Physics Problems Solution

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Normal Modes in 1D¹

a. Discrete

Consider a system of N particles which is subject to a restoring force proportional to the separation to the equilibrium position ($F \quad \alpha \quad x$). How many normal modes has this system?

N normal modes.

b. Continuous

Consider now a continuous system of particles confined to a length L and fixed borders. Which are normal wave lengths (λ_n) and the normal frequencies (ν_n) in terms of ν (the speed of waves in the medium)? Is it correct to think that this system has infinite normal modes?. If it does not have infinite normal modes, give a order of magnitude of the number of normal modes.

The normal wave lengths and frequencies are given by (1) and (2). The system does not have infinite normal modes, however the number of normal modes its big enough to consider it as infinite. The number of normal modes is in the order given by (3), where a is the average separation between particles in the system.

$$\lambda_n = \frac{2L}{n} \tag{1}$$

¹A. P. French. Vibrations and Waves. 2003

$$\nu_n = \frac{n\nu}{2L} \tag{2}$$

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$$N = \frac{L}{a} \tag{3}$$