SI 2014 Report

General Comments

More than 1800 candidates sat this paper, which represents another increase in uptake for this STEP paper. The impression given, however, is that many of these extra candidates are just not sufficiently well prepared for questions which are not structured in the same way as are the A-level questions that they are, perhaps, more accustomed to seeing. Although STEP questions try to give all able candidates “a bit of an intro.” into each question, they are not intended to be easy, and (at some point) imagination and real flair (as well as determination) are required if one is to score well on them. In general, it is simply not possible to get very far into a question without making some attempt to think about what is actually going on in the situation presented therein; and those students who expect to be told exactly what to do at each stage of a process are in for a shock. Too many candidates only attempt the first parts of many questions, restricting themselves to 3-6 marks on each, rather than trying to get to grips with substantial portions of work – the readiness to give up and try to find something else that is “easy pickings” seldom allows such candidates to acquire more than 40 marks (as was the case with almost half of this year’s candidature, in fact).

Poor preparation was strongly in evidence – curve-sketching skills were weak, inequalities very poorly handled, algebraic capabilities (especially in non-standard settings) were often pretty poor, and the ability to get to grips with extended bits of working lacking in the extreme; also, an unwillingness to be imaginative and creative, allied with a lack of thoroughness and attention to detail, made this a disappointing (and, possibly, very uncomfortable) experience for many of those students who took the paper.

On the other side of the coin, there was a very pleasing number of candidates who produced exceptional pieces of work on 5 or 6 questions, and thus scored very highly indeed on the paper overall. Around 100 of them scored 90+ marks of the 120 available, and they should be very proud of their performance – it is a significant and noteworthy achievement.

Comments on individual questions

[Examiner’s note: in order to extract the maximum amount of profit from this report, I would firmly recommend that the reader studies this report alongside the *Hints and Solutions* supplied separately.]

Q1 Traditionally, question 1 is intended to be the most generous and/or helpful question on the paper, in order to permit as many candidates as possible to get started in a reasonably friendly situation, and thereby pick up at least 10 marks on the paper; giving them a positive start to the examination. This year, however, despite the high rate of popularity (over 80% of the candidature attempted Q1), there were several surprises in store for the examiners.

Firstly, it was not nearly so popular as it appears from the proportion of attempts, as it turned out that many of these attempts were either weak or inconsequential, petering out the moment the work became algebraic rather than numerical. The other surprise was how poorly the very simple ideas were handled. Many candidates clearly did not know what constituted a proof in these settings, when little more than (say) a statement such as $2k+1 \equiv (k+1)^2 - k^2$ in (ii) was perfectly sufficient. Quite a few went on to attempt what was clearly intended to be an inductive proof, having already written the correct (and wholly adequate) result, sometimes in each of parts (ii), (iii) and (iv).

Furthermore these sorts of mistakes were often preceded by incorrect numerical work in part (i), including offerings that ignored the initial prompting regarding the use of non-negative integers, such as "12 = $7^2 - \left(\sqrt{37}\right)^2$. In other parts of the question, candidates would resort to providing counterexamples to results that had not been suggested; such as, in part (iv) producing a counterexample (such as "6") to refute the notion that "every number of the form $4k + 2$ can be written as the difference of two squares" when the question actually required them to show that ***no*** number of this form has the proposed property, so offering one example represented a considerable misunderstanding of mathematical ideas and terminology. Part (iv) also suffered from the common misconception that factorising $4k + 2$ as $2(2k + 1)$ immediately meant that $2k + 1$ had to be prime.

Candidates who had seen and used modular arithmetic had a bit of an advantage in (iv) but, in fact, there was very little evidence of such. Partly balancing the widespread lack of (pretty basic) number theoretic appreciation were the few candidates who found this all very straightforward, as had been intended to be the case. Overall, however, this question provided a very disappointing range of responses, and the mean score of under 5/20 underlines this fact.

Q2 This was another very popular question, attempted by around 80% of candidates, and producing much greater success, although it has to be noted that this usually extended to work up to the end of part (iii), after which the integration efforts had become so prolonged and involved that many candidates simply moved on elsewhere rather than plough on into (iv). This all led to a mean score on the question of almost exactly 10/20.

There were two perfectly acceptable approaches to part (i); integrating the LHS by parts , after writing $\ln(2 - x)$ as the product $\ln(2 - x).1$ or, instead, differentiating the RHS in order to verify the result. The first method is probably that which is most "in the spirit of the game", though the second is, perhaps, the shrewder tactic. For those adopting the first approach, some difficulties were encountered when the extra "$2 - x$" failed to appear naturally, indicating a failure to appreciate the nature of arbitrary constants (namely, that "$-x + C$" could equally well be written have been written as "$-x + 2 + C$").

The curve-sketching was handled in the usual mixed way, with many candidates clearly well-prepared and dealing admirably with crossing-points on the axes and asymptotes, while others at the other end of the ability range seemed capable only of plotting points and "joining the dots". There were also many who thought that, towards asymptotes, the graph of a function should only approach positive infinity, and this led to a $\cup$-shaped central portion. However, there were very few sketch-attempts that failed to pick up at least 3 or 4 point-scoring features of the graph at hand.

Following this, attempts at part (iii) again generally picked up quite a few of the available marks by using the correct limits, separating $\ln\left(4 - x^2\right)$ as $\ln(2 - x) + \ln(2 + x)$, and then adapting the given result of (i) to the second term (although this frequently took far longer than should have been the case). However, when it came to part (iv), most candidates decided they'd had enough and went elsewhere. Many who started (iv) were clearly thrown by the extra set of modulus-brackets (even though they were intended to make life easy by rendering every part of the first curve positive). The extra bits of area then came simply from the use of a new pair of limits and the given result for evaluating the integrand in the vicinity of the asymptote.

Q3 This was, by far, the most popular question on the paper, with around 90% of candidates making an attempt at it. The mean score on it was just under 9/20. There was much on this question that made it straightforward, although there were several points at which candidates either overlooked something or were not sufficiently careful in their explanation. Almost all candidates managed part (i) successfully and managed to obtain the quartic equation in (ii). Many of these, however, assumed they had made a mistake since the question gives a cubic instead, and some of them tried, often repeatedly, the same working again rather than try to remove the "obvious" factor of $(b - 1)$. Continuing with the given result again allowed candidates to employ some fairly routine skills, and many did so successfully (though often without important bits of explanation).

Part (iii) proved to be rather less successful for most candidates, as they simply substituted straightaway for p and q and then found themselves in difficulties with the ab term that arose. Finally, it was (once again) clear that the majority of candidates are really not very comfortable with inequalities and very few of them managed to establish both "halves" of the required result.

Q4 This question was very unpopular indeed, almost certainly due to the lack of given structure. It attracted the attention of less than half of all candidates, and many of these attempts got no further than the first two lines of working involving the use of the *Cosine Rule*. This led to a mean score of 4/20 on this question and made it the second lowest scoring question on the paper.

Those candidates who did then differentiate generally overlooked the fact that they should have been differentiating with respect to time; fortunately, with $\frac{d\theta}{dt}$ being constant, there was very little penalty in terms of both marks lost and in straying from a profitable path of progress through the working.

As mentioned already, attempts that went significantly beyond this point were few and far between, with relatively few of such candidates realising that they needed to turn the resulting equation into a quadratic in $\cos\theta$, and even fewer managing to factorise it appropriately. For the very final part of the question, quite a few managed to obtain the correct (given) answer legitimately, even though they had been unsuccessful with the bulk of the question's working until then. This, at least, demonstrates a shrewd grasp of examination technique and generally earned them 3 or 4 marks at the end.

Q5 As already indicated in the comments for Q3, candidates generally do not like working with inequalities and this question is riddled with them. Q5 thus turned out to be the second least popular of the pure questions, and scored poorly with a mean score of under 5/20, again largely due to a lack of progress beyond the first part of the question.

Even in (i), there was a tendency to dive straight in to the sketch without having worked out any useful points on the curve, including missing the obvious point $(a, 0)$. Using this would have led easily towards a factorisation of $f(x)$ into three linear factors, though most preferred to find the turning points instead (which approach works equally well). Though not crucial to the following working, the special case $a = 0$ remained almost universally unaddressed.

Each of parts (ii) and (iii) can be approached via the *AM-GM Inequality* though very few did so as we had instructed candidates to use the result of part (i). It was disappointing to see so few serious attempts at part (ii) since all that is required is to set $a = y$ and then the "$x + 2y$" is practically waving at you. Part (iii) required a bit more thought (setting $p = x$ and then $q + r = 2a$) but both imagination and determination seemed in short supply by this stage of the question.

Q6 Around half of the candidates attempted this question, though successful working was (yet again) almost entirely limited to the opening part. Even here, there were far too many candidates who were unable to turn $4\sin^2\theta\cos^2\theta$ into $\sin^2 2\theta$. Many of those who did spot this simplification had difficulties trying to find u_2 in a similar form – thereby completely overlooking the fact that starting with $\sin^2(A)$ **at any stage of the process** clearly had to yield $\sin^2(2A)$ at the next. This also applies to the inductive step, which thus requires almost no further working – a position which was carefully avoided by all those who ploughed on into the standard format of a *proof by induction* without thinking about what they had just established.

Part (ii), was not often attempted. Some candidates substituted for v_n but not v_{n+1}; others thought that u_n and u_n^2 were the same thing and collected their coefficients up together; and, generally, the algebra was clearly found very unappealling. The very last part of the question required a "hence" approach, so those candidates who simply set about the sequence numerically scored only one mark. Even those who played the game according to the "hence" overlooked the need to check that the given condition was satisfied.

Q7 Of the pure maths questions, this was by far the least popular, with attempts from only around a quarter of the candidature. Very few attempts proceeded beyond the opening stages. Surprisingly – despite the fact that the whole question can be done with little more than the knowledge of how to split a line segment in a given ratio, vector equations of lines joining two given points, and the finding of a point of intersection of two lines – most attempts began with incorrect statements of the position vectors **d** and **e** in terms of **b** and **a** (respectively). Most candidates wisely gave up at this point, though some persevered, but with little success. The mean score of under 3/20 on this question reflects the paucity and brevity of efforts.

Geometers amongst the readership may notice that this result follows from a combination of *Menelaus' Theorem* and *Ceva's Theorem*. A handful of candidates noticed it too, and scored most of the marks for relatively little effort.

Q8 In hindsight, this question was a little too straightforward, and could well have been placed earlier on in the paper. Nonetheless, around two-thirds of all candidates attempted it, and marks were generally very high, making it the second highest-scoring question on the paper (and only marginally behind Q2) at just under 10/20. Finding equations of lines and intersections in the coordinate geometry setting was clearly much more in candidates' comfort zone than the vector setting of Q7, although there were problems caused by the surfeit of minus signs, and many repeated their working for L_a when finding L_b rather than simply changing the a's into b's.

Parts (ii) and (iii) were also handled well, though slightly less confidently than (i). Part of this was due to the lack of clear explanations given by candidates as to what had been done or found, or a failure to realise that there was a need to justify that "$\sqrt{c}$" satisfied the same conditions as the a from earlier on. Sadly, some failed to give the x a new label (c here), and persisted to substitute x's as part of a gradient into what then became a non- linear formula. Overall, however, this was a good question for candidates and most managed to make substantial progress most of the way through it.

Qs.9-11 The mechanics questions in section B always seem to prove more popular than those in the probability & statistics section C, and this was again the case this year. Qs. 9 & 11 each drew around 700 attempts – I suspect because they look the more standard settings – while Q10 attracted the attention of almost 550. However, those who persevered with Q10 generally scored more marks (Q9's means score was just under 6½/20; Q11's a mark less; while Q10's was around 9½/20), largely because of the numerical "pay-off" at the end of the question.

The key point in Q9 was to realise that the given expression for T meant that T was the time when the particle "turned round" horizontally. Those candidates who spotted this then had the opportunity to score lots of marks, as the sketches related to the point at which the particle turned back relative to its highest point and its landing-point. Those who approached the problem from a purely algebraic direction usually struggled with the significances of the three (four) cases.

Those candidates who did well on Q10 were generally those who set their working out in a more structured manner. For instance, a diagram to illustrate the assigned (symbols for the) speeds and directions of the objects involved in the collision generally helps prevent mistakes involving signs when applying the principles of *Conservation of Linear Momentum* and *Newton's (Experimental) Law of Restitution*. Indeed, without such a clear indication, it is often very difficult indeed for the markers to follow working involving symbols that just appear from nowhere!

Unfortunately, Q11 suffered particularly from precisely this issue also, and most marks gained on it came from successful attempts at the single-pulley scenario given in part (i) – for which we allocated six marks. Efforts at part (ii) often saw candidates failing to produce diagrams indicating directions for the various accelerations (etc.) and simply resorting to writing down several vague statements based on vague interpretations of "resolving" ideas and/or $N2L$. The failure to grasp that there needed to be some notion of relative accelerations involved for the two masses attached to pulley P_1 meant that most efforts were fatally flawed anyhow. Strangely, yet illustrating again the shrewdness of exam. technique amongst some of the candidates, it was possible to attempt the very final part just by taking the two given answers and running with them (although many failed to appreciate that an 'if and only if' proof needed two directions of reasoning).

Qs12 & 13 These questions elicited the least amount of interest from candidates (250 and 320 attempts respectively), though marks were generally in line with those for Qs. 9 & 11 (also respectively) at around 6½/20 and 5½/20.

The big hurdle in Q12 was the modulus function, which many candidates simply ignored. Those who were happy to use it properly generally gained the required result and used it to find that $k = 7$ (though some failed to discount $k = 1$ along the way). A further obstacle arose as most candidates who continued into the second part of the question failed to account for all six outcomes when calculating $P(X > 25)$.

In Q13, apart from the usual sign errors, most candidates correctly found $g(x)$ and $h(x)$. A popular approach for the mean was to throw the word "centroid" at the problem and hope that this sufficed. For the median, around half of attempts failed to consider the easy (symmetric) case when $m = c = \frac{1}{2}(a + b)$. The case $c > \frac{1}{2}(a + b)$, corresponding to when m lies under the first line-segment, was handled very well; the other case very poorly, since most candidates tried to work from the LHS up rather than from the RHS down. A very few realised that working down from the top actually made this just a "write down" by switching a for b suitably.