

## STEP Mathematics III 2011: Report

The percentages attempting larger numbers of questions were higher this year than formerly. More than 90% attempted at least five questions and there were 30% that didn't attempt at least six questions. About 25% made substantive attempts at more than six questions, of which a very small number indeed were high scoring candidates that had perhaps done extra questions (well) for fun, but mostly these were cases of candidates not being able to complete six good solutions.

### Section A: Pure Mathematics

1. As might be expected, this was a very popular question, in fact the most popular being attempted by very nearly all the candidates. Fortunately, it was also generally well-attempted, with scores well above those for other questions. Apart from frequent algebraic errors and overlooking terms, especially when using results from a previous part that required adaptation, the main difficulties were in showing that (\*) in part (ii) did indeed lead to a first order differential equation in  $\frac{dz}{dx}$ , and the consequent solution of that equation. Part (iii) was generally well done. At the other end of the scale, some candidates did leave their answers to part (i) in the form  $\ln u =$  .

2. This was quite a popular question, being attempted by 70% of candidates. Scores were polarized, though overall the mean score was below half marks, much the same as half of the questions on the paper. Most candidates successfully dealt with the stem. Attempts at part (i) were in equal proportions, applying the stem or a variant of the standard proof of the irrationality of the square root of 2, though some of the latter overlooked the fact that it was the  $n$ th root being discussed. Parts (ii) and (iii) saw three methods employed. One method was to consider the location of the real roots then apply the stem, the second being to re-arrange the expression to equal the integer and consider factors (again applying the stem). In both these cases, failure to consider all cases lost marks, and there were frequent lacks of rigour. However, considering  $x$  being odd or even, when used, was particularly slick and successful.

3. The second most popular question, attempted by 80% of the cohort, with a similar level of success to question 2. The significance of the condition  $q^2 \neq 4p^3$  was ignored by many candidates, and the fact that it does not apply in the last part of the question was often similarly overlooked. Whilst  $a$  and  $b$  were generally found correctly, the rest of the first part was often missing. Though there were frequent numerical errors, many candidates correctly found the given solution of the equation, though the other two eluded most, with a common error being to assume that the other two were  $xw$  and  $xw^2$ .

4. About two thirds of the candidates tried this, with very slightly greater success than questions 2 and 3. They found part (i) tricky, especially understanding the integral of the inverse function. Also, commonly, they thought the condition was that  $b = a$  . However, part (ii) was done better, most errors being due to taking the inverse incorrectly, and of course, the verification frequently went wrong due to the false condition. Most realised the function to use in part (iii) but there was plenty of inaccuracy in working this part, though the final deduction caused few worries.

5. Less than a third of the candidates attempted this. There were quite a few perfect scores, however the vast majority scored less than a quarter of the marks, which was the

mean mark. The general result at the start of the question was the key to success. Those that stumbled with handling four variables in terms of the fifth one, and the consequent calculus, did not attempt to make further progress into the rest of the question.

6. This was quite popular, with attempts from three quarters of the candidates, and slightly more success than questions like 2 and 3. Needing to prove three equalities, many got close to doing two well and, with the others splitting half and half between getting close to all three or just one. A small number of candidates made several attempts without always having any sense of direction and often proved a particular pair equal both ways round. The other weaknesses were in dealing with the limits when changing variable and evaluating the definite term (which was zero!) when employing integration by parts.

7. The popularity and success rate of this was very similar to question 6. Quite a few failed to realise the importance of  $A_n^2 = a(a+1)B_n^2 + 1$  as part of the induction, and even if they did tripped up on that part of the working. Part (ii) generally went well and the result in  $C_n$  and  $D_n$  was found more easily. Very few had a problem with part (iii) but a small number failed totally to see what it was about.

8. The response rate of this was similar to question 4, but with success rate similar to question 2. Most students did reasonably well getting half to three quarters of the marks by finding  $u$  and  $v$  and doing part (i), and then getting hold of (ii) and (iii) or not. Part (iv) rightly discriminated the strong candidates from the generality. A few alternative methods were tried but mostly they had their limitations. Details like omitted points from loci and the negative sign that arises when using the cosine double angle formula frequently lost marks.

## **Section B: Mechanics**

9. About a sixth of candidates tried this, and on average with slightly less success than question 2. Of the attempts, about a third were close to completely correct, and nearly all the others were barely doing more than grasping at crumbs, reflecting the fact that candidates either did or did not know what they were doing. There was negligible middle ground.

10. Just under a quarter of candidates offered something on this, with relatively little success and less than 20 candidates earning good marks. As with question 9, it tended to be a case of “all or nothing”. Of the good solutions, half based their working on the motion of and relative to the centre of mass of the system, and the other half on setting up simultaneous differential equations for the displacements of the particles. Of the poor attempts, most usually drew some kind of diagram, but then didn’t use it to identify a sensible coordinate system, or positive direction, and there were common confusions over displacements  $x$  and extensions  $x$ . Energy approaches usually got nowhere.

11. The least popular question on the paper, attempted by about 4%, but with similar mean score to question 2 (and several others). Mostly, they did pretty well in finding the couple, and using the initial trigonometric relation and its consequences to do so. At that point they tended not to know how to proceed to the last part, though there were some very good and simple solutions from considering energy.

## Section C: Probability and Statistics

12. This question ran a close second to number 11 for unpopularity, but reflected the same level of success. Most attempts followed the method of the question, and if they got off on the right foot, often got most of the way through. Some struggled with the algebra for the variance result, and a few tripped up on the standard pgf for the number of tosses to the first head. Strangely, having found the pgf for  $Y$  successfully, and used it or the results of the question for expectation and variance, the final probabilities were often wrong, and not merely from overlooking the initial case.

13. This too was fairly unpopular, being attempted by about 10% of the candidates. Of these no more than a dozen got it largely correct, but there was only one totally correct solution as the detail for the non unique case frequently tripped even the better candidates. The mean score was only about a third of the marks available as most candidates got part (i) largely correct, barring some simplifying errors and not obtaining the non unique solution. Fewer candidates had the correct probabilities for part (ii) and so were unable to proceed, though a few were wrong merely by a constant which cancelled to give the correct ratio.