

ELECTRIC FIELDS

Positive Charge \oplus **Negative Charge** \ominus
 • protons • electrons
 • +ve ions • -ve ions

repulsive force

Unlike Charges **Like Charges**

$\oplus \rightarrow \leftarrow \ominus$ $\leftarrow \oplus \oplus \rightarrow$
 attractive force same magnitude opposite direction

$\begin{array}{c} \oplus \\ \ominus \\ \oplus \\ \ominus \end{array}$ } equal amounts of +ve & -ve charge
 $=$ electrically neutral

- * Electric Charge
- * Coulomb's Law
- * Concept of Electric Field
- * Electric Field of a Point Charge
- * Uniform Electric Field
- * Electric Potential



neutral materials are rubbed, electrons (experience friction!) are rubbed off

mass of electron $<$ proton
 (more easily!)

PRINCIPLE OF CONSERVATION OF CHARGE

in a closed / isolated system,

total charge before interaction = total charge after interaction

} transfer!
 x created
 x destroyed



ELECTRIC FIELD

a region where charged particles experience force

ELECTROSTATIC FORCE

Newton's Third Law { equal magnitude opposite direction

COULOMB'S LAW



$$F \propto Q_1 Q_2$$

$$F \propto \frac{1}{r^2}$$

$$F \propto \frac{Q_1 Q_2}{r^2}$$

depends on medium

$$k = \frac{1}{4\pi\epsilon_0}$$

① $F = k \frac{Q_1 Q_2}{r^2}$ ② permittivity of free space (vacuum)

relative to other medium $\epsilon_r > 1$

$$\frac{1}{4\pi\epsilon_0\epsilon_r} > \frac{1}{4\pi\epsilon_0\epsilon_0}$$

①, ②:

$\hookrightarrow 8.85 \times 10^{-12} C^2/Nm^2$ relative permittivity

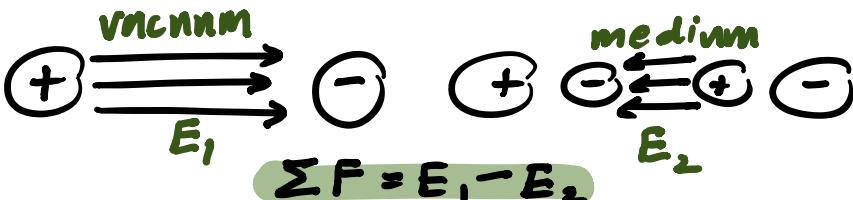
$$\text{minimum!} \uparrow$$

$$(9 \times 10^9) \frac{Q_1 Q_2}{r^2}$$

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$$

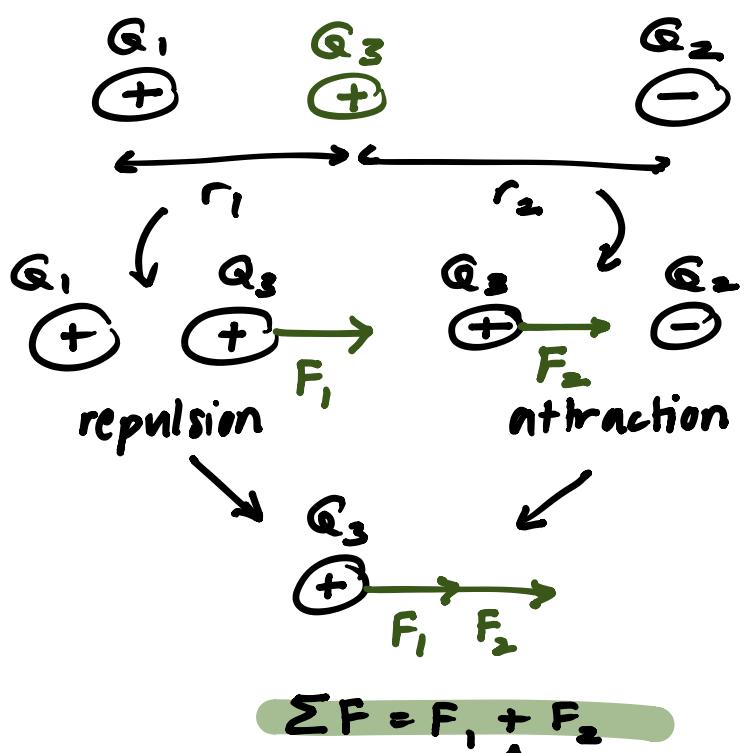
$$\epsilon_r = \frac{\epsilon_{\text{medium}}}{\epsilon_0 \text{ vacuum}}$$

ratio
 • the factor by which the Coulomb force decreased relative to in a vacuum



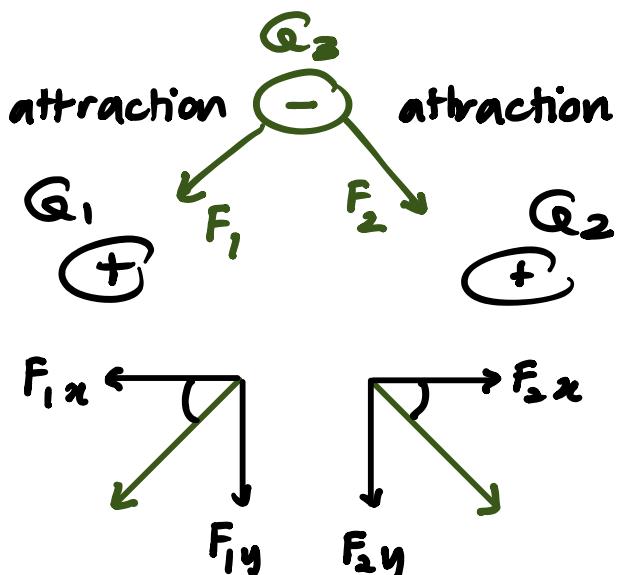
$$\sum F = E_1 - E_2$$

FORCES in Vector



$$\sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$$

in regards to direction
not charge! (vector)

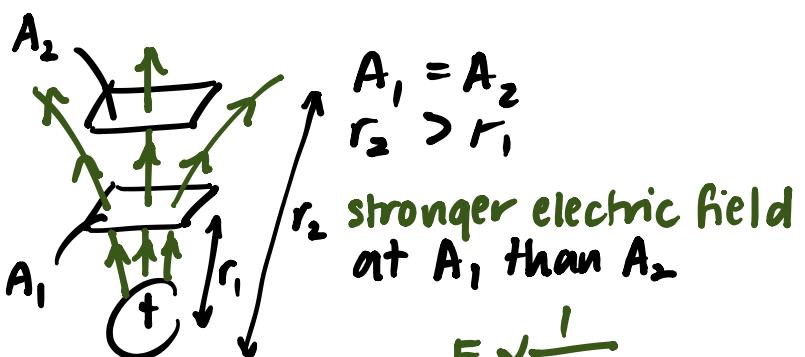
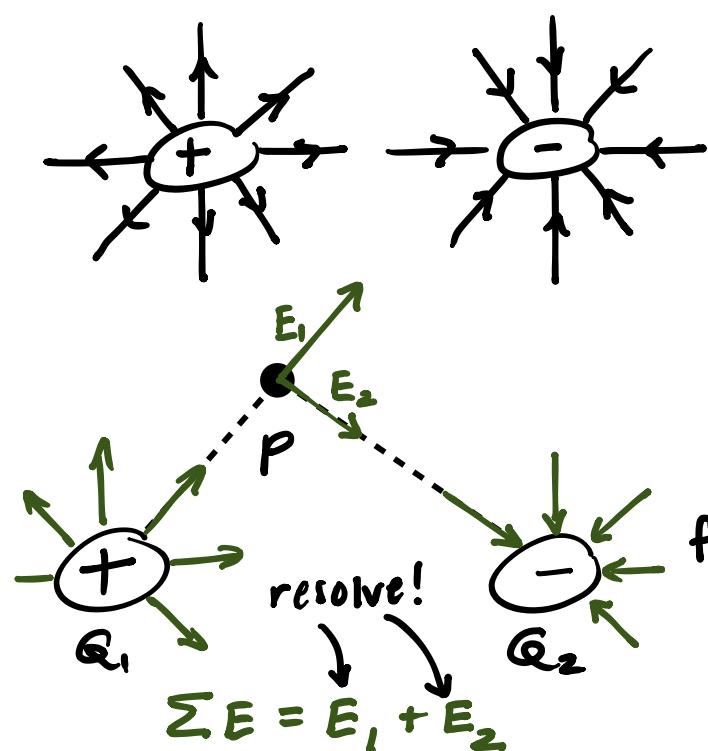


$$\sum F_x = -F_{1x} + F_{2x}$$

$$\sum F_y = -F_{1y} - F_{2y}$$

$$\sum \mathbf{F} = \sqrt{\sum F_x^2 + \sum F_y^2}$$

ELECTRIC FIELD



vector!

$$E = \frac{F}{Q_2}$$

towards negative
positive
test charge

$\sum E = E_1 + E_2$

force experienced by Q_2

$$E = k \frac{Q_1 Q_2}{r^2 Q_2} = k \frac{Q_1}{r^2}$$

strength of electric field generated by Q_1 at distance r

Q_1 is source of electric field

Uniform Electric Field

$$W_{EPE} = QV$$

Electric Potential Energy
 $\downarrow \uparrow$
 Kinetic Energy

$$W_{EXT} = Fd$$

$$W_{EXT} = W_{EPE}$$

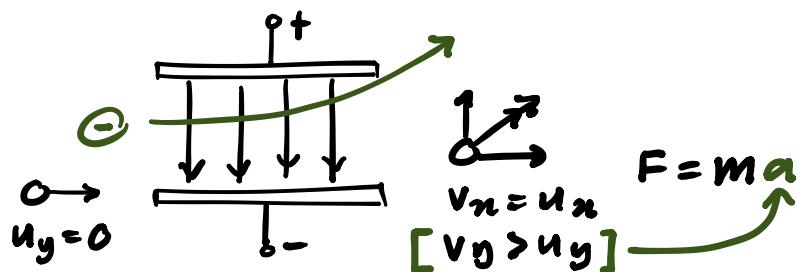
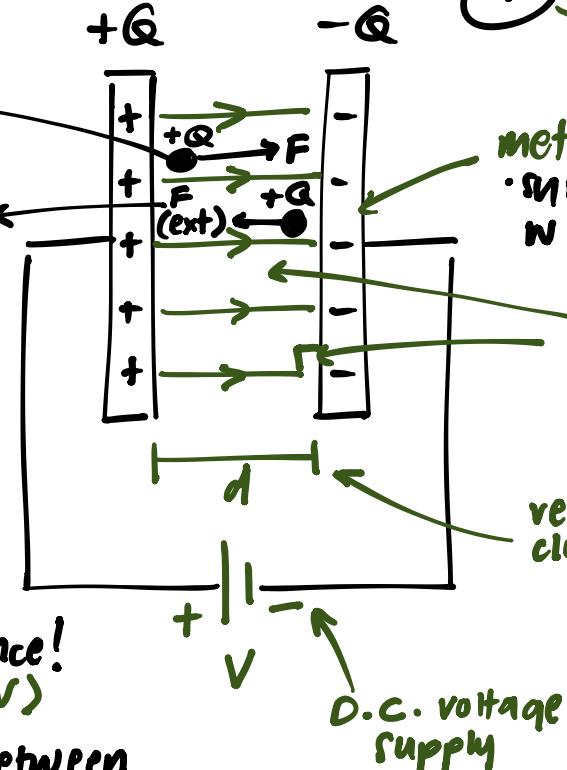
$$Fd = QV$$

$$\frac{F}{Q} = \frac{V}{d}$$

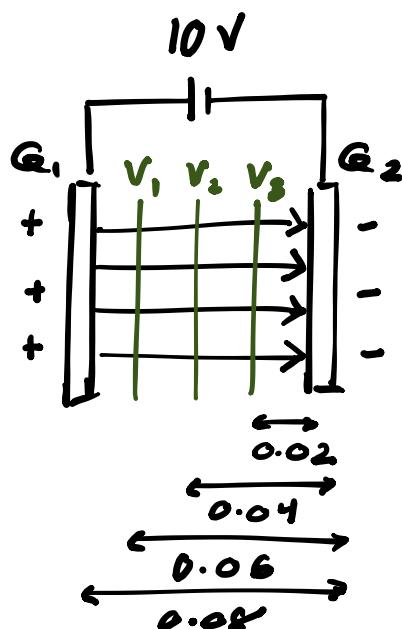
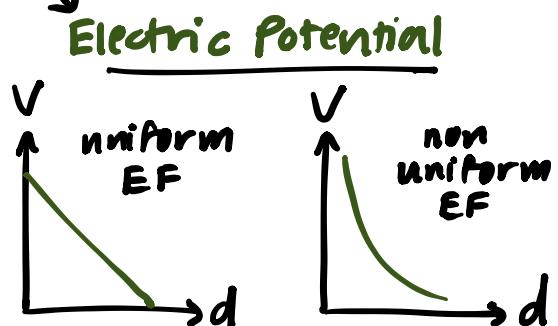
$$E = \frac{V}{d}$$

potential difference!
 $+V - (-V)$

distance between plates



Scalar!
 ↓
 I.S.T of energy supplied to 1C of charge



$$E = \frac{V}{d}$$

$$= \frac{10V}{0.08m}$$

$$= 125 \text{ NC}^{-1}$$

$$V_1 = (125)(0.06) = 7.5V$$

$$V_2 = (125)(0.04) = 5.0V$$

$$V_3 = (125)(0.02) = 2.5V$$

$$V_1 > V_2 > V_3$$

$$M = E = -\frac{V}{r}$$

by the test charge, q_2

$$q_2 V = W$$

$$V = \frac{W}{q}$$

$$E = k \frac{Q}{r^2}$$

$$V = k \frac{Q}{r}$$

set up by Q_1

E gained by q_2

consider +ve & -ve Q !

$$V = 0$$

$$+2 \mu C$$

(+)

$$-2 \mu C$$

(-)

$$V = +1 \quad V = -1$$

CURRENT OF ELECTRICITY

2

Circuit Symbols

| Symbol | Component Name | Symbol | Component Name | Symbol | Component Name |
|--------|--|--------|--------------------------------|--------|----------------------------|
| — | Connecting lead | ~~~~~ | Alternating signal (a.c.) | —m— | Inductor (solenoid) |
| -+ | Cell | V | Voltmeter | → | Diode |
| +--- | Battery of cells | A | Ammeter | →↑ | Light-emitting diode (LED) |
| —o— | Power supply / accumulator | G | Galvanometer | ⊗ | Filament lamp |
| — — | Switch | — — | Fixed resistor | — — | Fuse |
| —o— | Switch (alt.) | — — | Variable resistor | □ | Loudspeaker |
| — — | Junction of conductors (node) | — — | Light-dependent resistor (LDR) | ○ | Microphone |
| —+— | Crossing conductors (no connection) | — — | Thermistor (thermal-resistor) | ⊥ | Earth (ground) |
| —+— | Crossing conductors (no connection) (alt.) | — | Capacitor | ↑ | Current source |

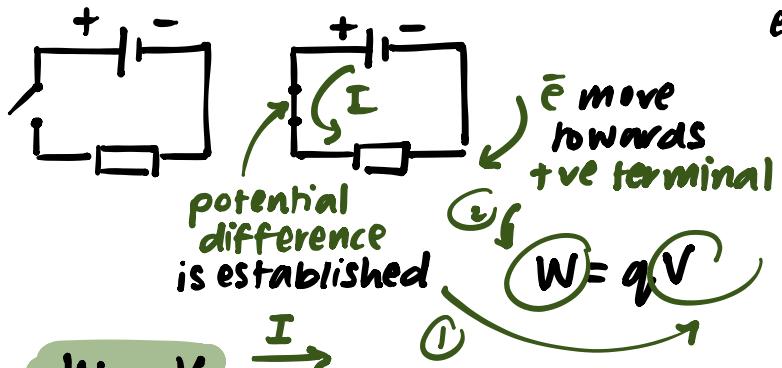
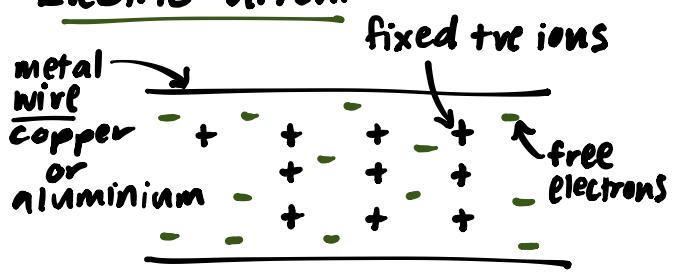
Circuit Symbols

- * Charge Carriers
- * Electric Current
- * Potential Difference
- * Resistance
- * Electric Power

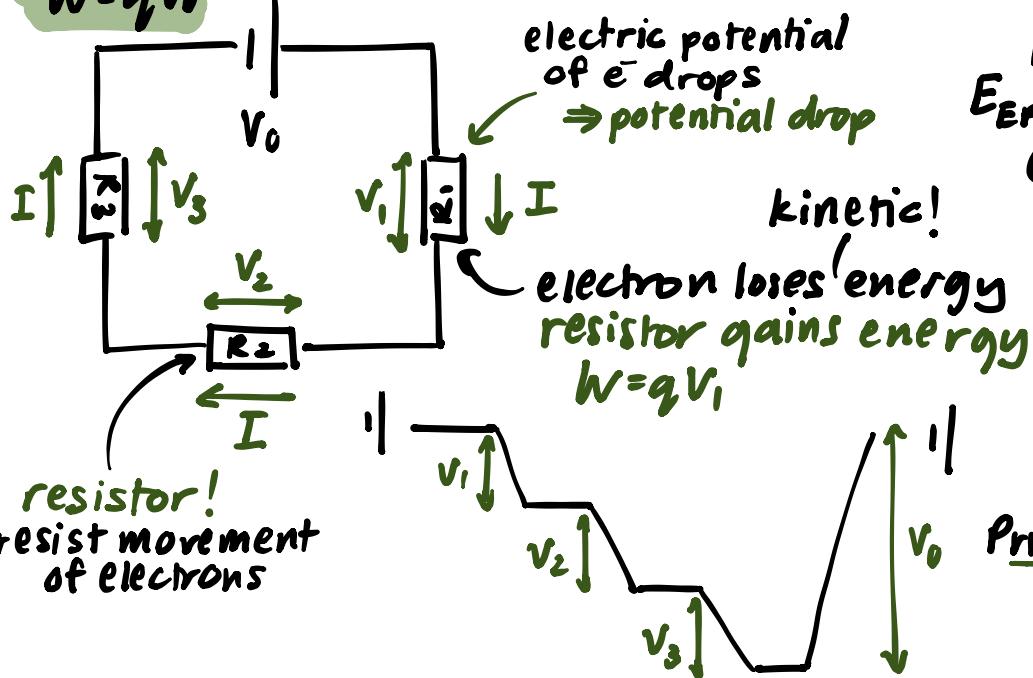
has to be free moving to conduct electricity!
Charge Carriers (charged particles)

- in metals → negatively charged free electrons
- in gas / non-metallic liquid → positive & negative ions

Electric Current



$$W = qV_0 \quad (1)$$



current → electrolyte $Q = It$
I Coulomb is 1 ampere in 1 second
 elementary charge (smallest!) $1.602 \times 10^{-19} C$
 electron proton produces current!
 $Q = Ne^q$ individual charge
 total charge no of particles

charge carriers in wires potential diff is Energy battery provide potential E

EPE converts to EKE
 charge carriers move = current!

$$I = \frac{dQ}{dt} = \frac{Q}{t}$$

Principle of Conservation of Energy

$$V_0 = V_1 + V_2 + V_3$$

Factors of Conduction

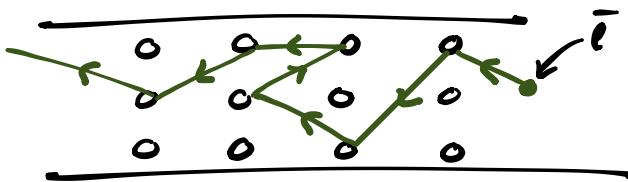
① Charge Carrier Density, n

diff metal / = diff no of element free electrons

$$n = \frac{N_e}{V} \quad \begin{matrix} \text{amount of} \\ \text{charge carriers} \\ \text{volume} \end{matrix}$$

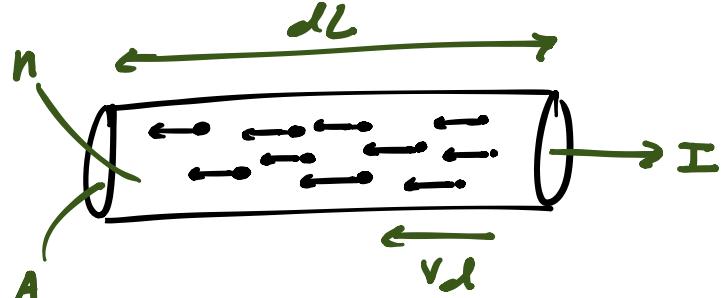
$$V_d = \frac{I}{nqA}$$

② Drift Velocity, V_d
electrons collide with atoms
electrons move in zigzag



③ Cross sectional Area of Conductor, A

$$I \propto A \quad J = \frac{I}{A} \quad \begin{matrix} \text{current} \\ \text{density} \end{matrix}$$



Number of Charge Carriers

$$nA \cdot dL = N \quad \frac{N}{V} \rightarrow v$$

$$\text{Charge} \quad dQ = Nq$$

Time taken for charge to pass through

$$dQ = (nA \cdot dL)q \rightarrow \textcircled{1} \quad dt = \frac{dL}{V_d} \rightarrow \textcircled{2}$$

$$\begin{matrix} \uparrow V_d \\ \uparrow A \\ \downarrow n \quad \uparrow V_d \end{matrix}$$

$$I = \frac{dQ}{dt}$$

Sub \textcircled{1} and \textcircled{2}:

$$I = (nA \cdot dL)q \cdot \left(\frac{V_d}{dL} \right)$$

$$I = nAqV_d$$

General Case!

Alloy

Metal A + Metal B

$\alpha\%$, n_A $\gamma\%$, n_B

$$I = I_A + I_B \quad \text{parallel current}$$

$$I = \frac{n_A q V_d \cdot (\alpha\% \cdot A)}{n_B q V_d \cdot (\gamma\% \cdot A)}$$

Conductor + Insulator

$\alpha\%$, n_A $\gamma\%$, no charge carrier

$$I = I_A$$

$$I = n_A q V_d \cdot (\alpha\% \cdot A)$$

Molecule A + Molecule B

$$I = I_A + I_B$$

$$I = (n_A + n_B) A q V_d$$

* volume of gas is negligible

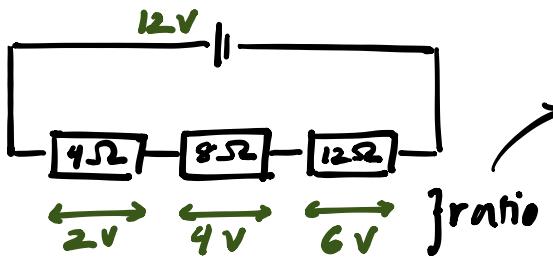
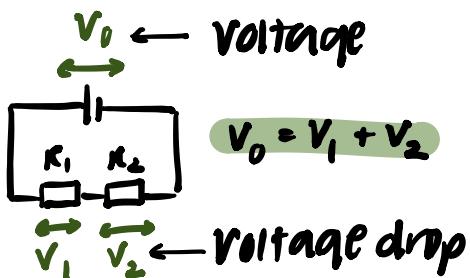
Potential Difference

Electrical energy (W) transferred by 1 Coulomb \downarrow charge

$$V = \frac{W}{Q}$$

Resistance

Ratio of potential diff to current



$$R = \frac{V}{I}$$

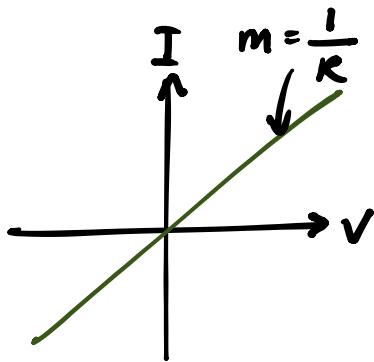
Metal!

$$V \uparrow \rightarrow I \uparrow \rightarrow e \uparrow$$

$\downarrow Q/t$

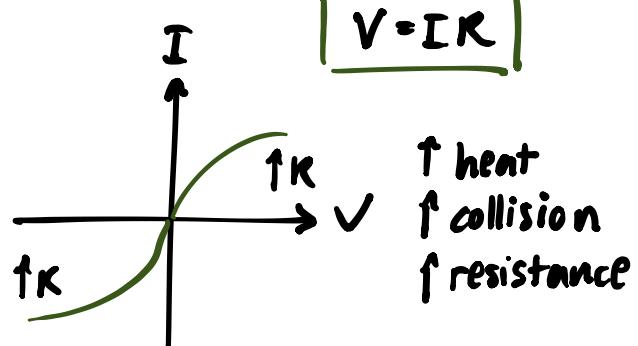
$R \uparrow \leftarrow$ collision \uparrow
 ↶ E in atom (lattice) $\rightarrow T \uparrow$
 until! vibration \uparrow \downarrow repeated
 vibration amp.
 vibration amplitude MAX
 ↶ resistance is max
 current saturated (constant)

I-V Characteristic



ohmic conductor

① pure metal / fixed resistor at constant temperature



Non-ohmic conductor

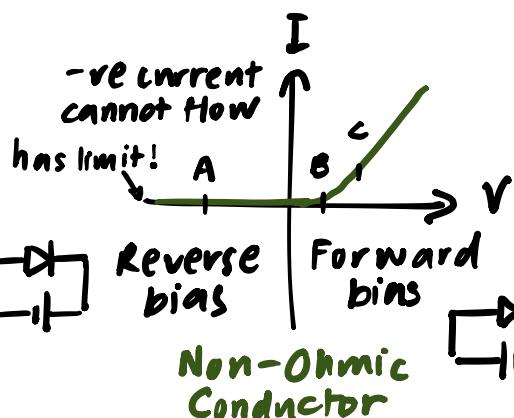
② filament lamp (tungsten)

$$V = IR$$

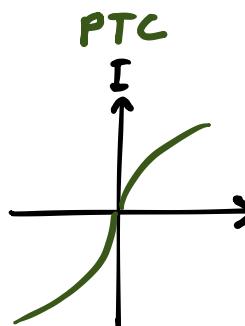
$\uparrow R$
 ↑ heat
 ↑ collision
 ↑ resistance

sensitive to temp (R not constant)

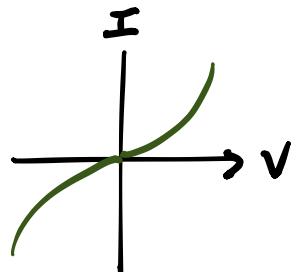
③ Semiconductor (Diode)



Non-Ohmic Conductor



PTC



NTC

Non-ohmic conductor

$T \uparrow, R \uparrow \rightarrow$ PTC : +ve temp coefficient
 $T \uparrow, R \downarrow \rightarrow$ NTC : -ve temp coefficient

Semiconductor!

$T \uparrow \rightarrow R \downarrow \rightarrow$ vibration \uparrow \rightarrow free $e \uparrow$

no free electrons!
 (covalent bonding)

$\therefore e \text{ gain energy}$

Electrical Power

$$P = \frac{W}{t} = \frac{QV}{t}$$

energy

rate at which
resistance converts
electrical E to other
forms of E

$$P = VI = I^2R = \frac{V^2}{R}$$

I
mostly
series

R
mostly
parallel

Efficiency

$$= \frac{\text{Useful output}}{\text{Input}} \times 100\%$$

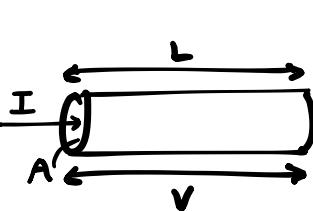
SERIES AND PARALLEL CIRCUITS

3

$$\text{Resistivity} \quad P = R \frac{A}{L}$$

$$R \propto L \rightarrow R = \rho \frac{L}{A}$$

Unit: $\Omega \text{ m}$ depends on temperature



Metal 10^{-8}
Semicon $10^{-5} \sim 10^1$
Insulator $> 10^{10}$

* Resistivity

* Resistors in series

* Resistors in Parallel

* Ammeters and Voltmeters

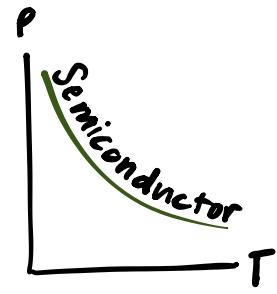
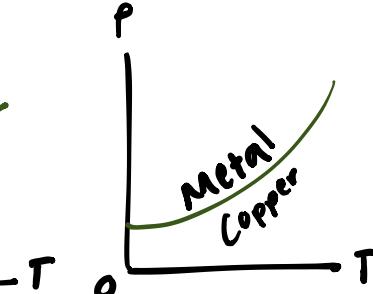
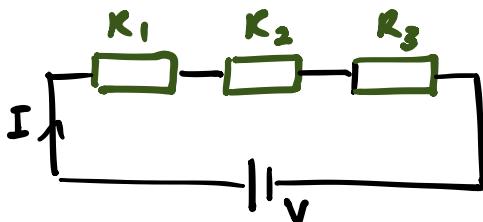
* Voltage Divider Rule

* Current Divider Rule

Conductivity

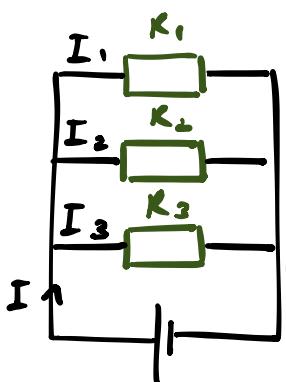
$$\sigma = \frac{1}{\rho} \quad \text{Unit: } \Omega^{-1} \text{ m}^{-1}$$

[Series]



$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 \\ V = I(R_1 + R_2 + R_3) = IK_{eq} \\ K_{eq} = R_1 + R_2 + R_3$$

[Parallel]



$$I = I_1 + I_2 + I_3 \\ I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \\ I = V \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\text{If } R_1 = R_2 = \dots = R_n, \quad K_{eq} = \frac{R}{n}$$

$$\text{Current Divider Rule} \quad I \propto \frac{1}{R} \quad \text{in parallel.}$$

$$I_x = \frac{K_{eq}}{R_x} I$$

Voltage Divider Rule

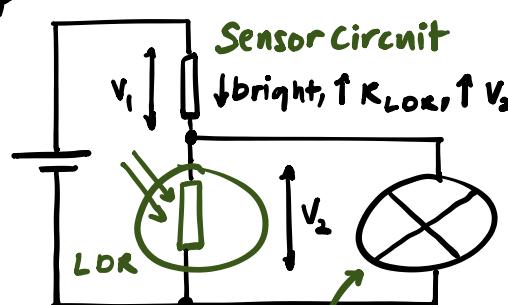
in series!

$V \propto R$

$$\frac{R_{target}}{R_{total}} = \frac{V_{target}}{V_{total}}$$

$$V_{tar} = \frac{R_{tar}}{R_{tot}} \times V_{tot}$$

$$V_x = \frac{R_{x0}}{R_{eq}} V_0$$



When $V_2 > \min V_{load}$
 \Rightarrow Load is activated

load w high K
 $\therefore V_{out}$ reduced less
 $R_{load} > 10R_2$

D.C. CIRCUITS I

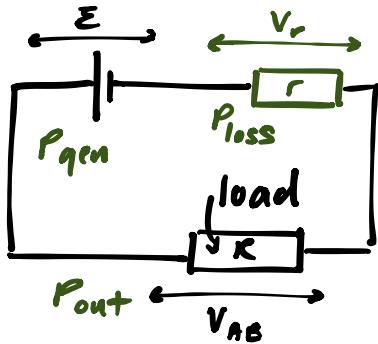
Chemical Cells have Internal Resistance

positive → negative runs } relative motion

- (1) Energy transfer From cell to charge
- (2) From charge to component

Electromotive Force

$$\boxed{E = \frac{W}{Q}} \text{ in V or } \text{JC}^{-1}$$



Ideal case!

$$\begin{aligned} P_{\text{gen}} &= P_{\text{out}} \\ I\epsilon &= IV_{AB} \\ \epsilon &= V_{AB} \end{aligned}$$

Actual case!

$$V_{AB} < \epsilon$$

$$\boxed{I = \frac{\epsilon}{R+r}}$$

$$\begin{aligned} P_{\text{gen}} &= P_{\text{out}} + P_{\text{loss}} \\ I\epsilon &= IV_{AB} + IR_r \end{aligned}$$

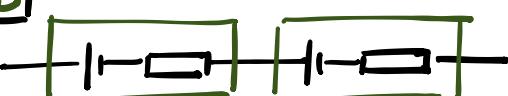
$$\epsilon = V_{AB} + V_r$$

$$\epsilon = IR + Ir$$

$$V_{AB} = \frac{R}{R+r} \epsilon \quad \left. \begin{array}{l} V \propto R \\ V \propto \epsilon \end{array} \right\}$$

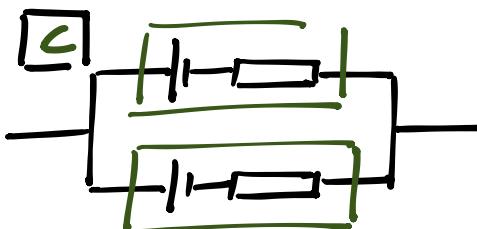
*terminal voltage!

B



$$\sum \epsilon = 2\epsilon$$

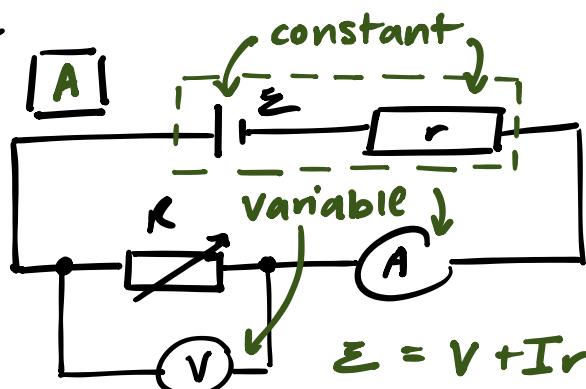
$$\sum r = 2r$$



$$\sum \epsilon = E$$

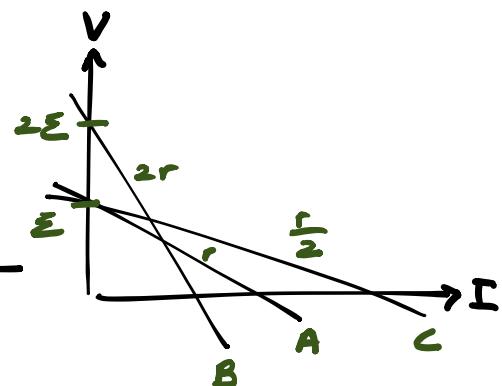
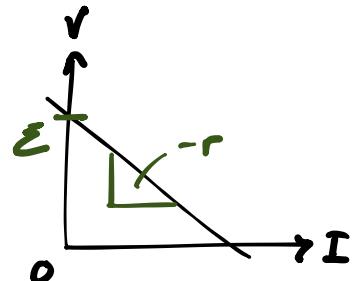
$$\sum r = \left(\frac{2}{r}\right)^{-1}$$

- * Electromotive Force
- * Internal Resistance
- * Determination of the E.m.f. and Internal Resistance



$$\begin{aligned} V &= -r(I) + \epsilon \\ Y &= mX + c \end{aligned}$$

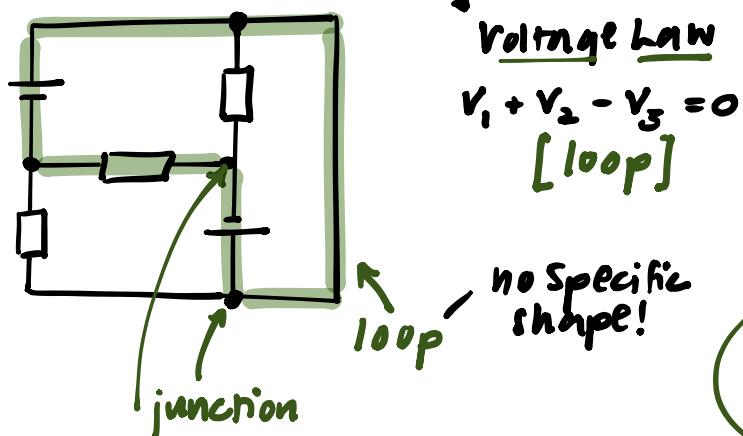
η -intercept = E
Gradient = r



D.C. CIRCUITS II

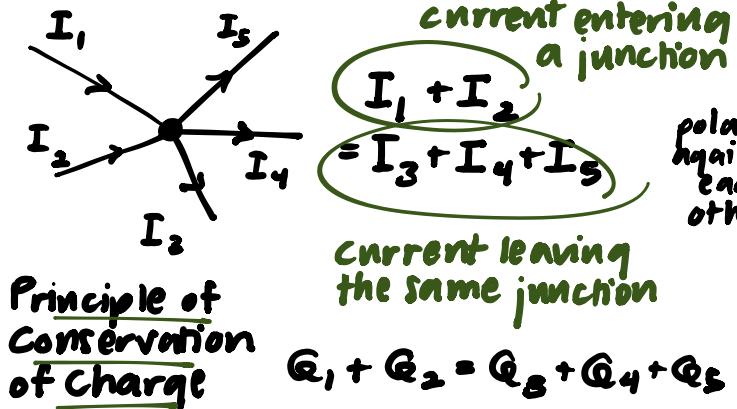
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Kirchoff's Law

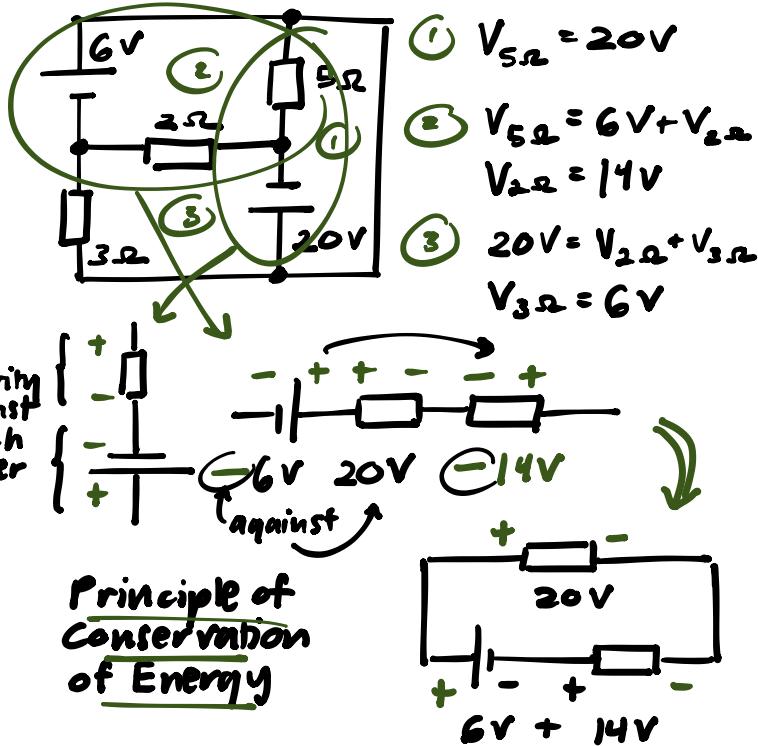


- * Kirchoff's Law
- * Kirchoff's Voltage Law
- * Kirchoff's Current Law
- * Applying Kirchoff's Law
- * Methods of Analysis

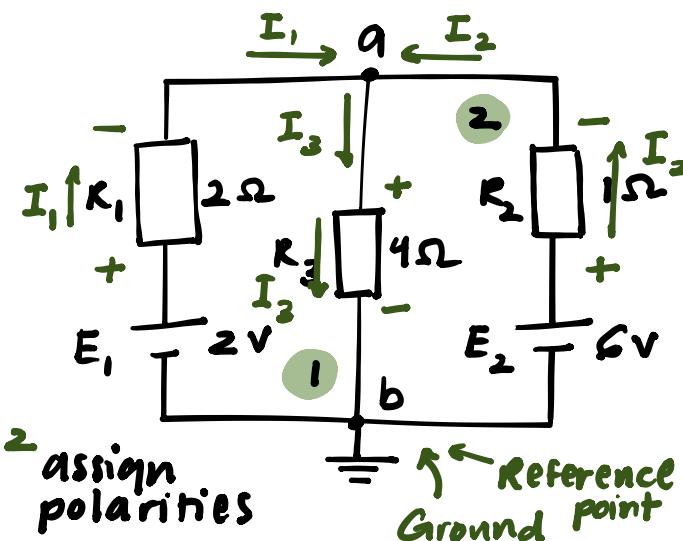
CURRENT LAW



VOLTAGE LAW



Branch-Current Analysis



$$I_1 = -1A (\leftarrow)$$

$$I_2 = 2A (\leftrightarrow)$$

$$I_3 = 1A (\downarrow)$$

4 find values, reassign direction

$$I_1 + I_2 - I_3 = 0 \rightarrow ①$$

$$E_1 - V_1 - V_3 = 0$$

$$E_1 - I_1 R_1 - I_3 R_3 = 0$$

$$2 - 2I_1 - 4I_3 = 0$$

use KVL in each loop

$$I_1 + 2I_3 = 1 \rightarrow ②$$

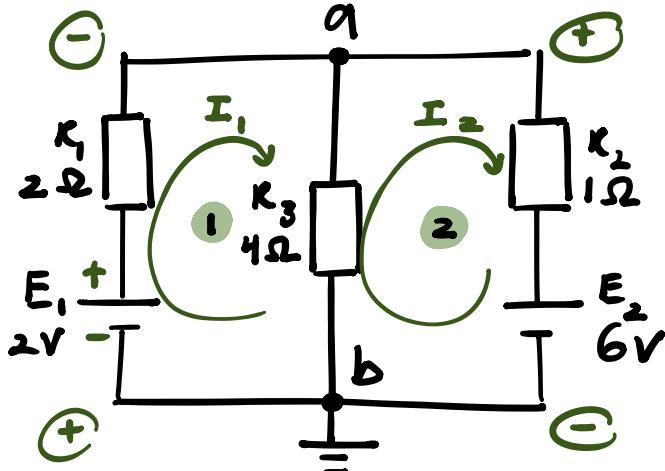
$$E_2 - V_2 - V_3 = 0$$

$$E_2 - I_2 R_2 - I_3 R_3 = 0$$

$$6 - I_2 - 4I_3 = 0$$

$$I_2 + 4I_3 = 6 \rightarrow ③$$

Mesh-Current Analysis



Loop I_1 :

$$(R_1 + R_3)I_1 - R_3 I_2 - E_1 = 0$$

$$(2 + 4)I_1 - 4I_2 - 2 = 0$$

$$6I_1 - 4I_2 = 2 \quad \rightarrow ①$$

Loop I_2 :

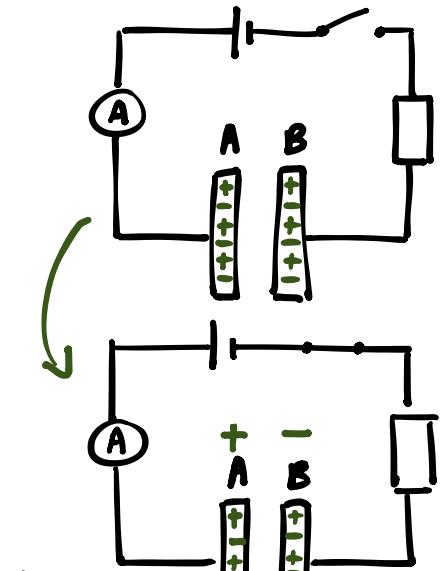
$$(4 + 1)I_2 - 4I_1 + 6 = 0$$

$$-4I_1 + 5I_2 = -6 \quad \rightarrow ②$$

$$I_1 = -1A, I_2 = -2A$$

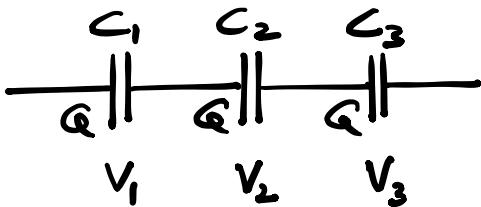
CAPACITORS

device to store charge energy



electron repelled by plate B

Capacitors in Series



$$Q = C_1 V_1 = C_2 V_2 = C_3 V_3$$

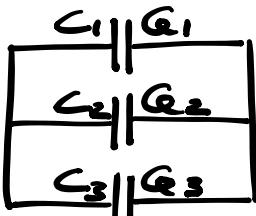
$$V_0 = V_1 + V_2 + V_3$$

$$V_0 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

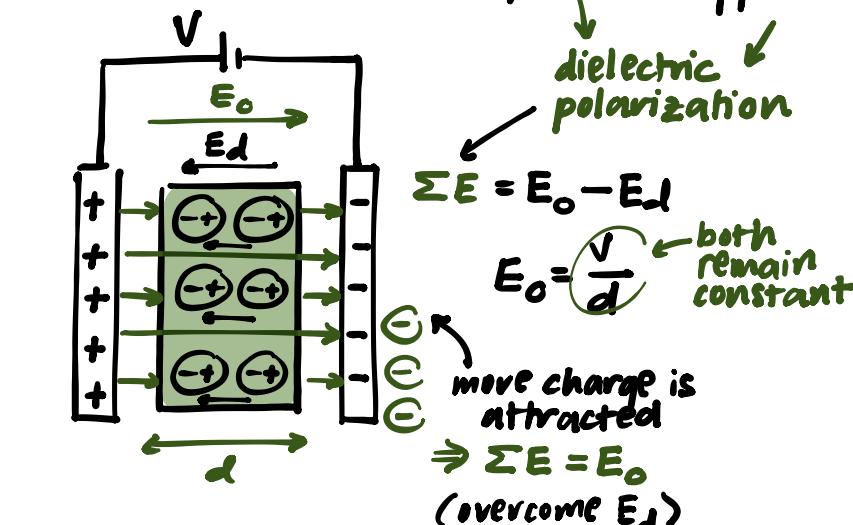
$$\frac{V_0}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Capacitors in Parallel



$$C_{eq} = C_1 + C_2 + C_3$$



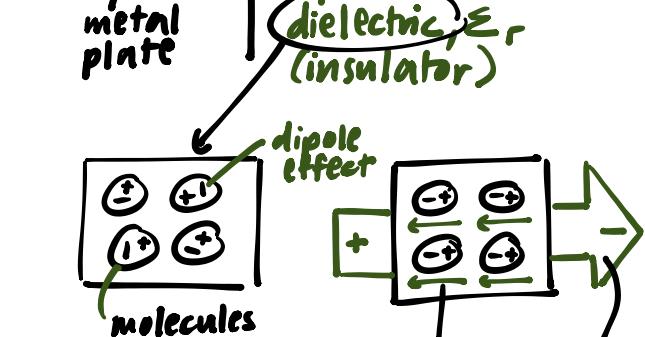
- * Capacitors
- * Capacitance
- * Capacitors in Series
- * Capacitors in Parallel
- * Energy Stored in a Capacitor
- * Charge Redistribution in Capacitors
- * Capacitance of Isolated Bodies
- * Charging Process of a Capacitor
- * Discharging Process of a Capacitor

Capacitance \rightarrow ↑ capacitance ≠ more charge stored
measure of ability to store charge

$$Q = CV$$

voltage

$$C = \epsilon_0 \epsilon_r \frac{A}{d}$$



molecules can't move but can orient

electric field generated

electric field applied

dielectric polarization

$$\Sigma E = E_0 - E_d$$

$$E_0 = \frac{V}{d}$$

both remain constant

more charge is attracted
 $\Rightarrow \Sigma E = E_0$
(overcome E_d)

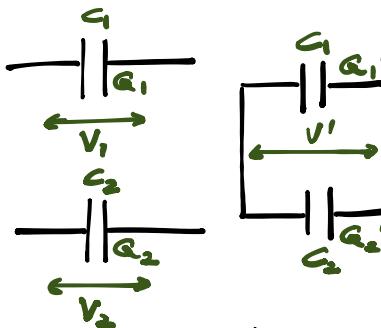
Energy in Capacitor

$$W = \frac{1}{2} C V^2$$

$$W = \frac{1}{2} \frac{Q^2}{C}$$

$$W = \frac{1}{2} QV$$

Charge Redistribution in Capacitors



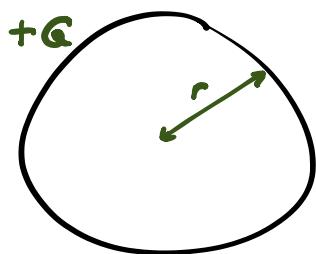
Principle of Conservation of Charge

$$Q_1 + Q_2 = Q'_1 + Q'_2$$

before parallel after parallel

$$C_1 V_1 + C_2 V_2 = (C_1 + C_2) V'$$

Isolated Bodies

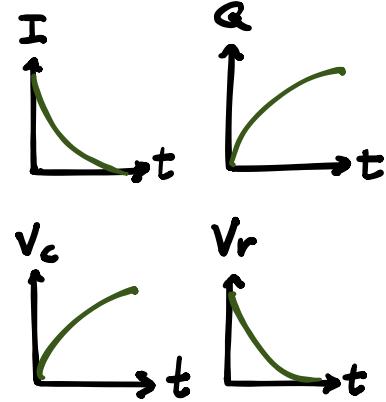
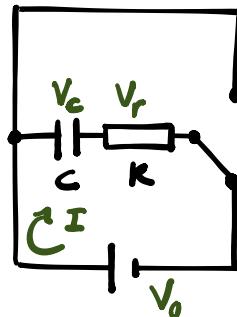


$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

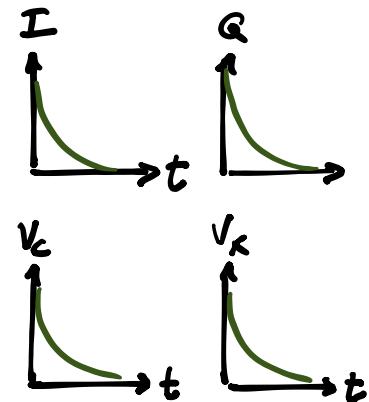
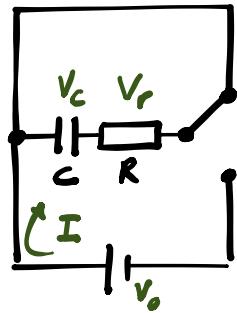
$$C = \frac{Q}{V}$$

$$\boxed{C = 4\pi\epsilon_0 r}$$

Charging

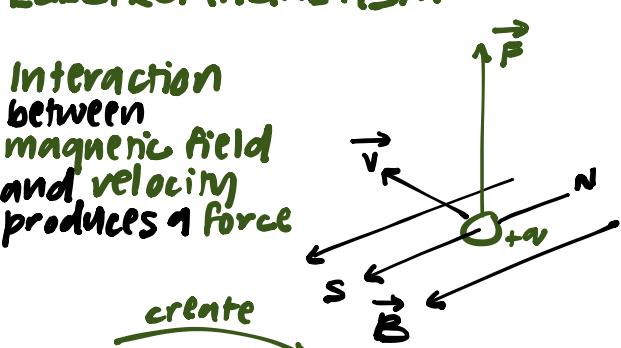


Discharging



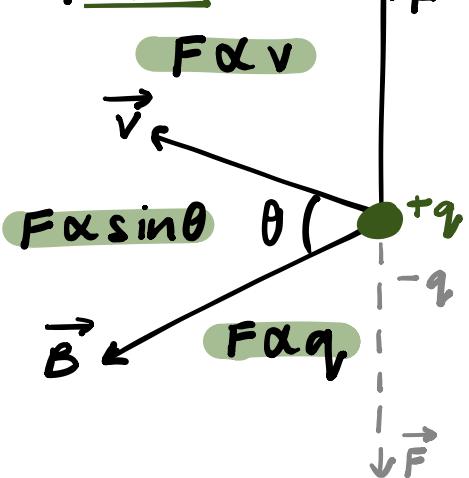
MAGNETIC FIELDS AND ELECTROMAGNETISM

Interaction between magnetic field and velocity produces a force



Moving charged particle creates Magnetic field

FORCE



$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$F = qvB \sin\theta$$

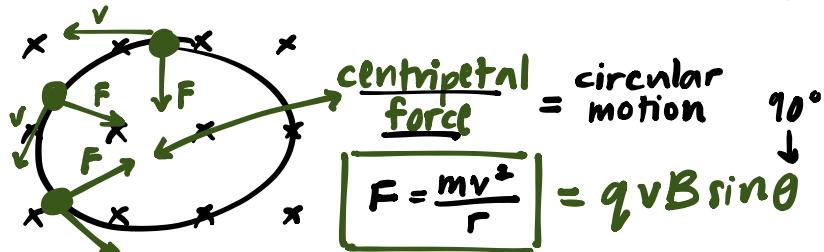
Unit: tesla (T)

For current-carrying wire

$$\vec{F} = I(\vec{L} \times \vec{B})$$

$$F = ILB \sin\theta$$

CIRCULATING CHARGE



Period

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

Frequency

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

Angular velocity

$$\omega = 2\pi f = \frac{qB}{m}$$

Magnetic Fields

Magnetic Field Lines

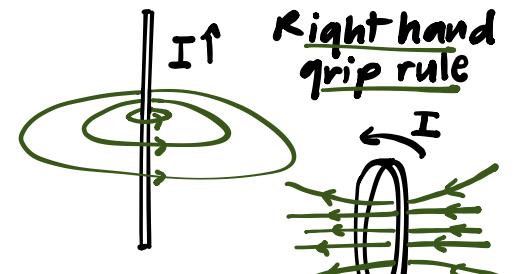
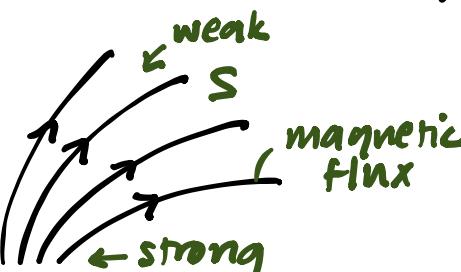
* Force on Moving Charged Particles

* Circulating charges

* Velocity selector

* Force on a Current-Carrying wire

* Meaning Magnetic Flux Density



out of the page
into the page

Magnetic Flux Density strength

$$q\left(\frac{L}{t} \times \vec{B}\right)$$

$$\frac{q}{t} (\vec{L} \times \vec{B})$$

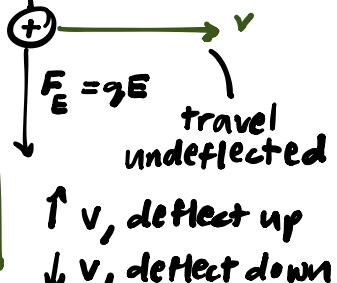
$$F_B = F_E$$

$$qvB = qE$$

$$vB = E$$

$$v = \frac{E}{B}$$

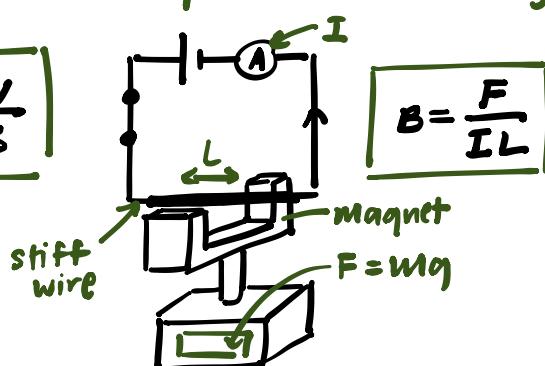
VELOCITY SELECTOR



$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

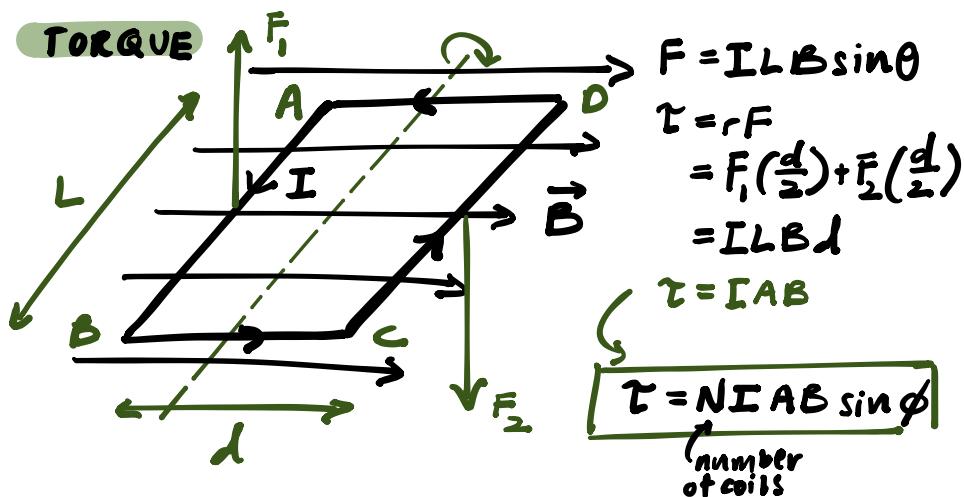
Magnetic Flux Density



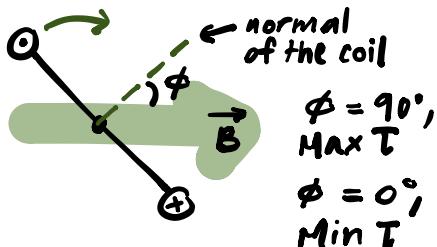
$$B = \frac{F}{IL}$$

ELECTROMAGNETISM APPLICATIONS

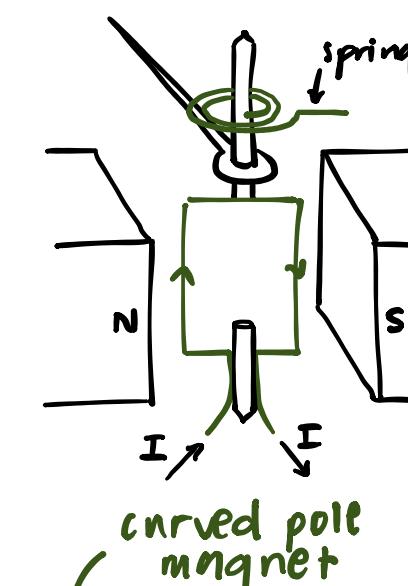
8



- * Torque on a Current-Carrying Loop
- * Galvanometer
- * Loudspeakers
- * Cyclotrons
- * The Hall Effect



GALVANOMETER



$$\tau_{loop} = NIAB \sin\phi$$

$$\tau_{spring} = k\theta$$

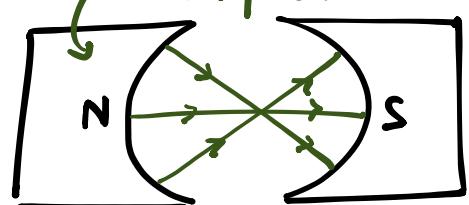
$$\tau_{loop} = \tau_{spring}$$

$$NIAB \sin\phi = k\theta$$

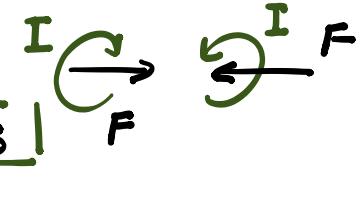
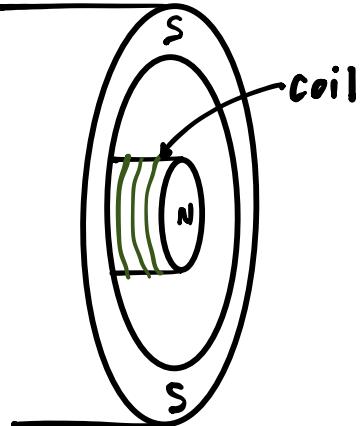
$$\theta = \frac{NIAB \sin\phi}{k}$$

$$\theta \propto I$$

$$\theta = \frac{NIAB}{k}$$

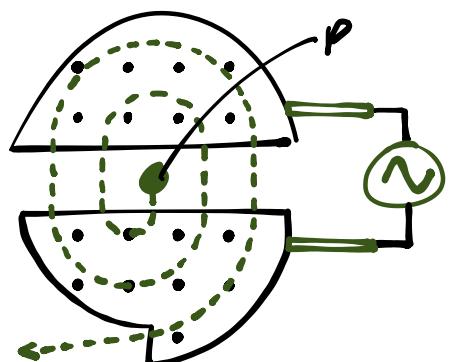


LOUDSPEAKER



number of turns \times circumference

CYCLOTRONS



Cyclotron Frequency

$$\omega = \frac{qB}{m} \quad (\text{rad s}^{-1})$$

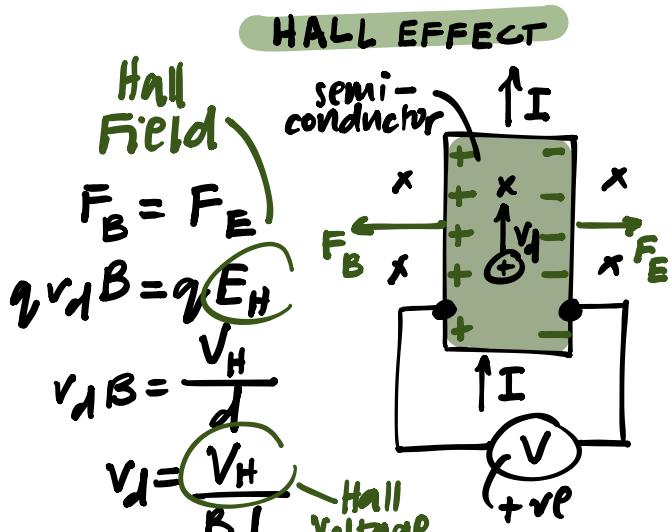
Oscillator Frequency

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

Kinetic Energy

$$K = \frac{1}{2}mv^2 = \frac{r^2 q^2 B^2}{2m}$$

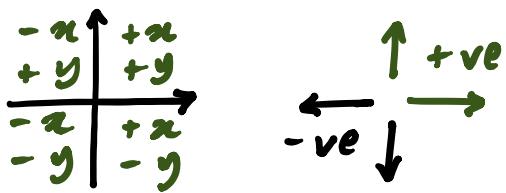
$$= \frac{qB}{2\pi m}$$



$$\text{from } V_d = \frac{I}{nAq}, \quad n = \frac{IB}{V_H e}$$

VECTORS QUICK GUIDE

0



Resolving vectors
Resultant of vectors

$$\sin \theta = \frac{W_y}{W}$$

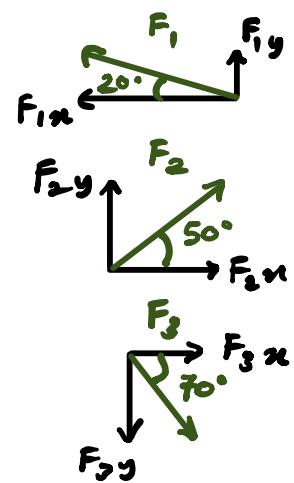
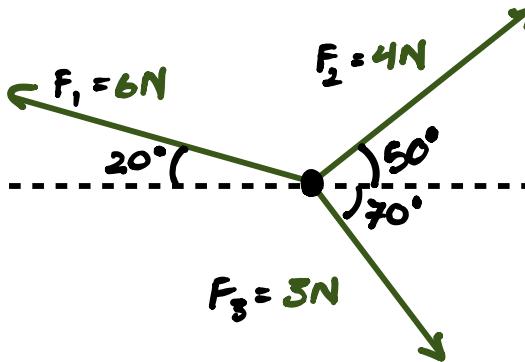
$$W_y = W \sin \theta$$

$$\cos \theta = \frac{W_x}{W}$$

$$\underline{F_1} \quad F_{1x} = -F_1 \cos 20^\circ \quad F_{1y} = F_1 \sin 20^\circ$$

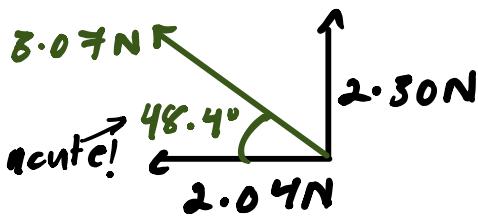
$$\underline{F_2} \quad F_{2x} = F_2 \cos 50^\circ \quad F_{2y} = F_2 \sin 50^\circ$$

$$\underline{F_3} \quad F_{3x} = F_3 \cos 70^\circ \quad F_{3y} = -F_3 \sin 70^\circ$$



$$\sum F = \sqrt{(\sum F_x)^2 + (\sum F_y)^2} = 3.07 N$$

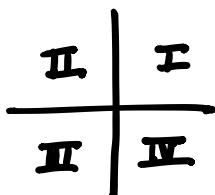
$$\theta = \tan^{-1} \frac{\sum F_y}{\sum F_x} = 48.4^\circ \text{ (in 2nd quadrant)}$$



angle is always with respect to the horizontal axis!

$$\left. \begin{aligned} \sum F_x &= F_{1x} + F_{2x} + F_{3x} \\ &= -6 \cos 20^\circ + 4 \cos 50^\circ + 3 \cos 70^\circ \\ &= -2.04 N \end{aligned} \right\} \text{left!}$$

$$\left. \begin{aligned} \sum F_y &= F_{1y} + F_{2y} + F_{3y} \\ &= +6 \sin 20^\circ + 4 \sin 50^\circ - 3 \sin 70^\circ \\ &= +2.80 N \end{aligned} \right\} \text{upwards!}$$



Bearing

