PHYSICAL QUANTITIES AND UNITS

physical unit, magnitude: $m = 23.8\,kg$ (unit)

* Magnitude and unit of physical quantities
* SI base quantities and their units

System of Units

- CGS (centimeter, gram, second)
- MKS (meter, kilogram, second) ✓
- FPS (foot, pound, second)

Base standards

1 length
2 time
3 mass
4 thermodynamics temp.
5 electric current
6 amount of substance
7 luminous intensity

Derived units

- from base units

eg. velocity $= \frac{length}{time}$

$1\,\text{Newton} = 1\,N = 1\,kg\,m\,s^{-2}$

HOMOGENEITY of Physical Equation

✓ $[v] = [v] + [at]$ ← dimension

$$ms^{-1} = ms^{-1} + \underbrace{(ms^{-2})(s)}_{ms^{-1}}$$

✗ $[v] = [v] + \frac{1}{2}[at^2]$

$$ms^{-1} = ms^{-1} + \underbrace{(ms^{-2})(s^2)}_{m}$$

PREFIXES

Submult	Prefix	Char	Mult	Prefix	Char
10^{-1}	deci	d	10	deca	da
10^{-2}	centi	c	10^{2}	hecto	h
10^{-3}	milli	m	10^{3}	kilo	k
10^{-6}	micro	u	10^{6}	mega	M
10^{-9}	nano	n	10^{9}	giga	G
10^{-12}	pico	p	10^{12}	tera	T
10^{-15}	femto	f	10^{15}	peta	P
10^{-18}	atto	a	10^{18}	exa	E
10^{-21}	zepto	z	10^{21}	zetta	Z
10^{-24}	yocto	y	10^{24}	yotta	Y

Unit Conversion

$$260\,cm = 260\,\cancel{cm} \times \frac{1\,m}{100\,\cancel{cm}} = 2.60\,m$$

$$5'3'' = 5\,ft\left(\frac{12\,in}{1\,ft}\right)\left(\frac{0.0254\,m}{1\,in}\right) + 3\,in\left(\frac{0.0254\,m}{1\,in}\right)$$

$$5'3'' = 1.6\,m$$

scientific notation

$28600 = \underbrace{2.86 \times 10^4}$

3 significant figures

2.860×10^4, 4 s.f.

300, 1 s.f.

300.0, 4 s.f.

0.0070, 2 s.f.

Multiplication/Division

$$\underset{3sf}{12.8} \times \underset{2sf}{5.1} = 65.28$$

→ 65

least significant figures

Addition/Substraction

→ $25.\underline{37} + 3.\underline{1} = 28.47$

2dp, 1dp

final answer = 28.5 (least decimal place)

$5.3 - 0.27 = 5.03$

→ 5.0

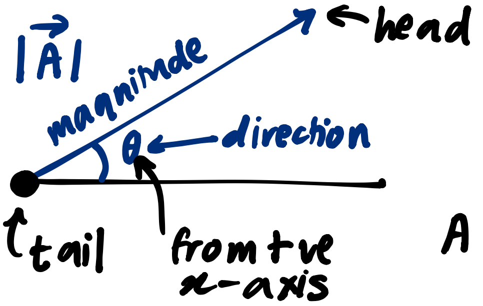
scalar
- physical quantity
- magnitude

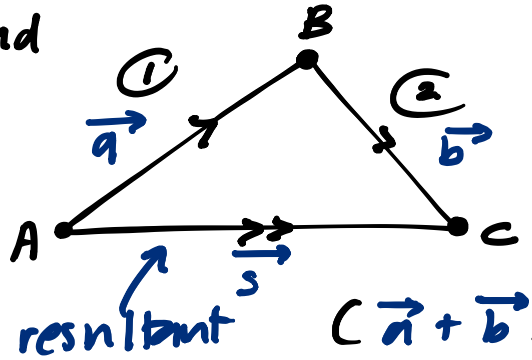
} temperature, pressure, energy

vector
- physical quantity
- magnitude
- direction

} displacement, velocity, acceleration

* Difference between scalar and vector quantities
* Resolution of vectors using trigonometry
* Manipulation of vectors
* Problem-solving using vectors



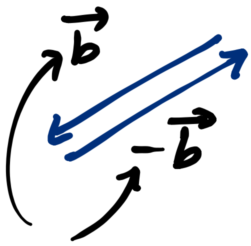


vector sum / resultant

$\vec{AB} + \vec{BC} = \vec{AC}$

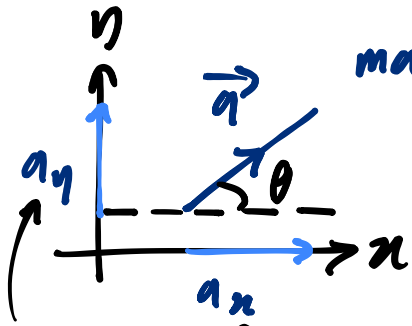
$\vec{a} + \vec{b} = \vec{s}$
$\vec{b} + \vec{a} = \vec{s}$ } commutative law

$\underbrace{(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})}_{\text{associative law}}$



$\vec{b} + (-\vec{b}) = 0$

same magnitude, opposite direction



components of vector a (resolved)

magnitude only

$a_x = a\cos\theta$

$a_y = a\sin\theta$

$$\left[a = \sqrt{a_x^2 + a_y^2}\right]$$

$$\left[\tan\theta = \frac{a_y}{a_x}\right]$$

UNIT VECTOR

- magnitude = 1 unit
- ✓ direction
- ✗ dimension

axis:	x	y	z
label:	$\hat{i}$	$\hat{j}$	$\hat{k}$

$\vec{a} = a_x\hat{i} + a_y\hat{j}$ ← vector component

magnitude (scalar!)

direction

Addition

1. resolve the vectors into scalar components
2. combine the scalar components
3. combine the components ⇒

$\vec{r} = \vec{a} + \vec{b}$

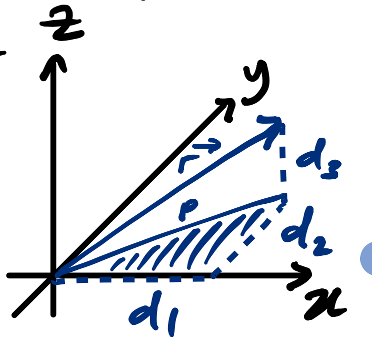
$\vec{r}_x = \vec{a}_x + \vec{b}_x$

$\vec{r}_y = \vec{a}_y + \vec{b}_y$

$\vec{r}_z = \vec{a}_z + \vec{b}_z$

} component of r ↕ component of (a+b)

3D Vector



$p^2 = d_1^2 + d_2^2$

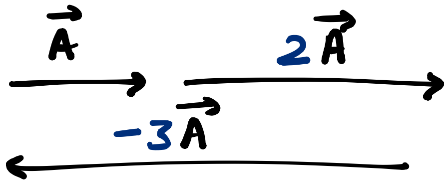
$r^2 = p^2 + d_3^2$

$r^2 = d_1^2 + d_2^2 + d_3^2$

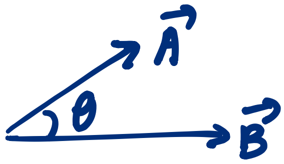
$r = \sqrt{d_1^2 + d_2^2 + d_3^2}$

[Vector Multiplication]

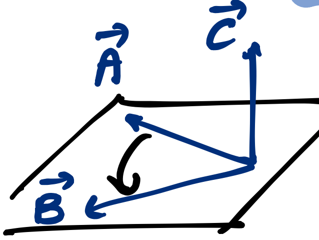
① Scalar × Vector



B Product is VECTOR (Cross Product)



$\vec{A}\times\vec{B}$
$= AB\sin\theta\,\hat{n}$
$= \vec{C}$ ← new vector!



right hand rule

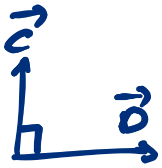
$\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$ } anti-comm

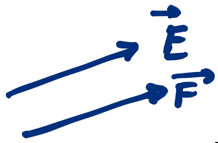
$\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$

distributive (addition)

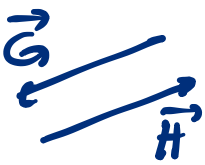
$|\vec{C}\times\vec{D}| = CD\sin 90° = CD$

max magnitude





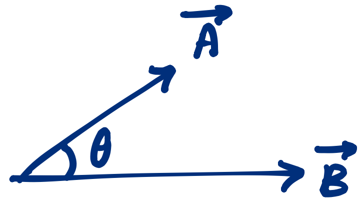
$\vec{E}\times\vec{F} = EF\sin 0°\,\hat{n} = \vec{0}$ ← null



$\vec{G}\times\vec{F} = GF\sin 180°\,\hat{n}$
$\vec{G}\times\vec{F} = \vec{0}$ ← null

② Vector × Vector

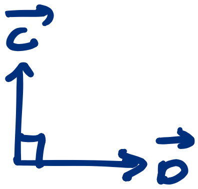
A Product is SCALAR (Dot product)



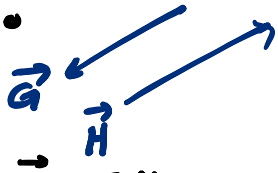
$\vec{A}\cdot\vec{B} = AB\cos\theta$ ← angle between the two vectors

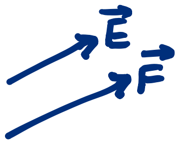
$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
$AB\cos\theta = BA\cos\theta$ } commutative law

$A\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$ } distributive law



$\vec{C}\cdot\vec{D} = CD\cos 90°$
$\vec{C}\cdot\vec{D} = 0$





$\vec{E}\cdot\vec{F} = EF\cos 0°$
$\vec{E}\cdot\vec{F} = EF$
max +ve

$\vec{G}\cdot\vec{H} = GH\cos 180°$
$\vec{G}\cdot\vec{H} = -GH$
max −ve

parallel { $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = (1)(1)\cos 0° = 1$

perpendicular { $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = (1)(1)\cos 90° = 0$

$\vec{a}\cdot\vec{b} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k})\cdot(b_x\hat{i} + b_y\hat{j} + b_z\hat{k})$

$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$

$a_x b_y(\hat{i}\hat{j}) = 0$ $a_x b_x(\hat{i}\hat{i}) = 1$

$a_x b_x$

$\boxed{\theta = \cos^{-1}\left[\frac{\vec{A}\cdot\vec{B}}{AB}\right]}$ ← angle between two lines

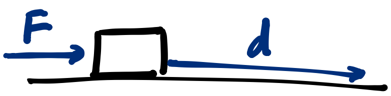
$\hat{i}\times\hat{j} = \hat{k}$ $\hat{j}\times\hat{i} = -\hat{k}$
$\hat{j}\times\hat{k} = \hat{i}$ $\hat{k}\times\hat{j} = -\hat{i}$
$\hat{k}\times\hat{i} = \hat{j}$ $\hat{i}\times\hat{k} = -\hat{j}$

$\vec{a}\times\vec{b} = (a_y b_z - b_y a_z)\hat{i} + (a_z b_x - b_z a_x)\hat{j} + (a_x b_y - b_x a_y)\hat{k}$

Dot Product

(work done)

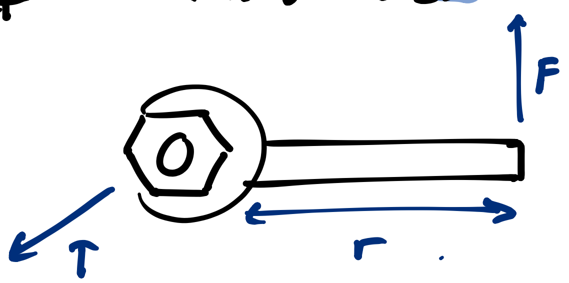
$W = F\cdot d$
scalar, vector



Cross Product

(torque)

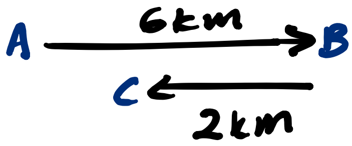
$\tau = r\times F$
vector



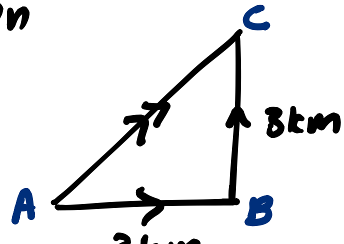
- Definitions of displacement, velocity, and accelaration
- Graphical representation of kinematic quantities
- Equation of uniformly accelerated motion
- Slope of $x-t$ graph to determine velocity
- Effect of gravity on moving objects
- Problem-solving related to projectile motion

Distance vs Displacement

Distance	Displacement
· scalar	· vector
· total length	· linear distance (straight line)
always +ve	+ve/−ve indicates direction



Distance = 8km
Displacement = 4km



Distance = 6km
Displacement $= \sqrt{3^2+3^2} = \sqrt{18}$
magnitude + direction
$\theta = \tan^{-1}\left(\frac{3}{3}\right)$
$\theta = 45°$ NoE

Average Speed VS Velocity

Average Speed $= \dfrac{\text{total distance}}{\text{total time}}$

Velocity → INSTANTANEOUS

$$\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

gradient of tangent!

AVERAGE

$$\frac{x_2 - x_1}{t_2 - t_1}$$

		Direction +ve	Direction −ve
Speed	↑	+	−
	↓	−	+

Acceleration

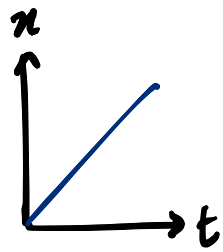
Average

$$\frac{v_2 - v_1}{t_2 - t_2}$$

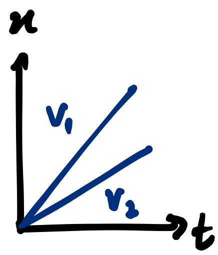
Instantaneous

$$\lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

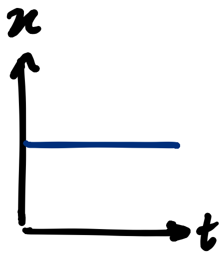
Displacement-time Graph



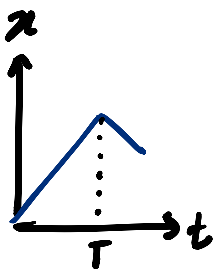
constant velocity



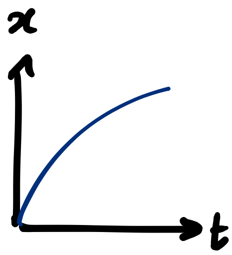
$v_1 > v_2$



$v = 0$ stationary

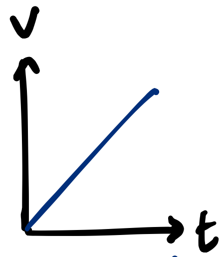


change direction at T

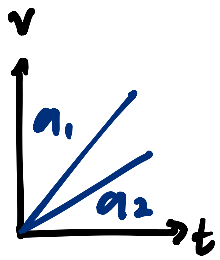


decreasing velocity

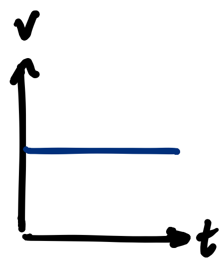
Velocity-time Graph



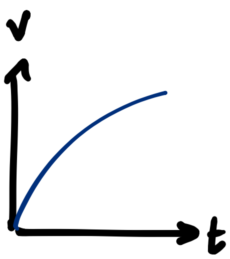
constant a



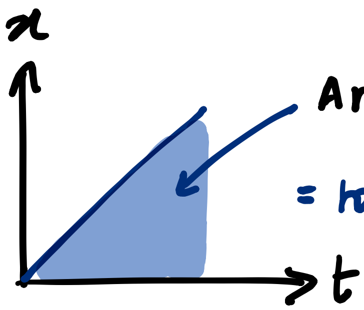
$a_1 > a_2$



$a = 0$ constant v



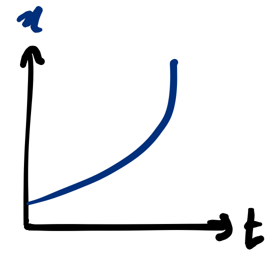
decreasing acceleration

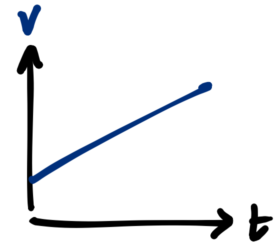


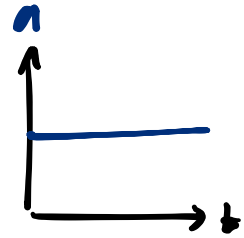
Area under graph = total distance

Uniformly Accelerated Motion

if acceleration is constant,
average acceleration = instantaneous acceleration







$$a = \bar{a} = \frac{v-u}{t-0}$$

$$\boxed{v = u + at}$$

$$\boxed{x - x_0 = ut + \frac{1}{2}at^2}$$

$$\boxed{\begin{array}{l} v^2 = u^2 + 2a(x - x_0) \\ x - x_0 = vt - \frac{1}{2}at^2 \end{array}}$$

$$\bar{v} = \frac{x - x_0}{t - 0}$$

$$x = x_0 + \bar{v}t$$

$$\bar{v} = \frac{1}{2}(u+v)$$

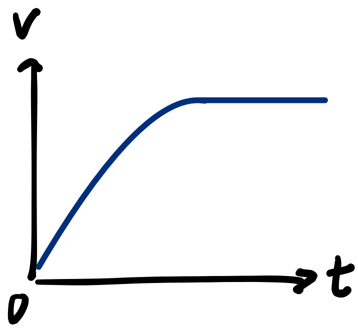
$$\boxed{x - x_0 = \frac{1}{2}(u+v)t}$$

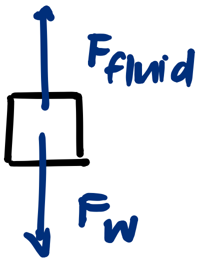
FREE FALL ACCELERATION

$$F_w = mg \leftarrow 9.81\,ms^{-2}$$

weight mass

Terminal velocity in fluids

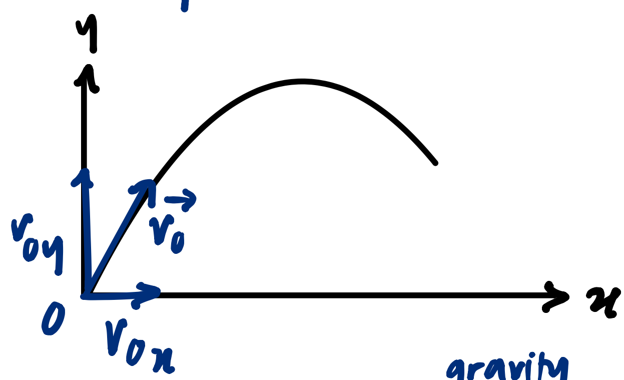




$F_{net} = 0$

MOTION IN 2D

Projectile Motion



Vertical Motion

$a_y = -g$

gravity only

$$\boxed{\vec{v} = v_x\hat{i} + v_y\hat{j}}$$

no resistance

Horizontal Motion

$a_x = 0$

$v_x = u_x + at$

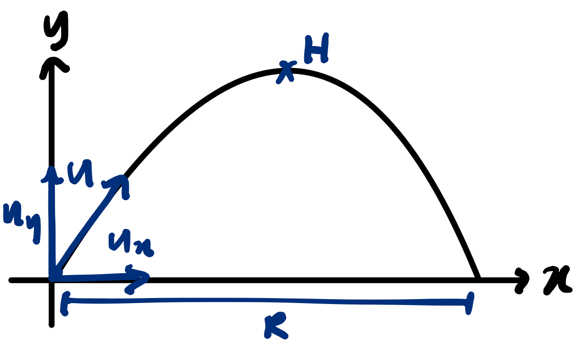
$v_x = u_x + 0t$

$v_x = u_x$

$x = u_x t + \frac{1}{2}at^2$

$x = u_x t + \frac{1}{2}(0)t^2$

$x = u_x t$



$x - x_0 = R, \quad y - y_0 = 0$

$R = u_x t = u\cos\theta\, t$

$0 = u_y t - \frac{1}{2}gt^2 = u\sin\theta\, t - \frac{1}{2}gt^2$

$R = \frac{2u^2}{g}\sin\theta\cos\theta = \frac{u^2}{g}\sin 2\theta$

$v_y^2 = (u\sin\theta)^2 - 2g(y - y_0)$

$0 = (u\sin\theta)^2 - 2gH$

$H = \frac{u^2}{2g}\sin^2\theta$

NEWTON'S LAWS AND LINEAR MOMENTUM

- Newton's Laws of motion
- Difference between weight and mass
- Concepts of impulse and momentum
- Law of conservation of momentum
- Problem solving in one dimension

① Newton's First Law

— state of an object at a moment

Every body will remain at rest or uniform motion in a straight line unless a net external force acts on it

aka. Law of Inertia

↑ mass

↑ inertia

↳ reluctance of a body to change its state of rest / motion

② Newton's Second Law

— a particular body

The rate of change of momentum of a body is directly proportional to the net force acting on it and moves in the direction of the force.

Momentum = mv

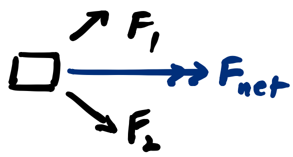
$$\boxed{\vec{F}_{net} = m\vec{a}}$$

↳ $a \propto F_{net}$

$$F = \frac{\Delta p}{t}$$

FORCE push/pull

Principle of superposition of forces



③ Newton's Third Law

— two different bodies

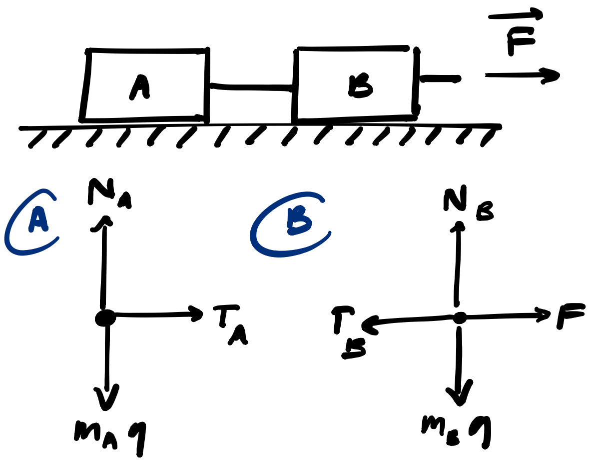
$F_1 = F_2$, $\vec{F}_1 = -\vec{F}_2$

WEIGHT gravitational attraction

$F_W = mg$

weight ≠ mass

FREE BODY DIAGRAM

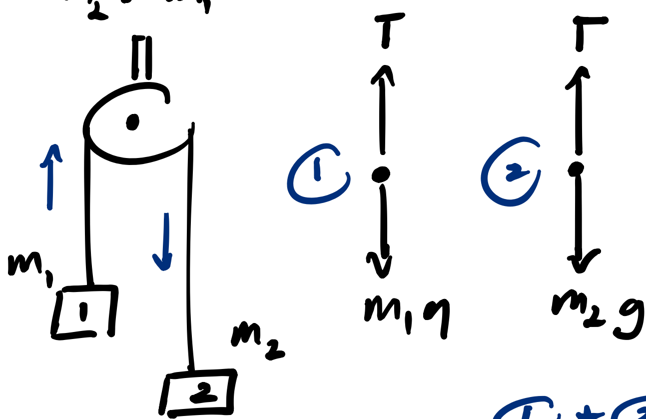


$$a = \frac{F}{m_A + m_B}$$

$$\begin{cases} T = m_A a \\ F - T = m_B a \end{cases}$$

$$T = m_A a$$

$m_2 > m_1$



1 : $T - m_1 g = m_1 a$

2 : $m_2 g - T = m_2 a$

① + ② :

$m_2 g - m_1 g = (m_1 + m_2) a$

Linear Momentum

product of mass and velocity

$$\boxed{\vec{p} = m\vec{v}} \quad kg\,ms^{-1}$$

same direction

Total momentum $= m_1\vec{v}_1 + m_2\vec{v}_2$

change of momentum $= m(v-u)$

Conservation of Momentum

$$\boxed{\vec{p}_i = \vec{p}_f}$$

in a closed, isolated system

Perfectly Inelastic Collision

- stick together after collision

Impulse

$$\boxed{J = \bar{F}\Delta t}$$

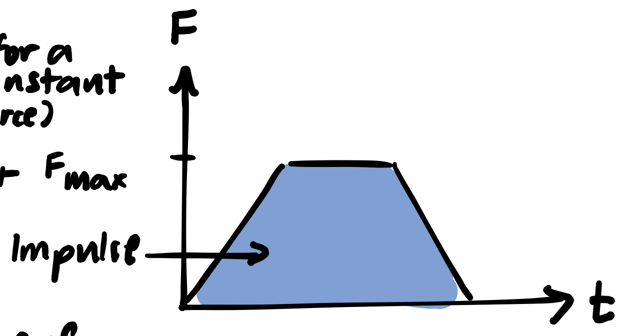
(for a constant force)

average force

contact time

$$\boxed{\bar{F}\Delta t = \Delta p}$$

change of momentum



Elastic and Inelastic Collision

both conserve momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \rightarrow (1)$$

kinetic energy is conserved

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \rightarrow (2)$$

From (1) and (2):

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2}{m_1 + m_2}u_2$$

$$v_2 = \frac{2m_1}{m_1 + m_2}u_1 + \frac{m_2 - m_1}{m_1 + m_2}u_2$$

① Gravitational Force (weight!) └ two masses attracted to each other

$$\boxed{F = \frac{G m_1 m_2}{r^2}}$$

→ $W = mg$

② Electrical Force └ between charges

$$\boxed{\frac{k Q_1 Q_2}{r^2}}$$

③ Magnetic Force

like poles repel
unlike poles attract

④ Viscous Force (fluid resistance)

↑ speed, ↑ force

⑤ Upthrust Force

Archimedes' Principle : = weight of fluid displaced

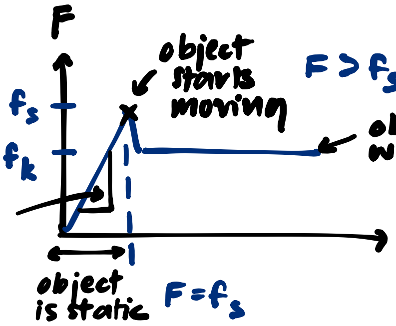
* Origin of upthrust acting on a body in a fluid
* Frictional force and viscous force
* vector triangle representation of forces in equilibrium
* Concept of the center of gravity
* Criteria required for equilibrium
* Moment of a force

⑦ Frictional Force (two surfaces in contact) → Static Frictional Force, f_s → Kinetic Frictional Force, f_k

↳ dissipate energy

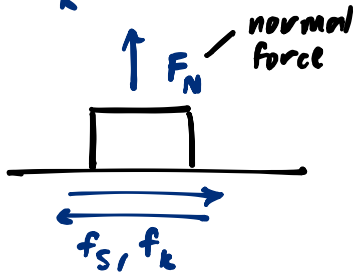
$\mu_s > \mu_k$

coefficient of f_s μ_s

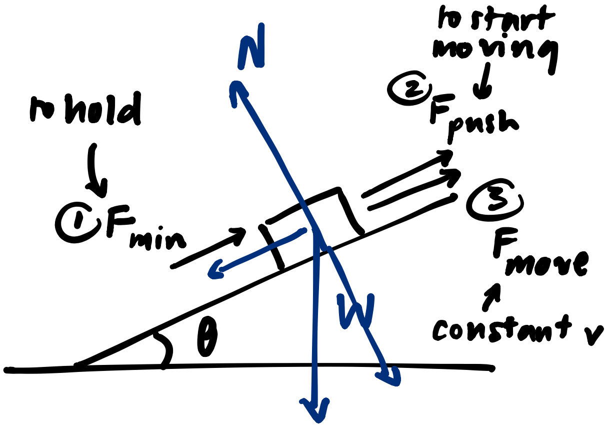


$F > f_{s,max}$

object moves w constant velocity $F = f_k$



$\mu_s = \frac{f_s}{F_N}$ $\mu_k = \frac{f_k}{F_N}$



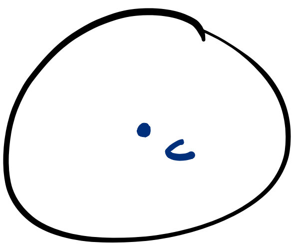
$F_{min} + f_s = F_W \sin\theta = 0$ $\mu_s F_N$

① $F_{min} = mg\sin\theta - \mu_s mg\cos\theta$

② $F_{push} = F_W \sin\theta + F_s$

③ $F_{move} = F_W \sin\theta + F_k$

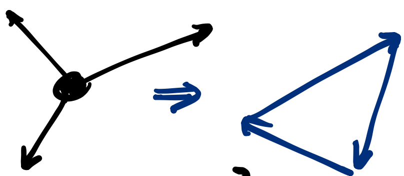
Centre of Gravity
└ point where the whole weight acts



Stability
└ low CoG
└ wide base

Coplanar Forces

$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$



closed triangle / polygon

Moments / Torques

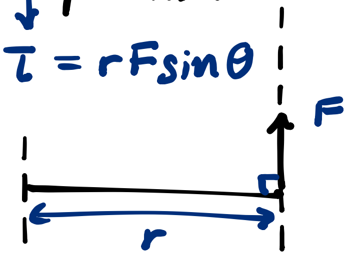
↳ turning effect

$\vec{\tau} = \vec{r} \times \vec{F}$ ~~$\vec{F} \times \vec{r}$~~

↳ Nm

cross product

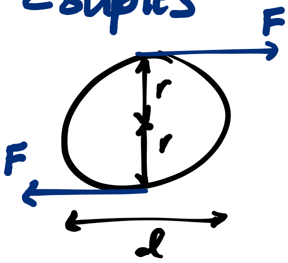
$\tau = rF\sin\theta$



anticlockwise

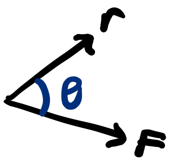
⇒ +ve

Couples



$F - F = dF$

$C_p = dF$



EQUILIBRIUM

Translational Equilibrium

$\Sigma \vec{F} = \vec{F}_{net} = 0$

Rotational Equilibrium

Σ clockwise moment = Σ anticlockwise moment

$\Sigma \vec{\tau} = \vec{\tau}_{net} = 0$

- Concepts of work and energy
- Potential and kinetic energy
- Conservation of Mechanical Energy
- Power

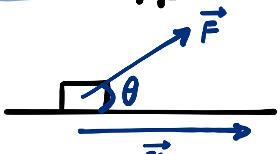
ENERGY (J)

└ ability to do work

- cannot be destroyed or created } **Principle of Conservation of Energy**

WORK (energy transfer to an object)

└ when a force displaces an object in the **direction of the force**



$W = Fx\cos\theta$

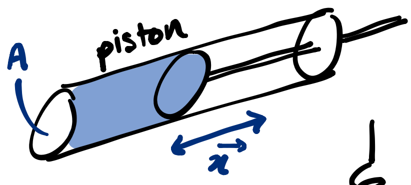
$$\boxed{W = \vec{F}\cdot\vec{x}}$$

Scalar!

+ve: work done by the force

−ve: work done on the force

Work Done by an Expanding Gas

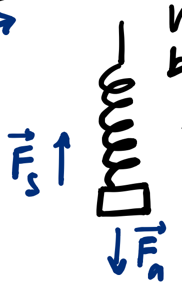


$W = Fx$

$W = PAx$ (constant)

$$\boxed{W = P\Delta V}$$

Work Done by a Spring



$\vec{F}_s = -k\vec{x}$ (spring constant)

$\vec{F}_a = k\vec{x}$

$$\boxed{W_a = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2}$$

$+x \leftrightarrow -F_s$

KINETIC ENERGY

└ in every object in motion

$$\boxed{K = \frac{1}{2}mv^2}$$

$W > 0 \rightarrow K\uparrow$

$W = 0 \rightarrow K\leftrightarrow$

$W < 0 \rightarrow K\downarrow$

W_{net}

$W_1 + W_2$

or

$\vec{F}_{net}\cdot\vec{x}$

$v^2 = u^2 + 2a_x x$

$W = F_x x$

$= ma_x x$

$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$$\boxed{W = K_f - K_i}$$

Gravitational P.E., U

$W_g = mgh$ (work done by gravity)

$$\boxed{U = mgh}$$

$U = -W_g$

Nonconservative Force

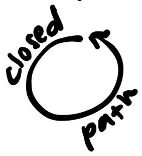
ie. kinetic energy

frictional force $\rightarrow$ thermal energy

Conservative Force

$W_1 = -W_2$

ie. gravitational force



$\Sigma W = W_1 + W_2$

$\Sigma W = mgh - mgh$

$\Sigma W = 0$

[POWER]

└ rate of work done

$$\boxed{P_{avg} = \frac{W}{\Delta t}}$$

(W: total work)

instantaneous

$$\boxed{P = \frac{dW}{dt} \quad \text{or} \quad P = \vec{F}\cdot\vec{v}}$$

electrical

$$\boxed{P = VI}$$

$1\,kWh = (1000W)(60\times60s)$

Energy Loss & Efficiency

E input = useful E out + wasted E

$$\boxed{\text{Efficiency} = \frac{E_{out}}{E_{in}}\times100\%}$$

Mechanical Energy

(in an isolated system)

$E_{mec} = K + U$ (kinetic energy, potential energy)

$$\boxed{\begin{gathered}K_1 + U_1 = K_2 + U_2\\ \Delta E_{mec} = \Delta K + \Delta U = 0\end{gathered}}$$

(no energy is lost)