Convertendo do Espaço de Estados para uma Função de Transferência

Fundamentos de Controle

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

$$s\mathbf{X}(s) = \mathbf{A}\mathbf{X}(s) + \mathbf{B}\mathbf{U}(s)$$

$$\mathbf{Y}(s) = \mathbf{C}\mathbf{X}(s) + \mathbf{D}\mathbf{U}(s)$$

$$(s\mathbf{I} - \mathbf{A})\mathbf{X}(s) = \mathbf{B}\mathbf{U}(s)$$

$$\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\mathbf{U}(s)$$

$$\mathbf{Y}(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\mathbf{U}(s) + \mathbf{D}\mathbf{U}(s) = \left[\mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}\right]\mathbf{U}(s)$$

$$\boxed{T(s) = \frac{Y(s)}{U(s)} = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}}$$

Exemplo 3.6

Representação no Espaço de Estados para Função de Transferência

PROBLEMA: Dado o sistema definido pelas Equações (3.74), obtenha a função de transferência $T(s) = Y(s)/U(s)$, em que $U(s)$ é a entrada e $Y(s)$ é a saída.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}u \qquad (3.74a)$$

$$y = [1 \quad 0 \quad 0]\mathbf{x} \qquad (3.74b)$$

$$(s\mathbf{I}-\mathbf{A}) = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 2 & s+3 \end{bmatrix}$$

$$(s\mathbf{I}-\mathbf{A})^{-1} = \frac{\operatorname{adj}(s\mathbf{I}-\mathbf{A})}{\det(s\mathbf{I}-\mathbf{A})} = \frac{\begin{bmatrix} (s^2+3s+2) & s+3 & 1 \\ -1 & s(s+3) & s \\ -s & -(2s+1) & s^2 \end{bmatrix}}{s^3+3s^2+2s+1}$$

$$\mathbf{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\operatorname{cof}(A) = \begin{bmatrix} +\begin{vmatrix} e & f \\ h & i \end{vmatrix} & -\begin{vmatrix} d & f \\ g & i \end{vmatrix} & +\begin{vmatrix} d & e \\ g & h \end{vmatrix} \\ -\begin{vmatrix} b & c \\ h & i \end{vmatrix} & +\begin{vmatrix} a & c \\ g & i \end{vmatrix} & -\begin{vmatrix} a & b \\ g & h \end{vmatrix} \\ +\begin{vmatrix} b & c \\ e & f \end{vmatrix} & -\begin{vmatrix} a & c \\ d & f \end{vmatrix} & +\begin{vmatrix} a & b \\ d & e \end{vmatrix} \end{bmatrix}$$

$$\operatorname{adj}(A) = \begin{bmatrix} +\begin{vmatrix} e & f \\ h & i \end{vmatrix} & -\begin{vmatrix} b & c \\ h & i \end{vmatrix} & +\begin{vmatrix} b & c \\ e & f \end{vmatrix} \\ -\begin{vmatrix} d & f \\ g & i \end{vmatrix} & +\begin{vmatrix} a & c \\ g & i \end{vmatrix} & -\begin{vmatrix} a & c \\ d & f \end{vmatrix} \\ +\begin{vmatrix} d & e \\ g & h \end{vmatrix} & -\begin{vmatrix} a & b \\ g & h \end{vmatrix} & +\begin{vmatrix} a & b \\ d & e \end{vmatrix} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{D} = 0$$

$$T(s) = \frac{10(s^2 + 3s + 2)}{s^3 + 3s^2 + 2s + 1}$$

```
syms s                              % Construct symbolic object for
                                    % frequency variable 's'.
A=[0 1 0;0 0 1;-1 -2 -3];           % Create matrix A.
B=[10;0;0];                         % Create vector B.
C=[1 0 0];                          % Create vector C.
D=0;                                % Create D.
I=[1 0 0;0 1 0;0 0 1];              % Create identity matrix.
'T(s)'                              % Display label.
T=C*((s*I-A)^-1)*B+D;               % Find transfer function.
pretty(T)                           % Pretty print transfer function.
pause
```

Exercício 3.4

PROBLEMA: Converta as equações de estado e de saída mostradas nas Equações (3.78) em uma função de transferência.

$$\mathbf{x} = \begin{bmatrix} -4 & -1{,}5 \\ 4 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 2 \\ 0 \end{bmatrix}u(t) \tag{3.78a}$$

$$y = \begin{bmatrix} 1{,}5 & 0{,}625 \end{bmatrix}\mathbf{x} \tag{3.78b}$$

RESPOSTA:

$$G(s) = \frac{3s + 5}{s^2 + 4s + 6}$$

```
A=[-4 -1.5;4 0];
B=[2 0]';
C=[1.5 0.625];
D=0;
T=ss(A,B,C,D);
T=tf(T)
```