Cambridge Assessment
Admissions Testing

Sixth Term Examination Paper [STEP]

Mathematics 3 [9475]

2018

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General Comments

The total entry was a record number, an increase of over 6% on 2017. Only question 1 was attempted by more than 90%, although question 2 was attempted by very nearly 90%. Every question was attempted by a significant number of candidates with even the two least popular questions being attempted by 9%. More than 92% restricted themselves to attempting no more than 7 questions with very few indeed attempting more than 8. As has been normal in the past, apart from a handful of very strong candidates, those attempting six questions scored better than those attempting more than six.

Question 1

As usual, this was the most popular question to be attempted with more than 93% of candidates doing so. However, scoring for it was only moderately good with a mean below 9/20. Most successfully differentiated and obtained a value for β from the cubic but ignored considering whether this was the only stationary point. Sketches frequently did not display the asymptote; some that did showed the negative branch of the curve touching rather than intersecting the asymptote at the maximum. Many did not appreciate that to sketch the second curve in part (i) it was not sufficient to just offer a drawing without the working; the horizontal point of inflection and asymptote were frequent casualties. Part (ii) was straightforward for most. Many recognised that part (iii) made use of the first function $f(\beta)$, provided that they used the condition to substitute for α . However, their justification suffered from ignoring the reality condition and using specious arguments, as a consequence. Part (iv) followed a similar trend to part (iii), except using (β) , and only differing in that of those that did apply the reality condition, quite a few overlooked $\beta = 1$ as a solution of the cubic inequality, and so their final answer was wrong.

Question 2

With about 89% attempting this, it was the second most popular question, and with a mean score of nearly 11/20, the second most successful. Part (i) was generally well-handled, and although most scored some marks on the proof by induction in part (ii), candidates often struggled to complete it, many of them because they attempted to use the original definition of the functions, which rarely led to success, rather than employ the result of part (i). Some candidates noticed that the proof by induction in part (ii) was equivalent to proving $\frac{dy_n}{dx} = 2ny_{n-1}$ by induction. This gave them a simpler base case but did not significantly simplify the inductive step. For the deduction in part (ii), it was very common for the first result of part (ii) to be squared, leading to pages of algebra, although they were often then successful. Part (iii) was surprisingly poorly attempted as few candidates realised that a proof by induction (or equivalent) was required.

Question 3

Only 65% attempted this, and it was the second weakest of the Pure questions with a mean score of about 7/20. It was imperative for candidates to demonstrate high levels of algebraic accuracy to score highly. Most successfully differentiated the initial expression, and equated coefficients but then failed to solve explicitly for a, b and c in terms of p and q (or demonstrate that such a, b and c existed). Candidates without these explicit expressions then often failed to spot one of the main strands of the question, integration of x^n in the two cases $n = -1$, and $n \neq -1$; some considered superfluous other cases. Some candidates fell at the final hurdle having used correct methods but then did not express their solutions in terms of p and q. Also, some were thrown by the two possible sets of solutions for a, b and c, successful candidates realising that these gave the same solutions to the differential equations.

Question 4

The third most popular question being attempted by just short of three quarters of the candidature, it was however the most successfully attempted with a mean score of not quite 12/20. The stem was usually correctly attempted either using parametric or implicit differentiation. Simultaneous equations were sensibly attempted for part (i), but sometimes they confused the two pairs of equations and as a result got the wrong answer. Some solutions elegantly achieved the correct result having found just one of the coordinates and arguing that as it lay on the tangent, it had to be the point P. Part (ii) was quite often abandoned partway through, giving up after obtaining x^2 and y^2 in the face of the algebra, although some forgot to answer the question at this point even though they had employed simultaneous equations to obtain x and y. Few managed to conclude the question, and it was very rare indeed that the non-zero nature of the denominator $(\sin\varphi - \sin\theta)$ was justified.

Question 5

A little under half the candidates attempted this, scoring marginally less on average than on question 1. Some candidates used the arithmetic mean/geometric mean inequality which was what the question was proving in part (iii), and so were heavily penalised as their arguments were thus circular. Part (i) was generally well done, with most justifying that their steps were reversible to obtain 'if and only if'. Marks were sometimes lost when justifying that $G_k > 0$ was not used to legitimise division by G_k and also that inequalities did not change sign. Part (ii) was well done too, though many candidates did not justify the 'only if', although some tried to use part (i) to prove (ii), which could not succeed. Part (iii) expressly required deduction, and only deduction, so those who had learned another proof of the inequality and just copied it out could not be rewarded. Many just used the results of (i) and (ii) without justifying why they could be used and some were imprecise with their induction arguments.

Question 6

The least popular of the Pure questions being attempted by under 40% of the candidates, it was the second least successfully attempted question in the whole paper with a mean score of only just better than 5/20. Many alternative solutions were successfully offered for the very first result, but then it was not an uncommon pitfall that q and c were treated as scalar multiples of a, and moreover, the * notation appeared to confuse some candidates. Those who moved onto part (ii), got started fairly well, but then found difficulties; those that used the fact that the points were on the unit circle, however, derived the final equation with ease, earning full marks at that stage. The few candidates who had the stamina to see the question through to part (iii) were generally very successful, though odd marks were lost when division by various factors was not justified as valid by their being non-zero.

Question 7

Marginally more popular than question 6, the mean score was 8/20. The first part of (i) created no problems, but far fewer got further, although those that expanded $(\cot\theta + i)^{2n+1}$ almost always managed to succeed, even if they did not always quite deal with the $\sin\theta \neq 0$ condition. Most that attempted part (ii) scored all the marks, though some forgot to divide $\binom{2n+1}{3}$ by $\binom{2n+1}{1}$.
Almost all that tried part (iii) proved the first result but applying the results to obtain the conclusion proved harder. A common mistake was to sum over θ rather than m and there were quite often mistakes in the algebra for the last part.

Question 8

The fifth most popular question, this was attempted by five eighths of the candidates, and was the fourth most successfully attempted with a mean score of a little over 9/20. The first result in (i) was generally very well done, but success in the second result was usually restricted to those that spotted the possibility of using partial fractions. The first integral in (ii) was usually done correctly. However the second integral attracted a range of mistakes such as failing to see that or justify how the first part could be used and not simplifying their answers. The nature of the periodicity and the need to integrate different expressions over the differing ranges were frequent stumbling blocks.

Question 9

Whilst being the most popular of the applied questions, it was much less popular than any of the Pure questions being attempted by only just over a quarter of the candidates. It was fairly weakly attempted with a mean score of 5.5/20, only slightly better than question 6. The low scores were mainly caused by sign errors in the velocities of P and Q, and candidates finding it difficult to know which variables to eliminate. However, most did attempt to express the momentum conservation and Newton's Experimental Law of Impact equations which were the starting points to the question. Most students considered the very first and second collisions, rather than the n^{th} and $(n-1)^{\text{th}}$; algebraically, these expressions were the same, but they did then need to justify the generalisation. The few candidates that got as far as part (ii), scored higher overall. In this case, often students showed that the given formula was a solution, which was not asked for, rather than finding u_0 and u_1. Those who found u_1 in terms of u_0 and v_0 generally managed to solve the question well. Very few attempted the last part, but those that did scored well, taking the limit correctly at the end.

Question 10

9% of the candidates attempted this question, exactly as many as attempted question 13, which meant that they were the joint least popular questions. With a mean score of just over 8/20, it was slightly more successfully done than question 7. Most candidates managed to draw the correct diagram, those that did not performed poorly in the rest of the question. The majority of candidates managed to find $\sin\beta$ and a wide variety of techniques were used to find AP, some of which led them astray due to their algebraic nature. They often struggled to justify the forms of each of the terms in the energy expression, in particular, the kinetic energy of the disc and the potential energy. In the latter case, often the zero potential energy level was not defined, too. Few were put off if their energy expression was incorrect and continued to attempt the question using either their expression or that given. Some candidates attempted to find the equation of motion by taking moments, but these solutions tended to be poor. In general, candidates struggled with the algebra and would often find their way to the solution erroneously from incorrect working.

Question 11

20% of the candidates attempted this, but it was the least successfully attempted question with a mean score of under 5/20. The vast majority managed only to express V correctly. Often, candidates failed to resolve forces correctly, and even those that did frequently abandoned the question at this point. Of those that did continue, roughly half then failed to obtain correct expressions for the initial coordinates of the particle in freefall, which led to incorrect expressions for the general freefall coordinates; candidates that did find these typically progressed well apart from any algebraic errors. Candidates that reached the third part often had a reasonable attempt at it; a significant minority confused displacements and velocities indicating a lack of physical understanding of the question. Although not many attempts were made at the last part, those that understood the conditions did well whilst those that did not could not complete it.

Question 12

Although it was only attempted by 14% of the candidates, it was moderately successfully done, just slightly less so than question 8 but still with a mean over 9/20. Part (i) was frequently poorly justified with candidates often attempting to describe the given expression without connecting it to the actual problem. However, binomial coefficients and factorials were generally successfully manipulated in part (ii), although care had to be taken to choose a form for the second expression which would be useful later. Most realised that it was necessary to differentiate the cdf and apply the previous result, though some failed to take care of the details. The deduction of the integral at the end of part (ii) was generally well done, and the majority correctly spotted that this a constant multiple of the integral of the pdf. Part (iii) was largely well done by those reaching this stage, either by recognising that this was of the form of the previous part or by direct integration by parts. A common mistake in this last part was to forget the constant term of the pdf when calculating the expectation.

Question 13

As already mentioned, this was the joint least popular question, although its success rate was only very marginally less than that for question 2. The first result of the stem was generally well done with clear explanation. Likewise, the pgf for the Poisson distribution was generally well calculated, although a few candidates merely quoted the result from the formula booklet and then performed some trivial rearrangements which, of course, did not satisfy what was required. In part (i), candidates recognised how to apply the stem to calculate k, and then either applied it further to express the pgf as $\frac{1}{2}\big(G_X(t) + G_X(-t)\big)$ or directly identified the cosh power series. Similarly, most justified the result for the expectation correctly. For part (ii), $G_Z(t) = \frac{1}{2}\big(G_Y(t) + G_Y(it)\big)$ neatly gave the pgf of Z (not asked but required to progress) from the previous result, although frequently, candidates worked from $G_X(t)$ or directly from power series again. Occasional attempts to use the same method again (i.e. considering $\frac{1}{2}\big(G_Y(t) + G_Y(-t)\big)$) failed. Candidates generally failed to complete the last stage, either not attempting it, or providing poor justifications of the existence of counterexamples as $\lambda \to \infty$ rather than simply furnishing an explicit counterexample.