S2 2014 Report

General Comments

There were good solutions presented to all of the questions, although there was generally less success in those questions that required explanations of results or the use of diagrams and graphs to reach the solution. Algebraic manipulation was generally well done by many of the candidates although a range of common errors such as confusing differentiation and integration and simple arithmetic slips were evident. Candidates should also be advised to use the methods that are asked for in questions unless it is clear that other methods will be accepted (such as by the use of the phrase “or otherwise”).

Comments on individual questions.

Question 1

While the first part of the question was successfully completed by many of the candidates, there were quite a few diagrams drawn showing the point P further from the line AB than Q. Those who established the expression for $x\cos\theta$ were usually able to find an expression for $x\sin\theta$ and good justifications of the quadratic equation were given. The case where P and Q lie on the lines AC produced and BC produced caused a lot of difficulty for many of the candidates, many of whom tried unsuccessfully to create an argument based on similar triangles.

The condition for (*) to be linear in x did not cause much difficulty, although a number of candidates did not give the value of $\cos^{-1}\left(-\frac{1}{2}\right)$. Many candidates realised that the justification that the roots were distinct would involve the discriminant, although some solutions included the case where the discriminant could be equal to 0 were produced. However, very few solutions were able to give a clear justification that the discriminant must be greater than 0.

In the final part some candidates sketched the graph of the quadratic rather than sketching the triangle in the two cases given. In the second case many candidates did not realise that Q was at the same point as C.

Question 2

This was one of the more popular questions of the paper. Most candidates successfully showed that the first inequality was satisfied, but when producing counterexamples, some failed to show that either $f(x) \neq 0$ or $f(\pi) \neq 0$ for their chosen functions. In the second part many candidates did not attempt to choose values of a, b and c, but substituted the general form of the quadratic function into the inequality instead. In the case where the function involved trigonometric functions, many of those who attempted it were able to deduce that $p = q = -r$, but several candidates made mistakes in the required integration. Those who established two inequalities were able to decide which gives the better estimate for π.

Question 3

Many candidates produced a correct solution to the first part of the question. There were a number of popular methods, such as the use of similar triangles, but an algebraic approach finding the intersection between the line and a perpendicular line through the origin was the most popular. Some candidates however, simply stated a formula for the shortest distance from a point to a line. Establishing the differential equation in the second part of the question was generally done well, but many candidates struggled with the solution of the differential equation. A common error was to ignore the case $y'' = 0$ and simply find the circle solution.

The final part of the question was attempted by only a few of the candidates, many of whom did not produce an example that satisfied all of the conditions stated in the question, in particular the condition that the tangents should not be vertical at any point was often missed.

Question 4

Many candidates were able to perform the given substitution correctly and then correctly explain how this demonstrates that the integral is equal to 1. The second part caused more difficulty, particularly with candidates not able to state the relationship between $\arctan x$ and $\arctan\left(\frac{1}{x}\right)$. Attempts to integrate with the substitution $v = \arctan\left(\frac{1}{u}\right)$ often resulted in an incorrect application of the chain rule when finding $\frac{dv}{du}$.

In the final part of the question many candidates attempted to use integration by parts to reach the given answer.

Question 5

This was the most popular question on the paper and the question which had the highest average score. Most candidates correctly solved the differential equation in the first part of the question, but many then calculated the constant term incorrectly. In the second part of the question most candidates were able to find the appropriate values of a and b, but then did not see how to apply the result from part (i) and so did the integration again or just copied the answer from the first part. Some candidates again struggled to obtain the correct constant for the integration and others did not substitute the correct values for the point on the curve (taking (X, Y) as $(1,1)$ rather than (x, y)).

Question 6

This was one of the less popular of the pure maths questions, but the average mark achieved on this paper was one of the highest for the paper. The first section did not present too much difficulty for the majority of candidates, with a variety of methods being used to show the first result such as proof by induction or use of $e^x = \cos x + i \sin x$. In the second part of the question many of the candidates struggled to explain the reasoning clearly to show the required result. Most candidates who reached the final part of the question realised that the previous part provides the basis for a proof by induction.

Question 7

This was another of the less popular pure maths questions. The nature of this question meant that many solutions involved a series of sketches of graphs with very little written explanation. Most candidates were able to identify that the sloping edges of $y = f(x)$ would have the same gradient as the sloping edges of $y = g(x)$, but many did not have both sloping edges overlapping for the two graphs. In some cases only one sloping edge of $y = g(x)$ was drawn. A large number of candidates who correctly sketched the graphs identified the quadrilateral as a rectangle, rather than a square. In the second part of the question, sketches of the case with one solution often did not have the graph of $y = |x - c|$ meeting the x-axis at one corner of the square identified in part (i), although many candidates were able to identify the different cases that could occur. Unfortunately in the final part of the question very few candidates used the result from the first part of the question and so considered a number of possibilities that do not exist for any values of a, b, c and d.

Question 8

This was the least popular of the pure maths questions and also the one with the lowest average score. Many of the candidates were able to show the required result at the start of the question, although very few candidates explained that m could be either of the two integers when the range included two integers. Parts (i) and (ii) were then quite straightforward for most candidates, although many calculated the range of values but did not justify their choice in the case where there were two possibilities. In the final two parts of the question some candidates mistakenly chose the value 0 when asked for a positive integer.

Question 9

This question was not attempted by a very large number of candidates and the average score achieved was the lowest on the paper. While there were a number of attempts that did not proceed beyond drawing a diagram to represent the situation, the first part of the question was done well by a large number of candidates. Many were also able to adjust the result for the case when the frictional force acts downwards. Unfortunately, in the final part of the question many candidates continued to use $F = \mu R$, not realising that this only applies in the critical case and so there were very few correct solutions to this part of the question.

Question 10

This was the most popular of the mechanics questions and also the one that had the best average score, although candidates did struggle to get very high marks on the question particularly on the final parts. The first part of the question asks for a derivation of the equation for the trajectory which was familiar to many candidates, although in some cases the result was obtained by stating that it is a parabola and knowledge of the maximum value and the range. Many candidates who successfully obtained the Cartesian equation then struggled with the differentiation with respect to λ, instead finding the maximum height for a constant value of λ. Unfortunately, this made the remainder of the question insoluble. Some candidates decided to differentiate with respect to θ instead, which did not cause any serious problems, although it did require more work. A few candidates used the discriminant rather than differentiation, but did not provide any justification of this method.

Candidates were able to draw the graph, but many did not label the area that was asked for in the question. Those who reached the final part of the question and considered the distance function for the position during the flight used differentiation to work out the greatest distance. However, many did not realise that the maximum value of a function can be achieved at an end-point of the domain even with a derivative that is non-zero.

Question 11

Many candidates who attempted this question struggled, particularly due to a difficulty in drawing a diagram to represent the situation. From these incorrect diagrams candidates often reached results where one of the signs did not match that given in the question. The calculation of the acceleration was found to be difficult by many of the candidates, although those who understood that differentiation of the coordinates of P would give the acceleration were then able to complete the rest of the question correctly. Those candidates that attempted the final part of the question were able to solve it correctly.

Question 12

This was the least popular question on the paper. A large number of candidates who attempted this question seemed unable to work out where to start on the first part of the question. Much of the rest of the question requires working with the hazard function defined at the start of the question and so many candidates who attempted these parts were able to do the necessary integration to solve the differential equations that arose. A common error among those who attempted part (iv) was to ignore the “if and only if” statement in the question and only show the result one way round.

Question 13

This was the more popular of the two questions on Probability and Statistics, but as in previous years it still only attracted answers from a very small number of candidates. The average mark for this question was also quite low, often due to a difficulty in explaining the reasoning behind some of the parts of the question. Many candidates were able to find the expression for $P(X = 4)$ and most were then able to obtain the general formula required in part (i) of the question, although a number of candidates did not include the correct number of factors in the answer. Parts (ii) and (iii) did not cause too much difficulty, but the final part required a clear explanation to gain full marks.