EDCO4B
ESTRUTURAS DE DADOS 2

Aula 07 - Heap Sort

Prof. Rafael G. Mantovani

UTFPR
UNIVERSIDADE TECNOLÓGICA FEDERAL DO PARANÁ

Apucarana - PR, Brasil

Universidade Tecnológica Federal do Paraná (UTFPR)
Engenharia de Computação

Roteiro

1. Introdução
2. Heaps
3. Heap Sort
4. Exemplo
5. Exercícios
6. Referências

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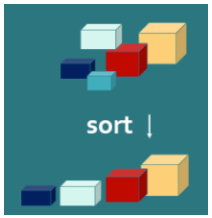
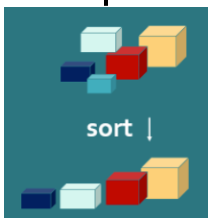
Introdução



Algoritmos de Ordenação

Introdução

Métodos Simples ⟷ Métodos Eficientes



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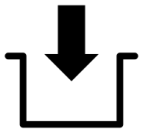
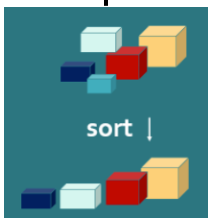
Métodos Eficientes

BubbleSort

InsertionSort



SelectionSort



Algoritmos de Ordenação

HeapSort

MergeSort

QuickSort

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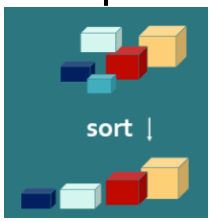
Métodos Eficientes

OK BubbleSort



Algoritmos de Ordenação

HeapSort

OK InsertionSort

MergeSort

OK

OK SelectionSort

QuickSort OK

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Heap

- **Heap:** estrutura de dados em array que pode ser vista como uma árvore binária quase completa

Heap

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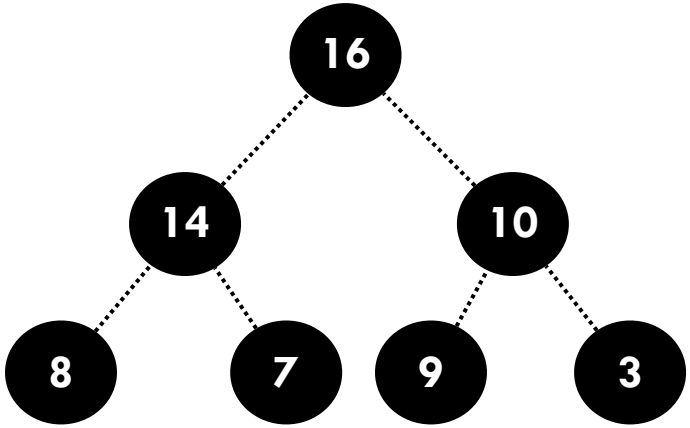
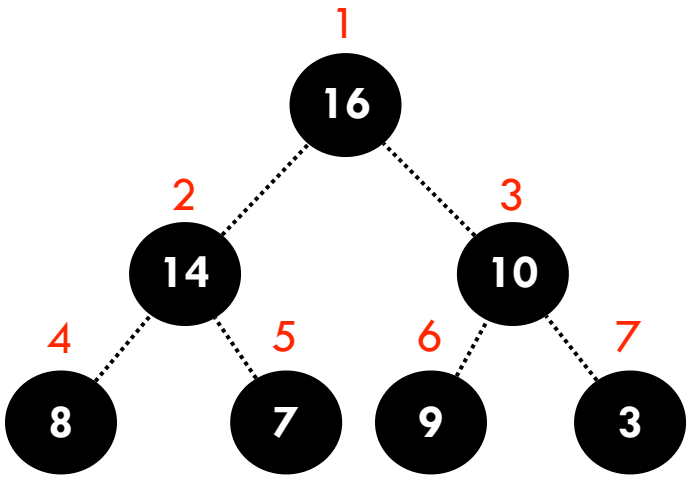


a) heap como árvore binária

Heap

- **Heap:** estrutura de dados em array que pode ser vista como uma árvore binária quase completa



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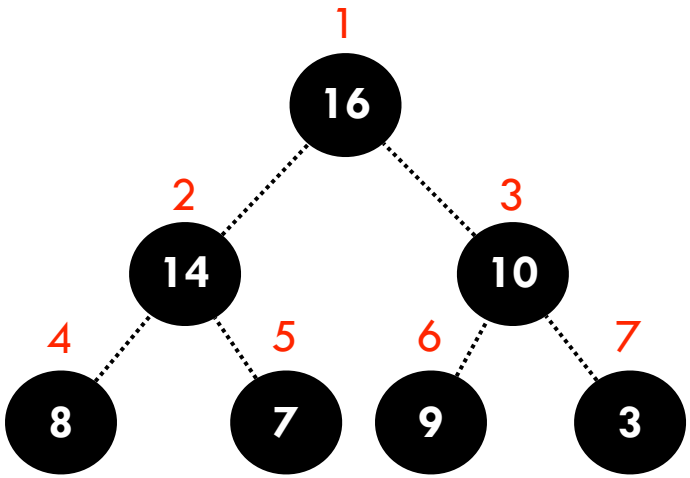

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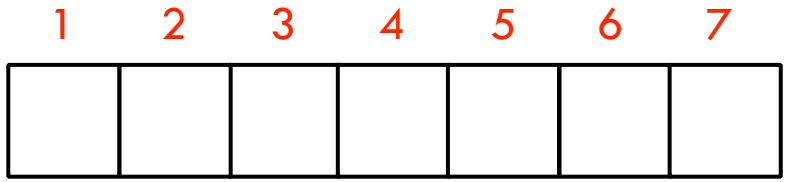
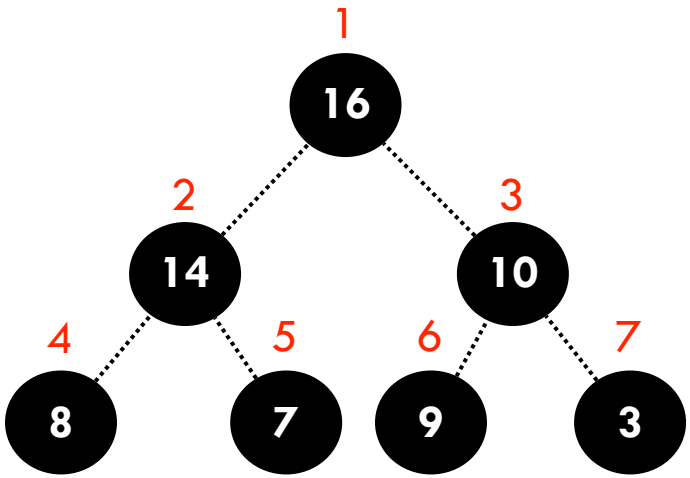


b) heap como array

Heap

- **Heap:** estrutura de dados em array que pode ser vista como uma árvore binária quase completa



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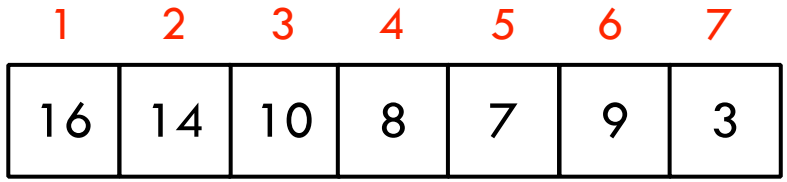

b) heap como array

Heap

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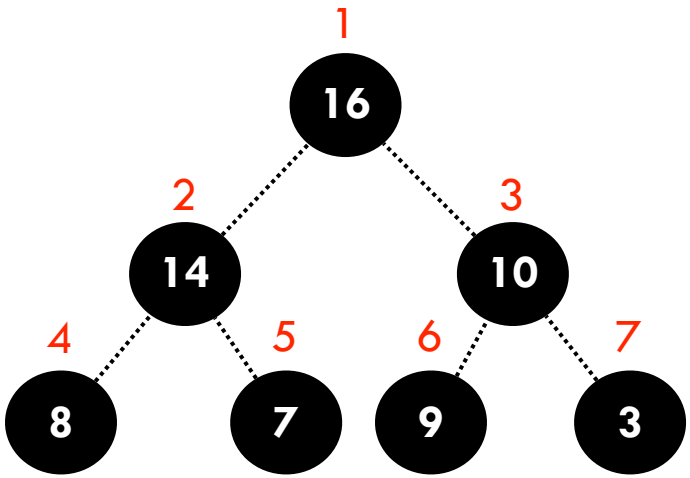

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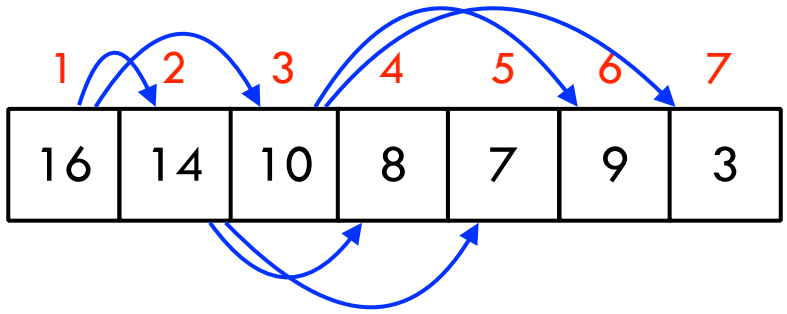

b) heap como array

Heap

* Pai(i) = floor(i/2)
* Esquerda(i) = 2*i
* Direita(i) = 2*i + 1

Heap

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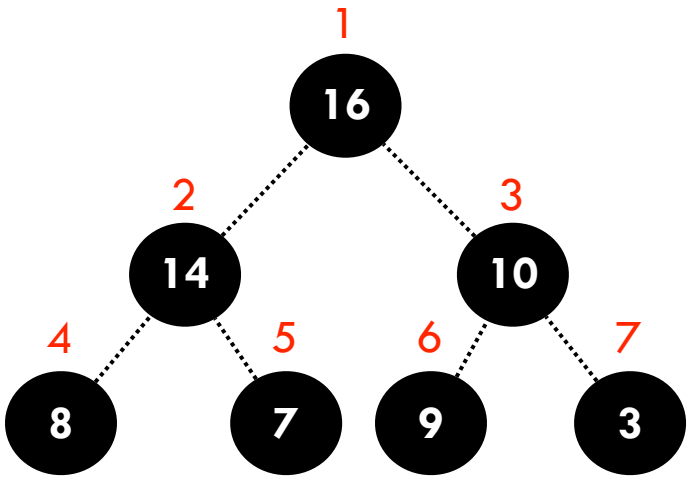
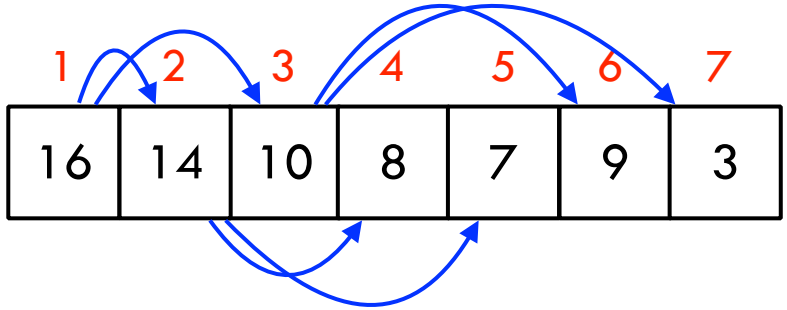


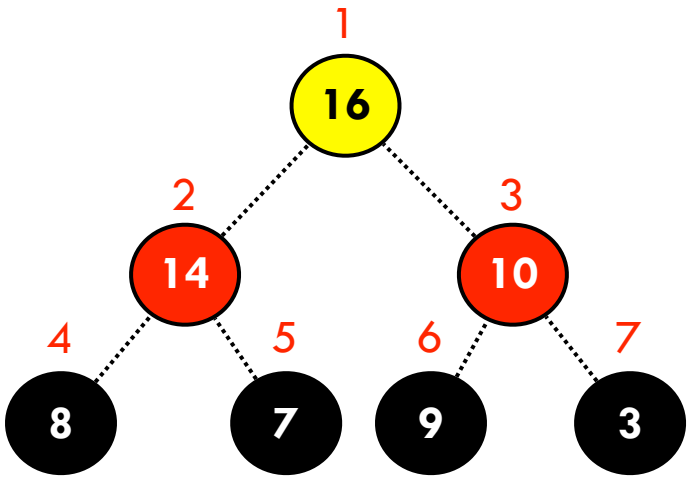
a) heap como árvore binária



b) heap como array

Heap

* Esquerda(1) = 2*1 = 2
* Direita(1) = 2*1 + 1 = 3



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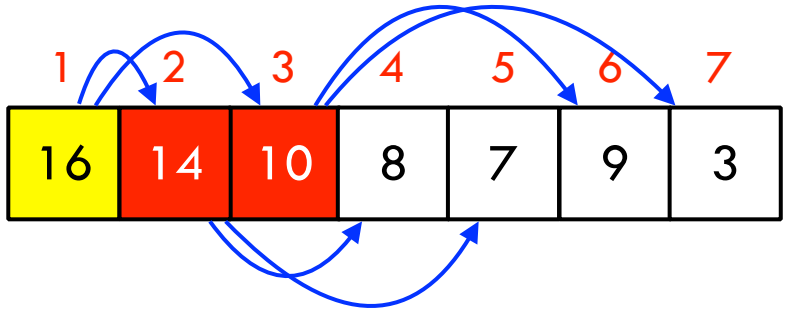
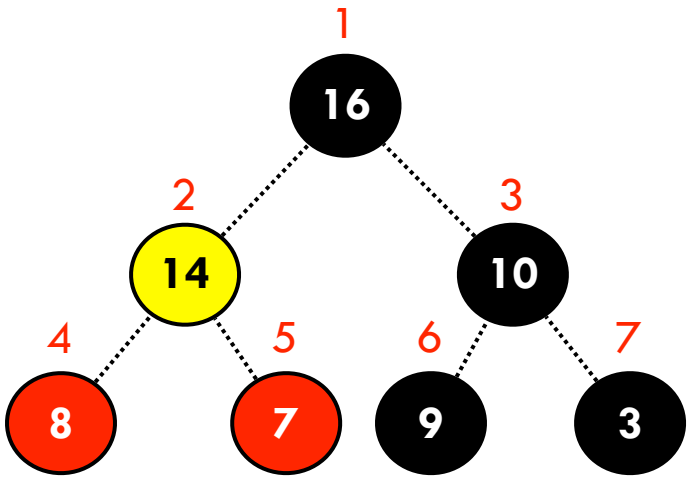


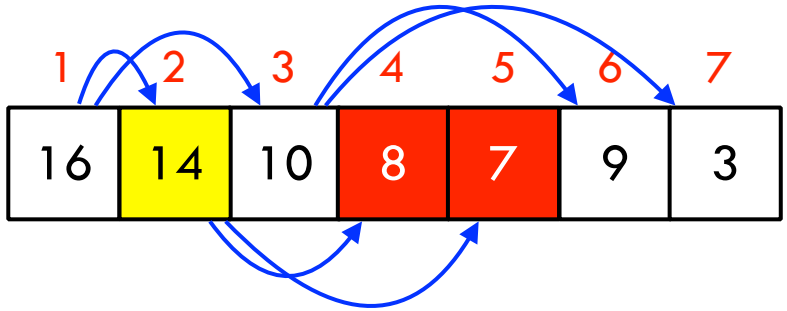
b) heap como array

Heap

* Pai(2) = floor(2/2) =1
* Esquerda(2) = 2*2 =4
* Direita(2) = 2*2 + 1 =5



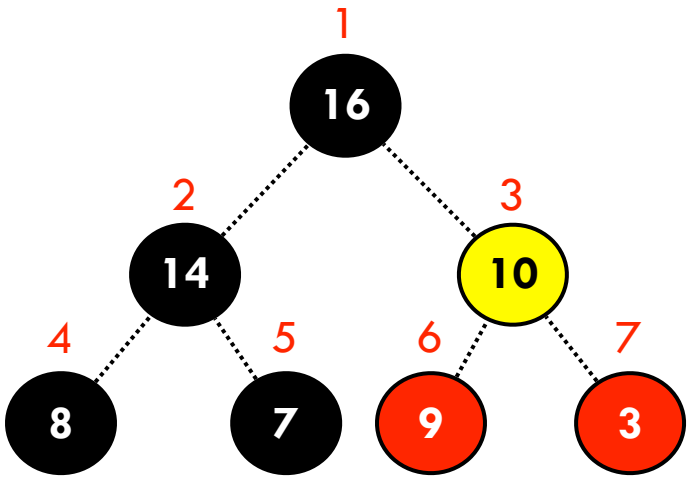
a) heap como árvore binária



b) heap como array

Heap

* Pai(3) = floor(3/2) =1
* Esquerda(3) = 2*3 =6
* Direita(3) = 2*3 + 1 =7



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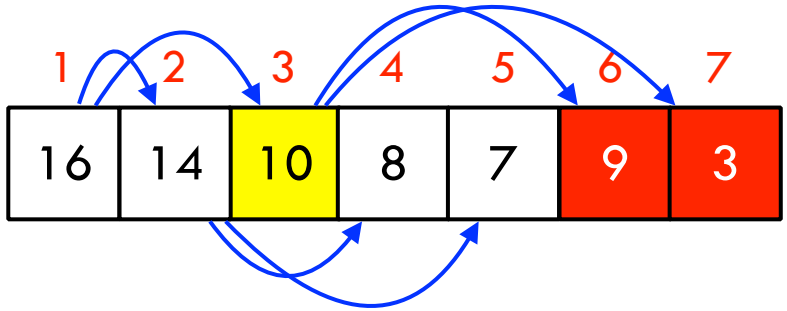
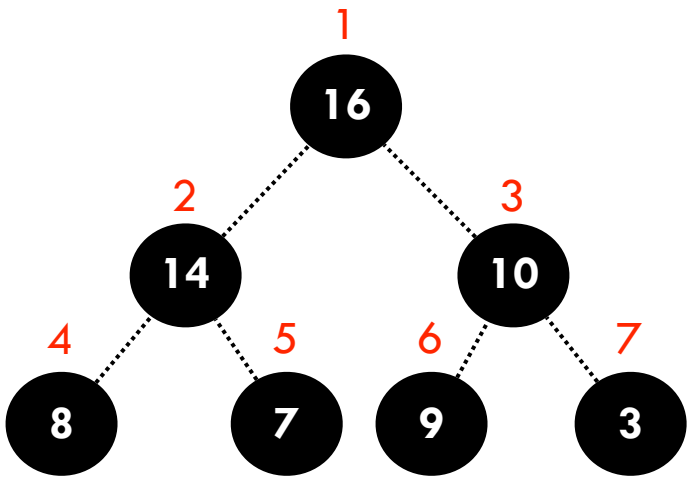


b) heap como array

Heap

- **Heap de máximo:**
  - A[parent(i)] >= A[i]



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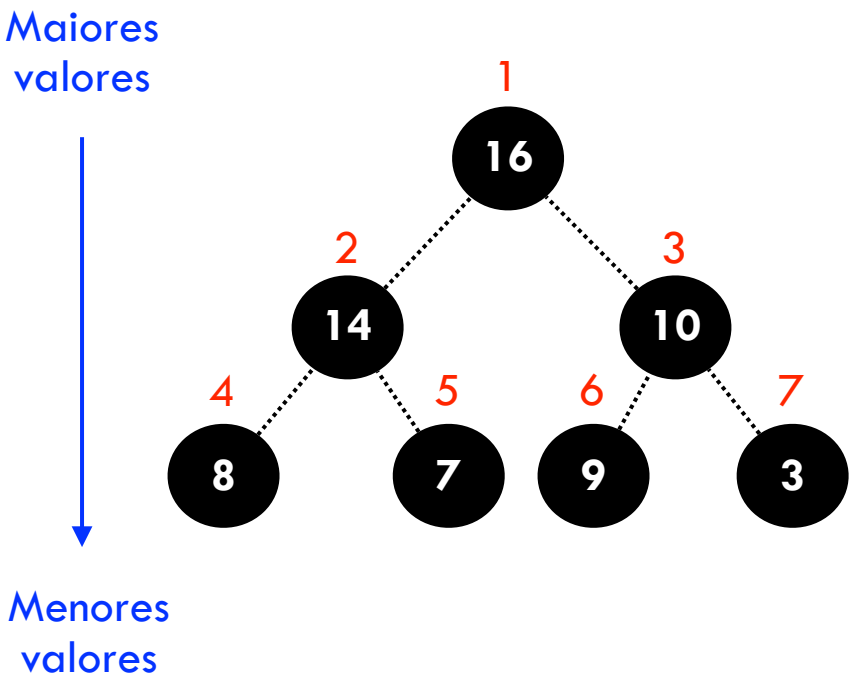

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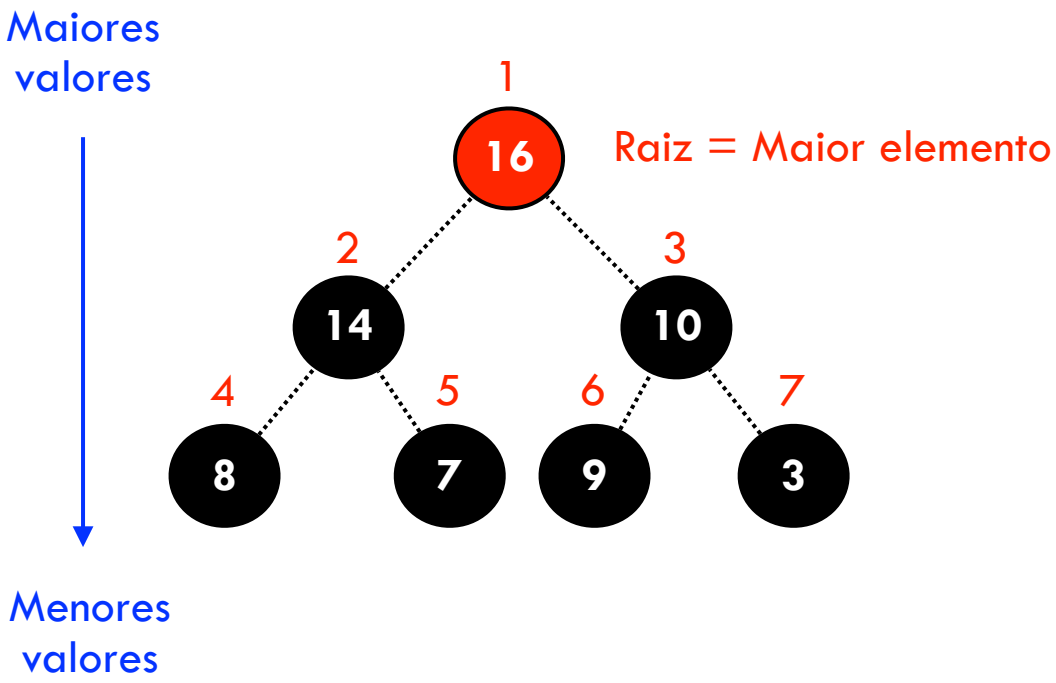

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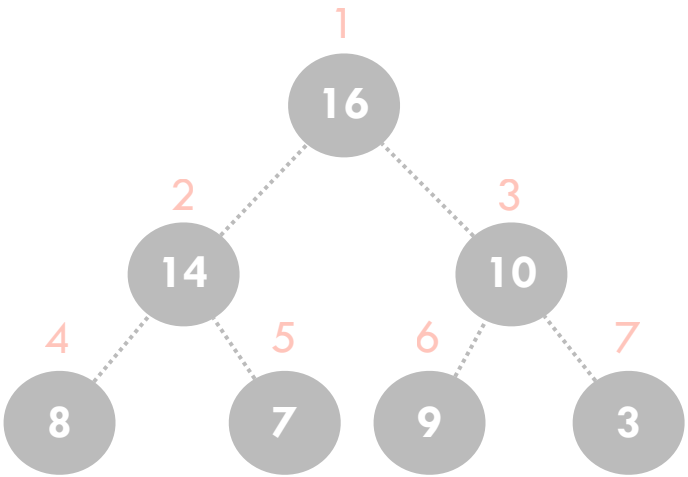
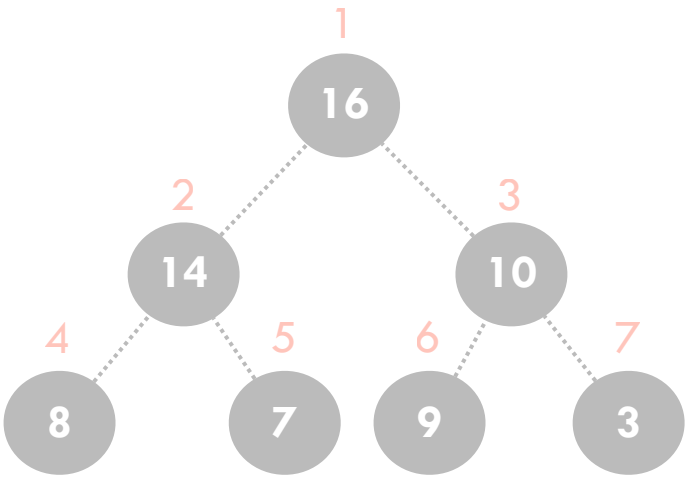
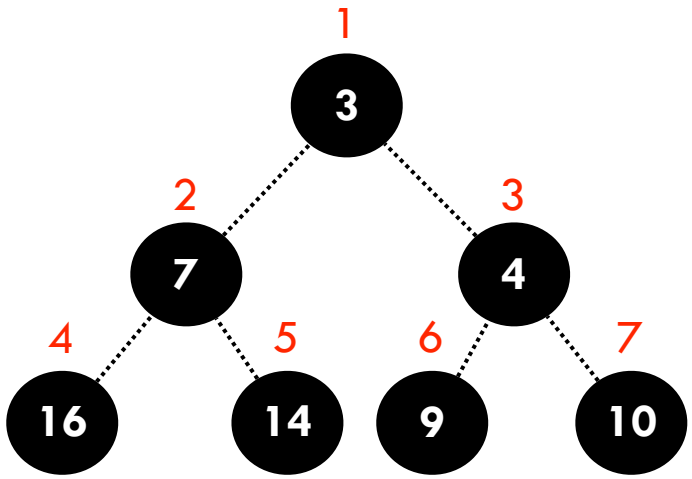


Heap

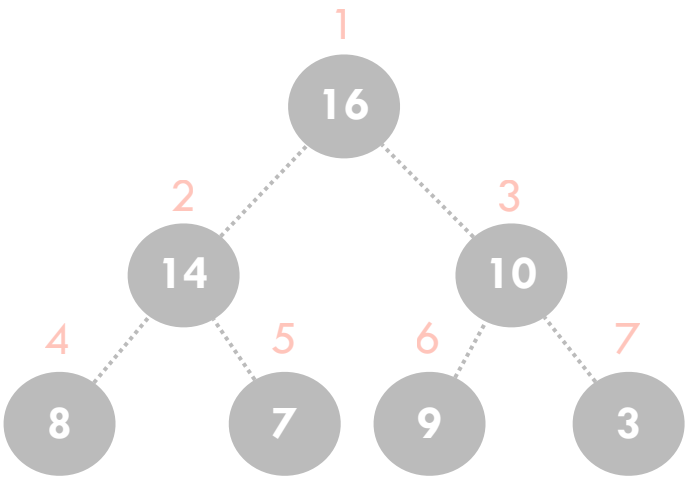
- **Heap de mínimo:**
  - A[Pai(i)] <= A[i]

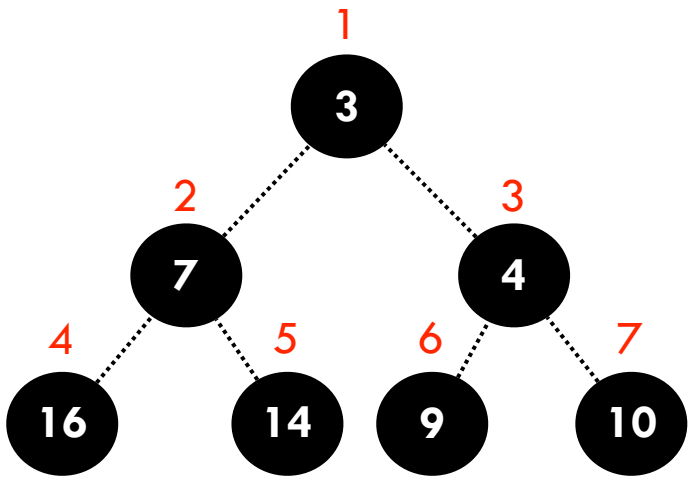




Heap

- **Heap de mínimo:**
  - A[Pai(i)] <= A[i]



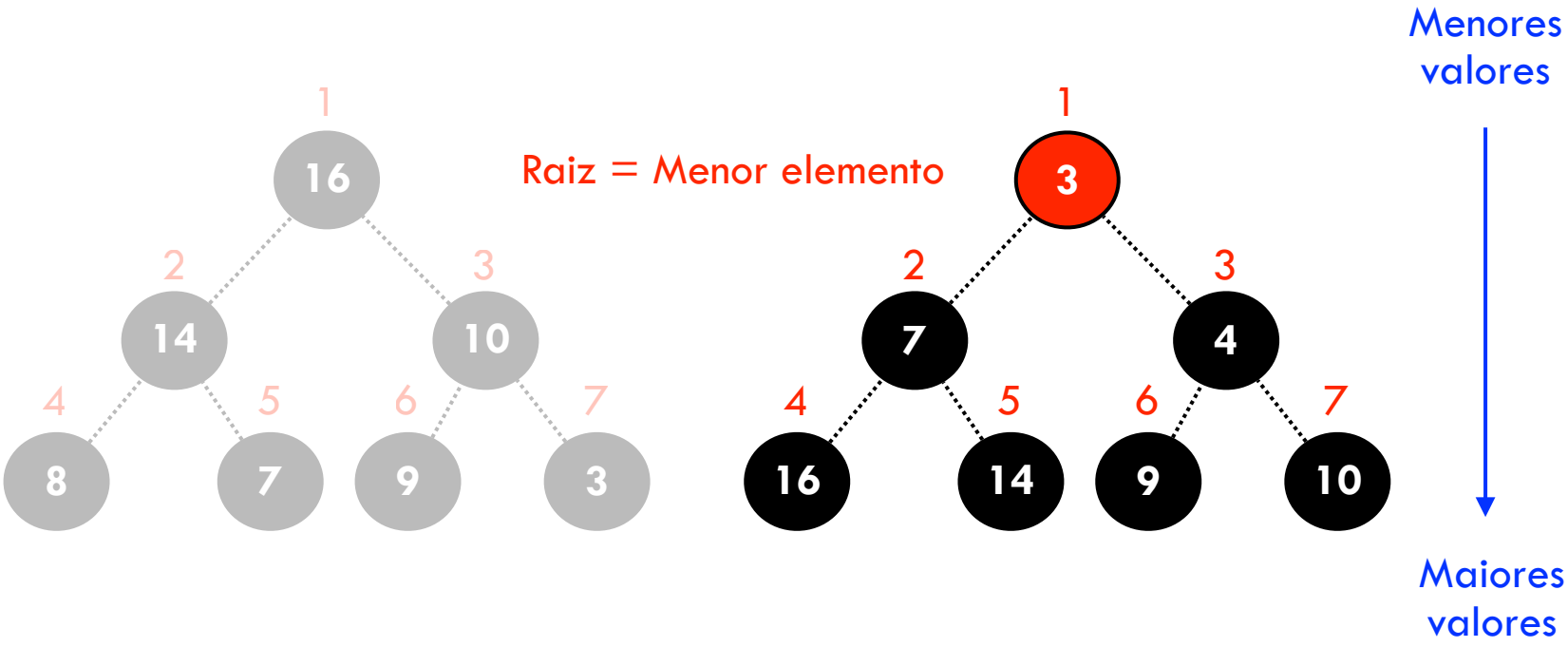


Menores valores

Maiores valores

Heap

- **Heap de mínimo:**
  - A[Pai(i)] <= A[i]



Roteiro

1. Introdução
2. Heaps
3. **Heap Sort**
4. Exemplo
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Heap Sort

Heap Sort

- **Funcionamento**

* Passos

1. Transformar o heap (vetor) em um max-heap
2. Mover a raiz (maior valor) para o final do vetor (última posição)
3. Reconstrói o heap do vetor restante

Heap Sort

- **Desempenho**

* **melhor caso:** O(N log N)

* **pior caso:** O(N log N)

* **caso médio:** O(N log N)

Heap Sort

- **Desempenho**

* **melhor caso:** O(N log N)

* **pior caso:** O(N log N)

* **caso médio:** O(N log N)

Obs: **Na prática, o HeapSort é mais lento que o QuickSort, exceto no pior caso.**

Heap Sort

- **Pseudocódigo**

1. **HEAPSORT:** ordena o vetor usando heap auxiliar

2. **BUILD-MAX-HEAP**: cria um heap de máximo a partir dos valores aleatórios de um vetor

3. **MAX-HEAPIFY**: mantem a propriedade de heap de máximo de um conjunto de valores

Heap Sort

- **Pseudocódigo** (mantém heap)

1. **MAX-HEAPIFY** (V, i)

- Entradas:
  - **V**: vetor de elementos
  - **i**: índice da posição para se criar max-heap

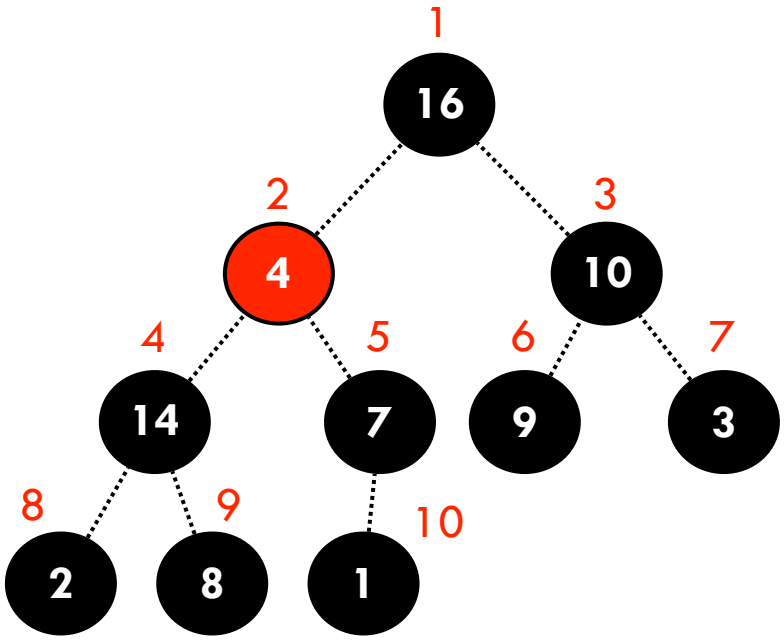
- Além disso, em V podemos obter:
  - **V.tamanho-heap**: tamanho a ser considerado para criar heap
  - **V.tamanho**: tamanho do vetor

Heap Sort

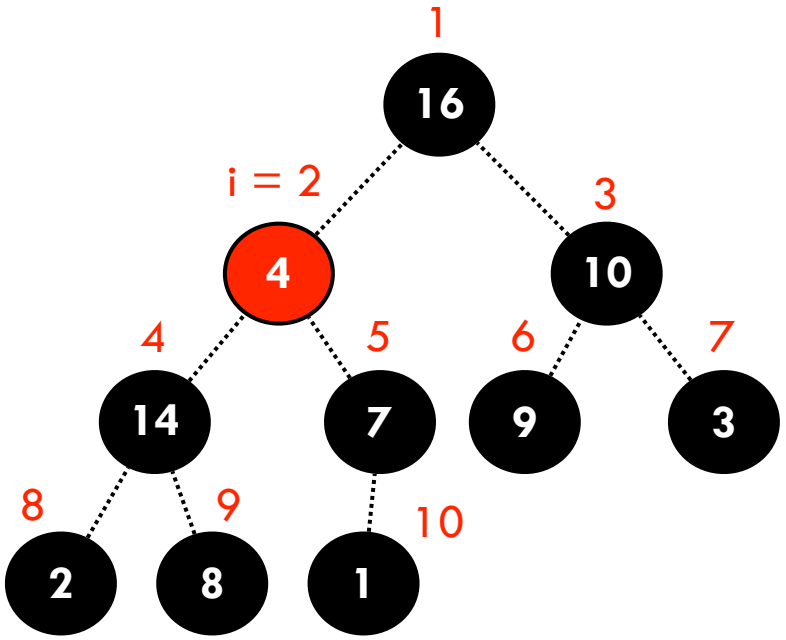
- **Pseudocódigo** (cria heap)

```
1.  MAX-HEAPIFY (V, i)
2.       L = Esquerda(i) // 2*i + 1
3.       R = Direita(i)    // 2*1 +2
4.       Maior = i
5.       se L <= V.tamanho-heap E V[L] > V[i]
6.                maior = L
7.       se R <= V.tamanho-heap E V[R] > V[maior]
8.                maior = R
9.       se maior != i
10.               troca(V[i], V[maior])
11.               MAX-HEAPIFY (V, maior)
```

MAX-HEAPIFY



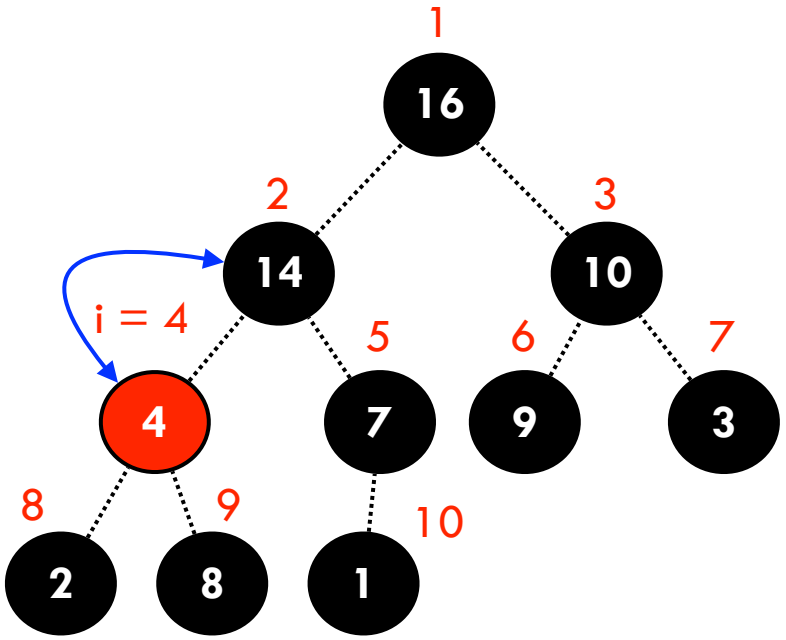
MAX-HEAPIFY



* Configuração inicial (i = 2)
  V[2] viola a propriedade de heap máximo:
  V[2] < V[4] e V[2] < V[5]

* **Solução**: SWAP(V[2], V[4])

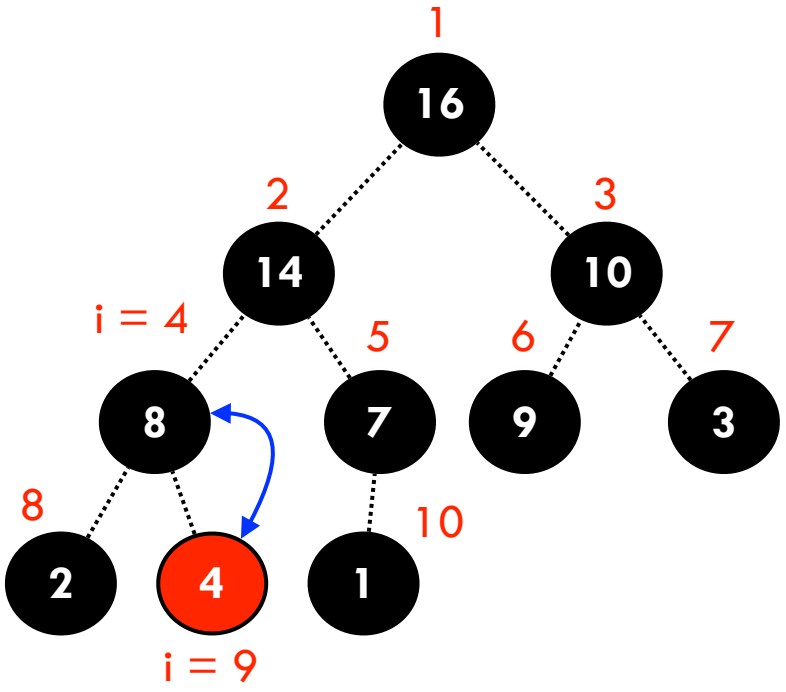
MAX-HEAPIFY



* Configuração atual (i = 4)
  v[4] viola a propriedade de heap máximo:
  V[4] < V[9]

  Solução: SWAP(V[4], V[9])

MAX-HEAPIFY



Tudo certo agora :)

Heap Sort

- **Pseudocódigo** (cria max-heap)

2. **BUILD-MAX-HEAP** (V)

- Entradas:
  - **V**: vetor de elementos
- Usa MAX-HEAPIFY em uma abordagem *bottom up* convertendo o array V em um max-heap
  - iterar de N/2 ... até 1

Heap Sort

- **Pseudocódigo** (cria heap)

```
1.  BUILD-MAX-HEAP (V)
2.      V.tamanho-heap = V.tamanho
3.      Para i = floor(V.tamanho/2) até 1
4.          MAX-HEAPIFY(V, i)
```

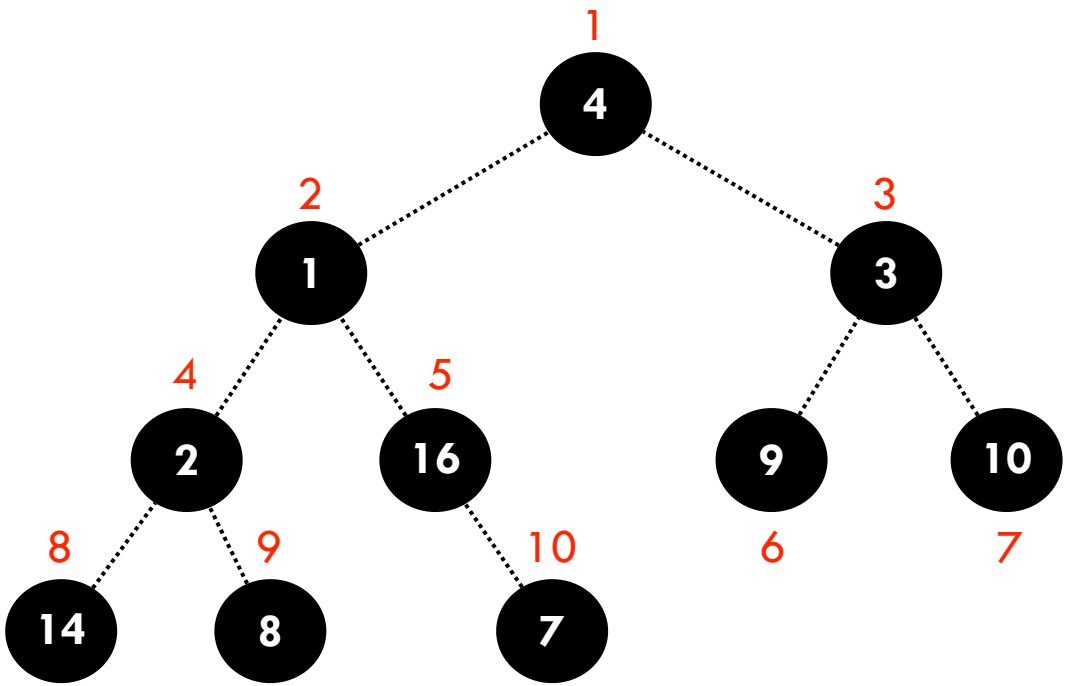
BUILD-MAX-HEAP

V =

1	2	3	4	5	6	7	8	9	10
4	1	3	2	16	9	10	14	8	7

BUILD-MAX-HEAP

	1	2	3	4	5	6	7	8	9	10
V =	4	1	3	2	16	9	10	14	8	7



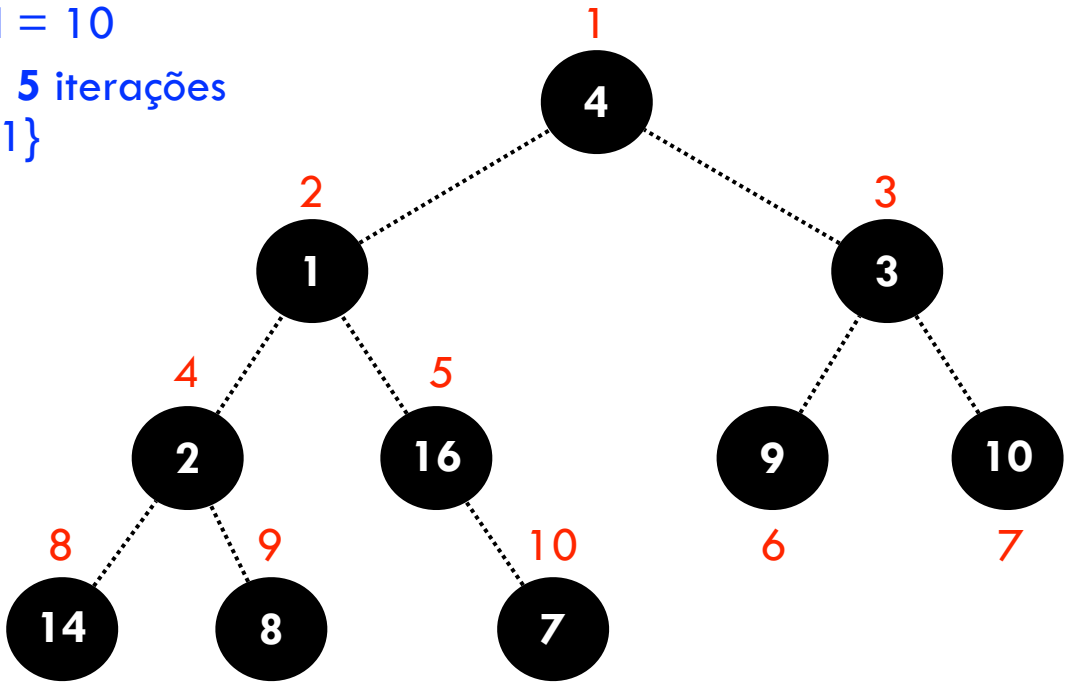
BUILD-MAX-HEAP

	1	2	3	4	5	6	7	8	9	10
V =	4	1	3	2	16	9	10	14	8	7

V.tamanho = **N** = 10

N/2 = 10/2 = **5** iterações

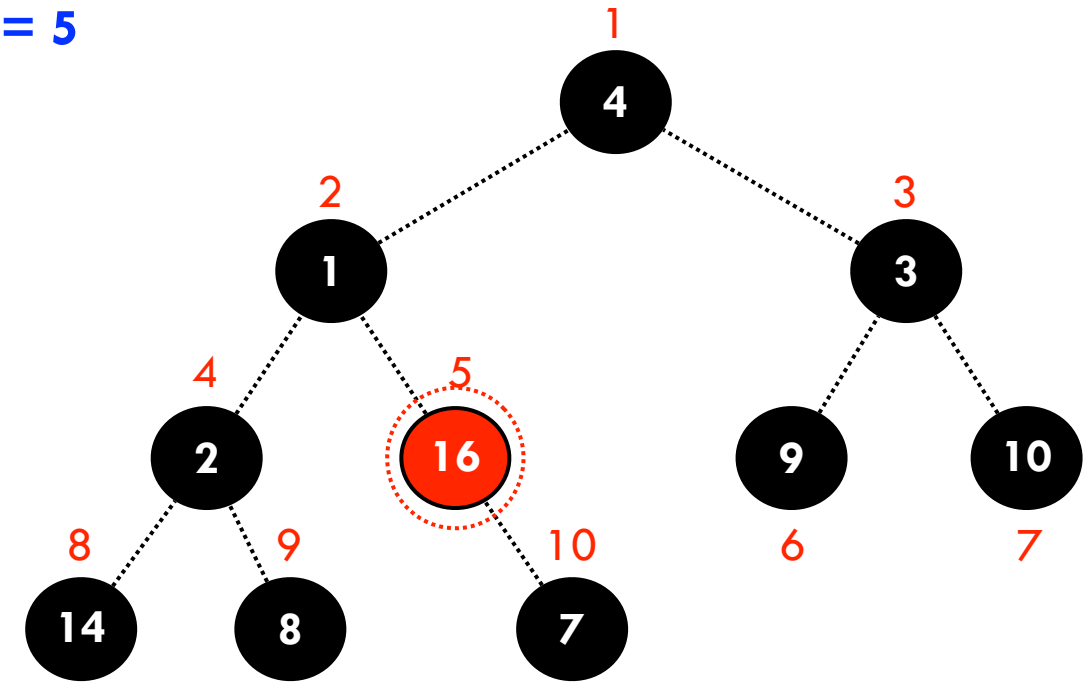
i = {5, 2, 3, 4, 1}



BUILD-MAX-HEAP

	1	2	3	4	5	6	7	8	9	10
V =	4	1	3	2	16	9	10	14	8	7

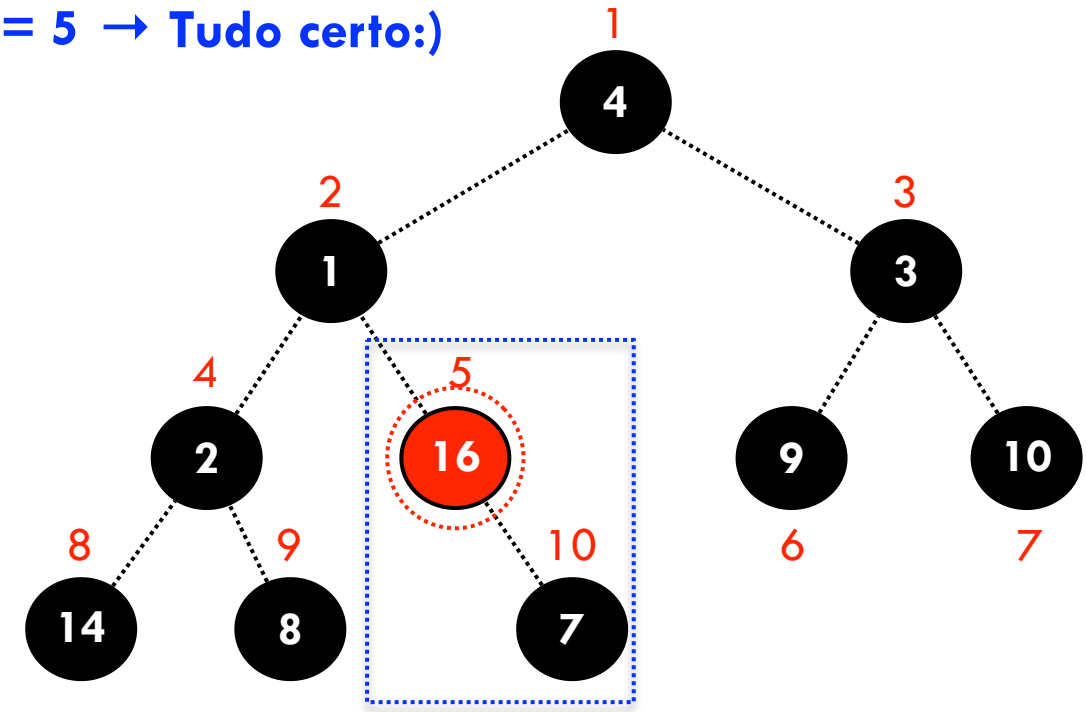
- iteração **i = 5**



BUILD-MAX-HEAP

	1	2	3	4	5	6	7	8	9	10
V =	4	1	3	2	16	9	10	14	8	7

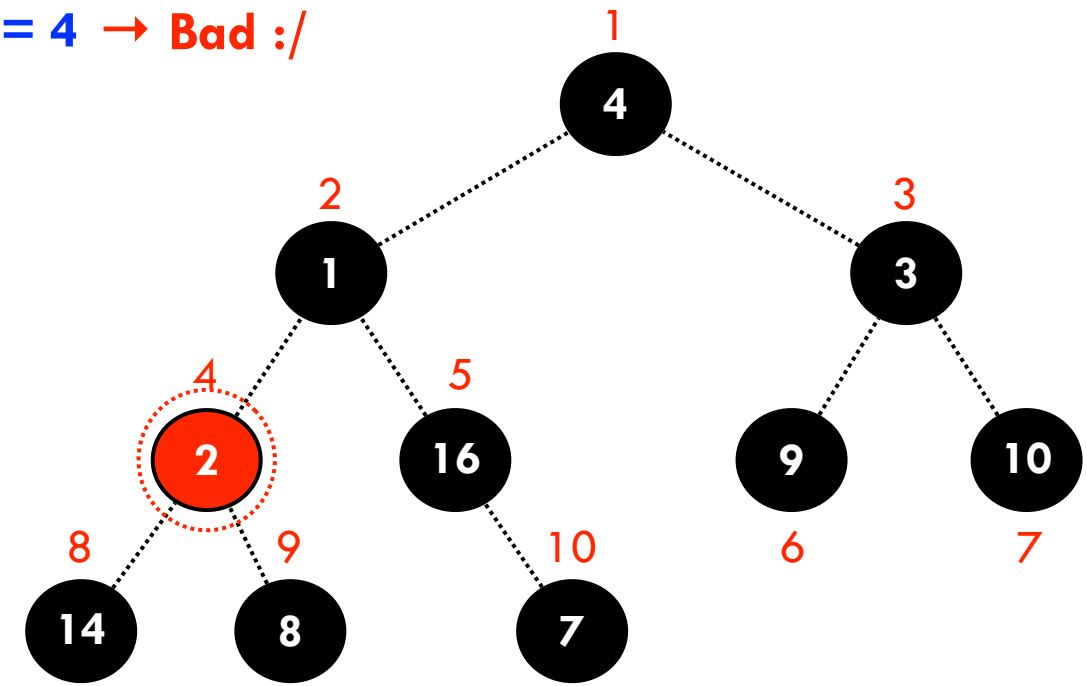
- iteração **i = 5 → Tudo certo:)**



BUILD-MAX-HEAP

	1	2	3	4	5	6	7	8	9	10
V =	4	1	3	2	16	9	10	14	8	7

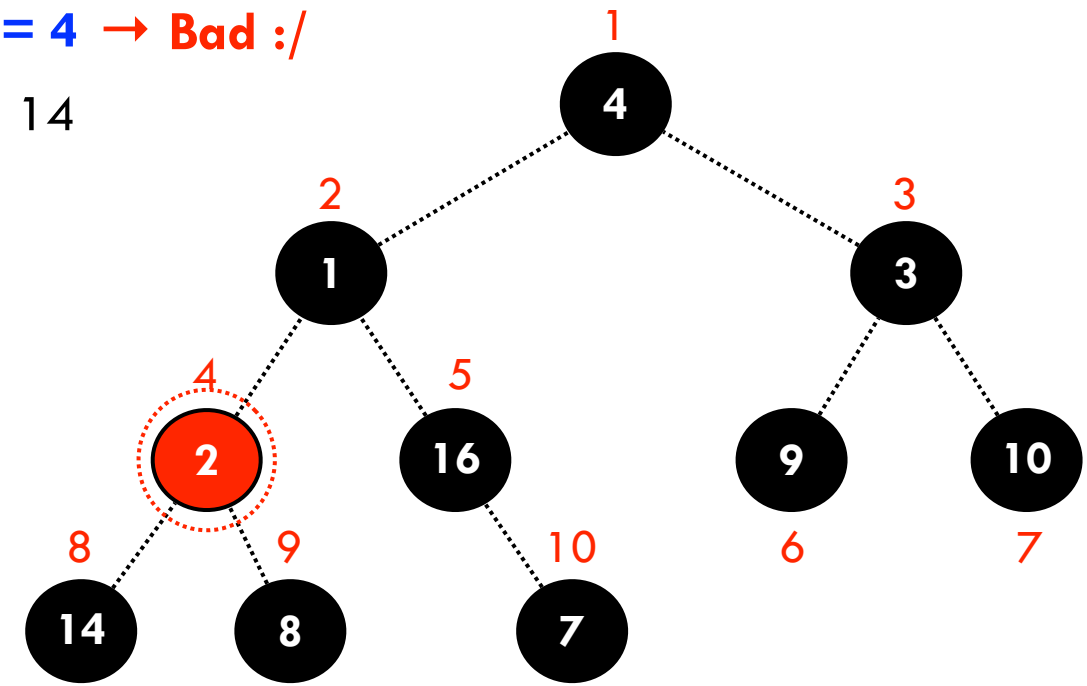
- iteração **i = 4** → **Bad :/**



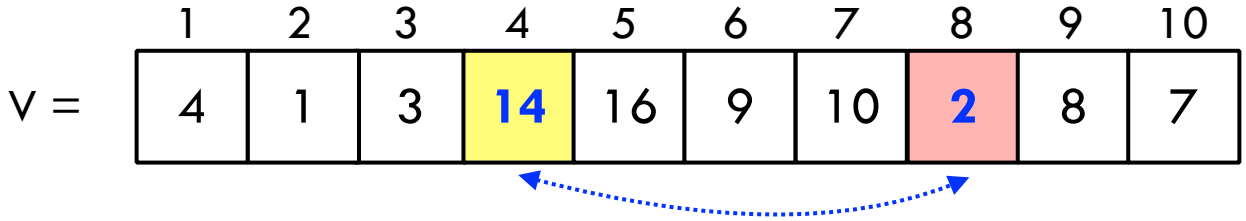
BUILD-MAX-HEAP

	1	2	3	4	5	6	7	8	9	10
V =	4	1	3	2	16	9	10	14	8	7

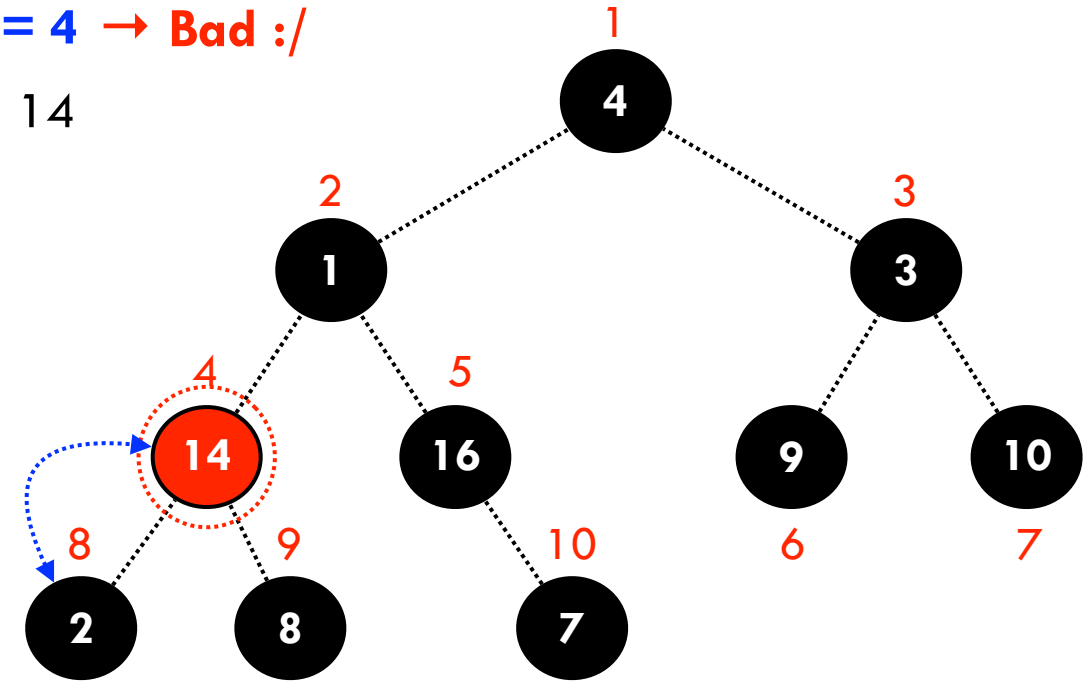
- iteração **i = 4** → **Bad :/**
- Trocar 2 e 14



BUILD-MAX-HEAP



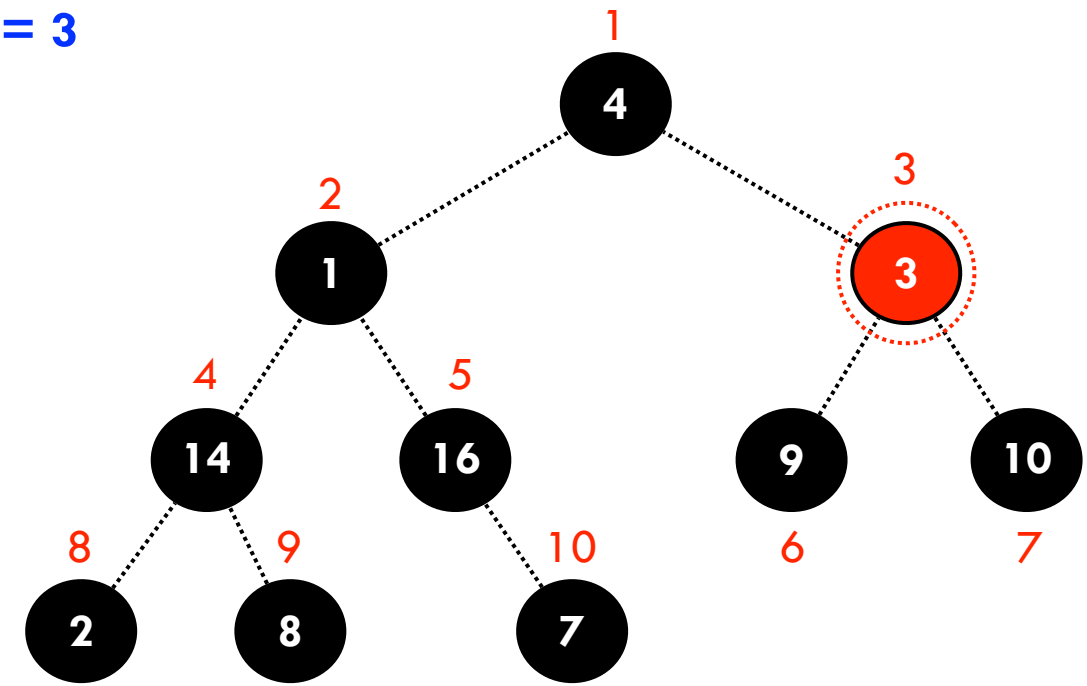
- iteração **i = 4** → **Bad :/**
- Trocar 2 e 14



BUILD-MAX-HEAP

	1	2	3	4	5	6	7	8	9	10
V =	4	1	3	14	16	9	10	2	8	7

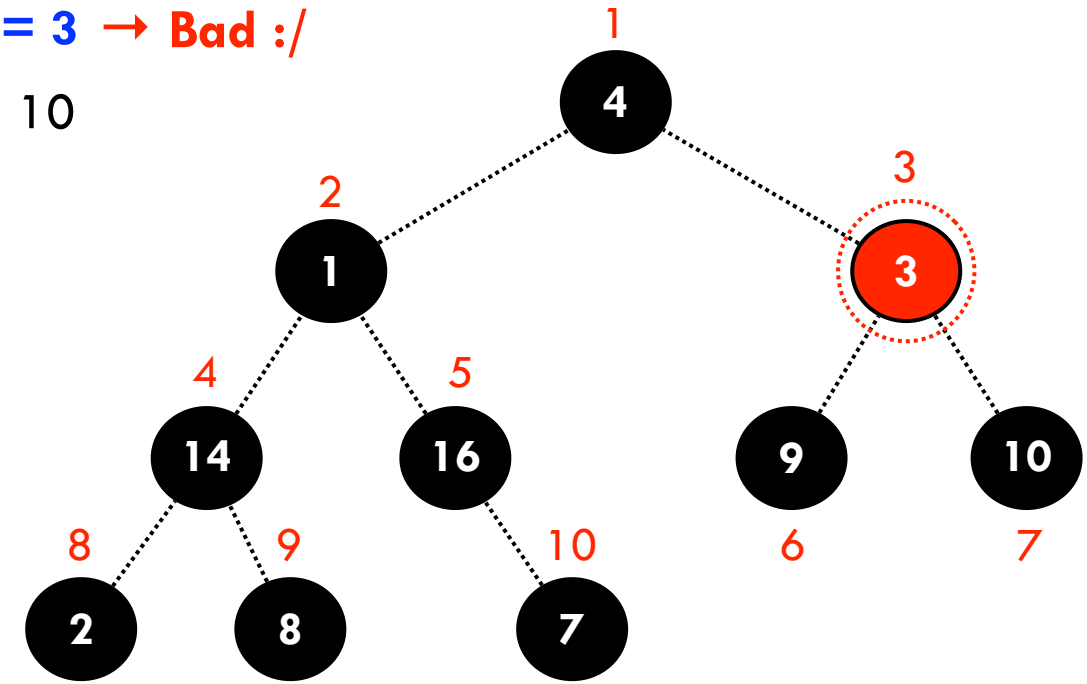
- iteração **i = 3**



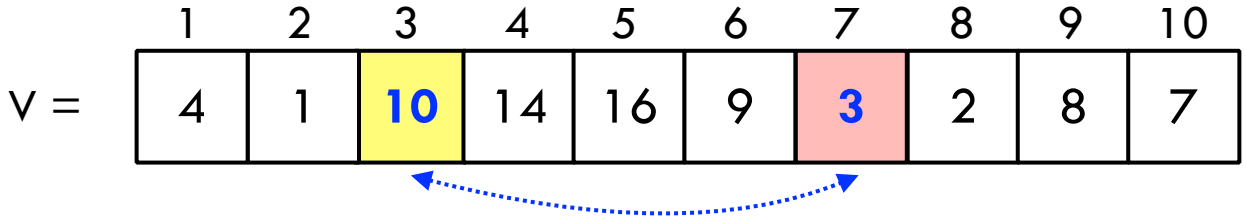
BUILD-MAX-HEAP

	1	2	3	4	5	6	7	8	9	10
V =	4	1	3	14	16	9	10	2	8	7

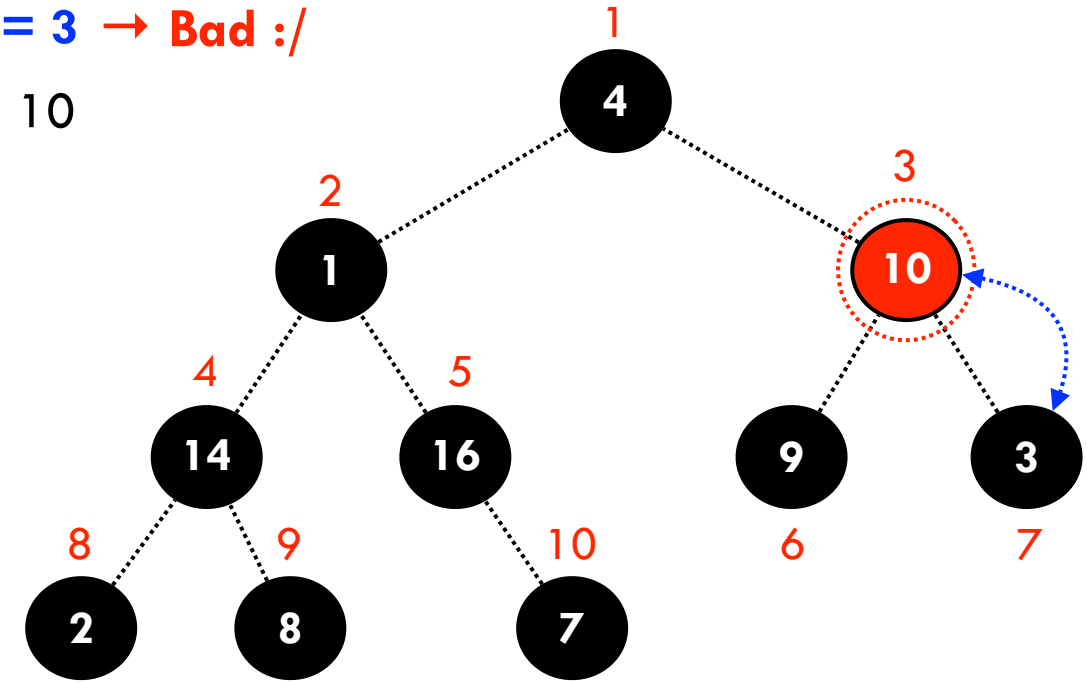
- iteração **i = 3** → **Bad :/**
- Trocar 3 e 10



BUILD-MAX-HEAP



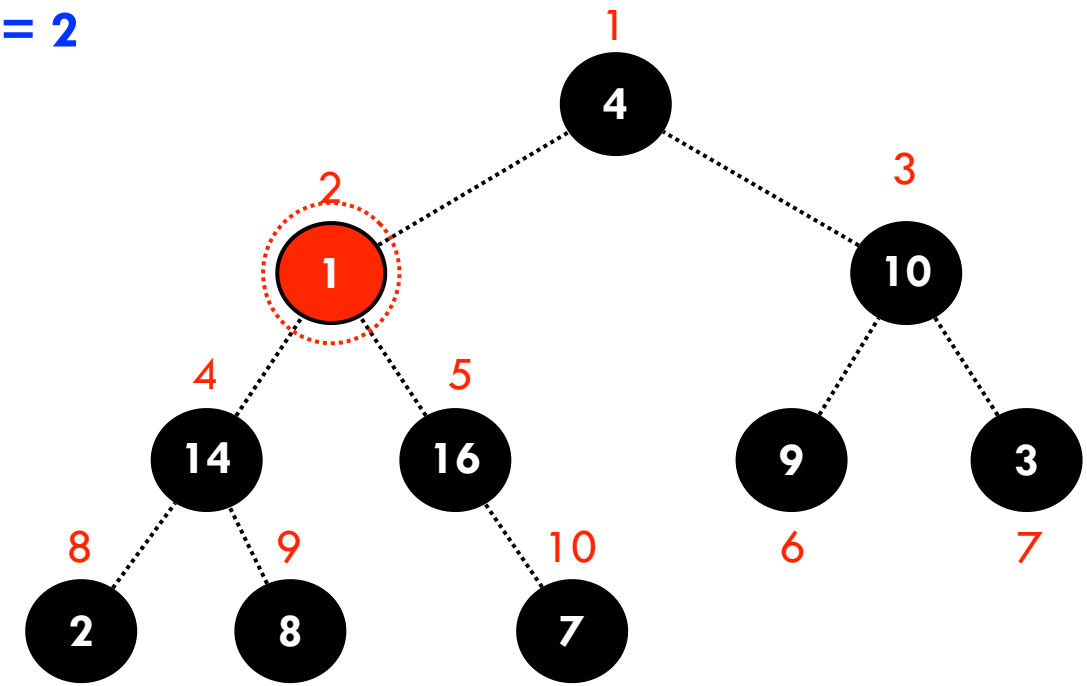
- iteração **i = 3** → **Bad :/**
- Trocar 3 e 10



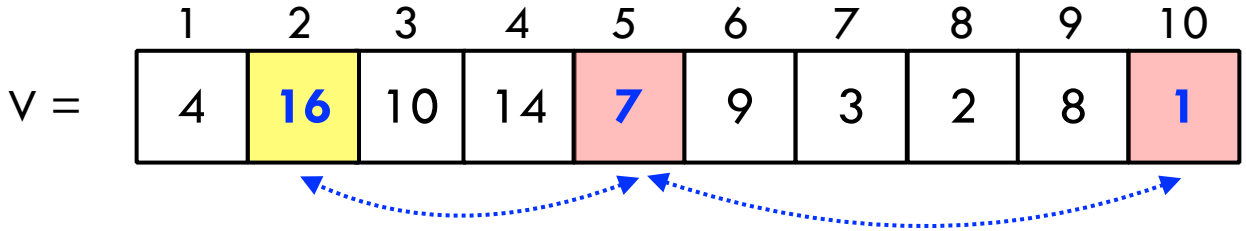
BUILD-MAX-HEAP

	1	2	3	4	5	6	7	8	9	10
V =	4	1	10	14	16	9	3	2	8	7

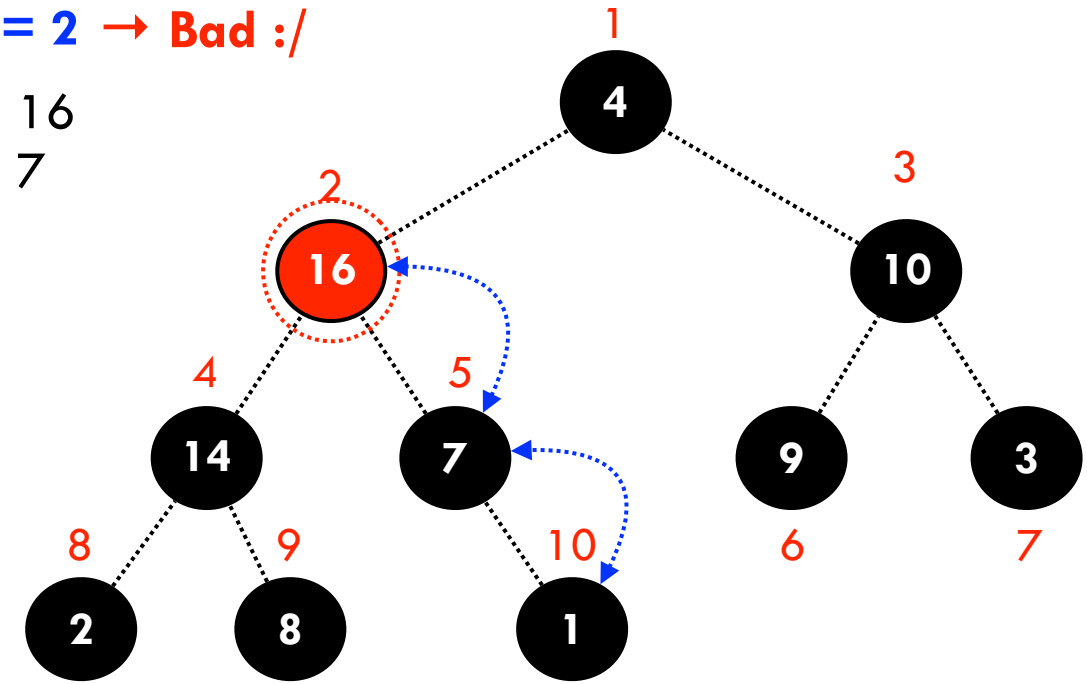
- iteração **i = 2**



BUILD-MAX-HEAP



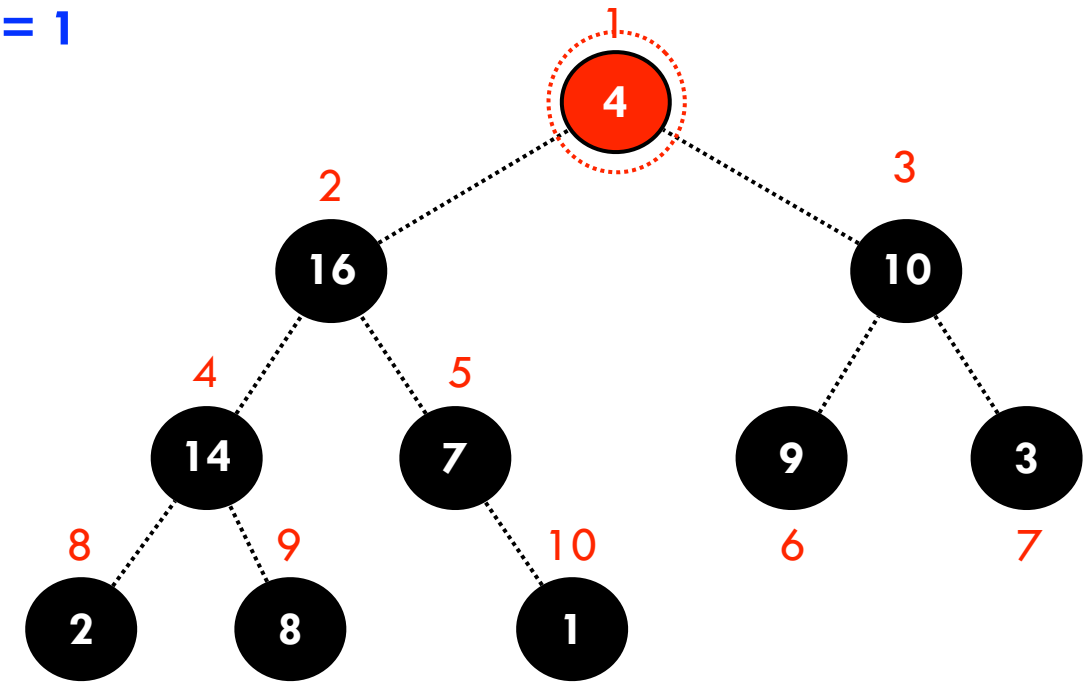
- iteração **i = 2** → **Bad :/**
- Trocar 1 e 16
- Trocar 1 e 7



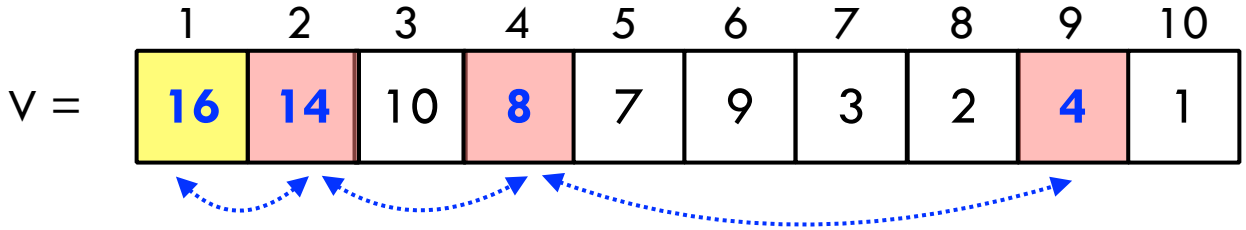
BUILD-MAX-HEAP

	1	2	3	4	5	6	7	8	9	10
V =	4	16	10	14	7	9	3	2	8	1

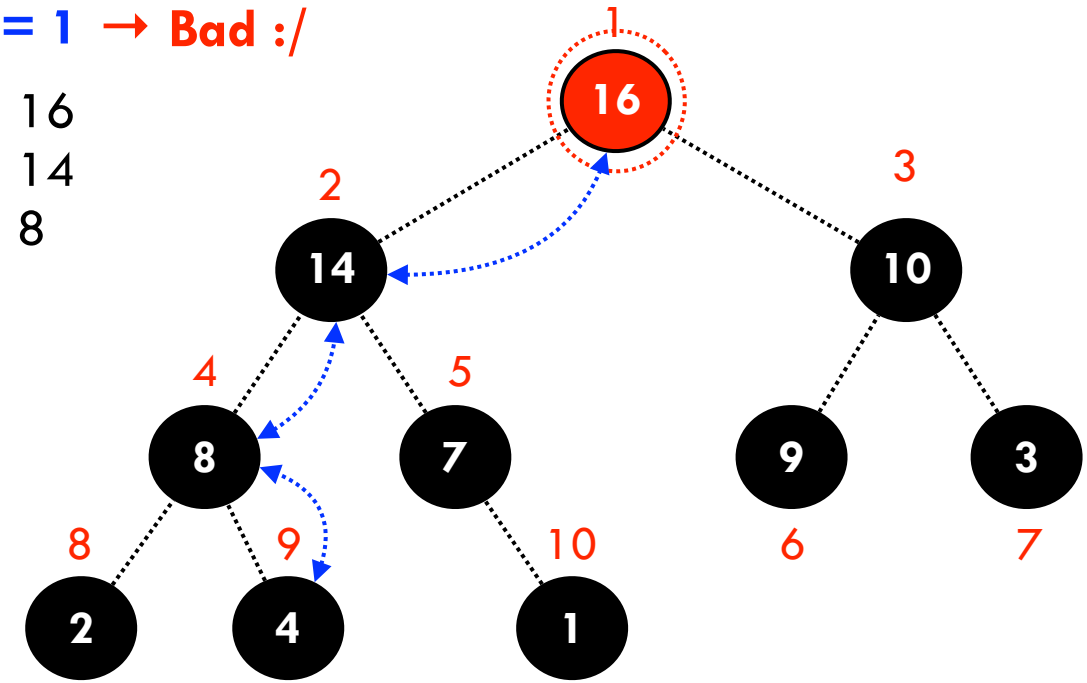
- iteração **i = 1**



BUILD-MAX-HEAP



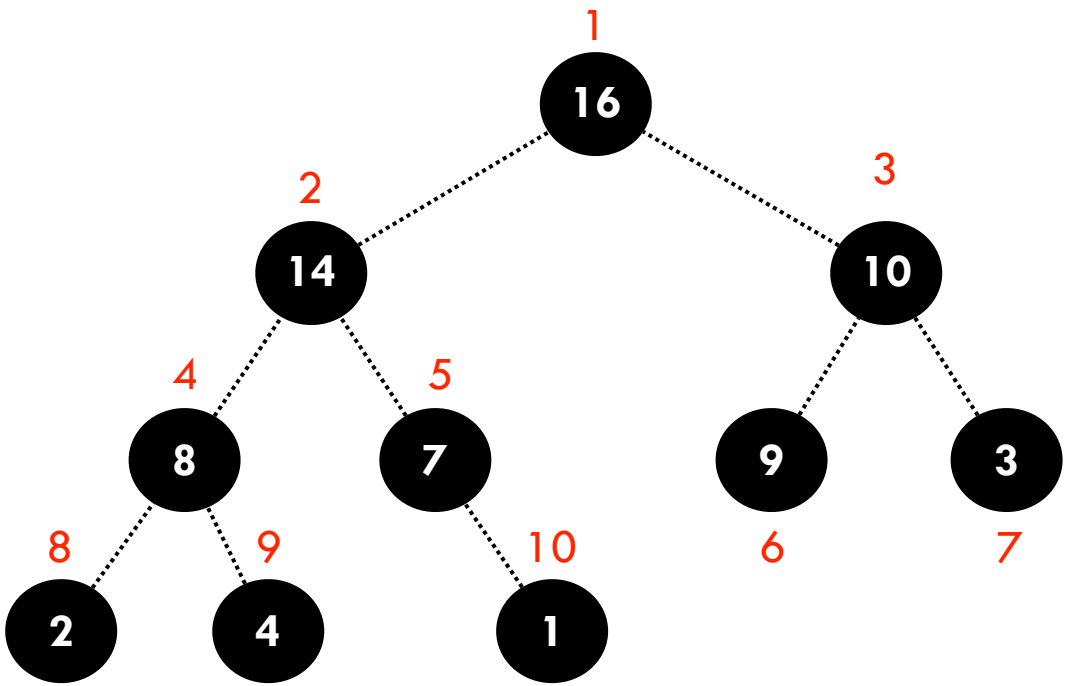
- iteração i = 1 → **Bad :/**
- Trocar 4 e 16
- Trocar 4 e 14
- Trocar 4 e 8



BUILD-MAX-HEAP

	1	2	3	4	5	6	7	8	9	10
V =	16	14	10	8	7	9	3	2	4	1

- Final!



Heap Sort

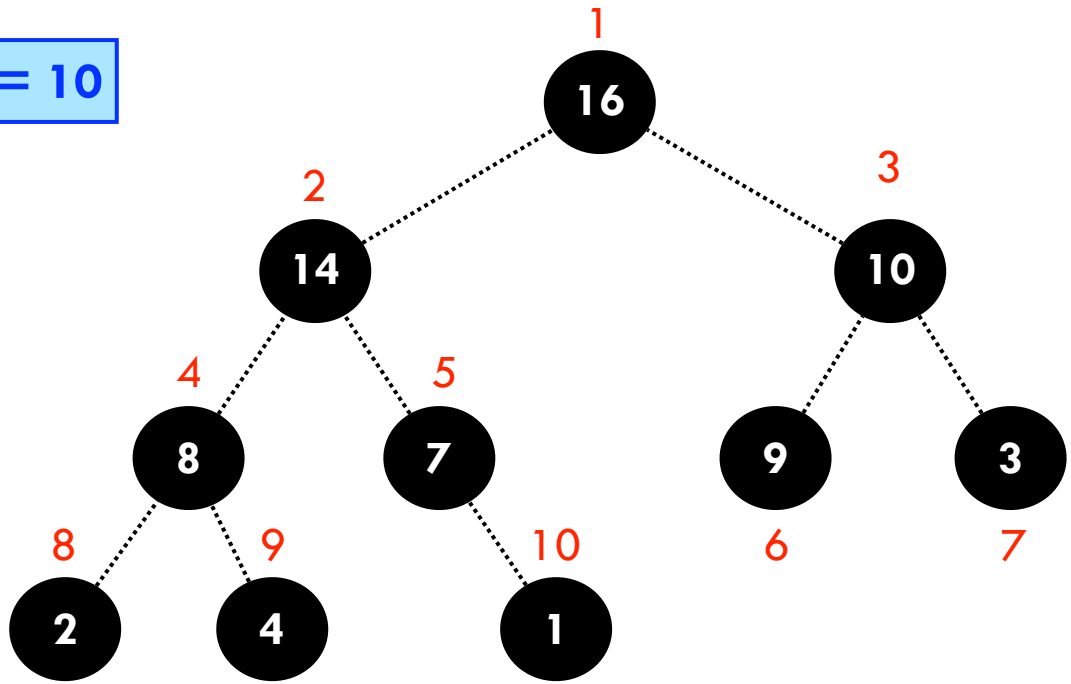
- **Pseudocódigo** (função principal)

```
1.  HEAPSORT (V)
2.      BUILD-MAX-HEAP(A)
3.      Para i = V.tamanho até 2
4.          Troca(V[1], V[i])
5.          V.tamanho-heap = V.tamanho-heap -1
6.          MAX-HEAPIFY(V, 1)
```

HEAP SORT

	1	2	3	4	5	6	7	8	9	10
V =	16	14	10	8	7	9	3	2	4	1

- iteração **i = 10**

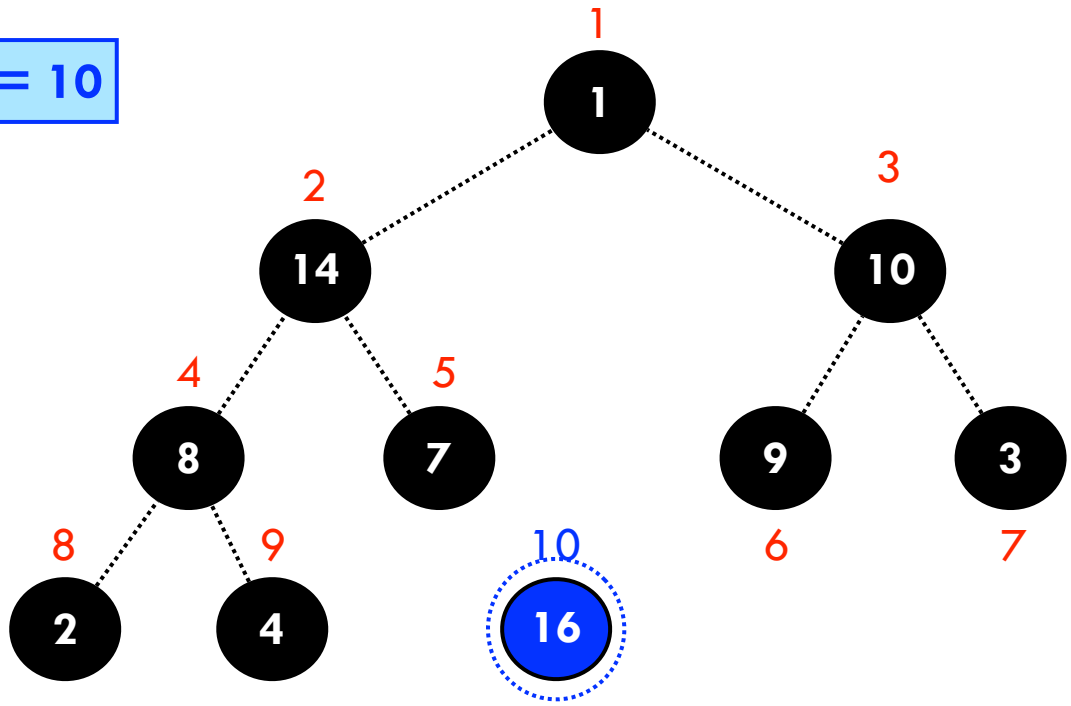


HEAP SORT

	1	2	3	4	5	6	7	8	9	10
V =	1	14	10	8	7	9	3	2	4	16

MAX-HEAPIFY ⟷

- iteração **i = 10**

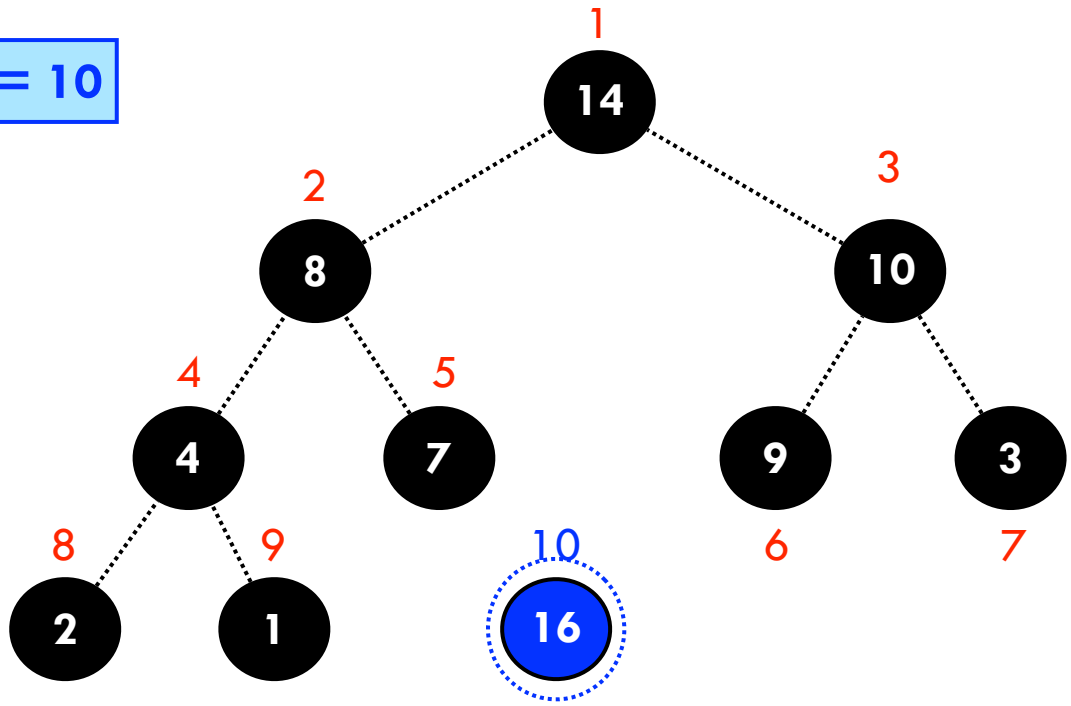


HEAP SORT

	1	2	3	4	5	6	7	8	9	10
V =	14	8	10	4	7	9	3	2	1	16

MAX-HEAPIFY ⟷

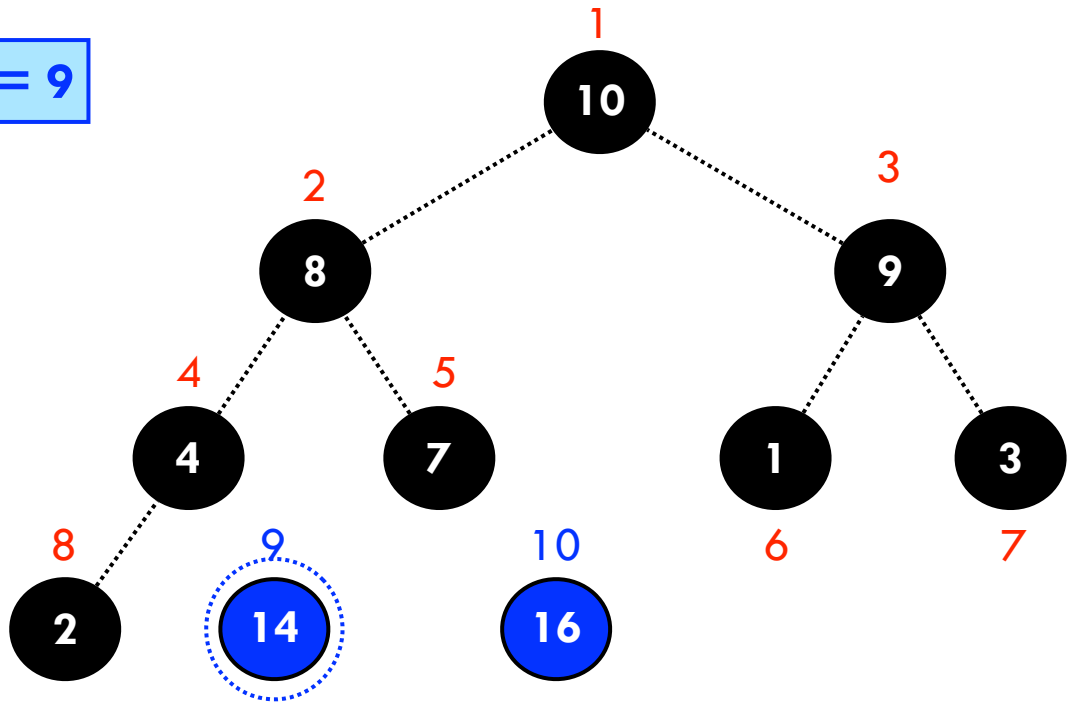
- iteração **i = 10**



HEAP SORT

	1	2	3	4	5	6	7	8	9	10
V =	10	8	9	4	7	1	3	2	14	16

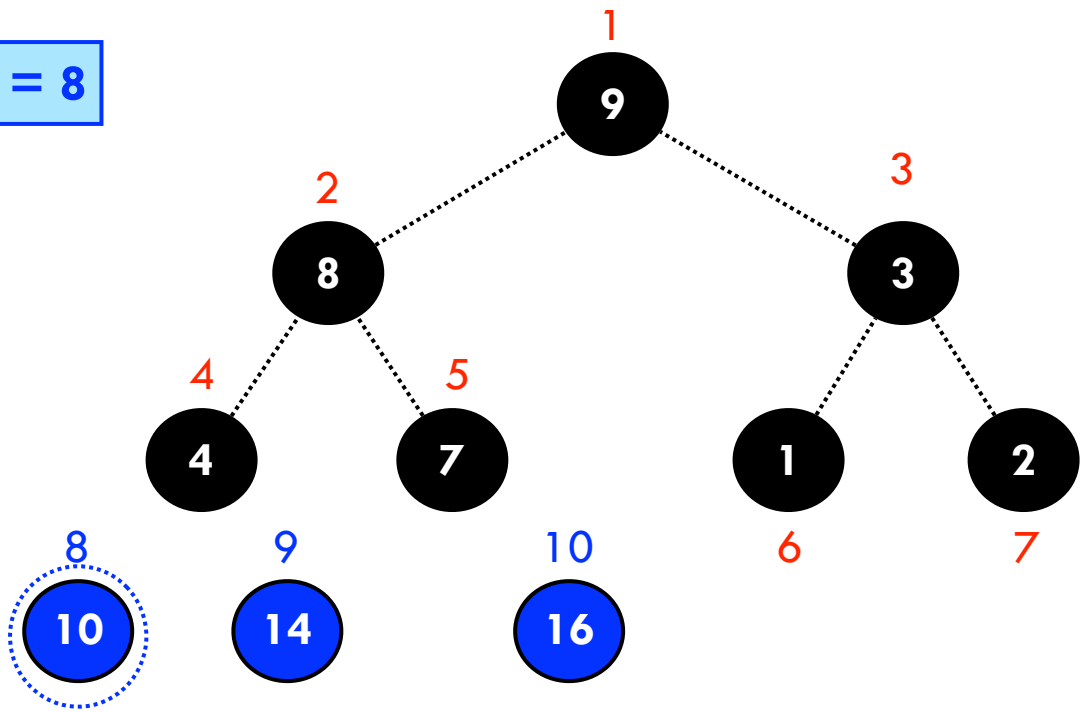
- iteração **i = 9**



HEAP SORT

	1	2	3	4	5	6	7	8	9	10
V =	9	8	3	4	7	1	2	10	14	16

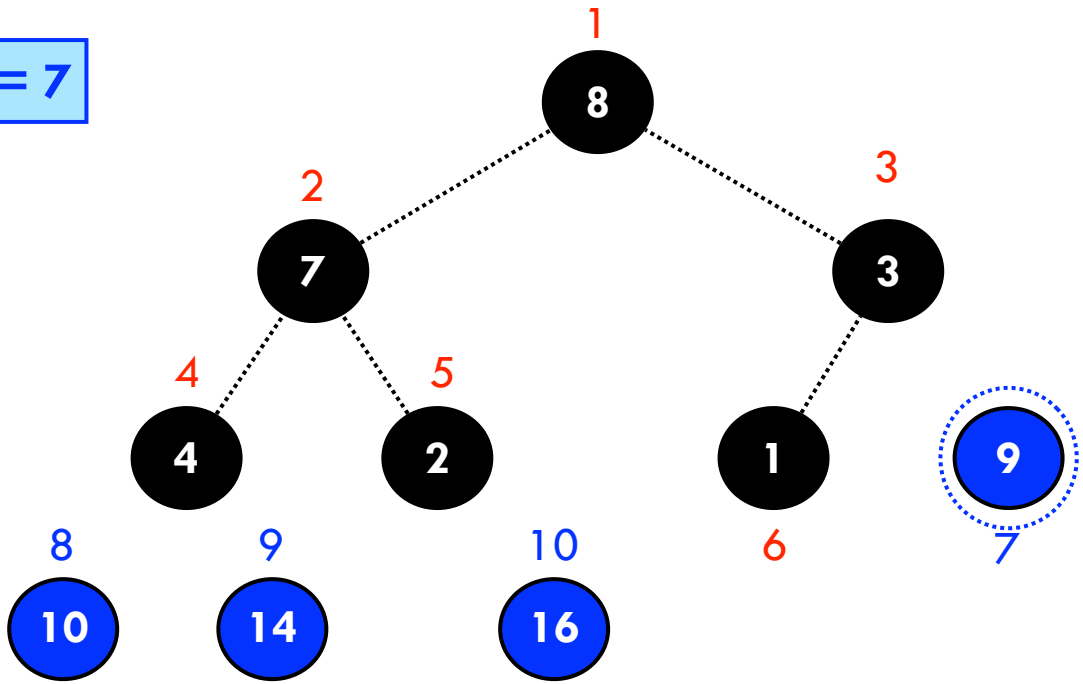
- iteração **i = 8**



HEAP SORT

	1	2	3	4	5	6	7	8	9	10
V =	9	8	3	4	7	1	9	10	14	16

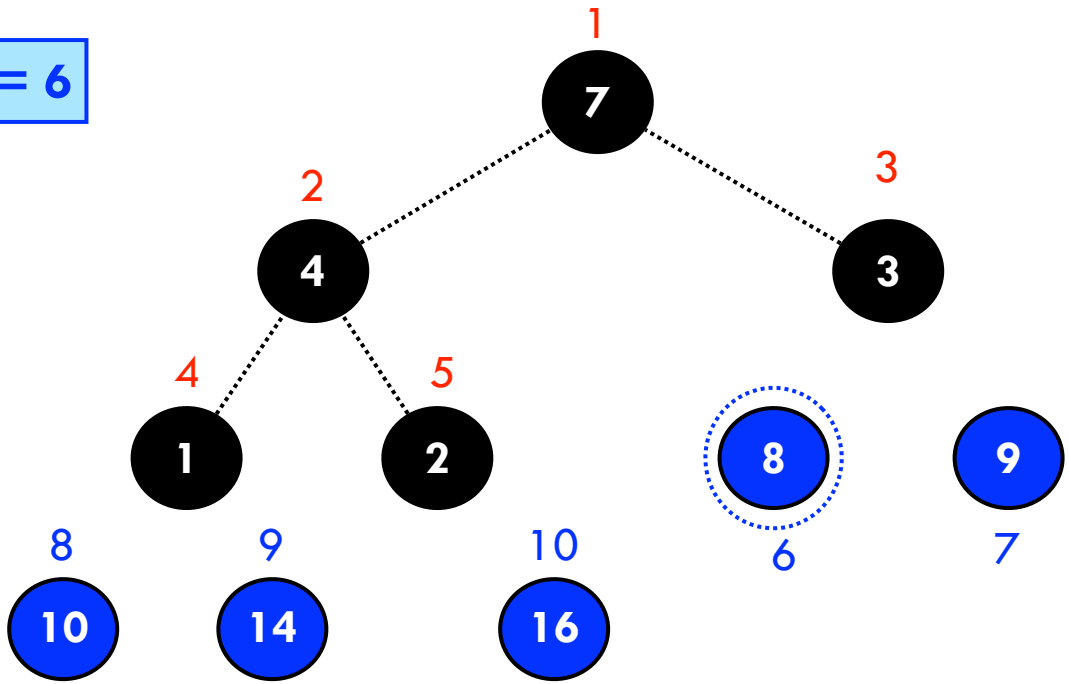
- iteração **i = 7**



HEAP SORT

	1	2	3	4	5	6	7	8	9	10
V =	7	4	3	1	2	8	9	10	14	16

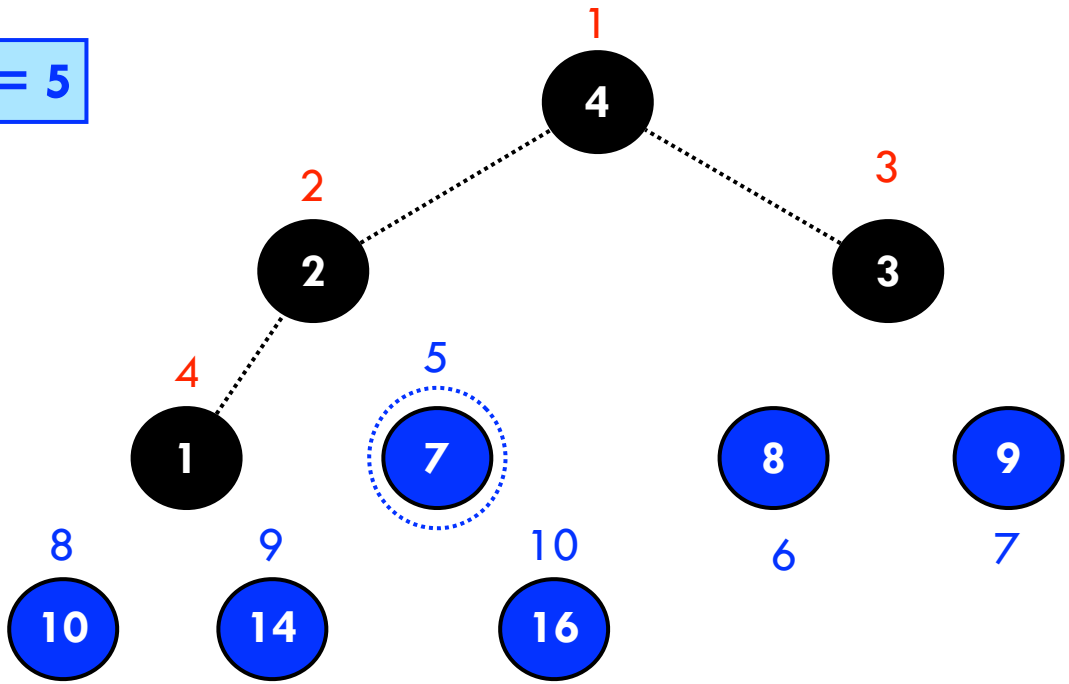
- iteração **i = 6**



HEAP SORT

	1	2	3	4	5	6	7	8	9	10
V =	4	2	3	1	7	8	9	10	14	16

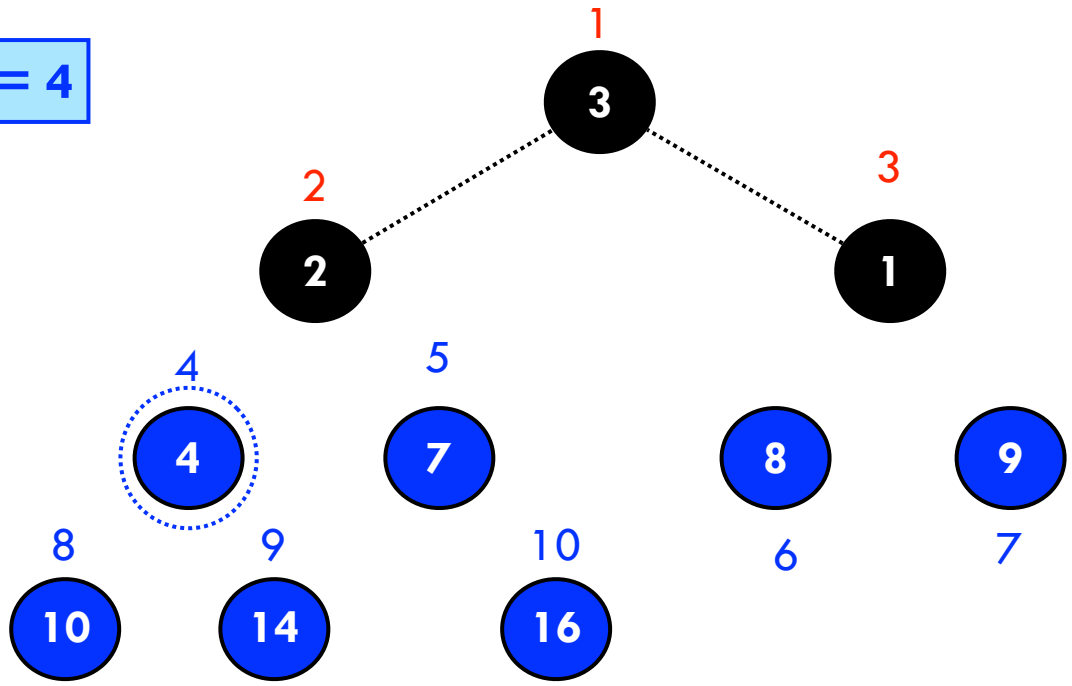
- iteração **i = 5**



HEAP SORT

	1	2	3	4	5	6	7	8	9	10
V =	3	2	1	4	7	8	9	10	14	16

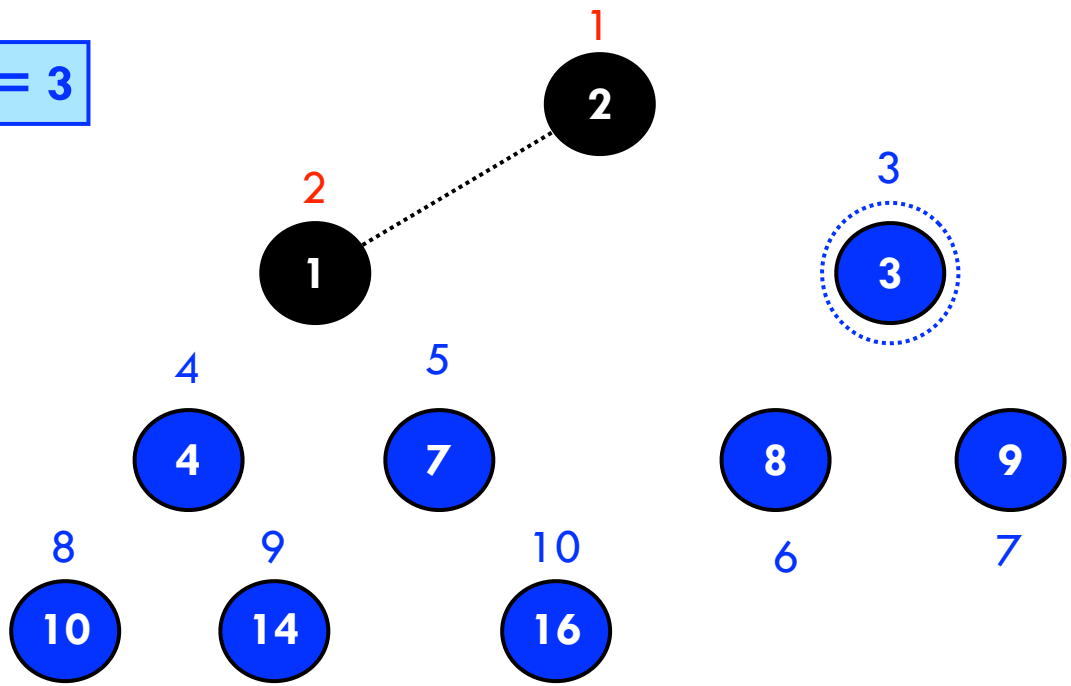
- iteração **i = 4**



HEAP SORT

	1	2	3	4	5	6	7	8	9	10
V =	2	1	3	4	7	8	9	10	14	16

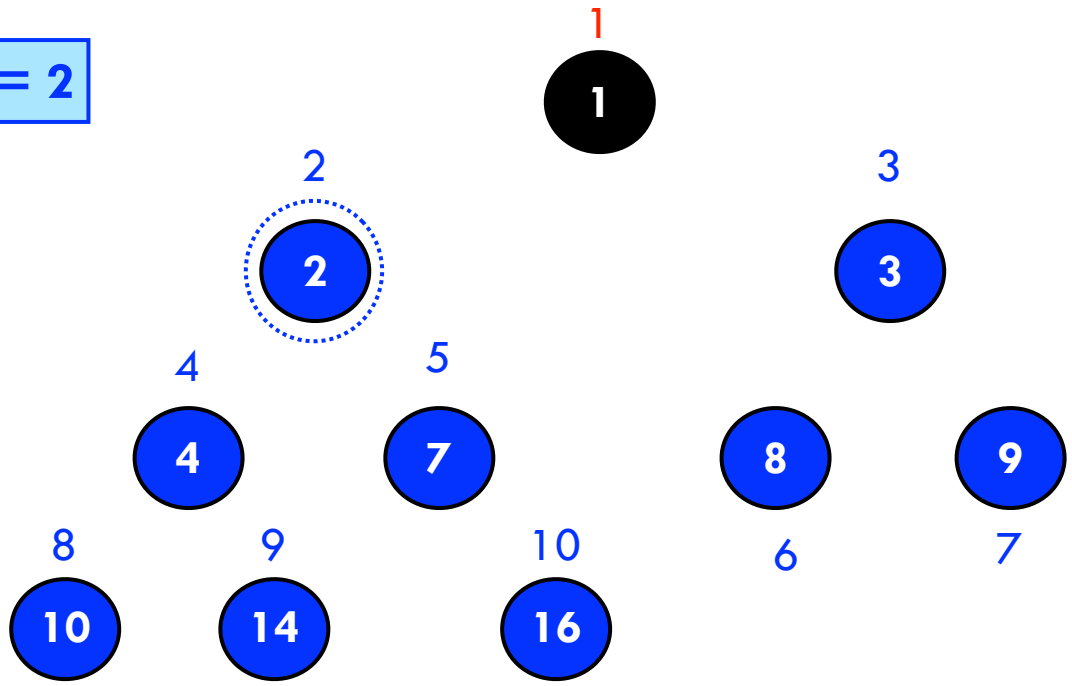
- iteração **i = 3**



HEAP SORT

	1	2	3	4	5	6	7	8	9	10
V =	1	2	3	4	7	8	9	10	14	16

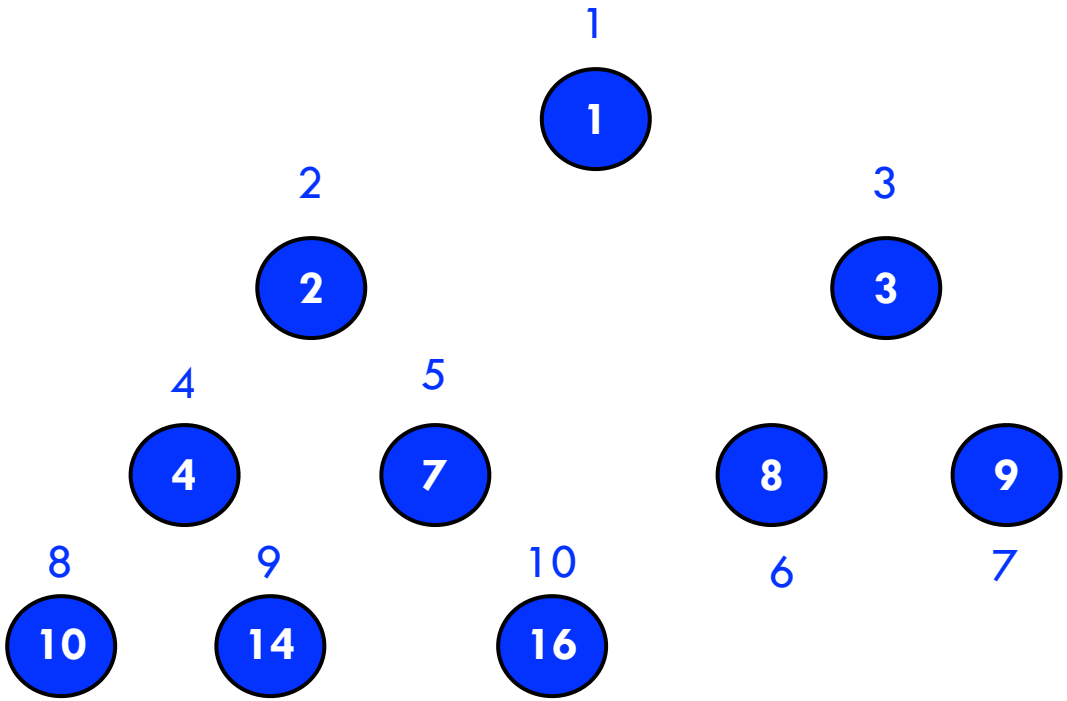
- iteração **i = 2**



HEAP SORT

V =

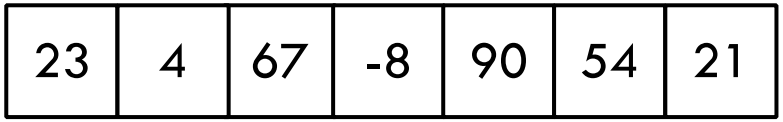
1	2	3	4	5	6	7	8	9	10
1	2	3	4	7	8	9	10	14	16



Roteiro

1. Introdução
2. Heaps
3. Heap Sort
4. Exemplo
5. Exercícios
6. Referências

Exemplo



vetor não ordenado

Roteiro

1. Introdução
2. Heaps
3. Heap Sort
4. Exemplo
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6. Referências

Exercícios

HANDS ON :)))

Exercícios

1) Reuna-se com seu grupo e execute o teste de mesa (simulação) do algoritmo para as suas sequências de números apresentadas

Exercícios

2) Implemente o **heapSort** em Python considerando as seguintes funções

```
/* função principal */
def heapSort(array);

/* constrói heap de máximo a partir do vetor */
def buildMaxHeap(array);

/* reconstrói o heap, desconsiderando o elemento já ordenado */
def maxHeapify(array, i, heapsize);
```

Roteiro

1. Introdução
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3. Heap Sort
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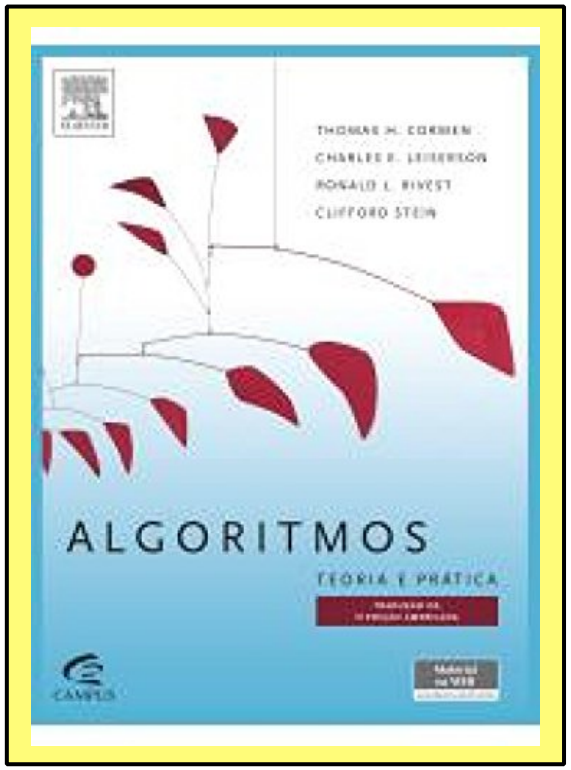
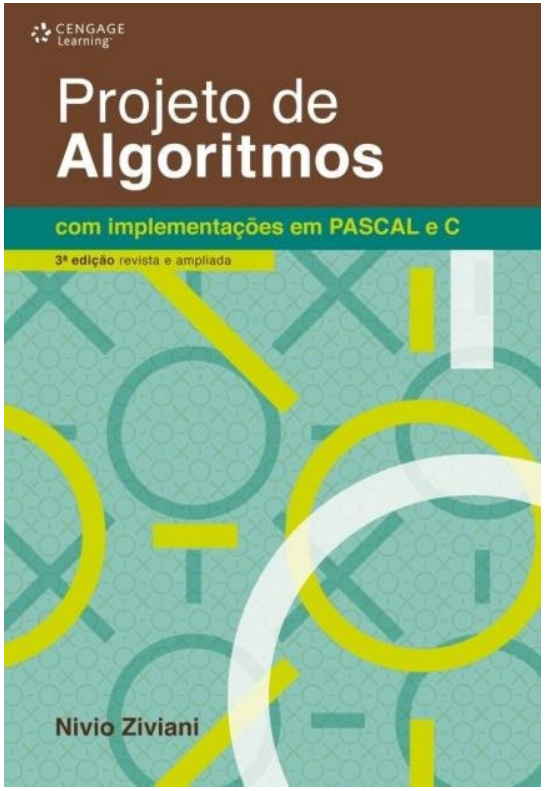
Referências sugeridas



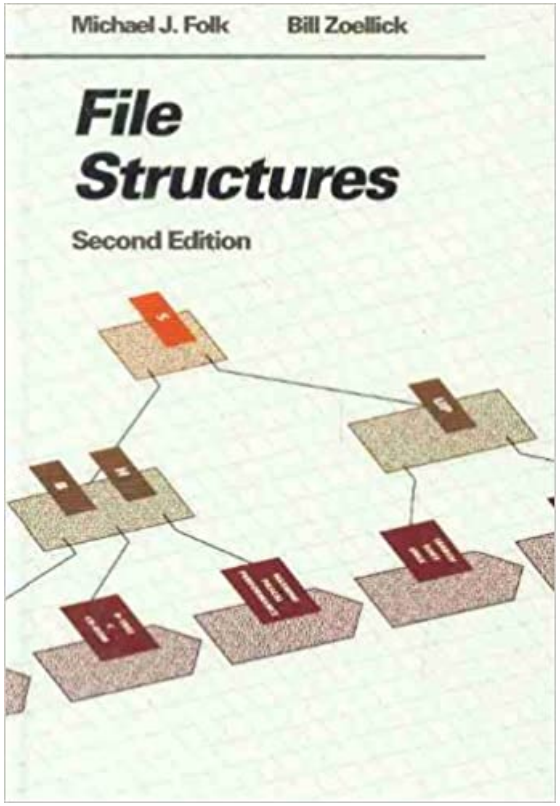
[Cormen et al, 2018]

[Drozdek, 2017]

Referências sugeridas



[Ziviani, 2010]



[Folk & Zoellick, 1992]

Perguntas?

Prof. Rafael G. **Mantovani**

rafaelmantovani@utfpr.edu.br