flagged, as is the fact that $g(x)$ must be something to do with the exponential function. One of the two functions to be used in (ii) is also given, as are the limits; an inspection of the **given** should lead to the (correct) conclusion that $g(x)$ *must* be $e^{-\frac{1}{4}x^2}$. Getting just this far takes the candidate to the 10-mark point, a perfectly good return for a candidate who has read the question through sufficiently carefully to realise that it has decent potential for mark-acquisition.
In the final part of the question some careful thought was needed, with only the required limits obvious at first glance. Most attempts, serious or otherwise, picked up the majority of their marks in (i) and (ii) and efforts at (iii) were very varied: many candidates simply gave up and moved on; many more picked up a few extra marks by setting $g(x) = \sqrt{\sin x}$ (which is a fairly obvious candidate to try) and working towards the right-hand half of the given result. Very few candidates indeed had the experience to realise that $\sqrt{\sin x}$ now needed to appear as the squared term, which also meant that a cosine term had to be involved.

Question 5

Attempts at this question were over the thousand figure, making it the third most popular question on the paper, with the second-highest mean score. Part (i) proved to be very routine; the calculus requirements in (ii) were obvious to most, though justifying the minimum distance was often poorly handled; for instance, finding the second derivative is a poor way to spend one's time when examining the sign of the first derivative is easily undertaken. The needs of part (iii) were also easily spotted though, again, a couple of marks were almost universally lost as the need to eliminate the two other cases that arise was largely ignored.

Question 6

This question was relatively popular, but it turned out to be one of the hardest of the pure questions. The first part was a reasonably standard example of induction but nearly all candidates failed to understand the subtlety of what was required in the last part.
Most candidates made significant progress with part (i), clearly being familiar with the process of induction. However, the algebra to complete the proof was too much for most candidates. Inequalities were frequently handled poorly and the general presentation of logical arguments was unclear with many candidates assuming what was required and not making implications clear. Attempts which brought in calculus were rarely relevant and even less frequently correct.

In the second part, candidates tended to overcomplicate the question. Squaring up (since both sides are clearly positive) and expanding brackets was all that was required. It would have been nice if students had shown some awareness that the squaring process was valid since both sides were positive, however if we had required this it would have effectively been a one mark penalty for all candidates.

In the final part candidates often considered S_1 to find a necessary bound on C. However, further work – usually an induction using their guess – was required to show that this bound works for all n. Many candidates seemed unaware that this final stage was required.

Question 7

Just over one thousand candidates attempted this question, but more than 400 of these attempts were not substantial; removing the large number of those scripts which got no further than part (i) raises the mean score from well under 8 to just over 12 out of 20.
The difficulty with questions like this is that it is very easy to make correct statements but much more difficult to support them with logically-crafted steps of reasoning based on results either given or known. Moreover, one needs to reason in such a way that the steps of working one writes down are justified … this was the principal barrier to anything more than the most faltering of starts. So, part (i) was an issue for candidates, with much written but not much of it coherently stated or supported. Of the few marks gained in the weaker attempts, part (ii) provided the majority of them, since most candidates were happy to take logs and then differentiate (the standard procedure for exponential equations of this kind).
It was slightly surprising to note that so few candidates attempted to establish the initial result in part (iv) using calculus; most of those that got this far presumably thought some other "inequality" technique was being tested.
Finally, even for those who had made good progress in several of the previous parts, the graphs at the end were frequently marred by a lack of labelling.

Question 8

The vectors question again proved extremely unpopular, despite the fact that it is perhaps the easiest question on the paper. It drew the least number of attempts from the Pure Maths questions (the only one under half of the entry) and two-thirds of these were not substantial attempts. In this case, it is easy to say what (almost invariably) appeared: candidates generally got no further than the first three marks, which could be gained by writing down two line equations, $\mathbf{r} = \mathbf{a} + \lambda\mathbf{u}$ and $\mathbf{r} = \mathbf{b} + \mu\,\mathbf{v}$, and then equating the two expressions for $\mathbf{r}$. Few made further progress, revealing a reluctance to engage with the algebraic manipulation of vectors (handling numerical vectors is, of course, a completely different matter altogether).

Question 9

This question was the least popular of the mechanics questions. Even amongst those who attempted this question only about a quarter made any meaningful attempt. As in so many mechanics questions a good, clearly labelled diagram often meant the difference between candidates making no progress and good attempts. It does seem that many students are reluctant to attempt these types of problems. It is hoped that looking at the hints and solutions should help candidates see that some resolving and taking of moments often leads to efficient solutions of problems like this.

Question 10

This was another very unpopular mechanics question. Many candidates who might have thought they made considerable progress did not score very well because they failed to communicate clearly the details required to “show” the given results.

Question 11

This was by far the most popular and most successfully answered mechanics question. The general concepts of energy and kinematics certainly seem to be familiar to most candidates, although there was a certain amount of “throwing SUVAT equations around” without any particular strategy, hoping something would miraculously appear.

The final part proved much more challenging. Several candidates attempted symmetry arguments, but these lacked the required rigour.

Question 12

The statistics questions were attempted by only a small fraction of the cohort, with question 12 the least popular question on the paper, receiving fewer than 200 attempts, of which only about a third made any meaningful progress. Although the question had a small wording ambiguity this did not seem to have bothered any but a very few candidates. It was disappointing that even the fairly standard analysis in part (i) proved difficult for most candidates, with several claiming that the mean and the variance being equal was a sufficient condition for $X + Y$ to follow a Poisson distribution.

Question 13

The first two parts of this question were reasonably straightforward, but this was only marginally more popular than question 12. A surprising number of candidates did not seem to be confident dealing with telescoping fractions – a fairly common tool in probability. The algebraic demands of the final part proved challenging for many candidates.