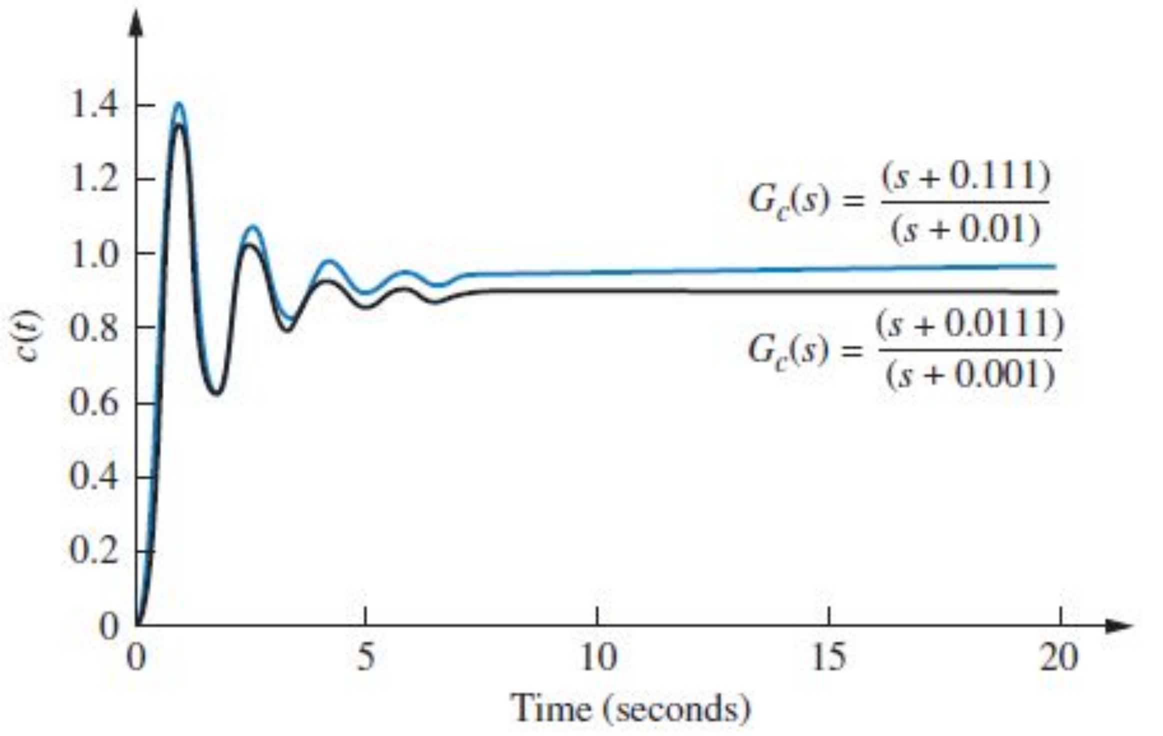
Melhorando a Resposta Transitória Via Compensação em Cascata

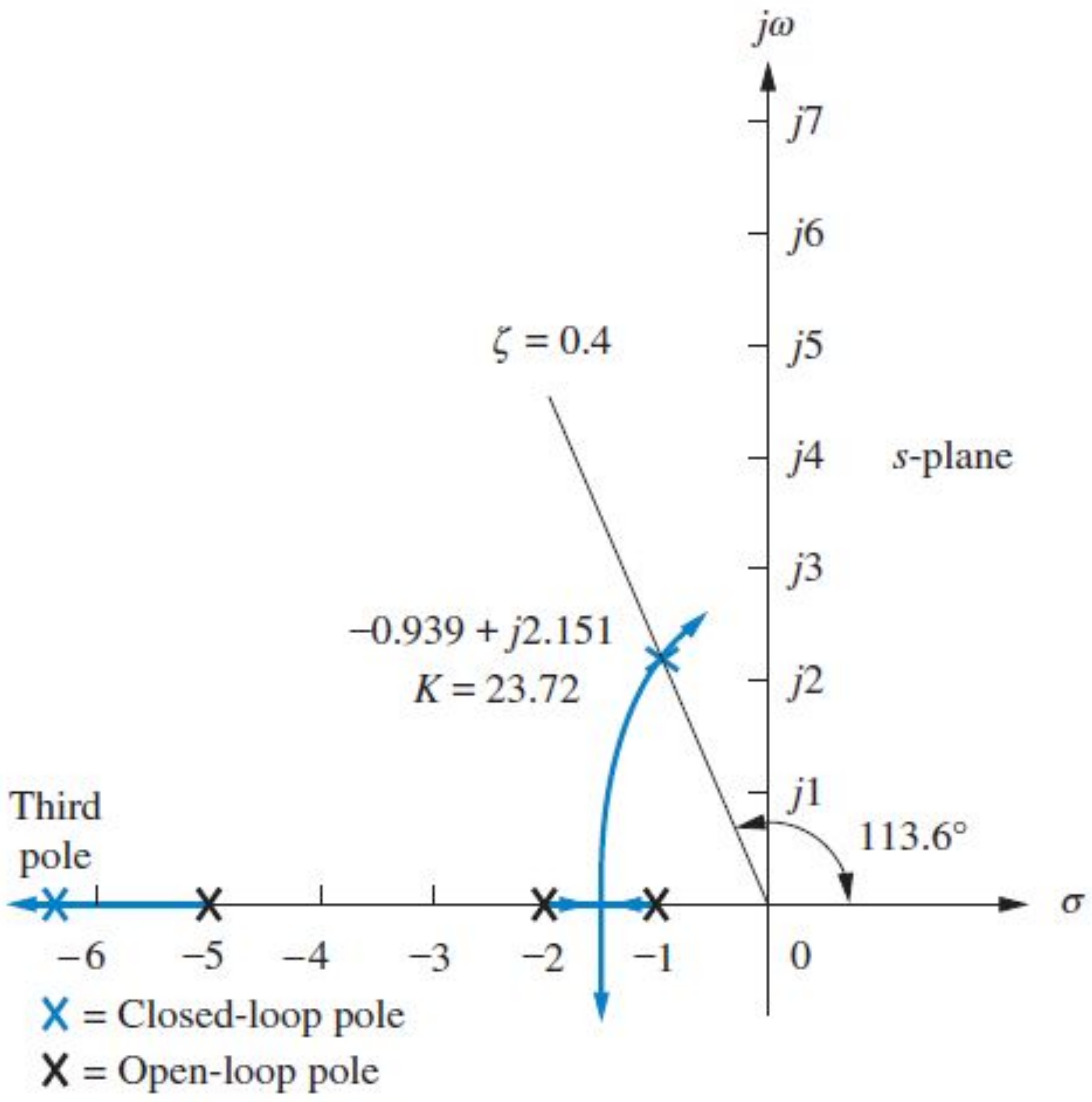
Fundamentos de Controle

Compensação Derivativa Ideal (PD)

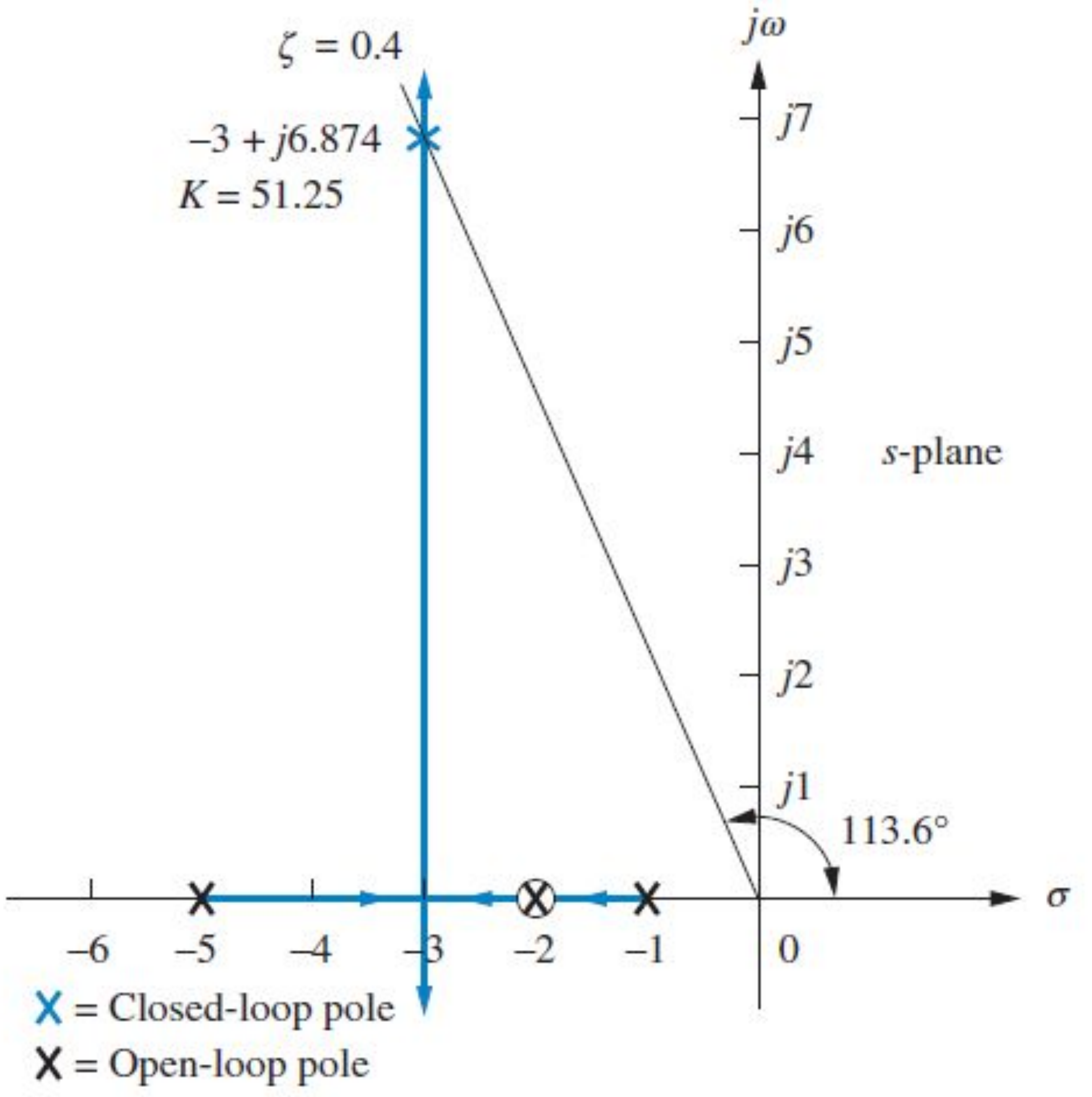
$$G_c(s) = s + z_c$$



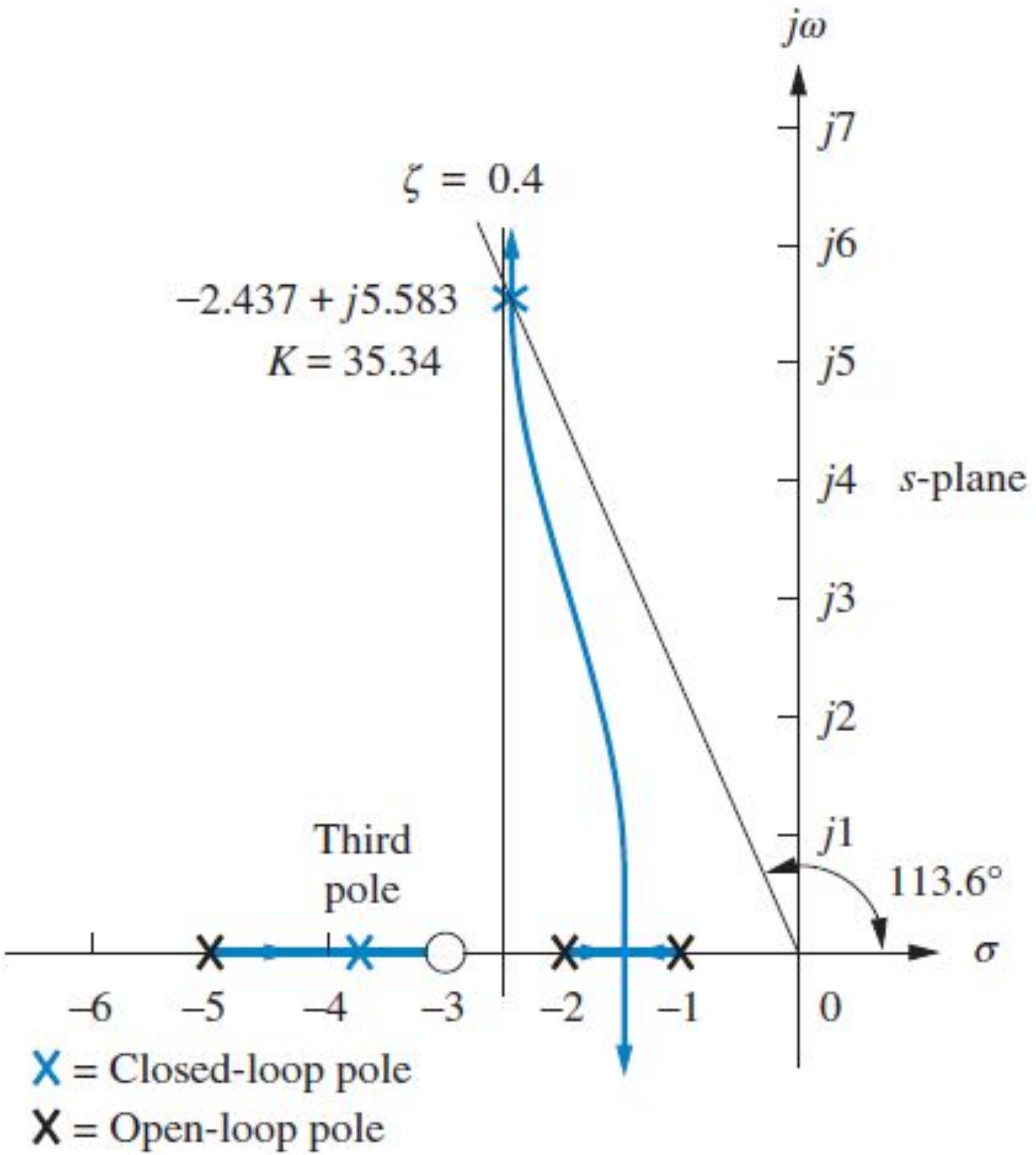
$$\frac{K}{(s+1)(s+2)(s+5)} \qquad \frac{K(s+2)}{(s+1)(s+2)(s+5)} \qquad \frac{K(s+3)}{(s+1)(s+2)(s+5)} \qquad \frac{K(s+4)}{(s+1)(s+2)(s+5)}$$



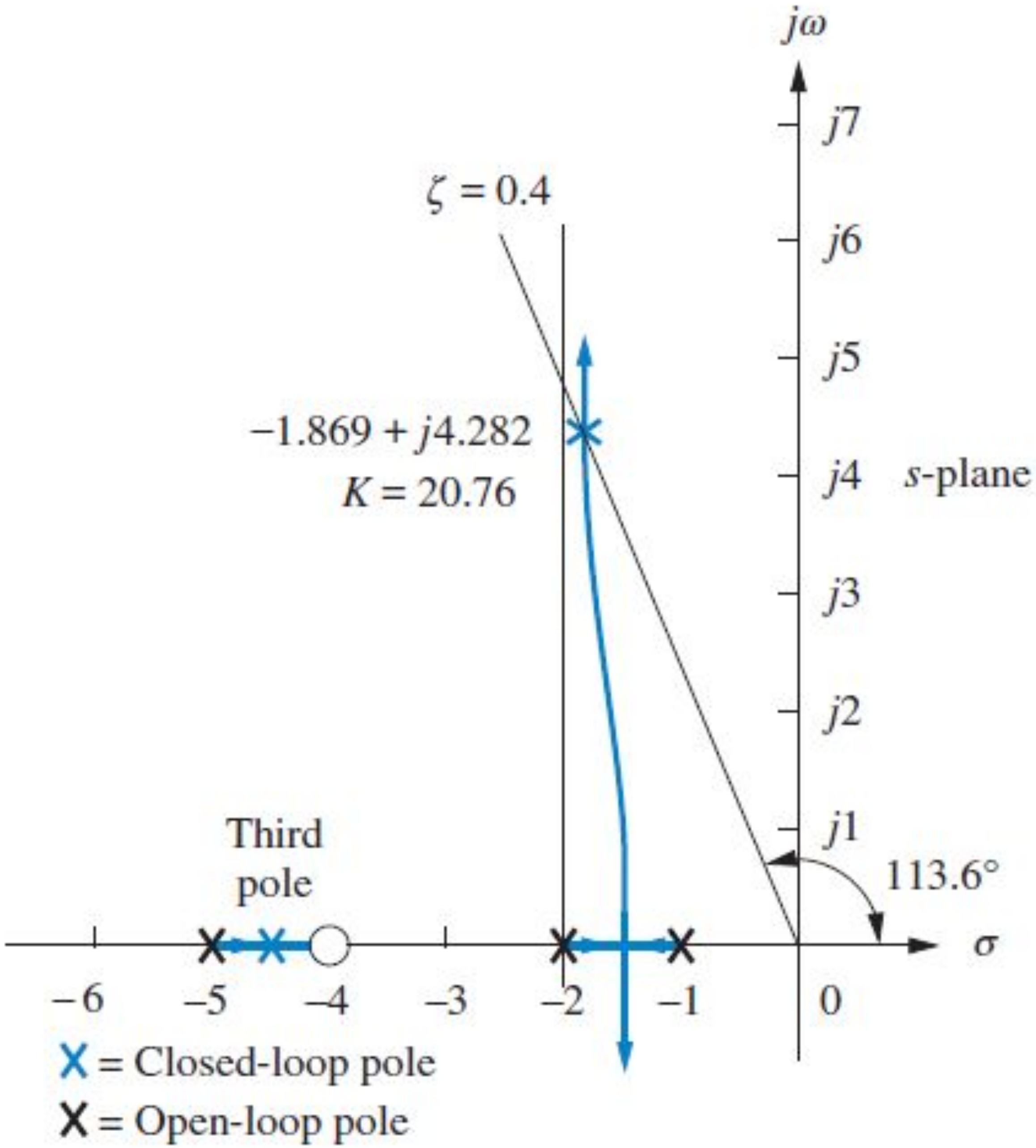
jω
j7
j6
j5
j4
j3
j2
j1
ζ = 0.4
s-plane
–0.939 + j2.151
K = 23.72
113.6°
Third
pole
σ
–6
–5
–4
–3
–2
–1
0
X = Closed-loop pole
X = Open-loop pole



jω
ζ = 0.4
–3 + j6.874
K = 51.25
j7
j6
j5
j4
j3
j2
j1
s-plane
113.6°
σ
–6
–5
–4
–3
–2
–1
0
X = Closed-loop pole
X = Open-loop pole

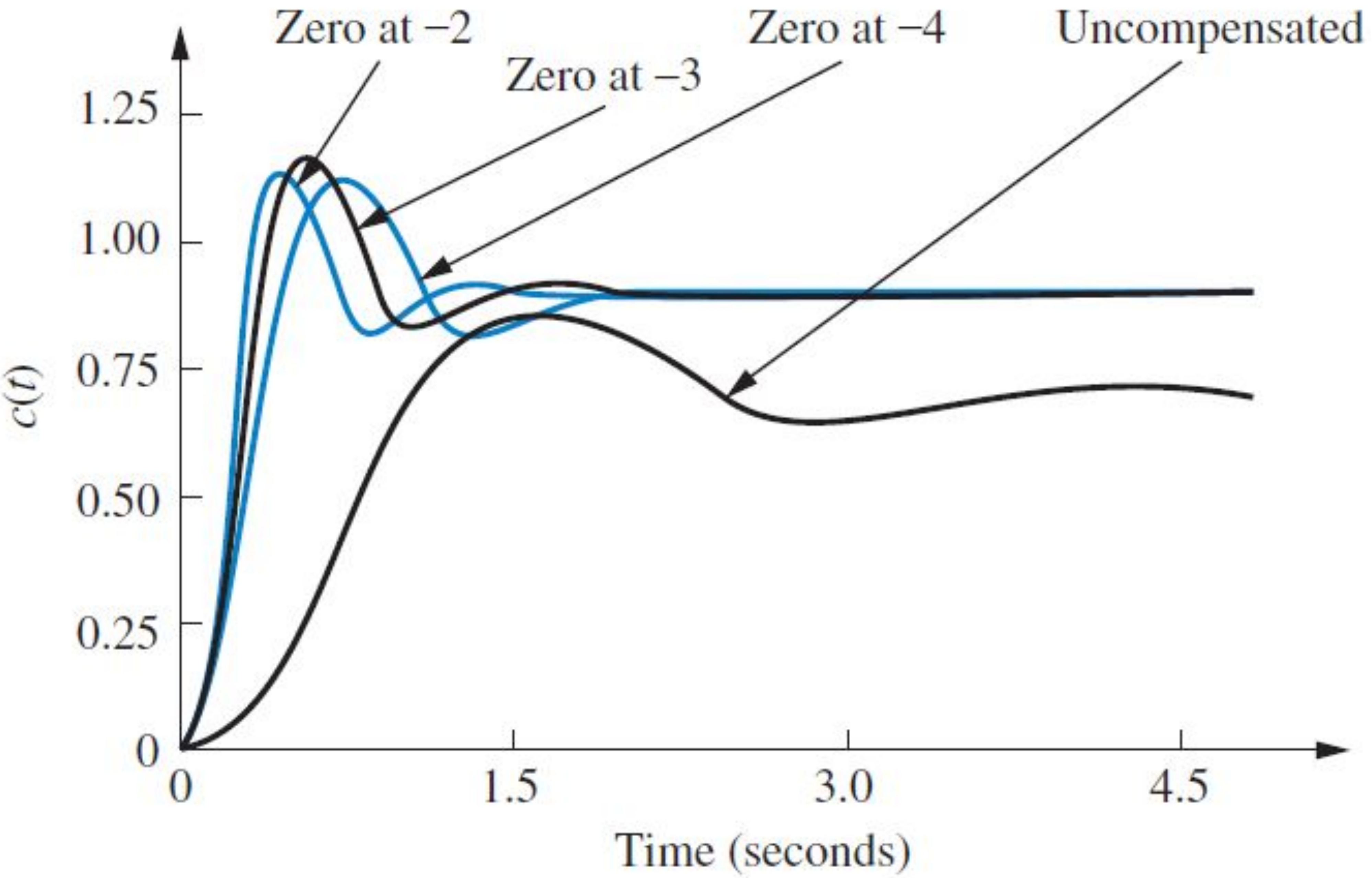


jω
j7
ζ = 0.4
j6
−2.437 + j5.583
K = 35.34
j5
j4
s-plane
j3
j2
j1
113.6°
Third
pole
σ
−6
−5
−4
−3
−2
−1
0
X = Closed-loop pole
X = Open-loop pole



jω
j7
ζ = 0.4
j6
j5
−1.869 + j4.282
K = 20.76
j4
s-plane
j3
j2
j1
113.6°
Third
pole
σ
−6
−5
−4
−3
−2
−1
0
X = Closed-loop pole
X = Open-loop pole

	Uncompensated	Compensation b	Compensation c	Compensation d
Plant and compensator	$\frac{K}{(s+1)(s+2)(s+5)}$	$\frac{K(s+2)}{(s+1)(s+2)(s+5)}$	$\frac{K(s+3)}{(s+1)(s+2)(s+5)}$	$\frac{K(s+4)}{(s+1)(s+2)(s+5)}$
Dom, poles	$-0.939 \pm j2.151$	$-3 \pm j6.874$	$-2.437 \pm j5.583$	$-1.869 \pm j4.282$
K	23.72	51.25	35.34	20.76
ζ	0.4	0.4	0.4	0.4
ω_n	2.347	7.5	6.091	4.673
$\%OS$	25.38	25.38	25.38	25.38
T_s	4.26	1.33	1.64	2.14
T_p	1.46	0.46	0.56	0.733
K_p	2.372	10.25	10.6	8.304
$e(\infty)$	0.297	0.089	0.086	0.107
Third pole	−6.123	None	−3.127	−4.262
Zero	None	None	−3	−4
Comments	Second-order approx. OK	Pure second-order	Second-order approx. OK	Second-order approx. OK


Zero at −2
Zero at −3
Zero at −4
Uncompensated
1.25
1.00
0.75
0.50
0.25
0
c(t)
0
1.5
3.0
4.5
Time (seconds)

Exemplo 9.3

Projeto de Compensador Derivativo Ideal

PROBLEMA: Dado o sistema da Figura 9.17, projete um compensador derivativo ideal para resultar em 16 % de ultrapassagem, com uma redução de três vezes no tempo de acomodação.

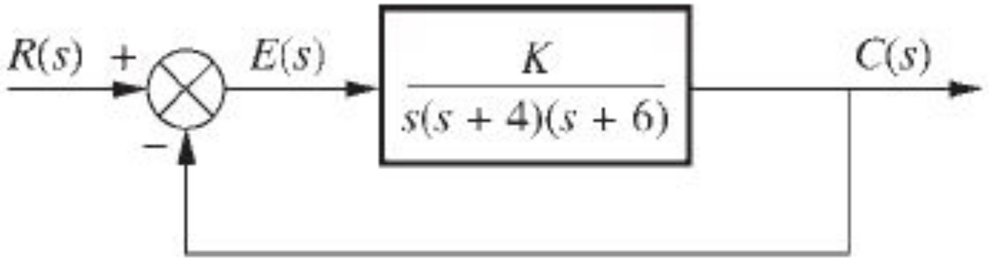
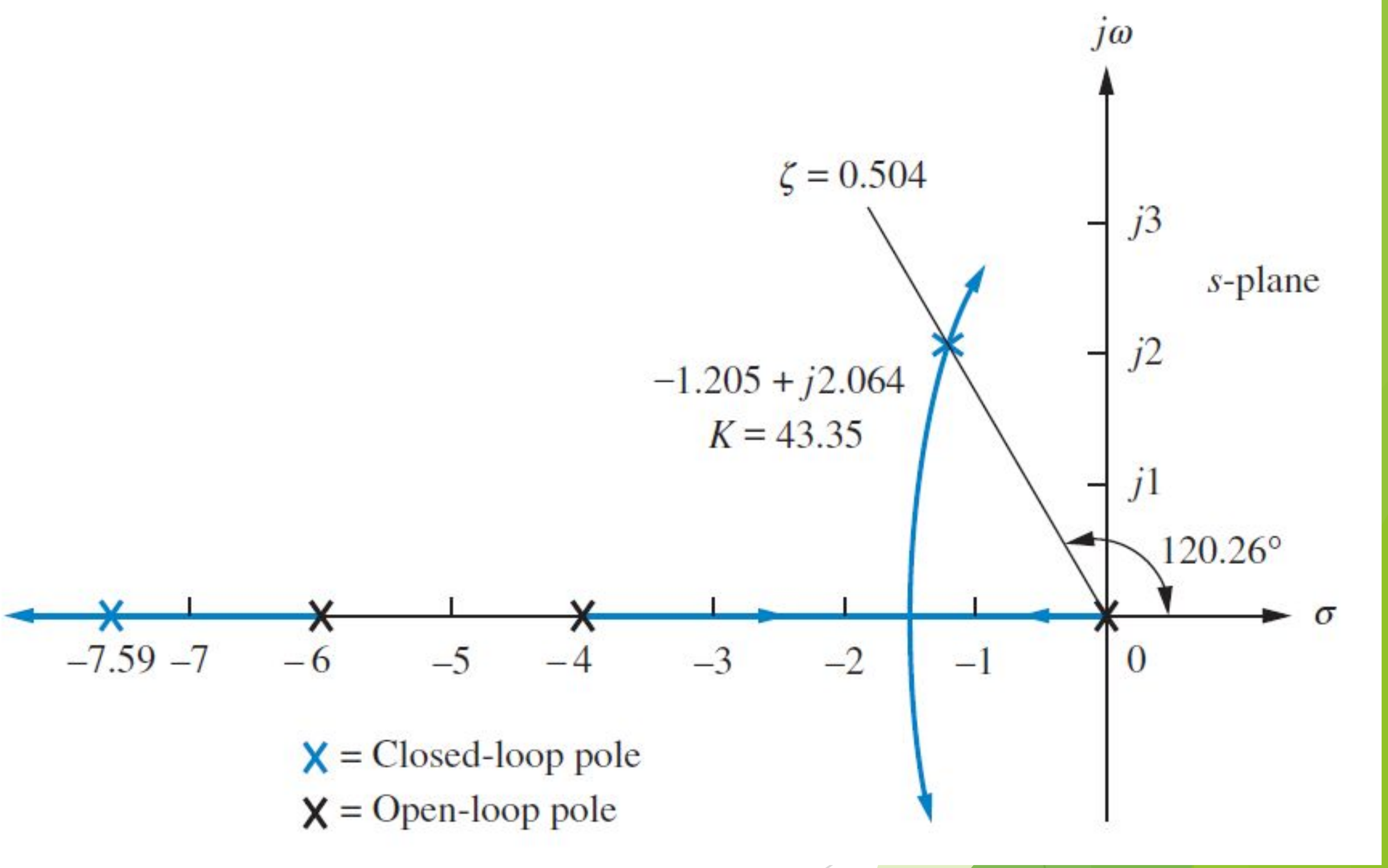


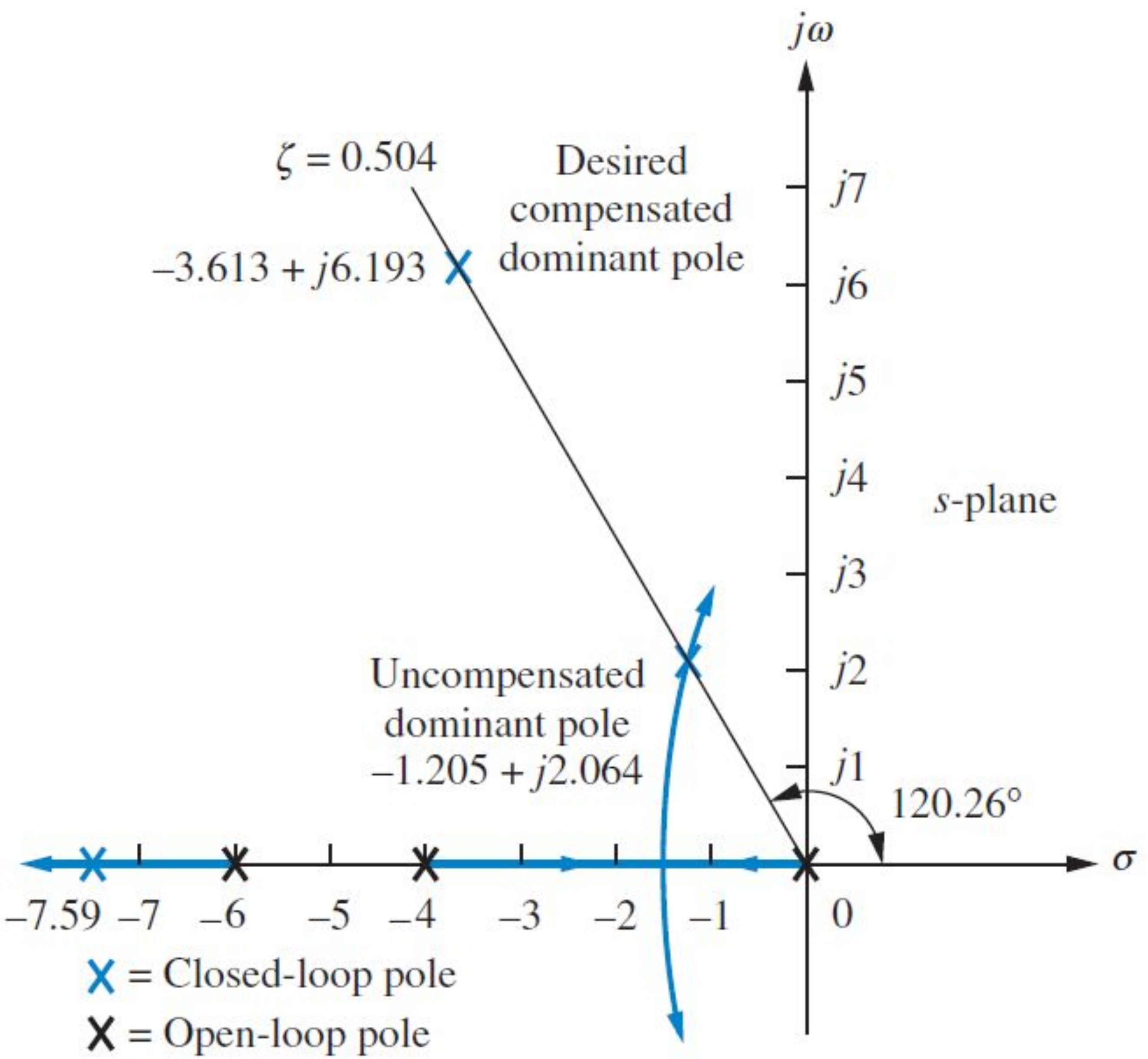
FIGURA 9.17 Sistema de controle com realimentação para o Exemplo 9.3.

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{1.205} = 3.320$$

$$\sigma = \frac{4}{T_s} = \frac{4}{1.107} = 3.613$$

$$\omega_d = 3.613\tan(180^\circ - 120.26^\circ) = 6.193$$



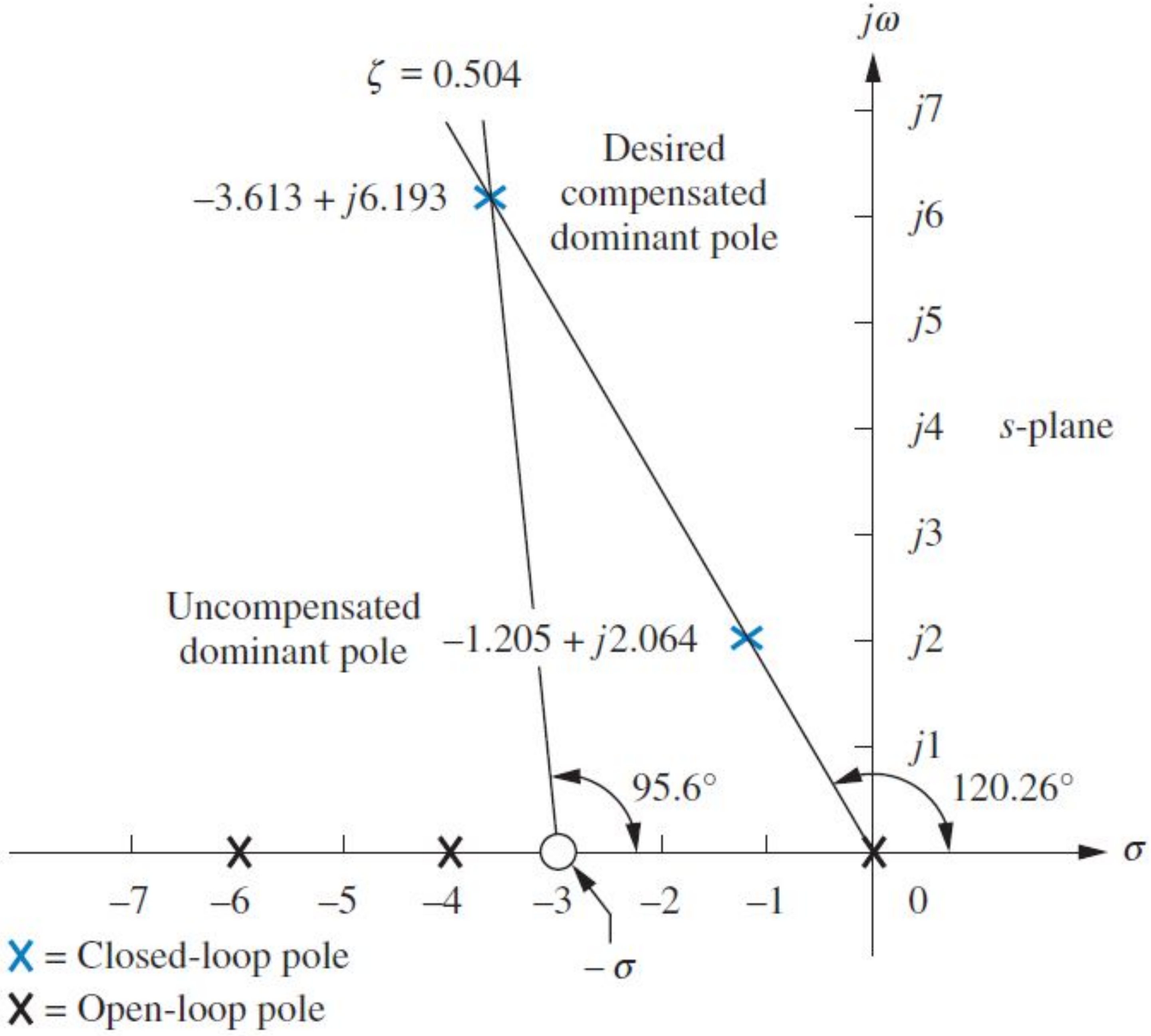


$j\omega$
$\zeta = 0.504$
Desired compensated dominant pole
$-3.613 + j6.193$
$j7$
$j6$
$j5$
$j4$
s-plane
$j3$
$j2$
$j1$
Uncompensated dominant pole
$-1.205 + j2.064$
120.26°
σ
–7.59 –7 –6 –5 –4 –3 –2 –1 0
X = Closed-loop pole
X = Open-loop pole

$$T(s) = \frac{KG(s)}{1 + KG(s)H(s)}$$

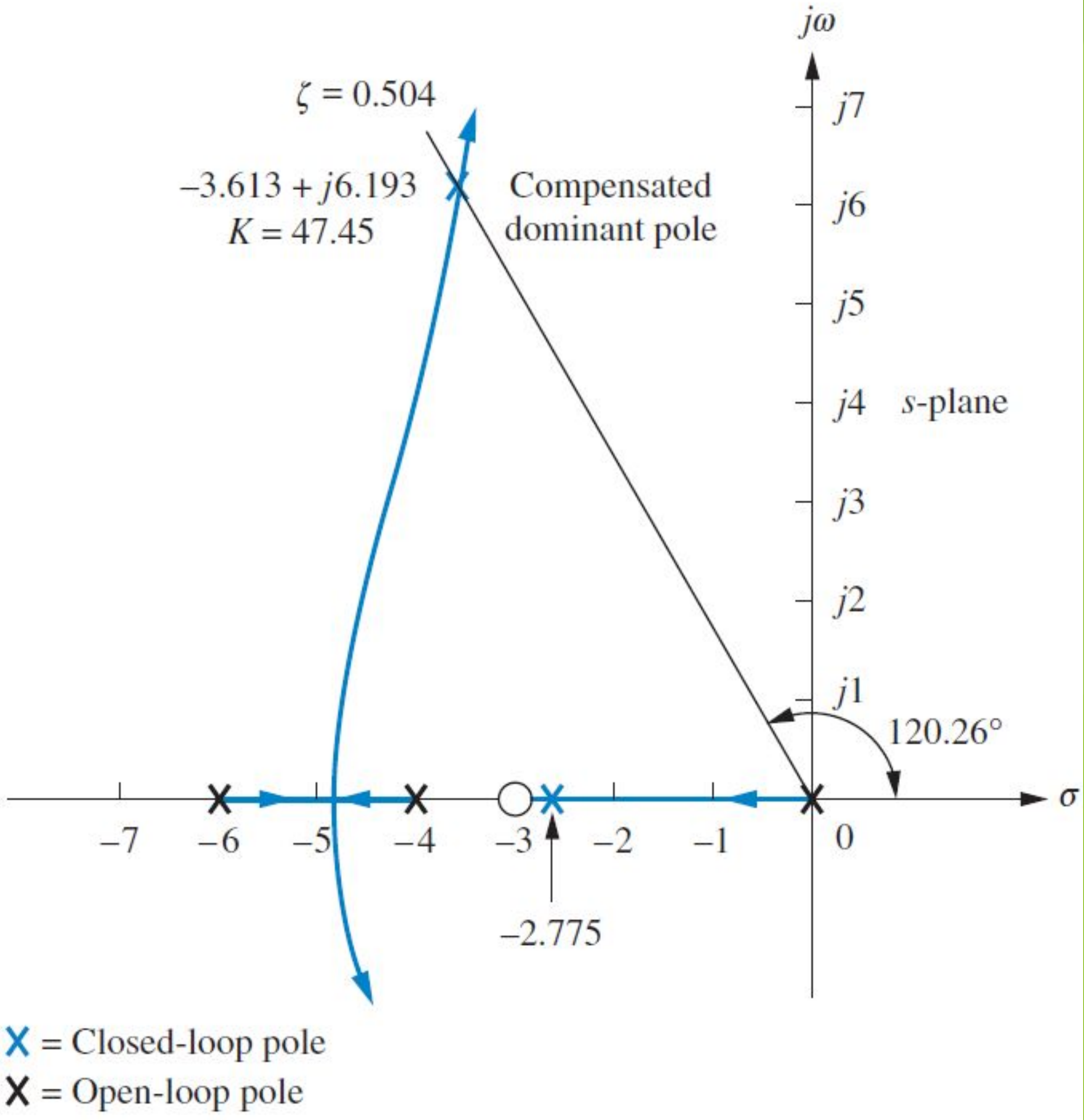
$$\angle KG(s)H(s) = (2k+1)180^\circ$$

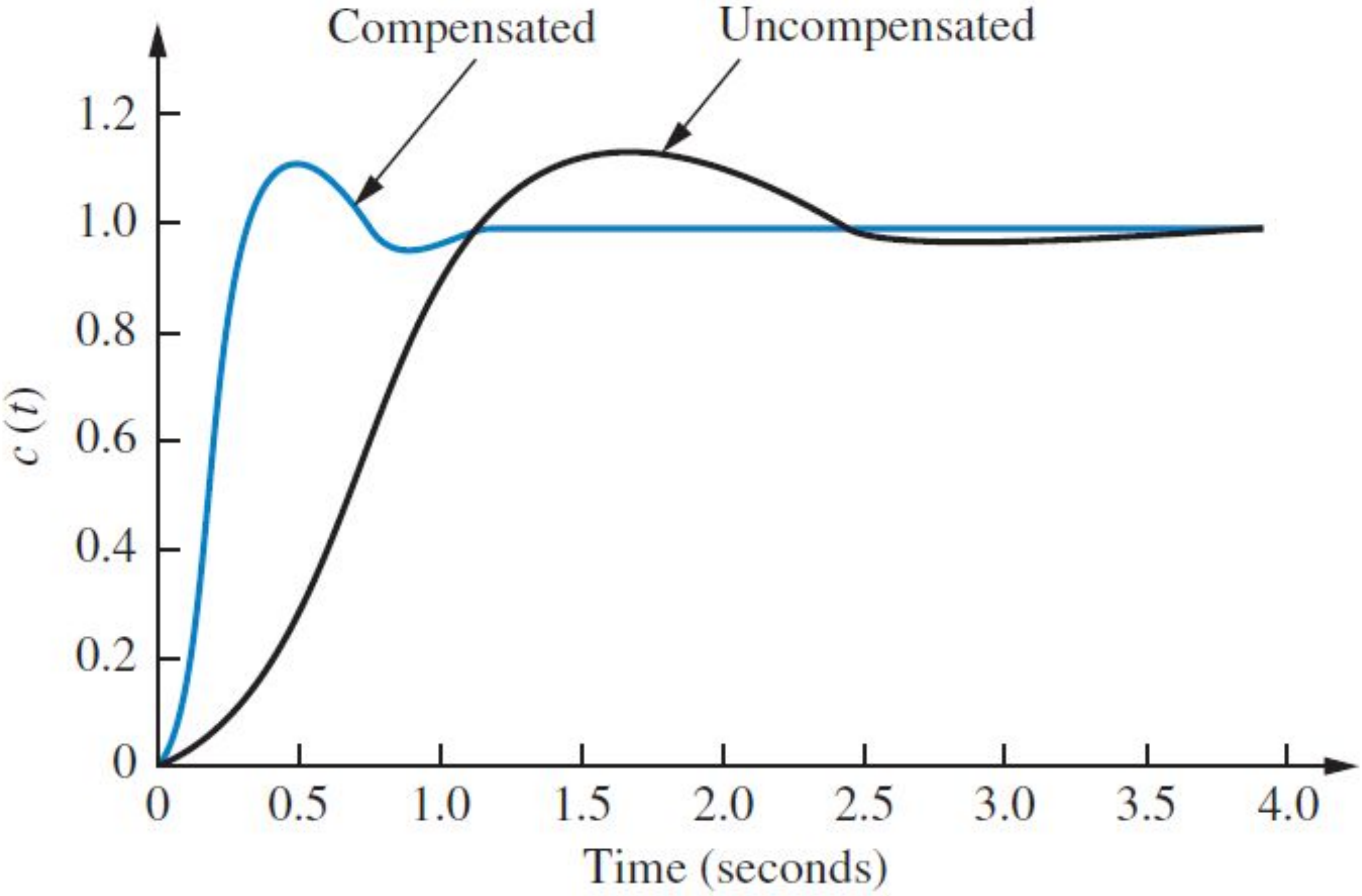
$$\begin{aligned}\theta &= \sum \text{zero angles} - \sum \text{pole angles}\\ &= \sum_{i=1}^{m} \angle(s+z_i) - \sum_{j=1}^{n} \angle(s+p_j)\end{aligned}$$



jω
ζ = 0.504
j7
Desired
compensated
dominant pole
–3.613 + j6.193
j6
j5
j4
s-plane
j3
Uncompensated
dominant pole
–1.205 + j2.064
j2
j1
95.6°
120.26°
σ
–7
–6
–5
–4
–3
–2
–1
0
– σ
X = Closed-loop pole
X = Open-loop pole

$$\frac{6.193}{3.613 - \sigma} = \tan(180^\circ - 95.6^\circ)$$




Compensated
Uncompensated
1.2
1.0
0.8
0.6
0.4
0.2
0
c (t)
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Time (seconds)

$$G_c(s) = K_2 s + K_1 = K_2\left(s + \frac{K_1}{K_2}\right)$$

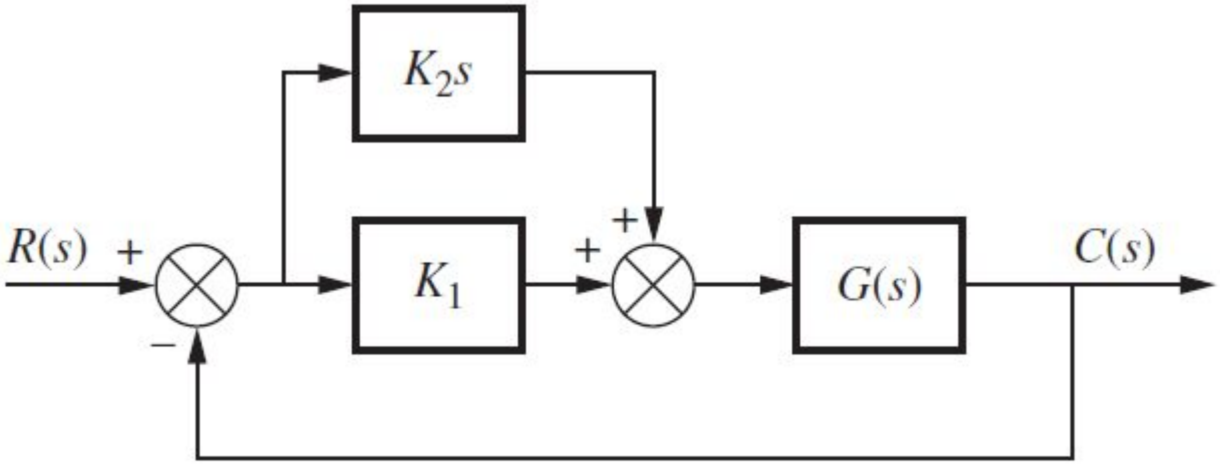
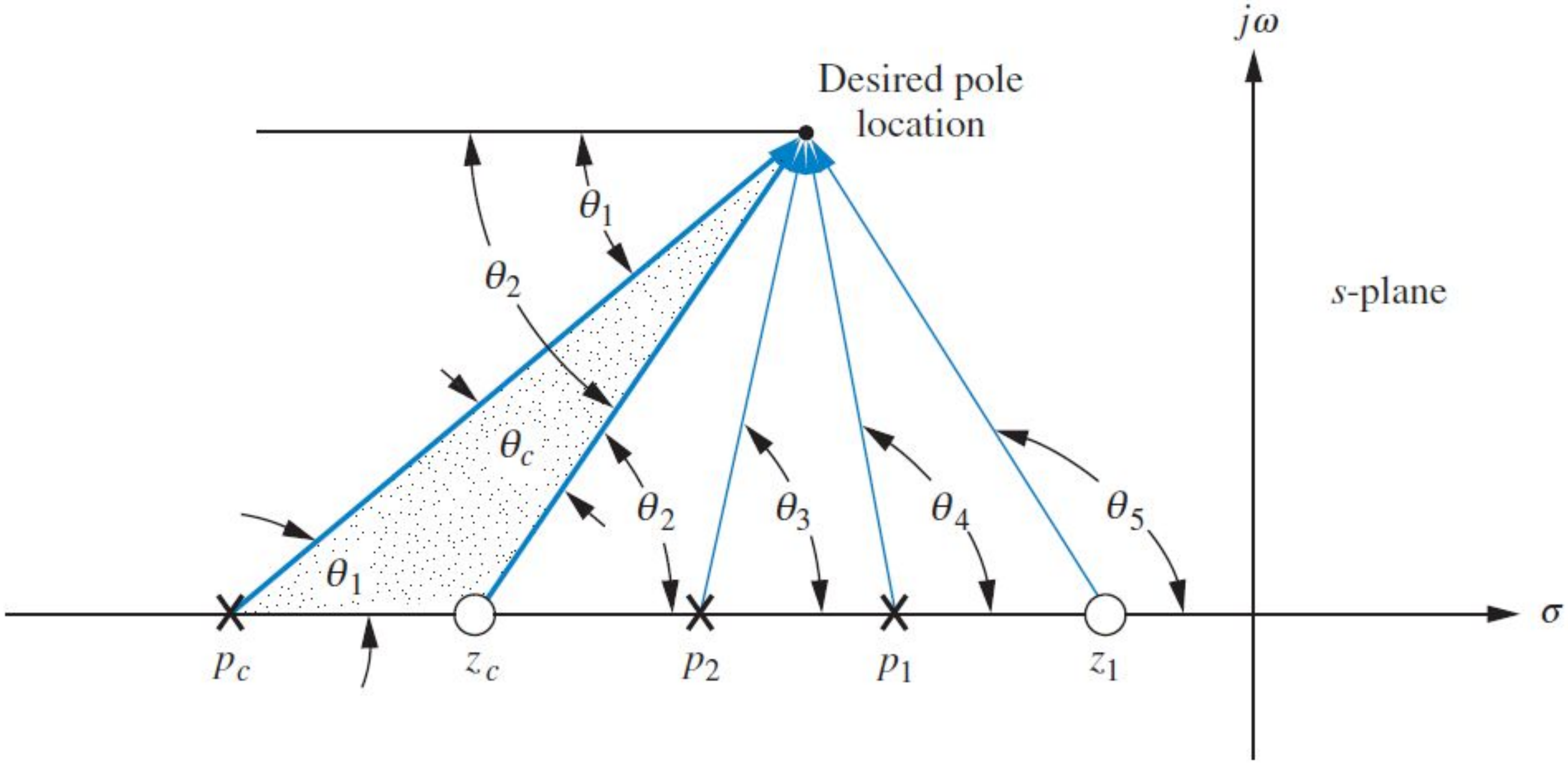


FIGURE 9.23 PD controller

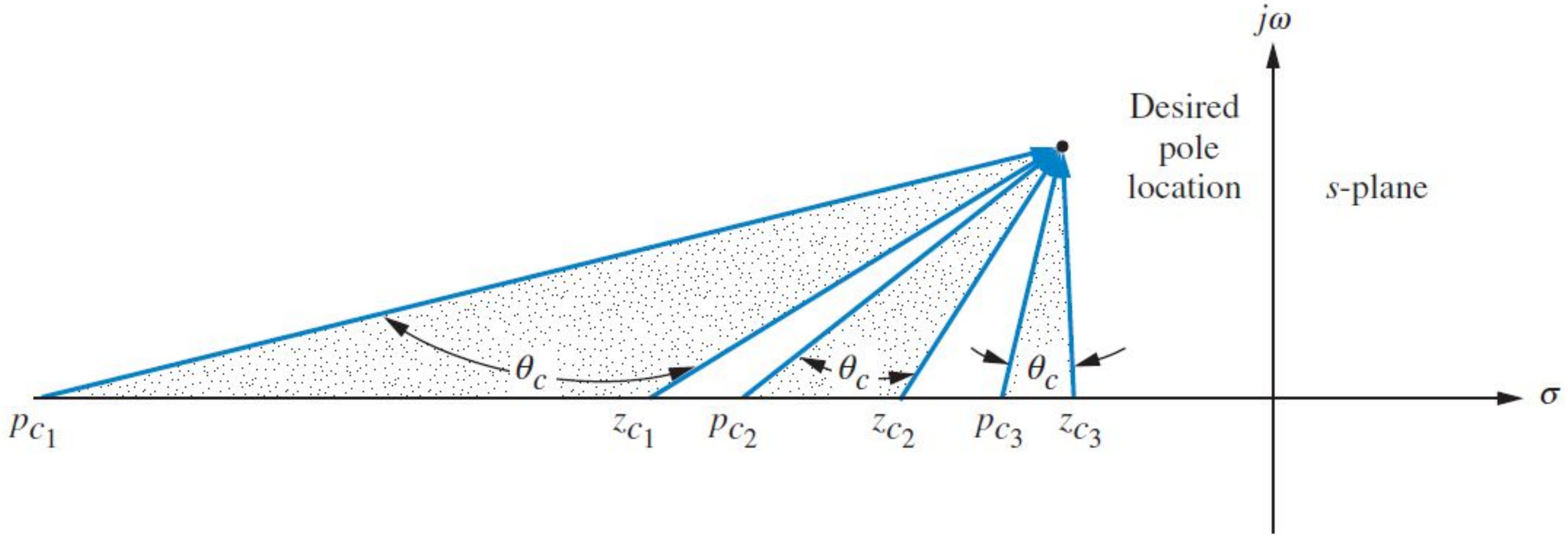
	Uncompensated	Simulation	Compensated	Simulation
Plant and compensator	$\dfrac{K}{s(s+4)(s+6)}$		$\dfrac{K(s+3.006)}{s(s+4)(s+6)}$	
Dominant poles	$-1.205 \pm j2.064$		$-3.613 \pm j6.193$	
K	43.35		47.45	
ζ	0.504		0.504	
ω_n	2.39		7.17	
$\%OS$	16	14.8	16	11.8
T_s	3.320	3.6	1.107	1.2
T_p	1.522	1.7	0.507	0.5
K_v	1.806		5.94	
$e(\infty)$	0.554		0.168	
Third pole	−7.591		−2.775	
Zero	None		−3.006	
Comments	Second-order approx. OK		Pole-zero not canceling	

Compensação de Avanço de Fase



$$\theta_2 - \theta_1 - \theta_3 - \theta_4 + \theta_5 = (2k+1)180^\circ$$

$$(\theta_2 - \theta_1) = \theta_c$$

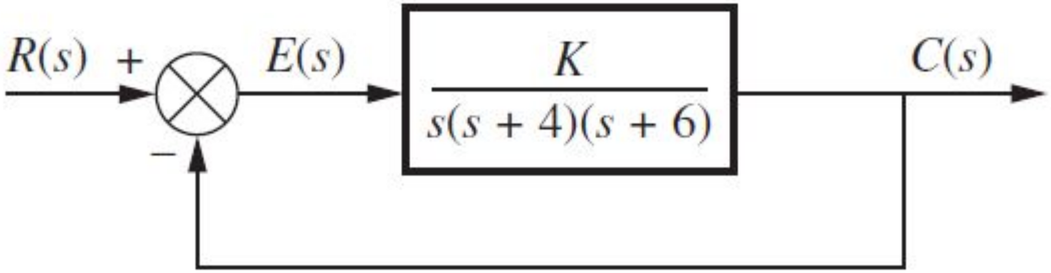


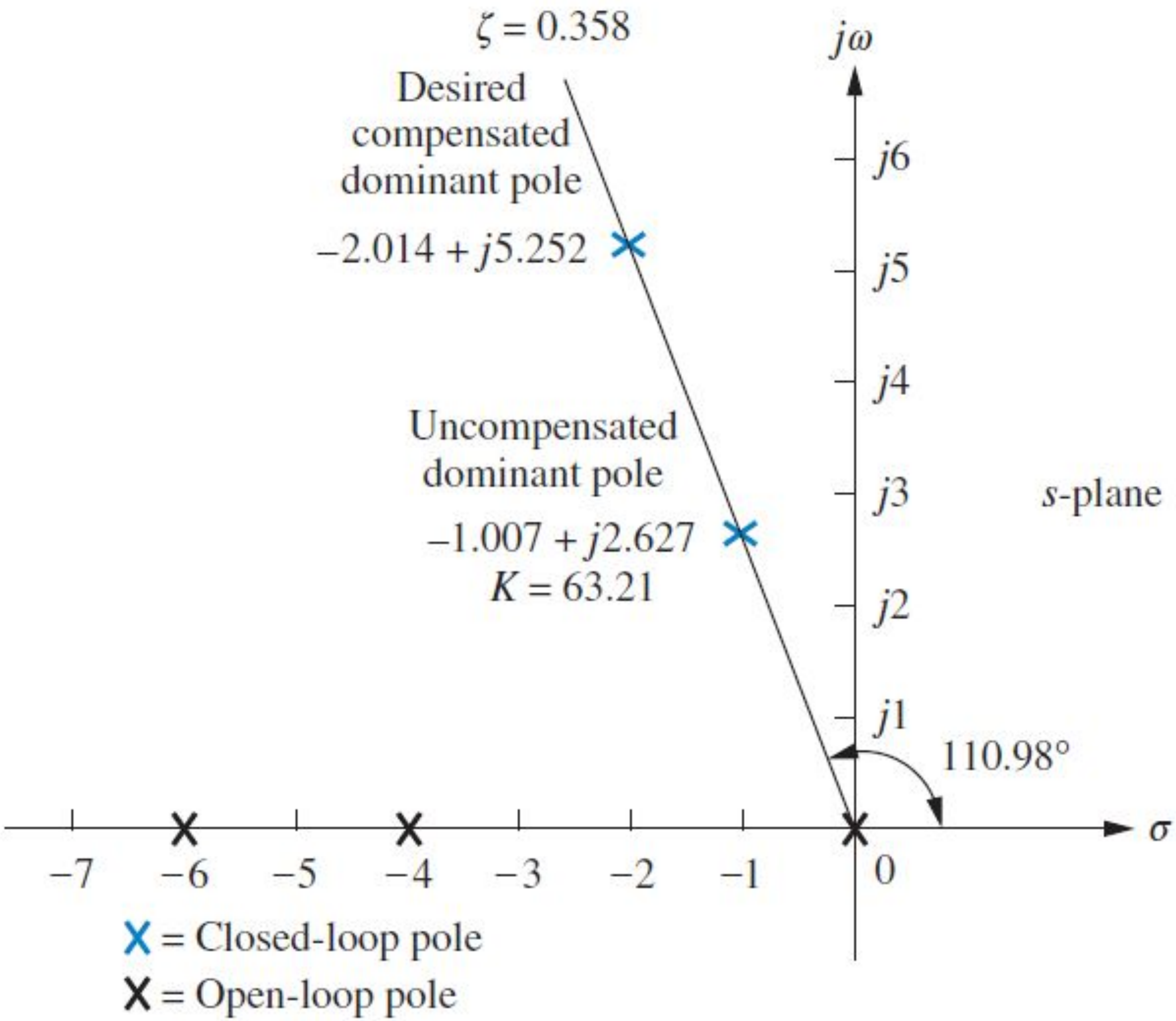
$j\omega$
Desired
pole
location
s-plane
θ_c
θ_c
θ_c
σ
p_{c_1}
z_{c_1}
p_{c_2}
z_{c_2}
p_{c_3}
z_{c_3}

Exemplo 9.4

Projeto de Compensador de Avanço de Fase

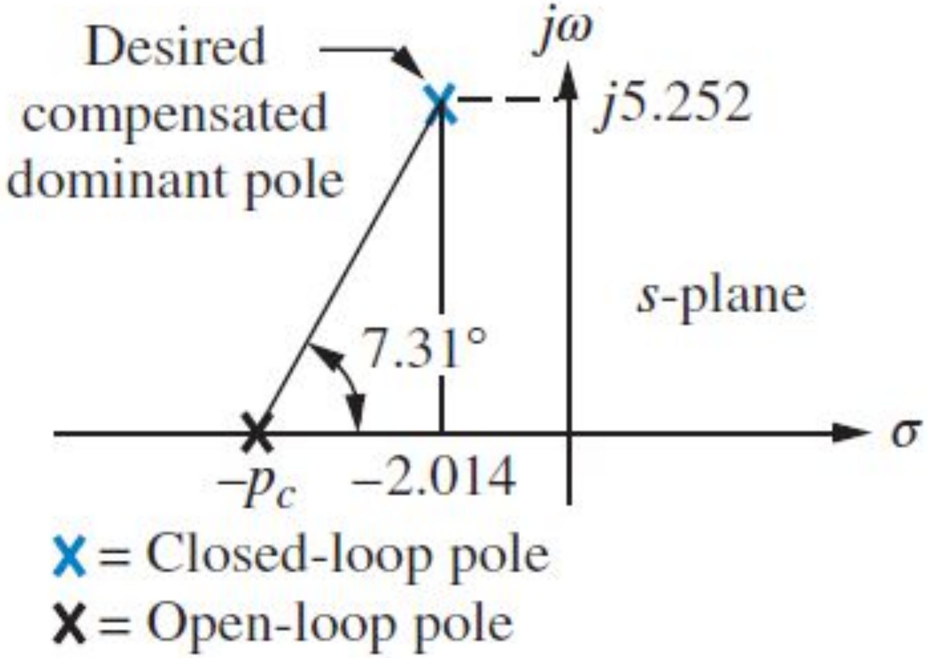
PROBLEMA: Projete três compensadores de avanço de fase para o sistema da Figura 9.17 que irão reduzir o tempo de acomodação por um fator de 2 enquanto mantém 30 % de ultrapassagem. Compare as características do sistema entre os três projetos.

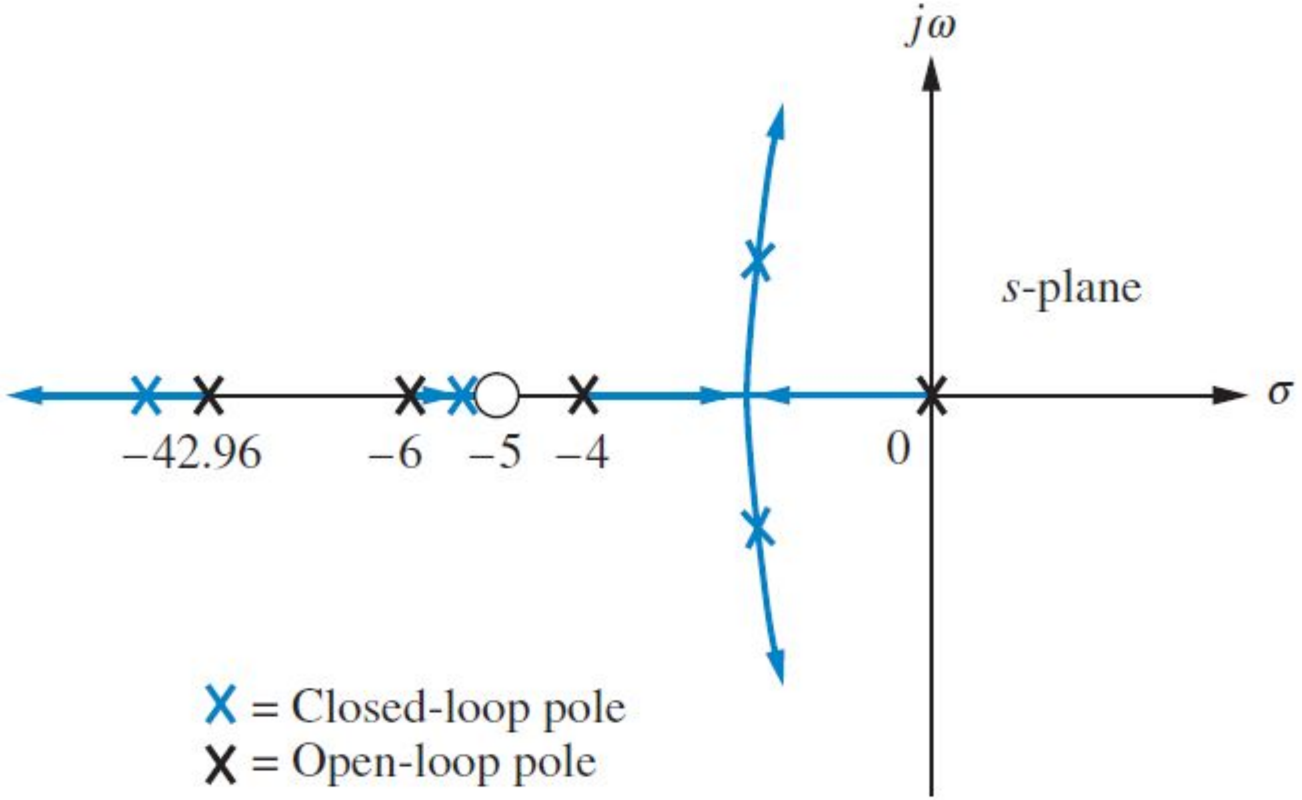




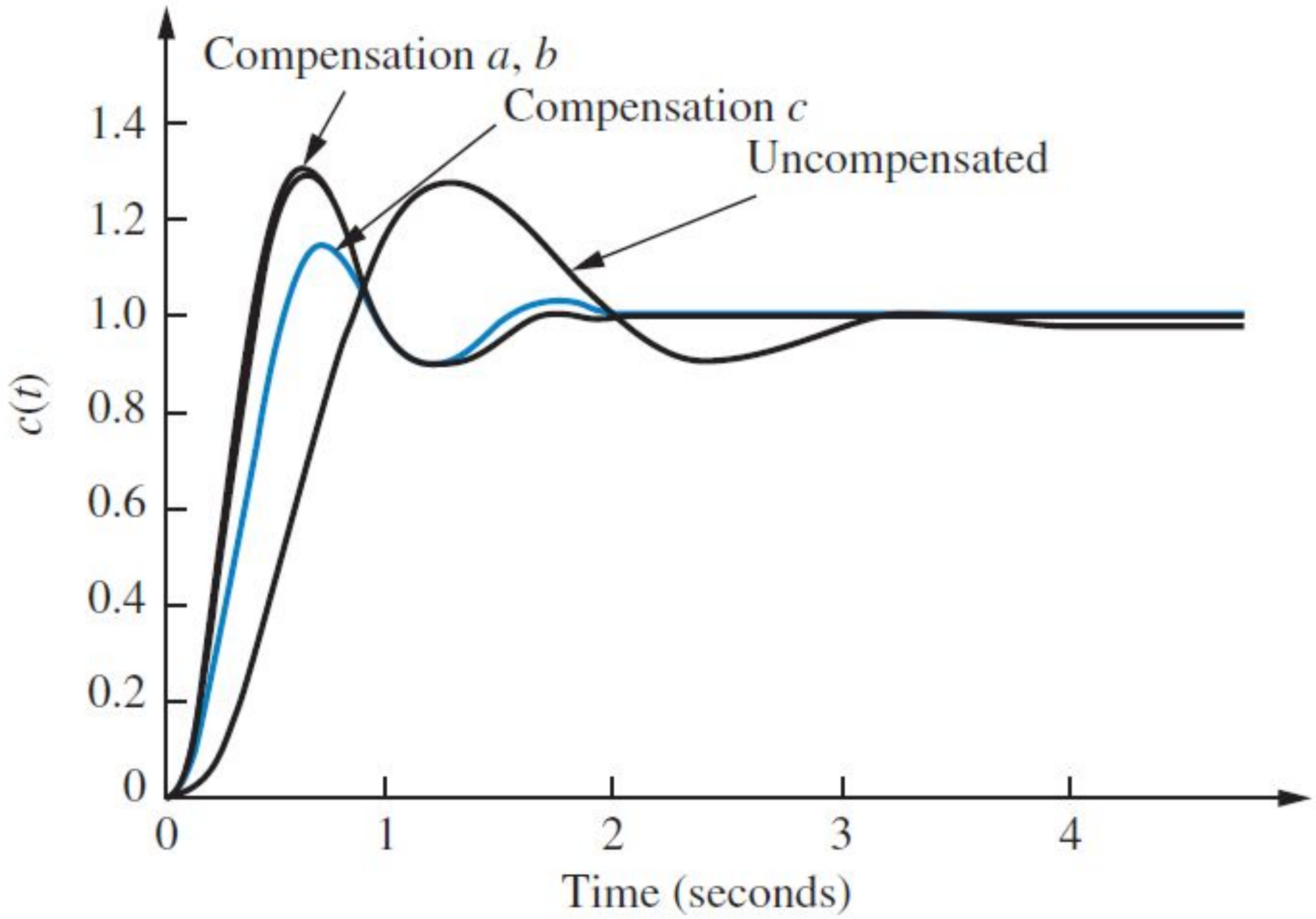
ζ = 0.358
jω
Desired compensated dominant pole
−2.014 + j5.252
j6
j5
j4
Uncompensated dominant pole
−1.007 + j2.627
K = 63.21
j3
s-plane
j2
j1
110.98°
σ
−7
−6
−5
−4
−3
−2
−1
0
= Closed-loop pole
= Open-loop pole

$$\frac{5.252}{p_c - 2.014} = \tan 7.31^\circ \qquad p_c = 42.96$$





	Uncompensated	Compensation a	Compensation b	Compensation c
Plant and compensator	$\frac{K}{s(s+4)(s+6)}$	$\frac{K(s+5)}{s(s+4)(s+6)(s+42.96)}$	$\frac{K(s+4)}{s(s+4)(s+6)(s+20.09)}$	$\frac{K(s+2)}{s(s+4)(s+6)(s+8.971)}$
Dominant poles	$-1.007 \pm j2.627$	$-2.014 \pm j5.252$	$-2.014 \pm j5.252$	$-2.014 \pm j5.252$
K	63.21	1423	698.1	345.6
ζ	0.358	0.358	0.358	0.358
ω_n	2.813	5.625	5.625	5.625
$\%OS^*$	30 (28)	30 (30.7)	30 (28.2)	30 (14.5)
T_s^*	3.972 (4)	1.986 (2)	1.986 (2)	1.986 (1.7)
T_p^*	1.196 (1.3)	0.598 (0.6)	0.598 (0.6)	0.598 (0.7)
K_v	2.634	6.9	5.791	3.21
$e(\infty)$	0.380	0.145	0.173	0.312
Other poles	−7.986	−43.8, −5.134	−22.06	−13.3, −1.642
Zero	None	−5	None	−2
Comments	Second-order approx. OK	Second-order approx. OK	Second-order approx. OK	No pole-zero cancellation


Compensation a, b
Compensation c
Uncompensated
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
c(t)
0
1
2
3
4
Time (seconds)

Exercício 9.2

PROBLEMA: Um sistema com realimentação unitária com a função de transferência à frente

$$G(s) = \frac{K}{s(s+7)}$$

está operando com uma resposta ao degrau em malha fechada que tem 15 % de ultrapassagem. Faça o seguinte:

a. Calcule o tempo de acomodação.

b. Projete um compensador de avanço de fase para reduzir o tempo de acomodação por um fator de três. Escolha o zero do compensador em –10.

RESPOSTAS:

a. $T_s = 1{,}143$ s

b. $G_{\text{avanço}}(s) = \dfrac{s+10}{s+25{,}52} \qquad K = 476{,}3$

A solução completa está disponível no GEN-IO, Ambiente de Aprendizagem do Grupo GEN.