STEP 3 2013 Examiners' report

With the number of candidates submitting scripts up by some 8% from last year, and whilst inevitably some questions were more popular than others, namely the first two, 7 then 4 and 5 to a lesser extent , all questions on the paper were attempted by a significant number of candidates. About a sixth of candidates gave in answers to more than six questions, but the extra questions were invariably scoring negligible marks. Two fifths of the candidates gave in answers to six questions.

1. Most candidates attempted this question, making it the most popular and it was also the most successful with a mean score of about two thirds marks. The first two standard results caused few problems, nor did the integration, but some struggled to simplify to the single inverse tan form. In the final part, common errors were failure to reduce to the $n = 0$ case, confusion with the index e.g. $I_n + 2I_{n-1} = \int_0^{\frac{1}{2}\pi} \sin^n x \, dx$ instead of the correct result, or for those that were more successful, algebraic inaccuracies let them down. Some attempted a recursive formula to evaluate $\int_0^{\frac{1}{2}\pi} \sin^n x \, dx$ with varying success. Most attempting the last part saw the connection between I_0 and the main result of the question.

2. This was the second most popular question, attempted by six out of every seven candidates, with only marginally less success than its predecessor. The first differential equation was proved correctly and many successfully completed the general result by induction, although there were some problems with the initial case. Some had difficulty finding the correct coefficients for the odd powers of x in the Maclaurin series but the last part produced a variety of errors and few correct answers. Such errors included $\sin^{-1}\frac{1}{2} = \frac{\pi}{3}$, forgetting to divide y by x , and attempting to evaluate the series using $x = 1$.

3. A seventh of the candidates attempted this, making this the second least popular Pure question, though with on average, half marks being scored, it was the third most successful of the Pure questions. Some candidates found the scalar product of $p_1 + p_2 + p_3 + p_4$ with itself to obtain the stem correctly, whilst some found its product with p_1 or p_i , in which case they did not always appreciate the importance of symmetry. Part (i) caused few problems. Part (ii) saw a few errors with consideration of $\pm$ signs, though some candidates used geometric considerations and then rotations correctly to obtain the results. The last part separated the sheep from the goats.

4. Just over two thirds of candidates attempted this with moderate success, approximately one third marks. Most succeeded with the opening result but even so, some lacked full explanation. Whilst most wrote down the correct form for the roots, few correctly expressed all the roots in the given range. Surprisingly, there was very limited understanding of the connection between the roots and the factors of $1 + z^{2n}$ so the general result was not well answered. Conversely, part (i) was well-answered with the exception of those who did not deal with the powers of i satisfactorily. Part (ii) was beyond most candidates mainly because they failed to cancel the factor $1 + z^2$. However, those that managed to deal with this aspect generally answered the whole question very well.

5. Nearly as many attempted this as question 4, but only achieving a quarter of the marks making it the least successfully answered question. Almost all missed the point of the question given in the first sentence, and made other assumptions, which frequently only applied to primes rather than integers in general. As a consequence, most did not satisfactorily justify their results.

They generally fared better tackling the second part of (i), though some tried to prove the statement in the wrong direction. They approached (ii) better though few gave a valid argument why $p^n \leq n$.

6. About half attempted this with marginally more success than question 4. Many candidates tried to write $z = x + iy$ or similar and likewise for w and then tried to expand which involved a lot more work than dealing with conjugates directly. Some tried to use the cosine rule rather than the triangle inequality from the diagram. In general, the first result and parts (i) and (ii) were well done but only the strongest candidates did better than pick up the odd mark here and there in trying to obtain the inequality. A lot of mistakes were made mishandling inequalities, but even those who could do this correctly overlooked the necessity of substantiating that th4 square roots are positive and that the denominator is non-zero.

7. Three quarters attempted this with more success than question 6 but less than question 3. Sadly, it was not uncommon for candidates to fail to differentiate $E(x)$ correctly. Many established that $\frac{dE}{dx} = 0$ but then $\frac{d^2y}{dx^2} = -1$, when $y = 1$, $\frac{dy}{dx} = 0$, and $x = 0$ giving a maximum which was not sufficient and missed the point of the squared $\frac{dy}{dx}$ term in $E(x)$, with consequences for the rest of the question. Many followed the stationary points line of logic correctly by considering the maximum and minimum values in part (i). Having established the constant value of $E(x)$, some candidates attempted to solve the differential equation, usually by incorrect methods. The errors of part (i) were largely replicated in part (ii). There were fewer attempts at part (iii), and a number fell at the first hurdle through not obtaining the correct $E(x)$. Further, numerous candidates assumed rather than proved that $5\cosh x - 4\sinh x - 3 \geq 0$.

8. A seventh answered this question, making it the second least attempted question scoring a third of the marks possible. The first result evaded many candidates who did not identify and calculate the geometric progression, although a few did employ the fact that the sum of the roots of unity is zero. The result for s caused few problems and was for many candidates the only success in the question. Those that attempted the length of the chord were comfortable with the algebra of trigonometry namely $\cos(\theta + \pi) = -\cos\theta$, and $2\cos^2\theta - 1 = \cos 2\theta$. There was mixed success with completing the final result.

9. About a fifth attempted this, with the same success as question 7. Common errors were false attempts for the volume at the beginning using hemisphere and cones, and in the last part approximating x small rather than $x - \frac{1}{2}R$ small. Many candidates were successful as far as the equilibrium but couldn't deal with the small oscillations successfully.

10. The number of candidates attempting this was almost identical to that attempting question 3 with marginally more success making it the third best attempted question. Most obtained the moment of inertia correctly, and many found the angular velocity correctly. Provided that they had correctly applied conservation of angular momentum, and Newton's law of elasticity, they almost all worked out the required result. Some attempted to use conservation of linear momentum whilst others did not use conservation of angular momentum correctly. Most then knew how to differentiate, but many made computation errors. Even if they got the correct quadratic equation at the end, many solve d it wrongly. Very few showed that the feasible solution did indeed generate a maximum.

11. A fifth of the candidates attempted this question, with marginally less success than question 3. Most that attempted this question managed to achieve the first two results successfully, unless they got the diagram wrong. However, the final result was found trickier as some forgot to include the gravitational potential energy, some failed to evaluate the correct elastic potential energy and there were many mistakes made handling the surds.

12. This was the least popular question, attempted by a ninth of the candidates, with slightly less success than question 8. The immediate problem was many made no mention of probabilities in order to calculate expectations. Throughout, there was very poor justification, which included treating the random variables as though they were independent and compensating errors which led to given results. Most progressed no further than part (a) of (ii) at best and many had $E(X_i) = \frac{ab}{n^2}$.

13. The number attempting this was very similar to that attempting question 3 with the same level of success as question 11. In general, candidates attempted both parts of (a) correctly, and then likewise part (i) of (b) then stopped. However, part (b) (ii) tripped up many. Some successfully dealt with part (iii) without having managed (ii).