

Homework #1

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Date: 17-09-2025

Class: CS435-003

1. Question 1:

A)

$$P(x) = 2 + x (4 + x (3 + x (6 + x (1 + x (5)))))$$

B)

Algorithm: polynomial_evaluation(A, power x)

Input: An array A, a power p

Output: Computation of each term of the array A

total \leftarrow 0

for i in range(input) do

 p \leftarrow 1

 for j=1 to i-1 do

 p \leftarrow p * x

 total \leftarrow total + p * A[i]

return total

C) Running time alg of b: $T(n) = n * O(n)$

$$= O(n^2)$$

D)

Algorithm: horners_rule(A, x)

Input: An array A, an x value

Output: Computation of each term of the array A

start \leftarrow 1

total \leftarrow A[0]

for i \leftarrow 1 to length(A) do

 total = total * x + A[i]

return total

E) Running time alg of c: $O(n)$

2. Running time: $O(n \log n)$

3.

a)

Theoretically Algorithm B grows faster than Algorithm A because it is linear, which means that as n becomes larger, it will grow slower, which leads to define that b has better asymptotic theoretical performance.

$j = i \rightarrow j = 2i \rightarrow j = 3i \rightarrow \dots \rightarrow j = ni$
 $j = I \rightarrow j = ki \rightarrow j = (k+1)i \rightarrow \dots \rightarrow (k+n)i$
 $j \leq n$ (Replace j with ki)
 $ki \leq n$
 $k \leq n/i$

Ex:

| | | | | | | | | |
|---------|--|-------|--|------------------|--|-------|--|-------|
| $i = x$ | | $j=i$ | | $j = i$ to n | | n/i | | n/i |
| $i = 1$ | | $j=1$ | | $j = 1, 2, 3, 4$ | | $4/1$ | | $n/1$ |
| $i = 2$ | | $j=2$ | | $j = 2-4$ | | $4/2$ | | $n/2$ |
| $i = 3$ | | $j=3$ | | $j = 3$ | | $4/3$ | | $n/3$ |
| $i = 4$ | | $j=4$ | | $j = 4-4$ | | $4/4$ | | $n/4$ |

Therefore:

$$\begin{aligned} T(n) &= n + n/2 + n/3 + \dots + 1 \\ &= n [1 + 1/2 + 1/3 + \dots + 1/n] \\ &= n [\sum_{i=1}^n \frac{1}{i}] = O(n \log n) \end{aligned}$$

b) $n = 5 * 10^6$

$$T_a(n) = T_b(n)$$

$$0.0001 n^2 / n = 500 n / n$$

$$0.0001 n = 500$$

$$n = 500 / 0.0001$$

$$n = 5000000 \rightarrow 5 * 10^6$$

c)

Algorithm A, because when we replace n as 10^6 for algorithm A, we end up having a time of 27.8 hours while if we replace with b , we end up having 138.8 hours. This means that definitely the value for A is smaller. However, on the other hands, if we decide to increase the value to 10^7 we can see that the value of A increases while B decreases. In other words, if $n > 10^6$ algorithm B begins to be faster as explained in literal a, while if $n < 10^6$ or small datasets, A is faster, this due to the threshold we got from the previous question.

$$T_a (10^6) = 0.0001 (10^6) = 27.8 \text{ hours}$$

$$T_b (10^6) = 500 (10^6) = 138.8 \text{ hours}$$

4.

n comparison

```
def find_index(arr, target):
    n = len(arr)
    i = 0

    for i in range(n):
        if arr[i] == target:
            return i
        else:
            return -1
```

Since the algorithm is divided into blocks, the worst case scenario happens when the target is inside the last block, because the algorithm has to go through all the previous blocks before getting to the last. In other words, it makes \sqrt{n} comparisons when jumping and inside each block to find the target, which in total they sum up at most $3\sqrt{n}$. Therefore the number of comparisons doesn't exceed this bound. There is one comparison per each element.

$< 3\sqrt{n}$ comparison

```
def find_index(arr, target):
    n = len(arr)
    jump = int(math.ceil(math.sqrt(n)))
    start_element = 0

    for start_element in range(0, n, jump):
        if start_element + jump < n:
            last_element = start_element + jump
        else:
            last_element = n

    if arr[last_element - 1] >= target:

        for j in range(start_element,
            last_element):
            if arr[j] == target:
                return j
        return -1
```

5)

Worst case scenario could occur when the target is inside the last and first block. Inside this snippet of the code, we encounter ourselves with binary search and exponential search, both having complexity time of $O(\log_2 n)$ which means that the algorithm keeps dividing itself until there is only one element of the array, therefore we consider each comparison reduces the space to half. Therefore, worst case scenario is $4\log_2 n$, because since there are these two searches, exponential and binary, we consider 2 comparisons per iteration.