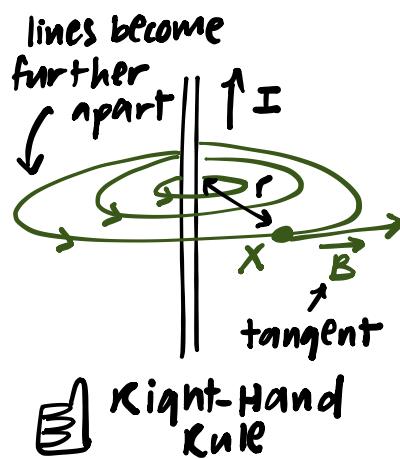


# MAGNETIC FIELD DUE TO CURRENT-CARRYING CONDUCTORS

1



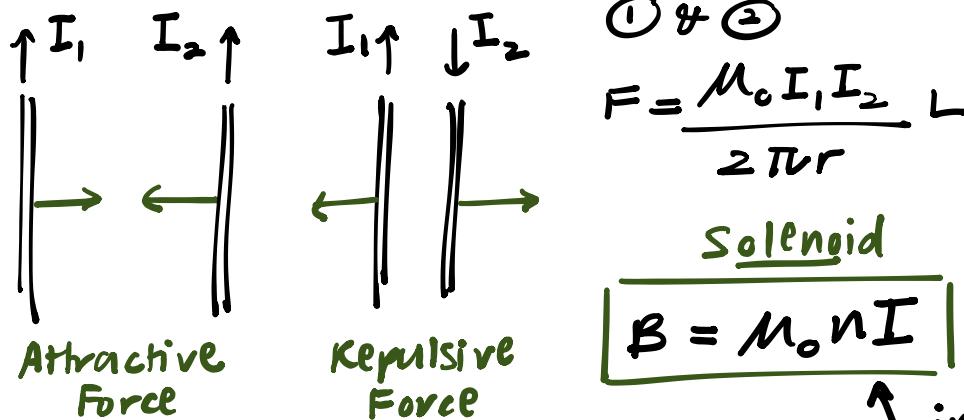
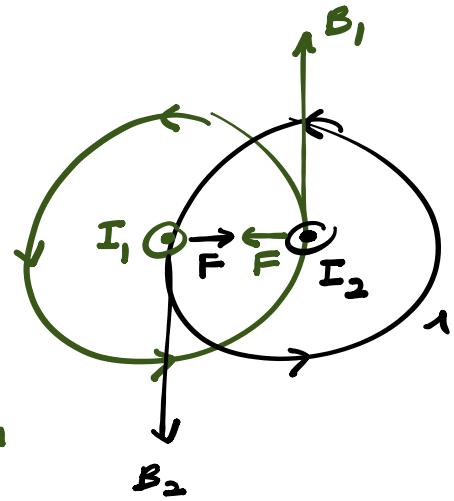
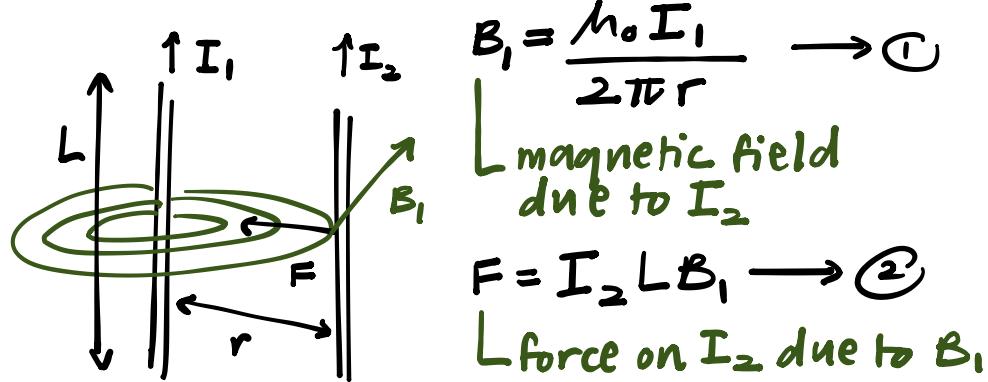
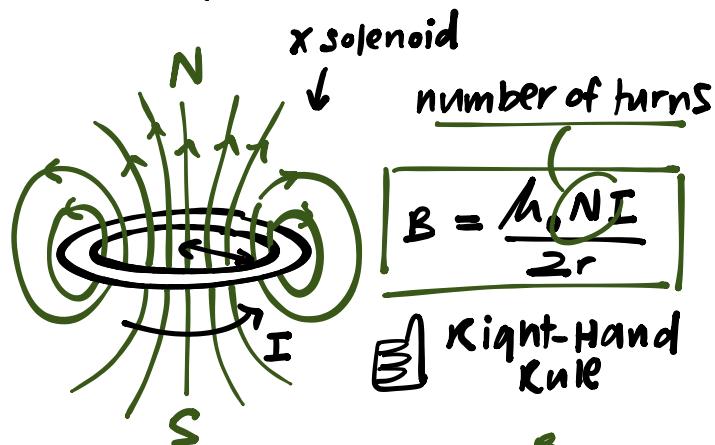
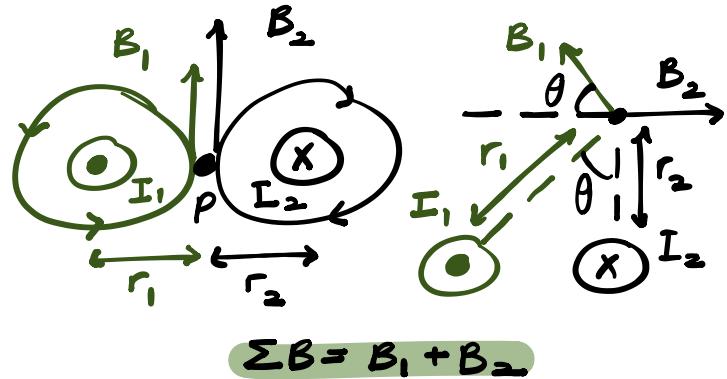
$$B \propto I \quad B \propto \frac{1}{r}$$

$$B = \frac{\mu_0 I}{2\pi r}$$

permeability of free space  
 $= 4\pi \times 10^{-7} \text{ H m}^{-1}$

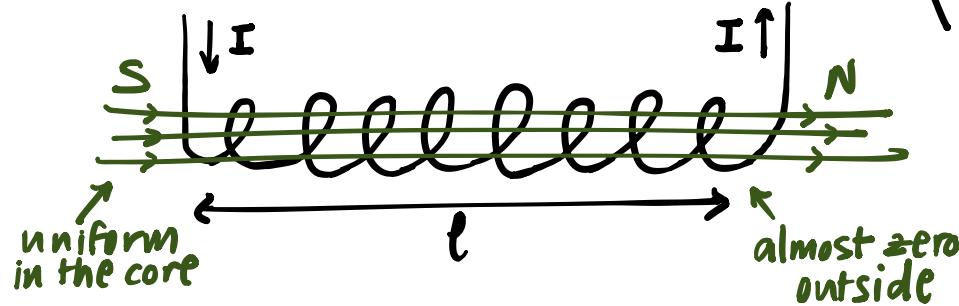
- Magnetic field due to a current in a very long straight wire
- in a circular arc of wire
- Forces between two parallel current-carrying wires
- Magnetic field due to a solenoid
- Ferromagnetism
- Electromagnet

Right-Hand Rule



Force per unit length

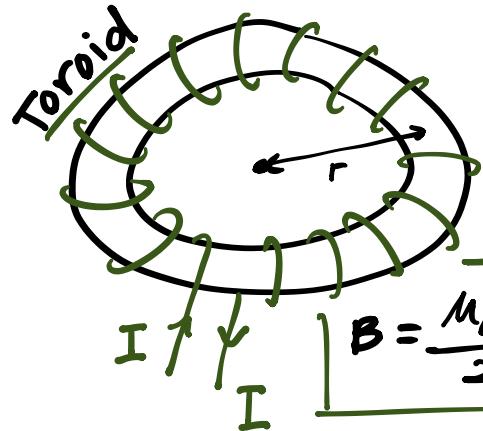
$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$



$B = \mu_0 \mu_r n I$

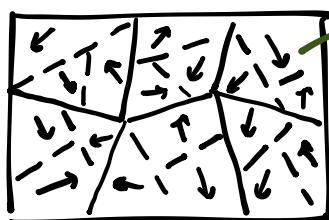
ferromagnetic material

- nickel
- iron
- cobalt



$$B = \frac{\mu_0 NI}{2\pi r}$$

## Ferromagnetism

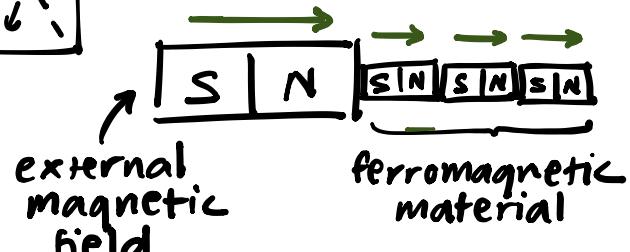


domain  
(elementary magnet)

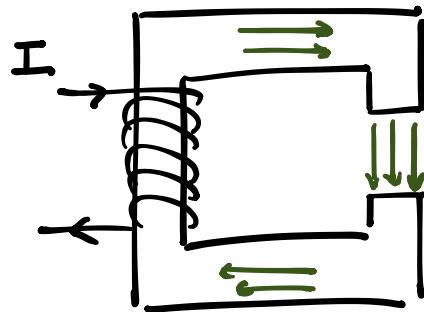
Electro-  
magnet

external  
magnetic  
field

$$B_o + B$$

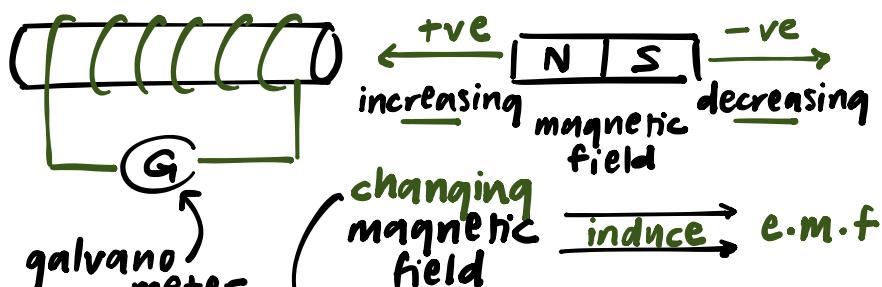


ferromagnetic  
material



# ELECTROMAGNETIC INDUCTION

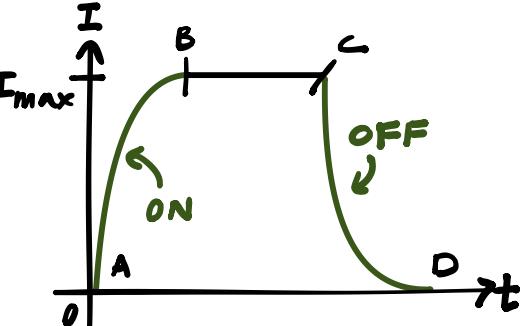
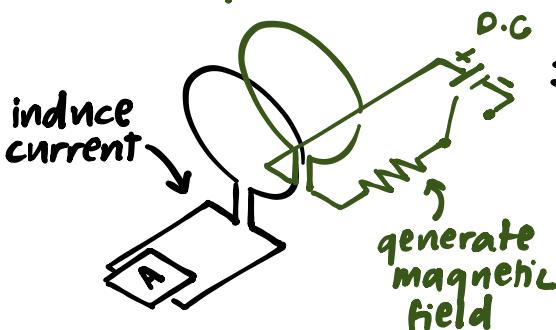
2



↑ induced current, if  
relative motion between the coil & magnetic field

↑ magnet strength  
↑ speed  
↑ area of coil  
↑ number of turns

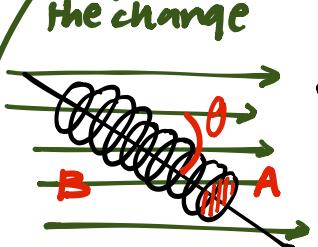
- \* Electromagnetic induction
- \* Faraday's Law of induction
- \* Lenz's Law
- \* Induced e.m.f. in a coil located in a magnetic field
- \* E.m.f. induced in a moving conductor
- \* Eddy currents



## Faraday's Law of Induction

$$E = -\frac{d\phi}{dt}$$

against the change

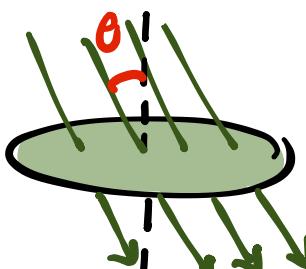


magnetic flux  
 $\phi = BA \cos \theta$

$$\phi = NBA \cos \theta$$

flux linkage

$$\therefore E = -\frac{Nd(BA \cos \theta)}{dt}$$



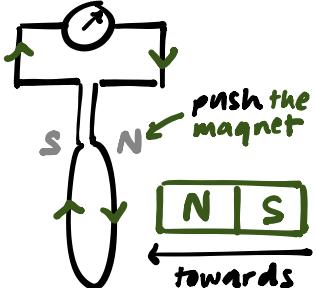
$$E \propto \frac{dB}{dt} \rightarrow$$

$$E \propto \frac{dI}{dt} \rightarrow$$

$$B \propto I$$

$$\phi = BA \Rightarrow \phi \propto I$$

## Lenz's Law



## Changing B

$$-A \cos \theta \frac{dB}{dt}$$

## Changing A

$$-B \cos \theta \frac{dA}{dt}$$

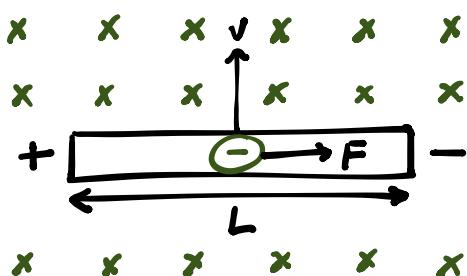
## Changing θ

$$-BA \frac{d(\cos \theta)}{dt}$$

$$BA \sin \theta \cdot \frac{d\theta}{dt}$$

$$BA \omega, \sin(\omega t)$$

Angular velocity



E.M.F Induced in a Moving Conductor

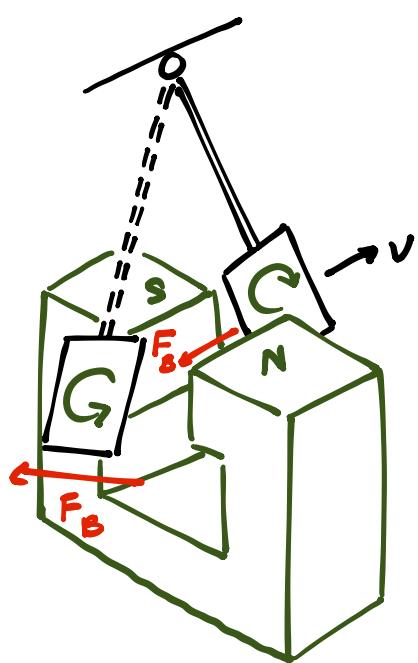
$$E = BLv$$

$$\begin{aligned} E &= \left| -\frac{d\phi}{dt} \right| \\ &= \frac{d(BA)}{dt} \\ &= BL \frac{dx}{dt} \\ E &= BLv \end{aligned}$$

$$\eta = \cos \theta$$

$$\frac{dy}{dt} = \frac{dy}{d\theta} \times \frac{d\theta}{dt}$$

## EDDY CURRENTS



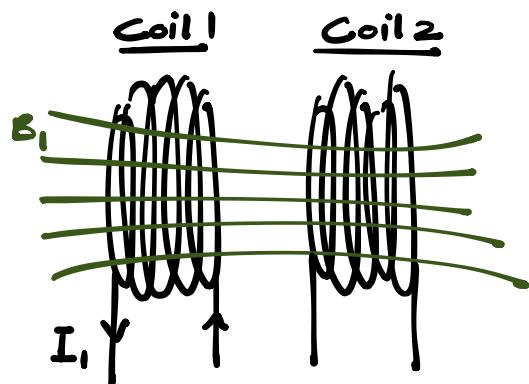
$\frac{dB}{dt} > 0$  when pendulum enters magnetic field

$\frac{dI}{dt} > 0 \Rightarrow$  eddy current

interaction between the eddy current and magnetic field  $\Rightarrow F_B$

# INDUCTANCE AND INDUCTORS

3



$$\frac{dI}{dt} \neq 0 \quad \frac{d\phi}{dt} \neq 0 \\ \frac{dB}{dt} \neq 0 \quad \Rightarrow \mathcal{E} \neq 0$$

$$\mathcal{E}_2 \propto -\frac{\Delta I_1}{\Delta t}$$

$$\mathcal{E}_2 = -M \frac{\Delta I_1}{\Delta t}$$

mutual inductance

$$\mathcal{E}_2 = -N_2 \frac{\Delta \phi}{\Delta t}$$

From 1 and 2:

$$M = N_2 \frac{\Delta \phi}{\Delta I_1}$$

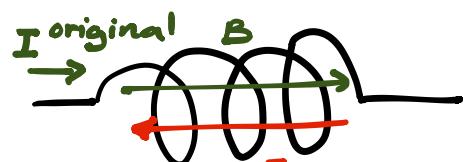
- \* Mutual inductance
- \* Self-inductance
- \* Storage and release process of an inductor
- \* Energy in a magnetic field
- \* RL circuits VS RC circuits

$$M = \frac{\mu_r A}{N} \quad M = N_2 \frac{(OB)A}{\Delta I_1},$$

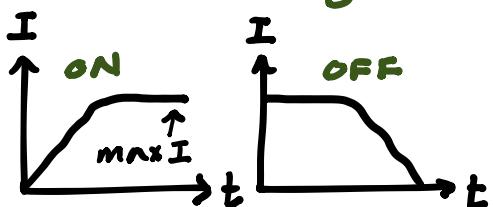
$$M = N_2 \frac{M_o N_1 \Delta I_1}{l} \frac{A}{\Delta I_1}$$

$$M = \frac{\mu_o N_1 N_2 A}{l}$$

## Self Inductance

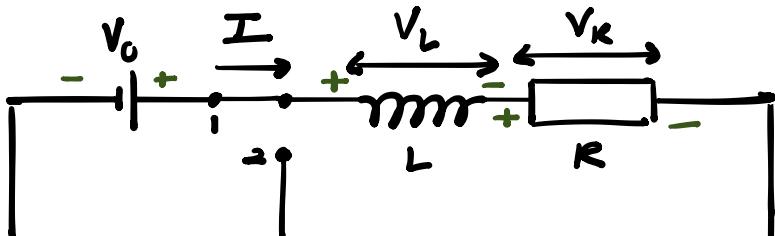


$I_{\text{induced}} < I_{\text{original}}$



$\frac{dI}{dt}$  is reduced  $\Rightarrow \mathcal{E}$  is reduced

## Storage Process



switch 1

$$\Rightarrow t=0$$

$$I=0 \quad \frac{dI}{dt}=\max \quad V_L=L \frac{dI}{dt} \quad V_L=\max$$

$$V_R$$

$$I=0 \quad V_R=0$$

$$t=t_1$$

$$I \uparrow, \frac{dI}{dt} \downarrow$$

$$V_L \downarrow$$

$$V_R \uparrow$$

$$t=t_2$$

$$I=\max \quad \frac{dI}{dt}=0$$

$$V_L=0$$

$$V_R=\max$$

$$\text{self inductance} \quad M_r \frac{A}{N}$$

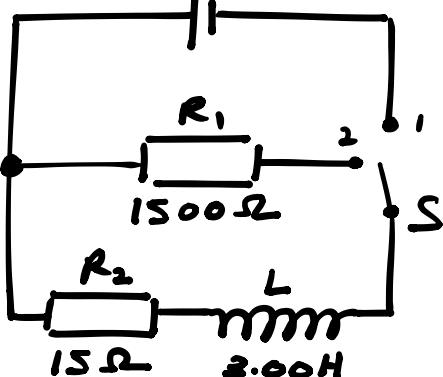
$$\mathcal{E} = -L \frac{\Delta I}{\Delta t}$$

$$-L \frac{\Delta I}{\Delta t} = -N \frac{\Delta \phi}{\Delta t}$$

$$L = N \frac{\Delta \phi}{\Delta I}$$

$$L = \frac{\mu_o N^2 A}{l}$$

$$V_0 = 18V$$

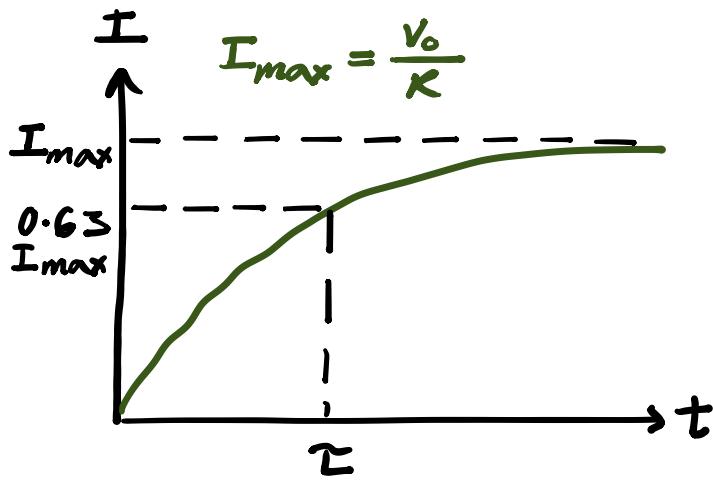


①

$$I = \frac{V_0}{R_2} = 1.2A$$

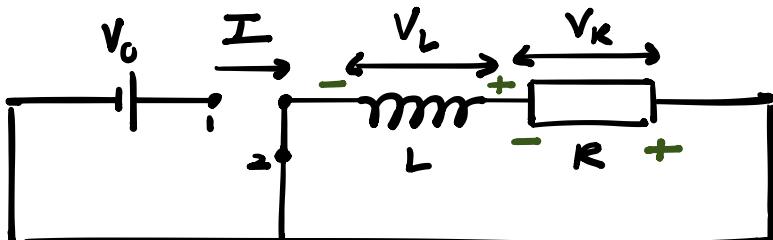
②

$$V_1 = IR_1 = 1750V$$



$$I = \left( \frac{V_o}{R} \right) (1 - e^{-\frac{t}{\tau}})$$

### Release Process



switch 2  
at  $t=0$

$$\begin{aligned} I &= \text{max} & V_L &= L \frac{dI}{dt} & I &= \text{max} \\ \frac{dI}{dt} &= \text{max} & V_L &= \text{max} & V_R &= \text{max} \end{aligned}$$

$$t = t_1 \quad I \downarrow, \frac{dI}{dt} \downarrow \quad V_L \downarrow \quad V_R \downarrow$$

$$t = t_2 \quad I = 0 \quad \frac{dI}{dt} = 0 \quad V_L = 0 \quad V_R = 0$$

### RC Circuits vs RL Circuits

$$\tau = RC$$

$$I = I_{\text{max}} e^{-\frac{t}{\tau}} \quad I = I_{\text{max}} (1 - e^{-\frac{t}{\tau}})$$

$$V_R = V_o e^{-\frac{t}{\tau}}$$

$$V_R$$

$$V_C = V_o (1 - e^{-\frac{t}{\tau}})$$

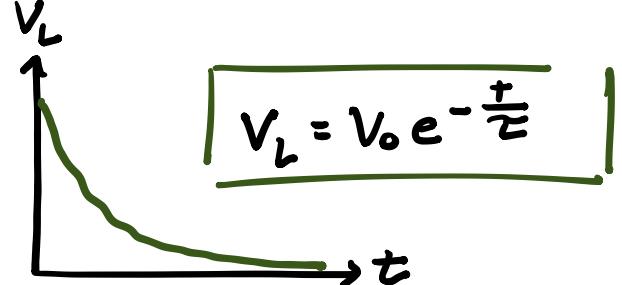
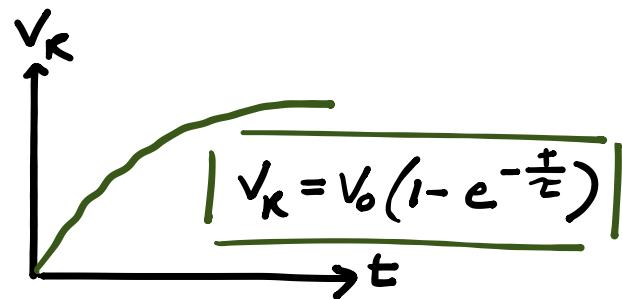
$$V_C = V_o e^{-\frac{t}{\tau}}$$

$$Q = Q_o (1 - e^{-\frac{t}{\tau}})$$

$$Q = Q_o e^{-\frac{t}{\tau}}$$

$$\tau = RC =$$

$t$  when  $I$  increases to 63%.  
OR decreases to 37%.



$$\tau = \frac{L}{R} = t \text{ when } I \text{ is 63% of } I_{\text{max}}$$

$$\begin{aligned} I &= I_{\text{max}} e^{-\frac{t}{\tau}} \\ V_R &= V_o e^{-\frac{t}{\tau}} \\ V_L &= V_o e^{-\frac{t}{\tau}} \end{aligned}$$

$$\tau = t \text{ when } I \text{ is 37% of } I_{\text{max}}$$

### ENERGY IN MAGNETIC FIELD

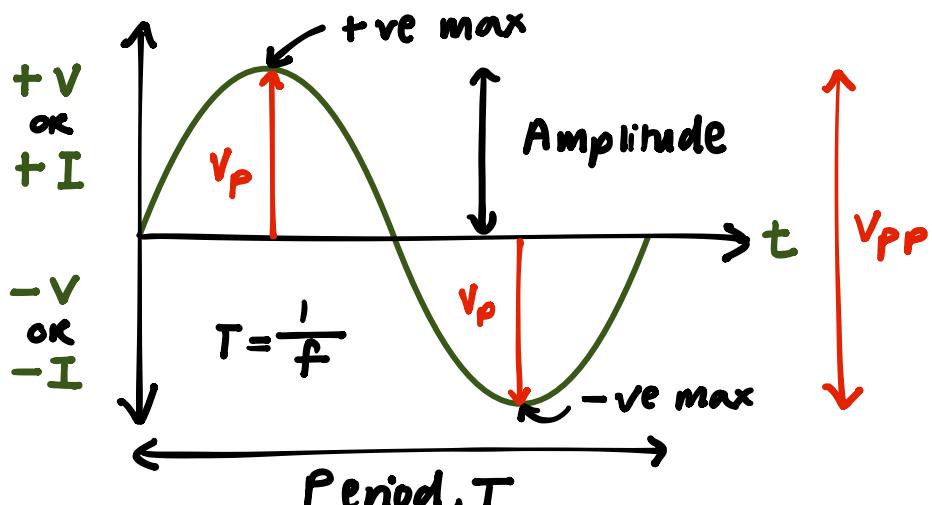
$$\begin{aligned} V_o &= V_L + V_R & V_o &= L \frac{dI}{dt} + IR \\ V_o - L \frac{dI}{dt} - IR &= 0 & P &= VI \\ IV_o - LI \frac{dI}{dt} - I^2 R &= 0 \end{aligned}$$

$$P = \frac{V}{t} \rightarrow \frac{dV}{dt} = LI \frac{dI}{dt}$$

$$U = Pt \rightarrow U = \int_0^I L I dI$$

$$U = \frac{1}{2} L I^2$$

## A.C. CIRCUITS I



- Peak Value,  $V_p$
- Peak-to-Peak Value,  $V_{pp}$

Instantaneous Value  
 $t_1 = V_t$

- \* The sine wave
- \* The sine wave formula
- \* Transformers
- \* Rectification
- \* Half-wave rectification
- \* Full-wave rectification
- \* Smoothing circuits

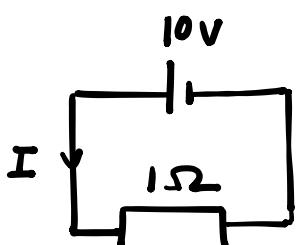
## Root Mean Square

r.m.s  
= effective value

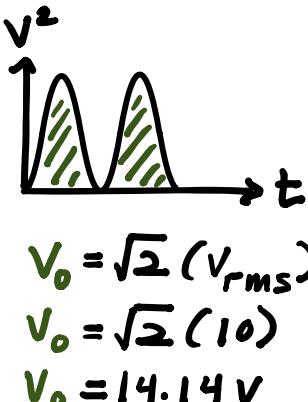
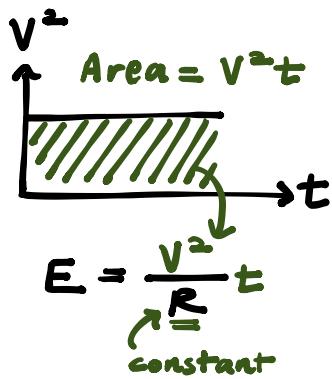
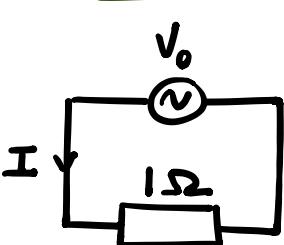
$$P_{AC} = P_{DC}$$

$$(I_{rms})(V_{rms}) = IV$$

### D.C. Circuit



### A.C. Circuit



only for sine function,

$$V_{rms} = \frac{V_p}{\sqrt{2}}, \quad I_{rms} = \frac{I_p}{\sqrt{2}}$$

$$(V_{rms})^2 = \frac{\int_0^T V^2 dt}{T}$$

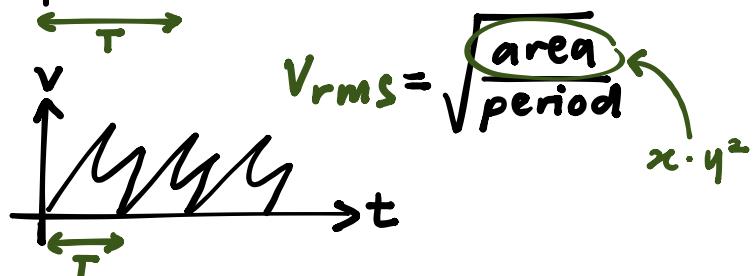
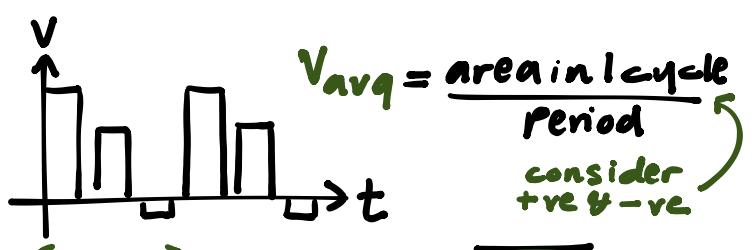
$\curvearrowleft$  average

$$= \frac{1}{T} \int_0^T V_0^2 \sin^2(\omega t) dt$$

$$V_{rms} = \frac{V_0}{\sqrt{2}} \quad \begin{matrix} \curvearrowleft \\ V = V_0 \sin \theta \end{matrix}$$

## Average Value

### Special Waveforms



$$V_{avg} = \frac{2}{\pi} V_p$$

$$I_{avg} = \frac{2}{\pi} I_p$$

min operational value

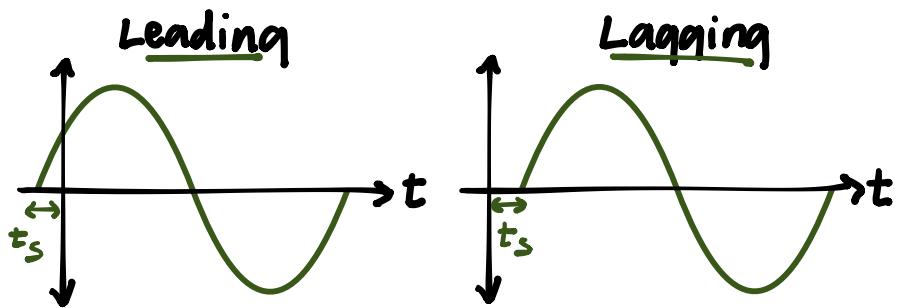
area under half cycle

only for sine wave

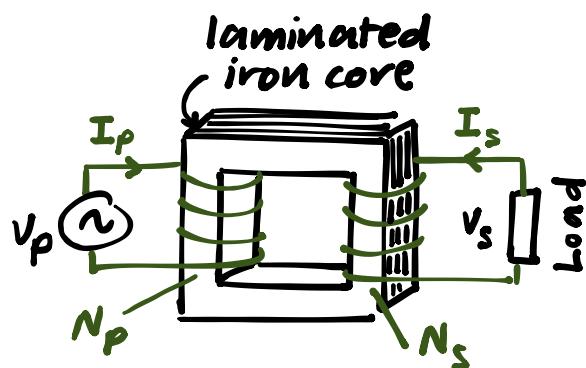
## Instantaneous Voltage

$$V = V_p \sin \theta$$

$$V = V_p \sin(\omega t) \quad \text{or} \quad 2\pi f$$



## TRANSFORMER



- same frequency
- phase difference of  $\pi$  or  $180^\circ$

$$\frac{V_s}{V_p} = \frac{N_s A \frac{dB}{dt}}{N_p A \frac{dB}{dt}} \quad \begin{matrix} \nearrow 100\% \text{ phase shift} \\ \text{efficiency} \end{matrix}$$

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \quad \begin{matrix} \text{secondary} \\ \text{primary} \end{matrix}$$

Turns-Ratio Equation

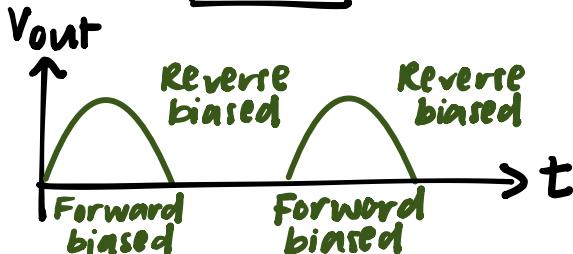
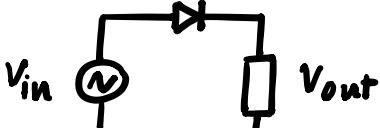
$$V_p I_p = V_s I_s$$

$$\frac{V_s}{V_p} = \frac{I_p}{I_s}$$

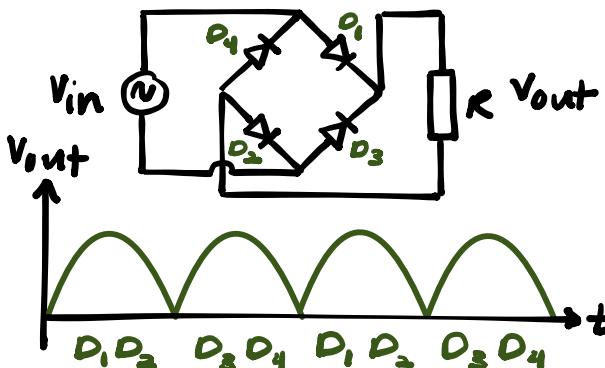
## RECTIFICATION

\* Efficiency affects the current only

### Half-Wave

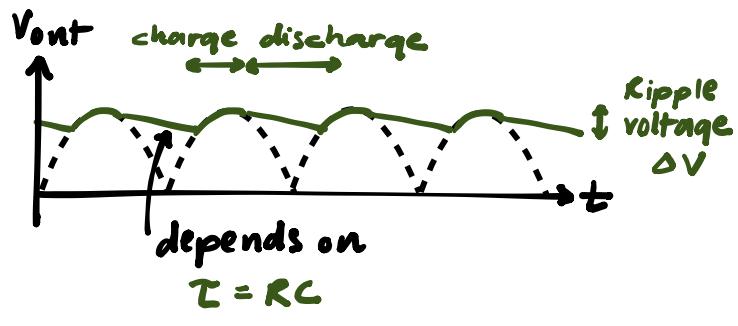
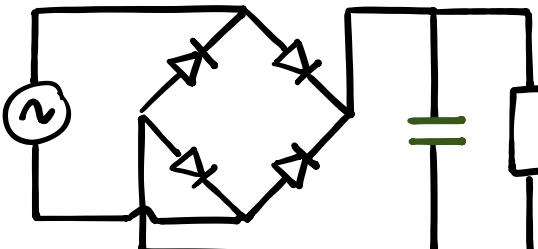


### Full-Wave



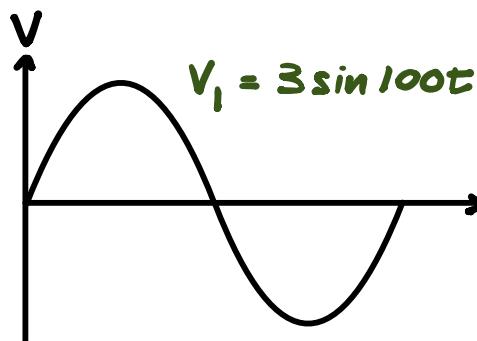
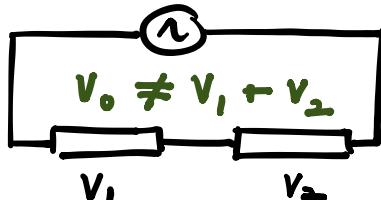
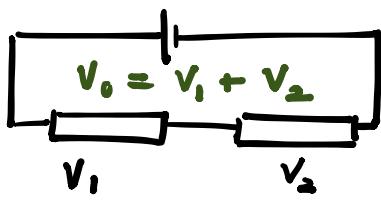
\* Efficiency affects the current only

## Smoothing Circuits



## A.C. CIRCUITS II

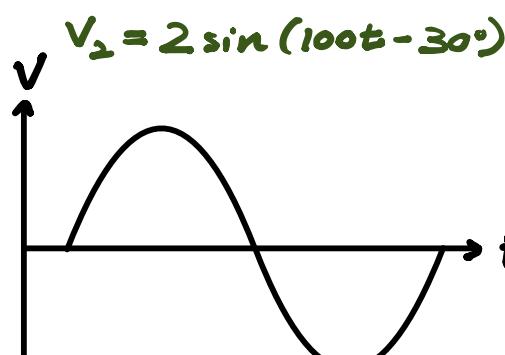
5



### PHASOR

$$V = V_p \sin \theta$$

- counter clockwise
- 1 complete rev
- vertical component = instantaneous  $V$



$$V_1 = 3 \angle 0^\circ$$

$$V_2 = 2 \angle -30^\circ$$

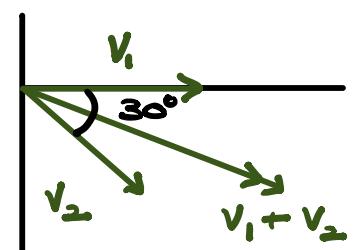
\* Phasors

\* Resistor, capacitor, and inductor in a.c. circuits

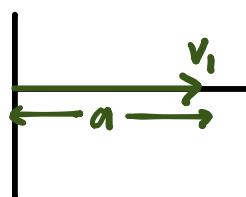
\* Impedance

\* RC circuits

\* LR circuits



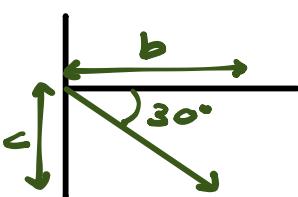
$$V = 4.82 \sin(100t - 11.97^\circ)$$



$$V_1 = a$$

$$V_1 = a \angle 0^\circ$$

$$\star j = 0$$



$$V_2 = b + j(-c)$$

$$= b - jc$$

$$V_2 = \sqrt{b^2 + c^2} \angle -30^\circ$$

$$V = V_1 + V_2$$

$$= a + (b - jc)$$

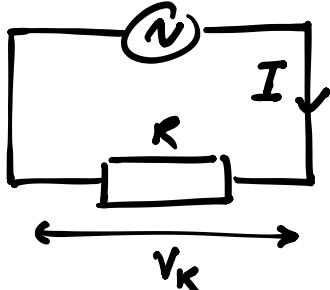
$$V = (a + b) - jc$$

$$V = 3 \angle 0^\circ + 2 \angle -30^\circ$$

$$V = 4.837 \angle -11.93^\circ$$

$$V = 4.837 \sin(100t - 11.93^\circ)$$

### Resistor



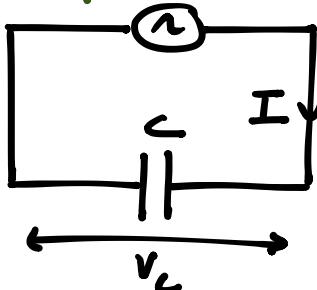
$$\begin{aligned} V_R &\angle 0^\circ \\ R &\angle 0^\circ \\ I &\angle 0^\circ \end{aligned}$$

$$V_R = V_0 \sin(\omega t)$$

$$I = \frac{V_R}{R}$$

$$I = I_0 \sin(\omega t)$$

### Capacitor



$$\begin{aligned} V_C &\angle 0^\circ \\ X_C &\angle -90^\circ \\ I_C &\angle 90^\circ \end{aligned}$$

$$V = IR$$

$$V_0 \cos(\omega t) = I \frac{1}{\omega C}$$

$X_C$ , capacitive reactance

$$V_C = V_0 \sin(\omega t)$$

$$Q = CV$$

$$I = \frac{dQ}{dt}$$

$$I = \omega \cdot C V_0 \cos(\omega t)$$

$$X_C = \frac{1}{2\pi f C}$$

## Inductor

$V_L < 0^\circ$   
 $X_L < 90^\circ$   
 $I < -90^\circ$   
 $V = IR$   
 $V_0 \cos(\omega t) = I(\omega L)$   
 $X_L$ , inductive reactance  
 $V_L = V_0 \sin(\omega t)$   
 $V_L = L \frac{dI}{dt}$   
 $I = -\left(\frac{1}{\omega L}\right) V_0 \cos(\omega t)$   
 $X_L = 2\pi f L$

## IMPEDANCE $\underline{z}$

(total resistance)

$$\underline{z} = R + X_L + X_C$$

if  $\text{Im}(\underline{z}) > 0$ , circuit is inductive current is lagging

if  $\text{Im}(\underline{z}) < 0$ , circuit is capacitive current is leading

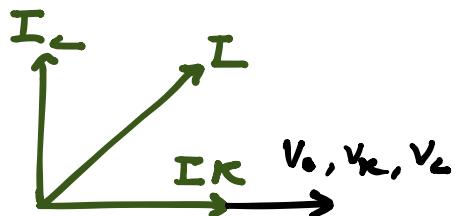
1. identify  $\omega = 2\pi f$
2. identify  $\phi$

## RL Circuit (Series)

$\underline{z} = R + X_L$   
 $= R + (-jX_L)$   
 $\underline{z} = R - j \frac{1}{\omega L}$   
 $\underline{z} = \sqrt{R^2 + X_L^2} \angle \tan^{-1}\left(-\frac{X_L}{R}\right)$   
 $V_0 = \sqrt{V_R^2 + V_L^2} \angle \tan^{-1}\left(-\frac{V_L}{V_R}\right)$   
 $V_R = IR, V_L = IX_L, V_0 = I\underline{z}$

## RC Circuit (Parallel)

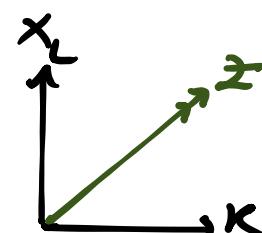
$$V_0 = V_L = V_R$$



f ↑,  $X_L \downarrow$ ,  $\underline{z} \downarrow$ ,  $\phi \downarrow$

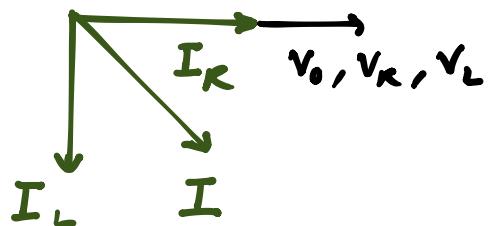
## LR Circuit (Series)

$\underline{z} = R + X_L$   
 $= R + jX_L$   
 $\underline{z} = R + j\omega L$   
 $\underline{z} = \sqrt{R^2 + X_L^2} \angle \tan^{-1}\left(\frac{X_L}{R}\right)$   
 $V_0 = \sqrt{V_R^2 + V_L^2} \angle \tan^{-1}\left(\frac{V_L}{V_R}\right)$



## LR Circuit (Parallel)

$$V_0 = V_L = V_R$$



f ↑,  $X_L \uparrow$ ,  $\underline{z} \uparrow$ ,  $\phi \uparrow$

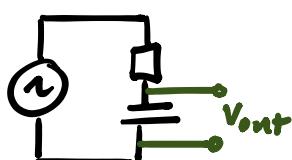
# A.C. CIRCUITS III

/ pass/reject frequencies

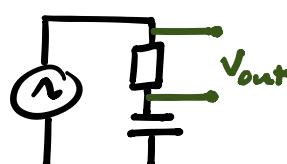
## Filter

- Active
- transistors
- amplifiers

### RC low pass



### RC high pass



\* Filters

\* LRC circuits

\* Series resonance

\* Power in passive components (a.c.)

\* Power in RC circuits

\* Power in LR circuits

## Passive

- $R, L,$
- and  $C$

$$X_C \propto \frac{1}{f}$$

low f,  $X_C \rightarrow \infty$

$$V_{out} = V_{in}$$

$$X_L \propto f$$

high f,  $X_L \rightarrow 0$

$$V_C \rightarrow 0$$

$$V_{out} = V_{in}$$

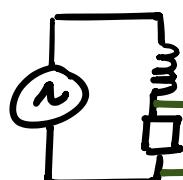
## critical frequency

$$f_c$$

$$X_C = R$$

$$f_c = \frac{1}{2\pi CR}$$

### RL low pass



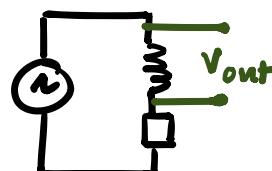
$$X_L \propto f$$

high f,  $X_L \rightarrow 0$

$$V_L \rightarrow 0$$

$$V_{out} = V_{in}$$

### RL high pass



$$X_L = R$$

$$f_c = \frac{2\pi L}{R}$$

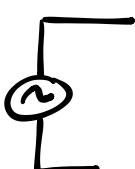
$$X_L \propto f$$

high f,  $X_L \rightarrow \infty$

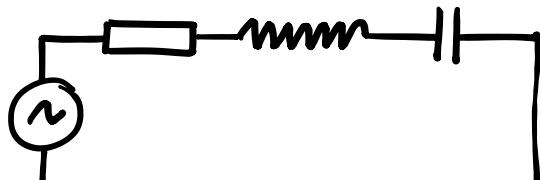
$$V_{out} = V_{in}$$

## Pass-Band Filter

## Stop-Band Filter



## LRC Circuit



$X_C > X_L$  (low f) capacitive

$X_L > X_C$  (high f) inductive

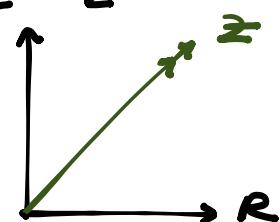
$X_L = X_C$  ( $f_r$ ) resistive

resonance

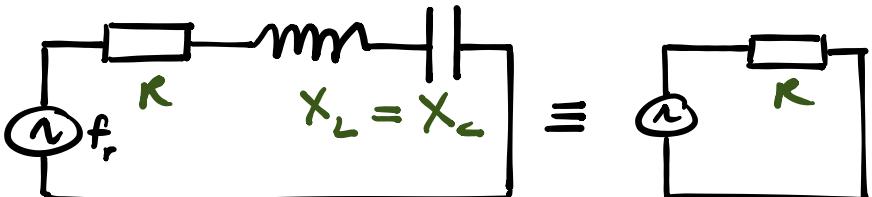
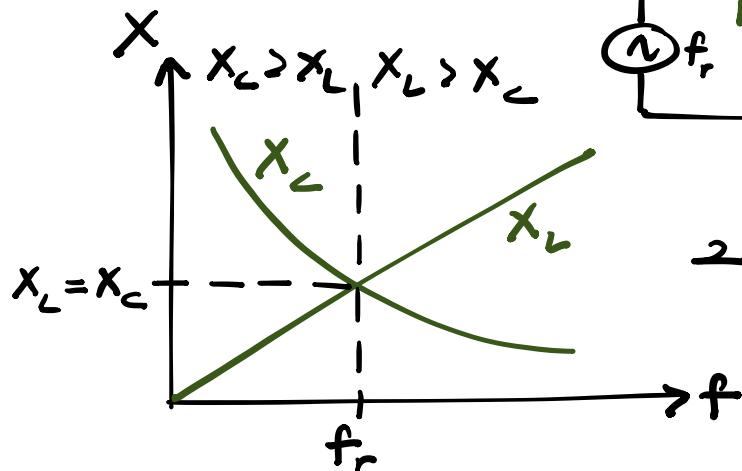
$$X_L - X_C$$

$$Z = R + jX_L - jX_C$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \angle \tan^{-1} \left( \frac{X_L - X_C}{R} \right)$$



## Series Resonance



$$X_L = X_C$$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r = \frac{1}{2\pi \sqrt{LC}}$$

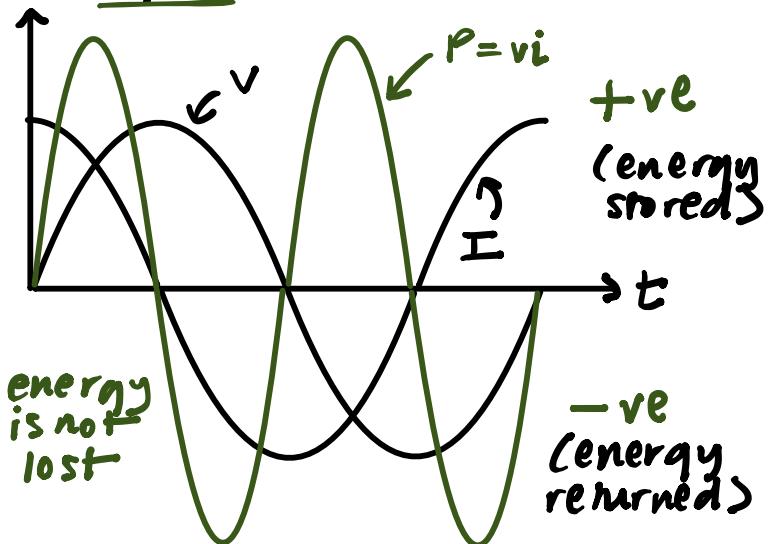
min  $\downarrow Z = R$

$$I = \frac{V_0}{Z}$$

$$I = \frac{V_0}{R}$$

## Power in Passive Components

### Capacitor



### Reactive power

rate at which a capacitor stores/returns energy

$$P_r = I_{\text{rms}}^2 X_C = \frac{V_{\text{rms}}^2}{X_C}$$

in VAR

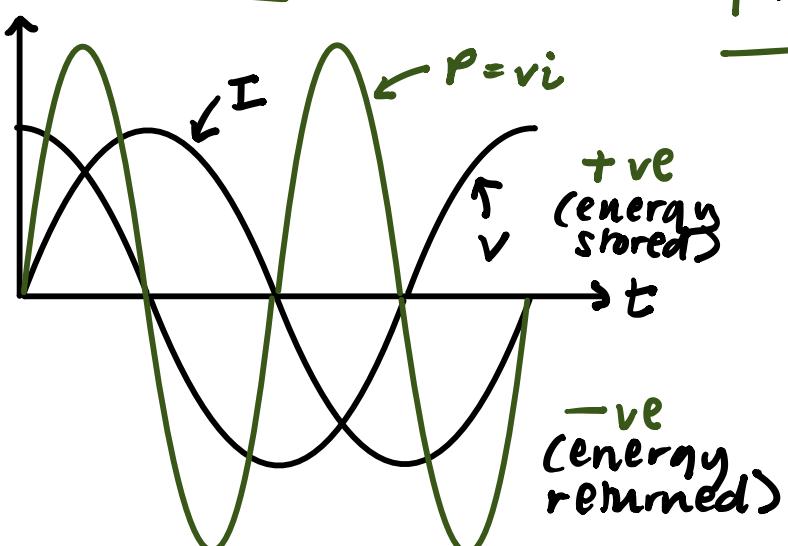
"furnish/deliver power" (-ve)

### Average Power

power dissipated in resistor

$$P_{\text{avg}} = V_{\text{rms}} I_{\text{rms}} = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R}$$

### Inductor



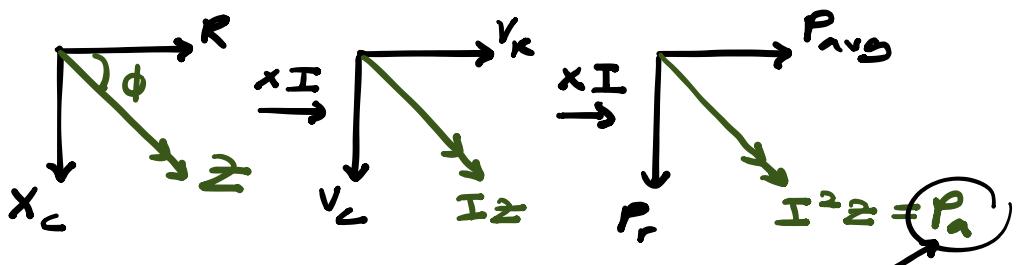
rate at which an inductor stores/returns energy

$$P_r = I_{\text{rms}}^2 X_L = \frac{V_{\text{rms}}^2}{X_L}$$

in VAR

"demand/absorb power" (+ve)

## RC Circuits



## Power Triangle

$\uparrow f, \downarrow P_r, \downarrow \phi$

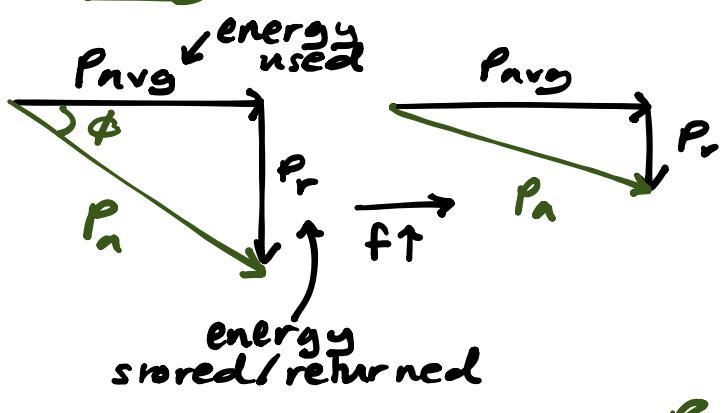
Apparent Power (in VA)

## Power Factor

$$PF = \frac{P_r}{P_{avg}}$$

$$|PF = \cos \phi|$$

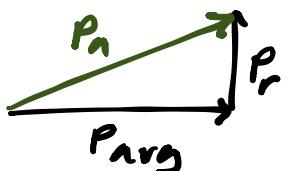
$0 \leftarrow PF \rightarrow 1$   
reactive resistive



$$P_{avg} = V_o I_o \cos \phi$$

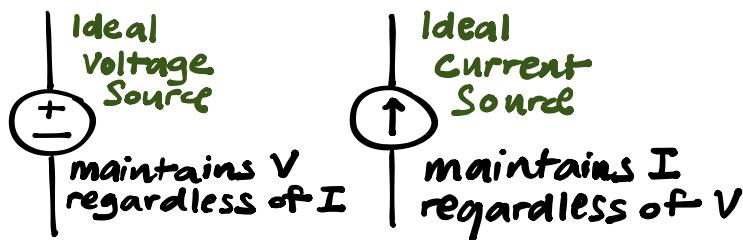
$$\begin{aligned} P_a &= I^2 R \\ &= \sqrt{P_r^2 + P_{avg}^2} \\ &= \frac{P_{avg}}{\cos \phi} \end{aligned}$$

## LK Circuits

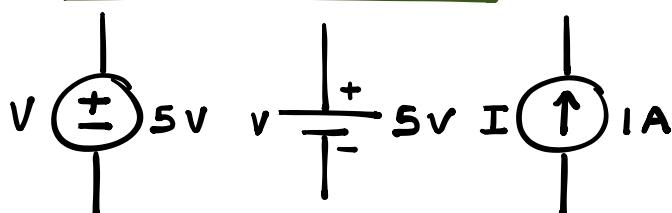


$\uparrow f, \uparrow P_r, \uparrow \phi$

## NODE VOLTAGE METHOD AND SUPERPOSITION THEOREM



### Independent Source



$$v_s = \mu v_x \quad v_s = \rho i_x \quad i_s = \alpha v_x \quad i_s = \beta i_x$$

### Dependent Source

$$v_s = \mu v_x \quad v_s = \rho i_x \quad i_s = \alpha v_x \quad i_s = \beta i_x$$

### NODE VOLTAGE METHOD

$K < L$ :

$$I_1 + I_2 = I_3 = 1A$$

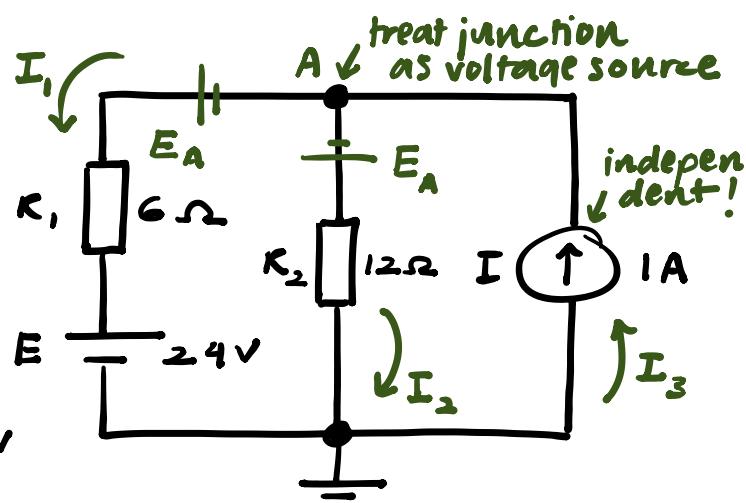
$$I = \frac{\Delta V}{R}$$

$$\frac{E_A - 24}{6} + \frac{E_A}{12} = 1$$

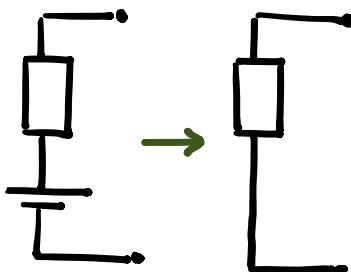
$$E_A = 20V$$

$$I_1 = -0.67A \uparrow$$

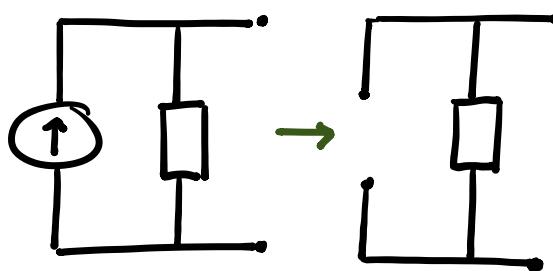
$$I_2 = 1.67A \downarrow$$



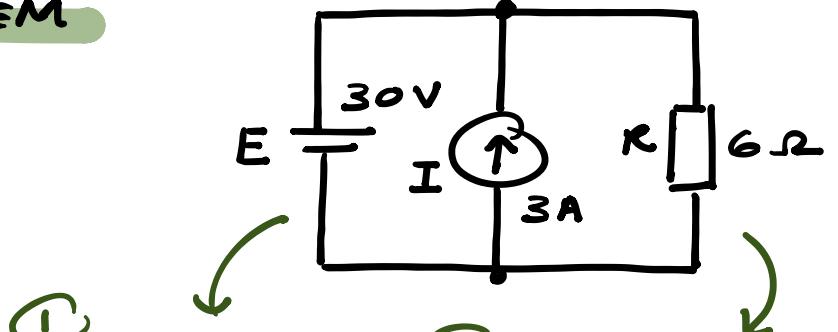
### SUPERPOSITION THEOREM



Voltage Source



Current Source



$$I'_1 = \frac{E}{R} = 5A \quad I'_2 = 0 \text{ (short circuit)}$$

$$I_1 = 5 + 0 = 5A$$

# DIRECT SENSING

8

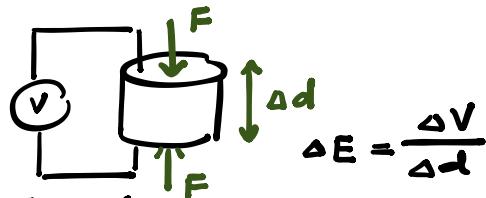
## Electronic sensor



change of physical quantity

### L ① Force

Piezo-Electric Transducers  
i.e. microphone, touch pad



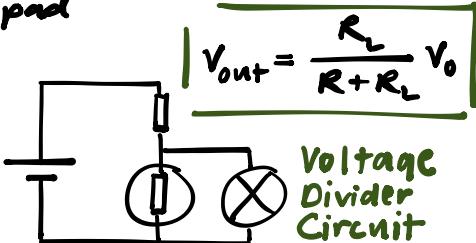
### \* Sensors

- \* The operational amplifier
- \* Open-loop configuration
- \* Negative feedback
- \* The inverting amplifier
- \* The non-inverting amplifier

### ② Light intensity

Light-Dependent Resistor

↑ light, ↓ resistance



### ③ Temperature

Thermistor

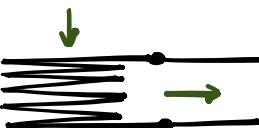
1. Positive Temperature Coefficient, PTC  
↑ temp, ↑ resistance

$$V_{out} = \frac{R_T}{R + R_T} V_0$$

2. Negative Temperature Coefficient, NTC  
↑ temp, ↓ resistance

### ④ Surface changes (expansion/contraction)

Metal-wire Strain Gauge



$$\text{From } \delta R = \frac{\rho \delta L}{A}, \quad \frac{\delta R}{R} = \frac{\delta L}{L}$$

$$\text{or } \frac{\Delta R_1}{R_2} = \frac{\Delta L_1}{\Delta L_2}$$

Inverting Input  $V_-$

+ve input, -ve output  
-ve input, +ve output

Non-inverting Input  $V_+$

+ve input output  
-ve input output

compared!

(unless one is grounded)

## Impedance

## Voltage Gain ( $G_v$ )

$$G_v = \frac{V_{out}}{V_{in}}$$

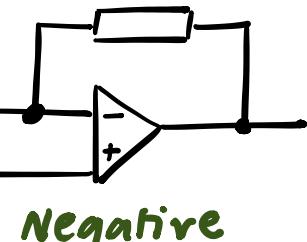
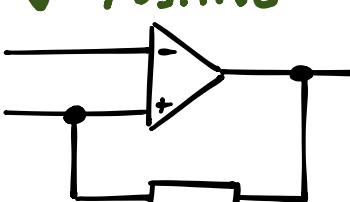
Default:  
 $10^5$

$R_{in}$  is infinite  $\Rightarrow$  small current

$R_{out}$  is zero  $\Rightarrow$  large current

increase amp feedback reduce amplification

Positive



Negative

### Ideal

- infinite  $R_{in}$
- zero  $R_{out}$
- infinite bandwidth & slew rate

### Practical

- high  $R_{in}$
- low  $R_{out}$
- limited bandwidth & slew rate

## OPEN LOOP CONFIG (no feedback)

$$V_{out} \propto V_{in}$$

$$V_{out} \propto (V_+ - V_-)$$

$$\boxed{V_{out} = G_r(V_+ - V_-)}$$

if  $V_{out} > V_s$

$\Rightarrow V_{out} = V_s$  (saturated)

if  $V_+ > V_-$ ,  $+V_s$

if  $V_+ < V_-$ ,  $-V_s$

## NEGATIVE FEEDBACK (reduce gain to 1)

$$V_{out} = G_r(V_+ - V_-)$$

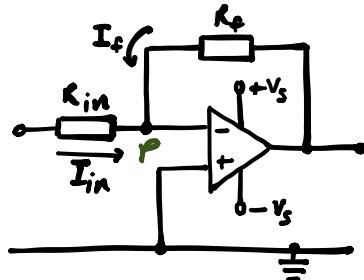
$$V_{out} = G_r(V_{in} - V_{out})$$

$$V_{out}(1 + G_r) = G_r V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{G_r}{1 + G_r}$$

$$\boxed{G_i' = \frac{G_r}{1 + G_r}}$$

## ① Inverting Amplifier



$$I_{in} + I_f = I_p$$

$I_{in} + I_f = 0$  ← virtual ground

$$\frac{V_{in} - V_-}{R_{in}} + \frac{V_{out} - V_-}{R_f} = 0$$

$G \rightarrow \infty$  ← very large

$$\frac{V_{out}}{V_+ - V_-} \rightarrow \frac{1}{0}$$

$$V_+ - V_- \approx 0$$

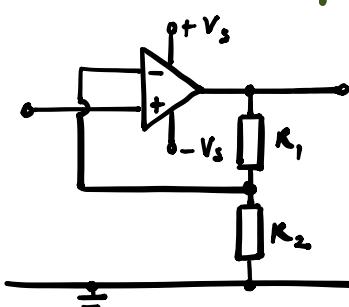
$$V_+ \approx V_-$$

$$V_+ = 0, V_- \approx 0, V_p \approx 0 \leftarrow \text{virtual ground}$$

$$\frac{V_{in}}{R_{in}} + \frac{V_{out}}{R_f} = 0$$

$$\boxed{\frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}}}$$

## ② Non-inverting Amp.



$G_r \rightarrow \infty$

$$V_+ \approx V_-$$

$$V_- \approx V_{in}$$

$$V_{in} = \frac{R_2}{R_1 + R_2} V_{out}$$

$$G_r = 1 + \frac{R_1}{R_2}$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{R_1 + R_2}{R_2}}$$