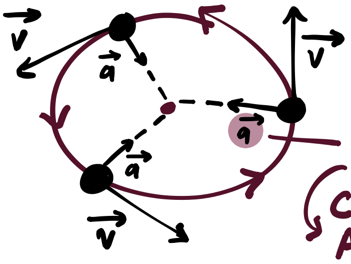


radially inward

Centripetal Acceleration

UNIFORM CIRCULAR MOTION

(uniform speed)

$$\boxed{a = \frac{v^2}{r}}$$

$$a \propto r$$

Period

$$\boxed{T = \frac{2\pi r}{v}}$$

circumference

Frequency $\boxed{f = \frac{1}{T}}$

* Uniform circular motion

Angular Speed / Frequency

(number of revolutions per second)

$$\boxed{\omega = \frac{2\pi}{T}}$$

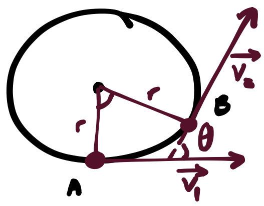
(rad/s)

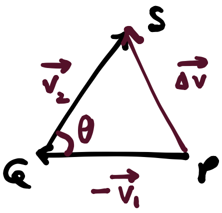
$$v = \frac{2\pi r}{T}$$

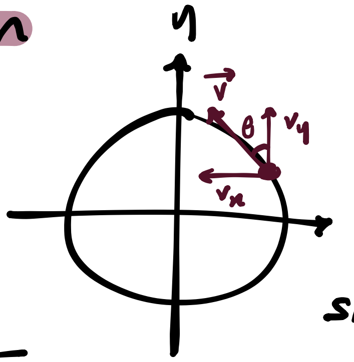
$$v = \omega r$$

$$\frac{v^2}{r} = \omega^2 r$$

Centripetal Acceleration







$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$
$$= (-v\sin\theta)\hat{i} + (v\cos\theta)\hat{j}$$

$$\sin\theta = \frac{y}{r}, \quad \cos\theta = \frac{x}{r}$$

$$\vec{a_c} = \left(-\frac{v^2}{r}\cos\theta\right)\hat{i} + \left(-\frac{v^2}{r}\sin\theta\right)\hat{j}$$

$$v_2 - v_1 = \Delta v$$

$$\frac{\Delta v}{AB} = \frac{v}{r}$$

$$AB = \text{arc } AB$$
$$= v\Delta t$$

$$\frac{\Delta v}{v\Delta t} = \frac{v}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

$$a = \frac{v^2}{r}$$

magnitude

direction: towards the center

Centripetal Force

$F = ma \longrightarrow (1)$

$a = \frac{v^2}{r} \longrightarrow (2)$

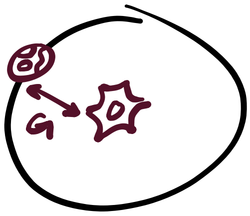
Net force!

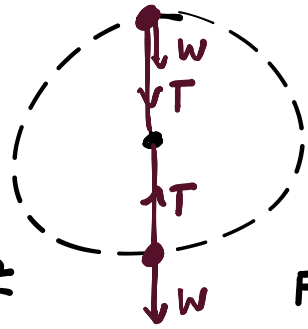
$$\boxed{F = m\frac{v^2}{r}}$$

not a force on its own!
it describes the source

* Centripetal force
* Application of circular motion
* Gravitational force

$F_c = G$
- gravitational
- electrostatic
- magnetic
- frictional
- tension

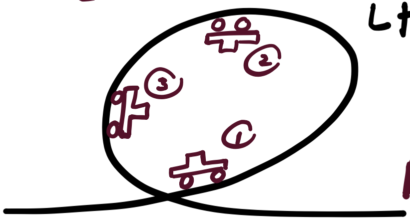




Top
$F_c = T + W$

Bottom
$F_c = T - W$

1 $F_c = F_N - W$
2 $F_c = F_N + W$
3 $F_c = F_N$



$$m\frac{v^2}{R} = \frac{Gm_1m_2}{r^2}$$

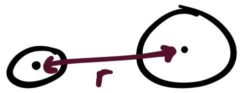
Gravitational Force

$$F \quad \boxed{\frac{GMm}{r^2} = m\frac{v^2}{r}}$$

gravitational constant
$6.67 \times 10^{-11}\,Nm^2kg^{-2}$

sub $F = ma$, gravitational acceleration

$$\boxed{a_g = \frac{GM}{r^2}}$$



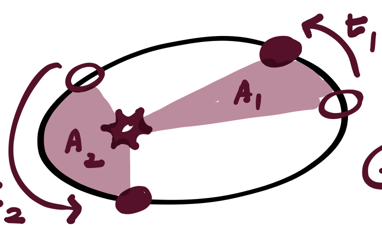
sub $v = \frac{2\pi r}{T}$,

$$\boxed{T^2 = \left(\frac{4\pi^2}{GM}\right)r^3}$$

Kepler's Laws

① Law of Orbits

All planets move in elliptical orbits, with the Sun at one focal point



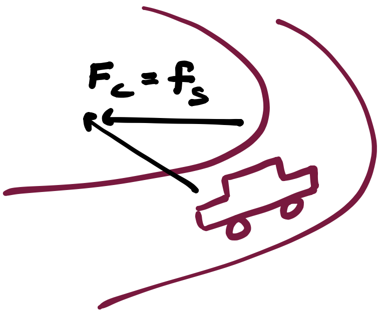
② Law of Areas

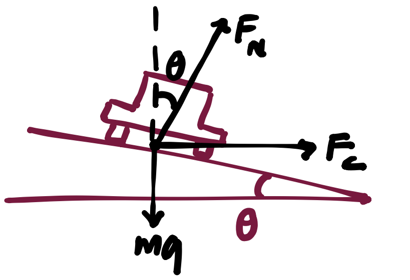
$A_1 = A_2$
$t_1 = t_2$ $\Rightarrow$ $\frac{dA}{dt}$ is constant

③ Law of Periods

$\frac{T^2}{r^3}$ is constant

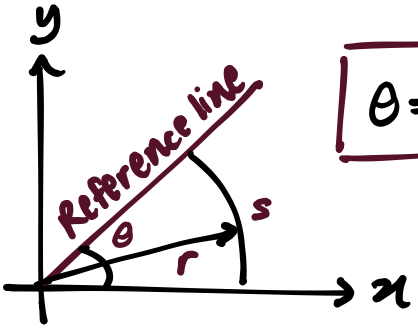
Rounding a corner on a tilted track





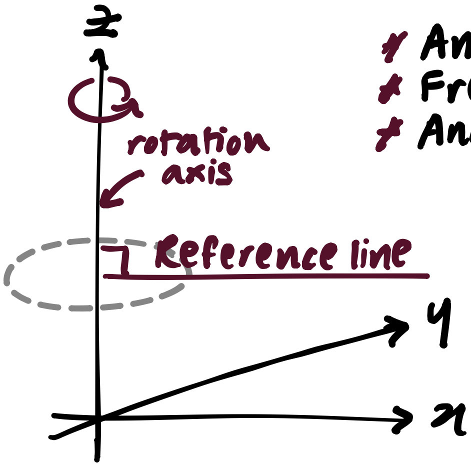
Angular Position

angle relative to a fixed direction (zero angular position)



$$\boxed{\theta = \frac{s}{r}}$$

rad!



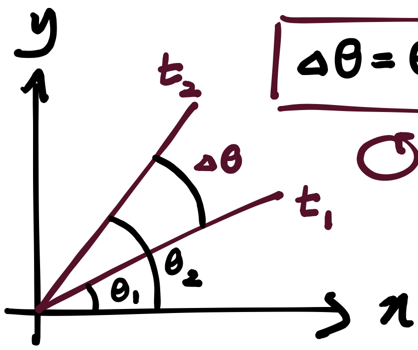
- Angular speed
- Frequency and period
- Angular acceleration

$1 \text{ rev} = 360^\circ$

$= \frac{2\pi r}{r}$

$= 2\pi \text{ rad}$

Angular Displacement



$$\boxed{\Delta\theta = \theta_2 - \theta_1}$$

↺ is +ve

Angular Velocity

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$$

$$\boxed{\omega = \frac{d\theta}{dt}}$$

rad s^{-1} or rev s^{-1} or rpm

Angular Acceleration

$$\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$$

$$\boxed{\alpha = \frac{d\omega}{dt}}$$

rad s^{-2} or rev s^{-2}

Acceleration in a Rotation

$$a = \sqrt{a_t^2 + a_c^2}$$

$$\theta = \tan^{-1}\frac{a_t}{a_c}$$

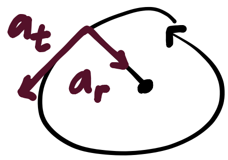
Translational		Rotational
$v = v_0 + at$	$\longrightarrow$	$\omega = \omega_0 + \alpha t$
$x = \frac{1}{2}(v_0 + v)t$	$\longrightarrow$	$\theta = \frac{1}{2}(\omega_0 + \omega)t$
$x = v_0 t + \frac{1}{2}at^2$	$\longrightarrow$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2ax$	$\longrightarrow$	$\omega^2 = \omega_0^2 + 2\alpha\theta$

In linear terms,

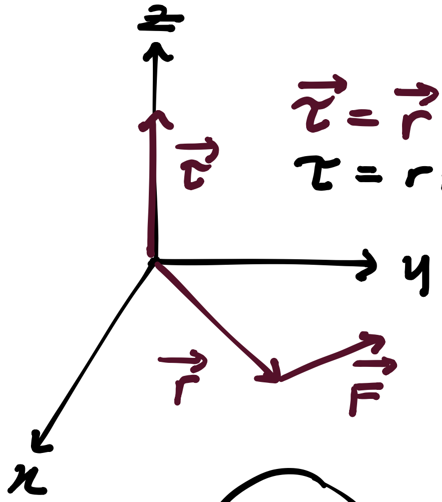
$$\left.\begin{aligned} s &= \theta r \\ v &= \omega r \\ a &= \alpha r \end{aligned}\right| \text{ in rad}$$

$$a_r = \frac{v^2}{r}$$

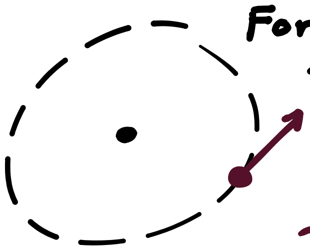
$$a_t = \omega^2 r$$



- Moment of Inertia
- calculation of the moment of inertia of solid bodies
- The parallel axis theorem
- Kinetic energy of rotation

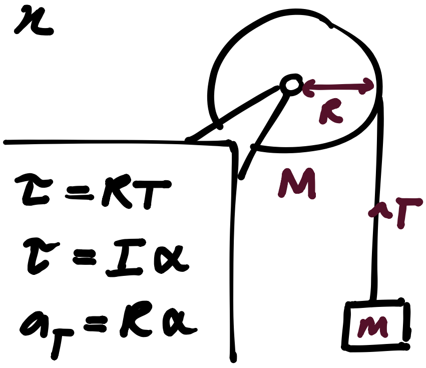


$$\vec{\tau} = \vec{r} \times \vec{F}$$
$$\tau = rF\sin\theta$$



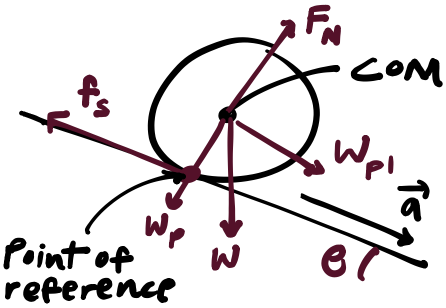
For a particle,
$$\tau = rF$$
$$= rma$$
$$= rm(r\alpha)$$
$$\tau = mr^2\alpha$$

For a body,
$$\tau_{net} = \tau_1 + \tau_2 + \ldots$$
$$\tau_{net} = (\Sigma m_i r_i^2)\alpha$$
$$\boxed{\tau_{net} = I\alpha}$$
center of mass



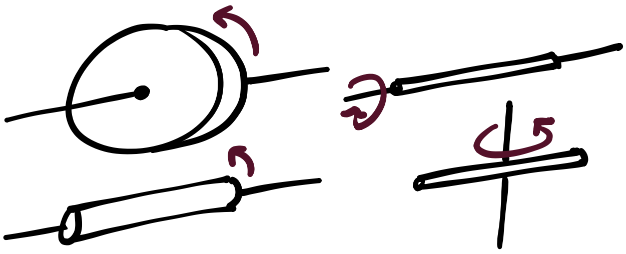
$$\tau = RT$$
$$\tau = I\alpha$$
$$a_T = R\alpha$$

$$RT = I\alpha$$
$$T = \frac{I\left(\frac{a_T}{R}\right)}{R}$$
$$T = \frac{I a_T}{R^2}$$



$$\tau = rF = rmg\sin\theta$$
$$I\alpha = I\frac{a}{r}$$
$$I\frac{a}{r} = rmg\sin\theta$$
$$a = \frac{r^2 mg\sin\theta}{I}$$

Moment of Inertia



different mass distribution

↓ inertia = easier to rotate

Rotational Inertia

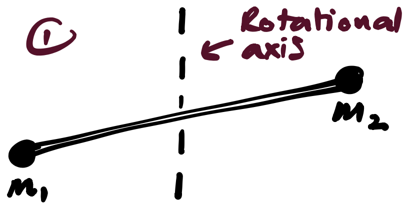
$$I = \Sigma m_i r_i^2$$

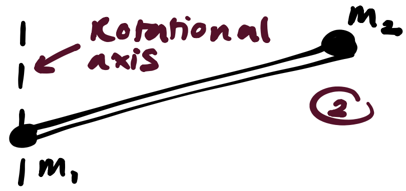
For a rigid body,

$$\boxed{I = \int r^2\,dm}$$

kg m²

①

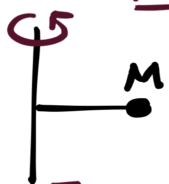




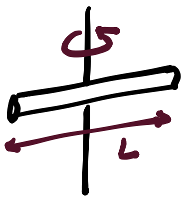
②

PARALLEL AXIS THEOREM

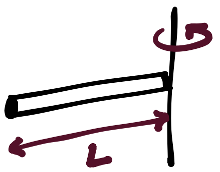
$$\boxed{I = I_{com} + Md^2}$$



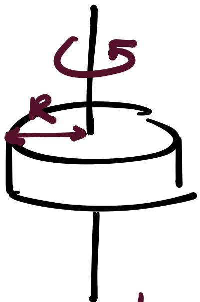
$I = MR^2$



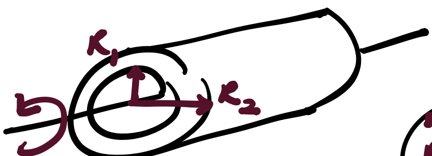
$I = \frac{1}{12}ML^2$



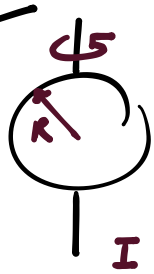
$I = \frac{1}{3}ML^2$



$I = \frac{1}{2}MR^2$



$I = \frac{1}{2}M(R_1^2 + R_2^2)$



$I = \frac{2}{5}MR^2$

①
$$I_{com} = m_1 r_1^2 + m_2 r_2^2$$
$$= m_1\left(\tfrac{1}{2}L\right)^2 + m_2\left(\tfrac{1}{2}L\right)^2$$
$$= \frac{m_1 L^2}{4} + \frac{m_2 L^2}{4}$$
$$I_{com} = \frac{mL^2}{2}$$

②
$$I_{end} = m_1 r_1^2 + m_2 r_2^2$$
$$= m_1(0) + m_2 L^2$$
$$I_{end} = mL^2$$

Rotational Kinetic Energy

$$K = \sum \frac{1}{2} m_i v_i^2$$

$$K = \frac{1}{2} m_i (\omega r_i)^2$$

$$\boxed{K = \frac{1}{2} I \omega^2}$$

$$\Delta K = W = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2$$

If τ is constant,

$$\boxed{W = \tau(\theta_f - \theta_i)}$$

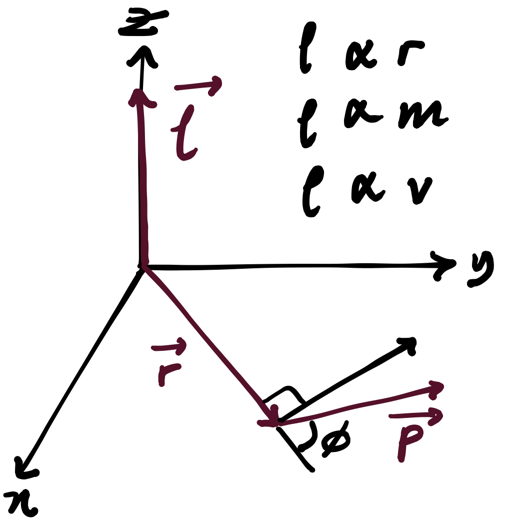
$$\boxed{P = \frac{dW}{dt} = \tau \omega}$$

Angular Momentum

$$\boxed{\vec{l} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})}$$

$$l = rmv\sin\phi$$

* Torque and rotational inertia
* Angular momentum
* Conservation of angular momentum
* Application of conservation of angular momentum



$l \propto r$

$l \propto m$

$l \propto v$

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

$$\Rightarrow \vec{\tau}_{net} = \frac{d\vec{l}}{dt}$$

$$\boxed{\tau = \frac{\Delta L}{\Delta t}}$$

Solid body / Particle

$$\boxed{L = I\omega}$$

Conservation of Angular Momentum

$$\boxed{\vec{L}_i = \vec{L}_f}$$

$$I_i \omega_i = I_f \omega_f$$

Simple Harmonic Motion

* oscillatory motion
* Simple Harmonic Motion (SHM)

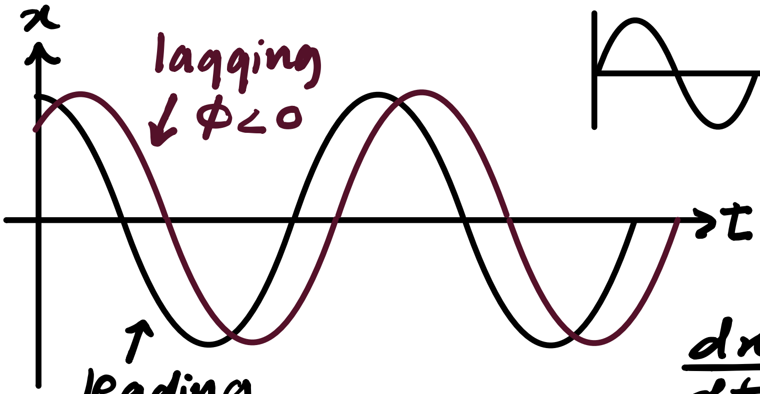
$x<0$ $x=0$ $x>0$

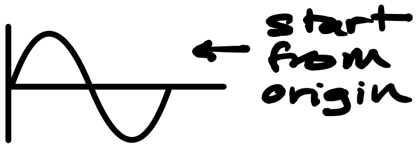
$v_x=0$ $v_x=v_{max}$ $v_x=0$

$a_x=a_{max}$ $a_x=0$ $a_x=-a_{max}$

$$\boxed{f=\frac{1}{T}}\ \text{Frequency}$$

$$\boxed{\omega=\frac{2\pi}{T}=2\pi f}\ \text{Angular frequency}$$





phase constant

$$\boxed{x(t)=x_m\cos(\omega t+\phi)}$$

$$\frac{dx}{dt}:\ \boxed{v(t)=-\omega x_m\sin(\omega t+\phi)}$$

$$\frac{dv}{dt}:\ a(t)=-\omega^2 x_m\cos(\omega t+\phi)$$

$$\boxed{a(t)=-\omega^2 x(t)}$$

Force

$$F=ma=(-\omega^2 x)m$$
$$=-m\omega^2 x$$
$$F=-kx$$
$$-kx=-m\omega^2 x$$

$$\boxed{T=2\pi\sqrt{\frac{m}{k}}}$$

$$\boxed{k=m\omega^2}\longrightarrow \omega=\sqrt{\frac{k}{m}}$$

Potential Energy

$$\boxed{U(t) = \frac{1}{2} k x_m^2 \cos^2(\omega t + \phi)}$$

Kinetic Energy

$$\boxed{K(t) = \frac{1}{2} k x_m^2 \sin^2(\omega t + \phi)}$$

* Energy in SHM

Total Energy

$$\boxed{\begin{aligned} E = U + K &= \frac{1}{2} k x_m^2 \\ &= \frac{1}{2} m v_m^2 \end{aligned}}$$

$$\tau = I\alpha$$

$$\alpha = -\frac{\tau}{I}$$

$$= -\frac{mgL\sin\theta}{I}$$

$$\boxed{\alpha = -\frac{mgL}{I}\theta}$$

$$\alpha(t) = -\omega^2 x(t)$$

$$T = 2\pi\sqrt{\frac{I}{mgL}} \longrightarrow \boxed{T = 2\pi\sqrt{\frac{I}{mgh}}}$$

Physical

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

Simple

$$\boxed{v = \sqrt{\frac{k}{m}(x_m^2 - x^2)}}$$