

MEASUREMENT AND ANALYSIS OF DATA

1

Accuracy

close to the true value

Precision

consistent, little spread about the mean

Accuracy and precision

- * Random, systematic, and zero errors

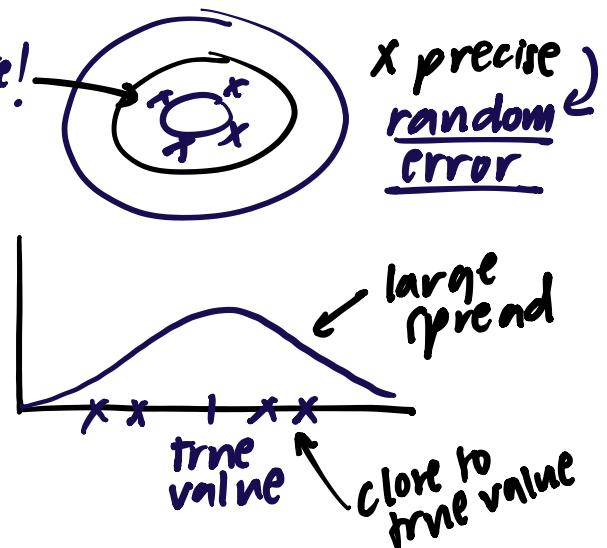
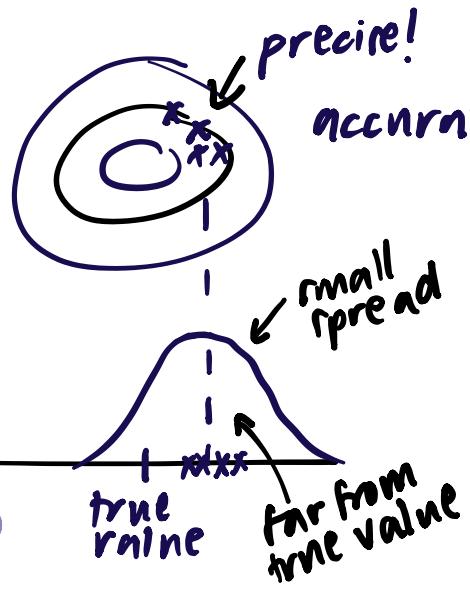
- * Recording of data in tables

- * Graph plotting with appropriate labels

- * Best fit straight-line graphs, intercepts, and gradients

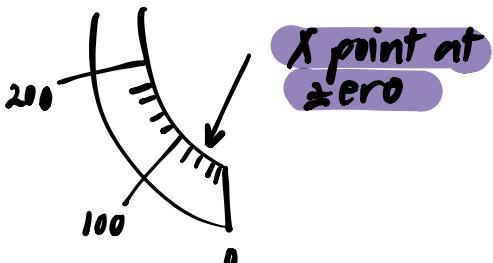
\times accurate
 \downarrow systematic error

precision is more important than accuracy



Systematic Error (predictable, can be corrected)

1. zero error / wrong calibration



2. wrong technique



3. faulty equipment / unsuitable lab conditions

Random Error (unpredictable)

[ANALYSIS OF DATA]

① Collecting data

L consider expected range of results

L take a fair spread of readings

! more readings in areas of special interest

② Choice of axes

$$T = 2\pi \sqrt{\frac{m}{k}}$$

dependent variable
= y -axis

independent variable
= x -axis

③ Recording data

L in TABLES!

(1) title	(2) name & symbol	(3) units & error
-----------	-------------------	-------------------

Frequency, f ($\pm 1 \text{ Hz}$)		
350		
380		
390		
400		
410		
420		
450		

(1) Table 1. Frequency vs intensity

$$T = 2\pi \sqrt{\frac{m}{k}}$$

NAME & SYMBOL
UNCERTAINTY
UNIT

$$T = 2\pi \sqrt{\frac{m}{k}} \rightarrow T^2 = \frac{4\pi^2}{k} \times m + 0 \quad (2)$$

(1) \uparrow \uparrow
 y-axis x-axis
 $= T \text{ vs } \sqrt{m}$

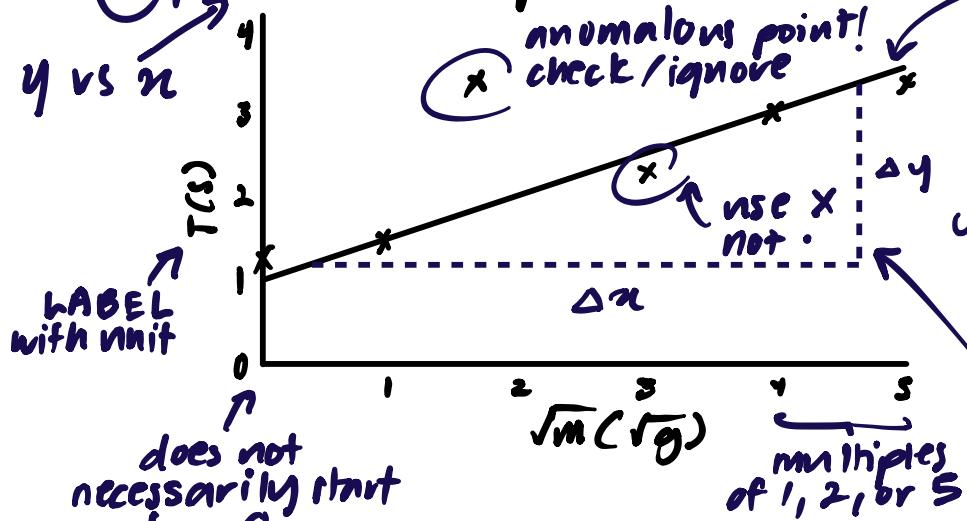
(convert to straight line equation) \rightarrow
 easier to analyse graph

Mass, m ($\pm 1g$)	Time for 20 oscillations, t ($\pm 1s$)	\sqrt{m} (\sqrt{g})	period, T ($\pm 0.05s$)	UNCERTAINTY (of derived data)
20	12	4.47 ± 0.11	0.60	
50	15	7.07 ± 0.07	0.75	
100	19	10.00 ± 0.05	0.95	

DECIMAL PLACES consistent with uncertainty

Table 1. Mass versus time of oscillations

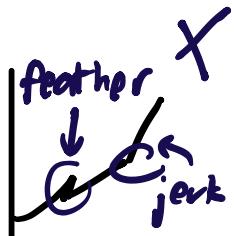
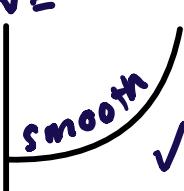
TITLE Time versus square root of mass



BEST FIT LINE

- points are evenly distributed above and below the line

CURVE



Analyzing Data

$m = \text{gradient}$

$$= \frac{\Delta y}{\Delta x} \quad \left. \begin{array}{l} \text{take largest} \\ \text{triangle} \end{array} \right\}$$

$$m = \frac{2\pi}{\sqrt{k}}$$

$$[\Delta y] \rightarrow s, [\Delta x] \rightarrow \sqrt{g}$$

$$[m] \rightarrow s g^{-\frac{1}{2}}$$

$c \neq 0$
 (should be 0) \nearrow systematic error!

$$\eta = a e^{kn} \rightarrow y = m x + c$$

Envelop's number

$$\ln \eta = \ln(a e^{kn})$$

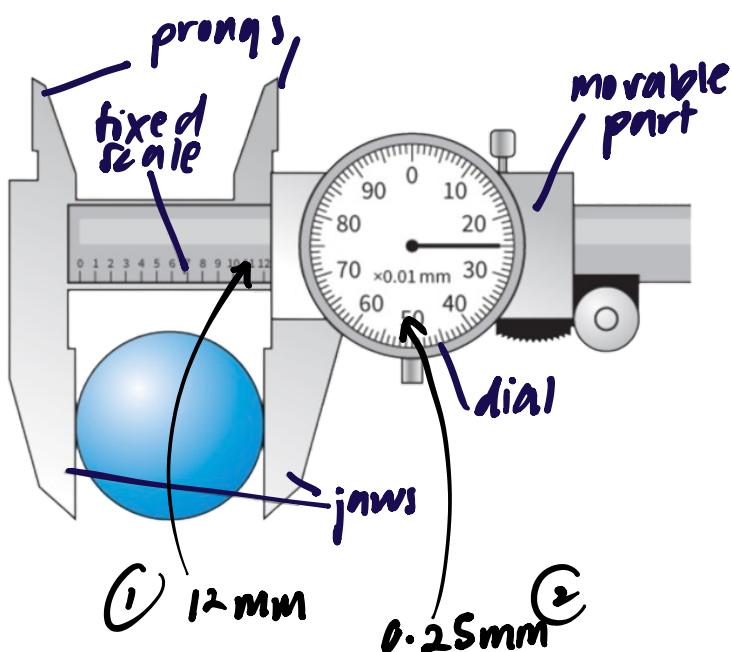
$$\ln \eta = \ln a + \ln e^{kn}$$

$$\ln \eta = k n \ln e + \ln a$$

$$\frac{\ln \eta}{y} = \underline{k n} + \underline{\ln a}$$

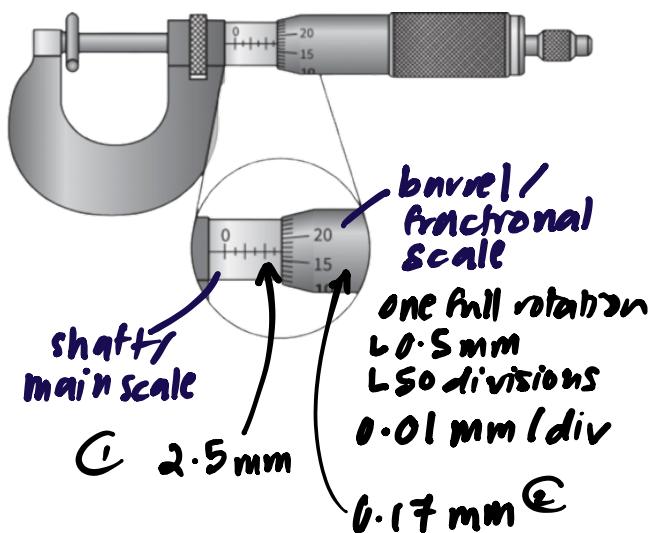
$$[\ln \eta] \rightarrow \ln(\eta / kg) \text{ axis unit w/o in / log}$$

VERNIER CALIPER



$$(1) + (2) = 12.25\text{ mm}$$

MICROMETER SCREW GAUGE



$$(1) + (2) = 2.67\text{ mm}$$

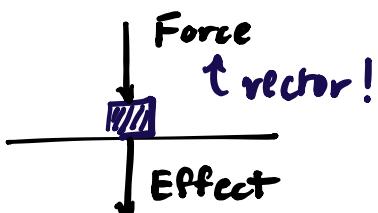
PRESSURE

$$P_{atm} = 10.3 \text{ m-H}_2\text{O}$$

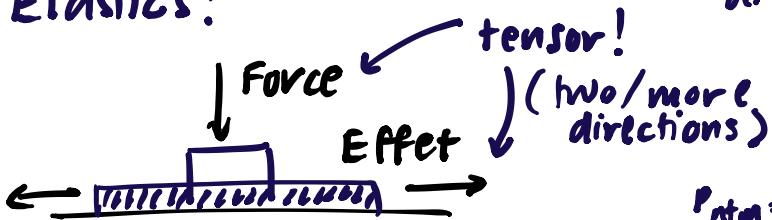
$$= 76 \text{ cm-Hg}$$

- * Density of materials
- * Pressure exerted by solids and fluids
- * Apparatuses for measuring pressure
- * Archimedes' principle and buoyant force

Solids!



Elastics!



Pressure in Static Fluids

$$P = \rho gh$$

h
A

$$P = \frac{F}{A}$$

$$= \frac{mg}{A}$$

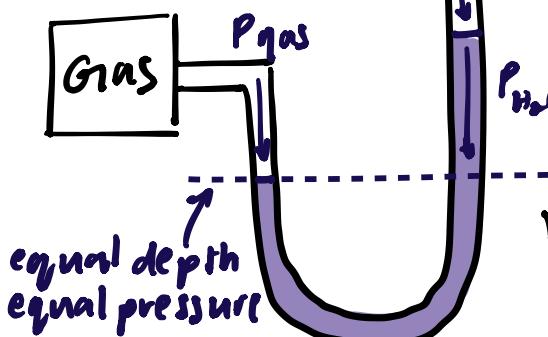
$$= \frac{\rho V g}{A}$$

$$P = h \rho g$$

weight of water
from
 $\rho = \frac{m}{V}$

$P = h \rho g$ ← does not depend on area

$$P_{gas} = P_{H_2O} + P_{atm}$$



Pressure ($N \text{ m}^{-2}$ or Pa)

average force over the size/area of an object

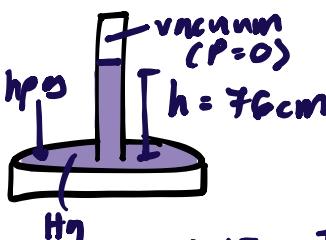
$$P = \frac{F}{A}$$

force
cross-sectional area

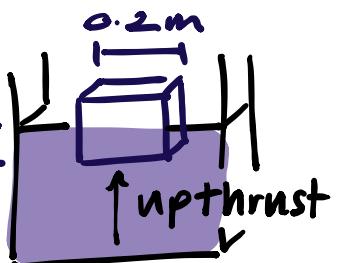
$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

Specific Gravity

$$s.g = \frac{P_{\text{substance}}}{P_{\text{water}}}$$



$$1000 \text{ kg m}^{-3}$$



$$P = h \rho g$$

$$= 0.15 \times 1000 \times 9.81$$

$$P = 1471.5 \text{ Pa}$$

$$F = PA$$

$$= 1471.5 \times 0.2^2$$

$$F = 59 \text{ N}$$

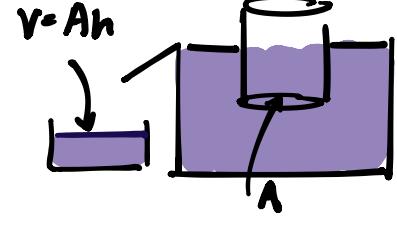
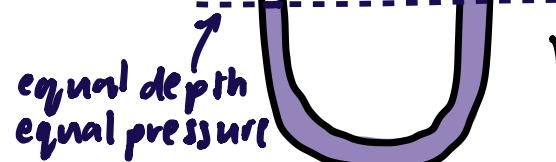
$$P_{gas} = P_{H_2O} + P_{atm}$$

Atmosphere

Buoyancy

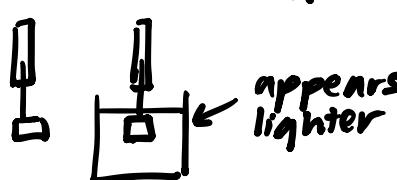
$$F_B = m_f d g$$

↑ mass of fluid displaced



Apparent Weight

$$W_{app} = W - F_B$$



$$F = Ap$$

$$F = A h \rho g$$

$$F = V \rho g$$

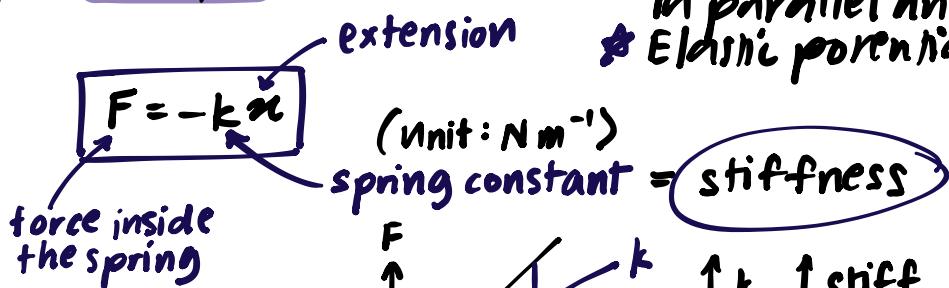
$$F = mg$$

$$F_1 = F_2 = F_{\max}$$

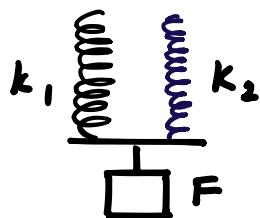
HOOKE'S LAW

Extension, α , produced is proportional to the applied force, F

F & α act in opposite directions



Parallel



$$k' = k_1 + k_2 \quad \text{(more stiff)}$$

Series

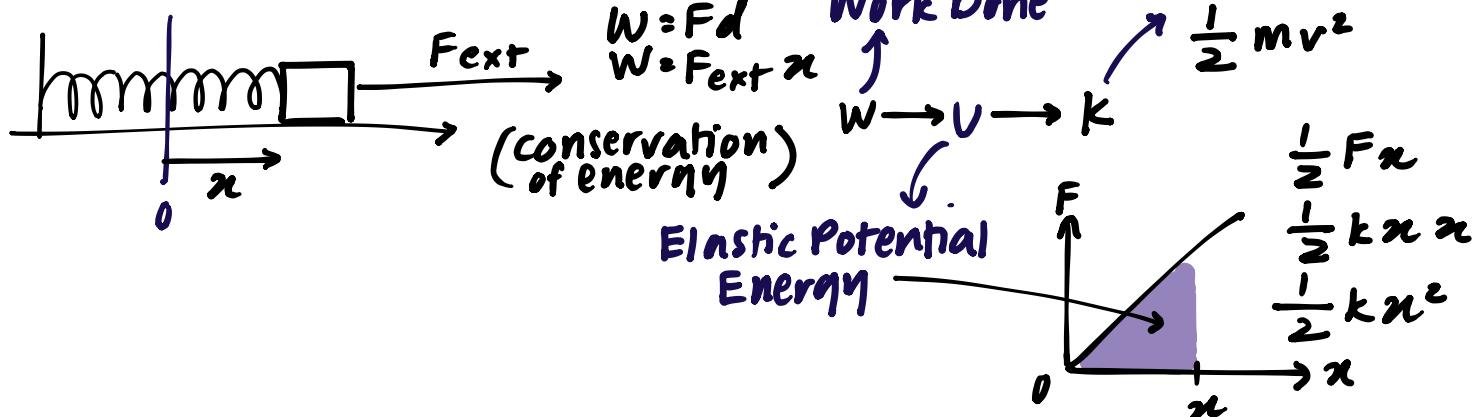


$$k' > k_1 \quad k' > k_2 \quad \text{more elastic}$$

$$\frac{\text{Total } \alpha}{F = k\alpha} \quad \alpha_1 = \frac{F}{k_1} \quad \frac{F}{k'} = \frac{F}{k_1} + \frac{F}{k_2}$$

$$\frac{1}{k'} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$\alpha' = \alpha_1 + \alpha_2$$

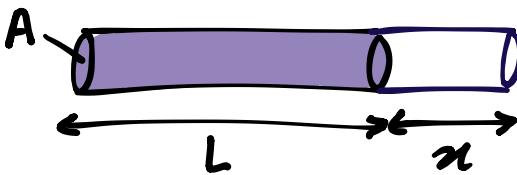
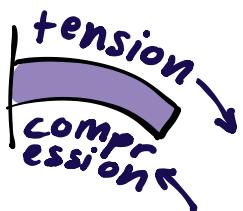


ELASTIC PROPERTIES OF SOLIDS

4



* Young's modulus and deformation in solids



STRESS

$$\sigma = \frac{F}{A}$$

in Pa / Nm⁻²

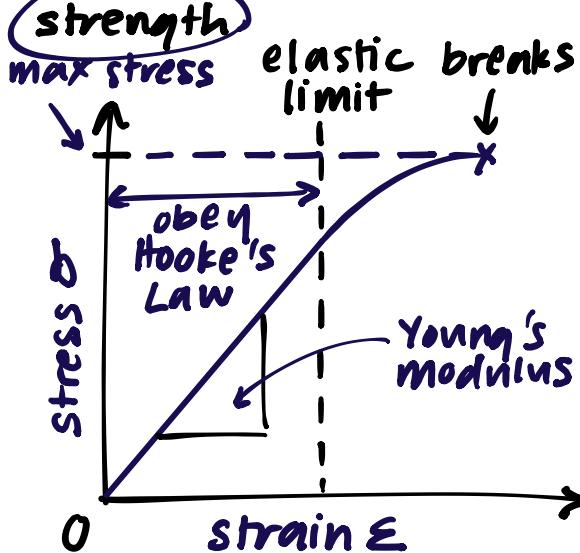
STRAIN

$$\epsilon = \frac{\propto}{L}$$

similar to pressure!

YOUNG'S MODULUS

$$E = \frac{\sigma}{\epsilon}$$



measure of stiffness

$\uparrow E, \uparrow$ stiffness

solid $\rightarrow \sigma \propto \epsilon$

$$\sigma = E \epsilon$$

$$F = k \propto$$

spring \therefore obeys

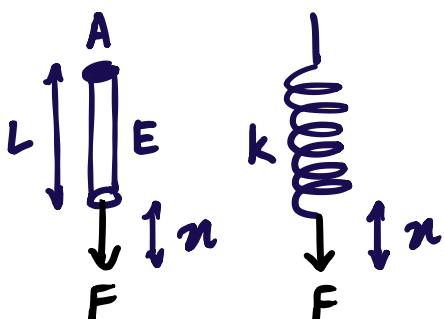
Hooke's Law

property of material
(like density!)

independent of size
and shape

in order of 10^9

Giga!



$$E = \frac{\sigma}{\epsilon}$$

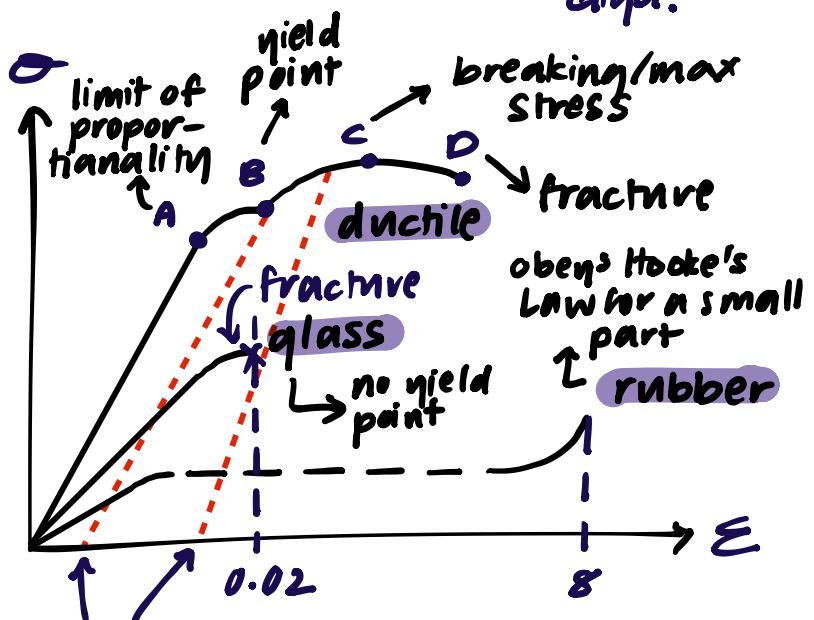
$$F = \left(\frac{EA}{L} \right) \propto$$

$$E = \frac{F}{A} \cdot \frac{\propto}{n}$$

$$E = \frac{FL}{An}$$

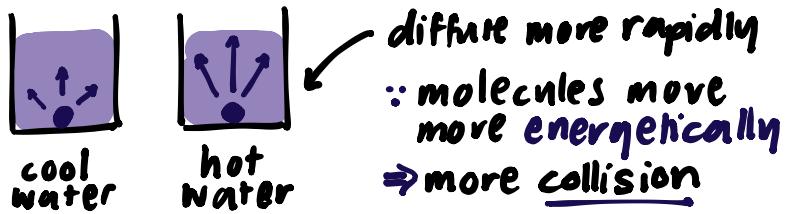
$$k = \frac{EA}{L}$$

contains
stiffness, diameter &
length of spring

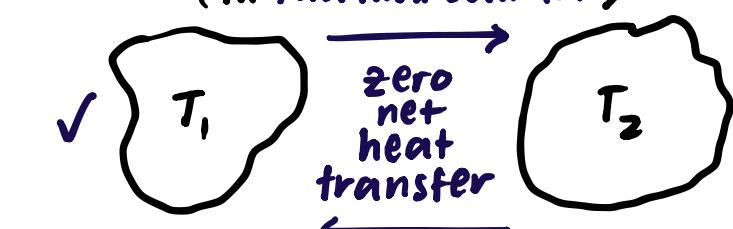
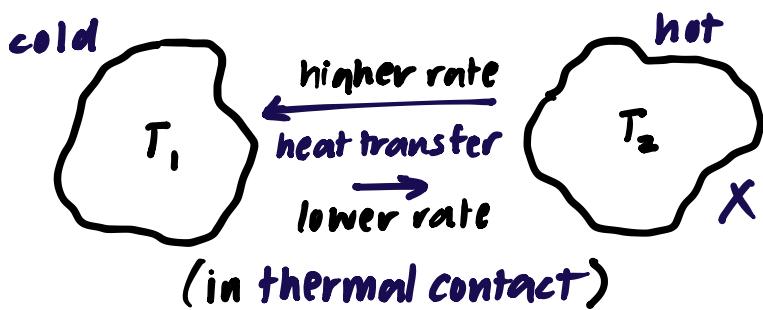


TEMPERATURE

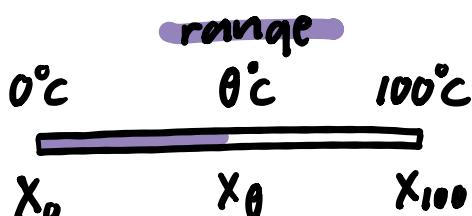
Measure of the average kinetic energy of molecules in a substance



Thermal Equilibrium



Celcius Scale



calibration

$$\frac{\theta - 0}{100 - 0} = \frac{X_\theta - X_0}{X_{100} - X_0}$$

linearity

$$\theta = \frac{X_\theta - X_0}{X_{100} - X_0} \times 100^\circ\text{C}$$

$$\Delta\theta \propto \Delta X$$

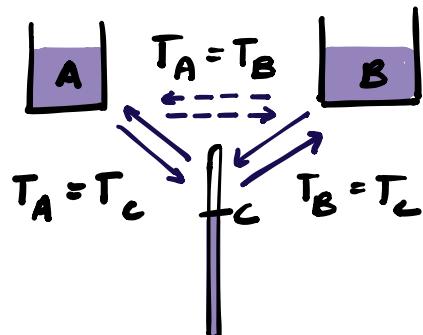
X : physical quantity that changes with temp.

$$\frac{\theta_F - 32}{212 - 32} = \frac{\theta_c - 0}{100 - 0}$$

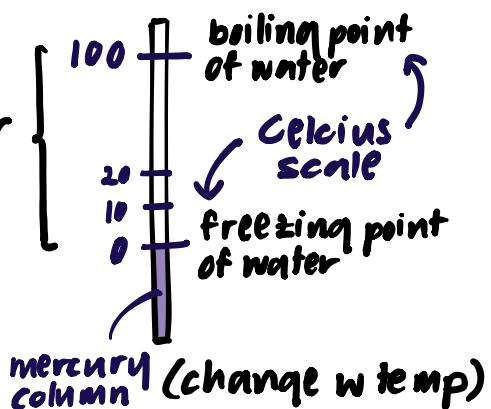
$$\theta_F = \frac{9}{5} \theta_c + 32$$

- * Concept of temperature
- * Empirical temperature scales and their application
- * Setup and advantages of different thermometers

Zeroth Law of Thermodynamics

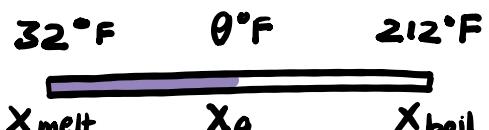


Calibration of Thermometer

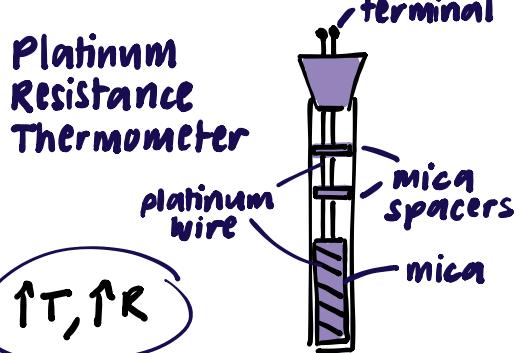


assume:
equal rises in temps give equal changes in output

Farenheit Scale



$$\frac{\theta_F - 32}{212 - 32} = \frac{X_\theta - X_{\text{melt}}}{X_{\text{boil}} - X_{\text{melt}}}$$



$\Delta T, \Delta R$

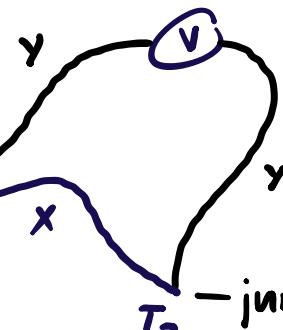
$$\theta = \frac{R_T - R_0}{R_{100} - R_0} \times 100^\circ C$$

Thermoelectric Thermometer (Thermocouple)

junction

$$\text{if } T_1 = T_2, V = 0$$

T_1 and T_2 create potential difference



$$\theta = \frac{V_T - V_0}{V_{100} - V_0} \times 100^\circ C$$

COMPARISON

Resistance VS Thermocouple

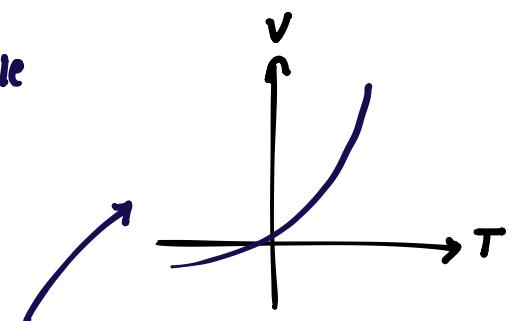
different similarity {
robustness
size
sensitivity
linearity}

wide range
robust

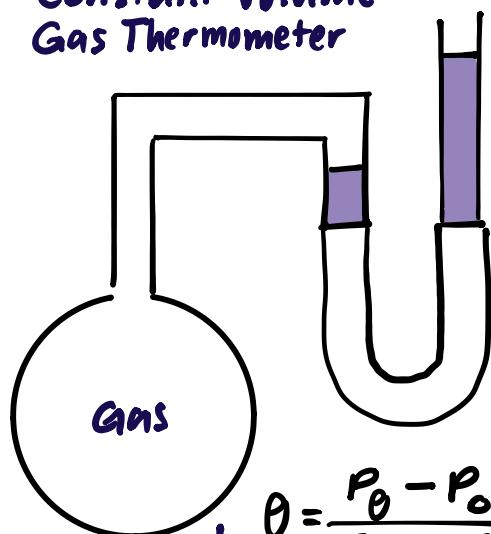
large
low
good

remote
operation

small
very
non-linear
↑ specific heat cap.
↑ require calibration



Constant Volume Gas Thermometer



$$\theta = \frac{P_T - P_0}{P_{100} - P_0} \times 100^\circ C$$

Thermodynamic Scale

• based on the average kinetic energy

At rest: $K_E = 0 : 0K$

Triple point of $H_2O: 273.16K$

Water exists as
610Pa

- solid
- liquid
- gas

Absolute(K) Celsius($^\circ C$)

373.15 steam point 100

273.16 triple point 0.01

273.15 ice point 0

0 -273.15

PHASES OF MATTER

6

SOLID	LIQUID	GAS
<ul style="list-style-type: none"> closely packed high density fixed shape & volume vibrate about equilibrium strong inter-molecular force 	<ul style="list-style-type: none"> slightly farther apart <u>lower density</u> fixed volume, variable shape has random restricted motion <u>weaker IM force</u> 	<ul style="list-style-type: none"> far apart <u>lower density</u> no fixed shape or volume move randomly at high speed negligible IM force
 melting		 boiling
 freezing		 condensation

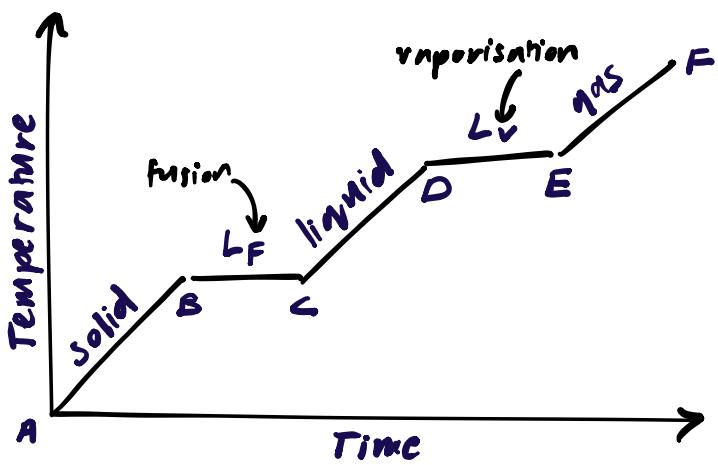
- * Three phases of matter
 - * Specific heat capacity and specific latent heat
 - * Boiling and evaporation

Internal Energy

$$= \text{potential energy} + \text{kinetic energy}$$

intermolecular force motion of molecules

PET when molecules move further apart



Phase transition at BC and DE

No temp change \neq No KE change

Heat transferred \Rightarrow Potential energy
(latent)

↓ intermolecular force
melting boiling
bonds loosen bonds break

Specific heat capacity, c

$$C = \frac{Q}{m \Delta \theta}$$

per unit mass

to raise the temp by 1K or 1°C

energy required

Specific latent heat, L

$$L = \frac{Q}{m}$$

energy required
per unit mass

To change its phase
w/o any change in temps

Evaporation

collision between liquid and air molecules

sunlight

gain enough KE

escapes

cools down (energy is removed)

- at any temp
- liquid cools
- only on H₂O
- temp may

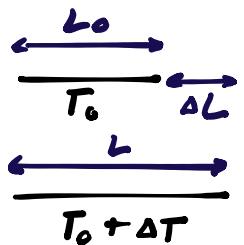
- at any temp
 - liquid cools down
 - only on the surface
 - temp may change

THERMAL EXPANSION

7

absorb heat \rightarrow expand
release heat \rightarrow shrink

* Linear law for linear, area, and volume expansion in thermal processes



$$\Delta L \propto \Delta T$$

linear expansion

$$\frac{\Delta L}{L_0} \propto \Delta T$$

coefficient of linear expansion
order: 10^{-6}

$$\frac{\Delta L}{L_0} = \alpha \Delta T \quad (\text{in } ^\circ\text{C})$$

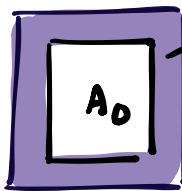
$\uparrow \alpha, \uparrow \Delta L$

$$L = L_0 (1 + \alpha \Delta T)$$

$$\Delta T \propto \Delta X$$

physical property

Area Expansion

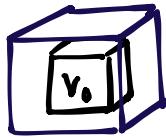


$$\frac{\Delta A}{A_0} = \beta \Delta T$$

2α

(differentiate)

Volume Expansion



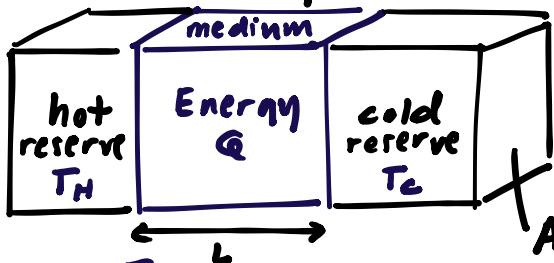
$$\frac{\Delta V}{V_0} = \gamma \Delta T$$

3α

HEAT TRANSFER

① lattice vibration
② collision of electrons } Conduction
in solids

reservoir: temp is constant



in solids

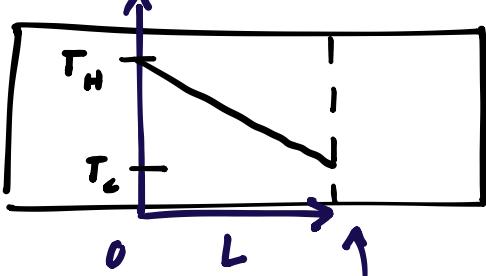
* Three heat transfer mechanisms

- conduction in solids
- convection in fluids
- radiation in vacuum

Temp gradient and cross sectional area

Rate of heat transfer

\propto



✗ thermal equilibrium!
(no heat transfer)

$$\Delta T = T_H - T_C$$

$$\text{Temp. gradient} = \frac{\Delta T}{L}$$

heat transfer at steady state

$$P_{\text{cond}} \downarrow$$

$$\frac{Q}{t} \propto A \frac{\Delta T}{L}$$

$$P_{\text{cond}} = k A \frac{T_H - T_C}{L}$$

$$Js^{-1}$$

$Js^{-1} m^{-1} C^{-1}$
depends on material

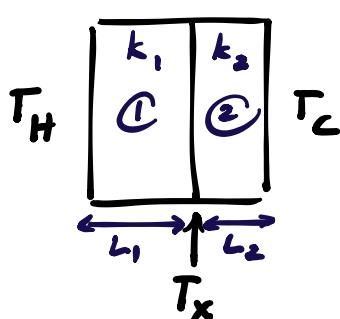
Steady state!

Rate of heat transfer is constant

Composite Material (different media)

$$P_{\text{cond}} = P_1 = P_2 \quad I = \frac{V}{R_1 + R_2} \quad P_1 = P_2$$

$$I_1 = I_2 \quad I = \frac{V}{\frac{L_1}{\sigma_1 A} + \frac{L_2}{\sigma_2 A}}$$

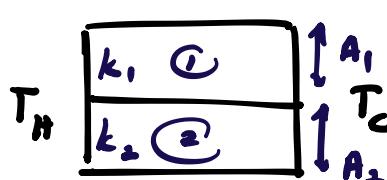


$$P_{\text{cond}} = \frac{A(T_H - T_C)}{\frac{L_1}{k_1} + \frac{L_2}{k_2}}$$

$$I = \frac{V}{R} \quad \begin{matrix} \nearrow \text{potential difference} \\ \searrow \text{resistance} \end{matrix}$$

$$R = \frac{L}{\sigma A} \quad \begin{matrix} \nearrow \text{rate of charge flow} \\ \nearrow \text{conductivity } (\frac{1}{\rho}) \end{matrix}$$

$$P = \frac{k A \Delta T}{L} = \frac{\Delta T}{\frac{L}{k A}} \quad \begin{matrix} \nearrow \text{rate of heat flow} \\ \nearrow \text{temp diff} \end{matrix}$$



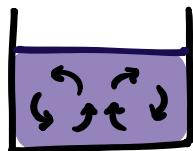
$$P = \left(\frac{k_1 A_1}{L} + \frac{k_2 A_2}{L} \right)^{-1} \Delta T$$

$$P = \left(\frac{k_1 A_1 + k_2 A_2}{L} \right)^{-1} \Delta T$$

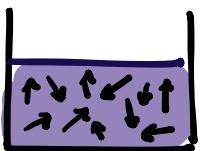
$$P = \frac{(T_H - T_C) L}{k_1 A_1 + k_2 A_2}$$

Convection

in fluids



weak \rightarrow ~~weak~~ laminar flow



strong turbulent flow

Radiation

in vacuum

non-zero temp = energy
↳ radiation

infra red R O Y G B I V ultra violet

increasing energy

in Kelvin

$$P_{rad} = \frac{Q}{t}$$

$$P_{rad} = e \sigma A T^4$$

Factors

$\propto T^4$ (in Kelvin)

$\propto A$

\propto emissivity, e

Stefan-Boltzmann

$0 \leq e \leq 1$

white/shiny $\rightarrow 0$
black/dull $\rightarrow 1$

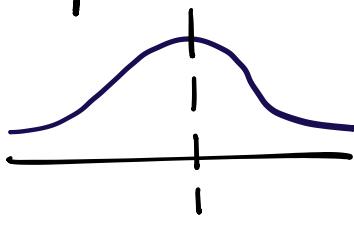
$$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$$

A diagram of a sphere labeled T_{obj} . Two arrows point away from the sphere, one labeled P_{rad} and another labeled P_{abs} . A third arrow points towards the sphere, labeled P_{net} , with a note "+ve" above it and "-ve" below it, indicating it is the net radiation exchange.

$$T_{sur} = \frac{e\sigma A}{(T_{sur}^4 - T_{obj}^4)}$$

UNCERTAINTY

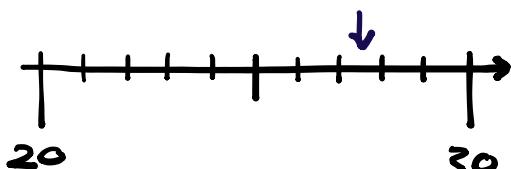
L estimate of the spread of values \pm



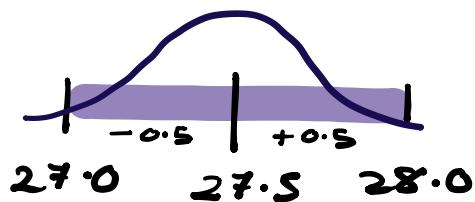
Absolute Uncertainty ΔA

1 s.f.

1. Smallest division



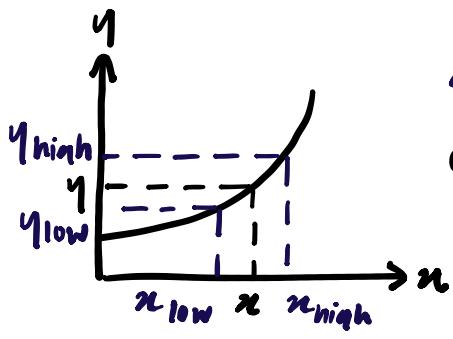
$27.5 \pm 0.5 \text{ mm}$



Uncertainty in Derived Quantities

$$y = f(x)$$

x manipulated variable



$$\Delta y = \frac{1}{2} |y_{\text{high}} - y_{\text{low}}|$$

$$\text{eq. } \Delta \rho = \frac{1}{2} |\rho_{\text{high}} - \rho_{\text{low}}|$$

$$m + \Delta m \rightarrow \frac{m_{\text{high}}}{m_{\text{low}}} \quad V - \Delta V \rightarrow \frac{V_{\text{high}}}{V_{\text{low}}}$$

* Calculation of uncertainties in measurements

- calibration of instrument (or half of it)
- our ability to observe
- errors by equipment and technique

Fractional Uncertainty

$$\frac{\Delta A}{A}$$

Percentage Uncertainty

$$\frac{\Delta A}{A} \times 100\%$$

measure of accuracy

→ Percentage Difference

difference between two values $\times 100\%$
one of the values OR their average

2. repeat readings

(statistical method)

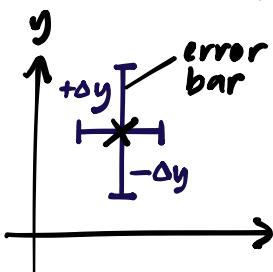
half of the range

$$\Delta x = \frac{1}{2} |x_{\text{high}} - x_{\text{low}}|$$

standard deviation

Error Analysis on Graphs

10



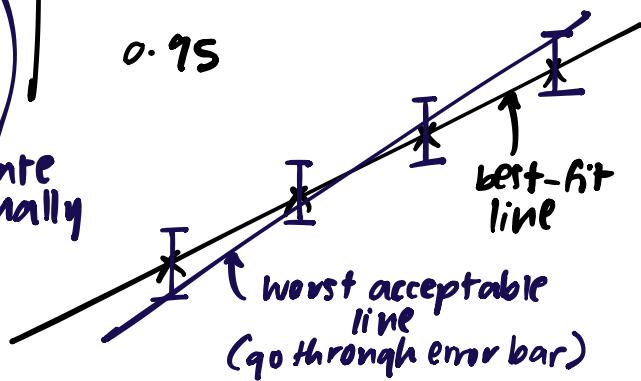
Worst acceptable line
steepest/shallowest line within error bar

- * Error bars in graphs
- * Uncertainty in gradient using the worst acceptable line and best fit line

Mass, m $(\pm 1g)$	Time for 20 oscillations, t $(\pm 1s)$	\sqrt{m} $(\pm \sqrt{g})$	Period, T $(\pm 0.05s)$
20	12	4.47 ± 0.11	0.60
50	15	7.07 ± 0.07	0.75
100	17	10.00 ± 0.05	0.95

exception to 1.s.t
(consistency)
same uncertainty
for the whole
column
(linear)

calculate
individually



$$\Delta m = |\text{best fit} - \text{worst fit}|$$