STEP MATHEMATICS 2

2019

Examiner's Report

Introduction

The Pure questions were again the most popular of the paper, with only one of the questions attempted by fewer than half of the candidates (of the remaining four questions, only question 9 was attempted by more than a quarter of the candidates). In many of the questions candidates were often unable to make good use of the results shown in the earlier parts of the question in order to solve the more complex later parts. Nevertheless, some good solutions were seen to all of the questions. For many of the questions, solutions were seen in which the results were reached, but without sufficient justification of some of the steps.

Question 1

This was the question answered by the largest proportion of candidates and many good solutions were seen. However, many candidates did not appreciate the importance of the phrase *if and only if* in parts of this question. As a result a large number of attempts failed to achieve full marks as it was not made clear that the reasoning presented also worked in the opposite direction.

Having shown the first result, many candidates were able to identify the appropriate choice of $g(x)$ when attempting part (i) and successfully showed that $2a = q + r$. Many were also able to find a correct expression for the gradient, although some did not find this expression in terms of the variables requested.

In part (ii) a pleasing number of candidates were able to recognise that the results from part (i) would be relevant here as well. Again, some of the solutions to this part failed to recognise that the question required the result to be shown in both directions.

Question 2

This question was another popular question that was generally well answered, achieving the second-best average mark of all of the questions and was also the question for which the largest number of solutions received full marks. Most candidates drew a convincing sketch to demonstrate that the two integrals make a rectangle. Arguments from sketches showing the inverse function and reflective symmetry were less successful and often candidates' diagrams assumed x to be a fixed point of $f(t)$.

By far the most common mistake in the first part was to notice the solution $g(2) = 1$ but not to factorise and use the quadratic discriminant to show that no other solutions were possible. The conceptually difficult part was to use $g^{-1}(y) = y^3 + y$, and many candidates stopped just before this point.

In the final part, many candidates tried to apply the stem identity in its original form, without noticing that $h(0) \neq 0$. This was the most difficult part, and those who modified it correctly generally did well. Candidates sometimes failed to check that $h'(t) > 0$, but this was not necessary for those who used $h(t) = g(t + 2)$.

Question 3

While this was a popular question it was also the one where the average mark achieved by candidates was the lowest. In this question many of the results to be reached were given in the question. Students therefore need to recognise that it is necessary for solutions to be presented very clearly, and it is for this reason that many solutions in the first parts did not achieve full marks. For example, justifications of the generalised result for a set of n real numbers expressed in the form of an inductive proof were the most successful.

For most candidates the majority of marks were scored in the sections up to and including part (i)(b). Many candidates were then unable to see how to work in the cases where $|x| \geq 1$ for part (i)(c). In the final part, candidates were often unable to put the equation into the form that had been used in the earlier parts of the questions and therefore did not manage to reduce the possible values of the integer roots to a sufficiently small set.

Question 4

This was a well-answered question, but also one in which a fairly large number of solutions scored very low marks. The majority of candidates were able to evaluate the first product using the identify provided and most were then able to apply the same technique to simplify the first expression in part (i). Many students then differentiated, but some then struggled to manage the notation correctly to reach the second result requested in part (ii).

Part (iii) required some care to ensure that the sums and products were over the correct range, but those who managed to adjust correctly for this were then able to reach the required results.

Question 5

It was difficult to get full marks on this question, with most candidates struggling to correctly prove 'if and only if' statements in both directions.

Mostly, the two constant sequences were successfully found and then correctly rejected for sequences of period 2, but few thought to check that the other two solutions to the quartic did not also coincide with the constant sequences. Most candidates were able to use the discriminant to produce bounds on p, but many could not justify the strictness of the inequality, which was best done by considering the boundary cases separately.

The first request of the second part was answered well, with most using only the fact that it was a positive quadratic and a minority delving into the details of $f(x)$. Most candidates who reached this part of the questions correctly used the result $f(x) > x$ to show that $f\big(f(x)\big) = x$ has no solutions, but many overlooked the connection between the final part and part (i).

Question 6

Of the Pure questions, this was the question that had the lowest average mark, mainly due to the large number of attempts that did not manage to score any marks.

Many candidates seemed uncomfortable with this question which asked them to look at what information can be gleaned about differential equations without directly solving them. Many candidates decided that the only way to proceed was to solve the differential equation, and almost invariably this led to long and convoluted methods. Candidates seemed to have very little idea that the differential equation can be interpreted as the gradient of a curve at different points – it was simply an object on which certain methods had to be applied. A surprisingly small number of candidates realised that setting $\frac{dy}{dx} = 0$ could (and should) be done directly in the differential equation to find the locus of stationary points.

This was also a question which required candidates to bring a lot of disparate information together in the final sketches. A large number of candidates said things like the gradient was negative between two lines, but their sketch showed something different. Some who said that there should be stationary points on the line $y = x - 1$ and $y = x - 3$ drew their curve tangentially to these two lines instead.

Overall this was a question which really benefitted candidates who took a moment to stop and think about what was being suggested, rather than blindly applying methods.

Question 7

This was the least popular of the Pure questions. Good solutions to this question often included clear diagrams to enable the angles being discussed to be identified easily. Many of the candidates were able to calculate the value of $\boldsymbol{a} \cdot \boldsymbol{b}$ correctly, but often did not fully justify that the triangle ABC was equilateral.

For the second part, many candidates were again able to establish the relationship between scalar products, but less success was seen in identifying the type of quadrilateral. In the final part there were a large number of different approaches taken and many of these were completed successfully by some of the candidates.

Question 8

Many good solutions were seen to this question, but solutions often lacked clear enough justification to be awarded full marks. However, there were also a surprising number of candidates who did not manage to invert the 2x2 matrices successfully. Candidates who claimed that the function f was the determinant of the matrix were not able to score high marks as the solutions did not then demonstrate that the results were true of *any* function satisfying the property given.

The first two parts of this question were largely done well. The third part was found more difficult, with few candidates realising that $\begin{pmatrix} a & b \\ ka & kb \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}\begin{pmatrix} a & b \\ a & b \end{pmatrix}$. Those who did were then often able to provide a full solution, although often these were not fully justified. Several candidates instead used $\begin{pmatrix} 1 & 0 \\ 0 & k \end{pmatrix}\begin{pmatrix} a & b \\ k^{-1}a & k^{-1}b \end{pmatrix} = \begin{pmatrix} a & b \\ a & b \end{pmatrix}$ to produce a solution which covered all cases apart from the one where $k = 0$. In some cases, candidates did not appear to consider $\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}$ to be an example of a matrix in which the second row was a multiple of the first.

In part (iv) many candidates made use of the fact that $f(P) \neq 0$ without showing that this must be the case.

Question 9

This was the most popular of the Mechanics and Statistics questions, but also one of the questions that attracted a large number of solutions that received no marks.

Students seemed relatively good at setting up the kinematics equations in this question and most had the useful idea of differentiating. Somewhat fewer thought about using either completing the square or the quadratic discriminant to decide where the derivative was positive. The logic of the question was very poorly understood, with many students seeing the given inequality as the end point rather than the starting point of the question.

In the second part of part (i) it was important that students demonstrated not just that a time existed where the distance is decreasing, but that this time was in the acceptable domain of the question.

Part (ii) was conceptually very similar to part (i) but most students found the increased algebraic demand too much.

Question 10

As with so many questions, the big stumbling block for students was drawing a good diagram from the information, including all the relevant forces.

With “show that” questions it is beholden on candidates to explain their working. Equations which just appear and lead to the correct answer are not sufficient. In mechanics, it would be very helpful for students to say, for example, “Taking moments about point A for the rod” or “Resolving for the string -rod system vertically” to give some sense of where an equation arises.

The flow of logic is a fundamental idea in mathematics, but it was clear in this question that it was not familiar to the vast majority of students. The questions effectively asked “if <given condition> show that <mechanical outcome>”. Most students reversed this to show that “if <mechanical outcome> then <given condition>”. In this question, most arguments were reversible, but it still demonstrated a fundamental misunderstanding of what was being asked.

The other issue which flummoxed students was dealing with inequalities. There are different rules of algebra associated with inequalities and this is something which is frequently tested in STEP. Candidates would benefit from thinking carefully about things like when can one inequality be substituted into another, or when can an inequality be squared. The intuition from equalities was too often applied without thinking.

Question 11

Candidates got the correct number of pairs in the special cases $n_3 = 9$ and $n_3 = 10$ but sometimes the working was very unclear. A large majority found the expressions for general n, the most common error being a shift $n \to n + 1$ in the answer.

Those who could obtain the result given for odd n in part (ii) were generally able to find the corresponding result for even n too. A common error was to double count the number of pairs of rods and not to double the number of pairs which made a triangle. Many candidates failed to explain why the conditions of part (i) were relevant for forming triangles.

The most successful candidates in part (iii) counted the number of triples which make a triangle using a sum, and divided by $\binom{2M+1}{3}$, while those who conditioned on the largest rod and used conditional probability did less well. A common conceptual error was to assume that each integer was equally likely to appear as the largest rod, and candidates making this assumption lost many marks. Otherwise, algebraic errors were the most common. Candidates should remember that when an answer is given in the question, they need to take care to fully justify their answers.

Question 12

Almost all candidates who attempted this question were able to achieve full marks on the first part. In the second part, the values of the interquartile range and 2σ were generally found correctly, but then many candidates did not realise that squaring would eliminate the square roots from the values to be compared.

In the final part of the question some candidates failed to recognise that the $(k+1)^{th}$ term of the expansion was the term in x^k and gave the term in x^{k+1} instead. A good number were successful in finding the lower quartile and the median, but only a minority realised that $\mu^{-n} = \left(1+\frac{1}{n}\right)^n$. Those that did were more successful in proving that $\mu > \left(\frac{1}{4}\right)^n$ than $\mu < \left(\frac{1}{2}\right)^n$.