SI 2015Report

General Comments

The aim of this report is to account for how well, or how poorly, the candidates performed this year while at the same time attempting to indicate their corresponding areas of strength, or weakness. If I should concentrate marginally more in the direction of the candidates' weaknesses, the reader should understand that it is with the hope that both future STEP candidates, and the teachers preparing them for this examination in years to come, will have the opportunity to focus on those areas of common weakness in an effort to ensure a better preparedness for what is unquestionably the most demanding of examinations in the UK for students of pre-university years.

For the record, the scripts are marked by a team of postgraduate mathematical students – many working towards a doctorate in this, or another, closely-related subject – who spend days poring over the scripts, working in small teams under the supervision of the Principal Examiner and carefully appointed "question captains". Their powers of concentration are truly phenomenal and they not only appreciate the need for mathematical rigour but (due to having once been in exactly this position themselves) are also deeply sympathetic towards the candidates; making every effort first to understand what has been presented to them by the candidates and then to reward genuinely good mathematics when it appears, no matter how hastily and/or messily it has been set down onto paper. Thus it is that the comments produced by the Principal Examiner within this Report are merely summaries of what these markers have passed on to him (or her) at the end of the marking period. Moreover, since the candidates' backgrounds are entirely unknown to the markers, any comments – critical or otherwise – cannot possibly be taken to have been directed towards specifically chosen targets.

More than 2000 candidates sat SI this year, which represents another increase of around 10% over last year's entry numbers. Once again, however, it is sadly the case that many of these candidates have simply not prepared sufficiently well to be in a position to emerge from the experience with any amount of positive feelings of success at the results of their efforts. In the first instance, many candidates (this year, more than half of the entry) attempt more than the recommended six questions. This automatically penalises them for the time that they have spent on extra questions whose marks will not count towards their final total – remember that only the highest-scoring six questions count towards a candidate's final total; note also that a grade 1 can usually be obtained from four questions which have been completed reasonably successfully, or from two questions done completely correctly plus four "halves", or from any in-between combination of question-scores. Thus, it is strongly advised that candidates spend a few minutes at some stage of the examination reading the questions carefully with a view to deciding which of them they would best attempt.

Overall, this year's paper worked out in very much the same way as had the 2014 paper, with a mean score of around 43-44% and with approximately 45% of candidates failing to exceed a total of 40 marks; though totals in excess of 100 marks were slightly down on 2014. Part of the reason for this is that the five applied maths questions were very much non-standard this year, and this prevented a lot of very easy marks (for routine beginnings) being picked up by candidates attempting these questions. The mean score for Qs.9-13 thus fell from 6½ in 2014 to 2½ in 2015. Another trend of recent years is the widespread dislike for the vectors questions, which have become both unpopular and very low-scoring for candidates.

Points of general application regarding candidates' attempts this year are little different to usual – far too many candidates produce only fragmentary attempts (often, as mentioned above, to almost every question they attempt) at solutions, with little apparent intent to persevere beyond the first obstacle. Presentation was also particularly poor this year, with most candidates making life hard both for themselves and for the markers who genuinely wish to find credit-worthy mathematics in order to award the marks available. In long questions such as these, with the barest minimum of structure provided, candidates need a lot of prior practice at past STEP questions in learning how to supply their own.

Curve-sketching skills continue to be a weakness, as candidates tend to veer away from justifying what they have drawn; algebra and calculus skills are very mixed, and it is was especially clear this year how little candidates like being required to formulate their own solution-strategies – no doubt being the result of an over-reliance on being told exactly what to do, as is customarily the case in AS- and A-level papers.

On the other side of the coin, there was a very pleasing number of candidates who produced exceptional pieces of work on 5 or 6 questions (or more), and thus scored very highly indeed on the paper overall. Around 80 of them scored 90+ marks of the 120 available, and they should be very proud of their performance – it is a significant and noteworthy achievement.

Comments on individual questions

[Examiner's note: in order to extract the maximum amount of profit from this report, I would firmly recommend that the reader studies this report alongside the *Hints and Solutions* or *Marking Scheme* supplied separately.]

Q1 Traditionally, question 1is intended to be the most generous and/or helpful question on the paper and, most years, it is attempted by almost everyone. This year – despite the fact that it is obviously the question most similar to one that might appear on an A-level paper – the lack of given structure, and the requirement for sketching, clearly put many candidates off, and only three-quarters of candidates even started it. Once started, however, it proved to be the highest-scoring question for candidates (as was the intention), eliciting a mean score of over 13½ out of 20. Most serious attempts were thus highly successful, and it was generally only in part (ii) that marks were commonly lost in bulk. Sadly, many who did move into (ii) with some measure of success, overlooked the fact that it was a fairly straightforward follow-on from all the information gained or used in part (i), and many attempts were thus successful but unnecessarily long-winded.

Q2 This was the most popular question of all, attempted by around 85% of candidates, and producing a mean success rate of just over 10 out of 20. Part (i) was almost always concluded successfully, and by the anticipated method of the use of the addition-formulae for sine and cosine; a method then used sensibly, along with the well-known double-angle formulae, in (ii) to establish that $\cos\alpha$ was indeed a root of the given cubic equation. For many candidates, though, the other roots of this cubic were often "found" by guesswork, and many candidates thought it appropriate to "cancel" x with $\cos\alpha$ in some strange way, rather than resort to the use of the quadratic formula. Part (iii) was frequently not attempted at all, and many who did boldly venture forth therein did so by **not** using the results of parts (i) and (ii) as instructed. A few rather presumptuously assumed that $\cos 15^{\circ}$ was a solution, when a little care would have revealed that it is, in fact, $2\cos 15^{\circ}$ that fits the bill; the key being that $2\cos 45^{\circ}$ is the $\sqrt{2}$ at the end of the given equation.

Q3 This question was attempted by just over half of the candidature, but produced a mean score of only 4 marks ... largely due (I suspect) to the twin difficulty of a lack of supplied structure and the poor ability of candidates to do their own modelling. In many instances, the two cases $b < 3a$ and $b > 3a$, given in the question, proved to be unhelpful as many candidates chose specific values of b in each of these ranges as "exemplar" values of b and then supposed that this sufficed in establishing the boundary cases; when, in fact, the given information was intended to guide where they were to end up rather than from where they should begin. It was also especially disappointing to see that so many candidates struggled to explain what they were doing, thinking that some poorly labelled diagram would 'do the trick'. The poor thinking behind the diagrams usually meant, for instance, that one of the two possible scenarios for when the guard stood at the midpoint of a side was completely unconsidered. Rather strangely – and disastrously in terms of scoring any marks at all – it was very common indeed for candidates to have considered the **area** of the courtyard that was visible to the guard, despite the very clear reference to the "length of the perimeter" in the question.

Q4 Amongst the pure maths questions, this one was least popular, with less than a third of candidates making an attempt at it, and producing a mean score of 4½ marks. Unlike Q3, there was a lot of helpful information given in the question, and key intermediate results also. Those candidates who realised that the gradient of the rod was $\tan\theta$ answered the first part quite acceptably, although it was relatively common to see the gradient of the curve, $\frac{1}{2}x$, being used as the "m" in the formula $y = mx + c$ for a straight line.

It is, unfortunately, often the case that when an answer is given with the view that it will prove helpful to candidates, that they then miraculously manage to obtain it through any means possible, and there were many wayward attempts to justify the given final answer without any clear supporting evidence: the biggest difficulty arose from the need to use $A_x = 0$ in order to eliminate b.

Q5 More than 60% of candidates attempted this question, and scores were relatively healthy, with a mean of almost 7½. Those candidates who realised that x was being treated as a constant within the integrals generally found it fairly straightforward to make good progress; those who didn't were doomed to failure from the outset. The sketches were generally completed fairly successfully, though few candidates managed to be entirely convincing, especially in (ii), where the modulus function needed to be employed (although some candidates thought the matter through sufficiently carefully without it by considering the various intervals of the domain of g).

Q6 This vectors question proved both unpopular and low-scoring, eliciting a mean of only 2.3 out of 20. In many cases, this was because candidates started their "solution" with a diagram before abandoning it altogether and moving on elsewhere. Most of the remaining attempts assumed that the quadrilateral was a square, rectangle or parallelogram to begin with – whether through a misreading of the question or through an inability to deal with a general scenario it is hard to say. Moreover, the usual convention of underlining vector quantities (thereby distinguishing them from scalar ones) was almost universally avoided, and this made it extremely difficult to give serious consideration to much of what was written, as candidates moved from scalar to vector and back again.

Q7 This was another popular question, since most candidates were able to make some progress with the ideas involved, though few seemed to have a particularly thorough grasp. The change in the variable being considered – from x to f(x) and then to a – was evidently the source of much of the confusion, though I am sure that candidates would have made better progress with a more carefully laid out plan for working through the different possibilities. In the end, it all boiled down to the fact that a (continuous) function takes its maximum value *on a finite interval* at either an endpoint or at a maximum turning-point. Thereafter, it was important to do some sensible comparisons using inequalities. Thus, there were easy enough marks to be had and the mean score of 7.7 out of 20 was the third highest on any question, after questions 1 & 2.

Q8 This was another very popular question, with three-quarters of the candidature making a start at it. However, the mean score of 5.4 almost certainly arose from the acquisition of the 3 marks allocated to the bit of introductory bookwork plus 2 or 3 marks gained by considering a suitable pairing of terms for considering S in (ii). Inductive proofs were unnecessary in (i), and almost invariably went wrong in (ii); this was a shame when the appearance of the term $(N-m)^k$ in the given expression really indicated for the use of the binomial theorem. It should have been relatively straightforward to apply the given initial result of (ii) in the two cases that followed, but each required the addition of an extra 0^k term … perversely, to make the odd number of terms into an even one, and then v.v. (since it is now the isolated middle term that is crucial), and this extra leap of intuition was clearly where the difficulty lay. The final arguments relied on a sound grasp of what had gone before, and most candidates had given up by this point.

Qs.9-11 These mechanics questions were – as mentioned earlier – very non-standard, and hence found very difficult. Of the relatively few attempts appearing from candidates, almost none of them got further than a hesitant start. There were two or three easy marks to be had in Q9 in finding the general time for any one bullet to land, but very few candidates were able to cope with replacing a specific launch angle with the variable angle given

In Q10, the real key to successful progress was to avoid worrying about any constants of proportionality (such as that introduced by the unstated width of the bus), so very few candidates managed to produce the early given result in a satisfactorily justifiable way. Introducing an extra proportionality relationship for the journey time was then a leap too far, even for those who had started well. Moreover, there were some candidates who only seem to be able to maximise or minimise a function by using calculus, and this provided an extra layer of unnecessary clutter here. Fewer than 200 candidates started this question, and most of these attempts had little more than a sketchy diagram for the markers to consider.

Q11 attracted double the number of attempts of Q10, but had a marginally lower mean score, and this was slightly surprising. A few years ago, statics questions such as this would have been gobbled up with glee by many candidates, happy to collect some very easy mechanics marks. The great hurdle for the weaker candidates remains the widespread inability to draw a good diagram with all relevant forces marked on it in appropriate directions. Sadly, here, almost all diagrams failed to include all of the relevant forces, and decent progress beyond that point was, therefore, essentially impossible. Remarkably few candidates managed even to explain satisfactorily that the two frictional forces were equal (by taking moments about the central axis of each of the two cylinders).

Qs12 & 13 These questions were also less routine than has usually been the case in recent years, although they were nowhere near as demanding as the 2-3 mark mean score might suggest. In Q12, despite the reference to the Poisson Distribution in the introduction, part (i) required a simple statement of a Binomial term. Part (ii) then proved difficult as it became clear that few candidates could manipulate a summation of terms in order to establish a result they might have anticipated being allowed to quote in an ordinary A-level examination. A few candidates managed part (iii) perfectly adequately without having gone very far with (ii), and this represented a shrewd use of "examination technique" on their part.

Most attempts at Q13 got little further on in the question than writing the simple, general term for P(A); namely $\left(\frac{5}{6}\right)^{n-1}\left(\frac{1}{6}\right)$. Even though the Geometric Distribution is not expected here, the probability of the run of independent events "n – 1 failures followed by a success" should be within the scope of any STEP candidate (who has studied any small amount of probability and/or statistics). Many candidates decided that (ii) and (iii) also required a general expression for each of P(B) and P($B \cap C$), whereas these were clearly intended to be numerical, and a brief '*symmetry*' argument quickly reveals their respective probabilities to be $\frac{1}{2}$ and $\frac{1}{3}$. Parts (iv) and (v) were tougher, but required the candidates to see that each was the sum of an infinite number of terms, and the helpfully given series expansion at the end of the question helped wrap these up. One third of all candidates made an attempt at this question, but almost none of them got around to either of these final two parts.