STEP Mathematics I 2007: Report

General comments

There were significantly more candidates attempting this paper this year (an increase of nearly 50%), but many found it to be very difficult and only achieved low scores. In particular, the level of algebraic skill required by the questions was often lacking. The examiners' express their concern that this was the case despite a conscious effort to make the paper more accessible than last year's. At this level, the fluent, confident and correct handling of mathematical symbols (and numbers) is necessary and is expected; many good starts to questions soon became unstuck after a simple slip. Graph sketching was usually poor: if future candidates wanted to improve one particular skill, they would be well advised to develop this.

There were of course some excellent scripts, full of logical clarity and perceptive insight. It was pleasing to note that the applied questions were more popular this year, and many candidates scored well on at least one of these. It was however surprising how rarely answers to questions such as 5, 9, 10, 11 and 12 began with a diagram.

However, the examiners were left with the overall feeling that some candidates had not prepared themselves well for the examination. The use of past papers to ensure adequate preparation is strongly recommended. A student's first exposure to STEP questions can be a daunting, demanding experience; it is a shame if that takes place during a public examination on which so much rides.

Further, and fuller, discussion of the solutions to these questions can be found in the *Hints and Answers* document.

Comments on specific questions

1 This question required little more than a clear head and some persistence: candidates had either ample or very little of both, and thus most scores were either high or very low. The examiners would like to stress that a solution to a question such as this must be written out methodically and coherently: many answers which began promisingly were soon hopelessly fragmented and incoherent, leaving the candidate unable to regain his or her train of thought. This was especially true when deriving the final expression given on the exam paper. Examiners follow closely a candidate's line of reasoning, and they have to be certain that the candidate has constructed a complete argument, and that he or she has not arrived at a printed result without full justification.

2 This was a popular question, and was usually well done. The argument at the end was often incomplete, though: many candidates simply stated that $t = 1$ or $t = 2$ without explaining why no other values were possible. To do so, use had to be made of the fact that s and t have no common factor other than 1.

3 This was the most popular question on the paper, and many different methods were seen. The intended method was to use the identities $\cos^4\theta - \sin^4\theta \equiv \cos 2\theta$ and $\cos^4\theta + \sin^4\theta \equiv 1 - ½ \sin^2 2\theta$ to evaluate the integrals of $\cos^4\theta - \sin^4\theta$ and $\cos^4\theta + \sin^4\theta$, and hence be able to write down

separately the values of the integrals of $\cos^4 \theta$ and $\sin^4 \theta$. A similar approach works well for $\cos^6 \theta - \sin^6 \theta$ and $\cos^6 \theta + \sin^6 \theta$. Other methods were, of course, acceptable, and many candidates received high marks for this question.

4 This question was found to be very difficult. The initial factorisation was beyond most candidates, even given the linear factor $x + b + c$. Anyone who wants to read Mathematics at university must be able to factorise quickly cubic expressions such as this one, and also $x^3 \pm y^3$. The *Hints and Answers* document discusses this in more detail.

Candidates who progressed to the second part of the question often deduced that $ak^2 + bk + c = 0$ and $bk^2 + ck + a = 0$, but then tried to eliminate k; given that the result they were asked to derive was still in terms of k, this was an unwise strategy.

5 Only a few candidates made much progress with this question, even though it only required GCSE Mathematics. Basic properties of triangles (for example, the sine and cosine rule, and the location of the centroid, the circumcentre and the incentre) are assumed knowledge at this level. It was surprising how many candidates tried to answer this question without a diagram.

6 This was a popular, straightforward question, which was often answered well. However, algebraic errors still occurred, for example when expanding $(x - y)^3$.

7 Part (i) was well done by most of those who attempted this question, but many then found it difficult to develop the strategy in part (ii). A certain amount of trial and error is needed to complete the squares in an expression in terms of both α and β, but the coefficients (in particular, $\mathbf{1}\alpha^2$, $\mathbf{1}\beta^2$ and $\mathbf{26}\beta^2k^2$) do not permit many possibilities. This question demanded some stamina, as Mathematics at university level also does.

8 This question was answered poorly; many candidates were unable to sketch the graphs correctly, even given the results derived earlier in the question. For example, many graphs did not touch at (2, 8). Also, many graphs were drawn with turning points, when a simple check of the derivative would have revealed that there were none. In part (iii), the effect of the negative coefficient of x^3 was often ignored.

Graph sketching is a very important skill in all mathematical subjects – from Economics to Engineering. STEP candidates are strongly advised to practise this skill as much as possible.

9 This was a popular question, and was usually well done. Not many candidates recognised that $\sin \theta \cos \theta \equiv \frac{1}{2} \sin 2\theta$, which makes the final inequality easier to obtain. Knowing identities "both ways" is important.

10 Only a few attempts at this question were seen, and those that did rarely made much headway; worryingly, the accurate simplification of the solutions of simple linear equations was found to be very difficult.

11 Hardly any attempts at this question were seen. It was remarkable how few diagrams were seen; it is always much easier for both the candidate and the examiner if answers begin with a labelled diagram.

12 Very few tree diagrams were seen here, and hence very few correct solutions were constructed; a clear tree diagram is invaluable when attempting a complicated probability question such as part (ii). Most candidates identified some (if not all) of the possible outcomes, but many mistakes were made (for example, writing a denominator of N rather than $N + 1$ or $N - 1$).

The subsequent algebraic simplification was found to be very demanding. Candidates would have probably made more progress if they had been more willing to factorise groups of terms which had obvious common factors, rather than (for example) attempting to write all the fractions with a common denominator.

13 A lot of attempts at this question were seen, but conceptual errors undermined many solutions. In particular, a lot of candidates seemed not to realise that they were being asked to calculate conditional probabilities in parts (ii) to (vi).

14 Only a few attempts at this question were seen. Poor graph sketching limited many candidates' progress; the importance of the ability to sketch accurately standard graphs such as $y = xe^{-x}$ cannot be overstated.