### simulated\_data

#### 2024-01-13

#### Load libraries

```
shhh = function(lib_name){ # It's a library, so shhh!
  suppressWarnings(suppressMessages(require(lib_name, character.only = TRUE)))
shhh("tidyverse")
shhh("ACutils")
shhh("mvtnorm")
shhh("salso")
shhh("FGM")
shhh("gmp")
shhh("mcclust")
shhh("mcclust.ext")
shhh("logr")
shhh("tidygraph")
shhh("ggraph")
shhh("igraph")
shhh("Rcpp")
shhh("RcppArmadillo")
shhh("RcppEigen")
## Load custom functions
source("utility_functions.R");
source("bulky_functions.R");
source("data_generation.R")
sourceCpp("wade.cpp")
Rcpp::sourceCpp('UpdateParamsGSL.cpp')
library('RcppGSL')
library(fda)
library(tidyverse)
library(coda)
library(lattice)
```

#### Simulated data

```
# Import simulated data
BaseMat = as.matrix(read_csv('BaseMat.csv'))
y_hat_true = as.matrix(read_csv('y_hat_true.csv'))
```

```
beta_true = as.matrix(read_csv('beta_true.csv'))
mu_true = as.matrix(read_csv('mu_true.csv'))

n <- dim(y_hat_true)[1]
r <- dim(y_hat_true)[2]
p <- dim(BaseMat)[2]</pre>
```

#### Initialization

```
# Define the starting matrix with error
# Beta = matrix(rnorm(n = p*n), nrow = p, ncol = n)
Beta = t(beta_true)
# Define the starting value of mu with error
# mu = rnorm(n=p)
mu = mu_true
# Fix tau_eps (squared)
tau_eps = 100
\# Define the precision matrix K
K = rWishart(n = 1, df = p+10, Sigma = diag(p))
K = K[,,1]
# ACutils::ACheatmap(K,center_value = NULL, remove_diag = T)
tbase_base = t(BaseMat)%*%BaseMat # p x p (phi_t * phi)
tbase_data = t(BaseMat)%*%t(y_hat_true) # p x n (phi_t * Y_t)
Sdata = sum(diag(y_hat_true%*%t(y_hat_true))) # inefficient calculation ((2b + Sdata) è il b di tau_eps
# Set True binary flag used to update values
Update_Beta <- FALSE</pre>
Update_Mu <- FALSE</pre>
Update_Tau <- FALSE</pre>
a_tau_eps <- 2000
b_tau_eps <- 2
sigma_mu <- 100
# Define variance of the Beta
beta_sig2 = 0.2
# Compute graph density
graph_density = 0.3
# Set the number of iterations
niter <- 10000
```

```
# Create a list for chains
chains <- list(</pre>
  Beta = vector("list", length = niter),
  mu = vector("list", length = niter),
 tau_eps = vector("list", length = niter),
 K = vector("list", length = niter),
 G = vector("list", length = niter),
 z = vector("list", length = niter),
 rho = vector("list", length = niter),
 time = vector("list", length = niter)
simKG <- readRDS("simKG.rds")</pre>
# Initialization of the chains
chains$Beta[[1]] <- Beta</pre>
chains$mu[[1]] <- mu</pre>
chains$tau_eps[[1]] <- tau_eps</pre>
chains$K[[1]] <- simKG$Prec</pre>
chains$G[[1]] <- simKG$Graph</pre>
chains$z[[1]] <- rep(1,p)
chains$time <- 0</pre>
chains$rho[[1]] <- p</pre>
sigma <-0.5
theta<-1
weights_a <- rep(1,p-1)</pre>
weights_d <- rep(1,p-1)</pre>
total_weights <- 0</pre>
total_K = simKG$Prec
total_graphs = simKG$Graph
graph_start = simKG$Graph
```

### Gibbs sampler

```
for(s in 2:niter) {
  fit = UpdateParamsGSL(
    chains$Beta[[s-1]],
    chains$mu[[s-1]],
    chains $tau_eps [[s-1]],
    chainsK[[s-1]],
    tbase_base,
    tbase_data,
    Sdata,
    a_tau_eps,
    b_tau_eps,
    sigma_mu,
    r,
    Update_Beta,
    Update Mu,
    Update_Tau
```

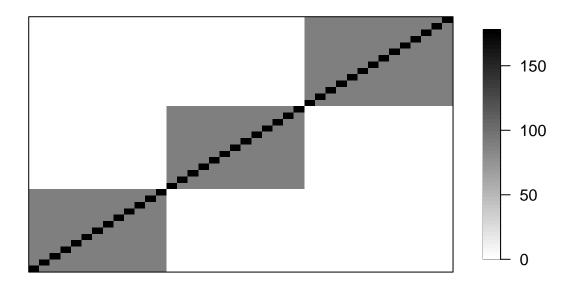
```
# Save Beta
chains$Beta[[s]] <- fit$Beta</pre>
# Save mu
chains$mu[[s]] <- fit$mu</pre>
# Save tau
chains$tau_eps[[s]] <- fit$tau_eps</pre>
# Set options for a single iteration of the Gibbs_sampler
options = set_options(
 sigma_prior_0=sigma,
 sigma_prior_parameters=list("a"=1,"b"=1,"c"=1,"d"=1),
 theta_prior_0=theta,
 theta_prior_parameters=list("c"=1,"d"=1),
 rho0=chains$rho[[s-1]],
 weights_a0=weights_a,
 weights_d0=weights_d,
 total_weights0=total_weights,
 total_K0 = total_K,
 total_graphs0 = total_graphs,
 graph = graph_start,
 alpha_target=0.234,
 beta_mu=graph_density, # expected value beta distr of the graph
                          # var beta distr del grafo, fra 0 e 0.25
 beta_sig2=beta_sig2,
 d=3,
                           # param della G wishart (default 3)
 alpha_add=0.5,
 adaptation_step=1/(p*1000),
 update_sigma_prior=FALSE,
 update_theta_prior=FALSE,
 update_weights=FALSE,
 update_partition=TRUE,
 update_graph=FALSE,
 perform_shuffle=TRUE
 )
# Run an iteration of the Gibbs Sampler
res <- Gibbs_sampler(</pre>
 data = t(fit$Beta - fit$mu),
 niter = 1, # niter finali, già tolto il burn in
 nburn = 0,
 thin = 1,
 options = options,
 seed = 123456,
 print = FALSE
z = do.call(rbind, lapply(res$rho, rho_to_z))
# Save rho
chains$rho[[s]] <- res$rho[[1]]</pre>
```

```
# Save K
chains$K[[s]] <- res$K [[1]]
# Save G
chains$G[[s]] <- res$G [[1]]
# Save z
chains$z[[s]] <- z
# Save times for each K
chains$time[[s]] <- res$execution_time</pre>
# Update quantities for the next iteration
# weights
weights_a <- res$weights_a[[1]]</pre>
weights_d <- res$weights_d[[1]]</pre>
total_weights <- res$total_weights</pre>
total_K <- res$total_K[[1]]</pre>
total_graphs <- res$total_graphs[[1]]</pre>
# graph
graph_start = res$bdgraph_start
```

#### Final plots: Plot of the obtained K\_fin

```
# Set the number burn in iterations
burn_in <- 1000 # TODO: decide which value to fix</pre>
# Update K as the sum of times*K/tot_time
sum <- 0
for(i in (burn_in+1):niter){
  sum <- sum + chains$time[[i]]*chains$K[[i]]</pre>
K_fin <- sum/sum(chains$time)</pre>
# Plot obtained K_fin
ACutils::ACheatmap(
 K_fin,
 use_x11_device = F,
 horizontal = F,
 main = "Estimated Precision matrix",
 center_value = NULL,
 col.upper = "black",
 col.center = "grey50",
  col.lower = "white"
```

#### **Estimated Precision matrix**



### Useful plots: 1. Plot smoothed curves

```
# Compute the mean of Beta in order to have data_post
sum_Beta <- matrix(0, p, n)
for(i in (burn_in+1):niter){
    sum_Beta <- sum_Beta + chains$Beta[[i]]
}
mean_Beta <- sum_Beta/(niter-burn_in)
data_post <- BaseMat %*% mean_Beta

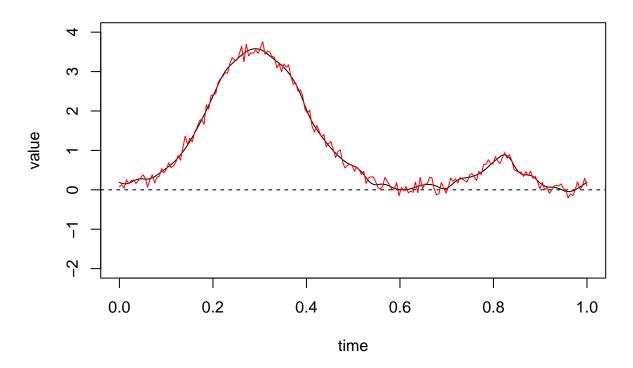
# Compute the x value, create the basis and the functional object
x <- seq(0, 1, length.out=r)
basis <- create.bspline.basis(rangeval=range(x), nbasis=40, norder=3)
data.fd <- Data2fd(y = data_post, argvals = x, basisobj = basis)

# Plot smoothed curves
plot.fd(data.fd[1,], main="smoothed curves", ylim=c(-2,4))

## [1] "done"

# plot(x, data_post[,1], type='l', ylim=c(-2,4))
lines(x,y_hat_true[1,], main="smoothed curves", col='red')</pre>
```

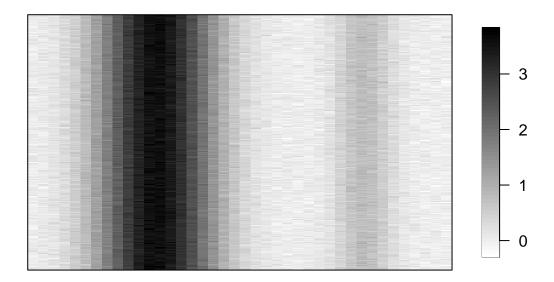
#### smoothed curves



# Useful plots: 2. Plot of the final Beta matrix

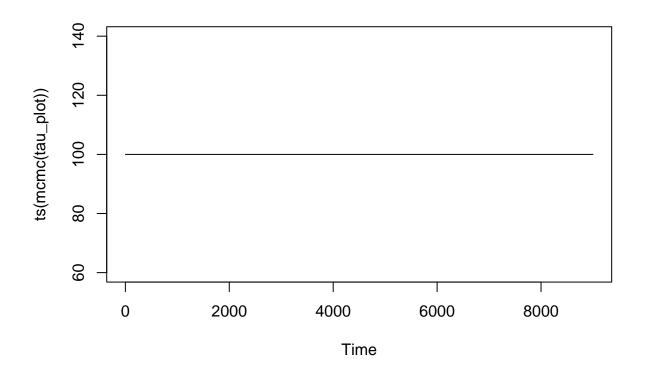
```
ACutils::ACheatmap(
  chains$Beta[[niter]],
  use_x11_device = F,
  horizontal = F,
  main = "Estimated Beta matrix",
  center_value = NULL,
  col.upper = "black",
  col.center = "grey50",
  col.lower = "white"
)
```

### **Estimated Beta matrix**



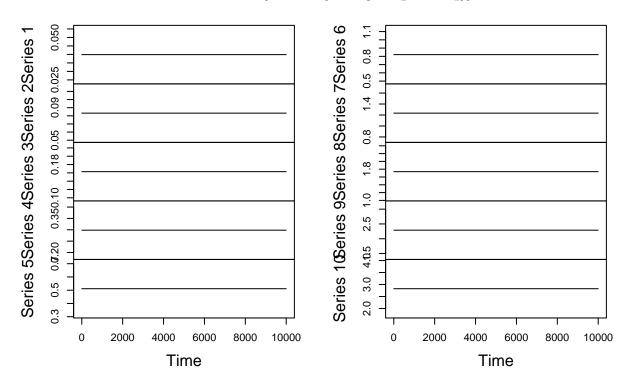
# Useful plots: 2. Traceplots (tau\_eps, mu)

```
tau_plot <- as.vector(chains$tau_eps)
tau_plot <- tau_plot[(burn_in+1):niter]
plot(ts(mcmc(tau_plot)))</pre>
```



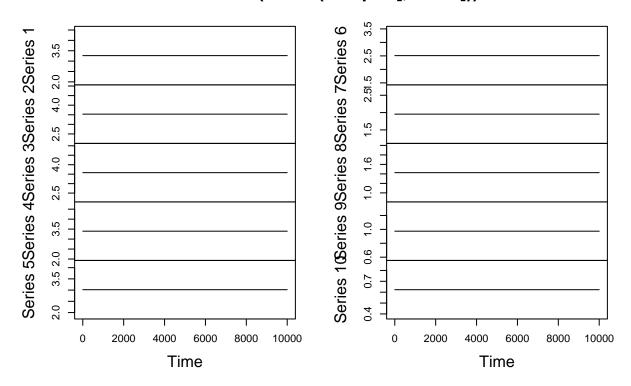
```
mu_plot <- matrix(0, niter, p)
for(i in 1:niter){
   mu_plot[i, ] <- chains$mu[[i]]
}
plot(ts(mcmc(mu_plot[, 1:10])))</pre>
```

# ts(mcmc(mu\_plot[, 1:10]))



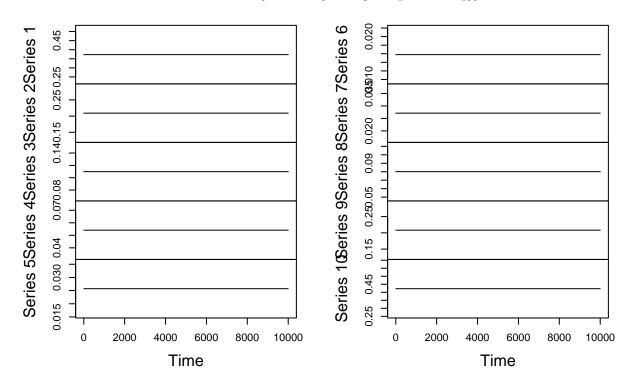
plot(ts(mcmc(mu\_plot[, 11:20])))

# ts(mcmc(mu\_plot[, 11:20]))



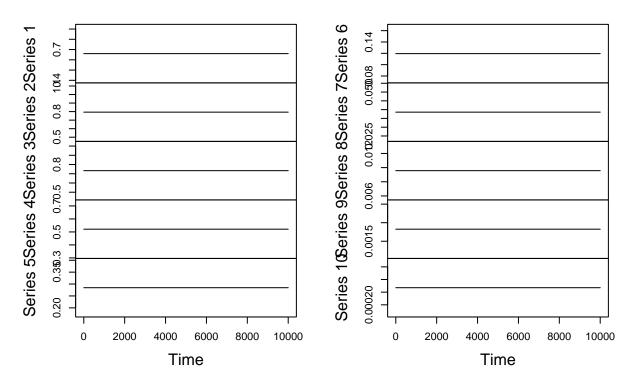
plot(ts(mcmc(mu\_plot[, 21:30])))

# ts(mcmc(mu\_plot[, 21:30]))

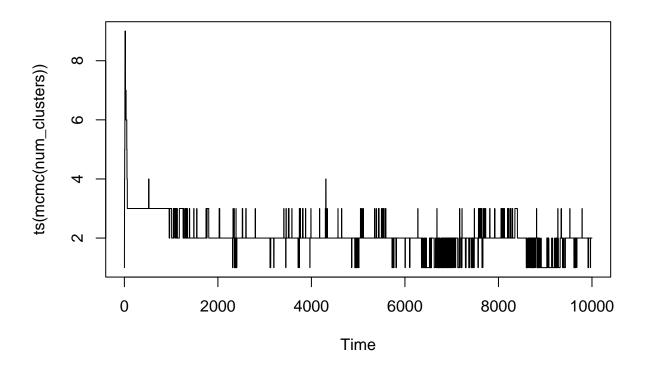


plot(ts(mcmc(mu\_plot[, 31:40])))

# ts(mcmc(mu\_plot[, 31:40]))



```
num_clusters = do.call(rbind, lapply(chains$rho, length))
num_clusters = as.vector(num_clusters)
plot(ts(mcmc(num_clusters)))
```

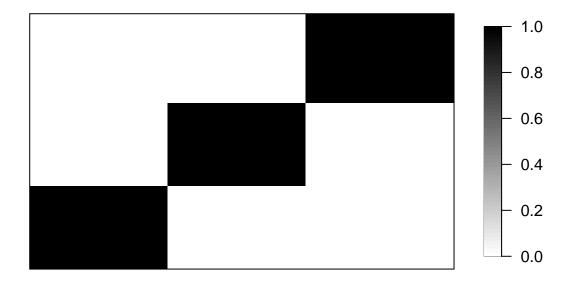


## Useful plots: 3. Plot of the plinks matrix

```
last_plinks = chains$G[[niter]]

ACutils::ACheatmap(
   last_plinks,
   use_x11_device = F,
   horizontal = F,
   main = "Estimated plinks matrix",
   center_value = NULL,
   col.upper = "black",
   col.center = "grey50",
   col.lower = "white"
)
```

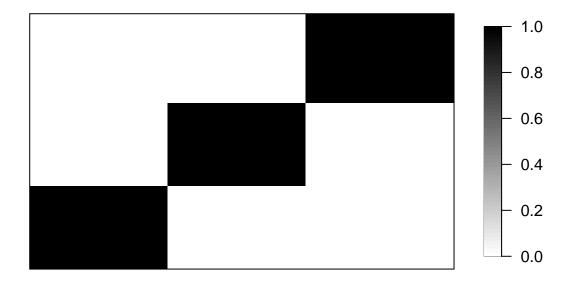
## **Estimated plinks matrix**



```
# Criterion 1 to select the threshold (should not work very well) and assign final graph
threshold = 0.5
G_est <- matrix(0,p,p)
G_est[which(last_plinks>threshold)] = 1

ACutils::ACheatmap(
    G_est,
        use_x11_device = F,
        horizontal = F,
        main = "Estimated Graph",
        center_value = NULL,
        col.upper = "black",
        col.center = "grey50",
        col.lower = "white"
)
```

### **Estimated Graph**



```
# Criterion 2 to select the threshold
bfdr_select = BFDR_selection(last_plinks, tol = seq(0.1, 1, by = 0.001))
# Inspect the threshold and assign final graph
bfdr_select$best_treshold
## [1] 1
G_est = bfdr_select$best_truncated_graph
ACutils::ACheatmap(
 G_est,
 use_x11_device = F,
 horizontal = F,
 main = "Estimated Graph",
 center_value = NULL,
 col.upper = "black",
 col.center = "grey50",
  col.lower = "white"
z <- matrix(0, niter-burn_in-1, p)</pre>
for(i in 1:(niter-burn_in-1)){
```

z[i,] <- as.vector(chains\$z[[i+burn\_in]])</pre>

```
z_true <- as.matrix(c(rep(1,13), rep(2,13), rep(3,14)))</pre>
\#z\_true \leftarrow as.matrix(c(rep(1,6), rep(2,6), rep(3,8)))
rand_index = apply(z, 1, mcclust::arandi, z_true)
# plotting the traceplot of the index
plot(
  x = seq_along(rand_index),
  y = rand_index,
  type = "n",
  xlab = "Iterations",
  ylab = "Rand Index",
  main = paste(
    "Rand Index - Traceplot\n",
    "Last:",
    round(tail(rand_index, n=1), 3),
    "- Mean:",
    round(mean(rand_index), 2)
  )
lines(x = seq_along(rand_index), y = rand_index)
abline(h = 1, col = "red", lwd = 4)
```

### Rand Index – Traceplot Last: 0.508 – Mean: 0.46

